The Origins Of Number Concepts In Five-, Six- And Seven-Year Old Anglo, Hispanic And Native American Children

Manon Pettit Charbonneau

Follow this and additional works at: https://digitalrepository.unm.edu/educ_teelp_etds

Part of the Educational Administration and Supervision Commons, Educational Leadership Commons, and the Teacher Education and Professional Development Commons
THE UNIVERSITY OF NEW MEXICO
ALBUQUERQUE, NEW MEXICO 87131

POLICY ON USE OF THESE AND DISSERTATIONS

Unpublished theses and dissertations accepted for master's and doctor's degrees and deposited in the University of New Mexico Library are open to the public for inspection and reference work. They are to be used only with due regard to the rights of the authors. The work of other authors should always be given full credit. Avoid quoting in amounts, over and beyond scholarly needs, such as might impair or destroy the property rights and financial benefits of another author.

To afford reasonable safeguards to authors, and consistent with the above principles, anyone quoting from theses and dissertations must observe the following conditions:

1. Direct quotations during the first two years after completion may be made only with the written permission of the author.

2. After a lapse of two years, theses and dissertations may be quoted without specific prior permission in works of original scholarship provided appropriate credit is given in the case of each quotation.

3. Quotations that are complete units in themselves (e.g., complete chapters or sections) in whatever form they may be reproduced and quotations of whatever length presented as primary material for their own sake (as in anthologies or books of readings) ALWAYS require consent of the authors.

4. The quoting author is responsible for determining "fair use" of material he uses.

This thesis/dissertation by [Manon Pettit Charbonneau] has been used by the following persons whose signatures attest their acceptance of the above conditions. (A library which borrows this thesis/dissertation for use by its patrons is expected to secure the signature of each user.)

NAME AND ADDRESS

-----------------------------------

DATE

-----------------------------------
Manon Pettit Charbonneau

Candidate

Curriculum and Instruction in Multicultural Teacher Education

Department

This dissertation is approved, and it is acceptable in quality and form for publication on microfilm:

Approved by the Dissertation Committee:

[Signatures of committee members]

Accepted:

Asst. Dean, Graduate School

January 4, 1988

Date
THE ORIGINS OF NUMBER CONCEPTS IN FIVE-, SIX- AND SEVEN-YEAR OLD ANGLO, HISPANIC AND NATIVE AMERICAN CHILDREN

BY

MANON PETTIT CHARBONNEAU
B.A., Bard College, 1965
M.A., University of New Mexico, 1982

DISSERTATION

Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Education

The University of New Mexico
Albuquerque, New Mexico

December, 1987
ACKNOWLEDGEMENTS

The writer's deepest appreciation is extended to Dr. David Darling, Chairman of the dissertation committee. This study was satisfactorily completed largely because of his guidance, patience, interest and encouragement.

Sincere gratitude is also expressed to the rest of the committee members, Drs. Patrick Scott, Catherine Loughlin, Rafael Diaz and David Hawkins. Their insight and constructive suggestions opened up many more possibilities in the study, and made it an exciting and challenging learning experience for the writer. Each has contributed uniquely to this work: Rick Scott, for many, many hours of discussion on children and the learning of mathematics; Catherine Loughlin, for her close analysis of the writing; Rafael Diaz, for advancing this writer's understanding of developmental learning and the research process; and David Hawkins, for his constant concern that the need to understand children and their understandings of mathematics is never forgotten.

The writer is also very grateful to Brian and Rethema Honyouti for their help with gaining support within the Native American communities, as well as in testing Native American children, and to Marie Edwina Martinez and Debbie Borrego, for their help in testing Hispanic children.
To everyone in the Department (CIMTE), many thanks for all the encouragement throughout the coursework and dissertation writing.

Especial thanks go to my family, my mother, Gwendolyn Pettit; my children, Peter and Karen, Michael and Mary, Cherie and Donald, Stephen, and my grandchildren, Christopher, Megan, Lindsay, Max and Caitlin, for moral as well as financial support through these long years. This accomplishment is, above all, for you and because of you.
THE ORIGINS OF NUMBER CONCEPTS IN FIVE-, SIX- AND SEVEN- YEAR OLD ANGLO, HISPANIC AND NATIVE AMERICAN CHILDREN

BY
MANON PETTIT CHARBONNEAU

ABSTRACT OF DISSERTATION

Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Education

The University of New Mexico
Albuquerque, New Mexico
December, 1987
THE ORIGINS OF NUMBER CONCEPTS IN FIVE-, SIX-, AND SEVEN-YEAR OLD ANGLO, HISPANIC AND NATIVE AMERICAN CHILDREN

Manon P. Charbonneau

B.A., Bard College, 1969
M.A., University of New Mexico 1982
Ph. D., University of New Mexico, 1987

This study investigated the origins of ordinal and cardinal number concepts in five-, six-, and seven-year old Anglo, Hispanic and Native American children, and the effects of that acquisition on the ability to manipulate the first four natural numbers. The research subjects were kindergarten and first grade students at rural public schools (Anglo and Hispanic) in New Mexico, and at a contract community school or an independent day school on the Hopi Reservation in Arizona. All the children were tested during the first two weeks of the school year to control for a minimum of school training, and were tested in their dominant language as determined by teachers and parents.

Ordinality is defined as a transitive asymmetric relationship (length and weight) and cardinality as a concept of manyness. The study looked at whether these concepts are acquired sequentially or simultaneously, as determined previously by Brainerd (sequential) and Piaget (simultaneous). Manipulation of number is defined as the ability to add and subtract the first four natural numbers. The study is an extension of the work done by Brainerd in 1973 and 1979.
It was hypothesized that: 1) Anglo and Hispanic children would acquire the two concepts simultaneously; 2) Native American children would acquire the two concepts sequentially (cardinal before ordinal); 3) children showing a simultaneous acquisition would achieve a higher level of addition and subtraction skills than children acquiring the concepts sequentially; 4) there would be a difference in the sequential population between boys and girls; and 5) children acquiring cardinality before ordinality would show a higher level of subtraction than addition.

Ninety kindergarten and first grade children were selected for testing either by this writer (English speakers) or their teachers (Hispanic and Hopi speakers). Each ethnic group of 30 subjects was evenly divided by sex and grade level. Each child was tested individually with a language pre-test, an ordinal task, a cardinal task, and a test of arithmetic skills.

Findings

It was found that Anglo and Hispanic children acquired the two concepts simultaneously and that Native American children acquired them sequentially, cardinal/ordinal. It was also found that simultaneous acquisition children achieved significantly higher addition but not subtraction scores. It was also found that more boys achieved the cardinal-before-ordinal sequence, but no difference was found between boys' and girls' skills in addition and subtraction within the sequential population.
In addition, other findings not directly related to the hypotheses showed that language seemed to play a significant role in both the Hispanic and Native American subjects' acquisition of the two concepts. Further, of the 21 conservers of number, 18 acquired the two concepts simultaneously. All showed significantly higher mean scores for both addition and subtraction than the non-conservers.

**Conclusions**

Conclusions of the study indicate: 1) most subjects acquired the two concepts simultaneously; 2) native language played a role in acquisition; 3) conservers achieved a higher rate of skill in arithmetic than non-conservers; and 4) boys and girls differed in their acquisition of ordinal/cardinal concepts.

**Implications and Recommendations**

The findings of this study indicate that there is a significant relationship between simultaneous or sequential acquisition of ordinality/cardinality and the ability to manipulate number. The findings further support the theory that language plays an important role in that acquisition. Recognition of these relationships is extremely important in initiating instruction in formal school arithmetic both in sequence and in materials used. Teacher-training programs, schools, and parents need to be aware of these differences and offer the child support in this natural acquisition in both the scope and sequence of formal instruction.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vi</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xiii</td>
</tr>
<tr>
<td><strong>CHAPTER</strong></td>
<td></td>
</tr>
<tr>
<td>I. THE NATURE AND SCOPE OF THE PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>General Problem Area</td>
<td>1</td>
</tr>
<tr>
<td>Specific Problem</td>
<td>4</td>
</tr>
<tr>
<td>Why The Topic Is Important</td>
<td>6</td>
</tr>
<tr>
<td>Research Approach to This Dissertation</td>
<td>8</td>
</tr>
<tr>
<td>Hypotheses</td>
<td>11</td>
</tr>
<tr>
<td>Limitations and Key Assumptions</td>
<td>12</td>
</tr>
<tr>
<td>Contribution To Be Made By This Research</td>
<td>19</td>
</tr>
<tr>
<td>Basic Questions of the Study</td>
<td>21</td>
</tr>
<tr>
<td>Summary of Chapter I</td>
<td>22</td>
</tr>
<tr>
<td>Remainder of the Study</td>
<td>24</td>
</tr>
<tr>
<td>II. REVIEW OF THE LITERATURE</td>
<td>25</td>
</tr>
<tr>
<td>Historical Perspective</td>
<td>25</td>
</tr>
<tr>
<td>Piaget's Theory of Ordinal/Cardinal Development</td>
<td>26</td>
</tr>
<tr>
<td>Brainerd's Theory of Ordinal/Cardinal Development</td>
<td>28</td>
</tr>
<tr>
<td>Other Studies on the Origin of Number Concepts</td>
<td>30</td>
</tr>
<tr>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Studies on Cardination</td>
<td>33</td>
</tr>
<tr>
<td>Studies on Equivalence and Order</td>
<td></td>
</tr>
<tr>
<td>Relationships</td>
<td>35</td>
</tr>
<tr>
<td>Studies of Counting as the Critical Element in the Development of Ordination and Cardination</td>
<td>36</td>
</tr>
<tr>
<td>Cross Cultural Studies</td>
<td>39</td>
</tr>
<tr>
<td>Brain Research and the Acquisition of Numbers</td>
<td>42</td>
</tr>
<tr>
<td>Summary of Chapter II</td>
<td>46</td>
</tr>
<tr>
<td>III. RESEARCH METHODOLOGY</td>
<td>49</td>
</tr>
<tr>
<td>Purpose of the Study</td>
<td>49</td>
</tr>
<tr>
<td>Design of the Study</td>
<td>49</td>
</tr>
<tr>
<td>The Research Instruments</td>
<td>58</td>
</tr>
<tr>
<td>Determination of Levels of Development</td>
<td>60</td>
</tr>
<tr>
<td>Procedure for the Study</td>
<td>62</td>
</tr>
<tr>
<td>Selection of the Subjects</td>
<td>64</td>
</tr>
<tr>
<td>Training of Examiners</td>
<td>65</td>
</tr>
<tr>
<td>Conducting the Study</td>
<td>67</td>
</tr>
<tr>
<td>Schedules for Testing</td>
<td>67</td>
</tr>
<tr>
<td>Data Gathering Procedures</td>
<td>68</td>
</tr>
<tr>
<td>Summary of Chapter III</td>
<td>70</td>
</tr>
<tr>
<td>IV. FINDINGS OF THE STUDY</td>
<td>72</td>
</tr>
<tr>
<td>Scoring</td>
<td>74</td>
</tr>
<tr>
<td>Development of Ordination</td>
<td>75</td>
</tr>
<tr>
<td>Development of Cardination</td>
<td>79</td>
</tr>
</tbody>
</table>
Ordinal and Cardinal Relationship by

Ethnic Group ............. 85
Manipulation of Natural Number ....... 87
Analysis of Arithmetic Proficiency .. 94
Relationship Between Ordination, Cardination and Arithmetic Proficiency ....... 95
Developmental Relationship between Ordination, Cardination and Arithmetic Proficiency ..... 100
Conservation of Number ............. 104
Statistical Treatment ............... 105
Testing the Hypotheses ............ 106
Conclusions ............... 114

V. FINDINGS, IMPLICATIONS AND RECOMMENDATIONS ....... 116

Summary ............... 116
Findings ............... 119
Discussion of the Hypotheses ............ 120
Conclusions ............... 122
Other Findings ............... 122
Findings of the Ordinal/Cardinal Tasks ..... 133
Implications and Recommendations ....... 137

APPENDICES

Appendix A Language Pre-test, Ordinal and Cardinal Tasks, Manipulation of Number Test 156
Appendix B Spanish Translation of Above .... 160
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Mean Ages of Population by Grade Level and Ethnic Group</td>
<td>73</td>
</tr>
<tr>
<td>2.</td>
<td>Numbers of Kindergarten and First-Grade Children Assigned to Three Ordination Levels</td>
<td>77</td>
</tr>
<tr>
<td>3.</td>
<td>Numbers of Anglo Children Assigned to Three Ordination Levels</td>
<td>78</td>
</tr>
<tr>
<td>4.</td>
<td>Numbers of Hispanic Children Assigned to Three Ordination Levels</td>
<td>78</td>
</tr>
<tr>
<td>5.</td>
<td>Numbers of Native American Children Assigned to Three Ordination Levels</td>
<td>79</td>
</tr>
<tr>
<td>6.</td>
<td>Numbers of Kindergarten and First-Grade Children Assigned to Three Cardination Levels</td>
<td>81</td>
</tr>
<tr>
<td>7.</td>
<td>Numbers of Anglo Children Assigned to Three Cardination Levels</td>
<td>82</td>
</tr>
<tr>
<td>8.</td>
<td>Numbers of Hispanic Children Assigned to Three Cardination Levels</td>
<td>82</td>
</tr>
<tr>
<td>9.</td>
<td>Numbers of Native American Children Assigned to Three Cardination Levels</td>
<td>83</td>
</tr>
<tr>
<td>10.</td>
<td>Numbers of Children Assigned to Three Levels of Ordination and Cardination: Developmental Relationship</td>
<td>83</td>
</tr>
<tr>
<td>11.</td>
<td>Numbers of Anglo Children Assigned to Three Levels of Ordination and Cardination: Developmental Relationship</td>
<td>85</td>
</tr>
</tbody>
</table>
12. Numbers of Hispanic Children Assigned to Three Levels of Ordination and Cardination: Developmental Relationship

13. Numbers of Native American Children Assigned to Three Levels of Ordination and Cardination: Developmental Relationship

14. Numbers of Kindergarten and First Grade Children Assigned to Three Levels of Arithmetic Proficiency: Addition

15. Numbers of Kindergarten and First-Grade Children Assigned to Three Levels of Arithmetic Proficiency: Subtraction

16. Numbers of Kindergarten and First-Grade Children, by Ethnicity, Assigned to Three Levels of Arithmetic Proficiency: Addition

17. Numbers of Kindergarten and First-Grade Children, by Ethnicity, Assigned to Three Levels of Arithmetic Proficiency: Subtraction

18. Numbers of Boys and Girls Assigned to Each of Three Levels of Arithmetic Proficiency


20. Numbers of Boys and Girls Assigned to Levels of Arithmetic Proficiency by Ethnic Group: Subtraction
21. Numbers of Children Assigned to Three Levels of Ordination, Cardination and Arithmetic Proficiency ........................................ 96

22. Numbers of Children Assigned to Three Levels of Ordination, Cardination and Arithmetic Proficiency: Anglo ........................................ 98

23. Numbers of Children Assigned to Three Levels of Ordination, Cardination and Arithmetic Proficiency: Hispanic ......................................... 99

24. Numbers of Children Assigned to Three Levels of Ordination, Cardination and Arithmetic Proficiency: Native American ..................................... 99

25. Numbers of Kindergarten and First-Grade Children Who Were Conservers of Number ........................................ 104

26. Numbers of Girls and Boys, by Ethnic Group, Who Were Conservers of Number .................................................. 105

27. Chi Square for Responses by Acquisition: Anglo ................ 106

28. Chi Square for Responses by Acquisition: Hispanic ................ 107

29. Chi Square for Responses by Acquisition: Native American .................................................. 108

30. Descriptive Statistics for Arithmetic Achievement of Two Groups: Simultaneous and Sequential Acquisition of Ordinality/Cardinality .............. 109

31. Observed Frequencies for Native American Children by Sex and Concept Acquisition ........................................ 111
32. Chi Square to Determine Significant Difference
   Between Boys and Girls and the Sequential Cardinal
   Acquisition of Native American Children ........ 11
33. Observed Frequencies for Native American Children
   by Concept Acquisition and Skills Development... 113
34. Chi Square to Determine Significant Difference
   Between Addition and Subtraction Levels of
   Native American Children ..................... 114
35. Summary Table .................................. 115
36. Acquisition of Ordinal/Cardinal Concepts by Sex... 133
CHAPTER I
NATURE AND SCOPE OF THE PROBLEM

General Problem Area

The acquisition of numeracy leading to substantive thinking and advanced work in the field of mathematics has rarely come under closer or more careful scrutiny than in this present decade. This examination is being considered from two perspectives: that of the learner and his/her natural capacity for acquisition (Baroody, 1987; Brainerd, 1973, 1979; Ginsburg, 1977; Steffe, von Glasersfeld, Richards and Cobb, 1983) as well as formal curricula and school experiences in reports such as A Nation at Risk (National Commission on Excellence in Education, April 1983), A Place Called School (John I. Goodlad, 1984), as well as the National Council of Teachers of Mathematics own Agenda For Action: Recommendations for School Mathematics of the 1980s (NCTM, 1980), one of NCTM’s major contributions to the "state of the art" in assessing current curricula and pedagogical practices in the field of mathematics.

The climate of opinion voiced in these reports point to a mathematics curriculum that is not accomplishing its goals: that of creating mathematically competent students, either in the majority culture or the many minorities. The problems confronted by minority students in the schools only sharpens the focus on the problems faced by many students in mathematics classrooms today (Leap, in press).

An editorial by Underhill in the Arithmetic Teacher (December 1985) made the following statement:
Many children benefit only nominally from much of the first and second grade curricula even under specially diagnosed conditions. After we diagnose children's characteristics, we must look at the curriculum and ask whether it fits the learner. The curriculum and the learner must have a two-way fit. It makes little sense to force children to master curricula that are inappropriate for them or to waste large blocks of instructional time on areas in which research has demonstrated the general futility of such an action.

(p.1)

Underhill further stated that:

The mathematics curriculum is viewed by many as a monolith, unchanging and unchangeable. These people are right in a certain sense. Curricula reflect society's wisdom and experience; curricula change slowly. But today we face a dilemma: we diagnose the learner's needs in the affective and cognitive domains and reflect the results against traditional curricula. A balance is needed: when the learners do not meet expectations we must examine carefully both the learner and the curriculum. When the cognitive outcomes demanded by the curriculum conflict with the capabilities of the learner, the learner may become anxious and develop a poor attitude; as a result both the cognitive and affective outcomes are thwarted. For such learners, the likelihood is great that the curriculum produces unintended outcomes in the
cognitive and affective domains that conflict with the intended outcomes.

Underhill finished his editorial with the following statement:

It is time to diagnose the curriculum. Evidence is accumulating that implies strongly that we are teaching many mathematical concepts and skills that many learners cannot grasp and some major ones that most of the learners cannot understand.

This "state of mind" and opinion is shared and expressed by several other current mathematics educators in editorials for the National Council of Teachers of Mathematics publications (Kansky, 1985; Vogeli, 1984; Altizer-Tuning, 1984) and other researchers in the field (Ginsberg, 1977; Kamii, 1985). These authors all espouse a careful and critical look at the current curricula in mathematics, and are calling for dramatic reforms in light of the demands of the advanced technology which exists now and will continue to color the future.

Altizer-Tuning (1984) drew particular attention to the current curricula of the elementary school, which, she stated, are concerned basically with complex computational algorithms using the four basic operations, as well as a fixed sequence of concept acquisition which is highly questionable in light of current research on how children learn. This opinion is strongly supported by Ginsburg (1977, 1983), Steffe (1971) and Kamii (1985). Further, a study of current conceptual work offered by the major
publishers in mathematics for the elementary years indicates a uniform approach to numbers across programs with a major emphasis on computation and counting. Even during the curricula reform years of the sixties which emphasized the structure of mathematical classes (cardinality), many elementary teachers, possessing only a minimal understanding of the "new math", continued to use an ordinal counting approach. This unilateral approach to young children's development of mathematical concepts has certainly not helped many students reach a significant level of mathematical understanding or proficiency (Underhill, 1985; Nichols, 1985; Altizer-Tuning, 1984; Vogeli, 1984; Kansky, 1985).

Specific Problem

The specific problem studied in this research is the natural acquisition of the two basic concepts of number, ordinality and cardinality, in kindergarten and first grade children of three varied cultural and ethnic backgrounds, and how this acquisition affects their ability to add and subtract the first four natural numbers.

If substantive changes in the curriculum are to be made which reflect the current knowledge of how children learn naturally and consider the differences in cultural influences and backgrounds (Macpherson, 1987; Stanic and Reyes, 1986, 1987, in press), building on and maximizing the innate ability of children to grasp and use basic concepts in their number work, it would seem wise to pursue further research into this basic acquisition of numeracy, and build appropriate curricula in mathematics around such findings.
This attitude is strongly supported by Brainerd (1973), who began studying the origin of number concepts in young children in the early seventies. Brainerd maintains that "number has been a concept of social importance since the dawn of recorded history" (Brainerd, 1973 p. 103), but few comprehensive studies have been undertaken to understand the origins of these number concepts in children.

In the acquisition of numerical principles, young children develop the concepts of ordination and cardination, as well as the ability to manipulate the natural numbers which grows out of the ordinal/cardinal understanding (Brainerd, 1973, 1979; Gelman and Gallistel, 1978; Hardeman, 1974; Kamii, 1985; Piaget, 1965; Steffe, Von Glasersfeld, Richards, & Cobb, 1983). The question of whether these two concepts of ordinality and cardinality are acquired simultaneously or sequentially, and if sequentially, in what order is critical to those concerned with the educational process if we are to make significant gains in the area of how children learn. Determination of this acquisition and its effects on the child's ability to understand the four basic operations of arithmetic should become the basis of future mathematics curriculum reform and revision as well as the training of teachers (Ginsburg, 1977; Steffe et al. 1983).

Further, careful consideration needs to be given to minority group children and their acquisition of these two critical concepts (Macpherson, 1987). Do all children acquire the ordinal and cardinal concepts in the same way,
or do ethnic and cultural differences and native language alter the order of acquisition in a way that would make a significant difference in the child's basic understanding of number? Epstein, researching on the brain and learning supports the need for a cognitive "match" between the learner and what is to be learned. He placed a major emphasis on the physical growth of the brain at specific times in human growth and development and what can reasonably be expected in the way of learning during growth spurt periods as opposed to what he referred to as the "plateau periods" (Epstein, 1978, 1981).

Why The Topic Is Important

Many children are not succeeding in school mathematics (Boyer, 1984; Goodlad, 1984; National Committee on Excellence in Education, 1983; Sizer, 1984; Vogeli, 1984), and while there are significantly more children from the mainstream culture who do reach advanced mathematics classes and who do succeed, there are, nevertheless many who do not. The number of minority students who reach any level of advanced mathematics study is staggeringly low (Croom, 1984; Matthews, Carpenter, Lindquist, Silver, 1984).

Mathematics educators have heretofore concerned themselves primarily with the subject matter of the discipline and not the learning process (Ginsburg, 1977; Hardeman, 1974; Kamii, 1985; Steffe and Carey, 1975) in creating mathematics curricula for schools. A need to change the focus from the discipline to the learning process is strongly indicated (Brainerd 1973, 1979; Davidson, 1983;
Epstein, 1981) if one is to improve the state of mathematics learning nationwide for all children from all ethnic backgrounds.

There is serious concern about the accessibility to higher mathematics and those jobs which entail the use of higher mathematics for minority students: Asians, Blacks, Native Americans, Hispanics and women (Bradley, 1982; Cheek, 1984; Cuevas, 1982; Fennema, 1982; Johnson, 1982; Valverde, 1982). Curricula developers need to know more about the natural acquisition of number concepts and skills of children of differing cultures in order to support optimum growth and development in schools. While mathematics learning for all children seems to be of great concern, the needs of minority children appear to be of greater magnitude (Clark, 1982). The literature indicates that there has not been sufficient research to provide supportable generalizations about the acquisition of ordinality and cardinality (Brainerd, 1973, 1979), which form the basis for all of mathematics. It is especially critical that the performance of minority group children in this very beginning and essential area of concern be studied and assessed. This statement is supported in the recent report of the Carnegie Forum Task Force on Teaching as a Profession (A Nation Prepared: Teachers For The 21st Century, 1986), where it is stated that "it would be fatal to assume that America can succeed if only a portion of our school children succeed". Demographically, this report also points out that by the year 2000, only thirteen years away, one out
of every three Americans will be a member of a minority group. The success of these children with the educational process, and especially in the fields of mathematics and science, is clearly a critical and crucial problem.

Research Approach to This Dissertation

This research study has undertaken an extension of the work begun by Dr. Charles Brainerd, University of Alberta, in the seventies. In his study, Brainerd devised tasks that tested the understanding of the ordinal and cardinal concepts, and the effects of this understanding on the child's ability to add and subtract the first four natural numbers.

Ordinal number is defined as the notion of progression, with a key element in the logical definition of progression being a relationship that is both transitive and asymmetrical. Ordinality may be defined somewhat more simply and concretely as the perception of everyday transitive-asymmetrical relations as "taller than", "larger than", "heavier than" which underlie the progressions found and used in the real world.

The logical, mathematical definition of cardinal number is the notion of "similarity", in turn being defined in terms of a correspondence relation. Quite simply for children, cardinality is an understanding of numerosity or "manyness"; the same as, more than, less than. These two critical beginning concepts of number, ordinality and cardinality, bear heavily on the child's later ability to understand and use the four basic operations of arithmetic,
addition, subtraction, multiplication and division.

Brainerd's findings determined that the two concepts of ordinality and cardinality were acquired sequentially, with ordinality developing considerably before cardinality. He further determined that the ordinal concept was the critical one affecting the child's ability to add and subtract, and that cardinality seemed to have relatively little influence on the child's growth and development in simple arithmetic. His work did not cite any study of differences between boys and girls, or include any ethnic groups beyond the mainstream culture in Edmonton, Alberta, Canada. Also, Brainerd's cardinal task was limited to two-dimensional pictures, which could have distorted his findings on children's understanding of the cardinal concept.

This writer's research also looked at the acquisition of the ordinal and cardinal concepts, whether acquired sequentially or simultaneously, and if sequentially, in what order. The subsequent effect of this acquisition on the ability of young children to manipulate the first four natural numbers was, as with Brainerd, a major part of the study. Further consideration was given to whether the acquisition of the ordinal and cardinal concepts and the ability to add and subtract might be different for boys or girls and different ethnic groups. Anglo, Hispanic and Native American children are the focus of this study instead of children solely from the mainstream culture as were the subjects in Brainerd's original study (personal communication, 1984).
Since language expresses and defines ideas, concepts and logic, it has been defined as a symbol of underlying thought (Lavatelli, 1970). Therefore, it would seem important that Hispanic and Native American children should be tested in their native language. Children from remote, rural communities were selected as subjects, as it is assumed that children from such communities, with limited contact with the mainstream culture, would, to a greater extent, speak their native language as the primary one, and can be considered to have received the least amount of influence from outside sources (Spolsky, 1974).

This research was concerned with the child's acquisition of the two basic concepts of ordinality and cardinality as much as possible in terms of natural development. While numbers are a product of man's invention, children bring to school concepts acquired naturally: an understanding of "how many" (cardination) and "which one" (ordination) (Gelman and Gallistel, 1978; Piaget, 1953, 1965). It seems obvious that the influence of culture and to some extent, direct teaching by parents, the media and the child's experiences in the community would be contributing factors. However, children from remote rural communities would seem to be subject to fewer outside influences such as television and preschool than those children growing up in or near larger cities. For these reasons, children were tested during the first two weeks of the school year to control as much as possible for school experience.
Hypotheses

Five hypotheses were developed to guide this study;

Hypothesis 1: Given a task of transitive-asymmetrical relations, and a task of comparison of manyness of two collections of objects, kindergarten and first grade Anglo and Hispanic children will perform the same, indicating a simultaneous acquisition of the ordinal and cardinal concepts.

Hypothesis 2: Given the same tasks of transitive-asymmetrical relationships and comparison of manyness, Native American children will show a sequential development, with cardination appearing before ordination.

Hypothesis 3: Given a test of addition and subtraction of the first four natural numbers, those children showing a simultaneous acquisition of the ordinal/cardinal concepts will achieve at a higher level than those children who show a sequential development, either ordination before cardination or cardination before ordination.

Hypothesis 4: If there is a sequential development of number concepts in most children, the majority of those children achieving the cardinal concept before the ordinal concept will be boys.

Hypothesis 5: Those boys who show a sequential acquisition of the ordinal and cardinal concepts, with cardinality appearing before ordinality, will show a higher level of subtraction skill than addition.
Limitations and Key Assumptions

Limitations

There are four basic limitations for this writer's study that need to be considered:

1) Limitation of language study. First and foremost, the question of language and how the structure of the native language can color the child's view and understanding of number. This linguistic aspect of the problem can only be touched upon in this research, as it is, in itself, an entire study. This work had to be confined to the child's acquisition and understanding of the two basic mathematical concepts and their influence on the child's ability to put together and take apart the natural numbers. However, the possibility of linguistic influence cannot be ignored. A startling assertion by hearing specialist Tadañobu Tsunoda (Sibatani, 1980), that language shapes the neurophysiological pathways of the brain supports the theory that language may be a key factor in how the brain operates. Tsunoda claims that the language one learns as a child influences the way in which the brain's right and left hemisphere develop their special talents. There is abundant evidence to confirm the Japanese students' superior performance in mathematics. The United States Department of Education's Second International Mathematics Study (1985) showed that for both eighth and twelfth grade students, mathematics scores for the Japanese students ranked them first among the countries compared. United States students at the 12th grade level ranked "dead last" among comparable
students in the 12 developed countries and Canadian provinces, while U.S. eighth graders scored third from the bottom amongst 14 developed countries and the Canadian provinces. There is further support for outstanding work in mathematics on the part of students of Oriental ethnic background: a perusal of the NCTM Bulletins which cite top scholars of all ages in the field, nationwide, a majority of whom are Oriental ethnic origin.

Hopi language might possibly influence the children’s understanding and use of number from the use of noun endings themselves. In place of a number word used as an adjective so frequently in English and other languages, Hopi nouns can have one of three endings; one ending indicating "one" or a singular quantity of whatever the noun represents, and ending indicating "two" of whatever the noun represents, and a third ending indicating "lots" of whatever the noun represents. Thus, children do not progress into the language using a great quantity of numerical words. Hopi language does have count words to 100 and some compound words which indicate larger collections; however, speakers of the language do not include many specific number word situations (Masayesva-Jeanne, personal communication, 1981). Also, the Hopi count word sequence is created around a base of twenty which must influence a child’s understanding of the English base ten sequence. From ten years of observation of Hopi children, it is this researcher’s opinion that because of the structure of the language, understanding of number could differ significantly from that
of non-Hopi children, and this is a possible situation for other Native American languages and cultures.

Further, access to scholars with a strong understanding of the intricacies of the linguistic structure and semantic overtones of Native American languages is limited. Also, the privacy of most Native American languages is carefully guarded. If Native American language seems to indicate a strong influence on the understanding of the two mathematical concepts under study, then a further research project is indicated; one incorporating a team, members of which should be linguists representing those languages involved.

A final language limitation is that the group of Native American children in this study were members of the Hopi Tribe, and it would not be prudent to make determinations for other Native American groups from this study. Native American languages differ greatly, and what might be true for Hopi might not be true for Navajo, Apache, Cherokee or any other Native American students.

2) Reliable cardinal tasks. A serious limitation for consideration is the difficulty in creating tasks that reliably test the child’s understanding of the concept of cardinality. While this research has basically made use of the tasks devised by Brainerd in his study, a change in the cardinal task was indicated. In his original study, Brainerd’s ordinal tasks involved the use of physical materials which the child is able to touch and feel and well as look at, while the cardinal task is constructed of two
dimensional "pictured" objects only. Support for a change in cardinal task dimensions was supported also by the work of Baroody (1982) in which he found that Brainerd's cardinality task significantly underestimated the understanding of cardinality in three age groups tested, including adults. McLaughlin (1981) also found a difference in children's responses to numerosity questions if the stimuli were three dimensional instead of two dimensions.

3) Exclusion of counting. Another limitation which must be discussed is the understanding and use of counting as a method of determining manyness. Brainerd considered counting as a strictly ordinal operation and asked that the child figure out "another way" to determine the manyness (by matching only). Van Engen (1970), Steffe et al. (1983), Gelman and Gallistel (1973) suggested that counting embodies an element of both cardinal and ordinal understanding.

Gelman and Gallistel cited the child who counts objects "one...two...three...four...five...six...seven...eight..." and repeats "oh! eight!: as having an understanding of the cardinal property of number, else the child would not have stopped and repeated the last number spoken as an indication of the manyness of the group counted.

Van Engen (1971) proposed two ways of counting:

\[
\begin{array}{cccc}
\text{one} & \text{two} & \text{three} & \text{four} & \text{five etc.} \\
\square & \square & \square & \square & \square
\end{array}
\]
as ordinal counting, and

\[
\begin{array}{cccccc}
\text{one} & \text{two} & \text{three} & \text{four} & \text{five} \\
\square & \square & \square & \square & \square \\
\square & \square & \square & \square & \square \\
\end{array}
\]

as a cardinal way of counting. Russac (1978) considered counting another effective strategy that can be used by the child to determine the numerosity of the set (cardinal number).

Limiting the child, as Brainerd did, to no counting in the cardinal task for determining manyness, could further underestimate the child's understanding of the cardinal concept. However, in order to maintain Brainerd's study with the fewest possible changes, both the ordinal and cardinal tasks devised by Brainerd (with the minimal change from two dimensional pictures to three dimensional cubes) were used. This study can be considered, then, to be an extension of his work to include two minority groups as well as mainstream culture children.

4. Influence of parents, media, pre-school. A further limitation that needs consideration is that many, many children are taught the count word sequence early on in their young lives; by parents, television, as well as Headstart and other preschool experiences (Fuson, Richards, and Briars, 1982; Gelman and Gallistel, 1978). Often this is done in a rote manner only, so that the child has little or no understanding of "two", "three" but only recites the
words. Children are also encouraged and taught to touch each item as they count, which is indicative of a beginning understanding of one-to-one correspondence. However, children are rarely taught to "match" items from one group with items from another group in a one-to-one matching without counting as a way of determining the manyness of one group over the other. The influence of counting to specify manyness, as determined by cultural practices cannot be dismissed.

Key Assumptions

One of the key assumptions that can be made is that all children have some number sense that is significantly developed before entering school (Gelman and Gallistel, 1978; Kamii, 1985; Walters, 1983). Those concepts of number which children develop early on are this understanding of manyness and order relationships, strongly influenced by perceptual cues and clues (Gibson, 1969; Piaget, 1953), although the actual experience a child may have had with determining manyness and ordering will have significant effect on the child's ability to understand the tasks being asked of him/her (Dienes, 1972; Piaget, 1953; Vygotsky, 1962). Gelman cites instances of children as young as 2 1/2 years using the cardinal principle, and "three- and four-year olds show clear evidence of being able to apply the cardinal principle". Gelman defined this "cardinal principle" as the ability to determine that the final "tag" in the series has a special significance. This tag, unlike any of the preceding tags, represents a property of the set
as a whole. The formal name for this property is the cardinal number of the set (Gelman and Gallistel, 1978). Brainerd (1979) cited the ability of very young children to see and use ordered progression, and Kamii (1984) further determined that young children’s ability to seriate is determined by the materials used and the nature of the task.

Recent and on-going research on brain organization and the role it plays in concept formation is a key assumption that can be made. That the organization of the brain contributes significantly to the child’s understanding and use of number cannot be underestimated, but again, the constraints of this dissertation will permit, as with language, only limited inclusion of the work that has been done in this field. Epstein’s work on brain growth spurts indicates some strong correlations between the cognitive stages defined by Piaget and some entirely new concepts of what can and cannot be expected of children during the growth spurt phase and also during the "plateau periods" of physical non-growth. This work on brain growth spurts of Epstein’s has just recently been supported by a team of neuroscientists headed by Thatcher at the University of Maryland-Eastern Shore (Trotter, 1987).

Further important work, focusing strictly on mathematics and the brain has been done by Davidson (1983). She has posited a theory of two cognitive styles as determined from a neurobiological model for learning which will be discussed later.

Levy’s work (1983) on hemispheric asymmetry has also
contributed knowledge of possible significant strengths and/or deficits originating from the right or left hemisphere of the brain which can affect the understanding of number and may have implications for sex differences in the student's basic understanding of number as well as his/her approach to problem solving.

Contribution To Be Made by This Research

Van Engen made a strong point in stating that regardless of the approach taken by any given mathematics text, teachers will teach in the manner that they themselves understand (Van Engen, 1971), and that, in many instances, children are not helped to understand the difference between ordinal and cardinal concepts, thus resulting in confusion for the child. It is also clear from a direct study of current mathematics textbooks, that it is only in the kindergärten year that any attention is directly given to cardinal number and the operation of matching as the appropriate strategy for determining manyness. First grade launches immediately into a use of cardinal number to add and subtract, assuming a fully operational understanding on the part of the child.

If, as Underhill has stated, many children in grade one do not profit from the instruction in arithmetic that they are currently receiving, then it seems critical that research is undertaken to explicate as fully as possible the development in the young child of these two basic concepts of number, ordinality and cardinality, so that curricula can be constructed which will support the natural acquisition
pattern, and benefit the child's learning instead of hindering or impeding it.

Brainerd (1973) has stated:

Considering the emphasis our society puts on numerical competence, it might be expected that extensive information would long ago have been collected on how the concept of number naturally unfolds in the course of a child's mental growth. The truth is we possess very few hard facts about the emergence of numerical ideas in children's thinking. p.104

Brainerd's extensive study of the development of the ordinal and cardinal concepts in children, as well as the effect of each on the ability to manipulate natural number, has been replicated only once, by Gonchar (1975) with a further study by Brainerd himself and Fraser (1975), with supportive findings. Yet, there has been no appreciable change in mathematics curricula for young children in the last twenty years (Brainerd, 1979; Ginsburg, 1977; Kamii, 1985; Underhill, 1985) except that the language and format of "sets" and set theory has been considerably toned down, if not eliminated, at the elementary level.

There is no research on the origins of the ordinal/cardinal concepts in Native American and Hispanic children, and considering the large numbers of such minority children in schools in the Southwest, this is seen as a serious gap. It would seem that a cross cultural study involving the growth and development of these two concepts is not only in order but highly desirable; and, possibly a
critical contribution to the literature on these minority children can be made, as well as further documentation of the growth and development in Anglo children.

The creation of curricula which will provide a cognitive "match" (Epstein, 1981), and build upon the natural development of children's use of number in their world (Ginsburg, 1977), and which takes into consideration basic differences, if such differences do exist, should result in students whose understanding of, and interest in, mathematics is clearly superior to children with the present system. Interest in and success with mathematics by minority students should result in significantly more of these young people entering into lifetime careers involving the use of mathematics.

**Basic Questions of the Study**

Among equivalence and order relations in the primary school curricula are set relations, based on the matching of finite sets of objects, and length and weight relations, determined by comparing relative lengths and weights of objects. An attempt was made in this study to see: 1) if these basic concepts of number are developed simultaneously, or 2) if there is a sequential development, and 3) does this acquisition (either simultaneous or sequential) affect the child's ability to manipulate the first four natural numbers? Further, 4) is the acquisition different for the two sexes and different ethnic groups?

Specifically, the following questions are basic to the study, where most, but not all, arise from Piaget's theory
of the origins of number concepts.

1. Do young children acquire the ordinal concept of number before the cardinal concept?
2. Do young children acquire the cardinal concept of number before the ordinal concept?
3. Are these two concepts acquired simultaneously as Piaget implies, but in developmental stages?
4. Is this pattern of acquisition (simultaneous or sequential) universal across cultures?
5. Is there a difference between boys and girls in the pattern of acquisition?
6. Does the order of acquisition affect the child's ability to manipulate natural numbers?
7. Is there a relationship between the pattern of acquisition and the ability to add and subtract?

Summary

Many children today, especially from the minority cultures, are finding only minimal success with the mathematics curricula in schools. A call for major reform of mathematics curricula includes an examination of the natural growth and development of number concepts in young children to provide a closer cognitive match with this natural development.

The concepts of ordinality and cardinality guide the basic understanding of number. Young preschool children develop the notion of these two concepts in the natural course of play, observation and usage; through the practical application of number in everyday living. Many researchers
believe that children enter the formal school situation with
very similar underlying understanding of number, even given
their varied experiences. However, it is not clear whether
the ordinal and cardinal concepts of number develop
simultaneously or sequentially, and if sequentially, in what
order. There have been several studies of the
simultaneous/sequential development of ordinality and
cardinality within the majority population; however, the
development of these concepts in minority children seems to
have been ignored.

Development of ordinality and cardinality may be
influenced by cultural experience, brain development, and
the child’s native language. Language, in turn, may further
influence the brain’s development and organization. It is
these three factors which influenced the formation of the
five hypotheses which guided this study.

This research looked at the development of these two
concepts in Anglo, Hispanic and Native American children in
the Southwest in order to increase what is known about the
origins of the concepts of ordinality and cardinality in
children from varied cultural and linguistic backgrounds.
Schools in this part of the country have considerable
populations of both Hispanic and Native American children as
well as Anglos, and it is well documented that the
achievement of both Hispanic and Native American students
falls well below that of the Anglo population.

If differences in acquisition are a major factor
between ethnic and cultural groups, then these differences
should be taken into consideration in the formal instruction presented in classrooms. Formal instruction, built upon the natural growth and development of such basic number concepts should result in increased participation and achievement in mathematics of all children.

The Remainder of the Study

A review of the literature is presented in Chapter II.

The study design and procedures are discussed in Chapter III.

The findings and conclusions based on the analysis of the data can be found in Chapter IV.

Chapter V concludes the study with a summary of the findings pertaining to the hypotheses as well as others which do not directly relate to the hypotheses. Also this chapter includes the implications and recommendations that resulted from the study.
CHAPTER II
REVIEW OF THE LITERATURE

Historical Perspective

"Number has been a concept of social importance since the dawn of recorded history" (Brainerd, 1973, p.103), but few comprehensive studies have been undertaken to understand the origins of number concepts in children (Brainerd, 1973).

The idea that natural numbers (the positive integers) were God-given entities prevailed from the time of Pythagoras until well into the nineteenth century. As attitudes changed, rigorous proofs began to appear in the form of step-wise derivations, generating number systems such as the rational, real and complex numbers. Two important mathematicians of that theoretical foundation were Karl Weierstrauß and Richard Dedekind, both of whom worked on the derivation of real numbers from the natural numbers. With the rise of symbolic logic came the notion that the natural numbers were definitely constructions of the human mind. Gottlob Frege, Guiseppe Peano and Bertrand Russell were the most famous and recent proponents of this idea, but took two distinct views in creating their theories.

Frege determined in 1884 that the natural numbers could be reduced to the notion of "classes" and that classes could be quantified by the operation of correspondence; that is, if two "classes" can be determined to be members of the same superordinate set (same number), it must be that a one-to-one correspondence between their members can be
established. Conversely, if a many-to-one correspondence exists, they are said to be examples of different numbers. Bertrand Russell "rediscovered" Frege's cardinal theory in 1901, which later became the basis for his Principles of Mathematics and two later publications.

In 1894 Guiseppi Peano published his theory, Formulaire de Mathematiques, positing five axioms explaining numbers in terms of a transitive, asymmetrical relationship, an ordinal concept of number as opposed to the Frege-Russell cardinal concept. Charles Brainerd cited the matching of items as the most common example of cardination, and counting as the most common example of ordination.

Piaget's Theory of Ordinal/Cardinal Development

Jean Piaget, in his extensive research into the origins of intelligence, concept formation and cognition, seemed to be the first to consider the origin of number concepts in psychological rather than logical terms, looking closely at the development of number concepts in young children. In so doing, he divorced the rational mathematical theories of the previously cited men, and minutely observed the natural development, the "construction" of the concept of number in the growing child. Piaget has left the world with detailed descriptions of the child's natural ability to construct an understanding of number in a parallel fashion; ordinal and cardinal concepts developing together but in a three-stage invariant progression. This parallel development eventually ends in a merging of the two concepts into what he described as an operational correspondence and seriation process that
gives the child a systematic and logical base for arithmetical manipulation: the ability to add, subtract, multiply and divide.

Piaget posed this parallel developmental process for both cardinal and ordinal number concepts in the young child, encompassing three distinct stages beginning at about 4.5 years and gradually merging at about 7 years: 1) the ability to compare or seriate globally ("globally" is the key word here), which becomes 2) an intuitive ability to make appropriate comparisons and seriations when the critical perceptual clues are available, to the 3) clearly functional, operational ability to understand that a relationship exists between order and quantity.

It was Piaget's position that children make no distinction between ordinal and cardinal number as they begin their acquisition of number concepts, and he further considered that the concept of cardinal number presupposed an ordinal relationship. Piaget also stated that children form their mathematical notions on a qualitative or logical basis (Piaget, 1953). For children at Stage I of mathematical development (about age 4.5), cardinal evaluation is merely global judgement, and seriation the juxtaposition of one object to another. At this stage of development, Piaget stated that the child, making the effort to seriate, ignores sets, and conversely, in his/her attempt to evaluate quantities, ignores order.

At Stage II, the child begins to show the ability to systematize his/her efforts, but within a range of what is
immediately perceived, and while there is an intermingling of the ordinal/cardinal processes, these processes are not yet truly coordinated. In each case (cardinal and ordinal), the child can make qualitative, intuitive decisions but only in light of the perceptual whole. Objects or items out of place or rearranged cannot be dealt with in an isolated manner, but require the entire "whole" to be reconstructed again (Piaget, 1965).

Stage III (reached at about age 7) sees operational correspondence and seriation rather firmly in place, independent of the field of perception, with the child able to solve ordinal and/or cardinal problems as dictated. Because Piaget saw this universal, invariant three-stage development of both ordinal and cardinal concepts surfacing in children's cognitive growth, he implied a parallel evolution as if the two concepts started on separate tracks, but gradually merged into one interrelated function.

**Brainerd's Theory of the Ordinal/Cardinal Development**

Because Charles Brainerd has published the only extensive study of the growth of the ordinal/cardinal concept development in young children since Piaget, and because this researcher is attempting a reasonable replication and extension of the study, careful consideration of Brainerd's work seems in order.

From the results of his research, Brainerd stated unequivocally that the development of the ordinal and cardinal concepts are not simultaneous, as Piaget contended, but sequential; the ordinal notion appears first, giving
rise to the child's ability to manipulate natural number (to add and subtract). He stated that the cardinal concept does not appear until considerably later, and that an understanding of cardinal number does not measurably affect the child's ability to do simple arithmetic.

Further, Brainerd stated that Piaget's error in parallel development stems from the fact that Piaget did not test the same children with both ordinal and cardinal tasks, but rather inferred this simultaneous development after observing the same three-stage progression in the children tested for ordinality and in those tested for cardinality. Brainerd implied that had Piaget tested the same children for both concepts, a sequential progression might have been found (Brainerd, 1979). Piaget used a code system of assigning a three letter "name" (i.e. Jen, Lev, Ora etc.) to each child, and careful reading of the chapters reveals that the same code names are described for many subjects in both ordinal and cardinal task elaboration. Duckworth (personal communication, 1985) confirmed that this meant that the same children were used for both tasks.

Brainerd also stated (1973) that the debate over the origins of number concepts cannot be ignored with safety, and that "the significance to society of number and number-related skills has increased tremendously with the rise of industrial civilization" (p.103).

Brainerd's study involved testing children aged five, six and seven (grades kindergarten and one) with what he refers to as the most "precise" embodiment of the 29
mathematical definition of ordinality: transitive, asymmetrical relationships, and the most precise embodiment of cardinality, the quantitiy comparison of two sets without counting, by "matching". He further tested for the effect of each concept on the ability of his young subjects to add and subtract the first four natural numbers. His findings strongly support the theory that, for most children, ordinal concepts emerge first, and that these children showed superior ability to add and subtract even over those children in whom the cardinal concept emerged first. His findings did not indicate in any way that the growth and development of the two concepts might be parallel and simultaneous. However, Brainerd stated that in his statistical analysis, he ignored all cases of children whose development was simultaneous. He looked only at those children whose acquisition of the ordinal/cardinal concepts were sequential.

All of Brainerd's work on these concepts involved children from a middle class community in Alberta, Canada, an implied middle class Anglo population. No mention is made of children of other cultures either in the literature or personal communication, in which a background on the children was requested and received.

Other Studies on the Origins of Number Concepts

The fact that there are very few studies in the literature on the origin of number concepts in young children would seem to imply either that the two major studies (Brainerd and Piaget) have basically satisfied those
concerned with the learning of young children, or that truly adequate ways to determine the level of understanding of the ordinal and cardinal concepts in young children have not been created, or possibly, that the question even seems trivial to many.

Gonchar replicated Brainerd's study in 1975 with 60 kindergarten children and 60 third-grade children. Each of the children received the ordination, cardination and natural number tests from the Brainerd study, with the results obtained paralleling those of Brainerd. Children were kindergarten and third grade students in four Beloit, Wisconsin elementary schools, with the majority of the students from the mainstream culture. Some few were Black.

Brainerd and Fraser (1975) also replicated the original study, but substituted Piaget's assessment of number, a conservation task, in place of Brainerd's addition and subtraction tasks. They also added a task of "numbers-as-classes", asking children to sort a packet of cards in several different ways; by number, by shape and by color. They obtained identical results as with Brainerd's original study; the ordinal concept emerged prior to the child's ability to conserve, which, in turn, emerged prior to cardination. Again, however, children whose acquisition was simultaneous were ignored in the statistical evaluation.

Brainerd (1974) also conducted research into the training of the ordinal and cardinal representations of the first five natural numbers. His subjects were 120 preschool children who showed no evidence of understanding either the
ordinal or cardinal properties of natural number symbols. The results of this study confirmed his theory that the ordinal property of the first five natural number symbols was more easily learned than the corresponding cardinal property. And further, the author found that ordinal training maintained over a longer period of time than did cardinal training. Brainerd defined "training" in this study as "feedback" given to the child using the same procedure for training as for pretest, only different stimuli. No teaching in the usual context was done. The author discussed the possibility that the demands of the cardinal task were greater than those of the ordinal task; however, this theory was dismissed in favor of one which posits a further needed step in development before cardinality could be understood.

One last study on the sequentiality of the ordinal and cardinal concepts was done by Kingma and Koops (1981). These researchers did a replication of Brainerd's work, but with "improvements" made on some of his tasks. Their data support Brainerd's contention that developmentally, the ordinal concept is acquired prior to the cardinal concept. However, they further stated that the relevance of the finding is doubtful because of the serious differences in the psychometric qualities of the different tasks used, and that Brainerd's definitions of cardinality and ordinality are too broad. Kingma and Koops also questioned Brainerd's between-subjects research design as less robust than a within-subjects design might have been.
Subjects were 266 children from grades kindergarten and primary one, from the mainstream culture in Holland.

Studies on Cardination

There are three major studies of young children's acquisition and understanding of cardinal number which have contributed substantial information to the literature; Russac (1978); Gullen (1978); and Kingma and Roeltinga (1982).

Russac's study, entitled "Relation between Two Strategies of Cardinal Number: Correspondence and Counting" was done with 60 kindergarten, first- and second-grade children. He stated that counting is, indeed, a developmental step in the child's progressively more sophisticated understanding of cardinal number. Russac also raised the question of basic directions to the children in the study; for while they were told to "count" for certain tasks, the children were not told to "match" on others. Russac implied that because the direction "match" is not a common one given to young children when determining the greater (or lesser) of two collections, the inclination to do so is not prevalent, and can be a limitation when evaluating the results. Because of this, children's understanding of cardinality could be underestimated.

Gullen's research (1978) was undertaken with kindergarten, first and second grade children from the Dearborn, Michigan public school system. These children were administered a set comparison test and a quantification test. Gullen was interested in finding out which of six
prevailent strategies for determining set comparison young children did, indeed, use most frequently: length, number, correspondence, heaped (high density), proximity, and guessing. None of the children in his study used "proximity", so this category was deleted from his analysis. The results of Gullen's study indicated that the strategy used depended more on the sizes of the sets to be compared. Those with a numerosity of five or less were most assessed by number (counting or subitizing). For sets with a numerosity of 8, 9, or 10, correspondence was the most used strategy, and those with a numerosity of 13, 14 or 15 were most often determined by length of the rows. However, overall, correspondence was used most often, with 44% of the assessments made with this strategy, compared to 32% by length and 19% by number. Less than 8% of the assessments were made with the "heaped" strategy (density) and less than 1% were determined by guessing.

Kingma and Roelinga (1982) worked with 525 kindergarten and grade 1 children in Holland, using three types of equivalent cardination tasks; linear, linear/non-linear, and nonlinear configurations. They found that non-linear comparison of sets was no more difficult for young children than linear, as opposed to Siegal (1982), and that density manipulation produced strong effects, though their results were contrary to those of Gelman and Gallistel (1978) and Brainerd (1973). These researchers also found that number reproduction and evoked correspondence tasks were easier than linear correspondence, but they admitted that
correspondence as a strategy as well as counting, was evoked as opposed to no clues given with the linear comparisons. Counting was the favored (80%) strategy of the children. Subitizing was favored by the grade 1 children.

The import of these three studies is basically in the data provided on the developmental stages of the cardinal concept in young children, and the authors' acceptance of counting as a legitimate strategy in making set comparisons, as opposed to Brainerd who considers counting as a strictly ordinal operation.

Studies on Equivalence and Order Relations

While not dealing directly with the order of acquisition of the ordinal and cardinal concepts, three studies did work on the underlying structures of the two concepts: equivalence and order relationships.

Steffe and Carey (1975), Johnson (1975) and Owens (1975) each did training studies concerned with the learning of equivalence and order relations in children aged four through seven (preschool through grade two). Their findings indicated that training significantly improved the ability of the children to classify and seriate, but that while experience fostered growth in these areas, it was not sufficient to foster the generalization of the concepts outside of the training situation. Owens subjects were Black children, and he found that growth and development of the equivalence concept was erratic, confirming the results of a previous study (Skypek, 1966) that the development of cardinal number conservation for low socio-economic status
children was unstable. All three of these studies found that children's use of the concept "longer than" and "shorter than" was easily learned, but that this ability did not imply understanding of the transitive, asymmetric property. Each study stressed the importance of the concepts "the same as", "more than" and "less than" as critical to the understanding of cardinal number.

Whitthuhn (1984) also found that numeration caused special problems for Native American and Black students, while geometry was an area of relative strength for both groups.

**Studies of Counting as the Critical Element in the Development of Ordination and Cardination**

One of the most extensive studies of the development of the number concept in young children has been done by Steffe, Von Glasersfeld, Richards and Cobb (1983). This work is comprehensive research into the counting behaviors of young children, and while the authors do not deal specifically with the acquisition of the ordinal and cardinal concepts, the issue of their development cannot be and is not avoided. These researchers posit a theory of five counting stages, three major periods and two (transitional) ones; children are counters of 1) perceptual unit items, which leads to the first transitional stage, being counters of (figural unit items; i.e. the counting of items not within the child's range of perception or action, or the ability to abstract a re-presentation of a perceptual item), progressing to 2) motoric unit items, which in turn
produces the second transitional stage, counters of (verbal unit items), finally achieving the ability to count 3) abstract unit items. These authors propose a theory of the gradual internalization of an intuitive understanding of number (both ordinal and cardinal) on the part of the child, growing from prenumerical understanding (counters of perceptual unit items through verbal unit items) to a fully numerical one (counters of abstract unit items). Reaching the numerical level brings together a workable understanding of both ordinal and cardinal concepts within the demands of the task involved. These authors also state that the issue of cardinal/ordinal number does not arise for children who are not "numerical" -- that is, for children who are not yet able to construct abstract unit items (the final stage of development). Steffe proposes that the child's understanding of the ordinal and cardinal concepts is much different than that of adults, and that close observation of children's use of number in both circumstances is a far better indicator of that understanding than the narrow constraints of transitive asymmetry and equivalence by matching. Steffe is also concerned that researchers must work to understand what it is that children make out of the tasks presented to them and how they attempt to solve these tasks, and what are the child's "intentions", above and beyond what the researcher is seeking to establish (personal communication, 1985).

In a discussion of their work, ten current scholars and researchers in the field of mathematics and young children,
reacted to the study with questions and comments at the Counting Types Symposium (1981). It is with the publication (1985) of the results of this symposium that some of the most pertinent arguments for a more comprehensive and developmental view of the ordinal and cardinal concepts are presented.

Fuson, Richards and Briars (1982) also have done extensive work on young children's learning of the number word sequence, and support Steffe et al in the principle that counting does indeed include the concept of cardinal number. These authors posed two basic phases of development, the acquisition phase and the elaboration phase, throughout which children acquire sequence meanings (number words in conventional sequence), counting meanings (use of conventional sequence in counting entities), cardinal meanings (use of a number word to refer to numerosity of a group), ordinal meanings (use of a number word to refer to a relative position of an entity), measure meanings (use of number words to refer to the numerosity of the units in some quantity), and quasi- or nonnumerical meanings such as street addresses and telephone numbers. This development occurs over several years, with the child reaching the ability to make connections between the various uses of number at about eight years of age. The work of these authors seems to support the work of Piaget, that the various concepts of number are acquired simultaneously, but in slow, developmental steps.

Kamii (1984) found that children's ability to seriate
objects depended on the materials used and the nature of the task itself. She further stated that the ability to seriate did not necessarily imply the child's understanding of transitive asymmetrical relations.

Walters (1983), in studying the origins of number concepts, found that children aged 4 and 5 could count accurately, and use counting effectively to investigate the world of numbers, as well as solve simple addition and subtraction problems. He stated that these children seemed to use a complex coordination of gestures, recitations and conceptual inferences in their use of counting. He also stated that there are few detailed explanations of the origins of number concepts in young children, and that many children have acquired some concepts of number that are not expected at these ages.

Cross Cultural Studies

While there seems to be no specific studies of the origins of the ordinal and cardinal concepts in young children from ethnic groups other than the Anglo majority, there is a considerable body of literature on minorities and mathematics.

One of the major focuses of research on minorities and mathematics is language; both the difficulties of learning mathematics in English as a second language (Cheek, 1984; Cuevas, 1984; De Avila, in press; Saxe, in press) and the specific difficulty of a very precise mathematical vocabulary (Cocking, in press; Earp and Tanner, 1980; Garbe, 1973, 1985).
Another finding of these studies is that minority group participation in mathematics classes at the high school and college level is considerably less than that of the majority, and that for these minority students, achievement in mathematics is well below that of the majority (Bradley, 1984; Cheek, 1984; Cuevas, 1984; Johnson, 1984; Tsang, 1984; Valverde, 1984). However, Garbe (1973, 1979) and Ginsburg (1972) indicate that as school beginners, there is no significant difference in children's understanding and use of number, majority or minority population.

Saxe and Posner (1983) have stated that much that is lacking in cross cultural research on mathematics is a lack of coordination between those who take a Vygotskian approach and those who take a Piagetian approach. The research taking Piaget's perspective takes logical operations as the major focus, and researchers have attempted to document universal changes in children's understandings. In contrast, those who support Vygotsky's approach have made the cultural context of problem solving the primary focus of the study. These researchers have tried to gain greater understanding of the general processes that foster culture-specific problem solving strategies. It is Saxe and Posner's feeling that either view alone is insufficient to allow a clear picture of the development of mathematical thinking in culturally different groups, and that a coordination of the Piagetian and Vygotskian theories should be achieved.

Matthews (1984) summarized the major factors which
figure directly in the mathematics learning of minority children. She cites as critical three broad areas; parents, the students themselves and the schools. Under the heading of parents and student she lists such factors as 1) "ascribed characteristics" as race, sex, age; 2) "cognitive factors" such as education and occupation, mathematics performance and enrollment; 3) "affective areas" such as expectations and aspirations, achievement orientation, locus of control, self-concept, support for mathematics performance, stereotyping, perceived utility, influence of significant others; 4) "cultural" aspects such as communication style, primary language spoken at home, cognitive learning style, and language proficiency. Under the heading of "school", Matthews lists 1) "climate" which includes discipline and attendance; 2) "organization" such as course offerings, sequence and prerequisites, curriculum placement and class size; 3) "resources" such as facilities and materials and 4) "personnel" who have ascribed characteristics, degrees of professionalism, varied instructional methods, attitudes and perceptions, and student interaction. Matthews states that changes in school variables can have the most immediate influence and impact on minority students, that studies of school variables seem to have been the most overlooked and, if indeed, young children across cultures come to school with the same basic understandings of number as indicated in the previously cited studies, then it seems likely that school variables are possible major factors in the lower
achievement and less participation of minority students.

**Brain Research and the Acquisition of Number Concepts**

Some of the recent research on brain functioning and organization also seem of importance to this study; in particular that of Davidson (1983), Levy (1984), Tsunoda (see Sibatani, 1980) and Epstein (1978, 1981).

Davidson, working on a neurobiological model for intellectual functioning in mathematics, hypothesized two learning styles, referred to, in her work, as Learning Style I and Learning Style II, broken down into four categories: achievement, math sense (problem solving), behaviors, and deficits. Learning Style I children are better at counting forward than backward, and at understanding addition and multiplication better than subtraction and division. Such children prefer a "recipe" approach to problem solving, in which they follow a step-by-step sequence of operations. These children seldom estimate, tend to remember parts rather than wholes, and have a strong need to talk themselves through procedures. Learning Style I children are very precise in carrying out their step-by-step approaches, but while arriving at the correct answer, they seem totally unaware of the logic that gives meaning to what they are doing and feel the necessity of asking the teacher if they are "right".

Learning Style II children are good at counting backwards, and at understanding subtraction and division better than addition and multiplication. These children are excellent estimators, and can often arrive at a correct
response without knowing how they achieved it. Large scale pattern recognition is usually a superior skill, and they are particularly good at picturing geometric situations and visualizing three-dimensional configurations.

These Learning Style II children seem to have a better general understanding of what is involved in a math problem; however, because of their impatience and imprecision, they are apt to get a wrong answer. These children often receive very little recognition for their superior talents and "math sense". Davidson found remarkable parallels with her learning styles and the hemispheric specialization theory.

It is this work of Davidson's, coupled with eight years of close observation of Hopi students learning mathematics that has led this author to hypothesize about the learning of Native American children, specifically Hopi children.

Levy's work has been primarily in the area of hemispheric specialization, in which she has found superior visual spatial skills in the right hemisphere and superior linear sequential skills in the left hemisphere. She has also found that males tend to have a larger right hemisphere than do females, and that they exhibit stronger visual spatial skills as well as wholistic approaches to problem solving. Females tend to have stronger left hemisphere skills and attributes.

Tsunoda's study of hearing difficulties has lead him to theorize that brains function differently because of the peculiarities of the language they learn. His work has been exclusively with Japanese/Caucasian comparisons, and he has
found fundamental differences in the way the Caucasian and Japanese brain divide up the labor of processing sensory data.

While Tsunoda's theories are still highly controversial, the importance of the effects of the native language on brain development and functioning cannot be ignored. As stated previously, there is certainly abundant evidence supporting the superior work in mathematics achieved by many Japanese students.

Epstein has concentrated on the physical growth of the brain, determining five growth spurt periods from infancy to the mid-teen years. Of interest to educators is his proposed theory of different kinds of learning which surely must occur during growth spurt periods as opposed to the "plateau" periods; during periods of brain growth, children need to be stimulated cognitively as much as possible, while during the plateau periods, more assimilation and accommodation is taking place. The first growth spurt, according to Epstein, occurs between the ages of three to ten months, with the second major growth spurt taking place in children between the ages of two to four years, and the third between the ages of six and eight, just as the years of formal school begins. The fourth growth spurt occurs between the ages of 10 and 12 years, and the fifth between 14 and 16 years (Epstein, 1978). A project of "brain compatible" curriculum was begun with a school district in 1981; however, publication of any formal evaluation has not been found by this writer.
Thirty years ago, Whorf (1956) also posited a theory of language competence as a vehicle for meaning, a theory which, today, seems largely ignored. Whorf seemed to grasp the relationship between human language and human thinking, assigning to language the task of shaping a person’s innermost thoughts. Whorf’s work is particularly applicable to Native American languages, specifically Hopi. He determined that a culture’s values, lifestyle, and all its unique features, expressed through language, would elicit from its members perceptions, which, in some situations, are drastically different from those who are not of that culture.

Every language is a vast pattern-system, different from others, in which are culturally ordained the forms and categories by which the personality not only communicates, but also analyzes nature, notices or neglects types of relationships and phenomena, channels his reasoning and builds the house of his consciousness.

(Whorf, 1956, p.252)

A prime example of this type of cultural approach to problem solving would be the way in which Hopi children determine that all of the sheep they have been herding are returned to the fold at night; instead of counting each sheep, the child is taught to recognize certain characteristics of each animal. Using these as a checking guide, the child knows if all the animals have been returned safely. Another example is the Hopi child’s (and often the
adult) interpretation of the mathematical expression "half of": when asked to give the answer to half of ten, for instance, the most frequently given response is "20"! Considerable conscious work and effort must be done on this concept to produce the correct mathematical response.

**Summary of Chapter II**

The literature shows that the acquisition of the ordinal and cardinal concepts has not been definitively researched for either the majority or minority populations.

Piaget's original work, which undertook to explain the natural or psychological growth and development of these two concepts was the first study to take this approach to the origins of number concepts in children. Previous to his work, the logical theories of either Frege, Peano or Russell prevailed. Piaget left the world with a considerable body of knowledge about the development of the concept of number in children as an acquisition which takes place over many years and in definable developmental stages. It is his theory that children develop the ordinal and cardinal concepts simultaneously but in three fixed developmental stages.

Brainerd undertook to replicate Piaget's work, but used the sophisticated mathematical definitions of ordinal and cardinal number as his criteria of acquisition. He found that children developed these concepts sequentially rather than simultaneously, with the concept of ordinality developing prior to the concept of cardinality. He also found that children developed the ability to manipulate
natural numbers (add and subtract) prior to their acquiring
the cardinal concept and that training in ordinality had
greater influence on growth and development in the ability
to add and subtract than did training in cardinality.

The literature indicates that there is considerable
disagreement over Brainerd's definition of ordinality and
cardinality, and his omission of the use of counting as a
method of determining manyness. Brainerd did not consider
counting as a legitimate strategy for determining manyness;
he only permitted the use of matching, in a one-to-one
correspondence as a correct procedure for such
determinations, as an understanding of cardinality.

Two recent major studies on counting each defined a
series of developmental steps in this counting process as
the child constructs his/her understanding of both the
ordinal and cardinal concepts, indicating support for
Piaget's theory of simultaneous acquisition.

In the literature on mathematics learning among other
cultures, study of the origin of the ordinal and cardinal
concepts has not been pursued. It appears that two basic
approaches to research have been taken: either a Vygotskian
perspective or a Piagetian one. Vygotskians have made the
cultural context of problem solving their primary focus and
have tried to understand the general processes that foster
these culture-specific problems solving strategies including
the influence of the native language. Piagetians, on the
other hand, have taken logical operations as their major
focus and have attempted to document the universal changes
in the understanding of number in children. In the eyes of some researchers, there needs to be a synthesis of the two approaches before any significant gains can be made in understanding the growth and development of number concepts.

Research on brain functioning is viewed as bearing directly on the acquisition of numerical concepts and the ability to become successful in the learning of mathematics. Hemispheric strengths and/or deficits may cause specific problem solving skills and approaches to be used by an individual learner, and further, the native language one learns may significantly contribute to and influence the brain's functioning.

With this background in mind, the next step in this author's work is the methodology used to construct the study.
CHAPTER III

RESEARCH METHODOLOGY

Purpose of the Study

The primary purpose of this study was to investigate the acquisition of the ordinal and cardinal concepts in young children, aged five, six and seven (kindergarten and grade I), from three ethnic groups: Anglo, Hispanic and Native American. Determination of whether these concepts are acquired simultaneously, as Piaget indicated, or sequentially, as Brainerd indicated, was a major concern. Further, this study looked at the effects of the acquisition of these two concepts on the ability to add and subtract, and on sex differences in both the acquisition process and manipulation of natural number. Another consideration was the effect of the child's dominant language on the acquisition of the two major concepts.

Design of the Study

Description of the Population

The subjects in this study were 90 students, 30 (15 kindergarten and 15 grade I children, as equitably divided between boys and girls as possible) from each ethnic group, from three rural communities. Two of these communities were located in New Mexico; the Hispanic one in the northern mountains and the Anglo one in the southwest desert. The Native American community was located in northeast Arizona. The subjects were members of kindergarten and first grade classrooms in their local public schools for the Anglo and Hispanic subjects and kindergarten and first grade children
from two community schools in the Native American villages. The dominant language of the Anglo community was English, for the Hispanic community, Spanish, and for the Native American community, Hopi.

Selection of these communities and schools was based on the language dominance and the shared attribute of a rural community setting. Also, the teachers in each of the local schools have as their home background the language and culture of the subjects. Two teachers from the Hispanic community school, (kindergarten and grade one) and two teachers from one of the Hopi community schools (founder and head teacher) tested the subjects whose dominant language was not English. This researcher tested the children from the Anglo community as well as the children in both the Hispanic and Native American communities whose primary and dominant language was English.

General Description of the Study

The study investigated the acquisition of the ordinal and cardinal concepts of number in five, six and seven year old children in kindergarten and first grade. Four tests were administered individually to each child; 1) a language pretest, 2) a task of ordinality, 3) a task of cardinality and 4) a task of addition and subtraction of the first four natural numbers. Each child was tested alone by either this researcher or his/her classroom teacher in the child’s dominant language. Approaches by individual children to solving the tasks were noted by the examiner, especially with the cardinal task and the addition and subtraction
task. Any behavior the examiners felt pertinent was noted, such as vocalization, subvocalization, counting methods and procedures, motoric behaviors and explanations by the child of his/her decisions.

All the children were tested during the first two weeks of their school year to negate the effects of school learning as much as possible.

The final version of the research instruments was based on a pilot study done by this author in the spring of 1984. In this pilot study the author replicated the tasks Brainerd created in his original research: the language pretest, the ordinal tasks of weight and length, the cardinal task of determining manyness by comparison of two groups and the task of addition and subtraction of the first four natural numbers. The only change made in this pilot study was to administer the subtraction section of the manipulation of number task to the kindergarten children. Brainerd had chosen not to do this because he felt subtraction was not a concept acquired before formal school learning.

It was during this pilot study that doubts about the cardinal task began to arise. Questions concerning equal difficulty between the ordinal and cardinal tasks were raised by the teacher in whose classroom the study was taking place. She noted that the ordinal task involved the use of three dimensional concrete objects (sticks and balls of clay) which the children were permitted to touch, hold and compare. The cardinal task consisted of cards with a row of red dots and a row of blue dots on them, "pictures"
so to speak. It was her feeling (as well as that of this author) that the children would react more positively to stimuli which were also three dimensional.

The cards were reconstructed with three dimensional red and blue cubes glued in rows exactly like the previous "pictures" of dots, and the task was administered again one month later to all the children, with a significant difference in performance.

Initially, the results obtained in the pilot study were quite similar to those of Brainerd: for most children, the ordinal concept did seem to appear before the cardinal concept.

Analysis of the tasks after the retest of the cardinal concept with three dimensional cubes indicated the majority of the children acquired the ordinal and cardinal concepts simultaneously. These findings, of course, could be directly affected by the "learning" factor (a second testing) as well as a maturational factor (one month's additional age and experience); however, the difference in performance was significant enough to convince this author of the need for this change in the cardinal task to materials which were comparable to those in the ordinal task.

Support for the use of concrete physical materials, instead of pictures, with young children can be found in Biggs, (1971); Bright, (1986); Choat, (1973); Dienes, (1965, 1967); Herbert, (1985); Piaget, (1953); Vargas, (1973); and Williams and Shuard, (1970).
The Research Instruments

The language pre-test and the three tasks of ordinality, cardinality and manipulation of number were replicated directly from Brainerd’s original study (1973) with a modification of the cardinal task.

Developmental researchers have known for some time that logical and mathematical concepts frequently develop independently of the capacity to use these concepts linguistically (Brainerd 1973; Dienes, 1972; Piaget, 1953). Consequently, when tests for such concepts involve understanding them linguistically, as the ordinal and cardinal tasks do, it is possible for children who actually possess the concepts in question to fail the tasks. To eliminate such errors, Brainerd employed a pretest to test for the children’s understanding of the terminology employed in the concept tasks. This pretest assessed the child’s grasp of certain key relational terms.

Language pre-tests

There were two separate tests for relational terms employed in the ordination task; a length and a weight pretest question. For the length test, the subject was shown a card on which two parallel lines are drawn; one 3 inches in length, the other 6 inches in length. Three questions were asked: 1) Are these lines the same length? 2) Is one of these lines longer (and if so, which one)? 3) Is one of these lines shorter (and if so, which one)? For the weight pretest, two green clay balls the same size, but weighing 16 and 24 ounces respectively, were placed in the
subject's hands and he/she was asked; 1) Do the balls weigh the same? 2) Does one of the balls weigh more (and if so, which one)? 3) Does one of the balls weigh less (and if so, which one)? Children who answered all the questions correctly were considered to have an adequate grasp of the terminology employed in the ordination task.

The cardination language pretests consisted of two story problems which the examiner told to the child. The first story was as follows: "Suppose that I had eight cookies and you had six cookies. Would one of us have more cookies (and if so, which one)? Would we both have the same number of cookies? The second story problem was "Suppose that I had four cookies and you had four cookies. Would one of us have more cookies (and if so, whom)? Would one of us have fewer cookies (and if so, whom)? Would we both have the same number of cookies? Cookies were laid out by the examiner when "telling" these stories. Children who answered all of the questions correctly were concluded to have sufficient grasp of the terminology employed in the cardination task.

**Ordinal task**

The ordination task was composed of two parts: a length test and a weight test. For the length test, two red sticks were placed on the table 24 inches apart. These sticks were 11 inches and 11 3/8 inches, and appeared, at the distance apart, to be the same length. Initially, the shorter stick was always placed to the subject's left. A third stick, painted yellow, was introduced. This stick also appeared to
be the same length as the two red sticks; however, it was 11
3/16 inches long, 3/16 inches longer than one red stick and
3/16 inches shorter than the other red stick. A comparison
between the yellow stick and the shorter red stick was made,
with the subject able to touch the ends of the two sticks.
Then a comparison between the yellow stick and the longer
red stick is made. (Again the subject was permitted to
touch the two sticks.) After each comparison the subject is
asked which of the two sticks is longer. Finally, to
determine whether or not the subject understood the
asymmetrical relation "longer than" is also transitive, the
examiner asks the following questions: 1) Are the two red
sticks the same length? 2) Is one of the red sticks longer
than the other (and if so, which one)? 3) Is one of the red
sticks shorter than the other (and if so, which one)? When
these questions are asked the subject is not permitted to
touch the sticks in any way. The ordination of length
procedure is then repeated in the reverse direction; that
is, the position of the two red sticks is reversed with the
longer one being on the subject's left, and the same three
questions also repeated.

The second ordination task (weight) was constructed
like the first. Three plastic plates, each 12 inches apart
were placed in front of the subject. Two red clay balls,
both the same size but one weighing eight ounces and one
weighing 24 ounces were placed on the two outside plates
with the lighter weight ball placed to the subject's left.
A third clay ball, again the same size as the other two, but
yellow in color and weighing 16 ounces was placed on the middle plate. Comparisons were made between the 8 ounce red ball and the 16 ounce yellow ball by placing them in the subject's left and right hand respectively and asking if the balls weigh the same. The yellow ball was then shifted to the subject's left hand and the 24 ounce red ball was placed in the subject's right hand and the same comparisons were made between the yellow ball and the red ball. After the balls are returned to the plates three questions similar to the length questions were asked; 1) Do the two red balls weigh the same? 2) Does one of the red balls weigh more (and if so, which one)? 3) Does one of the balls weigh less (and if so, which one)? Again, the subject was not permitted to touch or hold the clay balls during the questioning.

**Cardinal task**

The subject was first shown a card with a single row of eight evenly spaced red cubes glued onto it. The subject was told that he/she will be working with cards like this one. "All of the cards have a row of red cubes glued on just like this one; however, each card also has a row of blue cubes glued onto it. I will show you the cards one at a time. You take a good look at the red and blue cubes and try to figure out whether there are the same number of red cubes and blue cubes or whether one of the rows has more cubes. But there is a special rule for these cards; you cannot count any of the cubes - you have to figure out the answer another way."
The examiner then introduced the six test cards one at a time. Two questions are posed in connection with each of the six cards; 1) Are there just as many red cubes as blue cubes on this card? 2) Does one of the rows have more cubes than the other row (and if so, which one)? After each of the cards has been presented and the answers noted, the examiner then asked the subject "Did you find that you could answer all the questions without counting the cubes or did you have to count sometimes?" Examiners were also trained to watch for rhythmic hand movements, arm and foot movements, lip movements, and other overt motor signals that covert counting was going on. This check is an exceptionally effective one because below the age of seven or eight years, most children find it virtually impossible to count without some supporting motor response (Brainerd, 1973, 1979; Gelman and Gallistel, 1978; Steffe et al., 1983). Children who admitted to counting, or who were observed to count overtly were dropped from the analysis. There were only two children in this category.

As was indicated by the questions, each subject made a total of 12 judgements on both the ordination and the cardination tasks, and, on both tasks, the subject was given one point for each correct judgement.

**Manipulation of number**

To test the arithmetic proficiency of the subjects, 16 addition and 16 subtraction "facts" employing the first four natural numbers was presented to each subject. A combined verbal and written method was employed for this task. Each
arithmetic fact was written on a card (i.e. $1+1=2$) which was shown to the subject. The examiner asked, "How many apples are one apple plus one apple?" The child who could either write the symbol "2" or said "two" or did both, was judged to have passed the item.

A total of 32 problems, 16 each of addition and subtraction were administered to each child. The kindergarten children were again included because it has been this author's observation as well as that of other researchers that young children's ability to add and subtract minimal quantities does not depend on formal school instruction (Ginsburg, 1977; Kamii, 1985). The children were told that they were going to add and subtract some simple numbers. Each child was given a pencil and paper as they were tested and told they could either write or say the answer, or both. The children were also told they could count, use their fingers, use the counters (some colored chips) or the Cuisenaire rods which were placed on the table to help them find their answers.

At the beginning of each test item, the examiner placed a card on which the problem was printed in standard numerical symbols in front of the subject. The examiner then repeated a story problem which matched the problem printed on the card. After the examiner had posed the problem verbally, the child was allowed to respond by saying the answer aloud, by writing it down on the sheet of paper, or by doing both. Each examiner noted whether the child counted, overtly or covertly, used any of the materials, or
just "said" or "wrote" the answer. Any comments the child made about what they were doing were also noted by each examiner.

Any child who answered the problem either verbally or on paper, or both, passed the item. One point was awarded for each correct response on the addition and subtraction tasks for a total of 16 points for each part of the test.

Please see Appendix A for samples of the language pretest, the ordinal, cardinal and manipulation of number tasks. Spanish translation used is also provided in Appendix B. Permission to include the Native American translation was not obtained. Specific reasons for not permitting this inclusion were not given, however, preserving privacy (and especially with the language) is a major priority within Hopi, and honoring this privacy, for the privilege of testing the children, was important to this researcher. Currently, there is a team comprised of Anglo and Hispanic researchers who are compiling a dictionary of the Hopi language, a project which is heavily resented by the majority of the Hopi population and is "under fire" by Hopi scholars.

Appendix C contains the cardinal task card configurations and Appendix D the permission letter and form that was used.

Conservation of Number Task

Finally, a conservation of number task was administered to see if the ability to conserve played an important role in the development of the ordinal/cardinal concept.
acquisition as well as for the manipulation of number.

**Determination of Levels of Development**

Again, Brainerd’s study was followed in assigning levels of development to the ordinal, cardinal and manipulation of number task scores.

Three levels of development were established in the original study which showed up for this author as well in the pilot study and which was maintained for this research. As with Brainerd, these three categories also tended to be correlated with age; that is, children falling into the first category tended to be the youngest ones, subjects falling into the second category tended to be somewhat older, and subjects falling into the third category tended to be the oldest children in the sample. The important features of each of the three categories for ordination, cardination and the manipulation of number will be summarized briefly.

**Ordination**

**Level I: no ordering.** Children at this level made five or fewer correct judgements.

**Level II: partial ordering.** Children assigned to this level made more than five but fewer than eleven correct judgements.

**Level III: complete internal ordering.** Children at this level answered either eleven to twelve questions correctly.

**Cardination**

**Level I: no correspondence.** Children who made five
or fewer correct judgements were assigned to this level.

**Level II: no one-to-one correspondence.** These children answered more than five but fewer than eleven questions correctly.

**Level III: complete internal correspondence.** Children assigned to this level made eleven of twelve correct judgements.

**Manipulation of Number**

Again, Brainerd's three categories were used to determine the levels of achievement for the tests of addition and subtraction. These categories were "below average", "average" and "above average". To determine what, generally, is below average, average and above average arithmetic proficiency for children at this age level and school experience, Brainerd interviewed teachers and principals. On the basis of these interviews, the following criteria were formulated to assign children to the categories: children passing five or fewer of the items correctly were assigned to the below average category, those passing more than five but fewer than 12 of the items were assigned to the average category, and those children who passed between 12 and 16 items correctly were assigned to the above average category. These criteria were used for both the addition and subtraction tests.

As previously mentioned, Brainerd did not test the kindergarten subjects with subtraction, as he determined that these children had not had instruction with this operation of arithmetic. This author, however, did
administer the subtraction test to all the children in the study.

**Procedure For the Study**

**Permission for the Study**

Obtaining permission for the study from three sites required detailed presentations by mail and a follow-up personal interview/presentation in each case. Initially, two sites with the desired language/culture background were selected for each ethnic group. One Anglo, Hispanic and Native American group responded positively and followed up with personal interviews with the administrator(s) of the school/school district. The principal and the superintendent of schools from the rural community in Southwest New Mexico made the trip to Santa Fe to meet with this author; indication of such real interest in the study was very much appreciated.

Initially, both Hispanic communities refused permission, citing delicate political factors as reasons for the refusal. Intervention and support from William Trujillo, Mathematics Specialist for the State Department of Education, resulted in an acceptance from one of the previously mentioned communities in the northern mountains of New Mexico. The principal was very interested in the study, as were both the kindergarten and first grade teachers whose children were the subjects. Each of these teachers volunteered to test the children whose dominant language was Spanish.

"Official" permission from a small independent school
in the Native American community was gained with the initial
detailed letter. The founder had previously worked with
this author for five years at the local BIA school and later
as it became the first contract school on the Reservation.
He made it quite clear, however, that this author would have
to speak directly to the parents concerning the study.
Therefore, he and his wife who shares the responsibility of
teaching in their little school, hosted a parent meeting the
first day on site. To this meeting they had invited all of
the parents of kindergarten and first grade children in the
two adjacent villages served by this school and the contract
school. This author then explained the study and answered
questions for parents concerning the tasks which would be
administered to their children. The founder and his wife
translated into Hopi for anyone who did not understand the
concepts in English. Many of these parents were also
acquainted with this author from previous work at the
contract school. Anonymity was a major concern for these
parents, as they felt strongly that they had been "used"
badly by previous researchers, especially in publication of
the results. They also requested that, should this author
wish to publish any part of the findings, that they be
consulted first and be permitted to read the final
manuscript. This request was, of course, reasonable and
agreed to by this author. Permission slips were passed out
at this meeting and sufficient permissions were returned to
give the required number of subjects.

Teachers of children in grades kindergarten and first

63
grade in the Anglo and Hispanic communities sent home and received back the permission slips. Permissions from the Native American parents were gained as previously stated. Each child, whose parents had given permission, was told that this researcher was trying to find out how young children learned all about numbers so that we, as teachers, could become better teachers and help children more. The child was asked if he/she wanted to help with the study. The child was assured that it was quite alright to say no, even if their parents had given permission. Only one child, from the Anglo community, did not want to participate; this was a first grader, the teacher explained, who had taken the entire previous year to gain enough courage to speak in the classroom.

Selection of the Subjects

The study was concerned with the natural acquisition of the ordinal and cardinal concepts, so that children with a minimal amount of formal school influence were desired. Therefore, kindergarten and first grade children were identified as likely subjects, especially if they were tested during the first two weeks of the school year. It was fortunate for this researcher that each school opened at a different week, permitting her to be on site at each school while the testing was being done.

Training of Examiners

Because of the need to have children tested in their dominant language, it was necessary to recruit assistants who could carry out this task. The kindergarten and first
grade teachers in the Hispanic community volunteered to do this, as well as the founder of the community school on the Reservation, and his wife, who also teaches in the school. This couple had also taken a major role in the development of Hopi language in the classrooms at the contract school and are fluent speakers and writers of Hopi.

Before the start of school this author went to the Hispanic community to train the two teachers who had volunteered their help. The author spent the first two days on the Reservation working with the Native American teachers. In each case, the teachers were told that the study was concerned with the child's acquisition of the ordinal and cardinal concepts; whether these concepts were acquired simultaneously or sequentially, and if sequentially, in what order. "Ordinal" was defined as understanding that if A is less than B, and B is less than C, then A is less than C, and that this concept would be tested with length and weight materials. "Cardinal" was defined as comparing the manyness of two groups, and in this case, two groups of cubes (red and blue) involving the understanding and use of "more", "less" or "the same amount". None of the assistants were told of the hypotheses that had been formed.

Initially, in the training, each teacher took the part of the child, and this author administered the tasks to the adult exactly as she would have to a child. Then a "guest" child was recruited (one who would not have been a part of the study) and the assistant in turn administered the tasks
to this child for practice. Assistants were asked to make verbatim recordings of each child’s answers on each task sheet provided for all subjects, and note behaviors indicating how the child solved the problem(s), especially on the cardinal, addition and subtraction tasks.

Because this author was able to be on site in each community for an entire week (and on the Reservation for two weeks), she was able to watch each child being tested, and note behaviors that seemed pertinent to the child’s understanding. If the child was speaking either Spanish or Hopi, then translations of the child’s conversation were discussed and noted at the time of testing. Thus, it was possible to keep careful observations of each child as they were tested.

This author worked alone in the Anglo community, but because of the experience with the pilot study the previous year, had little difficulty with recording verbatim language (most children gave one or two word answers or short phrases), as well as noting behavior. It must be stated, however, that it would have been a major help to have been able to video tape each child for detailed record of all behavior, verbal and motor as well as facial expressions.

Because of the small size of the Hispanic and Native American communities, the two teachers at each location were well acquainted with all of the children in their classes. Extra time usually needed to put a child at ease before testing was generally not needed in these situations, as the children seemed to be entirely comfortable, even with this
Conducting the Study

Schedules for Testing

Each child was tested individually, and from experience, about 30 minutes was planned for each child. In the Anglo community, the school had two kindergarten and three first grade classrooms. Each teacher had obtained permissions from parents prior to this author’s arrival, and had arranged a schedule for taking the children from the room to be tested. Approximately 1/3 to 1/2 of the children in each classroom had returned permissions for a total of 37 children. A morning was spent with each classroom, testing the children whose parents had given permission and who wished to be in the study. The first afternoon after school hours, this author explained the study to the primary teachers in the school, and answered any questions they had. All responded enthusiastically to the work and indicated pleasure at having been selected as a site for the study.

In the Hispanic community, permissions from parents for participation on the part of their children were also gathered before this researcher arrived to begin testing. Each of the children in the two classes had received permission to participate, for a total of 33 children.

Again, the kindergarten and first grade teacher had arranged a schedule for testing each child, arranging for the few English dominant children to be tested by this author first, so that they (the teachers) could watch once again. Their own classroom was taken over by the principal on an
alternating basis while the teachers were testing children in Spanish with this author watching.

The testing procedure moved more slowly on the Reservation, as time, by the clock, is not a major issue for many Native Americans. Parents whose children attended the contract school brought their children to the community school by appointment in the afternoon after school, while children who attended the community school were tested during the morning session of school. The school day ends in both schools, for primary children, at noon so that afternoon testing was done shortly after that time while children were still quite alert. A total of 32 children were granted permission to participate and were tested. This community was the last to be tested as their school(s) did not open until well after the schools in New Mexico; therefore, a second week on the Reservation did not interfere with commitments at the other sites.

**Data Gathering Procedure**

At each site, a quiet, private location was provided for the examiner and the subject to work. A small table with two chairs was used, with subject and examiner seated opposite one another. Each subject was tested individually, with the examiner administering first the language pretest, followed by the ordinal, cardinal and addition and subtraction tasks. The ordinal, cardinal, addition and subtraction tasks were administered in random order. Finally, for each subject, a Piagetian task of number conservation was administered using colored chips.
A complete set of task sheets was provided for each subject so that examiners could easily record answers and behaviors. Extra paper and pencils were provided for the addition and subtraction tasks, so that subjects could write answers if they chose to do so.

As previously stated, a set of colored plastic chips for counters as well as Cuisenaire rods were also on the table for the child to use for addition and subtraction if they wished to use them. The child was told as they began this segment of the testing that they could count any way they wished, use their fingers or the materials. It was surprising that only seven children from the total number tested used materials at all!

Scoring was done by this author only, after testing was completed each day. There was no problem with ambiguity of answers, either the child made a correct judgement or he/she did not; a score of 1 was assigned to a correct response, 0 to an incorrect response.

Summary of Chapter III

Ninety five-, six- and seven-year old (kindergarten and first grade) children, about 30 each (boys and girls) from an Anglo, a Hispanic and a Native American community participated in a study examining the acquisition of the ordinal and cardinal concepts (simultaneous or sequential) and the effects of this acquisition on the ability to add and subtract. Each child was tested individually with a language pretest, a test of the ordinal concept (length and weight), the cardinal concept (comparison of manyness of two
groups of objects), an addition and subtraction test of the first four natural numbers and a Piagetian task of number conservation. Approximately thirty minutes were needed to test each child. Children whose dominant language was not English were tested in his/her native, stronger language. The kindergarten and first grade teacher from the Hispanic community, both native speakers of Spanish, tested those children for whom Spanish was the stronger language, and the founder of the small community school on the Reservation, and his wife, (also a teacher), natives of the community, tested those children for whom Hopi language was stronger.

The research instruments were taken directly from the original work of Brainerd (1979) with a change in the cardinal task from two dimensional pictures of dots to three dimensional cubes glued to a card. Identical row formations and numbers of cubes, as well as colors were preserved. All children, including the kindergartners, were given the subtraction test, in contrast to Brainerd who administered this task only to the first graders. A Piagetian task of number conservation was also administered to each child.

A week was spent on site in the Anglo community where this author did all of the testing. A week was spent on site in the Hispanic community where this author tested those children whose dominant language was English, and observed the testing of the Spanish dominant children. Two weeks were spent on the Reservation, as initially, permissions were slow to come in. This writer again tested those children whose dominant language was English, and
observed the testing of those children who spoke Hopi.

This chapter also contained a description of the design of the study as well as an explanation of the selection of subjects, the procedures for obtaining permission(s), the measurement instruments, and the assisting examiners' training process.

Chapter IV will present the data and a statistical analysis of the findings.
CHAPTER IV

FINDINGS OF THE STUDY

This study compared the development of the ordinal and cardinal concepts in five-, six- and seven- (kindergarten and first grade) year old children from three different cultures: Anglo, Hispanic and Native American. There were thirty children from each ethnic group, 15 kindergartners and 15 first graders in each group. For the Anglo and Hispanic children, there were 15 boys and 15 girls in each group, and for the Native American children, there were 14 girls and 16 boys. Mean ages for the entire population were five years, five months for the kindergarten children, and six years, five months for the first grade children. By ethnic origin, the mean ages for kindergarteners were: Anglo, five years, six months; Hispanic, five years, six months, and for Native American, five years, four months. Mean ages for first grade children by ethnic origin were: Anglo, six years, five months; for Hispanic children, six years, six months, and for Native American children, six years, four months.

Table 1 shows this information as follows:
Table 1
Mean Ages of Population by Grade Level and Ethnic Group

<table>
<thead>
<tr>
<th>Ethnicity</th>
<th>K</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anglo</td>
<td>5 yrs. 6 mo.</td>
<td>6 yrs. 5 mo.</td>
</tr>
<tr>
<td>Hispanic</td>
<td>5 yrs. 6 mo.</td>
<td>6 yrs. 6 mo.</td>
</tr>
<tr>
<td>Native Am.</td>
<td>5 yrs. 4 mo.</td>
<td>6 yrs. 4 mo.</td>
</tr>
<tr>
<td>Entire pop.</td>
<td>5 yrs. 5 mo.</td>
<td>6 yrs. 5 mo.</td>
</tr>
</tbody>
</table>

This study also looked at the effects of these two concepts on the child’s ability to add and subtract the first four natural numbers. The ordinal and cardinal tasks, as well as the manipulation of number tasks were replicated from an original study by Brainerd (1973), with a change in the cardinal task from two to three dimensional objects, making it more comparable to the ordinal task. A conservation of number task was also administered to each child.

The study examined the pattern of acquisition of the two concepts of ordinality and cardinality to see if they developed simultaneously, as Piaget stated, or sequentially, as Brainerd later stated. Both of these researchers found that the two concepts did evolve in developmental steps, over a period of time; the difference between them being that Piaget found the concepts developed simultaneously where Brainerd found that the concepts developed
sequentially, with ordinality appearing first. Brainerd also found that the ordinal concept affected the ability to add and subtract the first four natural numbers, rather than the cardinal concept. Therefore, this investigation also studied the developmental steps involved in the acquisition of ordinality and cardinality, and their effects on the child's ability to add and subtract these same four natural numbers. The inclusion of a conservation of number task was included so that a more careful comparison to Piaget's original work on these two concepts could be made. Hispanic and Native American children were tested in their native language(s) unless it was determined by teachers or parents that English was the dominant language. For the Hispanic population, 22 children were tested in Spanish with only eight children, five kindergartners and three first graders tested in English. Twenty Native American children were tested in their native language, leaving 10 tested in English, two of whom this researcher felt might have had greater success if they had been tested in the native language. Parents of these two children (brother and sister) had requested they be tested in English.

Scoring

The student's responses were measured as either 1 (for a correct response) or 0 (for an incorrect response) on the ordinal, cardinal and manipulation of number tasks. On the conservation task, the child was classified as either a conserver or a non-conserver of number.

Three developmental levels of achievement were
determined, using the Brainerd (1979) standard. For the ordinal and cardinal tasks, which asked twelve questions each, these were level 1; those children who made five or fewer correct judgements: level 2; those children who made more than five but less than eleven correct judgements: and level 3; those children who made eleven or twelve correct judgements.

**Development of Ordination**

When the children’s performance on the ordination tasks was examined, it was noted that the children in this study tended to fall into the three categories in much the same way that Brainerd had found; the three categories tended to be correlated with age. Children falling into the first category tended to be the youngest ones in the sample, children falling into the second category tended to be somewhat older and the children falling into the third category tended to be the oldest ones in the sample. The important features of the three categories can be briefly summarized as follows:

**Level 1: No ordering.** The children at this level showed little or no understanding that the practical, everyday asymmetrical relations "longer than" and "heavier than" are invariably transitive. The majority of children at this level thought that the two red sticks were the same length and the two red balls of clay were the same weight. These children appeared to be "perception bound"; that is, they were tied to what their eyes told them, rather than using any of the information available to them to construct an
ordered progression. Mean age for all children at this level was five years, four months. Broken down by ethnic origin, the ages were five years, six months for the Anglo children, five years, six months for the Hispanic children and five years, two months for the Native American children.

**Level II: Partial ordering.** The children in this category gave evidence of understanding that "longer than" and "heavier than" were transitive in some instances, but not in others. The majority of the children were able to answer the first half of the questions correctly, using the left-to-right progression, but had difficulty maintaining the transitive relationship when the progression was right-to-left. Mean age for the entire population at this level was six years, one month. Again, by ethnic origin, the mean age for Anglo children was six years, two months; for Hispanic children six years, and for Native American children six years, one month.

**Level III: Complete internal ordering.** The children in this category indicated an understanding that "longer than" and "heavier than" are always transitive operations by their consistent correct judgements. These Level III children had little difficulty with the concepts either in the left-to-right or right-to-left progression, and often made comments such as "You can't fool me. I know which one is longer/heavier!" Mean age for all subjects at this level was six years, two months. For Anglo children, the mean age was six years, for Hispanic children the mean age was six years, one month and the mean age for Native American
children was six years, five months.

The mean ages for ordination in the original Brainerd study were: level I, four years, eleven months; level II, five years, three months, and level III, five years, eight months (Brainerd 1979). These young ages would seem to indicate an earlier school entry age for Canadian school children.

The following tables (2 through 5) show the numbers of children assigned to each level of ordination, initially with the entire population, and secondly, by ethnic origin. The large clusters of children found at level II in the tables reflect a greater number of children from each group in the median age range:

Table 2

**Numbers of All Kindergarten and First-Grade Children Assigned to Each of the Three Ordination Levels**

<table>
<thead>
<tr>
<th>Grade</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>20</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>23</td>
<td>17</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>25</strong></td>
<td><strong>39</strong></td>
<td><strong>26</strong></td>
</tr>
</tbody>
</table>
### Table 3

**Numbers of Anglo Children Assigned to Each of the Three Ordination Levels**

<table>
<thead>
<tr>
<th>Grade</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>6</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

### Table 4

**Numbers of Hispanic Children Assigned to Each of the Three Ordination Levels**

<table>
<thead>
<tr>
<th>Grade</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>2</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>3</td>
<td>21</td>
<td>6</td>
</tr>
</tbody>
</table>
Table 5

Numbers of Native American Children Assigned to Each of Three Ordination Levels

<table>
<thead>
<tr>
<th>Grade</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>13</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Totals</td>
<td>16</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

Development of Cardination

The classification for the cardinal tasks also closely followed Brainerd's findings; that is, the youngest children tended to be found at Level I, the slightly older children at Level II, and the oldest children at Level III.

The important features of each of the categories for cardination is described as follows:

**Level I: No correspondence.** Children falling into this first category showed minimal evidence of understanding cardinal number by correspondence. There was little indication that these children understood that the type of correspondence that obtained between the two rows determined relative manyness. These children, again, relied on what their eyes told them, being heavily perception bound. Length and density of the rows were the determining factors rather than any attempt to match the red and blue cubes. Mean age for the entire population at Level I was five years, 4
months. By ethnic origin, the mean ages were: Anglo, five years, seven months; Hispanic, five years, four months, and Native American, five years, one month.

Level II: No one-to-one correspondence. Children falling into this category, while not understanding a matching one-to-one correspondence, clearly did not base their judgements on length cues. Their correct judgements were limited, however, to the cards with unequal rows of cubes. Their errors, mostly restricted to equal manyness cards, seemed to indicate a use of density as the main cue for judgement, as they almost always chose the row with the cubes glued closer together, and frequently accompanied their choice with a remark such as "they're are more because their closer together". Mean ages for children in this were: entire population, five years, nine months; Anglo, six years; Hispanic, six years, one month, and Native American, five years, six months.

Level III: Complete internal correspondence. The children who achieved this category appear to understand the connection between one-to-one correspondence and manyness. They did not depend on the perceptual cues of either length or density used so often by the children at Levels I and II. Many of these children indicated the use of one-to-one matching by their head or hand movements, matching a red with a blue cube and coming to a judgement after seeing if the card contained a one-to-one or a many-to-one correspondence. Mean age for the entire population sample was six years, three months. For Anglo children, the mean
age was six years, two months; for Hispanic children, six years, three months, and for Native American children, six years, five months.

Mean ages for cardination levels in the original Brainerd study were: level I, five years, ten months; level II, six years, eight months, and for level III, seven years, one month.

Tables 6 through 9 show the numbers of children assigned to each of the three levels of cardination, first by the entire population, followed by ethnic groups.

Table 6

Numbers of Kindergarten and First-Grade Children Assigned to Each of the Three Cardination Levels

<table>
<thead>
<tr>
<th>Level of Cardination</th>
<th>Grade I</th>
<th>Grade II</th>
<th>Grade III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11</td>
<td>30</td>
<td>4</td>
</tr>
<tr>
<td>K</td>
<td>2</td>
<td>22</td>
<td>21</td>
</tr>
<tr>
<td>Totals</td>
<td>13</td>
<td>52</td>
<td>25</td>
</tr>
</tbody>
</table>
Table 7

**Numbers of Anglo Children Assigned to Each of Three Cardination Levels**

<table>
<thead>
<tr>
<th>Grade</th>
<th>Level of Cardination</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>4</td>
<td>9</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>7</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>6</td>
<td>16</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Table 8

**Numbers of Hispanic Children Assigned to Each of Three Cardination Levels**

<table>
<thead>
<tr>
<th>Grade</th>
<th>Level of Cardination</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>6</td>
<td>7</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>12</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>14</td>
<td>19</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 9

Numbers of Native American Children Assigned to Each of Three Cardination Levels

<table>
<thead>
<tr>
<th>Grade</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>1</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>Totals</td>
<td>1</td>
<td>17</td>
<td>12</td>
</tr>
</tbody>
</table>

Cardinal - Ordinal Relationship

To study the developmental relationship between ordination and cardination, a table was set up as follows:

Table 10

Numbers of Children Assigned to Each of Three Levels of Ordination and Cardination; Developmental Relationship

<table>
<thead>
<tr>
<th>Level of Ordination</th>
<th>Level of Cardination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>I</td>
<td>4</td>
</tr>
<tr>
<td>II</td>
<td>6</td>
</tr>
<tr>
<td>III</td>
<td>2</td>
</tr>
<tr>
<td>Totals</td>
<td>13 (o/c)</td>
</tr>
</tbody>
</table>

The symbols c/o indicates those children whose cardinal
understanding exceeds their ordinal understanding; o/c indicates those children whose ordinal understanding exceeds their cardinal understanding, and c=o are those children whose understanding of both concepts was acquired simultaneously.

Looking at the figures in this table, it can be seen that 49 children showed a simultaneous acquisition of the ordinal/cardinal concepts, while 41 children showed a sequential acquisition. This sequential acquisition can be further broken down into two groups: those children for whom the ordinal concept preceded the cardinal concept (13 children) and those whose understanding of the cardinal concept preceded the ordinal concept (28 children).

Statistical significance of these differences will be discussed under Hypothesis I and II. However, discussion of the 4 children whose acquisition was simultaneous at Level I is in order. Because of the arbitrary cut-off for the three levels, it is entirely possible for a child to score 0 on one of the two concepts and as much as 5 on the other, and still be classified as "Level I" for both, indicating a simultaneous acquisition that might not truly exist as the child develops. This is, however, the classification that Brainerd used, and to keep as close as possible to his work for comparison of results, use of his categories was necessary. Brainerd also indicated in his description of the levels, that Level I children showed little or no understanding of either order or manyness. Further, for these data in this study, the four children who did achieve
this classification level, their scores on the ordinal/cardinal tasks would indicate a truly indicate a simultaneous acquisition. The two Anglo children who fell into this level achieved scores of "4" (ordinal) and "3" (cardinal) and "5" (ordinal) and "5" (cardinal). For the two Hispanic children at this level, the scores were: "5" (ordinal) and "5" (cardinal) and "5" (Ordinal) and "4" (cardinal). These close scores would seem to support a true picture of a simultaneous acquisition.

Ordinal and Cardinal Relationship by Ethnic Origin

If Table 10 is reconstructed for each ethnic group (Tables 11, 12, 13) a more clear-cut pattern can be seen.

Table 11

Numbers of Anglo Children Assigned to Each of Three Levels of Ordination and Cardination: Developmental Relationship

<table>
<thead>
<tr>
<th>Level of Ordination</th>
<th>Level of Cardination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>2</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
</tr>
<tr>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>Totals</td>
<td>6 (c/o)</td>
</tr>
</tbody>
</table>

85
Nineteen Anglo children showed a simultaneous development of the two concepts, while 11 showed a sequential development: ordinal before cardinal, 6 children, and cardinal before ordinal, 5 children.

Table 12

Numbers of Hispanic Children Assigned to Each of Three Levels of Ordination and Cardination: Developmental Relationship

<table>
<thead>
<tr>
<th>Level of Ordination</th>
<th>Level of Cardination</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>I</td>
<td>2</td>
</tr>
<tr>
<td>II</td>
<td>4</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
</tr>
<tr>
<td>Totals</td>
<td>5(c/o)</td>
</tr>
</tbody>
</table>

Twenty-four Hispanic children were clearly simultaneous in their acquisition of the concepts, with only 6 showing a sequential acquisition: 5 ordinal before cardinal, and only one showing cardinal before ordinal.
Table 13

**Numbers of Native American Children Assigned to Each of Three Ordination and Cardination Levels: Developmental Relationship**

<table>
<thead>
<tr>
<th>Level of Ordination</th>
<th>Level of Cardination</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
</tr>
</tbody>
</table>

**Totals**

3(c/c)  
6(c=0)

The developmental pattern here shifted dramatically, with only six children showing a simultaneous development and 24 showing a sequential acquisition of the concepts. And again, the sequential development was clearly different from the other groups; 20 children indicated a cardinal before ordinal acquisition, with only four showing an ordinal before cardinal sequence.

**Manipulation of Natural Number**

For the manipulation of natural number task, 16 addition and 16 subtraction problems were given. Three levels of proficiency were established, using the Brainerd standard for categorization: below average, average and above average.
The following criteria were employed by Brainerd to assign subjects to the previously mentioned categories: children passing five or fewer of the addition items were assigned to the below average category, those passing more than five but fewer than 12 of the addition items were assigned to the average category, and those who passed between 12 and 16 of the items were assigned to the above average category. The same cutoff points were used in this study to assign the children to below average, average, and above average subtraction categories.

Brainerd's justification for choosing these cutoff points to determine the proficiency levels for addition and subtraction was based on interviews with principals and teachers of kindergarten and first grade students, as well as making an assessment of the arithmetic curriculum for those two grades. After making a study of the current kindergarten and first-grade mathematics texts and talking with 25 kindergarten and first-grade teachers, this researcher agreed that the standards used still seemed appropriate today, and therefore maintained them in order to replicate the study as closely as possible.

Tables 14 through 16 show the numbers of children assigned to each level of proficiency for the manipulation of number tasks; again, the entire population first, then by ethnic origin.
Table 14

Numbers of Kindergarten and First-Grade Children Assigned to Each of the Three Levels of Arithmetic Proficiency: Add

<table>
<thead>
<tr>
<th>Grade</th>
<th>Below average</th>
<th>Average</th>
<th>Above Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>32</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>10</td>
<td>29</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>38</strong></td>
<td><strong>18</strong></td>
<td><strong>34</strong></td>
</tr>
</tbody>
</table>

TABLE 15

Numbers of Kindergarten and First Grade Children Assigned to Each of Three Levels of Arithmetic Proficiency: Subtraction

<table>
<thead>
<tr>
<th>Grade</th>
<th>Below average</th>
<th>Average</th>
<th>Above Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>21</td>
<td>17</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>10</td>
<td>31</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>25</strong></td>
<td><strong>27</strong></td>
<td><strong>38</strong></td>
</tr>
</tbody>
</table>

There is clearly a developmental trend in the overall number of children assigned to the three levels of arithmetic proficiency for both addition and subtraction, with a decreasing number of kindergartners at levels I, II and III of each operation of arithmetic and an increasing number of first graders at each level for both operations of arithmetic.
If, however, the tables are broken down by ethnic origin, there are variations in the patterns stated previously. For addition, the declines and increases are less gradual; rather more sharply differentiated, with the exception of Hispanic first graders and Native American kindergartners.

**TABLE 16**

**Numbers of Kindergarten and First Grade Children, by Ethnicity, Assigned to Each of Three Levels of Arithmetic Proficiency: Addition**

<table>
<thead>
<tr>
<th>Level</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anglo:K</td>
<td>11</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Hispanic:K</td>
<td>10</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Native American:K</td>
<td>11</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>Totals</td>
<td>38</td>
<td>18</td>
<td>34</td>
</tr>
</tbody>
</table>

The tables for subtraction again seem to indicate a strong developmental trend, with decreasing numbers at each level for kindergarten children and increasing for first graders. The breakdown in this pattern occurs with Anglo first grade children whose numbers show as many children at
level II as level III.

Table 17

Subtraction:

<table>
<thead>
<tr>
<th>Level</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anglo:K</td>
<td>8</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Hispanic:K</td>
<td>8</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>Native American:K</td>
<td>5</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>Totals</td>
<td>25</td>
<td>27</td>
<td>38</td>
</tr>
</tbody>
</table>

Addition and subtraction tables by sex

If the tables are reconstructed by sex instead of grade levels, the data shifts rather dramatically.

Table 18, following, shows the numbers of boys and girls assigned to each of the three levels of arithmetic proficiency for both addition and subtraction. Table 19 redistributes this information by ethnic group for addition, with Table 20 identifying the distribution for subtraction.
Table 18
Numbers of Boys and Girls Assigned to Each of Three Levels of Arithmetic Proficiency

<table>
<thead>
<tr>
<th></th>
<th>Addition Levels I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girls</td>
<td>20</td>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>Boys</td>
<td>18</td>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>Totals</td>
<td>38</td>
<td>18</td>
<td>34</td>
</tr>
</tbody>
</table>

Subtraction

<table>
<thead>
<tr>
<th></th>
<th>Girls</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girls</td>
<td>17</td>
<td>8</td>
<td>19</td>
</tr>
<tr>
<td>Boys</td>
<td>8</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>Totals</td>
<td>25</td>
<td>27</td>
<td>38</td>
</tr>
</tbody>
</table>

With addition, boys and girls both show clusters of children at either end of the scale, with fewer children at level II, while with subtraction, this pattern holds for girls only. Boys seem to be showing an increased ability for subtraction, in a more developmental pattern, at least sufficiently so to show as high a number at level II as level III.

When the data is broken down by ethnic group, the patterns are somewhat different.
Table 19

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition Levels</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anglo:girls</td>
<td>7</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>boys</td>
<td>7</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Hispanic:girls</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>boys</td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Native American:girls</td>
<td>7</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>boys</td>
<td>6</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>38</td>
<td>18</td>
<td>34</td>
</tr>
</tbody>
</table>

The pattern for Anglo children shows larger clusters at either ends of the scale (levels I and III), while for Hispanic and Native American children, the numbers are fairly evenly distributed at each level.

For subtraction, the frequency pattern of the number of children at each level (by ethnic group) again shifts slightly as indicated by Table 20:
Table 20
Numbers of Boys and Girls Assigned to Each Level of
Arithmetic Proficiency

<table>
<thead>
<tr>
<th>Subtraction Levels</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anglo:girls boys</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Hispanic:girls boys</td>
<td>7</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>Native American:girls boys</td>
<td>5</td>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

Totals ........................................... 25  27  38

Analysis of arithmetic proficiency

Anglo boys and girls are fairly evenly distributed between the three levels of proficiency, while Hispanic and Native American boys and girls seem to follow a quite different pattern. Hispanic boys clearly follow a developmental trend of increasing proficiency, while Native American boys are evenly divided between levels II and III. Both Hispanic and Native American girls follow the prevailing pattern seen with the overall population tables, that of having larger clusters of children at both ends of the spectrum.

As with Brainerd, the youngest children tended to fall
into Level I, the older children into Level II and the oldest children into Level III. Additionally, it was found that many kindergarten children were able to perform on the subtraction tasks at about the same level that they could with addition tasks.

Relationship Between Ordination, Cardination and Arithmetic Proficiency

The developmental relationship between ordination and cardination and arithmetic performance can be seen from tables 21 through 24. For each of these tables, the following symbols were used to facilitate reading: ordination exceeding addition (o/+), ordination equal to addition (+=o), addition exceeding ordination (+/o), ordination exceeding subtraction (o/-), ordination equal to subtraction (o=-), subtraction exceeding ordination (-/o), cardination exceeding addition (c/+), cardination equal to addition (c=+), addition exceeding cardination (+/c), subtraction exceeding cardination (-/c), subtraction equal to cardination (-=c), and subtraction exceeding cardination (-/-c). Data for arithmetic proficiency exceeding either ordination or cardination levels are found in the lower left triangles; data for ordination or cardination exceeding arithmetic proficiency are found in the upper right triangles; data for simultaneous acquisitions are found on the upper left to lower right diagonals.
Table 21

Numbers of Children Assigned to Each of Three Levels of Ordination, Cardination and Arithmetic Proficiency

<table>
<thead>
<tr>
<th>Arith. Prof.</th>
<th>Ordination</th>
<th>Cardination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I  II  III</td>
<td>I  II  III</td>
</tr>
<tr>
<td>Add: I</td>
<td>18  13  7  23(+/+)</td>
<td>10  26  2  32(c+/+)</td>
</tr>
<tr>
<td></td>
<td>II 6  9  3</td>
<td>1  15  4</td>
</tr>
<tr>
<td></td>
<td>III 4  14 16</td>
<td>2  11  19</td>
</tr>
<tr>
<td>Totals</td>
<td>24(+/o) 43(o=+) 14(+/c)</td>
<td>44(c=+)</td>
</tr>
<tr>
<td>Sub: I</td>
<td>7  13  5  25(o/-)</td>
<td>11  14  1  35(c/-)</td>
</tr>
<tr>
<td></td>
<td>II 12  7  7</td>
<td>2  19  20</td>
</tr>
<tr>
<td></td>
<td>III 8  17 14</td>
<td>0  19  20</td>
</tr>
<tr>
<td>Totals</td>
<td>37(-/o) 28(=-o) 21(-/c)</td>
<td>50(c=-)</td>
</tr>
</tbody>
</table>

For ordination/addition, there are 43 children whose achievement in arithmetic has developed simultaneously with their level of ordination, while 24 children showed a higher level of arithmetic proficiency than ordination, and 23 children showed a higher level of ordination than arithmetic proficiency.

Looking at the table for the cardination/addition relationship, it can be seen that 44 children showed a simultaneous development of these concepts, while 14
children showed a greater proficiency in arithmetic than
cardination, and 32 children showed a higher level of
cardination than arithmetic proficiency. It seems clear
that there are significantly more children whose arithmetic
proficiency develops in concert with their level of
ordinal/cardinal development than those for whom the ordinal
or cardinal concepts develop in sequence. However, it
should also be noted that more than twice as many children
showed the development of the cardinal concept proceeding
the development of arithmetic proficiency in addition than
the number of children for whom arithmetic proficiency
preceeded the development of cardinality, indicating to
this researcher that an understanding of the cardinal
concept is an essential component of achievement in
addition.

The ordination/subtraction relationship yielded the
following figures: children showed a simultaneous
development of the ordinal concept and subtraction
proficiency, while 37 children indicated a stronger
understanding of subtraction than ordination. Twenty-five
children had a stronger grasp on ordination than the ability
to subtract. The relationship between cardination and
subtraction showed that 50 children had a simultaneous
understanding of cardination and their ability to subtract,
while 21 were able to subtract at a higher level than they
could cardinate, and 19 children understood cardination at a
higher level than their ability to subtract. It seems clear
that an understanding of cardination is a necessary
prerequisite to the conceptual acquisition and use of subtraction. Tables 22, 23, and 24 show the numbers of children in this ordinal/cardinal arithmetic proficiency relationship for each ethnic group.

Table 22

**Numbers of Children Assigned to Each of Three Levels of Ordination, Cardination and Arithmetic Proficiency: Anglo**

<table>
<thead>
<tr>
<th>Arith. Prof.</th>
<th>Ordination</th>
<th>Cardination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Add: I</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Totals</td>
<td>5(+/o)</td>
<td>15(=o)</td>
</tr>
<tr>
<td>Sub: I</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>II</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>III</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Totals</td>
<td>6(+/o)</td>
<td>10(=o)</td>
</tr>
</tbody>
</table>

98
### TABLE 23: Hispanic

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add:</td>
<td>3</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Totals</td>
<td>7 (+/o)</td>
<td>14 (o=)</td>
<td>8 (+/c)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub:</td>
<td>2</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>Totals</td>
<td>13 (-/o)</td>
<td>10 (o=)</td>
<td>12 (-/c)</td>
</tr>
</tbody>
</table>

### TABLE 24: Native American

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
</table>
| Add: | 9 | 1 | 3 | 4 (+/o) | 1 | 11 | 1 | 13 (c/+)
|    | 6 | 0 | 0 | 0 | 5 | 1 |   |     |
|    | 3 | 3 | 5 | 0 | 1 | 10 |   |     |
| Totals | 12 (+/o) | 14 (+=o) | 1 (+/c) | 16 (c=) |

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub:</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Totals</td>
<td>18 (-/o)</td>
<td>8 (o=)</td>
<td>3 (-/c)</td>
</tr>
</tbody>
</table>

99
Developmental Relationship between Ordination, Cardination, and Arithmetic Proficiency

To study the developmental relationship between addition and ordination, addition and cardination, subtraction and ordination, and subtraction and cardination, it is necessary to look on the diagonal from top left to bottom right in each set of figures, which gives the numbers of children whose development of the two concepts is simultaneous; that is, their ability to add or subtract is developing together with their ability to ordinate or cardinate. The trio of figures in the bottom left "triangle" shows the numbers of children whose ability to add or subtract exceeds their developmental level for ordination or cardination, and the upper right hand "triangle" of figures indicates the numbers of children for whom the level of ordination or cardination exceeds their skill in arithmetic (addition and subtraction).

For both addition and subtraction, the number of Anglo children whose level of ordination and cardination exceeded the level of arithmetic competence is about 2 to 1. There are five children whose skill with addition exceeded their level of ordination, while ten children showed a competence with the ordinal level exceeding that of their arithmetic proficiency, a ratio of 10 to 5. 15 children showed a simultaneous development of ordination and addition skills. For addition competency and cardination, 13 children showed a simultaneous development while 5 showed addition skills which exceed their level of cardination, and 12 showed a
level of cardination which exceeded their addition skills.

For subtraction, 10 children developed their arithmetic skills in coordination with the ordinal concept, while 6 had subtraction skills which exceeded their level of ordination, and 14 showed a level of ordination which exceeded their level of skill with subtraction. For the subtraction/cardination relationship, 13 children developed cardination and subtraction proficiency in a simultaneous way, while 6 showed a skill with subtraction which exceeded their level of cardination, and 11 showed a level of cardination which exceeded their proficiency with subtraction. In each relationship, addition/ordination, addition/cardination and subtraction/ordination and subtraction/cardination, 20% or fewer of the Anglo children could gain arithmetic proficiency without significant understanding of the ordinal/cardinal concepts. There is no overwhelming evidence from these data that either the ordinal or the cardinal concepts are, alone, a necessary prerequisite to arithmetic proficiency.

For Hispanic children, the pattern shifted slightly. Addition skills/level of ordination showed 14 children with a simultaneous development, while 7 indicated a level of addition competence which exceeded ordination, and 9 children showed a level of ordination which exceeded their addition skill. Addition/cardination relationship showed 15 children with a simultaneous development, while 8 showed an addition proficiency exceeding the level of cardination, and 7 reached a higher level of cardination than addition skill.
For the subtraction/ordination relationship, 10 children showed a simultaneous development while 13 showed an ability to subtract which exceeded their level of ordinal understanding, and 7 children could ordinate in excess of their ability to subtract. These figures would seem to indicate that for Hispanic children, the ordinal concept is not critical to acquiring subtraction skills. The subtraction/cardination relationship yielded the following figures: 16 children \(6 + 5 + 5\) indicated a simultaneous development of subtraction proficiency and the ability to cardinate, 12 children could subtract in excess of their ability to cardinate, and 2 children were at a level of cardination which exceeded their ability to subtract.

Again, these figures might suggest that cardination alone is not a significant influence on the acquisition of subtraction skills, but, in fact, all 12 children whose ability to subtract exceeded their level of cardination, were at level III (above average) for subtraction skills, and level II for their understanding of the cardinal concept, which might suggest a closer relationship between subtraction and cardination for these Hispanic children than between subtraction and ordination, or addition/ordination and addition/cardination.

For the Hopi children, the pattern of relationships between arithmetic proficiency and levels of ordinal and cardinal understanding changed considerably. For the addition/ordinal relationship, 14 children showed a simultaneous development of the two concepts, while 12
children showed a proficiency with addition skills exceeding their ability to ordinate, and 4 understood ordination at a level greater than their addition proficiency, again indicating that ordination seemed to have little influence on their ability to learn addition. The addition/cardination relationship, however, showed 16 children indicating a simultaneous development, while only 1 child could add at a level greater than the ability to cardinate, and 13 children showed a level of cardinal understanding which exceeded their ability to add, indicating to this researcher, again, that cardination could be a necessary prerequisite to proficiency in addition for these children.

The subtraction/ordination relationship yielded the following data: only 8 children indicated a simultaneous development, while 18 children could subtract at a level which exceeded their ability to ordinate and only 4 had ordinal skills which exceeded their ability to subtract, again giving a strong indication that ordination seemed to have little influence on the ability to subtract. Subtraction/cardination figures showed that 21 children demonstrated a simultaneous development, while only 3 could subtract at a level which exceeded their ability to cardinate and 6 showed an ability to cardinate which exceeded their ability to subtract, further supporting a close developmental relationship between cardination and subtraction skills for these Hopi children.
Conservation of Number

On the conservation of number task, seven Anglo children, six Hispanic children and eight Native American children were fully conservers of number. For Anglo and Hispanic children, the conservers were those children whose ordinal and cardinal concepts developed simultaneously as well as having equal skill with addition and subtraction. With Native American children, of eight conservers, five showed a simultaneous development of ordinality, cardinality and an equal ability to add and subtract, while three showed an understanding of cardinality which exceeded their understanding of ordinality, but also, each of these children had above average scores for both addition and subtraction skills.

Table 25

Numbers of Kindergarten and First Grade Children Who Were

Conservers of Number

<table>
<thead>
<tr>
<th>Ethnic Group</th>
<th>K</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anglo</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Hispanic</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Native American</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>

The heavy preponderence of conservers at the first grade level was, of course, expected. Also, three of the four kindergarten children were going to be six years old.
within two or three weeks of testing, while the fourth was five years six months of age.

Of the total 21 children who were conservers of number, there were 11 boys and 10 girls. By ethnic groups, these data were as follows:

Table 26

Numbers of Girls and Boys, by Ethnic Group, Who Were Conservers of Number:

<table>
<thead>
<tr>
<th>Ethnic Group</th>
<th>Girls</th>
<th>Boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anglo</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Hispanic</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Native American</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>10</strong></td>
<td><strong>11</strong></td>
</tr>
</tbody>
</table>

Statistical Treatment

This study required the use of three statistical instruments to determine whether each dependent variable differed significantly under the conditions of ethnicity, sex or simultaneous or sequential acquisition of the ordinal and cardinal concepts.

Each of the four hypotheses were stated in a manner suggesting the predication of an anticipated outcome. Hypotheses one and two were tested for significant differences using a chi square distribution. Significant differences for hypothesis three were tested using a t-test, with simultaneous and sequential acquisition of the ordinal
and cardinal concepts as the independent variables. Hypotheses four and five were tested for significant difference using a 2 x 2 chi square test of independence for a two sample case (boys vs. girls).

Testing the hypotheses

Hypothesis one. This hypothesis was concerned with the simultaneous or sequential acquisition of the ordinal and cardinal concepts by Anglo and Hispanic children. The raw data for both groups were subjected to a chi square test of independence to determine whether or not significant differences did exist between those children who acquired the concepts simultaneously and those who acquired them sequentially. Table 27 shows the data for hypothesis one for Anglo children, and Table 28 for Hispanic children:

Table 27
Chi Square with df for Responses by Acquisition: Anglo

<table>
<thead>
<tr>
<th>Acquisition</th>
<th>O</th>
<th>E</th>
<th>O-E</th>
<th>(O-E)^2</th>
<th>(O-E)^2</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seq:0</td>
<td>6</td>
<td>10</td>
<td>-4</td>
<td>16</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>Seq.c</td>
<td>5</td>
<td>10</td>
<td>-5</td>
<td>25</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Sim.</td>
<td>19</td>
<td>10</td>
<td>+9</td>
<td>81</td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>30</td>
<td>30</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \chi^2 = 12.2 \]

\[ \alpha = .05 \]  df: 2  cv(5.991)
Table 28

Chi Square with df for Responses by Acquisition: Hispanic

<table>
<thead>
<tr>
<th>Acquisition</th>
<th>0</th>
<th>E</th>
<th>O-E</th>
<th>((O-E)^2)</th>
<th>((O-E)^2) E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seq:o</td>
<td>5</td>
<td>10</td>
<td>-5</td>
<td>25</td>
<td>2.5</td>
</tr>
<tr>
<td>Seq:c</td>
<td>1</td>
<td>10</td>
<td>-9</td>
<td>81</td>
<td>8.1</td>
</tr>
<tr>
<td>Sim.</td>
<td>24</td>
<td>10</td>
<td>14</td>
<td>196</td>
<td>19.6</td>
</tr>
</tbody>
</table>

Totals: 30 30 0 \(\chi^2=30.2\)

\(\alpha=.05\) df=2  \(cv=5.991\)

Hypothesis one stated, in the null form, that given a task of transitive-asymmetrical relations, and a task of comparison of manyness of two collections of objects, kindergarten and first grade Anglo and Hispanic children would show no difference in the simultaneous or sequential acquisition of the ordinal and cardinal concepts. In both cases, with Anglo children and Hispanic children, the difference was significant, with the probability less than 5% that these figures could have been caused by chance. The null hypothesis, that Anglo and Hispanic children showed no difference in the acquisition of the concepts of ordinality and cardinality, was rejected in favor of the research hypothesis.

**Hypothesis two.** This was concerned with the simultaneous or sequential acquisition of the ordinal and cardinal concepts of Native American children. Again, the
raw data were subjected to a chi square test of independence to determine any significant differences between such acquisition. Table 28 shows the chi square determination for the data on Hopi children.

Table 29

<table>
<thead>
<tr>
<th>Acquisition</th>
<th>0</th>
<th>E</th>
<th>0-E</th>
<th>(0-E)^2</th>
<th>(0-E)^2 / E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seq:o</td>
<td>3</td>
<td>10</td>
<td>-7</td>
<td>49</td>
<td>4.9</td>
</tr>
<tr>
<td>Seq:c</td>
<td>21</td>
<td>10</td>
<td>+11</td>
<td>121</td>
<td>12.1</td>
</tr>
<tr>
<td>Sim.</td>
<td>6</td>
<td>10</td>
<td>-4</td>
<td>16</td>
<td>1.6</td>
</tr>
<tr>
<td>Totals</td>
<td>30</td>
<td>30</td>
<td>0</td>
<td></td>
<td>( \chi^2 = 18.6 )</td>
</tr>
</tbody>
</table>

Hypothesis two stated, in null form, that given the same tasks of transitive-asymmetrical relationships and comparison of manyness, Native American children would show no difference in their acquisition of the ordinal and cardinal concepts. Again, the difference was significant, indicating that Native American children probably do acquire the two concepts sequentially, with the cardinal concept developing before the ordinal concept. The probability that these numbers could have occurred by chance was, again, less that 5%. Hypothesis two was also rejected in favor of the research hypothesis.
Hypothesis three. This hypothesis was concerned with the relationship between a simultaneous or sequential acquisition of the two concepts and arithmetic achievement. The addition and subtraction scores of those children indicating a simultaneous development of ordination/cardination were compared with the addition scores showing a sequential development of ordination/cardination by means of a t-test. Descriptive statistics for the two groups follow:

Table 30

Descriptive Statistics for Arithmetic Achievement of Two Groups: Simultaneous and Sequential Acquisition of Ordinality/Cardinality

<table>
<thead>
<tr>
<th>Acquisition</th>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simultaneous</td>
<td>$\bar{X}=9.94$</td>
<td>$\bar{X}=10.71$</td>
</tr>
<tr>
<td>Sequential</td>
<td>$\bar{X}=6.22$</td>
<td>$\bar{X}=8.80$</td>
</tr>
</tbody>
</table>

These combined data for all groups were subjected to a t-test for independent samples in order to determine whether a significant difference existed between these two groups, thus establishing or failing to establish a relationship between the arithmetic proficiency of those children who showed a simultaneous acquisition versus those children who indicated a sequential acquisition of the ordinal/cardinal concepts.

In null form, hypothesis three stated: given a test of
addition and subtraction of the first four natural numbers, there would be no difference in the skills level attained in addition and subtraction between those children acquiring the concepts simultaneously or sequentially, either ordinal before cardinal or cardinal before ordinal. This hypothesis was split in acceptance and rejection. Using a t-test for independent samples, the t value for addition scores (simultaneous vs. sequential) was 2.70. Critical value fell between 2.000 for df=60 and 1.980 for df=120. Df for this statistical test were 88, and not listed in the table of critical values. The t value of 2.70, however, exceeded either of these critical values. The probability that this figure could have occurred by chance was less than 5%. Therefore, for addition scores, the null hypothesis was rejected in favor of the research hypothesis.

For subtraction scores, the t value was 1.876. Again, the critical value for df=60 was 2.000 and for df=120, 1.980; therefore the critical value of df=88 would lie somewhere between these two. The t value 1.876 did not exceed the critical value for df=120, indicating no significant difference in subtraction scores between the children whose acquisition of the two concepts was simultaneous or sequential. Therefore, the second half of the null hypothesis three was retained.

Hypothesis four. This hypothesis was concerned with those children, if any, who showed a sequential development in their acquisition of the ordinal/cardinal concepts, and whether boys or girls showed a stronger tendency to acquire
one concept before the other. In this study, only Native American children, as a group, showed the developmental acquisition of cardinality before ordination; therefore only data from the Native American population were subjected to statistical treatment.

The hypothesis was first restated in the null form: there is no difference in the sequential acquisition of ordinality or cardinality between boys and girls. A chi square test of independence for a two sample case was then computed. The 2 x 2 table showed the following observed frequencies for Native American children (expected frequencies in parentheses):

Table 31

**Observed Frequencies for Native American Children by Sex and Concepts Acquisition: ("Expected" Frequencies in Parentheses)**

<table>
<thead>
<tr>
<th>Sex</th>
<th>Seq:Card.</th>
<th>Others</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>13(10.5)</td>
<td>2(4.5)</td>
<td>15</td>
</tr>
<tr>
<td>Female</td>
<td>8(10.5)</td>
<td>7(4.5)</td>
<td>15</td>
</tr>
</tbody>
</table>

Totals 21 9 30
The chi square computation yielded the following data:

Table 32

Chi Square to Determine Significant Difference Between Boys and Girls and the Sequential Cardinal Acquisition of Native American Children

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>O-E</th>
<th>(O-E)^2</th>
<th>(O-E)^2/E</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>10.5</td>
<td>2.5</td>
<td>6.25</td>
<td>0.5952</td>
</tr>
<tr>
<td>8</td>
<td>10.5</td>
<td>-2.5</td>
<td>6.25</td>
<td>0.5952</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
<td>-2.5</td>
<td>6.25</td>
<td>1.3888</td>
</tr>
<tr>
<td>7</td>
<td>4.5</td>
<td>2.5</td>
<td>6.25</td>
<td>1.3888</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>30</th>
<th>30</th>
<th>0</th>
<th>λ = 3.9690</th>
</tr>
</thead>
</table>

α = 0.05 df = 1 p < 0.05 cv = 3.841

The null hypothesis stated that there would be no difference in the sequential acquisition of boys and girls. The chi square exceeded the critical value, indicating that there was a difference in this acquisition between boys and girls. The chance that such numbers would occur by chance is less than 5%. Therefore the null hypothesis was rejected in favor of the research hypothesis; there is a difference between boys and girls in the acquisition of the cardinal—before-ordinal sequence in favor of the boys.

**Hypothesis five.** This hypothesis was concerned with the skills level of subtraction of the Native American children whose acquisition of the cardinal concept preceded
their acquisition of the ordinal concept. It was hypothesized in the research that these Native American children whose cardinal understanding exceeded their ordinal understanding would achieve a higher level of subtraction than addition.

Stated in the null form, for testing, this hypothesis read that there is no difference in the addition and subtraction skill level of these Native American children whose understanding of cardination exceeded ordination. Table 33 shows the frequencies for Native American children by concept acquisition and skills development.

Table 33

<table>
<thead>
<tr>
<th>Skills</th>
<th>Seq: card</th>
<th>Others</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>-/+</td>
<td>13(10.9)</td>
<td>2(4.1)</td>
<td>15</td>
</tr>
<tr>
<td>+=-</td>
<td>8(10.1)</td>
<td>6(3.9)</td>
<td>14</td>
</tr>
<tr>
<td>Totals</td>
<td>21</td>
<td>8</td>
<td>29</td>
</tr>
</tbody>
</table>

The chi square computation for the above data yielded the following results:
Table 34

Chi Square to Determine Significant Difference Between Addition and Subtraction levels of Native American Children

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>O-E</th>
<th>((O-E)^2)</th>
<th>(E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>10.9</td>
<td>2.1</td>
<td>4.41</td>
<td>0.4046</td>
</tr>
<tr>
<td>8</td>
<td>10.1</td>
<td>-2.1</td>
<td>4.41</td>
<td>0.4366</td>
</tr>
<tr>
<td>2</td>
<td>4.1</td>
<td>-2.1</td>
<td>4.41</td>
<td>1.0756</td>
</tr>
<tr>
<td>6</td>
<td>3.9</td>
<td>2.1</td>
<td>4.41</td>
<td>1.1308</td>
</tr>
</tbody>
</table>

\(\chi^2 = 3.0476\)
\(\infty = .05\) \(\text{df}=1\) \(p<.05\) \(cv=3.841\)

The chi square approached but did not exceed the critical value. Therefore the null hypothesis was retained; there is no significant difference between the addition and subtraction scores of Native American children whose concept acquisition was cardinal-before-ordinal.

Table 35, following, summarizes the data on the acquisition of the ordinal and cardinal concepts for all ethnic groups and clearly shows the differences for the entire population.
Table 35
Summary of Data on Acquisition of Concepts of Ordinality and Cardinality by Anglo, Hispanic and Native American Children

<table>
<thead>
<tr>
<th>Concept Acquisition</th>
<th>Numbers of Children by Ethnic Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Anglo</td>
</tr>
<tr>
<td>Simultaneous</td>
<td>19</td>
</tr>
<tr>
<td>Ordinal &gt; Cardinal</td>
<td>6</td>
</tr>
<tr>
<td>Cardinal &gt; Ordinal</td>
<td>5</td>
</tr>
<tr>
<td>Totals</td>
<td>30</td>
</tr>
</tbody>
</table>

These differences in approach to basic understanding of numbers in no way reflects inferior or superior thinking abilities in mathematics, only another way of thinking about number which might influence whether one is able to add or subtract with more skill. Much more study needs to be done before any definitive conclusions can be reached on how an ordinal or cardinal approach might determine skills in arithmetic. Native American children with a solid conceptual background in mathematics from the elementary years, and who study higher mathematics in high school, seem to be as successful as those from the majority culture (Hon'youli, personal communication, 1986).

Conclusions

The conclusions of the study indicated that Anglo and Hispanic children did acquire the concepts of of ordinality and cardinality simultaneously, and that Native American
children acquired the two concepts sequentially, with the cardinal concept developing first. This study also indicated that those children who acquired the two concepts simultaneously achieved a higher level of addition than those children who acquired the concepts sequentially, ordinal before cardinal or cardinal before ordinal. For Native American children, who showed a sequential acquisition of the two concepts with cardinality appearing first, there was a significant difference in the boys' and girls' acquisition of cardinality in favor of the boys. These children, however, did not achieve a significantly higher level of subtraction skills as was predicted.

Chapter V will present all of the findings of the study and give implications and recommendations.
CHAPTER V

FINDINGS, IMPLICATIONS, AND RECOMMENDATIONS

Summary

The purpose of this study was to investigate the acquisition of the ordinal and cardinal concepts of number by kindergarten and first grade Anglo, Hispanic and Native American children, and the effects of this acquisition on these children's ability to add and subtract the first four natural numbers. The rationale was that children's natural acquisition of these concepts should influence the approach taken by teachers and mathematics curricula in order to maximize the mathematical understanding and learning of all children in the school setting. Current curricula reflects only the concepts, and sequence of learning those concepts as predetermined and understood by adult mathematicians from the mainstream culture. Few enough of the students from the mainstream culture are achieving significantly in mathematics classes nationwide, the number of minorities (with the exception of Asian immigrants) is very small.

It was hypothesized that, using the child's native language:

(1) Anglo and Hispanic children would acquire the ordinal and cardinal concepts of number simultaneously in a developmental sequence, as determined by Piaget;

(2) Native American children would acquire the two concepts sequentially, with the cardinal concept developing before the ordinal concept;

(3) those children developing the two concepts
simultaneously would achieve a higher level of skill on
addition and subtraction of the first four natural
numbers than those acquiring the concepts sequentially;
(4) if a sequential development was seen in most
children, that those children acquiring the cardinal
concept before the ordinal concept would be boys;
(5) these boys would show a higher level of skill with
subtraction than addition.

The review of the literature included reviews of a) the
historical perspective of ordinality and acrdinality; b)
Piaget's theory of the ordinal/cardinal development; c)
Brainerd's theory of the ordinal/cardinal development; d)
studies of the development of the ordinal/cardinal concepts
by researchers other than Piaget and Brainerd; e) studies of
cordination only; f) studies of equivalence and order
relations; f) counting as a critical aspect of the
development of both the ordinal and cardinal concepts; g)
cross cultural studies of the acquisition of number
concepts, and; h) the impact of brain research on the
understanding of the acquisition of number concepts. The
literature indicates that few studies have been done, beyond
Piaget and Brainerd, to determine more definitively either
the sequential or simultaneous acquisition of the two
concepts in the mainstream culture, with even more limited
work having been done across cultures. The literature also
indicates that researchers are not satisfied with the kinds
of tasks so far devised to test the young child's
understanding of the two concepts, nor the impact of the use
of the child's native language for testing as opposed to the language used in school. Language and culture are considered to be closely intertwined and at the very core of human interpretation of reality - one's "world view", in Whorfian terms - and it seems only natural that the native language would and should color the child's understanding and use of number concepts.

Ninety kindergarten and first grade children participated in this study; thirty Anglo, thirty Hispanic and thirty Native American five, six and seven year olds. There were 15 girls and 15 boys per group, except for the Native American children where the ratio was 16 boys and 14 girls. The children were all from rural areas, two in New Mexico and one in Arizona. Each child was tested in his or her dominant language by native teachers. The Anglo and Hispanic children were enrolled in their local public schools, the Native American children were from a small private school on the Reservation as well as the community contract school.

Tasks used to determine ordinal and cardinal understanding, as well as the addition and subtraction of the first four natural numbers were taken directly from the original Brainerd study, with a modification of the cardinal task. For this task, Brainerd used rows of red and blue dots pasted to the task cards; these were changed to three-dimensional wooden cubes, red and blue, glued to the task cards. Numbers of cubes per row, as well as spacing between the cubes was replicated from the Brainerd cards.
Data were gathered by this researcher directly (all English speaking children were tested by this researcher), or by the native language speaker, in each case the child’s regular teacher with this researcher observing. These data were compared using a chi square for hypotheses one and two, a t-test for independent samples for hypothesis three and a 2x2 chi square test of independence for hypotheses four and five.

Findings

The data from the study support the following findings:

1. Anglo and Hispanic children significantly acquired the concepts of ordinality and cardinality simultaneously, whereas the Native American children acquired the two concepts in a sequential way, with cardinality acquired before ordinality.

2. The mean scores for addition of the first four natural numbers were significantly higher for those children who acquired the concepts simultaneously than those children who acquired the two concepts sequentially. As opposed to the research prediction, there was no significant difference for subtraction scores.

3. For the population who did who did show a sequential, cardinal-before-ordinal, development (Native American), there were significantly more boys and girls who acquired the concepts in the cardinal-before-ordinal sequence.

4. Within this same Native American population, there
was no significant difference in the ability to add and subtract.

Discussion of the Hypotheses

In hypothesis one, 60% the Anglo and 80% of the Hispanic population acquired the concepts of ordinality and cardinality simultaneously in accordance with Piaget's work on the acquisition of the two concepts.

This was in direct contrast with the original study, where Brainerd showed only 25% of his population achieving a simultaneous level of development for the two concepts, while 71% achieved ordinality before cardinality and only 4% achieved cardinality before ordinality, indicating an overwhelming majority in the ordinal-before-cardinal classification. Such a drastic difference in the acquisition of the two concepts would seem to indicate that the original cardinal task might well have been more difficult than the ordinal one, especially since the pilot work for this current study, replicating the Brainerd tasks exactly, yielded results closely comparable to those of Brainerd.

In hypothesis two, 70% of the Hopi children acquired the concepts sequentially, with cardination appearing before ordination. This was, again, in direct contrast with Brainerd's findings, where the significant numbers were in the group who acquired ordinality before cardinality. With these Hopi population, it seems likely that both the change in cardinal task as well as the influence of the native language on cognitive understanding could be the major
factors.

In hypothesis three, the skills level of those children whose acquisition was simultaneous was significantly higher for addition than those children who acquired the concepts sequentially, either ordinal before cardinal or cardinal before ordinal. The level of subtraction skills for those with a simultaneous acquisition as opposed to a sequential acquisition was not significant. It is interesting to note, however, that when comparing the level of ordination with the level of addition skills, 27% of the children could add on a level greater than their ability to ordinate, whereas comparing addition skills levels with cardination, only 16% of the children could add on a level greater than their ability to cardinate. Comparing subtraction levels with ordination, 41% of the children could subtract on a higher level than they could ordinate, while only 23% could subtract on a level greater than their ability to cardinate, and 21% had a cardination level which exceeded their level of subtraction skills. It seems that ordination has less to do with the ability to add and subtract than does cardination. For each comparison (ordinal/addition and subtraction and cardinal/addition and subtraction) about 50% of the total population maintained a skills level comparable to that of their concept level.

Hypothesis four did show a significant difference between boys and girls within the population acquiring cardination before ordination (Native American), but that significance only slightly exceeded the critical value,
indicating, perhaps, a stronger influence of language and culture on the understanding of number than sex difference.

Hypothesis five indicated that there was no significant difference between the ability of these Native American children in their ability to add and their ability to subtract the first four natural numbers. However, the chi squares for both hypothesis four and five were closely clustered around the critical value, one just above (hypothesis four) and one just below (hypothesis five). A larger population might cause these figures to go either direction.

Conclusions

On the basis of the data found in this study, the following conclusions are possible:

1. Most children acquire the two basic concepts of number simultaneously as stated by Piaget.

2. Children whose acquisition of the two number concepts is simultaneous rather than sequential achieve higher levels of arithmetic skills in addition than children whose acquisition is sequential. Again, the t value closely approached the critical value, and perhaps a larger population would indicate significance.

3. Because only the Hopi population had a significant number of children whose acquisition of the two concepts was sequential, with cardinal appearing before ordinal, it can be assumed that the native language and/or cultural practices and culturally patterned experiences have an influence on the understanding of basic concepts.
4. Boys may have a cardinal/subtraction advantage.

Other Findings

Some discoveries which were not directly related to the hypotheses offer interesting insights into the acquisition of basic number concepts and the skills of arithmetic. These incidental occurrences were consistent and meaningful when viewed in context with current research into how children acquire and make use of their numeracy skills. As with many other studies, these other discoveries may prove to be just as valuable as the information yielded by the hypotheses. Of first consideration is the ability to conserve number.

Conservation

Out of a total population of 90 children, 21 were able to conserve number, 7 Anglo children, 6 Hispanics and 8 Native Americans, 11 boys and 10 girls. Of these 21 children, 18 showed a simultaneous acquisition of ordinality and cardinality. The remaining 3 children showed a sequential development with cardinal appearing before ordinal. It is interesting to note that none of these conservers had acquired the ordinal concept before the cardinal one. Eighteen of these 21 conservers were at level III of both concepts and skills of arithmetic, addition and subtraction. Of the remaining three, two were at level II of concepts and level III of arithmetic skills, and the third was at level III for concepts and level II for arithmetic skills. The mean score for addition for the 21 conservers was 14.0 as opposed to 8.01 for the remaining 69
non-conservers; for subtraction, the mean score for conservers was 14.7 as opposed to 10.21 for the non-conservers. In each case, the difference was highly significant at the .05 level, indicating that the ability to conserve number seems to be a critical factor in achieving the skills of arithmetic, addition and subtraction.

Hypotheses four and five also take on a different aspect if all children who showed a sequential development of the ordinal and cardinal concepts are considered in the statistical analysis, rather than just the Native American population. In hypothesis four, as predicted, there continues to be a significant difference between boys and girls, in favor of the boys, for those acquiring the cardinal concept before the ordinal one. And further, as predicted in hypothesis five, these boys did achieve a significantly higher skills level in subtraction than addition. A study using a larger total population, or a larger Native American population might support these differences. Testing hypothesis five as originally planned, using only that population which showed a significant sequential acquisition, the chi square closely approached but did not exceed the critical value.

Brain organization

Much of the logic of these deductions find support in the work of Davidson (University of Massachusetts Harbor Campus and the Learning Disabilities Clinic of Boston Childrens' Hospital) which has shown a strong connection between right and left hemisphere dominance and the ability
to add and subtract. Her work has indicated that those children with a stronger right hemisphere tend to be able to subtract and divide better than they can add or multiply, while those children with a stronger left hemisphere, add and multiply better than they can subtract or divide. And this ability to add/multiply or subtract/divide is closely tied to the more favored ability of counting forwards (add/multiply) or backwards (subtract/divide): for those children starting with the parts and proceeding to the whole (+, x), or for those children starting from the whole and proceeding to the parts (−, ÷). An understanding of the ordinal concept seems to require consideration of one part at a time, then the entire group, as does addition (and multiplication) which also requires going from parts (addends) to the whole (sum). Understanding the cardinal concept involves taking the entire group and making a judgement about a successful one-to-one matching of the elements to make a determination of more, less or the same number. Use of visual cues (density, length of lines etc.) clearly involve a consideration of the whole over each individual part, and for those children who are more successful with subtraction, this consideration of whole-to-part would seem to play an important role.

**Language**

While a detailed analysis of the effects of language on the formation of concepts is not within the scope of this dissertation, consideration of the role of language on the organization of the brain and its subsequent effect on the
development of concepts in general and number in particular cannot be ignored. The very large number of Hispanic children (80%) who showed a balanced, simultaneous acquisition of ordinality and cardinality was not expected. A figure more closely allied with the Anglo population (63%) was the expectation. Such an overwhelming percentage in the Hispanic population would seem to indicate a strong influence of Spanish language on these children's developing view of number. In actual numbers, 80% of the Hispanic population was 24 children out of 30. Of those Hispanic children (6) who acquired the concepts in a sequential fashion, 5 showed the ordinal/cardinal sequence with 4 of these being girls. Only one child (boy) showed the cardinal/ordinal sequence.

The effect of language on the Native American population also must be considered here. With this group, while 80% of the children showed a like development, this development was sequential (24 out of 30), with 21 showing cardinal before ordinal and 3 showing ordinal before cardinal. Breaking down these figures by sex, of the 21 whose development was cardinal before ordinal, 14 were boys and 7 were girls, whereas in the ordinal/cardinal sequence, 2 of the 3 children were girls. While statistical significance was achieved in a comparison of males/females and not with addition/subtraction skills level within the cardinal/ordinal Native American population (hypotheses four and five), the figures indicate serious consideration and further study, especially in context of the work on
mathematics and the brain done by Davidson and her neurobiological model for intellectual functioning.

**Kindergarten children**

Another finding of this study was that most kindergarten children in all three ethnic groups were able to subtract at a level closely commensurate with their ability to add, and out of 45 kindergarten children, a total of 15, or 1/3 of them, could subtract at a higher level than they could add. No direct comparison can be made with Brainerd’s study for these scores, as he did not test kindergarten children on subtraction, feeling that the subtraction process was a learned process at the first grade level, and would not be within the scope of their development. This study found, however, that these younger children had virtually no problem with the subtraction process, and that what limited their ability in any way was the magnitude of the number(s) with which they were working.

**Arithmetic achievement**

Again, looking at the arithmetic achievement of children as measured by the Brainerd levels, only 9 out of 90 (10%) reached an addition level which exceeded that of subtraction, while 29 out of 90, or 32%, attained a subtraction level which exceeded that of addition. The greatest proportion of the children, however, (58%) showed identical levels of achievement in addition and subtraction. Broken down by sex, 4 boys and 5 girls comprised the addition-over-subtraction category, 19 boys and 10 girls composed the subtraction-over-addition category, and 23 boys
and 29 girls made up the addition-equal-to-subtraction group. These numbers would seem to indicate support for much of Baroody's work on natural number concepts in pre-schoolers; the rudiments of understanding number and its uses are in place. Children make use of them in practical ways, daily in play and interactions with peers.

Further findings in the area of arithmetic were careful observations of how the children arrived at their "answers". For the arithmetic tasks, the children were told that they could just say the answer, write the answer, or both. They were also told they could count, use their fingers, or either of the manipulative materials available to them (Cuisenaire rods and counting chips). Of the entire population of ninety children, only seven used materials; two used Cuisenaire rods, and five used the chips. None used paper and pencil, either to write the answer or to make counting marks. Twenty-three used their fingers, with 58 of the remaining children just, somehow, working mentally. Two children deserve special mention for the unusual way in which they solved the arithmetic problems. These two were Native American children, a boy and a girl, both first graders who had been using Cuisenaire rods for several years in a play/building situation as well as for arithmetic. They did not, however, use the rods during testing. Each "estimated" the length of the rod between thumb and forefinger and gave their answer from an estimated length. Not only was this an unusual technique to use in the first place, but as each child was tested separately and in a
private room, others were not able to see or hear what the child being tested was doing. In this case, also, these two children followed one another in the testing order, so that discussion was not a possibility either. This was the first such "use" of rods, or length as a criteria, for problem solving this writer had seen in over 28 years of teaching and observing children with and without materials! Whether this approach was precipitated by previous familiarity with rods, or whether knowing how to solve addition and subtraction problems with rods reinforced a length approach to number already in place would be hard to determine. Both children quite accurately approximated the length of the rods associated with the numbers they were working with; the girl scored a perfect 16 on both addition and subtraction, the boy scored 15 on addition and a perfect 16 on subtraction. Neither of the children had been to Head Start.

**Counting and thinking strategies**

Both the finger counters and the "thinkers" also merit some discussion about observed techniques. Of the 23 finger counters, five seemed to use their fingers in an ordinal way: extending one finger at a time and counting until both addends had been extended and counted. These children also counted each addend separately:

```
  one  two  three     one  two  three  four
```

![Finger gesture images](image-url)

130
and then counted again from the beginning:

one two three four five six seven

Baroody (1987) refers to this approach as the "concrete counting all" method, and states that it is a basic method usually figured out and used by most pre-school children without direct teaching.

For subtraction, these children would again extend one finger at a time until the minuend was reached, then fold down one finger at a time until the number to be subtracted was counted out, then returned to counting by ones to determine "what was left". One child exhibited an unusual, if faulty, approach to subtraction with fingers. She would exhibit the minuend on one hand and the subtrahend on the other, and count the fingers that were left unused on the "subtrahend" hand. This technique worked fine for any problem which began "5 - ? = "; for any other combination, it was flawed.

The remaining 18 who used their fingers, when needing "4", extended four fingers at a time, then the correct number of fingers for the second addend, and either counted on from the first to reach a total, or subitized the answer. For subtraction, the technique was used in reverse: they extended the correct number of fingers either all at once (if the minuend was "5" or less) or as "5" + the number of
fingers from the second hand which would give the correct
minuend, next fold down all at once the number of fingers
denoting the subtrahend, then subitized the "answer". This
technique (of immediately showing two fingers for "2" or
four fingers for "4" etc.) seemed to indicate a more
cardinal understanding and approach to number. Four of the
five "ordinal" counters were Kindergarteners, while of the
18 cardinal counters, 5 were Kindergarteners and 13 were
first graders. All five "ordinal" counters had achieved an
ordinal understanding of number before a cardinal one,
whereas the "cardinal" counters were evenly divided between
simultaneous and cardinal/ordinal development of the two
concepts. It should be noted, however, that the
so-designated "ordinal" counters had the "cardinal
understanding" as described by Gelman; that the last named
number is the cardinal value of the set. One child showed
only an understanding of order, or the "ordinal" approach to
number: because the first problem presented to her was 1 + 1
=? (for which she correctly answered "2"), she subsequently
answered "3", "4", "5", "6", keeping the count word sequence
going correctly without any obvious understanding of the
operations of addition or subtraction.

Those who just "thought out" their answers also
presented some interesting findings. With these children,
it would be virtually impossible to state with any certainty
how much mental counting was occurring, but presumably there
was some, at least with the 3+4, 4+4, 8-4, 7-3 level of
problems. However, those referred to as "thinkers" showed
no overt counting procedures, subvocalization, or kinesthetic movement of the head or hands. The youngest children exhibited the tendency to rapidly guess when the numbers went beyond 1+1, 1+2, 1+3 etc. although almost all children knew the doubles (1+1, 2+2, 3+3, 4+4), supporting Ginsberg’s work. Some of the younger children also exhibited a phenomena never observed before: when the first card of addition or subtraction was presented, the tester would ask the question "How much is one apple and two apples?" or "If you had two apples and gave 1 to your best friend, how many would you have left?", he/she would simultaneously show the correct number of fingers for the addends, the minuend and the subtrahend, to indicate to the child that this kind of manipulation was quite alright. It would seem that the child would eagerly use these cues and clues to get started. Mostly, however, these children just gazed off into space, then responded, as if to indicate they had their own way of determining the outcome and for the adult not to interfere with that "way". Some of these kinds of "gazers" did not have the correct response, while again, some did.

These findings, on techniques for solving simple arithmetic problems, reinforce the research that young children differ greatly in their approach to using number, although there is strong support for a somewhat universal and developmental approach to counting. However, trying to funnel them all down the same pathway, as is the trend in standardized curricula and texts, could ignore some innate
strengths already in place, and possibly even distort or
damage the intuitive, "spontaneous" concepts which form the
basis of the later refined "scientific" concepts as
described by Vygotsky. Suina (personal communication, 1984)
has indicated that for many Native American children
beginning the study of mathematics, the more correct verb
would be "destroy" rather than "distort" or "damage".

Findings on Ordinal/Cardinal Tasks

Overall picture

A close scrutiny of responses to the ordinal and
cardinal tasks also produced some pertinent findings. While
the statistical analyses yielded some significant
differences in the overall trends of the three ethnic
groups, a look at individual responses showed some trends
and raised some questions that deserve consideration. The
following table shows the number of children, by sex who
acquired the two concepts either sequentially or
simultaneously:

Table 36:

Acquisition of Ordinal/Cardinal Concepts by Sex

<table>
<thead>
<tr>
<th>Sex</th>
<th>Sequential:Ord.</th>
<th>Sequential:Card.</th>
<th>Simultaneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>3</td>
<td>18</td>
<td>25</td>
</tr>
<tr>
<td>Female</td>
<td>11</td>
<td>9</td>
<td>24</td>
</tr>
<tr>
<td>Totals</td>
<td>14</td>
<td>27</td>
<td>49</td>
</tr>
</tbody>
</table>

Almost four times as many girls showed the
ordinal-before-cardinal acquisition. Only three boys, one from each ethnic group, acquired the ordinal concept before the cardinal concept, while 11 girls showed this acquisition; five Anglos, 4 Hispanics and 2 Native Americans. Looking at the sequential, cardinal before ordinal, acquisition, twice as many boys indicated this acquisition as girls. However, again, broken down by ethnic group, 14 Hopi, one Hispanic and 3 Anglo boys made up this group. The simultaneous acquisition is quite balanced in the Anglo and Hispanic populations (11 boys and 9 girls from the Anglo group and 13 boys and 11 girls from the Hispanic group). Only the Hopi population showed considerably fewer numbers: one boy and 5 girls.

Looking at these raw data gives indications that there does seem to be differences between boys and girls and the way they acquire the two concepts. Again, there is support for this assumption in the work of Davidson as described previously. The ordinal concept is much more involved with a linear, sequential, step-by-step, parts-to-whole approach to number, while the cardinal concept is more concerned with the ability to work from the whole to the parts. Parts to whole requires left hemisphere strengths while whole to parts, the right hemisphere, and it is more often the girls who show these left hemisphere strengths while boys show strengths from the right hemisphere.

The overall numbers themselves also present an interesting picture; while 49 children showed a simultaneous acquisition, 41 showed a sequential one, and if viewed in
the light of just these two categories, there is no
significant difference in statistical analysis. Considered
from this perspective, it could be said that the population
is about equal in its acquisition of the two concepts;
simultaneous or sequential. It would, however, be deceptive
to view the acquisition in such a way. The ordinal approach
differs sufficiently from the cardinal approach that lumping
them together negates or reduces the importance of the
different orientation required by the brain in determining a
transitive asymmetric relationship and one of manyness.
Viewed from such a two-category perspective also negates the
differences seen in the three ethnic populations,
differences which would seem to be of importance for
beginning learners in the classroom.

**Ordinal tasks**

Looking at the responses on the ordinal tasks, length
and weight, also yielded some pertinent information. There
were a total of 57.5% correct responses given on the ordinal
tasks of length, while a total of 71% correct responses were
scored on the ordinal task of weight. This variation could
be the result of differences in materials used for these
tasks. The weight task was completed by direct comparison
(holding the appropriate balls of clay), whereas the length
task, while permitting the child to touch the sticks, did
not involve the child as directly as did the weight task.
Previous research supports the need for young children to be
directly and actively involved with materials: the actual
manipulation of whatever they are working with.
Of the total population, 25 children (or 28% of the total population) scored at level III of ordination, a raw score of 11 or 12 on the tasks. Of these 25, 11 were Anglos, 6 Hispanic and 8 Native Americans. The emerging trend would seem to indicate that Anglo children understand ordinality at twice the rate of the other two ethnic groups.

**Cardinal tasks**

On the cardinal task (see Appendix C for card configurations), responses for each card yielded valuable information. On card A (more red cubes), 88% of the population achieved a correct response, with density being the appropriate clue used. On card B (more blue cubes), 83% of the children were correct, again with density being the correct appropriate cue used. On card C (same number of red cubes as blue cubes, but blue cubes clustered close together), only 50% of the children achieved a correct response. Of those children who achieved an incorrect response, length was the over-riding perception. Card D (more blue cubes than red ones, again with blues clustered close together), 51% of the children achieved a correct response; here again length was the over-riding perceptual feature causing error. On card E, only 29% of the population achieved a correct response. This was the second card where the number of red cubes equaled the number of blue cubes but the reds were clustered close together and the blues were spaced out. Two-thirds (66%) of the children were beguiled by perception and chose the blue cubes as more. The final card, card F (more red cubes than blue
cubes, but the red were clustered close together and the blue cubes spread out) showed 57% of the children achieving a correct response, with 42% being seduced by the length of the blue line of cubes.

On the four cards where appreciable errors were made, length was the overriding perceptual cue which caused the error, supporting the Piagetian theory that conservation of number is a critical factor in the correct determination of manyess. There were a total of 27 children (30% of the total population) who scored 11 or 12 correct responses (level III) on the cardinal task; 8 Anglos, 6 Hispanics and 13 Native Americans.

Implications and Recommendations

The implications of this study are many and varied; some for future research, some for teacher training, some for changes in the mathematics curriculum, and some for practical application in the classroom. More questions have been raised than have been answered in any definitive manner. The first implications to be discussed will pertain to future research.

Objective testing, such as was done in this research project, has yielded critical data and useful information. From the tasks used in this study, taken from the original study by Brainerd in 1973 and which he expanded into others (1976, 1979), it is possible to open a small window into the developmental thinking of young children on the two critical concepts of number, ordination and cardination. These tasks cannot take into consideration the differences in
experiences which each of the children have had in their
background, all of which are culturally patterned whether
from the mainstream or a minority population. And further,
the tasks themselves are limited and limiting; a factor only
first glimpsed in the pilot study. The study itself more
clearly displayed potentially serious flaws as follows:

1) The ordinal tasks are not equal in involvement by
the children.

**Ordinal Tasks of Length and Weight**

The ordinal task of length used three sticks, differing in
length by 3/16 of an inch; two red ones (shortest and
longest) and one yellow one. The red sticks were glued onto
a board at a distance apart of 24 inches. The yellow stick
was used as the "criteria" or "measuring" stick, and was
placed, by the examiner, first next to the shorter stick,
then next to the longer one. The child was then asked the
questions. The child was not forbidden to touch the two
sticks being compared; however, most did not. Heavy
reliance is being placed on visual perception in this
comparison, which, we are told by Piaget's original research
and subsequently supported by others, is the overriding
consideration at this pre-operational stage of development.

The ordinal task of weight, however, directly involved the
child through the use of more direct comparison. Again,
three balls of clay, two red and one yellow, exactly the
same size were used for this task of transitive asymmetry.
One red ball contained a weight of 8 ounces, the other a
weight of 24 ounces. The yellow "criterion ball" contained
a weight of 16 ounces. The examiner first placed the
8-ounce red ball in the child’s left hand, and the yellow
ball in the child’s right hand, and asked a comparison
question of estimated weight (more, less, the same). The
yellow ball was then transferred to the child’s left hand,
and the second red ball weighing 24 ounces was placed in the
child’s right hand and again the comparison question of
weight was asked. Finally, without any of the clay balls in
hand, the child was asked the question to determine
transitive asymmetry. The fact that there was an increase
of approximately 25% in correct responses for this task of
ordinality would indicate that actually being able to handle
the clay balls increased the ability to make a correct
response, and that the two tasks were not equivalent in
difficulty because of this difference.

2) The cardinal task could grossly underestimate the
child’s cardinal understanding.

Cardinal Task

The cardinal task also presented problems that need
further examination. Not only did this task not involve the
children with any direct manipulation, but as explained
previously, the original task from the Brainerd studies
involved red and blue dots pasted on cards in two rows:
two-dimensional pictures. The rows contained at most ten
dots, with the fewest being six dots, and arranged in rows,
with red at the top, so that either the rows gave the
appearance of the same length, different lengths or
different densities. The change made for this study, as a
result of the pilot study, was to change the "pictures" of dots to three dimensional cubes, using the same colors and configuration of rows as the Brainerd task. Precisely how this helped would be difficult to determine, except that the results support the research findings that young children do, indeed, function at a higher level of understanding with physical materials over pictures. The use of the cubes made a significant difference in the pilot study, and the scores obtained in this work supported the scores obtained in the pilot study.

3) The importance and role of counting in the child's cardinal understanding.

The Need to Consider Counting

While this change from pictured dots to 3-D cubes did, in fact, yield responses more in line with those obtained on the ordinal tasks, the direction to the child that he/she could not count but must devise a way of testing manyness without resorting to a count presents a problem which is diametrically opposed to the experiences young children have had with "manyness" and number. One of the first things pre-schoolers are taught, either at home, or in pre-school or a Head Start experience is counting: both rote and rational, one-to-one correspondence with items. Rarely are children asked to estimate manyness of two groups of items, and if they have had such exposure, the "check" for correctness is usually a rational, one-to-one correspondence count of the two groups. In general, the majority culture counts in one of two ways;
either ordinally:

\[
\begin{array}{cccc}
\square & \square & \square & \square \\
\text{one} & \text{two} & \text{three} & \text{four}
\end{array}
\]

or cardinally:

\[
\begin{array}{cccc}
\square & \square & \square & \square \\
\square & \square & \square & \square \\
\text{one} & \text{two} & \text{three} & \text{four}
\end{array}
\]

The mathematical understanding and experience of cardinality strictly as "matching" the items of two groups in a one-to-one correspondence, rather than counting, is not a commonplace encounter for most students, let alone young children. Part of the process of learning one-to-one correspondence is that of giving a correct number name to each item counted, and, in the end, using Gelman's "cardinal principle" (the last named number) to determine the amount of the set. Baroody’s study, previously cited, which replicated Brainerd’s cardinal task with a population of varying ages to include adults, indicated that most of the adults balked at the notion of finding "another way" to determine manyness other than counting. Their reluctance even included comments such as "How can you possibly find out which is more (or less or the same) without counting?" The idea of matching alone is not one that is a culturally common experience, at least for the majority population.

4) Consideration must be given to creating tasks which
are developmentally appropriate, including the materials used.

**Need for a Developmental Approach to Tasks**

The implications for future research in the area of the ordinal/cardinal development of number concepts indicates the need for more insightful and appropriate kinds of tasks in both areas, ordinal and cardinal, for the young child. Specifically, while these tasks of Brainerd's do indeed test the mature mathematical definition of "ordinal" and "cardinal", they do not meet the developmental understanding of young children.

Implications from this study are that Piaget was basically correct in his determination that the two concepts do develop simultaneously, in an ever-increasingly complex fashion. Tasks need to be developed that reflect these developmental stages, as well as ones that involve the child directly, and on an equal basis, with the materials used. Also, more careful attention needs to be given to the place of counting in both the ordinal and cardinal understanding of number. Totally ruling out "counting" as a cardinal determiner of manyness would seem to be negating much of what culture imposes on everyone daily, from the very earliest of experiences. Steffe's work on children's counting types needs to be explored more fully as a possible avenue to understanding the developing concepts of ordinality and cardinality. While he and his research team did not deal directly with these concepts, they did not ignore them either. A closer study of the tasks devised and
used in this work could be of practical help to future research.

5) Language as a critical determinant of concept formation.

The Role of Language

Implications are clearly present in the need for a better understanding of the role that language and cultural practices play in the conceptual understanding of numbers. All of the populations studied yielded data quite different from that of Brainerd. The Anglo and Hispanic populations were simultaneous in acquisition, while the Native American population was sequential with cardinality appearing before ordinality. Brainerd's population showed a strongly sequential acquisition with ordinal before cardinal. He did not, however, have a population that included minorities.

That the Native American population in this study showed a cardinal understanding of and acquisition of number over the ordinal or simultaneous acquisition can possibly be attributed to two factors: the influence of the native language (the "deep" meaning of language in Chomsky terms) and/or the fact that this tribe does not basically use count words or the act of counting to determine manyness. Children are taught to ascertain the completeness of the flock (of sheep), for instance, not by counting, but by looking for, or "matching" certain critical characteristics found on each animal. Children are used to estimating; manyness, distance, time, space. It is a common cultural characteristic of Native Americans; certainly for this group
which maintains its distance physically and mentally from the mainstream culture. Visual-spatial problem solving is a more natural phenomena in this culture than in the majority culture.

Noun endings in this Native American language are either singular, indicating "one" of something; dual, indicating "two" of something; or plural, indicating "lots" of something. There is no specific word for 1000 in this language; the largest actual count word is 100. The words for 200, 300, 400 etc. have been created by modern need, and are compounded from the words for "two", "three", "four" and the word for "hundred. The actual count word sequence gives indication of an original base twenty system of numeration, and gives strong support to a system of thinking in groups, rather than individual consideration of numbers.

Reference specifically has been made here to "this Native American language" because it would seem rash to generalize the findings of this study's Native American population to other Native American groups. Some groups have assimilated into the mainstream population to a greater extent, and above all, Native American languages differ greatly in origin, structure and grammatical pattern (Masayesva-Jeanne, personal communication, 1981). Certainly samples from the various Native American language groups would need to be tested in order to determine if this pattern of cardinal acquisition before the ordinal one exists for most Native American groups rather than just for one.
Further implications in the area of language is the extent to which the native language, whatever it is, is an influence on the cognitive development of mathematics for school beginners. Does the cognitive concept of number developed in the native language carry over to formal school work conducted in English, or does the conceptual thinking shift with a shift in language? And which language groups tend to be affected by such a conceptual difference?

This situation could be tested by using three classifications of children from each language group: one whose primary language is the native one, one group of balanced bilinguals (children who have equal strength in both the native language and English), and the third group, speakers of English only or whose English is the strongest language. Current research on the brain and language seems to indicate that the brain activity closely matches the pattern demanded by the stronger language, and that balanced bilinguals are showing larger and more integrated brain structures (Sibatani, 1981).

The information yielded in such a situation could be of immense help in organizing teaching/learning strategies for children in the classroom. If Native American children do indeed start out with and keep this cardinal view of number over the ordinal one, significant changes and modifications would need to be made in mathematics curricula for these children.

6. Children's intentions need to be considered over and beyond the tasks.
Children's intentions

Lastly, in the area of implications for future research, is an issue raised by Steffe in much of his work, and Baroody, Ginsberg and Hawkins in much of theirs: that of children's "intentions" and children's own invented approaches as opposed to the formal number work of school curriculum.

Steffe indicates that children's underlying intentions are so often different from what is indicated in an "answer" to a test question or situation, and that researchers need to become more involved in looking into the deeper meaning of what the child did in performance, a more ethnographic approach. Close observation of children in live, interactive mathematical situations could yield critical information about the development of children's mathematical thinking, in addition to specific tests of specific concepts in highly structured and selective testing situations.

Understanding how young children think about numbers, arrive at the conclusions they do, and further, how they actually use numbers daily can provide invaluable help in determining appropriate next steps in the learning/teaching process. It is clear from the studies of the above cited researchers that children often do not see the connection between their own informal, intuitive understanding and use of number and that of formal school arithmetic. That each child is different and learns in a different way is often heard throughout the educational community. That anything is actually done about it is rare, as witnessed by Goodlad in
his description and complaint of the sameness of American classrooms across the country.

Implications for Teacher Training

Implications for teacher training institutions are that they need to extend beyond the standard training currently provided to the standard mainstream culture elementary classroom teacher. Mathematics has always been a much disliked or feared subject (Becker, 1986; Begle, 1979; Bulmahn and Young, 1982; Fennema, Wollenet, Pedro and Becker, 1981; NCTM, 1982) for many elementary teachers and teachers-in-preparation even from the mainstream culture, let alone ethnic and language minorities. Teachers need to be empowered, as children must, with an attitude of mathematical investigation. Teachers must learn to look at children's "intentions" rather than just their answers. Why and how an error was made should be the focus of correction. Teachers must learn to explore numbers, "play" with them as children do, experiment with new ways to solve problems, especially "invented" algorithms. Many children have unique and efficient ways to do arithmetic calculations which do not even begin to resemble the traditional textbook algorithmic method but which are more efficient and understandable for them than the standard one. Teachers must begin to look for and understand these techniques, rather than force the child to hide their "unusual" approaches. Teachers need to be comfortable with these new ways of looking at numbers, if they are to help all children, majority as well as minority, meet the demands of
the scientific and technologic twentieth century. Teachers who are not comfortable with divergent thinking in mathematical processes can hardly be expected to make any visible and major changes in what actually takes place day to day in the classroom. The critical key to change is the teacher; his/her basic understanding of the subject matter and the learning styles and cognitive complexities of the students in their charge. The successful teacher looks at the students first, and makes the curriculum fit the needs of the children. This in no way implies lowering the standards or the goals and objectives especially for minority children; only approaching the subject matter through the strengths with which children come equipped as well as appropriate materials (which will be discussed later).

Dossey (1987) has cited data that entering U.S. kindergarteners are on a par with their peers worldwide but that the gap in achievement begins immediately and widens steadily. Within the United States, this is also the picture between the children from the mainstream culture and those of many minorities. Dossey also cited figures that find the number of students majoring in mathematical sciences has dropped in 10 years from 1.5% to 0.5% of the college undergraduate population. The figure of 1.5% (of 10 years ago) is certainly not one of earthshaking proportions to begin with; a drop to 0.5% cannot be accepted.

Mathematical knowledge, Dossey contended, as have others, is the critical key to entering the skilled trades and related
Vocations.

Dossey's data that entering U.S. kindergartners enter kindergarten on a par nationwide and worldwide also raises some interesting questions. Does this mean they are on a par with counting ability? an understanding of ordinality? cardinality? knowing the count word sequence? This writer's data tends to indicate that the Hopi children included in this study have a different understanding of number and its use in daily life than the mainstream populations.

If, as has been stated, that by the year 2000, 1 out of every 3 children in school will be from a minority group, and that minorities will be our leaders of the future, it will be the school's responsibility to turn out mathematically literate students. The enormous failure rate of the present can no longer be tolerated.

Implications for Use of Materials in the Classroom

Concurrent with a change in teacher training is the need to develop an understanding not only of the necessity for physical materials in the mathematics classroom, but an understanding of just what each material does for the child in the way of concept formation. Scott's (1983) survey of teachers currently using materials in the elementary classroom revealed that over half of them used only the textbook and rulers, and those who did use materials decreased that use as the grade level increased. Further, the biggest bulk of those materials used even in the primary grades were discrete objects: counters of some variety. Discrete objects (counters) reinforce the left hemispheric,
ordinal counting approach to number. They do not help or encourage the child who has a measurement approach to number, a more right hemispheric approach; one supported more by the cardinal concept.

Children should have experience with all types of materials; discrete objects to encourage the development of both ordinal and cardinal counting, continuous models (Cuisenaire rods) which enhance the visual, spatial understanding, and "bridging" materials (beans and beansticks, unifix cubes etc.) which encompass both hemispheres and approaches to number.

Teachers must develop a basic but thorough understanding of these kinds of materials and see that models of each variety are in the classroom and available for use on a daily basis. Those who have worked extensively with materials and the neurobiological model for intellectual functioning in mathematics stress the need for entry into a new concept through the child's strength, then exposure to, and practical experience with all types of materials which model that concept. Their work to date has indicated that those students whose hemispheres can work in a highly integrated fashion (simultaneous functioning) achieve at higher levels than those with right or left hemisphere dominance and/or deficit.

Teachers must also understand the often subtle differences in concept formation underlined by specific materials. A critical example, beyond the basic first steps in understanding number (ordinality and cardinality), is the
base ten system of numeration. This concept is one of documented daily and continuing difficulty for most children, but especially so for minority children with the exception of Orientals. It has been stated that Oriental languages structurally support the base ten numeration system, while other language groups, in particular English, do not (Muir, 1986). It has also been documented (Leap, in press; Whitthun, 1984) that Native American children have particular difficulty with numeration concepts. If, as is indicated in this study, Native American children have a cardinal understanding of number as their initial strength, then a continuous model rather than a discrete one, for number and numeration, is indicated. This type of material is infrequently seen or used in today's classrooms.

Introducing number and numeration through this strength, then work with discrete objects could supply the solid foundation needed for future success in mathematics. For children with an ordinal understanding preceding a cardinal one, discrete objects, counters of all types, are the excellent first materials, but these children, too, need experience with continuous models if they are to equalize their understanding and use of ordinality and cardinality.

Base ten concepts viewed through the model of Cuisenaire rods is vastly different than base ten viewed through the model of colored chips or popsicle sticks; all models do not undergird the same understanding. This is the kind of difference in materials teachers must learn in order to maximize the learning situation for all children. One
way, one model, one material, one text cannot take into consideration the multiple approaches devised and understood by young children in today's classrooms.

Implications for a Change in Mathematics Curricula

These citations and many others indicate the need for better and more thorough teacher preparation and use of materials in the classroom. A further critical factor, and one supported by the findings of this study as well as others, is the need for radical change in the mathematics curricula of formal schooling.

A study of the kindergarten and first grade segments of currently published mathematics programs by major publishing houses has yielded data which confirms a one-sided approach to the concept of number: an ordinal one. An analysis of the pages devoted to number concepts indicates that these basal programs devote better than 75% of these pages to ordinal topics or to looking at number via an ordinal counting approach. Fewer than 6 pages of any kindergarten workbook deals with determining manyness by one-to-one matching only (more, less or the same amounts of two groups). Counting, and in particular ordinal counting, is the critical concept developed in any given printed program, and this is a concept which continues into the grade 1 workbook as well. That there is little, if any, change in emphasis from text to text would not be overstating the situation. Further, discussions with over 50 kindergarten and first grade teachers during the past two years has also yielded the information that even if the text and teacher's
guide describes matching as the intended activity, most
teachers encourage the children to count, and basically in
an ordinal fashion as pictures are very difficult to count
cardinally. Children can achieve the one-to-one
correspondence by touching each picture as they give it a
number name; however, grouping the objects together as they
count them is impossible. Teachers also give very little
evidence of understanding the difference between ordinal and
cardinal counting, or an ordinal versus a cardinal
understanding of number.

All currently published mathematics programs also avoid
teaching any subtraction in kindergarten, and each begin
their first grade workbook with addition only. Implications
of this study have indicated that kindergarten children are
quite capable of handling the subtraction concept and
process, and indeed, if children do have a stronger cardinal
understanding of number, subtraction can be the stronger
operation of arithmetic for some children. The simultaneous
children also showed equal strengths with both operations,
implying the need for working both forwards (addition) and
backwards (subtraction) simultaneously rather than the
traditional sequence of addition, then subtraction.

Of particular concern are Native American children who
do seem to have a different orientation to numbers from that
of the mainstream population. Implications are strong that
curricula will have to undergo radical change to meet the
needs of these children in the formative elementary years of
mathematics study. It has been stated that Native Americans
often have their initial interest and/or success in mathematics stifled or smothered in the traditional classroom. There is, of course, a basic body of knowledge which all children must acquire if they are to be mathematically literate; but the influence of the standard procedures and algorithms, as well as the traditional sequence in which they are taught should not distract from the importance of modifications to meet the needs of these different students. If only 0.5% of the undergraduate population of this nation is choosing to major in the mathematical sciences, changes must be made; in teacher attitudes and training, materials in the classroom and basic textbook curriculum, beginning with the kindergarten.

This study has shown that children vary in their basic understanding of the two critical concepts of ordinality and cardinality, as well as their approach to number both within and between ethnic groups. These factors are seen as critical first steps in the development of school mathematics and arithmetic skills. In order for meaningful changes to occur in mathematics education for all children, majority and minority, information such as this must be obtained and used by the educational community; it is of particular value for those working exclusively or primarily with ethnic and language minority children. Such information is also important at this time when there is such strong evidence of low achievement nationwide in mathematics and the call for radical reform in curriculum and teacher training is being heard from many sources.
APPENDIX A

LANGUAGE PRE-TEST, ORDINAL AND CARDINAL TASKS, TEST OF ARITHMETIC PROFICIENCY

Story problems:

1. Suppose that I had four cookies and you had six cookies. Would one of us have more cookies than the other?
   
   If so, who has more?

   Would we both have the same number of cookies?

2. Suppose that I had four cookies and you had five cookies. Would one of us have fewer cookies?

   If so, who has fewer?

   Would we both have the same number of cookies?
Ordinal tasks

Length: (shorter to longer)
1) Are the two red sticks the same length?
2) Is one of the red sticks longer than the other?
   Which one?
3) Is one of the red sticks shorter than the other?
   Which one?

(longer to shorter)
1) Are the two red sticks the same length?
2) Is one of the red sticks longer than the other?
   Which one?
3) Is one of the red sticks shorter than the other?
   Which one?

Weight: (light to heavy)
1) Do the two red balls weigh the same?
2) Does one of the red balls weigh more?
   Which one?
3) Does one of the red balls weigh less?
   Which one?

(heavy to light)
1) Do the two red balls weigh the same?
2) Does one of the red balls weigh more?
   Which one?
3) Does one of the red balls weigh less?
   Which one?
Cardinal task

Card A: Are there just as many red blocks as blue blocks on this card? Does one of the rows have more blocks than the other row? Which one?

Card B: Are there just as many red blocks as blue blocks on this card? Does one of the rows have more blocks than the other row? Which one?

Card C: Are there just as many red blocks as blue blocks on this card? Does one of the rows have more blocks than the other row? Which one?

Card D: Are there just as many red blocks as blue blocks on this card? Does one of the rows have more blocks than the other row? Which one?

Card E: Are there just as many red blocks as blue blocks on this card? Does one of the rows have more blocks than the other row? Which one?

Card F: Are there just as many red blocks as blue blocks on this card? Does one of the rows have more blocks than the other row? Which one?

Did you find that you could answer all the questions without counting the blocks, or did you have to count sometimes?
<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 + 1 =</td>
<td>2 - 1 =</td>
</tr>
<tr>
<td>1 + 2 =</td>
<td>3 - 1 =</td>
</tr>
<tr>
<td>1 + 3 =</td>
<td>4 - 1 =</td>
</tr>
<tr>
<td>1 + 4 =</td>
<td>5 - 1 =</td>
</tr>
<tr>
<td>2 + 1 =</td>
<td>5 - 2 =</td>
</tr>
<tr>
<td>2 + 2 =</td>
<td>6 - 2 =</td>
</tr>
<tr>
<td>2 + 3 =</td>
<td>4 - 2 =</td>
</tr>
<tr>
<td>2 + 4 =</td>
<td>3 - 2 =</td>
</tr>
<tr>
<td>3 + 1 =</td>
<td>4 - 3 =</td>
</tr>
<tr>
<td>3 + 2 =</td>
<td>5 - 3 =</td>
</tr>
<tr>
<td>3 + 3 =</td>
<td>6 - 3 =</td>
</tr>
<tr>
<td>3 + 4 =</td>
<td>7 - 3 =</td>
</tr>
<tr>
<td>4 + 1 =</td>
<td>5 - 4 =</td>
</tr>
<tr>
<td>4 + 2 =</td>
<td>6 - 4 =</td>
</tr>
<tr>
<td>4 + 3 =</td>
<td>7 - 4 =</td>
</tr>
<tr>
<td>4 + 4 =</td>
<td>8 - 4 =</td>
</tr>
</tbody>
</table>
APPENDIX B

SPANISH TRANSLATION: LANGUAGE PRE-TEST, ORDINAL AND CARDINAL TASKS
Language pre-test

A. Ordinal

Linias:
1) ¿Estan estas linias lo mismo de largas?
2) ¿Esta una de estas linias mas larga que la otra? ¿A cual?
3) ¿Esta una de estas linias mas corta? ¿A cual?

Bolas de barro:
1) ¿Pesan la misma esta bolas de barro?
2) ¿Pesa mas una de estas bolas que las demas? ¿A cual?
3) ¿Pesa menos una de estas bolas? ¿A cual?

B. Cardinal

Story Problems:
1) ¿Vamos a suponer que yo tengo ocho biscochitos y tu tienes seis biscochitos. ¿Tiene mas biscochitos uno de nosotros?
   ¿Quien tiene mas?
   ¿Tenemos el mismo numero los dos?

2) ¿Vamos a suponer que yo tengo quatro biscochitos y tu tienes quatro biscochitos. ¿Tiene mas pocos uno de nosotros?
   ¿Quien tiene mas pocos?
   ¿Tenemos el mismo numero de biscochitos los dos?
Ordinal tasks
Length: (shorter to longer)

1) ¿Están los dos palitos rojos lo mismo de largo?

2) ¿Esta uno de los palitos más largo que el otro?
   ¿A cual?

3) ¿Esta uno de los palitos rojos más chico que el otro?
   ¿A cual?

(longer to shorter)

1) ¿Están los dos palitos rojos lo mismo de largo?

2) ¿Esta uno de los palitos rojos más largo que el otro?
   ¿A cual?

3) ¿Esta uno de las palitos rojos más chico que el otro?
   ¿A cual?

Weight: (light to heavy)

1) ¿Pesan lo mismo las dos bolas rojas?

2) ¿Pesa más una bola roja que la otra?
   ¿A cual?

3) ¿Pesa menos una bola roja que la otra?
   ¿A cual?

(heavy to light)

1) ¿Pesan lo mismo las dos bolas rojas?

2) ¿Pesa más una bola roja que la otra?
   ¿A cual?

3) ¿Pesa menos una bola roja que la otra?
   ¿A cual?
Cardinal task

Card A: ¿Hay tantos bloques rojos como azules en esta tarjeta?
  ¿Tiene más una carrera de bloques que la otra?
  ¿A cual?

Card B: ¿Hay tantos bloques rojos como azules en esta tarjeta?
  ¿Tiene más una carrera de bloques que la otra?
  ¿A cual?

Card C: ¿Hay tantos bloques rojos como azules en esta tarjeta?
  ¿Tiene más una carrera de bloques que la otra?
  ¿A cual?

Card D: ¿Hay tantos bloques rojos como azules en esta tarjeta?
  ¿Tiene más una carrera de bloques que la otra?
  ¿A cual?

Card E: ¿Hay tantos bloques rojos como azules en esta tarjeta?
  ¿Tiene más una carrera de bloques que la otra?
  ¿A cual?

Card F: ¿Hay tantos bloques rojos como azules en esta tarjeta?
  ¿Tiene más una carrera de bloques que la otra?
  ¿A cual?

¿Has podido responder todas las preguntas sin contar los bloques, o tuviste que contar a veces?
Subtraction

¿dos menos uno es?
¿tres menos uno son?
¿cuatro menos uno son?
¿cinco menos uno son?
¿cinco menos dos son?
¿seis menos dos son?
¿cuatro menos dos son?
¿tres menos dos son?
¿cuatro menos tres son?
¿cinco menos tres son?
¿seis menos tres son?
¿siete menos tres son?
¿cinco menos cuatro son?
¿seis menos cuatro son?
¿siete menos cuatro son?
¿ocho menos cuatro son?
APPENDIX C

CARDINAL TASK CARD CONFIGURATIONS
APPENDIX D

....................................PARENTAL PERMISSION FORM....................................
August 1985

Dear Parents:

I am a doctoral student at the University of New Mexico in the Department of Curriculum and Instruction in Multicultural Teacher Education, elementary level, and am doing my doctoral dissertation research on the origins of number concepts in Anglo, Hispanic and Native American children ages five, six and seven (grades Kindergarten and 1). I have presented my study to the administration of your school and have obtained their permission to use this school as a data collection site.

I will be giving each child four short tests: two ordinal tasks ("ordinal" meaning ordering or seriating objects according to size, weight etc.), one cardinal task ("cardinal" meaning quantification and the comparison of quantities), and one task involving addition and subtraction of the first four numbers. These tasks should take no more than 30 minutes per child, who will be tested individually and in private. If your child is more fluent and comfortable in a language other than English, a native speaker will do the testing under my training and guidance. Because these tasks are basic classroom procedures, there are no foreseeable risks or discomforts to the children. All results will be held in strictest confidence.

The children will be told that their participation can help teachers learn more about how children learn numbers, and that this information can help make good teachers into better teachers. Each child will then be asked if s(he) wishes to help. Verbal assent will be needed from each child, and no child will be made to feel uncomfortable in any way if s(he) does not want to participate. Each child will be identified by a code number so that all responses will be anonymous, and analysis will be of group responses.

If you have any questions, inquiries or problems about the study, you may call the UNM College of Education Assistant Dean for Research Dr. Peggy J. Blackwell, at 1-505-277-3638. If you have any legal questions, contact the Office of Risk Management, Lamy Building, Santa Fe, New Mexico 87501. My faculty advisor on this dissertation is Dr. David Darling, Department Chair, Department of Curriculum and Instruction in Multicultural Teacher Education, 1-505-277-411
I hope you will permit your child to participate in this study, the results of which will be available after January 1, 1986. Your school administration will be supplied a summary of the results; if you wish a personal copy, please feel free to contact me.

Sincerely,

Manon P. Charbonneau (Mrs. U.A.)
417 Washington Avenue
Santa Fe, New Mexico 87501
(505) 983-9609
PERMISSION:

I understand that my child has been asked to participate in this study. I also understand that my child’s participation is voluntary and that s/he may withdraw from the study at any time with no penalty. If I have any concerns about the study, I understand that I should call the College of Education Assistant Dean Dr. Peggy J. Blackwell at 277-3638. Legal information regarding human subjects in research is available from the Risk Management Division, Room 24, Lamy Building, Santa Fe, New Mexico 87503. I have read all 3 pages of this consent form and understand what my child has been asked to do.

I hereby (give/do not give) my consent for my child to participate in this study. I have received two copies of this consent form, one to return to my child’s school and one for me to keep.

Signed: 

Date: 

........................................

168
BIBLIOGRAPHY

second grade. New York: Teachers College.

Altizer-Tunnicliff, Carol. (1984). Crisis in arithmetic
teaching: the future is here. The Arithmetic Teacher,
1, 30-31. September.

Apologetic, May. (1982). The missing link: reflections on
mathematical beginnings. Outlook, 32, 0-23. Autumn.

Baroody, Arthur. (1980). Evaluating the variety of
research in mathematics education. 19, 12, 41-44.

March

of mathematical structures. Journal for Research in
Mathematics Education 13, 37-46.


school mathematics teachers. The Arithmetic Teacher, 13, (3),


BIBLIOGRAPHY


Duckworth, Eleanor. A case study about some depths and perplexities of elementary arithmetic. Unpublished manuscript.


1, (8), pp.22-27.

Siegal, L. (1982). The development of quantity concepts:
percepts and linguistic factors. In C.J. Brainerd (Ed.).
Children's Logical and Mathematical Cognition (pp.

Sizer, Theodore. (1984). Horace's compromise, the dilemma of

status to the development of conservation of number.
Unpublished doctoral dissertation, University of
Wisconsin, Madison.

TESOL Quarterly, 8, 17-26.

Stanic, George & L. Reyes. (1986). Gender and race
differences in mathematics: a case study of
seventh-grade classroom. For the Learning of Mathematics,

Stanic, George & L. Reyes. (1987). Excellence and equity in
mathematics classrooms. For the Learning of Mathematics,

and order relations by four- and five-year old children.
In L. Steffe (Ed.). Research on mathematical thinking of
young children (pp. 19-460). Reston, VA: NCTM.


