

10-12-2019

Neutrosophic Semi-Baire Spaces

R. Vijayalakshmi

A. Savitha Mary

S. Anjalmoose

Follow this and additional works at: https://digitalrepository.unm.edu/nss_journal

Recommended Citation

Vijayalakshmi, R.; A. Savitha Mary; and S. Anjalmoose. "Neutrosophic Semi-Baire Spaces." *Neutrosophic Sets and Systems* 30, 1 (). https://digitalrepository.unm.edu/nss_journal/vol30/iss1/10

This Article is brought to you for free and open access by UNM Digital Repository. It has been accepted for inclusion in Neutrosophic Sets and Systems by an authorized editor of UNM Digital Repository. For more information, please contact amywinter@unm.edu, lsloane@salud.unm.edu, sarahrk@unm.edu.



Neutrosophic Semi-Baire Spaces

R. Vijayalakshmi^{1*}, A. Savitha Mary² and S. Anjalmoose³

¹PG & Research Department of Mathematics, Arignar Anna Government Arts College, Namakkal, Tamilnadu, India.

E.Mail: viji_lakshmi80@rediffmail.com

^{2,3}. Department of Mathematics, St. Joseph's College of Arts & Science (Autonomous), Manjakuppam, Cuddalore, Tamilnadu, India.

E.Mail: savitha.marry139@gmail.com, ansalmoose@gmail.com

* Correspondence: viji_lakshmi80@rediffmail.com

Abstract: In this paper, we introduce the concept of Neutrosophic Semi Baire spaces in Neutrosophic Topological Spaces. Also we define Neutrosophic Semi-nowhere dense, Neutrosophic Semi-first category and Neutrosophic Semi-second category sets. Some of its characterizations of Neutrosophic Semi-Baire spaces are also studied. Several examples are given to illustrate the concepts

Keywords: Neutrosophic semi-open set, Neutrosophic semi-nowhere dense set, Neutrosophic semi-first category, Neutrosophic semi-second category and Neutrosophic semi-Baire spaces

1. Introduction and Preliminaries

The fuzzy idea has invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [29]. The important concept of fuzzy topological space was offered by C. L. Chang [9] and from that point forward different ideas in topology have been reached out to fuzzy topological space. The concept of "intuitionistic fuzzy set" was first presented by Atanassov [5]. He and his associates studied this useful concept [6 - 8]. Afterward, this idea was generalized to "intuitionistic L – fuzzy sets" by Atanassov and Stoeva [6]. The idea of somewhat fuzzy continuous functions and somewhat fuzzy open hereditarily irresolvable were introduced and investigated by G. Thangaraj and G. Balasubramanian in [25]. The idea of intuitionistic fuzzy nowhere dense set in intuitionistic fuzzy topological space presented and studied by Dhavaseelan and et al. in [16]. The concepts of neutrosophy and Neutrosophic set were introduced by F. Smarandache [[22], [23]]. Afterwards, the works of Smarandache inspired A. A. Salama and S. A. Alblowi [21] to introduce and study the concepts of Neutrosophic crisp set and Neutrosophic crisp topological spaces. The Basic definitions and Proposition related to Neutrosophic topological spaces was introduced and discussed by Dhavaseelan et al. [17]. The concepts of Neutrosophic Baire spaces are introduced by R. Dhavaseelan, S. Jafari, R. Narmada Devi, Md. Hanif Page [16]

Definition 1.1. [22, 23] Let T, I, F be real standard or non standard subsets of $]0^-, 1^+[$, with

$$\sup_T = t_{\sup}; \inf_T = t_{\inf}$$

$$\text{Sup}_I = i_{\sup}; \text{inf}_I = i_{\inf}$$

$$\text{Sup}_F = f_{\sup}; \text{inf}_F = f_{\inf}$$

$$n - \sup = t_{\sup} + i_{\sup} + f_{\sup}$$

$$n - \inf = t_{\inf} + i_{\inf} + f_{\inf}. \quad T, I, F \text{ are Neutrosophic components.}$$

Definition 1.2. [22, 23] Let X is a nonempty fixed set. A Neutrosophic set [briefly Ne.S] K is an object having the form $K = \{ \langle x, \mu_K(x), \sigma_K(x), \gamma_K(x) \rangle : x \in X \}$ where $\mu_K(x), \sigma_K(x)$ and $\gamma_K(x)$ which represents the degree of membership function (namely $\mu_K(x)$), the degree of indeterminacy (namely $\sigma_K(x)$) and the degree of non-membership (namely $\gamma_K(x)$) respectively of each element $x \in X$ to the set K .

Remark 1.2. [22, 23]

- (1) A Ne.S $K = \{ \langle x, \mu_K(x), \sigma_K(x), \gamma_K(x) \rangle : x \in X \}$ can be identified to an ordered triple $\langle \mu_K, \sigma_K, \gamma_K \rangle$ in $]0^-, 1^+[$ on X .
- (2) For the sake of simplicity, we shall use the symbol $K = \langle \mu_K, \sigma_K, \gamma_K \rangle$ for the Ne.S $K = \{ \langle x, \mu_K(x), \sigma_K(x), \gamma_K(x) \rangle : x \in X \}$

Definition 1.3. [22, 23] Let X be a nonempty set and the Ne.Sets K and L in the form

$K = \{ \langle x, \mu_K(x), \sigma_K(x), \gamma_K(x) \rangle : x \in X \}$, $L = \{ \langle x, \mu_L(x), \sigma_L(x), \gamma_L(x) \rangle : x \in X \}$. Then

- (a) $K \subseteq L$ iff $\mu_K(x) \leq \mu_L(x), \sigma_K(x) \leq \sigma_L(x), \gamma_K(x) \geq \gamma_L(x)$ for all $x \in X$;
- (b) $K = L$ iff $K \subseteq L$ and $L \subseteq K$;
- (c) $\bar{K} = \{ \langle x, \gamma_L(x), \sigma_K(x), \mu_L(x) \rangle : x \in X \}$; [Complement of K]
- (d) $K \cap L = \{ \langle x, \mu_K(x) \wedge \mu_L(x), \sigma_K(x) \wedge \sigma_L(x), \gamma_K(x) \vee \gamma_L(x) \rangle : x \in X \}$;
- (e) $K \cup L = \{ \langle x, \mu_K(x) \vee \mu_L(x), \sigma_K(x) \vee \sigma_L(x), \gamma_K(x) \wedge \gamma_L(x) \rangle : x \in X \}$;
- (f) $[K] = \{ \langle x, \mu_K(x), \sigma_K(x), 1 - \mu_K(x) \rangle : x \in X \}$;
- (g) $\langle \rangle K = \{ \langle x, 1 - \gamma_K(x), \sigma_K(x), \gamma_K(x) \rangle : x \in X \}$

Definition 1.4. [22, 23] Let $\{K_i : i \in J\}$ be an arbitrary family of Ne.Sets in X . Then

- (a) $\bigcap K_i = \{ \langle x, \bigwedge \mu_{K_i}(x), \bigwedge \sigma_{K_i}(x), \bigvee \gamma_{K_i}(x) \rangle : x \in X \}$,
- (b) $\bigcup K_i = \{ \langle x, \bigvee \mu_{K_i}(x), \bigvee \sigma_{K_i}(x), \bigwedge \gamma_{K_i}(x) \rangle : x \in X \}$,

Since our main purpose is to construct the tools for developing Ne.T.Spaces, we introduce the Ne.Sets 0_N and 1_N in X as follows:

Definition 1.5. [22, 23]

$$0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \} \text{ and } 1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$$

Definition 1.6. [21]

A Neutrosophic topology (Ne.T) on a nonempty set X is a family N_T of Ne.Sets in X satisfying the following axioms:

- (i) $0_N, 1_N \in N_T$,
- (ii) $G_1 \cap G_2 \in N_T$ for any $G_1, G_2 \in N_T$.
- (iii) $\bigcup G_i$ for arbitrary family $\{G_i | i \in \Lambda\}$.

In this case the ordered pair (X, N_T) or simply X is called a Neutrosophic Topological Space (briefly Ne.T.S) and each Ne.S in N_T is called a Neutrosophic open set (briefly Ne.O.S). The complement K of a Ne.O.S K in X is called a Neutrosophic closed set (briefly Ne.C.S) in X .

Definition 1.7. [9]

Let K be a Ne.S in a Ne.T.S X . Then

$$\text{Ne.int}(K) = \bigcup \{ G \mid G \text{ is Neutrosophic open set in } X \text{ and } G \subseteq K \}$$

is called the Neutrosophic interior of K ;

$$Ne.cl(K) = \cap \{G \mid G \text{ is Neutrosophic closed set in } X \text{ and } G \supseteq K\}$$

is called the Neutrosophic closure of K .

Definition 1.8: [13] A Ne.S K in a Ne.T.S X is said to a Neutrosophic Semi Open set (Ne.S.O.S) if $K \subseteq Ne.cl(Ne.int(K))$ and Neutrosophic Semi Closed set (Ne.S.C.S) if $Ne.int(Ne.cl(K)) \subseteq K$.

Definition 1.9:[13] Let K be a Ne.S in a Ne.T.S X . Then

$$Ne.S.int(K) = \cup \{G \mid G \text{ is Neutrosophic semi open set in } X \text{ and } G \subseteq K\}$$

is called the Neutrosophic semi interior of K ;

$$Ne.S.cl(K) = \cap \{G \mid G \text{ is Neutrosophic semi closed set in } X \text{ and } G \supseteq K\}$$

is called the Neutrosophic semi closure of K ;

Result: 1.9 Let K be a Ne.S in a Ne.T.S X . Then

$$Ne.S.cl(K) = K \cup Ne.int(Ne.cl(K))$$

$$Ne.S.int(K) = K \cap Ne.cl(Ne.int(K))$$

2. Neutrosophic Semi-nowhere dense sets

Definition 2.1 A Ne.S K in Ne.T.S (X, N_T) is called Neutrosophic semi nowhere dense (briefly Ne.S.N.D) if there exists no non-zero Ne.S.O.S L in $(X; N_T)$ such that $L \subset Ne.S.cl(K)$. That is $Ne.S.int(Ne.S.cl(K)) = 0_N$

Example 2.1 Let $X = \{k, l\}$. Define the Ne.S K, L and M on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.6}\right), \left(\frac{k}{0.5}, \frac{l}{0.2}\right), \left(\frac{k}{0.4}, \frac{l}{0.5}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.2}, \frac{l}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.1}\right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L, K \cup L, K \cap L\}$ is Ne.T on X . Thus (X, N_T) is a Ne.T.S. Now the sets $\overline{K}, \overline{L}, \overline{K \cup L}$ are Ne.S.N.D set

Proposition 2.1. If K is a Ne.S.N.D set in $(X; N_T)$, then \overline{K} is a Ne.S.D set in (X, T)

Proposition 2.2. Let K be a set. If K is a Ne.S.C.S in (X, N_T) with $Ne.S.int(K) = 0_N$, then K is a Ne.S.N.D set in $(X; N_T)$.

Definition 2.2. Let K be a Neutrosophic semi first category set (Ne.S.F.C.) in (X, N_T) . Then \overline{K} is called a Neutrosophic residual set in $(X; N_T)$.

Proposition 2.3. The complement of a Ne.S.N.D. set in a Ne.T.S (X, N_T) need not be Ne.S.N.D. set.

Proof: For, in example 2.1, \overline{K} is a Ne.S.N.D. set in (X, N_T) whereas K is not a Ne.S.N.D. set in (X, N_T) .

Proposition 2.4. If K & L are Ne.S.N.D. sets in a Ne.T.S (X, N_T) , then $K \cup L$ need not be Ne.S.N.D. set in (X, N_T) .

Proof: For, in example 2.1, $\overline{K} \& \overline{L}$ is Ne.S.N.D. sets in (X, N_T) . But $\overline{K} \cup \overline{L}$ implies that $Ne.S.int(Ne.S.cl(\overline{K} \cup \overline{L})) \neq 0_N$. Therefore union of Ne.S.N.D. sets need not be Ne.S.N.D. set in (X, N_T) .

Proposition 2.5: If the Ne.Sets K and L are Ne.S.N.D. sets in a Ne.T.S (X, N_T) then $K \cap L$ is a Ne.S.N.D. set in (X, N_T) .

Proof: Let the fuzzy sets K and L be Ne.S.N.D. sets in (X, N_T) . Now $Ne.S.int(Ne.S.cl(K \cap L)) \subseteq Ne.S.int(Ne.S.cl(K)) \cap Ne.S.int(Ne.S.cl(L)) = 0_N \cap 0_N$ (since $Ne.S.int(Ne.S.cl(K)) = 0_N$ and $Ne.S.int(Ne.S.cl(L)) = 0_N$). That is, $Ne.S.int(Ne.S.cl(K \cap L)) = 0_N$. Hence $(K \cap L)$ is a Ne.S.N.D. set in (X, N_T) .

Proposition 2.6: If K is a Ne.S.N.D. set in a Ne.T.S (X, N_T) then $Ne.S.int(K) = 0_N$.

Proof: Let K be a Ne.S.N.D. set in (X, N_T) . Then, we have $Ne.S.int(Ne.S.cl(K)) = 0_N$. Now $K \subseteq Ne.S.cl(K)$ we have $Ne.S.int(K) \subseteq Ne.S.int(Ne.S.cl(K)) = 0_N$. Hence $Ne.S.int(K) = 0_N$

Proposition 2.7:

If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T) then $Ne.int(Ne.S.cl(K)) = 0$.

Proof: Let K be a Ne.S.N.D. sets in (X, N_T) . Then, we have $Ne.int(Ne.cl(K)) = 0_N$ and $Ne.int(K) = 0_N$. Now $Ne.S.cl(K) = K$, since K is fuzzy semi-closed set in (X, N_T) implies that $Ne.int(Ne.S.cl(K)) = Ne.int(K) = 0_N$. Hence $Ne.int(Ne.S.cl(K)) = 0_N$.

Proposition 2.8: If K is a Ne.S.N.D. set and L is any Ne.Set in a Ne.T.S. (X, N_T) , then $(K \cap L)$ is a Ne.S.N.D. set in (X, N_T) .

Proof: Let K be a Ne.S.N.D. set in (X, N_T) . Then, $Ne.S.int(Ne.S.cl(K)) = 0$. Now $Ne.S.int(Ne.S.cl(K \cap L)) \subseteq Ne.S.int(Ne.S.cl(K)) \cap Ne.S.int(Ne.S.cl(L)) \subseteq 0_N \cap Ne.S.int(Ne.S.cl(L)) = 0_N$. That is, $Ne.S.int(Ne.S.cl(K \cap L)) = 0_N$. Hence $(K \cap L)$ is a Ne.S.N.D. set in (X, N_T) .

Definition 2.3 A Ne.S. K in Ne.T.S. $(X; N_T)$ is called Neutrosophic semi dense(Ne.S.D.) if there exists no Ne.S.C.set L in $(X; N_T)$ such that $K \subset L \subset 1_N$. That is $Ne.S.cl(K) = 1_N$

Proposition 2.9 If K is a Ne.S.D. and Ne.S.O. set in a Ne.T.S. (X, N_T) and if $L \subseteq 1 - K$ then L is a Ne.S.N.D. set in (X, N_T) .

Proof: Let K be a Ne.S.D. set in (X, N_T) . Then we have $Ne.S.cl(K) = 1_N$ and $Ne.S.int(K) = K$. Now $L \subseteq 1 - K$ implies that $Ne.S.cl(L) \subseteq Ne.S.cl(1 - K)$. Then $Ne.S.cl(L) \subseteq 1 - Ne.S.int(K) = 1 - K$. Hence $Ne.S.cl(L) \subseteq (1 - K)$, which implies that $Ne.S.int(Ne.S.cl(L)) \subseteq Ne.S.int(1 - K) = 1 - Ne.S.cl(K) = 1 - 1 = 0_N$. That is, $Ne.S.int(Ne.S.cl(L)) = 0_N$. Hence L is a Ne.S.N.D. set in (X, N_T) .

Proposition 2.10: If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T) , then $1 - K$ is a Ne.S.D. set in (X, N_T) .

Proof: Let K be a Ne.S.N.D. set in (X, N_T) . Then, $Ne.S.int(Ne.S.cl(K)) = 0_N$. Now $K \subseteq Ne.S.cl(K)$ implies that $Ne.S.int(K) \subseteq Ne.S.int(Ne.S.cl(K)) = 0_N$. Then $Ne.S.int(K) = 0_N$ and $Ne.S.cl(1 - K) = 1 - Ne.S.int(K) = 1 - 0_N = 1_N$ and hence $1 - K$ is a fuzzy semi-dense set in (X, N_T) .

Proposition 2.11: If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T) , then $Ne.S.cl(K)$ is also a Ne.S.N.D. set in (X, N_T) .

Proof: Let K be a Ne.S.N.D. set in (X, N_T) . Then, $Ne.S.int (Ne.S.cl (K)) = 0_N$. Now $Ne.S.cl (Ne.S.cl (K)) = Ne.S.cl (K)$. Hence $Ne.S.int (Ne.S.cl (Ne.S.cl (K))) = Ne.S.int (Ne.S.cl (K)) = 0_N$. Therefore $Ne.S.cl (K)$ is also a Ne.S.N.D. set in (X, N_T) .

Proposition 2.12: If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T) , then $1 - Ne.S.cl (K)$ is a Ne.S.D. set in (X, N_T) .

Proof: Let K be a Ne.S.N.D. set in (X, N_T) . Then, by proposition 2.11, $Ne.S.cl (K)$ is a Ne.S.N.D. set in (X, T) . Also by proposition 2.10, $1 - Ne.S.cl (K)$ is a Ne.S.D. set in (X, N_T) .

Proposition 2.13: Let K be a Ne.S.D. set in a Ne.T.S. (X, N_T) . If L is any Ne. set in (X, N_T) , then L is a Ne.S.N.D. set in (X, N_T) if and only if $K \cap L$ is a Ne.S.N.D. set in (X, N_T) .

Proof: Let L be a Ne.S.N.D. set in (X, N_T) . Then, $Ne.S.int (Ne.S.cl (L)) = 0_N$. Now $Ne.S.int (Ne.S.cl (K \cap L)) \subseteq Ne.S.int (Ne.S.cl (K) \cap Ne.S.int (Ne.S.cl (L))) \subseteq Ne.S.int (Ne.S.cl (K)) \cap 0_N = 0_N$. That is, $Ne.S.int (Ne.S.cl (K \cap L)) = 0_N$. Hence $(K \cap L)$ is a Ne.S.N.D. set in (X, N_T) . Conversely, let $(K \cap L)$ be a Ne.S.N.D. set in (X, N_T) . Then $Ne.S.int Ne.S.cl (K \cap L) = 0_N$. Then, $Ne.S.int (Ne.S.cl (K)) \cap Ne.S.int (Ne.S.cl (L)) = 0_N$. Since K is a Ne.S.D. set in (X, N_T) , $Ne.S.cl (K) = 1_N$. Then, $Ne.S.int (1_N) \cap Ne.S.int (Ne.S.cl (L)) = 0_N$. That is, $(1_N) \cap Ne.S.int (Ne.S.cl (L)) = 0_N$. Hence $Ne.S.int (Ne.S.cl (L)) = 0_N$, which means that L is a Ne.S.N.D. set in (X, N_T) .

3. Neutrosophic Semi Baire Spaces

Definition 3.1. Let (X, N_T) be a Ne.T.S. A Ne. Set K in (X, N_T) is called Neutrosophic semi first category (Ne.S.F.C.) if $A = \bigcup_{i=1}^{\infty} A_i$ where A_i 's are Ne.S.N.D. sets in (X, N_T) . Any other Ne. set in (X, N_T) is said to be of Neutrosophic semi second category (Ne.S.S.C.).

Example 3.1: Let $X = \{k, l\}$. Define the Ne. set K, L, M and N on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5} \right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5} \right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3} \right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5} \right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.6} \right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4} \right) \right\rangle$$

$$M = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4} \right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4} \right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.5} \right) \right\rangle$$

$$N = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3} \right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3} \right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.7} \right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L\}$ is Ne.T. on X . Thus (X, N_T) is a Ne.T.S.. Now the sets

$$\bar{K}, \bar{L}, M, N \text{ are Ne.S.N.D. set and } \left[\bar{K} \cup \bar{L} \cup M \cup N \right] = \bar{L} \text{ is Ne.S.F.C. set in } (X, N_T)$$

Definition 3.2: Let K be a Ne.S.F.C. set in a Ne.S. (X, N_T) . Then $1 - K$ is called a Neutrosophic semi-residual (Ne.S.R.) set in (X, N_T) .

Proposition 3.1: If K is a Ne.S.F.C. set in a Ne.T.S. (X, N_T) , then $1 - K = \bigcap_{i=1}^{\infty} K_i$, where $Ne.S.cl (L_i) = 1_N$.

Proof: Let K be a Ne.S.F.C. set in (X, N_T) . Then we have $K = \bigcup_{i=1}^{\infty} K_i$, where K_i 's are Ne.S.N.D. in (X, N_T) . Now $1-K = \bigcap_{i=1}^{\infty} (1-K_i)$. Let $L_i = 1 - K_i$. Then $1-K = \bigcap_{i=1}^{\infty} L_i$. Since K_i 's are Ne.S.N.D. sets in (X, N_T) , by proposition 2.10, we have $1-K$'s are Ne.S.D. sets in (X, N_T) . Hence $Ne.S.cl (L_i) = Ne.S.cl (1-K_i) = 1_N$. Therefore we have $1-K = \bigcap_{i=1}^{\infty} L_i$ where $Ne.S.cl (L_i) = 1_N$.

Definition 3.3: A Ne.T.S. (X, N_T) is called a Ne.S.F.C. space if the Ne. set 1_N is a Ne.S.F.C. set in (X, N_T) . That is, $1_N = \bigcup_{i=1}^{\infty} K_i$ where K_i 's are Ne.S.N.D. sets in (X, N_T) . Otherwise (X, N_T) will be called a Ne.S.S.C. space.

Proposition 3.2: If K is a Ne.S.C. set in a Ne.T.S. (X, N_T) and if $Ne.S.int (K) = 0_N$, then K is a Ne.S.N.D. set in (X, N_T) .

Proof: Let K be a Ne.S.C. set in (X, N_T) . Then we have $Ne.S.cl (K) = K$. Now $Ne.S.int (Ne.S.cl (K)) = Ne.S.int (K)$ and $Ne.S.int(K) = 0_N$, implies that $Ne.S.int(Ne.S.cl(K)) = 0_N$. Hence K is a Ne.S.N.D. set in (X, N_T) .

Definition 3.4: Let (X, N_T) be a Ne.T.S.. Then (X, N_T) is called a Neutrosophic semi-Baire space (Ne.S.B.) if $Ne.S.int [\bigcup_{i=1}^{\infty} K_i] = 0_N$, where K_i 's are Ne.S.N.D. sets in (X, N_T) .

Example 3.2: Let $X = \{k, l\}$. Define the Ne. set k, L, M and N on X as follows:

$$\begin{aligned}
 K &= \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right) \right\rangle \\
 L &= \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.6}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right) \right\rangle \\
 M &= \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.5}\right) \right\rangle \\
 N &= \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.7}\right) \right\rangle
 \end{aligned}$$

Then the families $N_T = \{0_N, 1_N, K, L\}$ is Ne.T. on X . Thus (X, N_T) is a Ne.T.S.. Now the sets

\bar{K}, \bar{L}, M, N are Ne.S.N.D. set and $[\bar{K} \cup \bar{L} \cup M \cup N] = Ne.S.int (\bar{L}) = 0_N$ is Ne.S.B. space.

Example 3.3: Let $X = \{k, l\}$. Define the Ne.Sets K, L and M on X as follows:

$$\begin{aligned}
 K &= \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.6}\right), \left(\frac{k}{0.5}, \frac{l}{0.2}\right), \left(\frac{k}{0.4}, \frac{l}{0.5}\right) \right\rangle \\
 L &= \left\langle x, \left(\frac{k}{0.2}, \frac{l}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.1}\right) \right\rangle
 \end{aligned}$$

Then the families $N_T = \{0_N, 1_N, K, L, K \cup L, K \cap L\}$ is Ne.T on X . Thus (X, N_T) is a Ne.T.S. Now the sets $\overline{K}, \overline{L}, \overline{K \cup L}$ are Ne.S.N.D set and $Ne.S.int(\overline{K \cup L} \cup \overline{K \cap L}) = Ne.S.int(\overline{K \cap L}) \neq 0_N$. Hence the Ne.T.S. (X, N_T) is not Ne.S.B. space.

Proposition 3.3: If $Ne.S.int(\bigcup_{i=1}^{\infty} K_i) = 0_N$, where $Ne.S.int(K_i) = 0_N$ and K_i 's are Ne.S.C. sets in a Ne.T.S. (X, N_T) , then (X, N_T) is a Ne.S.B. space.

Proof: Let K_i 's be Ne.S.C. sets in (X, N_T) . Since $Ne.S.int(K_i) = 0_N$, by proposition 3.2, the K_i 's are Ne.S.N.D. sets in (X, N_T) . Therefore we have $Ne.S.int(\bigcup_{i=1}^{\infty} K_i) = 0_N$, where K_i 's are fuzzy semi-nowhere dense sets in (X, N_T) . Hence (X, N_T) is a Ne.S.B. space.

Proposition 3.4:

If $Ne.S.cl(\bigcap_{i=1}^{\infty} K_i) = 1_N$, where K_i 's are Ne.S.D. and Ne.S.O. sets in a Ne.T.S. (X, N_T) , then (X, N_T) is a Ne.S.B. space.

Proof:

Now $Ne.S.cl(\bigcap_{i=1}^{\infty} K_i) = 1_N$ implies that $1-Ne.S.cl(\bigcap_{i=1}^{\infty} K_i) = 0_N$. Then we have

$Ne.S.int(1-\bigcap_{i=1}^{\infty} K_i) = 0_N$, which implies that $Ne.S.int(\bigcup_{i=1}^{\infty} (1-K_i)) = 0_N$. Since K_i 's are Ne.S.D. sets in (X, N_T) ,

$Ne.S.cl(K_i) = 1_N$ and $Ne.S.int(1-K_i) = 1-Ne.S.cl(K_i) = 1-1_N = 0_N$. Hence we have $Ne.S.int(\bigcup_{i=1}^{\infty} (1-K_i)) =$

0_N , where $Ne.S.int(1-K_i) = 0$ and $(1-K_i)$'s are Ne.S.C. sets in (X, N_T) . Then, by proposition 3.3, (X, N_T) is a Ne.S.B. space.

Proposition 3.5: Let (X, N_T) be a Ne.T.S. The $\bigcup_{i=1}^{\infty} K_i$ in the following are equivalent:

- (1). (X, N_T) is a Ne.S.B. space.
- (2). $Ne.S.int(K) = 0_N$ for every Ne.S.F.C. set K in (X, N_T) .
- (3). $Ne.S.cl(L) = 1_N$ for every Ne.S.R. set in (X, N_T) .

Proof: (1) \rightarrow (2). Let K be a Ne.S.F.C. set in (X, N_T) . Then $K = \bigcup_{i=1}^{\infty} K_i$, where K_i 's are Ne.S.N.D. sets in

(X, N_T) . Now $Ne.S.int(K) = Ne.S.int(\bigcup_{i=1}^{\infty} K_i) = 0_N$ (since (X, N_T) is a Ne.S.B. space). Therefore

$Ne.S.int(K) = 0_N$.

(2) \rightarrow (3). Let L be a Ne.S.R. set in (X, N_T) . Then $1-L$ is a Ne.S.F.C set in (X, N_T) . By hypothesis, $Ne.S.int(1-L) = 0_N$ which implies that $1-Ne.S.cl(L) = 0_N$.

Hence we have $Ne.S.cl(L) = 1_N$.

(3) \rightarrow (1). Let K be a Ne.S.F.C.set in (X, N_T) . Then $K = \bigcup_{i=1}^{\infty} K_i$ where K_i 's are Ne.S.N.D.sets in (X, N_T) . $1-K$

is a Ne.S.R. set in (X, N_T) . Since K is a Ne.S.F.C. set in (X, N_T) , By hypothesis, we have $Ne.S.cl(1-K)$

$K) = 1_N$. Then $1\text{-Ne.S.int}(K) = 1_N$, which implies that $\text{Ne.S.int}(K) = 0_N$. Hence $\text{Ne.S.int}(\bigcup_{i=1}^{\infty} K_i) = 0_N$

where K_i 's are Ne.S.N.D. sets in (X, N_T) . Hence (X, N_T) is a Ne.S.B. space.

Proposition 3.6: If a fuzzy topological space (X, N_T) is a Ne.S.B. space, then (X, N_T) is a Ne.S.S.C.space.

Proof: Let (X, N_T) be a Ne.S.B. space. Then $\text{Ne.S.int}(\bigcup_{i=1}^{\infty} K_i) = 0_N$ where K_i 's are Ne.S.N.D. sets in $(X,$

$N_T)$. Then $\bigcup_{i=1}^{\infty} K_i \neq 1_N$. (Suppose, $\bigcup_{i=1}^{\infty} K_i = 1_N$ implies that $\text{Ne.S.int}(\bigcup_{i=1}^{\infty} K_i) = \text{Ne.S.int}(1_N)$ which implies that $0_N = 1_N$, a contradiction). Hence (X, N_T) is a Ne.S.S.C. space.

Remarks 3.6: The converse of the above proposition need not be true. A Ne.S.S.C. space need not be Ne.S.B. space.

Example 3.4: Let $X = \{k, l\}$. Define the Ne.Sets K and L on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.6}\right), \left(\frac{k}{0.5}, \frac{l}{0.2}\right), \left(\frac{k}{0.4}, \frac{l}{0.5}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.2}, \frac{l}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.1}\right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L, K \cup L, K \cap L\}$ is Ne.T on X . Thus (X, N_T) is a Ne.T.S. Now the sets

$\overline{K}, \overline{L}, \overline{K \cup L}$ are Ne.S.N.D set and $(\overline{K \cup L} \cap \overline{K \cap L}) = \overline{K \cap L} \neq 1_N$ & $\text{Ne.S.int}(\overline{K \cap L}) \neq 0_N$.

Hence the Ne.S.S.C. space need not be Ne.S.B.space.

Proposition 3.7: If a Ne.T.S. (X, N_T) is a Ne.S.B. space, then no non-zero Ne.S.O. set in (X, N_T) is a fuzzy semi-first category set in (X, N_T) .

Proof: Suppose that K is a non-zero Ne.S.O. set in (X, N_T) such that $K = \bigcup_{i=1}^{\infty} K_i$, where K_i 's are

Ne.S.N.D. sets in (X, N_T) . Then we have $\text{Ne.S.int}(K) = \text{Ne.S.int}(\bigcup_{i=1}^{\infty} K_i)$. Since K is a non-zero Ne.S.O.

set in (X, N_T) $\text{Ne.S.int}(K) = K$. Then $\text{Ne.S.int}(\bigcup_{i=1}^{\infty} K_i) = K \neq 0$. But this is a contradiction to (X, N_T)

being a Ne.S.B. space, in which $\text{Ne.S.int}(\bigcup_{i=1}^{\infty} K_i) = 0$, where K_i 's are Ne.S.N.D. sets in (X, N_T) . Hence

we must have $A \neq (\bigcup_{i=1}^{\infty} K_i)$.

Therefore no non-zero Ne.S.O. set in (X, N_T) is a Ne.S.F.C. set in (X, N_T) .

Proposition 3.8: A Ne.S.B. space is a Ne.B. space. For consider the following example:

Example 3.5: Let $X = \{k, l, m\}$. Define the Ne. set K, L, M and N on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.6}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right) \right\rangle$$

$$M = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4} \right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4} \right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.5} \right) \right\rangle$$

$$N = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3} \right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3} \right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.7} \right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L\}$ is Ne.T. on X . Thus (X, N_T) is a Ne.T.S. Now the sets \bar{K}, \bar{L}, M, N are Ne.S.N.D. set and $\text{Ne.S.int}[\bar{K} \cup \bar{L} \cup M \cup N] = \text{Ne.S.int}(\bar{L}) = 0_N$. Hence the Ne.T.S. (X, N_T) is Ne.S.B. space.

Here the sets \bar{K}, \bar{L}, M, N are Ne.N.D. set and $\text{Ne.int}[\bar{K} \cup \bar{L} \cup M \cup N] = \text{Ne.int}(\bar{L}) = 0_N$. Hence Ne.S.B. space is a Ne.B. space

Conclusions

Many different forms of closed sets have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences. In this paper, we introduced the concept of Neutrosophic Semi Baire spaces in Neutrosophic Topological Spaces. Also we define Neutrosophic Semi-nowhere dense, Neutrosophic Semi-first category and Neutrosophic Semi-second category sets. Some of its characterizations of Neutrosophic Semi-Baire spaces are also studied. This shall be extended in the future Research with some applications

Acknowledgements

The authors are highly grateful to the Referees for their constructive suggestions.

Conflicts of Interest

The authors declare no conflict of interest.

References

1. Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2019). A group decision making framework based on Neutrosophic TOPSIS approach for smart medical device selection. *Journal of medical systems*, 43(2), 38.
2. Abdel-Baset, M., Chang, V., & Gamal, A. (2019). Evaluation of the green supply chain management practices: A novel Neutrosophic approach. *Computers in Industry*, 108, 210-220.
3. Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2018). A hybrid approach of Neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*, 1-22.
4. K. Atanassov, Intuitionistic fuzzy sets, V. Sgurev, Ed., VII ITKR's Session, Sofia (June 1983 Central Sci. and Techn. Library, Bulg. Academy of Sciences, 1984).
5. K. Atanassov, intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 1986, 20, 87-96.
6. K. Atanassov, Review and new results on intuitionistic fuzzy sets, *Preprint IM-MFAIS*, , Sofia, 1988, 1-88.

7. K. Atanassov and S. Stoeva, intuitionistic fuzzy sets, *Polish Syrup. on Interval & Fuzzy Mathematics, Poznan, August 1983*, 23-26.
8. K. Atanassov and S. Stoeva, intuitionistic L-fuzzy sets, *R. Trappl, Ed., Cybernetics and System Research, Elsevier, Amsterdam, 1984, Vol. 2*, 539-540.
9. C.L.Chang, Fuzzy topological spaces, *J. Math. Anal. Appl.*, **1968**, 24, 182-190.
10. D. Coker, An introduction to intuitionistic fuzzy topological spaces, *Fuzzy Sets and Systems.*, **1997**, 88, 81-89.
11. R.Dhavaseelan, E.Roja and M.K.Uma, Intuitionistic Fuzzy Resolvable and Intuitionistic Fuzzy Irresolvable spaces, *Scientia Magna* , **2011**,7, 59-67.
12. R.Dhavaseelan and S.Jafari, Generalized Neutrosophic closed sets, (Submitted).
13. R. Dhavaseelan, S. Jafari, C. Ozel and M. A. Al-Shumrani, Generalized Neutrosophic Contra-Continuity(submitted).
14. R.Dhavaseelan, R.Narmada Devi and S. Jafari, Characterization of Neutrosophic Nowhere Dense Sets,(Submitted).
15. V. Banu priya S.Chandrasekar: Neutrosophic α s Continuity and Neutrosophic α s Irresolute Maps, *Neutrosophic Sets and Systems*, vol. 28, **2019**, pp. 162-170. DOI: 10.5281/zenodo.3382531
16. R. Dhavaseelan, S. Jafari ,R. Narmada Devi, Md. Hanif Page, Neutrosophic Baire Space, *Neutrosophic Sets and Systems*, **2017**, Vol. 16.
17. D. Jayanthi ,Generalized Closed Sets in Neutrosophic α Topological Spaces, *International Journal of Mathematics Trends and Technology (IJMTT)*, **2018**, 88- 91.
18. C.Maheswari, M.Sathyabama, S.Chandrasekar,Neutrosophic generalized b-closed Sets In Neutrosophic Topological Spaces,*Journal of physics Conf. Series* 1139 (**2018**) 012065. doi:10.1088/1742-6596/1139/1/012065
19. T. Rajesh Kannan , S. Chandrasekar, Neutrosophic $\omega\alpha$ - Closed Sets in Neutrosophic Topological Spaces, *Journal of Computer and Mathematical Sciences*, Vol.9(10),1400-1408 October **2018**.
20. T.Rajesh Kannan , S.Chandrasekar, Neutrosophic α -Continuity Multifunction In Neutrosophic Topological Spaces, *The International journal of analytical and experimental modal analysis* ,Volume XI, Issue IX, September/**2019** ISSN NO: 0886- 9367 PP.1360-1368
21. A.A.Salama and S.A.Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces, *IOSR Journal of Mathematics*, Sep-Oct. **2012**, Volume 3 Issue 4, PP 31-35.
22. F. Smarandache, Neutrosophy and Neutrosophic Logic, *First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA 2002*, smarand@unm.edu.
23. F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, *American Research Press, Rehoboth, NM, 1999*.
24. V.K.Shanthi.V.K.,S.Chandrasekar, K.SafinaBegam, Neutrosophic Generalized Semi closed Sets In Neutrosophic Topological Spaces, *International Journal of Research in Advent Technology*, Vol.(ii),6, No.7, , 1739-1743,July (**2018**)
25. G.Thangaraj and G.Balasubramanian, On Somewhat Fuzzy Continuous Functions, *J.Fuzzy. Math.*, **2003**, 11,No.2, 725-736.
26. G.Thangaraj and S.Anjalmoose, On Fuzzy Baire space, *J. Fuzzy Math.*, **2013**, Vol.21 (3), 667-676.
27. S.S.Thakur and R.Dhavaseelan, Nowhere Dense sets in intuitionistic fuzzy topological spaces, *Proceedings of National Seminar on Recent Developments in Topology*,11-12 February, **2015**.

28. Wadei F. Al-Omeri , Saeid Jafari: Neutrosophic pre-continuity multifunctions and almost pre-continuity multifunctions, *Neutrosophic Sets and Systems*, vol. 27, 2019, pp. 53-69 . DOI: 10.5281/zenodo.3275368
29. L.A. Zadeh, Fuzzy sets, *Inform. and Control*, **1965**, 8, 338-353.

Received: Sep 03, 2019. Accepted: Dec 01, 2019