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A Reactor Transfer Function Analyzer

Earl O. Swickard Jr

A REACTOR TRANSFER FUNCTION ANALYZER - SWICKARD

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By

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MASTER OF SCIENCE

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DATE June 3, 1959

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A REACTOR TRANSFER FUNCTION ANALYZER

I. Introduction

This thesis describes the design of a device for determining the transfer function of a nuclear reactor. A number of different methods have been successfully employed for this purpose but were, in some cases, tedious or required the use of delicately balanced components.[1,2,3,4,5] The equipment described computes values which are simply related to the values of the transfer function and is operable over a frequency range of about 1000:1. The time required to obtain data for one frequency point is two minutes or less. No precise adjustment is necessary during operation.

The analyzer makes use of a technique developed several years ago by engineers studying complex feedback systems.[6,7] The technique is to introduce a small, sinusoidal disturbance at the input of a system and measure the relation of the output to the input. Repeating the measurement at points over the frequency range of interest yields the frequency response of the system which is the transfer function.

II. The Nuclear Reactor as a System

A nuclear reactor could be a serious hazard if, through improper design or operation, there were a power excursion of sufficient magnitude to rupture containment and release highly radioactive material to the atmosphere. Determination of a reactor's transfer function as various parameters, primarily power, are varied, gives an insight into its stability and permits detection of incipient instabilities before operating under conditions which would result in an excursion. By checking frequency response as power is increased, it is possible to start up a

new, untested reactor safely.[8]

The kinetic equations which describe approximately the behavior of the neutron chain reaction in a reactor are[9]

$$\frac{dn}{dt} = \frac{\delta k - \beta}{l^*} n + \sum_{i=1}^{6} \lambda_i c_i \qquad (1)$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{l^*} n - \lambda_i c_i \qquad (2)$$

where n = neutron level (power is proportional to n)

$\delta k = K_1 \sin \omega t$ = amount the multiplication factor differs from unity.

β = total delayed neutron fraction

β_i = delayed neutron fraction of the i-th group

l^* = mean prompt neutron lifetime

λ_i = decay constant of the i-th delayed neutron group

c_i = concentration of the i-th delayed neutron group emitters

It is assumed in Eqs. (1) and (2) that $k \ll \beta$ and that delayed neutrons have the same effectiveness as prompt neutrons in causing fission.

The equations do not have constant coefficients, a necessary condition for developing a transfer function. However, if it is assumed that the variation of n is sufficiently small, Eqs. (1) and (2) can be approximated by ones with constant coefficients.

The value of n can be considered as consisting of two parts; N_0, a steady state value, and δn, the departure from the steady state. For very small δk's, $\delta n = N_1 \sin(\omega t - \alpha)$. In the same way c_i can be

separated into C_{i_o} and δc_i. Making these substitutions in Eqs. (1) and (2) and some further manipulations yield the reactor kinetics equations

$$\frac{d\delta n}{dt} = \frac{\delta k}{l^*} N_o - \sum_{i=1}^{6} \frac{d\delta c_i}{dt} \qquad (3)$$

$$\frac{d\delta c_i}{dt} = \frac{\beta_i}{l^*} \delta n - \lambda_i \delta c_i \qquad (4)$$

Reducing Eqs. (3) and (4) to the Laplace transform operational form gives

$$s\delta n(s) = \frac{N_o}{l^*} \delta k(s) - s \sum_{i=1}^{6} \delta c_i(s) \qquad (5)$$

$$s\delta c_i(s) = \frac{\beta_i}{l^*} \delta n(s) - \lambda_i \delta c_i(s) \qquad (6)$$

For the purpose of transfer function analysis, $s = j\omega$. The initial condition transforms have been dropped for they describe the transient behavior of the system when sinusoidal excitation is suddenly begun at time, $t = o$. It is the equilibrium response after the starting transient has died away that is of interest.

$$\frac{\delta n(s)}{\delta k(s)} = \frac{N_o}{l^*} \frac{1}{s\left[1 + \sum_{i=1}^{6} \frac{\beta_i}{l^*(s + \lambda_i)}\right]} \qquad (7)$$

which, for the assumptions that were made, is the transfer function of a simple chain reacting assembly.

Since the magnitude of $\frac{\delta n(s)}{\delta k(s)}$ is dependent not only on the bracketed term of Eq. (7) but also on the power level, N_o, it is usual to remove the N_o from the right side of the expression and define the reactor transfer

function as

$$\frac{\delta n(s)}{\delta k(s) N_0} = \frac{1}{l^*} \frac{1}{s\left[1 + \sum_{i=1}^{6} \frac{\beta_i}{l^*(s + \lambda_i)}\right]} \tag{8}$$

A nuclear reactor may be considered to be a system having the block diagram of Fig. 1.

It is possible, if the reactor transfer function and kinetic equations are known, to calculate the transfer function of the feedback box of Fig. 1 and, knowing this, one may be able to assign the cause of an instability to a particular time constant. If this time constant can be related to a particular component of the reactor, modification of the component could remove the instability.

III. General Description of the Analyzer

The purpose of the analyzer is to apply a small, sinusoidally varying δk of known magnitude to a reactor and measure the amplitude of $\frac{N_1}{N_0}$ and the phase angle between δk and δn. The frequency range over which the analyzer was designed to operate is .01 to 10 cycles per second, a range including the frequencies where instabilities could be expected. Figure 2 is a schematic diagram of the analyzer or reactor oscillator as it is often called.

A rotating oscillator rod supplies the sinusoidally varying δk because of its asymmetry about the axis of rotation. Figure 3 shows the cross section of a possible oscillator rod which changes reactivity during rotation as the cylindrical void changes its distance from the core. The rotating rod is part of the core reflector. When the void is nearest the core, leakage of neutrons from the assembly is greater than when the void

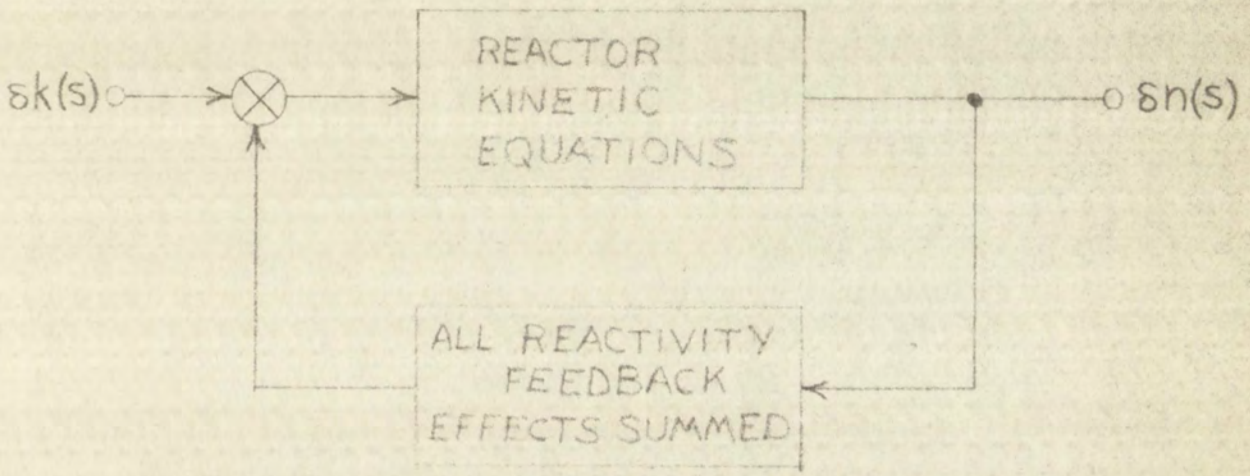
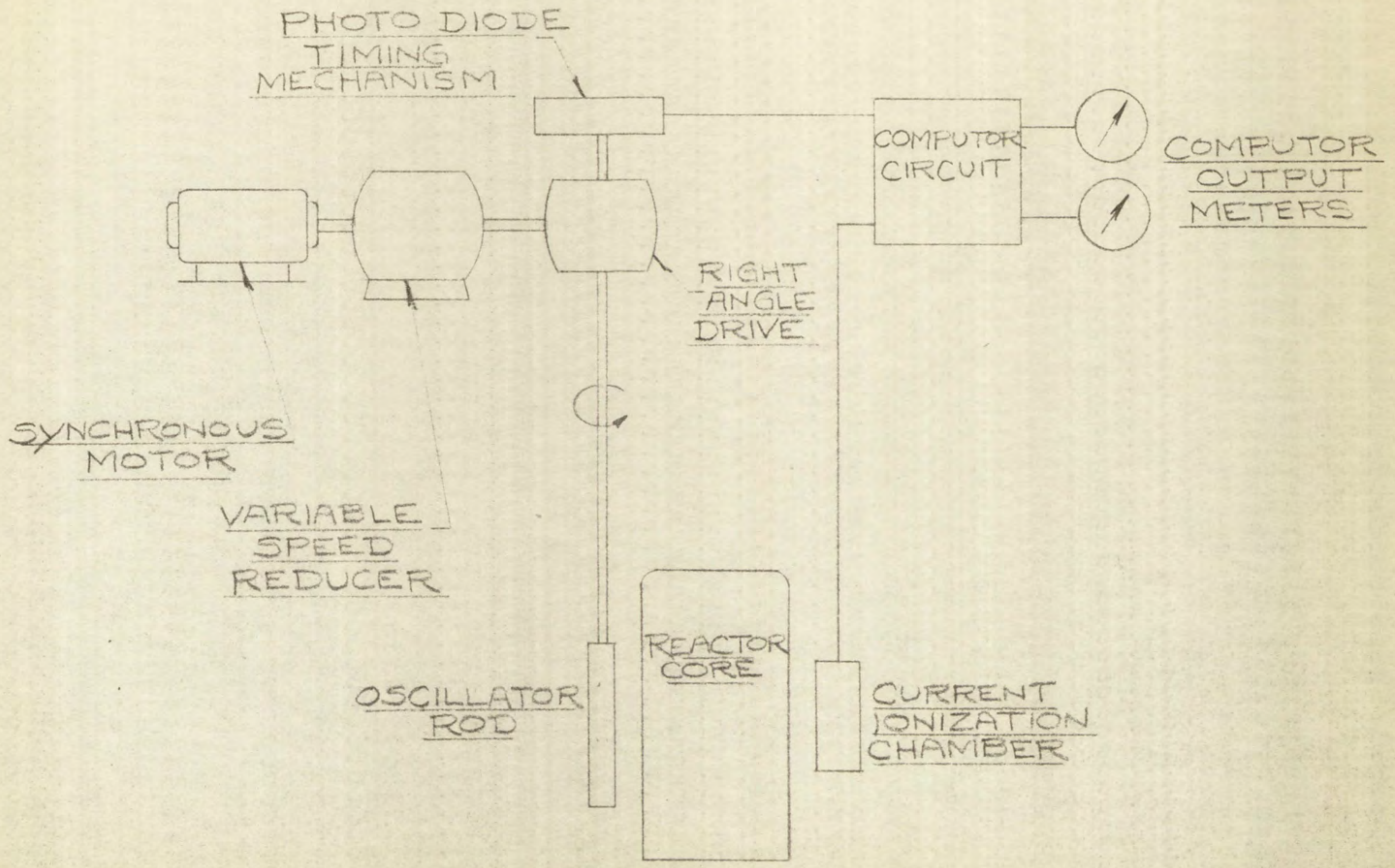

δk(s)
REACTOR
KINETIC
EQUATIONS
δn(s)
ALL REACTIVITY
FEEDBACK
EFFECTS SUMMED

FIG. 1



REACTOR TRANSFER FUNCTION ANALYZER

FIG. 2

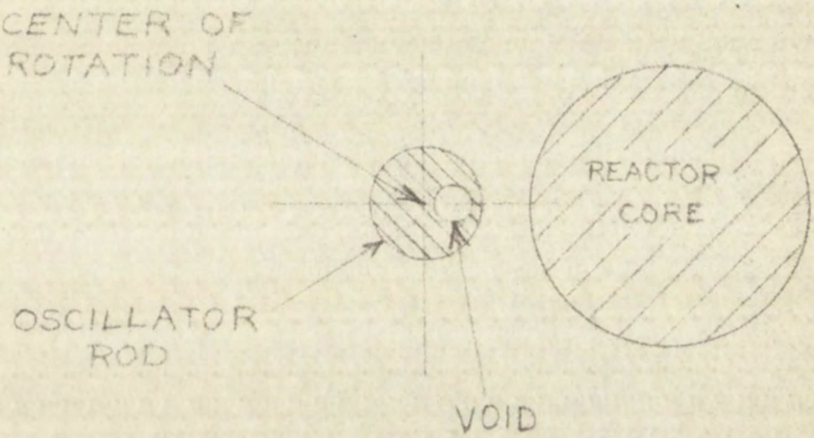


FIG. 3

is further away. Hence the rod's contribution to reactivity is greatest when the void furthest from the core and least when it is nearest. Several rod geometries have been tried by experimenters to achieve sinusoidal reactivity disturbances.[2] The rod used in test of this analyzer will be described in Section VI.

The use of the analyzer assumes the Laplace operator, s, to be $j\omega$ and that $\delta k = K_1 \sin \omega t$. If δk contains any harmonics of the fundamental frequency, the method of arriving at the transfer function is much more complex. It is expedient to obtain a rod geometry such that the harmonic content is small enough that its effect on results can be neglected.

The motor and variable speed reducer shown in Fig. 2 is a Graham transmission unit. The motor is a 1/3 horsepower, 1800 rpm synchronous type. The output speed of the reducer may be varied from 0 to 650 rpm. Prior to incorporation into the equipment, output speed drift over the operating range was checked and found to be less than .2%. A small electric motor driving the speed changing mechanism permits changes in speed to be made remotely. The reducer is connected to the oscillator rod and timing mechanism through a right angle drive. Light sources, a slotted disc and photo diodes generate pulses at certain rod positions and the pulses are fed to the computer circuit.

An ionization chamber in the neutron flux of the reactor delivers a current to the computer circuit which is proportional to the neutron flux and hence to power level. The chamber used was of the boron coated variety. A gamma ray scintillator was tried briefly and appeared to give satisfactory results.

The computer circuit operates on its input signals and displays on two output voltmeters values from which the relation of $\delta n/N_0$ to δk

may be readily obtained. Operation of the computer circuit is described in the following section.

IV. Operation of the Computer Circuit

The five amplifiers shown in the schematic diagram, Fig. 4, are high gain operational amplifiers. The particular ones used are George A. Philbrick Researches model USA-3. They have a gain of about 10^7. The following discussion briefly describes the behavior of the amplifiers when used in an analog computer application.[10]

When connected as shown in Fig. 5a, the amplifiers have the output-input relation $e_o = \frac{-Z_o}{Z_1} e_1$. If Z_o and Z_1 are resistances, then $e_o = \frac{-R_o}{R_1} e_1$. If Z_o is a capacitor and Z_1 a resistor, $e_o = \frac{-1}{R_1 C_o} \int e_1 \, dt$.

An operational amplifier with two inputs as shown in Fig. 5b sums the inputs.

$$e_o = - \left(\frac{R_o e_1}{R_1} + \frac{R_o e_2}{R_2} \right).$$

In the case of an amplifier connected as in Fig. 5c, $e_o = - i_o R_o$.

In Fig. 4 the chamber current, i_n, is made up of components as given by Eq. (9).

$$i_n = I_o + I_1 \sin(\omega t - \alpha) + \text{noise} \tag{9}$$

where I_o = current from the average neutron flux

I_1 = amplitude of the sinusoidally varying current

ω = frequency of oscillation

α = lag angle of neutron flux sinusoid relative to input reactivity sinusoid

noise is the random fluctuation of current from any cause but primarily from the statistical variation of power level.

R_o is adjusted so that the component of e_1 due to I_o is -5 volts.

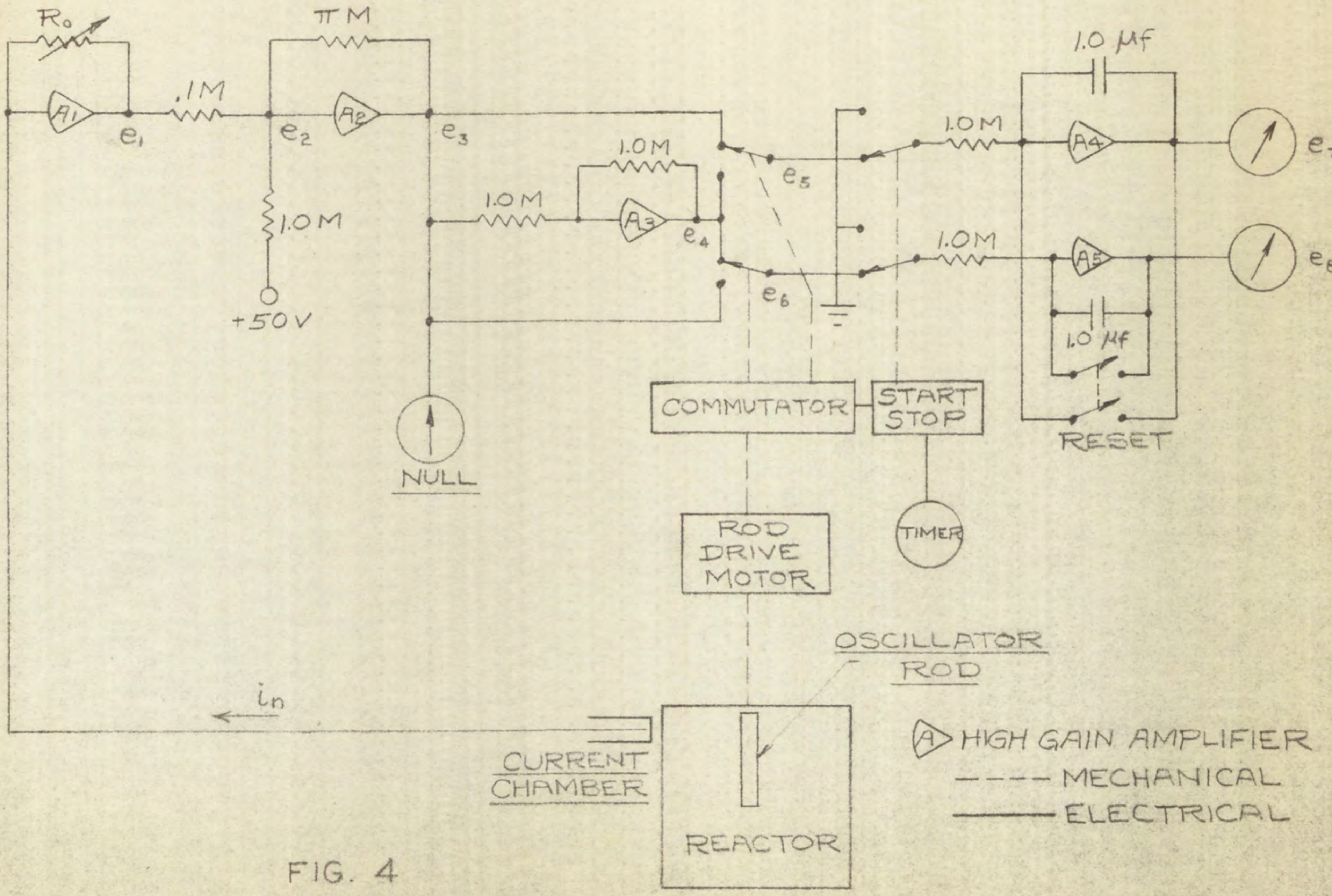

R₀
.1M
πM
e_1
e_2
e_3
1.0M
+50V
1.0M
1.0M
e_4
e_5
e_6
NULL
1.0M
1.0M
1.0 μf
1.0 μf
e_7
e_8
RESET
COMMUTATOR
START STOP
TIMER
ROD DRIVE MOTOR
OSCILLATOR ROD
i_n
CURRENT CHAMBER
REACTOR
HIGH GAIN AMPLIFIER
MECHANICAL
ELECTRICAL

FIG. 4

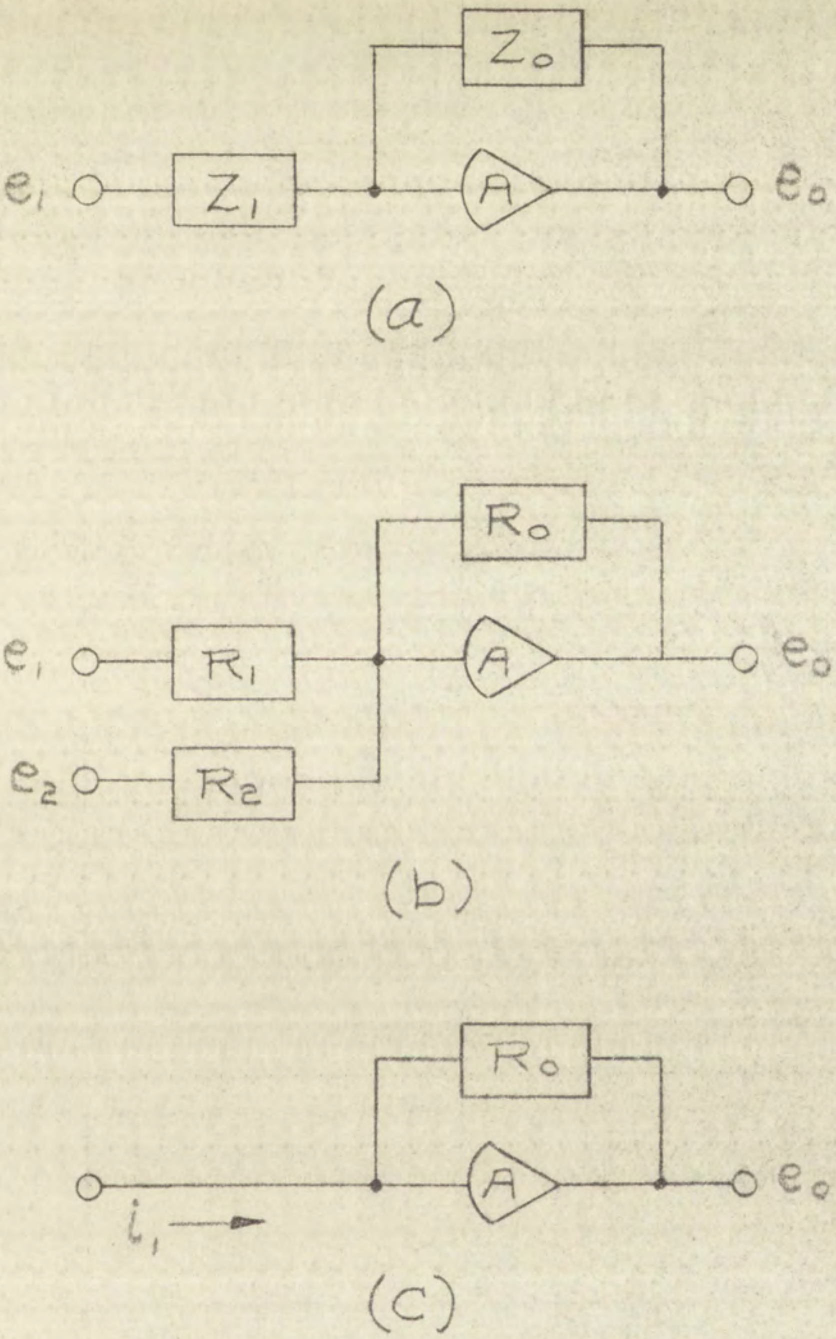


FIG. 5

The explanation for the particular value of -5 volts is made when the operation of A_2 amplifier is described. Proper adjustment of R_o is made by changing R_o until oscillations of the null meter are centered about the meter null. As will be brought out in Section V, very precise adjustment of R_o is not required.

$$I_oR_o = -(-5)$$

$$R_o = \frac{5}{I_o} \tag{10}$$

$$e_1 = i_n R_o \tag{11}$$

Substituting Eqs. (9) and (10) in Eq. (11) gives

$$e_1 = -\frac{5}{I_o}\left[I_o + I_1 \sin(\omega t - \alpha) + \text{noise}\right] = -5 - \frac{5I_1}{I_o}\sin(\omega t - \alpha) \tag{12}$$

The noise term has been dropped for its contribution to computer output is very small. The reason that noise can be neglected is because of later integration of the signal. The integral of truly random noise over a long time is zero; the integral over a short time might not be exactly zero, but will be small. Conceivably it might be necessary to rerun a point that lay outside the expected range because of the small probability that there is correlation between noise and ω.

A_2 is a summing amplifier and is used to buck out the I_o component of the input signal. The cancelling is done by changing the gain of the input amplifier, A_1, (by changing R_o) until $10\ I_oR_o = 50$ volts. The figure of 50 volts was arbitrarily chosen as a comfortable point at which to operate an amplifier with a maximum allowable input voltage of 100 volts. Since the 50 volt reference input resistor to A_2 is 10 times the input resistor to A_2 is 10 times the input resistor of e_1, then e_1 should be -5 volts for cancellation.

$$e_3 = -\left[\frac{\pi}{.1} e_1 + \frac{\pi}{1}(50)\right] \quad . \qquad (13)$$

where resistances are in megohms. Substituting the expression for e_1 in Eq. (13) gives

$$e_3 = \left[\pi\left[50 + \frac{50\, I_1}{I_0} \sin(\omega t - \alpha) - 50\right]\right]$$

$$= \frac{50\pi I_1}{I_0} \sin(\omega t - \alpha) \qquad (14)$$

The reason for the factor of π in the A_2 feedback resistor is that subsequent integration introduces a $\frac{1}{\pi}$ factor in the results, and by inserting π in the gain along the way, the $\frac{1}{\pi}$ is cancelled. The factor π was eliminated from the final output in order to simplify the equations used to calculate $\frac{I_1}{I_0}$ and α.

$$e_4 = -e_3 \qquad (15)$$

e_3 and e_4, after commutation, are fed into the integrating amplifiers A_4 and A_5. By proper selection of the commutation scheme, the outputs of A_4 and A_5 are voltages proportional to (time in seconds) x $\frac{I_1}{I_0}$ cos α and (time in seconds) x $\frac{I_1}{I_0}$ sin α respectively.

Figure 6 shows timing of the commutation for the A_4 input. The heavy line, e_5, is obtained by switching alternately between e_3 and $-e_3$ at times when the reactivity sinusoid $\delta k = K_1 \sin \omega t = 0$, i.e. commutation occurs when ωt is an integral multiple of π. Similarly, the A_5 input, shown as the heavy line in Fig. 7, is obtained by commutation of e_3 and $-e_3$ when ωt is an integral multiple of π plus $\frac{\pi}{2}$.

$$e_7 = \frac{-1}{(10^6 \text{ ohms})(10^{-6} \text{ farads})} \int e_5\, dt = -\int e_5\, dt \qquad (16)$$

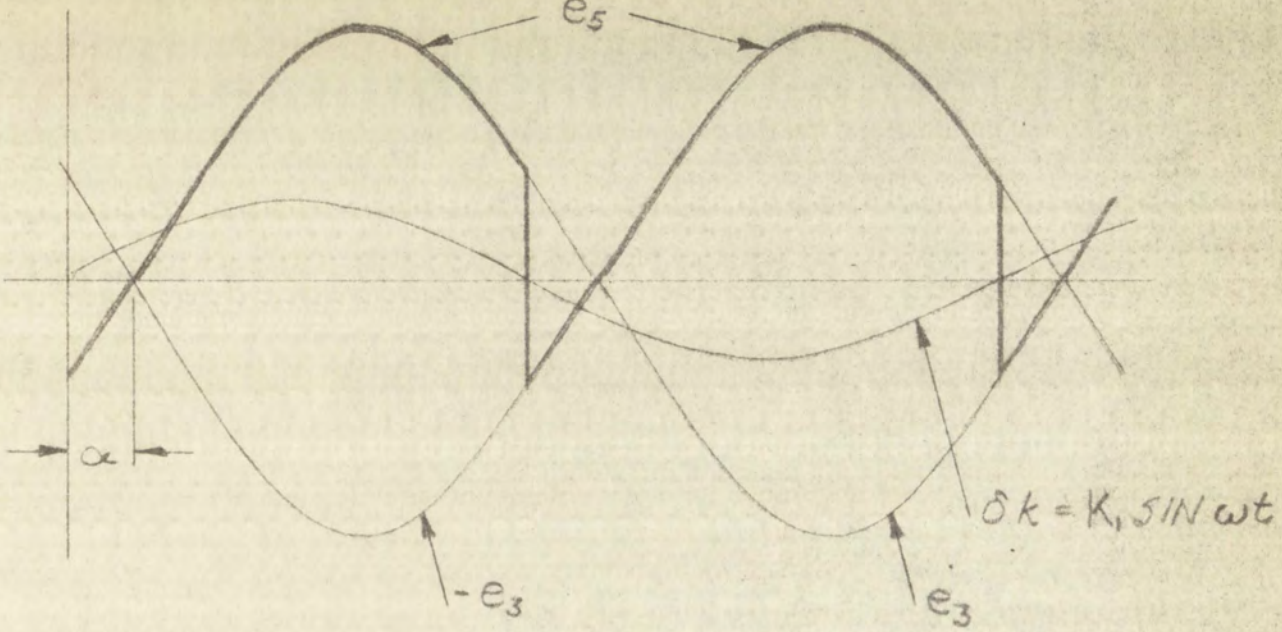

e_5
α
$\delta k = K_1 \sin \omega t$
$-e_3$
e_3

FIG. 6

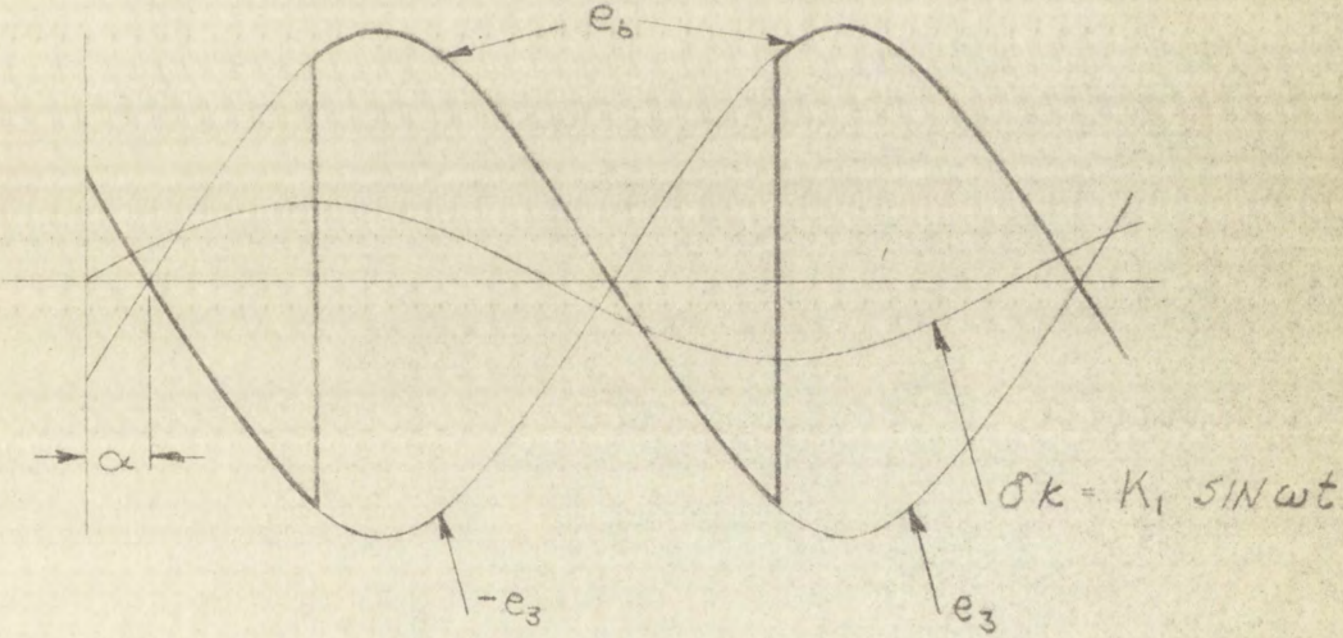

e_6
α
$\delta k = K_1 \sin \omega t$
$-e_3$
e_3

FIG. 7

$$e_8 = \frac{-1}{(10^6 \text{ ohms})(10^{-6} \text{ farads})} \int e_6 \, dt = - \int e_6 \, dt \qquad (17)$$

The constant of integration is represented by voltage across the feedback capacitor and is made zero by shorting the capacitor immediately prior to making a run.

Integration of both e_5 and e_6 is carried out over an integral number of cycles, m, of $K_1 \sin \omega t$. The start-stop circuit connects A_4 and A_5 to e_5 and e_6 respectively at the start of a cycle of $K_1 \sin \omega t$ and disconnects them m cycles later.

$$e_7 = - \int_0^{\frac{2m\pi}{\omega}} e_5 dt = - 2m \int_0^{\frac{\pi}{\omega}} \frac{50\pi I_1}{I_0} \sin (\omega t - \alpha) \, dt$$

$$= - \frac{100\pi m I_1}{I_0} \int_0^{\frac{\pi}{\omega}} (\cos \alpha \sin \omega t - \sin \alpha \cos \omega t) \, dt$$

$$= - \frac{100\pi m I_1}{\omega I_0} \Big[- \cos \alpha \cos \omega t - \sin \alpha \sin \omega t \Big]_0^{\frac{\pi}{\omega}}$$

$$= - \frac{100\pi m I_1}{\omega I_0} \Big[2 \cos \alpha \Big]$$

$$= - 100t \frac{I_1}{I_0} \cos \alpha \qquad (18)$$

where t = time in seconds = $\frac{2\pi m}{\omega}$

In similar fashion,

$$e_8 = -\int_0^{\frac{2m\pi}{\omega}} e_6 \, dt = -\frac{100\pi m I_1}{\omega I_0} - \left[\cos\alpha \cos\omega t - \sin\alpha \sin\omega t\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= -\frac{100\pi m I_1}{\omega I_0} \left[2 \sin\alpha\right]$$

$$= -100\, t \frac{I_1}{I_0} \sin\alpha \qquad (19)$$

From Eqs. (18) and (19)

$$\frac{I_1}{I_0} \cos\alpha = -\frac{e_7}{100t} \qquad (20)$$

$$\frac{I_1}{I_0} \sin\alpha = -\frac{e_8}{100t} \qquad (21)$$

A timer, operated by the start-stop circuit, measures in seconds the time elapsed for m cycles of $K_1 \sin \omega t$. Division of meter readings e_7 and e_8 by 100 t gives the two vector components of $\frac{I_1}{I_0}$ and α.

Assuming, then, that $\delta k = K_1 \sin \omega t$, that the value of K_1 is known and that I_1 and I_0 are proportional to n_1 and N_0, the transfer function has been obtained.

Two methods of commutation were considered, but only one has been tried to date. The method used employed light sources, a slotted disc and photo diodes. Figure 8 is a pictorial representation of the commutation circuit.

Photo diodes A, B, C and D are equally spaced above the edge of a slotted disc. Each diode has a light source below it but the disc masks the diode from the light except during passage of the slot. A and C are connected to one flip-flop circuit; B and D to the other. Each flip-flop

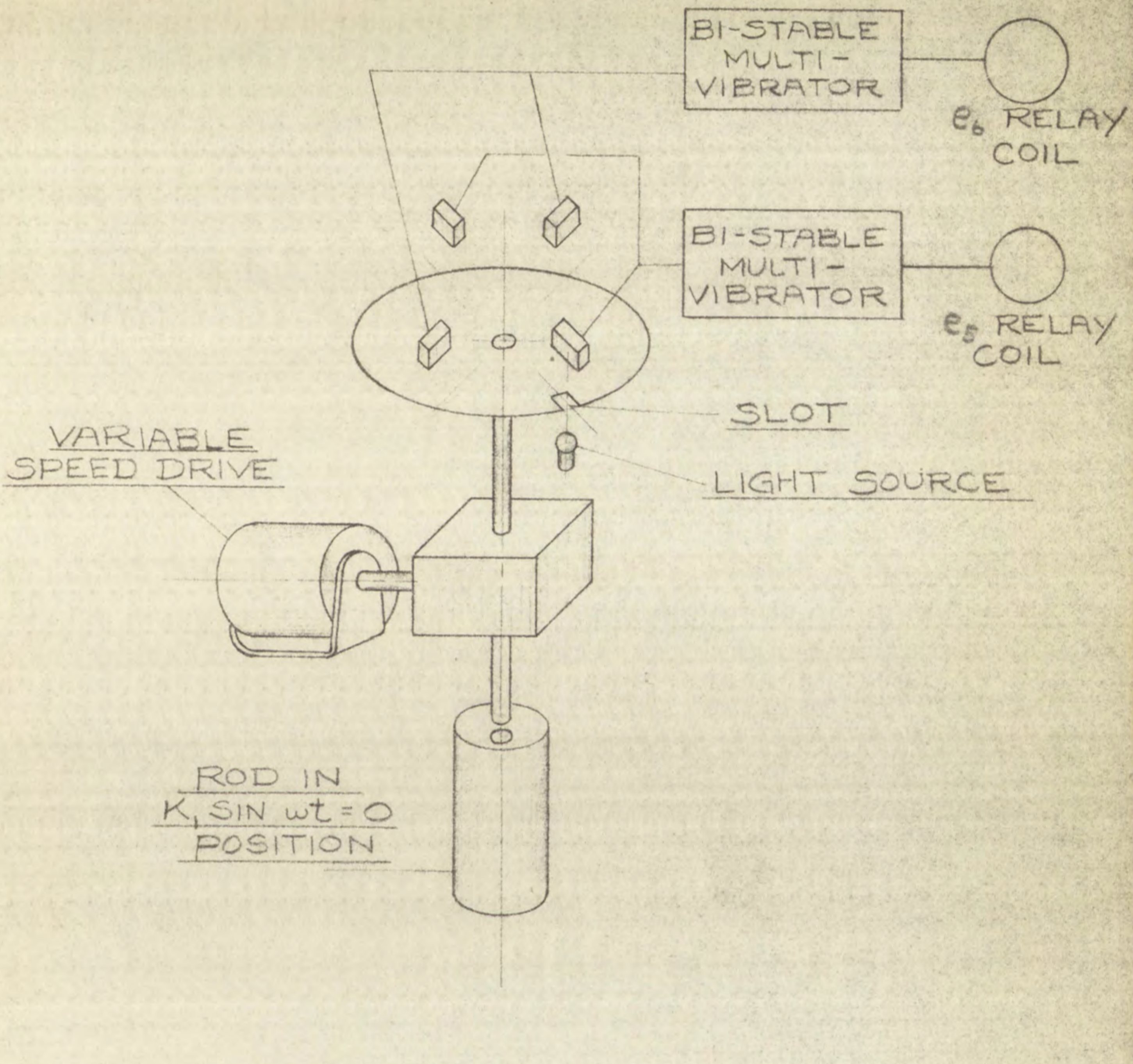

BI-STABLE MULTI-VIBRATOR
e_6 RELAY COIL
BI-STABLE MULTI-VIBRATOR
e_5 RELAY COIL
SLOT
LIGHT SOURCE
VARIABLE SPEED DRIVE
ROD IN $K_1 \sin \omega t = 0$ POSITION

FIG. 8

drives a relay which alternately connects its moving contact to e_3 and $-e_3$. It is necessary to align the timing slot so that A and C produce pulses when $K_1 \sin \omega t = 0$ and B and D produce pulses when $|K_1 \sin \omega t|$ is a maximum.

A second, and probably better, method will use a rotating, brush-type commutator connected to the right angle drive. This type of commutator will eliminate diode and light source alignment problems, the flip-flop circuits, and the relays.

The start-stop circuit is composed of logic elements and operates the integrator input relays. The analyzer operator pushes a start switch when he is ready to make a run, but the relay is not actuated until the circuit receives the next pulse from photo diode A. In like fashion, at the termination of a run, the relay is de-energized by the first pulse from A received after the operator has pushed the stop switch. This scheme is employed to ensure that integration takes place only over an integral number of cycles. Figure 9 is a photograph of the variable speed drive and photo diode timing assembly; Fig. 10 a photograph of the computer circuit cabinet.

The computer circuit diagram is included as Appendix B.

V. Advantages and Limitations

The chief advantage of the analyzer just described is the relative ease and speed with which points of the transfer function curve can be obtained. Maximum integration time is 100 seconds -- the period of a .01 cps oscillation.

Precise adjustment of R_o for cancelling out the I_o component of i_n is not necessary since integration of a plus signal for half a cycle followed by integration of a minus signal for the next half cycle cancels

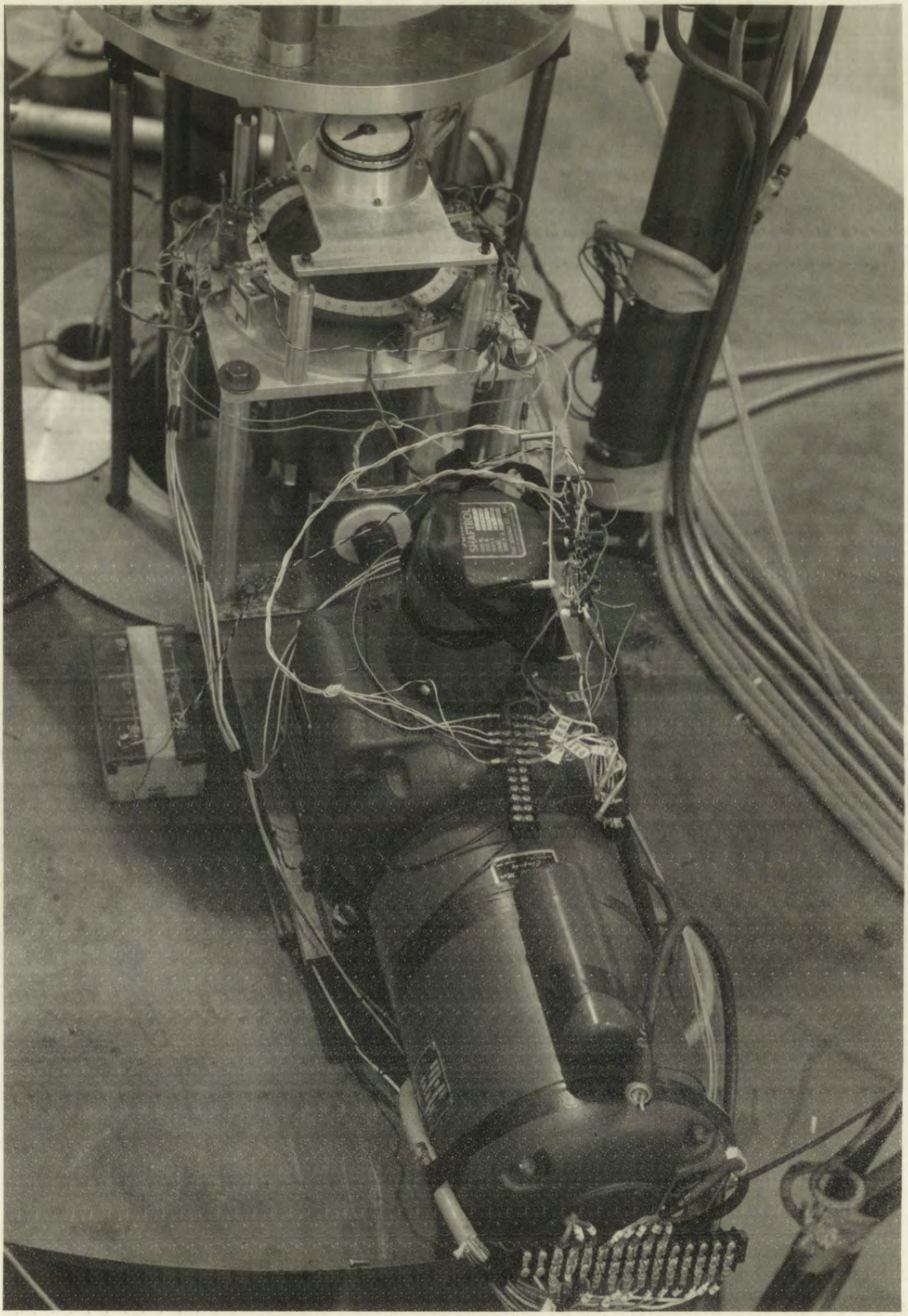


Fig. 9.

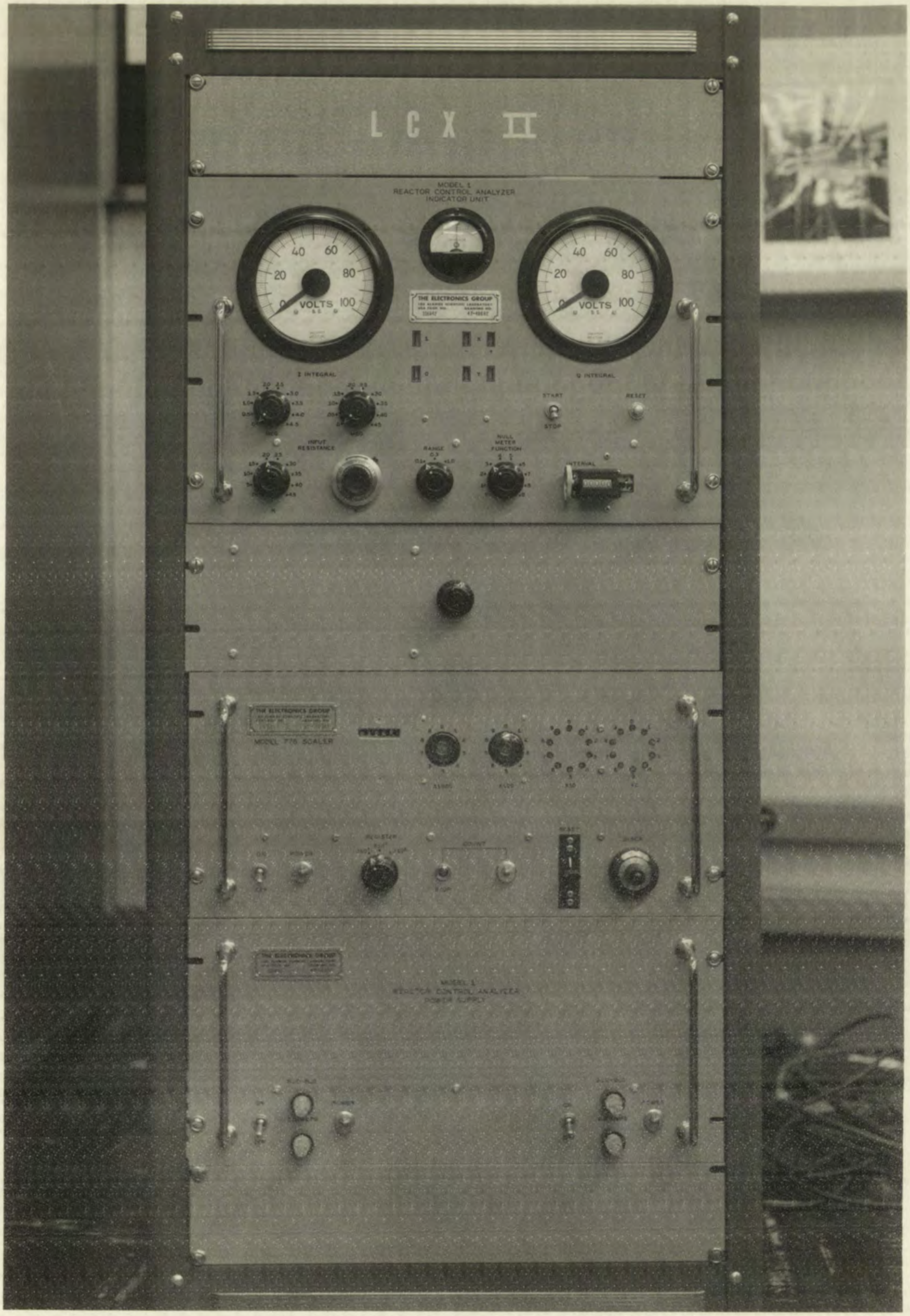

LCX II
MODEL 1
REACTOR CONTROL ANALYZER
INDICATOR UNIT
VOLTS
THE ELECTRONICS GROUP
VOLTS
I INTEGRAL
Q INTEGRAL
START
STOP
RESET
INPUT
RESISTANCE
RANGE
NULL
METER
FUNCTION
THE ELECTRONICS GROUP
THE ELECTRONICS GROUP

Fig. 10.

any d.c. component. For the same reason, variation of I_0 during a run is cancelled to a first order approximation. Random fluctuations in reactor power, i.e. "noise", does not affect the output appreciably because integration will eliminate random components in the signal i_n. There is the slight probability that the noise during a particular run is similar in character to the ω frequency and this correlation would cause error.

The possibility of harmonics in the reactivity input has been mentioned. Such harmonics will show up in the i_n expression and, if large enough, make analysis of the results difficult. Integration eliminates even harmonics from the signal since integration of sin $2\omega t$, sin $4\omega t$, etc., over a half period of the fundamental frequency yields zero. Odd harmonics could contribute to error, but experimental results indicate that they are not present to a bothersome degree with the type of oscillator rod used.

The use of an operational amplifier on the output of the ionization current chamber results in negligible phase shift due to chamber capacitance. Since the input of an operational amplifier is virtually at ground potential, the R of the RC product is just the resistance of the connecting cable. There is no large input resistor in the amplifier as is usual with many d.c. amplifiers.

There are several limitationsinherent in the analyzer. In any mechanically oscillated system, the range of frequencies is limited at the upper end by bearing and vibration difficulties. With a dynamically balanced rod, frequencies of 100 cps or above should be attainable. The balancing problem can be solved in some applications by using a rod which introduces two cycles of reactivity disturbance for one mechanical

revolution; for instance, a round rod with diametrically opposite milled flats.

The operation of the circuitry will accommodate, with one exception, frequencies far in excess of those attainable with the mechanical drive. The exception is the commutating relay. The relay used has about a one millisecond dead time during switching and can, at 10 cps, introduce about 3% error for some α's. Use of brush-type commutators will reduce error from this source. A discussion or error caused by switching dead time appears in Appendix A.

The operational amplifiers, provided they are of quality design and manufacture, will give rise to no appreciable error due to moderate changes in vacuum tube characteristics. Feedback and input resistors as well as feedback capacitors can, however, cause error if they are unstable or have large temperature coefficients. Capacitor leakage can be another source of error. High quality components are available which are satisfactory.

VI. Experimental Results

Operation of the computer circuit was checked by connecting a sine potentiometer to the right angle drive and substituting the output of the potentiometer for the ionization chamber current at the A_1 input. Several angles set into the potentiometer over the range of 0 to 90° were computed by the circuit with an error or less than 1/2°. The test runs were made at several frequencies between .02 and 3 cps.

The analyzer was tested on a critical assembly at the Pajarito site of the Los Alamos Scientific Laboratory. The assembly, LCX II, was a mockup of a reactor under development by the Laboratory's K-Division.

Conclusions and Recommend...

...

The operation...

...

analyzer and ...

% error for ...

from this source...

appears in Appendix ...

The operational...

and manufacturer...

changes in ambient...

will be ...

or have large temperature...

source of error. ...

theory.

VI. <u>Experimental Results</u>

Operation of the ...

potentiometer to the ...

potentiometer for the ...

engine was into the ...

by the ...

several ...

The analyzer was ...

of the ...

vapor of ...

The fissionable material was in the form of plutonium-iron alloy discs. Sandwiched between the fuel discs were discs of tantalum and aluminum to simulate core structure and coolant. It was a fast assembly with a calculated mean prompt neutron lifetime of 10^{-8} seconds. Surrounding the core was a stationary reflector and outside the stationary reflector was a movable iron annular reflector used for reactivity control. Rods moving independently in holes in the annular reflector provided vernier reactivity control. One of these rods was replaced by an oscillator rod and the assembly's transfer function was measured from .025 to 2.8 cps. Mechanical difficulties have prevented operation beyond this range to date. Figure 11 shows geometry of the oscillator rod.

The reactivity versus angular position of the rod was measured and the results are plotted in Fig. 12. It is seen that the reactivity input is appreciably different from a sinusoid. The addition of 18% second harmonic to a fundamental gives a curve which closely fits that of Fig. 12. Comparison of test data with theory would indicate that the second harmonic probably had little influence on results. Harmonics higher than the second did not seem to be present.

Figure 13 is a plot of the experimental data and also the curve computed from theory by Keepin and Wimett.[11]

VII. Conclusion

The analyzer behaved as predicted and the taking of data from which a reactor transfer function could be calculated was easily and quickly accomplished. It is believed that the computer is an advance over previous methods of determining reactor transfer functions.

The fluctuation ...

eliminated because ...

be complete ...

eliminated both groups of ...

more than a millisecond ...

a circuit from ...

independently in relation to ...

control. One of three ...

assembly a transfer ...

difficulties have prevented ...

II shows recovery ...

The analytic method ...

and the results are given in ...

input is appreciably diff...

second harmonic as a function ...

of Fig. 12. Comparison of ...

second harmonic probably ...

higher than the second ...

Figure 13 is a plot ...

computed from theory.

VII. Conclusion

The analytic method ...

which a reactor transfer function ...

entirely accomplished. ...

over previous methods of ...

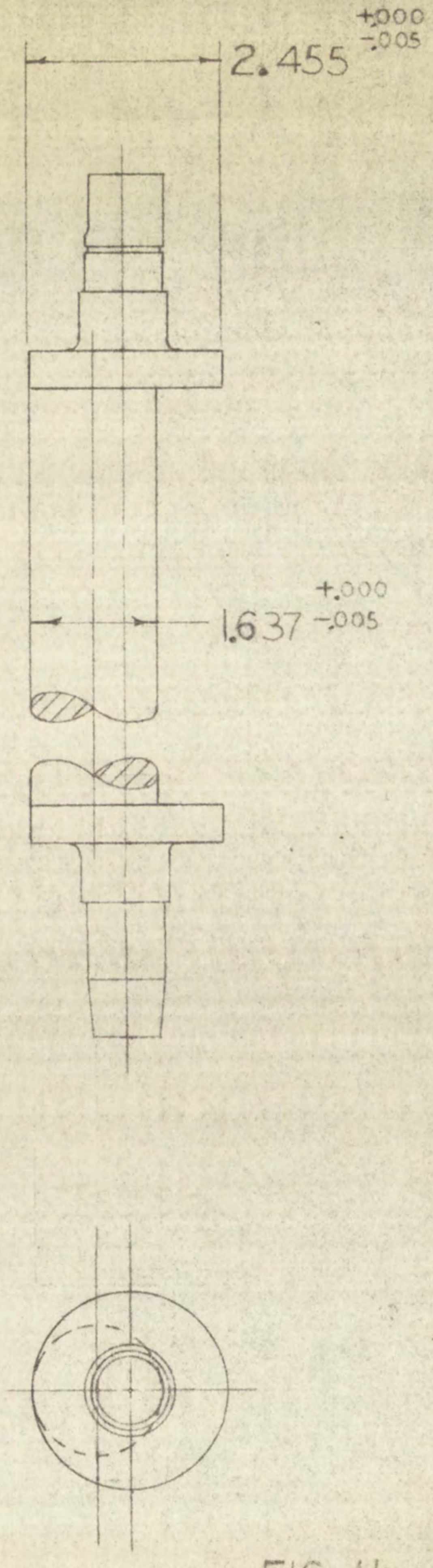


FIG. 11

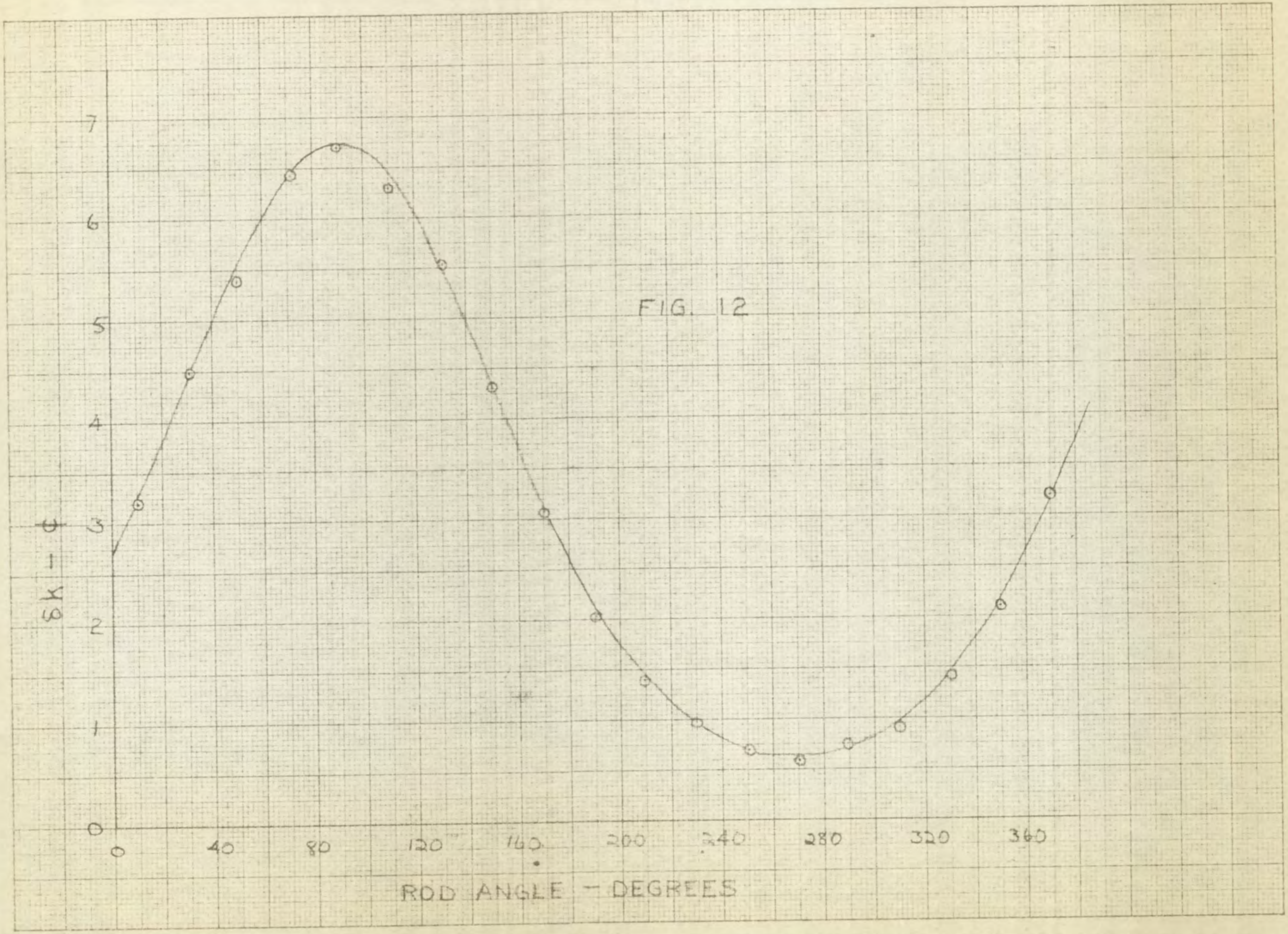


FIG. 12

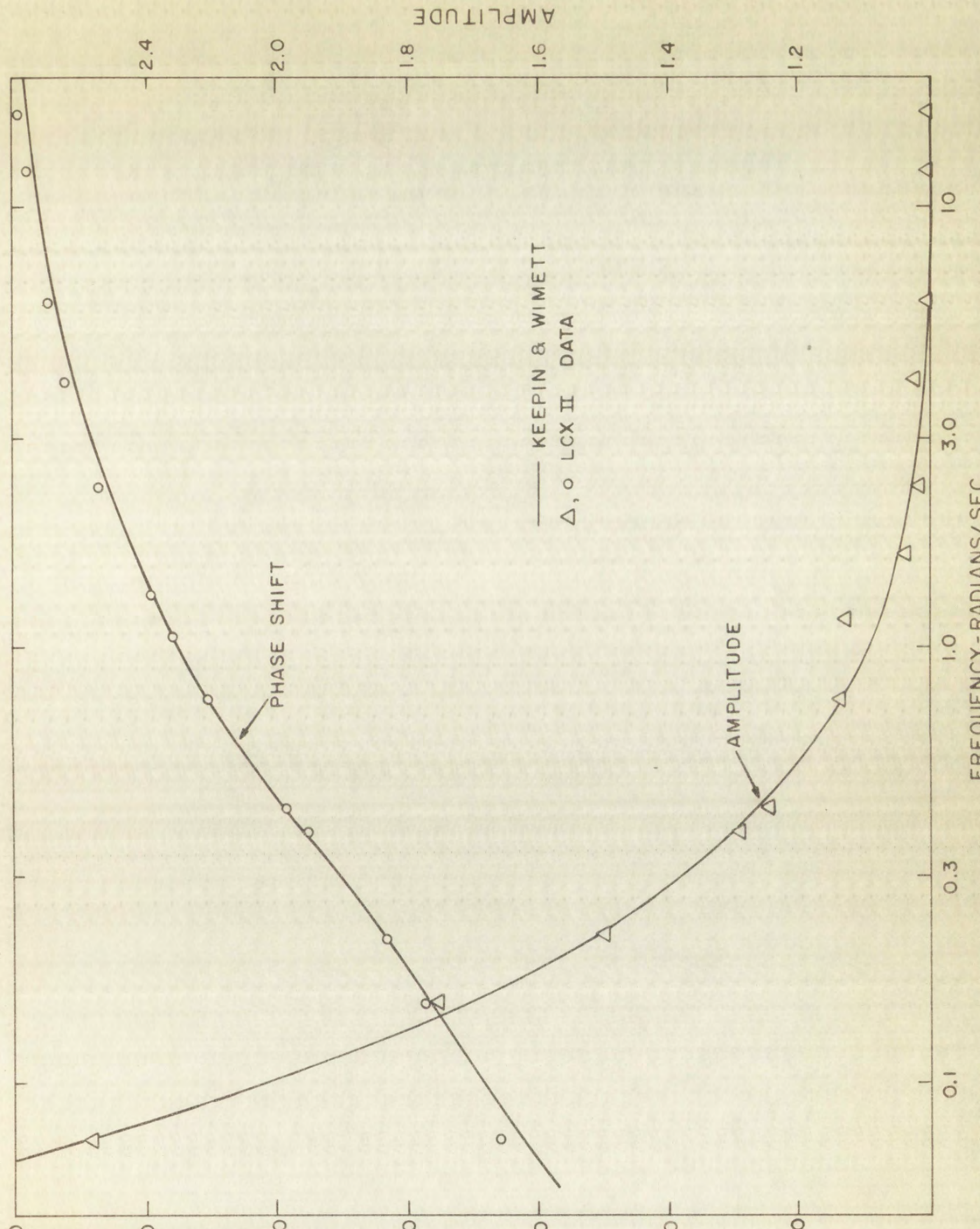
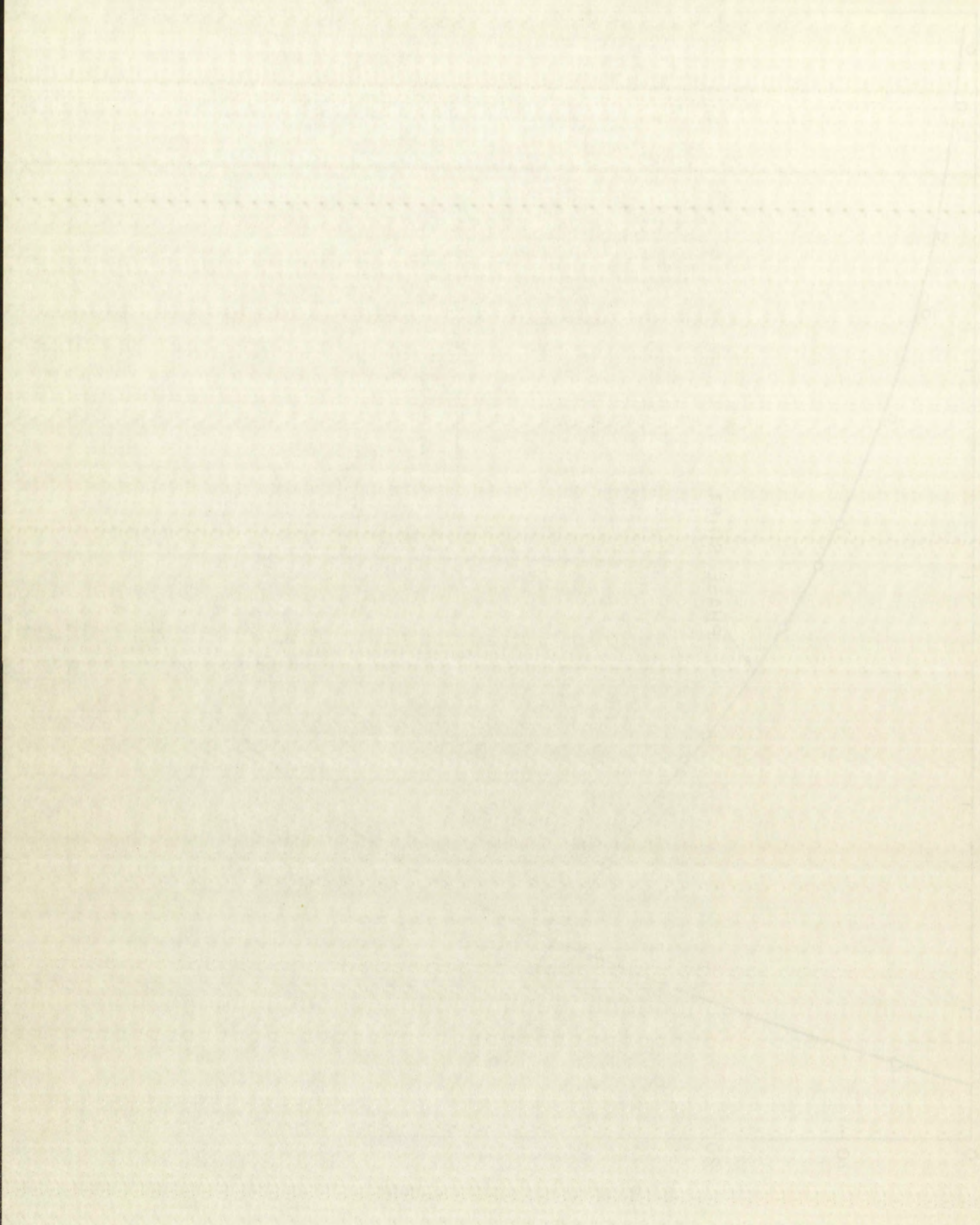

AMPLITUDE
2.4
2.0
1.8
1.6
1.4
1.2
— KEEPIN & WIMETT
Δ, ○ LCX II DATA
PHASE SHIFT
AMPLITUDE
FREQUENCY - RADIANS/SEC
0.1
0.3
1.0
3.0
10
PHASE SHIFT - DEGREES
0
10
20
30
40
50
60

FIG. 13



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The [illegible] is [illegible] to [illegible]

analyzer. [illegible] who designed and [illegible]

of the photo[illegible] system and the [illegible]

[illegible] functions [illegible]

[illegible]

[illegible]

deflected the oscillator rod, [illegible]

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Appendix A

The relays used for commutation have a switching dead-time of about one millisecond. During this millisecond the integrator input is connected to neither e_3 nor $-e_3$. This situation introduces an error in both the amplitude and phase figures. For a constant dead-time it is apparent that the error increases with frequency because the dead-time becomes a larger proportion of the period. The error resulting from relay switching is calculated below.

Figure 14 shows the commutation of the A_4 input and includes a dead-time of t_s seconds. e_7 will be derived to include t_s.

$$e_7 = - \frac{100\pi m I_1}{i_o} \int_{t_s}^{\frac{\pi}{\omega}} (\cos\alpha \sin\omega t - \sin\alpha \cos\omega t)\, dt$$

$$= - \frac{100\pi m I_1}{\omega I_o} \Big[- \cos\alpha\ \omega t - \sin\alpha \sin\omega t \Big]_{t_s}^{\pi}$$

$$= - \frac{100\pi m I_1}{\omega I_o} \Big[- \cos\alpha\, (-1 - \cos\omega t_s) - \sin\alpha \sin\omega t_s \Big]$$

$$= - \frac{100\pi m I_1}{\omega I_o} \Big[\cos\alpha\, (1 + \cos\omega t_s) - \sin\alpha \sin\omega t_s \Big] \qquad (22)$$

similarly,

$$e_8 = \frac{100\pi m i}{\omega i_o} \Big[\sin\alpha\, (1 + \cos\omega t_s) - \cos\alpha \sin\omega t_s \Big] \qquad (23)$$

Since it is only the bracketed expression that contains the terms causing error, define e_7' and e_8' as

[illegible]

[illegible]

shows one millisecond. During this millisecond, the integrator input is connected to [illegible] as [illegible]. This [illegible] introduces an error in both the amplitude and phase figures. For a constant dead-time it is apparent that the error increases with frequency because the dead time becomes a larger proportion of the period. The error [illegible] from the [illegible] is calculated below.

Figure 13 shows the calculation of the e_n terms and indicates a dead time of 5 seconds. e_n will be derived to include

[illegible]

[illegible]

[illegible]

[illegible]

similarly,

[illegible]

Since it is only the [illegible] that concerns the [illegible], half of e_n and e_s [illegible]

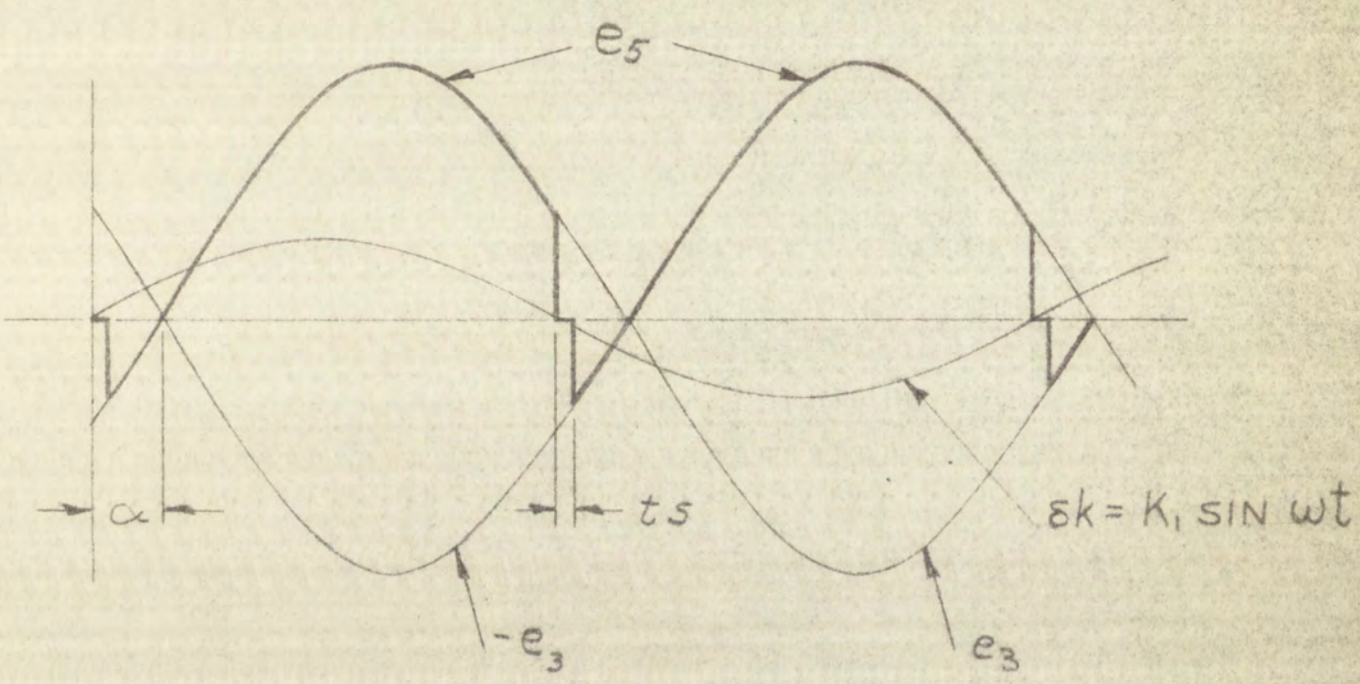

FIG. 14

$$e_7' = \cos\alpha\,(1 + \cos\omega t_s) - \sin\alpha\,\sin\omega t_s \tag{24}$$

$$e_8' = \sin\alpha\,(1 + \cos\omega t_s) - \cos\alpha\,\sin\omega t_s \tag{25}$$

For a frequency of 10 cps and a dead-time of .001 second, Eqs. (24) and (25) become

$$e_7' = \cos\alpha\,(1 + \cos .0628) - \sin\alpha\,\sin .0628$$

$$= 1.998\cos\alpha - .0628\sin\alpha \tag{26}$$

$$e_8' = 1.998\sin\alpha - .0628\cos\alpha \tag{27}$$

Tabulated below are errors for several α's resulting from the use of relays for commutation. Equations (26) and (27) were used to calculate the values in the table

True α	$\tan^{-1}\left(\frac{e_8'}{e_7'}\right)$	$\sqrt{(e_7')^2 + (e_8')^2}$	% Amplitude Error
0°	- 1.80	1.998	- 0.1
10	8.2	1.979	- 1.0
20	18.6	1.960	- 2.0
30	29.0	1.942	- 2.9
40	39.7	1.938	- 3.1
50	50.3	1.938	- 3.1
60	61.0	1.942	- 2.9
70	71.4	1.960	- 2.0
80	81.8	1.979	- 1.0
90	91.8	1.998	- 0.1

For values of frequency less than 10 cps, the angle and amplitude errors are proportionately reduced, e.g. they are one-half of those in the table for 5 cps. For switching times not greatly different from 1 millisecond, error is proportional to switching time.

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