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# Neutrosophic Supra Topological Applications in Data Mining Process

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**Abstract:** The primary aim of this paper is to introduce the neutrosophic supra topological spaces. Neutrosophic subspaces and neutrosophic mappings are presented by which some contradicting examples of the statements of Abd-Monsef and Ramadan<sup>[9]</sup> in fuzzy supra topological spaces are derived. Finally, a new method is proposed to solve medical diagnosis problems by using single valued neutrosophic score function.

**Keywords:** Fuzzy topology, Intuitionistic topology, Neutrosophic topology, Neutrosophic subspaces, Neutrosophic supra topology.

## 1 Introduction

The concept of fuzzy set was introduced by A. Zadeh [1] in 1965 which is a generalization of crisp set to analyse imprecise mathematical information. Adlassnig [2] applied fuzzy set theory to formalize medical relationships and fuzzy logic to computerized diagnosis system. This theory [3, 4, 5] has been used in the fields of artificial intelligence, probability, biology, control systems and economics. C.L Chang [6] introduced the fuzzy topological spaces and further the properties of fuzzy topological spaces are studied by R. Lowen [7]. By relaxing one topological axiom, Mashhour et al. [8] introduced supra topological space in 1983 and discussed its properties. Abd-Monsef and Ramadan [9] introduced fuzzy supra topological spaces and its continuous mappings. K. Atanassov [10] considered the degree of non-membership of an element along with the degree of membership and introduced intuitionistic fuzzy sets. Dogan Coker [11] introduced intuitionistic fuzzy topology. Saadati [12] further studied the basic concept of intuitionistic fuzzy point. S.K.De et al. [13] was the first one to develop the applications of intuitionistic fuzzy sets in medical diagnosis. Several researchers [14, 15, 16] further studied intuitionistic fuzzy sets in medical diagnosis. Hung and Tuan [17] noted that the approach in [13] has some questionable results on false diagnosis of patients' symptoms. Generally it is recognized that the available information about the patient and medical relationships is inherently uncertain. There may be indeterminacy components in real life problems for data mining and neutrosophic logic can be used in this regard. Neutrosophic logic is a generalization of fuzzy, intuitionistic, boolean, paraconsistent logics etc. Compared to all other logics, neutrosophic logic introduces a percentage of "indeterminacy" and this logic allows

each component  $t$  true,  $i$  indeterminate,  $f$  false to "boil over" 100 or "freeze" under 0. Here no restriction on  $T, I, F$ , or the sum  $n = t + i + f$ , where  $t, i, f$  are real values from the ranges  $T, I, F$ . For instance, in some tautologies  $t > 100$ , called "overtrue". As a generalization of Zadeh's fuzzy set and Atanassov's intuitionistic fuzzy set, Florentin Smarandache [18] introduced neutrosophic set. Neutrosophic set  $A$  consists of three independent objects called truth-membership  $\mu_A(x)$ , indeterminacy-membership  $\sigma_A(x)$  and falsity-membership  $\gamma_A(x)$  whose values are real standard or non-standard subset of unit interval  $]^{-0}, 1^{+}[$ . In data analysis, many methods have been introduced [19, 20, 21] to measure the similarity degree between fuzzy sets. But these are not suitable for the similarity measures of neutrosophic sets. The single-valued neutrosophic set is a neutrosophic set which can be used in real life engineering and scientific applications. The single valued neutrosophic set was first initiated by Smarandache [22] in 1998 and further studied by Wang et al. [23]. Majumdar and Samanta [24] defined some similarity measures of single valued neutrosophic sets in decision making problems. Recently many researchers [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45] introduced several similarity measures and single-valued neutrosophic sets in medical diagnosis. The notion of neutrosophic crisp sets and topological spaces were introduced by A. A. Salama and S. A. Alblowi [46,47].

In section 2 of this paper, we present some basic preliminaries of fuzzy, intuitionistic, neutrosophic sets and topological spaces. The section 3 introduces the neutrosophic subspaces with its properties. In section 4, we define the concept of neutrosophic supra topological spaces. In section 5, we introduce neutrosophic supra continuity,  $S^*$ -neutrosophic continuity and give some contradicting examples in fuzzy supra topological spaces<sup>[9]</sup>. As a real life application, a common method for data analysis under neutrosophic supra topological environment is presented in section 6. In section 7, we solve numerical examples of above proposed method and the last section states the conclusion and future work of this paper.

## 2 Preliminary

This section studies some of the basic definitions of fuzzy, intuitionistic, neutrosophic sets and respective topological spaces which are used for further study.

**Definition 2.1.** [1] Let  $X$  be a non empty set, then  $A = \{(x, \mu_A(x)) : x \in X\}$  is called a fuzzy set on  $X$ , where  $\mu_A(x) \in [0, 1]$  is the degree of membership function of each  $x \in X$  to the set  $A$ . For  $X$ ,  $I^X$  denotes the collection of all fuzzy sets of  $X$ .

**Definition 2.2.** [10] Let  $X$  be a non empty set, then  $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$  is called an intuitionistic set on  $X$ , where  $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$  for all  $x \in X$ ,  $\mu_A(x), \gamma_A(x) \in [0, 1]$  are the degree of membership and non membership functions of each  $x \in X$  to the set  $A$  respectively. The set of all intuitionistic sets of  $X$  is denoted by  $I(X)$ .

**Definition 2.3.** [23] Let  $X$  be a non empty set, then  $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$  is called a neutrosophic set on  $X$ , where  $^{-0} \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3^{+}$  for all  $x \in X$ ,  $\mu_A(x), \sigma_A(x)$  and  $\gamma_A(x) \in ]^{-0}, 1^{+}[$  are the degree of membership (namely  $\mu_A(x)$ ), the degree of indeterminacy (namely  $\sigma_A(x)$ ) and the degree of non membership (namely  $\gamma_A(x)$ ) of each  $x \in X$  to the set  $A$  respectively. For  $X$ ,  $N(X)$  denotes the collection of all neutrosophic sets of  $X$ .

**Definition 2.4.** [18] The following statements are true for neutrosophic sets  $A$  and  $B$  on  $X$ :

- (i)  $\mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x)$  and  $\gamma_A(x) \geq \gamma_B(x)$  for all  $x \in X$  if and only if  $A \subseteq B$ .
- (ii)  $A \subseteq B$  and  $B \subseteq A$  if and only if  $A = B$ .
- (iii)  $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \min\{\sigma_A(x), \sigma_B(x)\}, \max\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$ .
- (iv)  $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \max\{\sigma_A(x), \sigma_B(x)\}, \min\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$ .

More generally, the intersection and the union of a collection of neutrosophic sets  $\{A_i\}_{i \in \Lambda}$ , are defined by  $\bigcap_{i \in \Lambda} A_i = \{(x, \inf_{i \in \Lambda}\{\mu_{A_i}(x)\}, \inf_{i \in \Lambda}\{\sigma_{A_i}(x)\}, \sup_{i \in \Lambda}\{\gamma_{A_i}(x)\}) : x \in X\}$  and  $\bigcup_{i \in \Lambda} A_i = \{(x, \sup_{i \in \Lambda}\{\mu_{A_i}(x)\}, \sup_{i \in \Lambda}\{\sigma_{A_i}(x)\}, \inf_{i \in \Lambda}\{\gamma_{A_i}(x)\}) : x \in X\}$ .

**Notation 2.5.** Let  $X$  be a non empty set. We consider the fuzzy, intuitionistic, neutrosophic empty set as  $\emptyset = \{(x, 0) : x \in X\}$ ,  $\emptyset = \{(x, 0, 1) : x \in X\}$ ,  $\emptyset = \{(x, 0, 0, 1) : x \in X\}$  respectively and the fuzzy, intuitionistic, neutrosophic whole set as  $X = \{(x, 1) : x \in X\}$ ,  $X = \{(x, 1, 0) : x \in X\}$ ,  $X = \{(x, 1, 1, 0) : x \in X\}$  respectively.

**Definition 2.6.** [24] A neutrosophic set  $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$  is called a single valued neutrosophic set on a non empty set  $X$ , if  $\mu_A(x), \sigma_A(x)$  and  $\gamma_A(x) \in [0, 1]$  and  $0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3$  for all  $x \in X$  to the set  $A$ . For each attribute, the single valued neutrosophic score function (shortly SVNSF) is defined as  $SVNSF = \frac{1}{3m} [\sum_{i=1}^m [2 + \mu_i - \sigma_i - \gamma_i]]$ .

**Definition 2.7.** [6] Let  $X$  be a non empty set. A subcollection  $\tau_f$  of  $I^X$  is said to be fuzzy topology on  $X$  if the sets  $X$  and  $\emptyset$  belong to  $\tau_f$ ,  $\tau_f$  is closed under arbitrary union and  $\tau_f$  is closed under finite intersection. Then  $(X, \tau_f)$  is called fuzzy topological space ( shortly fts ), members of  $\tau_f$  are known as fuzzy open sets and their complements are fuzzy closed sets.

**Definition 2.8.** [11] Let  $X$  be a non empty set and a subfamily  $\tau_i$  of  $I(X)$  is called intuitionistic fuzzy topology on  $X$  if  $X$  and  $\emptyset \in \tau_i$ ,  $\tau_i$  is closed under arbitrary union and  $\tau_i$  is closed under finite intersection. Then  $(X, \tau_i)$  is called intuitionistic fuzzy topological space ( shortly ifts ), elements of  $\tau_i$  are called intuitionistic fuzzy open sets and their complements are intuitionistic fuzzy closed sets.

**Definition 2.9.** [46, 47] Let  $X$  be a non empty set. A neutrosophic topology on  $X$  is a subfamily  $\tau_n$  of  $N(X)$  such that  $X$  and  $\emptyset$  belong to  $\tau_n$ ,  $\tau_n$  is closed under arbitrary union and  $\tau_n$  is closed under finite intersection. Then  $(X, \tau_n)$  is called neutrosophic topological space ( shortly nts ), members of  $\tau_n$  are known as neutrosophic open sets and their complements are neutrosophic closed sets. For a neutrosophic set  $A$  of  $X$ , the interior and closure of  $A$  are respectively defined as:  $int_n(A) = \cup\{G : G \subseteq A, G \in \tau_n\}$  and  $cl_n(A) = \cap\{F : A \subseteq F, F^c \in \tau_n\}$ .

**Corollary 2.10.** [18] The following statements are true for the neutrosophic sets  $A, B, C$  and  $D$  on  $X$ :

- (i)  $A \cap C \subseteq B \cap D$  and  $A \cup C \subseteq B \cup D$ , if  $A \subseteq B$  and  $C \subseteq D$ .
- (ii)  $A \subseteq B \cap C$ , if  $A \subseteq B$  and  $A \subseteq C$ .  $A \cup B \subseteq C$ , if  $A \subseteq C$  and  $B \subseteq C$ .
- (iii)  $A \subseteq C$ , if  $A \subseteq B$  and  $B \subseteq C$ .

**Definition 2.11.** [48] Let  $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ ,  $B = \{(y, \mu_B(y), \sigma_B(y), \gamma_B(y)) : y \in Y\}$  be two neutrosophic sets and  $f : X \rightarrow Y$  be a function.

- (i)  $f^{-1}(B) = \{(x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)) : x \in X\}$  is a neutrosophic set on  $X$  called the pre-image of  $B$  under  $f$ .
- (ii)  $f(A) = \{(y, f(\mu_A)(y), f(\sigma_A)(y), (1 - f(1 - \gamma_A))(y)) : y \in Y\}$  is a neutrosophic set on  $Y$  called the image of  $A$  under  $f$ , where

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$f(\sigma_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \sigma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$(1 - f(1 - \gamma_A))(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

For the sake of simplicity, let us use the symbol  $f_-(\gamma_A)$  for  $(1 - f(1 - \gamma_A))$ .

### 3 Neutrosophic Subspaces

This section introduces differences of two fuzzy, intuitionistic and neutrosophic sets on  $X$ . We also introduce neutrosophic subspaces with its properties.

**Definition 3.1.** The difference of neutrosophic sets  $A$  and  $B$  on  $X$  is a neutrosophic set on  $X$ , defined as  $A \setminus B = \{(x, |\mu_A(x) - \mu_B(x)|, |\sigma_A(x) - \sigma_B(x)|, 1 - |\gamma_A(x) - \gamma_B(x)|) : x \in X\}$ . Clearly  $X^c = X \setminus X = (x, 0, 0, 1) = \emptyset$  and  $\emptyset^c = X \setminus \emptyset = (x, 1, 1, 0) = X$ .

**Definition 3.2.** Let  $A, B$  be two intuitionistic fuzzy sets of  $X$ , then the difference of  $A$  and  $B$  is an intuitionistic fuzzy set on  $X$ , defined as  $A \setminus B = \{(x, |\mu_A(x) - \mu_B(x)|, 1 - |\gamma_A(x) - \gamma_B(x)|) : x \in X\}$ . Clearly  $X^c = X \setminus X = (x, 0, 1) = \emptyset$  and  $\emptyset^c = X \setminus \emptyset = (x, 1, 0) = X$ .

**Definition 3.3.** Let  $A, B$  be two fuzzy sets of  $X$ , then the difference of  $A$  and  $B$  is a fuzzy set on  $X$ , defined as  $A \setminus B = \{(x, |\mu_A(x) - \mu_B(x)|) : x \in X\}$ . Clearly  $X^c = X \setminus X = (x, 0) = \emptyset$  and  $\emptyset^c = X \setminus \emptyset = (x, 1) = X$ .

**Corollary 3.4.** The following statements are true for the neutrosophic sets  $\{A\}_{i=1}^{\infty}, A, B$  on  $X$ :

- (i)  $(\bigcap_{i \in \Lambda} A_i)^c = \bigcup_{i \in \Lambda} A_i^c, (\bigcup_{i \in \Lambda} A_i)^c = \bigcap_{i \in \Lambda} A_i^c$ .
- (ii)  $(A^c)^c = A. B^c \subseteq A^c, \text{ if } A \subseteq B$ .

*Proof.* : **Part(i):**  $(\bigcap_{i \in \Lambda} A_i)^c = \{(x, |1 - \inf_{i \in \Lambda} \{\mu_{A_i}(x)\}|, |1 - \inf_{i \in \Lambda} \{\sigma_{A_i}(x)\}|, 1 - |0 - \sup_{i \in \Lambda} \{\gamma_{A_i}(x)\}|\} : x \in X\} = \{(x, \sup_{i \in \Lambda} (|1 - \mu_{A_i}(x)|), \sup_{i \in \Lambda} (|1 - \sigma_{A_i}(x)|), \inf_{i \in \Lambda} (|1 - \gamma_{A_i}(x)|) : x \in X\} = \bigcup_{i \in \Lambda} A_i^c$ . Similarly we can prove  $(\bigcup_{i \in \Lambda} A_i)^c = \bigcap_{i \in \Lambda} A_i^c$  and part(ii).  $\square$

Generally, in the sense of Chang<sup>[6]</sup> every fuzzy topology is intuitionistic fuzzy topology as well as neutrosophic topology. The following lemmas show that every intuitionistic fuzzy topology  $\tau_i$  induce two fuzzy topologies on  $X$  and every neutrosophic topology  $\tau_n$  induce three fuzzy topologies on  $X$ .

**Lemma 3.5.** In an intuitionistic fuzzy topological space  $(X, \tau_i)$ , each of the following collections form fuzzy topologies on  $X$ :

- (i)  $\tau_{f_1} = \{A = (x, \mu_A(x)) : (x, \mu_A(x), \gamma_A(x)) \in \tau_i\}$ .
- (ii)  $\tau_{f_2} = \{A = (x, 1 - \gamma_A(x)) : (x, \mu_A(x), \gamma_A(x)) \in \tau_i\}$ .

*Proof.* : Here we shall prove part (ii) only and similarly we can prove part (i). Clearly  $\emptyset = (x, 0)$  and  $X = (x, 1)$  are belong to  $\tau_{f_2}$ . If  $\{A_j\}_{j \in \Lambda} \in \tau_{f_2}$ , then  $\{(x, \mu_{A_j}(x), \gamma_{A_j}(x))\}_{j \in \Lambda} \in \tau_i$  and  $(x, \sup_{j \in \Lambda} \{\mu_{A_j}(x)\}, \inf_{j \in \Lambda} \{\gamma_{A_j}(x)\}) \in \tau_i$ . Therefore  $(x, \sup_{j \in \Lambda} \{1 - \gamma_{A_j}(x)\}) = (x, 1 - \inf_{j \in \Lambda} \{\gamma_{A_j}(x)\}) \in \tau_{f_2}$  and so  $\cup_{j \in \Lambda} A_j \in \tau_{f_2}$ . If  $\{A_j\}_{j=1}^m \in \tau_{f_2}$ , then  $\{(x, \mu_{A_j}(x), \gamma_{A_j}(x))\}_{j=1}^m \in \tau_i$  and  $(x, \inf_j \{\mu_{A_j}(x)\}, \sup_j \{\gamma_{A_j}(x)\}) \in \tau_i$ . Therefore  $(x, \inf_j \{1 - \gamma_{A_j}(x)\}) = (x, 1 - \sup_j \{\gamma_{A_j}(x)\}) \in \tau_{f_2}$  and so  $\cap_{j=1}^m A_j \in \tau_{f_2}$ .  $\square$

**Lemma 3.6.** In a neutrosophic topological space  $(X, \tau_n)$ , each of the following collections form fuzzy topologies on  $X$ :

- (i)  $\tau_{f_1} = \{A = (x, \mu_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n\}$ .
- (ii)  $\tau_{f_2} = \{A = (x, \sigma_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n\}$ .
- (iii)  $\tau_{f_3} = \{A = (x, 1 - \gamma_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n\}$ .

*Proof.* : **Part (i):** Clearly  $\emptyset = (x, 0)$  and  $X = (x, 1)$  are belong to  $\tau_{f_1}$ . If  $\{A_j\}_{j \in \Lambda} \in \tau_{f_1}$ , then  $\{(x, \mu_{A_j}(x), \sigma_{A_j}(x), \gamma_{A_j}(x))\}_{j \in \Lambda} \in \tau_n$  and  $(x, \sup_{j \in \Lambda} \{\mu_{A_j}(x)\}, \sup_{j \in \Lambda} \{\sigma_{A_j}(x)\}, \inf_{j \in \Lambda} \{\gamma_{A_j}(x)\}) \in \tau_n$ . Therefore  $(x, \sup_{j \in \Lambda} \{\mu_{A_j}(x)\}) \in \tau_{f_1}$  and so  $\cup_{j \in \Lambda} A_j \in \tau_{f_1}$ . If  $\{A_j\}_{j=1}^m \in \tau_{f_1}$ , then  $\{(x, \mu_{A_j}(x), \sigma_{A_j}(x), \gamma_{A_j}(x))\}_{j=1}^m \in \tau_n$  and  $(x, \inf_j \{\mu_{A_j}(x)\}, \inf_j \{\sigma_{A_j}(x)\}, \sup_j \{\gamma_{A_j}(x)\}) \in \tau_n$ . Therefore  $(x, \inf_j \{\mu_{A_j}(x)\}) \in \tau_{f_1}$  and so  $\cap_{j=1}^m A_j \in \tau_{f_1}$ . In similar manner we can prove part (ii) and (iii).  $\square$

**Corollary 3.7.** Let  $A$  be a neutrosophic set of  $(X, \tau_n)$ , then the collection  $(\tau_n)_A = \{A \cap O : O \in \tau_n\}$  is a neutrosophic topology on  $A$ , called the induced neutrosophic topology on  $A$  and the pair  $(A, (\tau_n)_A)$  is called neutrosophic subspace of nts  $(X, \tau_n)$ . The elements of  $(\tau_n)_A$  are called  $(\tau_n)_A$ -open sets and their complements are called  $(\tau_n)_A$ -closed sets.

*Proof.* : Obviously  $\emptyset = (x, \min(\mu_A(x), 0), \min(\sigma_A(x), 0), \max(\gamma_A(x), 1)) = A \cap \emptyset \in (\tau_n)_A$  and  $A = (x, \min(\mu_A(x), 1), \min(\sigma_A(x), 1), \max(\gamma_A(x), 0)) = A \cap X \in (\tau_n)_A$ . Take  $\{A_j\}_{j \in \Lambda} \in (\tau_n)_A$ , there exist  $O_j \in \tau_n, j \in \Lambda$ , such that  $A_j = A \cap O_j$  for each  $j \in \Lambda$ . Then  $A' = \cup_{j \in \Lambda} A_j = \{(x, \sup_{j \in \Lambda} \{\mu_{A_j}(x)\}, \sup_{j \in \Lambda} \{\sigma_{A_j}(x)\}, \inf_{j \in \Lambda} \{\gamma_{A_j}(x)\})\} = \{(x, \sup_{j \in \Lambda} (\min\{\mu_A(x), \mu_{O_j}(x)\}), \sup_{j \in \Lambda} (\min\{\sigma_A(x), \sigma_{O_j}(x)\}), \inf_{j \in \Lambda} (\max\{\gamma_A(x), \gamma_{O_j}(x)\}))\} = \{(x, \min(\sup_{j \in \Lambda} \{\mu_A(x), \mu_{O_j}(x)\}), \min(\sup_{j \in \Lambda} \{\sigma_A(x), \sigma_{O_j}(x)\}), \max(\inf_{j \in \Lambda} \{\gamma_A(x), \gamma_{O_j}(x)\}))\} \in (\tau_n)_A$ . Therefore  $(\tau_n)_A$  is closed under arbitrary union on  $A$ . If we take  $\{A_j\}_{j=1}^m \in (\tau_n)_A$ , there exist  $O_j \in \tau_n, j = 1, 2, \dots, m$ , such that  $A_j = A \cap O_j$  for each  $j \in \Lambda$ . Then  $A' = \cap_{j=1}^m A_j = \{(x, \inf_j \{\mu_{A_j}(x)\}, \inf_j \{\sigma_{A_j}(x)\}, \sup_j \{\gamma_{A_j}(x)\})\} = \{(x, \inf_j (\min\{\mu_A(x), \mu_{O_j}(x)\}), \inf_j (\min\{\sigma_A(x), \sigma_{O_j}(x)\}), \sup_j (\max\{\gamma_A(x), \gamma_{O_j}(x)\}))\} = \{(x, \min(\inf_j \{\mu_A(x), \mu_{O_j}(x)\}), \min(\inf_j \{\sigma_A(x), \sigma_{O_j}(x)\}), \max(\sup_j \{\gamma_A(x), \gamma_{O_j}(x)\}))\} = \{(x, \mu_{A \cap (\cap_{j \in \Lambda} O_j)}(x), \sigma_{A \cap (\cap_{j \in \Lambda} O_j)}(x), \gamma_{A \cap (\cap_{j \in \Lambda} O_j)}(x))\} \in (\tau_n)_A$ . Therefore  $(\tau_n)_A$  is a neutrosophic topology on  $A$ .  $\square$

**Corollary 3.8.** Let  $A$  be a fuzzy set (resp. intuitionistic fuzzy set) of fts  $(X, \tau_f)$  (resp. ifts  $(X, \tau_i)$ ), then the collection  $(\tau_f)_A = \{A \cap O : O \in \tau_f\}$  (resp.  $(\tau_i)_A = \{A \cap O : O \in \tau_i\}$ ) is a fuzzy topology (resp. intuitionistic fuzzy topology) on  $A$ , called the induced fuzzy topology (resp. induced intuitionistic fuzzy topology) on  $A$  and the pair  $(A, (\tau_f)_A)$  (resp.  $(A, (\tau_i)_A)$ ) is called fuzzy subspace (resp. intuitionistic fuzzy subspace).

*Proof.* : Proof follows from the above corollary. □

**Lemma 3.9.** Let  $(A, (\tau_n)_A)$  be a neutrosophic subspace of nts  $(X, \tau_n)$  and  $B \subseteq A$ . If  $B$  is  $(\tau_n)_A$ -open in  $(A, (\tau_n)_A)$  and  $A$  is neutrosophic open in nts  $(X, \tau_n)$ , then  $B$  is neutrosophic open in  $(X, \tau_n)$ .

*Proof.* : Since  $B$  is  $(\tau_n)_A$ -open in  $(A, (\tau_n)_A)$ ,  $B = A \cap O$  for some neutrosophic open set  $O$  in  $(X, \tau_n)$  and so  $B$  is neutrosophic open in  $(X, \tau_n)$ . □

**Lemma 3.10.** Let  $(A, (\tau_f)_A)$  (resp.  $(A, (\tau_i)_A)$ ) be a fuzzy subspace (resp. intuitionistic fuzzy subspace) of fts  $(X, \tau_f)$  (resp. of ifts  $(X, \tau_i)$ ) and  $B \subseteq A$ . If  $B$  is  $(\tau_f)_A$ -open (resp.  $(\tau_i)_A$ -open) in  $(A, (\tau_f)_A)$  (resp.  $(A, (\tau_i)_A)$ ) and  $A$  is fuzzy open (resp. intuitionistic fuzzy open) in fts  $(X, \tau_f)$  (resp. ifts  $(X, \tau_i)$ ), then  $B$  is fuzzy open (resp. intuitionistic fuzzy open) in  $(X, \tau_f)$  (resp. ifts  $(X, \tau_i)$ ).

*Proof.* : Proof is similar as above lemma. □

**Remark 3.11.** In classical topology, we know that if  $(A, \tau_A)$  is a subspace of  $(X, \tau)$  and  $B \subseteq A$ , then

- (i)  $B = A \cap F$ , where  $F$  is closed in  $X$  if and only if  $B$  is closed in  $A$ .
- (ii)  $B$  is closed in  $X$ , if  $B$  is closed in  $A$  and  $A$  is closed in  $X$ .

The following examples illustrate that these are not true in fuzzy, intuitionistic fuzzy and neutrosophic topological spaces.

**Example 3.12.** Let  $X = \{a, b, c\}$  with  $\tau_n = \{\emptyset, X, ((1, 1, 1), (0, 0, 0), (0.7, 0.7, 0.7)), ((0.6, 0.6, 0.6), (0, 0, 0), (0, 0, 0)), ((1, 1, 1), (0, 0, 0), (0, 0, 0)), ((0.6, 0.6, 0.6), (0, 0, 0), (0.7, 0.7, 0.7))\}$ . Then  $(\tau_n)^c = \{X, \emptyset, ((0, 0, 0), (1, 1, 1), (0.3, 0.3, 0.3)), ((0.4, 0.4, 0.4), (1, 1, 1), (1, 1, 1)), ((0, 0, 0), (1, 1, 1), (1, 1, 1)), ((0.4, 0.4, 0.4), (1, 1, 1), (0.3, 0.3, 0.3))\}$ . Let  $A = ((0.6, 0.6, 0.2), (1, 0, 1), (0.8, 0.7, 0.6))$ , then  $(\tau_n)_A = \{\emptyset, A, ((0.6, 0.6, 0.2), (0, 0, 0), (0.8, 0.7, 0.7)), ((0.6, 0.6, 0.2), (0, 0, 0), (0.8, 0.7, 0.6))\}$  and  $((\tau_n)_A)^c = \{A, \emptyset, ((0, 0, 0), (1, 0, 1), (1, 1, 0.9)), ((0, 0, 0), (1, 0, 1), (1, 1, 1))\}$ . Clearly  $B = ((0, 0, 0), (1, 0, 1), (1, 1, 0.9))$  is  $(\tau_n)_A$ -closed in  $(A, (\tau_n)_A)$  and  $B \neq A \cap F$  for every neutrosophic closed set  $F$  in  $(X, \tau_n)$ . Since  $A = ((0, 0, 0), (1, 1, 1), (0.3, 0.3, 0.3))$  is neutrosophic closed in  $(X, \tau_n)$ , then  $(\tau_n)_A = \{\emptyset, A, ((0, 0, 0), (0, 0, 0), (0.7, 0.7, 0.7)), ((0, 0, 0), (0, 0, 0), (0.3, 0.3, 0.3))\}$  and  $((\tau_n)_A)^c = \{A, \emptyset, ((0, 0, 0), (1, 1, 1), (0.6, 0.6, 0.6)), ((0, 0, 0), (1, 1, 1), (1, 1, 1))\}$ . Clearly  $B = ((0, 0, 0), (1, 1, 1), (0.6, 0.6, 0.6))$  is  $(\tau_n)_A$ -closed in  $(A, (\tau_n)_A)$ , but it is not neutrosophic closed in  $(X, \tau_n)$ .

**Example 3.13.** Let  $X = \{a, b, c\}$  with  $\tau_i = \{\emptyset, X, ((0.4, 0.4, 0.3), (0.6, 0.6, 0.7)), ((0.3, 0.8, 0.1), (0.7, 0.2, 0.9)), ((0.3, 0.4, 0.1), (0.7, 0.6, 0.9)), ((0.4, 0.8, 0.3), (0.6, 0.2, 0.7))\}$ . Then  $(\tau_i)^c = \{X, \emptyset, ((0.6, 0.6, 0.7), (0.4, 0.4, 0.3)), ((0.7, 0.2, 0.9), (0.3, 0.8, 0.1)), ((0.7, 0.6, 0.9), (0.3, 0.4, 0.1)), ((0.6, 0.2, 0.7), (0.4, 0.8, 0.3))\}$ . Since  $A = ((0.7, 0.2, 0.9), (0.3, 0.8, 0.1))$  is intuitionistic fuzzy closed in  $(X, \tau_i)$ , then  $(\tau_i)_A = \{\emptyset, A, ((0.4, 0.2, 0.3), (0.6, 0.8, 0.7)), ((0.3, 0.2, 0.1), (0.7, 0.8, 0.9))\}$  and  $((\tau_i)_A)^c = \{A, \emptyset, ((0.3, 0, 0.6), (0.7, 1, 0.4)), ((0.4, 0, 0.8), (0.6, 1, 0.2))\}$ . Clearly  $B = ((0.3, 0, 0.6), (0.7, 1, 0.4))$  is  $(\tau_i)_A$ -closed in  $(A, (\tau_i)_A)$ , but  $B \neq A \cap F$  for every intuitionistic fuzzy closed set  $F$  in  $(X, \tau_i)$  and  $B$  is not intuitionistic fuzzy closed in  $(X, \tau_i)$ .

**Example 3.14.** Let  $X = \{a, b, c\}$  with  $\tau_f = \{\emptyset, X, (0.2, 0.3, 0.1), (0.7, 0.1, 0.8), (0.2, 0.1, 0.1), (0.7, 0.3, 0.8)\}$ . Then  $(\tau_f)^c = \{X, \emptyset, (0.8, 0.7, 0.9), (0.3, 0.9, 0.2), (0.8, 0.9, 0.9), (0.3, 0.7, 0.2)\}$ . Since  $A = (0.8, 0.7, 0.9)$  is fuzzy closed in  $(X, \tau_f)$ , then  $(\tau_f)_A = \{\emptyset, A, (0.2, 0.3, 0.1), (0.7, 0.1, 0.8), (0.2, 0.1, 0.1), (0.7, 0.3, 0.8)\}$  and  $((\tau_f)_A)^c = \{A, \emptyset, (0.6, 0.4, 0.8), (0.1, 0.6, 0.1), (0.6, 0.6, 0.8), (0.1, 0.4, 0.1)\}$ . Clearly  $B = (0.6, 0.6, 0.8)$  is  $(\tau_f)_A$ -closed in  $(A, (\tau_f)_A)$ , but  $B \neq A \cap F$  for every fuzzy closed set  $F$  in  $(X, \tau_f)$  and  $B$  is not fuzzy closed in  $(X, \tau_f)$ .

## 4 Neutrosophic Supra Topological Spaces

In this section, we introduce neutrosophic supra topological spaces and also establish its properties.

**Definition 4.1.** A subcollection  $\tau_n^*$  of neutrosophic sets on a non empty set  $X$  is said to be a neutrosophic supra topology on  $X$  if the sets  $\emptyset, X \in \tau_n^*$  and  $\bigcup_{i=1}^\infty A_i \in \tau_n^*$ , for  $\{A_i\}_{i=1}^\infty \in \tau_n^*$ . Then  $(X, \tau_n^*)$  is called neutrosophic supra topological space on  $X$  ( for short nsts). The members of  $\tau_n^*$  are known as neutrosophic supra open sets and its complement is called neutrosophic supra closed. A neutrosophic supra topology  $\tau_n^*$  on  $X$  is said to be an associated neutrosophic supra topology with neutrosophic topology  $\tau_n$  if  $\tau_n \subseteq \tau_n^*$ . Every neutrosophic topology on  $X$  is neutrosophic supra topology on  $X$ .

**Remark 4.2.** The following table illustrates the comparision of fuzzy supra topological spaces, intuitionistic supra topological spaces, neutrosophic supra topological spaces.

Comparison Table

S.No	Fuzzy supra topological spaces	Intuitionistic supra topological spaces	Neutrosophic supra topological spaces
1	It deals with fuzzy sets	It deals with intuitionistic sets	It deals with neutrosophic sets
2	A subcollection $\tau_f^*$ of fuzzy sets on a non empty set $X$ is said to be a fuzzy supra topology on $X$ if the sets $\emptyset, X \in \tau_f^*$ and $\bigcup_{i=1}^\infty A_i \in \tau_f^*$ , for $\{A_i\}_{i=1}^\infty \in \tau_f^*$ .	A subcollection $\tau_i^*$ of intuitionistic sets on a non empty set $X$ is said to be a intuitionistic supra topology on $X$ if the sets $\emptyset, X \in \tau_i^*$ and $\bigcup_{i=1}^\infty A_i \in \tau_i^*$ , for $\{A_i\}_{i=1}^\infty \in \tau_i^*$ .	A subcollection $\tau_n^*$ of neutrosophic sets on a non empty set $X$ is said to be a neutrosophic supra topology on $X$ if the sets $\emptyset, X \in \tau_n^*$ and $\bigcup_{i=1}^\infty A_i \in \tau_n^*$ , for $\{A_i\}_{i=1}^\infty \in \tau_n^*$ .
3	A non empty set $X$ together with the collection $\tau_f^*$ is called fuzzy supra topological space on $X$ ( for short fsts) denoted by the ordered pair $(X, \tau_f^*)$ .	A non empty set $X$ together with the collection $\tau_i^*$ is called intuitionistic supra topological space on $X$ ( for short ists) denoted by the ordered pair $(X, \tau_i^*)$ .	A non empty set $X$ together with the collection $\tau_n^*$ is called neutrosophic supra topological space on $X$ ( for short nsts) denoted by the ordered pair $(X, \tau_n^*)$ .
4	The members of $\tau_f^*$ are known as fuzzy supra open sets.	The members of $\tau_i^*$ are known as intuitionistic supra open sets.	The members of $\tau_n^*$ are known as neutrosophic supra open sets.
5	It is a generalization of classical supra topological spaces.	It is a generalization of fuzzy supra topological spaces.	It is a generalization of intuitionistic supra topological spaces.
6	Every fuzzy topology is fuzzy supra topology.	Every intuitionistic topology is intuitionistic supra topology.	Every neutrosophic topology is neutrosophic supra topology.

**Proposition 4.3.** The collection  $(\tau_n^*)^c$  of all neutrosophic supra closed sets in  $(X, \tau_n^*)$  satisfies:  $\emptyset, X \in (\tau_n^*)^c$  and  $(\tau_n^*)^c$  is closed under arbitrary intersection.

*Proof.* : The proof is obvious. □

**Lemma 4.4.** As Proposition 3.4, every neutrosophic supra topology  $\tau_n^*$  induce three fuzzy supra topologies  $\tau_{f_1}^* = \{A = (x, \mu_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n^*\}$ ,  $\tau_{f_2}^* = \{A = (x, \sigma_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n^*\}$  and  $\tau_{f_3}^* = \{A = (x, 1 - \gamma_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n^*\}$  on  $X$ .

**Definition 4.5.** The neutrosophic supra topological interior  $int_{\tau_n^*}(A)$  and closure  $cl_{\tau_n^*}(A)$  operators of a neutrosophic set  $A$  are respectively defined as:  $int_{\tau_n^*}(A) = \cup\{G : G \subseteq A \text{ and } G \in \tau_n^*\}$  and  $cl_{\tau_n^*}(A) = \cap\{F : A \subseteq F \text{ and } F^c \in \tau_n^*\}$ .

**Theorem 4.6.** The following are true for neutrosophic sets  $A$  and  $B$  of nsts  $(X, \tau_n^*)$ :

- (i)  $A = cl_{\tau_n^*}(A)$  if and only if  $A$  is neutrosophic supra closed.
- (ii)  $A = int_{\tau_n^*}(A)$  if and only if  $A$  is neutrosophic supra open.
- (iii)  $cl_{\tau_n^*}(A) \subseteq cl_{\tau_n^*}(B)$ , if  $A \subseteq B$ .
- (iv)  $int_{\tau_n^*}(A) \subseteq int_{\tau_n^*}(B)$ , if  $A \subseteq B$ .
- (v)  $cl_{\tau_n^*}(A) \cup cl_{\tau_n^*}(B) \subseteq cl_{\tau_n^*}(A \cup B)$ .
- (vi)  $int_{\tau_n^*}(A) \cup int_{\tau_n^*}(B) \subseteq int_{\tau_n^*}(A \cup B)$ .
- (vii)  $cl_{\tau_n^*}(A) \cap cl_{\tau_n^*}(B) \supseteq cl_{\tau_n^*}(A \cap B)$ .
- (viii)  $int_{\tau_n^*}(A) \cap int_{\tau_n^*}(B) \supseteq int_{\tau_n^*}(A \cap B)$ .
- (ix)  $int_{\tau_n^*}(A^c) = (cl_{\tau_n^*}(A))^c$ .

*Proof.* : Here we shall prove parts (iii), (v) and (ix) only. The remaining parts similarly follows. Part (iii):  $cl_{\tau_n^*}(B) = \cap\{G : G^c \in \tau_n^*, B \subseteq G\} \supseteq \cap\{G : G^c \in \tau_n^*, A \subseteq G\} = cl_{\tau_n^*}(A)$ . Thus,  $cl_{\tau_n^*}(A) \subseteq cl_{\tau_n^*}(B)$ . Part (v): Since  $A \cup B \supseteq A, B$ , then  $cl_{\tau_n^*}(A) \cup cl_{\tau_n^*}(B) \subseteq cl_{\tau_n^*}(A \cup B)$ . Part (ix):  $cl_{\tau_n^*}(A) = \cap\{G : G^c \in \tau_n^*, G \supseteq A\}$ ,  $(cl_{\tau_n^*}(A))^c = \cup\{G^c : G^c \text{ is a neutrosophic supra open in } X \text{ and } G^c \subseteq A^c\} = int_{\tau_n^*}(A^c)$ . Thus,  $(cl_{\tau_n^*}(A))^c = int_{\tau_n^*}(A^c)$ . □

**Remark 4.7.** In neutrosophic topological space, we have  $cl_{\tau_n}(A \cup B) = cl_{\tau_n}(A) \cup cl_{\tau_n}(B)$  and  $int_{\tau_n}(A \cap B) = int_{\tau_n}(A) \cap int_{\tau_n}(B)$ . These equalities are not true in neutrosophic supra topological spaces as shown in the following examples.

**Example 4.8.** Let  $X = \{a, b, c\}$  with neutrosophic topology  $\tau_n^* = \{\emptyset, X, ((0.5, 1, 0), (0.5, 1, 0), (0.5, 0, 1)), ((0.25, 0, 1), (0.25, 0, 1), (0.75, 1, 0)), ((0.5, 1, 1), (0.5, 1, 1), (0.5, 0, 0))\}$ . Then  $(\tau_n^*)^c = \{X, \emptyset, ((0.5, 0, 1), (0.5, 0, 1), (0.5, 1, 0)), ((0.75, 1, 0), (0.75, 1, 0), (0.25, 0, 1)), ((0.5, 0, 0), (0.5, 0, 0), (0.5, 1, 1))\}$ . Let  $C = ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1))$  and  $D = ((0.5, 0, 0.5), (0.5, 0, 0.5), (0.5, 1, 0.5))$ , then  $cl_{\tau_n^*}(C) = ((0.75, 1, 0), (0.75, 1, 0), (0.25, 0, 1))$  and  $cl_{\tau_n^*}(D) = ((0.5, 0, 1), (0.5, 0, 1), (0.5, 1, 0))$ , so  $cl_{\tau_n^*}(C) \cup cl_{\tau_n^*}(D) = ((0.75, 1, 1), (0.75, 1, 1), (0.25, 0, 0))$ . But  $C \cup D = ((0.5, 0.5, 0.5), (0.5, 0.5, 0.5), (0.5, 0.5, 0.5))$  and  $cl_{\tau_n^*}(C \cup D) = ((1, 1, 1), (1, 1, 1), (0, 0, 0)) = X$ . Therefore  $cl_{\tau_n^*}(C \cup D) \neq cl_{\tau_n^*}(C) \cup cl_{\tau_n^*}(D)$ .

Let  $E = ((0.5, 1, 0.25), (0.5, 1, 0.25), (0.5, 0, 0.75))$  and  $F = ((0.5, 0.5, 1), (0.5, 0.5, 1), (0.5, 0.5, 0))$ . Then  $int_{\tau_n^*}(E) = ((0.5, 1, 0), (0.5, 1, 0), (0.5, 0, 1))$  and  $int_{\tau_n^*}(F) = ((0.25, 0, 1), (0.25, 0, 1), (0.75, 1, 0))$ , so  $int_{\tau_n^*}(E) \cap int_{\tau_n^*}(F) = ((0.25, 0, 0), (0.25, 0, 0), (0.75, 1, 1))$ . But  $E \cap F = ((0.5, 0.5, 0.25), (0.5, 0.5, 0.25), (0.5, 0.5, 0.75))$  and  $int_{\tau_n^*}(E \cap F) = ((0, 0, 0), (0, 0, 0), (1, 1, 1)) = \emptyset$ . Therefore  $int_{\tau_n^*}(E \cap F) \neq int_{\tau_n^*}(E) \cap int_{\tau_n^*}(F)$ .

## 5 Mappings of Neutrosophic Spaces

In this section, we define and establish the properties of some mappings in neutrosophic supra topological spaces and neutrosophic subspaces.

**Definition 5.1.** Let  $\tau_n^*$  and  $\sigma_n^*$  be associated neutrosophic supra topologies with respect to  $\tau_n$  and  $\sigma_n$ . A mapping  $f$  from a nts  $(X, \tau_n)$  into nts  $(Y, \sigma_n)$  is said to be  $S^*$ -neutrosophic open if the image of every neutrosophic open set in  $(X, \tau_n)$  is neutrosophic supra open in  $(Y, \sigma_n^*)$  and  $f : X \rightarrow Y$  is said to be  $S^*$ -neutrosophic continuous if the inverse image of every neutrosophic open set in  $(Y, \sigma_n)$  is neutrosophic supra open in  $(X, \tau_n^*)$ .

**Definition 5.2.** Let  $\tau_n^*$  and  $\sigma_n^*$  be associated neutrosophic supra topologies with respect to nts's  $\tau_n$  and  $\sigma_n$ . A mapping  $f$  from a nts  $(X, \tau_n)$  into a nts  $(Y, \sigma_n)$  is said to be supra neutrosophic open if the image of every neutrosophic supra open set in  $(X, \tau_n^*)$  is a neutrosophic supra open in  $(Y, \sigma_n^*)$  and  $f : X \rightarrow Y$  is said to be supra neutrosophic continuous if the inverse image of every neutrosophic supra open set in  $(Y, \sigma_n^*)$  is neutrosophic supra open in  $(X, \tau_n^*)$ .

A mapping  $f$  of nts  $(X, \tau_n)$  into nts  $(Y, \sigma_n)$  is said to be a mapping of neutrosophic subspace  $(A, (\tau_n)_A)$  into neutrosophic subspace  $(B, (\sigma_n)_B)$  if  $f(A) \subset B$ .

**Definition 5.3.** A mapping  $f$  of neutrosophic subspace  $(A, (\tau_n)_A)$  of nts  $(X, \tau_n)$  into neutrosophic subspace  $(B, (\sigma_n)_B)$  of nts  $(Y, \sigma_n)$  is said to be relatively neutrosophic continuous if  $f^{-1}(O) \cap A \in (\tau_n)_A$  for every  $O \in (\sigma_n)_B$ . If  $f(O') \in (\sigma_n)_B$  for every  $O' \in (\tau_n)_A$ , then  $f$  is said to be relatively neutrosophic open.

**Theorem 5.4.** If a mapping  $f$  is neutrosophic continuous from nts  $(X, \tau_n)$  into nts  $(Y, \sigma_n)$  and  $f(A) \subset B$ . Then  $f$  is relatively neutrosophic continuous from neutrosophic subspace  $(A, (\tau_n)_A)$  of nts  $(X, \tau_n)$  into neutrosophic subspace  $(B, (\sigma_n)_B)$  of nts  $(Y, \sigma_n)$ .

*Proof.* : Let  $O \in (\sigma_n)_B$ , then there exists  $G \in \sigma_n$  such that  $O = B \cap G$  and  $f^{-1}(G) \in \tau_n$ . Therefore  $f^{-1}(O) \cap A = f^{-1}(B) \cap f^{-1}(G) \cap A = f^{-1}(G) \cap A \in (\tau_n)_A$ . □

**Remark 5.5.** (i) Every neutrosophic continuous (resp. neutrosophic open) mapping is  $S^*$ -neutrosophic continuous (resp.  $S^*$ -neutrosophic open), but converse need not be true.

(ii) Every supra neutrosophic continuous (resp. supra neutrosophic open) mapping is  $S^*$ -neutrosophic continuous (resp.  $S^*$ -neutrosophic open), but converse need not be true.

(iii) Supra neutrosophic continuous and neutrosophic continuous mappings are independent each other.

(iv) Supra neutrosophic open and neutrosophic open mappings are independent each other.

*Proof.* : The proof follows from the definition, the converse and independence are shown in the following example. □

**Example 5.6.** Let  $Y = \{x, y, z\}$ ,  $X = \{a, b, c\}$  with neutrosophic topologies  $\tau_n = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1))\}$  and  $\sigma_n = \{\emptyset, Y, ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))\}$ . Let  $\tau_n^* = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1)), ((0.5, 0.5, 0.5), (0.5, 0.5, 0.5), (0.5, 0.5, 0.5)), ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))\}$  and  $\sigma_n^* = \sigma_n$  be associated neutrosophic supra topologies with respect to  $\tau_n$  and  $\sigma_n$ . Define a mapping  $f : X \rightarrow Y$  by  $f(c) = z, f(b) = y, f(a) = x$ , then  $f^{-1}(((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))) = ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5)) \in \tau_n^*$ . Clearly  $f$  is supra neutrosophic continuous and  $S^*$ -neutrosophic continuous but not neutrosophic continuous.

Let  $Y = \{x, y, z\}$ ,  $X = \{a, b, c\}$  with neutrosophic topologies  $\tau_n = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1)), ((0.5, 0.25, 0), (0.5, 0.25, 0), (0.5, 0.75, 1))\}$  and  $\sigma_n = \{\emptyset, Y, ((0.5, 0.25, 0), (0.5, 0.25, 0), (0.5, 0.75, 1))\}$ . Let  $\tau_n^* = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1)), ((0.5, 0.25, 0), (0.5, 0.25, 0), (0.5, 0.75, 1)), ((1, 0.5, 0), (1, 0.5, 0), (0, 0.5, 1))\}$  and  $\sigma_n^* = \{\emptyset, Y, ((0.5, 0.25, 0), (0.5, 0.25, 0), (0.5, 0.75, 1)), ((0.3, 0.25, 0.5), (0.3, 0.25, 0.5), (0.7, 0.75, 0.5)), ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))\}$  be associated neutrosophic supra topologies with respect to  $\tau_n$  and  $\sigma_n$ . Consider a mapping  $f : X \rightarrow Y$  by  $f(c) = z, f(b) = y, f(a) = x$ , then  $f^{-1}(((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))) = ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5)) \notin \tau_n^*$ . Therefore  $f$  is neutrosophic continuous and  $S^*$ -neutrosophic continuous but not supra neutrosophic continuous. If we consider a mapping  $g : Y \rightarrow X$  by  $g(z) = c, g(y) = b, g(x) = a$ , then  $g$  is neutrosophic open and  $S^*$ -neutrosophic open but not supra neutrosophic open.

Let  $Y = \{x, y, z\}$ ,  $X = \{a, b, c\}$  with neutrosophic topologies  $\tau_n = \{\emptyset, X, ((1, 0.5, 0.3), (1, 0.5, 0.3), (0, 0.5, 0.7))\}$  and  $\sigma_n = \{\emptyset, Y, ((1, 0.3, 0.5), (1, 0.3, 0.5), (0, 0.7, 0.5))\}$ . Let  $\sigma_n^* = \{\emptyset, Y, ((1, 0.3, 0.5), (1, 0.3, 0.5), (0, 0.7, 0.5)), ((1, 0.5, 0.3), (1, 0.5, 0.3), (0, 0.5, 0.7)), ((1, 0.5, 0.5), (1, 0.5, 0.5), (0, 0.5, 0.5))\}$  and  $\tau_n^* = \tau_n$  be associated neutrosophic supra topologies with respect to  $\sigma_n$  and  $\tau_n$ . Then  $f : X \rightarrow Y$  defined by  $f(c) = z, f(b) = y, f(a) = x$  is  $S^*$ -neutrosophic open and supra neutrosophic open but not neutrosophic open.

**Observation 5.7.** The following are the examples of contradicting the statements of Abd-Monsef and Ramadan<sup>[9]</sup>. In fuzzy supra topological space, consider  $Y = \{x, y, z\}$ ,  $X = \{a, b, c\}$  with fuzzy topologies  $\tau_f = \{\emptyset, X, (0.5, 0.5, 0), (0.5, 0.25, 0)\}$  and  $\sigma_f = \{\emptyset, Y, (0.5, 0.25, 0)\}$ . Let  $\tau_f^* = \{\emptyset, X, (0.5, 0.5, 0), (0.5, 0.25, 0), (1, 0.5, 0)\}$  and  $\sigma_f^* = \{\emptyset, Y, (0.5, 0.25, 0), (0.3, 0.25, 0.5), (0.5, 0.25, 0.5)\}$  be associated fuzzy supra topologies with respect to  $\tau_f$  and  $\sigma_f$ . Consider a mapping  $h : X \rightarrow Y$  by  $h(c) = z, h(b) = y, h(a) = x$ , then  $h^{-1}((0.5, 0.25, 0.5)) = (0.5, 0.25, 0.5) \notin \tau_f^*$ . Then  $h$  is fuzzy continuous but not supra fuzzy continuous. If we define a mapping  $g : Y \rightarrow X$  by  $g(z) = c, g(y) = b, g(x) = a$ , then  $g$  is fuzzy open but not supra fuzzy open.

**Theorem 5.8.** The following statements are equivalent for the mapping  $f$  of nts  $(X, \tau_n)$  into nts  $(Y, \sigma_n)$ :

- (i) The mapping  $f : X \rightarrow Y$  is  $S^*$ -neutrosophic continuous.
- (ii) The inverse image of every neutrosophic closed set in  $(Y, \sigma_n)$  is neutrosophic supra closed in  $(X, \tau_n^*)$ .
- (iii) For each neutrosophic set  $A$  in  $Y$ ,  $cl_{\tau_n^*}(f^{-1}(A)) \subseteq f^{-1}(cl_{\sigma_n}(A))$ .
- (iv) For each neutrosophic set  $B$  in  $X$ ,  $f(cl_{\tau_n^*}(B)) \subseteq cl_{\sigma_n}(f(B))$ .

(v) For each neutrosophic set  $A$  in  $Y$ ,  $int_{\tau_n^*}(f^{-1}(A)) \supseteq f^{-1}(int_{\sigma_n}(A))$ .

*Proof.* : (i)  $\Rightarrow$  (ii): Let  $f$  be a  $S^*$ -neutrosophic continuous and  $A$  be a neutrosophic closed set in  $(Y, \sigma_n)$ ,  $f^{-1}(Y - A) = X - f^{-1}(A)$  is neutrosophic supra open in  $(X, \tau_n^*)$  and so  $f^{-1}(A)$  is neutrosophic supra closed in  $(X, \tau_n^*)$ .

(ii)  $\Rightarrow$  (iii):  $cl_{\sigma_n}(A)$  is neutrosophic closed in  $(Y, \sigma_n)$ , for each neutrosophic set  $A$  in  $Y$ , then  $f^{-1}(cl_{\sigma_n}(A))$  is neutrosophic supra closed in  $(X, \tau_n^*)$ . Thus  $f^{-1}(cl_{\sigma_n}(A)) = cl_{\tau_n^*}(f^{-1}(cl_{\sigma_n}(A))) \supseteq cl_{\tau_n^*}(f^{-1}(A))$ .

(iii)  $\Rightarrow$  (iv):  $f^{-1}(cl_{\sigma_n}(f(B))) \supseteq cl_{\tau_n^*}(f^{-1}(f(B))) \supseteq cl_{\tau_n^*}(B)$ , for each neutrosophic set  $B$  in  $X$  and so  $f(cl_{\tau_n^*}(B)) \subseteq cl_{\sigma_n}(f(B))$ .

(iv)  $\Rightarrow$  (ii): Let  $B = f^{-1}(A)$ , for each neutrosophic closed set  $A$  in  $Y$ , then  $f(cl_{\tau_n^*}(B)) \subseteq cl_{\sigma_n}(f(B)) \subseteq cl_{\sigma_n}(A) = A$  and  $cl_{\tau_n^*}(B) \subseteq f^{-1}(f(cl_{\tau_n^*}(B))) \subseteq f^{-1}(A) = B$ . Therefore  $B = f^{-1}(A)$  is neutrosophic supra closed in  $X$ .

(ii)  $\Rightarrow$  (i): Let  $A$  be a neutrosophic open set in  $Y$ , then  $X - f^{-1}(A) = f^{-1}(Y - A)$  is neutrosophic supra closed in  $X$ , since  $Y - A$  is neutrosophic closed in  $Y$ . Therefore  $f^{-1}(A)$  is neutrosophic supra open in  $X$ .

(i)  $\Rightarrow$  (v):  $f^{-1}(int_{\sigma_n}(A))$  is neutrosophic supra open in  $X$ , for each neutrosophic set  $A$  in  $Y$  and  $int_{\tau_n^*}(f^{-1}(A)) \supseteq int_{\tau_n^*}(f^{-1}(int_{\sigma_n}(A))) = f^{-1}(int_{\sigma_n}(A))$ .

(v)  $\Rightarrow$  (i):  $f^{-1}(A) = f^{-1}(int_{\sigma_n}(A)) \subseteq int_{\tau_n^*}(f^{-1}(A))$ , for each neutrosophic open set  $A$  in  $Y$  and so  $f^{-1}(A)$  is neutrosophic supra open in  $X$ . □

**Theorem 5.9.** The following statements are equivalent for the mapping  $f$  of nts  $(X, \tau_n)$  into nts  $(Y, \sigma_n)$ :

(i) A mapping  $f : (X, \tau_n^*) \rightarrow (Y, \sigma_n^*)$  is neutrosophic supra continuous.

(ii) The inverse image of every neutrosophic supra closed set in  $(Y, \sigma_n^*)$  is neutrosophic supra closed in  $(X, \tau_n^*)$ .

(iii) For each neutrosophic set  $A$  in  $Y$ ,  $cl_{\tau_n^*}(f^{-1}(A)) \subseteq f^{-1}(cl_{\sigma_n^*}(A)) \subseteq f^{-1}(cl_{\sigma_n}(A))$ .

(iv) For each neutrosophic set  $B$  in  $X$ ,  $f(cl_{\tau_n^*}(B)) \subseteq cl_{\sigma_n^*}(f(B)) \subseteq cl_{\sigma_n}(f(B))$ .

(v) For each neutrosophic set  $A$  in  $Y$ ,  $int_{\tau_n^*}(f^{-1}(A)) \supseteq f^{-1}(int_{\sigma_n^*}(A)) \supseteq f^{-1}(int_{\sigma_n}(A))$ .

*Proof.* : The proof is straightforward from theorem 5.8. □

**Theorem 5.10.** If  $f : X \rightarrow Y$  is  $S^*$ -neutrosophic continuous and  $g : Y \rightarrow Z$  is neutrosophic continuous, then  $g \circ f : X \rightarrow Z$  is  $S^*$ -neutrosophic continuous.

*Proof.* : The proof follows directly from the definition. □

**Theorem 5.11.** If  $f : X \rightarrow Y$  is supra neutrosophic continuous and  $g : Y \rightarrow Z$  is  $S^*$ -neutrosophic continuous (or neutrosophic continuous), then  $g \circ f : X \rightarrow Z$  is  $S^*$ -neutrosophic continuous.

*Proof.* : It follows from the definition. □

**Theorem 5.12.** If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are supra neutrosophic continuous (resp. supra neutrosophic open) mappings, then  $g \circ f : X \rightarrow Z$  is supra neutrosophic continuous (resp. supra neutrosophic open).

*Proof.* : The proof follows obviously from the definition. □

**Remark 5.13.** Abd-Monsef and Ramadan<sup>[9]</sup> stated that if  $g : X \rightarrow Y$  is supra fuzzy continuous and  $h : Y \rightarrow Z$  is fuzzy continuous, then  $h \circ g : X \rightarrow Z$  is supra fuzzy continuous. But in general this is not true, for example consider  $Z = \{p, q, r\}$ ,  $Y = \{x, y, z\}$ , and  $X = \{a, b, c\}$  with fuzzy topologies  $\tau_f = \{\emptyset, X, (1, 0.5, 0), (0.3, 0.3, 0)\}$ ,  $\sigma_f = \{\emptyset, Y, (0.5, 0.5, 0), (0.5, 0.25, 0)\}$  and  $\eta_f = \{\emptyset, Z, (0.5, 0.25, 0)\}$  on  $X, Y$  and  $Z$  respectively. Let  $\tau_f^* = \{\emptyset, X, (0.5, 0.5, 0), (0.5, 0.25, 0), (1, 0.5, 0), (0.3, 0.3, 0)\}$ ,  $\sigma_f^* = \{\emptyset, Y, (0.5, 0.5, 0), (0.5, 0.25, 0), (1, 0.5, 0)\}$  and  $\eta_f^* = \{\emptyset, Z, (0.5, 0.25, 0), (0.3, 0.25, 0.5), (0.5, 0.25, 0.5)\}$  be associated fuzzy supra topologies with respect to  $\tau_f, \sigma_f$  and  $\eta_f$ . Then the mapping  $g : X \rightarrow Y$  defined by  $g(c) = z, g(b) = y, g(a) = x$  is supra fuzzy continuous and the mapping  $h : Y \rightarrow Z$  by  $h(z) = r, h(y) = q, h(x) = p$  is fuzzy continuous. But  $h \circ g : X \rightarrow Z$  is not supra fuzzy continuous, since  $(g \circ h)^{-1}((0.3, 0.25, 0.5)) = (0.3, 0.25, 0.5) \notin \tau_f^*$ .

**Remark 5.14.** In general the composition of two supra neutrosophic continuous mappings is again supra neutrosophic continuous, but the composition of two  $S^*$ -neutrosophic continuous mappings may not be  $S^*$ -neutrosophic continuous. Let  $Z = \{p, q, r\}$ ,  $Y = \{x, y, z\}$ , and  $X = \{a, b, c\}$  with neutrosophic topologies  $\tau_n = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1))\}$ ,  $\sigma_n = \{\emptyset, Y, ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))\}$  and  $\eta_n = \{\emptyset, Z, ((0.3, 0.7, 0.5), (0.3, 0.7, 0.5), (0.7, 0.3, 0.5))\}$  on  $X, Y$  and  $Z$  respectively. Let  $\tau_n^* = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1)), ((0.5, 0.5, 0.5), (0.5, 0.5, 0.5), (0.5, 0.5, 0.5)), ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))\}$  and  $\sigma_n^* = \{\emptyset, Y, ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5)), ((0.3, 0.7, 0.5), (0.3, 0.7, 0.5), (0.7, 0.3, 0.5)), ((0.5, 0.7, 0.5), (0.5, 0.7, 0.5), (0.5, 0.3, 0.5))\}$  be associated neutrosophic supra topologies with respect to  $\tau_n$  and  $\sigma_n$ . Then the mappings  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are defined respectively by  $f(c) = z, f(b) = y, f(a) = x$  and  $g(z) = r, g(y) = q, g(x) = p$  are  $S^*$ -neutrosophic continuous. But  $g \circ f : X \rightarrow Z$  is not  $S^*$ -neutrosophic continuous, since  $(g \circ f)^{-1}(((0.3, 0.7, 0.5), (0.3, 0.7, 0.5), (0.7, 0.3, 0.5))) = ((0.3, 0.7, 0.5), (0.3, 0.7, 0.5), (0.7, 0.3, 0.5)) \notin \tau_n^*$ .

**Theorem 5.15.** If mappings  $f : (A, (\tau_n)_A) \rightarrow (B, (\sigma_n)_B)$  from neutrosophic subspace  $(A, (\tau_n)_A)$  of nts  $(X, \tau_n)$  into neutrosophic subspace  $(B, (\sigma_n)_B)$  of nts  $(Y, \sigma_n)$  and  $g : (B, (\sigma_n)_B) \rightarrow (C, (\eta_n)_C)$  from neutrosophic subspace  $(B, (\sigma_n)_B)$  of nts  $(Y, \sigma_n)$  into neutrosophic subspace  $(C, (\eta_n)_C)$  of nts  $(Z, \eta_n)$  are relatively neutrosophic continuous (resp. relatively neutrosophic open) mappings, then  $g \circ f : (A, (\tau_n)_A) \rightarrow (C, (\eta_n)_C)$  is relatively neutrosophic continuous (resp. relatively neutrosophic open) from neutrosophic subspace  $(A, (\tau_n)_A)$  of nts  $(X, \tau_n)$  into neutrosophic subspace  $(C, (\eta_n)_C)$  of nts  $(Z, \eta_n)$ .

*Proof.* : Let  $O \in (\eta_n)_C$ , then  $g^{-1}(O) \cap B \in (\sigma_n)_B$  and  $f^{-1}(g^{-1}(O) \cap B) \cap A \in (\tau_n)_A$ . Since  $B \supset f(A)$ , then  $(g \circ f)^{-1}(O) \cap A = f^{-1}(g^{-1}(O) \cap B) \cap A$ . Therefore  $g \circ f$  is relatively neutrosophic continuous. Let  $U \in (\tau_n)_A$ , then  $f(U) \in (\sigma_n)_B$  and  $g(f(U)) = (g \circ f)(U) \in (\eta_n)_C$ . Therefore  $g \circ f$  is relatively neutrosophic open.  $\square$

## 6 Neutrosophic Supra Topology in Data Mining

In this section, we present a methodical approach for decision-making problem with single valued neutrosophic information. The following necessary steps are proposed the methodical approach to select the proper attributes and alternatives in the decision-making situation.

### Step 1: Problem field selection:

Consider multi-attribute decision making problems with  $m$  attributes  $A_1, A_2, \dots, A_m$  and  $n$  alternatives  $C_1, C_2, \dots, C_n$  and  $p$  attributes  $D_1, D_2, \dots, D_p$ , ( $n \leq p$ ).

	$C_1$	$C_2$	.	.	.	$C_n$
$A_1$	$(a_{11})$	$(a_{12})$	.	.	.	$(a_{1n})$
$A_2$	$(a_{21})$	$(a_{22})$	.	.	.	$(a_{2n})$
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
$A_m$	$(a_{m1})$	$(a_{m2})$	.	.	.	$(a_{mn})$

	$A_1$	$A_2$	.	.	.	$A_m$
$D_1$	$(d_{11})$	$(d_{12})$	.	.	.	$(d_{1m})$
$D_2$	$(d_{21})$	$(d_{22})$	.	.	.	$(d_{2m})$
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
$D_p$	$(d_{p1})$	$(d_{p2})$	.	.	.	$(d_{pm})$

Here all the attributes  $a_{ij}$  and  $d_{ki}$  ( $i = 1, 2, \dots, m, j = 1, 2, \dots, n$  and  $k = 1, 2, \dots, p$ ) are single valued neutrosophic numbers.

**Step 2: Form neutrosophic supra topologies for  $(C_j)$  and  $(D_k)$ :**

- (i)  $\tau_j^* = A \cup B$ , where  $A = \{1_N, 0_N, a_{1j}, a_{2j}, \dots, a_{mj}\}$  and  $B = \{a_{1j} \cup a_{2j}, a_{1j} \cup a_{3j}, \dots, a_{m-1j} \cup a_{mj}\}$ .
- (ii)  $\nu_k^* = C \cup D$ , where  $C = \{1_N, 0_N, d_{k1}, d_{k2}, \dots, d_{km}\}$  and  $D = \{d_{k1} \cup d_{k2}, d_{k1} \cup d_{k3}, \dots, d_{km-1} \cup d_{km}\}$ .

**Step 3: Find Single valued neutrosophic score functions:**

Single valued neutrosophic score functions (shortly SVNSF) of  $A, B, C, D, C_j$  and  $D_k$  are defined as follows.

- (i)  $SVNSF(A) = \frac{1}{3(m+2)} [\sum_{i=1}^{m+2} [2 + \mu_i - \sigma_i - \gamma_i]]$ , and  $SVNSF(B) = \frac{1}{3q} [\sum_{i=1}^q [2 + \mu_i - \sigma_i - \gamma_i]]$ , where  $q$  is the number of elements of  $B$ . For  $j = 1, 2, \dots, n$ ,
 
$$SVNSF(C_j) = \begin{cases} SVNSF(A) & \text{if } SVNSF(B) = 0 \\ \frac{1}{2}[SVNSF(A) + SVNSF(B)] & \text{otherwise} \end{cases}$$
- (ii)  $SVNSF(C) = \frac{1}{3(m+2)} [\sum_{i=1}^{m+2} [2 + \mu_i - \sigma_i - \gamma_i]]$  and  $SVNSF(D) = \frac{1}{3r} [\sum_{i=1}^r [2 + \mu_i - \sigma_i - \gamma_i]]$ , where  $r$  is the number of elements of  $D$ . For  $k = 1, 2, \dots, p$ ,
 
$$SVNSF(D_k) = \begin{cases} SVNSF(C) & \text{if } SVNSF(D) = 0 \\ \frac{1}{2}[SVNSF(C) + SVNSF(D)] & \text{otherwise} \end{cases}$$

**Step 4: Final Decision**

Arrange single valued neutrosophic score values for the alternatives  $C_1 \leq C_2 \leq \dots \leq C_n$  and the attributes  $D_1 \leq D_2 \leq \dots \leq D_p$ . Choose the attribute  $D_p$  for the alternative  $C_1$  and  $D_{p-1}$  for the alternative  $C_2$  etc. If  $n < p$ , then ignore  $D_k$ , where  $k = 1, 2, \dots, n - p$ .

## 7 Numerical Example

Medical diagnosis has increased volume of information available to physicians from new medical technologies and comprises of uncertainties. In medical diagnosis, very difficult task is the process of classifying different set of symptoms under a single name of a disease. In this section, we exemplify a medical diagnosis problem for effectiveness and applicability of above proposed approach.

**Step 1: Problem field selection:**

Consider the following tables giving informations when consulted physicians about four patients  $P_1, P_2, P_3, P_4$  and symptoms are Temperature, Cough, Blood Plates, Joint Pain, Insulin. We need to find the patient and to find the disease such as Tuberculosis, Diabetes, Chikungunya, Swine Flu, Dengue of the patient. The data in Table 1 are explained by the membership, the indeterminacy and the non-membership functions. From Table 2, we can observe that for tuberculosis, cough is high ( $\mu = 0.9, \sigma = 0.1, \gamma = 0.1$ ), but for chikungunya, cough is low ( $\mu = 0, \sigma = 0.1, \gamma = 0.9$ ).

Table 1.

Patients \ Symptoms	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>
Temperature	(0.8,0,0.2)	(0.1,0,0.7)	(0.9,0.1,0)	(0,0.1,0.9)
Cough	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0,0.3,0.7)	(0.8,0.1,0.2)
Blood Plates	(0.8,0,0.2)	(0.2,0.1,0.6)	(0.3,0.1,0.6)	(0.3,0.1,0.6)
Joint Pain	(0.4,0.2,0.5)	(0.4,0.2,0.5)	(0.9,0,0.1)	(0.2,0.2,0.7)
Insulin	(0.3,0.2,0.5)	(0.9,0,0.1)	(0.2,0.1,0.7)	(0.4,0.3,0.2)

Table 2.

Symptoms \ Diagnosis	Temperature	Cough	Blood Plates	Joint Pain	Insulin
Tuberculosis	(0.6,0.3,0.1)	(0.9,0.1,0.1)	(0,0.2,0.8)	(0,0.1,0.8)	(0,0.1,0.9)
Diabetes	(0.1,0.1,0.8)	(0.1,0.1,0.8)	(0.2,0.2,0.1)	(0.1,0.4,0.6)	(0.9,0,0.1)
Chikungunya	(0.9,0,0.1)	(0,0.1,0.9)	(0.7,0.2,0.1)	(0.9,0.1,0.1)	(0.2,0,0.8)
Swine Flu	(0.2,0.5,0.3)	(0.1,0.4,0.3)	(0.2,0.4,0.1)	(0.1,0.3,0.5)	(0.2,0.4,0.1)
Dengue	(0.9,0,0.1)	(0.2,0.6,0.4)	(0.2,0.6,0.4)	(0.3,0.1,0.6)	(0.2,0.1,0.7)

**Step 2: Form neutrosophic supra topologies for  $(C_j)$  and  $(D_k)$ :**

- (i)  $\tau_1^* = A \cup B$ , where  $A = \{(1, 1, 0), (0, 0, 1), (0.8, 0, 0.2), (0.1, 0.2, 0.7), (0.4, 0.2, 0.5), (0.3, 0.2, 0.5)\}$  and  $B = \{(0.8, 0.2, 0.2)\}$ .
- (ii)  $\tau_2^* = A \cup B$ , where  $A = \{(1, 1, 0), (0, 0, 1), (0.1, 0, 0.7), (0.1, 0.1, 0.8), (0.2, 0.1, 0.6), (0.4, 0.2, 0.5), (0.9, 0, 0.1)\}$  and  $B = \{(0.1, 0.1, 0.7), (0.9, 0.1, 0.1), (0.9, 0.2, 0.1)\}$ .
- (iii)  $\tau_3^* = A \cup B$ , where  $A = \{(1, 1, 0), (0, 0, 1), (0.9, 0.1, 0), (0, 0.3, 0.7), (0.3, 0.1, 0.6), (0.9, 0, 0.1), (0.2, 0.1, 0.7)\}$  and  $B = \{(0.9, 0.3, 0), (0.3, 0.3, 0.6), (0.9, 0.3, 0.1), (0.2, 0.3, 0.7), (0.9, 0.1, 0.1)\}$ .
- (iv)  $\tau_4^* = A \cup B$ , where  $A = \{(1, 1, 0), (0, 0, 1), (0, 0.1, 0.9), (0.8, 0.1, 0.2), (0.3, 0.1, 0.6), (0.2, 0.2, 0.7), (0.4, 0.3, 0.2)\}$  and  $B = \{(0.8, 0.2, 0.2), (0.8, 0.3, 0.2), (0.3, 0.2, 0.6)\}$ .
- (i)  $\nu_1^* = C \cup D$ , where  $C = \{(1, 1, 0), (0, 0, 1), (0.6, 0.3, 0.1), (0.9, 0.1, 0.1), (0, 0.2, 0.8), (0, 0.1, 0.8), (0, 0.1, 0.9)\}$  and  $D = \{(0.9, 0.3, 0.1), (0.9, 0.2, 0.1)\}$ .

- (ii)  $\nu_2^* = C \cup D$ , where  $C = \{(1, 1, 0), (0, 0, 1), (0.1, 0.1, 0.8), (0.2, 0.2, 0.1), (0.1, 0.4, 0.6), (0.9, 0, 0.1)\}$  and  $D = \{(0.9, 0.1, 0.1), (0.2, 0.4, 0.1), (0.9, 0.2, 0.1), (0.9, 0.4, 0.1)\}$ .
- (iii)  $\nu_3^* = C \cup D$ , where  $C = \{(1, 1, 0), (0, 0, 1), (0.9, 0, 0.1), (0, 0.1, 0.9), (0.7, 0.2, 0.1), (0.9, 0.1, 0.1), (0.2, 0, 0.8)\}$  and  $D = \{(0.9, 0.2, 0.1), (0.2, 0.1, 0.8)\}$ .
- (iv)  $\nu_4^* = C \cup D$ , where  $C = \{(1, 1, 0), (0, 0, 1), (0.2, 0.5, 0.3), (0.1, 0.4, 0.3), (0.2, 0.4, 0.3), (0.2, 0.4, 0.1), (0.1, 0.3, 0.5)\}$  and  $D = \{(0.2, 0.5, 0.1)\}$ .
- (v)  $\nu_k^* = C \cup D$ , where  $C = \{(1, 1, 0), (0, 0, 1), (0.9, 0, 0.1), (0.2, 0.6, 0.4), (0.3, 0.1, 0.6), (0.2, 0.1, 0.7)\}$  and  $D = \{(0.9, 0.6, 0.1), (0.9, 0.1, 0.1), (0.3, 0.6, 0.4)\}$ .

### Step 3: Find Single valued neutrosophic score functions:

- (i)  $SVNSF(A) = 0.5611$  and  $SVNSF(B) = 0.8$ , where  $q = 1$ .  $SVNSF(C_1) = 0.6801$ .
- (ii)  $SVNSF(A) = 0.5524$  and  $SVNSF(B) = 0.7333$ , where  $q = 3$ .  $SVNSF(C_2) = 0.6428$ .
- (iii)  $SVNSF(A) = 0.6$  and  $SVNSF(B) = 0.6933$ , where  $q = 5$ .  $SVNSF(C_3) = 0.6466$ .
- (iv)  $SVNSF(A) = 0.5381$  and  $SVNSF(B) = 0.6888$ , where  $q = 3$ .  $SVNSF(C_4) = 0.6135$ .
- (i)  $SVNSF(C) = 0.5238$  and  $SVNSF(D) = 0.85$ , where  $r = 2$ .  $SVNSF(D_1) = 0.6869$ .
- (ii)  $SVNSF(C) = 0.5555$  and  $SVNSF(D) = 0.7833$ , where  $r = 4$ .  $SVNSF(D_2) = 0.6694$ .
- (iii)  $SVNSF(C) = 0.6333$  and  $SVNSF(B) = 0.65$ , where  $r = 2$ .  $SVNSF(D_3) = 0.6416$ .
- (iv)  $SVNSF(C) = 0.4888$  and  $SVNSF(B) = 0.5333$ , where  $r = 1$ .  $SVNSF(D_4) = 0.5111$ .
- (v)  $SVNSF(C) = 0.5555$  and  $SVNSF(B) = 0.6888$ , where  $r = 3$ .  $SVNSF(D_5) = 0.6222$ .

### Step 4: Final Decision:

Arrange single valued neutrosophic score values for the alternatives  $C_1, C_2, C_3, C_4$  and the attributes  $D_1, D_2, D_3, D_4, D_5$  in ascending order. We get the following sequences  $C_4 \leq C_2 \leq C_3 \leq C_1$  and  $D_4 \leq D_5 \leq D_3 \leq D_2 \leq D_1$ . Thus the patient  $P_4$  suffers from tuberculosis, the patient  $P_2$  suffers from diabetes, the patient  $P_3$  suffers from chikungunya and the patient  $P_1$  suffers from dengue.

## 8 Conclusion and Future Work

Neutrosophic topological space is one of the research areas in general fuzzy topological spaces to deal the concept of vagueness. This paper introduced neutrosophic supra topological spaces and its real life application. Moreover we have discussed some mappings in neutrosophic supra topological spaces and derived some contradicting examples in fuzzy supra topological spaces. This theory can be develop and implement to other research areas of general topology such as rough topology, digital topology and so on.

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