Winter 2-19-2018

Novel Algorithms for Ultra Scale Electromagnetic Problems in the Supercomputing Era

Brian MacKie-Mason
University of New Mexico

Follow this and additional works at: https://digitalrepository.unm.edu/ece_etds

Part of the Electrical and Computer Engineering Commons

Recommended Citation
https://digitalrepository.unm.edu/ece_etds/410

This Dissertation is brought to you for free and open access by the Engineering ETDs at UNM Digital Repository. It has been accepted for inclusion in Electrical and Computer Engineering ETDs by an authorized administrator of UNM Digital Repository. For more information, please contact disc@unm.edu.
Brian MacKie-Mason

Candidate

Electrical and Computer Engineering

Department

This dissertation is approved, and it is acceptable in quality and form for publication:  Approved

by the Dissertation Committee:

Professor Zhen Peng, Chair

Professor Daniel Appelö, Member

Dean Christos Christodoulou, Member

Doctor Yang Shao, Member
Novel Algorithms for Ultra Scale Electromagnetic Problems in the Supercomputing Era

by

Brian MacKie-Mason

B.S.E., Nuclear Engineering, University of Michigan, 2011
M.S., Nuclear Engineering, University of Wisconsin, 2013

DISSERTATION

Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy Engineering

The University of New Mexico
Albuquerque, New Mexico

May 2018
Dedication

This dissertation is dedicated to the memory of Daniel A. Polovich.
Acknowledgments

I would like to thank my research advisor, Zhen Peng, for accepting me as his first PhD student. The opportunity he gave me to do my doctoral studies with him was truly a blessing and I couldn’t have asked for a better and more supportive mentor-mentee relationship while completing such a rigorous educational experience. His attention to detail in my growth is something that I will remember forever.

This dissertation could not have come to fruition without the help of many others involved in my career. I would like to thank my other committee members, Daniel Appelö, Christos Christodoulou, and Yang Shao, for taking their time to offer suggestions on my research. I also would not have been able to develop and test many of the algorithms in this dissertation without the modeling assistance provided by Yang Shao. I would also like to thank Andrew Greenwood, Peter Mardahl, Nathaniel Lockwood, and Wilkin Tang for introducing me to the field of computational electromagnetics. The warm welcome I received to applied electromagnetics at UNM from Edl Schamiloglu and Mark Gilmore was also very important in choosing UNM for my doctoral studies.

I have enjoyed countless discussions on all matters CEM, antennas, electromagnetic radiation, and applied electromagnetics with my fellow students in the CEM Lab and those in other Applied Electromagnetics research groups.

Year 1 of my PhD was financially supported by a departmental graduate assistantship and the final 3+ years were supported by research assistantships through a DoD HPCMP PETTT Program grant.
Novel Algorithms for Ultra Scale Electromagnetic Problems in the Supercomputing Era

by

Brian MacKie-Mason

B.S.E., Nuclear Engineering, University of Michigan, 2011
M.S., Nuclear Engineering, University of Wisconsin, 2013
Ph.D., Electrical Engineering, University of New Mexico, 2018

Abstract

The need for increasing fidelity and accuracy in mission-critical electromagnetic applications have pushed the problem sizes toward extreme computational scales. Therefore, there is a large premium placed on the investigation of parallel and scalable integral equation simulators, a particularly useful class of computational electromagnetics, to meet this demand. This dissertation will focus on three interrelated areas: i) quasi-optimal, well-conditioned integral equation based domain decomposition methods, ii) geometry-adaptive, multi-scale discontinuous Galerkin boundary element methods, and iii) high-performance and scalable algorithms to reduce the computational complexity of extreme-scale simulations with the aid of parallel computing architectures.
This dissertation will first develop the mathematical underpinnings of a geometry-adaptive, integral equation domain decomposition method. Then the parallelization technique will be discussed along with many numerical experiments to profile its performance. Finally, two real-world applications will be discussed that demonstrate the utility of a high performance-enabled solution. First, a solution to the challenging problem of the solution and prototyping of antennas on large platforms will be discussed. Finally, the code will be used to solve a channel modeling problem that has many important applications for the fifth generation (5G) of wireless telecommunication.
Contents

List of Figures xi

List of Tables xv

1 Introduction 1

1.1 Why is CEM important? 1

1.2 Challenges addressed 3

1.3 Example problems 3

1.4 Research Contributions 5

1.5 Outline 6

2 Notation 7

3 Geometry aware integral equation domain decomposition method 10
Contents

3.1 Theory & Formulation ........................................ 10
   3.1.1 Model problem ......................................... 10
   3.1.2 Discontinuous Galerkin formulation .................. 11
   3.1.3 Domain decomposition solver ......................... 14

3.2 Benchmarking: Numerical Study .......................... 16
   3.2.1 Eigenspectrum ..................................... 16
   3.2.2 Convergence ....................................... 17
   3.2.3 Accuracy .......................................... 19

4 HPC-enabled integral equation domain decomposition 21
   4.1 Scalable parallelism .................................... 21
      4.1.1 Pre-processing .................................. 22
      4.1.2 Parallel Computation .......................... 24
      4.1.3 Post-processing ................................ 28
   4.2 Benchmarking: Performance Study ..................... 28
      4.2.1 Weak scalability ............................... 29
      4.2.2 Strong scalability ............................. 32
      4.2.3 Parallel computing challenges ................. 33
List of Figures

1.1 High-definition model of an aircraft carrier. ........................................ 4
1.2 High-definition model of a city block in downtown Albuquerque. ... 5
3.1 Electromagnetic planewave scattering from a PEC object. ............ 11
3.2 Domain decomposition of Figure 6.1 into three sub-domains. ....... 12
3.3 Eigenspectrum for a PEC sphere. .......................................................... 17
3.4 Iterative solver convergence with respect to operating frequency. .. 18
3.5 Accuracy of PEC sphere with respect to operating frequency. ....... 19
3.6 Bistatic Radar Cross Section of a PEC sphere at 16 GHz. ............ 20
4.1 A framework for scalable parallelism of GA-IE-DDM. ................. 22
4.2 Component-based domain decomposition of an aircraft carrier. ... 23
4.3 Computational domain decomposition of an aircraft carrier. ......... 24
4.4 Distance-based separable radiation coupling. ................................. 27
List of Figures

4.5 Domain partitioning for F16 aircraft. .................................. 30
4.6 Parallelization efficiency for F16 scattering problems. ............. 31
4.7 Electric surface current for F16 scattering problems. .................. 31
4.8 Strong scaling for F16 scattering problems. ............................. 32
4.9 Dimensions of a full-scale aircraft carrier. .............................. 33
4.10 Geometry-based decomposition of an aircraft carrier. ................. 34
4.11 Computational decomposition of an aircraft carrier. ................... 35
4.12 Electric surface current on aircraft carrier at $f = 400 MHz$. ........ 36
4.13 Plot of time spent in each of three major tasks: i) serialized tasks,
    ii) sub-domain solver, iii) sub-domain coupling. ....................... 37

5.1 Example of a real-world wireless channel environment in downtown
    Albuquerque, NM. ......................................................... 41
5.3 Description of the two-ray model. ...................................... 48
5.4 Reference Figure for LOS power predictions [72]. ....................... 50
5.5 Comparison of the computational result and the semi-analytical 2-
    ray model. ................................................................. 51
5.6 The computational setup of the validation example. ..................... 54
5.7 The electric field distribution. .......................................... 55
5.8 Comparison of the simulation result and the measurement data. ...... 55
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.9</td>
<td>Example of a real-world wireless channel environment: downtown, Albuquerque, NM.</td>
<td>57</td>
</tr>
<tr>
<td>5.10</td>
<td>Geometry partitioning of Albuquerque downtown at 400 MHz.</td>
<td>58</td>
</tr>
<tr>
<td>5.11</td>
<td>Antennas in their environments.</td>
<td>59</td>
</tr>
<tr>
<td>5.12</td>
<td>Graphical depiction of the separation of boundary conditions.</td>
<td>60</td>
</tr>
<tr>
<td>5.13</td>
<td>Electric surface current at $f = 400 \text{MHz}$.</td>
<td>61</td>
</tr>
<tr>
<td>5.14</td>
<td>Zoomed in view of electric surface current.</td>
<td>62</td>
</tr>
<tr>
<td>5.15</td>
<td>Convergence of ABQ city block problem at $f = 400 \text{MHz}$.</td>
<td>63</td>
</tr>
<tr>
<td>6.1</td>
<td>Simplified FEBI antenna model.</td>
<td>66</td>
</tr>
<tr>
<td>6.2</td>
<td>FEBI antenna model with traces.</td>
<td>67</td>
</tr>
<tr>
<td>6.3</td>
<td>Simplified FEBI antenna model on PEC platform.</td>
<td>69</td>
</tr>
<tr>
<td>6.4</td>
<td>M Antennas on 1 PEC platform.</td>
<td>73</td>
</tr>
<tr>
<td>6.5</td>
<td>1 Antenna on N PEC platforms.</td>
<td>74</td>
</tr>
<tr>
<td>6.6</td>
<td>M Antennas on N PEC platforms.</td>
<td>76</td>
</tr>
<tr>
<td>6.7</td>
<td>Schur complement algorithm.</td>
<td>81</td>
</tr>
<tr>
<td>6.8</td>
<td>Antenna models used for validation of Schur complement technique.</td>
<td>83</td>
</tr>
<tr>
<td>6.9</td>
<td>Two antennas on PEC platform with different solution techniques.</td>
<td>85</td>
</tr>
<tr>
<td>6.10</td>
<td>Skeletonization example</td>
<td>86</td>
</tr>
</tbody>
</table>
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.11</td>
<td>75MHz antenna on aircraft carrier</td>
<td>86</td>
</tr>
<tr>
<td>6.12</td>
<td>Effect of skeletonization on S11 parameters.</td>
<td>87</td>
</tr>
<tr>
<td>8.1</td>
<td>Computational domain decomposition of an aircraft carrier</td>
<td>93</td>
</tr>
<tr>
<td>8.2</td>
<td>FAFFA numerical experiment setup.</td>
<td>95</td>
</tr>
<tr>
<td>8.3</td>
<td>FAFFA surface current results; $f = 300 MHz$.</td>
<td>96</td>
</tr>
<tr>
<td>8.4</td>
<td>Error for FAFFA</td>
<td>96</td>
</tr>
<tr>
<td>8.5</td>
<td>FAFFA Criteria Study</td>
<td>97</td>
</tr>
</tbody>
</table>
# List of Tables

4.1 Computational Statistics for scaling experiments of EM scattering from a full-scale F16. ........................................... 30

5.1 Physical parameters for ground. ........................................... 49

5.2 Antenna heights in downtown Albuquerque model. ............... 57

5.3 $S_{11}$ parameters for antennas in downtown Albuquerque. ....... 60

5.4 Pathloss for the receiving antennas in downtown Albuquerque. ... 61

6.1 $S_{11}$ parameters for single antenna on PEC plate. .................. 84

6.2 $S_{11}$ parameters for single antenna on PEC plate. .................. 85

6.3 $S_{11}$ parameters for single antenna on aircraft carrier platform; f = 75MHz. ................................................................. 87

B.1 Principle of Duality Substitutions. ..................................... 108
Chapter 1

Introduction

1.1 Why is CEM important?

The solution to the equations of electricity and magnetism dates back to 1865 [1]. The study of computational electromagnetics (CEM) began about one century later with the development of the Yee cell [2], primarily for the finite-difference time-domain, and later the development of RWG functions [3], which is primarily for integral equation methods. Surface integral equation (SIE) methods have become very popular for the solution of large, complex geometries especially for electromagnetic scattering [4]. Concurrently with the development of high performance computing systems, the development of fast algorithms [5] and iterative solvers [6] have provided electrical engineers with increasingly powerful tools for the modeling, simulation, and design of a variety of systems and applications.
Chapter 1. Introduction

The finite element method (FEM) is used in computational electromagnetics due to its high level of accuracy and direct representation of the model under design [7]. Unfortunately, the degrees of freedom (DOFs) in FEM are prohibitive to the solution of extremely large problems. The surface integral equation is used to solve electromagnetic problems where the unknowns reside entirely on the surface of a three-dimensional object; this occurs when the object under study is a perfect electric conductor (PEC) due to the surface equivalence theorem [8]. The surface of this object can be discretized into a mesh which results in a low number of unknowns for a given problem size. The matrix, however, is dense, and many fast algorithms have been used to speed up its solution [9–12], which reduce a $O(N^2)$ calculation to $O(N\log N)$.

The use of domain decomposition methods [13] in CEM has led to a set of algorithms well-suited to solve increasingly complex problems. The surface integral equation method coupled with domain decomposition techniques has been used to solve problems in electromagnetic scattering from periodic structures [14], radar communications [15], electromagnetic compatibility and interference [16], and in bioelectromagnetic analysis [17]. The largest electromagnetic scattering problem solved to date is reported as 2.5 billion degrees-of-freedom [18]. Some of the earliest problems solved in CEM involved the scattering of electromagnetic radiation from a sphere of about one thousand DOFs. In addition to the size and scope of problems that can be solved, some additional improvements to the field of CEM involve the introduction of other fields of study [19], and improved discretization strategies [20].
Chapter 1. Introduction

1.2 Challenges addressed

As the power of supercomputing is growing constantly, so is the need to adapt to such a changing environment. Progress in many fields of fundamental science and engineering will rely upon on the ability to perform advanced simulations and to process and analyze the massive amount of resulting data. The U.S. DOE issued a report in 2014 that addressed the ten major research challenges facing the scientific community in the supercomputing era [21], which include improved computing and networking technologies, better scalable software and algorithms, and more scientific productivity. At both the U.S. Departments of Energy [22] and Defense [23] this need for high performance computing solutions to challenging engineering problems has been recognized and is an active area of research. This work will address one of those challenges: the design of exascale algorithms. Algorithms will be designed to meet a few key objectives, rapid, flexible, and compatible with extreme-scale computing. In this dissertation, not only will algorithms be used to achieve more advanced computational capabilities, but problems will also be viewed from different perspectives than before. All computational algorithms will be demonstrated on real-world science and engineering applications.

1.3 Example problems

In this dissertation we will address both the development of the underlying mathematics as well as demonstrate its capabilities in a few different real-world applications. The aircraft carrier shown in Figure 1.1 is an example of the complicated geometries that the methods described in this work are suited for. This problem will
Chapter 1. Introduction

be studied for electromagnetic planewave scattering as well as antenna radiation.

![High-definition model of an aircraft carrier.](image)

**Figure 1.1:** High-definition model of an aircraft carrier.

Not only is the electrical size of the carrier at an extremely high level, but the multi-scale nature of the carrier necessitates different mesh densities in different regions of the carrier. The solution of any real-world problems, such as antenna optimization, EMC/EMI, and other electromagnetic engineering design challenges on such a complex geometry require first the solution of the unknown electric surface currents when a plane wave is incident upon the platform. In order to do so, a high-fidelity discretization must be used to maintain the geometrical complexity, a variety of fast algorithms are employed, and scalable parallelism is harnessed to take advantage of advanced high performance computing environments.

Another problem discussed in this dissertation will be that of millimeter wave channel modeling in dense, urban environments. Figure 1.2 shows a downtown city block of Albuquerque, NM that is used to study this phenomena.

This study will involve the inclusion of material parameters for the ground and
Figure 1.2: High-definition model of a city block in downtown Albuquerque.

the buildings. Though this model is greatly simplified it is aiming to be the first of its kind as a full-wave simulation of communication channel modeling.

1.4 Research Contributions

This dissertation advances the field of computational electromagnetics in three important ways. First, we develop a novel discontinuous Galerkin weak formulation for Maxwell’s equations in integral form. Second, we present a one-level, non-overlapping Schwarz preconditioner domain decomposition method that leads to superior convergence properties. Finally, we develop several software packages involving different hybrid solvers that have high utility in analysis- and design-centered engineering applications. These ingredients lead to a framework that can solve the largest real-world computational electromagnetics problem with greater accuracy, fewer iterations, and a faster solution time. These research contributions have been
Chapter 1. Introduction

disseminated through journal publications, conference presentations, and technical reports submitted to funding agencies.

1.5 Outline

This dissertation is organized as follows. Chapter 2 provides a basic set of notation used throughout the dissertation. Chapter 3 will discuss the geometry-adaptive integral equation domain decomposition method used, and then Chapter 4 will present the parallel implementation of that algorithm. In Chapters 5 and 6 we dive into two real-world applications: communication channel modeling and rapid antenna prototyping. Finally, Chapter 7 will summarize the completed work. Chapter 8 provides a summary of the future direction of this work.
Chapter 2

Notation

This brief chapter outlines the notation used throughout this dissertation. Matrices, $\mathbf{A}$, and vectors, $\mathbf{x}$, will both be defined in boldface throughout. Superscripts will often be used to denote the physical significance of the variable, $\mathbf{E}^{\text{inc}}$ for the incident electric field, and subscripts are used for bookkeeping purposes, $j_m$, $i = 1, M$. A generic volume is denoted by $\Omega$.

A generic surface is given by $\mathcal{S}$. The outward unit normal vector to a surface is given by $\mathbf{\hat{n}}$. The tangential trace operator (2.1) and the twisted tangential trace operator (2.2) are used to simplify many of the formulations used.

\begin{align}
\pi_\tau (f) & := \mathbf{\hat{n}} \times (f \times \mathbf{\hat{n}}) |_{\mathcal{S}}. \\
\pi_\times (f) & := \mathbf{\hat{n}} \times \pi_\tau (f).
\end{align}

The incident surface electric field and incident electric surface current are defined
Chapter 2. Notation

by Equations 2.3 and 2.4, respectively. The scaled surface current on the scatterer is defined in Equation 2.5.

\[
e^{\text{inc}} := \pi_\tau \left( E^{\text{inc}} \right).
\]

(2.3)

\[
j^{\text{inc}} := \eta_0 \pi_x \left( H^{\text{inc}} \right).
\]

(2.4)

\[
j := -\frac{1}{ik_0} \pi_x (\nabla \times E).
\]

(2.5)

Two integral operators are defined by 2.6 and 2.7 on the surface of the scatterer.

\[
\mathcal{L} (f; S) (r) := -ik_0 \Psi_A (f; S) (r) + \frac{1}{ik_0} \nabla \Psi_F (\nabla_\tau \cdot f; S) (r).
\]

(2.6)

\[
\mathcal{K} (f; S) (r) := \nabla \times \Psi_A (f; S) (r).
\]

(2.7)

Where the single- and double-layer potential integrals are given in Equations 2.8 and 2.9.

\[
\Psi_A (f; S) (r) := \int_S f (r') G (r, r') \, dr'.
\]

(2.8)

\[
\Psi_F (\rho; S) (r) := \int_S \rho (r') G (r, r') \, dr'.
\]

(2.9)

The free space Green’s function used in this dissertation are defined by Equations 2.10 and 2.11.

\[
G (r; r') := \left( I + \frac{\nabla \nabla}{k^2} \right) g (r; r').
\]

(2.10)

\[
g (r; r') = \frac{\exp^{-ik_0 |r-r'|}}{4\pi |r-r'|}.
\]

(2.11)

In Chapter 3.1.2 the domain \( \Omega \) will be split up until constituent sub-domains where the jump of the surface electric current across sub-domains is given in Equation
Chapter 2. Notation

2.12.

\[ [j^h]_{mn} := \hat{t}_m \cdot j^h_m + \hat{t}_n \cdot j^h_n. \]  \hspace{1cm} (2.12)

Finally, the inner product is defined in Equation 2.13.

\[ \langle A, B \rangle_S = \int_S (A^* B) \, dS. \]  \hspace{1cm} (2.13)
Chapter 3

Geometry aware integral equation
domain decomposition method

3.1 Theory & Formulation

3.1.1 Model problem

The discussion of the methodology used in this dissertation begins with a study of the time-harmonic solution to electromagnetic scattering from the finite 3D PEC object shown in Figure 6.1. The exterior region, $\Omega_{\text{ext}}$, is assumed to be free space.

The combined field integral equation (CFIE) formulation will be employed to
avoid internal resonance solutions \cite{24} and is defined by Equation 3.1.

\[
\frac{\alpha}{2} \mathbf{j} - \alpha \pi_x (\mathbf{K} (\mathbf{j}; S)) - (1 - \alpha) \pi_\tau (\mathbf{L} (\mathbf{j}; S)) = \alpha \mathbf{j}^{\text{inc}} + (1 - \alpha) \mathbf{e}^{\text{inc}}. \tag{3.1}
\]

In this CFIE formulation the principle value of the magnetic field operator is used, and the coupling parameter is chosen as \( \alpha = \frac{1}{2} \) for the simulations in this work.

### 3.1.2 Discontinuous Galerkin formulation

Equation 3.1 is solved by the Galerkin method, which is based on a variational formulation in suitable trial and testing function spaces. Usually conformal boundary elements are required; however, in many practical engineering applications generating such conformal discretizations is cumbersome. Some recent attention has been given to this challenge \cite{25–29}. These methods accomplish the challenging problem of matching the boundary conditions by creating an overlapping boundary, creating
Chapter 3. Geometry aware integral equation domain decomposition method

a contour boundary, or by requiring in-situ mesh refinement, all of which are not scalable to large problem sizes.

This dissertation uses an interior penalty discontinuous Galerkin method which solves the CFIE using $L^2$ trial and test function spaces [30, 31]. Due to the local characteristics of such function spaces, non-conformal surface discretizations can be used to great effect to tackle the problems mentioned above. Initial investigations have been made on this method, which show promising accuracy, convergence, and stability behavior [31]. The method has the same order of accuracy and convergence as traditional Galerkin CFIE methods that use $H^{-1/2}(\text{div}_t, S)$ boundary element spaces. In this work, we use half-RWG basis functions, which are linear [3]. However, unlike traditional RWG basis functions, we do not require that they are conformal at the boundary between triangles.

Consider a tessellation of $S$ made up of $M$ triangles $S^h = \cup_{m=1}^{M} \{S_m\}$. The triangles are not required to match vertices or edges at their contour boundary. For any $S_m \in S^h$ the boundary of the sub-domain is $C_{mn}$ and the outward unit normal $\hat{t}_m$, as shown in Figure 3.2.

![Figure 3.2: Domain decomposition of Figure 6.1 into three sub-domains.](image-url)
Chapter 3. Geometry aware integral equation domain decomposition method

The approximation of the current $j^h$ is given by Equation 3.2.

$$j^h (r) = \bigoplus_{m=1}^{M} j^h_m (r).$$  \hfill (3.2)

The local approximation of the surface electric current is given by $j^h_m \in X^h_m$ and $X^h_m$ is the space spanned by the vector basis functions. Let $T_m$ and $T_n$ be two adjacent triangles sharing the contour boundary $C_{mn}$. Then the bilinear form is defined as:

$$a_h (v^h, j^h) := \frac{i k_0}{2} \sum_{m=1}^{M} \left< v^h_m, \sum_{n=1}^{M} \Psi_A \left( j^h_n; T_n \right) \right>_{T_m} + \frac{1}{2 i k_0} \sum_{m=1}^{M} \left< \nabla \cdot v^h_m, \sum_{n=1}^{M} \Psi_F \left( \nabla \cdot j^h_n; T_n \right) \right>_{T_m} + \frac{1}{4 i k_0} \sum_{C_{mn} \in C} \left< [v^h]_{mn}, [\Psi_F]_{mn} \right>_{C_{mn}} - \frac{1}{4 i k_0} \sum_{C_{mn} \in C} \left< [v^h]_{mn}, [\nabla \cdot j^h_n]_{mn} \right>_{C_{mn}} + \frac{\beta}{2 k_0} \sum_{C_{mn} \in C} \left< [v^h]_{mn}, [j^h_{mn}]_{mn} \right>_{C_{mn}} + \frac{1}{4} \sum_{m=1}^{M} \left< v^h_m, \bar{J}_m \right>_{T_m} + \frac{1}{2} \sum_{m=1}^{M} \left< \nabla \times (v^h_m), \sum_{n=1}^{M} \bar{\cal K} (j^h_n; T_n) \right>_{T_m}.$$

(3.3)

The stabilization parameter is chosen to be $\beta = |\log \tilde{h}| / 10$, where $\tilde{h}$ is the global average element size [31]. Finally, the variational weak formulation of Equation 3.1 can be stated as: Seek $j^h = \bigoplus_{m=1}^{M} m^h$, $j^h_m \in X^h_m$ such that

$$a_h (v^h, j^h) = \frac{1}{2} \left< v^h, e^{inc} \right>_{S^h} + \frac{1}{2} \left< v^h, j^{inc} \right>_{S^h},$$

(3.4)
Chapter 3. Geometry aware integral equation domain decomposition method

\[ \forall v^h = \bigoplus_{m=1}^{M} v^h_m, \quad v^h_m \in X^h_m. \] The last term \( \frac{1}{2} \langle v^h ; j^{inc} \rangle_{\mathcal{S}^h} \) results from the principle value integral contained within the magnetic field boundary potential operator \( \mathcal{K} \). After the surface electric current is expanded in terms of the basis functions the resulting discrete system can be cast as a matrix equation as: \( Ax = b \), where \( x \) is the unknown coefficient vector, \( b \) is the right hand side vector, and \( A \) contains both the CFIE impedance matrix for each sub-domain along the identity diagonal and the coupling matrix between each sub-domain on each off-diagonal. Unlike the hybridized discontinuous Galerkin formulation we do not use different order basis functions across interfaces. The use of higher-order basis functions leads to larger patch sizes, ultimately reducing the computational requirements; however, due to the time to implement such basis functions this will be done as future work. The consistency of Equation 3.3 is shown in Appendix A.

3.1.3 Domain decomposition solver

For most problems of interest the dimensions of the matrix equation created by Equation 3.3 is too large to solve using direct methods. Preconditioned Krylov subspace iterative methods are often employed to solve such problems, but efficient and parallel computation of preconditioners is an immense challenge. In this work a one-level, non-overlapping additive Schwarz domain decomposition preconditioner is used [32]. The two main advantages to this method are that it leads to an efficient parallelization suitable for distributed computing architectures, and that different solvers can be used in different sub-domain problems.

To apply the domain decomposition method the global mesh is partitioned into
Chapter 3. Geometry aware integral equation domain decomposition method

N sub-domains. The resulting matrix equation is given in Equation 3.5.

\[
\begin{bmatrix}
A_1 & C_{12} & \ldots & C_{1N} \\
C_{21} & A_2 & \ldots & C_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
C_{N1} & C_{N2} & \ldots & A_N
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_N
\end{bmatrix}
= 
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_N
\end{bmatrix}.
\] (3.5)

\(A_i\) is the sub-domain CFIE impedance matrix on \(S_i^h\) and \(C_{ij}\) is the coupling matrix between sub-domains \(S_i^h\) and \(S_j^h\). These sub-domains are obtained by a direct partitioning of the surface without the introduction of an artificial contour [33, 34] or auxiliary unknowns [35, 36]. The preconditioned matrix equation is obtained by row-multiplying Equation 3.5 with the inverse of the sub-domain solution. The preconditioned matrix equation is given in Equation 3.6.

\[
\begin{bmatrix}
I & C_{12}A_2^{-1} & \ldots & C_{1N}A_N^{-1} \\
C_{21}A_1^{-1} & I & \ldots & C_{2N}A_N^{-1} \\
\vdots & \vdots & \ddots & \vdots \\
C_{N1}A_1^{-1} & C_{N2}A_2^{-1} & \ldots & I
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
\vdots \\
u_N
\end{bmatrix}
= 
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_N
\end{bmatrix}.
\] (3.6)

The unknown coefficient vector in the preconditioned system becomes \(u_i = A_i x_i\).

The matrix equation is solved in two steps, first by iteratively solving Equation 3.6 to satisfy the residual norm, in Equation 3.7,

\[
\varepsilon := \frac{\|P^{-1}(Ax - b)\|}{\|P^{-1}b\|},
\] (3.7)
and then the solution for each sub-domain is recovered from $x_i = A_i^{-1}u_i$.

This method is an effective preconditioning scheme that reduces the condition number and provides an efficient parallelizable solution for systems of large equations. Many numerical experiments are performed in Chapters 3.2 and 4.2 that show good convergence and scalable performance. Therefore, this method is suitable for applications in extreme-scale electromagnetic engineering and also when flexible solution methods are necessary.

### 3.2 Benchmarking: Numerical Study

#### 3.2.1 Eigenspectrum

The spread of eigenvalues for the preconditioned system matrix (Eq. 3.6) is an important metric in determining the convergence properties of the method proposed in Chapter 3.1.3. The metric we use to measure the spread of eigenvalues for the antisymmetric matrix $A$ is defined by $\kappa = \frac{\|\lambda_{\text{max}}\|}{\|\lambda_{\text{min}}\|}$ [6]. In order to measure the eigenspectrum, a numerical experiment is performed on a spherical PEC target at two different frequencies, 180 MHz and 240 MHz, each with a radius of 0.5m and a fixed discretization size of $h = \lambda_0/10$. The resulting triangular meshes, as well as the eigenvalue distributions, are shown in Figure 3.3.

All of the eigenvalues reside within the unit circle shifted along the real axis and are well separated from the origin. These results indicate a well-conditioned system and therefore a rapid convergence of the Krylov solver is expected [37,38].
Chapter 3. Geometry aware integral equation domain decomposition method

3.2.2 Convergence

The convergence behavior of the iterative method is studied in this section, in particular the methods capabilities of handling larger problems is considered. A PEC cube of dimensions 1m-by-1m-by-1m is discretized and evaluated at five different frequencies. The frequencies studied range from 0.6-9.6 GHz, which correspond to
Chapter 3. Geometry aware integral equation domain decomposition method

cubes of an electrical size between $1-16\lambda_0$, and a sub-domain count from 2 to 512. The mesh size is fixed at $\lambda_0/10$ for all frequencies. In this dissertation we use linear RWG basis functions to perform the numerical experiments. At this time, we have not explored other basis functions, such as higher-order or curved elements. The iterative solver convergence for these simulations is plotted in Figure 3.4.

![Graph showing iterative solver convergence with respect to operating frequency.](image)

Figure 3.4: Iterative solver convergence with respect to operating frequency.

It is observed that the convergence behavior is not sensitive to an increase in the number of sub-domains indicating that as the problem size is increased the unknown matrix remains well-conditioned. If the smallest and largest problem sizes are compared the iteration count does not even double from 2 to 512 sub-domains.
Chapter 3. Geometry aware integral equation domain decomposition method

3.2.3 Accuracy

The final numerical study performed is that of the accuracy of the proposed solution method. The electromagnetic scattering from a PEC sphere of 0.5m is considered and the analytic solution from a Mie series [39] is compared to the computational solution. The error of the numerical solution is calculated using the $L^2$ norm as defined in Equation 3.8.

$$\|j^h - j^{\text{ref}}\|_{L^2(S^h)} = \left[ \int_{S^h} (j^h(r) - j^{\text{ref}}(r)) \cdot (j^h(r) - j^{\text{ref}}(r))^* \, dr \right]^{1/2} \quad (3.8)$$

The results of this numerical experiment are plotted in Figure 3.5.

![Figure 3.5: Accuracy of PEC sphere with respect to operating frequency.](image)

It is observed that the error asymptotes at higher frequencies to $4 \cdot 10^{-4}$. Finally, the bistatic radar cross section (RCS) is shown in Figure 3.6 when an electromagnetic planewave with a frequency of 16 GHz illuminates a perfect electric conductor (PEC) sphere. Again, the computational result agrees well with the Mie series solution.
Figure 3.6: Bistatic Radar Cross Section of a PEC sphere at 16 GHz.
Chapter 4

HPC-enabled integral equation domain decomposition

4.1 Scalable parallelism

The mathematical techniques discussed in Chapter 3.1 lead to an adaptive, scalable, and parallel framework suitable for modern distributed computing systems. The simulation flow can be divided into three phases of development, shown in Figure 4.1: pre-processing, parallel computation, and post-processing. All three stages can be formulated with the mathematical and computational ingredients previously discussed. This section will provide an overview of these stages.
Chapter 4. HPC-enabled integral equation domain decomposition

4.1.1 Pre-processing

Geometry-based domain decomposition

The first stage for pre-processing is the creation of a CAD-generated surface model on a component-by-component basis. This allows for a flexible mesh generation process where each component can have its own unique characteristics and properties. As an example, the component-based domain decomposition of an aircraft carrier is shown in Figure 4.2 with aircraft, an integrated island, and the flight deck. These modules can be further decomposed into a collection of smaller components, such as navigation, control, and weapons systems. Each component has its own geometrical...
Chapter 4. HPC-enabled integral equation domain decomposition

description.

Figure 4.2: Component-based domain decomposition of an aircraft carrier.

**Geometry-adaptive mesh generation**

Individual components can be discretized concurrently into triangular elements. The maximum discretization size is determined according to the wavelength of the operating frequency, and, if necessary, smaller discretizations are used to represent fine geometrical details. Each sub-domain contains its own collection of triangles, edges, and nodes. Therefore, the mesh generation process is trivially parallel and allows for the rapid generation of high-fidelity models of complicated geometries. Since each component is computationally interchangeable it provides a high level of flexibility and convenience for numerical experiments and engineering design problems.
Chapter 4. HPC-enabled integral equation domain decomposition

Graph partitioning

In this section an automatic spatial decomposition method will be discussed that provides a load-balanced mesh. The algorithmic graph partitioning tool, METIS, is employed to partition the global DG discretization into many local sub-domains based on the high performance computing architecture requirements [40]. Specifically, the partitioning is based on the number of available processors and local memory available to each process. The automatic computational domain decomposition of an aircraft carrier is presented in Figure 8.1. After applying this spatial decomposition strategy we have a load-balanced spatial decomposition suitable for massively parallel HPC systems.

![Figure 4.3: Computational domain decomposition of an aircraft carrier.](image)

4.1.2 Parallel Computation

In order to fully exploit modern high performance computing technology a hybrid openMP/MPI parallelization strategy is employed for distributed computing systems. There are three main parts to solving Equation 3.6, a parallel preconditioned
Chapter 4.  HPC-enabled integral equation domain decomposition

Krylov subspace iterative method, the use of the additive Schwarz preconditioner to solve local sub-domains, and separable radiation coupling.

Iterative Krylov Method

Since the global matrix equation is non-symmetric due to the CFIE formulation a truncated GCR method [41] is employed to solve Equation 3.6. The truncated GCR method obtains an approximation $u^n$ to the solution $u$ in the Krylov subspace $\mathcal{K}^n((AP^{-1}), r^0)$. The Krylov subspace is defined by Equation 4.1 and the initial preconditioned residual is given in Equation 4.2.

\begin{equation}
\mathcal{K}^n (B, y) := \text{Span} \{ y, By, \ldots, B^{n-1} y \}.
\end{equation}

\begin{equation}
r^0 = (b - AP^{-1} u^0).
\end{equation}

This iterative method can easily be extended to solve problems with $M$ sub-domains. In a distributed memory programming model, each sub-domain vector will stored in the local memory associated with individual MPI processes. Algorithms 1 and 2 briefly describe the key algorithms used in the implementation of the iterative Krylov method.
Chapter 4. HPC-enabled integral equation domain decomposition

Algorithm 1 Computation of matrix-vector multiplication $r = (AP^{-1})v$

1: Subdomain solution $t_i = A_i^{-1}v_i$, $\forall i = 1, M$

2: for $i = 1, M$ do

3: $r_i = \sum C_{ij} t_j$, $\forall j = 1, M$ and $j \neq i$

4: $r_i = v_i + r_i$

Algorithm 2 Computation of solution vector $x = P^{-1}u$

1: Initialize $x_i = 1$, $\forall i = 1, M$

2: for $i = 1, M$ do

3: $x_i = A_i^{-1}u_i$

Data parallel sub-domain solution

As shown in Chapter 4.1.2, the additive Schwarz preconditioner requires the solution of individual sub-domain problems. A big advantage of this preconditioner is the ability to solve all the sub-domain problems simultaneously in each iteration. A task-parallel implementation has been used for the sub-domain solutions, which allows each sub-domain to choose its own sub-domain solver based on the local wave characteristics and geometrical features of individual components. Each sub-domain solution is considered an independent task in a distributed MPI programming model. Individual MPI processes execute different tasks simultaneously. Finally, a queue-based task balancing strategy has been employed. The time for each task is measured, the tasks are sorted into a task queue, each MPI process is assigned a group of tasks, and this leads to a dynamic load balancing environment.
Distance-based radiation coupling

The final parallel computing stage is the radiation coupling among sub-domains where each surface current is radiated to all the other sub-domains. There are two primary technical ingredients: a hierarchical multi-level fast multipole method (H-MLFMM) [42, 43] and a primal-dual octree partitioning algorithm for separable sub-domain coupling [44] where a local octree is created for each sub-domain. A graphical depiction of separable radiation coupling is given in Figure 4.4.

Figure 4.4: Distance-based separable radiation coupling.

H-LFMM leads to the seamless integration of a multi-level skeletonization technique which provides an effective matrix compression for non-uniform discretizations. Just like the sub-domain solution a task-based parallel architecture is used in sub-
domain radiation coupling. The coupling can be performed without any communication or synchronization between MPI processes. Parallelization is also attained using openMP within each task to exploit the fast memory access of the multi-core processors. For a very large number of sub-domains, highly scalable parallel performance can be obtained with a hybrid MLFMA-FFT algorithm [45, 46].

4.1.3 Post-processing

Once the simulation is complete the surface electric current for each sub-domain is calculated and translated into separate Silo files. Silo is a popular mesh and field library developed by Lawrence Livermore National Laboratory. To process the extreme-scale Silo data files efficiently the ViSiT tool is used, which is a distributed, parallel visualization and graphical analysis tool [47]. For the largest of mesh files advanced computing architectures, such as GPU acceleration, are available for use. Finally, a far-field calculator to compute the scattered far-field from the surface electric currents has been developed.

4.2 Benchmarking: Performance Study

In this section, numerical experiments are presented that profile the parallel performance of the implementation of the proposed methods. Results were obtained on a Cray XE6m DoD HPCMP Open Research System cluster named Copper. Each compute node contains two different sixteen-core processors that have 60 GBytes of accessible DDR3 memory. Each node consists of two 2.3-GHz AMD 6200 series
Chapter 4. HPC-enabled integral equation domain decomposition

processors. The system has a total of 239 TBytes of formatted parallel disk storage, of which 224 TBytes resides within the the work directory [48]. All of the reported times are measured using the MPI routine MPI_Wtime().

4.2.1 Weak scalability

Weak scaling is a measure of the time-to-solution as more computational nodes are used with a fixed ratio of degrees-of-freedom per core. We consider plane wave scattering from a mock-up F16. The dimensions are approximately 14.5m-by-9.6m-by-5.0m. A plane wave is launched towards the aircraft, $-\hat{x}$, and is polarized in an upwards, $+\hat{z}$, direction. The discretization is first chosen for the lowest frequency using DG boundary elements to accurately represent the many complex surfaces, which often results in different mesh sizes of different parts of the F16 aircraft. For higher frequencies the meshes for each sub-domain are each refined independently to the level of accuracy needed to satisfy the Nyquist sampling rate.

In this experiment a constant number, 3.5 million, of DOFs per sub-domain are used. The discretization is then partitioned into a number of sub-domains using METIS [40] to maintain that ratio. The resulting partitions for the weak scaling experiments are shown in Figures 4.5(a)-(d).

Each sub-domain is assigned to one MPI process, which in turn is assigned thirty-two openMP threads corresponding to the number of cores per node. The parallelization efficiency is plotted in Figure 4.6. The results indicate a very efficient parallelization for all computations above or at 90%. Additional computational results are presented in Table 4.1. We observe very small increases in the iteration count
Chapter 4. HPC-enabled integral equation domain decomposition

as the frequency increases, and also very fast convergence in terms of wall clock time. Finally, the surface electric currents are plotted in Figures 4.7(a)-(d), and a smooth current with no noticeable discontinuities across the sub-domain boundaries is observed.

Table 4.1: Computational Statistics for scaling experiments of EM scattering from a full-scale F16.

<table>
<thead>
<tr>
<th>Num. of cores</th>
<th>Num. of subdomains</th>
<th>Total time per iteration (minutes)</th>
<th>Num. of iterations</th>
<th>Num. of DOFs (million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>2</td>
<td>57</td>
<td>4</td>
<td>6.95</td>
</tr>
<tr>
<td>256</td>
<td>8</td>
<td>55</td>
<td>5</td>
<td>27.4</td>
</tr>
<tr>
<td>576</td>
<td>18</td>
<td>65</td>
<td>7</td>
<td>62.3</td>
</tr>
<tr>
<td>1024</td>
<td>32</td>
<td>57</td>
<td>8</td>
<td>111.3</td>
</tr>
</tbody>
</table>
Chapter 4. HPC-enabled integral equation domain decomposition

Figure 4.6: Parallelization efficiency for F16 scattering problems.

Figure 4.7: Electric surface current for F16 scattering problems.
Figure 4.8: Strong scaling for F16 scattering problems.

4.2.2 Strong scalability

Strong scaling is a measure of the solution time as more MPI processes are used to solve the same problem. This experiment was performed on the aircraft discretized for 3 GHz partitioned into 2 to 32 sub-domains. As in the weak scaling experiment each sub-domain is assigned to one MPI process with thirty-two openMP threads per MPI process. The scalability results are presented in Figure 4.8. The speed-up is super-linear between 2 and 8 MPI processes, and then sub-linear from 8 to 32 due to an increase in iteration count. This indicates that there is an ideal partitioning for each given problem that is based on the condition number of the matrix. The speed-up relative to 2 nodes at 32 nodes is approximately 12 times.
Chapter 4. \textit{HPC-enabled integral equation domain decomposition}

Figure 4.9: Dimensions of a full-scale aircraft carrier.

4.2.3 Parallel computing challenges

Real-world Application: Aircraft Carrier

To demonstrate the utility of the methods proposed in the previous sections the real-world example of planewave scattering from an aircraft carrier (Figure 1.1) will be demonstrated in this section. The dimensions of the particular carrier model used are shown in Figure 4.9. There are a number of small components on this carrier including radar antennas, weapons systems, landing systems, which are all located on an integrated island. Many mockup aircraft have also been included on the landing platform (Figure 4.10). In other words, there is a very high multi-scale nature to the geometrical features modeled on this aircraft carrier.

At a frequency of 400 MHz, this aircraft carrier has $\sim$35 million DOFs, which is quite large. Both the multi-scale and extreme-scale nature of this problem present a significant challenge for conventional IE methods. To simulate planewave scattering
Chapter 4. HPC-enabled integral equation domain decomposition

from this geometry, the model is divided into 24 sub-domains. The computational partitioning is shown in Figure 4.11. Each sub-domain is assigned to one MPI process with 32 computing cores.

The simulation requires 8 iterations to reach a relative residual of $10^{-2}$, and the surface electric current is plotted in Figure 4.12. Additionally, a simulation at 1.2 GHz was run requiring 254 million DOFs and 62 sub-domains. The entire simulation takes 12 iterations, with an average time per iteration of 1 hour and 51 minutes, to reach a relative residual of $10^{-2}$. In Figure 4.13 the run-time for each sub-domain is presented. There are two key observations to make. First, the time spent during serialized tasks is practically negligible compared to the time spent in the sub-domain and coupling solvers. Finally, the computational load distribution, especially in the sub-domain solution stage, is unbalanced across the entire computational domain. In other words, many MPI processes wait a long time for other processes to finish their tasks. This challenge is addressed in Chapter 4.2.3.
Chapter 4. HPC-enabled integral equation domain decomposition

Figure 4.11: Computational decomposition of an aircraft carrier.

Dynamic load-balancing

As observed in Figure 4.13, the automated computational partitioning for a complex, real-world problem is not load balanced. Much work has been done to address this issue through the development of dynamic load balancing [49, 50]. Among the possible choices one in which a centralized dynamic task queue is employed will result in the highest efficiency [51]. Tasks will be timed and sorted in descending order. Then, tasks will be assigned to MPI processes on a next available basis. As MPI processes finish their previous task they will execute the next one from the queue. During each DD iteration, different tasks could be assigned to different MPI processes. This is made possible from the task-based parallelism employed in the algorithms.

One requirement of this method is that the number of sub-domains greatly outweighs the number of MPI processes. If it does not, then such an algorithm will
never result in the best assignment of tasks to MPI processes since the run-time is not long enough for different processes to receive a different number of tasks. To demonstrate the effectiveness of this load balancing algorithm it will be applied to the same problem presented in Chapter 4.2.3.
Chapter 4. *HPC-enabled integral equation domain decomposition*

Figure 4.13: Plot of time spent in each of three major tasks: i) serialized tasks, ii) sub-domain solver, iii) sub-domain coupling.
Chapter 5

Channel Modeling in Dense Urban Environments

5.1 Introduction

5.1.1 Literature Review

The study of channel modeling for the fifth-generation (5G) of cellular communications has gained recent attention [52]. As the number of currently available communication channels become more and more congested antenna engineers must design new communication bands. Fifth-generation communication has been studied for a variety of different communication links, such as wearables [53], vehicular networks [54–57], UAV-ground over a variety of environments [58–64], GPS [65],
but perhaps most importantly cell phone communication in dense urban street canyons [52, 53, 56, 62, 63, 66–74]. The frequencies currently being investigated for 5G communication are in the mm-wave regime. At such a small wavelength the free space path loss (FSPL) will be prohibitive to long-distance communication, but at a short-range it will open up new bands of communication.

The growing sophistication in wireless communication systems and the increasing demand for network capacity have driven the evolution of channel models. Consider cellular networks as an example [75–77]. Rapidly increasing features, higher spatial resolutions, and expended use cases are all developed in order to better approximate real world propagation scenarios. The channel models are consistently evolving towards higher fidelity predictions.

The most popular approaches in the literature are ray-based high-frequency approximations [76–80]. One first creates a simplified 3D geometric description of the propagation environment. The propagation of the electromagnetic (EM) wave from the transmitter (Tx) to the receiver (Rx) is approximated by many discrete rays. Non-line-of-sight (NLOS) models have been developed [54] and line-of-sight (LOS) path loss has been extensively studied in a variety of environmental conditions [54, 58, 60–63, 66]. Additional factors include the Ricean K-factor, for small-scale fading, spatial and interband correlation, RMS delay spread, and tapped line delay [58].

Ray-tracing based analysis is useful to investigate high-frequency propagation mechanisms of wireless channels, such as reflection and diffraction, diffuse scattering, blocking, and so on. However, there are many open issues and difficulties in dealing with the precise modeling of antennas, non-specular scattering effects, hu-
man body shadowing, and vegetation loss. The study of these issues is important to gain a fundamental understanding of the attenuation, blockage, interference and transmission range of wireless signals.

The objective of this work is to investigate a high-resolution, first-principles computational methodology for wireless propagation channels in realistic usage scenarios. A variety of parallel and scalable algorithms are proposed in this dissertation to overcome key challenges in extreme-scale channel modeling and simulation. This work provides much greater channel model resolution than existing deterministic channel modeling technologies. Large antenna arrays, effects of human body shadowing, reflections due to vehicles, and/or attenuation by dense vegetation can be analyzed rigorously. The advancements of this research are expected to improve the knowledge of wireless channel characteristics and propagation behavior.

5.1.2 Challenges to be addressed

Despite its apparent importance to wireless communication networks, the development of EM field-based full wave channel models faces significant mathematical and computational challenges. One representative application is the outdoor wireless mobile network in a 3D urban environment (Downtown Albuquerque, NM), as shown in Figure 5.1. The computational domain includes dozens of buildings and structures that are used to analyze the propagation of EM signals in this outdoor setting. A number of antennas are placed in a variety of locations to evaluate the signal coverage and transmission range for line-of-sight (LOS) and non-line-of-sight (NLOS) links. Furthermore, in order to examine the blockage, attenuation, and interference issues, high-definition vehicles and dense vegetation modes need to be considered.
The full wave, high-fidelity analysis of such an EM problem is very challenging due to the enormous level of physical detail and complexity. A system-level simulation of the environment in Figure 5.1 in the UHF band requires billions of mesh points in the spatial domain, and involves a wide spectrum of spatial scales, a variation beyond ten orders of magnitude. Even if such an analysis can be performed, it is expected to take weeks to months for one simulation using existing full wave methods. Clearly, fundamental research into mathematics and computational algorithms are required to turn the problem from infeasible to feasible.
Chapter 5. Channel Modeling in Dense Urban Environments

5.2 Formulation

Recently, a geometry-aware integral equation domain decomposition method [81, 82] (GA-IE-DDM) was investigated for the rigorous and parallel full wave solution of large multi-scale EM problems. It has demonstrated three major benefits: (i) it results in a robust and effective preconditioning technique that reduces the condition number of very large systems of equations; (ii) it provides a flexible and natural way to set up the mathematical models, to create the problem geometries and to discretize the computational domain; (iii) it leads to highly efficient and naturally parallelizable computational algorithms on distributed memory many-core parallel computing systems. The proposed work builds on and significantly extends the GA-IE-DDM framework with the numerical ingredients discussed below.

5.2.1 A geometry-aware hybrid finite element (FE) and boundary integral (BI) formulation

Large antenna arrays have been considered as an effective way to reduce the wave propagation losses due to their ability to steer and collect the beam energy. To accurately predict in-situ radiation performance, mutual interactions of antenna arrays, the repositioner, the surrounding regions, and the platform must be considered simultaneously. The objective of this work is to develop a parallel and scalable hybrid FE-BI method for the in-situ antenna array analysis. The technical ingredients include a volume-based Schwarz FE domain decomposition (DD) method [83,84] and a surface-based interior penalty BI DD method [81,82]. Compared to existing FE-BI algorithms, this work features geometry-aware decomposition, a preconditioner with
linear computational complexity with respect to the number of sub-domains, and scalable iterative solver convergence.

### 5.2.2 Coarse-graining approach via hierarchical skeletonization

The skeletonization scheme can be viewed as a compressed representation of structured rank-deficient matrices. It has been applied to the compression of low rank matrices via interpolative decomposition \([42, 43, 85]\), and the development of fast direct solvers \([86–88]\). This work employs multi-level skeletonization to build effective “coarse-grid” basis functions building on the GA-IE-DDM. These “coarse-grid” bases are selected from the original basis functions on the surface discretizations using interpolative decomposition \([89]\), without the need to generate nested meshes in multi-grid methods \([90–93]\) or to construct hierarchical bases \([94,95]\). Then, the original domain is mapped to the skeletonized problem, solved according to the techniques discussed in this dissertation, and finally the skeletonized result is mapped back to the whole domain. Hence, they are termed skeleton basis functions, which can be considered the largest contributors to the final solution. Huygens’ surfaces will be introduced for individual sub-domains to facilitate the selection of the skeleton basis functions. Thus, the skeletonization can be achieved locally per sub-domain and embarrassingly in parallel. Subsequently, the interactions between sub-domains will be directly computed using selected skeletons, and the DD iteration will be performed on the coarse-grain compressed system.
Chapter 5. Channel Modeling in Dense Urban Environments

5.2.3 A parallel, separable, and scalable coupling scheme

The parallel computations in GA-IE-DDM requires the solution of individual sub-domain problems and coupling among multiple sub-domains. To achieve low computational costs, we employ separate representations based on the relative spatial positions of two sub-domains. Specifically, the coupling between sub-domains can be classified into three types: skeletonized representation for the near-field region, the fast multipole method for the mid-field region, and the fast far-field approximation for the far-field region. Furthermore, to achieve high parallelization efficiency, we utilize a primal-dual octree partitioning algorithm aiming for separable sub-domain coupling. Both the skeletonization and aggregation/disaggregation can be computed locally per sub-domain and in parallel.

5.3 Free Space Verification

In this section, the proposed algorithms are tested against the analytical free space propagation model.

Dipole antennas will be used throughout this numerical study.

The first numerical simulation will simply be a freespace calculation of the point-to-point communication of the two antennas.

Each numerical study will require the calculation of the path loss based on the S parameters. Since each antenna is the same we know that the $S_{11}$ is the same for the transmitting antenna and the receiving antenna. Path loss is defined as the ratio of the power received to the transmitted power [96, 97]. If we assume that all power
Chapter 5. Channel Modeling in Dense Urban Environments

injected into the antenna is either radiated or reflected, then the total power can be written as Equation 5.1 with the trivial assumption that the total power is 1 Watt.

\[ P_{\text{tot}} = 1 = P_{\text{ref}} + P_{\text{rad}}. \] (5.1)

Since the reflected power is proportional to the square of the magnitude of the reflection coefficient the power radiated can be written in terms of the \( S_{11} \) parameter in Equation 5.2.

\[ P_{\text{rad}} = 1 - |S_{11}|^2. \] (5.2)

The simulation results give us the \( S_{12} \) parameter for the receiving antenna. Therefore, the power received can be written in Equation 5.3.

\[ P_{\text{received}} = |S_{12}|_{\text{receiver}}^2. \] (5.3)

\[ PL = \frac{|S_{12}|_{\text{receiver}}^2}{(1 - |S_{11}|_{\text{receiver}}^2)}. \] (5.4)

The mesh density for the simulation is \( 5 \cdot 10^{-4} m \approx 0.16 \lambda \).

### 5.3.1 Free Space Measurement

The numerical results obtained from the free space measurement will be compared to analytical calculations. The Friis' Transmission Equation (Eq. 5.5) gives the
Chapter 5. Channel Modeling in Dense Urban Environments

path loss if we are given certain information about the receiving and transmitting antennas [97].

\[
\frac{P_r}{P_t} = e_{c_{dt}} e_{c_{dr}} (1 - |\Gamma_t|^2) (1 - |\Gamma_r|^2) \left( \frac{\lambda}{4\pi R} \right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) |\hat{\rho}_t \cdot \hat{\rho}_r|^2 .
\] (5.5)

If we assume that the antennas are reflection and polarization-matched, have an efficiency of one, and are isotropic monopoles, then the only term that remains is dependent on the distance between the receiver and transmitter, and Equation 5.5 reduces to 5.6.

\[
\frac{P_r}{P_t} = \left( \frac{\lambda}{4\pi R} \right)^2 .
\] (5.6)

Another free space simulation was conducted to match the experiment in [74]. The plot in Figure 5.2(a) is the path loss when the transmitting and receiving antenna are about 80cm apart and the receiving antenna is moved ±5mm from the centerline. The simulated results are compared to the analytical results. We observe a difference of 3dB, and this is plotted in Figure 5.2(a).

5.4 Denver Experiment Validation

5.4.1 Free Space Measurements

This section outlines the validation of GA-IE-DDM against a channel modeling experiment conducted in Denver, CO [72]. The purpose of this experiment was to
develop communication channel models for urban street canyons. In this study, the scientists first performed a series of measurements to validate the theory developed by ray tracing theory. This dissertation will present a validation scheme for GA-IE-DDM using the experimentally validated model presented in [72]. When an antenna radiates in free space above a ground plane there are two parts of the radiation that reach the receiver. This is depicted in Figure 5.3.

The transmitting antenna (Tx) is at a height of $h_1$ and the receiver (Rx) is at a height of $h_2$ and are separated by a distance $d$. Due to Snell’s Law the angle of incidence and reflection are the same and is $\theta = \tan^{-1}\left(\frac{d}{h_1+h_2}\right)$. The line-of-sight (LOS) ray is represented by $\mathbf{r}_1$ and the reflected ray is represented by $\mathbf{r}_2$. The power received by the receiver from both the direct ray and the reflected ray is given by Equation 5.7 [98].

$$P_{2R}(R) = P_{Tx} \left(\frac{\lambda}{4\pi}\right)^2 \frac{e^{-j2\pi \sqrt{R^2+({h_{Tx}-h_{Rx}})^2}/\lambda}}{\sqrt{R^2+(h_{Tx}-h_{Rx})^2}} + \Gamma(\theta_i) \frac{e^{-j2\pi \sqrt{R^2+({h_{Tx}+h_{Rx}})^2}/\lambda}}{\sqrt{R^2+(h_{Tx}+h_{Rx})^2}}.$$
Note that if $\Gamma(\theta) = 0$ (total transmission) we recover the equation for no ground plane present. Additionally, if the two antennas are at the same height then the first term is recovered as simple point-to-point communication. The expression for $\Gamma(\theta)$ is given in Equation 5.8 [57].

$$\Gamma(\theta) = \frac{\varepsilon_r \cos \theta_i - \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}}.$$  \hspace{1cm} (5.8)$$

The reflection coefficient can be composed into a TE (5.10) and a TM (5.9) mode,
Chapter 5. Channel Modeling in Dense Urban Environments

where the TE mode has no electric field in the incident plane and the TM mode has no magnetic field in the incident plane.

\[ \Gamma_{TM} = \frac{-\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}} \]  \hspace{1cm} (5.9)

\[ \Gamma_{TE} = \frac{\cos \theta_i - \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}} \]  \hspace{1cm} (5.10)

To validate GA-IE-DDM against this theory, which has been validated by experimental results, we design a simulation of two antennas radiating in free space at 900 MHz with material properties for the ground similar to the experiment. The material properties used for the ground are in Table 5.1 [73].

<table>
<thead>
<tr>
<th>(\varepsilon)</th>
<th>15 - (j) 60 (\sigma\lambda)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma)</td>
<td>0.005 S/m</td>
</tr>
<tr>
<td>(f)</td>
<td>900 MHz</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>15 - (j) 0.1</td>
</tr>
</tbody>
</table>

The electromagnetic wave incident on the material interface is decomposed into TE and TM components, each of those components are reflected from the material interface, and then propagated to Rx. The plot of received power for the TE mode is presented in Figure 5.5 where the electromagnetic planewave has been decomposed into its transverse electric and magnetic components and these results are compared to 5.4 [72].
Figure 5.4: Reference Figure for LOS power predictions [72].

\[
\begin{bmatrix}
\hat{\rho} \\
\hat{\theta} \\
\hat{\phi}
\end{bmatrix}
= \begin{bmatrix}
\sin \theta \cos \phi & \sin \theta \sin \phi & \cos \\
\cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\
- \sin \phi & \cos \phi & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}.
\] (5.11)

This leads to the expressions for \( \hat{\theta} \) and \( \hat{\phi} \) in Equation 5.13.

\[
\hat{\theta} = \cos \theta \cos \phi \hat{x} + \cos \theta \sin \phi \hat{y} - \sin \theta \hat{z}.
\] (5.12)

\[
\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}.
\] (5.13)
Chapter 5. Channel Modeling in Dense Urban Environments

Figure 5.5: Comparison of the computational result and the semi-analytical 2-ray model.

When setting $\phi = 90^\circ$ the expressions reduce to Equation 5.15.

\[
\hat{\theta} = \cos \theta \hat{y} - \sin \theta \hat{z}. \\
\hat{\phi} = -\hat{x}. 
\] (5.14) (5.15)

The far-field solver used in this experiment provides the real and imaginary components of the electric field in both the $\hat{\theta}$ and $\hat{\phi}$ directions. The incident electric field can be decomposed into both TE and TM parts as shown in Equation 5.18.
\[ \mathbf{E}_{\text{inc}}^{\text{TE}} = -\left( \Re \{ \mathbf{E}_\phi \} + \Im \{ \mathbf{E}_\phi \} \right). \]  
\[ \mathbf{E}_{\text{inc}}^{\text{TM}} = \left( \Re \{ \mathbf{E}_\theta \} + \Im \{ \mathbf{E}_\theta \} \right) \left( \cos \theta \mathbf{\hat{y}} - \sin \theta \mathbf{\hat{z}} \right). \]  
\[ \mathbf{E}^{\text{inc}} = \mathbf{E}_{\text{inc}}^{\text{TE}} + \mathbf{E}_{\text{inc}}^{\text{TM}}. \]  

The reflected electric field is then computed by applying Equations 5.9 and 5.10 to Equation 5.18 as shown in Equation 5.19.

\[ \mathbf{E}^{\text{ref}}_{\text{TE}} = \Gamma_{\text{TE}} \mathbf{E}_{\text{inc}}^{\text{TE}}. \]  
\[ \mathbf{E}^{\text{ref}}_{\text{TM}} = \Gamma_{\text{TM}} \mathbf{E}_{\text{inc}}^{\text{TM}}. \]  
\[ \mathbf{E}^{\text{ref}} = \mathbf{E}^{\text{ref}}_{\text{TE}} + \mathbf{E}^{\text{ref}}_{\text{TM}}. \]  

With the incident and reflected electric field available, the computation of the reflection coefficient is shown in Equation 5.20.

\[ \Gamma (\theta) = \left| \frac{\mathbf{E}^{\text{ref}}}{\mathbf{E}^{\text{inc}}} \right| = \left| \frac{\Gamma_{\text{TE}} \mathbf{E}_{\text{inc}}^{\text{TE}} + \Gamma_{\text{TM}} \mathbf{E}_{\text{inc}}^{\text{TM}}}{\mathbf{E}_{\text{inc}}^{\text{TE}} + \mathbf{E}_{\text{inc}}^{\text{TM}}} \right|. \]  

Finally, we expect the wave impedance to depend on the angle of incidence. This wave impedance is found by taking the ratio of transmitted electric and magnetic fields in the tangential direction, as shown in Equation 5.21.

\[ \eta (\theta) = \left| \frac{\mathbf{E}^{\text{tran}}_{\text{TE}} \mathbf{\hat{x}} + \mathbf{E}^{\text{tran}}_{\text{TM}} \cos \theta \mathbf{\hat{y}}}{\mathbf{H}^{\text{tran}}_{\text{TM}} \mathbf{\hat{x}} - \mathbf{H}^{\text{tran}}_{\text{TE}} \cos \theta \mathbf{\hat{y}}} \right|. \]
magnetic field, and applying Snell’s Law to the angle of transmission, the above equation can be simplified to one that is only based on the material parameters and the angle of incidence, as shown in Equation 5.22.

\[
\eta(\theta) = \sqrt{\frac{T_{TE}^2 + T_{TM}^2\left(1 - \frac{1}{\varepsilon_r}\sin^2 \theta_i\right)}{\left(\frac{T_{TE}}{\eta_2}\right)^2 \left(1 - \frac{1}{\varepsilon_r}\sin^2 \theta_i\right) + \left(\frac{T_{TM}}{\eta_2}\right)^2}}. \tag{5.22}
\]

Finally, to include the effects of the antenna we introduce the antenna gain to Equation 5.7 and form Equation 5.26.

\[
P_{2R}(R) = P_{Tx}\left(\frac{\lambda}{4\pi}\right)^2 \left| G_1 e^{-j2\pi\sqrt{R^2 + (h_{Tx} - h_{Rx})^2}/\lambda} \right|^2 + \left| G_2 \frac{e^{-j2\pi\sqrt{R^2 + (h_{Tx} + h_{Rx})^2}/\lambda}}{\sqrt{R^2 + (h_{Tx} + h_{Rx})^2}} \right|^2 \tag{5.23}
\]

\[
+ \Gamma(\theta_i) \left| G_2 e^{-j2\pi\sqrt{R^2 + (h_{Tx} + h_{Rx})^2}/\lambda} \right|^2 \tag{5.24}
\]

\[
G_1(\theta, \phi) = \sqrt{G_t(90^\circ, 90^\circ) G_r(90^\circ, 270^\circ)}. \tag{5.25}
\]

\[
G_2(\theta, \phi) = \sqrt{G_t(180^\circ - \theta_{inc}, 90^\circ) G_r(180^\circ - \theta_{inc}, 270^\circ)}. \tag{5.26}
\]

This power received should be compared to Equation 5.4.

### 5.4.2 Validation Study

We proceed to validate the work with measurements conducted at the University of Illinois at Chicago (Section III.B in [99]). The experiment was original designed
to validate the Fresnel-Kirchhoff integral, which is a useful tool to predict wave diffraction, reflection, and path loss from 3-D buildings in urban areas.

The geometry and computational setup are illustrated in Fig. 5.6. In the simulation, the WR-34 standard gain horn antenna is used as the transmitter, whose configuration is given in [100]. The operating frequency is 25GHz. The calculated electric field distribution in the horizontal plane is plotted in Fig. 5.7, where we can visualize the complete propagation physics of reflection, diffraction, and blocking. Next, we evaluate the normalized receiving electric field, $\tilde{E} = E/E^{\text{inc}}$ along a prescribed trajectory. The simulation result is compared with the measurement data in Fig. 5.8. The computational prediction is in good agreement with the measurements.

Figure 5.6: The computational setup of the validation example.
Chapter 5. Channel Modeling in Dense Urban Environments

Figure 5.7: The electric field distribution.

Figure 5.8: Comparison of the simulation result and the measurement data.
5.5 Real-world example: ABQ city block at 2.4 GHz

In this section, we will present the computational solution of an antenna radiation problem in a very large, dense urban environment. There is an Unmanned Aerial Vehicle (UAV) placed over a city block with an antenna mounted on its underbelly. The antenna is excited at a frequency of 400 MHz. The methodology described in the previous section is then used to iteratively solve the antenna radiation problem. Four receiving antennas are placed throughout the city block at frequencies of 500 MHz, 1 GHz (2 antennas), and 2 GHz.

As a representative example, we consider the EM full wave simulation of wireless channel propagation in a dense urban setting. In the simulation, we use the automatic partitioning tool of METIS to sub-divide the city domain into 30 sub-domains, along with a sub-domain for the UAV, and five sub-domains for the antennas. Each city sub-domain has 3.8 million degrees-of-freedom (DOFs). The total number of DOFs is about 115 millions. A plot of the computational partitioning is shown in Fig. 5.10. A scaled 3D city CAD model of downtown Albuquerque is presented in Figure 5.9, where we have employed monopole antennas as transmitters (Tx) and receivers (Rx) located on either building rooftops or mobile heights of 1.5 m above ground. A small modification was made to the coupling code to handle the increased demands on the memory.

The total area mapped out is $300m \times 300m$, and the tallest building is 80m high; the UAV is at a height of 60m. The five antennas in this simulation, 4 Rx and 1 Tx mounted to a UAV, are plotted against their closest sub-domain in Figure 5.11 and
Chapter 5. Channel Modeling in Dense Urban Environments

Figure 5.9: Example of a real-world wireless channel environment: downtown, Albuquerque, NM.

their heights are given in Table 5.2.

Table 5.2: Antenna heights in downtown Albuquerque model.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Furthermore, each triangle element in the discretization can be assigned a unique electromagnetic wave impedance. This is shown in Figure 5.12 with an impedance for the building and one for the ground. The wave impedance for concrete is $177.7 + j9.8$ [101] and the wave impedance for loam (a mixture of sand, silt, and other minerals) is $188 + j7.04$ [102].
The computation were performed on Excalibur, an SGI ICE X System in the DoD HPCMP. The standard compute node has two 2.3-GHz Intel Xeon 3G-2698 16-core processors, for a total of 32 cores per compute node, and 128 GB of DDR3 memory (126 of which is available to users). Excalibur uses both shared- and distributed-memory models. Memory is shared amongst all cores on a node, but not across nodes.
Chapter 5. Channel Modeling in Dense Urban Environments

(a) 1.5 GHz monopole in sub-domain ID 799. 
(b) 3 GHz monopole in sub-domain ID 1059. 
(c) 1.5 GHz monopole in sub-domain ID 596. 
(d) 3 GHz monopole in sub-domain ID 106. 
(e) 3 GHz monopole mounted on UAV.

Figure 5.11: Antennas in their environments.

Figure 5.13 shows the electric surface current on the city buildings, and Figure 5.14 shows a zoomed in view of the UAV “source” antenna within the dense urban environment.

The convergence of this simulation is plotted in Figure 5.15. The simulation took a total of 10 hours and 15 minutes of wallclock time to complete on 36 compute nodes with 32 cores. The total number of core-hours is 11.8k. It converged to a relative residual of 0.0094474 in 9 iterations. Each iteration took 1 hour, and the remainder of the time (about 75 minutes) was spent copying large data files into temporary directories. The time to perform the coupling and the sub-domain solution are about the same within each DD iteration.

Table 5.3 shows the $S_{11}$ parameters for both the receiving and transmitting antennas.

The pathloss for the links from the UAV to each receiving antenna is listed in
Figure 5.12: Graphical depiction of the separation of boundary conditions.

Table 5.3: $S_{11}$ parameters for antennas in downtown Albuquerque.

<table>
<thead>
<tr>
<th>Antenna</th>
<th>Frequency [MHz]</th>
<th>S-parameter [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>UAV (transmitting)</td>
<td>500</td>
<td>-9.8</td>
</tr>
<tr>
<td>City Monopole (receiving)</td>
<td>1000</td>
<td>-84.38</td>
</tr>
<tr>
<td>City Monopole (receiving)</td>
<td>1000</td>
<td>-85.86</td>
</tr>
<tr>
<td>City Monopole (receiving)</td>
<td>2000</td>
<td>-100.29</td>
</tr>
<tr>
<td>City Monopole (receiving)</td>
<td>500</td>
<td>-77.11</td>
</tr>
</tbody>
</table>

Table 5.4.
Chapter 5. Channel Modeling in Dense Urban Environments

Figure 5.13: Electric surface current at $f = 400 MHz$.

Table 5.4: Pathloss for the receiving antennas in downtown Albuquerque.

<table>
<thead>
<tr>
<th>Antenna</th>
<th>Frequency [MHz]</th>
<th>Pathloss [dB]</th>
<th>Distance to Tx [m]</th>
<th>Free Space Path Loss [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopole (recieving)</td>
<td>1000</td>
<td>-86.6</td>
<td>128</td>
<td>-66.7</td>
</tr>
<tr>
<td>Monopole (recieving)</td>
<td>1000</td>
<td>-87.6</td>
<td>126</td>
<td>-66.5</td>
</tr>
<tr>
<td>Monopole (recieving)</td>
<td>2000</td>
<td>-102.1</td>
<td>65</td>
<td>-63.6</td>
</tr>
<tr>
<td>Monopole (recieving)</td>
<td>500</td>
<td>-78.2</td>
<td>60</td>
<td>-60.7</td>
</tr>
</tbody>
</table>
Figure 5.14: Zoomed in view of electric surface current.
Figure 5.15: Convergence of ABQ city block problem at $f = 400\, MHz$. 
Chapter 6

Data-sparse Rapid Antenna

Prototyping by the Schur Complement

6.1 Introduction

Radio frequency assignment in complex electromagnetic environments has been a focus of study for many decades due to its importance for modern communication systems [103]. It was initially a focus for military communications [103–112] and more recent attention in cellular communications problems [113–116]. One solution is to use a simplified model [104], and another solution is to do full-wave simulations [106, 107, 113], but these are very computationally demanding. There has also been
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

work on using advanced computing technologies [108] and domain decomposition methods [109] to speed up computation to the full-wave solution.

The purpose of this research is to merge the best of two worlds: speed and completeness of solution. The full-wave solutions that have been used are very computationally expensive and cannot be used to solve many antenna designs. The simplified model approach has very limited usage and is very model-dependent.

When antenna engineers design antennas in complex electromagnetic environments the best design is typically found by parameter scans. By varying the design parameters over a range of possible values the optimal configuration is found. However, each time the antenna is modified the simulation to obtain the output parameter must be re-done. We will merge the GA-IE-DDM method with a hybrid finite-element boundary-integral method. For the purposes of this chapter, we will make the following assumptions, that the platform never changes and that the antenna has a very limited number of locations for placement.

6.2 Formulation

6.2.1 Hybrid finite-element boundary-integral formulation

Although not a central component to this research, it is important to discuss the modeling techniques used for the antennas in this line of work. A hybrid finite-element boundary-integral (FEBI) method has been developed that is both suitable for a discontinuous Galerkin formulation, as well as being easily parallelizable. The model problem to be solved is shown in Figure 6.1.
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

In FEBI, we have to account for both the electric fields and currents. The equations for the boundary value problem are given in Equations 6.1-6.5.

\[ \nabla \times \nabla \times E^{\text{sca}} - k_0^2 E^{\text{sca}} = 0 \in \Omega_{\text{ext}}. \quad (6.1) \]

\[ \nabla \times \mu_r^{-1} \nabla \times E - k_0^2 \varepsilon_r E = 0 \in \Omega. \quad (6.2) \]

\[ \hat{n} \times E^{\text{sca}} \times \hat{n} + \hat{n} \times E^{\text{inc}} \times \hat{n} = \hat{n} \times E \times \hat{n} \in \partial \Omega. \quad (6.3) \]

\[ \hat{n} \times \nabla \times E^{\text{sca}} + \hat{n} \times \nabla \times E^{\text{inc}} = \hat{n} \times \frac{1}{\mu_r} \nabla \times E \in \partial \Omega. \quad (6.4) \]

\[ \lim_{|r| \to \infty} |r| \left( \nabla \times E^{\text{sca}} - i k_0 \hat{r} \times E^{\text{sca}} \right) = 0. \quad (6.5) \]

The traces on this model problem are shown in Figure 6.2.

The traces are defined in Equations 6.6-6.9.
In addition, we introduce the boundary conditions on the interface (i.e. between BI and FE discretizations) as described in Equations 6.10-6.11.

\begin{align*}
\mathbf{j}^+ &= \frac{1}{-i\kappa_0} \hat{n}^+ \times \nabla \times \mathbf{E} \in \mathbf{H}^{-1/2}(\text{div}, \partial \Omega^+) \quad (6.6) \\
\mathbf{j}^- &= \frac{1}{-i\kappa_0} \hat{n}^- \times \nabla \times \mathbf{E} \in \mathbf{H}^{-1/2}(\text{div}, \partial \Omega^-) \quad (6.7) \\
\mathbf{e}^+ &= \hat{n}^+ \times \mathbf{E} \times \hat{n}^+ \in \mathbf{H}^{-1/2}(\text{curl}, \partial \Omega^+) \quad (6.8) \\
\mathbf{e}^- &= \hat{n}^- \times \mathbf{E} \times \hat{n}^- \in \mathbf{H}^{-1/2}(\text{curl}, \partial \Omega^-) \quad (6.9)
\end{align*}

- \eta \mathbf{j}^- + \mathbf{e}^- = \eta \mathbf{j}^+ + \mathbf{e}^+ \in \partial \Omega^- \quad (6.10)
- \mathbf{j}^+ + \mathbf{j}^+ = \mathbf{j}^- + \mathbf{e}^- \in \partial \Omega^+ \quad (6.11)
The multi-trace CFIE formulation is shown in Equation 6.12.

\[
\alpha e^+ + (1 - \alpha) j^+ - C^k_\alpha (j^+; \partial \Omega^+) - \hat{n} \times C^k_{(1-\alpha)} (e^+ \times \hat{n}; \partial \Omega^+) = y^{inc} \in \partial \Omega^+. \quad (6.12)
\]

The matrix equation in Equation 6.13 shows concisely how the final solution is computed.

\[
\begin{bmatrix}
A^{\text{FEM}} & N \\
N^T & A^{\text{BI}}
\end{bmatrix}
\begin{bmatrix}
x^{\text{FEM}} \\
x^{\text{BI}}
\end{bmatrix} =
\begin{bmatrix}
0 \\
b^{\text{BI}}
\end{bmatrix}. \quad (6.13)
\]

There are many benefits to the FEBI formulation. In the context of this research the two biggest benefits are its ability to easily match boundary conditions with a GA-IE-DDM formulation as well as the ease with which the Schur complement technique can be applied, which is described in Section 6.2.2.

### 6.2.2 Schur Complement

The antenna is modeled using a hybrid finite element-boundary integral (FEBI) discretization discussed in Section 6.2.1. Specifically, the antenna is modeled with finite elements, and then a large airbox encloses the antenna whose surface is modeled with boundary elements. The box is much larger than the antenna so that when the antenna is changed the field propagated to the edge of the box does not change. [CHECK THIS]. A simplified model of an FEBI antenna mounted on a PEC platform is shown in Figure 6.3.
The antenna is contained within $\Omega_1$ and BI/FEM denotes the boundary element surface or the interior finite element antenna. The sub-regions $\Omega_2$ and $\Omega_3$ are both PEC platforms. There are three fundamental physical interactions that must be considered or neglected in our model:

- Every PEC sub-domain couples to every BI sub-domain
- Every FE antenna *only* couples only to its “partner” BI sub-domain
- No PEC sub-domains may couple to FE antennas (directly)

Therefore, the antenna radiation coupling problem shown in Figure 6.3 can be solved according to Equation 6.14.

$$
\begin{bmatrix}
B_1 & N_{12} & 0 & 0 \\
N_{21} & A_2 & C_{23} & C_{24} \\
0 & C_{32} & A_3 & C_{34} \\
0 & C_{42} & C_{43} & A_4
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
=
\begin{bmatrix}
b_1 \\
b_2 \\
b_3 \\
b_4
\end{bmatrix}.
$$

(6.14)
Chapter 6. *Data-sparse Rapid Antenna Prototyping by the Schur Complement*

The sub-domain solver for the finite element antenna is denoted by $B_1$, $N_{ij}$ represents the coupling between the finite element antenna and the boundary integral airbox, $C_{ij}$ is the coupling between any boundary integral sub-domain, and $A_i$ is the sub-domain solution for any boundary integral sub-domain. The 4-by-4 matrix in Equation 6.14 can be reduced to the 3-by-3 matrix in Equation 6.15.

$$
\begin{bmatrix}
B_1 & N_{12} & 0 & 0 \\
N_{21} & A_2 & C_{23} & C_{24} \\
0 & C_{32} & A_3 & C_{34} \\
0 & C_{42} & C_{43} & A_4
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} =
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}.
$$

(6.15)

After using the techniques of Schur complement to subtract the PEC problem from the antenna problem the matrix equation in Equation 6.15 can be reduced to Equation 6.16, a 2-by-2 matrix.

$$
\begin{bmatrix}
B_1 & N_{12} \\
N_{21}^T & A_2 - [C_{23} & C_{24}]
\end{bmatrix}
\begin{bmatrix}
A_3 & C_{34} \\
C_{43} & A_4
\end{bmatrix}^{-1}
\begin{bmatrix}
C_{32} \\
C_{42}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} =
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}.
$$

(6.16)

Similar modifications have been made to the right hand side of the equation. We have decoupled the large static PEC platform from the hybrid finite-element boundary-integral antenna. Note that the $C^T A^{-1} C$ term is solved using the work in [81] and can be readily accomplished in an HPC-enabled environment. The ultimate goal is to not have to repeat computations. Note that due to a DG formulation, when the antenna inside the box is changed, the coupling between the box and platform is
not changed. Once the coupling between the antenna and platform is found it can be used in an FEBI solution routine to solve for $x_1$. 

Equation 6.16 is solved in two steps: an offline stage to solve $C^TA^{-1}C$ and an on-line stage to solve for $B_1$. The benefits of this formulation are two-fold: i) solution of the matrix equation only needs to be performed once if the airbox remains unchanged, ii) the finite element antenna can be changed without needing another expensive computation.

From a physical interpretation the algorithm is as follows:

1. Feed the boundary element box around the antenna with a uniform, arbitrary magnitude

2. For each boundary element:
   (a) Couple outgoing field to platform
   (b) Iteratively solve for the platform solution
   (c) Couple platform solution back to the boundary element
   (d) Output result as a column vector

3. Compute field pattern of antenna based on current on antenna

### 6.2.3 Skeletonization

This work builds off the work in Section 6.2.2. In this section, the goal is to reduce the number of degrees-of-freedom for an antenna modeled by FEBI. Let’s begin with Equation 6.16. The simple research question to be answered is, can the
computations of \( C^T A^{-1} C \), or the PEC platform contribution, be further sped up? The answer is, yes, we can find a skeleton of the antenna, as shown in Equation 6.17.

\[
\begin{bmatrix}
C_{21} \\
C_{31}
\end{bmatrix} = 
\begin{bmatrix}
\tilde{C}_{21} \\
\tilde{C}_{31}
\end{bmatrix}
\begin{bmatrix}
R_1
\end{bmatrix} + \mathcal{O}(\varepsilon).
\]

(6.17)

This uses interpolative decomposition to compress the matrix. Only a subset of the DOFs of the antenna are used. Algorithmically, a mapping between the full discretization and the skeletonized discretization is created and an extra step is performed to map the skeletonized discretization back to the original discretization.

### 6.2.4 Generalized formulation

What follows is a generalized formulation for three cases: i) M antennas on 1 PEC platform; ii) 1 antenna on N PEC platforms; iii) M antennas on N PEC platforms.

**M antennas on 1 PEC sub-domain**

Consider an arbitrary number of antennas on a PEC platform as shown in Figure 6.4. Each antenna is modeled with the hybrid FEBI method, and the PEC platforms use a traditional surface integral equation formulation.

The PEC platform is contained entirely within one sub-domain, \( \Omega_1 \). Each antenna contains two sub-domains, a boundary element airbox \( \Omega_{2m} \) and a finite element antenna \( \Omega_{2m+1} \). The interactions between the constituent sub-domains is given in
Once the Schur complement technique is applied and the PEC platform part of the problem is subtracted from the boundary element airboxes we reach Equation 6.19. On the surface it appears that this will require solving the PEC problem $M^2$ times, but due to the symmetry of the problem and the sequencing of the algorithm
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

d this can be reduced to $2M$ times.

\[
\begin{bmatrix}
A_1 & \cdots & C_{1(2m)} & 0 & \cdots & C_{1(2M)} & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & A_{2m} - C_{(2m)1}A_1^{-1}C_{1(2m)} & N_{(2m)(2m+1)} & \cdots & C_{(2m)(2M)} - C_{(2m+1)A_1^{-1}C_{1(2M)}} & 0 \\
0 & \cdots & N_{(2m+1)(2m)} & B_{(2m+1)} & \cdots & 0 & 0 \\
0 & \cdots & C_{(2M)(2m)} - C_{(2M)1}A_1^{-1}C_{1(2m)} & 0 & \cdots & A_{2M} - C_{(2M)1}A_1^{-1}C_{1(2M)} & N_{(2M)(2M+1)} \\
0 & \cdots & 0 & 0 & \cdots & N_{(2M+1)(2M)} & B_{(2M+1)}
\end{bmatrix}
\]

(6.19)

1 antenna on N PEC platforms

Consider one antenna on a PEC platform divided into N PEC sub-domains, as shown in Figure 6.5. Each antenna is modeled with the hybrid FEBI method, and the PEC platforms use a traditional surface integral equation formulation.

![Figure 6.5: 1 Antenna on N PEC platforms.](image)

The PEC platform is divided in N sub-domains. The antenna is modeled with the hybrid finite element boundary integral equation. The matrix shown in Equation
6.20 describes the interactions between the various constituent sub-regions.

\[
\begin{bmatrix}
B_1 & N_{12} & 0 & \ldots & 0 & \ldots & 0 \\
N_{21} & A_2 & C_{23} & \ldots & C_{2(n+2)} & \ldots & C_{2(N+2)} \\
0 & C_{32} & A_3 & \ldots & C_{3(n+2)} & \ldots & C_{3(N+2)} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & C_{(n+2)2} & C_{(n+2)3} & \ldots & A_{n+2} & \ldots & C_{(n+2)(N+2)} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & C_{(N+2)2} & C_{(N+2)3} & \ldots & C_{(N+2)(n+2)} & \ldots & A_{(N+2)}
\end{bmatrix}.
\]

As with the previous formulation, the PEC platform part of the problem is subtracted, resulting in Equation 6.21.

\[
\begin{bmatrix}
B_1 & N_{12} \\
N_{21} & A_2 - C_{2(N)}A^{-1}C_{N(2)}^T
\end{bmatrix},
\]

where \( A \) is the PEC problem and the coupling matrices are the BI surfaces coupling to the PEC surface they are mounted on.
M antennas on N PEC platforms

Consider M antennas on a PEC platform divided into N PEC sub-domains, as shown in Figure 6.6. As usual the antennas are modeled with FEBI and the PEC platform is modeled with a traditional boundary integral formulation.

The PEC platform is divided in N sub-domains. The M antennas are all modeled with the hybrid finite element boundary integral equation. The matrix shown in Equation 6.22 describes the interactions between the various constituent sub-regions. Once the PEC platform part of the problem is subtracted, the resulting matrix is shown in Equation 6.2.4.
\[
\begin{bmatrix}
B_1 & N_{12} & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\
N_{21} & A_2 & \ldots & 0 & C_{2(2m)} & \ldots & 0 & C_{2(2M)} & C_{2(2M+1)} & \ldots & C_{2(2M+n)} & \ldots & C_{2(2M+N)} \\
0 & 0 & \ldots & B_2 & N_{2(2m-1)} & \ldots & 0 & 0 & 0 & \ldots & 0 & \ldots & 0 \\
0 & C_{(2m)2} & N_{(2m)(2m-1)} & A_{2m} & \ldots & 0 & C_{(2m)(2M)} & C_{(2m)(2M+1)} & \ldots & C_{(2m)(2M+n)} & \ldots & C_{(2m)(2M+N)} \\
0 & 0 & \ldots & 0 & 0 & \ldots & B_{2M-1} & N_{(2M-1)(2M)} & 0 & \ldots & 0 & \ldots & 0 \\
0 & C_{(2M)2} & \ldots & 0 & C_{(2M)(2M)} & \ldots & N_{(2M)(2M-1)} & A_{2M} & C_{(2M)(2M+1)} & \ldots & C_{(2M)(2M+n)} & \ldots & C_{(2M)(2M+N)} \\
0 & C_{(2M+1)2} & \ldots & 0 & C_{(2M+1)(2M)} & \ldots & 0 & C_{(2M+1)(2M)} & A_{2M+1} & \ldots & C_{(2M+1)(2M+n)} & \ldots & C_{(2M+1)(2M+N)} \\
0 & C_{(2M+n)2} & \ldots & 0 & C_{(2M+n)(2M)} & \ldots & 0 & C_{(2M+n)(2M)} & C_{(2M+n)(2M+1)} & \ldots & A_{(2M+n)} & \ldots & C_{(2M+n)(2M+N)} \\
0 & C_{(2M+N)2} & \ldots & 0 & C_{(2M+N)(2M)} & \ldots & 0 & C_{(2M+N)(2M)} & C_{(2M+N)(2M+1)} & \ldots & C_{(2M+N)(2M+n)} & \ldots & A_{(2M+N)} \\
\end{bmatrix}
\]

(6.22)
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

The key steps to the problem formulation and reduction can be described as follows:

1. Setup the full matrix problem
   
   (a) Place all of the antenna sub-domains in the upper-left-hand quadrant
   
   (b) Place the PEC platform sub-domains, and corresponding coupling entries, in the lower-right-hand quadrant
   
   (c) The off-diagonal entries then become the coupling between the PEC platform and the antenna air boxes

2. Consider only the upper-left-hand sub-matrix

3. From each $C_{ij}$ and $A_i$, subtract appropriate PEC problem

4. Make similar adjustments for the right-hand-side of the matrix equation

The most simple version of the matrix equation is presented in Equation 6.23.

$$
\begin{bmatrix}
\text{Antenna Problem} & C_{(\text{Antenna})(\text{PEC})} \\
C_{(\text{PEC})(\text{Antenna})} & \text{PEC Problem}
\end{bmatrix}
\begin{bmatrix}
x_{\text{Antenna}} \\
x_{\text{PEC}}
\end{bmatrix} =
\begin{bmatrix}
b_{\text{Antenna}} \\
b_{\text{PEC}}
\end{bmatrix}.
$$ (6.23)

The antenna problem quadrant is a 2M-by-2M sub-matrix, and the PEC problem is a 2N-by-2N sub-matrix.
The subtraction of the PEC part of the problem can be further described by Equation 6.24.

\[
    X_{ij} \rightarrow X_{ij} = \left[ C_{i(2M+1)} \ldots C_{i(2M+n)} \ldots C_{i(2M+N)} \right] \left[ \begin{array}{c} \text{PEC} \\ \text{Problem} \end{array} \right]^{-1} \begin{bmatrix} C_{(2M+1)j} \\ \vdots \\ C_{(2M+n)j} \\ \vdots \\ C_{(2M+N)j} \end{bmatrix}.
\]

(6.24)

The term \( X_{ij} \) can refer to either a sub-domain solution term, or a coupling term. As a reminder, the number of antennas is \( M \) and the number of PEC platforms is \( N \). Each antenna has two associated sub-domains, a boundary integral airbox and a finite element antenna. The index \( i \) is used to track the antennas and the index \( j \) is used to track the PEC sub-domains.

6.3 Computational Methodology

We begin our discussion of the computational solution to antenna prototyping problems by focusing our attention on the term \( A_{2m} - C_{(2m)1}A_1^{-1}C_{1(2m)} \) in Equation 6.19. This term is a measure of how much electric surface current is returned to a
radiating antenna on a PEC platform. Furthermore, the second term \((A^{-1}_1C_{1(2m)})\) is nothing more than the solution to the problem discussed throughout this dissertation where the source term is a radiating antenna on the PEC platform. We therefore, use the following algorithm to solve this problem on a high performance supercomputing machine:

1. Excite the boundary integral surface of the antenna to a uniform current
2. Couple the boundary integral surface to the PEC surface
3. Iteratively solve for the electric surface current on the PEC surface.
4. Couple the electric current on the PEC surface back to the boundary integral surface of the antenna

A graphical description of the Schur complement algorithm presented above is provided in Figures 6.7(a)-6.7(d).

Step 3 is the same computation as discussed throughout this dissertation. Steps 2 and 4 are easily performed prior to and after step 3 by some minor modifications to the existing code structure, primarily in the file I/O algorithms. Step 1 is also easily accomplished by a simple modification of the existing code structure. The results of the above algorithm will produce a set of results that can be used in an FEBI code to solve an antenna radiation problem in free space. Equation 6.25 shows
Chapter 6. *Data-sparse Rapid Antenna Prototyping by the Schur Complement*

Figure 6.7: Schur complement algorithm.

the unmodified FEBI system matrix, and Equation 6.26 shows how the results of the above described algorithm are used if the PEC platform can be described by one sub-domain $\Omega_3$. The below algorithms can all be performed on a local workstation as long as the complexity of the antenna is relatively low.
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

\[
\begin{bmatrix}
B_1 & N_{12} \\
N_{21} & A_2 - C_{23}A_3^{-1}C_{32}
\end{bmatrix}
\] \hspace{1cm} (6.26)

Finally, to implement the skeletonization of the Schur complement is also a simple task of bookending the Schur complement algorithm with some minor changes.

1. Identify the elements of the skeleton
2. Perform the Schur complement algorithm on only the skeleton
3. Map the result back to the original discretization

6.4 Results

A validation of the proposed methodology for rapid prototyping is presented below. We also present results to demonstrate the capability and utility of this method.

6.4.1 Validation of Methodology

In the first validation experiment we consider the antenna radiation problems of both a monopole and patch antenna on a PEC box. The operating frequency
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

considered is 1.5 GHz. This validation process assumes that the methods developed in GA-IE-DDM are accurate. A view of the finite element antenna is shown in Figures 6.8(a) and 6.8(c) and the boundary element box surrounding the antennas is shown in Figures 6.8(b) and 6.8(d).

Figure 6.8: Antenna models used for validation of Schur complement technique.
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

The patch and monopole antenna problems were both simulated at 1.5 GHz using three different solution techniques: i) GA-IE-DDM; ii) Schur complement; iii) Data-sparse Schur complement. All three simulations converged and then postprocessing codes were run to compute the radiation characteristics. The S-parameters were computed and the results are presented in Table 6.1. We see close agreement across all solution techniques.

<table>
<thead>
<tr>
<th></th>
<th>GA [dB]</th>
<th>Schur [dB]</th>
<th>Skeleton (1e-3) [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopole @ 1.5GHz</td>
<td>-12.65</td>
<td>-12.68</td>
<td>-12.69</td>
</tr>
<tr>
<td>Patch @ 1.5GHz</td>
<td>-12.90</td>
<td>-12.46</td>
<td>-12.46</td>
</tr>
</tbody>
</table>

Table 6.1: $S_{11}$ parameters for single antenna on PEC plate.

In a similar manner, we have performed the same experiment on a model with two antennas on one PEC platform. This time we plot the surface currents in Figures 6.9(b) and 6.9(c). There is good agreement in the current pattern between the Schur complement method and the GA-IE-DDM method.

In the next experiment we test the validity of the skeletonization compression technique. At three different compression levels the data-sparse Schur complement technique was used to compute the electric surface current on the surface of the antenna and to compute the S parameters. Figures 6.10(a)-6.10(c) show the skeletonization model used and Figures 6.10(d)-6.10(f).

The S parameters and compression factors are given in Table 6.2. The compres-
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

![Two antenna model.](image)

(a) Two antenna model.

![Schur method.](image)

(b) Schur method; $\|S_{11}\| = 0.382$.

![GA-IE-DDM method.](image)

(c) GA-IE-DDM method; $\|S_{11}\| = 0.377$.

Figure 6.9: Two antennas on PEC platform with different solution techniques.

Compression factor is defined as the ratio of the number of total DOFs in the FEBI antenna discretization to the number of DOFs in the skeletonized discretization.

<table>
<thead>
<tr>
<th>Compression Error</th>
<th>Compression Factor</th>
<th>$S_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-8}$</td>
<td>0</td>
<td>0.382</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>3.5</td>
<td>0.409</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>6</td>
<td>0.413</td>
</tr>
</tbody>
</table>

Table 6.2: $S_{11}$ parameters for single antenna on PEC plate.

Finally, we have also tested the Schur complement technique on a large aircraft carrier at 75 MHz. The surface current is plotted in Figure 6.11(a) and a zoomed in
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

Figure 6.10: Skeletonization example

(a) Two antenna model; (b) Two antenna model; (c) Two antenna model; compression error = $10^{-8}$. compression error = $10^{-3}$. compression error = $10^{-2}$.

(d) Current on two antennas; (e) Current on two antennas; compression error = $10^{-8}$. compression error = $10^{-3}$. compression error = $10^{-2}$.

Figure 6.10: Skeletonization example

view of the antenna is in Figure 6.11(b). The current pattern matches both physical expectations as well as the solution from GA-IE-DDM.

Figure 6.11: 75MHz antenna on aircraft carrier

(a) Monopole antenna pattern on aircraft carrier @ 75MHz. (b) Zoomed in view of monopole antenna on aircraft carrier.

Figure 6.11: 75MHz antenna on aircraft carrier

The $S_{11}$ parameters for the aircraft carrier problem solved under different algo-
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

Figure 6.12: Effect of skeletonization on S11 parameters.

<table>
<thead>
<tr>
<th>Monopole</th>
<th>GA [dB]</th>
<th>Schur [dB]</th>
<th>Skeleton (1e-3) [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-21.15</td>
<td>-24.27</td>
<td>-24.31</td>
</tr>
</tbody>
</table>

Table 6.3: $S_{11}$ parameters for single antenna on aircraft carrier platform; $f = 75$MHz.

Finally, a numerical experiment was conducted to test the validity of the skeletonization algorithm. As the compression error is increased the $S_{11}$ is measured. The results are plotted in Figure 6.12.

It is observed that as the number of DOFs used decreases, the $S_{11}$ both increases and has a higher error. We expect both of these results because when the skeleton is chosen, the basis functions that contribute the most to the current on the antenna are chosen. Therefore, when a postprocessing code is used to compute the $S_{11}$ which averages over all the basis functions a higher than expected average current is obtained.
Chapter 6. Data-sparse Rapid Antenna Prototyping by the Schur Complement

6.4.2 Plug and play antenna design

One goal of this research is to develop a method that allows the antenna design engineer to compute the field pattern on a large platform one time, and then be able to change the antenna type, size, or location within the boundary integral box. A monopole antenna was mounted on a PEC box, and an entire simulation was run in the GA-IE-DDM framework, which included outputting the coupling of the PEC platform to the antenna. These results were then used with a patch antenna design that fit within the same PEC box. The S-parameter for the patch antenna was computed to be -12.46 dB, which is the same as previous GA-IE-DDM based calculations. With a large design space, this provides a much faster way of solving antenna design challenges as the electric currents on the platform need only be calculated one time.
Chapter 7

Conclusions

In this dissertation we have developed a parallel implementation of the combined field integral equation for extreme-scale, time-harmonic computational electromagnetics utilizing the domain decomposition method. We also investigated several important applications, such as electromagnetic scattering, antenna radiation, co-site interference, and communications problems, that require the use of high performance computing algorithms and their accompanying systems.

In Chapter 3 we introduced the basic problem of electromagnetic scattering from a single PEC sphere, and developed the weak discontinuous Galerkin formulation. We proceeded to decompose the domain into sub-domains and form the system matrix. We then performed a series of numerical experiments to demonstrate various
Chapter 7. Conclusions

numerical properties, such as the matrix conditioning, convergence, and accuracy of this method. All of the experiments, except for some of the convergence tests, could be solved on a single workstation. Chapter 4 focuses more on the capabilities of the algorithms than the underlying formulations. We developed a strategy for how high performance computing systems could be brought to bear on extreme-scale CEM by parallelizing every stage of the computation. Where feasible, we used physics-based modeling insights to consider certain fast algorithms. We conducted numerical experiments to benchmark the scalability of the algorithm. Finally, we concluded by solving the electromagnetic scattering problem from an aircraft carrier.

In Chapter 5 and 6 we showed how this parallel framework can be used to solve problems of great importance to the antennas and propagation community. In 5 we introduce a problem of cellular communications in dense, urban environments for fifth generation technology. We then tested our algorithms against two experiments from literature in order to verify our results. Validation against the first experiment was unsuccessful due to a large error in the experiment that could not be accounted for. We then solve a problem of radiowave propagation in a downtown city block of Albuquerque, NM. In 6 we introduced a method by which an antenna can be decoupled from its large, static platform to greatly reduce the time complexity of many antenna prototyping problems. Furthermore, after application of this problem we have the surface electric current on the platform and the radiation pattern for any
Chapter 7. Conclusions

antenna can be solved for down the road as long as it fits into the pre-determined airbox.

The future directions of this work include improvements to the computational algorithms, including a full implementation of FAFFA and a load balancing scheme. In rapid antenna prototyping the next step would be to introduce large, complex antenna arrays rather than just the monopoles that were used in this dissertation. For the channel modeling work, the next step is to study cell phone communication in an office building for 5G. The future directions of this work are discussed in much greater detail in Chapter 8.
Chapter 8

Future Work

This chapter will present the next immediate steps of the project as well as a longterm vision for extreme-scale CEM.

8.1 Fast far-field approximation algorithm

The primary motivation of this work is to speed up the computation of long-range antenna coupling problems by reducing a $O(C_1 M^2)$ operation to a $O(C_2 M^2)$ operation where $C_1 \gg C_2$. This is accomplished by replacing the translation stage of MLFMA with a Taylor expansion of the plane wave. In Figure 8.1 we see the computational sub-domain partitioning of a large aircraft carrier.
Chapter 8. Future Work

Figure 8.1: Computational domain decomposition of an aircraft carrier.

It should be noted that sub-domain A and B can be considered in the far-field at high frequencies. Formally, this can be stated as $R_{mn} \gg r_{im} \cup r_{nj}$ where $r_{im}$ and $r_{nj}$ are the FMM group radii. The original work on the underlying theory is presented in [12]. The development of the theory begins by expanding the planewave function in Equation 8.1.

\[
e^{i k |\mathbf{r}_i - \mathbf{r}_j|} \approx \frac{e^{ik_0(\mathbf{r}_{im}+\mathbf{r}_{nj})} e^{i k_0 \cdot \mathbf{r}_{mn}}}{r_{mn}}.
\] (8.1)

The planewave function has been expanded about $r_{mn} \gg r_{im} + r_{nj}$. We can re-write Equation 8.1 in order to gain some physical insight into the Taylor expansion, as shown in Equation 8.2.

\[
e^{i k |\mathbf{r}_i - \mathbf{r}_j|} \approx \frac{tk e^{i k_0(\mathbf{r}_{im}+\mathbf{r}_{nj})}}{4\pi} \frac{4\pi e^{i k_0 \cdot \mathbf{r}_{mn}}}{tk} \frac{r_{mn}}{r_{mn}}.
\] (8.2)

The coupling term of the DDM matrix can be succinctly described by Equation
Chapter 8. Future Work

8.3.

\[ Z_{ij} = \frac{ik}{4\pi} V_{im}(k_0) \cdot \alpha_{mn}^{far}(k_0) V_{nj}(k_0). \] (8.3)

If \( V_{im}(k_0) \) and \( V_{nj}(k_0) \) are available, then we may directly use them. Due to the discretization they are rarely, if ever, available directly. This means that alternative points must be used. The simplest choice is to use the \( r_{mn} \) that most closely follows the center-to-center vector from sub-domain A to B. We can accomplish this using the following algorithm:

1. Compute \( \hat{k}_i \cdot \hat{d} \forall i \)

2. Sort into list in descending order.

3. Select \( k_i \) that yields the largest dot product.

We can then use \( k_i \) in place of \( k_0 \) with reasonable accuracy, especially in the far-field. If we wish we can choose multiple \( k_i \)s and then use an appropriate interpolation scheme. The higher number of \( k_i \)s used, the lower the payoff in increased computational speed will be.

The above algorithm has already been implemented in the code. However, it still requires testing for accuracy and speed-up. The current implementation uses the criteria \( R_{mn} > A \cdot r_{im} \bigcup r_{nj} \) where \( A \) is a really large number.
Chapter 8. Future Work

A numerical experiment was conducted to measure the error as the separation of two spheres increases. The experimental setup is shown in Figure 8.2.

It should be noted that \( r_{mn} \gg R \) for the FAFFA method to be accurate; the numerical scale on the x-axis should be ignored. The surface current when \( f = 300 \text{ MHz} \), \( r_{mn} = 6 \text{ m} \), and \( R = 1 \text{ m} \) is plotted in Figure 8.3.

The error of this solution technique is evaluated in two ways. We first compare the error from a FAFFA-only method to an MLFMA-only method (8.4(a)), and then we compare the error when different numbers of interpolation points are used for the FAFFA translation (8.4(b)).

Finally, a numerical experiment was conducted to find the ideal FAFFA criteria, \( A \) from the criteria statement above, and those results are plotted in Figure 8.5.

These preliminary results are promising and demonstrate the accuracy of FAFFA. Additional numerical experiments should be conducted, which include timing the
Chapter 8. Future Work

Figure 8.3: FAFFA surface current results; $f = 300 \text{MHz}$.

(a) DD error convergence for two well-separated spheres. (b) L2-Norm Comparison for two different methods.

Figure 8.4: Error for FAFFA

FAFFA computations and comparing to MLFMA computations, as well as implementing the proposed algorithms for larger-scale computations. This technique only
has realizable payoffs for large problems when the sub-domain count is in the few hundreds.

### 8.2 Load balancing

One observation made when solving some of the extreme-scale problems within this dissertation is that the sub-domain solver is not always well load balanced. The astute reader will recall that an automatic partitioning scheme has been used where each sub-domain contains roughly the same number of DOFs. It turns out that although this works reasonably well for well-formed sub-domains, it does not in sub-domains where there is a large variation in discretization size or where there are sharp
Chapter 8. Future Work

edges in the geometry. Figure 4.13 highlights this problem as several sub-domains can be viewed as requiring different amounts of time for convergence to be reached.

This can be solved through a load balancing scheme in one of two ways: i) the “problem” sub-domains can be further partitioned; ii) a task queue can be created and sub-domains can be solved according to the time predicted for solution. Solution $i$ requires that the simulation be terminated after the pre-conditioning stage and then submitted to the queue again; this can cause computer-hours to be wasted if the load balancing is not very good to begin with. Solution $ii$ requires that $N_{\text{sub-domains}} \gg N_{\text{CPUs}}$ otherwise any performance gains cannot be realized in improved efficiency. Both solutions are valid, and both can be useful under different scenarios.

The efficiency of any load balancing scheme is an important metric in determining which method is most efficient. If we assume that there exists an ideal time for execution, then the efficiency can be defined by Equation 8.4, and the ideal wallclock time is given by Equation 8.5; $N$ is the number of tasks to complete and $M$ is the number of CPUs available for computation.

\[
\eta := \frac{t_{\text{ideal}}}{t_{\text{actual}}}. \quad (8.4)
\]
Chapter 8. Future Work

\[ t_{\text{wall}} = \frac{1}{M} \sum_{i=1}^{N} t_i. \] (8.5)

There are three general categories for a load balancing algorithm: i) naive; ii) static; iii) dynamic. The current method is a naive one. It contains the underlying assumption that each degree-of-freedom takes the same amount of CPU time to solve. As Figure 4.13 demonstrates this is not the case. Among the static options that exist include running the pre-conditioner stage and then further partitioning sub-domains that are far from the average time to solution. The other static option is to provide a different execution order for every iteration, and then assign every sub-domain to a CPU prior to the iteration. The final option, a dynamic one, is to order the sub-domains according to the time for the last iteration and then dynamically accept sub-domains as a CPU becomes available.

8.3 Channel Modeling at 5G

In Chapter 5 we were able to compute the full-wave solution to compute the pathloss in a dense urban environment at 2.4GHz. While impressive, this does not match current state-of-the-art solutions. It is possible that the solutions proposed within this dissertation could be used to calculate the EM propagation within a single office building at fifth generation frequencies (≈ 90 GHz). Additionally, including the
wave-stochastic work already done in this field could provide valuable in predicting
EM wave behavior within a dense urban environment. A common experiment that
communication engineers perform is to measure channel characteristics in an office
environment, and this could be accomplished at mm-wave frequencies.
Appendix A

Proof of consistency for IEDG

This appendix will prove the consistency of Equation 3.3 when the exact solution of $j$ is used, which is mathematically stated in Equation A.1.

$$a_h (v^h, j) = \frac{1}{2} \langle v^h, e^{inc} \rangle_{S^h} + \frac{1}{2} \langle v^h, j^{inc} \rangle_{S^h} \forall v^h \in X^h$$  \hspace{1cm} (A.1)

When the exact solution of the surface electric current is used there is no jump across the sub-domain boundaries and two terms can be eliminated as shown in Equation A.2.

$$a_h (v^h, j^h) := \frac{i k_0}{2} \sum_{m=1}^{M} \langle v^h_m, \sum_{n=1}^{M} \Psi_A (J^h_n; T_n) \rangle_{T_m} + \frac{1}{2 i k_0} \sum_{m=1}^{M} \langle \nabla_T \cdot v^h_m, \sum_{n=1}^{M} \Psi_F (\nabla_T \cdot J^h_n; T_n) \rangle_{T_m}$$

$$+ \frac{1}{4 i k_0} \sum_{m=1}^{M} \langle v^h_m, J^h_m \rangle_{T_m} + \frac{1}{2} \sum_{m=1}^{M} \langle \pi_x (v^h_m) \cdot \sum_{n=1}^{M} \sigma (J^h_n; T_n) \rangle_{T_m}$$  \hspace{1cm} (A.2)
Appendix A. Proof of consistency for IEDG

Using the definition of the jump operator, the term involving discontinuous testing functions can be re-written as shown in Equation A.3.

\[-\frac{1}{4ik_0} \sum_{C_{mn} \in \mathcal{C}} \left( \mathbf{v}^h_{m,n} \cdot \sum_{n=1}^{M} \Psi_F (\nabla_\tau \cdot \mathbf{j}_n ; \mathbf{T}_n) \right)_{C_{mn}} = -\frac{1}{2ik_0} \sum_{n=1}^{M} \left( \mathbf{v}_m \cdot \mathbf{t}_m, \sum_{n=1}^{M} \Psi_F (\nabla_\tau \cdot \mathbf{j}_n ; \mathbf{S}_n) \right)_{C_m} \tag{A.3} \]

After applying the surface Green’s formula the bilinear form reduces to Equation A.4.

\[
a_h (\mathbf{v}^h, \mathbf{j}) = \frac{ik_0}{2} \sum_{m=1}^{M} \left( \mathbf{v}_m^h, \sum_{n=1}^{M} \Psi_A (\mathbf{j}_n ; \mathbf{T}_n) \right)_{\mathbf{T}_m} - \frac{1}{2ik_0} \sum_{m=1}^{M} \left( \mathbf{v}_m^h, \sum_{n=1}^{M} \nabla_\tau \Psi_F (\nabla_\tau \cdot \mathbf{j}_n ; \mathbf{T}_n) \right)_{\mathbf{T}_m} \\
+ \frac{1}{4} \sum_{m=1}^{M} \left( \mathbf{v}_m^h, \frac{1}{4} \mathbf{j}_m \right)_{\mathbf{T}_m} + \frac{1}{2} \sum_{m=1}^{M} \left( \pi \times (\mathbf{v}_m^h), \sum_{n=1}^{M} \bar{K} (\mathbf{j}_n ; \mathbf{T}_n) \right)_{\mathbf{T}_m} \\
= \sum_{m=1}^{M} \left( \mathbf{v}_m^h, \frac{1}{2} \mathbf{j}^{\text{inc}} \right)_{\mathbf{T}_m} + \sum_{m=1}^{M} \left( \mathbf{v}_m^h, \frac{1}{2} \mathbf{e}^{\text{inc}} \right)_{\mathbf{T}_m} \tag{A.4} \]

Therefore, the weak formulation presented in this dissertation proposal is consistent. Equation 3.1 has been used in the final step of this proof.
Appendix B

CFIE Derivation

The derivation begins with the basic statement of the time-harmonic Maxwell’s equations:

\[
\nabla \cdot \mathbf{D} = \rho \quad \text{(B.1)}
\]

\[
\nabla \cdot \mathbf{B} = 0 \quad \text{(B.2)}
\]

\[
\nabla \times \mathbf{E} = \mathbf{j} \omega \mu \mathbf{H} \quad \text{(B.3)}
\]

\[
\nabla \times \mathbf{H} = \mathbf{j} - \omega \varepsilon \mathbf{E} \quad \text{(B.4)}
\]

We then proceed by taking the curl of Equation B.3. \(\nabla \times (\text{B.3})\):
Appendix B. CFIE Derivation

\[ \nabla \times \nabla \times \mathbf{E} = \omega \mu (\nabla \times \mathbf{H}) \]
\[ \nabla \times \nabla \times \mathbf{E} = \omega \mu \mathbf{J} + \omega^2 \mu \varepsilon \mathbf{E} \]  \hspace{1cm} (B.5)
\[ \nabla \times \nabla \times \mathbf{E} - k^2 \mathbf{E} = ik \eta \mathbf{J} \]

Where \( k = \omega \sqrt{\mu \varepsilon} \) and \( \eta = \sqrt{\frac{\mu}{\varepsilon}} \). Equation B.5 is called the vector wave equation, and it relates an electric field to an electric current. This equation can be solved more completely by setting up an equivalent problem using Green’s functions.

\[ \nabla \times \nabla \times \bar{\mathbf{G}} - k^2 \bar{\mathbf{G}} = \bar{\mathbf{I}} \delta (\mathbf{r} - \mathbf{r}') \]  \hspace{1cm} (B.6)

where,

\[ \bar{\mathbf{G}} (\mathbf{r}; \mathbf{r}') = \left( \bar{\mathbf{I}} + \frac{\nabla \nabla}{k^2} \right) g (\mathbf{r}; \mathbf{r}') \]
\[ g (\mathbf{r}; \mathbf{r}') = \frac{e^{-ik|\mathbf{r} - \mathbf{r}'|}}{4\pi |\mathbf{r} - \mathbf{r}'|} \]

We perform the operation \( \mathbf{E} \cdot (\bar{\mathbf{G}}) - \bar{\mathbf{G}} \cdot \mathbf{E} \):
Appendix B. CFIE Derivation

\[ E \cdot \left( \nabla \times \nabla \times \vec{G} \right) - k^2 E \cdot \vec{G} - \left( \nabla \times \nabla \times E \right) \cdot \vec{G} + k^2 E \cdot \vec{G} = E \delta (r - r') - \eta k J \cdot \vec{G} \]

We first work on the left-hand side of the equation

\[ E \cdot \left( \nabla \times \nabla \times \vec{G} \right) - \left( \nabla \times \nabla \times E \right) \cdot \vec{G} = E \delta (r - r') - \eta k J \cdot \vec{G} \]

We then take \( \iiint_{\Omega_{\text{ext}}} \text{(B.7)} \) dV, and work on the right-hand side first:
Appendix B. CFIE Derivation

\[ \iiint_{\Omega_{ext}} E \delta (r - r') \, dV = E (r) \]
\[ \iiint_{\Omega_{ext}} \eta k J (r') \cdot \vec{G} (r; r') \, dV = E^{inc} (r') \]

Working on the left side now:

\[ \iiint_{\Omega_{ext}} \nabla \cdot \left[ \nabla \times E \times \vec{G} - E \times \nabla \times \vec{G} \right] \, dV \]
\[ = \iint_{S} \left[ \nabla \times E \times \vec{G} - E \times \nabla \times \vec{G} \right] \cdot dS \]
\[ = \iint_{S} \hat{n} \cdot \left[ \nabla \times E \times \vec{G} - E \times \nabla \times \vec{G} \right] \, dS \]
Appendix B. CFIE Derivation

\[ \hat{n} \cdot \nabla \times E \times \vec{G} = \hat{n} \cdot E \times \nabla \times \vec{G} \]

\[ A = \nabla \times E \quad \quad \quad \quad \quad \quad \quad \quad \quad B = \nabla \times \vec{G} \]

\[ \hat{n} \cdot A \times \vec{G} = \hat{n} \cdot E \times \vec{B} \]

\[ A = \vec{G} \cdot \hat{n} \quad \quad \quad \quad \quad \quad \quad \quad \quad E = \vec{B} \cdot \hat{n} \]

\[ \hat{n} \times A \times \vec{G} = -E \times \hat{n} \cdot B \]

\[ \hat{n} \times \vec{A} \times \vec{G} = \hat{n} \times E \cdot \vec{B} \]

\[ \hat{n} \times \nabla \times E \cdot \vec{G} = \hat{n} \times \vec{E} \cdot \nabla \times \vec{G} \]

\[ \hat{n} \times \vec{G} \cdot \vec{H} = -M \cdot \nabla \times \vec{G} \]

\[ \hat{n} \times \vec{E} = \vec{E} \times \hat{n} \]

Where,

\[ J = \hat{n} \times H \]

\[ M = E \times \hat{n} \]

Plugging these simplifications into the original equation we obtain Equation B.8.

\[ E (r) = E^{inc} (r') + \iint_{S} \left[ \eta \mu J (r) \cdot \vec{G} (r; r') - M (r') \cdot \left( \nabla \times \vec{G} (r; r') \right) \right] dS \quad \text{(B.8)} \]
Appendix B. CFIE Derivation

Equation B.8 is known as the electric field integral equation (EFIE). To derive the magnetic field integral equation (MFIE) we apply the principle of duality to substitute magnetic quantities for electric quantities. In particular, the following four substitutions are used:

<table>
<thead>
<tr>
<th>EFIE Quantity</th>
<th>MFIE Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>H</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$\frac{1}{\eta}$</td>
</tr>
<tr>
<td>J</td>
<td>M</td>
</tr>
<tr>
<td>M</td>
<td>-J</td>
</tr>
</tbody>
</table>

Using the substitutions in Table B.1, we can obtain the magnetic field integral equation, as shown in Equation B.11.

$$H(r) = H^{\text{inc}}(r') + \iint_S \left[ \frac{ik}{\eta} M(r) \cdot \tilde{G}(r; r') + J(r') \cdot \nabla \times \tilde{G}(r; r') \right] dS$$ (B.9)

On a perfect electric conducting (PEC) surface this equation can be greatly simplified. Since the tangential fields are always continuous, the electric fields are zero; a PEC surface indicates the use of Dirichlet boundary conditions. We also apply the principal value integration technique.
Appendix B. CFIE Derivation

\[ \hat{n} \times H(r) = \hat{n} \times \left[ H_{\text{inc}}(r') \right] \\
+ \oint_{S} \left[ \frac{ik}{\eta} M(r) - \tilde{G}(r; r') + J(r') \cdot \nabla \times \tilde{G}(r; r') \right] dS \]

\[ = \hat{n} \times H_{\text{inc}}(r') + \hat{n} \times \oint_{S} J(r') \cdot \nabla \times \tilde{G}(r; r') dS \]

\[ \nabla \times \tilde{G} \cdot J = \nabla \times \left( \hat{I} + \frac{\nabla \nabla'}{k^2} g(r; r') \right) \cdot J \]

\[ = \nabla g \times J + g \nabla \times J \]

\[ \hat{n} \times \oint_{S} J(r') \cdot \nabla \times \tilde{G}(r; r') dS = \frac{1}{2} J(r) + \hat{n} \times \text{P.V.} \oint_{S} \frac{ik}{\eta} \nabla g(r; r') \times J(r') \]

(B.10)

Making the above substitutions leads to the MFIE for a PEC boundary, as shown in Equation B.11.

\[ \frac{1}{2} J(r) = J_{\text{inc}}(r') + \hat{n} \times \text{P.V.} \oint_{S} \frac{ik}{\eta} \nabla g (r; r') \times J (r') \] (B.11)

To eliminate internal resonance problems the EFIE and MFIE are combined to form the CFIE in the following manner:

CFIE := \alpha \text{EFIE} + (1 - \alpha) \text{MFIE}
References


References


References


References


References


References


[80] Channel Modeling and Characterization. FP7-ICT-608637, MiWEBA, Deliverable 5.1, v1.0, June 2014.

References


References


References


References

