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Neutrosophic Almost Contra α -Continuous Functions

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Abstract: This study utilizes the notions of \aleph - α -open set to introduce and study new form of \aleph -continuity termed as \aleph -almost contra α -continuous function. Besides, we also introduce $\aleph\alpha$ -connected space, \aleph -weakly Hausdorff space, separation axioms, $\aleph\alpha$ -normal and \aleph -strong normal spaces. Characterizations of \aleph -almost contra α -continuous functions is also discussed.

Keywords: \aleph -almost contra α -continuous function; $\aleph\alpha$ -connected space; \aleph -weakly Hausdorff space; \aleph -locally α -indiscrete space; $\aleph\alpha$ -normal space; \aleph -strong normal space.

1. Introduction

Many real-world problems in Finance, Medical sciences, Engineering and Social sciences deals with uncertainties. There are difficulties in solving the uncertainties in these data by traditional mathematical models. There are approaches such as fuzzy sets [28], intuitionistic fuzzy sets [10], vague sets [13], and rough sets [18] which can be treated as mathematical tools to avert obstacles dealing with ambiguous data. But all these approaches have their implicit crisis in solving the problems involving indeterminate and inconsistent data due to inadequacy of parameterization tools. Smarandache [24] studied the idea of neutrosophic set as an approach for solving issues that cover unreliable, indeterminacy and persistent data. Neutrosophic topological space was introduced by Salama et.al. [19] in 2012. Further Neutrosophic topological spaces are studied in [20]. Applications of neutrosophic topology depend upon the properties of neutrosophic open sets, neutrosophic closed sets, neutrosophic interior operator and neutrosophic closure operator. Topologists studied the sets that are near to neutrosophic open sets and neutrosophic closed sets. In this order, Arokiarani et.al.[9] defined neutrosophic semi-open (resp. pre-open and α -open) functions and investigated their relations. In [9], the characterizations of characterizations of neutrosophic pre continuous (resp. α -continuous) functions is also discussed.

The idea of almost continuous functions is done in 1968 [21] in topology. Similarly, the notion of fuzzy almost contra continuous and fuzzy almost contra α -continuous functions were discussed in [16]]. Recently, Al-Omeri and Smarandache [26, 27] introduced and studied a number of the definitions of neutrosophic closed sets, neutrosophic mapping, and obtained several preservation properties and some characterizations about neutrosophic of connectedness and neutrosophic connectedness continuity. More recently, in [1, 8] authors have given how new trend of Neutrosophic theory is applicable in the field of Medicine and multimedia with a novel and powerful model.

In this paper, we define Almost contra-continuity in the context of neutrosophic topology such as Neutrosophic Almost α -contra-continuous function. We also discuss some characterizations of this concept. Moreover $\aleph\alpha$ -connected space, \aleph - weakly Hausdorff space, separation axioms and $\aleph\alpha$ -normal spaces are presented and investigated some properties.

2. Preliminaries

Definition: 2.1 [22, 23] Allow T,I,F as real standard or non standard members of $]0^-, 1^+[$, with

$$sup_T = t_{sup}, inf_T = t_{inf},$$

$$sup_I = i_{sup}, inf_I = i_{inf},$$

$$sup_F = f_{sup}, inf_F = f_{inf}$$

$$n - sup = t_{sup} + i_{sup} + f_{sup}$$

$$n - inf = t_{inf} + i_{inf} + f_{inf}. T,I,F \text{ are neutrosophic components.}$$

Definition: 2.2 [22, 23] Let S_1 be a non-empty fixed set. A definition set (in short N -set) Λ is an object such that $\Lambda = \{ \langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \gamma_\Lambda(x) \rangle : x \in S_1 \}$ wherein $\mu_\Lambda(x), \sigma_\Lambda(x)$ and $\gamma_\Lambda(x)$ which represents the degree of membership function (viz $\mu_\Lambda(x)$), the degree of indeterminacy (viz $\sigma_\Lambda(x)$) as well as the degree of non-membership (viz $\gamma_\Lambda(x)$) respectively of each element $x \in S_1$ to the set Λ .

Remark: 2.3[22, 23]

- I. An N -set $\Lambda = \{ \langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \gamma_\Lambda(x) \rangle : x \in S_1 \}$ can be identified to an ordered triple $\langle \mu_\Lambda, \sigma_\Lambda, \gamma_\Lambda \rangle$ in $]0^-, 1^+[$ on S_1 .
- II. In this paper, we use the symbol $\Lambda = \langle \mu_\Lambda, \sigma_\Lambda, \gamma_\Lambda \rangle$ for the N -set $\Lambda = \{ \langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \gamma_\Lambda(x) \rangle : x \in S_1 \}$.

Definition: 2.4[12] Let $S_1 \neq \emptyset$ and the N -sets Λ and Γ be defined as

$$\Lambda = \{ \langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \gamma_\Lambda(x) \rangle : x \in S_1 \}, \Gamma = \{ \langle x, \mu_\Gamma(x), \sigma_\Gamma(x), \gamma_\Gamma(x) \rangle : x \in S_1 \}. \text{ Then}$$

- I. $\Lambda \subseteq \Gamma$ iff $\mu_\Lambda(x) \leq \mu_\Gamma(x), \sigma_\Lambda(x) \leq \sigma_\Gamma(x)$ and $\gamma_\Lambda(x) \geq \gamma_\Gamma(x)$ for all $x \in S_1$;
- II. $\Lambda = \Gamma$ iff $\Lambda \subseteq \Gamma$ and $\Gamma \subseteq \Lambda$;
- III. $\bar{\Lambda} = \{ \langle x, \gamma_\Lambda(x), \sigma_\Lambda(x), \mu_\Lambda(x) \rangle : x \in S_1 \}$; [Complement of Λ]
- IV. $\Lambda \cap \Gamma = \{ \langle x, \mu_\Lambda(x) \wedge \mu_\Gamma(x), \sigma_\Lambda(x) \wedge \sigma_\Gamma(x), \gamma_\Lambda(x) \vee \gamma_\Gamma(x) \rangle : x \in S_1 \}$;
- V. $\Lambda \cup \Gamma = \{ \langle x, \mu_\Lambda(x) \vee \mu_\Gamma(x), \sigma_\Lambda(x) \vee \sigma_\Gamma(x), \gamma_\Lambda(x) \wedge \gamma_\Gamma(x) \rangle : x \in S_1 \}$;
- VI. $|\Lambda = \{ \langle x, \mu_\Lambda(x), \sigma_\Lambda(x), 1 - \mu_\Lambda(x) \rangle : x \in S_1 \}$;
- VII. $\langle \rangle \Lambda = \{ \langle x, 1 - \gamma_\Lambda(x), \sigma_\Lambda(x), \gamma_\Lambda(x) \rangle : x \in S_1 \}$.

Definition: 2.5[12] Let $\{ \Lambda_i : i \in J \}$ be an arbitrary family of N -sets in S_1 . Thereupon

- I. $\cap \Lambda_i = \{ \langle p, \wedge \mu_{\Lambda_i}(p), \wedge \sigma_{\Lambda_i}(p), \vee \gamma_{\Lambda_i}(p) \rangle : p \in S_1 \}$;
- II. $\cup \Lambda_i = \{ \langle p, \vee \mu_{\Lambda_i}(p), \vee \sigma_{\Lambda_i}(p), \wedge \gamma_{\Lambda_i}(p) \rangle : p \in S_1 \}$.

The main theme is to construct the tools for developing NTS, so we establish the neutrosophic sets 0_\aleph along with 1_\aleph in X as follows:

Definition: 2.6[12] $0_\aleph = \{ \langle q, 0, 0, 1 \rangle : q \in X \}$ and $1_\aleph = \{ \langle q, 1, 1, 0 \rangle : q \in X \}$.

Definition: 2.7[12] A definition topology (in short, \aleph -topology) on $S_1 \neq \emptyset$ is a family ξ_1 of N -sets in S_1 satisfying the laws given below:

- I. $0_N, 1_N \in \xi_1$,
- II. $W_1 \cap W_2 \in T$ being $W_1, W_2 \in \xi_1$,
- III. $\cup W_i \in \xi_1$ for arbitrary family $\{ W_i | i \in \Lambda \} \subseteq \xi_1$.

In this case the ordered pair (S_1, ξ_1) or simply S_1 is termed as NTS and each N -set in ξ_1 is named as neutrosophic open set (in short, \aleph -open set) . The complement $\bar{\Lambda}$ of an \aleph -open set Λ in S_1 is known as neutrosophic closed set (briefly, \aleph -closed set) in S_1 .

Definition: 2.8[12] Let Λ be an \aleph -set in an $NTSS_1$. Thereupon

$\aleph int(\Lambda) = \cup \{G | G \text{ is an } \aleph\text{-open set in } S_1 \text{ and } G \subseteq \Lambda\}$ is termed as neutrosophic interior (in brief \aleph -interior) of Λ ;

$\aleph cl(\Lambda) = \cap \{G | G \text{ is an } \aleph\text{-closed set in } S_1 \text{ and } G \supseteq \Lambda\}$ is termed as neutrosophic closure (shortly $\aleph cl$) of Λ .

Definition: 2.9[12] Let X be a nonempty set. Whenever r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ then the neutrosophic set $x_{r,t,s}$ is termed as neutrosophic point (in short NP) in X

given by $x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$ for $x_p \in X$ is termed as the support of $x_{r,t,s}$, wherein r

indicates the degree of membership value, t indicates the degree of indeterminacy along with s as the degree of non-membership value of $x_{r,t,s}$.

Definition: 2.10[12] Allow (S_1, ξ_1) be a NTS. A neutrosophic set Λ in (S_1, ξ_1) is termed as $g\aleph$ closed set if $Ncl(\Lambda) \subseteq \Gamma$ whenever $\Lambda \subseteq \Gamma$ and Γ is a \aleph -open set. The complement of a $g\aleph$ -closed set is named as $g\aleph$ -open set.

Definition: 2.11[12] Let (X, T) be a NTS and Λ be a neutrosophic set in X . Subsequently, the neutrosophic generalized closure and neutrosophic generalized interior of Λ are defined by,

$$\begin{aligned} (i)NGcl(\Lambda) &= \cap \{G : G \text{ is a generalized neutrosophic closed set in } S_1 \text{ and } \Lambda \subseteq G\}. \\ (ii)NGint(\Lambda) &= \cup \{G : G \text{ is a generalized neutrosophic open set in } S_1 \text{ and } \Lambda \supseteq G\}. \end{aligned}$$

3. Neutrosophic Almost Contra α -Continuous Functions.

A new form of $\aleph\alpha$ -continuity termed as \aleph -almost contra α -continuity is discussed along with some of their properties.

Definition 3.1 Let (S_1, ξ_1) and (S_2, ξ_2) be any two NTS. A function $g: (S_1, \xi_1) \rightarrow (S_2, \xi_2)$ is named as \aleph -almost contra α -continuous if inverse image of each \aleph -regular open set in S_2 is $\aleph\alpha$ -closed set in S_1 .

Recall that, for a function $f: S_1 \rightarrow S_2$, the subset $G_f = \{x, f(x) : x \in S_1\} \subset S_1 \times S_2$ is said to be graph of f .

Theorem 3.2 Let $f: S_1 \rightarrow S_2$ be a function along with $g: S_1 \rightarrow S_1 \times S_2$ be the graph function defined by $g(x) = (x, f(x))$ being each $x \in S_1$. Whenever g is \aleph -almost contra α -continuous function, thereupon f is \aleph -almost contra α -continuous function.

Proof. Let M be a \aleph -regular closed set in S_2 accordingly $S_1 \times M$ is a \aleph -regular closed set in $S_1 \times S_2$. In view of g is \aleph -almost contra α -continuous, so that $f^{-1}(M) = g^{-1}(S_1 \times M)$ is a $\aleph\alpha$ -open in S_1 . Thus f is \aleph -almost contra α -continuous.

Definition: 3.3

1. A nonempty family \mathbb{F} of \aleph -open sets on (S_1, ξ_1) is known as \aleph -filter if
 - I. $0_\aleph \notin \mathbb{F}$
 - II. If $A, B \in \mathbb{F}$ then $A \cap B \in \mathbb{F}$
 - III. If $A \in \mathbb{F}$ and $A \subset B$ then $B \in \mathbb{F}$
2. A nonempty family \mathbb{B} of \aleph -open sets on \mathbb{F} is named as \aleph -filter base if
 - I. $0_\aleph \notin \mathbb{B}$

- II. If $B_1, B_2 \in \mathbb{B}$ then $B_3 \subset B_1 \cap B_2$ for some $B_3 \in \mathbb{B}$
3. A \mathfrak{N} -filter \mathbb{F} is known as \mathfrak{N} -convergent to a \mathfrak{N} -point $x_{r,s,t}$ of a NTS(S_1, ξ_1) if for each \mathfrak{N} -open set A of (S_1, ξ_1) containing $x_{r,s,t}$, there exists a \mathfrak{N} -set $B \in \mathbb{F}$ so as $B \subseteq A$.
 4. A \mathfrak{N} -filter \mathbb{F} is said to be $\mathfrak{N}\alpha$ -convergent to a \mathfrak{N} -point $x_{r,s,t}$ of a NTS(S_1, ξ_1) if for each $\mathfrak{N}\alpha$ -open set A of (S_1, ξ_1) containing $x_{r,s,t}$, there exists a \mathfrak{N} -set $B \in \mathbb{F}$ thereby $B \subseteq A$.
 5. A \mathfrak{N} -filter \mathbb{F} is said to be \mathfrak{N} rc-convergent to a \mathfrak{N} -point $x_{r,s,t}$ of a NTS(S_1, ξ_1) if for each \mathfrak{N} regular closed set A of (S_1, ξ_1) containing $x_{r,s,t}$, there exists a \mathfrak{N} -set $B \in \mathbb{F}$ so as $B \subseteq A$.

Proposition 3.4 If a function $\mu: S_1 \rightarrow S_2$ is \mathfrak{N} -almost contra α -continuous function and each \mathfrak{N} -filter base \mathbb{F} in S_1 is $\mathfrak{N}\alpha$ -converging to $x_{r,s,t}$, the \mathfrak{N} -filter base $\mu(\mathbb{F})$ is \mathfrak{N} rc-convergent to $\mu(x_{r,s,t})$.

Proof. Let $x_{r,s,t} \in S_1$ and \mathbb{F} be any \mathfrak{N} -filter base in S_1 is $\mathfrak{N}\alpha$ -converging to $x_{r,s,t}$. As μ is \mathfrak{N} -almost contra α -continuous, subsequently for any \mathfrak{N} regular closed R in S_2 including $\mu(x_{r,s,t})$, there exists $\mathfrak{N}\alpha$ -open W in S_1 involving $x_{r,s,t}$ so as $\mu(W) \subset R$. As \mathbb{F} is $\mathfrak{N}\alpha$ -convergent to $x_{r,s,t}$, there occurs $A \in \mathbb{F}$ thereby $A \subset W$. This means that $\mu(A) \subset R$ and consequently the \mathfrak{N} -filter base $\mu(\mathbb{F})$ is \mathfrak{N} rc-convergent to $\mu(x_{r,s,t})$.

Definition: 3.5

1. A space S_1 is termed as $\mathfrak{N}\alpha$ -connected if S_1 can't be expressed as union of two disjoint non-empty $\mathfrak{N}\alpha$ -open sets.
2. A space S_1 is named as \mathfrak{N} -connected if S_1 cannot be written as union of two disjoint non-empty \mathfrak{N} -open sets.

Theorem 3.6 If $f: S_1 \rightarrow S_2$ is a \mathfrak{N} -almost contra α -continuous surjection along with S_1 is $\mathfrak{N}\alpha$ -connected space, then S_2 is \mathfrak{N} -connected.

Proof. Let $f: S_1 \rightarrow S_2$ be a \mathfrak{N} -almost contra α -continuous surjection with S_1 is $\mathfrak{N}\alpha$ -connected space. Assuming S_2 is a not \mathfrak{N} -connected space. Accordingly, there exist disjoint \mathfrak{N} -open sets W and R such that $S_2 = W \cup R$. Then, W and R are \mathfrak{N} -clopen in S_2 . As f is \mathfrak{N} -almost contra α -continuous, $f^{-1}(W)$ and $f^{-1}(R)$ are $\mathfrak{N}\alpha$ -open sets in S_1 . In addition $f^{-1}(W)$ and $f^{-1}(R)$ are disjoint non-empty and $S_1 = f^{-1}(W) \cup f^{-1}(R)$. It is contradiction to the fact that S_1 is $\mathfrak{N}\alpha$ -connected space. Hence, S_2 is \mathfrak{N} -connected.

Definition 3.6 A space S_1 is named as \mathfrak{N} -locally α -indiscrete if every $\mathfrak{N}\alpha$ -open set is \mathfrak{N} -closed in S_1 .

Definition 3.7 A function $g: S_1 \rightarrow S_2$ is termed as \mathfrak{N} -almost continuous if $g^{-1}(V)$ is \mathfrak{N} -open in S_1 for each \mathfrak{N} -regular open set V in S_2 .

Definition 3.8 A function $f: S_1 \rightarrow S_2$ is known as \mathfrak{N} -almost α -continuous if $f^{-1}(V)$ is $\mathfrak{N}\alpha$ -open in S_1 for each \mathfrak{N} -regular open set V in S_2 .

Theorem 3.9 If a function $\eta: S_1 \rightarrow S_2$ is \mathfrak{N} -almost contra α -continuous function and S_1 is \mathfrak{N} -locally α -indiscrete space, then f is \mathfrak{N} -almost continuous function.

Proof. Let W be a \mathfrak{N} -regular closed set in S_2 . Since η is \mathfrak{N} -almost contra α -continuous function, $\eta^{-1}(W)$ is $\mathfrak{N}\alpha$ -open set in S_1 and S_1 is \mathfrak{N} -locally α -indiscrete space, which implies $\eta^{-1}(W)$ is a \mathfrak{N} -closed set in S_1 . Hence, η is \mathfrak{N} -almost continuous function.

Definition 3.10 A space S_1 named as \aleph -weakly Hausdorff if each element of S_1 is an intersection of \aleph -regular closed sets.

Definition 3.11 A space S_1 is named as

1. $\aleph\alpha$ - T_0 if for each pair of distinct \aleph -points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1 , there exists $\aleph\alpha$ -open set U such that $x_{r,s,t} \in U$, $y_{r,s,t} \notin U$ or $x_{r,s,t} \notin U$, $y_{r,s,t} \in U$.
2. $\aleph\alpha$ - T_1 if for each pair of distinct \aleph -points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1 , there exist $\aleph\alpha$ -open sets U and V containing $x_{r,s,t}$ and $y_{r,s,t}$ respectively, so as $y_{r,s,t} \notin U$ and $x_{r,s,t} \notin V$.
3. $\aleph\alpha$ - T_2 if for each pair of distinct \aleph -points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1 , there exists $\aleph\alpha$ -open set U containing $x_{r,s,t}$ and $\aleph\alpha$ -open set V containing $y_{r,s,t}$ so as $U \cap V = 0_N$.
4. A space S_1 is termed as $\aleph\alpha$ -normal if each pair of non-empty disjoint \aleph -closed sets can be separated by disjoint $\aleph\alpha$ -open sets.
5. A space S_1 is termed as \aleph -strongly-normal if each pair of disjoint non-empty \aleph -closed sets U and V there exists disjoint \aleph -open sets W and R such that $U \subset W$, $V \subset R$ and $\aleph cl(W) \cup \aleph cl(R) = 0_{\aleph}$.
6. A space S_1 is called a \aleph -ultra normal if each pair of non-empty disjoint \aleph -closed sets can be separated by disjoint \aleph -clopen sets.

Theorem 3.12 If $f: S_1 \rightarrow S_2$ is an \aleph -almost contra α -continuous injection and S_2 is \aleph -weakly Hausdorff space, then S_1 is $\aleph\alpha$ - T_1 .

Proof. Let S_2 be a \aleph -weakly Hausdorff space. For any distinct \aleph points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1 , there exist V and W , \aleph -regular closed sets in S_2 such that $f(x_{r,s,t}) \in V$, $f(y_{r,s,t}) \notin V$, $f(y_{r,s,t}) \in W$ and $f(x_{r,s,t}) \notin W$. As f is \aleph -almost contra α -continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $\aleph\alpha$ -open subsets of S_1 such that $x_{r,s,t} \in f^{-1}(V)$, $y_{r,s,t} \notin f^{-1}(V)$, $y_{r,s,t} \in f^{-1}(W)$ and $x_{r,s,t} \notin f^{-1}(W)$. Hence, S_1 is $\aleph\alpha$ - T_1 .

Theorem 3.13 If $h: S_1 \rightarrow S_2$ is a \aleph -almost contra α -continuous injective mapping from space S_1 into a \aleph -Ultra Hausdorff space S_2 , then S_1 is $\aleph\alpha$ - T_2 .

Proof. Let $x_{r,s,t}$ and $y_{r,s,t}$ be any two distinct \aleph elements in S_1 . As f is an injective $h(x_{r,s,t}) \neq h(y_{r,s,t})$ and S_2 is \aleph -Ultra Hausdorff space, there exist disjoint \aleph -clopen sets U and V of S_2 containing $h(x_{r,s,t})$ and $h(y_{r,s,t})$ respectively. Subsequently, $x_{r,s,t} \in h^{-1}(U)$ and $y_{r,s,t} \in h^{-1}(V)$, wherein $h^{-1}(U)$ and $h^{-1}(V)$ are disjoint $\aleph\alpha$ -open sets in S_1 . Then, S_1 is $\aleph\alpha$ - T_2 .

Proposition 3.14 If S_2 is \aleph strongly-normal and $\mu: S_1 \rightarrow S_2$ is a \aleph almost contra- α -continuous closed injection, then S_1 is $\aleph\alpha$ -normal.

Proof. Suppose J and K are disjoint \aleph -closed members of S_1 . Let μ is \aleph -closed and injective $f(J)$ and $f(K)$ are disjoint \aleph -closed sets in S_2 . As S_2 is \aleph strongly-normal, there exist \aleph -open sets W and R in Y so that $\mu(J) \subset W$ and $\mu(K) \subset R$ and $\aleph cl(W) \cap \aleph cl(R) = 0_{\aleph}$. Then, since $\aleph cl(W)$ and $\aleph cl(V)$ are \aleph regular closed, and μ is an \aleph almost contra α -continuous, $\mu^{-1}(\aleph cl(W))$ and $\mu^{-1}(\aleph cl(R))$ are $\aleph\alpha$ -open sets in S_1 . This implies $J \subseteq \mu^{-1}(\aleph cl(W))$, $K \subseteq \mu^{-1}(\aleph cl(R))$ and $\mu^{-1}(\aleph cl(W))$ and $\mu^{-1}(\aleph cl(R))$ are disjoint, so S_1 is $\aleph\alpha$ -normal.

Theorem 3.15 If $f: S_1 \rightarrow S_2$ is a \aleph -almost contra α -continuous, \aleph -closed injection along with S_2 is \aleph -ultra normal, then S_1 is $\aleph\alpha$ -normal.

Proof. Let P and Q be disjoint \aleph -closed sets of S_1 . As f is \aleph -closed as well as injective, $f(P)$ along with $f(Q)$ are disjoint \aleph -closed sets in S_2 . Since S_2 is \aleph -ultra normal, there exist disjoint \aleph -clopen sets U and V in S_2 such that $f(P) \subset U$ and $f(Q) \subset V$. This implies $P \subset f^{-1}(U)$ with $Q \subset f^{-1}(V)$. As f is a \aleph -almost contra α -continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $\aleph\alpha$ -open sets in S_1 . Therefore, S_1 is $\aleph\alpha$ -normal.

Definition 3.16 A function $f: S_1 \rightarrow S_2$ is called \aleph -weakly almost contra α continuous if for each \aleph -point $x_{r,s,t}$ in S_1 and each \aleph regular closed set V of S_2 containing $f(x_{r,s,t})$, there exists a $\aleph\alpha$ -open set U in S_1 , such that $\aleph cl(f(U)) \subseteq V$.

Definition 3.17 A function $f: S_1 \rightarrow S_2$ is termed as $\aleph(\alpha, S)$ -open if the image of each \aleph -open set is \aleph -semi open.

Theorem 3.18 If $f: S_1 \rightarrow S_2$ is a \aleph -weakly almost contra α -continuous and $\aleph(\alpha, S)$ -open then, f is \aleph -almost contra α continuous.

Proof. Let $x_{r,s,t}$ be a \aleph point in S_1 and V be a \aleph -regular closed set containing $f(x_{r,s,t})$. Since f is \aleph -weakly almost contra α continuous, there exist a $\aleph\alpha$ -open set U in S_1 containing $x_{r,s,t}$ so as $\aleph cl(f(U)) \subseteq V$. Since f is a $\aleph(\alpha, S)$ -open, $f(U)$ is a \aleph -semi open set in S_2 and $f(U) \subseteq \aleph cl(\aleph int(f(U))) \subseteq V$. This shows f is \aleph almost contra α continuous.

4. Conclusions

In this paper, we have introduced and studied the concepts like, Neutrosophic Almost α -contra-continuous function, $\aleph\alpha$ -connected space, \aleph -weakly Hausdorff space, separation axioms and $\aleph\alpha$ -normal spaces and investigated some properties. Some preservation theorems are also discussed. It will be necessary to carry out more theoretical research to establish a general framework for decision-making and to define patterns for complex network conceiving and practical application.

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