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NEW CLASS OF SOFT LINEAR ALGEBRAIC CODES AND THEIR PROPERTIES USING SOFT SETS

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ABSTRACT. Algebraic codes play a significant role in the minimization of data corruption which caused by **defects** such as inference, noise channel, crosstalk, and packet **lost**. In this paper, we introduced soft codes (soft linear codes) through the application of soft sets which is an approximated collection of codes. We also discussed several types of soft codes such as type-1 soft codes, complete soft codes etc. The innovative idea of soft codes is advantageous, for it can simultaneously transmit n-distinct messages to n-set of receivers. Further this new technique makes use of bi-matrices or to be more general uses the concept of n-matrices. Certainly this notion will save both time and economy. Moreover, we develop two techniques for the decoding of soft codes. At the end, we present a soft communication process and develop a model for this soft **communication** process. The disticntions and comparison of soft linear codes and linear codes are also presented.

1. INTRODUCTION

The transmission and storage of large amounts of data reliably and without error is a significant part of the modern communication systems. **Algebraic** codes are used for data compression, cryptography, error correction and for network coding. The theory of codes was first focused by Shanon in 1948 and then gradually developed by time to time by different **researchers**. There are many types of codes which is important to its **algebraic** structures such as Linear block codes, Hamming codes, BCH codes [46] and so on. The most common type of code is a linear code over the field F_q. Recently a variety of codes over finite rings have been studied. The linear codes over finite rings are initiated by Blake in a series of papers [13, 14] and Spiegel [49, 50] . Huber defined codes over Gaussian integers [26, 27, 28]. Shankar studied BCH codes over rings of residue integers [46]. Satyanarayana consider analyses of codes over $\mathbb{Z}_n$ by viewing their properties under the Lee metric [44]. Some more literature can be studied in [18, 19, 20, , 21, 23, 30, 31, 35, 38, 43, 44, 46] .

Zadeh in his seminal paper introduced the innovative concept of fuzzy sets in 1965 [53] . A fuzzy set is characterized by a membership function whose values are defined in the unit interval $[0, 1]$ and thus fuzzy set perhaps is the most suitable **framework** to model uncertain data. Fuzzy sets have a several interesting applications in the areas such as signal processing, decision making, control theory, reasoning, pattern recognition, computer version and so on. The theory of fuzzy set is a significantly used in medical diagnosis, social science, engineering etc. The algebraic **structures** in the context of fuzzy sets have been studied such as fuzzy groups, fuzzy rings, fuzzy semigroups fuzzy codes etc. Alcantud et al. studied

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real applications of hesitant fuzzy sets to improvement of teaching performance assessments and construction of meta-rankings for ranking universities [4]. **Gonzalez et al. discussed application of interval type-2 fuzzy systems to edge detection** [25].Some more study on fuzzy set can be found in [52, 54].

The theory of rough sets was first introduced by Pawlak in 1982 which is another significant mathematical tool to **handle** vague data and information [22, 39, 40] . The theory of rough sets mainly based upon equivalence classes to approximate crisp sets. Rough sets has several applications in data mining, machine learning, medicine, data analysis, expert systems and cognitive analysis etc. Some more literature can be found on rough sets in [15, 17, 22, 29, 41, 42, 47, 51]. Algebraic structures can also be studied in the context of rough set such as **rough** groups [12], rough semigroups and so on. **Chen et al. gave the application of rough sets to graph theory** [16].

The complexities of modelling uncertain data is the main problem in engineering, environmental science, economics, social sciences, health and medical sciences etc. Classical theories are not always successful as the uncertainties are of several types which appearing in these domains. The fuzzy set theory [53], probability theory, rough set theory [22, 39, 40] etc are well known and useful mathematical tools which describe uncertainty but each of them has its own limitation pointed out by Molodstov. Therefore, Molodstov **introduced** the theory of soft sets to model vague and uncertain information [36]. A soft set a parameterized collection of subsets of a universe of discourse. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. Soft set theory has been applied successfully in several areas such as, smoothness of functions, game theory, operation **research**, Riemann integration, Perron integration, and probability. Maji et al. gave the application of soft sets in decision making problem [32, 33, 34]. Recently soft set theory attained much attention of the researchers since its appearance and start studying soft algebraic **structures**. Aktas and Cagman introduced soft groups which laid down the foundations to study algebraic structures in the **context** of soft sets [1]. Some properties and algebra may be found in [8] . Feng et al. studied soft semigroups in [24]. **Alcantud discussed some formal relationships among soft sets, fuzzy sets, and their extensions** [5]**. Alcantud et al. and Muthukumar presented the application of soft sets in medical diagnosis** [3, 37]. A huge amount of literature can be seen in [6, 7, 8, 9, 10, 16, 37, 45, 48] .

The main purpose of this paper is to introduce **algebraic** soft coding theory which **extends** the notion of a code to soft sets. A soft code is a parameterized collection of codes. Different types of error correcting codes have been extend to **construct** soft error correcting codes. A variety of soft codes can be found by applying soft sets to codes. Soft linear codes have been discussed mainly in this paper. The novel concept of soft dimension have been introduced which **in fac**t a generalization of the dimension of a code and the concept of soft minimum distance is introduced here. Soft codes of *type* 1 have been established in this paper. The important notions of soft generator matrix as well as soft parity check matrix have been constructed to study more features of soft linear codes. Further, the notions of soft complete codes are introduced in this paper and in the end two soft decoding algorithm has been costructed in this paper.

The organization of this paper is as follows: In section 2, basic concepts of soft sets and codes are presented. In section 3, the important notions of soft codes are given with the study of some of their basic properties and features. In section 4, soft generator matrix (generator n-matrix) and soft parity check matrix (parity check n-matrix) are presented. Section 5 is about the soft decoding of soft codes. We developed two techniques for the decoding of soft codes. Further, in this section, two examples are presented for the verification of soft decoding process. In section 6, we gave the idea of soft communication process and we also constructed a model for this soft communication process. In section 7, we presented the main disticntion and comparison of soft linear codes with linear codes. Conclusion is given in section 8.

2. Basic concepts

This section has two subsections. First subsection recalls all the basic notions of linear algebraic codes. The second subsection gives the basic definition and properties of soft sets.

2.1. Codes.

Definition 1. [30]*Let H be an $n-k\times n$ matrix with elements in K_q. The set of all n-dimensional vectors satisfying $Hx^T=(0)$ over K_q is called a linear code(block code) C over K_q of block* ***length*** *n. C is also known as linear (n,k) code.*

Definition 2. [43]. ***Each vector of the subspace C is termed as codeword of the lenght n. A codeword is used for secrecy or convenience instead of the usual name for something.***

Definition 3. [43]. *Let K^n be a vector space over the field K, and $x,y\in K^n$ where $x=x_1x_2...x_n$,$y=y_1y_2...y_n$. The Hamming distance between the vectors x and y is denoted by $d\left(x,y\right)$, and is defined as $d\left(x,y\right)=\left|i:x_i\neq \mathbf{y}_i\right|$.*

Definition 4. [43]. *The minimum distance of a code C is the smallest distance between any two distinct codewords in C which is denoted by $d\left(C\right)$; that is $d\left(C\right)=\min\left\{d\left(x,y\right):x,y\in C,\ x\neq y\right\}$.*

Definition 5. [43]. *Let K be a finite field and n be a positive integer. Let C be a subspace of the vector space $V=K^n$. Then C is called a linear code over K.*

Definition 6. [43]. *The linear code C is called linear $[n,k]$-code if $dim(C)=k$.*

Definition 7. [43]. *Let C be a linear $[n,k]$-code. Let G be a $k\times n$ matrix whose rows form basis of C. Then G is called generator matrix of the code C.*

Definition 8. [43]. *Let C be an $[n,k]$-code over K. Then the dual code of C is defined to be*

$$C^{\perp}=\{y\in\mathbf{K}^n:x\cdot y=0\text{ for all }x\in C\}$$

Definition 9. [43]. *Let C be an $[n,k]$-code and let H be the generator matrix of the dual code $C^{\perp}$. Then H is called a parity-check matrix of the code C.*

Definition 10. [43]. *A code C is called self-orthogonal code if $C\subset C^{\perp}$.*

Definition 11. [43]. *Let C be a code over the field K and for every $x\in K^n$, the coset of C is defined to be*

$$C_c=\{x+c:c\in C\}$$

Definition 12. [43]. *Let C be a linear code over K. The coset leader of a given coset C is defined to be the vector with least weight in that coset.*

Definition 13. [43]. *If a codeword x is transmitted and the vector y is received, then $e = y - x$ is called error vector. Therefore a coset leader is the error vector for each vector y lying in that coset.*

2.2. **Soft set.** Throughout this subsection U refers to an initial universe, N is a set of parameters, $P(U)$ is the power set of U, and $A \subset N$. Molodtsov [31]. defined the soft set in the following manner:

Definition 14. [31]. *A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \longrightarrow P(U)$.*

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(a)$ may be considered as the set of a-elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Definition 15. [28]. *For two soft sets (F, A) and (H, B) over U, (F, A) is called a soft subset of (H, B) if*

(1) $A \subseteq B$ and
(2) $F(e) \subseteq G(e)$, for all $e \in A$.

This relationship is denoted by $(F, A) \tilde{\subset} (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \tilde{\supset} (H, B)$.

Definition 16. [28]. *Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).*

3. Soft Linear Code

In this section for the first time the notion of soft code and soft code of *type* 1 are introduced. Examples of these codes are provided.

Definition 17. *Let K be a finite field and $V = K^n$ be a vector space over K where n is a positive integer. Let $P(V)$ be the power set of V and (F, A) be a soft set over V. Then (F, A) is called soft linear code over V if and only if each $F(a)$ is a linear code (subspace) of V* ***for all*** *$a \in A$. In the rest of the paper, from a soft code, we mean a soft linear code (F, A).*

Example 1. *Let $K = K_2$ and $V = K_2^3$ is a vector space over K_2 and let (F, A) be a soft set over $V = K_2^3$. Then clearly (F, A) is a soft linear code over $V = K_2^3$, where*

$$\begin{aligned} F(a_1) &= \{000, 111\}, \\ F(a_2) &= \{000, 110, 101, 011\}. \end{aligned}$$

Definition 18. ***Let (F, A) be a soft code over the field K. Then the soft element $F(a)$ is called soft codeword of (F, A) for all $a \in A$. Here we denote the soft codeword by Y_s. In other words a soft codeword is a parameterized set of codewords of C.***

Definition 19. *Let (F, A) be a soft code over $V = K^n$. Then D_s is called soft dimension of (F, A) if*

$$D_s = \{\dim(F(a)),\ \textit{for all } a \in A\}.$$

The soft dimension D_s of the soft code is simply an n-tuple, where n is the number of parameters in the parameter set A.

Example 2. *Let (F, A) be a soft code defined in above example. Then the soft dimension is as follows,*

$$\begin{aligned} D_s &= \{\dim(F(a_1)) = 1, \dim(F(a_2)) = 2\}, \\ &= \{1, 2\}. \end{aligned}$$

Definition 20. *A soft linear code (F, A) over V of soft dimension D_s is called soft linear $[n, D_s]$-code.*

Definition 21. *Let (F, A) be a soft code over V. Then the soft minimum distance of (F, A) is denoted by $S_d(F, A)$ and is defined to be*

$S_d(F, A) = \{d(F(a)) : \textit{for all } a \in A\}$, ***where*** $d(F(a))$ ***is the minimum distance of the code*** $F(a)$.

Example 3. ***Let*** (F, A) ***be a soft code dfined in Example 1.*** *Then the minimum distance of the code $F(a_1) = 3$ and $F(a_2) = 2$. Thus the soft* ***minimum*** *distance of the soft code (F, A) is given as*

$$\begin{aligned} S_d(F, A) &= \{d(F(a_1)) = 3, d(F(a_2)) = 2\}, \\ &= \{3, 2\}. \end{aligned}$$

Definition 22. *A soft code (F, A) in V over the field K is called soft code of type 1, if the dimension of $F(a)$ is same, for all $a \in A$.*

Example 4. *Let (F, A) be a soft code in V over the field K_2^2, where*

$$F(a_1) = \{00, 01\},\ F(a_2) = \{00, 10\}.$$

The soft dimension D_s of (F, A) is as follows,

$$D_s = \{1, 1\}.$$

Thus clearly (F, A) is a *type* 1 soft code.

Theorem 1. *Every soft code of type 1 is trivially a soft code but the converse is not true.*

For converse, we take the following example.

Example 5. ***Let*** (F, A) ***be a soft code dfined in Example 1.*** *Then clearly (F, A) is not a type 1 soft code.*

4. Soft Generator Matrix and Soft Parity Check Matrix

In this section the soft generator n-matrix and the parity check n-matrix of the soft linear code is introduced and examples are given. When $n = 2$, it reduces to the bimatrix. In this paper, we used the convnetion of replacing $\cup$ by $|$.

Definition 23. *Let (F, A) be a soft linear $[n, D_s]$-code. Let G_s be the n-matrix whose elements are the generator matrices of the soft code (F, A), corresponding to each $a \in A$ where n is the number of parameters in A. Then G_s is termed as the soft generator matrix of the soft linear code (F, A).*

Example 6. ***Let*** (F, A) ***be a soft code defined in Example 1.*** *Clearly we have,*

$F(a_1)$ has generator matrix $G_{F(a_1)} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ and $F(a_2)$ has generator matrix $G_{F(a_2)} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

Then the soft generator **matrix** of the soft code (F, A) is a bimatrix given as follows.

$$G_s = \left[\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \middle| \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \right].$$

Definition 24. *Let* (F, A) *be a soft* $[n, D_s]$*-code over the field* K *and the vector space* V*. Then the soft dual code of* (F, A) *is defined to be*

$$(F, A)_{dual} = \{F(a)_{dual} : \textit{ for all } a \in A\},$$

wher $F(a)_{dual}$ **is the dual code of** $F(a)$**.**

Example 7. ***Let*** (F, A) ***be a soft code defined in Example 1.*** *Then the soft dual code of* (F, A) *is* $(F, A)_{dual}$*, where*

$$\begin{aligned} F(a_1)_{dual} &= \{000, 110, 101, 011\}, \\ F(a_2)_{dual} &= \{000, 111\}. \end{aligned}$$

Definition 25. *A soft linear code* (F, A) *in* $V = K^n$ *over the field* K *is called complete-soft code if for all* $a \in A$*, the dual of* $F(a)$ *also exist in* (F, A)*.*

Example 8. ***Let*** (F, A) ***be a soft code defined in Example 1.*** *Then clearly* (F, A) *is a complete-soft code because the dual of* $F(a_1)$ *is* $F(a_2)$ *and also the dual of* $F(a_2)$ *is* $F(a_1)$*.*

Theorem 2. *All complete-soft codes are trivially soft codes but the converse is not true in general.*

Theorem 3. *A complete-soft code* (F, A) *over the field* K *and the vector space* V *is the parametrized collection of the codes* C *with its dual code* $C^{\perp}$*.*

Definition 26. *Let* (F, A) *be a soft code over the field* K *and the vector space* V *and* $(F, A)_{dual}$ *be the soft dual code of* (F, A)*.Then the soft dimension of the soft dual code* $(F, A)_{dual}$ *is denoted by* $(D_s)_{dual}$ *and is defined as*

$(D_s)_{dual} = \{\dim(F(a)_{dual}) : \textit{for all } a \in A\}$, ***where*** $\dim(F(a)_{dual})$ ***is the dimenesion of the dual code*** F

Example 9. *In* ***previous*** *example the soft dimension of the dual code* $(F, A)_{dual}$ *is following*

$$\begin{aligned} (D_s)_{dual} &= \{\dim(F(a_1)_{dual}) = 2, \dim(F(a_1)_{dual}) = 1\} \\ &= \{2, 1\}. \end{aligned}$$

Definition 27. *Let* (F, A) *be a soft over the field* K *and the vector space* V *and let* H_s *be the soft generator* ***matrix*** *of the soft dual code* $(F, A)_{dual}$*. Then* H_s *is called the soft parity check matrix of the soft code* (F, A)*.*

Example 10. *In above example the soft dual code of* (F, A) *is* $(F, A)_{dual}$*, where*

$$\begin{aligned} F(a_1)_{dual} &= \{000, 110, 101, 011\}, \\ F(a_2)_{dual} &= \{000, 111\}. \end{aligned}$$

The soft generator bimatrix of $(F,A)_{dual}$ is $S(H)$, where

$$H_s = \left[\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} | \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \right],$$

where $H_{F(a_1)} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ is a parity check matrix of $G_{F(a_1)}$ and $H_{F(a_2)} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ is a parity check matrix of $G_{F(a_2)}$.

Theorem 4. *Let (F,A) be a soft code over the field K and the vector space V. Let G_s and H_s be the soft generator matrix and soft parity check* ***matrix*** *of the soft code (F,A). Then*

$$G_s H_s^T = 0.$$

Definition 28. *A soft linear code (F,A) in V over the field K is called soft self dual code if $(F,A)_{dual} = (F,A)$.*

Definition 29. *Let (F,A) be a soft code over the field K and the vector space V. Let G_s be the soft generator matrix of (F,A). Then the soft canonical generator matrix of (F,A) is denoted by G_s^* and is defined as*

$$G_S^* = \begin{bmatrix} G_{F(a)}^* \end{bmatrix},$$

where $G_{F(a)}^*$ is the canonical generator matrix of $F(a)$, for all $a \in A$. In fact a soft canonical generator matrix is a n-canonical generator matrix.

Example 11. ***Let (F,A) be a soft code over $V = K_2^5$, where***

$F(a_1) = \{00000, 10010, 01001, 00110, 11011, 10100, 01111, 11101\}, F(a_2) = \{00000, 11111, 10110, 01001\}.$

The soft canonical generator bimatrix of (F,A) is as under,

$$G_s^* = \left[\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \middle| \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \right],$$

where $G_{F(a_1)}^* = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}, G_{F(a_2)}^* = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$ are respectively the canonical generator matrices of $F(a_1)$ and $F(a_2)$.

Theorem 5. *Let (F,A) be a soft code. If (F,A) has a soft canonical generator n-matrix*

$$G_s^* = \left[G_{F(a)}^* = \begin{bmatrix} I_k \vdots A \end{bmatrix}, \textit{ for all } a \in A \right].$$

Then

$$H_s^* = \left[H_{F(a)}^* = \left[-A^T \vdots I_{m-k} \right], \text{ for all } a \in A \right]$$

is the soft canonical parity check n-matrix of (F,A). Conversely, if

$$H_s^* = \left[H_{F(a)}^* = \begin{bmatrix} B \vdots I_{m-k} \end{bmatrix}, \text{ for all } a \in A \right]$$

is the soft canonical parity check matrix of (F,A), then

$$G_s^* = \left[G_{F(a)}^* = \begin{bmatrix} I_k \vdots -B^T \end{bmatrix}, \text{ for all } a \in A \right].$$

5. Soft Decoding Algorithms

In this section we provide a soft decoding algorithm for soft linear codes which forms subsection one of this section. This concept is illustrated with examples. In subsection two soft syndrome decoding is introduced. This helps in error correction of soft codes.

5.1. Soft Standard Array Decoding.

Definition 30. *Let (F, A) be a soft linear code or soft subspace of $V = K^n$ over the field K. Then for every $x \in K^n$,* ***the soft coset of*** (F, A) ***is defined as following.***

$$(F, A)_C = \{x + F(a), \text{ for all } a \in A\}.$$

The soft coset of (F, A) **is denoted by** $(F, A)_C$.

Definition 31. *Let (F, A) be a soft linear code in $V = K^n$ over the field K. The soft coset leader of a given soft coset $(F, A)_C$ is denoted by E_i and is defined to be*

$$E_i = \{u : \text{ for all } a \in A\}, \text{ where } u \text{ is the coset leader of } F(a).$$

Example 12. ***Let*** (F, A) ***be a soft code defined in Example 1.*** *Then the soft cosets of the soft code (F, A) are as follows:*

$$(F, A)_{C_1} = \{000 + F(a_1), 000 + F(a_2)\} = (F, A),$$

$$(F, A)_{C_2} = \{100 + F(a_1), 100 + F(a_2)\} = \{\{100, 011\}, \{100, 010, 001, 111\}\},$$

$$(F, A)_{C_3} = \{010 + F(a_1), 010 + F(a_2)\} = \{\{010, 101\}, \{100, 010, 001, 111\}\},$$

$$(F, A)_{C_4} = \{001 + F(a_1), 001 + F(a_2)\} = \{\{001, 110\}, \{100, 010, 001, 111\}\}.$$

Following are the soft coset leaders of the soft coset (F, A),

$$\begin{aligned} E_1 &= \{000, 000\}, E_2 = \{100, 100\}, \\ E_3 &= \{010, 010\}, E_4 = \{001, 001\}, \text{ and so on.} \end{aligned}$$

Definition 32. *A set of standard arrays of the corresponding parametrized codes is called soft standard array for the soft linear code (F, A).*

Example 13. ***Let*** (F, A) ***be a soft code defined in Example 1.*** *Then the soft cosets of (F, A) are as follows.*

$$(F, A)_{C_1} = \{000 + F(a_1), 000 + F(a_2)\} = (F, A),$$

$$(F, A)_{C_2} = \{100 + F(a_1), 100 + F(a_2)\} = \{\{100, 011\}, \{100, 010, 001, 111\}\},$$

$$(F, A)_{C_3} = \{010 + F(a_1), 010 + F(a_2)\} = \{\{010, 101\}, \{100, 010, 001, 111\}\},$$

$$(F,A)_{C_4} = \{001 + F(a_1), 001 + F(a_2)\} = \{\{001, 110\}, \{100, 010, 001, 111\}\}.$$

The following are the coset leaders of the soft linear code (F, A),

$$E_1 = \{000, 000\}, E_2 = \{100, 100\},$$

$$E_3 = \{010, 010\}, E_4 = \{001, 001\}.$$

The soft standard array for the soft linear code (F, A) is following,

$$\left\{ \begin{array}{cc} 000 & 111 \\ 100 & 011 \\ 010 & 101 \\ 001 & 110 \end{array}, \begin{array}{cccc} 000 & 110 & 101 & 011 \\ 100 & 010 & 001 & 111 \\ 010 & 100 & 111 & 001 \\ 001 & 111 & 100 & 010 \end{array} \right\}.$$

5.2. **Soft Syndrome Decoding.**

Definition 33. *Let (F, A) be a soft code over K with soft parity check matrix H_S. For any vector $Y_s \subseteq K^n$, the soft syndrome of Y_s is defined as $S(Y_s) = Y_s (H_s)^T$.*

Theorem 6. *Let (F, A) be a soft code over the field K. For $Y_s \subseteq K^n$, the soft codeword nearest to Y_s is given by $X_s = Y_s - E_s$, where E_s is the soft coset leader.*

Let (F, A) be a soft code over the field K with soft parity check matrix H_S. For soft syndrome decoding, we first find all the soft coset of the soft code (F, A) and then find the soft coset leaders E_S which are **in fact** the collection of coset leaders corresponding to each parameterized code. Then, we compute the soft syndrome for all the soft coset leaders and then make a table of soft coset leaders with their soft syndroms. To decode a soft codeword say Y_S, we simply find the soft syndrome of that soft codeword and then compare their soft syndrome with soft coset leader syndrome. After comparing their soft syndromes, we then subtract the soft coset leader from the soft decoded word. Hence Y_S is soft decoded as $X_S = Y_S - E_s$.

Example 14. ***Let*** (F, A) ***be a soft code defined in Example 1.*** *The soft parity check matrix of (F, A) is H_s, where*

$$H_s = \left[\left[\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 1 \end{array} \right] | \left[\begin{array}{ccc} 1 & 1 & 1 \end{array} \right] \right].$$

The transpose of H_s is following,

$$(H_s)^T = \left[\left[\begin{array}{cc} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{array} \right] \middle| \left[\begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right] \right].$$

Then the soft cosets of (F, A) are as follows.

$$(F,A)_{C_1} = \{000 + F(a_1), 000 + F(a_2)\} = (F, A),$$

$$(F,A)_{C_2} = \{100 + F(a_1), 100 + F(a_2)\} = \{\{100, 011\}, \{100, 010, 001, 111\}\},$$

$$(F,A)_{C_3} = \{010 + F(a_1), 010 + F(a_2)\} = \{\{010, 101\}, \{100, 010, 001, 111\}\},$$

$(F,A)_{C_4} = \{001 + F(a_1), 001 + F(a_2)\} = \{\{001, 110\}, \{100, 010, 001, 111\}\}.$

The following are the soft coset leaders E_i of the soft code (F, A),

$$E_1 = \{000, 000\}, E_2 = \{100, 100\},$$

$$E_3 = \{010, 010\}, E_4 = \{001, 001\}.$$

Soft syndrome table of soft coset leader is given as follows.

soft coset leader	soft syndrome
$\{000, 000\}$	$\{00, 0\}$
$\{100, 100\}$	$\{10, 1\}$
$\{010, 010\}$	$\{11, 1\}$
$\{001, 001\}$	$\{01, 1\}$

We want to **decode** a soft codeword $Y_s = \{110, 101\}$. First we find the soft syndrom of the soft codeword.

$$S(Y_s) = y(H_s)^T = [110, 100] \left[\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \middle| \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right] = [01, 1].$$

Since $S(\{110, 100\}) = \{01, 1\} = S(\{001, 001\})$.
The decoded soft codeword is

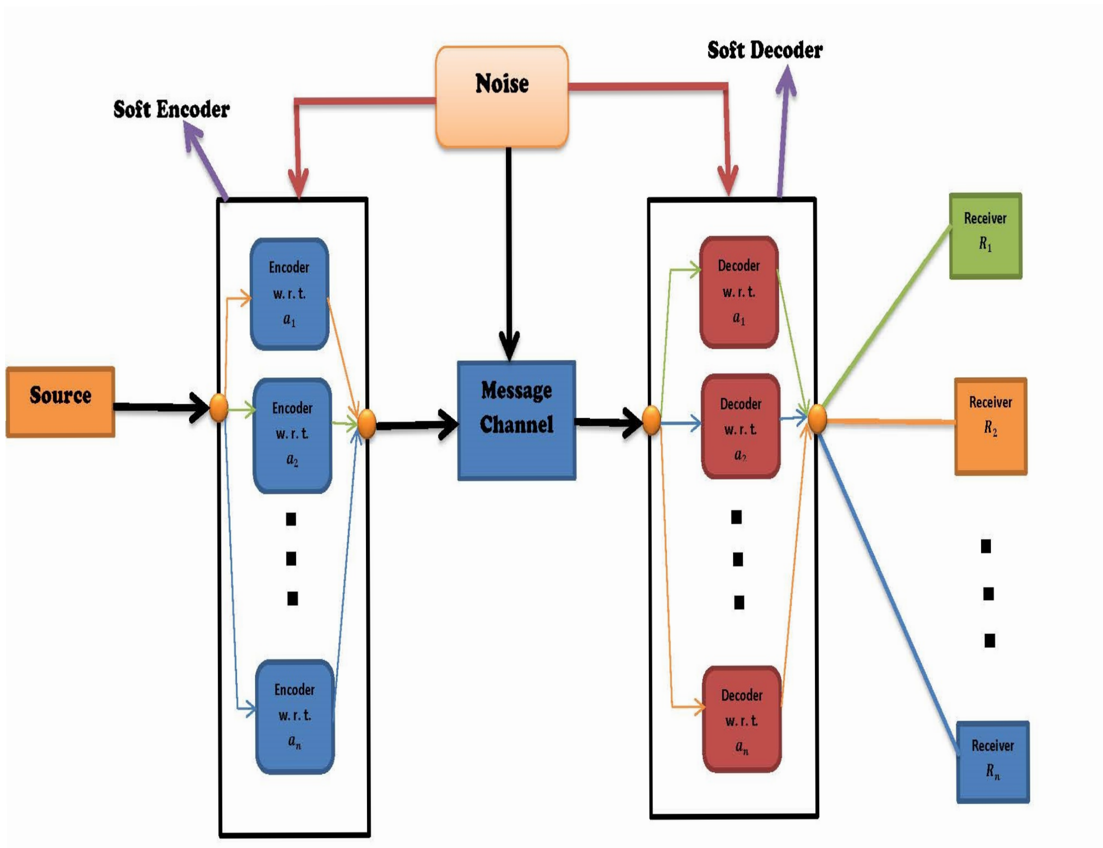
$$\begin{aligned} S(Y_s) - S(E_4) &= X_s \\ S(\{110, 100\}) - S(\{001, 001\}) &= \{110 - 001, 100 - 001\} \\ &= \{111, 101\}. \end{aligned}$$

Hence the soft **codeword** $Y_s = \{110, 100\}$ is decoded as $X_s = \{111, 101\}$. By similar **fashion,** we can find all the soft decoded codewords.

6. Soft Communication Transmission

In this section, we present a soft communication transmission with the help of a model. This model consists of a soft encoder which is the collection of encoders. Therefore, **corresponding** to each parameter a in A, we have an encoder in the soft encoder. Furthermore, we have also a soft decoder which is the collection of decoders and thus to each parameter a in A, we have a decoder in the soft decoder. There are n parameters in the parameter set A, therefore, we have n encoders in the soft encoder as well as n decoders in the soft decoder. Consequently, we have a collection of n messages and n receivers to recieve their desired message.

The signals from the source cannot be transmitted directly by the channel. Therefore the n encoders perform the important work of data reduction and suitably transforms the n-messages into usable form. Thus there is a difference between source encoding and channel encoding. The former reduces the messages to **recognizable** parts and the latter adds redundant information to enable deduction and correction of possible errors in transmission. Similarly on receiving we have to distinguish between channel decoding and source decoding, which invert the corresponding channel and source encoding besides deleting and correcting errors. If we have $n = 1$, the soft communication transmission reduces to classical communication transmission. The model of soft communication transmission is given in the following Figure 1.



$Figure 1$: Soft communication transmission model

7. Comparison of Soft Linear Codes with linear Codes

In this section, we have presented some of the main differences of soft linear codes with linear codes by comparison. The following are the main distinguishing features of our proposed soft linear codes.

(1) The main distinction between linear code and soft linear code is that for the soft linear code each soft codeword has some flavors, i.e. each soft codeword is characterized by some attributes, while the non-soft codewords are characterized by no attributes. Then one can manipulate the attributes of a soft codewords, for example an attribute "a_1" can some some attribute that 'trick' the code hackers, or may be chances that the soft codeword has a smaller chance to belong to the message (i.e. included just to deceive the hackers). and so on. Hence soft linear codes are more secure than the classical codes due to parameterization.

(2) The soft linear code differ in structure as a linear code C is just a one subspace, but a soft linear code is a collection of subspaces. At least a

soft linear code has two subspaces, each subspace depending on the set of parameters used. Therefore, soft linear codes are more generalized than the linear codes.

(3) Soft linear codes can simultaneously transmit n distinct messages to n set of receivers whereas linear codes can transmit only one message to a receiver.
(4) This new technique makes use of bi-matrices or to be more general uses the concept of n-matrices. Certainly this notion will save both time and economy.
(5) In soft decoding process, one can decode a set of code words (soft code word) at a time while it is not possible in linear decoding process.
(6) Soft linear codes can provide us several types of new codes such as type-1 codes, complete codes etc., which is not possible in linear codes.

8. Conclusion

Algebraic codes play a significant role in the minimization of data corruption which caused by **defects** such as inference, noise channel, crosstalk, and packet lost. In this paper for the first time we have introduced the new notions of soft linear codes using soft sets. The advantages of this new class of codes are that it can send simultaneously n-messages to n-persons. Hence this new codes can save both time and economy. Soft generator matrix (generator n-matrix) and soft parity check matrix (parity check n-matrix) are presented. Two techniques are developed for the decoding of soft linear codes. The channel transmission is also illustrated. Finally, the main disticntion and comparison of soft linear codes with linear codes are presented.

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