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Neutrosophic Generalized Pre Regular Closed Sets

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Abstract: As a generalization of fuzzy sets and intuitionistic fuzzy sets, Neutrosophic sets have been developed by Smarandache to represent imprecise, incomplete and inconsistent information existing in the real world. A neutrosophic set is characterized by a truth value, an indeterminacy value and a falsity value. In this paper, we introduce and study a new class of Neutrosophic generalized closed set, namely Neutrosophic generalized pre regular closed sets and Neutrosophic generalized pre regular open sets in Neutrosophic topological spaces. Also we study the separation axioms of Neutrosophic generalized pre regular closed sets, namely Neutrosophic pre regular $T_{1/2}$ space and Neutrosophic pre regular $T^*_{1/2}$ space and their properties are discussed.

Keywords: Neutrosophic generalized pre regular closed sets, Neutrosophic generalized pre regular open sets, $NprT_{1/2}$ space and $NprT^*_{1/2}$ space.

1. Introduction

In 1970, Levine [12] introduced the concept of g-closed sets in general topology. Generalized closed sets play a very important role in general topology and they are now the research topics of many researchers worldwide. In 1965, Zadeh [19] introduced the notion of fuzzy sets [FS]. Later, fuzzy topological space was introduced by Chang [6] in 1968 using fuzzy sets. In 1986, Atanassov [5] introduced the notion of intuitionistic fuzzy sets [IFS], where the degree of membership and degree of non-membership of an element in a set X are discussed. In 1997, Intuitionistic fuzzy topological spaces were introduced by Coker [7] using intuitionistic fuzzy sets.

Neutrality the degree of indeterminacy as an independent concept was introduced by Florentine Smarandache [8]. He also defined the Neutrosophic set on three components, namely Truth (membership), Indeterminacy, Falsehood (non-membership) from the fuzzy sets and intuitionistic fuzzy sets. Smarandache's Neutrosophic concepts have wide range of real time applications for the fields of [1, 2, 3&4] Information systems, Computer science, Artificial Intelligence, Applied Mathematics and Decision making.

In 2012, Salama A. A and Alblowi [14] introduced the concept of Neutrosophic topological spaces by using Neutrosophic sets. Salama A. A. [15] introduced Neutrosophic closed set and Neutrosophic continuous functions in Neutrosophic topological spaces. Further the basic sets like Neutrosophic regular-open sets, Neutrosophic semi-open sets, Neutrosophic pre-open sets, Neutrosophic α -open sets and Neutrosophic generalized closed sets are introduced in Neutrosophic topological space and their properties are studied by various authors [10], [15], [17], [13]. In this direction, we introduce and analyze a new class of Neutrosophic generalized closed set called Neutrosophic generalized pre regular closed sets and Neutrosophic generalized pre regular open sets in Neutrosophic topological spaces. Also we study the separation axioms of Neutrosophic generalized pre regular closed sets, namely Neutrosophic pre regular $T_{1/2}$ space and Neutrosophic

pre regular $T^*_{1/2}$ space in Neutrosophic topological spaces. Many examples are given to justify the results.

2. Preliminaries

We recall some basic definitions that are used in the sequel.

Definition 2.1: [14] Let X be a non-empty fixed set. A Neutrosophic set (NS for short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$ where the functions $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A .

Remark 2.2: [14] A Neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$ can be identified to an ordered triple $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ in non-standard unit interval $]^{-0}, 1^{+}$ on X .

Remark 2.3: [14] For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \nu_A \rangle$ for the neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$.

Example 2.4: [14] Every IFS A is a non-empty set in X is obviously on NS having the form $A = \{ \langle x, \mu_A(x), 1 - (\mu_A(x) + \nu_A(x)), \nu_A(x) \rangle : x \in X \}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the NS 0_N and 1_N in X as follows:

0_N may be defined as:

$$(0_1) 0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$$

$$(0_2) 0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$$

$$(0_3) 0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$$

$$(0_4) 0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$$

1_N may be defined as:

$$(1_1) 1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$$

$$(1_2) 1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$$

$$(1_3) 1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$$

$$(1_4) 1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$$

Definition 2.5: [14] Let $A = \langle \mu_A, \sigma_A, \nu_A \rangle$ be a NS on X , then the complement of the set A [$C(A)$ for short] may be defined as three kind of complements:

$$(C_1) C(A) = \{ \langle x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \nu_A(x) \rangle : x \in X \}$$

$$(C_2) C(A) = \{ \langle x, \nu_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}$$

$$(C_3) C(A) = \{ \langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}$$

Definition 2.6: [14] Let X be a non-empty set and Neutrosophic sets A and B in the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X \}$. Then we may consider two possible definitions for subsets ($A \subseteq B$).

$$(1) A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x) \text{ and } \mu_A(x) \geq \mu_B(x) \quad \forall x \in X$$

$$(2) A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x) \text{ and } \mu_A(x) \geq \mu_B(x) \quad \forall x \in X$$

Proposition 2.7: [14] For any Neutrosophic set A , the following conditions hold:

$$0_N \subseteq A, 0_N \subseteq 0_N$$

$$A \subseteq 1_N, 1_N \subseteq 1_N$$

Definition 2.8: [14] Let X be a non-empty set and $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$, $B = \{ \langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X \}$ are NSs. Then $A \cap B$ may be defined as:

$$(I_1) A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x) \text{ and } \nu_A(x) \vee \nu_B(x) \rangle$$

$$(I_2) A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x) \text{ and } \nu_A(x) \vee \nu_B(x) \rangle$$

$A \cup B$ may be defined as:

$$(U_1) A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x) \text{ and } \nu_A(x) \wedge \nu_B(x) \rangle$$

$$(U_2) A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x) \text{ and } \nu_A(x) \wedge \nu_B(x) \rangle$$

We can easily generalize the operations of intersection and union in Definition 2.8., to arbitrary family of NSs as follows:

Definition 2.9: [14] Let $\{A_j: j \in J\}$ be an arbitrary family of NSs in X , then

$\cap A_j$ may be defined as:

$$(i) \quad \cap A_j = \langle x, \wedge_{j \in J} \mu_{A_j}(x), \wedge_{j \in J} \sigma_{A_j}(x), \vee_{j \in J} \nu_{A_j}(x) \rangle$$

$$(ii) \quad \cap A_j = \langle x, \wedge_{j \in J} \mu_{A_j}(x), \vee_{j \in J} \sigma_{A_j}(x), \vee_{j \in J} \nu_{A_j}(x) \rangle$$

$\cup A_j$ may be defined as:

$$(i) \quad \cup A_j = \langle x, \vee_{j \in J} \mu_{A_j}(x), \vee_{j \in J} \sigma_{A_j}(x), \wedge_{j \in J} \nu_{A_j}(x) \rangle$$

$$(ii) \quad \cup A_j = \langle x, \vee_{j \in J} \mu_{A_j}(x), \wedge_{j \in J} \sigma_{A_j}(x), \wedge_{j \in J} \nu_{A_j}(x) \rangle$$

Proposition 2.10: [14] For all A and B are two Neutrosophic sets then the following conditions are true:

$$C(A \cap B) = C(A) \cup C(B); C(A \cup B) = C(A) \cap C(B).$$

Definition 2.11: [14] A Neutrosophic topology [NT for short] is a non-empty set X is a family τ of Neutrosophic subsets in X satisfying the following axioms:

$$(NT_1) \quad 0_N, 1_N \in \tau,$$

$$(NT_2) \quad G_1 \cap G_2 \in \tau \text{ for any } G_1, G_2 \in \tau,$$

$$(NT_3) \quad \cup G_i \in \tau \text{ for every } \{G_i : i \in J\} \subseteq \tau.$$

Throughout this paper, the pair (X, τ) is called a Neutrosophic topological space (NTS for short). The elements of τ are called Neutrosophic open sets [NOS for short]. A complement $C(A)$ of a NOS A in NTS (X, τ) is called a Neutrosophic closed set [NCS for short] in X .

Example 2.12: [14] Any fuzzy topological space (X, τ) in the sense of Chang is obviously a NTS in the form $\tau = \{A: \mu_A \in \tau\}$ wherever we identify a fuzzy set in X whose membership function is μ_A with its counterpart.

The following is an example of Neutrosophic topological space.

Example 2.13: [14] Let $X = \{x\}$ and $A = \{\langle x, 0.5, 0.5, 0.4 \rangle: x \in X\}$, $B = \{\langle x, 0.4, 0.6, 0.8 \rangle: x \in X\}$, $C = \{\langle x, 0.5, 0.6, 0.4 \rangle: x \in X\}$, $D = \{\langle x, 0.4, 0.5, 0.8 \rangle: x \in X\}$. Then the family $\tau = \{0_N, A, B, C, D, 1_N\}$ of NSs in X is Neutrosophic topological space on X .

Now, we define the Neutrosophic closure and Neutrosophic interior operations in Neutrosophic topological spaces:

Definition 2.14: [14] Let (X, τ) be NTS and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle: x \in X\}$ be a NS in X . Then the Neutrosophic closure and Neutrosophic interior of A are defined by

$$NCl(A) = \cap \{K : K \text{ is a NCS in } X \text{ and } A \subseteq K\}$$

$$NInt(A) = \cup \{G : G \text{ is a NOS in } X \text{ and } G \subseteq A\}$$

It can be also shown that $NCl(A)$ is NCS and $NInt(A)$ is a NOS in X .

$$a) \quad A \text{ is NOS if and only if } A = NInt(A),$$

$$b) \quad A \text{ is NCS if and only if } A = NCl(A).$$

Proposition 2.15: [14] For any Neutrosophic set A is (X, τ) we have

$$a) \quad NCl(C(A)) = C(NInt(A)),$$

$$b) \quad NInt(C(A)) = C(NCl(A)).$$

Proposition 2.16: [14] Let (X, τ) be NTS and A, B be two Neutrosophic sets in X . Then the following properties are holds:

- a) $NInt(A) \subseteq A$,
- b) $A \subseteq NCl(A)$,
- c) $A \subseteq B \Rightarrow NInt(A) \subseteq NInt(B)$,
- d) $A \subseteq B \Rightarrow NCl(A) \subseteq NCl(B)$,
- e) $NInt(NInt(A)) = NInt(A)$,
- f) $NCl(NCl(A)) = NCl(A)$,
- g) $NInt(A \cap B) = NInt(A) \cap NInt(B)$,
- h) $NCl(A \cup B) = NCl(A) \cup NCl(B)$,
- i) $NInt(0_N) = 0_N$,
- j) $NInt(1_N) = 1_N$,
- k) $NCl(0_N) = 0_N$,
- l) $NCl(1_N) = 1_N$,
- m) $A \subseteq B \Rightarrow C(A) \subseteq C(B)$,
- n) $NCl(A \cap B) \subseteq NCl(A) \cap NCl(B)$,
- o) $NInt(A \cup B) \supseteq NInt(A) \cup NInt(B)$.

Definition 2.17: [9] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be

- (i) Neutrosophic regular closed set (NRCS for short) if $A = NCl(NInt(A))$,
- (ii) Neutrosophic regular open set (NROS for short) if $A = NInt(NCl(A))$,
- (iii) Neutrosophic semi closed set (NSCS for short) if $NInt(NCl(A)) \subseteq A$,
- (iv) Neutrosophic semi open set (NSOS for short) if $A \subseteq NCl(NInt(A))$,
- (v) Neutrosophic pre closed set (NPCS for short) if $NCl(NInt(A)) \subseteq A$,
- (vi) Neutrosophic pre open set (NPOS for short) if $A \subseteq NInt(NCl(A))$,
- (vii) Neutrosophic α - closed set (NSCS for short) if $NCl(NInt(NCl(A))) \subseteq A$,
- (viii) Neutrosophic α - open set (NSOS for short) if $A \subseteq NInt(NCl(NInt(A)))$.

Definition 2.18: [18] Let (X, τ) be NTS and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ be a NS in X . Then the Neutrosophic pre closure and Neutrosophic pre interior of A are defined by

$$NPCL(A) = \bigcap \{K : K \text{ is a NPCS in } X \text{ and } A \subseteq K\},$$

$$NPInt(A) = \bigcup \{G : G \text{ is a NPOS in } X \text{ and } G \subseteq A\}.$$

Definition 2.18: [13] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be a Neutrosophic generalized closed set (NGCS for short) if $NCl(A) \subseteq U$ whenever $A \subseteq U$ and U is a NOS in (X, τ) . A NS A of a NTS (X, τ) is called a Neutrosophic generalized open set (NGOS for short) if $C(A)$ is a NGCS in (X, τ) .

Definition 2.20: [11] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be a Neutrosophic α - generalized closed set (NaGCS for short) if $NaCl(A) \subseteq U$ whenever $A \subseteq U$ and U is a NOS in (X, τ) . A NS A of a NTS (X, τ) is called a Neutrosophic α - generalized open set (NaGOS for short) if $C(A)$ is a NaGCS in (X, τ) .

Definition 2.21: [16] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be a Neutrosophic ω closed set ($N\omega$ CS for short) if $NCl(A) \subseteq U$ whenever $A \subseteq U$ and U is a NSOS in (X, τ) . A NS A of a NTS (X, τ) is called a Neutrosophic ω open set ($N\omega$ OS for short) if $C(A)$ is a $N\omega$ CS in (X, τ) .

Definition 2.22: [9] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be a Neutrosophic regular generalized closed set (NRGCS for short) if $NCl(A) \subseteq U$ whenever $A \subseteq U$ and U is a NROS in (X, τ) . A NS A of a NTS (X, τ) is called a Neutrosophic regular generalized open set (NRGOS for short) if $C(A)$ is a NRGCS in (X, τ) .

Definition 2.23: [18] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be a Neutrosophic generalized pre closed set (NGPCS for short) if $NPCI(A) \subseteq U$ whenever $A \subseteq U$ and U is a NOS in (X, τ) . A NS A of a NTS (X, τ) is called a Neutrosophic generalized pre open set (NGPOS for short) if $C(A)$ is a NGPCS in (X, τ) .

Definition 2.24: [9] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be a Neutrosophic regular α generalized closed set (NR α GCS for short) if $N\alpha CI(A) \subseteq U$ whenever $A \subseteq U$ and U is a NROS in (X, τ) . A NS A of a NTS (X, τ) is called a Neutrosophic regular α generalized open set (NR α GOS for short) if $C(A)$ is a NRGCS in (X, τ) .

3. Neutrosophic Generalized Pre Regular Closed Sets

In this section we introduce Neutrosophic generalized pre regular closed sets in the Neutrosophic topological space and study some of their properties.

Definition 3.1: A NS A in a NTS (X, τ) is said to be a Neutrosophic generalized pre regular closed set (NGPRCS for short) if $NPCI(A) \subseteq U$ whenever $A \subseteq U$ and U is a NROS in (X, τ) . The family of all NGPRCSs of a NTS (X, τ) is denoted by $NGPRC(X)$.

Example 3.2: Let $X = \{a, b\}$ and $\tau = \{0_N, U, V, 1_N\}$ where $U = \langle (0.5, 0.3, 0.6), (0.4, 0.4, 0.7) \rangle$ and $V = \langle (0.7, 0.5, 0.3), (0.7, 0.5, 0.2) \rangle$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \langle (0.2, 0.1, 0.7), (0.4, 0.4, 0.7) \rangle$ is a NGPRCS in (X, τ) . Since $A \subseteq U$ and U is a NROS, we have $NPCI(A) = A \subseteq U$.

Theorem 3.3: Every NCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is NCS in (X, τ) , we have $NCl(A) = A$. Therefore $NPCI(A) \subseteq NCl(A) = A \subseteq U$, by hypothesis. Hence A is a NGPRCS in (X, τ) .

Example 3.4: In Example 3.2., the NS $A = \langle (0.2, 0.1, 0.7), (0.4, 0.4, 0.7) \rangle$ is a NGPRCS but not NCS in (X, τ) .

Theorem 3.5: Every $N\alpha CS$ in (X, τ) is an NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is $N\alpha CS$ in (X, τ) , we have $NCl(NInt(NCl(A))) \subseteq A$, now $A \subseteq NCl(A)$, $NCl(NInt(A)) \subseteq NCl(NInt(NCl(A))) \subseteq A$. Therefore $NPCI(A) = A \cup NCl(NInt(A)) \subseteq A \cup A = A \subseteq U$. Hence A is a NGPRCS in (X, τ) .

Example 3.6: In Example 3.2., the NS $A = \langle (0.2, 0.1, 0.7), (0.4, 0.4, 0.7) \rangle$ is a NGPRCS but not $N\alpha CS$ in (X, τ) .

Theorem 3.7: Every $N\omega CS$ in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is $N\omega CS$ in (X, τ) , we have $NCl(A) \subseteq U$ because every NROS is NSOS in (X, τ) . Therefore $NPCI(A) \subseteq NCl(A) \subseteq U$, by hypothesis. Hence A is a NGPRCS in (X, τ) .

Example 3.8: Let $X = \{a, b\}$ and $\tau = \{0_N, U, V, 1_N\}$ where $U = \langle (0.6, 0.5, 0.2), (0.7, 0.5, 0.1) \rangle$ and $V = \langle (0.5, 0.4, 0.7), (0.4, 0.5, 0.6) \rangle$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \langle (0.4, 0.3, 0.7), (0.3, 0.2, 0.6) \rangle$ is a NGPRCS in (X, τ) . Since $A \subseteq V$ and V is a NROS, we have $NPCI(A) = A \subseteq V$. But A is not $N\omega CS$ in (X, τ) . Since $A \subseteq V$ and V is a NSOS, we have $NCl(A) = C(V) \not\subseteq V$.

Theorem 3.9: Every NPCS in (X, τ) is an NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is NPCS in (X, τ) , we have $NCl(NInt(A)) \subseteq A$. Therefore $NPCI(A) = AU \cap NCl(NInt(A)) \subseteq AUA = A \subseteq U$. Hence A is a NGPRCS in (X, τ) .

Example 3.10: Let $X = \{a, b\}$ and $\tau = \{0_N, U, V, 1_N\}$ where $U = \{(0.3, 0.2, 0.6), (0.1, 0.2, 0.7)\}$ and $V = \{(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)\}$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \{(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)\}$ is a NGPRCS in (X, τ) . Since $A \subseteq 1_N$, we have $NPCI(A) = 1_N \subseteq 1_N$. But A is not NPCS in (X, τ) . Since $NCl(NInt(A)) = 1_N \not\subseteq A$.

Theorem 3.11: Every NGCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is NGCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ) . Therefore $NPCI(A) \subseteq NCl(A) \subseteq U$, by hypothesis. Hence A is a NGPRCS in (X, τ) .

Example 3.12: Let $X = \{a, b\}$ and $\tau = \{0_N, U, V, 1_N\}$ where $U = \{(0.3, 0.5, 0.7), (0.4, 0.5, 0.6)\}$ and $V = \{(0.8, 0.5, 0.2), (0.7, 0.5, 0.3)\}$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \{(0.3, 0.5, 0.7), (0.3, 0.5, 0.7)\}$ is a NGPRCS in (X, τ) . Since $A \subseteq U$ and U is a NROS, we have $NPCI(A) = A \subseteq U$. But A is not NGCS in (X, τ) . Since $A \subseteq U$ and U is a NOS, we have $NCl(A) = C(U) \not\subseteq U$.

Theorem 3.13: Every $N\alpha$ GCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is $N\alpha$ GCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ) . Therefore $NPCI(A) \subseteq N\alpha Cl(A) \subseteq U$, by hypothesis. Hence A is a NGPRCS in (X, τ) .

Example 3.14: Let $X = \{a, b\}$ and $\tau = \{0_N, U, V, 1_N\}$ where $U = \{(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)\}$ and $V = \{(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)\}$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \{(0.4, 0.3, 0.6), (0.3, 0.4, 0.7)\}$ is a NGPRCS in (X, τ) . Since $A \subseteq U$ and U is a NROS, we have $NPCI(A) = A \subseteq U$. But A is not $N\alpha$ GCS in (X, τ) . Since $A \subseteq U$ and U is a NOS, we have $N\alpha Cl(A) = C(U) \not\subseteq U$.

Theorem 3.15: Every $NR\alpha$ GCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is $NR\alpha$ GCS in (X, τ) . Therefore $NPCI(A) \subseteq N\alpha Cl(A) \subseteq U$, by hypothesis. Hence A is a NGPRCS in (X, τ) .

Example 3.16: In Example 3.14., the NS $A = \{(0.4, 0.3, 0.6), (0.3, 0.4, 0.7)\}$ is a NGPRCS but not $NR\alpha$ GCS in (X, τ) .

Theorem 3.17: Every NGPCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is NGPCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ) . Therefore $NPCI(A) \subseteq U$, by hypothesis. Hence A is a NGPRCS in (X, τ) .

Example 3.18: In Example 3.10., the NS $A = \{(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)\}$ is a NGPRCS in (X, τ) . Since $A \subseteq 1_N$, we have $NPCI(A) = 1_N \subseteq 1_N$. But A is not NGPCS in (X, τ) . Since $A \subseteq V$ and V is a NOS, we have $NPCI(A) = 1_N \not\subseteq V$.

Theorem 3.19: Every NRGCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is NRGCS in (X, τ) . Therefore $NPCI(A) \subseteq NCl(A) \subseteq U$, by hypothesis. Hence A is a NGPRCS in (X, τ) .

Example 3.20: In Example 3.8., the NS $A = \langle (0.4, 0.3, 0.7), (0.3, 0.2, 0.6) \rangle$ is a NGPRCS but not NRGCS in (X, τ) .

Theorem 3.21: Every $N\alpha$ GCS in (X, τ) is a $NR\alpha$ GCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is $N\alpha$ GCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ) . Therefore $N\alpha Cl(A) \subseteq U$, by hypothesis. Hence A is a $NR\alpha$ GCS in (X, τ) .

Example 3.22: In Example 3.10., the NS $A = \langle (0.7, 0.2, 0.3), (0.8, 0.2, 0.2) \rangle$ is a $NR\alpha$ GCS but not $N\alpha$ GCS in (X, τ) .

Theorem 3.23: Every NGCS in (X, τ) is a $N\alpha$ GCS in (X, τ) but not conversely.

Proof: Let U be a NOS in (X, τ) such that $A \subseteq U$. Since A is NGCS in (X, τ) . Therefore $N\alpha Cl(A) \subseteq NCl(A) \subseteq U$, by hypothesis. Hence A is a $N\alpha$ GCS in (X, τ) .

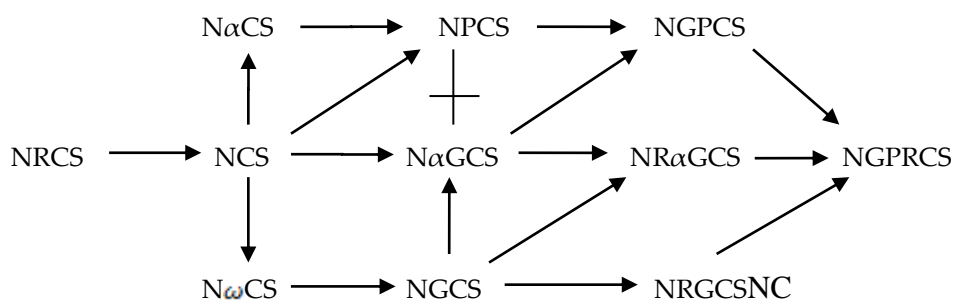
Example 3.24: Let $X = \{a\}$ and $\tau = \{0_N, U, V, 1_N\}$ where $U = \langle 0.5, 0.4, 0.7 \rangle$ and $V = \langle 0.8, 0.5, 0.2 \rangle$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \langle 0.2, 0.2, 0.8 \rangle$ is a $N\alpha$ GCS in (X, τ) . Since $A \subseteq U$ and U is a NOS, we have $N\alpha Cl(A) = A \subseteq U$. But A is not NGCS in (X, τ) . Since $A \subseteq U$, we have $NCl(A) = C(V) \not\subseteq U$.

Theorem 3.25: Every NGCS in (X, τ) is a NRGCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that $A \subseteq U$. Since A is NGCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ) . Therefore $NCl(A) \subseteq U$, by hypothesis. Hence A is a NRGCS in (X, τ) .

Example 3.26: Let $X = \{a, b, c\}$ and $\tau = \{0_N, U, 1_N\}$ where $U = \langle (0.6, 0.4, 0.3), (0.8, 0.5, 0.2), (0.7, 0.4, 0.8) \rangle$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \langle (0.5, 0.6, 0.6), (0.3, 0.5, 0.3), (0.5, 0.4, 0.3) \rangle$ is a NRGCS in (X, τ) . Since $A \subseteq 1_N$, we have $NCl(A) = 1_N \subseteq 1_N$. but A is not NGCS in (X, τ) . Since $A \subseteq U$ and U is a NOS, we have $NCl(A) = 1_N \not\subseteq U$.

The following diagram, we have provided the relation between NGPRCS and the other existed NSs.



In this diagram by $A \longrightarrow B$ means A implies B but not conversely and $A \perp B$ means A & B are independent.

Remark 3.27: The union of any two NGPRCSs in (X, τ) is not an NGPRCS in (X, τ) in general as seen from the following example.

Example 3.28: Let $X = \{a, b\}$ and $\tau = \{0_N, U, V, 1_N\}$ where $U = \langle (0.5, 0.3, 0.6), (0.4, 0.4, 0.7) \rangle$ and $V = \langle (0.7, 0.5, 0.3), (0.7, 0.5, 0.2) \rangle$. Then the NSs $A = \langle (0.2, 0.1, 0.7), (0.4, 0.4, 0.7) \rangle$ and $B = \langle (0.5, 0.3, 0.6), (0.4, 0.4, 0.7) \rangle$.

$(0.2, 0.2, 0.8)$ are NGPRCSs in (X, τ) but $A \cup B = \langle (0.5, 0.3, 0.6), (0.4, 0.4, 0.7) \rangle$ is not a NGPRCS in (X, τ) . Since $A \cup B \subseteq U$ but $\text{NPCl}(A \cup B) = C(U) \not\subseteq U$.

Remark 3.29: The intersection of any two NGPRCSs in (X, τ) is not an NGPRCS in (X, τ) in general as seen from the following example.

Example 3.30: Let $X = \{a, b\}$ and $\tau = \{0_N, U, V, 1_N\}$ where $U = \langle (0.5, 0.3, 0.6), (0.4, 0.4, 0.7) \rangle$ and $V = \langle (0.7, 0.5, 0.3), (0.7, 0.5, 0.2) \rangle$. Then the NSs $A = \langle (0.5, 0.5, 0.4), (0.7, 0.6, 0.7) \rangle$ and $B = \langle (0.6, 0.3, 0.6), (0.4, 0.4, 0.3) \rangle$ are NGPRCSs in (X, τ) but $A \cap B = \langle (0.5, 0.3, 0.6), (0.4, 0.4, 0.7) \rangle$ is not a NGPRCS in (X, τ) . Since $A \cap B \subseteq U$ but $\text{NPCl}(A \cap B) = C(U) \not\subseteq U$.

Theorem 3.31: Let (X, τ) be a NTS. Then for every $A \in \text{NGPRC}(X)$ and for every NS $B \in \text{NS}(X)$, $A \subseteq B \subseteq \text{NPCl}(A)$ implies $B \in \text{NGPRC}(X)$.

Proof: Let $B \subseteq U$ and U is a NROS in (X, τ) . Since $A \subseteq B$, then $A \subseteq U$. Given A is a NGPRCS, it follows that $\text{NPCl}(A) \subseteq U$. Now $B \subseteq \text{NPCl}(A)$ implies $\text{NPCl}(B) \subseteq \text{NPCl}(\text{NPCl}(A)) = \text{NPCl}(A)$. Thus, $\text{NPCl}(B) \subseteq U$. This proves that $B \in \text{NGPRC}(X)$.

Theorem 3.32: If A is a NROS and a NGPRCS in (X, τ) , then A is a NPCS in (X, τ) .

Proof: Since $A \subseteq A$ and A is a NROS in (X, τ) , by hypothesis, $\text{NPCl}(A) \subseteq A$. But since $A \subseteq \text{NPCl}(A)$. Therefore $\text{NPCl}(A) = A$. Hence A is a NPCS in (X, τ) .

Theorem 3.33: Let (X, τ) be a NTS and $\text{NPC}(X)$ (resp. $\text{NRO}(X)$) be the family of all NPCSs (resp. NROSs) of X . If $\text{NPC}(X) = \text{IRO}(X)$ then every Neutrosophic subset of X is NGPRCS in (X, τ) .

Proof: If $\text{NPC}(X) = \text{IRO}(X)$ and A is any Neutrosophic subset of X such that $A \subseteq U$ where U is NROS in X . Then by hypothesis, U is NPCS in X which implies that $\text{NPCl}(U) = U$. Then $\text{NPCl}(U) \subseteq \text{NPCl}(U) = U$. Therefore A is NGPRCS in (X, τ) .

Definition 3.34: Let (X, τ) be a NTS and $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$ be the subset of X . Then
 $\text{NGPRCl}(A) = \cap \{ K : K \text{ is a NGPRCS in } X \text{ and } A \subseteq K \}$ and
 $\text{NGPRInt}(A) = \cup \{ G : G \text{ is a NGPROS in } X \text{ and } G \subseteq A \}$.

Lemma 3.35: Let A and B be subsets of (X, τ) . Then the following results are obvious.

- a) $\text{NGPRCl}(0_N) = 0_N$.
- b) $\text{NGPRCl}(1_N) = 1_N$.
- c) $A \subseteq \text{NGPRCl}(A)$.
- d) $A \subseteq B \Rightarrow \text{NGPRCl}(A) \subseteq \text{NGPRCl}(B)$.

4. Neutrosophic Generalized Pre Regular Open Sets

In this section we introduce Neutrosophic generalized pre regular open sets in Neutrosophic topological space.

Definition 4.1: A NS A in a NTS (X, τ) is said to be a Neutrosophic generalized pre regular open set (NGPROS for short) if $\text{NPInt}(A) \supseteq U$ whenever $A \supseteq U$ and U is a NRCS in (X, τ) . Alternatively, A NS A is said to be a Neutrosophic generalized pre regular open set (NGPROS for short) if the complement of $C(A)$ is a NGPRCS in (X, τ) .

The family of all NGPROSs of a NTS (X, τ) is denoted by $\text{NGPRO}(X)$.

Example 4.2: Let $X = \{a, b\}$ and $\tau = \{0_N, U, V, 1_N\}$ where $U = \langle(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)\rangle$ and $V = \langle(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)\rangle$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \langle(0.8, 0.9, 0.2), (0.9, 0.6, 0.1)\rangle$ is a NGPROS in (X, τ) . Since $A \supseteq C(U)$ and $C(U)$ is a NRCS, we have $NPInt(A) = A \supseteq C(U)$.

Theorem 4.3: Every NOS is a NGPROS in (X, τ) but the converses may not be true in general.

Proof: Let U be a NRCS in (X, τ) such that $A \supseteq U$. Since A is NOS, $NInt(A) = A$. By hypothesis, $NPInt(A) = A \cap NInt(NCl(A)) = A \cap NCl(A) \supseteq A \cap A = A \supseteq U$. Therefore A is a NGPROS in (X, τ) .

Example 4.4: In Example 4.2., the NS $A = \langle(0.8, 0.9, 0.2), (0.9, 0.6, 0.1)\rangle$ is an NGPROS in (X, τ) but not a NOS in (X, τ) .

Theorem 4.5: Every $N\alpha OS$, NWOS, NPOS, NGOS, $N\alpha GOS$, NGPOS, NRGOS, $NR\alpha GOS$ is a NGPROS in (X, τ) but the converses are not true in general.

Example 4.6: Let $X = \{a, b\}$ and $\tau = \{0_N, U, 1_N\}$ where $U = \langle(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)\rangle$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \langle(0.2, 0.8, 0.6), (0.6, 0.4, 0.9)\rangle$ is a NGPROS in (X, τ) . Since $A \supseteq 0_N$, we have $NPInt(A) = 0_N \supseteq 0_N$. but A is not a $N\alpha OS$, NWOS, NPOS in (X, τ) .

Example 4.7: Let $X = \{a, b\}$ and $\tau = \{0_N, U, 1_N\}$ where $U = \langle(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)\rangle$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \langle(0.3, 0.8, 0.4), (0.7, 0.4, 0.8)\rangle$ is a NGPROS in (X, τ) . Since $A \supseteq 0_N$, we have $NPInt(A) = 0_N \supseteq 0_N$. but A is not a NGOS, $N\alpha GOS$, NGPOS in (X, τ) .

Example 4.8: Let $X = \{a, b\}$ and $\tau = \{0_N, U, V, 1_N\}$ where $U = \langle(0.6, 0.5, 0.2), (0.7, 0.5, 0.1)\rangle$ and $V = \langle(0.5, 0.4, 0.7), (0.4, 0.5, 0.6)\rangle$. Then (X, τ) is a Neutrosophic topological space. Here the NS $A = \langle(0.8, 0.8, 0.2), (0.7, 0.9, 0.3)\rangle$ is a NGPROS in (X, τ) . Since $A \supseteq C(V)$ and $C(V)$ is a NRCS, we have $NPInt(A) = A \supseteq C(V)$. but A is not NRGOS, $NR\alpha GOS$ in (X, τ) .

Theorem 4.9: Let (X, τ) be a NTS. Then for every $A \in NGPRO(X)$ and for every $B \in NP(X)$, $NPInt(A) \subseteq B \subseteq A$ implies $B \in NGPRO(X)$.

Proof: Let A be any NGPROS of (X, τ) and B be any NS of X . By hypothesis $NPInt(A) \subseteq B \subseteq A$. Then $C(A)$ is an NGPRCS in (X, τ) and $C(A) \subseteq C(B) \subseteq NPCl(C(A))$. By Theorem 3.31., $C(B)$ is an NGPRCS in (X, τ) . Therefore B is an NGPROS in (X, τ) . Hence $B \in NGPRO(X)$.

Theorem 4.10: A NS A of a NTS (X, τ) is a NGPROS in (X, τ) if and only if $F \subseteq NPInt(A)$ whenever F is a NRCS in (X, τ) and $F \subseteq A$.

Proof: Necessity: Suppose A is a NGPROS in (X, τ) . Let F be a NRCS in (X, τ) such that $F \subseteq A$. Then $C(F)$ is a NROS and $C(A) \subseteq C(F)$. By hypothesis $C(A)$ is a NGPRCS in (X, τ) , we have $NPCl(C(A)) \subseteq C(F)$. Therefore $F \subseteq NPInt(A)$.

Sufficiency: Let U be a NROS in (X, τ) such that $C(A) \subseteq U$. By hypothesis, $C(U) \subseteq NPInt(A)$. Therefore $NPCl(C(A)) \subseteq U$ and $C(A)$ is a NGPRCS in (X, τ) . Hence A is a NGPROS in (X, τ) .

Theorem 4.11: Let (X, τ) be a NTS and $NPO(X)$ (resp. $NGPRO(X)$) be the family of all NPOS (resp. NGPROSs) of X . Then $NPO(X) \subseteq NGPRO(X)$.

Proof: Let $A \in NPO(X)$. Then $C(A)$ is NPCCS and so NGPRCS in (X, τ) . This implies that A is NGPROS in (X, τ) . Hence $A \in NGPRO(X)$. Therefore $NPO(X) \subseteq NGPRO(X)$.

5. Separation Axioms of Neutrosophic Generalized Pre Regular Closed Sets

In this section we have provide some applications of Neutrosophic generalized pre regular closed sets in Neutrosophic topological spaces.

Definition 5.1: If every NGPRCS in (X, τ) is a NPCS in (X, τ) , then the space (X, τ) can be called a Neutrosophic pre regular $T_{1/2}$ (NPRT $_{1/2}$ for short) space.

Theorem 5.2: An NTS (X, τ) is a NPRT $_{1/2}$ space if and only if $NPOS(X) = NGPRO(X)$.

Proof: Necessity: Let (X, τ) be a NPRT $_{1/2}$ space. Let A be a NGPROS in (X, τ) . By hypothesis, $C(A)$ is a NGPRCS in (X, τ) and therefore A is a NPOS in (X, τ) . Hence $NPO(X) = NGPRO(X)$.

Sufficiency: Let $NPO(X, \tau) = NGPRO(X, \tau)$. Let A be a NGPRCS in (X, τ) . Then $C(A)$ is a NGPROS in (X, τ) . By hypothesis, $C(A)$ is a NPOS in (X, τ) and therefore A is a NPCS in (X, τ) . Hence (X, τ) is a NPRT $_{1/2}$ space.

Definition 5.3: A NTS (X, τ) is said to be a Neutrosophic pre regular $T^*_{1/2}$ space (NPRT $^*_{1/2}$ space for short) if every NGPRCS is a NCS in (X, τ) .

Remark 5.4: Every NPRT $^*_{1/2}$ space is a NPRT $_{1/2}$ space but not conversely.

Proof: Assume be a NPRT $^*_{1/2}$ space. Let A be a NGPRCS in (X, τ) . By hypothesis, A is an NCS. Since every NCS is a NPCS, A is a NPCS in (X, τ) . Hence (X, τ) is a NPRT $_{1/2}$ space.

Example 5.8: Let $X = \{a, b\}$ and let $\tau = \{0N, U, 1N\}$ where $U = \{(0.5, 0.4, 0.7), (0.4, 0.5, 0.6)\}$. Then (X, τ) is a NPRT $_{1/2}$ space, but it is not NPRT $^*_{1/2}$ space. Here the NS $A = \{(0.2, 0.3, 0.8), (0.3, 0.4, 0.8)\}$ is a NGPRCS but not a NCS in (X, τ) .

Theorem 5.9: Let (X, τ) be a NPRT $^*_{1/2}$ space then,

- (i) the union of NGPRCSs is NGPRCS in (X, τ)
- (ii) the intersection of NGPROSs is NGPROS in (X, τ)

Proof: (i) Let $\{A_i\}_{i \in I}$ be a collection of NGPRCSs in a NPRT $^*_{1/2}$ space (X, τ) . Thus, every NGPRCSs is a NCS. However, the union of NCSs is a NCS in (X, τ) . Therefore the union of NGPRCSs is NGPRCS in (X, τ) . (ii) Proved by taking the complement in (i).

6. Conclusion

In this paper, we have defined new class of Neutrosophic generalized closed sets called Neutrosophic generalized pre regular closed sets; Neutrosophic generalized pre regular open sets and studied some of their properties in Neutrosophic topological spaces. Furthermore, the work was extended as the separation axioms of Neutrosophic generalized pre regular closed sets, namely Neutrosophic pre regular $T_{1/2}$ space and Neutrosophic pre regular $T^*_{1/2}$ space and discussed their properties. Further, the relation between Neutrosophic generalized pre regular closed set and existing Neutrosophic closed sets in Neutrosophic topological spaces were established. Many examples are given to justify the results.

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