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Applications of Extenics to 2D-Space and 3D-Space

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Abstract. In this article one proposes several numerical examples for applying the extension set to 2D- and 3D-spaces. While rectangular and prism geometrical figures can easily be decomposed from 2D and 3D into 1D linear problems, similarly for the circle and the sphere, it is not possible in general to do the same for other geometrical figures.

Key words. Extentics, extension engineering, contradictory problems, computational and artificial intelligence

I. Introduction

EXTENICS has been used since 1983 by Cai Wen and many other Chinese scholars in solving contradictory problems.

The distance between a number and a set, and the degree of dependence of a point with respect to a set were defined for the one-dimensional space, and later for higher dimensional spaces. We present below several examples in 2D and 3D spaces.

2. APPLICATION 1.

We have a factory piece whose desired *2D*-dimensions should be *20 cm* × *30 cm*, and acceptable *2D*-dimensions *22 cm* × *34 cm*. We define the extension 2D-distance, and then we compute the extension 2D-dependent function. Let's do an extension diagram:

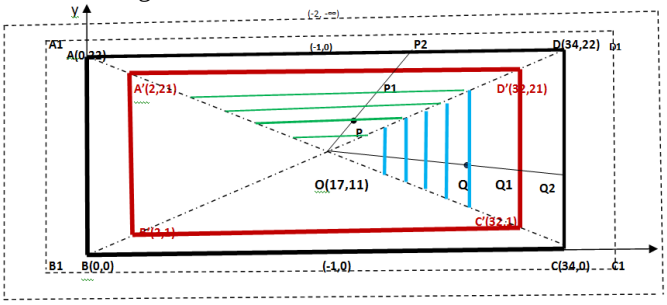

Diagram 1.

We have a desirable factory piece A'B'C'D' and an acceptable factory piece ABCD. The optimal point for both of them is O(17,11).

a) The region determined by the rays OA and OD.

The extension 2D-distance ρ between a point P and a set is the $\pm$ distance from P to the closest frontier of the set, distance measured on the line OP. Whence

$$\rho\,(P, A'B'C'D') = -|PP_1| \qquad (1)$$

and

$$\rho\,(P, ABCD) = -|PP_2|. \qquad (2)$$

The extension 2D-dependent function *k* of a point P which represents the dependent of the point of the nest of the two sets is:

$$k(P) = \pm\frac{\rho(P, bigger\_set)}{\rho(P, bigger\_set) - \rho(P, smaller\_set)} = \pm\frac{\rho(P, ABCD)}{\rho(P, ABCD) - \rho(P, A'B'C')} = \pm\frac{|PP_2|}{|PP_2| - |PP_1|} = \pm\frac{|PP_2|}{|P_1P_2|}. \qquad (3)$$

In other words, the extension 2D-dependent function k of a point P is the 2D-extension distance between the point and the closest frontier of the larger set, divided by the 2D-extension distance between the frontiers of the two nested sets; all these 2D-extension distances are taken along the line OP.

In our application one has:

$$k(P) = +\frac{|PP_2|}{|P_1P_2|} \qquad (4)$$

since P is inside of the larger set. If P was outside of the larger set, then k(P) would be negative.

Let's consider the coordinates of $P(x_0,y_0)$, where P is between the rays OA and OD in order to make sure OP intersects the line segments AD and A'D' which are closest frontiers of the rectangles ABCD and respectively A'B'C'D'. {The problem would be similar if P was in between the rays OB and OC.}

Hence $y_0 \in (11, \infty]$ but such y_0 that remains in between the rays OA and OD.

Let's find the coordinates of P_1.

In analytical geometry the equation of line OP passing through two points, O(17,11) and $P(x_0,y_0)$, is:

$$y - 11 = \frac{y_0 - 11}{x_0 - 17}(x - 17). \qquad (5)$$

Since the y-coordinate of P_1 is 21, we replace y = 21 in the above equation and we get the x-coordinate of P_1.

Whence one has

$$P_1\left(\frac{10x_0 + 17y_0 - 357}{y_0 - 11}, 21\right).$$

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Let's find the coordinates of P_2. The y-coordinate of P_2 is 22. Replace $y = 22$ in equation (2) and solve for the x-coordinate of P_2.

One gets

$$P_2(\frac{11x_0+17y_0-374}{y_0-11},22).$$

The classical distance in *2D*-space between two points $M(m_1, m_2)$, $N(n_1, n_2)$ is

$$d(M,N)=\sqrt{(m_1-n_1)^2+(m_2-n_2)^2}. \quad (6)$$

We compute the classical *2D*-distances $d(P, P_2)$ and $d(P_1, P_2)$.

$$k(P)=\pm\frac{|PP_2|}{|P_1P_2|}=\pm\frac{\sqrt{\left(\frac{11x_0+17y_0-374}{y_0-11}-x_0\right)^2+(22-y_0)^2}}{\sqrt{\left(\frac{11x_0+17y_0-374}{y_0-11}-\frac{10x_0+17y_0-357}{y_0-11}\right)^2+(22-21)^2}}$$

$$=\pm\frac{\sqrt{\left(\frac{22x_0+17y_0-x_0y_0-374}{y_0-11}\right)^2+(y_0-22)^2}}{\sqrt{\left(\frac{x_0-17}{y_0-11}\right)^2+1}}=\pm\sqrt{\frac{(x_0-17)^2(y_0-22)^2}{(y_0-11)^2}+(y_0-22)^2}{\frac{(x_0-17)^2}{(y_0-11)^2}+1}}$$

$$=\pm|y_0-22|=\begin{cases}22-y_0, & y_0\in(11,22]\\ 22-y_0, & y_0>22\end{cases}=22-y_0,\ y_0>11 \quad (7)$$

and *P* in between the rays *OA* and *OD*.

Since the extension *2D*-dependent function $k(x_0,y_0) = 22-y_0$, for $y_0>11$, does not depend on x_0 for the region between rays *OA* and *OD*, one has classes of points lying on horizontal lines parallel to *A'D'* (see the green line segments on *Diagram 1*) whose extension *2D*-dependent function value is the same. For example, the green horizontal line segment passing thought *P* is the class of points having the same extension *2D*-dependent function value as point *P*.

b) The region determined by the rays *OC* and *OD*. {Similar result would obtain if one gets the opposite region determined by the rays *OA* and *OB*.}

If one takes another region determined by the rays OC and OD and a point $Q(x_1,y_1)$ in between one gets

$$k(Q)=k(x_1,y_1)=\pm\frac{|QQ_2|}{|Q_1Q_2|} \quad (8)$$

By a similar method we find the Cartesian coordinates of the points Q_1 and Q_2.

In analytical geometry the equation of line OQ passing through two points, O(17,11) and $Q(x_1,y_1)$, is:

$$y-11=\frac{y_1-11}{x_1-17}(x-17). \quad (9)$$

Since the x-coordinate of Q_1 is 32, we replace x = 32 in the above equation and we get the y-coordinate of P_1.

Whence one has $Q_1(32,\frac{11x_1+15y_1-352}{x_1-17})$ (10)

Let's find the coordinates of Q_2.

The x-coordinate of P_2 is 34. Replace x=22 in equation (3) and solve for the y-coordinate of Q_2. One gets

$$Q_2(34,\frac{11x_1+17y_1-374}{x_1-17}). \quad (11)$$

We compute the classical 2D-distances d(Q, Q_2) and d(Q_1, Q_2).

$$k(P)=\pm\frac{|QQ_2|}{|Q_1Q_2|}=\pm\frac{\sqrt{(34-x_1)^2+\left(\frac{11x_1+17y_1-374}{x_1-17}-y_1\right)^2}}{\sqrt{(34-32)^2+\left(\frac{11x_1+17y_1-374}{x_1-17}-\frac{11x_1+15y_1-352}{x_1-17}\right)^2}}$$

$$=\pm\frac{\sqrt{(x_1-34)^2+\frac{(x_1-34)^2(y_1-11)^2}{(x_1-17)^2}}}{\sqrt{4+\frac{4(y_1-11)^2}{(x_1-17)^2}}}=\pm\frac{|x_1-34|}{2}=\frac{34-x_1}{2},\ x_1>17 \quad (12)$$

and Q in between the rays OC and OD.

Since the extension 2D-dependent function $k(x_1,y_1) = \frac{34-x_1}{2}$, for $x_1 > 17$, does not depend on y_1 for the region between rays OC and OD, one has classes of points lying on vertical lines parallel to C'D' (see the red line segments on Diagram 1) whose extension 2D-dependent function value is the same. For example, the blue vertical line segment passing thought Q is the class of points having the same extension 2D-dependent function value as point Q.

2. SPLITTING AN EXTENSION 2D-PROBLEM INTO TWO 1D-PROBLEMS.

Remarkably, for rectangular shapes one can decompose a 2D-problem into two 1D-problems. Yet, for other geometrical figures it is not possible. The more irregular geometrical figure, the less chance to decompose a 2D-problem into 1D-problems.

In our case, we separately consider the factory piece's width and length.

1) The width of a factory piece is desirable to be 20 cm and acceptable up to 22 cm.
2) And the length of a factory piece is desirable to be 30 cm and acceptable up to 34 cm.

In the first 1D-problem one makes the diagram:

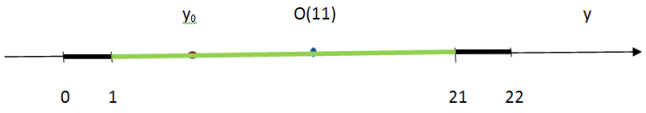

Diagram 2.

One computes, using Prof. Cai Wen's extention 1D-dependent function:

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$$k(y_0)=\frac{|y_0-11|-\frac{22-0}{2}}{|y_0-11|-\frac{22-0}{2}-(|y_0-11|-\frac{21-1}{2})}=\frac{|y_0-11|-11}{-11+10}=11-|y_0-11| \quad (13)$$

If $y_0 > 11$ as in our 2D-space problem, then $k(y_0) = 22-y_0$ which is consistent with what we got in the 2D case.
In the second 1D-problem one makes the diagram:

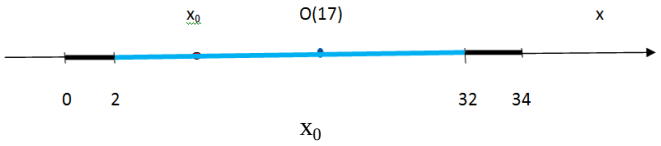

Diagram 3.

One computes, using Prof. Cai Wen's extension 1D-dependent function:

$$k(x_0)=\frac{|x_0-17|-\frac{34-0}{2}}{|x_0-17|-\frac{34-0}{2}-(|x_0-17|-\frac{32-2}{2})}=\frac{|x_0-17|-17}{-17+15}=\frac{|x_0-17|-17}{-2}=\frac{17-|x_0-17|}{2} \quad (14)$$

If $x_0 > 17$ as in our 2D-space problem, then $k(x_0)=\frac{34-x_0}{2}$, which is consistent with what we got in the 2D-case.
Therefore, a 2D-extension problem involving rectangles is equivalent with two 1D-extension problems. Certainly this equivalence is not valid any longer if instead of rectangles we have more irregular geometrical figures representing factory pieces.
Similarly will be possible for splitting a 3D-application for prisms into three 1D-applications, or into one 2D-application and one 1D-application.

3. REGION CRITICAL ZONE.

Critical Zone is the region of points where the degree of dependence of a point *P* with respect to a nest of two intervals $k(P) \in (-1, 0)$.
In the above figure, it is all area between the rectangles ABCD and $A_1B_1C_1D_1$.
$A_1B_1C_1D_1$ was constructed by drawing parallels to the sides of the rectangle ABCD, such that:

- The distance between the parallel lines A'D' and AD, be the same with the distance between the parallel lines AD and A_1D_1;
- The distance between the parallel lines A'B' and AB, be the same with the distance between the parallel lines AB and A_1B_1;
- The distance between the parallel lines B'C' and BC, be the same with the distance between the parallel lines BC and B_1C_1;
- The distance between the parallel lines C'D' and CD, be the same with the distance between the parallel lines CD and C_1D_1.

One then extend the construction of a net of included rectangles $A_iB_iC_iD_i \subset A_{i+1}B_{i+1}C_{i+1}D_{i+1}$
and for the points P_{i+I} lying on surface in between the rectangles $A_iB_iC_iD_i$ and $A_{i+1}B_{i+1}C_{i+1}D_{i+1}$ the dependent function $k(P_{i+I}) \in (-i-1, -i)$.

4. APPLICATION IN THE 3D-SPACE.

A factory piece has the desirable dimensions 20x30x7 but the acceptable factory piece can be 22x34x10 (in centimeters).
The red prism is the desirable form, and the green prism is the acceptable form.
We consider a Cartesian system XYZ and the vertexes of these two prisms are:
A(0,22,0), B(0,0,0), C(34,0,0), D(34,22,0), E(0,22,10), F(0,0,10), G(34,0,10), H(34,22,10);
A'(2,21,3), B'(2,1,3), C'(32,1,3), D'(32,21,3), E'(2,21,7), F'(2,1,7), G'(32,1,7), H'(32,21,7).
O(17,11,5); $P(x_0,y_0,z_0)$, $P'(x_1,y_1,7)$, $P''(x_2,y_2,10)$;
$Q(17,11,z_0)$, Q'(17,11,7), Q''(17,11,10). (15)
The following triangles are similar: $\triangle$ QOP, $\triangle$ Q'OP', $\triangle$ Q''OP''.

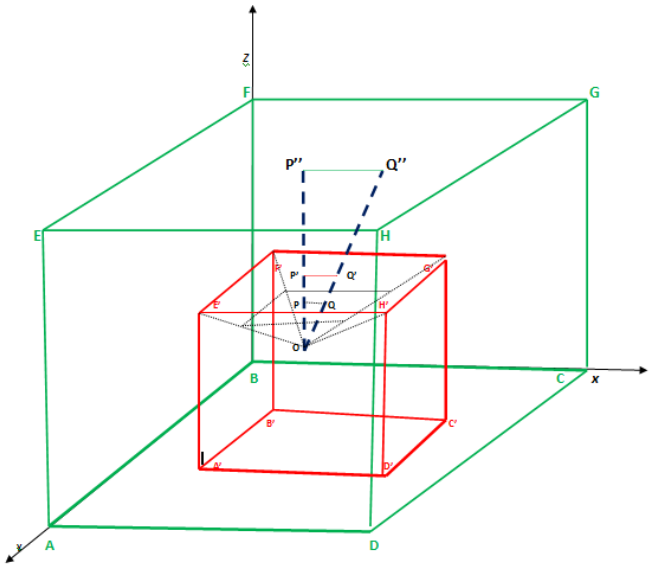

Diagram 4.

Using similarity of triangles, Thales Theorem, and proportionalizations we get that:

$$\frac{|PP"|}{|P'P"|}=\frac{|QQ"|}{|Q'Q"|}$$

which is equivalent to the equality of dependent function values

$$\text{of } k(P)=k(Q),$$

since

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$$k(P)=\pm\frac{\rho(P,ABCDEFGH)}{\rho(P,ABCDEFGH)-\rho(P,A'B'C'D'E'F'G'H')}=\pm\frac{|PP"|}{|PP"|-|PP'|}=\pm\frac{|PP"|}{|P'P"|}$$

(16)

and similarly:

$$k(Q)=\pm\frac{\rho(Q,ABCDEFGH)}{\rho(Q,ABCDEFGH)-\rho(Q,A'B'C'D'E'F'G'H')}=\pm\frac{|QQ"|}{|QQ"|-|QQ'|}=\pm\frac{|QQ"|}{|Q'Q"|}$$

. (17)

Therefore, the plane which passes through the point *P* and is parallel with the planes *EFGH* and *E'F'G'H'* (limited by the lines *OE', OF', OG', OH'*) is the locus of points having the same dependent function value.

$k(P)=\dfrac{z_0-10}{3}$ for $z_0 > 5$ and point *P* inside the reversed pyramid *OEFGH*.

5. CRITICAL ZONE.

The Critical Zone, whose dependent function of each point in this zone belongs to (-1, 0), will be a larger prism $A_1B_1C_1D_1E_1F_1G_1H_1$ which envelopes the prism *ABCDEFGH* at the same distance from each face as it was between the prisms *A'B'C'D'E'F'G'H'* and *ABCDEFGH*. Therefore, the distance between faces A'B'C'D' and ABCD is the same as the distance between faces ABCD and $A_1B_1C_1D_1$; and the faces A'B'C'D' and ABCD and $A_1B_1C_1D_1$ are parallel. Similarly for all six faces of the prism $A_1B_1C_1D_1E_1F_1G_1H_1$: the distance between faces A'E'H'D' and AEHD is the same as the distance between faces AEHD and $A_1E_1H_1D_1$; and the faces A'E'H'D' and AEHD and $A_1E_1H_1D_1$ are parallel, etc.

One can construct a net of such prisms: $A_{i+1}B_{i+1}C_{i+1}D_{i+1}E_{i+1}F_{i+1}G_{i+1}H_{i+1} \supset A_iB_iC_iD_iE_iF_iG_iH_i$ where the value of the dependent function for the points which belong to $Int(A_{i+1}B_{i+1}C_{i+1}D_{i+1}E_{i+1}F_{i+1}G_{i+1}H_{i+1} - A_iB_iC_iD_iE_iF_iG_iH_i)$ is in the interval *(-i-1, -i)*, while for the points lying on the $Fr(A_{i+1}B_{i+1}C_{i+1}D_{i+1}E_{i+1}F_{i+1}G_{i+1}H_{i+1})$ the dependent function is *–i-1*. One considers *ABCDEFGH* as $A_0B_0C_0D_0E_0F_0G_0H_0$, and *A'B'C'D'E'F'G'H'* as $A_{-1}B_{-1}C_{-1}D_{-1}E_{-1}F_{-1}G_{-1}H_{-1}$ for the rule to work for all included prisms.

6. SPLITTING A 3D-PROBLEM INTO THREE 1D-PROBLEM.

Similarly to the previous 2D-problem, we separately consider the factory piece's width, length, and height.

1) The width of a factory piece is desirable to be 20 cm and acceptable up to 22 cm.
2) And the length of a factory piece is desirable to be 30 cm and acceptable up to 34 cm.
3) And the height of a piece factory is desirable to be 7 cm and acceptable 10 cm.

In the first 1D-problem one makes the diagram:

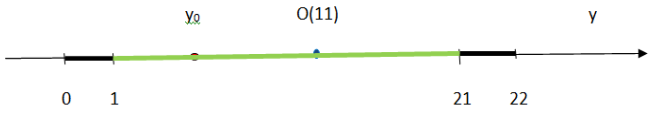

Diagram 5.

One computes, using Prof. Cai Wen's extention 1D-dependent function:

$$k(y_0)=11-|y_0-11| \quad (18)$$

In the second 1D-problem one makes the diagram:

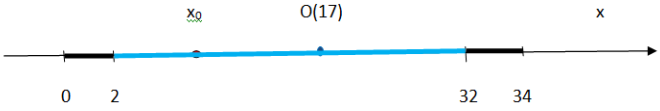

Diagram 6.

In the third 1D-problem one makes the diagram:

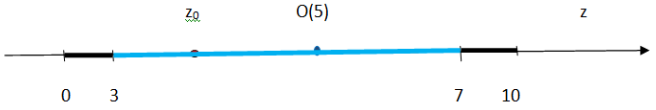

Diagram 7.

One computes, using Prof. Cai Wen's extention 1D-dependent function:

$$k(z_0)=\frac{z_0-10}{3} \quad (19)$$

7. SPLITTING A 3D-PROBLEM INTO A 2D-PROBLEM AND A 1D-PROBLEM.

Similarly to the previous 2D-problem, we separately consider the factory piece's width, length, and height.

1) The factory 2D-piece is desirable to be 20x30 cm and acceptable up to 22x34 cm.
2) And the height of a piece factory is desirable to be 7 cm and acceptable 10 cm.

8. A 2D-PROBLEM WHICH IS SPLIT INTO ONLY ONE 1D-PROBLEM.

Assume the desirable circular factory piece radius is 6 cm and acceptable is 8 cm.

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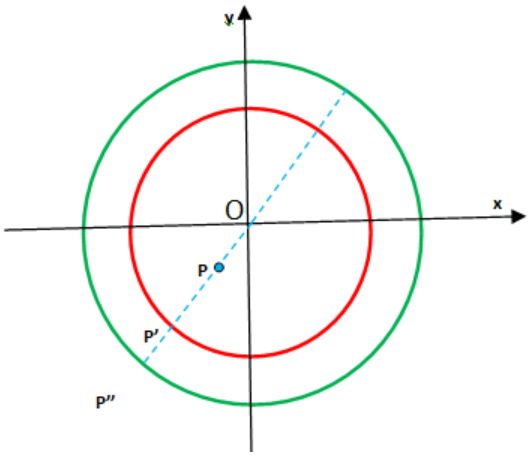

Diagram 8.

It is equivalent to a 1D-problem which has the diagram:

Diagram 9.

One computes, using Prof. Cai Wen's extension 1D-dependent function:

$$k(x_0) = \frac{x_0}{2} \qquad (20)$$

9. A 2D-PROBLEM WHICH CANNOT BE SPLIT INTO 1D-PROBLEMS.

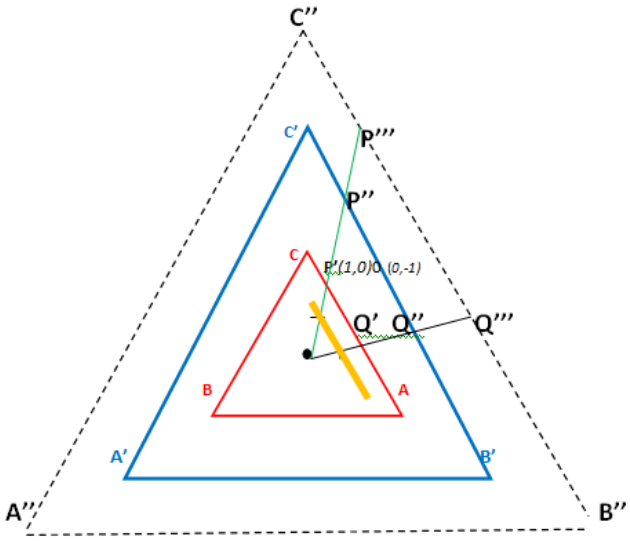

Diagram 10.

1. The **Critical Zone** is between the blue triangle *A'B'C'* and the black dotted triangle *A''B''C''*. Points lying on lines parallel to the red triangle's sides have the same dependence function value (for example the points lying on the orange line segment).

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