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Abstract: Neutrosophic sets (NSs) handle uncertain information while fuzzy sets (FSs) and intuitionistic fuzzy sets (IFs) fail to handle indeterminate information. Soft set theory, neutrosophic set theory, and rough set theory are different mathematical models for handling uncertainties and they are mutually related. The neutrosophic soft rough set (NSRS) model is a hybrid model by combining neutrosophic soft sets with rough sets. We apply neutrosophic soft rough sets to graphs. In this research paper, we introduce the idea of neutrosophic soft rough graphs (NSRGs) and describe different methods of their construction. We consider the application of NSRG in decision-making problems. In particular, we develop efficient algorithms to solve decision-making problems.

Keywords: neutrosophic soft rough sets; neutrosophic soft rough graphs; decision-making; algorithm

1. Introduction

Smarandache [1] initiated the concept of neutrosophic set (NS). Smarandache's NS is characterized by three parts: truth, indeterminacy, and falsity. Truth, indeterminacy and falsity membership values behave independently and deal with problems having uncertain, indeterminant and imprecise data. Wang et al. [2] gave a new concept of single valued neutrosophic sets (SVNSs) and defined the set theoretic operators on an instance of NS called SVNS. Peng et al. [3] discussed the operations of simplified neutrosophic numbers and introduced an outranking idea of simplified neutrosophic numbers.

Molodtsov [4] introduced the notion of soft set (SS) as a novel mathematical approach for handling uncertainties. Molodtsov's SSs gave us a new technique for dealing with uncertainty from the viewpoint of parameters. Maji et al. [5–7] introduced neutrosophic soft sets (NSSs), intuitionistic fuzzy soft sets and fuzzy soft sets (FSSs). In [8], Sahin and Kucuk presented NSS in the form of neutrosophic relations.

Theory of rough set (RS) was proposed by Pawlak [9] in 1982. Rough set theory is used to study the intelligence systems containing incomplete, uncertain or inexact information. The lower and upper approximation operators of RSs are used for managing hidden information in a system. Feng et al. [10] took a significant step to introduce parametrization tools in RSs. Meng et al. [11] provide further discussion of the combination of SSs, RSs and FSs. The existing results of RSs and other extended RSs such as rough fuzzy sets, generalized rough fuzzy sets, soft fuzzy rough sets and intuitionistic fuzzy rough sets based decision-making models have their advantages and limitations [12,13]. In a different way, rough set approximations have been constructed into the intuitionistic fuzzy environment and are known as intuitionistic fuzzy rough sets and rough intuitionistic fuzzy sets [14,15]. Zhang et al. [16,17] presented the notions of soft rough sets, soft rough intuitionistic fuzzy sets, intuitionistic fuzzy soft rough sets, its application in decision-making, and also introduced generalized intuitionistic fuzzy soft rough sets. Broumi et al. [18,19] developed a hybrid structure by combining

RSs and NSs, called RNSs, they also presented interval valued neutrosophic soft rough sets by combining interval valued neutrosophic soft sets and RSs. Yang et al. [20] proposed single valued neutrosophic rough sets (SVNRSs) by combining SVNSs and RSs and defined SVNRSs on two universes and established an algorithm for a decision-making problem based on SVNRSs on two universes. Akram and Nawaz [21] have introduced the concept of soft graphs and some operation on soft graphs. Certain concepts of fuzzy soft graphs and intuitionistic fuzzy soft graphs are discussed in [22–24]. Akram and Shahzadi [25] have introduced neutrosophic soft graphs. Zafar and Akram [26] introduced a rough fuzzy digraph and several basic notions concerning rough fuzzy digraphs. In this research paper, a neutrosophic soft rough set is a generalization of a neutrosophic rough set, and we introduce the idea of neutrosophic soft rough graphs (NSRGs) that are made by combining NSRSs with graphs and describe different methods of their construction. We consider the application of NSRG in decision-making problems and resolve the problem. In particular, we develop efficient algorithms to solve decision-making problems.

For other notations, terminologies and applications not mentioned in the paper, the readers are referred to [27–35].

2. Neutrosophic Soft Rough Information

In this section, we will introduce the notions of neutrosophic soft rough relation (NSRR), and NSRGs.

Definition 1. *Let Y be an initial universal set, $\mathbb{P}$ a universal set of parameters and $\mathbb{M} \subseteq \mathbb{P}$. For an arbitrary neutrosophic soft relation Q over $Y \times \mathbb{M}$, $(Y, \mathbb{M}, Q)$ is called neutrosophic soft approximation space (NSAS).*

For any NS $A \in \mathcal{N}(\mathbb{M})$, we define the upper neutrosophic soft rough approximation (UNSRA) and the lower neutrosophic soft rough approximation (LNSRA) operators of A with respect to $(Y, \mathbb{M}, Q)$ denoted by $\overline{Q}(A)$ and $\underline{Q}(A)$, respectively as follows:

$$\begin{aligned}\overline{Q}(A) &= \{(u, T_{\overline{Q}(A)}(u), I_{\overline{Q}(A)}(u), F_{\overline{Q}(A)}(u)) \mid u \in Y\},\\ \underline{Q}(A) &= \{(u, T_{\underline{Q}(A)}(u), I_{\underline{Q}(A)}(u), F_{\underline{Q}(A)}(u)) \mid u \in Y\},\end{aligned}$$

where

$$\begin{aligned}T_{\overline{Q}(A)}(u) &= \bigvee_{e\in\mathbb{M}} \big(T_{Q(A)}(u,e) \wedge T_A(e)\big), \quad I_{\overline{Q}(A)}(u) = \bigwedge_{e\in\mathbb{M}} \big(I_{Q(A)}(u,e) \vee I_A(e)\big),\\ F_{\overline{Q}(A)}(u) &= \bigwedge_{e\in\mathbb{M}} \big(F_{Q(A)}(u,e) \vee F_A(e)\big); \quad T_{\underline{Q}(A)}(u) = \bigwedge_{e\in\mathbb{M}} \big(F_{Q(A)}(u,e) \vee T_A(e)\big),\\ I_{\underline{Q}(A)}(u) &= \bigvee_{e\in\mathbb{M}} \big((1 - I_{Q(A)}(u,e)) \wedge I_A(e)\big),\ F_{\underline{Q}(A)}(u) = \bigvee_{e\in\mathbb{M}} \big(T_{Q(A)}(u,e) \wedge F_A(e)\big).\end{aligned}$$

The pair $(\underline{Q}(A), \overline{Q}(A))$ is called NSRS of A w.r.t $(Y, \mathbb{M}, Q)$, *$\underline{Q}$ and $\overline{Q}$ are referred to as the LNSRA and the UNSRA operators, respectively.*

Example 1. *Suppose that $Y = \{w_1, w_2, w_3, w_4\}$ is the set of careers under consideration, and Mr. X wants to select the best suitable career. $\mathbb{M} = \{e_1, e_2, e_3\}$ is a set of decision parameters. Mr. X describes the "most suitable career" by defining a neutrosophic soft set $(Q, \mathbb{M})$ on Y that is a neutrosophic relation from Y to $\mathbb{M}$ as shown in Table 1.*

Table 1. Neutrosophic soft relation Q.

Q	w_1	w_2	w_3	w_4
e_1	(0.3, 0.4, 0.5)	(0.4, 0.2, 0.3)	(0.1, 0.5, 0.4)	(0.2, 0.3, 0.4)
e_2	(0.1, 0.5, 0.4)	(0.3, 0.4, 0.6)	(0.4, 0.4, 0.3)	(0.5, 0.3, 0.8)
e_3	(0.3, 0.4, 0.4)	(0.4, 0.6, 0.7)	(0.3, 0.5, 0.4)	(0.5, 0.4, 0.6)

Now, Mr. X gives the most favorable decision object A, which is an NS on $\mathbb{M}$ *defined as follows:* $A = \{(e_1, 0.5, 0.2, 0.4), (e_2, 0.2, 0.3, 0.1), (e_3, 0.2, 0.4, 0.6)\}$. *By Definition 1, we have*

$$T_{\overline{Q}(A)}(w_1) = 0.3,\quad I_{\overline{Q}(A)}(w_1) = 0.4,\quad F_{\overline{Q}(A)}(w_1) = 0.4,$$
$$T_{\overline{Q}(A)}(w_2) = 0.4,\quad I_{\overline{Q}(A)}(w_2) = 0.2,\quad F_{\overline{Q}(A)}(w_2) = 0.4,$$
$$T_{\overline{Q}(A)}(w_3) = 0.2,\quad I_{\overline{Q}(A)}(w_3) = 0.4,\quad F_{\overline{Q}(A)}(w_3) = 0.3,$$
$$T_{\overline{Q}(A)}(w_4) = 0.2,\quad I_{\overline{Q}(A)}(w_4) = 0.3,\quad F_{\overline{Q}(A)}(w_4) = 0.4.$$

Similarly,

$$T_{\underline{Q}(A)}(w_1) = 0.4,\quad I_{\underline{Q}(A)}(w_1) = 0.4,\quad F_{\underline{Q}(A)}(w_1) = 0.3,$$
$$T_{\underline{Q}(A)}(w_2) = 0.5,\quad I_{\underline{Q}(A)}(w_2) = 0.4,\quad F_{\underline{Q}(A)}(w_2) = 0.4,$$
$$T_{\underline{Q}(A)}(w_3) = 0.4,\quad I_{\underline{Q}(A)}(w_3) = 0.4,\quad F_{\underline{Q}(A)}(w_3) = 0.3,$$
$$T_{\underline{Q}(A)}(w_4) = 0.5,\quad I_{\underline{Q}(A)}(w_4) = 0.4,\quad F_{\underline{Q}(A)}(w_4) = 0.5.$$

Thus, we obtain

$$\begin{aligned}
\overline{Q}(A) &= \{(w_1, 0.3, 0.4, 0.4), (w_2, 0.4, 0.2, 0.4), (w_3, 0.2, 0.4, 0.3), (w_4, 0.2, 0.3, 0.4)\},\\
\underline{Q}(A) &= \{(w_1, 0.4, 0.4, 0.3), (w_2, 0.5, 0.4, 0.4), (w_3, 0.4, 0.4, 0.3), (w_4, 0.5, 0.4, 0.5)\}.
\end{aligned}$$

Hence, $(\underline{Q}(A), \overline{Q}(A))$ *is an NSRS of A.*

The conventional neutrosophic soft set is a mapping from a parameter to the neutrosophic subset of the universe and letting $(Q, \mathbb{M})$ be neutrosophic soft set. Now, we present the constructive definition of neutrosophic soft rough relation by using a neutrosphic soft relation S from $\mathbb{M} \times \mathbb{M} = \acute{\mathbb{M}}$ to $\mathcal{N}(Y \times Y = \acute{Y})$, where Y is a universal set and $\mathbb{M}$ be a set of parameters.

Definition 2. *A neutrosophic soft rough relation* $(\underline{S}(B), \overline{S}(B))$ *on* Y *is an NSRS,* $S : \acute{\mathbb{M}} \to \mathcal{N}(\acute{Y})$ *is a neutrosophic soft relation on* Y *defined by*

$$\begin{aligned}
S(e_ie_j) &= \{u_iu_j \mid \exists u_i \in Q(e_i), u_j \in Q(e_j)\},\ u_iu_j \in \acute{Y},\ \textit{such that}\\
T_S(u_iu_j, e_ie_j) &\leqslant \min\{T_Q(u_i, e_i), T_Q(u_j, e_j)\}\\
I_S(u_iu_j, e_ie_j) &\leqslant \max\{I_Q(u_i, e_i), I_Q(u_j, e_j)\}\\
F_S(u_iu_j, e_ie_j) &\leqslant \max\{F_Q(u_i, e_i), F_Q(u_j, e_j)\}.\\
\textit{For any } B \in \mathcal{N}(\acute{\mathbb{M}}),\ B &= \{\big(e_ie_j, T_B(e_ie_j), I_B(e_ie_j), F_B(e_ie_j)\big)\ u_iu_j \in \acute{\mathbb{M}}\},\\
T_B(e_ie_j) &\leqslant \min\{T_A(e_i), T_A(e_j)\},\\
I_B(e_ie_j) &\leqslant \max\{I_A(e_i), I_A(e_j)\},\\
F_B(e_ie_j) &\leqslant \max\{F_A(e_i), F_A(e_j)\}.
\end{aligned}$$

The UNSA and the LNSA of B w.r.t $(\acute{Y}, \acute{\mathbb{M}}, S)$ *are defined as follows:*

$$\overline{S}(B) = \{(u_iu_j, T_{\overline{S}(B)}(u_iu_j), I_{\overline{S}(B)}(u_iu_j), F_{\overline{S}(B)}(u_iu_j)) \mid u_iu_j \in \acute{Y}\},$$

$$\underline{S}(B) = \{(u_iu_j, T_{\underline{S}(B)}(u_iu_j), I_{\underline{S}(B)}(u_iu_j), F_{\underline{S}(B)}(u_iu_j)) \mid u_iu_j \in \acute{Y}\},$$

where

$$\begin{aligned} T_{\overline{S}(B)}(u_iu_j) &= \bigvee_{e_ie_j \in \acute{\mathbb{M}}} \big(T_S(u_iu_j, e_ie_j) \wedge T_B(e_ie_j)\big), \\ I_{\overline{S}(B)}(u_iu_j) &= \bigwedge_{e_ie_j \in \acute{\mathbb{M}}} \big(I_S(u_iu_j, e_ie_j) \vee I_B(e_ie_j)\big), \\ F_{\overline{S}(B)}(u_iu_j) &= \bigwedge_{e_ie_j \in \acute{\mathbb{M}}} \big(F_S(u_iu_j, e_ie_j) \vee F_B(e_ie_j)\big); \end{aligned}$$

$$\begin{aligned} T_{\underline{S}(B)}(u_iu_j) &= \bigwedge_{e_ie_j \in \acute{\mathbb{M}}} \big(F_S(u_iu_j, e_ie_j) \vee T_B(e_ie_j)\big), \\ I_{\underline{S}(B)}(u_iu_j) &= \bigvee_{e_ie_j \in \acute{\mathbb{M}}} \big((1 - I_S(u_iu_j, e_ie_j)) \wedge I_B(e_ie_j)\big), \\ F_{\underline{S}(B)}(u_iu_j) &= \bigvee_{e_ie_j \in \acute{\mathbb{M}}} \big(T_S(u_iu_j, e_ie_j) \wedge F_B(e_ie_j)\big). \end{aligned}$$

The pair $(\underline{S}(B), \overline{S}(B))$ is called NSRR and $\underline{S}, \overline{S} : \mathcal{N}(\acute{\mathbb{M}}) \to \mathcal{N}(\acute{Y})$ are called the LNSRA and the UNSRA operators, respectively.

Remark 1. *Consider an NS B on $\acute{\mathbb{M}}$ and an NS A on $\mathbb{M}$, according to the definition of NSRR, we get*

$$\begin{aligned} T_{\overline{S}(B)}(u_iu_j) &\leqslant \min\{T_{\overline{S}(A)}(u_i), T_{\overline{S}(A)}(u_j)\}, \\ I_{\overline{S}(B)}(u_iu_j) &\leqslant \max\{I_{\overline{S}(A)}(u_i), I_{\overline{S}(A)}(u_j)\}, \\ F_{\overline{S}(B)}(u_iu_j) &\leqslant \max\{F_{\overline{S}(A)}(u_i).F_{\overline{S}(A)}(u_j)\}. \end{aligned}$$

Similarly, for LNSRA operator $\underline{S}(B)$,

$$\begin{aligned} T_{\underline{S}(B)}(u_iu_j) &\leqslant \min\{T_{\underline{S}(A)}(u_i), T_{\underline{S}(A)}(u_j)\}, \\ I_{\underline{S}(B)}(u_iu_j) &\leqslant \max\{I_{\underline{S}(A)}(u_i), I_{\underline{S}(A)}(u_j)\}, \\ F_{\underline{S}(B)}(u_iu_j) &\leqslant \max\{F_{\underline{S}(A)}(u_i).F_{\underline{S}(A)}(u_j)\}. \end{aligned}$$

Example 2. *Let $Y = \{u_1, u_2, u_3\}$ be a universal set and $\mathbb{M} = \{e_1, e_2, e_3\}$ a set of parameters. A neutrosophic soft set $(Q, \mathbb{M})$ on Y can be defined in tabular form in Table 2 as follows:*

Table 2. Neutrosophic soft set $(Q, \mathbb{M})$.

Q	u_1	u_2	u_3
e_1	$(0.4, 0.5, 0.6)$	$(0.7, 0.3, 0.2)$	$(0.6, 0.3, 0.4)$
e_2	$(0.5, 0.3, 0.6)$	$(0.3, 0.4, 0.3)$	$(0.7, 0.2, 0.3)$
e_3	$(0.7, 0.2, 0.3)$	$(0.6, 0.5, 0.4)$	$(0.7, 0.2, 0.4)$

Let $E = \{u_1u_2, u_2u_3, u_2u_2, u_3u_2\} \subseteq \acute{Y}$ and $L = \{e_1e_3, e_2e_1, e_3e_2\} \subseteq \acute{\mathbb{M}}$.
Then, a soft relation S on E (from L to E) can be defined in Table 3 as follows:

Table 3. Neutrosophic soft relation S.

S	u_1u_2	u_2u_3	u_2u_2	u_3u_2
e_1e_3	(0.4, 0.4, 0.5)	(0.6, 0.3, 0.4)	(0.5, 0.4, 0.2)	(0.5, 0.4, 0.3)
e_2e_1	(0.3, 0.3, 0.4)	(0.3, 0.2, 0.3)	(0.2, 0.3, 0.3)	(0.7, 0.2, 0.2)
e_3e_2	(0.3, 0.3, 0.2)	(0.5, 0.3, 0.2)	(0.2, 0.4, 0.4)	(0.3, 0.4, 0.4)

Let $A = \{(e_1, 0.2, 0.4, 0.6), (e_2, 0.4, 0.5, 0.2), (e_3, 0.1, 0.2, 0.4)\}$ *be an NS on* $\mathbb{M}$*, then*
$\overline{S}(A) = \{(u_1, 0.4, 0.2, 0.4), (u_2, 0.3, 0.4, 0.3), (u_3, 0.4, 0.2, 0.3)\}$,
$\underline{S}(A) = \{(u_1, 0.3, 0.5, 0.4), (u_2, 0.2, 0.5, 0.6), (u_3, 0.4, 0.5, 0.6)\}$.
Let $B = \{(e_1e_3, 0.1, 0.3, 0.5), (e_2e_1, 0.2, 0.4, 0.3), (e_3e_2, 0.1, 0.2, 0.3)\}$ *be an NS on L, then*
$\overline{S}(B) = \{(u_1u_2, 0.2, 0.3, 0.3), (u_2u_3, 0.2, 0.3, 0.3), (u_2u_2, 0.2, 0.4, 0.3), (u_3u_2, 0.2, 0.4, 0.3)\}$,
$\underline{S}(B) = \{(u_1u_2, 0.2, 0.4, 0.4), (u_2u_3, 0.2, 0.4, 0.5), (u_2u_2, 0.3, 0.4, 0.5), (u_3u_2, 0.2, 0.4, 0.5)\}$.
Hence, $S(B) = (\underline{S}(B), \overline{S}(B))$ *is NSRR.*

Definition 3. *A neutrosophic soft rough graph (NSRG) on a non-empty* V *is an 4-ordered tuple* $(V, \mathbb{M}, Q(A), S(B))$ *such that*

(i) $\mathbb{M}$ *is a set of parameters,*
(ii) Q *is an arbitrary neutrosophic soft relation over* $V \times \mathbb{M}$,
(iii) S *is an arbitrary neutrosophic soft relation over* $\acute{V} \times \acute{\mathbb{M}}$,
(vi) $Q(A) = (\underline{Q}A, \overline{Q}A)$ *is an NSRS of* A,
(v) $S(B) = (\underline{S}B, \overline{S}B)$ *is an NSRR on* $\acute{V} \subset V \times V$,
(iv) $G = (Q(A), S(B))$ *is a neutrosophic soft rough graph, where* $\underline{G} = (\underline{Q}A, \underline{S}B)$ *and* $\overline{G} = (\overline{Q}A, \overline{S}B)$ *are lower neutrosophic approximate graph (LNAG) and upper neutrosophic approximate graph (UNAG), respectively of neutrosophic soft rough graph (NSRG)* $G = (Q(A), S(B))$.

Example 3. *Let* $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ *be a vertex set and* $\mathbb{M} = \{e_1, e_2, e_3\}$ *a set of parameters. A neutrosophic soft relation over* $V \times \mathbb{M}$ *can be defined in tabular form in Table 4 as follows:*

Table 4. Neutrosophic soft relation Q.

Q	v_1	v_2	v_3	v_4	v_5	v_6
e_1	(0.4, 0.5, 0.6)	(0.7, 0.3, 0.5)	(0.6, 0.2, 0.3)	(0.4, 0.4, 0.2)	(0.5, 0.5, 0.6)	(0.4, 0.5, 0.6)
e_2	(0.5, 0.4, 0.2)	(0.6, 0.4, 0.5)	(0.7, 0.3, 0.4)	(0.5, 0.3, 0.2)	(0.4, 0.5, 0.4)	(0.6, 0.5, 0.4)
e_3	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.2)	(0.5, 0.4, 0.3)	(0.6, 0.2, 0.3)	(0.5, 0.4, 0.4)	(0.7, 0.3, 0.5)

Let $A = \{(e_1, 0.5, 0.4, 0.6), (e_2, 0.7, 0.4, 0.5), (e_3, 0.6, 0.2, 0.5)\}$ *be an NS on* $\mathbb{M}$*, then*

$$\begin{aligned} \overline{S}(A) &= \{(v_1, 0.5, 0.4, 0.5), (v_2, 0.6, 0.3, 0.5), (v_3, 0.7, 0.4, 0.5), (v_4, 0.6, 0.2, 0.5), (v_5, 0.5, \\ &\quad 0.4, 0.5), (v_6, 0.6, 0.3, 0.5)\}, \\ \underline{S}(A) &= \{(v_1, 0.6, 0.4, 0.5), (v_2, 0.5, 0.4, 0.6), (v_3, 0.5, 0.4, 0.6), (v_4, 0.5, 0.4, 0.5), (v_5, 0.6, \\ &\quad 0.4, 0.5), (v_6, 0.6, 0.4, 0.5)\}. \end{aligned}$$

Let $E = \{v_1v_1, v_1v_2, v_2v_1, v_2v_3, v_4v_5, v_3v_4, v_5v_2, v_5v_6\} \subseteq \acute{V}$ *and* $L = \{e_1e_3, e_2e_1, e_3e_2\} \subseteq \acute{\mathbb{M}}$. *Then, a neutrosophic soft relation S on E (from L to E) can be defined in Tables 5 and 6 as follows:*

Table 5. Neutrosophic soft relation S.

S	v_1v_1	v_1v_2	v_2v_1	v_2v_3
e_1e_2	(0.4, 0.4, 0.2)	(0.4, 0.4, 0.5)	(0.4, 0.4, 0.5)	(0.6, 0.3, 0.4)
e_2e_3	(0.5, 0.4, 0.1)	(0.4, 0.3, 0.2)	(0.4, 0.3, 0.2)	(0.5, 0.3, 0.2)
e_1e_3	(0.4, 0.4, 0.1)	(0.4, 0.2, 0.2)	(0.4, 0.2, 0.2)	(0.5, 0.3, 0.3)

Table 6. Neutrosophic soft relation S.

S	v_3v_4	v_4v_5	v_5v_2	v_5v_6
e_1e_2	$(0.4, 0.2, 0.2)$	$(0.4, 0.4, 0.2)$	$(0.4, 0.3, 0.4)$	$(0.3, 0.2, 0.3)$
e_2e_3	$(0.6, 0.2, 0.4)$	$(0.3, 0.2, 0.1)$	$(0.4, 0.3, 0.2)$	$(0.4, 0.3, 0.4)$
e_1e_3	$(0.4, 0.2, 0.3)$	$(0.4, 0.3, 0.1)$	$(0.5, 0.3, 0.2)$	$(0.5, 0.3, 0.5)$

$$
\begin{aligned}
\text{Let } B &= \{(e_1e_2, 0.4, 0.4, 0.5;), (e_2e_3, 0.5, 0.4, 0.5), (e_1e_3, 0.5, 0.2, 0.5)\} \text{ be an NS on } \mathrm{L}, \text{ then} \\
\overline{S}B &= \{(v_1v_1, 0.5, 0.4, 0.5), (v_1v_2, 0.4, 0.2, 0.5), (v_2v_1, 0.4, 0.2, 0.5), (v_2v_3, 0.5, 0.3, 0.5), \\
&\quad (v_3v_4, 0.5, 0.2, 0.5), (v_4v_5, 0.4, 0.3, 0.5), (v_5v_2, 0.5, 0.3, 0.5), (v_5v_6, 0.5, 0.3, 0.5)\}, \\
\underline{S}B &= \{(v_1v_1, 0.4, 0.4, 0.5)(v_1v_2, 0.5, 0.4, 0.4), (v_2v_1, 0.5, 0.4, 0.4), (v_2v_3, 0.4, 0.4, 0.5), \\
&\quad (v_3v_4, 0.4, 0.4, 0.5), (v_4v_5, 0.4, 0.4, 0.4), (v_5v_2, 0.4, 0.4, 0.5), (v_5v_6, 0.4, 0.4, 0.5)\}. \\
\text{Hence, } S(B) &= (\underline{S}B, \overline{S}B) \text{ is NSRR on } \acute{V}.
\end{aligned}
$$

Thus, $\underline{G} = (\underline{Q}A, \underline{S}B)$ *and* $\overline{G} = (\overline{Q}A, \overline{S}B)$ *are LNAG and UNAG, respectively, are shown in Figure 1.*

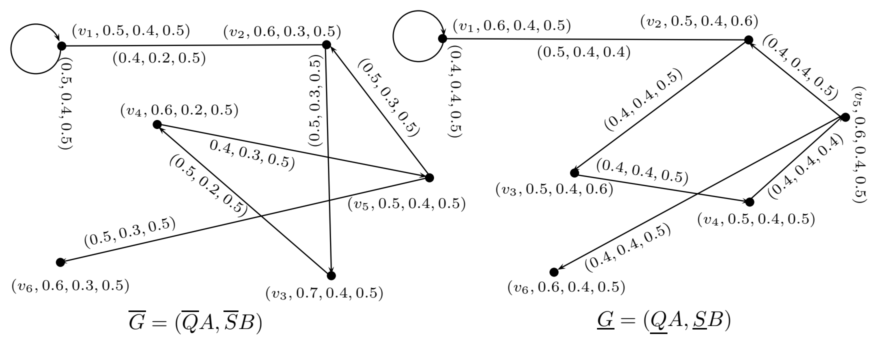


Figure 1. Neutrosophic soft rough graph $G = (\underline{G}, \overline{G})$

Hence, $G = (\underline{G}, \overline{G})$ *is NSRG.*

Definition 4. *Let* $G = (V, \mathbb{M}, Q, S)$ *be a neutrosophic soft rough graph on a non-empty set* V*. The order of* G *can be denoted by* $\mathbf{O}(G)$*, defined by*

$$
\begin{aligned}
\mathbf{O}(G) &= \mathbf{O}(\overline{G}) + \mathbf{O}(\underline{G}), \text{ where} \\
\mathbf{O}(\overline{G}) &= \sum_{v \in V} \overline{Q}A(v), \mathbf{O}(\underline{G}) = \sum_{v \in V} \underline{Q}A(v).
\end{aligned}
$$

The size of neutrosophic soft rough graph G*, denoted by* $\mathbf{S}(G)$*, defined by*

$$
\begin{aligned}
\mathbf{S}(G) &= (\mathbf{S}\overline{G} + \mathbf{S}\underline{G}), \text{ where} \\
\mathbf{S}(\overline{G}) &= \sum_{uv \in E} \overline{S}B(uv), \mathbf{S}(\underline{G}) = \sum_{uv \in E} \underline{S}B(uv).
\end{aligned}
$$

Example 4. *Let* G *be a neutrosophic soft rough graph as shown in Figure 1. Then,*

$$
\begin{aligned}
\mathbf{O}(\overline{G}) &= (3.5, 2.0, 3.0), \ \mathbf{O}(\underline{G}) = (3.3, 2.4, 3.2), \\
\mathbf{O}(G) &= \mathbf{O}(\overline{G}) + \mathbf{O}(\underline{G}) = (6.8, 4.4, 6.2), \text{ and} \\
\mathbf{S}(\overline{G}) &= (3.2, 1.8, 3.0) \ \mathbf{S}(\underline{G}) = (2.5, 2.4, 2.8) \\
\mathbf{S}(G) &= \mathbf{S}(\overline{G}) + \mathbf{S}(\underline{G}) = (5.7, 4.2, 5.8).
\end{aligned}
$$

Definition 5. *Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic soft rough graphs on V. The union of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1 \cup G_2 = (\underline{G}_1 \cup \underline{G}_2, \overline{G}_1 \cup \overline{G}_2)$, where $\underline{G}_1 \cup \underline{G}_2 = (\underline{Q}A_1 \cup \underline{Q}A_2, \underline{S}B_1 \cup \underline{S}B_2)$ and $\overline{G}_1 \cup \overline{G}_2 = (\overline{Q}A_1 \cup \overline{Q}A_2, \overline{S}B_1 \cup \overline{S}B_2)$ are neutrosophic graphs, such that*

(i) $\forall v \in QA_1$ *but* $v \notin QA_2$.

$$T_{\overline{Q}A_1 \cup \overline{Q}A_2}(v) = T_{\overline{Q}A_1}(v), T_{\underline{Q}A_1 \cup \underline{Q}A_2}(v) = T_{\underline{Q}A_1}(v),$$
$$I_{\overline{Q}A_1 \cup \overline{Q}A_2}(v) = I_{\overline{Q}A_1}(v), \; I_{\underline{Q}A_1 \cup \underline{Q}A_2}(v) = I_{\underline{Q}A_1}(v),$$
$$F_{\overline{Q}A_1 \cup \overline{Q}A_2}(v) = F_{\overline{Q}A_1}(v), F_{\underline{Q}A_1 \cup \underline{Q}A_2}(v) = F_{\underline{Q}A_1}(v).$$

(ii) $\forall v \notin QA_1$ *but* $v \in QA_2$.

$$T_{\overline{Q}A_1 \cup \overline{Q}A_2}(v) = T_{\overline{Q}A_2}(v), T_{\underline{Q}A_1 \cup \underline{Q}A_2}(v) = T_{\underline{Q}A_2}(v),$$
$$I_{\overline{Q}A_1 \cup \overline{Q}A_2}(v) = I_{\overline{Q}A_2}(v), \; I_{\underline{Q}A_1 \cup \underline{Q}A_2}(v) = I_{\underline{Q}A_2}(v),$$
$$F_{\overline{Q}A_1 \cup \overline{Q}A_2}(v) = F_{\overline{Q}A_2}(v), F_{\underline{Q}A_1 \cup \underline{Q}A_2}(v) = F_{\underline{Q}A_2}(v).$$

(iii) $\forall v \in QA_1 \cap \underline{Q}A_2$

$$T_{\overline{Q}A_1 \cup \overline{Q}A_2}(v) = \max\{T_{\overline{Q}A_1}(v), T_{\overline{Q}A_2}(v)\}, T_{\underline{Q}A_1 \cup \underline{Q}A_2}(v) = \max\{T_{\underline{Q}A_1}(v), T_{\underline{Q}A_2}(v)\},$$
$$I_{\overline{Q}A_1 \cup \overline{Q}A_2}(v) = \min\{I_{\overline{Q}A_1}(v), I_{\overline{Q}A_2}(v)\}, \; I_{\underline{Q}A_1 \cup \underline{Q}A_2}(v) = \min\{I_{\underline{Q}A_1}(v), I_{\underline{Q}A_2}(v)\},$$
$$F_{\overline{Q}A_1 \cup \overline{Q}A_2}(v) = \min\{F_{\overline{Q}A_1}(v), F_{\overline{Q}A_2}(v)\}, F_{\underline{Q}A_1 \cup \underline{Q}A_2}(v) = \min\{F_{\underline{Q}A_1}(v), F_{\underline{Q}A_2}(v)\}.$$

(iv) $\forall vu \in SB_1$ *but* $vu \notin SB_2$.

$$T_{\overline{S}B_1 \cup \overline{S}B_2}(vu) = T_{\overline{S}B_1}(vu), T_{\underline{S}B_1 \cup \underline{S}B_2}(vu) = T_{\underline{S}B_1}(vu),$$
$$I_{\overline{S}B_1 \cup \overline{S}B_2}(vu) = I_{\overline{S}B_1}(vu), \; I_{\underline{S}B_1 \cup \underline{S}B_2}(vu) = I_{\underline{S}B_1}(vu),$$
$$F_{\overline{S}B_1 \cup \overline{S}B_2}(vu) = F_{\overline{S}B_1}(vu), F_{\underline{S}B_1 \cup \underline{S}B_2}(vu) = F_{\underline{S}B_1}(vu).$$

(v) $\forall vu \notin SB_1$ *but* $vu \in SB_2$

$$T_{\overline{S}B_1 \cup \overline{S}B_2}(vu) = T_{\overline{S}B_2}(vu), T_{\underline{S}B_1 \cup \underline{S}B_2}(vu) = T_{\underline{S}B_2}(vu),$$
$$I_{\overline{S}B_1 \cup \overline{S}B_2}(vu) = I_{\overline{S}B_2}(vu), \; I_{\underline{S}B_1 \cup \underline{S}B_2}(vu) = I_{\underline{S}B_2}(vu),$$
$$F_{\overline{S}B_1 \cup \overline{S}B_2}(vu) = F_{\overline{S}B_2}(vu), F_{\underline{S}B_1 \cup \underline{S}B_2}(vu) = F_{\underline{S}B_2}(vu).$$

(vi) $\forall vu \in SB_1 \cap \underline{S}B_2$

$$T_{\overline{S}B_1 \cup \overline{S}B_2}(vu) = \max\{T_{\overline{S}B_1}(vu), T_{\overline{S}B_2}(vu)\}, T_{\underline{S}B_1 \cup \underline{S}B_2}(vu) = \max\{T_{\underline{S}}B_1(vu), T_{\underline{S}B_2}(vu)\},$$
$$I_{\overline{S}B_1 \cup \overline{S}B_2}(vu) = \min\{I_{\overline{S}B_1}(vu), I_{\overline{S}B_2}(vu)\}, \; I_{\underline{S}B_1 \cup \underline{S}B_2}(vu) = \min\{I_{\underline{S}B_1}(vu), I_{\underline{S}B_2}(vu)\},$$
$$F_{\overline{S}B_1 \cup \overline{S}B_2}(vu) = \min\{F_{\overline{S}B_1}(vu), F_{\overline{S}B_2}(vu)\}, F_{\underline{S}B_1 \cup \underline{S}B_2}(vu) = \min\{F_{\underline{S}B_1}(vu), F_{\underline{S}B_2}(vu)\}.$$

Example 5. *Let $V = \{v_1, v_2, v_3, v_4\}$ be a set of universes, and $\mathbb{M} = \{e_1, e_2, e_3\}$ a set of parameters. Then, a neutrosophic soft relation over $V \times \mathbb{M}$ can be written as in Table 7.*

Table 7. Neutrosophic soft relation Q.

Q	v_1	v_2	v_3	v_4
e_1	(0.5, 0.4, 0.3)	(0.7, 0.6, 0.5)	(0.7, 0.6, 0.4)	(0.5, 0.7, 0.4)
e_2	(0.3, 0.5, 0.6)	(0.4, 0.5, 0.1)	(0.3, 0.6, 0.5)	(0.4, 0.8, 0.2)
e_3	(0.7, 0.5, 0.8)	(0.2, 0.3, 0.8)	(0.7, 0.3, 0.5)	(0.6, 0.4, 0.3)

Let $A_1 = \{(e_1, 0.5, 0.7, 0.8), (e_2, 0.7, 0.5, 0.3), (e_3, 0.4, 0.5, 0.3)\}$, *and* $A_2 = \{(e_1, 0.6, 0.3, 0.5), (e_2, 0.5, 0.8, 0.2), (e_3, 0.5, 0.7, 0.2)\}$ *are two neutrosophic sets on* $\mathbb{M}$*, Then,* $Q(A_1) = (\underline{Q}(A_1), \overline{Q}(A_1))$ *and* $Q(A_2) = (\underline{Q}(A_2), \overline{Q}(A_2))$ *are NSRSs, where*

$$\begin{aligned}
\underline{Q}(A_1) &= \{(v_1, 0.5, 0.6, 0.5), (v_2, 0.5, 0.5, 0.7)(v_3, 0.5, 0.5, 0.7), (v_4 0.4, 0.5, 0.5)\}, \\
\overline{Q}(A_1) &= \{(v_1, 0.5, 0.5, 0.6), (v_2, 0.5, 0.5, 0.3), (v_3, 0.5, 0.5, 0.5), (v_4 0.5, 0.5, 0.3)\}, \\
\underline{Q}(A_2) &= \{(v_1, 0.6, 0.5, 0.5), (v_2, 0.5, 0.7, 0.5), (v_3, 0.5, 0.7, 0.5), (v_4, 0.5, 0.6, 0.5)\}, \\
\overline{Q}(A_2) &= \{(v_1, 0.5, 0.4, 0.5), (v_2, 0.6, 0.6, 0.2), (v_3, 0.6, 0.6, 0.5), (v_4, 0.5, 0.7, 0.2)\}.
\end{aligned}$$

Let $E = \{v_1v_2, v_1v_4, v_2v_2, v_2v_3, v_3v_3, v_3v_4\} \subseteq V \times V$, *and* $L = \{e_1e_2, e_1e_3, e_2e_3\} \subset \acute{\mathbb{M}}$. *Then, a neutrosophic soft relation on E can be written as in Table 8.*

Table 8. Neutrosophic soft relation S.

S	v_1v_2	v_1v_4	v_2v_2	v_2v_3	v_3v_3	v_3v_4
e_1e_2	(0.3, 0.4 ,0.1)	(0.4, 0.4, 0.2)	(0.4, 0.5, 0.1)	(0.3, 0.5, 0.4)	(0.3, 0.4, 0.4)	(0.4, 0.5, 0.2)
e_1e_3	(0.2 ,0.3 ,0.3)	(0.4, 0.3, 0.2)	(0.2, 0.3, 0.5)	(0.4, 0.3, 0.3)	(0.5, 0.3, 0.3)	(0.5, 0.4, 0.3)
e_2e_3	(0.2,0.3,0.5)	(0.3, 0.3, 0.3)	(0.2, 0.3, 0.1)	(0.4, 0.3, 0.1)	(0.3, 0.3, 0.5)	(0.3, 0.4, 0.3)

Let $B_1 = \{(e_1e_2, 0.5, 0.4, 0.5), (e_1e_3, 0.3, 0.4, 0.5), (e_2e_3, 0.4, 0.4, 0.3)\}$, *and* $B_2 = \{(e_1e_2, 0.5, 0.3, 0.2), (e_1e_3, 0.4, 0.3, 0.3), (e_2e_3, 0.4, 0.6, 0.2)\}$ *are two neutrosophic sets on L, Then,* $S(B_1) = (\underline{S}(B_1), \overline{S}(B_1))$ *and* $S(B_2) = (\underline{S}(B_2), \overline{S}(B_2))$ *are NSRRs, where*

$$\begin{aligned}
\underline{S}(B_1) &= \{(v_1v_2, 0.3, 0.4, 0.3), (v_1v_4, 0.3, 0.4, 0.4), (v_2v_2, 0.4, 0.4, 0.4), (v_2v_3, 0.3, 0.4, 0.4), \\
&\quad (v_3v_3, 0.3, 0.4, 0.5), (v_3v_4, 0.3, 0.4, 0.5)\}, \\
\overline{S}(B_1) &= \{(v_1v_2, 0.3, 0.4, 0.5), (v_1v_4, 0.4, 0.4, 0.3), (v_2v_2, 0.4, 0.4, 0.3), (v_2v_3, 0.4, 0.4, 0.3), \\
&\quad (v_3v_3, 0.3, 0.4, 0.5), (v_3v_4, 0.4, 0.4, 0.3)\}; \\
\underline{S}(B_2) &= \{(v_1v_2, 0.4, 0.6, 0.2), (v_1v_4, 0.4, 0.6, 0.3), (v_2v_2, 0.4, 0.6, 0.2), (v_2v_3, 0.4, 0.6, 0.3), \\
&\quad (v_3v_3, 0.4, 0.6, 0.3), (v_3v_4, 0.4, 0.6, 0.3)\}, \\
\overline{S}(B_2) &= \{(v_1v_2, 0.3, 0.3, 0.2), (v_1v_4, 0.4, 0.3, 0.2), (v_2v_2, 0.4, 0.3, 0.2), (v_2v_3, 0.4, 0.3, 0.2), \\
&\quad (v_3v_3, 0.4, 0.3, 0.3), (v_3v_4, 0.4, 0.4, 0.2)\}.
\end{aligned}$$

Thus, $G_1 = (\underline{G}_1, \overline{G}_1)$ *and* $G_2 = (\underline{G}_2, \overline{G}_2)$ *are NSRGs, where* $\underline{G}_1 = (\underline{Q}(A_1), \underline{S}(B_1))$, $\overline{G}_1 = (\overline{Q}(A_1), \overline{S}(B_1))$ *as shown in Figure 2.*

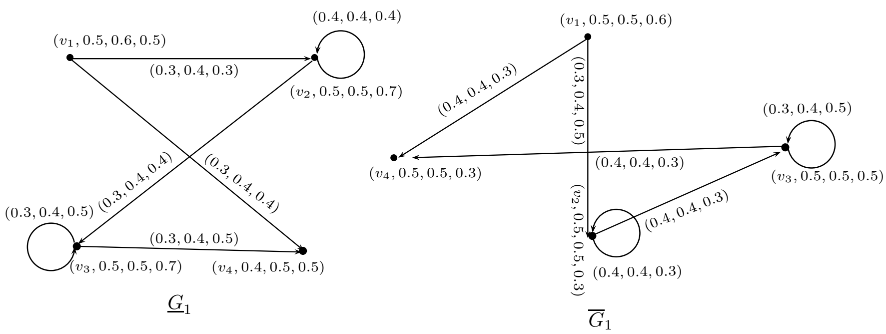


Figure 2. Neutrosophic soft rough graph $G_1 = (\underline{G}_1, \overline{G}_1)$

$\underline{G}_2 = (\underline{Q}(A_2), \underline{S}(B_2))$, $\overline{G}_2 = (\overline{Q}(A_2), \overline{S}(B_2))$ *as shown in Figure 3.*

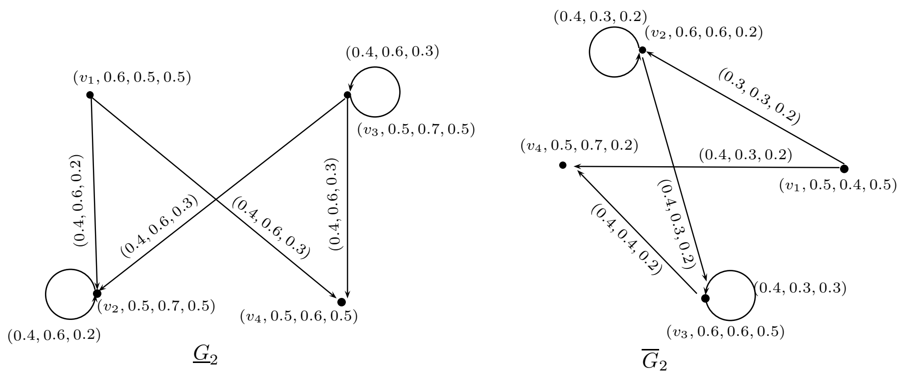


Figure 3. Neutrosophic soft rough graph $G_2 = (\underline{G}_2, \overline{G}_2)$

The union of $G_1 = (\underline{G}_1, \overline{G}_1)$ *and* $G_2 = (\underline{G}_2, \overline{G}_2)$ *is NSRG* $G = G_1 \cup G_2 = (\underline{G}_1 \cup \underline{G}_2, \overline{G}_1 \cup \overline{G}_2)$ *as shown in Figure 4.*

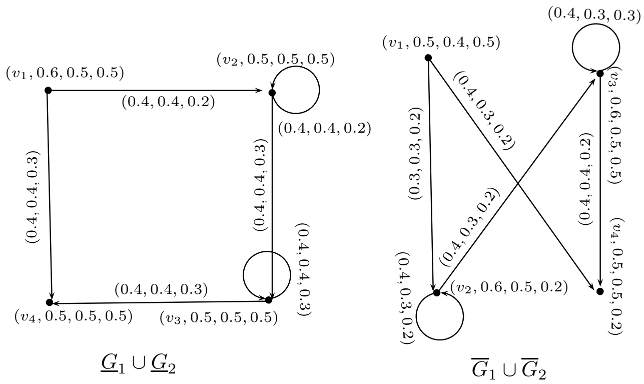


Figure 4. Neutrosophic soft rough graph $G_1 \cup G_2 = (\underline{G}_1 \cup \underline{G}_2, \overline{G}_1 \cup \overline{G}_2)$

Definition 6. *Let* $G_1 = (\underline{G}_1, \overline{G}_1)$ *and* $G_2 = (\underline{G}_2, \overline{G}_2)$ *be two NSRGs on* V*. The intersection of* G_1 *and* G_2 *is a neutrosophic soft rough graph* $G = G_1 \cap G_2 = (\underline{G}_1 \cap \underline{G}_2, \overline{G}_1 \cap \overline{G}_2)$*, where* $\underline{G}_1 \cap \underline{G}_2 = (\underline{Q}A_1 \cap \underline{Q}A_2, \underline{S}B_1 \cap \underline{S}B_2)$ *and* $\overline{G}_1 \cap \overline{G}_2 = (\overline{Q}A_1 \cap \overline{Q}A_2, \overline{S}B_1 \cap \overline{S}B_2)$ *are neutrosophic graphs, respectively, such that*

(i) $\forall v \in QA_1$ *but* $v \notin QA_2$.

$$
\begin{aligned}
T_{\overline{Q}A_1 \cap \overline{Q}A_2}(v) =& T_{\overline{Q}A_1}(v), T_{\underline{Q}A_1 \cap \underline{Q}A_2}(v) = T_{\underline{Q}A_1}(v),\\
I_{\overline{Q}A_1 \cap \overline{Q}A_2}(v) =& I_{\overline{Q}A_1}(v),\ I_{\underline{Q}A_1 \cap \underline{Q}A_2}(v) = I_{\underline{Q}A_1}(v),\\
F_{\overline{Q}A_1 \cap \overline{Q}A_2}(v) =& F_{\overline{Q}A_1}(v), F_{\underline{Q}A_1 \cap \underline{Q}A_2}(v) = F_{\underline{Q}A_1}(v).
\end{aligned}
$$

(ii) $\forall v \notin QA_1$ *but* $v \in QA_2$.

$$
\begin{aligned}
T_{\overline{Q}A_1 \cap \overline{Q}A_2}(v) =& T_{\overline{Q}A_2}(v), T_{\underline{Q}A_1 \cap \underline{Q}A_2}(v) = T_{\underline{Q}A_2}(v),\\
I_{\overline{Q}A_1 \cap \overline{Q}A_2}(v) =& I_{\overline{Q}A_2}(v),\ I_{\underline{Q}A_1 \cap \underline{Q}A_2}(v) = I_{\underline{Q}A_2}(v),\\
F_{\overline{Q}A_1 \cap \overline{Q}A_2}(v) =& F_{\overline{Q}A_2}(v), F_{\underline{Q}A_1 \cap \underline{Q}A_2}(v) = F_{\underline{Q}A_2}(v).
\end{aligned}
$$

(iii) $\forall v \in QA_1 \cap \underline{Q}A_2$

$$\begin{aligned}
T_{\overline{Q}A_1\cap\overline{Q}A_2}(v) =& \min\{T_{\overline{Q}A_1}(v), T_{\overline{Q}A_2}(v)\},\ T_{\underline{Q}A_1\cap\underline{Q}A_2}(v) = \min\{T_{\underline{Q}A_1}(v), T_{\underline{Q}A_2}(v)\},\\
I_{\overline{Q}A_1\cap\overline{Q}A_2}(v) =& \max\{I_{\overline{Q}A_1}(v), I_{\overline{Q}A_2}(v)\},\ \ I_{\underline{Q}A_1\cap\underline{Q}A_2}(v) = \max\{I_{\underline{Q}A_1}(v), I_{\underline{Q}A_2}(v)\},\\
F_{\overline{Q}A_1\cap\overline{Q}A_2}(v) =& \max\{F_{\overline{Q}A_1}(v), F_{\overline{Q}A_2}(v)\}, F_{\underline{Q}A_1\cap\underline{Q}A_2}(v) = \max\{F_{\underline{Q}A_1}(v), F_{\underline{Q}A_2}(v)\}.
\end{aligned}$$

(iv) $\forall vu \in SB_1$ *but* $vu \notin SB_2$.

$$\begin{aligned}
T_{\overline{S}B_1\cap\overline{S}B_2}(vu) =& T_{\overline{S}B_1}(vu), T_{\underline{S}B_1\cap\underline{S}B_2}(vu) = T_{\underline{S}B_1}(vu),\\
I_{\overline{S}B_1\cap\overline{S}B_2}(vu) =& I_{\overline{S}B_1}(vu),\ I_{\underline{S}B_1\cap\underline{S}B_2}(vu) = I_{\underline{S}B_1}(vu),\\
F_{\overline{S}B_1\cap\overline{S}B_2}(vu) =& F_{\overline{S}B_1}(vu), F_{\underline{S}B_1\cap\underline{S}B_2}(vu) = F_{\underline{S}B_1}(vu).
\end{aligned}$$

(v) $\forall vu \notin SB_1$ *but* $vu \in SB_2$

$$\begin{aligned}
T_{\overline{S}B_1\cap\overline{S}B_2}(vu) =& T_{\overline{S}B_2}(vu), T_{\underline{S}B_1\cap\underline{S}B_2}(vu) = T_{\underline{S}B_2}(vu),\\
I_{\overline{S}B_1\cap\overline{S}B_2}(vu) =& I_{\overline{S}B_2}(vu),\ I_{\underline{S}B_1\cap\underline{S}B_2}(vu) = I_{\underline{S}B_2}(vu),\\
F_{\overline{S}B_1\cap\overline{S}B_2}(vu) =& F_{\overline{S}B_2}(vu), F_{\underline{S}B_1\cap\underline{S}B_2}(vu) = F_{\underline{S}B_2}(vu).
\end{aligned}$$

(vi) $\forall vu \in SB_1 \cap \underline{S}B_2$

$$\begin{aligned}
T_{\overline{S}B_1\cap\overline{S}B_2}(vu) =& \min\{T_{\overline{S}B_1}(vu), T_{\overline{S}B_2}(vu)\},\ T_{\underline{S}B_1\cap\underline{S}B_2}(vu) = \min\{T_{\underline{S}}B_1(vu), T_{\underline{S}B_2}(vu)\},\\
I_{\overline{S}B_1\cap\overline{S}B_2}(vu) =& \max\{I_{\overline{S}B_1}(vu), I_{\overline{S}B_2}(vu)\},\ \ I_{\underline{S}B_1\cap\underline{S}B_2}(vu) = \max\{I_{\underline{S}B_1}(vu), I_{\underline{S}B_2}(vu)\},\\
F_{\overline{S}B_1\cap\overline{S}B_2}(vu) =& \max\{F_{\overline{S}B_1}(vu), F_{\overline{S}B_2}(vu)\}, F_{\underline{S}B_1\cap\underline{S}B_2}(vu) = \max\{F_{\underline{S}B_1}(vu), F_{\underline{S}B_2}(vu)\}.
\end{aligned}$$

Definition 7. *Let* $G_1 = (\underline{G}_1, \overline{G}_1)$ *and* $G_2 = (\underline{G}_2, \overline{G}_2)$ *be two neutrosophic soft rough graphs on* V. *The join of* G_1 *and* G_2 *is a neutrosophic soft rough graph* $G = G_1 + G_2 = (\underline{G}_1 + \underline{G}_2, \overline{G}_1 + \overline{G}_2)$, *where* $\underline{G}_1 + \underline{G}_2 = (\underline{Q}A_1 + \underline{Q}A_2, \underline{S}B_1 + \underline{S}B_2)$ *and* $\overline{G}_1 + \overline{G}_2 = (\overline{Q}A_1 + \overline{Q}A_2, \overline{S}B_1 + \overline{S}B_2)$ *are neutrosophic graph, respectively, such that*

(i) $\forall v \in QA_1$ *but* $v \notin QA_2$.

$$\begin{aligned}
T_{\overline{Q}A_1+\overline{Q}A_2}(v) =& T_{\overline{Q}A_1}(v), T_{\underline{Q}A_1+\underline{Q}A_2}(v) = T_{\underline{Q}A_1}(v),\\
I_{\overline{Q}A_1+\overline{Q}A_2}(v) =& I_{\overline{Q}A_1}(v),\ I_{\underline{Q}A_1+\underline{Q}A_2}(v) = I_{\underline{Q}A_1}(v),\\
F_{\overline{Q}A_1+\overline{Q}A_2}(v) =& F_{\overline{Q}A_1}(v), F_{\underline{Q}A_1+\underline{Q}A_2}(v) = F_{\underline{Q}A_1}(v).
\end{aligned}$$

(ii) $\forall v \notin QA_1$ *but* $v \in QA_2$.

$$\begin{aligned}
T_{\overline{Q}A_1+\overline{Q}A_2}(v) =& T_{\overline{Q}A_2}(v), T_{\underline{Q}A_1+\underline{Q}A_2}(v) = T_{\underline{Q}A_2}(v),\\
I_{\overline{Q}A_1+\overline{Q}A_2}(v) =& I_{\overline{Q}A_2}(v),\ I_{\underline{Q}A_1+\underline{Q}A_2}(v) = I_{\underline{Q}A_2}(v),\\
F_{\overline{Q}A_1+\overline{Q}A_2}(v) =& F_{\overline{Q}A_2}(v), F_{\underline{Q}A_1+\underline{Q}A_2}(v) = F_{\underline{Q}A_2}(v).
\end{aligned}$$

(iii) $\forall v \in QA_1 \cap \underline{Q}A_2$

$$\begin{aligned}
T_{\overline{Q}A_1+\overline{Q}A_2}(v) =& \max\{T_{\overline{Q}A_1}(v), T_{\overline{Q}A_2}(v)\}, T_{\underline{Q}A_1+\underline{Q}A_2}(v) = \max\{T_{\underline{Q}A_1}(v), T_{\underline{Q}A_2}(v)\},\\
I_{\overline{Q}A_1+\overline{Q}A_2}(v) =& \min\{I_{\overline{Q}A_1}(v), I_{\overline{Q}A_2}(v)\},\ \ I_{\underline{Q}A_1+\underline{Q}A_2}(v) = \min\{I_{\underline{Q}A_1}(v), I_{\underline{Q}A_2}(v)\},\\
F_{\overline{Q}A_1+\overline{Q}A_2}(v) =& \min\{F_{\overline{Q}A_1}(v), F_{\overline{Q}A_2}(v)\}, F_{\underline{Q}A_1+\underline{Q}A_2}(v) = \min\{F_{\underline{Q}A_1}(v), F_{\underline{Q}A_2}(v)\}.
\end{aligned}$$

(iv) $\forall vu \in SB_1$ *but* $vu \notin SB_2$.

$$T_{\overline{S}B_1+\overline{S}B_2}(vu) = T_{\overline{S}B_1}(vu), T_{\underline{S}B_1+\underline{S}B_2}(vu) = T_{\underline{S}B_1}(vu),$$
$$I_{\overline{S}B_1+\overline{S}B_2}(vu) = I_{\overline{S}B_1}(vu), \; I_{\underline{S}B_1+\underline{S}B_2}(vu) = I_{\underline{S}B_1}(vu),$$
$$F_{\overline{S}B_1+\overline{S}B_2}(vu) = F_{\overline{S}B_1}(vu), F_{\underline{S}B_1+\underline{S}B_2}(vu) = F_{\underline{S}B_1}(vu).$$

(v) $\forall vu \notin SB_1$ *but* $vu \in SB_2$

$$T_{\overline{S}B_1+\overline{S}B_2}(vu) = T_{\overline{S}B_2}(vu), T_{\underline{S}B_1+\underline{S}B_2}(vu) = T_{\underline{S}B_2}(vu),$$
$$I_{\overline{S}B_1+\overline{S}B_2}(vu) = I_{\overline{S}B_2}(vu), \; I_{\underline{S}B_1+\underline{S}B_2}(vu) = I_{\underline{S}B_2}(vu),$$
$$F_{\overline{S}B_1+\overline{S}B_2}(vu) = F_{\overline{S}B_2}(vu), F_{\underline{S}B_1+\underline{S}B_2}(vu) = F_{\underline{S}B_2}(vu).$$

(vi) $\forall vu \in SB_1 \cap \underline{S}B_2$

$$T_{\overline{S}B_1+\overline{S}B_2}(vu) = \max\{T_{\overline{S}B_1}(vu), T_{\overline{S}B_2}(vu)\}, T_{\underline{S}B_1+\underline{S}B_2}(vu) = \max\{T_{\underline{S}}B_1(vu), T_{\underline{S}B_2}(vu)\},$$
$$I_{\overline{S}B_1+\overline{S}B_2}(vu) = \min\{I_{\overline{S}B_1}(vu), I_{\overline{S}B_2}(vu)\}, \; I_{\underline{S}B_1+\underline{S}B_2}(vu) = \min\{I_{\underline{S}B_1}(vu), I_{\underline{S}B_2}(vu)\},$$
$$F_{\overline{S}B_1+\overline{S}B_2}(vu) = \min\{F_{\overline{S}B_1}(vu), F_{\overline{S}B_2}(vu)\}, F_{\underline{S}B_1+\underline{S}B_2}(vu) = \min\{F_{\underline{S}B_1}(vu), F_{\underline{S}B_2}(vu)\}.$$

(vii) $\forall vu \in \tilde{E}$, *where* $\tilde{E}$ *is the set of edges joining vertices of* QA_1 *and* QA_2.

$$T_{\overline{S}B_1+\overline{S}B_2}(vu) = \min\{T_{\overline{Q}A_1}(v), T_{\overline{Q}A_2}(u)\}, \; T_{\underline{S}B_1+\underline{S}B_2}(vu) = \min\{T_{\underline{Q}A_1}(v), T_{\underline{Q}A_2}(u)\},$$
$$I_{\overline{S}B_1+\overline{S}B_2}(vu) = \max\{I_{\overline{Q}A_1}(v), I_{\overline{Q}A_2}(u)\}, \; I_{\underline{S}B_1+\underline{S}B_2}(vu) = \max\{I_{\underline{Q}A_1}(v), I_{\underline{Q}A_2}(u)\},$$
$$F_{\overline{S}B_1+\overline{S}B_2}(vu) = \max\{F_{\overline{Q}A_1}(v), F_{\overline{Q}A_2}(u)\}, F_{\underline{S}B_1+\underline{S}B_2}(vu) = \max\{F_{\underline{Q}A_1}(v), F_{\underline{Q}A_2}(u)\}.$$

Definition 8. *The Cartesian product of* G_1 *and* G_2 *is a* $G = G_1 \ltimes G_2 = (\underline{G}_1 \ltimes \underline{G}_2, \overline{G}_1 \ltimes \overline{G}_2)$, *where* $\underline{G}_1 \ltimes \underline{G}_2 = (\underline{Q}A_1 \ltimes \underline{Q}A_2, \underline{S}B_1 \ltimes \underline{S}B_2)$ *and* $\overline{G}_1 \ltimes \overline{G}_2 = (\overline{Q}A_1 \ltimes \overline{Q}A_2, \overline{S}B_1 \ltimes \overline{S}B_2)$ *are neutrosophic digraph, such that*

(i) $\forall\, (v_1, v_2) \in QA_1 \times QA_2$.

$$T_{(\overline{Q}A_1 \ltimes \overline{Q}A_2)}(v_1, v_2) = \min\{T_{\overline{Q}A_1}(v_1), T_{\overline{Q}A_2}(v_1)\}, \; T_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v_1, v_2) = \min\{T_{\underline{Q}A_1}(v_1), T_{\underline{Q}A_2}(v_1)\},$$
$$I_{(\overline{Q}A_1 \ltimes \overline{Q}A_2)}(v_1, v_2) = \max\{I_{\overline{Q}A_1}(v_1), I_{\overline{Q}A_2}(v_1)\}, \; I_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v_1, v_2) = \max\{I_{\underline{Q}A_1}(v_1), I_{\underline{Q}A_2}(v_1)\},$$
$$F_{(\overline{Q}A_1 \ltimes \overline{Q}A_2)}(v_1, v_2) = \max\{F_{\overline{Q}A_1}(v_1), F_{\overline{Q}A_2}(v_1)\}, F_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v_1, v_2) = \max\{F_{\underline{Q}A_1}(v_1), F_{\underline{Q}A_2}(v_1)\}.$$

(ii) $\forall v_1 v_2 \in SB_2, v \in QA_1$.

$$T_{(\overline{S}B_1 \ltimes \overline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \min\{T_{\overline{Q}A_1}(v), T_{\overline{S}B_2}(v_1 v_2)\},$$
$$T_{(\underline{S}B_1 \ltimes \underline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \min\{T_{\underline{Q}A_1}(v), T_{\underline{S}B_2}(v_1 v_2)\},$$
$$I_{(\overline{S}B_1 \ltimes \overline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{I_{\overline{Q}A_1}(v), I_{\overline{S}B_2}(v_1 v_2)\},$$
$$I_{(\underline{S}B_1 \ltimes \underline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{I_{\underline{Q}A_1}(v), I_{\underline{S}B_2}(v_1 v_2)\},$$
$$F_{(\overline{S}B_1 \ltimes \overline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{F_{\overline{Q}A_1}(v), F_{\overline{S}B_2}(v_1 v_2)\},$$
$$F_{(\underline{S}B_1 \ltimes \underline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{F_{\underline{Q}A_1}(v), F_{\underline{S}B_2}(v_1 v_2)\}.$$

(iii) $\forall v_1v_2 \in SB_1, v \in QA_2$.

$$\begin{aligned}
T_{(\underline{S}B_1 \ltimes \underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \min\{T_{\underline{S}B_1}(v_1v_2), T_{\underline{Q}A_2}(v)\},\\
T_{(\overline{S}B_1 \ltimes \overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \min\{T_{\overline{S}B_1}(v_1v_2), T_{\overline{Q}A_2}(v)\},\\
I_{(\overline{S}B_1 \ltimes \overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{I_{\overline{S}B_1}(v_1v_2), I_{\overline{Q}A_2}(v)\},\\
I_{(\underline{S}B_1 \ltimes \underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{I_{\underline{S}B_1}(v_1v_2), I_{\underline{Q}A_2}(v)\},\\
F_{(\overline{S}B_1 \ltimes \overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{F_{\overline{S}B_1}(v_1v_2), F_{\overline{Q}A_2}(v)\},\\
F_{(\underline{S}B_1 \ltimes \underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{F_{\underline{S}B_1}(v_1v_2), F_{\underline{Q}A_2}(v)\}.
\end{aligned}$$

Definition 9. *The cross product of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1 \circledcirc G_2 = (\underline{G}_1 \circledcirc \underline{G}_2, \overline{G}_1 \circledcirc \overline{G}_2)$, where $\underline{G}_1 \circledcirc \underline{G}_2 = (\underline{Q}A_1 \circledcirc \underline{Q}A_2, \underline{S}B_1 \circledcirc \underline{S}B_2)$ and $\overline{G}_1 \circledcirc \overline{G}_2 = (\overline{Q}A_1 \circledcirc \overline{Q}A_2, \overline{S}B_1 \circledcirc \overline{S}B_2)$ are neutrosophic graphs, respectively, such that*

(i) $\forall\ (v_1,v_2) \in QA_1 \times QA_2$.

$$\begin{aligned}
T_{(\overline{Q}A_1 \circledcirc \overline{Q}A_2)}(v_1,v_2) &= \min\{T_{\overline{Q}A_1}(v_1), T_{\overline{Q}A_2}(v_1)\},\ T_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(v_1,v_2) = \min\{T_{\underline{Q}A_1}(v_1), T_{\underline{Q}A_2}(v_1)\},\\
I_{(\overline{Q}A_1 \circledcirc \overline{Q}A_2)}(v_1,v_2) &= \max\{I_{\overline{Q}A_1}(v_1), I_{\overline{Q}A_2}(v_1)\},\ I_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(v_1,v_2) = \max\{I_{\underline{Q}A_1}(v_1), I_{\underline{Q}A_2}(v_1)\},\\
F_{(\overline{Q}A_1 \circledcirc \overline{Q}A_2)}(v_1,v_2) &= \max\{F_{\overline{Q}A_1}(v_1), F_{\overline{Q}A_2}(v_1)\}, F_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(v_1,v_2) = \max\{F_{\underline{Q}A_1}(v_1), F_{\underline{Q}A_2}(v_1)\}.
\end{aligned}$$

(ii) $\forall v_1u_1 \in SB_1, v_2u_2 \in SB_2$.

$$\begin{aligned}
T_{(\overline{S}B_1 \circledcirc \overline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \min\{T_{\overline{S}B_1}(v_1u_1), T_{\overline{S}B_2}(v_1u_2)\},\\
T_{(\underline{S}B_1 \circledcirc \underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \min\{T_{\underline{S}B_1}(v_1u_1), T_{\underline{S}B_2}(v_1u_2)\},\\
I_{(\overline{S}B_1 \circledcirc \overline{S}B_2}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{I_{\overline{S}B_1}(v_1u_1), I_{\overline{S}B_2}(v_1u_2)\},\\
I_{(\underline{S}B_1 \circledcirc \underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{I_{\underline{S}B_1}(v_1u_1), I_{\underline{S}B_2}(v_1u_2)\},\\
F_{(\overline{S}B_1 \circledcirc \overline{S}B_2}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{F_{\overline{S}B_1}(v_1u_1), F_{\overline{S}B_2}(v_1u_2)\},\\
F_{(\underline{S}B_1 \circledcirc \underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{F_{\underline{S}B_1}(v_1u_1), F_{\underline{S}B_2}(v_1u_2)\}.
\end{aligned}$$

Definition 10. *The rejection of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1|G_2 = (\underline{G}_1|\underline{G}_2, \overline{G}_1|\overline{G}_2)$, where $\underline{G}_1|\underline{G}_2 = (\underline{S}A_1|\underline{S}A_2, \underline{S}B_1|\underline{S}B_2)$ and $\overline{G}_1|\overline{G}_2 = (\overline{S}A_1|\overline{S}A_2, \overline{S}B_1|\overline{S}B_2)$ are neutrosophic graphs such that*

(i) $\forall\ (v_1,v_2) \in QA_1 \times QA_2$.

$$\begin{aligned}
T_{(\overline{Q}A_1|\overline{Q}A_2)}(v_1,v_2) &= \min\{T_{\overline{Q}A_1}(v_1), T_{\overline{Q}A_2}(v_2)\}, T_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,v_2) = \min\{T_{\underline{Q}A_1}(v_1), T_{\underline{Q}A_2}(v_2)\},\\
I_{(\overline{Q}A_1|\overline{Q}A_2)}(v_1,v_2) &= \max\{I_{\overline{Q}A_1}(v_1), I_{\overline{Q}A_2}(v_2)\},\ I_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,v_2) = \max\{I_{\underline{Q}A_1}(v_1), I_{\underline{Q}A_2}(v_2)\},\\
F_{(\overline{Q}A_1|\overline{Q}A_2)}(v_1,v_2) &= \max\{F_{\overline{Q}A_1}(v_1), F_{\overline{Q}A_2}(v_2)\}, F_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,v_2) = \max\{F_{\underline{Q}A_1}(v_1), F_{\underline{Q}A_2}(v_2)\}.
\end{aligned}$$

(ii) $\forall v_2u_2 \notin SB_2,\ v \in QA_1$.

$$\begin{aligned}
T_{(\overline{S}B_1|\overline{S}B_2)}\big((v,v_2)(v,u_2)\big) &= \min\{T_{\overline{Q}A_1}(v), T_{\overline{Q}A_2}(v_2), T_{\overline{Q}A_2}(u_2)\},\\
T_{(\underline{S}B_1|\underline{Q}B_2)}\big((v,v_2)(v,u_2)\big) &= \min\{T_{\underline{Q}A_1}(v), T_{\underline{Q}A_2}(v_2), T_{\underline{Q}A_2}(u_2)\},\\
(I_{\overline{S}B_1|\overline{S}B_2)}\big((v,v_2)(v,u_2)\big) &= \max\{I_{\overline{Q}A_1}(v), I_{\overline{Q}A_2}(v_2), I_{\overline{Q}A_2}(u_2)\},\\
(I_{\underline{S}B_1|\underline{S}B_2)}\big((v,v_2)(v,u_2)\big) &= \max\{I_{\underline{Q}A_1}(v), I_{\underline{Q}A_2}(v_2), I_{\underline{Q}A_2}(u_2)\},\\
(F_{\overline{S}B_1|\overline{S}B_2)}\big((v,v_2)(v,u_2)\big) &= \max\{F_{\overline{Q}A_1}(v), F_{\overline{Q}A_2}(v_2), F_{\overline{Q}A_2}(u_2)\},\\
(F_{\underline{S}B_1|\underline{S}B_2)}\big((v,v_2)(v,u_2)\big) &= \max\{F_{\underline{Q}A_1}(v), F_{\underline{Q}A_2}(v_2), F_{\underline{Q}A_2}(u_2)\}.
\end{aligned}$$

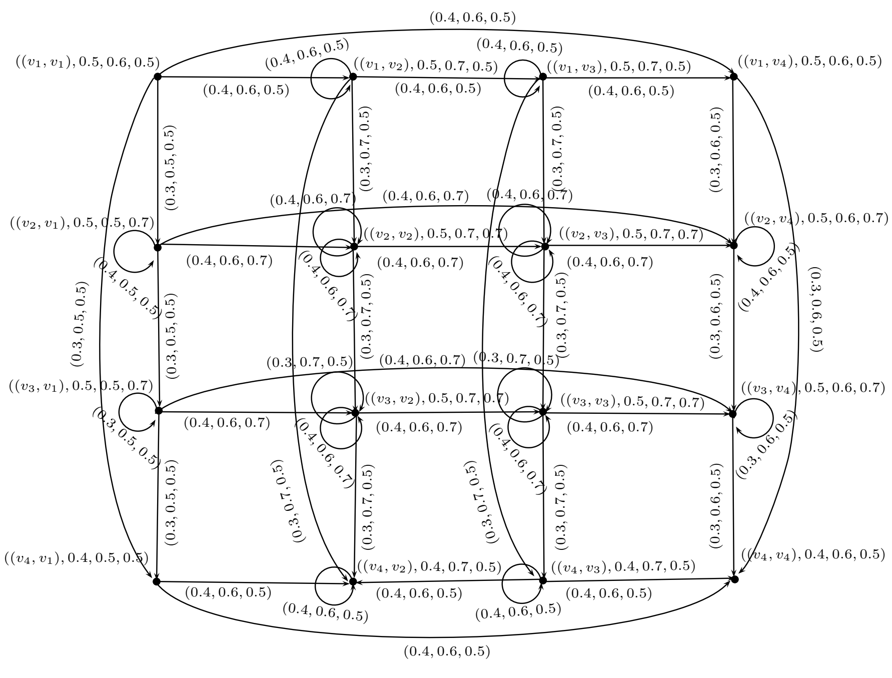
(iii) $\forall v_1u_1 \notin SB_1,\ v \in QA_2,$

$$
\begin{aligned}
T_{(\underline{S}B_1|\underline{S}B_2)}\big((v_1,v)(u_1,v)\big) &= \min\{T_{\underline{Q}A_1}(v_1), T_{\underline{Q}A_1}(u_1), T_{\underline{Q}A_2}(v)\},\\
I_{(\underline{S}B_1|\underline{S}B_2)}\big((v_1,v)(u_1,v)\big) &= \max\{I_{\underline{Q}A_1}(v_1), I_{\underline{Q}A_1}(u_1), I_{\underline{Q}A_2}(v)\},\\
F_{(\underline{S}B_1|\underline{S}B_2)}\big((v_1,v)(u_1,v)\big) &= \max\{F_{\underline{Q}A_1}(v_1), F_{\underline{Q}A_1}(u_1), F_{\underline{Q}A_2}(v)\},\\
T_{(\overline{S}B_1|\overline{S}B_2)}\big((v_1,v)(u_1,v)\big) &= \min\{T_{\overline{Q}A_1}(v_1), T_{\overline{Q}A_1}(u_1), T_{\overline{Q}A_2}(v)\},\\
I_{(\overline{S}B_1|\overline{S}B_2)}\big((v_1,v)(u_1,v)\big) &= \max\{I_{\overline{Q}A_1}(v_1), I_{\overline{Q}A_1}(u_1), I_{\overline{Q}A_2}(v)\},\\
F_{(\overline{S}B_1|\overline{S}B_2)}\big((v_1,v)(u_1,v)\big) &= \max\{F_{\overline{Q}A_1}(v_1), F_{\overline{Q}A_1}(u_1), F_{\overline{Q}A_2}(v)\}.
\end{aligned}
$$

(iv) $\forall v_1u_1 \notin \overline{S}B_1,\ v_2u_2 \notin \overline{S}B_2,\ v_1 = u_1.$

$$
\begin{aligned}
T_{(\underline{S}B_1|\underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \min\{T_{\underline{Q}A_1}(v_1), T_{\underline{Q}A_1}(u_1), T_{\underline{Q}A_2}(v_2), T_{\underline{Q}A_2}(u_2)\},\\
I_{(\underline{S}B_1|\underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{I_{\underline{Q}A_1}(v_1), I_{\underline{Q}A_1}(u_1), I_{\underline{Q}A_2}(v_2), I_{\underline{Q}A_2}(u_2)\},\\
F_{(\underline{S}B_1|\underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{F_{\underline{Q}A_1}(v_1), F_{\underline{Q}A_1}(u_1), F_{\underline{Q}A_2}(v_2), F_{\underline{Q}A_2}(u_2)\},\\
T_{(\overline{S}B_1|\overline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \min\{T_{\overline{Q}A_1}(v_1), T_{\overline{Q}A_1}(u_1), T_{\overline{Q}A_2}(v_2), T_{\overline{Q}A_2}(u_2)\},\\
I_{(\overline{S}B_1|\overline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{I_{\overline{Q}A_1}(v_1), I_{\overline{Q}A_1}(u_1), I_{\overline{Q}A_2}(v_2), I_{\overline{Q}A_2}(u_2)\},\\
F_{(\overline{S}B_1|\overline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{F_{\overline{Q}A_1}(v_1), F_{\overline{Q}A_1}(u_1), F_{\overline{Q}A_2}(v_2), F_{\overline{Q}A_2}(u_2)\},
\end{aligned}
$$

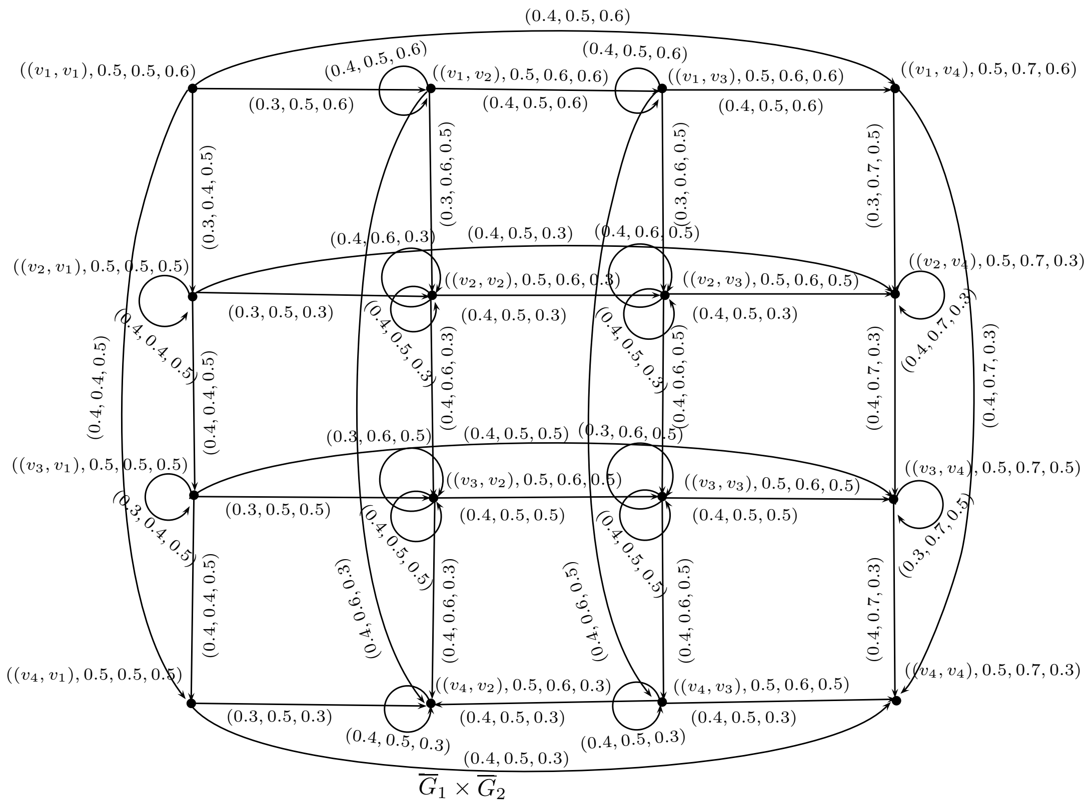
Example 6. *Let* $G_1 = (\underline{G}_1, \overline{G}_1)$ *and* $G_2 = (\underline{G}_2, \overline{G}_2)$ *be two neutrosophic soft rough graphs on* V*, where* $\underline{G}_1 = (\underline{Q}A_1, \underline{S}B_1)$ *and* $\overline{G}_1 = (\overline{Q}A_1, \overline{S}B_1)$ *are neutrosophic graphs as shown in Figure 2 and* $\underline{G}_2 = (\underline{Q}A_2, \underline{S}B_2)$ *and* $\overline{G}_2 = (\overline{Q}A_2, \overline{S}B_2)$ *are neutrosophic graphs as shown in Figure 3. The Cartesian product of* $G_1 = (\underline{G}_1, \overline{G}_1)$ *and* $G_2 = (\underline{G}_2, \overline{G}_2)$ *is NSRG* $G = G_1 \times G_2 = (\underline{G}_1 \times \underline{G}_2, \overline{G}_1 \times \overline{G}_2)$ *as shown in Figure 5.*



$\underline{G}_1 \times \underline{G}_2$

(a)

Figure 5. *Cont.*



(b)

Figure 5. Cartesian product of two neutrosophic soft rough graphs $G_1 \times G_2$

Definition 11. *The symmetric difference of* G_1 *and* G_2 *is a neutrosophic soft rough graph* $G = G_1 \oplus G_2 = (\underline{G}_1 \oplus \underline{G}_2, \overline{G}_1 \oplus \overline{G}_2)$, *where* $\underline{G}_1 \oplus \underline{G}_2 = (\underline{Q}A_1 \oplus \underline{Q}A_2, \underline{S}B_1 \oplus \underline{S}B_2)$ *and* $\overline{G}_1 \oplus \overline{G}_2 = (\overline{Q}A_1 \oplus \overline{Q}A_2, \overline{S}B_1 \oplus \overline{S}B_2)$ *are neutrosophic graphs, respectively, such that*

(i) $\forall\ (v_1, v_2) \in QA_1 \times QA_2$.

$$T_{(\overline{Q}A_1 \oplus \overline{Q}A_2)}(v_1, v_2) = \min\{T_{\overline{Q}A_1}(v_1), T_{\overline{Q}A_2}(v_2)\},\ T_{(\underline{Q}A_1 \oplus \underline{Q}A_2)}(v_1, v_2) = \min\{T_{\underline{Q}A_1}(v_1), T_{\underline{Q}A_2}(v_2)\},$$
$$I_{(\overline{Q}A_1 \oplus \overline{Q}A_2)}(v_1, v_2) = \max\{I_{\overline{Q}A_1}(v_1), I_{\overline{Q}A_2}(v_2)\},\ I_{(\underline{Q}A_1 \oplus \underline{Q}A_2)}(v_1, v_2) = \max\{I_{\underline{Q}A_1}(v_1), I_{\underline{Q}A_2}(v_2)\},$$
$$F_{(\overline{Q}A_1 \oplus \overline{Q}A_2)}(v_1, v_2) = \max\{F_{\overline{Q}A_1}(v_1), F_{\overline{Q}A_2}(v_2)\}, F_{(\underline{Q}A_1 \oplus \underline{Q}A_2)}(v_1, v_2) = \max\{F_{\underline{Q}A_1}(v_1), F_{\underline{Q}A_2}(v_2)\}.$$

(ii) $\forall v_1 v_2 \in SB_2,\ v \in QA_1$.

$$T_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \min\{T_{\overline{Q}A_1}(v), T_{\overline{S}B_2}(v_1 v_2)\},$$
$$T_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \min\{T_{\underline{Q}A_1}(v), T_{\underline{S}B_2}(v_1 v_2)\},$$
$$I_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{I_{\overline{Q}A_1}(v), I_{\overline{S}B_2}(v_1 v_2)\},$$
$$I_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{I_{\underline{Q}A_1}(v), I_{\underline{S}B_2}(v_1 v_2)\},$$
$$F_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{F_{\overline{Q}A_1}(v), F_{\overline{S}B_2}(v_1 v_2)\},$$
$$F_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{F_{\underline{Q}A_1}(v), F_{\underline{S}B_2}(v_1 v_2)\}.$$

(iii) $\forall v_1v_2 \in SB_1, v \in QA_2.$

$$
\begin{aligned}
T_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \min\{T_{\overline{S}B_1}(v_1v_2), T_{\overline{Q}A_2}(v)\},\\
T_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \min\{T_{\underline{S}B_1}(v_1v_2), T_{\underline{Q}A_2}(v)\},\\
I_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{I_{\overline{S}B_1}(v_1v_2), I_{\overline{Q}A_2}(v)\},\\
I_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{I_{\underline{S}B_1}(v_1v_2), I_{\underline{Q}A_2}(v)\},\\
F_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{F_{\overline{S}B_1}(v_1v_2), F_{\overline{Q}A_2}(v)\},\\
F_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{F_{\underline{S}B_1}(v_1v_2), F_{\underline{Q}A_2}(v)\}.
\end{aligned}
$$

(iv) $\forall v_1u_1 \notin SB_1,\ v_2u_2 \in SB_2.$

$$
\begin{aligned}
T_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \min\{T_{\overline{S}B_1}(v_1u_1), T_{\overline{Q}A_2}(v_2), T_{\overline{Q}A_2}(u_2)\},\\
T_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \min\{T_{\underline{S}B_1}(v_1u_1), T_{\underline{Q}A_2}(v_2), T_{\underline{Q}A_2}(u_2)\},\\
I_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{I_{\overline{S}B_1}(v_1u_1), I_{\overline{Q}A_2}(v_2), I_{\overline{Q}A_2}(u_2)\},\\
I_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{I_{\underline{S}B_1}(v_1u_1), I_{\underline{Q}A_2}(v_2), I_{\underline{Q}A_2}(u_2)\},\\
F_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{F_{\overline{S}B_1}(v_1u_1), F_{\overline{Q}A_2}(v_2), F_{\overline{Q}A_2}(u_2)\},\\
F_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{F_{\underline{S}B_1}(v_1u_1), F_{\underline{Q}A_2}(v_2), F_{\underline{Q}A_2}(u_2)\}.
\end{aligned}
$$

(v) $\forall v_1u_1 \notin SB_1,\ v_2u_2 \in SB_2.$

$$
\begin{aligned}
T_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \min\{T_{\overline{Q}A_1}(v_1), T_{\overline{Q}A_1}(u_1), T_{\overline{S}B_2}(v_2u_2)\},\\
T_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \min\{T_{\underline{Q}A_1}(v_1), T_{\underline{Q}A_1}(u_1), T_{\underline{S}B_2}(v_2u_2)\},\\
I_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{I_{\overline{Q}A_1}(v_1), I_{\overline{Q}A_1}(u_1), I_{\overline{S}B_2}(v_2u_2)\},\\
I_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{I_{\underline{Q}A_1}(v_1), I_{\underline{Q}A_1}(u_1), I_{\underline{S}B_2}(v_2u_2)\},\\
F_{(\overline{S}B_1 \oplus \overline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{F_{\overline{Q}A_1}(v_1), F_{\overline{Q}A_1}(u_1), F_{\overline{S}B_2}(v_2u_2)\},\\
F_{(\underline{S}B_1 \oplus \underline{S}B_2)}\big((v_1,v_2)(u_1,u_2)\big) &= \max\{F_{\underline{Q}A_1}(v_1), F_{\underline{Q}A_1}(u_1), F_{\underline{S}B_2}(v_2u_2)\}.
\end{aligned}
$$

Example 7. *Let* $G_1 = (\underline{G}_1, \overline{G}_1)$ *and* $G_2 = (\underline{G}_2, \overline{G}_2)$ *be two neutrosophic soft rough graphs on* V*, where* $\underline{G}_1 = (\underline{Q}A_1, \underline{S}B_1)$ *and* $\overline{G}_1 = (\overline{Q}A_1, \overline{S}B_1)$ *are neutrosophic graphs as shown in Figure 6 and* $\underline{G}_2 = (\underline{Q}A_2, \underline{S}B_2)$ *and* $\overline{G}_2 = (\overline{Q}A_2, \overline{S}B_2)$ *are neutrosophic graphs as shown in Figure 7.*

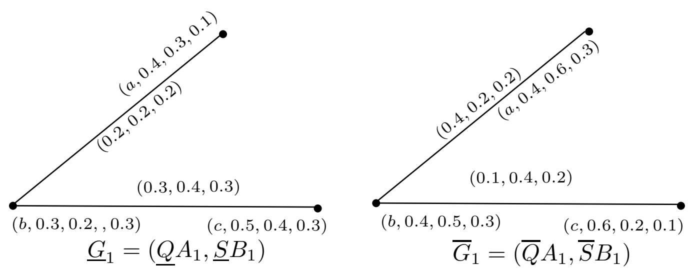


Figure 6. Neutrosophic soft rough graph $G_1 = (\underline{G}_1, \overline{G}_1)$

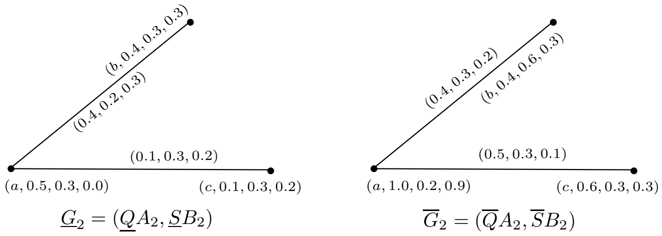


Figure 7. Neutrosophic soft rough graph $G_2 = (\underline{G}_2, \overline{G}_2)$

The symmetric difference of G_1 *and* G_2 *is* $G = G_1 \oplus G_2 = (\underline{G}_1 \oplus \underline{G}_2, \overline{G}_1 \oplus \overline{G}_2)$*, where* $\underline{G}_1 \oplus \underline{G}_2 = (\underline{Q}A_1 \oplus \underline{Q}A_2, \underline{S}B_1 \oplus \underline{S}B_2)$ *and* $\overline{G}_1 \oplus \overline{G}_2 = (\overline{Q}A_1 \oplus \overline{Q}A_2, \overline{S}B_1 \oplus \overline{S}B_2)$ *are neutrosophic graphs as shown in Figure 8.*

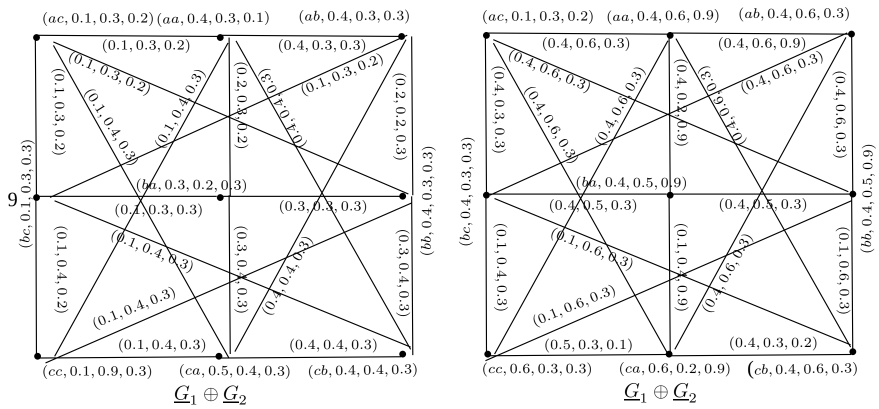


Figure 8. Neutrosophic soft rough graph $G_1 \oplus G_2 = (\underline{G}_1 \oplus G_2, \overline{G}_1 \oplus \overline{G}_2)$

Definition 12. *The lexicographic product of* G_1 *and* G_2 *is a neutrosophic soft rough graph* $G = G_1 \odot G_2 = (G_{1*} \odot G_{2*}, G_1^* \odot G_2^*)$*, where* $G_{1*} \odot G_{2*} = (\underline{Q}A_1 \odot \underline{Q}A_2, \underline{S}B_1 \odot \underline{S}B_2)$ *and* $G_1^* \odot G_2^* = (\overline{Q}A_1 \odot \overline{Q}A_2, \overline{S}B_1 \odot \overline{S}B_2)$ *are neutrosophic graphs, respectively, such that*

(i) $\forall\ (v_1, v_2) \in QA_1 \times QA_2.$

$$T_{(\overline{Q}A_1 \odot \overline{Q}A_2)}(v_1, v_2) = \min\{T_{\overline{Q}A_1}(v_1), T_{\overline{Q}A_2}(v_2)\}, T_{(\underline{Q}A_1 \odot \underline{Q}A_2)}(v_1, v_2) = \min\{T_{\underline{Q}A_1}(v_1), T_{\underline{Q}A_2}(v_2)\},$$
$$I_{(\overline{Q}A_1 \odot \overline{Q}A_2)}(v_1, v_2) = \max\{I_{\overline{Q}A_1}(v_1), I_{\overline{Q}A_2}(v_2)\},\ I_{(\underline{Q}A_1 \odot \underline{Q}A_2)}(v_1, v_2) = \max\{I_{\underline{Q}A_1}(v_1), I_{\underline{Q}A_2}(v_2)\},$$
$$F_{(\overline{Q}A_1 \odot \overline{Q}A_2)}(v_1, v_2) = \max\{F_{\overline{Q}A_1}(v_1), F_{\overline{Q}A_2}(v_2)\}, F_{(\underline{Q}A_1 \odot \underline{Q}A_2)}(v_1, v_2) = \max\{F_{\underline{Q}A_1}(v_1), F_{\underline{Q}A_2}(v_2)\}.$$

(ii) $\forall v_1 v_2 \in SB_2,\ v \in QA_1.$

$$T_{(\overline{S}B_1 \odot \overline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \min\{T_{\overline{Q}A_1}(v), T_{\overline{S}B_2}(v_1 v_2)\},$$
$$T_{(\underline{S}B_1 \odot \underline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \min\{T_{\underline{Q}A_1}(v), T_{\underline{S}B_2}(v_1 v_2)\},$$
$$I_{(\overline{S}B_1 \odot \overline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{I_{\overline{Q}A_1}(v), I_{\overline{S}B_2}(v_1 v_2)\},$$
$$I_{(\underline{S}B_1 \odot \underline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{I_{\underline{Q}A_1}(v), I_{\underline{S}B_2}(v_1 v_2)\},$$
$$F_{(\overline{S}B_1 \odot \overline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{F_{\overline{Q}A_1}(v), F_{\overline{S}B_2}(v_1 v_2)\},$$
$$F_{(\underline{S}B_1 \odot \underline{S}B_2)}\big((v, v_1)(v, v_2)\big) = \max\{F_{\underline{Q}A_1}(v), F_{\underline{S}B_2}(v_1 v_2)\}.$$

(iii) $\forall v_1u_1 \in SB_1, v_1u_2 \in SB_2$.

$$\begin{aligned}
T_{(\overline{S}B_1 \odot \overline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \min\{T_{\overline{S}B_1}(v_1u_1), T_{\overline{S}B_2}(v_1u_2)\},\\
T_{(\underline{S}B_1 \odot \underline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \min\{T_{\underline{S}B_1}(v_1u_1), T_{\underline{S}B_2}(v_1u_2)\},\\
I_{(\overline{S}B_1 \odot \overline{S}B_2}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{I_{\overline{S}B_1}(v_1u_1), I_{\overline{S}B_2}(v_1u_2)\},\\
I_{(\underline{S}B_1 \odot \underline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{I_{\underline{S}B_1}(v_1u_1), I_{\underline{S}B_2}(v_1u_2)\},\\
F_{(\overline{S}B_1 \odot \overline{S}B_2}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{F_{\overline{S}B_1}(v_1u_1), F_{\overline{S}B_2}(v_1u_2)\},\\
F_{(\underline{S}B_1 \odot \underline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{F_{\underline{S}B_1}(v_1u_1), F_{\underline{S}B_2}(v_1u_2)\}.
\end{aligned}$$

Definition 13. *The strong product of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1 \otimes G_2 = (G_{1*} \otimes G_{2*}, G_1^* \otimes G_2^*)$, where $G_{1*} \otimes G_{2*} = (\underline{Q}A_1 \otimes \underline{Q}A_2, \underline{S}B_1 \otimes \underline{S}B_2)$ and $G_1^* \otimes G_2^* = (\overline{Q}A_1 \otimes \overline{Q}A_2, \overline{S}B_1 \otimes \overline{S}B_2)$ are neutrosophic graphs, respectively, such that*

(i) $\forall\ (v_1,v_2) \in QA_1 \times QA_2$.

$$\begin{aligned}
T_{(\overline{Q}A_1 \otimes \overline{Q}A_2)}(v_1,v_2) &= \min\{T_{\overline{Q}A_1}(v_1), T_{\overline{Q}A_2}(v_2)\},\ T_{(\underline{Q}A_1 \otimes \underline{Q}A_2)}(v_1,v_2) = \min\{T_{\underline{Q}A_1}(v_1), T_{\underline{Q}A_2}(v_2)\},\\
I_{(\overline{Q}A_1 \otimes \overline{Q}A_2)}(v_1,v_2) &= \max\{I_{\overline{Q}A_1}(v_1), I_{\overline{Q}A_2}(v_2)\},\ \ I_{(\underline{Q}A_1 \otimes \underline{Q}A_2)}(v_1,v_2) = \max\{I_{\underline{Q}A_1}(v_1), I_{\underline{Q}A_2}(v_2)\},\\
F_{(\overline{Q}A_1 \otimes \overline{Q}A_2)}(v_1,v_2) &= \max\{F_{\overline{Q}A_1}(v_1), F_{\overline{Q}A_2}(v_2)\}, F_{(\underline{Q}A_1 \otimes \underline{Q}A_2)}(v_1,v_2) = \max\{F_{\underline{Q}A_1}(v_1), F_{\underline{Q}A_2}(v_2)\}.
\end{aligned}$$

(ii) $\forall v_1v_2 \in SB_2,\ v \in QA_1$.

$$\begin{aligned}
T_{(\overline{S}B_1 \otimes \overline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \min\{T_{\overline{Q}A_1}(v), T_{\overline{S}B_2}(v_1v_2)\},\\
T_{(\underline{S}B_1 \otimes \underline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \min\{T_{\underline{Q}A_1}(v), T_{\underline{S}B_2}(v_1v_2)\},\\
I_{(\overline{S}B_1 \otimes \overline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \max\{I_{\overline{Q}A_1}(v), I_{\overline{S}B_2}(v_1v_2)\},\\
I_{(\underline{S}B_1 \otimes \underline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \max\{I_{\underline{Q}A_1}(v), I_{\underline{S}B_2}(v_1v_2)\},\\
F_{(\overline{S}B_1 \otimes \overline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \max\{F_{\overline{Q}A_1}(v), F_{\overline{S}B_2}(v_1v_2)\},\\
F_{(\underline{S}B_1 \otimes \underline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \max\{F_{\underline{Q}A_1}(v), F_{\underline{S}B_2}(v_1v_2)\}.
\end{aligned}$$

(iii) $\forall v_1v_2 \in SB_1,\ v \in QA_2$.

$$\begin{aligned}
T_{(\overline{S}B_1 \otimes \overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \min\{T_{\overline{S}B_1}(v_1v_2), T_{\overline{Q}A_2}(v)\},\\
T_{(\underline{S}B_1 \otimes \underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \min\{T_{\underline{S}B_1}(v_1v_2), T_{\underline{Q}A_2}(v)\},\\
I_{(\overline{S}B_1 \otimes \overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{I_{\overline{S}B_1}(v_1v_2), I_{\overline{Q}A_2}(v)\},\\
I_{(\underline{S}B_1 \otimes \underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{I_{\underline{S}B_1}(v_1v_2), I_{\underline{Q}A_2}(v)\},\\
F_{(\overline{S}B_1 \otimes \overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{F_{\overline{S}B_1}(v_1v_2), F_{\overline{Q}A_2}(v)\},\\
F_{(\underline{S}B_1 \otimes \underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{F_{\underline{S}B_1}(v_1v_2), F_{\underline{Q}A_2}(v)\}.
\end{aligned}$$

(iv) $\forall v_1u_1 \in SB_1, v_1u_2 \in SB_2$.

$$\begin{aligned}
T_{(\overline{S}B_1\otimes\overline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \min\{T_{\overline{S}B_1}(v_1u_1), T_{\overline{S}B_2}(v_1u_2)\},\\
T_{(\underline{S}B_1\otimes\underline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \min\{T_{\underline{S}B_1}(v_1u_1), T_{\underline{S}B_2}(v_1u_2)\},\\
I_{(\overline{S}B_1\otimes\overline{S}B_2}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{I_{\overline{S}B_1}(v_1u_1), I_{\overline{S}B_2}(v_1u_2)\},\\
I_{(\underline{S}B_1\otimes\underline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{I_{\underline{S}B_1}(v_1u_1), I_{\underline{S}B_2}(v_1u_2)\},\\
F_{(\overline{S}B_1\otimes\overline{S}B_2}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{F_{\overline{S}B_1}(v_1u_1), F_{\overline{S}B_2}(v_1u_2)\},\\
F_{(\underline{S}B_1\otimes\underline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{F_{\underline{S}B_1}(v_1u_1), F_{\underline{S}B_2}(v_1u_2)\}.
\end{aligned}$$

Definition 14. *The composition of G_1 and G_2 is a neutrosophic soft rough graph $G = G_1[G_2] = (G_{1*}[G_{2*}], G_1^*[G_2^*])$, where $G_{1*}[G_{2*}] = (\underline{Q}A_1[\underline{Q}A_2], \underline{S}B_1[\underline{S}B_2])]$ and $G_1^*[G_2^*] = (\overline{Q}A_1[\overline{Q}A_2], \overline{S}B_1[\overline{S}B_2])$ are neutrosophic graphs, respectively, such that*

(i) $\forall (v_1,v_2) \in QA_1 \times QA_2$.

$$\begin{aligned}
T_{(\overline{Q}A_1\times\overline{Q}A_2)}(v_1,v_2) &= \min\{T_{\overline{Q}A_1}(v_1), T_{\overline{Q}A_2}(v_2)\},\ T_{(\underline{Q}A_1\times\underline{Q}A_2)}(v_1,v_2) = \min\{T_{\underline{Q}A_1}(v_1), T_{\underline{Q}A_2}(v_2)\},\\
I_{(\overline{Q}A_1\times\overline{Q}A_2)}(v_1,v_2) &= \max\{I_{\overline{Q}A_1}(v_1), I_{\overline{Q}A_2}(v_2)\},\ \ I_{(\underline{Q}A_1\times\underline{Q}A_2)}(v_1,v_2) = \max\{I_{\underline{Q}A_1}(v_1), I_{\underline{Q}A_2}(v_2)\},\\
F_{(\overline{Q}A_1\times\overline{Q}A_2)}(v_1,v_2) &= \max\{F_{\overline{Q}A_1}(v_1), F_{\overline{Q}A_2}(v_2)\}, F_{(\underline{Q}A_1\times\underline{Q}A_2)}(v_1,v_2) = \max\{F_{\underline{Q}A_1}(v_1), F_{\underline{Q}A_2}(v_2)\}.
\end{aligned}$$

(ii) $\forall v_1v_2 \in SB_2,\ v \in QA_1$.

$$\begin{aligned}
T_{(\overline{S}B_1\times\overline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \min\{T_{\overline{Q}A_1}(v), T_{\overline{S}B_2}(v_1v_2)\},\\
T_{(\underline{S}B_1\times\underline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \min\{T_{\underline{Q}A_1}(v), T_{\underline{S}B_2}(v_1v_2)\},\\
I_{(\overline{S}B_1\times\overline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \max\{I_{\overline{Q}A_1}(v), I_{\overline{S}B_2}(v_1v_2)\},\\
I_{(\underline{S}B_1\times\underline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \max\{I_{\underline{Q}A_1}(v), I_{\underline{S}B_2}(v_1v_2)\},\\
F_{(\overline{S}B_1\times\overline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \max\{F_{\overline{Q}A_1}(v), F_{\overline{S}B_2}(v_1v_2)\},\\
F_{(\underline{S}B_1\times\underline{S}B_2)}\big((v,v_1)(v,v_2)\big) &= \max\{F_{\underline{Q}A_1}(v), F_{\underline{S}B_2}(v_1v_2)\}.
\end{aligned}$$

(iii) $\forall v_1v_2 \in SB_1,\ v \in QA_2$.

$$\begin{aligned}
T_{(\overline{S}B_1\times\overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \min\{T_{\overline{S}B_1}(v_1v_2), T_{\overline{Q}A_2}(v)\},\\
T_{(\underline{S}B_1\times\underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \min\{T_{\underline{S}B_1}(v_1v_2), T_{\underline{Q}A_2}(v)\},\\
I_{(\overline{S}B_1\times\overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{I_{\overline{S}B_1}(v_1v_2), I_{\overline{Q}A_2}(v)\},\\
I_{(\underline{S}B_1\times\underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{I_{\underline{S}B_1}(v_1v_2), I_{\underline{Q}A_2}(v)\},\\
F_{(\overline{S}B_1\times\overline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{F_{\overline{S}B_1}(v_1v_2), F_{\overline{Q}A_2}(v)\},\\
F_{(\underline{S}B_1\times\underline{S}B_2)}\big((v_1,v)(v_2,v)\big) &= \max\{F_{\underline{S}B_1}(v_1v_2), F_{\underline{Q}A_2}(v)\}.
\end{aligned}$$

(iv) $\forall v_1u_1 \in SB_1,\ v_1 \neq u_2 \in QA_2.$

$$\begin{aligned}
T_{(\overline{S}B_1\times\overline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \min\{T_{\overline{S}B_1}(v_1u_1), T_{\overline{S}B_2}(v_1u_2)\},\\
T_{(\underline{S}B_1\times\underline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \min\{T_{\underline{S}B_1}(v_1u_1), T_{\underline{S}B_2}(v_1u_2)\},\\
I_{(\overline{S}B_1\times\overline{S}B_2}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{I_{\overline{S}B_1}(v_1u_1), I_{\overline{S}B_2}(v_1u_2)\},\\
I_{(\underline{S}B_1\times\underline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{I_{\underline{S}B_1}(v_1u_1), I_{\underline{S}B_2}(v_1u_2)\},\\
F_{(\overline{S}B_1\times\overline{S}B_2}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{F_{\overline{S}B_1}(v_1u_1), F_{\overline{S}B_2}(v_1u_2)\},\\
F_{(\underline{S}B_1\times\underline{S}B_2)}\big((v_1,v_1)(u_1,u_2)\big) &= \max\{F_{\underline{S}B_1}(v_1u_1), F_{\underline{S}B_2}(v_1u_2)\}.
\end{aligned}$$

Definition 15. *Let $G = (\underline{G}, \overline{G})$ be a neutrosophic soft rough graph. The complement of G, denoted by $\acute{\underline{G}} = (\acute{\underline{G}}, \acute{\overline{G}})$ is a neutrosophic soft rough graph, where $\acute{\underline{G}} = (\underline{Q}\acute{}A, \underline{S}\acute{}B)$ and $\acute{\overline{G}} = (\overline{Q}\acute{}A, \overline{S}\acute{}B)$ are neutrosophic graphs such that*

(i) $\forall v \in QA.$

$$\begin{aligned}
T^{\acute{}}_{\overline{Q}A}(v) = T_{\overline{Q}A(v)},\ \ I^{\acute{}}_{\overline{Q}A}(v) = I_{\overline{Q}A(v)},\ \ F^{\acute{}}_{\overline{Q}A}(v) = F_{\overline{Q}A(v)},\\
T^{\acute{}}_{\underline{Q}A}(v) = T_{\underline{Q}A(v)},\ \ I^{\acute{}}_{\underline{Q}A}(v) = I_{\underline{Q}A(v)},\ \ F^{\acute{}}_{\underline{Q}A}(v) = F_{\underline{Q}A(v)}.
\end{aligned}$$

(ii) $\forall\ v, u \in QA.$

$$\begin{aligned}
T^{\acute{}}_{\overline{S}B}(vu) &= \min\{T_{\overline{Q}A}(v), T_{\overline{Q}A}(u)\} - T_{\overline{S}B}(vu),\\
I^{\acute{}}_{\overline{S}B}(vu) &= \max\{I_{\overline{Q}A}(v), I_{\overline{Q}A}(u)\} - I_{\overline{S}B}(vu),\\
F^{\acute{}}_{\overline{S}B}(vu) &= \max\{F_{\overline{Q}A}(v), F_{\overline{Q}A}(u)\} - F_{\overline{S}B}(vu),\\
T^{\acute{}}_{\underline{S}B}(vu) &= \min\{T_{\underline{Q}A}(v), T_{\underline{Q}A}(u)\} - T_{\underline{S}B}(vu),\\
I^{\acute{}}_{\underline{S}B}(vu) &= \max\{I_{\underline{Q}A}(v), I_{\underline{Q}A}(u)\} - I_{\underline{S}B}(vu),\\
F^{\acute{}}_{\underline{S}B}(vu) &= \max\{F_{\underline{Q}A}(v), F_{\underline{Q}A}(u)\} - F_{\underline{S}B}(vu).
\end{aligned}$$

Example 8. *Consider an NSRGs G as shown in Figure 9.*

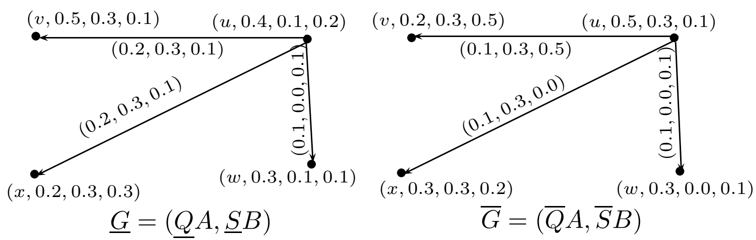


Figure 9. Neutrosophic soft rough graph $G = (\underline{G}, \overline{G})$

The complement of G is $\acute{G} = (\acute{\underline{G}}, \acute{\overline{G}})$ is obtained by using the Definition 15, where $\acute{\underline{G}} = (\underline{Q}\acute{}A, \underline{S}\acute{}B)$ and $\acute{\overline{G}} = (\overline{Q}\acute{}A, \overline{S}\acute{}B)$ are neutrosophic graphs as shown in Figure 10.

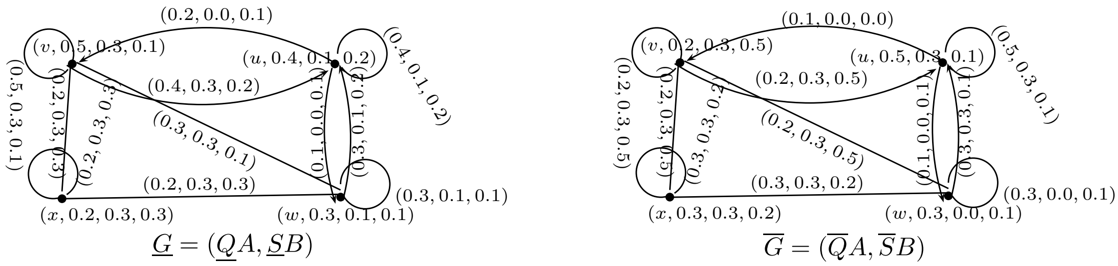


Figure 10. Neutrosophic soft rough graph $\acute{G} = (\underline{\acute{G}}, \overline{\acute{G}})$

Definition 16. *A graph G is called self complement, if* $G = \acute{\mathbf{G}}$, *i.e.,*

(i) $\forall v \in QA$.

$$T^{\acute{}}_{\overline{Q}A}(v) = T_{\overline{Q}A(v)},\ \ I^{\acute{}}_{\overline{Q}A}(v) = I_{\overline{Q}A(v)}, F^{\acute{}}_{\overline{Q}A}(v) = F_{\overline{Q}A(v)},$$
$$T^{\acute{}}_{\underline{Q}A}(v) = T_{\underline{Q}A(v)},\ I^{\acute{}}_{\underline{Q}A}(v) = I_{\underline{Q}A(v)},\ F^{\acute{}}_{\underline{Q}A}(v) = F_{\underline{Q}A(v)}.$$

(ii) $\forall\ v, u \in QA$.

$$T^{\acute{}}_{\overline{S}B}(vu) = T_{\overline{S}B}(vu),\ I^{\acute{}}_{\overline{S}B}(vu) = I_{\overline{S}B}(vu),\ F^{\acute{}}_{\overline{S}B}(vu) = F_{\overline{S}B}(vu),$$
$$T^{\acute{}}_{\underline{S}B}(vu) = T_{\underline{S}B}(vu),\ I^{\acute{}}_{\underline{S}B}(vu) = I_{\underline{S}B}(vu),\ F^{\acute{}}_{\underline{S}B}(vu) = F_{\underline{S}B}(vu).$$

Definition 17. *A neutrosophic soft rough graph G is called strong neutrosophic soft rough graph if* $\forall uv \in SB$,

$$T_{\overline{S}B}(vu) = \min\{T_{\overline{Q}A}(v), T_{\overline{Q}A}(u)\},\ I_{\overline{S}B}(vu) = \max\{I_{\overline{Q}A}(v), I_{\overline{Q}A}(u)\}),\ F_{\overline{S}B}(vu) = \max\{F_{\overline{Q}A}(v), F_{\overline{Q}A}(u)\},$$
$$T_{\underline{S}B}(vu) = \min\{T_{\underline{Q}A}(v), T_{\underline{Q}A}(u)\},\ I_{\underline{S}B}(vu) = \max\{I_{\underline{Q}A}(v), I_{\underline{Q}A}(u)\},\ F_{\underline{S}B}(vu) = \max\{F_{\underline{Q}A}(v), F_{\underline{Q}A}(u)\}.$$

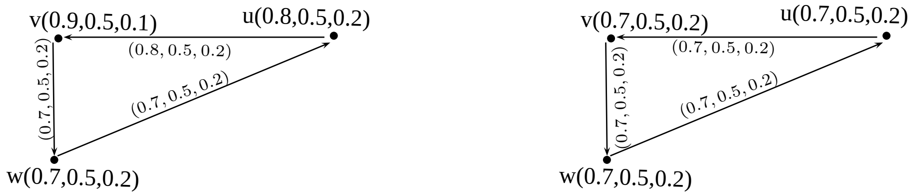
Example 9. *Consider a graph G such that* $V = \{u, v, w\}$ *and* $E = \{uv, vw, wu\}$. *Let QA be a neutrosophic soft rough set of V and let SB be a neutrosophic soft rough set of E defined in the Tables 9 and 10, respectively.*

Table 9. Neutrosophic soft rough set on V.

V	$\overline{Q}A$	$\underline{Q}A$
u	(0.8, 0.5, 0.2)	(0.7, 0.5, 0.2)
v	(0.9, 0.5, 0.1)	(0.7, 0.5, 0.2)
w	(0.7, 0.5, 0.1)	(0.7, 0.5, 0.2)

Table 10. Neutrosophic soft rough set on E.

E	$\overline{S}B$	$\underline{S}B$
uv	(0.8, 0.5, 0.2)	(0.7, 0.5, 0.2)
vw	(0.7, 0.5, 0.1)	(0.7, 0.5, 0.2)
wu	(0.7, 0.5, 0.2)	(0.7, 0.5, 0.2)



$\overline{G} = (\overline{Q}A, \overline{S}B)$ $\underline{G} = (\underline{Q}A, \underline{S}B)$

Figure 11. Strong neutrosophic soft rough graph $G = (QA, SB)$

Hence, $G = (QA, SB)$ is a strong neutrosophic soft rough graph.

Definition 18. *A neutrosophic soft rough graph G is called a complete neutrosophic soft rough graph if $\forall\ vu \in QA$,*

$$T_{\overline{S}B}(vu) = \min\{T_{\overline{Q}A}(v), T_{\overline{Q}A}(u)\}, I_{\overline{S}B}(vu) = \max\{I_{\overline{Q}A}(v), I_{\overline{Q}A}(u)\}, F_{\overline{S}B}(vu) = \max\{F_{\overline{Q}A}(v), F_{\overline{Q}A}(u)\},$$
$$T_{\underline{S}B}(vu) = \min\{T_{\underline{Q}A}(v), T_{\underline{Q}A}(u)\}, I_{\underline{S}B}(vu) = \max\{I_{\underline{Q}A}(v), I_{\underline{Q}A}(u)\}, F_{\underline{S}B}(vu) = \max\{F_{\underline{Q}A}(v), F_{\underline{Q}A}(u)\}.$$

Remark 2. *Every complete neutrosophic soft rough graph is a strong neutrosophic soft rough graph. However, the converse is not true.*

Definition 19. *A neutrosophic soft rough graph G is isolated, if $\forall x, y \in QA$.*

$$T_{\underline{S}B}(vu) = 0,\ I_{\underline{S}B}(vu) = 0,\ F_{\underline{S}B}(vu) = 0,\ T_{\overline{S}B}(vu) = 0,\ I_{\overline{S}B}(vu) = 0,\ F_{\overline{S}B}(vu) = 0.$$

Theorem 1. *The rejection of two neutrosophic soft rough graphs is a neutrosophic soft rough graph.*

Proof. Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two NSRGs. Let $G = G_1|G_2 = (\underline{G}_1|\underline{G}_2, \overline{G}_1|\overline{G}_2)$ be the rejection of G_1 and G_2, where $\underline{G}_1|\underline{G}_2 = (\underline{Q}A_1|\underline{Q}A_2, \underline{S}B_1|\underline{S}B_2)$ and $\overline{G}_1|\overline{G}_2 = (\overline{Q}A_1|\overline{Q}A_2, \overline{S}B_1|\overline{S}B_2)$. We claim that $G = G_1|G_2$ is a neutrosophic soft rough graph. It is enough to show that $\underline{S}B_1|\underline{S}B_2$ and $\overline{S}B_1|\overline{S}B_2$ are neutrosophic relations on $\underline{Q}A_1|\underline{Q}A_2$ and $\overline{Q}A_1|\overline{Q}A_2$, respectively. First, we show that $\underline{S}B_1|\underline{S}B_2$ is a neutrosophic relation on $\underline{Q}A_1|\underline{Q}A_2$.
If $v \in \underline{Q}A_1$, $v_1v_2 \notin \underline{S}B_2$, then

$$\begin{aligned}
T_{(\underline{S}B_1|\underline{S}B_2)}((v,v_1)(v,v_2)) &= (T_{\underline{Q}A_1}(v) \wedge (T_{\underline{Q}A_2}(v_2) \wedge T_{\underline{Q}A_2}(v_2))) \\
&= (T_{\underline{Q}A_1}(v) \wedge T_{\underline{Q}A_2}(v_2)) \wedge (T_{\underline{Q}A_1}(v) \wedge T_{\underline{Q}A_2}(v_2)) \\
&= T_{(\underline{Q}A_1|\underline{Q}A_2)}(v,v_1) \wedge T_{(\underline{Q}A_1|\underline{Q}A_2)}(v,v_2) \\
T_{(\underline{S}B_1|\underline{S}B_2)}((v,v_1)(v,v_2)) &= T_{(\underline{Q}A_1|\underline{Q}A_2)}(v,v_1) \wedge T_{(\underline{Q}A_1|\underline{Q}A_2)}(v,v_2) \\
\text{Similarly, } I_{(\underline{S}B_1|\underline{S}B_2)}((v,v_1)(v,v_2)) &= I_{(\underline{Q}A_1|\underline{Q}A_2)}(v,v_1) \vee I_{(\underline{Q}A_1|\underline{Q}A_2)}(v,v_2) \\
F_{(\underline{S}B_1|\underline{S}B_2)}((v,v_1)(v,v_2)) &= F_{(\underline{Q}A_1|\underline{Q}A_2)}(v,v_1) \vee F_{(\underline{Q}A_1|\underline{Q}A_2)}(v,v_2).
\end{aligned}$$

If $v_1v_2 \notin \underline{S}B_1$, $v \in \underline{Q}A_2$, then

$$\begin{aligned}
T_{(\underline{S}B_1|\underline{S}B_2)}((v_1,v)(v_2,v)) &= ((T_{\underline{Q}A_1}(v_1) \wedge T_{\underline{Q}A_1}(v_2)) \wedge T_{\underline{Q}A_2}(v)) \\
&= ((T_{\underline{Q}A_1}(v_1) \wedge T_{\underline{Q}A_2}(v)) \wedge (T_{\underline{Q}A_1}(v_2) \wedge T_{\underline{Q}A_2}(v))) \\
&= T_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,v) \wedge T_{(\underline{Q}A_1|\underline{Q}A_2)}(v_2,v) \\
T_{(\underline{S}B_1|\underline{S}B_2)}((v_1,v)(v_2,v)) &= T_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,v) \wedge T_{(\underline{Q}A_1|\underline{Q}A_2)}(v_2,v) \\
\text{Similarly, } I_{(\underline{S}B_1|\underline{S}B_2)}((v_1,v)(v_2,v)) &= I_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,v) \vee I_{(\underline{Q}A_1|\underline{Q}A_2)}(v_2,v) \\
F_{(\underline{S}B_1|\underline{S}B_2)}((v_1,v)(v_2,v)) &= F_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,v) \vee F_{(\underline{Q}A_1|\underline{Q}A_2)}(v_2,v).
\end{aligned}$$

If $v_1v_2 \notin \underline{S}B_1$, $u_1, u_2 \notin \underline{S}B_2$, then

$$\begin{aligned}
T_{(\underline{S}B_1|\underline{S}B_2)}((v_1,u_1)(v_2,u_2)) &= ((T_{\underline{Q}A_1}(v_1) \wedge T_{\underline{Q}A_1}(v_2)) \wedge (T_{\underline{Q}A_2}(u_1) \wedge T_{\underline{Q}A_2}(u_2))) \\
&= (T_{\underline{Q}A_1}(v_1) \wedge T_{\underline{Q}A_2}(u_1)) \wedge (T_{\underline{Q}A_1}(v_2) \wedge T_{\underline{Q}A_2}(u_2)) \\
&= T_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,u_1) \wedge T_{(\underline{Q}A_1|\underline{Q}A_2)}(v_2,u_2) \\
T_{(\underline{S}B_1|\underline{S}B_2)}((v_1,u_1)(v_2,u_2)) &= T_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,u_1) \wedge T_{(\underline{Q}A_1|\underline{Q}A_2)}(u_1,u_2) \\
\text{Similarly, } I_{(\underline{S}B_1|\underline{S}B_2)}((v_1,u_1)(v_2,u_2)) &= I_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,u_1) \vee I_{(\underline{Q}A_1|\underline{Q}A_2)}(u_1,u_2) \\
F_{(\underline{S}B_1|\underline{S}B_2)}((v_1,u_1)(v_2,u_2)) &= F_{(\underline{Q}A_1|\underline{Q}A_2)}(v_1,u_1) \vee F_{(\underline{Q}A_1|\underline{Q}A_2)}(u_1,u_2).
\end{aligned}$$

Thus, $\underline{S}B_1|\underline{S}B_2$ is a neutrosophic relation on $\underline{Q}A_1|\underline{Q}A_2$. Similarly, we can show that $\overline{S}B_1|\overline{S}B_2$ is a neutrosophic relation on $\overline{Q}A_1|\overline{Q}A_2$. Hence, G is a neutrosophic soft rough graph. □

Theorem 2. *The Cartesian product of two NSRGs is a neutrosophic soft rough graph.*

Proof. Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two NSRGs. Let $G = G_1 \ltimes G_2 = (\underline{G}_1 \ltimes \underline{G}_2, \overline{G}_1 \ltimes \overline{G}_2)$ be the Cartesian product of G_1 and G_2, where $\underline{G}_1 \ltimes \underline{G}_2 = (\underline{Q}A_1 \ltimes \underline{Q}A_2, \underline{S}B_1 \ltimes \underline{S}B_2)$ and $\overline{G}_1 \ltimes \overline{G}_2 = (\overline{Q}A_1 \ltimes \overline{Q}A_2, \overline{S}B_1 \ltimes \underline{S}B_2)$. We claim that $G = G_1 \ltimes G_2$ is a neutrosophic soft rough graph. It is enough to show that $\underline{S}B_1 \ltimes \underline{S}B_2$ and $\overline{S}B_1 \ltimes \overline{S}B_2$ are neutrosophic relations on $\underline{Q}A_1 \ltimes \underline{Q}A_2$ and $\overline{Q}A_1 \ltimes \overline{Q}A_2$, respectively. We have to show that $\underline{S}B_1 \ltimes \underline{S}B_2$ is a neutrosophic relation on $\underline{Q}A_1 \ltimes \underline{Q}A_2$.

If $v \in \underline{Q}A_1$, $v_1u_1 \in \underline{S}B_2$, then

$$\begin{aligned}
T_{(\underline{S}B_1 \ltimes \underline{S}B_2)}((v,v_1)(v,u_1)) &= T_{(\underline{Q}A_1)}(v) \wedge T_{(\underline{S}B_2)}(v_1u_1) \\
&\leqslant T_{(\underline{Q}A_1)}(v) \wedge (T_{(\underline{Q}A_2)}(v_1) \wedge T(\underline{Q}A_2)(u_1)) \\
&= (T_{(\underline{Q}A_1)}(v) \wedge T_{(\underline{Q}A_2)}(v_1)) \wedge (T_{(\underline{Q}A_1)}(v) \wedge T_{(\underline{Q}A_2)}(u_1)) \\
&= T_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v,v_1) \wedge T_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v,u_1) \\
T_{(\underline{S}B_1 \ltimes \underline{S}B_2)}((v,v_1)(v,u_1)) &\leqslant T_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v,v_1) \wedge T_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v,u_1) \\
\text{Similarly, } I_{(\underline{S}B_1 \ltimes \underline{S}B_2)}((v,v_1)(v,u_1)) &\leqslant I_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v,v_1) \vee I_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v,u_1) \\
F_{(\underline{S}B_1 \ltimes \underline{S}B_2)}((v,v_1)(v,u_1)) &\leqslant F_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v,v_1) \vee F_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v,u_1).
\end{aligned}$$

If $v_1u_1 \in \underline{S}B_1$, $z \in \underline{Q}A_2$, then

$$\begin{aligned}
T_{(\underline{S}B_1 \ltimes \underline{S}B_2)}((v_1,z)(u_1,z)) &= T_{(\underline{S}B_1)}(v_1u_1) \wedge T_{(\underline{Q}A_2)}(z) \\
&\leqslant (T_{(\underline{Q}A_1)(v_1) \wedge (\underline{Q}A_1)}(u_1)) \wedge T_{(\underline{Q}A_2)}(z) \\
&= T_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v_1,z) \wedge T_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(u_1,z) \\
T_{(\underline{S}B_1 \ltimes \underline{S}B_2)}((v_1,z)(u_1,z)) &\leqslant T_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v_1,z) \wedge T_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(u_1,z) \\
\text{Similarly, } I_{(\underline{S}B_1 \ltimes \underline{S}B_2)}((v_1,z)(u_1,z)) &\leqslant I_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v_1,z) \vee I_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(u_1,z) \\
F_{(\underline{S}B_1 \ltimes \underline{S}B_2)}((v_1,z)(u_1,z)) &\leqslant F_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(v_1,z) \vee F_{(\underline{Q}A_1 \ltimes \underline{Q}A_2)}(u_1,z).
\end{aligned}$$

Therefore, $\underline{S}B_1 \ltimes \underline{S}B_2$ is a neutrosophic relation on $\underline{Q}A_1 \ltimes \underline{Q}A_2$. Similarly, $\overline{S}B_1 \ltimes \overline{S}B_2$ is a neutrosophic relation on $\overline{Q}A_1 \ltimes \overline{Q}A_2$. Hence, G is a neutrosophic rough graph. □

Theorem 3. *The cross product of two neutrosophic soft rough graphs is a neutrosophic soft rough graph.*

Proof. Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two NSRGs. Let $G = G_1 \circledcirc G_2 = (\underline{G}_1 \circledcirc \underline{G}_2, \overline{G}_1 \circledcirc \overline{G}_2)$ be the cross product of G_1 and G_2, where $\underline{G}_1 \circledcirc \underline{G}_2 = (\underline{Q}A_1 \circledcirc \underline{Q}A_2, \underline{S}B_1 \circledcirc \underline{S}B_2)$ and $\overline{G}_1 \circledcirc \overline{G}_2 = (\overline{Q}A_1 \circledcirc \overline{Q}A_2, \overline{S}B_1 \circledcirc \overline{S}B_2)$. We claim that $G = G_1 \circledcirc G_2$ is a neutrosophic soft rough graph. It is enough to show that $\underline{S}B_1 \circledcirc \underline{S}B_2$ and $\overline{S}B_1 \circledcirc \overline{S}B_2$ are neutrosophic relations on $\underline{Q}A_1 \circledcirc \underline{Q}A_2$ and $\overline{Q}A_1 \circledcirc \overline{Q}A_2$, respectively. First, we show that $\underline{S}B_1 \circledcirc \underline{S}B_2$ is a neutrosophic relation on $\underline{Q}A_1 \circledcirc \underline{Q}A_2$.

If $v_1u_1 \in \underline{S}B_1$, $v_1u_2 \in \underline{S}B_2$, then

$$\begin{aligned}
T_{(\underline{S}B_1 \circledcirc \underline{S}B_2)}((v_1, v_1)(u_1, u_2)) &= T_{(\underline{S}B_1)}(v_1u_1) \wedge T_{(\underline{S}B_2)}(v_1u_2) \\
&\leqslant (T_{(\underline{Q}A_1)}(v_1) \wedge T_{(\underline{Q}A_1)}(u_1) \wedge (T_{(\underline{Q}A_2)}(v_1) \wedge T_{(\underline{Q}A_2)}(u_2)) \\
&= (T_{(\underline{Q}A_1)}(v_1) \wedge T_{(\underline{Q}A_2)}(v_1)) \wedge (T_{(\underline{Q}A_1)}(u_1) \wedge T_{(\underline{Q}A_2)}(u_2)) \\
&= T_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(v_1, v_1) \wedge T_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(u_1, u_2) \\
T_{(\underline{S}B_1 \circledcirc \underline{S}B_2)}((v_1, v_1)(u_1, u_2)) &\leqslant T_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(v_1, v_1) \wedge T_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(v, u_2) \\
\text{Similarly, } I_{(\underline{S}B_1 \circledcirc \underline{S}B_2)}((v_1, v_1)(u_1, u_2)) &\leqslant I_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(v_1, v_1) \vee I_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(v, u_2) \\
F_{(\underline{S}B_1 \circledcirc \underline{S}B_2)}((v_1, v_1)(u_1, u_2)) &\leqslant F_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(v_1, v_1) \vee F_{(\underline{Q}A_1 \circledcirc \underline{Q}A_2)}(v, u_2).
\end{aligned}$$

Thus, $\underline{S}B_1 \circledcirc \underline{S}B_2$ is a neutrosophic relation on $\underline{Q}A_1 \circledcirc \underline{Q}A_2$. Similarly, we can show that $\overline{S}B_1 \circledcirc \overline{S}B_2$ is a neutrosophic relation on $\overline{Q}A_1 \circledcirc \overline{Q}A_2$. Hence, G is a neutrosophic soft rough graph. □

3. Application

In this section, we apply the concept of NSRSs to a decision-making problem. In recent times, the object recognition problem has gained considerable importance. The object recognition problem can be considered as a decision-making problem, in which final identification of objects is founded on a given set of information. A detailed description of the algorithm for the selection of most suitable objects based on an available set of alternatives is given, and purposed decision-making method can be used to calculate lower and upper approximation operators to progress deep concerns of the problem. The presented algorithms can be applied to avoid lengthy calculations when dealing with large number of objects. This method can be applied in various domains for multi-criteria selection of objects.

Selection of Most Suitable Generic Version of Brand Name Medicine

In the pharmaceutical industry, different pharmaceutical companies develop, produce and discover pharmaceutical medicine (drugs) for use as medication. These pharmaceutical companies deals with "brand name medicine" and "generic medicine". Brand name medicine and generic medicine are bioequivalent, and have a generic medicine rate and element of absorption. Brand name medicine and generic medicine have the same active ingredients, but the inactive ingredients may differ. The most important difference is cost. Generic medicine is less expensive as compared to brand name comparators. Usually, generic drug manufacturers face competition to produce cost less products. The product may possibly be slightly dissimilar in color, shape, or markings. The major difference is cost. We consider a brand name drug "u = Loratadine" used for seasonal allergies medication. Consider

$$\begin{aligned}
V = \{ & u_1 = \text{Triamcinolone}, u_2 = \text{Cetirizine/Pseudoephedrine}, \\
& u_3 = \text{Pseudoephedrine}, u_4 = \text{loratadine/pseudoephedrine}, \\
& u_5 = \text{Fluticasone}\}
\end{aligned}$$

is a set of generic versions of "Loratadine". We want to select the most suitable generic version of Loratadine on the basis of parameters e_1 = Highly soluble, e_2 = Highly permeable, e_3 = Rapidly dissolving. $\mathbb{M} = \{e_1, e_2, e_3\}$ be a set of paraments. Let Q be a neutrosophic soft relation from V to parameter set M, and describe truth-membership, indeterminacy-membership and false-membership degrees of generic version medicine corresponding to the parameters as shown in Table 11.

Table 11. Neutrosophic soft set (Q, M).

Q	u_1	u_2	u_3	u_4	u_5
e_1	(0.4, 0.5, 0.6)	(0.5, 0.3, 0.6)	(0.7, 0.2, 0.3)	(0.5, 0.7, 0.5)	(0.6, 0.5, 0.4)
e_2	(0.7, 0.3, 0.2)	(0.3, 0.4, 0.3)	(0.6, 0.5, 0.4)	(0.8, 0.4, 0.6)	(0.7, 0.8, 0.5)
e_3	(0.6, 0.3, 0.4)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.4)	(0.8, 0.7, 0.6)	(0.7, 0.3, 0.5)

Suppose $A = \{(e_1, 0.2, 0.4, 0.5), (e_2, 0.5, 0.6, 0.4), (e_3, 0.7, 0.5, 0.4)\}$ is the most favorable object that is an NS on the parameter set M under consideration. Then, $(\underline{Q}(A), \overline{Q}(A))$ is an NSRS in NSAS (V, M, Q), where

$$\begin{aligned} \overline{Q}(A) &= \{(u_1, 0.6, 0.5, 0.4), (u_2, 0.7, 0.4, 0.4), (u_3, 0.7, 0.4, 0.4), (u_4, 0.7, 0.6, 0.5), (u_5, 0.7, 0.5, 0.5)\}, \\ \underline{Q}(A) &= \{(u_1, 0.5, 0.6, 0.4), (u_2, 0.5, 0.6, 0.5), (u_3, 0.3, 0.3, 0.5), (u_4, 0.5, 0.6, 0.5), (u_5, 0.4, 0.5, 0.5)\}. \end{aligned}$$

Let $E = \{u_1v_2, u_1u_3, u_4u_1, u_2u_3, u_5u_3, u_2u_4, u_2u_5\} \subseteq \acute{V}$ and $L = \{e_1e_3, e_2e_1, e_3e_2\} \subseteq \acute{\mathbb{M}}$. Then, a neutrosophic soft relation S on E (from L to E) can be defined as follows:

Table 12. Neutrosophic soft relation S.

S	u_1u_2	u_1u_3	u_4u_1	u_2u_3	u_5u_3	u_2u_4	u_2u_5
e_1e_2	(0.3, 0.4 ,0.2)	(0.4, 0.4, 0.5)	(0.4, 0.4, 0.5)	(0.6, 0.3, 0.4)	(0.4, 0.2, 0.2)	(0.4, 0.4, 0.2)	(0.4, 0.3, 0.4)
e_2e_3	(0.5 ,0.4 ,0.1)	(0.4, 0.3, 0.2)	(0.4, 0.3, 0.2)	(0.3, 0.3, 0.2)	(0.6, 0.2, 0.4)	(0.3, 0.2, 0.1)	(0.3, 0.3, 0.2)
e_1e_3	(0.4,0.4,0.1)	(0.4, 0.2, 0.2)	(0.4, 0.2, 0.2)	(0.5, 0.3, 0.3)	(0.4, 0.2, 0.3)	(0.4, 0.3, 0.1)	(0.5, 0.3, 0.2)

Let $B = \{(e_1e_2, 0.2, 0.4, 0.5), (e_2e_3, 0.5, 0.4, 0.4), (e_1e_3, 0.5, 0.2, 0.5)\}$ be an NS on L that describes some relationship between the parameters under consideration; then, $SB = (\underline{S}B, \overline{S}B)$ is an NSRR, where

$$\begin{aligned} \overline{S}B &= \{(u_1u_2, 0.5, 0.4, 0.4), (u_1u_3, 0.4, 0.2, 0.4), (u_4u_1, 0.4, 0.2, 0.4), (u_2u_3, 0.5, 0.3, 0.4), \\ &\quad (u_5u_3, 0.5, 0.2, 0.4), (u_2u_4, 0.4, 0.3, 0.4), (u_2u_5, 0.5, 0.3, 0.4)\}, \\ \underline{S}B &= \{(u_1u_2, 0.2, 0.4, 0.4)(u_1u_3, 0.5, 0.4, 0.4), (u_4u_1, 0.5, 0.4, 0.4), (u_2u_3, 0.4, 0.4, 0.5), \\ &\quad (u_5u_3, 0.2, 0.4, 0.4), (u_2u_4, 0.2, 0.4, 0.4), (u_2u_5, 0.4, 0.4, 0.5)\}. \end{aligned}$$

Thus, $G = (\underline{G}, \overline{G})$ is an NSRG as shown in Figure 12.

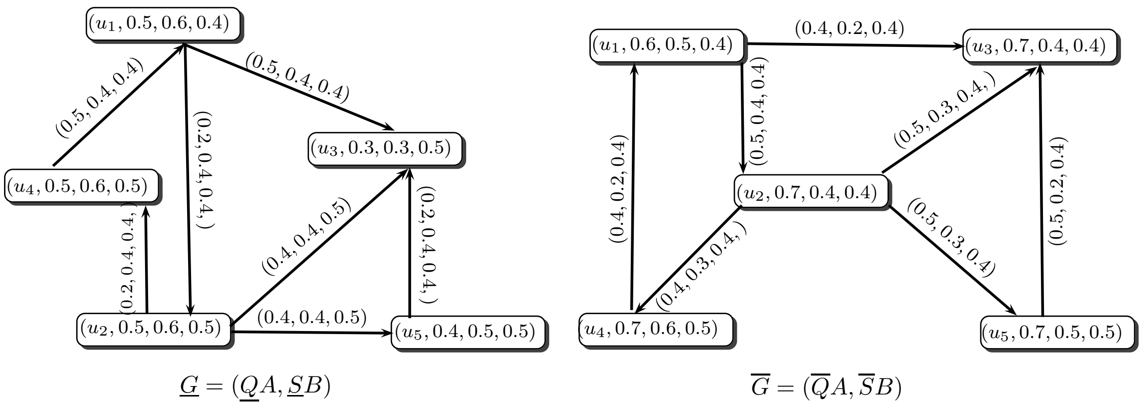


Figure 12. Neutrosophic soft rough graph $G = (\underline{G}, \overline{G})$

In [3], the sum of two neutrosophic numbers is defined.

Definition 20. *[3] Let C and D be two single valued neutrosophic numbers, and the sum of two single valued neutrosophic number is defined as follows:*

$$C \oplus D =< T_C + T_D - T_C \times T_D, I_C \times I_D, F_C \times F_D > . \tag{1}$$

Algorithm 1: Algorithm for selection of most suitable objects

1. Input the number of elements in vertex set $V = \{u_1, u_2, \ldots, u_n\}$.
2. Input the number of elements in parameter set $\mathbb{M} = \{e_1, e_2, \ldots, e_m\}$.
3. Input a neutrosophic soft relation Q from V to $\mathbb{M}$.
4. Input a neutrosophic set A on M.
5. Compute neutrosophic soft rough vertex set $QA = (\underline{Q}A, \overline{Q}(A))$.
6. Input the number of elements in edge set $E = \{u_1u_1, u_1u_2, \ldots, u_ku_1\}$.
7. Input the number of elements in parameter set $\acute{\mathbb{M}} = \{e_1e_1, e_1e_2, \ldots, e_le_1\}$.
8. Input a neutrosophic soft relation S from $\acute{V}$ to $\acute{\mathbb{M}}$.
9. Input a neutrosophic set B on $\acute{\mathbb{M}}$.
10. Compute neutrosophic soft rough edge set $SB = (\underline{S}B, \overline{S}(B))$.
11. Compute neutrosophic set $\alpha = (T_\alpha(u_i), I_\alpha(u_i), F_\alpha(u_i))$, where

$$\begin{aligned} T_\alpha(u_i) &= T_{\overline{Q}(A)}(u_i) + T_{\underline{Q}(A)}(u_i) - T_{\overline{Q}(A)}(u_i) \times T_{\underline{Q}(A)}(u_i), \\ I_\alpha(u_i) &= T_{\overline{Q}(A)}(u_i) \times T_{\underline{Q}(A)}(u_i), \\ F_\alpha(u_i) &= F_{\overline{Q}(A)}(u_i) \times F_{\underline{Q}(A)}(u_i). \end{aligned}$$

12. Compute neutrosophic set $\beta = (T_\beta(u_iu_i), I_\beta(u_iu_j), F_\beta(u_iu_j))$, where

$$\begin{aligned} T_\beta(u_iu_j) &= T_{\overline{S}(B)}(u_iu_j) + T_{\underline{S}(B)}(u_iu_j) - T_{\overline{S}(B)}(u_iu_j) \times T_{\underline{S}(B)}(u_iu_j), \\ I_\beta(u_iu_j) &= T_{\overline{S}(B)}(u_iu_j) \times T_{\underline{S}(B)}(u_iu_j), \\ F_\beta(u_iu_j) &= F_{\overline{S}(B)}(u_iu_j) \times F_{\underline{S}(B)}(u_iu_j). \end{aligned}$$

13. Calculate the score values of each object u_i, and the score function is defined as follows:

$$\tilde{S}(u_i) = \sum_{u_iu_j \in E} \frac{T_\alpha(u_j) + I_\alpha(u_j) - F_\alpha(u_j)}{3 - (T_\beta(u_iu_j) + I_\beta(u_iu_j) - F_\beta(u_iu_j))}.$$

14. The decision is S_i if $S_i = \max_{i=1}^{n} \tilde{S}_i$.
15. If i has more than one value, then any one of S_i may be chosen.

The sum of UNSRS $\overline{Q}A$ and the LNSRS $\underline{Q}A$ and sum of LNSRR $\underline{S}B$ and the UNSRR $\overline{S}B$ are NSs $\overline{Q}A \oplus \underline{Q}A$ and $\overline{S}B \oplus \underline{S}B$, respectively defined by

$$\begin{aligned} \alpha = \overline{Q}A \oplus \underline{Q}A &= \{(u_1, 0.8, 0.3, 0.16), (u_2, 0.85, 0.24, 0.2), (u_3, 0.79, 0.2, 0.2), (u_4, 0.85, 0.36, 0.25), \\ &\quad (u_5, 0.82, 0.25, 0.25)\}, \\ \beta = \overline{S}B \oplus \underline{S}B &= \{(u_1u_2, 0.6, 0.16, 0.16), (u_1u_3, 0.7, 0.8, 0.16), (u_4u_1, 0.7, 0.8, 0.16), (u_2u_3, 0.7, \\ &\quad 0.12, 0.2), (u_5u_3, 0.6, 0.08, 0.16), (u_2u_4, 0.52, 0.12, 0.16), (u_2u_5, 0.7, 0.12, 0.2)\}. \end{aligned}$$

The score function $\tilde{S}(u_k)$ defines for each generic version medicine $u_i \in V$,

$$\tilde{S}(u_i) = \sum_{u_iu_j \in E} \frac{T_\alpha(u_j) + I_\alpha(u_j) - F_\alpha(u_j)}{3 - (T_\beta(u_iu_j) + I_\beta(u_iu_j) - F_\beta(u_iu_j))} \tag{2}$$

and u_k with the larger score value $u_k = \max_i S(u_i)$ is the most suitable generic version medicine. By calculations, we have

$$\tilde{S}(u_1) = 0.88, \tilde{S}(u_2) = 0.69, \ \tilde{S}(u_3) = 0.26 \ \tilde{S}(u_4) = 0.57, \text{ and } \tilde{S}(u_5) = 0.33. \tag{3}$$

Here, u_1 is the optimal decision, and the most suitable generic version of "Loratadine" is "Triamcinolone". We have used software MATLAB (version 7, MathWorks, Natick, MA, USA) for calculating the required results in the application. The algorithm is given in Algorithm 1. The algorithm of the program is general for any number of objects with respect to certain parameters.

4. Conclusions

Rough set theory can be considered as an extension of classical set theory. Rough set theory is a very useful mathematical model to handle vagueness. NS theory, RS theory and SS theory are three useful distinguished approaches to deal with vagueness. NS and RS models are used to handle uncertainty, and combining these two models with another remarkable model of SSs gives more precise results for decision-making problems. In this paper, we have presented the notion of NSRGs and investigated some properties of NSRGs in detail. The notion of NSRGs can be utilized as a mathematical tool to deal with imprecise and unspecified information. In addition, a decision-making method based on NSRGs is proposed. This research work can be extended to (1) Rough bipolar neutrosophic soft sets; (2) Bipolar neutrosophic soft rough sets, (3) Interval-valued bipolar neutrosophic rough sets, and (4) Soft rough neutrosophic graphs.

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