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Mohana K,

Princy R

F. Smarandache

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An Introduction to Neutrosophic Bipolar Vague Topological Spaces

Mohana K[1], Princy R[2,*], Florentin Smarandache[3]

[1] Assistant Professor, Department of Mathematics, Nirmala College for Women, India. Email: riyaraju1116@gmail.com

[2] Research Scholar, Department of Mathematics, Nirmala College for Women, India. Email: princy.pjs@gmail.com

[3] Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA.Email: fsmarandache@gmail.com

*Correspondence: princy.pjs@gmail.com.

Abstract: The main objective of this paper is to make known to a new concept of generalised neutrosophic bipolar vague sets and also defined neutrosophic bipolar vague topology in topological spaces. Also, we introduce generalized neutrosophic bipolar vague closed sets and conferred its properties.

Keywords: Bipolar set, Vague set, Neutrosophic set, Neutrosophic Bipolar Vague set, Neutrosophic Bipolar Vague Topological Spaces.

1. Introduction

Levine [24] studied the Generalized closed sets in general topology. Several investigations were conducted on the generalizations of the notion of the fuzzy set, after the introduction of the concept of fuzzy sets by Zadeh [34]. In the traditional fuzzy sets, the membership degree of component ranges over the interval [0, 1]. Few types of fuzzy set extensions in the fuzzy set theory are present, for example, intuitionistic fuzzy sets[12], interval-valued fuzzy sets[32], vague sets[30] etc. As a generalization of Zadeh's fuzzy set, the notion of vague set theory was first introduced by Gau W.L and Buehrer D.J [22]. In 1996, H.Bustince & P.Burillo indicated that vague sets are intuitionistic fuzzy sets [15].

Intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets can handle only unfinished information but not the indeterminate and unreliable information which happens normally in actual circumstances. Hence, the conception of a neutrosophic set is very common, and then it can overcome the aforesaid issues on the intuitionistic fuzzy set and the interval-valued intuitionistic fuzzy set. In 1995, the definition of Smarandache's neutrosophic set, neutrosophic sets and neutrosophic logic have been useful in many real applications to handle improbability. Neutrosophy is a branch of philosophy which studies the source, nature and scope of neutralities, as well as their interactions with different ideational scales [31]. The neutrosophic set uses one single value to indicate the truth-membership grade, indeterminacy-membership degree and falsity membership grade of an element in the universe X. The theory has been brought into extensive application in varieties of field [1-6, 8, 10, 11, 14, 17, 23, 27, 33, 35] for dealing with indeterminate and unreliable information in actual domain. The conception of Neutrosophic Topological space was introduced by A.A.Salama and S.A.Alblowi [29].

Bipolar-valued fuzzy sets, which was introduced by Lee [25, 26] is an extension of fuzzy sets whose membership degree range is extended from the interval [0, 1] to [-1, 1]. The membership degrees of the Bipolar valued fuzzy sets signify the degree of satisfaction to the property analogous to a fuzzy set and its counter-property in a bipolar valued fuzzy set, if the membership degree is 0 it means that the elements are unrelated to the corresponding property. Furthermore if the membership degree is on (0, 1] it indicates that the elements somewhat fulfil the property, and if the membership degree is on

[-1,0) it indicates that elements somewhat satisfy the entire counter property. After that, Deli et al. [21] announced the concept of bipolar neutrosophic sets, as an extension lead of neutrosophic sets. In the bipolar neutrosophic sets, the positive membership degree $T^{+}(x), I^{+}(x), F^{+}(x)$ signifies the truth membership, indeterminate membership and false membership of an element x ∈ X analogous to a bipolar neutrosophic set A and the negative membership degree $T^{-}(x),\ I^{-}(x), F^{-}(x)$ signifies the truth membership, indeterminate membership and false membership of an element x ∈ X to some implied counter-property analogous to a bipolar neutrosophic set A. There are quite a few extensions of Neutrosophic Bipolar sets such as Neutrosophic Bipolar Soft sets [7] and Rough Neutrosophic Bipolar sets [28].

Neutrosophic vague set is a combination of neutrosophic set and vague set which was well-defined by Shawkat Alkhazaleh [30]. Neutrosophic vague theory is a useful tool to practise incomplete, indeterminate and inconsistent information. In this paper, we introduced the perception of a neutrosophic bipolar vague set as a combination of neutrosophic set, Bipolar set and vague set and we also define the concept of generalised Neutrosophic Bipolar Vague set.

2. Preliminaries

Definition 2.1[16]: Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function μ_A^+: X→[0,1] and a negative membership function μ_A^-: X→[-1,0]. For sake of easiness, we shall practice the symbol A= {< x, μ_A^+(x), μ_A^-(x)>: x ∈ X}.

Definition 2.2[18]: Let A and B be two bipolar valued fuzzy sets then their union, intersection and complement are well-defined as follows:

(i)$\mu_{A\cup B}^+(x) = \max\{\mu_A^+(x), \mu_B^+(x)\}$.

(ii) $\mu_{A\cup B}^-(x) = \min\{\mu_A^-(x), \mu_B^-(x)\}$.

(iii) $\mu_{A\cap B}^+(x) = \min\{\mu_A^+(x), \mu_B^+(x)\}$.

(iv) $\mu_{A\cap B}^-(x) = \max\{\mu_A^-(x), \mu_B^-(x)\}$.

(v) $\mu_{\bar{A}}^+(x)$= 1-μ_A^+(x) and $\mu_{\bar{A}}^-(x)$= -1-μ_A^+(x) for all x∈ X.

Definition 2.3[15]: A vague set A in the universe of discourse U is a pair (t_A , f_A) where t_A : U→[0,1], f_A : U→[0,1] denote the mapping such that $t_A + f_A \leq 1$ for all u∈ U .The function t_A and f_A are called true membership function and false membership function respectively. The interval [t_A ,1-f_A] is called the vague value of u in A, and denoted by v_A(u), i.e v_A(u)=[t_A ,1-f_A].

Definition 2.4[15]: Let A be a non-empty set and the vague set A and B in the form A= {<x, t_A ,1-f_A >:x∈ X}, B={<x, t_B ,1-f_B >:x∈ X}.

Then

(i)A⊆ B if and only if $t_A(x) \leq t_A(x)$ and 1-$f_B(x) \leq$ 1-$f_B(x)$.

(ii)A∪ B={<max($t_A(x)$,$t_B(x)$), max(1-$f_A(x)$,1-$f_B(x)$)>/x∈X}.

(iii) A∩ B={<min($t_A(x)$,$t_B(x)$), min(1-$f_A(x)$,1-$f_B(x)$)>/x∈X}.

(iv)$\bar{A}$={<x, $f_A(x)$,1-$t_A(x)$>:x∈X}.

Definition 2.5[14]:Let X be a universe of discourse. Then a neutrosophic set is well-defined as: A = {⟨x, $T_A(x)$, $I_A(x)$, $F_A(x)$⟩:x ∈ X}, which is categorized by a truth-membership function T_A:X →]0−,1+[, an indeterminacy membership function I_A:X →]0−,1+[and a falsity-membership function F_A:X →]0−,1+[. There is no restriction to the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) \leq \sup I_A(x) \leq \sup F_A(x) \leq 3^+$.

Definition 2.6[30]: A neutrosophic vague set A_{NBV}(NVS in short) on the universe of discourse X written as,

$A_{NBV} = \{< \hat{T}_{NBV}(x),\ \hat{I}_{NBV}(x),\ \hat{F}_{NBV}(x) >: x \in X\}$ whose truth-membership, indeterminacy-membership and falsity-membership functions is defined as,

$\hat{T}_{NBV}(x) = [T^{-}, T^{+}]$, $\hat{I}_{NBV}(x) = [I^{-}, I^{+}]$, $\hat{F}_{NBV}(x) = [F^{-}, F^{+}]$ where

$T^{+} = 1 - F^{-}$, $F^{+} = 1 -$and T^{-}, $^{-}0 \leq T^{-} + I^{-} + F^{-} \leq 2^{+}$.

3. Bipolar Neutrosophic Vague Set:

Under this division, we present and well-defined the notion of neutrosophic bipolar vague set and its operations.

Definition 3.1: If A={< x, $[T_A^-,T_A^+]^+,[I_A^-,I_A^+]^+,[F_A^-,F_A^+]^+,[T_A^-,T_A^+]^-,[I_A^-,I_A^+]^-,[F_A^-,F_A^+]^-$ >} and B={< x, $[T_B^-,T_B^+]^+,[I_B^-,I_B^+]^+,[F_B^-,F_B^+]^+,[T_B^-,T_B^+]^-,[I_B^-,I_B^+]^-,[F_B^-,F_B^+]^-$>} where $(T^+)^+=1-(F^-)^+$, $(F^+)^+=1-(T^-)^+$ and $(T^+)^-=-1-(F^-)^-$, $(F^+)^-=-1-(T^-)^-$ T^+ , I^+ , F^+ : X→[0,1] and T^-,I^-,F^-:X→[-1,0] are two neutrosophic bipolar vague sets then their union, intersection and complement are well-defined as follows:

1.A∪B= { $\max[T_A^-,T_B^-]^+$, $\max[T_A^+,T_B^+]^+$,
$\min[I_A^-,I_B^-]^+$, $\min[I_A^+,I_B^+]^+$,
$\min[F_A^-,F_B^-]^+$, $\min[F_A^+,F_B^+]^+$,
$\min[T_A^-,T_B^-]^-$, $\min[T_A^+,T_B^+]^-$,
$\max\,[I_A^-,I_B^-]^-$, $\max[I_A^+,I_B^+]^-$,
$\max[F_A^-,F_B^-]^-$, $\max[F_A^+,F_B^+]^-$}.

2.A∩B={ $\min[T_A^-,T_B^-]^+$, $\min[T_A^+,T_B^+]^+$,
$\max[I_A^-,I_B^-]^+$,$\max[I_A^+,I_B^+]^+$,
$\max[F_A^-,F_B^-]^+$, $\max[F_A^+,F_B^+]^+$,
$\max[T_A^-,T_B^-]^-$, $\max[T_A^+,T_B^+]^-$,
$\min\,[I_A^-,I_B^-]^-$, $\min[I_A^+,I_B^+]^-$,
$\min[F_A^-,F_B^-]^-$, $\min[F_A^+,F_B^+]^-$}.

3. $\bar{A}$={<$[F_A^-,F_A^+]^+,[1-I_A^-,1-I_A^+]^+,[T_A^-,T_A^+]^+,[F_A^-,F_A^+]^-,[1-I_A^-,1-I_A^+]^-,[T_A^-,T_A^+]^-$>}.

Definition 3.2: Suppose A and B be two neutrosophic bipolar vague sets defined over a universe of disclosure X. We say that A ⊆ B if and only if $[T_A^-\le T_B^-]^+$, $[T_A^+\le T_B^+]^+$, $[I_A^-\ge I_B^-]^+$, $[I_A^+\ge I_B^+]^+$, $[F_A^-\ge F_B^-]^+$, $[F_A^+\ge F_B^+]^+$, $[T_A^-\ge T_B^-]^-$, $[T_A^+\ge T_B^+]^-$, $[I_A^-\le I_B^-]^-$, $[I_A^+\le I_B^+]^-$, $[F_A^-\le F_B^-]^-$, $[F_A^+\le F_B^+]^-$.

Definition 3.3: A bipolar vague topology NBVT on a nonempty set X is a family NBV$_\tau$ of Neutrosophic bipolar vague set in X sustaining the following axioms:

1. 0, 1 ∈ NBV_τ.
2. $G_1 \cap G_2 \in NBV_\tau$, for any $G_1,G_2 \in NBV_\tau$.
3. $\cup G_i \in NBV_\tau$ for any arbitrary family { G_i: $G_i \in NBV_\tau$, i∈I}.

Under such case the pair (X, NBV_τ) is known as the neutrosophic bipolar vague topological space and any NBVS in NBV_τ is known as bipolar vague open set in X . The complement $\bar{A}$ of a neutrosophic bipolar vague open set (NBVOS) A in a neutrosophic bipolar vague topological space (X, NBV_τ) is referred as a neutrosophic bipolar vague closed (NBVCS) in X.

Example 3.4: Assume X={u,v},

A_{NBV}= {$\frac{u}{[0.5,0.7][0.5,0.5][0.3,0.5][-0.4,-0.1][-0.5,-0.6][-0.9,-0.6]}, \frac{v}{[0.3,0.6][0.4,0.4][0.4,0.7][-0.2,-0.2][-0.6,-0.8][-0.8,-0.8]}$} ,

B_{NBV}= {$\frac{u}{[0.5,0.9][0.3,0.3][0.1,0.5][-0.4,-0.3][-0.4,-0.4][-0.7,-0.6]}, \frac{v}{[0.4,0.6][0.2,0.2][0.4,0.6][-0.5,-0.3][-0.5,-0.5][-0.7,-0.5]}$}.

Then the family NBV_τ={0,1,A,B} of neutrosophic bipolar vague sets in X is a NBVT on X.

Definition 3.5: Suppose (X, NBV_τ) is a neutrosophic bipolar vague topological space and A={< x, $[T_A^-,T_A^+]^+,[I_A^-,I_A^+]^+,[F_A^-,F_A^+]^+,[T_A^-,T_A^+]^-,[I_A^-,I_A^+]^-,[F_A^-,F_A^+]^-$ >} be a NBVS inX . Then the neutrosophic bipolar vague interior and neutrosophic bipolar vague closure of A are well-defined by,

NBVcl(A)=∩{K:K is a NBVCS in X and A⊆K},

NBVint(A)=∪{G:G is a NBVOS in X and G⊆A}.

Note that NBVcl(A) is a NBVCS and NBVint(A) is a NBVOS in X . Further,

1. A is a NBVCS in X iff NBVcl(A)=A
2. A is a NBVOS in X iff NBVint(A)=A.

Example 3.6: Assume that X={a, b},

A={x,$\frac{a}{[0.5,0.7][0.5,0.5][0.3,0.5][-0.4,-0.1][-0.5,-0.6][-0.9,-0.6]}, \frac{b}{[0.3,0.6][0.4,0.4][0.4,0.7][-0.2,-0.2][-0.6,-0.8][-0.8,-0.8]}$}

B= {$\frac{a}{[0.5,0.9][0.3,0.3][0.1,0.5][-0.4,-0.3][-0.4,-0.4][-0.7,-0.6]}, \frac{b}{[0.4,0.6][0.2,0.2][0.4,0.6][-0.5,-0.3][-0.5,-0.5][-0.7,-0.5]}$}.

Then the family NBV_τ={0,1,A,B} of a neutrosophic bipolar vague sets in X is NBVT on X. If,

F={<x,$\frac{a}{[0.5,0.4][0.5,0.5][0.6,0.5][-0.6,-0.4][-0.3,-0.3][-0.6,-0.4]}, \frac{b}{[0.5,0.7][0.1,0.1][0.3,0.5][-0.3,-0.4][-0.2,-0.2][-0.6,-0.7]}$ >}

Then, NBVint(A)=∪{G:G is a NBVOS in X and G⊆F}=0 and NBVcl(A)=∩{K:K is a NBVCS in X and F⊆K}=1.

Proposition 3.7: For any NBVS A in (X, NBV_τ) we have,

1. NBVcl($\bar{A}$)=$\overline{\text{NBVint(A)}}$
2. NBVint($\bar{A}$)=$\overline{\text{NBVcl(A)}}$.

Proof: Let A={< x, $[T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^-$ >} and suppose that NBVOS's contained in A are indexed by the family

{<x, $[T_{G_i}^-, T_{G_i}^+]^+, [I_{G_i}^-, I_{G_i}^+]^+, [F_{G_i}^-, F_{G_i}^+]^+, [T_{G_i}^-, T_{G_i}^+]^-, [I_{G_i}^-, I_{G_i}^+]^-, [F_{G_i}^-, F_{G_i}^+]^-$ >: $i \in J$}. Then

NBVint(A)= <$x, \cup[T_{G_i}^-, T_{G_i}^+]^+, \cap[I_{G_i}^-, I_{G_i}^+]^+, \cap[F_{G_i}^-, F_{G_i}^+]^+, \cap[T_{G_i}^-, T_{G_i}^+]^-, \cup[I_{G_i}^-, I_{G_i}^+]^-, \cup[F_{G_i}^-, F_{G_i}^+]^-$ > and hence

$\overline{\text{NBVint(A}}$) =< $x, \cap[F_{G_i}^-, F_{G_i}^+]^+, \cup[1-I_{G_i}^-, 1-I_{G_i}^+]^+, \cup[T_{G_i}^-, T_{G_i}^+]^+, \cup[F_{G_i}^-, F_{G_i}^+]^-, \cap[1-I_{G_i}^-, 1-I_{G_i}^+]^-, \cap[T_{G_i}^-, T_{G_i}^+]^-$ > --------- (1)

Since,

$\bar{A}$={<$[F_A^-, F_A^+]^+, [1-I_A^-, 1-I_A^+]^+, [T_A^-, T_A^+]^+, [F_A^-, F_A^+]^-, [1-I_A^-, 1-I_A^+]^-, [T_A^-, T_A^+]^-$>}. Where

$[T_{G_i}^-, T_{G_i}^+]^+ \leq [T_A^-, T_A^+]^+$, $[I_{G_i}^-, I_{G_i}^+]^+ \geq [I_A^-, I_A^+]^+$, $[F_{G_i}^-, F_{G_i}^+]^+ \geq [F_A^-, F_A^+]^+$, $[T_{G_i}^-, T_{G_i}^+]^- \geq [T_A^-, T_A^+]^-$,

$[I_{G_i}^-, I_{G_i}^+]^- \leq [I_A^-, I_A^+]^-$, $[F_{G_i}^-, F_{G_i}^+]^- \leq [F_A^-, F_A^+]^-$ for every i∈J we obtain that

{< $x, [F_{G_i}^-, F_{G_i}^+]^+, [1-I_{G_i}^-, 1-I_{G_i}^+]^+, [T_{G_i}^-, T_{G_i}^+]^+, [F_{G_i}^-, F_{G_i}^+]^-, [1-I_{G_i}^-, 1-I_{G_i}^+]^-, [T_{G_i}^-, T_{G_i}^+]^-$ > i∈J}

Is the family of NBVS's containing$\bar{A}$, that is ,

NBVcl($\bar{A}$)= < $x, \cap[F_{G_i}^-, F_{G_i}^+]^+, \cup[1-I_{G_i}^-, 1-I_{G_i}^+]^+, \cup[T_{G_i}^-, T_{G_i}^+]^+, \cup[F_{G_i}^-, F_{G_i}^+]^-, \cap[1-I_{G_i}^-, 1-I_{G_i}^+]^-, \cap[T_{G_i}^-, T_{G_i}^+]^-$ > ------------ (2).

Hence from (1) and (2) we get NBVcl($\bar{A}$)=$\overline{\text{NBVint(A)}}$

(2) follows from (1).

Proposition 3.8: If (X, NBV_τ) is a NBVTS and A,B be are NBVS's in X. Then the following properties hold:

1. NBVint(A)⊆A
2. A⊆ NBvcl(A)
3. A⊆ B ⇒ NBVint(A) ⊆ NBVint(B)
4. A⊆ B ⇒ NBVcl(A) ⊆ NBVcl(B)
5. NBVint(NBVint(A))= NBVint(A)
6. NBVcl(NBVcl(A)= NBVcl(A)
7. NBVint($A \cap B$)= NBVint(A) ∩ NBVint(B)
8. NBVcl(A∪ B)= NBVcl(A)∪ NBVcl(B)
9. NBVint(1)=1
10. NBVcl(0)=0

Definition 3.9: Suppose (X, NBV_τ) and (Y, NBV_σ) be two neutrosophic bipolar vague topological spaces and $\psi: X \to Y$ be a function. Then ψ is referred to be a neutrosophic bipolar vague continuous iff the preimage of each neutrosophic bipolar vague open set in Y is a neutrosophic bipolar vague open set in X.

Proposition 3.10: Suppose A, {A_i: i∈J} be a neutrosophic bipolar vague set in X, and B, {B_j: j∈K} be a neutrosophic bipolar vague set in Y, and let $\psi: X \to Y$ be a function. Then,

(a) $A_1 \subseteq A_2 \Leftrightarrow \psi(A_1) \subseteq \psi(A_2)$
(b) $B_1 \subseteq B_2 \Leftrightarrow \psi^{-1}(B_1) \subseteq \psi^{-1}(B_2)$
(c) $\psi^{-1}(\cup B_i) = \cup\, \psi^{-1}(B_i)$ and $\psi^{-1}(\cap B_i) = \cap\, \psi^{-1}(B_i)$

Proof: Obvious.

Proposition 3.11: The subsequent are equivalent to each other.

1. $\psi: X \to Y$ is neutrosophic bipolar vague continuous.
2. ψ^{-1}(NBVint(B)) ⊆ NBVint(ψ^{-1}(B)) for each NBVOS B in Y.
3. NBVcl(ψ^{-1}(B)) ⊆ ψ^{-1}(NBVcl(B)) for each NBVOS B in Y.

Proof: (1)⟹(2) Given $\psi: X \to Y$ is neutrosophic bipolar vague continuous.

Then we have to show that ψ^{-1}(NBVint(B)) $\subseteq$ NBVint(ψ^{-1}(B)) for each NBVOS B in Y.
Let B={< y, $[T_B^-, T_B^+]^+$, $[I_B^-, I_B^+]^+$, $[F_B^-, F_B^+]^+$, $[T_B^-, T_B^+]^-$, $[I_B^-, I_B^+]^-$, $[F_B^-, F_B^+]^-$>} be NBVOS in Y.
NBVint(B)=
$\{ < y, \cup[T_{H_i}^-, T_{H_i}^+]^+, \cap[I_{H_i}^-, I_{H_i}^+]^+, \cap[F_{H_i}^-, F_{H_i}^+]^+, \cap[T_{H_i}^-, T_{H_i}^+]^-, \cup[I_{H_i}^-, I_{H_i}^+]^-, \cup[F_{H_i}^-, F_{H_i}^+]^- >$:i∈I}
Where,
$[T_{H_i}^-, T_{H_i}^+]^+ \le [T_B^-, T_B^+]^+$, $[I_{H_i}^-, I_{H_i}^+]^+ \ge [I_B^-, I_B^+]^+$, $[F_{H_i}^-, F_{H_i}^+]^+ \ge [F_B^-, F_B^+]^+$, $[T_{H_i}^-, T_{H_i}^+]^- \ge [T_B^-, T_B^+]^-$, $[I_{H_i}^-, I_{H_i}^+]^- \le [I_B^-, I_B^+]^-$, $[F_{H_i}^-, F_{H_i}^+]^- \le [F_B^-, F_B^+]^-$ for every i ∈ I. By the definition of continuity ψ^{-1}(NBVint(B)) is a neutrosophic bipolar vague open set in NBV_τ. Now,
ψ^{-1}(NBVint(B)) ={
$\psi^{-1}(< y, \cup[T_{H_i}^-, T_{H_i}^+]^+, \cap[I_{H_i}^-, I_{H_i}^+]^+, \cap[F_{H_i}^-, F_{H_i}^+]^+, \cap[T_{H_i}^-, T_{H_i}^+]^-, \cup[I_{H_i}^-, I_{H_i}^+]^-, \cup[F_{H_i}^-, F_{H_i}^+]^- >)$}
= {(<x, $\psi^{-1}(\cup[T_{H_i}^-, T_{H_i}^+]^+)$, $\psi^{-1}(\cap[I_{H_i}^-, I_{H_i}^+]^+)$, $\psi^{-1}\left(\cap[F_{H_i}^-, F_{H_i}^+]^+\right)$, $\psi^{-1}(\cap[T_{H_i}^-, T_{H_i}^+]^-)$, $\psi^{-1}(\cup[I_{H_i}^-, I_{H_i}^+]^-)$, $\psi^{-1}(\cup[F_{H_i}^-, F_{H_i}^+]^-)$ >)}.
= {<x, $\cup[\psi^{-1}[[T_{H_i}^-, T_{H_i}^+]^+]$, $\cap[\psi^{-1}[I_{H_i}^-, I_{H_i}^+]^+]$, $\cap[\psi^{-1}[F_{H_i}^-, F_{H_i}^+]^+]$, $\cap[\psi^{-1}[T_{H_i}^-, T_{H_i}^+]^-]$, $\cup[\psi^{-1}[[I_{H_i}^-, I_{H_i}^+]^-]$, $\cup[\psi^{-1}[[F_{H_i}^-, F_{H_i}^+]^-]$>}.
$\subseteq$ NBVint(ψ^{-1}(B))
(2)⟹(1). Given ψ^{-1}(NBVint(B)) $\subseteq$ NBVint(ψ^{-1}(B)) for each NBVOS B in Y. Let
B={< y, $[T_B^-, T_B^+]^+$, $[I_B^-, I_B^+]^+$, $[F_B^-, F_B^+]^+$, $[T_B^-, T_B^+]^-$, $[I_B^-, I_B^+]^-$, $[F_B^-, F_B^+]^-$>} be NBVOS in Y. We know that B is a neutrosophic bipolar vague open in Y if and only if NBVint(B)=B and hence ψ^{-1}(NBVint(B))= ψ^{-1}(B). But according to our supposition ψ^{-1}(NBVint(B)) $\subseteq$ NBVint(ψ^{-1}(B)), therefore we get ψ^{-1}(B) $\subseteq$ NBVint(ψ^{-1}(B), i.e., ψ^{-1}(B) is a NBVS in X and thus ψ is a neutrosophic bipolar vague continuous.
(1)⟹ (3) Given $\psi: X \to Y$ is neutrosophic bipolar vague continuous.
Suppose B={< y, $[T_B^-, T_B^+]^+$, $[I_B^-, I_B^+]^+$, $[F_B^-, F_B^+]^+$, $[T_B^-, T_B^+]^-$, $[I_B^-, I_B^+]^-$, $[F_B^-, F_B^+]^-$>} be NBVOS in Y.
Also suppose NBVcl(B) =
{<y,$\cap[T_{K_i}^-, T_{K_i}^+]^+, \cup[I_{K_i}^-, I_{K_i}^+]^+, \cup[F_{K_i}^-, F_{K_i}^+]^+, \cup[T_{K_i}^-, T_{K_i}^+]^-, \cap[I_{K_i}^-, I_{K_i}^+]^-, \cap[F_{K_i}^-, F_{K_i}^+]^-$>:i∈ I},where
$[T_{K_i}^-, T_{K_i}^+]^+ \le [T_B^-, T_B^+]^+$,$[I_{K_i}^-, I_{K_i}^+]^+ \ge [I_B^-, I_B^+]^+$,$[F_{K_i}^-, F_{K_i}^+]^+ \ge [F_B^-, F_B^+]^+$,
$[T_{K_i}^-, T_{K_i}^+]^- \ge [T_B^-, T_B^+]^-$,$[I_{K_i}^-, I_{K_i}^+]^- \le [I_B^-, I_B^+]^-$, $[F_{K_i}^-, F_K^+]^- \le [F_B^-, F_B^+]^-$ for every i ∈ I. Since ψ is a neutrosophic bipolar vague continuous iff the inverse image of each NBVCS in Y is a NBVCS in X, therefore ψ^{-1}(NBVcl(B)) is a NBVCS in X.
Now, ψ^{-1} (NBVcl(B))= {$\psi^{-1}(< y, \cap[T_{K_i}^-, T_{K_i}^+]^+, \cup[I_{K_i}^-, I_{K_i}^+]^+, \cup[F_{K_i}^-, F_{K_i}^+]^+, \cup[T_{K_i}^-, T_{K_i}^+]^-, \cap[I_{K_i}^-, I_{K_i}^+]^-, \cap[F_{K_i}^-, F_{K_i}^+]^- >)$}
={(<x, $\psi^{-1}(\cap[T_{K_i}^-, T_{K_i}^+]^+)$, $\psi^{-1}(\cup[I_{K_i}^-, I_{K_i}^+]^+)$, $\psi^{-1}\left(\cup[F_{K_i}^-, F_{K_i}^+]^+\right)$, $\psi^{-1}(\cup[T_{K_i}^-, T_{K_i}^+]^-)$, $\psi^{-1}(\cap[I_{K_i}^-, I_{K_i}^+]^-)$, $\psi^{-1}(\cap[F_{K_i}^-, F_{K_i}^+]^-)$ >)}.
= {<x, $\cap[\psi^{-1}[[T_{K_i}^-, T_{K_i}^+]^+]$, $\cup[\psi^{-1}[I_{K_i}^-, I_{K_i}^+]^+]$, $\cup[\psi^{-1}[F_{K_i}^-, F_{K_i}^+]^+]$, $\cup[\psi^{-1}[T_{K_i}^-, T_{K_i}^+]^-]$, $\cap[\psi^{-1}[[I_{K_i}^-, I_{K_i}^+]^-]$, $\cap[\psi^{-1}[[F_{K_i}^-, F_{K_i}^+]^-]$>}
$\supseteq$ NBVcl(ψ^{-1}(B))
(3)⟹(1)
Given NBVcl(ψ^{-1}(B))⊆ ψ^{-1}(NBVcl(B)), for each NBVOS B in Y. Let
B={< y, $[T_B^-, T_B^+]^+$, $[I_B^-, I_B^+]^+$, $[F_B^-, F_B^+]^+$, $[T_B^-, T_B^+]^-$, $[I_B^-, I_B^+]^-$, $[F_B^-, F_B^+]^-$>} be NBVCS in Y. Since NBVcl(B)=B. But it is given that NBVcl(ψ^{-1}(B))⊆ ψ^{-1}(NBVcl(B)), hence NBVcl(ψ^{-1}(B))⊆ ψ^{-1}(B). Hence ψ^{-1}(B)= NBVcl(ψ^{-1}(B)), i.e., ψ^{-1}(B) is a NBVCS in X and this proves that ψ is a neutrosophic bipolar vague continuous.

4. Generalized Neutrosophic Bipolar Vague Closed Sets:

Definition 4.1: Suppose if (X, NBV_τ) be a neutrosophic bipolar vague topological space. A neutrosophic bipolar vague set A in (X, NBV_τ) is referred to be a generalized neutrosophic bipolar vague closed set if NBVcl(A) $\subseteq G$ whenever A⊆G and G is a neutrosophic bipolar vague open. The complement of a generalized neutrosophic bipolar vague closed set is generalized neutrosophic bipolar vague open set.

Definition 4.2: Suppose let (X, NBV_τ) be a neutrosophic bipolar vague topological space and let A be a neutrosophic bipolar vague set in X. The generalized neutrosophic bipolar vague closure (GNBVcl for short) and the generalized neutrosophic bipolar vague interior (GNBVint for short) of A are well-defined by,

1) GNBVcl(A)=⋂{G:G is a generalized neutrosophic bipolar vague closed sets in X and A⊆ G},
2) GNBVint(A)=⋃{G:G is a generalized neutrosophic bipolar vague open sets in X and A⊇ G}.

Remark 4.3: Every NBVCS is generalized neutrosophic bipolar vague closed but not conversely.

Example 4.4: Assume that X= {u,v} and NBV_τ={0,1,F} is a NBVT on X where,

F=< $x, \frac{u}{[0.5,0.9][0.3,0.3][0.1,0.5][-0.4,-0.3][-0.4,-0.4][-0.7,-0.6]}, \frac{v}{[0.4,0.6][0.2,0.2][0.4,0.6][-0.5,-0.3][-0.5,-0.5][-0.7,-0.5]}$>

Then the neutrosophic bipolar vague set,

A=< $x, \frac{u}{[0.5,0.7][0.5,0.5][0.3,0.5][-0.4,-0.1][-0.5,-0.6][-0.9,-0.6]}, \frac{v}{[0.3,0.6][0.4,0.4][0.4,0.7][-0.2,-0.2][-0.6,-0.8][-0.8,-0.8]}$ > is a generalized neutrosophic bipolar vague closed but not NBVC in X.

Proposition 4.5: Suppose that (X, NBV_τ) be a neutrosophic bipolar vague topological space. If A is a generalized neutrosophic bipolar vague closed set and A⊆B⊆NBVcl(A), then B is a generalized neutrosophic bipolar vague closed set.

Proof: Suppose let G be a neutrosophic bipolar vague open set in (X, NBV_τ), such that B⊆G. Since A⊆B, A⊆ G. Now A is a generalized neutrosophic bipolar vague closed set and NBVcl(A) ⊆G. But NBVcl(B) ⊆ NBVcl(A). Since NBVcl(B) ⊆ NBVcl(A) ⊆G, NBVcl(B) ⊆G. Hence B is a generalized neutrosophic bipolar vague closed set.

Proposition 4.6: Suppose if A is a neutrosophic bipolar vague open set and generalized neutrosophic bipolar vague closed set in (X, NBV_τ), then A is said to be a neutrosophic bipolar vague closed set in X.

Proof: Assume that A is a neutrosophic bipolar vague open set in X. Since A⊆A, by hypothesis NBVcl(A) ⊆A. Then from definition A⊆ NBVcl(A). Therefore NBVcl(A)= A. Hence A is neutrosophic bipolar vague closed set in X.**Proposition 4.7**: Suppose that NBVint(A) ⊆B⊆A and assume A is a generalized neutrosophic bipolar vague open set then B is also a generalized neutrosophic bipolar vague open set.

Proof: Now, $\bar{A} \subseteq \bar{B} \subseteq \overline{\mathrm{NBVint(A)}}$= NBVcl($\bar{A}$). As A is a generalized neutrosophic bipolar vague open, $\bar{A}$ is a generalized neutrosophic bipolar vague closed set. By proposition 4.5, $\bar{B}$ generalized neutrosophic bipolar vague closed set. That is, B is also a generalized neutrosophic bipolar vague open set.

Definition 4.8: Suppose (X, NBV_τ) and (Y, NBV_σ)be any two neutrosophic bipolar vague topological spaces.

1. A map ψ: (X, NBV_τ)→(Y, NBV_σ) is referred to be a generalized neutrosophic bipolar vague continuous if the inverse image of every neutrosophic bipolar vague open set in (Y, NBV_σ) is a generalized neutrosophic bipolar vague open set in (X, NBV_τ).
2. A map ψ: (X, NBV_τ)→(Y, NBV_σ) is called as a generalized neutrosophic bipolar vague irresolute if the inverse image of every generalized neutrosophic bipolar vague open set in (Y, NBV_σ) is a generalized neutrosophic bipolar vague open set in (X, NBV_τ).

Proposition 4.9: Suppose (X, NBV_τ) and (Y, NBV_σ)be any two neutrosophic bipolar vague topological spaces. A mapping ψ: (X, NBV_τ)→(Y, NBV_σ) is referred to be generalized neutrosophic bipolar vague continuous function mapping. Then for every neutrosophic bipolar vague set A in X, ψ(GNBVcl(A)) ⊆ NBVcl($\psi(A)$).

Proof: Assume A to be a neutrosophic bipolar vague set in (X, NBV_τ). Since NBVcl($\psi(A)$) is a neutrosophic bipolar vague closed set and since ψ is a generalized neutrosophic bipolar vague continuous mapping, the set ψ^{-1}(NBVcl($\psi(A)$)) is a generalized neutrosophic bipolar vague closed set and thus ψ^{-1}(NBVcl($\psi(A)$)) ⊇A.

Now, GNBVcl(A))⊆ $\psi^{-1}\left(\mathrm{NBVcl}(\psi(A))\right)$. Therefore ψ(GNBVcl(A)) ⊆ NBVcl($\psi(A)$).

Proposition 4.10: If (X, NBV_τ) and (Y, NBV_σ) are two neutrosophic bipolar vague topological spaces. Let the mapping ψ : (X, NBV_τ) →(Y, NBV_σ) be a generalized neutrosophic bipolar vague continuous mapping. Then for every neutrosophic bipolar vague set A in Y, GNBVcl(ψ^{-1}(A)) ⊆ ψ^{-1}(NBVcl(A).

Proof: Assume A to be a neutrosophic bipolar vague set in (Y, NBV_σ). Let B= ψ^{-1}(A). Then, ψ(B)= $\psi(\psi^{-1}$(A)) ⊆A. By proposition 4.10, ψ(GNBVcl(ψ^{-1}(A))) ⊆NBV cl($\psi(\psi^{-1}$(A))). Thus, GNBVcl(ψ^{-1}(A)) ⊆ ψ^{-1}(NBVcl(A).

Proposition 4.11: Suppose let (X, NBV_τ) and (Y, NBV_σ) be any two neutrosophic bipolar vague topological spaces. Let ψ : (X, NBV_τ) →(Y, NBV_σ) is referred to be a neutrosophic bipolar vague continuous mapping, then it is a generalized neutrosophic bipolar vague continuous mapping.

Proof: Suppose let A be a neutrosophic bipolar vague open set in (Y, NBV_σ). Since the mapping ψ is a neutrosophic bipolar vague continuous mapping, $\psi^{-1}(A)$ is a neutrosophic bipolar vague open set in (X, NBV_τ). Every neutrosophic bipolar vague open set is a generalized neutrosophic bipolar vague open set. Now, $\psi^{-1}(A)$ is a generalized neutrosophic bipolar vague open set in (X, NBV_τ). Hence ψ is thus a generalized neutrosophic bipolar vague continuous mapping.

The converse of the proposition need not be true as shown in example.

Example 4.12: Assume that X={a,b}, Y={u,v} and,

A=<x, $\frac{a}{[0.5,0.4][0.5,0.5][0.6,0.5][-0.6,-0.4][-0.3,-0.3][-0.6,-0.4]}, \frac{b}{[0.6,0.7][0.1,0.1][0.3,0.4][-0.3,-0.4][-0.2,-0.2][-0.6,-0.7]}$>,

B=<x, $\frac{a}{[0.5,0.3][0.5,0.5][0.7,0.5][-0.4,-0.2][-0.4,-0.4][-0.8,-0.6]}, \frac{b}{[0.5,0.4][0.2,0.2][0.6,0.5][-0.3,-0.4][-0.2,-0.2][-0.6,-0.7]}$>.

Then NBV_τ={0,1,A} and NBV_σ={0,1,B} are NBVT on X and Y respectively. Define a mapping ψ: (X, NBV_τ)→(Y, NBV_σ) by ψ(a)=u and ψ(b)=v. then ψ is a generalized neutrosophic bipolar vague continuous mapping but not bipolar vague continuous mapping.

Proposition 4.13: Suppose let (X, NBV_τ) and (Y, NBV_σ) be any two neutrosophic bipolar vague topological spaces. A mapping ψ: (X, NBV_τ)→(Y, NBV_σ) is said to be a generalized neutrosophic bipolar vague irresolute mapping, then it is a generalized neutrosophic bipolar vague continuous mapping.

Proof: Let A be a neutrosophic bipolar vague open set in (Y, NBV_σ). Since every neutrosophic bipolar vague open set is a generalized neutrosophic bipolar vague open set in (Y, NBV_σ), but ψ is a generalized neutrosophic bipolar vague irresolute mapping, $\psi^{-1}(A)$ is a generalized neutrosophic bipolar vague open set in (X, NBV_τ). Thus ψ is a generalized neutrosophic bipolar vague continuous mapping.

Proposition 4.14: Suppose let (X, NBV_τ), (Y, NBV_σ) and (Z, NBV_ρ) be any three bipolar vague topological spaces. Let ψ: (X, NBV_τ)→(Y, NBV_σ) be a generalized neutrosophic bipolar vague irresolute mapping and ψ_1: (Y, NBV_σ)→(Z, NBV_ρ) be a generalized neutrosophic bipolar vague continuous mapping. Then $\psi_1 \circ \psi$ is a generalized neutrosophic bipolar vague continuous mapping.

Proof: Let A be a neutrosophic bipolar vague open set in (Z, NBV_ρ). Since ψ_1 is a generalized neutrosophic bipolar vague continuous mapping, ψ_1^{-1} (A) is a generalized neutrosophic bipolar vague open set in (Y, NBV_σ) . Since ψ is a generalized neutrosophic bipolar vague irresolute mapping, $\psi^{-1}(\psi_1^{-1}$ (A)) is a generalized neutrosophic bipolar vague open set in (X, NBV_τ). Thus $\psi_1 \circ \psi$ is a generalized neutrosophic bipolar vague continuous mapping.

Conclusion:

This paper presented the new concept of Neutrosophic Bipolar Vague sets and studied some basic operational relation of Neutrosophic Bipolar Vague set. Then a generalization of NBVSs in closed set is done. As a future work, we shall continue to work in the application of NBVS to other domains, such as medical diagnosis, pattern recognition and decision making.

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