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Generalized Neutrosophic Exponential map

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Abstract: The concept of $\mathfrak{g}\aleph$ compact open topology is introduced. Some characterization of this topology are discussed.

Keywords: $\mathfrak{g}\aleph$ locally Compact Hausdorff space; $\mathfrak{g}\aleph$ product topology; $\mathfrak{g}\aleph$ compact open topology; $\mathfrak{g}\aleph$ homeomorphism; $\mathfrak{g}\aleph$ evaluation map; $\mathfrak{g}\aleph$ Exponential map.

1 Introduction

Ever since the introduction of fuzzy sets by Zadeh [12] and fuzzy topological space by Chang [5], several authors have tried successfully to generalize numerous pivot concepts of general topology to the fuzzy setting. The concept of intuitionistic fuzzy set was introduced are studied by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [[2],[3],[4]]. The concepts of generalized intuitionistic fuzzy closed set was introduced by Dhavaseelan et al[6]. The concepts of Intuitionistic Fuzzy Exponential Map Via Generalized Open Set by Dhavaseelan et al[8]. After the introduction of the neutrosophic set concept [[10], [11]]. The concepts of Neutrosophic Set and Neutrosophic Topological Spaces was introduced by A.A.Salama and S.A.Alblowi[9].

In this paper the concept of $\mathfrak{g}\aleph$ compact open topology are introduced. Some interesting properties are discussed. In this paper the concepts of $\mathfrak{g}\aleph$ local compactness and generalized \aleph - product topology are developed. We have Throughout this paper neutrosophic topological spaces (briefly *NTS*) $(S_1, \xi_1), (S_2, \xi_2)$ and (S_3, ξ_3) will be replaced by S_1, S_2 and S_3 , respectively.

2 Preliminaries

Definition 2.1. [10, 11] Let T, I, F be real standard or non standard subsets of $]0^-, 1^+[$, with $sup_T = t_{sup}, inf_T = t_{inf}$
 $sup_I = i_{sup}, inf_I = i_{inf}$
 $sup_F = f_{sup}, inf_F = f_{inf}$
 $n - sup = t_{sup} + i_{sup} + f_{sup}$
 $n - inf = t_{inf} + i_{inf} + f_{inf}$. T, I, F are \aleph - components.

Definition 2.2. [10, 11] Let S_1 be a non-empty fixed set. A \aleph -set (briefly N -set) Λ is an object such that $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \gamma_\Lambda(x) \rangle : x \in S_1\}$ where $\mu_\Lambda(x)$, $\sigma_\Lambda(x)$ and $\gamma_\Lambda(x)$ which represents the degree of membership function (namely $\mu_\Lambda(x)$), the degree of indeterminacy (namely $\sigma_\Lambda(x)$) and the degree of non-membership (namely $\gamma_\Lambda(x)$) respectively of each element $x \in S_1$ to the set Λ .

Remark 2.1. [10, 11]

- (1) An N -set $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1\}$ can be identified to an ordered triple $\langle \mu_\Lambda, \sigma_\Lambda, \Gamma_\Lambda \rangle$ in $]0^-, 1^+[$ on S_1 .
- (2) In this paper, we use the symbol $\Lambda = \langle \mu_\Lambda, \sigma_\Lambda, \Gamma_\Lambda \rangle$ for the N -set $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1\}$.

Definition 2.3. [7] Let $S_1 \neq \emptyset$ and the N -sets Λ and Γ be defined as

$\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1\}$, $\Gamma = \{\langle x, \mu_\Gamma(x), \sigma_\Gamma(x), \Gamma_\Gamma(x) \rangle : x \in S_1\}$. Then

- (a) $\Lambda \subseteq \Gamma$ iff $\mu_\Lambda(x) \leq \mu_\Gamma(x)$, $\sigma_\Lambda(x) \leq \sigma_\Gamma(x)$ and $\Gamma_\Lambda(x) \geq \Gamma_\Gamma(x)$ for all $x \in S_1$;
- (b) $\Lambda = \Gamma$ iff $\Lambda \subseteq \Gamma$ and $\Gamma \subseteq \Lambda$;
- (c) $\bar{\Lambda} = \{\langle x, \Gamma_\Lambda(x), \sigma_\Lambda(x), \mu_\Lambda(x) \rangle : x \in S_1\}$; [Complement of Λ]
- (d) $\Lambda \cap \Gamma = \{\langle x, \mu_\Lambda(x) \wedge \mu_\Gamma(x), \sigma_\Lambda(x) \wedge \sigma_\Gamma(x), \Gamma_\Lambda(x) \vee \Gamma_\Gamma(x) \rangle : x \in S_1\}$;
- (e) $\Lambda \cup \Gamma = \{\langle x, \mu_\Lambda(x) \vee \mu_\Gamma(x), \sigma_\Lambda(x) \vee \sigma_\Gamma(x), \Gamma_\Lambda(x) \wedge \Gamma_\Gamma(x) \rangle : x \in S_1\}$;
- (f) $[\]\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), 1 - \mu_\Lambda(x) \rangle : x \in S_1\}$;
- (g) $\langle \rangle \Lambda = \{\langle x, 1 - \Gamma_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1\}$.

Definition 2.4. [7] Let $\{\Lambda_i : i \in J\}$ be an arbitrary family of N -sets in S_1 . Then

- (a) $\bigcap \Lambda_i = \{\langle x, \wedge \mu_{\Lambda_i}(x), \wedge \sigma_{\Lambda_i}(x), \vee \Gamma_{\Lambda_i}(x) \rangle : x \in S_1\}$;
- (b) $\bigcup \Lambda_i = \{\langle x, \vee \mu_{\Lambda_i}(x), \vee \sigma_{\Lambda_i}(x), \wedge \Gamma_{\Lambda_i}(x) \rangle : x \in S_1\}$.

Since our main purpose is to construct the tools for developing NTS, we must introduce the \aleph -sets 0_N and 1_N in X as follows:

Definition 2.5. [7] $0_N = \{\langle x, 0, 0, 1 \rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0 \rangle : x \in X\}$.

Definition 2.6. [7] A \aleph -topology (briefly N -topology) on $S_1 \neq \emptyset$ is a family ξ_1 of N -sets in S_1 satisfying the following axioms:

- (i) $0_N, 1_N \in \xi_1$,
- (ii) $G_1 \cap G_2 \in \xi_1$ for any $G_1, G_2 \in \xi_1$,
- (iii) $\bigcup G_i \in \xi_1$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq \xi_1$.

In this case the ordered pair (S_1, ξ_1) or simply S_1 is called an NTS and each N -set in ξ_1 is called a \aleph -open set (briefly N -open set). The complement $\bar{\Lambda}$ of an N -open set Λ in S_1 is called a \aleph -closed set (briefly N -closed set) in S_1 .

Definition 2.7. [7] Let Λ be an N -set in an $NTS S_1$. Then

$Nint(\Lambda) = \bigcup\{G \mid G \text{ is an } N\text{-open set in } S_1 \text{ and } G \subseteq \Lambda\}$ is called the \aleph - interior (briefly N -interior) of Λ ;
 $Ncl(\Lambda) = \bigcap\{G \mid G \text{ is an } N\text{-closed set in } S_1 \text{ and } G \supseteq \Lambda\}$ is called the \aleph - closure (briefly N -cl) of Λ .

Definition 2.8. [7] Let X be a nonempty set. If r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ then the \aleph - set $x_{r,t,s}$ is called a \aleph - point(in short NP)in X given by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$.where r denotes the degree of membership value, t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

Definition 2.9. [7] Let (S_1, ξ_1) be a NTS . A \aleph - set Λ in (S_1, ξ_1) is said to be a $g\aleph$ closed set if $Ncl(\Lambda) \subseteq \Gamma$ whenever $\Lambda \subseteq \Gamma$ and Γ is a \aleph - open set. The complement of a $g\aleph$ closed set is called a $g\aleph$ open set.

Definition 2.10. [7] Let (X, T) be a \aleph - topological space and Λ be a \aleph - set in X . Then the \aleph - generalized closure and \aleph - generalized interior of Λ are defined by,

- (i) $NGcl(\Lambda) = \bigcap\{G: G \text{ is a generalized } \aleph\text{- closed set in } S_1 \text{ and } \Lambda \subseteq G\}$.
- (ii) $NGint(\Lambda) = \bigcup\{G: G \text{ is a generalized } \aleph\text{- open set in } S_1 \text{ and } \Lambda \supseteq G\}$.

3 Neutrosophic Compact Open Topology

Definition 3.1. Let S_1 and S_2 be any two NTS . A mapping $f : S_1 \rightarrow S_2$ is generalized neutrosophic[briefly $g\aleph$] continuous iff for every $g\aleph$ open set V in S_2 , there exists a $g\aleph$ open set U in S_1 such that $f(U) \subseteq V$.

Definition 3.2. A mapping $f : S_1 \rightarrow S_2$ is said to be $g\aleph$ homeomorphism if f is bijective, $g\aleph$ continuous and $g\aleph$ open.

Definition 3.3. Let S_1 be a NTS . S_1 is said to be $g\aleph$ Hausdorff space or T_2 space if for any two \aleph - sets A and B with $A \cap B = 0_\sim$,there exist $g\aleph$ open sets U and V , such that $A \subseteq U, B \subseteq V$ and $U \cap V = 0_\sim$.

Definition 3.4. A $NTS S_1$ is said to be $g\aleph$ locally compact iff for any \aleph set A , there exists a $g\aleph$ open set G , such that $A \subseteq G$ and G is $g\aleph$ compact. That is each $g\aleph$ open cover of G has a finite subcover.

Remark 3.1. Let S_1 and S_2 be two NTS with S_2 \aleph - compact. Let $x_{r,t,s}$ be any \aleph - point in S_1 . The \aleph - product space $S_1 \times S_2$ containing $\{x_{r,t,s}\} \times S_2$. It is cleat that $\{x_{r,t,s}\} \times S_2$ is \aleph - homeomorphic to S_2

Remark 3.2. Let S_1 and S_2 be two NTS with S_2 \aleph - compact. Let $x_{r,t,s}$ be any \aleph - point in S_1 . The \aleph - product space $S_1 \times S_2$ containing $\{x_{r,t,s}\} \times S_2$. $\{x_{r,t,s}\} \times S_2$ is \aleph - compact.

Remark 3.3. A \aleph - compact subspace of a \aleph - Hausdorff space is \aleph - closed.

Proposition 3.1. A $g\aleph$ Hausdorff topological space S_1 ,the following conditions are equivalent.

- (a) S_1 is $g\aleph$ locally compact

(b) for each \aleph set A , there exists a $\mathfrak{g}\aleph$ open set G in S_1 such that $A \subseteq G$ and $NGcl(G)$ is $\mathfrak{g}\aleph$ compact

Proof. (a) \Rightarrow (b) By hypothesis for each \aleph - set A in S_1 , there exists a $\mathfrak{g}\aleph$ open set G , such that $A \subseteq G$ and G is $\mathfrak{g}\aleph$ compact. Since S_1 is $\mathfrak{g}\aleph$ Hausdorff, by Remark 3.3 ($\mathfrak{g}\aleph$ compact subspace of $\mathfrak{g}\aleph$ Hausdorff space is $\mathfrak{g}\aleph$ closed), G is $\mathfrak{g}\aleph$ closed, thus $G = NGcl(G)$. Hence $A \subseteq G = NGcl(G)$ and $NGcl(G)$ is $\mathfrak{g}\aleph$ compact.

(b) \Rightarrow (a) Proof is simple.

Proposition 3.2. Let S_1 be a $\mathfrak{g}\aleph$ Hausdorff topological space. Then S_1 is $\mathfrak{g}\aleph$ locally compact on an \aleph - set A in S_1 iff for every $\mathfrak{g}\aleph$ open set G containing A , there exists a $\mathfrak{g}\aleph$ open set V , such that $A \subseteq V$, $NGcl(V)$ is $\mathfrak{g}\aleph$ compact and $NGcl(V) \subseteq G$.

Proof. Suppose that S_1 is $\mathfrak{g}\aleph$ locally compact on an \aleph - set A . By Definition 3.4, there exists a $\mathfrak{g}\aleph$ open set G , such that $A \subseteq G$ and G is $\mathfrak{g}\aleph$ compact. Since S_1 is $\mathfrak{g}\aleph$ Hausdorff space, by Remark 3.3 ($\mathfrak{g}\aleph$ compact subspace of $\mathfrak{g}\aleph$ Hausdorff space is $\mathfrak{g}\aleph$ closed), G is $\mathfrak{g}\aleph$ closed, thus $G = NGcl(G)$. Consider an \aleph - set $A \subseteq G$. Since S_1 is $\mathfrak{g}\aleph$ Hausdorff space, by Definition 3.3, for any two \aleph - sets A and B with $A \cap B = 0_\sim$, there exist a $\mathfrak{g}\aleph$ open sets C and D , such that $A \subseteq C$, $B \subseteq D$ and $C \cap D = 0_\sim$. Let $V = C \cap G$. Hence $V \subseteq G$ implies $NGcl(V) \subseteq NGcl(G) = G$. Since $NGcl(V)$ is $\mathfrak{g}\aleph$ closed and G is $\mathfrak{g}\aleph$ compact, by Remark 3.3 (every $\mathfrak{g}\aleph$ closed subset of a $\mathfrak{g}\aleph$ compact space is $\mathfrak{g}\aleph$ compact) it follows that $NGcl(V)$ is \aleph - compact. Thus $A \subseteq NGcl(V) \subseteq G$ and $NGcl(G)$ is $\mathfrak{g}\aleph$ compact.

The converse follows from Proposition 3.1(b).

Definition 3.5. Let S_1 and S_2 be two NTS. The function $T : S_1 \times S_2 \rightarrow S_2 \times S_1$ defined by $T(x, y) = (y, x)$ for each $(x, y) \in S_1 \times S_2$ is called a \aleph - switching map.

Proposition 3.3. The \aleph - switching map $T : S_1 \times S_2 \rightarrow S_2 \times S_1$ defined as above is $\mathfrak{g}\aleph$ continuous.

We now introduce the concept of $\mathfrak{g}\aleph$ compact open topology in the set of all $\mathfrak{g}\aleph$ continuous functions from a NTS S_1 to a NTS S_2 .

Definition 3.6. Let S_1 and S_2 be two NTS and let $S_2^{S_1} = \{f : S_1 \rightarrow S_2 \text{ such that } f \text{ is } \mathfrak{g}\aleph \text{ continuous}\}$. We give this class $S_2^{S_1}$ a topology called the $\mathfrak{g}\aleph$ compact open topology as follows: Let $\mathcal{K} = \{K \in I_1^S : K \text{ is } \mathfrak{g}\aleph \text{ compact } S_1\}$ and $\mathcal{V} = \{V \in I_2^S : V \text{ is } \mathfrak{g}\aleph \text{ open in } S_2\}$. For any $K \in \mathcal{K}$ and $V \in \mathcal{V}$, let $S_{K,V} = \{f \in S_2^{S_1} : f(K) \subseteq V\}$.

The collection of all such $\{S_{K,V} : K \in \mathcal{K}, V \in \mathcal{V}\}$ generates an \aleph - structure on the class $S_2^{S_1}$.

4 Generalized Neutrosophic Evaluation Map and Generalized Neutrosophic Exponential Map

We now consider the $\mathfrak{g}\aleph$ product topological space $S_2^{S_1} \times S_1$ and define a $\mathfrak{g}\aleph$ continuous map from $S_2^{S_1} \times S_1$ into S_2 .

Definition 4.1. The mapping $e : S_2^{S_1} \times S_1 \rightarrow S_2$ defined by $e(f, A) = f(A)$ for each \aleph - set A in S_1 and $f \in S_2^{S_1}$ is called the $\mathfrak{g}\aleph$ evaluation map.

Definition 4.2. Let S_1, S_2 and S_3 be three NTS and $f : S_3 \times S_1 \rightarrow S_2$ be any function. Then the induced map $\hat{f} : S_1 \rightarrow S_2^{S_3}$ is defined by $(\hat{f}(A_1))(A_2) = f(A_2, A_1)$ for \aleph - sets A_1 of S_1 and A_2 of S_3 .

Conversely, given a function $\hat{f} : S_1 \rightarrow S_2^{S_3}$, a corresponding function f can be also be defined by the same rule.

Proposition 4.1. Let S_1 be a $g\aleph$ locally compact Hausdorff space. Then the $g\aleph$ evaluation map $e : S_2^{S_1} \times S_1 \rightarrow S_2$ is $g\aleph$ continuous.

Proof. Consider $(f, A_1) \in S_2^{S_1} \times S_1$, where $f \in S_2^{S_1}$ and \aleph - set A_1 of S_1 . Let V be a $g\aleph$ open set containing $f(A_1) = e(f, A_1)$ in S_2 . Since S_1 is $g\aleph$ locally compact and f is $g\aleph$ continuous, by Proposition 3.2, there exists a $g\aleph$ open set U in S_1 , such that $A_1 \subseteq NGcl(U)$ and $NGcl(U)$ is $g\aleph$ compact and $f(NGcl(U)) \subseteq V$.

Consider the $g\aleph$ open set $S_{NGcl(U),V} \times U$ in $S_2^{S_1} \times S_1$. (f, A_1) is such that $f \in S_{NGcl(U),V}$ and $A_1 \subseteq U$. Let (g, A_2) be such that $g \in S_{NGcl(U),V}$ and $A_2 \subseteq U$ be arbitrary, thus $g(NGcl(U)) \subseteq V$. Since $A_2 \subseteq U$, we have $g(A_2) \subseteq V$ and $e(g, A_2) = g(A_2) \subseteq V$. Thus $e(S_{NGcl(U),V} \times U) \subseteq V$. Hence e is $g\aleph$ continuous.

Proposition 4.2. Let S_1 and S_2 be two NTS with S_2 is $g\aleph$ compact. Let A_1 be any \aleph - set in S_1 and N be a $g\aleph$ open set in the $g\aleph$ product space $S_1 \times S_2$ containing $\{A_1\} \times S_2$. Then there exists some $g\aleph$ open W with $A_1 \subseteq W$ in S_1 , such that $\{A_1\} \times S_2 \subseteq W \times S_2 \subseteq N$.

Proof. It is clear that by Remark 3.1, $\{A_1\} \times S_2$ is $g\aleph$ homeomorphism to S_2 and hence by Remark 3.2, $\{A_1\} \times S_2$ is $g\aleph$ compact. We cover $\{A_1\} \times S_2$ by the basis elements $\{U \times V\}$ (for the $g\aleph$ product topology) lying in N . Since $\{A_1\} \times S_2$ is $g\aleph$ compact, $\{U \times V\}$ has a finite subcover, say a finite number of basis elements $U_1 \times V_1, \dots, U_n \times V_n$. Without loss of generality we assume that $\{A_1\} \subseteq U_i$ for each $i = 1, 2, \dots, n$. Since otherwise the basis elements would be superfluous.

Let $W = \bigcap_{i=1}^n U_i$. Clearly W is $g\aleph$ open and $A_1 \subseteq W$. We show that $W \times S_2 \subseteq \bigcup_{i=1}^n (U_i \times V_i)$. Let (A_1, B) be an \aleph - set in $W \times S_2$. Now $(A_1, B) \subseteq U_i \times V_i$ for some i , thus $B \subseteq V_i$. But $A_1 \subseteq U_i$ for every $i = 1, 2, \dots, n$ (because $A_1 \subseteq W$). Therefore, $(A_1, B) \subseteq U_i \times V_i$ as desired. But $U_i \times V_i \subseteq N$ for all $i = 1, 2, \dots, n$ and $W \times S_2 \subseteq \bigcup_{i=1}^n (U_i \times V_i)$, therefore $W \times S_2 \subseteq N$.

Proposition 4.3. Let S_3 be a $g\aleph$ locally compact Hausdorff space and S_1, S_2 be arbitrary NTS. Then a map $f : S_3 \times S_1 \rightarrow S_2$ is $g\aleph$ continuous iff $\hat{f} : S_1 \rightarrow S_2^{S_3}$ is $g\aleph$ continuous, where \hat{f} is defined by the rule $(\hat{f}(A_1))(A_2) = f(A_2, A_1)$.

Proof. Suppose that \hat{f} is $g\aleph$ continuous. Consider the functions $S_3 \times S_1 \xrightarrow{i_Z} \times \hat{f} S_3 \times S_2^{S_3} \xrightarrow{t} S_2^{S_3} \times S_3 \xrightarrow{e} S_2$, where i_Z denote the \aleph - identity function on Z , t denote the \aleph - switching map and e denote the $g\aleph$ evaluation map. Since $et(i_Z \times \hat{f})(A_2, A_1) = et(A_2, \hat{f}(A_1)) = e(\hat{f}(A_1), A_2) = (\hat{f}(A_1))(A_2) = f(A_2, A_1)$ it follows that $f = et(i_Z \times \hat{f})$ and f being the composition of $g\aleph$ continuous functions is itself $g\aleph$.

Conversely, suppose that f is $g\aleph$ continuous, let A_1 be any arbitrary \aleph - set in S_1 . We have $\hat{f}(A_1) \in S_2^{S_3}$. Consider $S_{K,U} = \{g \in S_2^{S_3} : g(K) \subseteq U, K \in I^{S_3} \text{ is } g\aleph \text{ compact and } U \in I^{S_2} \text{ is } g\aleph \text{ open}\}$, containing $\hat{f}(A_1)$. We need to find a $g\aleph$ open W with $A_1 \subseteq W$, such that $\hat{f}(A_1) \subseteq S_{K,U}$; this will suffice to prove \hat{f} to be a $g\aleph$ continuous map.

For any \aleph - set A_2 in K , we have $(\hat{f}(A_1))(A_2) = f(A_2, A_1) \in U$ thus $f(K \times \{A_1\}) \subseteq U$, that is $K \times \{A_1\} \subseteq f^{-1}(U)$. Since f is $g\aleph$ continuous, $f^{-1}(U)$ is a $g\aleph$ open set in $S_3 \times S_1$. Thus $f^{-1}(U)$ is a $g\aleph$ open set $S_3 \times S_1$ containing $K \times \{A_1\}$. Hence by Proposition 4.2, there exists a $g\aleph$ open W with $A_1 \subseteq W$ in S_1 , such that $K \times \{A_1\} \subseteq K \times W \subseteq f^{-1}(U)$. Therefore $f(K \times W) \subseteq U$. Now for any $A_1 \subseteq W$ and $A_2 \subseteq K$, $f(A_2, A_1) = (\hat{f}(A_1))(A_2) \subseteq U$. Therefore $\hat{f}(A_1)(K) \subseteq U$ for all $A_1 \subseteq W$. That is $\hat{f}(A_1) \in S_{K,U}$ for all $A_1 \subseteq W$. Hence $\hat{f}(W) \subseteq S_{K,U}$ as desired.

Proposition 4.4. Let S_1 and S_3 be two $g\aleph$ locally compact Hausdorff spaces. Then for any NTS S_2 , the function $E : S_2^{S_3 \times S_1} \rightarrow (S_2^{S_3})^{S_1}$ defined by $E(f) = \hat{f}$ (that is $E(f)(A_1)(A_2) = f(A_2, A_1) = (\hat{f}(A_1))(A_2)$) for all $f : S_3 \times S_1 \rightarrow S_2$ is a $g\aleph$ homeomorphism.

Proof.

- (a) Clearly E is onto.

- (b) For E to be injective. Let $E(f) = E(g)$ for $f, g : S_3 \times S_1 \rightarrow S_2$. Thus $\widehat{f} = \widehat{g}$, where \widehat{f} and \widehat{g} are the induced maps of f and g respectively. Now for any \aleph - set A_1 in S_1 and any \aleph - set A_2 in S_3 , $f(A_2, A_1) = (\widehat{f}(A_1))(A_2) = (\widehat{g}(A_1))(A_2) = g(A_2, A_1)$; thus $f = g$.
- (c) For proving the $\mathfrak{g}\aleph$ continuity of E , consider any $\mathfrak{g}\aleph$ subbasis neighbourhood V of \widehat{f} in $(S_2^{S_3})^{S_1}$, that is V is of the form $S_{K,W}$ where K is a $\mathfrak{g}\aleph$ compact subset of S_1 and W is $\mathfrak{g}\aleph$ open in $S_2^{S_3}$. Without loss of generality we may assume that $W = S_{L,U}$, where L is a $\mathfrak{g}\aleph$ compact subset of S_3 and U is a $\mathfrak{g}\aleph$ open set in S_2 . Then $\widehat{f}(K) \subseteq S_{L,U} = W$ and this implies that $\widehat{f}(K)(L) \subseteq U$. Thus for any \aleph - set $A_1 \subseteq K$ and for all \aleph - sets $A_2 \subseteq L$. We have $(\widehat{f}(A_1))(A_2) \subseteq U$, that is $f(A_2, A_1) \subseteq U$ and therefore $f(L \times K) \subseteq U$. Now since L is $\mathfrak{g}\aleph$ compact in S_3 and K is $\mathfrak{g}\aleph$ compact in S_1 , $L \times K$ is also $\mathfrak{g}\aleph$ compact in $S_3 \times S_1$ [6] and since U is a $\mathfrak{g}\aleph$ open set in S_2 , we conclude that $f \in S_{L \times K, U} \subseteq S_{L \times K, U}^{S_3 \times S_1}$. We assert that $E(S_{L \times K, U}) \subseteq S_{K,W}$. Let $g \in S_{L \times K, U}$ be arbitrary. Thus $g(L \times K) \subseteq U$, that is $g(A_2, A_1) = (\widehat{g}(A_1))(A_2) \subseteq U$ for all \aleph - sets $A_2 \subseteq L$ in S_3 and for all \aleph - sets $A_1 \subseteq K$ in S_1 . So $(\widehat{g}(A_1))(L) \subseteq U$ for all \aleph - sets $A_1 \subseteq K$ in S_1 , that is $\widehat{g}(A_1) \subseteq S_{L,U} = W$ for all \aleph - sets $A_1 \subseteq K$ in U . Hence we have $\widehat{g}(K) \subseteq W$, that is $\widehat{g} = E(g) \in S_{K,W}$ for any $g \in S_{L \times K, U}$. Thus $E(S_{L \times K, U}) \subseteq S_{K,W}$. This proves that E is $\mathfrak{g}\aleph$ continuous.
- (d) For proving the $\mathfrak{g}\aleph$ continuity of E^{-1} , we consider the following $\mathfrak{g}\aleph$ evaluation maps: $e_1 : (S_2^{S_3})^{S_1} \times S_1 \rightarrow S_2^{S_3}$ defined by $e_1(\widehat{f}, A_1) = \widehat{f}(A_1)$ where $\widehat{f} \in (S_2^{S_3})^{S_1}$ and A_1 is an \aleph - set in S_1 and $e_2 : S_2^{S_3} \times S_3 \rightarrow S_2$ defined by $e_2(g, A_2) = g(A_2)$ where $g \in S_2^{S_3}$ and A_2 is a \aleph - set in S_3 . Let ψ denote the composition of the following $\mathfrak{g}\aleph$ continuous functions $\psi : (S_3 \times S_1) \times (S_2^{S_3})^{S_1} \xrightarrow{T} (S_2^{S_3})^{S_1} \times (S_3 \times S_1) \xrightarrow{i \times t} (S_2^{S_3})^{S_1} \times (S_1 \times S_3) \xrightarrow{\cong} ((S_2^{S_3})^{S_1} \times S_1) \times S_3 \xrightarrow{e_1 \times i_Z} (S_2^{S_3}) \times S_3 \xrightarrow{e_2} S_2$, where i, i_Z denote the \aleph - identity maps on $(S_2^{S_3})^{S_1}$ and S_3 respectively and T, t denote the \aleph - switching maps. Thus $\psi : (S_3 \times S_1) \times (S_2^{S_3})^{S_1} \rightarrow S_2$ that is $\psi \in S_2^{(S_3 \times S_1) \times (S_2^{S_3})^{S_1}}$. We consider the map $\widetilde{E} : S_2^{(S_3 \times S_1) \times (S_2^{S_3})^{S_1}} \rightarrow (S_2^{(S_3 \times S_1)})^{(S_2^{S_3})^{S_1}}$ (as defined in the statement of the proposition in fact it is E). So $\widetilde{E}(\psi) : (S_2^{S_3})^{S_1} \rightarrow S_2^{(S_3 \times S_1)}$. Now for any \aleph - sets A_2 in S_3, A_1 in S_1 and $f \in S_2^{(S_3 \times S_1)}$, again to check that $(\widetilde{E}(\psi) \circ E)(f)(A_2, A_1) = f(A_2, A_1)$; hence $\widetilde{E}(\psi) \circ E = \text{identity}$. Similarly for any $\widehat{g} \in (S_2^{S_3})^{S_1}$ and \aleph - sets A_1 in S_1, A_2 in S_3 , again to check that $(E \circ \widetilde{E}(\psi))(\widehat{g})(A_1, A_2) = (\widehat{g}(A_1))(A_2)$; hence $E \circ \widetilde{E}(\psi) = \text{identity}$. Thus E is a $\mathfrak{g}\aleph$ homeomorphism.

Definition 4.3. The map E in Proposition 4.4 is called the $\mathfrak{g}\aleph$ exponential map.

As easy consequence of Proposition 4.4 is as follows.

Proposition 4.5. Let S_1, S_2 and S_3 be three $\mathfrak{g}\aleph$ locally compact Hausdorff spaces. Then the map $N : S_2^{S_1} \times S_3^{S_2} \rightarrow S_3^{S_1}$ defined by $N(f, g) = g \circ f$ is $\mathfrak{g}\aleph$ continuous.

Proof. Consider the following compositions: $S_1 \times S_2^{S_1} \times S_3^{S_2} \xrightarrow{T} S_2^{S_1} \times S_3^{S_2} \times S_1 \xrightarrow{t \times i_X} S_3^{S_2} \times S_2^{S_1} \times S_1 \xrightarrow{\cong} S_3^{S_2} \times (S_2^{S_1} \times S_1) \xrightarrow{i \times e_2} S_3^{S_2} \times S_2 \xrightarrow{e_2} S_3$ where T, t denote the \aleph - switching maps, i_X, i denote the \aleph - identity functions on S_1 and $S_3^{S_2}$ respectively and e_2 denote the $\mathfrak{g}\aleph$ evaluation maps. Let $\varphi = e_2 \circ (i \times e_2) \circ (t \times i_X) \circ T$. By proposition 4.4, we have an exponential map. $E : S_3^{S_1 \times S_2^{S_1} \times S_3^{S_2}} \rightarrow (S_3^{S_1})^{S_2^{S_1} \times S_3^{S_2}}$. Since $\varphi \in S_3^{S_1 \times S_2^{S_1} \times S_3^{S_2}}, E(\varphi) \in (S_3^{S_1})^{S_2^{S_1} \times S_3^{S_2}}$. Let $N = E(\varphi)$, that is $N : S_2^{S_1} \times S_3^{S_2} \rightarrow S_3^{S_1}$ is an $\mathfrak{g}\aleph$ continuous. For $f \in S_2^{S_1}, g \in S_3^{S_2}$ and for any \aleph - set A_1 in S_1 , it is easy to see that $N(f, g)(A_1) = g(f(A_1))$.

5 Conclusions

In this paper, we introduced the concept of g_N compact open topology and Some characterization of this topology are discussed.

References

- [1] K. Atanassov. Intuitionistic fuzzy sets, in: V. Sgurev, Ed., VII ITKR's Session, Sofia (June 1983 Central Sci. and Techn. Library, Bulg. Academy of Sciences., 1984).
- [2] K. Atanassov. Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*,20 (1986) 87 - 96.
- [3] K. Atanassov. Review and new results on Intuitionistic fuzzy sets, Preprint IM-MFAIS-1-88, Sofia, 1988.
- [4] K. Atanassov and S. Stoeva. Intuitionistic fuzzy sets, in: Polish Syrup. on Interval & Fuzzy Mathematics, Poznan,(August 1983) 23 - 26.
- [5] C.L. Chang. Fuzzy topological spaces, *J. Math. Anal. Appl.*, 24 (1968) 182 - 190.
- [6] R.Dhavaseelan,E.Roja and M.K.Uma. Generalized intuitionistic Fuzzy Closed Sets, *Advances in Fuzzy Mathematics*, 5, (2010) 152-172.
- [7] R.Dhavaseelan and S.Jafari. Generalized Neutrosophic closed sets, *New Trends in Neutrosophic Theory and Applications*,Volume II, 2017,261–273.
- [8] R. Dhavaseelan, E. Roja and M. K. Uma. Intuitionistic Fuzzy Exponential Map Via Generalized Open Set, *International Journal of Mathematical Archive*,3(2), 2012, Page: 636-643
- [9] A.A.Salama and S.A.Alblowi. Neutrosophic Set and Neutrosophic Topological Spaces,*IOSR Journal of Mathematics*,Volume 3, Issue 4 (Sep-Oct. 2012), PP 31-35.
- [10] F. Smarandache. Neutrosophy and Neutrosophic Logic , First International Conference on Neutrosophy , Neutrosophic Logic , Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA(2002) , smarand@unm.edu
- [11] F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Research Press, Rehoboth, NM, 1999.
- [12] L.A. Zadeh. Fuzzy sets, *Inform. and Control.*,8 (1965),338-353.

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