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A Novel on $\aleph\mathbb{S}\Re$ Contra Strong Precontinuity

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Abstract: In this paper, the concept of $\aleph\mathbb{S}\Re$ contra continuous function is introduced. Several types of contra continuous functions in $\aleph\mathbb{S}\Re$ spaces are discussed. Some interesting properties of $\aleph\mathbb{S}\Re$ contra strongly precontinuous function is established.

Keywords: $\aleph\Re$, $\aleph\mathbb{S}\Re-\mathfrak{C}\mathcal{CF}$, $\aleph\mathbb{S}\Re-\mathfrak{C}\alpha\mathcal{CF}$, $\aleph\mathbb{S}\Re-\mathfrak{C}pre\mathcal{CF}$ and $\aleph\mathbb{S}\Re-\mathfrak{C}Strpre\mathcal{CF}$.

1 Introduction

L. A. Zadeh introduced the idea of fuzzy sets in 1965[16] and later Atanassov [1] generalized it and offered the concept of intuitionistic fuzzy sets. Intuitionistic fuzzy set theory has applications in many fields like medical diagnosis, information technology, nanorobotics, etc. The idea of intutionistic L-fuzzy subring was introduced by K. Meena and V. Thomas [9]. R. Narmada Devi et al. [10, 11, 12] introduced the concept of contra strong precontinuity with respect to the intuitionistic fuzzy structure ring spaces and B. Krteska and E. Ekici [5, 7, 8] introduced the idea of intuitionistic fuzzy contra continuity. The concept of α continuity in intuitionistic fuzzy topological spaces was introduced by J. K. Jeon et al. [6]. F. Smarandache introduced the important and useful concepts of neutrosophy and neutrosophic set [[14], [15]]. A. A. Salama and S. A. Alblowi were established the concepts of neutrosophic crisp set and neutrosophic crisp topological space[13]. In this paper, the concept of $\aleph\mathbb{S}\Re$ contra continuous function is introduced. Several types of contra-continuous functions in $\aleph\mathbb{S}\Re$ spaces are discussed. Some interesting properties of $\aleph\mathbb{S}\Re$ contra strongly precontinuous function is established.

2 Preliminiaries

Definition 2.1. [14, 15] Let T, I, F be real standard or non standard subsets of $]0^-, 1^+[$, with

(i) $sup_T = t_{sup}, inf_T = t_{inf}$

(ii) $sup_I = i_{sup}, inf_I = i_{inf}$

(iii) $sup_F = f_{sup}, inf_F = f_{inf}$

(iv) $n - sup = t_{sup} + i_{sup} + f_{sup}$

(v) $n - inf = t_{inf} + i_{inf} + f_{inf}$.

Observe that T, I, F are neutrosophic components.

Definition 2.2. [14, 15]Let S_1 be a non-empty fixed set. A neutrosophic set (briefly N-set) Λ is an object such that $\Lambda = \{\langle u, \mu_{\Lambda}(u), \sigma_{\Lambda}(u), \gamma_{\Lambda}(u)\rangle : u \in S_1\}$ where $\mu_{\Lambda}(u), \sigma_{\Lambda}(u)$ and $\gamma_{\Lambda}(u)$ which represents the degree of membership function (namely $\mu_{\Lambda}(u)$), the degree of indeterminacy (namely $\sigma_{\Lambda}(u)$) and the degree of non-membership (namely $\gamma_{\Lambda}(u)$) respectively of each element $u \in S_1$ to the set Λ.

Definition 2.3. [13] Let $S_1 \neq \emptyset$ and the N-sets Λ and Γ be defined as
$\Lambda = \{\langle u, \mu_{\Lambda}(u), \sigma_{\Lambda}(u), \Gamma_{\Lambda}(u)\rangle : u \in S_1\}$, $\Gamma = \{\langle u, \mu_{\Gamma}(u), \sigma_{\Gamma}(u), \Gamma_{\Gamma}(u)\rangle : u \in S_1\}$. Then

(a) $\Lambda \subseteq \Gamma$ iff $\mu_{\Lambda}(u) \leq \mu_{\Gamma}(u)$, $\sigma_{\Lambda}(u) \leq \sigma_{\Gamma}(u)$ and $\Gamma_{\Lambda}(u) \geq \Gamma_{\Gamma}(u)$ for all $u \in S_1$;

(b) $\Lambda = \Gamma$ iff $\Lambda \subseteq \Gamma$ and $\Gamma \subseteq \Lambda$;

(c) $\bar{\Lambda} = \{\langle u, \Gamma_{\Lambda}(u), \sigma_{\Lambda}(u), \mu_{\Lambda}(u)\rangle : u \in S_1\}$; [Complement of Λ]

(d) $\Lambda \cap \Gamma = \{\langle u, \mu_{\Lambda}(u) \wedge \mu_{\Gamma}(u), \sigma_{\Lambda}(u) \wedge \sigma_{\Gamma}(u), \Gamma_{\Lambda}(u) \vee \Gamma_{\Gamma}(u)\rangle : u \in S_1\}$;

(e) $\Lambda \cup \Gamma = \{\langle u, \mu_{\Lambda}(u) \vee \mu_{\Gamma}(u), \sigma_{\Lambda}(u) \vee \sigma_{\Gamma}(u), \Gamma_{\Lambda}(u) \wedge \gamma_{\Gamma}(u)\rangle : u \in S_1\}$;

(f) $[\ ]\Lambda = \{\langle u, \mu_{\Lambda}(u), \sigma_{\Lambda}(u), 1 - \mu_{\Lambda}(u)\rangle : u \in S_1\}$;

(g) $\langle\rangle\Lambda = \{\langle u, 1 - \Gamma_{\Lambda}(u), \sigma_{\Lambda}(u), \Gamma_{\Lambda}(u)\rangle : u \in S_1\}$.

Definition 2.4. [13] Let $\{\Lambda_i : i \in J\}$ be an arbitrary family of N-sets in S_1. Then

(a) $\bigcap \Lambda_i = \{\langle u, \wedge\mu_{\Lambda_i}(u), \wedge\sigma_{\Lambda_i}(u), \vee\Gamma_{\Lambda_i}(u)\rangle : u \in S_1\}$;

(b) $\bigcup \Lambda_i = \{\langle u, \vee\mu_{\Lambda_i}(u), \vee\sigma_{\Lambda_i}(u), \wedge\Gamma_{\Lambda_i}(u)\rangle : u \in S_1\}$.

Definition 2.5. [13] $0_N = \{\langle u, 0, 0, 1\rangle : u \in S\}$ and $1_N = \{\langle u, 1, 1, 0\rangle : u \in S\}$.

Definition 2.6. [4] A neutrosophic topology (briefly N-topology) on $S_1 \neq \emptyset$ is a family ξ_1 of N-sets in S_1 satisfying the following axioms:

(i) $0_{N}, 1_{N} \in \xi_1$,

(ii) $H_1 \cap H_2 \in \xi_1$ for any $H_1, H_2 \in \xi_1$,

(iii) $\cup H_i \in \xi_1$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq \xi_1$.

In this case the ordered pair (S_1, ξ_1) or simply S_1 is called an NTS and each N-set in ξ_1 is called a neutrosophic open set (briefly N-open set) . The complement $\overline{\Lambda}$ of an N-open set Λ in S_1 is called a neutrosophic closed set (briefly N-closed set) in S_1.

Definition 2.7. [13] Let D be any neutrosophic set in an neutrosophic topological space S. Then the neutrosophic interior and neutrosophic closure of D are defined and denoted by

(i) $Nint(D) = \bigcup\{H \mid H$ is an NS open set in $S$and $H \subseteq D\}$.

(ii) $Ncl(D) = \bigcap\{H \mid H$ is a neutrosophic closed set in S and $H \supseteq D\}$.

Proposition 2.1. [13] For any neutrosophic set D in (S, τ) we have
$Ncl(C(D)) = C(Nint(D))$ and $Nint(C(D)) = C(Ncl(D))$.

Corollary 2.1. [4] Let $D, D_i(i \in J)$ and $U, U_j(j \in K)$ IFSs in be S_1 and S_2 and $\phi : S_1 \to S_2$ a function. Then

(i) $D \subseteq \phi^{-1}(\phi(D))$ (If ϕ is injective,then $D = \phi^{-1}(\phi(D))$),

(ii) $\phi(\phi^{-1}(U)) \subseteq U$ (If ϕ is surjective,then $\phi(\phi^{-1}(D)) = D$),

(iii) $\phi^{-1}(\bigcup U_j) = \bigcup \phi^{-1}(U_j)$ and $\phi^{-1}(\bigcap U_j) = \bigcap \phi^{-1}(U_j)$,

(iv) $\phi^{-1}(1_\sim) = 1_\sim$ and $\phi^{-1}(0_\sim) = 0_\sim$,

(v) $\phi^{-1}(\overline{U}) = \overline{\phi^{-1}(U)}$.

Definition 2.8. [5]
An IFS D of an $IFTS$ is called an intuitionistic fuzzy α-open set (IFαOS) if $D \subseteq int(cl(int(D)))$. The complement of an IFαOSis called an intuitionistic fuzzy α-closed set(IFαCS).

Definition 2.9. [3] A $\phi : X \to Y$ be a function.

(i) If $B = \{\langle v, \mu_B(v), \gamma_B(v)\rangle : v \in Y\}$ is an IFS in Y, then the preimage of B under ϕ (denoted by $\phi^{-1}(B)$) is defined by $\phi^{-1}(B) = \{\langle u, \phi^{-1}(\mu_B)(u), \phi^{-1}(\gamma_B)(u)\rangle : u \in X\}$.

(ii) If $A = \{\langle u, \lambda_A(u), \vartheta_A(u)\rangle : u \in X\}$ is an IFS in X, then the image of A under ϕ (denoted by $\phi(A)$) is defined by $\phi(A) = \{\langle v, \phi(\lambda_A(v)), (1 - \phi(1 - \vartheta_A))(v)\rangle : v \in Y\}$.

Definition 2.10. [3] Let (X, τ) and (Y, σ) be two $IFTS$s and $\phi : X \to Y$ be a function. Then ϕ is said to be intuitionistic fuzzy continuous if the preimage of each IFS in σ is an IFS in τ.

Definition 2.11. [8,9] Let R be a ring. An intuitionistic fuzzy set $A = \langle u, \mu_A, \gamma_A\rangle$ in R is called an intuitionistic fuzzy ring on R if it satisfies the following conditions:

(i) $\mu_A(u + v) \geq \mu_A(u) \wedge \mu_A(v)$ and $\mu_A(uv) \geq \mu_A(u) \wedge \mu_A(v)$.

(ii) $\gamma_A(u + v) \leq \gamma_A(u) \vee \gamma_A(v)$ and $\gamma_A(uv) \leq \mu_A(u) \vee \gamma_A(v)$.

for all $u, v \in R$.

3 Neutrosophic structure ring contra strong precontinuous function

In this section, the concepts of neutrosophic ring, neutrosophic structure ring space are introduced. Also some interesting properties of neutrosophic structure ring contra strong precontinuous function and their characterizations are studied.

Definition 3.1. Let $\Re$ be a ring. A neutrosophic set $\Lambda = \{\langle u, \mu_{\Lambda(u)}, \sigma_{\Lambda(u)}, \gamma_{\Lambda(u)}\rangle : u \in R\}$ in $\Re$ is called a neutrosophic ring[briefly $\aleph\Re$] on $\Re$ if it satisfies the following conditions:

(i) $\mu_{\Lambda(u+v)} \geq \mu_{\Lambda(u} \wedge \mu_{\Lambda(v)}$ and $\mu_{\Lambda(uv)} \geq \mu_{\Lambda(u)} \wedge \mu_{\Lambda(v)}$.

(ii) $\sigma_{\Lambda(u+v)} \geq \sigma_{\Lambda(u)} \wedge \sigma_{\Lambda(v)}$ and $\sigma_{\Lambda(uv)} \geq \sigma_{\Lambda(u)} \wedge \sigma_{\Lambda(v)}$.

(iii) $\gamma_{\Lambda(u+v)} \leq \gamma_{\Lambda(u)} \vee \gamma_{\Lambda(v)}$ and $\gamma_{\Lambda(uv)} \leq \gamma_{\Lambda(x)} \vee \gamma_{\Lambda(y)}$.

for all $u, v \in \Re$.

Definition 3.2. Let $\Re$ be a ring. A family $\mathscr{S}$ of a $\aleph\Re$'s in $\Re$ is said to be neutrosophic structure ring on $\Re$ if it satisfies the following axioms:

(i) $0_N, 1_N \in \mathscr{S}$.

(ii) $H_1 \cap H_2 \in \mathscr{S}$ for any $H_1, H_2 \in \mathscr{S}$.

(iii) $\cup H_k \in \mathscr{S}$ for arbitrary family $\{H_k \mid k \in J\} \subseteq \mathscr{S}$.

The ordered pair $(\Re, \mathscr{S})$ is called a neutrosophic structure ring($\aleph\mathbb{S}\Re$) space. Every member of $\mathscr{S}$ is called a $\aleph$ open ring (briefly $\aleph\mathbb{O}\Re$) in $(\Re, \mathscr{S})$. The complement of a $\aleph\mathbb{O}\Re$ in $(\Re, \mathscr{S})$ is a $\aleph$ closed ring ($\aleph\mathbb{C}\Re$)in $(\Re, \mathscr{S})$.

Definition 3.3. Let D be a $\aleph$ ring in $\aleph\mathbb{S}\Re$ space $(\Re, \mathscr{S})$. Then $\aleph\mathbb{S}\Re$ interior and $\aleph\mathbb{S}\Re$ closure of D are defined and denoted by

(i) $\aleph int_{\Re}(D) = \bigcup\{H \mid H$ is a $\aleph\mathbb{O}\Re$ in $\Re$ and $H \subseteq D\}$.

(ii) $\aleph cl_{\Re}(D) = \bigcap\{H \mid H$ is a $\aleph\mathbb{C}\Re$ in $\Re$ and $H \supseteq D\}$.

Proposition 3.1. For any $\aleph\Re$ D in $(\Re, \mathscr{S})$ we have

(i) $\aleph cl_{\Re}(C(D)) = C(\aleph int_{\Re}(D))$

(ii) $\aleph int_{\Re}(C(D)) = C(\aleph cl_{\Re}(D))$

Definition 3.4. A $\aleph\Re$ D of a $\aleph\mathbb{S}\Re$ space $(\Re, \mathscr{S})$ is said be a

(i) $\aleph$ regular open structure ring ($\aleph Reg\mathbb{O}\mathbb{S}\Re$), if $D = \aleph int_{\Re}(\aleph cl_{\Re}(D))$

(ii) $\aleph\alpha$-open structure ring ($\aleph\alpha\mathbb{O}\mathbb{S}\Re$), if $D \subseteq \aleph int_{\Re}(\aleph cl_{\Re}(\aleph int_{\Re}(D)))$

(iii) $\aleph$ semiopen structure ring ($\aleph Semi\mathbb{O}\mathbb{S}\Re$), if $D \subseteq \aleph cl_{\Re}(\aleph int_{\Re}(D))$

(iv) $\aleph$ preopen structure ring ($\aleph Pre\mathbb{O}\mathbb{S}\Re$), if $D \subseteq \aleph int_{\Re}(\aleph cl_{\Re}(D))$

(v) $\aleph\beta$-open structure ring ($\aleph\beta\mathbb{OS}\Re$), if $D \subseteq \aleph cl_{\Re}(\aleph int_{\Re}(\aleph cl_{\Re}(D)))$

Note 3.1. Let $(\Re, \mathscr{S})$ be a $\aleph\mathbb{S}\Re$ space. Then the complement of a $\aleph Reg\mathbb{OS}\Re$ (resp. $\aleph\alpha\mathbb{OS}\Re$, $\aleph Semi\mathbb{OS}\Re$, $\aleph Pre\mathbb{OS}\Re$ and $\aleph\beta\mathbb{OS}\Re$) is a $\aleph$ regular closed structure ring($\aleph Reg\mathbb{CS}\Re$) (resp. $\aleph\alpha$-closed structure ring($\aleph\alpha\mathbb{CS}\Re$), $\aleph$ semiclosed structure ring($\aleph Semi\mathbb{CS}\Re$), $\aleph$ preclosed structure ring($\aleph Pre\mathbb{CS}\Re$), $\aleph\beta$-closed structure ring($\aleph\beta\mathbb{CS}\Re$)).

Definition 3.5. The $\aleph\mathbb{S}\Re$ preinterior and $\aleph\mathbb{S}\Re$ preclosure of $\aleph\Re$ D of a $\aleph\mathbb{S}\Re$ space are defined and denoted by

(i) $\aleph pint_{\Re}(D) = \bigcup\{H : H$ is a $\aleph Pre\mathbb{OS}\Re$ in $(R, \mathscr{S})$ and $H \subseteq D\}$.

(ii) $\aleph pcl_{\Re}(D) = \bigcap\{H : H$ is a $\aleph Pre\mathbb{CS}\Re$ in $(R, \mathscr{S})$ and $D \subseteq H\}$.

Remark 3.1. For any $\aleph\Re$ D of a $\aleph\mathbb{S}\Re$ space $(\Re, \mathscr{S})$, then

(i) $\aleph pint_{\Re}(D) = D$ if and only if D is a $\aleph Pre\mathbb{OS}\Re$.

(ii) $\aleph pcl_{\Re}(D) = D$ if and only if D is a $\aleph Pre\mathbb{CS}\Re$.

(iii) $\aleph int_{\Re}(D) \subseteq \aleph pint_{\Re}(D) \subseteq D \subseteq \aleph pcl_{\Re}(D) \subseteq \aleph cl_{\Re}(D)$

Definition 3.6. A $\aleph\Re$ D of a $\aleph\mathbb{S}\Re$ space $(\Re, \mathscr{S})$ is called a $\aleph$ strongly preopen structure ring ($\aleph stronglyPre\mathbb{OS}\Re$), if $D \subseteq \aleph int_{\Re}(\aleph pcl_{\Re}(D))$. The complement of a $\aleph stronglyPre\mathbb{OS}\Re$ is a $\aleph$ strongly preclosed structure ring(briefly $\aleph stronglyPre\mathbb{CS}\Re$).

Definition 3.7. The $\aleph\mathbb{S}\Re$ strongly preinterior and $\aleph\mathbb{S}\Re$ strongly preclosure of of $\aleph\Re$ D of a $\aleph\mathbb{S}\Re$ space are defined and denoted by

(i) $\aleph spint_{\Re}(D) = \bigcup\{H : H$ is a $\aleph stronglyPre\mathbb{OS}\Re$ in $(R, \mathscr{S})$ and $H \subseteq D\}$.

(ii) $\aleph spcl_{\Re}(D) = \bigcap\{H : H$ is a $\aleph stronglyPre\mathbb{CS}\Re$ in $(R, \mathscr{S})$ and $D \subseteq H\}$.

Remark 3.2. For any $\aleph\Re$ D of a $\aleph\mathbb{S}\Re$ space $(\Re, \mathscr{S})$, then

(i) $\aleph spint_{\Re}(D) = D$ if and only if D is a $\aleph stronglyPre\mathbb{OS}\Re$.

(ii) $\aleph spcl_{\Re}(D) = D$ if and only if D is a $\aleph stronglyPre\mathbb{CS}\Re$.

(iii) $\aleph int_{\Re}(D) \subseteq \aleph spint_{\Re}(D) \subseteq D \subseteq \aleph spcl_{\Re}(D) \subseteq \aleph cl_{\Re}(D)$

Proposition 3.2. A $\aleph\Re$ of a $\aleph\mathbb{S}\Re$ space $(\Re, \mathscr{S})$ is a $\aleph\alpha\mathbb{OS}\Re$ if and only if it is both $\aleph Semi\mathbb{OS}\Re$ and $\aleph stronglyPre\mathbb{OS}\Re$.

Definition 3.8. Let $(\Re_1, \mathscr{S}_1)$ and $(\Re_2, \mathscr{S}_2)$ be any two $\aleph\mathbb{S}\Re$ spaces. A function $\phi : (\Re_1, \mathscr{S}_1) \to (\Re_2, \mathscr{S}_2)$ is called a $\aleph\mathbb{S}\Re$

(i) contra continuous function($\aleph\mathbb{S}\Re - \mathfrak{C}\mathcal{CF}$) if $\phi^{-1}(U)$ is a $\aleph\mathbb{O}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$.

(ii) contra α-continuous function ($\aleph\mathbb{S}\Re - \mathfrak{C}\alpha\mathcal{CF}$) if $\phi^{-1}(U)$ is a $\aleph\alpha\mathbb{O}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$.

(iii) contra precontinuous function($\aleph\mathbb{S}\Re - \mathfrak{C}pre\mathcal{CF}$) if $\phi^{-1}(U)$ is a $\aleph Pre\mathbb{OS}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$.

(iv) contra strongly precontinuous function ($\aleph\mathbb{S}\Re - \mathfrak{C}StrpreC\mathcal{F}$) if $\phi{-}1(U)$ is a $\aleph stronglyPre\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$.

Proposition 3.3. Let $(\Re_1, \mathscr{S}_1)$ and $(\Re_2, \mathscr{S}_2)$ be any two $\aleph\mathbb{S}\Re$ spaces. Let $\phi : (\Re_1, \mathscr{S}_1) \to (\Re_2, \mathscr{S}_2)$ be a function. If ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}C\mathcal{F}$, then ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}\alpha C\mathcal{F}$.

Proof:

Let U be a $\aleph\mathbb{C}\Re$ in $(\Re_2, \mathscr{S}_2)$. Since ϕ is $\aleph\mathbb{S}\Re - \mathfrak{C}C\mathcal{F}$, $\phi^{-1}(U)$ is a $\aleph\mathbb{O}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$. By Remark 3.1(iii), $\phi^{-1}(U) \subseteq \aleph cl_{\Re_1}(\phi^{-1}(U))$.

Since $\phi^{-1}(U)$ is a $\aleph\mathbb{O}\Re$ in $(\Re_1, \mathscr{S}_1)$, $\phi^{-1}(U) \subseteq \aleph cl_{\Re_1}(\aleph int_{\Re_1}(\phi^{-1}(U)))$. Hence, $\phi^{-1}(U)$ is a $\aleph Semi\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$. By Remark 3.1 (iii), $\phi^{-1}(U) \subseteq \aleph pcl_{\Re_1}(\phi^{-1}(U))$. Taking interior on both sides, $\aleph int_{\Re_1}(\phi^{-1}(U)) \subseteq \aleph int_{Re_1}(\aleph pcl_{\Re_1}(\phi^{-1}(U)))$. Since $\phi^{-1}(U)$ is a $\aleph\mathbb{O}\Re$ in $(\Re_1, \mathscr{S}_1)$, $\phi^{-1}(U) \subseteq \aleph int_{\Re_1}(\aleph pcl_{\Re_1}(\phi^{-1}(U)))$. Hence, $\phi^{-1}(U)$ is a $\aleph stronglyPre\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$. Therefore, $\phi^{-1}(U)$ is both $\aleph Semi\mathbb{O}\mathbb{S}\Re$ and $\aleph stronglyPre\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$. By Proposition 3.1, $\phi^{-1}(\phi)$ is a $\aleph\alpha\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$. Hence, ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}\alpha C\mathcal{F}$.

Proposition 3.4. Let $(\Re_1, \mathscr{S}_1)$ and $(\Re_2, \mathscr{S}_2)$ be any two $\aleph\mathbb{S}\Re$ spaces. Let $\phi : (\Re_1, \mathscr{S}_1) \to (\Re_2, \mathscr{S}_2)$ be a function. If ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}\alpha C\mathcal{F}$, then ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}StrpreC\mathcal{F}$.

Proof:

Let U be any $\aleph\mathbb{C}\Re$ in $(\Re_2, \mathscr{S}_2)$. Since ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}\alpha C\mathcal{F}$, $\phi^{-1}(U)$ is a $\aleph\alpha\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$. By Proposition 3.1, $\phi^{-1}(U)$ is both $\aleph Semi\mathbb{O}\mathbb{S}\Re$ and $\aleph stronglyPre\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$. Therefore, $\phi^{-1}(U)$ is a $\aleph stronglyPre\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$. Hence ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}StrpreC\mathcal{F}$.

Proposition 3.5. Let $(\Re_1, \mathscr{S}_1)$ and $(\Re_2, \mathscr{S}_2)$ be any two $\aleph\mathbb{S}\Re$ spaces. Let $\phi : (\Re_1, \mathscr{S}_1) \to (\Re_2, \mathscr{S}_2)$ be a function. If ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}StrpreC\mathcal{F}$, then ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}preC\mathcal{F}$.

Proof:

Let U be any $\aleph\mathbb{C}\Re$ in $(\Re_2, \mathscr{S}_2)$. Since ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}StrpreC\mathcal{F}$, $\phi^{-1}(U)$ is a $\aleph stronglyPre\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$, that is,

$$\phi^{-1}(U) \subseteq \aleph int_{\Re_1}(\aleph pcl_{\Re_1}(\phi^{-1}(U))) \tag{3.1}$$

By Remark 3.1(iii),

$$\aleph pcl_{\Re_1}(\phi^{-1}(U)) \subseteq \aleph cl_{\Re_1}(\phi^{-1}(U)) \tag{3.2}$$

Substitute (3.2) in (3.1), we get $\phi^{-1}(U) \subseteq \aleph int_{\Re_1}(\aleph cl_{\Re_1}(\phi^{-1}(U)))$. Therefore, $\phi^{-1}(U)$ is a $\aleph Pre\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$. Hence, ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}preC\mathcal{F}$.

Proposition 3.6. Let $(\Re_1, \mathscr{S}_1)$ and $(\Re_2, \mathscr{S}_2)$ be any two $\aleph\mathbb{S}\Re$ spaces. Let $\phi : (\Re_1, \mathscr{S}_1) \to (\Re_2, \mathscr{S}_2)$ be a function. If ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}C\mathcal{F}$, then ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}StrpreC\mathcal{F}$.

Proof:

Let U be any $\aleph\mathbb{C}\Re$ in $(\Re_2, \mathscr{S}_2)$. Since ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}C\mathcal{F}$, $\phi^{-1}(U)$ is a $\aleph\mathbb{O}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$, that is, $\phi^{-1}(U) = \aleph int_{\Re_1}(\phi^{-1}(U))$. By Remark 3.1(iii),

$$\phi^{-1}(U) \subseteq \aleph pcl_{\Re_1}(\phi^{-1}(U)) \tag{3.3}$$

Taking interior on both sides in (3.3),

$$\phi^{-1}(U) = \aleph int_{\Re_1}(\phi^{-1}(U)) \subseteq \aleph int_{\Re_1}(\aleph pcl_{\Re_1}(\phi^{-1}(U))).$$

Hence, $\phi^{-1}(U)$ is a $\aleph stronglyPre\mathbb{OS}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$. Thus, ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}StrpreC\mathcal{F}$.

Proposition 3.7. Let $(\Re_1, \mathscr{S}_1)$ and $(\Re_2, \mathscr{S}_2)$ be any two $\aleph\mathbb{S}\Re$ spaces. Let $\phi : (\Re_1, \mathscr{S}_1) \to (\Re_2, \mathscr{S}_2)$ be a function. If ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}C\mathcal{F}$, then ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}preC\mathcal{F}$.

Proof:

Let U be any $\aleph\mathbb{C}\Re$ in $(\Re_2, \mathscr{S}_2)$. Since ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}C\mathcal{F}$, $\phi^{-1}(U)$ is a $\aleph\mathbb{O}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$, that is, $\phi^{-1}(U) = \aleph int_{\Re_1}(\phi^{-1}(U))$. By Remark 3.1(iii),

$$\phi^{-1}(B) \subseteq \aleph cl_{\Re_1}(\phi^{-1}(U)) \tag{3.4}$$

Taking interior on both sides in (3.4),

$$\phi^{-1}(U) = \aleph int_{\Re_1}(\phi^{-1}(U)) \subseteq \aleph int_{\Re_1}(\aleph cl_{\Re_1}(\phi^{-1}(U))).$$

Hence, $\phi^{-1}(U)$ is a $\aleph Pre\mathbb{OS}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$. Thus, ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}preC\mathcal{F}$.

Remark 3.3. The converses of the Proposition 3.2, Proposition 3.3, Proposition 3.4, Proposition 3.5 and Proposition 3.6 need not be true as it is shown in the following example.

Example 3.1. Let $\Re = \{a, b, c\}$ be a nonempty set with two binary operations as follows:

+	u	v	w
u	u	v	w
v	v	w	u
w	w	u	v

and

*	u	v	w
u	u	u	u
v	u	v	w
w	u	w	v

Then $(\Re, +, *)$ is a ring. Define $\aleph\Re$'s L, M and P as follows:

$$\mu_L(u) = 0.4, \mu_L(v) = 0.8, \mu_L(w) = 0.2;$$
$$\mu_M(u) = 0.7, \mu_M(v) = 0.9, \mu_M(w) = 0.4;$$
$$\mu_P(u) = 0.5, \mu_P(v) = 0.7, \mu_P(w) = 0.3;$$
$$\sigma_L(u) = 0.4, \sigma_L(v) = 0.8, \sigma_L(w) = 0.2;$$
$$\sigma_M(u) = 0.7, \sigma_M(v) = 0.9, \sigma_M(w) = 0.4;$$
$$\sigma_P(u) = 0.5, \sigma_P(v) = 0.7, \sigma_P(w) = 0.3;$$
$$\gamma_L(u) = 0.1, \gamma_L(v) = 0.1, \gamma_L(w) = 0.1;$$
$$\gamma_M(u) = 0.1, \gamma_M(v) = 0.1, \gamma_M(w) = 0.1; \text{ and}$$
$$\gamma_P(u) = 0.1, \gamma_P(v) = 0.1, \gamma_P(w) = 0.1.$$

Then $\mathscr{S}_1 = \{0_N, 1_N, L, M\}$, $\mathscr{S}_2 = \{0_N, 1_N, P\}$, $\mathscr{S}_3 = \{0_N, 1_N, C(L)\}$ and $\mathscr{S}_4 = \{0_N, 1_N, P\}$ are the $\aleph\mathbb{S}\Re$'s on $\Re$.

Then the identity function $\phi : (\Re, \mathscr{S}_2) \to (\Re, \mathscr{S}_3)$ is a $\aleph\mathbb{S}\Re - \mathfrak{C}preC\mathcal{F}$, but ϕ is neither $\aleph\mathbb{S}\Re - \mathfrak{C}C\mathcal{F}$ nor $\aleph\mathbb{S}\Re - \mathfrak{C}StrpreC\mathcal{F}$.

Similarly the identity function $\phi : (\Re, \mathscr{S}_1) \rightarrow (\Re, \mathscr{S}_4)$ is a $\aleph\mathbb{S}\Re - \mathfrak{C}Strpre\mathcal{CF}$ but ϕ is neither $\aleph\mathbb{S}\Re - \mathfrak{C}\mathcal{CF}$ nor $\aleph\mathbb{S}\Re - \mathfrak{C}\alpha\mathcal{CF}$

Remark 3.4. Clearly the following diagram holds.

$$\begin{tikzcd}
& \boxed{\aleph\mathbb{S}\Re - \mathfrak{C}\mathcal{CF}} \arrow[dl] \arrow[dd] \arrow[dr] & \\
\boxed{\aleph\mathbb{S}\Re - \mathfrak{C}\alpha\mathcal{CF}} \arrow[dr] & & \boxed{\aleph\mathbb{S}\Re - \mathfrak{C}pre\mathcal{CF}} \arrow[dl] \\
& \boxed{\aleph\mathbb{S}\Re - \mathfrak{C}Strpre\mathcal{CF}} &
\end{tikzcd}$$

Proposition 3.8. Let $(\Re_1, \mathscr{S}_1)$ and $(\Re_2, \mathscr{S}_2)$ be any two $\aleph\mathbb{S}\Re$ spaces. Let $\phi : (\Re_1, \mathscr{S}_1) \rightarrow (\Re_2, \mathscr{S}_2)$ be a function. Then the following are equivalent.

(i) ϕ is $\aleph\mathbb{S}\Re - \mathfrak{C}Strpre\mathcal{CF}$.

(ii) $\phi^{-1}(U)$ is a $\aleph stronglyPre\mathbb{CS}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{O}\Re$ U in $(\Re_2, \mathscr{S}_2)$.

Proof:
(i)⇒(ii)
Let U be any $\aleph\mathbb{O}\Re$ in $(\Re_2, \mathscr{S}_2)$, then $C(U)$ is a $\aleph\mathbb{C}\Re$ in $(\Re_2, \mathscr{S}_2)$. Since ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}Strpre\mathcal{CF}$, $\phi^{-1}(C(U))$ is a $\aleph stronglyPre\mathbb{OS}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ $C(U)$ in $(\Re_2, \mathscr{S}_2)$. By Remark 3.2(i), $\phi^{-1}(C(U)) = C(\phi^{-1}(U)) \subseteq \aleph int_{\Re_1}(\aleph pcl_{\Re_1}(C(\phi^{-1}(U))))$. Therefore, $\phi^{-1}(U)$ is a $\aleph stronglyPre\mathbb{CS}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{O}\Re$ U in $(\Re_2, \mathscr{S}_2)$.
(ii)⇒(i)
Let $C(U)$ be any $\aleph\mathbb{O}\Re$ in $(\Re_2, \mathscr{S}_2)$. Then U is a $\aleph\mathbb{C}\Re$ in $(\Re_2, \mathscr{S}_2)$. Since $\phi^{-1}(C(U))$ is a $\aleph stronglyPre\mathbb{CS}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{O}\Re$ $C(U)$ in $(\Re_2, \mathscr{S}_2)$. We have, $\phi^{-1}(U)$ is a $\aleph stronglyPre\mathbb{OS}\Re$in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ U in $(\Re_2, \mathscr{S}_2)$. Hence, ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}Strpre\mathcal{CF}$.

Proposition 3.9. Let $(\Re_1, \mathscr{S}_1)$ and $(\Re_2, \mathscr{S}_2)$ be any two $\aleph\mathbb{S}\Re$ spaces. Let $\phi : (\Re_1, \mathscr{S}_1) \rightarrow (\Re_2, \mathscr{S}_2)$ be a function. Suppose if one of the following statement hold.

(i) $\phi^{-1)}(\aleph cl_{\Re_2}(V)) \subseteq \aleph int_{\Re_1}(\aleph pcl_{\Re_1}(\phi^{-1}(V)))$, for each $\aleph\Re$ V in $(\Re_2, \mathscr{S}_2)$.

(ii) $\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(\phi^{-1}(V))) \subseteq \phi^{-1}(\aleph int_{\Re_2}(V))$, for each $\aleph\Re$ V in $(\Re_2, \mathscr{S}_2)$.

(iii) $\phi(\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(V))) \subseteq \aleph int_{\Re_2}(\phi(V))$, for each $\aleph\Re$ V in $(\Re_1, \mathscr{S}_1)$.

(iv) $\phi(\aleph cl_{\Re_1}(V)) \subseteq \aleph int_{\Re_2}(\phi V))$, for each $\aleph Pre\mathbb{OS}\Re$ V in $(\Re_1, \mathscr{S}_1)$.

Then, ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}Strpre\mathcal{CF}$.
Proof:
(i)⇒(ii)
Let U be any $\aleph\Re$ in $(\Re_2, \mathscr{S}_2)$. Then, $\phi^{-1}(\aleph cl_{\Re_2}(U)) \subseteq \aleph int_{\Re_1}(\aleph pcl_{\Re_1}(\phi^{-1}(U)))$ By taking complement on both sides,

$$C(\aleph int_{\Re_1}(\aleph pcl_{\Re_1}(\phi^{-1}(U)))) \subseteq C(\phi^{-1}(\aleph cl_{\Re_2}(U)))$$

$$\aleph cl_{\Re_1}(C(\aleph pcl_{\Re_1}(\phi^{-1}(U)))) \subseteq \phi^{-1}(C(\aleph cl_{\Re_2}(U)))$$
$$\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(C(\phi^{-1}(U)))) \subseteq \phi^{-1}(\aleph int_{\Re_2}(C(U)))$$

Therefore, $\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(\phi^{-1}(V))) \subseteq \phi^{-1}(\aleph int_{\Re_2}(V))$, for each $\aleph\Re$ $V = C(U)$ in $(\Re_2, \mathscr{S}_2)$.
(ii)⇒(iii)
Let U be any $\aleph\Re$ in $(\Re_2, \mathscr{S}_2)$. Let V be any $\aleph\Re$ in $(\Re_1, \mathscr{S}_1)$ such that $U = \phi(V)$. Then $V \subseteq \phi^{-1}(U)$. By (ii), $\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(\phi^{-1}(U))) \subseteq \phi^{-1}(\aleph int_{\Re_2}(U))$. We have

$$\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(V)) \subseteq \aleph cl_{\Re_1}(\aleph pint_{\Re_1}(\phi^{-1}(U))) \subseteq \phi^{-1}(\aleph int_{\Re_2}(\phi(V)))$$

Therefore, $\phi(\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(V))) \subseteq \aleph int_{\Re_2}(\phi(V))$, for each $\aleph\Re$ V in $(\Re_1, \mathscr{S}_1)$.
(iii)⇒(iv)
Let V be any $\aleph Pre\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$. Then $\aleph pint_{\Re_1}(V) = V$. By (iii),

$$\phi(\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(V))) = \phi(\aleph cl_{\Re_1}(V)) \subseteq \aleph int_{\Re_2}(\phi(V)).$$

Therefore, $\phi(\aleph cl_{R_1}(V)) \subseteq \aleph int_{R_2}(\phi(V))$, for each $\aleph Pre\mathbb{O}\mathbb{S}\Re$ V in $(\Re_1, \mathscr{S}_1)$.

Suppose that (iv) holds. Let U be any $\aleph\mathbb{O}\Re$ in $(\Re_2, \mathscr{S}_2)$. Then $\aleph pint_{R_1}(\phi^{-1}(U))$ is a $\aleph Pre\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$. By (iv),

$$\begin{aligned}\phi(\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(\phi^{-1}(U)))) &\subseteq \aleph int_{\Re_2}(\phi(\aleph pint_{\Re_1}(\phi^{-1}(U))))\\ &\subseteq \aleph int_{\Re_2}(\phi(\phi^{-1}(U))) \subseteq \aleph int_{\Re_2}(U) = U.\end{aligned}$$

We have, $\phi^{-1}(\phi(\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(\phi^{-1}(U))))) \subseteq \phi^{-1}(U)$.
Then $\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(\phi^{-1}(U))) \subseteq \phi^{-1}(U)$. This implies that $\phi^{-1}(U)$ is a $\aleph Pre\mathbb{C}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$. Taking complement on both sides, $C(\phi^{-1}(U)) \subseteq C(\aleph cl_{\Re_1}(\aleph pint_{\Re_1}(\phi^{-1}(U))))$. This implies that $\phi^{-1}(C(B)) \subseteq \aleph int_{\Re_1}(\aleph pcl_{\Re_1}(\phi^{-1}(C(U))))$. Therefore $\phi^{-1}(C(U))$ is a $\aleph stronglyPre\mathbb{O}\mathbb{S}\Re$ in $(\Re_1, \mathscr{S}_1)$, for each $\aleph\mathbb{C}\Re$ $C(U)$ in $(\Re_2, \mathscr{S}_2)$. Hence, ϕ is a $\aleph\mathbb{S}\Re - \mathfrak{C}Strpre\mathcal{CF}$.

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