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# BMBJ-neutrosophic ideals in $BCK/BCI$ -algebras

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**Abstract:** The concepts of a BMBJ-neutrosophic  $\circ$ -subalgebra and a (closed) BMBJ-neutrosophic ideal are introduced, and several properties are investigated. Conditions for an MBJ-neutrosophic set to be a BMBJ-neutrosophic ideal in  $BCK/BCI$ -algebras are provided. Characterizations of BMBJ-neutrosophic ideal are discussed. Relations between a BMBJ-neutrosophic subalgebra, a BMBJ-neutrosophic  $\circ$ -subalgebra and a (closed) BMBJ-neutrosophic ideal are considered.

**Keywords:** MBJ-neutrosophic set; BMBJ-neutrosophic subalgebra; BMBJ-neutrosophic ideal; BMBJ-neutrosophic  $\circ$ -subalgebra.

## 1 Introduction

Smarandache introduced the notion of neutrosophic set which is a more general platform that extends the notions of classic set, (intuitionistic) fuzzy set and interval valued (intuitionistic) fuzzy set (see [11, 12]). Neutrosophic set theory is applied to various part which is referred to the site

<http://fs.gallup.unm.edu/neutrosophy.htm>.

Jun and his colleagues applied the notion of neutrosophic set theory to  $BCK/BCI$ -algebras (see [4, 5, 6, 7, 10, 13, 14]). Borzooei et al. [2] studied commutative generalized neutrosophic ideals in  $BCK$ -algebras. Mohseni et al. [9] introduced the notion of MBJ-neutrosophic sets which is another generalization of neutrosophic set. They introduced the concept of MBJ-neutrosophic subalgebras in  $BCK/BCI$ -algebras, and investigated related properties. They gave a characterization of MBJ-neutrosophic subalgebra, and established a new MBJ-neutrosophic subalgebra by using an MBJ-neutrosophic subalgebra of a  $BCI$ -algebra. They considered the homomorphic inverse image of MBJ-neutrosophic subalgebra, and discussed translation of MBJ-neutrosophic subalgebra. Bordbar et al. [1] introduced the notion of BMBJ-neutrosophic subalgebras, and investigated related properties.

In this paper, we apply the notion of MBJ-neutrosophic sets to ideals of  $BCK/BI$ -algebras. We introduce the concepts of a BMBJ-neutrosophic  $\circ$ -subalgebra and a (closed) BMBJ-neutrosophic ideal, and investigate several properties. We provide conditions for an MBJ-neutrosophic set to be a BMBJ-neutrosophic ideal in  $BCK/BCI$ -algebras, and discuss characterizations of BMBJ-neutrosophic ideal. We consider relations between a BMBJ-neutrosophic subalgebra, a BMBJ-neutrosophic  $\circ$ -subalgebra and a (closed) BMBJ-neutrosophic ideal.

## 2 Preliminaries

By a *BCI-algebra*, we mean a set  $X$  with a binary operation  $*$  and a special element  $0$  that satisfies the following conditions:

$$(I) ((x * y) * (x * z)) * (z * y) = 0,$$

$$(II) (x * (x * y)) * y = 0,$$

$$(III) x * x = 0,$$

$$(IV) x * y = 0, y * x = 0 \Rightarrow x = y$$

for all  $x, y, z \in X$ . If a *BCI-algebra*  $X$  satisfies the following identity:

$$(V) (\forall x \in X) (0 * x = 0),$$

then  $X$  is called a *BCK-algebra*.

By a *weakly BCK-algebra* (see [3]), we mean a *BCI-algebra*  $X$  satisfying  $0 * x \leq x$  for all  $x \in X$ .

Every *BCK/BCI-algebra*  $X$  satisfies the following conditions:

$$(\forall x \in X) (x * 0 = x), \tag{2.1}$$

$$(\forall x, y, z \in X) (x \leq y \Rightarrow x * z \leq y * z, z * y \leq z * x), \tag{2.2}$$

$$(\forall x, y, z \in X) ((x * y) * z = (x * z) * y), \tag{2.3}$$

$$(\forall x, y, z \in X) ((x * z) * (y * z) \leq x * y) \tag{2.4}$$

where  $x \leq y$  if and only if  $x * y = 0$ . Any *BCI-algebra*  $X$  satisfies the following conditions (see [3]):

$$(\forall x, y \in X) (x * (x * (x * y)) = x * y), \tag{2.5}$$

$$(\forall x, y \in X) (0 * (x * y) = (0 * x) * (0 * y)). \tag{2.6}$$

A *BCI-algebra*  $X$  is said to be *p-semisimple* (see [3]) if

$$(\forall x \in X) (0 * (0 * x) = x). \tag{2.7}$$

In a *p-semisimple BCI-algebra*  $X$ , the following holds:

$$(\forall x, y \in X) (0 * (x * y) = y * x, x * (x * y) = y). \tag{2.8}$$

A *BCI-algebra*  $X$  is said to be *associative* (see [3]) if

$$(\forall x, y, z \in X) ((x * y) * z = x * (y * z)). \tag{2.9}$$

By an *(S)-BCK-algebra*, we mean a *BCK-algebra*  $X$  such that, for any  $x, y \in X$ , the set

$$\{z \in X \mid z * x \leq y\}$$

has the greatest element, written by  $x \circ y$  (see [8]).

A nonempty subset  $S$  of a  $BCK/BCI$ -algebra  $X$  is called a *subalgebra* of  $X$  if  $x * y \in S$  for all  $x, y \in S$ . A subset  $I$  of a  $BCK/BCI$ -algebra  $X$  is called an *ideal* of  $X$  if it satisfies:

$$0 \in I, \tag{2.10}$$

$$(\forall x \in X) (\forall y \in I) (x * y \in I \Rightarrow x \in I). \tag{2.11}$$

A subset  $I$  of a  $BCI$ -algebra  $X$  is called a *closed ideal* of  $X$  (see [3]) if it is an ideal of  $X$  which satisfies:

$$(\forall x \in X)(x \in I \Rightarrow 0 * x \in I). \tag{2.12}$$

By an *interval number* we mean a closed subinterval  $\tilde{a} = [a^-, a^+]$  of  $I$ , where  $0 \leq a^- \leq a^+ \leq 1$ . Denote by  $[I]$  the set of all interval numbers.

Let  $X$  be a nonempty set. A function  $A : X \rightarrow [I]$  is called an *interval-valued fuzzy set* (briefly, an *IVF set*) in  $X$ . Let  $[I]^X$  stand for the set of all IVF sets in  $X$ . For every  $A \in [I]^X$  and  $x \in X$ ,  $A(x) = [A^-(x), A^+(x)]$  is called the *degree* of membership of an element  $x$  to  $A$ , where  $A^- : X \rightarrow I$  and  $A^+ : X \rightarrow I$  are fuzzy sets in  $X$  which are called a *lower fuzzy set* and an *upper fuzzy set* in  $X$ , respectively. For simplicity, we denote  $A = [A^-, A^+]$ .

Let  $X$  be a non-empty set. A *neutrosophic set* (NS) in  $X$  (see [11]) is a structure of the form:

$$A := \{ \langle x; A_T(x), A_I(x), A_F(x) \rangle \mid x \in X \}$$

where  $A_T : X \rightarrow [0, 1]$  is a truth membership function,  $A_I : X \rightarrow [0, 1]$  is an indeterminate membership function, and  $A_F : X \rightarrow [0, 1]$  is a false membership function. For the sake of simplicity, we shall use the symbol  $A = (A_T, A_I, A_F)$  for the neutrosophic set

$$A := \{ \langle x; A_T(x), A_I(x), A_F(x) \rangle \mid x \in X \}.$$

We refer the reader to the books [3, 8] for further information regarding  $BCK/BCI$ -algebras, and to the site “<http://fs.gallup.unm.edu/neutrosophy.htm>” for further information regarding neutrosophic set theory.

Let  $X$  be a non-empty set. By an *MBJ-neutrosophic set* in  $X$  (see [9]), we mean a structure of the form:

$$\mathcal{A} := \{ \langle x; M_A(x), \tilde{B}_A(x), J_A(x) \rangle \mid x \in X \}$$

where  $M_A$  and  $J_A$  are fuzzy sets in  $X$ , which are called a truth membership function and a false membership function, respectively, and  $\tilde{B}_A$  is an IVF set in  $X$  which is called an indeterminate interval-valued membership function.

For the sake of simplicity, we shall use the symbol  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  for the MBJ-neutrosophic set

$$\mathcal{A} := \{ \langle x; M_A(x), \tilde{B}_A(x), J_A(x) \rangle \mid x \in X \}.$$

Let  $X$  be a  $BCK/BCI$ -algebra. An MBJ-neutrosophic set  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  in  $X$  is called a *BMBJ-neutrosophic subalgebra* of  $X$  (see [1]) if it satisfies:

$$(\forall x \in X)(M_A(x) + B_A^-(x) \leq 1, B_A^+(x) + J_A(x) \leq 1) \tag{2.13}$$

and

$$(\forall x, y \in X) \left( \begin{array}{l} M_A(x * y) \geq \min\{M_A(x), M_A(y)\} \\ B_A^-(x * y) \leq \max\{B_A^-(x), B_A^-(y)\} \\ B_A^+(x * y) \geq \min\{B_A^+(x), B_A^+(y)\} \\ J_A(x * y) \leq \max\{J_A(x), J_A(y)\} \end{array} \right). \quad (2.14)$$

### 3 BMBJ-neutrosophic ideals

**Definition 3.1.** Let  $X$  be a  $BCK/BCI$ -algebra. An  $MBJ$ -neutrosophic set  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  in  $X$  is called a *BMBJ-neutrosophic ideal* of  $X$  if it satisfies (2.13) and

$$(\forall x \in X) \left( \begin{array}{l} M_A(0) \geq M_A(x) \\ B_A^-(0) \leq B_A^-(x) \\ B_A^+(0) \geq B_A^+(x) \\ J_A(0) \leq J_A(x) \end{array} \right), \quad (3.1)$$

$$(\forall x, y \in X) \left( \begin{array}{l} M_A(x) \geq \min\{M_A(x * y), M_A(y)\} \\ B_A^-(x) \leq \max\{B_A^-(x * y), B_A^-(y)\} \\ B_A^+(x) \geq \min\{B_A^+(x * y), B_A^+(y)\} \\ J_A(x) \leq \max\{J_A(x * y), J_A(y)\} \end{array} \right). \quad (3.2)$$

A *BMBJ-neutrosophic ideal*  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  of a  $BCI$ -algebra  $X$  is said to be *closed* if

$$(\forall x \in X) \left( \begin{array}{l} M_A(0 * x) \geq M_A(x) \\ B_A^-(0 * x) \leq B_A^-(x) \\ B_A^+(0 * x) \geq B_A^+(x) \\ J_A(0 * x) \leq J_A(x) \end{array} \right). \quad (3.3)$$

**Example 3.2.** Consider a set  $X = \{0, 1, 2, a\}$  with the binary operation  $*$  which is given in Table 1. Then

Table 1: Cayley table for the binary operation “ $*$ ”

$*$	0	1	2	$a$
0	0	0	0	$a$
1	1	0	0	$a$
2	2	2	0	$a$
$a$	$a$	$a$	$a$	0

$(X; *, 0)$  is a  $BCI$ -algebra (see [3]). Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be an  $MBJ$ -neutrosophic set in  $X$  defined by Table 2. It is routine to verify that  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a closed  $MBJ$ -neutrosophic ideal of  $X$ .

Table 2: MBJ-neutrosophic set  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$

$X$	$M_A(x)$	$\tilde{B}_A(x)$	$J_A(x)$
0	0.7	[0.02, 0.08]	0.2
1	0.5	[0.02, 0.06]	0.2
2	0.4	[0.02, 0.06]	0.7
$a$	0.3	[0.02, 0.06]	0.7

**Proposition 3.3.** *Let  $X$  be a BCK/BCI-algebra. Then every BMBJ-neutrosophic ideal  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  of  $X$  satisfies the following assertion.*

$$x * y \leq z \Rightarrow \begin{cases} M_A(x) \geq \min\{M_A(y), M_A(z)\}, \\ B_A^-(x) \leq \max\{B_A^-(y), B_A^-(z)\}, \\ B_A^+(x) \geq \min\{B_A^+(y), B_A^+(z)\}, \\ J_A(x) \leq \max\{J_A(y), J_A(z)\} \end{cases} \tag{3.4}$$

for all  $x, y, z \in X$ .

*Proof.* Let  $x, y, z \in X$  be such that  $x * y \leq z$ . Then

$$M_A(x * y) \geq \min\{M_A((x * y) * z), M_A(z)\} = \min\{M_A(0), M_A(z)\} = M_A(z),$$

$$B_A^-(x * y) \leq \max\{B_A^-((x * y) * z), B_A^-(z)\} = \max\{B_A^-(0), B_A^-(z)\} = B_A^-(z),$$

$$B_A^+(x * y) \geq \min\{B_A^+((x * y) * z), B_A^+(z)\} = \min\{B_A^+(0), B_A^+(z)\} = B_A^+(z),$$

and

$$J_A(x * y) \leq \max\{J_A((x * y) * z), J_A(z)\} = \max\{J_A(0), J_A(z)\} = J_A(z).$$

It follows that

$$M_A(x) \geq \min\{M_A(x * y), M_A(y)\} = \min\{M_A(y), M_A(z)\},$$

$$B_A^-(x) \leq \max\{B_A^-(x * y), B_A^-(y)\} = \max\{B_A^-(y), B_A^-(z)\},$$

$$B_A^+(x) \geq \min\{B_A^+(x * y), B_A^+(y)\} = \min\{B_A^+(y), B_A^+(z)\},$$

and

$$J_A(x) \leq \max\{J_A(x * y), J_A(y)\} = \max\{J_A(y), J_A(z)\}.$$

This completes the proof. □

We provide conditions for an MBJ-neutrosophic set to be a BMBJ-neutrosophic ideal in  $BCK/BCI$ -algebras.

**Theorem 3.4.** *Every MBJ-neutrosophic set in a  $BCK/BCI$ -algebra  $X$  satisfying (3.1) and (3.4) is a BMBJ-neutrosophic ideal of  $X$ .*

*Proof.* Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be an MBJ-neutrosophic set in  $X$  satisfying (3.1) and (3.4). Note that  $x * (x * y) \leq y$  for all  $x, y \in X$ . It follows from (3.4) that

$$M_A(x) \geq \min\{M_A(x * y), M_A(y)\},$$

$$B_A^-(x) \leq \max\{B_A^-(x * y), B_A^-(y)\},$$

$$B_A^+(x) \geq \min\{B_A^+(x * y), B_A^+(y)\},$$

and

$$J_A(x) \leq \max\{J_A(x * y), J_A(y)\}.$$

Therefore  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic ideal of  $X$ . □

Given an MBJ-neutrosophic set  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  in a  $BCK/BCI$ -algebra  $X$ , we consider the following sets.

$$\begin{aligned} U(M_A; t) &:= \{x \in X \mid M_A(x) \geq t\}, \\ L(B_A^-; \alpha^-) &:= \{x \in X \mid B_A^-(x) \leq \alpha^-\}, \\ U(B_A^+; \alpha^+) &:= \{x \in X \mid B_A^+(x) \geq \alpha^+\}, \\ L(J_A; s) &:= \{x \in X \mid J_A(x) \leq s\} \end{aligned}$$

where  $t, s, \alpha^-, \alpha^+ \in [0, 1]$ .

**Theorem 3.5.** *An MBJ-neutrosophic set  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  in a  $BCK/BCI$ -algebra  $X$  is an MBJ-neutrosophic ideal of  $X$  if and only if the non-empty sets  $U(M_A; t)$ ,  $L(B_A^-; \alpha^-)$ ,  $U(B_A^+; \alpha^+)$  and  $L(J_A; s)$  are ideals of  $X$  for all  $t, s, \alpha^-, \alpha^+ \in [0, 1]$ .*

*Proof.* Suppose that  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is an MBJ-neutrosophic ideal of  $X$ . Let  $t, s, \alpha^-, \alpha^+ \in [0, 1]$  be such that  $U(M_A; t)$ ,  $L(B_A^-; \alpha^-)$ ,  $U(B_A^+; \alpha^+)$  and  $L(J_A; s)$  are non-empty. Obviously,  $0 \in U(M_A; t) \cap L(B_A^-; \alpha^-) \cap U(B_A^+; \alpha^+) \cap L(J_A; s)$ . For any  $x, y, a, b, p, q, u, v \in X$ , if  $x * y \in U(M_A; t)$ ,  $y \in U(M_A; t)$ ,  $a * b \in L(B_A^-; \alpha^-)$ ,  $b \in L(B_A^-; \alpha^-)$ ,  $p * q \in U(B_A^+; \alpha^+)$ ,  $q \in U(B_A^+; \alpha^+)$ ,  $u * v \in L(J_A; s)$  and  $v \in L(J_A; s)$ , then

$$\begin{aligned} M_A(x) &\geq \min\{M_A(x * y), M_A(y)\} \geq \min\{t, t\} = t, \\ B_A^-(a) &\leq \max\{B_A^-(a * b), B_A^-(b)\} \leq \max\{\alpha^-, \alpha^-\} = \alpha^-, \\ B_A^+(p) &\geq \min\{B_A^+(p * q), B_A^+(q)\} \geq \min\{\alpha^+, \alpha^+\} = \alpha^+, \\ J_A(u) &\leq \max\{J_A(u * v), J_A(v)\} \leq \min\{s, s\} = s, \end{aligned}$$

and so  $x \in U(M_A; t)$ ,  $a \in L(B_A^-; \alpha^-)$ ,  $p \in U(B_A^+; \alpha^+)$  and  $u \in L(J_A; s)$ . Therefore  $U(M_A; t)$ ,  $L(B_A^-; \alpha^-)$ ,  $U(B_A^+; \alpha^+)$  and  $L(J_A; s)$  are ideals of  $X$ .

Conversely, assume that the non-empty sets  $U(M_A; t)$ ,  $L(B_A^-; \alpha^-)$ ,  $U(B_A^+; \alpha^+)$  and  $L(J_A; s)$  are ideals of  $X$  for all  $t, s, \alpha^-, \alpha^+ \in [0, 1]$ . Assume that  $M_A(0) < M_A(a)$ ,  $B_A^-(0) > B_A^-(a)$ ,  $B_A^+(0) < B_A^+(a)$  and  $J_A(0) > J_A(a)$  for some  $a \in X$ . Then  $0 \notin U(M_A; M_A(a)) \cap L(B_A^-; B_A^-(a)) \cap U(B_A^+; B_A^+(a)) \cap L(J_A; J_A(a))$ , which is a contradiction. Hence  $M_A(0) \geq M_A(x)$ ,  $B_A^-(0) \leq B_A^-(x)$ ,  $B_A^+(0) \geq B_A^+(x)$  and  $J_A(0) \leq J_A(x)$  for all  $x \in X$ . If  $M_A(a_0) < \min\{M_A(a_0 * b_0), M_A(b_0)\}$  for some  $a_0, b_0 \in X$ , then  $a_0 * b_0 \in U(M_A; t_0)$  and  $b_0 \in U(M_A; t_0)$  but  $a_0 \notin U(M_A; t_0)$  for  $t_0 := \min\{M_A(a_0 * b_0), M_A(b_0)\}$ . This is a contradiction, and thus  $M_A(a) \geq \min\{M_A(a * b), M_A(b)\}$  for all  $a, b \in X$ . Similarly, we can show that  $J_A(a) \leq \max\{J_A(a * b), J_A(b)\}$  for all  $a, b \in X$ . Suppose that  $B_A^-(a_0) > \max\{B_A^-(a_0 * b_0), B_A^-(b_0)\}$  for some  $a_0, b_0 \in X$ . Taking  $\alpha^- = \max\{B_A^-(a_0 * b_0), B_A^-(b_0)\}$  implies that  $a_0 * b_0 \in L(B_A^-; \alpha^-)$  and  $b_0 \in L(B_A^-; \alpha^-)$  but  $a_0 \notin L(B_A^-; \alpha^-)$ . This is a contradiction. Thus  $B_A^-(x) \leq \max\{B_A^-(x * y), B_A^-(y)\}$  for all  $x, y \in X$ . Similarly, we obtain  $B_A^+(x) \geq \min\{B_A^+(x * y), B_A^+(y)\}$  for all  $x, y \in X$ . Consequently  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic ideal of  $X$ . □

**Theorem 3.6.** An MBJ-neutrosophic set  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  in a BCK/BCI-algebra  $X$  is a BMBJ-neutrosophic ideal of  $X$  if and only if  $(M_A, B_A^-)$  and  $(B_A^+, J_A)$  are intuitionistic fuzzy ideals of  $X$ .

*Proof.* Straightforward. □

**Theorem 3.7.** Given an ideal  $I$  of a BCK/BCI-algebra  $X$ , let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be an MBJ-neutrosophic set in  $X$  defined by

$$M_A(x) = \begin{cases} t & \text{if } x \in I, \\ 0 & \text{otherwise,} \end{cases} \quad B_A^-(x) = \begin{cases} \alpha^- & \text{if } x \in I, \\ 1 & \text{otherwise,} \end{cases}$$

$$B_A^+(x) = \begin{cases} \alpha^+ & \text{if } x \in I, \\ 0 & \text{otherwise,} \end{cases} \quad J_A(x) = \begin{cases} s & \text{if } x \in I, \\ 1 & \text{otherwise,} \end{cases}$$

where  $t, \alpha^+ \in (0, 1]$ ,  $s, \alpha^- \in [0, 1)$ . Then  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic ideal of  $X$  such that  $U(M_A; t) = L(B_A^-; \alpha^-) = U(B_A^+; \alpha^+) = L(J_A; s) = I$ .

*Proof.* It is clear that  $U(M_A; t) = L(B_A^-; \alpha^-) = U(B_A^+; \alpha^+) = L(J_A; s) = I$ . Let  $x, y \in X$ . If  $x * y \in I$  and  $y \in I$ , then  $x \in I$  and so

$$M_A(x) = t = \min\{M_A(x * y), M_A(y)\}$$

$$B_A^-(x) = \alpha^- = \max\{B_A^-(x * y), B_A^-(y)\},$$

$$B_A^+(x) = \alpha^+ = \min\{B_A^+(x * y), B_A^+(y)\},$$

$$J_A(x) = s = \max\{J_A(x * y), J_A(y)\}.$$

If any one of  $x * y$  and  $y$  is contained in  $I$ , say  $x * y \in I$ , then  $M_A(x * y) = t$ ,  $B_A^-(x * y) = \alpha^-$ ,  $J_A(x * y) = s$ ,  $M_A(y) = 0$ ,  $B_A^-(y) = 1$ ,  $B_A^+(y) = 0$  and  $J_A(y) = 1$ . Hence

$$M_A(x) \geq 0 = \min\{t, 0\} = \min\{M_A(x * y), M_A(y)\}$$

$$B_A^-(x) \leq 1 = \max\{B_A^-(x * y), B_A^-(y)\},$$

$$B_A^+(x) \geq 0 = \min\{B_A^+(x * y), B_A^+(y)\},$$

$$J_A(x) \leq 1 = \max\{s, 1\} = \max\{J_A(x * y), J_A(y)\}.$$



If  $x * y, y \notin I$ , then  $M_A(x * y) = 0 = M_A(y)$ ,  $B_A^-(x * y) = 1 = B_A^-(y)$ ,  $B_A^+(x * y) = 0 = B_A^+(y)$  and  $J_A(x * y) = 1 = J_A(y)$ . It follows that

$$\begin{aligned} M_A(x) &\geq 0 = \min\{M_A(x * y), M_A(y)\} \\ B_A^-(x) &\leq 1 = \max\{B_A^-(x * y), B_A^-(y)\}, \\ B_A^+(x) &\geq 0 = \min\{B_A^+(x * y), B_A^+(y)\}, \\ J_A(x) &\leq 1 = \max\{J_A(x * y), J_A(y)\}. \end{aligned}$$

It is obvious that  $M_A(0) \geq M_A(x)$ ,  $B_A^-(0) \leq B_A^-(x)$ ,  $B_A^+(0) \geq B_A^+(x)$  and  $J_A(0) \leq J_A(x)$  for all  $x \in X$ . Therefore  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic ideal of  $X$ .  $\square$

**Theorem 3.8.** For any non-empty subset  $I$  of  $X$ , let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be an MBJ-neutrosophic set in  $X$  which is given in Theorem 3.7. If  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic ideal of  $X$ , then  $I$  is an ideal of  $X$ .

*Proof.* Obviously,  $0 \in I$ . Let  $x, y \in X$  be such that  $x * y \in I$  and  $y \in I$ . Then  $M_A(x * y) = t = M_A(y)$ ,  $B_A^-(x * y) = \alpha^- = B_A^-(y)$ ,  $B_A^+(x * y) = \alpha^+ = B_A^+(y)$  and  $J_A(x * y) = s = J_A(y)$ . Thus

$$\begin{aligned} M_A(x) &\geq \min\{M_A(x * y), M_A(y)\} = t, \\ B_A^-(x) &\leq \max\{B_A^-(x * y), B_A^-(y)\} = \alpha^-, \\ B_A^+(x) &\geq \min\{B_A^+(x * y), B_A^+(y)\} = \alpha^+, \\ J_A(x) &\leq \max\{J_A(x * y), J_A(y)\} = s, \end{aligned}$$

and hence  $x \in I$ . Therefore  $I$  is an ideal of  $X$ .  $\square$

**Theorem 3.9.** In a BCK-algebra, every BMBJ-neutrosophic ideal is a BMBJ-neutrosophic subalgebra.

*Proof.* Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be a BMBJ-neutrosophic ideal of a BCK-algebra  $X$ . Since  $(x * y) * x \leq y$  for all  $x, y \in X$ , it follows from Proposition 3.3 that

$$\begin{aligned} M_A(x * y) &\geq \min\{M_A(x), M_A(y)\}, \\ B_A^-(x * y) &\leq \max\{B_A^-(x), B_A^-(y)\}, \\ B_A^+(x * y) &\geq \min\{B_A^+(x), B_A^+(y)\}, \\ J_A(x * y) &\leq \max\{J_A(x), J_A(y)\} \end{aligned}$$

for all  $x, y \in X$ . Hence  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic subalgebra of a BCK-algebra  $X$ .  $\square$

The converse of Theorem 3.9 may not be true as seen in the following example.

**Example 3.10.** Consider a BCK-algebra  $X = \{0, 1, 2, 3\}$  with the binary operation  $*$  which is given in Table 3. Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be an MBJ-neutrosophic set in  $X$  defined by Table 4. Then  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic subalgebra of  $X$ , but it is not a BMBJ-neutrosophic ideal of  $X$  since

$$B_A^+(1) \not\geq \min\{B_A^+(1 * 2), B_A^+(2)\}.$$

We provide a condition for a BMBJ-neutrosophic subalgebra to be a BMBJ-neutrosophic ideal in a BCK-algebra.

Table 3: Cayley table for the binary operation “\*”

*	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	1	0	2
3	3	3	3	0

Table 4: MBJ-neutrosophic set  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$

$X$	$M_A(x)$	$\tilde{B}_A(x)$	$J_A(x)$
0	0.7	[0.03, 0.08]	0.2
1	0.4	[0.02, 0.06]	0.3
2	0.4	[0.03, 0.08]	0.4
3	0.6	[0.02, 0.06]	0.5

**Theorem 3.11.** *Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be a BMBJ-neutrosophic subalgebra of a BCK-algebra  $X$  satisfying the condition (3.4). Then  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic ideal of  $X$ .*

*Proof.* For any  $x \in X$ , we get

$$M_A(0) = M_A(x * x) \geq \min\{M_A(x), M_A(x)\} = M_A(x),$$

$$B_A^-(0) = B_A^-(x * x) \leq \max\{B_A^-(x), B_A^-(x)\} = B_A^-(x),$$

$$B_A^+(0) = B_A^+(x * x) \geq \min\{B_A^+(x), B_A^+(x)\} = B_A^+(x),$$

and

$$J_A(0) = J_A(x * x) \leq \max\{J_A(x), J_A(x)\} = J_A(x).$$

Since  $x * (x * y) \leq y$  for all  $x, y \in X$ , it follows from (3.4) that

$$\begin{aligned} M_A(x) &\geq \min\{M_A(x * y), M_A(y)\}, \\ B_A^-(x) &\leq \max\{B_A^-(x * y), B_A^-(y)\}, \\ B_A^+(x) &\geq \min\{B_A^+(x * y), B_A^+(y)\}, \\ J_A(x) &\leq \max\{J_A(x * y), J_A(y)\} \end{aligned}$$

for all  $x, y \in X$ . Therefore  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic ideal of  $X$ . □

Theorem 3.9 is not true in a *BCI*-algebra as seen in the following example.

**Example 3.12.** Let  $(Y, *, 0)$  be a *BCI*-algebra and let  $(\mathbb{Z}, -, 0)$  be an adjoint *BCI*-algebra of the additive group  $(\mathbb{Z}, +, 0)$  of integers. Then  $X = Y \times \mathbb{Z}$  is a *BCI*-algebra and  $I = Y \times \mathbb{N}$  is an ideal of  $X$  where  $\mathbb{N}$  is the set of all non-negative integers (see [3]). Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be an *MBJ*-neutrosophic set in  $X$  which is given in Theorem 3.7. Then  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a *BMBJ*-neutrosophic ideal of  $X$  by Theorem 3.7. But it is not a *BMBJ*-neutrosophic subalgebra of  $X$  since

$$M_A((0, 0) * (0, 1)) = M_A((0, -1)) = 0 < t = \min\{M_A((0, 0)), M_A(0, 1)\},$$

$$B_A^-((0, 0) * (0, 2)) = B_A^-((0, -2)) = 1 > \alpha^- = \max\{B_A^-((0, 0)), B_A^-(0, 2)\},$$

$$B_A^+((0, 0) * (0, 2)) = B_A^+((0, -2)) = 0 < \alpha^+ = \min\{B_A^+((0, 0)), B_A^+(0, 2)\},$$

and/or

$$J_A((0, 0) * (0, 3)) = J_A((0, -3)) = 1 > s = \max\{J_A((0, 0)), J_A(0, 3)\}.$$

**Definition 3.13.** A *BMBJ*-neutrosophic ideal  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  of a *BCI*-algebra  $X$  is said to be *closed* if

$$(\forall x \in X)(M_A(0 * x) \geq M_A(x), B_A^-(0 * x) \leq B_A^-(x), B_A^+(0 * x) \geq B_A^+(x), J_A(0 * x) \leq J_A(x)). \quad (3.5)$$

**Theorem 3.14.** In a *BCI*-algebra, every closed *BMBJ*-neutrosophic ideal is a *BMBJ*-neutrosophic subalgebra.

*Proof.* Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be a closed *BMBJ*-neutrosophic ideal of a *BCI*-algebra  $X$ . Using (3.2), (2.3), (III) and (3.3), we have

$$M_A(x * y) \geq \min\{M_A((x * y) * x), M_A(x)\} = \min\{M_A(0 * y), M_A(x)\} \geq \min\{M_A(y), M_A(x)\},$$

$$B_A^-(x * y) \leq \max\{B_A^-((x * y) * x), B_A^-(x)\} = \max\{B_A^-(0 * y), B_A^-(x)\} \leq \max\{B_A^-(y), B_A^-(x)\},$$

$$B_A^+(x * y) \geq \min\{B_A^+((x * y) * x), B_A^+(x)\} = \min\{B_A^+(0 * y), B_A^+(x)\} \geq \min\{B_A^+(y), B_A^+(x)\},$$

and

$$J_A(x * y) \leq \max\{J_A((x * y) * x), J_A(x)\} = \max\{J_A(0 * y), J_A(x)\} \leq \max\{J_A(y), J_A(x)\}$$

for all  $x, y \in X$ . Hence  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a *BMBJ*-neutrosophic subalgebra of  $X$ .  $\square$

**Theorem 3.15.** In a weakly *BCK*-algebra, every *BMBJ*-neutrosophic ideal is closed.

*Proof.* Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be a *BMBJ*-neutrosophic ideal of a weakly *BCK*-algebra  $X$ . For any  $x \in X$ , we obtain

$$M_A(0 * x) \geq \min\{M_A((0 * x) * x), M_A(x)\} = \min\{M_A(0), M_A(x)\} = M_A(x),$$

$$B_A^-(0 * x) \leq \max\{B_A^-((0 * x) * x), B_A^-(x)\} = \max\{B_A^-(0), B_A^-(x)\} = B_A^-(x),$$

$$B_A^+(0 * x) \geq \min\{B_A^+((0 * x) * x), B_A^+(x)\} = \min\{B_A^+(0), B_A^+(x)\} = B_A^+(x),$$

and

$$J_A(0 * x) \leq \max\{J_A((0 * x) * x), J_A(x)\} = \max\{J_A(0), J_A(x)\} = J_A(x).$$

Therefore  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a closed BMBJ-neutrosophic ideal of  $X$ . □

**Corollary 3.16.** *In a weakly BCK-algebra, every BMBJ-neutrosophic ideal is a BMBJ-neutrosophic subalgebra.*

The following example shows that any BMBJ-neutrosophic subalgebra is not a BMBJ-neutrosophic ideal in a BCI-algebra.

**Example 3.17.** Consider a BCI-algebra  $X = \{0, a, b, c, d, e\}$  with the  $*$ -operation in Table 5.

Table 5: Cayley table for the binary operation “ $*$ ”

$*$	0	a	b	c	d	e
0	0	0	c	b	c	c
a	a	0	c	b	c	c
b	b	b	0	c	0	0
c	c	c	b	0	b	b
d	d	b	a	c	0	a
e	e	b	a	c	a	0

Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be an MBJ-neutrosophic set in  $X$  defined by Table 6.

Table 6: MBJ-neutrosophic set  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$

$X$	$M_A(x)$	$\tilde{B}_A(x)$	$J_A(x)$
0	0.7	[0.14, 0.19]	0.3
a	0.4	[0.04, 0.45]	0.6
b	0.7	[0.14, 0.19]	0.3
c	0.7	[0.14, 0.19]	0.3
d	0.4	[0.04, 0.45]	0.6
e	0.4	[0.04, 0.45]	0.6

It is routine to verify that  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic subalgebra of  $X$ . But it is not a BMBJ-neutrosophic ideal of  $X$  since

$$M_A(d) < \min\{M_A(d * c), M_A(c)\},$$

$$B_A^-(d) > \max\{B_A^-(d * c), B_A^-(c)\},$$

$$B_A^+(d) < \min\{B_A^+(d * c), B_A^+(c)\},$$

and/or

$$J_A(d) > \max\{J_A(d * c), J_A(c)\}.$$

**Theorem 3.18.** *In a  $p$ -semisimple BCI- $\tilde{B}$ -algebra  $X$ , the following are equivalent.*

- (1)  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a closed BMBJ-neutrosophic ideal of  $X$ .
- (2)  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic subalgebra of  $X$ .

*Proof.* (1)  $\Rightarrow$  (2). See Theorem 3.14.

(2)  $\Rightarrow$  (1). Suppose that  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic subalgebra of  $X$ . For any  $x \in X$ , we get

$$M_A(0) = M_A(x * x) \geq \min\{M_A(x), M_A(x)\} = M_A(x),$$

$$B_A^-(0) = B_A^-(x * x) \leq \max\{B_A^-(x), B_A^-(x)\} = B_A^-(x),$$

$$B_A^+(0) = B_A^+(x * x) \geq \min\{B_A^+(x), B_A^+(x)\} = B_A^+(x),$$

and

$$J_A(0) = J_A(x * x) \leq \max\{J_A(x), J_A(x)\} = J_A(x).$$

Hence  $M_A(0 * x) \geq \min\{M_A(0), M_A(x)\} = M_A(x)$ ,  $B_A^-(0 * x) \leq \max\{B_A^-(0), B_A^-(x)\} = B_A^-(x)$ ,  $B_A^+(0 * x) \geq \min\{B_A^+(0), B_A^+(x)\} = B_A^+(x)$  and  $J_A(0 * x) \leq \max\{J_A(0), J_A(x)\} = J_A(x)$  for all  $x \in X$ . Let  $x, y \in X$ . Then

$$\begin{aligned} M_A(x) &= M_A(y * (y * x)) \geq \min\{M_A(y), M_A(y * x)\} \\ &= \min\{M_A(y), M_A(0 * (x * y))\} \\ &\geq \min\{M_A(x * y), M_A(y)\}, \end{aligned}$$

$$\begin{aligned} B_A^-(x) &= B_A^-(y * (y * x)) \leq \max\{B_A^-(y), B_A^-(y * x)\} \\ &= \max\{B_A^-(y), B_A^-(0 * (x * y))\} \\ &\leq \max\{B_A^-(x * y), B_A^-(y)\} \end{aligned}$$

$$\begin{aligned} B_A^+(x) &= B_A^+(y * (y * x)) \geq \min\{B_A^+(y), B_A^+(y * x)\} \\ &= \min\{B_A^+(y), B_A^+(0 * (x * y))\} \\ &\geq \min\{B_A^+(x * y), B_A^+(y)\} \end{aligned}$$

and

$$\begin{aligned} J_A(x) &= J_A(y * (y * x)) \leq \max\{J_A(y), J_A(y * x)\} \\ &= \max\{J_A(y), J_A(0 * (x * y))\} \\ &\leq \max\{J_A(x * y), J_A(y)\}. \end{aligned}$$

Therefore  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a closed BMBJ-neutrosophic ideal of  $X$ . □

Since every associative BCI-algebra is  $p$ -semisimple, we have the following corollary.

**Corollary 3.19.** *In an associative BCI-algebra  $X$ , the following are equivalent.*

- (1)  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a closed BMBJ-neutrosophic ideal of  $X$ .
- (2)  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic subalgebra of  $X$ .

**Definition 3.20.** Let  $X$  be an  $(S)$ -BCK-algebra. An MBJ-neutrosophic set  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  in  $X$  is called a BMBJ-neutrosophic  $\circ$ -subalgebra of  $X$  if the following assertions are valid.

$$\begin{aligned} M_A(x \circ y) &\geq \min\{M_A(x), M_A(y)\}, \\ B_A^-(x \circ y) &\leq \max\{B_A^-(x), B_A^-(y)\}, \\ B_A^+(x \circ y) &\geq \min\{B_A^+(x), B_A^+(y)\}, \\ J_A(x \circ y) &\leq \max\{J_A(x), J_A(y)\} \end{aligned} \tag{3.6}$$

for all  $x, y \in X$ .

**Lemma 3.21.** *Every BMBJ-neutrosophic ideal of a BCK/BCI-algebra  $X$  satisfies the following assertion.*

$$(\forall x, y \in X) (x \leq y \Rightarrow M_A(x) \geq M_A(y), B_A^-(x) \leq B_A^-(y), B_A^+(x) \geq B_A^+(y), J_A(x) \leq J_A(y)). \tag{3.7}$$

*Proof.* Assume that  $x \leq y$  for all  $x, y \in X$ . Then  $x * y = 0$ , and so

$$M_A(x) \geq \min\{M_A(x * y), M_A(y)\} = \min\{M_A(0), M_A(y)\} = M_A(y),$$

$$B_A^-(x) \leq \max\{B_A^-(x * y), B_A^-(y)\} = \max\{B_A^-(0), B_A^-(y)\} = B_A^-(y),$$

$$B_A^+(x) \geq \min\{B_A^+(x * y), B_A^+(y)\} = \min\{B_A^+(0), B_A^+(y)\} = B_A^+(y),$$

and

$$J_A(x) \leq \max\{J_A(x * y), J_A(y)\} = \max\{J_A(0), J_A(y)\} = J_A(y).$$

This completes the proof. □

**Theorem 3.22.** *In an  $(S)$ -BCK-algebra, every BMBJ-neutrosophic ideal is a BMBJ-neutrosophic  $\circ$ -subalgebra.*

*Proof.* Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be a BMBJ-neutrosophic ideal of an  $(S)$ -BCK-algebra  $X$ . Note that  $(x \circ y) * x \leq y$  for all  $x, y \in X$ . Using Lemma 3.21 and (3.2) implies that

$$M_A(x \circ y) \geq \min\{M_A((x \circ y) * x), M_A(x)\} \geq \min\{M_A(y), M_A(x)\},$$

$$B_A^-(x \circ y) \leq \max\{B_A^-((x \circ y) * x), B_A^-(x)\} \leq \max\{B_A^-(y), B_A^-(x)\},$$

$$B_A^+(x \circ y) \geq \min\{B_A^+((x \circ y) * x), B_A^+(x)\} \geq \min\{B_A^+(y), B_A^+(x)\},$$

and

$$J_A(x \circ y) \leq \max\{J_A((x \circ y) * x), J_A(x)\} \leq \max\{J_A(y), J_A(x)\}.$$

Therefore  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic  $\circ$ -subalgebra of  $X$ . □

We provide a characterization of a BMBJ-neutrosophic ideal in an  $(S)$ -BCK-algebra.

**Theorem 3.23.** *Let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be an MBJ-neutrosophic set in an  $(S)$ -BCK-algebra  $X$ . Then  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic ideal of  $X$  if and only if the following assertions are valid.*

$$\begin{aligned} M_A(x) &\geq \min\{M_A(y), M_A(z)\}, B_A^-(x) \leq \max\{B_A^-(y), B_A^-(z)\}, \\ B_A^+(x) &\geq \min\{B_A^+(y), B_A^+(z)\}, J_A(x) \leq \max\{J_A(y), J_A(z)\} \end{aligned} \tag{3.8}$$

for all  $x, y, z \in X$  with  $x \leq y \circ z$ .

*Proof.* Assume that  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic ideal of  $X$  and let  $x, y, z \in X$  be such that  $x \leq y \circ z$ . Using (3.1), (3.2) and Theorem 3.22, we have

$$\begin{aligned} M_A(x) &\geq \min\{M_A(x * (y \circ z)), M_A(y \circ z)\} \\ &= \min\{M_A(0), M_A(y \circ z)\} \\ &= M_A(y \circ z) \geq \min\{M_A(y), M_A(z)\}, \end{aligned}$$

$$\begin{aligned} B_A^-(x) &\leq \max\{B_A^-(x * (y \circ z)), B_A^-(y \circ z)\} \\ &= \max\{B_A^-(0), B_A^-(y \circ z)\} \\ &= B_A^-(y \circ z) \leq \max\{B_A^-(y), B_A^-(z)\}, \end{aligned}$$

$$\begin{aligned} B_A^+(x) &\geq \min\{B_A^+(x * (y \circ z)), B_A^+(y \circ z)\} \\ &= \min\{B_A^+(0), B_A^+(y \circ z)\} \\ &= B_A^+(y \circ z) \geq \min\{B_A^+(y), B_A^+(z)\}, \end{aligned}$$

and

$$\begin{aligned} J_A(x) &\leq \max\{J_A(x * (y \circ z)), J_A(y \circ z)\} \\ &= \max\{J_A(0), J_A(y \circ z)\} \\ &= J_A(y \circ z) \leq \max\{J_A(y), J_A(z)\}. \end{aligned}$$

Conversely, let  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  be an MBJ-neutrosophic set in an  $(S)$ -BCK-algebra  $X$  satisfying the condition (3.8) for all  $x, y, z \in X$  with  $x \leq y \circ z$ . Since  $0 \leq x \circ x$  for all  $x \in X$ , it follows from (3.8) that

$$M_A(0) \geq \min\{M_A(x), M_A(x)\} = M_A(x),$$

$$B_A^-(0) \leq \max\{B_A^-(x), B_A^-(x)\} = B_A^-(x),$$

$$B_A^+(0) \geq \min\{B_A^+(x), B_A^+(x)\} = B_A^+(x),$$

and

$$J_A(0) \leq \max\{J_A(x), J_A(x)\} = J_A(x).$$

Note that  $x \leq (x * y) \circ y$  for all  $x, y \in X$ . Hence we have

$$\begin{aligned} M_A(x) &\geq \min\{M_A(x * y), M_A(y)\}, B_A^-(x) \leq \max\{B_A^-(x * y), B_A^-(y)\}, \\ B_A^+(x) &\geq \min\{B_A^+(x * y), B_A^+(y)\} \text{ and } J_A(x) \leq \max\{J_A(x * y), J_A(y)\}. \end{aligned}$$

Therefore  $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$  is a BMBJ-neutrosophic ideal of  $X$ . □

## 4 Conclusions

As a generalization of neutrosophic set, Mohseni et al. [9] have introduced the notion of MBJ-neutrosophic sets, and have applied it to BCK/BCI-algebras. BMBJ-neutrosophic set has been introduced in [1] with an application in BCK/BCI-algebras. In this article, we have applied the notion of MBJ-neutrosophic sets to ideals of BCK/BI-algebras. We have introduced the concepts of a BMBJ-neutrosophic  $\circ$ -subalgebra and a (closed) BMBJ-neutrosophic ideal, and have investigated several properties. We have provided conditions for an MBJ-neutrosophic set to be a BMBJ-neutrosophic ideal in BCK/BCI-algebras, and have discussed characterizations of BMBJ-neutrosophic ideal. We have considered relations between a BMBJ-neutrosophic subalgebra, a BMBJ-neutrosophic  $\circ$ -subalgebra and a (closed) BMBJ-neutrosophic ideal. Using the results and ideas in this paper, our future work will focus on the study of several algebraic structures and substructures.

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## References

- [1] H. Bordbar, M. Mohseni Takallo, R.A. Borzooei and Y.B. Jun, BMBJ-neutrosophic subalgebras of  $BCK/BCI$ -algebras, preprint
- [2] R.A. Borzooei, X.H. Zhang, F. Smarandache and Y.B. Jun, Commutative generalized neutrosophic ideals in  $BCK$ -algebras, *Symmetry* 2018, 10, 350; doi:10.3390/sym10080350.
- [3] Y.S. Huang,  $BCI$ -algebra, Beijing: Science Press (2006).
- [4] Y.B. Jun, Neutrosophic subalgebras of several types in  $BCK/BCI$ -algebras, *Ann. Fuzzy Math. Inform.* 14(1) (2017), 75–86.
- [5] Y.B. Jun, S.J. Kim and F. Smarandache, Interval neutrosophic sets with applications in  $BCK/BCI$ -algebra, *Axioms* 2018, 7, 23; doi:10.3390/axioms7020023
- [6] Y.B. Jun, F. Smarandache and H. Bordbar, Neutrosophic  $\mathcal{N}$ -structures applied to  $BCK/BCI$ -algebras, *Information* 2017, 8, 128; doi:10.3390/info8040128
- [7] Y.B. Jun, F. Smarandache, S.Z. Song and M. Khan, Neutrosophic positive implicative  $\mathcal{N}$ -ideals in  $BCK/BCI$ -algebras, *Axioms* 2018, 7, 3; doi:10.3390/axioms7010003
- [8] J. Meng and Y.B. Jun,  $BCK$ -algebras, Kyung Moon Sa Co., Seoul (1994).
- [9] M. Mohseni Takallo, R.A. Borzooei and Y.B. Jun, MBJ-neutrosophic structures and its applications in  $BCK/BCI$ -algebras, *Neutrosophic Sets and Systems*, 23 (2018), 72–84. DOI: 10.5281/zenodo.2155211
- [10] M.A. Öztürk and Y.B. Jun, Neutrosophic ideals in  $BCK/BCI$ -algebras based on neutrosophic points, *J. Inter. Math. Virtual Inst.* 8 (2018), 1–17.
- [11] F. Smarandache, A unifying field in logics. *Neutrosophy: Neutrosophic probability, set and logic*, Rehoboth: American Research Press (1999).
- [12] F. Smarandache, Neutrosophic set, a generalization of intuitionistic fuzzy sets, *International Journal of Pure and Applied Mathematics*, 24(5) (2005), 287–297.
- [13] S.Z. Song, M. Khan, F. Smarandache and Y.B. Jun, A novel extension of neutrosophic sets and its application in  $BCK/BI$ -algebras, *New Trends in Neutrosophic Theory and Applications (Volume II)*, Pons Editions, Brussels, Belgium, EU 2018, 308–326, by Florentin Smarandache and Surapati Pramanik (Editors).
- [14] S.Z. Song, F. Smarandache and Y.B. Jun, , Neutrosophic commutative  $\mathcal{N}$ -ideals in  $BCK$ -algebras, *Information* 2017, 8, 130; doi:10.3390/info8040130

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