Spring 5-14-2022

Perceptions of Current Secondary Mathematics Teachers about their Teacher Preparation

Peterson Chiliromango MOYO
The University of New Mexico

Peterson Chiliromango Moyo

Candidate

Teacher Education, Education Leadership & Policy

Department

This dissertation is approved, and it is acceptable in quality and form for publication:

Approved by the Dissertation Committee:

Dr. Cheryl Torrez, Chairperson

Dr. Marjori Krebs

Dr. Leila Flores Duenas

Dr. Patricia Oakley

Dr. Maria Cristina Pereyra

PERCEPTIONS OF CURRENT SECONDARY MATHEMATICS TEACHERS ABOUT THEIR TEACHER PREPARATION

by

PETERSON C. MOYO

B. A., General Education and Mathematics, Lakeland College, 2002
M. S., Mathematics, The University of New Mexico, 2009

DISSERTATION

Submitted in Partial Fulfillment of the
Requirements for the Degree of

Doctor of Philosophy
Teaching, Learning, and Teacher Education

The University of New Mexico
Albuquerque, New Mexico

May 2022

DEDICATION

This dissertation is dedicated to my wife Wezzie and the whole family for their perseverance with the challenges that they encountered along the journey of the PhD. You persevered a lot with God holding your hands until the end of this journey. It looked too far, but God has been faithful to us until it is over. We all celebrate this achievement with smiles on our faces. All the glory be to God for this achievement.

This dissertation is also dedicated to my mother, Witness who passed away three years before I earned this Doctor of Philosophy in Teaching, Learning, and Teacher Education. She had been an inspiration to me from the beginning of my school age. She wanted me to achieve higher in the circles of education while she only went as far as second grade. It is because of her encouragements that I have achieved the PhD.

My special memory for you mother is when my father passed away when I was in 9th grade. There was a big financial gap that my father left, which was a big mountain to climb. Many people came saying that I should stop schooling, but your answer was simple and clear, “he cannot leave school because it is his future’s life.” Your word has really come to pass though you have not enjoyed much of the benefits of my schooling.

I nearly dropped out of school when I was in 10th grade, but mum you fought the battles for me, and persevered a lot for me to continue school to the extent that I wrote the national examinations and passed with flying colors. Mother, here I am graduating with a PhD. I wished you were here to see me parade on the hallways of the State University 1 with PhD regalia earning the highest academic qualification on the planet. Mother, rest in eternal peace!

ACKNOWLEDGEMENTS

I would like to thank my dissertation chair Dr. Cheryl Torrez for the support that she has given me throughout the process of writing this dissertation. In fact, I started working with her on my comprehensive examinations. She has given me the required assistance all along, and she has been on my side in all spheres. She has been giving me assistance where it was duly needed. Thank you, many times, Dr. Cheryl Torrez.

I also thank you my dissertation committee members Dr. Marjori Krebs, Dr. Leila Flores Duenas, and Dr. Patricia Oakley for the support that you have given me during the dissertation process. Your support during the dissertation proposal defense gave me strength and encouragement that I could finish this dissertation. Thank you so much for the constructive comments that shaped my dissertation to the final product

I thank you Dr. Maria Cristina Pereyra for coming in to satisfy the graduate studies requirement. I thank you sincerely because you have seen me grow academically. You started being in my committee when I was doing my master's degree, and you have now done a final blast! Thank you so much, Dr. Maria Cristina Pereyra.

I thank all of you my committee members for good work that you have done to me. May God bless you abundantly.

PERCEPTIONS OF CURRENT SECONDARY MATHEMATICS TEACHERS ON THEIR TEACHER PREPARATION

by

PETERSON C. MOYO

B. A., General Education and Mathematics, Lakeland College, 2002
M. S., Mathematics, The University of New Mexico, 2009
PhD., Teaching, Learning, and Teacher Education, The University of New Mexico, 2022

ABSTRACT

The purpose of this sequential explanatory mixed methods case study was to examine how current secondary mathematics (6-12) teachers (SMTs) perceived their teacher preparation. The population consisted of 27 current SMTs who taught mathematics in a southwestern state of the United States. The first research question asked how SMTs perceived that mathematics methods courses influenced mathematical knowledge for teaching (MKT). The second research question examined ways in which content and instruction in mathematics methods courses contributed to the teaching of mathematics.

During quantitative phase, 27 SMTs completed the questionnaire. Chi-Square statistical analysis showed no significant relationship between participants' mathematics methods course and readiness to teach mathematics. During qualitative phase, an analysis of semi-structured interviews revealed that mathematics methods course(s) had little influence on the development of MKT, and teachers considered themselves not ready to teach mathematics effectively.

The findings might lead to positive social change such as curriculum revisions to develop mathematics teachers' MKT to improve future instruction. The findings suggested possibilities of informing preparation of secondary mathematics teachers and a framework of coursework that could influence the development of MKT during mathematics teacher preparation for secondary schools.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

Introduction

This dissertation research reports the findings of a mixed-methods case study that examined perceptions of current secondary mathematics (6-12) teachers (SMTs) about their mathematics teacher preparation. In this context, secondary mathematics (6-12) teachers are referred to as middle and high school mathematics. The dissertation research used a survey instrument and semi-structured interviews as a means of data collection. The dissertation research used descriptive statistics, contingency tables, and Chi-Square statistical analyses to examine quantitative data. The study used categorization, coding, and conceptualization to analyze qualitative data. The participants consisted of 27 current SMTs teaching mathematics in public schools in a southwestern state of the United States.

The first chapter of the dissertation research presents the background knowledge of the study, specifies the statement of the problem, defines the purpose of the study, describes the significance of the study, and presents an overview of the methodology used in the study. The first chapter also presents limitations and delimitations of the study, defines key terms used in the study, and concludes with a description of the organization of the dissertation.

Background of the Study

Current SMTs must have a strong understanding of the mathematics taught in secondary schools (Ferrini-Mundy & Findell, 2001). The current SMTs should develop specialized knowledge for teaching mathematics (Ball et al., 2008). Teacher educators in the United States and Canada have been investigating the influence of mathematics teacher preparation programs on the teaching of mathematics (Ball et al., 2008; Darling-Hammond &

Oates, 2019; Kajander & Holm, 2016). Therefore, teacher preparation should emphasize that secondary mathematics teachers must be prepared and trained before teaching their classes.

Accordingly, Darling-Hammond (2000) emphasized that SMTs should get teacher preparation and training before teaching. Ball et al. (2008) commented on the need for mathematics teacher preparation to gain content and pedagogical knowledge required for teaching mathematics. Teacher preparation should help the SMTs to acquire a variety of ways to represent mathematical content and to assist students in deepening their mathematical understanding (Mahajan, 2014; National Research Council [NRC], 2001; Superfine & Li, 2014).

In the United States, there have been a concern about students' performance in school mathematics and international mathematics competitions (Ball, 2003; Ball et al., 2005; Hoover & Ball, 2014; Hoover et al., 2016). The students' performance in mathematics has not improving significantly because most teachers did not possess the requisite subject-matter knowledge to implement quality instruction in mathematics (Ball, 1990; Ball & Bass, 2000; Conference Board for Mathematical Sciences (CBMS), 2001, 2012; Hill et al., 2004, Ma, 1999; Superfine & Li, 2014; Stockton & Wasserman, 2017; Wasserman et al., 2019). Americans have long complained about the quality of mathematics education in the wake-up calls that spawned the *New Math* of Sputnik era, *No Child Left Behind* (NCLB), and the warnings of *A Nation at Risk* (National Commission on Excellence in Education, 1983). This included the wake-up calls of the *Math Wars* in the 1990s about the reforms in mathematics education in the United States (Klein, 2003).

An Associated Press conducted a study in 2005 in the USA that showed 40% of adults hated mathematics when they were in school (Lester, 2005). The same study showed that the percentage of mathematics haters was at least double the percentage of those who hated other subjects. In general, the percentages in the news poll indicated that people hated mathematics more than other subjects. This poor performance in mathematics has been a concern in the United States for a long time.

The 2009 Program for International Student Assessment (PISA) survey showed that 23 percent of American youth scored below the baseline of mathematics proficiency level (Fleischman et al. 2010). By 2015, the performance deteriorated, with 29 percent of American youth scoring below the baseline of mathematics proficiency score (Kastberg et al., 2016; Organization for Economic Cooperation and Development [OECD], 2016). Students performed poorly on PISA because it aimed at assessing whether students could extrapolate from what they knew, not simply reproducing knowledge (Mahajan, 2014). This Students needed to understand mathematics to extrapolate and transfer their knowledge to new situations. Besides that, most students learned mathematics using rote learning, which was a "fundamental obstacle to PISA's goal, that mathematics should help to prepare students for full participation in society" (Mahajan, 2014, p. 2).

Another study on the 2015 National Assessment Education Program (NAEP) showed that 60 percent of fourth and eighth-grade students scored below the proficient level in mathematics (National Center for Education Statistics [NCES], 2015). Students in grades four, eight, and twelve in the United States showed a decrease in mathematical performance in Trends in Mathematics and Science Study (TIMSS) compared to other countries over 24 years from 1995 to 2019 (Mullis et al., 2020). This indicated that low performance in mathematics meant students understood mathematics less comprehensively, and therefore, they scored very poorly on the international examinations. The teachers in the mathematics classes focused on drilling formulas and procedures rather than teaching students to think creatively (Richards, 2020). In many mathematics classrooms, there was a reliance on shallow teaching syndrome (Stacey, 2003) where students completed many repetitive low complexity problems often by blindly copying procedures at the direction of their teachers (Presccott & Cavanagh, 2008).

With these results, it was not surprising that "young adults in the United States lacked not only the quantitative and problem-solving skills necessary for success in the workplace and postsecondary education but also the numeracy and problem-solving skills necessary for meaningful participation in our demographic institutions" (Goodman et al., 2015, p. 5). Brelias (2015) stated that students should leave high school with quantitative literacy and critical thinking processes necessary to determine the validity of scientific, economic, social, and political arenas. These skills in mathematics should be acquired from the teaching and learning of secondary school mathematics.

According to Drew (2011), 40 percent of the students planning to study engineering and science majors ended up switching to other majors or failing to get any degree because they performed poorly in mathematics. Mathematics acted as a barrier for students opting to study engineering and science majors. Therefore, improving mathematics teaching could increase the pass rate of students in mathematics and could increase the number of students who wanted to study engineering and science majors. That said, many other career fields required a solid mathematical foundation. These fields were architecture, accounting, banking, business, medicine, ecology, and aerospace; mathematics was vital to economics and finance, as well as the computing technology and software development underlying our technologically advanced world (Mullis et al., 2016).

Mathematics served multiple purposes, and “it should be taught in ways that prepare students to flourish as human beings” (Su, 2017, p. 483). The truth was that when taught effectively, with an emphasis on critical thinking and mathematical reasoning, secondary school mathematics has the potential to help combat the increasing problem of truth decay in the American society (Kavanagh & Rich, 2018). Hence, secondary school mathematics should be taught effectively to prepare and empower students for different roles in the society.

Although mathematics teacher education programs aimed to assist mathematics teachers in acquiring the basic skills and pedagogical content knowledge (Shulman, 1986) to learn how to teach mathematics intentionally and systematically (Morris et al., 2009), most mathematics teachers have completed teacher education preparation programs with less mathematical knowledge for teaching, and they were not confident to teach mathematics in secondary schools (Hine & Thai, 2019, Patterson, 2020; Wasserman, 2018).

Ball and Bass (2000) agreed that most current mathematics teachers did not receive adequate experiences from their teacher education programs to develop a deep conceptual knowledge of mathematics to be taught in secondary schools. Due to a shallow understanding of mathematics, mathematics teachers could not teach mathematics effectively in secondary schools to cater for the individual needs of the students (Hine, 2015). Agreeably, “…middle school and high school teachers possessed a limited knowledge of mathematics including the mathematics they teach since the mathematics that they received as K-12 students and teacher preparation have not provided them with sufficient opportunities to learn mathematics” (Conference Board of Mathematical Sciences [CBMS], 2012, p. 372). As a result, further research in secondary mathematics (6-12) teachers’ preparation was needed to examine how mathematics methods courses influenced MKT to teach mathematics and investigate their readiness to teach mathematics.

According to Wasserman (2018), "the content that teachers should know should be directly related to the knowledge that gets used in the various aspects of teaching" (p. 116). As such, the content knowledge for high school mathematics teachers should be related to high school mathematics. With this situation on the ground, there was a vicious cycle: mathematics teachers entered college with insufficient understanding of school mathematics and then entered the classrooms inadequately prepared to teach mathematics to the students (CBMS, 2012). This could not improve the performance of students in mathematics because mathematics teachers were not engaged deeply with mathematics content which was considered essential in mathematics teaching (Miller & Davidson, 2006).

A study by Hurrel (2013) argued that many mathematics teachers exhibited weaknesses and lacked deep conceptual understanding of mathematics when teaching mathematics. This required a need to examine secondary school mathematics teaching so that mathematics made sense to the students and the society. This should start in teacher preparation programs in the universities and colleges of education to improve the quality of mathematics teacher preparation.

A report by CBMS (2012) stated that there should be a rethinking of the mathematics education of mathematics teachers where mathematics departments should emphasize on the intellectual substance of school mathematics and the special nature of the mathematical knowledge needed for teaching. The report focused on teacher preparation and the design of mathematics courses for mathematics teachers since few mathematics teachers were well prepared to teach secondary mathematics (CBMS, 2012; Superfine & Li, 2014).

There have been attempts to change the curriculum, but high school mathematics has nearly been the same today as it were a century ago. Understandably, the high school mathematics curriculum has continued to begin with a year of Algebra I, a year of Geometry, a year of Algebra II, and some electives in the senior year (Waggener, 1996; Kilpatrick, 1992). Historically, this sequence of mathematics was first recommended by the committee of 10 in 1892, and it has remained the course of the pathway at more than 90% of high schools in the United States (Dossey et al., 2016; Kilpatrick, 1992).

The mathematics curriculum has not changed much for a little more than a century, so it could not adequately reflect the changing of mathematics, science, and technology. It could be expected that the curriculum changed to meet the current needs of the society for mathematics teachers to get well-prepared to teach mathematics. Developing the content knowledge and mathematical knowledge for teaching was essential to promote successful teaching and learning of mathematics (Ball et al., 2008). Therefore, this dissertation research examined current secondary mathematics teachers' perceptions about how mathematics methods courses influenced the teachers' MKT and their readiness to teach mathematics in secondary schools.

Statement of the Problem

In the United States, the primary concern for many policymakers in mathematics education was whether teachers possess strong enough content knowledge to teach mathematics effectively in secondary schools (Greenberg & Walsh, 2008; Murray et al., 2017; National Mathematics Advisory Panel, 2008,). Teacher education programs were directly connected and implicated in this matter because content knowledge for teaching was a critical component of teacher education preparation programs (Feuer et al., 2013). Teacher education programs must design programs that help mathematics teachers to understand a wide array of concepts and skills about teaching and learning (Darling-Hammond, 2006; Darling-Hammond & Oates, 2019). The extant literature (Ball et al., 2008; Ball & Forzani, 2009; CBMS, 2012; Hine, 2015; Hine & Thai, 2019; Lai, 2019; Sullivan, 2018) on the teacher preparation of current mathematics teachers indicated that mathematics teachers entered the teaching profession with insufficient mathematical knowledge for teaching. The most pressing issues were whether teacher preparation programs made a difference in teacher preparation and whether they provided teacher preparation and training that was essential for teaching and learning of mathematics (Cochran-Smith et al., 2015; Feuer et al., 2013; Greenberg, McKee, & Walsh, 2013; Lai, 2019). These were questions that were critical to current educational policy debates that involved mathematics teacher education preparation programs.

Understanding current mathematics teachers' perceptions about how their mathematics methods courses influenced their MKT and teacher development was essential because mathematics continued to be the foundation for economic growth through science, technology, and engineering as the basis of innovation (Mahajan, 2016). Besides that, little research had been conducted on how content and instruction of mathematics methods courses has contributed to the effective teaching of secondary school mathematics (6-12). There was little research that was conducted on how mathematics teachers' MKT got influenced for effective teaching of mathematics in secondary schools (6-12). As such, the problem was that mathematics teachers who entered the teaching profession encountered many challenges in mathematics teaching and learning (CBMS, 2012; Lai, 2019). This created a knowledge gap in the teaching and learning of mathematics since mathematics teachers started teaching mathematics with insufficient mathematical knowledge for teaching, yet they completed teacher preparation programs in mathematics (CBMS, 2012; Hine, 2015; Hine & Thai, 2019).

Although there was lack of understanding on how best to prepare mathematics teachers (Hine, 2015), an analysis of this study might be insightful for future efforts in the mathematics teacher preparation programs. A survey conducted by Cox et al. (2013) stated that 18% of the participants cited pedagogical concerns after completing their teacher preparation programs. Lai and Paterson (2017) agreed that current mathematics teaching weakly supported the development of mathematical knowledge for teaching in teacher preparation programs. This created some knowledge gaps in the teacher preparation programs. This dissertation study focused on current mathematics teachers' perceptions about how their teacher preparation influenced their MKT for effective mathematics teaching.

The study focused on mathematics teachers' perceptions about their mathematics teacher preparation, teacher knowledge, and mathematical knowledge for teaching (MKT). Teacher knowledge and MKT are fundamental in the preparation of teachers to teach mathematics in secondary schools (Wasserman et al., 2017). The study also focused on the readiness to teach mathematics in the context of mathematics methods courses providing the skills and techniques of how mathematics teachers could teach mathematics. Mathematics teacher preparation programs should focus on preparing mathematics teachers how to teach mathematics in secondary schools, and this work should start working in the teacher education preparation programs where the secondary mathematics teachers are being prepared and trained to teach mathematics in secondary schools.

Purpose of the Study

The purpose of this sequential explanatory mixed-methods study was to examine mathematics teachers' perceptions about their teacher preparation for effective teaching of mathematics. This was conducted by surveying a sample of 27 current secondary mathematics teachers and then follow-up with four voluntarily selected mathematics teachers for semi-structured interviews. The four case study teachers were chosen teachers on a typical response and explored in-depth the results from the statistical tests.

Research Questions

The study used a sequential explanatory mixed-methods case study. For the quantitative phase of the study, the guiding question was: To what extent did current mathematics teachers perceive that mathematics methods courses influenced their MKT? For the second qualitative part of the study, the guiding question was: In what ways did current mathematics teachers perceive that content and instruction in mathematics methods courses contributed to their effective teaching of school mathematics?

Significance of the Study

This dissertation study drew attention to teacher education preparation and presented a significant potential about how mathematics methods courses influenced MKT to teach mathematics in secondary schools. The need for increasing mathematical competencies among citizens has been a point of focus over the past decades (California Space Education and Work Institute, 2008; Gardner, 1983). There was an identified lack of mathematical literacy in the United States, a significant factor driving the focus of the study. For example, Phillip (2007) stated that high numbers of adults struggled with daily tasks involving mathematics, including computing interest paid on loan (78%), calculating miles per gallon when traveling (71%), and determining a 10% gratuity for a lunch bill (58%). Such deficiencies were due to the mathematics education taught in middle and high schools that was responsible for teaching mathematics needed for the computations.

Despite these alarming percentages of low mathematics potential, students should learn mathematics in profound, conceptual ways that lead to mathematical literacy (NCTM, 2000). More importantly, “every student should learn the essential concepts to expand professional opportunities, understand and critique the world and experience the joy, wonder, and beauty of mathematics” (NCTM, 2018, p. 9). With more research and effort, the teaching of mathematics could improve tremendously for the benefit of all students in the United States.

This dissertation study may add to the growing body of literature on SMTs' perceptions about their knowledge of teaching mathematics during teachers’ education preparation programs. The study described teacher education as it was done during teacher preparation and training. It was essential to represent the teachers’ mathematical knowledge journey for teaching during the teacher preparation programs. This study also described how mathematics teachers translated their understanding of mathematics into readiness to teaching mathematics.

The results of this study, examining the perceptions of mathematics teachers about their mathematics teacher preparation, may have a significant influence in mathematics performance of students in secondary schools since “mathematics appears to be the subject in which accomplishment in secondary school was particularly significant for both an individual’s and a country’s economic well-being” (Hanushek et al., 2010, p. 8). A study done by the American Diploma project as cited in Hanushek et al. (2010) estimated that 62%

of American jobs needed proficiency in algebra, geometry, data interpretation, probability, and statistics over the next 10 years. Hence, to get employed in these jobs, Americans must be well educated in algebra, geometry, data interpretation, probability, and statistics, which are the mathematics that are offered at secondary school (6-12).

Ball (2003) stated that raising MKT for teachers was important for improving the quality of teaching and learning of mathematics. Masingila et al. (2018) argued that teachers' mathematical knowledge significantly influenced how and what teachers teach, how and what their students learned. As a result, teachers' mathematical knowledge needed to be strong to deal with the teaching of mathematics, and it should start in the teacher preparation programs to improve the quality of teacher preparation.

The dissertation research may add to the mathematical understanding of how mathematics teachers used their perceptions to make daily classroom decisions to design learning experiences for students. It may help teacher education programs and other educational reformists who look forward to improving mathematics teaching for secondary mathematics/ teachers as most mathematics teachers entered classrooms with insufficient MKT (Lai, 2019; Murray et al., 2017).

According to Ball et al. (2001), MKT was needed because SMTs used it to interpret, to reform ideas, to manage the challenges of change, to enact new practices, and to teach new content that all depended on teachers' knowledge of mathematics. RAND and Ball (2003) agreed that teachers' knowledge of mathematical thinking combined with knowledge of mathematical content to shape their presentation and representation, use of materials, and ability to understand their students. Hence, it was very important for SMTs to acquire MKT for effective teaching of mathematics.

Overview of the Methodology

The purpose of this sequential explanatory mixed-methods case study was to examine the mathematics teachers' perceptions about their teacher preparation for effective teaching of mathematics. The population consisted of 27 current secondary mathematics teachers teaching in public secondary schools in a southwestern state of the United States. Survey methodology was used to answer the quantitative part of the study, and semi-structured interview questions were used to answer the question in the qualitative part of the study.

Descriptive statistics, contingency tables, and Chi-Square statistical analyses were employed in the study to analyze quantitative data. Frequencies and percentages were computed for each survey question. The results had been compared with the results that were in the extant literature (Ball et al., 2008, Shulman, 1986; Wasserman et al., 2018). These studies indicated that most mathematics teachers started teaching with insufficient mathematical knowledge. Contingency tables and Chi-Square statistical analysis were used to test the relationships between the predictor variables and criterion variables in the quantitative part of the study.

The qualitative part of the study used semi-structured interviews, which were done via Zoom. The interviews were recorded on Zoom and transcribed using categorization, coding, and conceptualization to analyze the qualitative data. Semi-structured interviews were used because they use open-ended questions, which allowed follow-up questions as the interviews progressed (Leavy, 2017). This allowed more information to be collected from emergent questions, and data were categorized into insights from the interviews. The methodologies used in this study are further explained in Chapter Three of the dissertation research.

Limitations and Delimitations of the Study

The instrument that was used in this study was a survey questionnaire that relied on self-reporting of the responses from the participants. The design of the questionnaire and the protocol followed for emailing the participants reduced limitations. The contact emails of the participants were the email addresses that were publicly available on the public schools' websites. In some instances, the email addresses were not current, and some surveys got returned, and follow-up contacts were not received by the current mathematics teachers in their schools.

At the time of research, the participants were in their respective schools teaching mathematics in the Spring 2021 semester; therefore, sending out emails was the most convenient means of sending survey questionnaires to the participants due to COVID-19 pandemic. Also, emails were suitable to reduce contacts with paper and pencil in the context of COVID-19. The study was delimited to current mathematics teachers who were teaching mathematics in a southwestern state of the United States.

By using current mathematics teachers during the Spring 2021 semester, data were collected despite COVID-19 pandemic, which was a major limitation to the study. COVID-19 restricted movements and schools were forced to virtual learning so the researcher could not visit the mathematics teachers in the schools. Despite this limitation, data that were collected, analyzed, and synthesized enabled the researcher to understand and share each participant's story of how mathematics methods courses influenced MKT and find ways in which the content and instruction in mathematics methods courses contributed to the development of MKT for the effective teaching of mathematics of mathematics in secondary schools.

Definition of Key Terms

In this section, all the key terms used in this dissertation were defined to give their meanings as they were used in the dissertation research.

C*ontent Knowledge*: Knowledge of subject matter needed for teaching that entails that specialized content knowledge is different from that of other professions (Shulman, 1986).

Pedagogical Knowledge: Knowledge of methods of teaching a subject and how the content can be made comprehensible to others (Shulman, 1987).

Pedagogical Content Knowledge: A combined knowledge of content and pedagogical the knowledge that is required for teaching any subject (Shulman, 1986).

Mathematical Competence: It is the ability to develop and apply mathematical thinking to solve a range of problems in everyday situations (National Research Council [NRC], 2001).

Mathematics Literacy: This is mathematical knowledge put into functional use in different situations in varied, reflective, and insightful ways (Mahajan et al., 2016)

Mathematical Knowledge for Teaching: This is the content and pedagogical knowledge for teaching mathematics (Ball et al., 2008).

Truth Decay: This means the diminishing role of facts and analysis in American public life (Kavanagh & Rich, 2018).

Rote Learning: A memorization technique based on repletion to quickly recall the meaning of the material (Mahajan, 2014)

Extrapolate: This means to apply the known mathematical knowledge to solve mathematical knowledge without reproducing the knowledge (Mahajan, 2014).

Capstone Course: These are courses that target the preservice teachers only, usually in the final year of their preparation program (Wasserman, 2018; Lai, 2019; Sullivan, 2018).

Emergent Codes: codes that come up as the coding process of data analysis is taking place (Miles & Haberman, 1994).

Mathematics for Teachers: These are mathematics content courses that are offered to mathematics teachers during teacher preparation to familiarize them with secondary mathematics content that they will learn how to teach, mostly offered in their final year of their teacher preparation (Holm & Kajander, 2012).

Mathematics Methods Courses: These are methods offered to mathematics teachers during teacher preparation so that they can learn how to teach mathematics in secondary schools (6-12), mostly offered in their final year of teacher preparation (Lai, 2019).

Organization of the Dissertation

In this section, the researcher presents the organization of the remaining chapters. Chapter Two of the dissertation provides a review of the relevant literature of the study. It reviews the literature by considering themes that emerged frequently in the relevant literature. Chapter Three discusses the procedures and methods used for designing and conducting the study, data collection, and data analyses that have been used in the study. Chapter Four presents the results of the study and their analyses from the quantitative and qualitative phases of the dissertation. Chapter Five presents a summary and discussion of the study. It includes an interpretation of the results as well as their applications and suggestions for further research.

CHAPTER 2

Review of the Related Literature

This section provides a synthesis of the research literature on the development of MKT used for effective teaching of mathematics (Ball- et al., 2008). It included an overview of the components of MKT, according to Ball et al. (2008), Hill et al., (2008), and Silverman and Thompson (2008). The impact of mathematics methods coursework on influencing MKT for SMTs had been discussed, and the need to understand the future SMTs' knowledge of teaching mathematics has also been discussed.

There was substantial research on MKT about the teaching of mathematics. Since mathematics was crucial to succeeding in engineering and sciences, there had been much research conducted in this area. Most of the published works has been in effective teaching of mathematics and how effective teaching improved the performance of the students in mathematics (Ball et al., 2008; Wasserman, 2018; Hine & Thai, 2019). While many previous studies offered valuable insights into mathematics teaching, they provided partial solutions on effectively preparing mathematics teachers who could be well-equipped to teach mathematics in secondary schools (6-12) (Prescott & Cavanagh, 2008). A limited body of knowledge existed regarding the preparation of secondary mathematics teachers, especially on the mathematics teachers' perceptions of how mathematics methods courses influenced their MKT for effective teaching of mathematics.

The aim of this literature review was to summarize the theoretical framework underpinning this study. The literature review also summarized themes that appeared frequently in the literature about the development of MKT. These themes included: an overview of mathematics teacher education, teachers' knowledge of mathematics, knowledge bases of teaching mathematics, relevant coursework for teacher preparation, effective teaching of mathematics, teachers' readiness to teach mathematics, and the impact of mathematics courses in MKT development. The themes have been discussed extensively to provide an understanding of the research topic of the dissertation.

Search Strategy

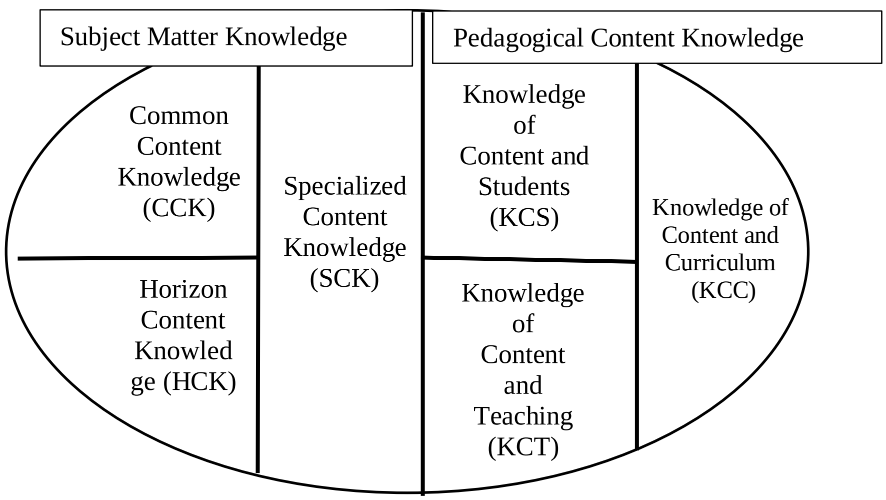
The search strategy for this study started with establishing a literature review component outline, which guided the keywords used in the search databases. Keywords included but were not limited to secondary mathematics teachers, content knowledge, pedagogical knowledge, pedagogical content knowledge, mathematical knowledge for teaching, and effective mathematics teaching. It also included coursework for teacher preparation. The ProQuest, ERIC, EBSCOHOST, and SAGE databases were searched, and Google Scholar was also used to search for the needed information.

Sources of information included peer-reviewed journal articles, books, government statistics, theses, and dissertations. Many articles, dating from the 1980s to the present, were identified with relevant material. Most of the articles were published within the last ten years, with a good number within the recent five years. Older sources were included to provide a perspective of the longevity and history of MKT and the topic under study. As listed in the references section of this dissertation, a subset of the sources retrieved was identified as the most relevant sources for this study and provided the foundation of the literature review.

Theoretical Framework

The theoretical framework for this study focused on the development of MKT for SMTs. Shulman (1986) first proposed pedagogical content knowledge (PCK) with different categories of teacher knowledge that included knowledge of content, pedagogy, curriculum, learners, and educational contexts. Ball et al. (2008) refined Shulman's model and proposed the MKT framework as a construct to conceptualize mathematical knowledge specific to the teaching of mathematics. The researcher used Ball et al. (2008) framework to study perceptions of SMTs about their teacher preparation. Figure 1 shows the domains of MKT according to Ball et al. (2008) and their relationship to Shulman's (1986) categories of subject matter knowledge (SMK) and pedagogical content knowledge (PCK).

Figure 1. Domains of Mathematical Knowledge for Teaching (Ball et al., 2008)



Ball addressed two broad domains in the construct of MKT namely: subject matter knowledge (SMK) and pedagogical content knowledge (PCK). The subject matter knowledge domains were listed as common content knowledge (CCK), horizon content knowledge (HCK), and specialized content knowledge (SCK). They represented the complexity of content knowledge (CK) necessary to teach mathematics. Ball et al. (2008) defined common content knowledge (CCK) as "the mathematical knowledge and skill used in settings other than teaching" (p. 399). CCK was defined as a measure of an individual's ability to obtain or recognize correct answers to mathematics problems. Ball et al. (2008) identified knowledge that extended beyond obtaining correct solutions as specialized content knowledge (SCK). SCK was defined as “the mathematical knowledge and skill unique to teaching” (p. 400). SCK highlighted the work that teachers do when identifying student errors or evaluating the merit of a student’s approach to a problem. Lastly, Ball et al. (2008) recognized horizon content knowledge as “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (p. 403). Horizon content knowledge was useful in helping teachers understand the mathematical foundation they were setting with their students and what pedagogical approaches might assist in allowing a student to build upon their knowledge in future learning experiences.

The pedagogical content knowledge domains were listed as knowledge of content and students (KCS), knowledge of content and teaching (KCT), knowledge of content and curriculum (KCC). They represented a teacher’s ability to blend their knowledge of mathematics and instruction to advance students’ understanding of mathematics. Ball et al. (2008) defined knowledge of content and students (KCS) as “knowledge that combines knowing about students and knowing about mathematics” (p. 401). KCS was represented in a teacher’s ability to identify mathematical tasks that students would find interesting along with anticipating common errors students were most likely to make. Ball et al. (2008) described the knowledge of content and teaching (KCT) as the combination of “knowing about teaching and knowing about mathematics” (p. 401). KCT was described as the knowledge that teachers used to design instruction with a focus on the impact of student learning. Investigating the changes in MKT across the subdomains might be useful in understanding how MKT developed for mathematics teachers who got enrolled in the mathematics methods courses. The theoretical framework was suitable for the dissertation research because it connects mathematics content and mathematics pedagogy.

The framework asserted that MKT develops when SMTs connected content and pedagogical knowledge to create a new understanding of how to support the teaching and learning of mathematics. To assist the development of MKT, teacher educators must be intentional in designing learning experiences that engaged SMTs in the process of exploring content and pedagogy while considering how their future students might approach similar tasks of teaching mathematics. Developing MKT was a process that blended a teacher's understanding of content, teaching, methods, and students (Shulman, 1987).

The interviews with teachers helped to explain SMTs perceptions of how the mathematics methods courses provided opportunities to consider how mathematics teachers might develop different mathematical approaches of teaching. Therefore, the framework that was started by Shulman (1986), and later refined by Ball et al. (2008) related well to the use of a mixed methods approach blending a quantitative assessment of changes in MKT with SMTs perceptions of specific learning experiences that supported their development of MKT. Next, the themes that were outlined in the literature are explained in detail.

An Overview of Mathematics Teacher Education in the United States

In the 19th century, mathematics teacher education programs were for mathematical content knowledge (Shulman, 1986). For example, the California State Board Examination for elementary school teachers in March 1875 comprised ten items on mental arithmetic, of which all the items were in the content area with one methods question. In the mid-20th century, the mathematics teacher education programs privileged pedagogy (Shulman, 1986). This was what Shulman called a missing paradigm because content knowledge was missing in the teacher education programs. At that time, the assumption was that pre-service mathematics teachers already knew mathematics content from secondary school (6-12), so they just needed mathematics methods to learn how to teach mathematics, and they needed class management skills.

Shulman (1986) suggested that the lessons' content was missing. The questions and explanations offered during classes were also missing. This left Shulman mind-boggling as to "where do teachers' explanations come from? How do teachers decide what to teach, how to represent it, how to question students about it, and how to deal with questions of misunderstanding?" (1986, p. 8). Shulman (1986), then, got interested in the sources of teachers' knowledge and the consequences of varying degrees of subject matter competence and incompetence. This sparked a lot of research in this area, and it is still being researched at present. This type of training described by Shulman (1986) produced teachers of different competencies since different mathematics teachers were subjected to varying types of content knowledge at different levels of their mathematics education.

Teacher education in the USA was experiencing extraordinary challenges as competing versions of how teachers were educated and promoted by the government, professional societies, and others (Imig, Wiseman, & Imig, 2011). There was little evidence that suggested the right way to prepare teachers, and this had invited extraordinary efforts to experiment with alternative models of teacher preparation (National Research Council, 2001). Ball et al. (2005) stated that

> Until and unless we, as educators, are willing to claim that there is professional knowledge that matters for the quality of instruction and can back that claim with evidence, we will continue to be no more than one voice among many competing to assert what teachers should know and how they might learn that, and why (p. 45).

If educators accepted the domains of professional knowledge to be sufficient and necessary, then there should be one voice in the teacher education preparation programs that should unify the teacher education programs.

Teachers' Knowledge of Mathematics

Teachers' knowledge of mathematics was prominent in the discussions of how to improve mathematics teaching and learning (NRC, 2001). Hoover et al. (2016) stated there was a growing sense that mathematics was essential for improving teaching and learning. According to Patterson (2020), teacher preparation programs should improve teachers' mathematics knowledge to use it to develop students' mathematical abilities during teaching. Practitioners had turned attention to the increasing understanding of relevant mathematics needed for teaching. In general, Kilic (2011) stated that "teacher education programs should provide several contents, general pedagogy, and content-specific methods courses to support the development of professional knowledge for teaching" (p. 17). Secondary mathematics (6-12) teachers should acquire MKT in these mathematics methods courses to improve their knowledge base for effective teaching of mathematics.

Masingila (2018) stated that teachers' mathematical knowledge significantly influenced how teachers taught and how and what their students learned. Ball and Bass (2000) proposed that teachers' mathematical knowledge needed to be strong to deal with the complexity of teaching mathematics to diverse student populations. RAND and Ball (2003) added that the need for teachers' knowledge of mathematics was obvious to explain and answer questions like (a) $a^0 = 1$, (b) when solving inequalities, dividing by a negative number reverses the inequality sign. Whitehead (1948) stated that teachers' knowledge of mathematics should allow students to develop an appreciation for the power of mathematics as a system of human thought. NRC (2012) stated that teachers' mathematical knowledge was vital in deciding if a student's solution was mathematically valid, spotting an error in a textbook, finding alternative ways of getting answers, and selecting good examples for the lessons. In this respect, teachers' knowledge of mathematics was essential in the teaching and learning process, and it could not be overemphasized. Next, the researcher discusses knowledge bases that was another component in the themes of the related literature.

The Knowledge Bases for Teaching Mathematics

Like any other profession, teaching has its knowledge base for education (Darling-Hammond & Oakes, 2019). Shulman (1987) also stressed that teachers needed to complete teacher education preparation to have the required knowledge base for teaching mathematics. Many professions could not do what lawyers, doctors, or engineers did. Similarly, other people could not do what teachers did because teaching had its knowledge base for teaching mathematics (Rowland, 2014). Darling-Hammond and Oakes (2019) agreed that teachers have a well-established knowledge base to teach that nobody, who did not have this knowledge base, could teach effectively. As a result, teaching mathematics requires a well-grounded knowledge base for teaching mathematics.

Mathematics teachers also had their knowledge base for teaching mathematics because there was a particular body of knowledge that was specific to mathematics teaching. Teachers needed to acquire this knowledge base to teach mathematics effectively. Teacher education programs were responsible for developing the knowledge base in the first place, and secondary mathematics teachers could gain this knowledge during teacher education preparation programs. This was because teacher education programs allowed SMTs to engage deeply with mathematics content courses to get an understanding of the content knowledge for teaching mathematics (Sullivan, 2018).

The knowledge bases for teaching mathematics could be strengthened because some researchers have argued that the knowledge base of mathematics of many teachers was rule-bound, and it severely lacked the meaning necessary to provide adequate explanations to its students (Ball & Wilson, 1990; Ball et al., 2008). It was well documented that many teachers exhibited weaknesses and lacked a deep knowledge base of mathematics (Ball et al., 2005; Hill et al., 2008). Ball et al. (2008) stressed that the ability to use mathematics and apply it to teaching mattered a lot for the quality of instruction that they could produce. According to NRC (2001), the knowledge base for teaching mathematics included knowledge of mathematics, knowledge of students, and knowledge of instructional practices. Shulman (1986) stated that content knowledge, pedagogical knowledge, and pedagogical content knowledge were knowledge bases for the teaching any subject. Next, each knowledge base for teaching mathematics is discussed concerning how it affects mathematics teaching.

Content Knowledge for Teaching Mathematics (CKTM)

Content knowledge was an essential component of knowledge base for teaching mathematics (Ball & McDiarmid, 1990). Content knowledge was also the fundamental knowledge impacting student achievement (National Council of Teachers of Mathematics [NCTM], 2000). Teachers' in-depth and accurate information about mathematics increased the effectiveness of teaching (Ball, 1990). Being competent in content knowledge helped teachers to know what kind of prerequisite knowledge was necessary to teach a certain subject, the appropriate examples, homework, and what kind of illustrations could be used (Shulman, 1986). Hill et al. (2005) described that the components of mathematical knowledge for teaching were essentially related to content knowledge of mathematics, and it should be strengthened to support the teaching of mathematics in secondary schools.

According to Shulman (1986, 1987), content knowledge was the deep knowledge of the structures of the subject beyond procedural and factual knowledge. There was almost uniform agreement among researchers that knowledge of mathematical content was central to mathematics teaching (Ball et al., 2005; Norton, 2010). The importance of teachers' content knowledge has been articulated by the U. S. Department of Education (2008) that "teachers must know in detail the mathematics content they are responsible for teaching, and its connections to other important mathematics, both prior and beyond the level they are assigned to teach" (p. 36). As such, secondary mathematics teachers must be very knowledgeable to guide students in such processes. The content knowledge for teaching mathematics should start during teacher preparation programs because they are responsible for the work of mathematics teacher preparation for the secondary school mathematics teachers.

Banner and Cannon (1997) summarized the critical importance of teacher content knowledge that "in order to teach they must know what they teach and know how to teach it; and in order to teach effectively, they know deeply and well" (p. 7), but there was considerable debate as to what mathematics could develop in secondary teacher education. According to Baumert et al. (2010), there was disagreement on the necessary breadth and depth of teachers' mathematical knowledge that was required for teaching mathematics in secondary schools. This created a gap in the body of mathematical knowledge for teaching because there was a need for a well-established knowledge base for mathematics teachers' preparation.

According to the CBMS (2012), teachers at any grade level needed to know how their mathematics relationships to that of prior and later grades. CBMS (2012) commented that, "all teachers of mathematics need to be able to detect flaws in students' arguments and to help students the nature of those errors" (p. 1). Content knowledge alone was insufficient to support mathematics teaching, and a lack of pedagogical content knowledge (PCK) negatively affected a teacher's instructional practices (Baki & Arslan, 2016; Maher & Muir, 2013). The development of content knowledge and pedagogical content knowledge of SMTs was essential to promote mathematics successful teaching and learning. Therefore, mathematics preparation programs must ensure that curriculum requirements intentionally addressed the mathematical knowledge for teaching.

Pedagogical Knowledge for Teaching (PKT)

In addition to content knowledge for teaching, pedagogical knowledge for teaching was also vital for teachers to know how to transfer mathematical content for others to understand. Thus, teachers should have a good strategy and symbolic knowledge to teach mathematics effectively. Shulman (1987) considered the knowledge of teaching strategies and methods as a transformation; in other words, it was the presentation of the subject in the forms that students could understand well. Shulman (1987) further discussed them under the headings of knowing the most functional representation of topics and concepts; knowing what facilitated learning the subject or what made it complicated; and knowing simulations, illustrations, examples, and explanations for better understanding concepts and eliminating misconceptions. What was essential for effective teaching of mathematics was that teachers looked through the window of the students to make mathematical knowledge convenient for them (NCTM, 2000). Knowledge of instructional strategies also included learning activities, materials, and representations such as explanations, metaphors, examples, illustrations, and analogies that facilitated students' understanding (Grossman, 1990; NRC, 2001; Park & Oliver, 2008).

Researchers defined and interpreted pedagogy in mathematics teaching differently to focus on how it was significantly essential for quality teaching. Lovat (2003) described pedagogy as "a highly complex blend of theoretical understanding and practical skill" (p. 12). Mathematical pedagogy explicitly emphasized the substance of mathematics and its nature and epistemology (Ball & Forzani, 2009; Davis, 1967), which assumed that students must be actively involved in constructing their understandings in discovering and inventing mathematics.

The pedagogical strategies were always a determinant factor of effective classroom teaching. Teachers' quality was the single most significant factor in explaining student achievement that was more important than classroom-related issues (Lovat, 2003). According to Darling-Hammond & Oates (2019), the quality of students learning outcomes was directly dependent on the quality of the teacher to implement effective pedagogical practices. The quality of the teachers due to pedagogical knowledge allowed them to know students' common difficulties, errors, conceptions, and misconceptions. They also knew the problematic mathematical concepts for students to grasp and possible sources of students' errors in solving mathematics (An et al., 2000; Wu, 2004).

Authors have asserted that teachers require the development of PK (Chick, 2012; Shulman 1987) to teach mathematics effectively. In other words, PK could be understood as knowing various ways to present mathematics content and assisting students in deepening their understanding (Ma, 1999). Understandably, teachers with a strong PK have rich repertoires of teaching activities, and they could choose tasks, examples, representations, and teaching strategies that were appropriate for their students (Borko & Purton, 1996). The teachers with sound knowledge of PK knew how to facilitate discourse and manage classroom activities effectively.

It was essential for teachers to know how to transfer mathematical content for students to understand when teaching mathematics**.** Grossman (1990) stated that knowledge of instructional strategies included learning activities and use of learning materials in mathematics. Park and Oliver (2008) added that learning materials included representations such as explanations, metaphors, examples, illustrations, and analogies that facilitate students' understanding of mathematics during teaching.

Pedagogical Content Knowledge (PCK)

Pedagogical content knowledge (PCK) was one of the teacher's knowledge bases, but specifically for teaching and learning of a specific subject like mathematics. Probably, the most influential reconceptualization of teachers' knowledge bases within mathematics education was done by the constructs of mathematical knowledge for teaching (Ball et al., 2008; Hill et al., 2008) that covered content knowledge (CK) and pedagogical knowledge (PK). Thus, MKT referred to the knowledge base that was used to teach mathematics. This knowledge base distinguished a mathematician from a mathematics teacher because a mathematics teacher has special knowledge for teaching, which a mathematician do not have (Ball et al., 2005). Ball (2003) stated that mathematics teaching was around teacher content knowledge because teachers could not explain well what they did not know.

Hill et al. (2008) and Schilling and Zopf (2008) used a construct of MKT as initially proposed by Shulman (1986), those of subject matter knowledge and pedagogical content knowledge. Depaepe et al. (2013) cited two merits of MKT (a) it was borne out of empirical research on the understanding that teachers required to teach mathematics (b) MKT took Shulman's (1986) heuristic and turned it into a valid measure of teachers' mathematical knowledge for teaching.

In the last two decades, researchers and mathematics educators increasingly emphasized the significance of mathematical knowledge for teaching (Hoover et al., 2016). Darling-Hammond (2000) and Conference Board of Mathematical Sciences (2012) stressed that teacher education programs needed to focus on distinct courses that expanded future teachers' conceptual and pedagogical knowledge in the teaching of mathematics. Most teacher education programs demanded that their teachers developed a deep and flexible understanding of secondary mathematics content (Wasserman et al., 2017) through some designated mathematics courses.

To build this knowledge, most secondary mathematics teachers were required to complete a substantial number of courses in advanced mathematics (CBMS, 2001, 2012; Stacey, 2008). Hill (2011) documented that many secondary mathematics teachers completed a mathematics major to develop an understanding of mathematics. This was done with an experience that mathematics covered in advanced university courses was connected to secondary mathematics and, therefore, relevant for teaching secondary mathematics (Wasserman, 2017). For example, the notions of additive and multiplicative inverses, commutative and associative properties of a binary operation, and the zero-product property of real numbers were discussed extensively in abstract algebra (Bosse et al., 2012). Similar arguments could be made about the content in other university mathematics courses such as real analysis, number theory, and college geometry. The assumption was that taking these university courses helped secondary mathematics teachers to understand secondary mathematics content (Sullivan, 2019).

There was little evidence that completing these courses influences SMTs' instruction in mathematics (Darling-Hammond, 2000; Monk, 1994; Zazkis & Leikin, 2010) because secondary mathematics teachers might not have understood the basic concepts in the advanced mathematics courses. Although mathematical organizations said that mastery of advanced mathematics was vital for teaching secondary mathematics (CBMS, 2012), the actual secondary mathematics teachers frequently viewed such courses as both unnecessary and unrelated to their teaching (Goulding et al., 2000). This was because secondary mathematics teachers did not see a clear link between advanced mathematics and secondary mathematics (Hine & Thai, 2019).

There should be some courses that are needed for secondary mathematics teachers to teach mathematics effectively (Hoover et al., 2016). In the past decades, researchers and mathematics educators emphasized the significance of subject-specific type of mathematical knowledge that was different from mathematics typically taught in most college mathematics and other professionals other than teachers. This implied that secondary mathematics teachers should understand mathematics at a level that was higher than students. That is, they must know more mathematics than their students to teach mathematics effectively.

Menon (2006) complimented that some secondary mathematics teachers had not transformed their learner knowledge to teacher knowledge. This meant that the teachers needed much deeper knowledge to teach mathematics. They needed to be given opportunities to reflect on the actual mathematics behind whatever mathematics topics they were supposed to teach (NRC, 2001). According to Sullivan (2018), mathematics capstone courses provided an avenue to explore mathematics content and addressed MKT domains. It was therefore required that teacher education programs should design some of these capstone courses for secondary mathematics teachers.

Relevant Coursework for Teacher Preparation

In teacher preparation, capstone courses engaged secondary mathematics teachers in the exploration of mathematical concepts that they were expected to teach from the teacher and students' perspectives in the context of content knowledge (Holm & Kajander, 2012). Capstone courses prepared secondary mathematics teachers to advance their MKT in later mathematics coursework compared with traditional mathematics courses (Kajander & Holm, 2016; Cardetti & Truxaw, 2014). As such, Holm et al. (2016) advocated for the need of capstone courses to support MKT to maximize learning in the methods coursework. Such training allowed mathematics teachers to engage deeply with mathematics content which was considered essential in mathematics teaching (Miller & Davidson, 2006; Norton, 2010; Wasserman, 2018).

In the mathematics methods courses, secondary mathematics teachers were supposed to learn a variety of ways to represent mathematical content and to assist students in deepening their understanding in mathematics (Ma, 1999; Shulman 1987, 1999; Silverman & Thompson, 2008). Wasserman et al (2017) stated that "most teacher education programs demanded that their prospective teachers developed a deep and flexible understanding of secondary mathematics content" (p.560). Ideally, methods courses impacted the development of MKT across domains. Mathematics methods courses could improve content knowledge (CK) and pedagogical content knowledge (PCK) (Auslander et al., 2016; Qian & Youngs, 2016). Auslander et al. (2016) stressed that mathematics methods coursework focused on connecting teaching, learning, and student thinking that resulted in stronger MKT than that of traditional courses. Thus, mathematics methods coursework appeared to provide SMTs with the opportunity to deepen their conceptual understanding of mathematics while exploring how to teach mathematics content in a manner that was conducive to student learning.

There was a common ground across studies that PCK dealt with teachers' knowledge, connected content knowledge and pedagogical knowledge, and it was specific to the teaching of a particular subject matter (Lai, 2019). Klein (1932) advocated the connections between school mathematics and advanced level mathematics to be made explicit during teacher preparation. This would help teachers to learn mathematics teaching techniques. Ball (1990) supported that SMTs would look forward to mathematics methods because they expected to learn how to teach specific mathematics topics. As a result, the relevant courses could be able to develop MKT in the teachers so that they were ready for mathematics teaching.

Effective Teaching of Mathematics

Effective teaching utilized practice-based opportunities. Practice-based opportunities are those that afford preservice opportunities to integrate both content and pedagogy acquired through coursework into instruction (Erickson, 2014). This means that the skills learned in coursework are put into practice in the classrooms. According to Benedict et al. **(**2006**),** the science of effective teaching is informed by a rich, deep body of scholarship that remains promising. Effective teaching incorporated modeling into practice-based opportunities in the teaching of mathematics. Benedict et al. (2006) defined modeling as "how teacher educators provide preservice teachers examples of what expert performance looks like in teaching by making visible the underlying knowledge base and through a process being drawn while enacting the skill" (p. 12).

According to Russ-Eft (2002), space learning opportunities provided experiences to apply knowledge and skills acquired through coursework. They increased secondary mathematics teachers' overall effectiveness in teaching. Thus, for SMTs to learn to be effective teachers, they needed high-quality opportunities and skills to practice in their teaching. According to the National Council of Teachers of Mathematics (NCTM) (2000), "effective teaching required knowing and understanding mathematics, students as learners, and pedagogical strategies" (p. 17). Darling-Hammond (2000) described an effective teacher as one who learned from teaching rather than one who had finished learning how to teach. The National Research Council (NRC) (2001) stated that "despite the common myth that teaching was a little more than common sense or that some people are just born teachers, effective teaching practice can be learned" (p. 369). This meant that SMTs could learn to be effective teachers through effective teacher preparation and practice.

Effective teachers needed to know the mathematics that is taught as well as the horizons of that mathematics. Teachers should know how to use their knowledge flexibly in practice to appraise and adapt instructional materials to represent the content in honest and accessible ways (NRC, 2001). That is why Masters (2009) stated that

> Highly effective teachers understand the subjects they teach. These teachers have studied the content they teach in greater depth than the depth at which they currently teach, and they have high levels of confidence in the subjects they teach. Their deep content knowledge allows them to focus on teaching underlying methods, concepts, principles, and big ideas in a subject matter, rather than on factual and procedural knowledge alone (p. 4).

The description of mathematics teaching and learning required SMTs to have a deep and flexible understanding of mathematics content, understanding students as learners, knowledge of instructional strategies, and curricula resources (Darling-Hammond, 2008; NCTM, 2000). Of the factors that affected the quality of teaching mathematics, the first factor was teachers' knowledge of mathematics. Johnson et al. (2002) agreed that "of all the factors in schooling, the quality and caliber of teachers have the greatest effect on student learning" (p. 9). This implied that effective teaching mainly relied on the knowledgeability of the teachers in the content and pedagogical knowledge of teaching.

Secondary mathematics (6-12) teachers were more likely to be effective where their preparation experiences were connected to classroom practice (Boyd et al. 2009; Ronfeldt & Reininger, 2012). This type of knowledge and skill was not developed from reading books, but it is cultivated through high-quality opportunities to practice, coupled with support and feedback (Ball & Forzani, 2009; Phelps, 2009). NRC (2001) believed that the more a teacher knew about subject matter, the more effective he became as a teacher.

Teachers' Readiness to Teach Mathematics

Adequate teacher preparation required that secondary mathematics teachers were fully equipped with CK and PK which could bring about MKT needed for a real classroom environment. The importance attached to teacher education preparation was noted in the study conducted by Ball and Forzani (2009). This study argued that secondary mathematics teachers needed to be well-prepared to teach well because teaching is unnatural as it needed special skills to be done effectively. They explained that effective teaching required acquiring specialized values, attitudes, knowledge, and skills, which were necessary for teachers' readiness to teach mathematics.

Ball and Forzani (2009) pointed out that "there [was] need for the general public to acknowledge that teaching is hard work that many people need to learn to do well and build a system of reliable, professional preparation" (p.509). Based on these assertions, the researcher investigated secondary mathematics teachers' perceptions about how mathematics methods courses influenced the teachers' MKT to teach mathematics effectively.

Considering the role of teacher educators in teaching mathematics to secondary mathematics teachers, Shulman (1987) noted that

> To the extent that they are likely to teach both what and as they have been taught, unlike any other subjects in your classes, the future teachers are, if you will, carriers. Whatever understandings or misunderstandings you infect them with, both about the content and regarding the pedagogy, they will carry to generations of young people whom they will carry to generations of young people whom they will subsequently teach, and who themselves will eventually appear at your doorstep (p.406).

The argument by Shulman was valid because if teachers were not effectively prepared to teach, they were expected to teach ineffectively to the students, and students would proceed to tertiary institutions of learning without sound knowledge of mathematics. Researchers of the second TIMSS attributed the poor performance of the United States students to uneven exposure of students' teachers to the mathematics topics that were taught in secondary school. This was partly that the curriculum was decentralized, and it was approached differently in different secondary schools (6-12). Like other scholars, Beisiegel et al (2013) asserted that mathematics and statistics departments have the responsibility to ensure that future teachers of mathematics have a deep and connected understanding of the mathematics to be taught in secondary schools.

In teacher education programs, there was an assumption that teachers who completed the required mathematics content and pedagogical courses could be ready to teach mathematics at a secondary school. As such, most of the studies investigated teachers' MKT in relation to teachers' effectiveness in teaching (Ball et al., 2008; Beswick & Goos, 2012; Norton, 2010). However, it was becoming increasingly apparent that the performance during teacher education programs might not be directly correlated with classroom readiness to teach (Burges & Geach, 2011; Tatto et al., 2008).

The Impact of Mathematics Methods Courses in Teachers' MKT

Secondary mathematics teachers required a bachelor's degree to teach mathematics in a southwestern state of the United States, and they were required to take some advanced mathematics courses (Wasserman et al., 2018). On the other hand, the impact of the advanced mathematics courses done in undergraduate mathematics was unclear (Ball et al., 2008). Qian and Youngs (2016) stated that university mathematics did not affect MKT development. Traditional content courses did have a positive impact on the CCK of SMTs but failed to impact the MKT of the SMTs (Kajander & Kolm, 2016). A potential reason for the minimal impact traditional courses had on the MKT might be the relevancy of the course contents of the traditional courses. Secondary mathematics teachers found the traditional mathematics coursework as disconnected from their future mathematics classroom work (Sullivan, 2019). However, mathematics content courses designed specifically for teachers provided an avenue to explore mathematics content and addressed MKT. The mathematics content courses were meant to improve the quality of mathematics teacher preparation.

Special content courses often engaged SMTs in the exploration of mathematical concepts they were expected to teach from the teacher and student perspectives. Special content courses were seen to raise MKT (Holm & Kajander, 2012). Special content courses prepared SMTs for furthering their MKT in later methods courses especially when compared to traditional mathematics courses (Cardetti &Truxaw, 2014; Kajander & Holm, 2016). Thus, Holm et al. (2016) advocated the need of special content courses to support SCK development to best maximize learning in mathematics methods courses.

Mathematics methods courses were vital to the development of MKT for SMTs. Smith et al. (2012) provided evidence that the MKT levels of SMTs were not influenced by additional content coursework, but by additional mathematics methods coursework. The development of MKT was unrelated to the number of content coursework taken, but it was related to the content covered where SMTs concentrated to the school mathematics and pedagogical knowledge (Qian & Youngs, 2016; Schmidt et al., 2017). Therefore, mathematics methods courses have a significant impact on the development of MKT in the SMTs for teaching mathematics.

Like special content courses, mathematics methods courses improved MKT for the secondary mathematics teachers (Auslander et al., 2016; Qian & Youngs, 2016). Auslander et al. (2016) stressed that mathematics methods courses focused on connecting teaching, learning, and student thinking which resulted in stronger SCK when compared to traditional content courses. Kajander and Holm et al. (2016) suggested that the mathematics methods coursework helped to support stronger MKT for the teachers once in their teaching schools. Thus, mathematics methods courses provided SMTs with the opportunity to deepen their conceptual understanding of mathematics. Mathematics methods coursework helped the secondary mathematics teachers to explore how to teach mathematics in a manner that stimulated student thinking and learning of mathematics.

The research results from mathematics methods courses aligned with Silverman and Thompson's (2008) transformative model where teachers must consider content from a learner's point of view to transform their understanding into new mathematical knowledge for teaching. This literature review identified how specific coursework might impact the development of MKT in SMTs applicable within the mathematics methods coursework.

Summary

The role of teacher preparation programs is to prepare secondary mathematics teachers for the demands of teaching and learning of mathematics. The teaching of mathematics entails a unique blend of knowledge coursework for SMTs that had the potential to improve MKT especially when presented in a blended format. However, more research methodology was necessary to understand how MKT develops in SMTs. Therefore, the purpose of this study was to examine how SMTs perceived that mathematics methods courses influenced MKT for effective teaching of mathematics.

Also, the dissertation research may add to the understanding of how secondary mathematics teachers used their perceptions to make daily decisions in their mathematics classrooms and design learning experiences for students. The dissertation study may help teacher education programs and other educational reformists looking forward to improve mathematics teaching for SMTs. This is because research has pointed out that most SMTs entered mathematics classrooms with insufficient knowledge of teaching mathematics (NRC, 2012). As a result, mathematics teacher education programs should focus on the preparation of mathematics teachers to improve the quality of mathematics teacher preparation.

CHAPTER 3

Methodology

This chapter presents the methods and procedures that were used in this dissertation study, and it provides a general overview of the dissertation. This general overview comprises the research design, case study methodology, and research paradigm. The general overview also includes participants, instrumentation, methods of data collection, methods of data analysis, and a summary of the chapter.

General Overview

The purpose of this study was to examine perceptions of current SMTs about how mathematics methods courses influenced their mathematical knowledge for teaching (MKT). The research examined the perceptions of current SMTs about how the content and instruction of the mathematics methods courses contributed to their MKT to teach mathematics effectively. The dissertation sought to understand current SMTs' perceptions about their teacher education preparation that they have done to teach mathematics in secondary schools.

Mathematical knowledge for teaching (MKT) was defined as "mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students" (Ball et al., 2008, p. 399). This was the mathematical knowledge that Shulman (1986) called the mathematical knowledge that teachers should possess in deciding how best to represent mathematical ideas to be understood by others. Speer et al. (2015) stated that MKT played its role in shaping current secondary mathematics teachers in the teaching of mathematics. According to Andreas et al. (2014), MKT was the body of mathematics that was important for secondary mathematics teachers to know to teach secondary school mathematics (6-12).

Research Questions

This study sought to answer the following research questions:

(1) To what extent did current secondary mathematics teachers perceive that their mathematics methods course(s) influenced their MKT?

(2) In what ways did current secondary mathematics perceive that content and instruction in their mathematics methods course(s) contributed to their effective teaching of school mathematics?

Research Design

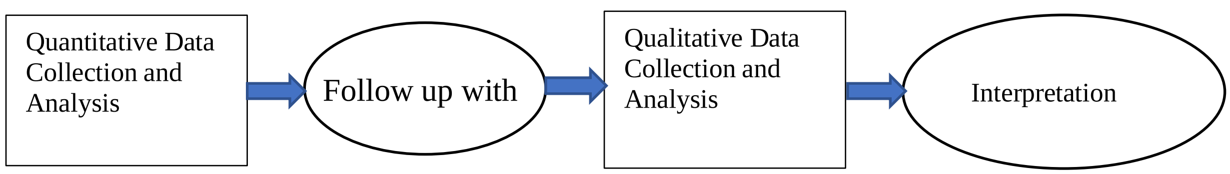
The researcher used a mixed-methods research case study, and it used the sequential explanatory design of mixed methods, which typically involved two phases (Almalki, 2016). An initial quantitative phase was followed by a qualitative phase. The qualitative phase built directly on the results from the quantitative phase (Gay et al., 2009). In this way, quantitative results were explained in more detail through qualitative data. According to Leavy (2017), this process provided a better understanding of the research problem than each of the methods could do independently. Creswell and Plano Clark (2011) stated that it was practical because it permitted the usage of multiple techniques and approaches to look at the research problem.

The sequential explanatory design used semi-structured interview results to explain statistical patterns generated from survey analysis (Creswell, 2014). Descriptive statistics were used for data analysis to explore the perceptions of the participants, and inferential statistics were used to explore the responses of the participant sample. The data were collected using a one-time online survey instrument, which was preceded by the consent form as shown in Appendix B. This study was therefore classified as a cross-sectional study, which had the advantage of measuring the current attitudes of the participants according to Creswell (2015).

In this dissertation, current SMTs' perceptions about their teacher education preparation were expressed numerically, and then their perceptions were explained verbally because interviews could be used to explain the numbers that were produced (Creswell, 2014). That is, the qualitative phase explained why the numbers appeared in that pattern. The description of quantitative data followed by qualitative data adopted the sequential explanatory design by Creswell and Plano Clark (2011). The diagram below shows how the sequential explanatory design was laid out during the process of data collection and analysis.

Figure 1

Sequential Explanatory Model (Creswell & Plano Clark, 2011)



The population for this study was 27 current secondary mathematics teachers in a southwestern state of the United States. Survey methodology was conducted using an online questionnaire due to the scattered nature of the study population and the benefits inherent in internet survey research. Dillman et al. (2009) noted that it was easy to get responses over a short period, and research could be conducted at a low cost. Descriptive and inferential analysis were utilized to interpret the data and drew conclusions on the relative importance of the domains of the survey instruments. Attention was paid to a variety of characteristics within the sample including years of teaching experience, teaching subjects, type of teaching license, type of degree (BS/BA/BEd), and the highest level of education attained

Case Study Methodology

The sequential explanatory design was used with the potential for the qualitative data to provide a detailed explanation and the descriptive findings of participants' perceptions about the mathematics methods courses that they took during the teacher education preparations. Ravitch and Carl (2016) stated that a qualitative case study must explore a real-life event. Gay et al. (2009) stated that a case study was a qualitative research approach in which researchers focused on a unit of study known as a bounded system. Merriam and Tisdell (2016) defined a case study as "…an in-depth description and analysis of a bounded system" (p. 37). This implied that a case study was a bounded system that the researcher could fence in what the researcher wanted to study. For example, a case study could be a school, a community, an institution, and or a program of study (Merriam & Tisdell, 2016). In this case, a case study were the four secondary mathematics teachers who volunteered to take part in the semi-structured interviews.

The researcher used semi-structured interviews for the secondary mathematics teachers that volunteered to be interviewed. The focus of the interviews was on how content and instruction in mathematics methods courses contributed to the development of mathematical knowledge for teaching for effective teaching of mathematics. The individual interviews with the secondary mathematics teachers provided insight into the instructional components of the mathematics methods courses. Integration of the data sets occurred after the analysis of the quantitative data was completed. According to Creswell (2014), the integration and interpretation of data types was used to explain findings in more detail. The interpretation of the results in this study focused on how the qualitative findings explained and extended the results of the quantitative data analysis.

Constructivism

This qualitative case study used the constructivist worldview as the research paradigm. Constructivism suggested that learners used their experiences to actively construct an understanding that made sense to them, rather than acquiring understanding by having it presented in an already organized fashion (Eggen & Kauchek, 1994). This contemporary view drew from the constructivist perspectives of Piaget (1954) and Vygotsky (1978) and emphasized the social and cultural nature of knowledge construction.

Constructivism, with its emphasis on exploring the life and work setting of individuals, aligned with the research question and with the purpose that the researcher explored (Creswell, 2013). In asking the respondents to reflect on their perceptions of teacher education preparation and their MKT, both in questionnaire form and semi-structured interviews, the researcher expected to see significant variability in the experiences and responses. While some of the participants viewed themselves as mathematically competent to teach mathematics, others felt uncertain when they were asked to describe their level of comfort with teaching certain mathematics subjects.

Constructivism held the view that "all knowledge, and therefore all meaningful reality as such, is contingent upon human practices, being constructed in and out of the interaction between human beings and the world" (Crotty, 2003, p. 42). Therefore, the constructivist perspective was best suited for investigating how individuals incorporated new knowledge into their existing knowledge and then made sense of this new construct (Ferguson, 2007).

Participants of the Study

The study took place in public secondary schools in a southwestern state of the United States. The dissertation research started in Spring 2021 with current SMTs who were teaching mathematics in the public schools in the southwestern state. The study took place according to the suggested timeline shown in Appendix F**.**

The population consisted of the 27 secondary mathematics teachers from the public schools in the southwestern state. All participants were fluent in English language, but English did not have to be their native language. Due to COVID-19, some secondary mathematics teachers did not participate. This affected data collection since mathematics methods courses and special mathematics courses in which current mathematics teachers were enrolled at their respective universities, served as examples of mathematics methods courses that provided mathematical knowledge for teaching to the teachers to enhance mathematical competencies for effective teaching of mathematics**.**

The sampling method for this mixed-methods study was purposive sampling. Merriam and Tisdell (2016) defined purposeful sampling as the selection of participants based on their ability to answer a specific question. The current secondary mathematics (6-12) teachers who were teaching in a southwestern state served as a purposeful sample for the quantitative study. Purposeful sampling was used to select participants by their ability to provide specific knowledge to answer research questions and address the purpose of a study (Patton, 2015). To explain the quantitative findings, purposive sampling was used to follow up on the results of the quantitative phase and interview individuals who volunteered to be interviewed. Glaser and Strauss (1967) described saturation as exploring all perspectives to the point that additional inquiry failed to provide new information. So, in the qualitative phase, data collection and data analysis took place concurrently.

Efforts were made from the beginning of the study to protect privacy, built relationships, minimized harm, and respected the experiences of all participants. The relationships with participants and their privacy were protected using an informed consent form (Dooly et al., 2017; Ravitch & Carl, 2016). All current SMTs in the southwestern state were provided with a consent form at the beginning of the study with detailed information about the study to make an informed decision about participating in the study. The consent form described the plan to protect the participants' privacy and how the data were used to avoid any harmful effects. This consent form is shown in Appendix B.

Participants were informed that no identifying information was recorded or reported. No names were collected to protect privacy and no information was shared with others that could identify participants by name. To build relationships with participants, the researcher conducted individual interviews during times convenient to the teachers' schedule. They were required to agree and sign the consent form before proceeding to the questionnaire hosted by SurveyMonkey.

The research study was sent to the Institutional Review Board (IRB) where the researcher was doing studies for approval before the research was conducted. That is, the IRB approved this study before the dissemination of the survey questionnaire.

Instrumentation

After the review of the research literature, a survey instrument shown in Appendix C was used to collect data from the current mathematics teachers regarding their acquiring mathematical knowledge for teaching and the importance of teacher education preparation for current secondary mathematics (6-12) teachers. A tested mathematics instrument used in Trends in Mathematics and Science Study (TIMSS) (Provasnik et al., 2016) was used to develop the survey instrument for this dissertation research. The instruments were used in wide-scale surveys in the United States and world-wide to check the mathematical knowledge for teaching in the current mathematics teachers who taught mathematics in public schools that participated in TIMSS worldwide.

Survey questions were designed to address mathematics teachers' perceptions about how mathematics methods courses influenced their MKT, teacher preparation and training, and effective teaching of mathematics. Dillman et al. (2009) was used in the design of the survey instrument. This protocol included a criterion for question development and order. The questionnaire asked participants to provide demographic data about the highest level of education. Besides that, the participants were also asked to provide data for their current teaching status, teaching licensure level, the pathway to licensure, years of teaching experience, and the school level that they were teaching. After that, participants were asked to indicate on a four-item Likert scale the extent to which the participants used MKT in the teaching of mathematics, and how well they were prepared to teach mathematics from their respective teacher education preparation programs.

To establish validity, the researcher presented the constructed survey instrument to the dissertation committee during the dissertation proposal defense. The researcher considered the changes suggested by the committee members. The researcher then scheduled pilot interviews with two secondary mathematics teachers with master's degrees in mathematics education and experience to review the draft of the instrument and provide feedback on the perceived validity, accuracy, and grammatical clarity of the survey items. Further revisions were made to the survey instrument based on the feedback from the secondary mathematics educators.

To address reliability, the researcher estimated Cronbach's alpha reliability coefficient, a correlational measure of the reliability of the items in a scale to ensure that the items were measuring aspects of the same thing and that it was appropriate to add up items for an overall rating scale (Vogt, 2007). This made sure that the survey questions estimated the consistency of the scores on the survey instrument.

Methods of Data Collection

The dissertation research integrated quantitative and qualitative methods. That is, the dissertation research used quantitative methods and qualitative methods of data collection and analysis. It started with quantitative methods of data collection followed by qualitative methods of data collection according to Creswell and Plano Clark (2007, 2011).

Phase I Quantitative Methods of Data Collection

The first phase of the dissertation research was quantitative data collection. This phase examined current SMTs' perceptions about how mathematics methods courses influenced their mathematical knowledge for teaching. Data collection for this study used the tailor-design approach. According to Dillman et al. (2009), the "tailored design involves using multiple motivational features incompatible and mutually supportive ways to encourage high quantity and quality of response to the surveyor's request" (p. 16). A tailored design could be applied in the development of all aspects of a survey to reduce the total survey error to acceptable levels and to motivate all types of sample members to respond within time constraints (Dillman et al., 2009).

Using these guidelines**,** an online survey format was used in the administration of the survey. After the approval from the IRB, the researcher started soliciting the email addresses of the current SMTs from the websites of the public schools in the southwestern state of the United States. The researcher sent a link to the questionnaire hosted by SurveyMonkey to the email addresses of the current SMTs. The current secondary mathematics teachers first viewed an informed consent before accessing the questionnaire. They would not proceed with the questionnaire unless they agreed to the terms of the study as outlined in the consent form. They had the option to disagree on the consent form and exit the survey.

The email addresses for participants were collected from the websites of the public schools in the southwestern state of the United States. The recruitment email was sent to all secondary mathematics teachers whose contact emails were publicly available on the websites of the schools. Finally, some schools did not have published email addresses, but they provided a messaging system on their websites. This messaging system was used to contact the secondary mathematics (6-12) teachers directly.

To encourage potential participants to participate in the survey, questionnaire completers were offered an opportunity to enter a drawing for 20 $25 gift cards. As noted in Dillman et al. (2009), compensation had shown to notably increase the response rate of surveys, both online and in-person surveys. Questionnaire completers who chose to participate in the study were led to a separate page where they submitted their name and email address. This was where names and email addresses were collected for the drawing to ensure the anonymity of the participants to the research survey.

Data Set Revision

After exporting 58 variables from SurveyMonkey to SPSS version 28, the researcher created five new variables. The researcher computed variable 59 readiness to teach mathematics (RTTrc) after the researcher decided to assign a “1” to nay participant who responded “moderately extent” or “large extent” answer and “0” to any participant who responded with a “not at all” or “small extent” to answer well prepared items in the questionnaire. The researcher computed variable 60 “importance of mathematics methods instructors” by adding each participants’ answers to the questions related to mathematics methods instructors. The researcher assigned a “1” for who agreed that mathematics methods instructors are important and a “0” for those who disagreed. The researcher created variable 61 “importance of teacher preparation programs” by adding each participant’s answer to the teacher preparation programs. The researcher assigned a “1” for those who agreed that teacher preparation programs are important and “0” for those who did not. The researcher created variable 62 “importance of mathematics methods courses” by adding each participant answers to the mathematics methods courses responses. The researcher assigned a “1” for those who agreed that mathematics methods were important and “0” for those who did not. The researcher created variable 63 “importance of appropriate methods of teaching” by adding each participant answers to the appropriate methods of teaching responses. The researcher assigned a “1” for those who agreed that appropriate methods of teaching were important and “0” for those who did not. These categorical variables were included in the codebook for analysis.

Methods of Data Analysis

Phase I Quantitative Data Analysis Methods

The researcher calculated descriptive statistics for all variables. This included frequencies and percentages using SPSS version 28. The researcher estimated the Chi-Square statistics. Although most of the questionnaire items addressed secondary teachers' mathematical perceptions about the teacher preparation programs, there were some other questions related to teacher preparation, and there were some questions related to mathematics teacher preparations of secondary schools (6-12). There were some questions on the professional development for the current secondary mathematics teachers.

Phase II Qualitative Data Collection

The second phase of the dissertation consisted of qualitative data collection. This phase sought to understand how current SMTs' perceptions about the teaching of mathematics in the teacher education preparation programs. The researcher individually conducted semi-structured interviews (Leavy, 2017) via Zoom as one of the primary methods of data collection for those who took part in the interviews after the survey. There was a question at the end of the survey asking the participants to take part in a 20–30-minute individual interview. The questionnaire for individual semi-structured interviews is as shown in Appendix D. It was constructed using Leavy (2017), and they were pretested with two experienced secondary mathematics (6-12) teachers for the reliability of the questions.

In the design of the instrument for semi-structured interviews, the researcher sought to understand the participants' perceptions of how the teacher preparation programs were utilized in the construction of mathematical knowledge for teaching to support secondary teachers in the teaching and learning of mathematics. The questionnaire consisted of thirteen questions that were sufficient to answer the second research question that focused on participant's perceptions about teacher education programs regarding their MKT for effective mathematics teaching in public schools.

Throughout the interview process, the researcher took notes to record observations and emergent questions as the interview went on, and it helped to clarify and enrich the data transcription process. The researcher kept reflective memos to document the coding process and capture emergent findings. The interviews were video recorded on Zoom, and they were transcribed manually using the procedure outlined in Miles and Haberman (1994). The researcher verified the accuracy of the transcription by comparing the audio recordings to the transcription. After transcription was completed, the data were ready for analysis.

Definition and Importance of Interview Method

The method of qualitative data collection used an individual interview method. According to Gay et al. (2009), "an interview is a purposeful interaction in which one person obtains information from another" (p. 371). Specifically, the researcher used individual semi-structured interview questions shown in Appendix D. They employed open-ended questions, which allowed follow-up questions during the process of the interview (Johnson, 2017; Gay et al., 2009). This gave the researcher chance to ask emergent questions as the interview progressed. According to Cohen et al. (2011), open-ended questions "invite an honest, personal comment from respondents…" (p. 392). Hence, open-ended questions attracted honest, personal comments from the participants about their experiences with the mathematics methods courses as they were teaching in public schools.

While other methods could not provide data from past events, the interview method did provide data from the past events quite well because questions could be asked about past events, which the participants could remember and give answers. The researcher was able to set up interviews as a follow-up activity from the survey. This clarified areas that were not understood clearly during the survey data collection (Cohen, 2011, Yin, 2014).

Interviews were used to examine attitudes, interests, concerns, and values more easily than through observation and questionnaires (Leavy, 2017; Gay et al., 2009). In this case, semi-structured interviews were used to describe perceptions of secondary mathematics teachers about teacher education programs regarding their teacher preparation to teach mathematics effectively.

The interview was one of the best methods because the researcher found out perceptions of the participants about how the mathematics content and methods courses contributed to the mathematical knowledge for teaching (Cohen et al., 2011). Using the interview method, the researcher collected data by using videotaping on Zoom because it provided a verbatim account of the session, and it ensured that the original data was available at any time (Gay et al., 2009).

All the participants were allowed to indicate on the survey questionnaire whether they wanted to participate in individual semi-structured interviews or not. This method of data collection was very much recommended at that time due to the COVID-19 pandemic, which did not allow meetings to take place, and teaching was done virtually. This affected the number of teachers that participated in semi-structured interviews. The interviews were scheduled for half an hour for the participants that volunteered to participate in the individual interviews.

During the quantitative phase of the study, 10 secondary mathematics teachers volunteered to take part in the interviews, but only four secondary teachers were available for the interviews. This was due to COVID-19 related issues that happened. The collection of data was done on Zoom, which made it problematic as the interviews could not be done in person. Some of the secondary mathematics teachers that could have participated failed because of the COVID-19 pandemic. This explains why the number of secondary school teachers who were interviewed got reduced to only four teachers.

Methods of Data Analysis

Qualitative data analysis was a range of processes and procedures whereby the qualitative data required some form of explanation, understanding, and interpretation of the people and situations that were being investigated (Yin, 2009). The main idea was to examine the meaningful and symbolic content of qualitative data that had been collected (Cohen et al., 2011). Data analysis was important so that data made sense to the people once the research was done.

Data analysis was done using conceptualization, coding, and categorizing method (Stake 1995, Yin 2009). First, the researcher organized the data in table matrices that could be sorted out by participant identification number (ID#), question number (Q#), and other important characteristics. The table looked like the one below considering that pseudo-names were used for the anonymity of the participants.

Table 1

A Table Matrix showing recorded Information from the Questionnaire Respondents

Participant ID #	Participant Name	Interview Done
1	Mbachie	D
2	Eneka	D
3	Yochie	D
4	Mackie	D

The table lists names in alphabetical order and participant identification numbers. The numbers start from one up to four participants in alphabetical order. In the third column, the researcher checked (D) if the interview were done or not. For recording purposes, as each interview was done, the participant identification number (ID #) was written at the top of the first page of the questionnaire and the letter (D) was written in the third column.

Second, the researcher created a four-column table that looked like the one in Table 2 below. The first column had a participant identification number (ID #), then question number

(Q#), Response (R), and column four was for the code the researcher assigned during data analysis. The researcher began entering the responses from the questionnaires by entering the ID# first, the question number under Q#, followed by the transcribed response in column three. The researcher continued entering data like that until all the participants were done. That is, for each response the researcher entered participant ID#, then question number (Q#), and then the transcribed response (R). If there was no response to that question, then the researcher entered no response (NR). The researcher continued in this manner until all responses to all individual semi-structured interview questions were entered. The table matrix looked like the one below.

Table 2

A Table Matrix Showing (ID#), Question Numbers (Q#), Response (R), and Code

ID#	**Q#**	**Response**	**Code**
4	1	xxxxxxxxxxxxx	
4	2	xxxxxxxxxxxxxxxxxxxxxxx	
4	3	xxxxxxxxxxxxxxxxxxxxx	
4	4	xxxxxxxxxxxxxxxxxxxxxxxxxxxxx	
4	5	xxxxxxxxxxxxxxx	
3	1	xxxxxxxxxxxxxxxxxxx	
3	2	xxxxxxxxxxxxx	
3	3	xxxxxxxxxxxxxxxxxxxx	
3	4	xxxxxxxxxxxxxxxxxxx	
3	5	xxxxxxxxxxxxxxxxxxxxxxx	

Third, the interview responses were sorted out to make the information more useful than before. The researcher sorted column 1 (ID#) to put all participants in alphabetical order by ID#. The researcher then sorted column 2 so that the researcher had all the responses to each question together. The table matrix looked like the one below.

Table 3

Data Sorted out According to Question Numbers

ID#	Q#	Response	Code
1	1	Xxxxxxxxxxxxxxxxxxxxxxxxxx	
2	1	Xxxxxxxxxxxxxxxxxxxxx	
3	1	Xxxxxxxxxxxxxxxxxxxx	
4	1	Xxxxxxxxxx	
1	2	Yyyyyyyyyyyyyyy	
2	2	Yyyyyyyyyyyyyyyy	
3	2	Yyyyyyyyyyyyyyyyyy	
4	2	Yyyyyyyyyyyy	

At this juncture, the data was ready to code and analyze the question by question. This was a very easy coding process using question by question because the questions were arranged close to each other so that it was easy to read the same question from different participants. The type of analysis described above was a simplified and adapted version of what Miles and Huberman (1994) explained.

Conceptualization, Coding, and Categorization

Due to the numbers of secondary mathematics teachers interviewed, the coding process was not complicated as the researcher thought it would be. There were just four secondary mathematics teachers that participated in the interviews. The researcher developed a set of codes using predefined codes and the ones that emerged from the data. In this case, predefined codes were categories and insights that the researcher expected to see based on the review of the literature and prior knowledge. This was recorded in a two-column table listing all the categories and the codes that were assigned to them.

According to Cohen et al. (2011), coding is a process of reducing data into smaller groupings so that they are more manageable and easier to understand. It enabled the researcher to begin to see relationships that were emerging across the data. Hence, the researcher looked for similarities and differences in different sets of data to see what different groupings were saying. The researcher conducted qualitative data analysis manually as had been outlined above. This was because the interviewed mathematics teachers were not many, and that the researcher wanted to get familiarized with the data analysis process.

Summary

The goal of this chapter was to outline the research methods that were used to answer the research questions. A discussion of the procedure, study participants, data collection, and interview questions outlined the specifics of how the study was conducted and who participated in the study. A constructivist case study methodology was used to describe secondary mathematics (6-12) teachers' perceptions about how mathematics methods courses contributed to the mathematical knowledge for teaching.

Data were analyzed quantitatively and qualitatively. For quantitative data collection, a survey questionnaire was used to collect data and analyzed using SPSS version 28. In this study, the method of qualitative data collection was individual semi-structured interviews. Thus, data analysis was conducted using conceptualization, coding, and categorization.

To sum up the mixed methods methodology, the quantitative research questions were determined, and the sample was also determined. Qualitative data collection and analysis followed up to assist in answering quantitative research questions. This purposively facilitated the selection of participants in the qualitative phase of the study. The quantitative results were used to refine the qualitative research question and determine which participants were selected for further qualitative sample (Creswell & Plano Clark, 2011). In this case, the researcher interviewed the secondary mathematics teachers that volunteered themselves.

The last part of the study was to summarize and interpret quantitative and qualitative results, to discuss to what extent the qualitative results helped to explain the quantitative results (Fetters et al., 2013). The plan included what questions needed further probing and which individuals could best help to explain the quantitative results (Bryman, 2006). This was how the dissertation research process was conducted. In the next chapter, the researcher presents the results of the study from the quantitative and qualitative data analysis.

CHAPTER 4

Results

As stated in Chapter One, the purpose of this sequential explanatory mixed-methods case study was to examine the perceptions of current SMTs about their teacher preparation for effective teaching of secondary school mathematics. This chapter is organized in terms of the two specific research questions that were posed in Chapter One. In the first place, it reports the extent to which current secondary mathematics teachers perceive that mathematics methods courses influenced their MKT. It then examines ways in which current secondary mathematics teachers perceive that content and instruction in the mathematics methods courses contributed to the effective teaching of secondary school mathematics.

This chapter also presents the results of data analysis from the responses to the survey instrument described in Chapter Three. First, the information was presented on the sample that was used for data collection and analysis and the demographic information collected from the questionnaire. The researcher used descriptive analysis contingency tables, and Chi-Square statistics to address the quantitative research question outlined in Chapter One. The researcher also used semi-structured interviews in the qualitative data analysis. Data were transcribed and analyzed qualitative data to infer insights that the data revealed.

Survey Instrument Response

Before administering the questionnaire, the researcher identified 220 unique email addresses for current SMTs. These email addresses were publicly available on the school websites of the public schools in the southwestern state of the United States, as outlined in Chapter One**.** The researcher sent a recruitment email shown in Appendix A to the 220 email addresses in the first week of May 2021, but only two participants responded and completed the survey.

The researcher used the techniques of increasing the response rate of online surveys as outlined in Dillman et al. (2009). After applying those techniques, the researcher received 17 responses at the end of the second week of May 2021. The researcher received eight responses in the third week of May 2021, followed by seven responses in the fourth week of May 2021. After May, the researcher received three responses in June 2021. The slow response was because most of the teachers were on summer break.

Of the 220 emails that the researcher sent, 17 were returned as undeliverable, leaving 203 sent emails. Out of 203 delivered emails, only 39 participants responded to the emails as described in the previous paragraph. Of the 39 replied emails, seven were incomplete, with only the first 11 questions answered, while five submitted emails without any of the questions answered. The researcher eliminated the responses from the data analysis, and this left 27 valid responses that were suitable for data analysis.

Respondent Demographic Information

This section reports the demographic information of the respondents. The information about the respondents will be presented in tables for easy data analysis. The first demographic information to be reported was the number of years of teaching experience. Other items of the demographic information were also reported.

As noted in Table 4, the largest number of respondents were reported between 6-10 years of teaching experience, with 10 respondents. This was followed by the respondents with 11-15 years of teaching experience, with six teachers. The smallest number of respondents were reported between 21-25 years of experience with one teacher. Four respondents who had 25 years or above were also reported. The number of years of teaching experience for other groups of teaching experiences are as shown in Table 4.

Table 4

The Distribution of Frequencies and Percentages for the Number of Years of Teaching Experience of Current Secondary School Mathematics Teachers (N = 27)

Teaching Experience	Frequency	Percentage
1—5 years	4	14.8%
6—10 years	10	37.0%
11—15 years	6	22.2%
16—20 years	2	7.4%
21—25 years	1	3.7%
25 + years	4	14.8%
Totals	27	100%

Note: The teaching experience for the mathematics teachers involved in the survey grouped as 1-5 years, 6-10 years…,25+ years.

The respondents reported the grades that they were teaching in their respective schools. The most represented teaching grades were grades 11 and 12, with 15 teachers. They were seconded by grade 10 with 12 teachers who were teaching in that grade in the public schools. The numbers of respondents teaching in other grades are shown in Table 5 although the least number of teachers were reported in grade sic and seven with three teachers teaching in the grades.

The frequency in Table 5 is more than 27 because some teachers taught in more than one grade level. As a result, the frequency indicating the number of respondents was 66, greater than 27, the total number of respondents in the survey. Table 5 summarizes the results of the respondents that were teaching in the grade levels as described in the study.

Table 5

Frequencies and Percentages for Current Secondary Mathematics Teachers by Their Teaching Grades

Teaching Grade	Frequency	Percentage
Grade 6	3	10.7%
Grade 7	3	10.7%
Grade 8	7	25.0%
Grade 9	11	39.3%
Grade 10	12	44.4%
Grade 11	15	53.6%
Grade 12	15	53.6%
Totals	66	237.3%

Note: The frequencies and percentages for the number of teachers teaching in grades 6-12.

The respondents also reported the subjects they were teaching in the grade levels at the respective schools. Of the 27 respondents, 12 of them were teaching Algebra I, nine were teaching Algebra II, and eight teachers were teaching General Mathematics. The least number of respondent teachers were reported teaching in grade six and seven with three teachers. The distribution of the respondents that were teaching the subjects is indicated in Table 3.

The frequencies that were reported in Table 6 add up to 33, which is more than 27. This was because some respondent teachers were teaching more than one subject in their respective schools. It was like the situation reported in Table 5, where the respondents taught in more than one grade level in their respective public schools.

Table 6

Frequencies and Percentages for Mathematics Teachers by Their Teaching Subjects

Teaching Subject	Frequency	Percentage
General Mathematics	8	29.6%
Algebra I	12	44.4%
Algebra II	9	33.3%
Geometry	8	29.6%
AP Mathematics	1	3.6%
Precalculus	1	3.6%
Calculus	4	14.3%
Totals	33	158.4%

Note: The number of teachers teaching each mathematics subject in schools.

Table 7

Distribution of Frequencies and Percentages for the Type of Teaching License Held by Mathematics Teachers

Characteristic	Frequency	Percentage
Secondary Vocational (7-12)	1	3.7%
Secondary (6-12)	21	77.8%
Middle Level (5-9)	2	7.4%
Elementary (K-8)	1	3.7%
Missing	2	7.4%
Total	27	100%

Note: Types of licenses for teaching in public schools

The other issue about teaching licensure was the teaching licensure level. The state has a licensure level system with four tiers. The tiers are categorized as level I, alternative level I, level II, and level III. Level I and alternative level I are for early career teachers, are valid for five years, and cannot be renewed. Level II and level III are professional level licenses that are renewed indefinitely, and level III requires a master's degree or National Board Certification. As summarized in Table 8, six level I teachers were reported. Nine level II teachers were reported, and ten level III teachers were reported.

Table 8

Frequencies and Percentages for Teaching License Level of Mathematics Teachers

License Level	Frequency	Percentage
Level I	6	22.2%
Level II	9	33.3%
Level III	11	40.7%
Alternative Level I	1	3.8%
Totals	27	100%

Note: The levels of teaching licensure for the secondary teachers in the study

Finally, the respondents reported the pathways they used to obtain their teaching licenses. Of the four pathways, 15 respondents reported the four-year degree pathway, and nine did the alternative pathway. Two did the master's degree pathway, and one respondent did not indicate the licensure pathway. education, and they could get licensed through the alternative licensure pathway. The alternative licensure pathway was a second pathway from the traditional license pathway with a lot of teachers trained in that pathway. All the license pathways are shown in Table 9.

Table 9

Frequencies and Percentages for Teaching license Pathways for the Secondary School Teachers

License Pathway	Frequency	Percentage
Alternative Level	9	33.3%
Bachelor’s Degree Route	15	55.6%
Master's Degree Route	2	7.4%
Other	1	3.7%
Totals	27	100%

Note: The licensure pathways for s mathematics teachers to get a teaching license.

The other demographic item on the questionnaire explored the highest academic qualification of the respondents. As outlined in Table 10, 15 respondents had a bachelor's degree, and eleven respondents had a master's degree. One respondent indicated a doctorate degree as the highest academic qualification.

Table 10

Distribution of Frequencies and Percentages of the Highest Academic Qualification of the Mathematics Teachers

Highest Qualification	Frequency	Percentage
Bachelor’s Degree	15	55.6%
Master’s Degree	11	40.7%
Doctoral Degree	1	3.7%
Total	27	100%

Note: The highest academic degree that the teachers earned to teach mathematics

Another demographic item asked where the respondents earned their highest academic qualification. Sixteen respondents reported they earned their highest educational qualification at State University 1, and Table 11 summarizes all the results about the universities where the respondents earned their highest academic education.

Table 11

Frequencies and Percentages for the University Where the Teachers Earned Their Degrees

University	Frequency	Percentage
State University 1	16	59.3%
State University 2	2	7.4%
Other Universities	9	33.3%
Total	27	100%

Note: The frequencies and percentages where the teachers earned their degrees

Table 12

Frequencies and Percentages for Type of Degree that the Teachers Earned at their Universities

Type of Degree	Frequency	Percentage
BS Mathematics Education	4	14.8%
BS in Education	9	33.3%
BA in Education	2	7.4%
BS in Mathematics	2	7.4%
Other	10	37.0%
Totals	27	100%

Note: The type of degree that the respondents earned to teach mathematics

Lastly, the respondents reported the majors that they did in their respective universities. Most of the respondents indicated that they majored in mathematics, with nine participants. The other 10 respondents indicated that they majored in other majors apart from mathematics. Four respondents indicated that they majored in mathematics education. One reported that he majored in pure mathematics, and the two missing participants did not reveal their majors in the questionnaire. The results indicated that there were 10 respondents that majored other majors apart from mathematics. These were such as Chemical Engineering and Electrical Engineering, which had a large component of mathematics. The mathematics component allowed the teachers to get teaching licenses and be able to teach mathematics in secondary school.

Table 13

Distribution of Frequencies and Percentages of the Majors of the Current Secondary School Mathematics Teachers

Degree Major	Frequency	Percentage
Mathematics	9	33.3%
Mathematics Education	4	14.8%
Pure Mathematics	1	3.7%
Applied Mathematics	1	3.7%
Other majors	10	37.0%
Missing	2	7.4%
Totals	27	100%

Note: The degree majors of the secondary mathematics teachers that enabled them to teach mathematics in secondary school

Results

All respondents provided the demographic information as indicated in the survey instrument Appendix C. The following section addresses the first research question of the study, and this research question is on the quantitative part of the study.

Research Question 1. To what extent did current secondary mathematics public school teachers perceive that their mathematics methods courses influenced their MKT?

The results of this research question are outlined in four sections. First, the results about the teaching and learning of mathematics are reported. Second, the results of teacher preparation for teaching mathematics are presented. This is followed by the results for effective teaching of mathematics. Finally, the results for professional development are presented.

Teaching and Learning of Mathematics

The current secondary school mathematics teachers indicated that they moderately related mathematics lessons to the students' daily lives with 74 percent. In comparison, 18.5 percent of the respondents showed that they related mathematics to the students' daily lives to a large extent. In addition to the relation of mathematics to the students' daily lives, the respondents also reported that they asked students to explain their answers to check their understanding, and 66.7 percent of the teachers were able to ask students to explain their answers to a large extent. Further, 63 percent of the respondents indicated that they encouraged discussions in their classrooms to a large extent. Also, 81.5 percent of the respondents stated that they linked new content to students' prior knowledge to a large extent. These teaching techniques were well used by the mathematics teachers. Table 14 summarizes the information about teaching mathematics at secondary school.

Table 14

Frequencies of the Teachers' Techniques Working with Students in the Classroom

Teaching Techniques	Not at all	Small Extent	Moderately Extent	Large Extent
Relate the lesson to students' daily lives	0	2	20	5
Ask students to explain their answers	0	1	8	18
Encourage discussions among students	0	3	7	17
Link new content to students' prior knowledge	0	1	4	22

Note: The frequencies and percentages of how the teachers using their teaching techniques.

Table 15, the participants reported how mathematics methods courses were conducted during their teacher preparation using tables, charts, and graphs.

Table 15

Frequencies and Percent Scores for Extent of Preparation with Mathematics Methods Courses in the University during Teacher Preparation

Characteristic	Frequency	Percentage
Not at all	3	11.1
Small Extent	19	70.4
Moderate Extent	2	7.4
Large Extent	2	7.4
Missing	1	3.7
Total	27	100%

Note: Table 15 shows the extent of teacher preparation done in mathematics methods courses during their teacher preparation.

The results in Table 15 indicate how mathematics methods courses represented the course content using tables, charts, or graphs to help secondary mathematics teachers understand mathematics.

Effective Teaching of Mathematics

The respondents overwhelmingly reported that it was essential for mathematics teachers to have a sound knowledge of MKT, with 96.3 percent showing moderate or large extent. The respondents also overwhelmingly said that secondary mathematics teachers must use appropriate teaching methods, with 92.6 percent of the respondents to a moderate or large extent**.** Table 16 below shows the results.

Table 16

Distribution of Percentage of the Importance of MKT and Appropriate Methods of Teaching

Characteristic	Not important	Less important	Moderate important	Very Important
Importance of teachers having MKT	0	1	6	20
Importance of using appropriate methods of teaching	0	2	2	23

Note: The numbers of the teachers who rated the importance of MKT and appropriate methods of teaching

After the respondents reported that they agreed to a large extent or moderate extent on the importance of MKT in the teaching and learning of mathematics. They also agreed on the importance of using appropriate teaching methods, and the**y** indicated how well-prepared they were to teach mathematics in secondary schools. This showed that 55.5% of the secondary mathematics teachers were prepared moderately or to a large extent. Table 17 summarizes the results that the respondents presented.

Table 17

Distribution of Frequencies and Percentages for the Extent of Preparation

Characteristic	Frequency	Percentage
Not at all	2	7.4%
Small Extent	9	33.3%
Moderate Extent	8	29.6%
Large Extent	7	26.0%
Missing	1	3.7%
Totals	27	100%

Note: The preparedness of the respondents for the teaching and learning of mathematics

Teacher Preparation for Teaching Mathematics

After looking at the teaching and learning of mathematics, the respondents reported how well-prepared they were to teach the mathematics subjects that were offered in secondary schools. The subjects were Algebra I, Algebra II, Geometry, General Mathematics, AP Mathematics, Precalculus, Calculus, and Coordinate Geometry. Given that the eight subjects were ordinal variables, the researcher calculated the percentage of participants who were not at all prepared, prepared to a small extent, prepared moderately, and prepared to a large extent.

The current secondary mathematics teachers indicated that they were prepared moderately or to a large extent to teach General Mathematics with 74.1%, followed by Algebra I with 69.2%. The least number of percentage preparation were reported in AP mathematics 34.6% and Calculus 26.9%. Table 18 summarizes all the information about their readiness to teach mathematics subjects that are mostly offered in secondary schools.

Table 18

Distribution of Percentages of Secondary School Teachers' Responses to eight mathematics subjects in readiness to teach mathematics (N = 27)

Mathematics Subjects	Not at all Prepared	Small Extent	Moderate Extent	Large Extent	Missing Percent
General Mathematics	3.7	11.1	29.2	44.9	11.1
Algebra I	3.7	15.4	26.9	42.3	11.1
Algebra II	7.7	11.5	34.6	26.9	19.2
Geometry	15.4	7.7	26.9	26.9	23.1
Coordinate Geometry	19.2	15.4	19.2	23.1	23.1
AP Mathematics	34.6	11.5	19.2	7.7	26.9
Precalculus	23.1	15.4	19.2	15.4	26.8
Calculus	30.8	19.2	11.5	15.4	23.1

Note: The percentages of how well-prepared respondents are to teach different mathematics subjects offered in secondary schools.

The results in Table 18 required more analysis. Hence, a more complex story was evident when examining the relationship between teaching preparedness by subject and the readiness to teach mathematics. As such, the researcher conducted a cross-tabulation analysis between teaching preparedness by subject and readiness to teach mathematics (RTTrc). Table 19 summarizes the percentage of participants who were prepared to teach mathematics subjects and those who agreed that they were prepared to teach mathematics (coded as the Readiness to Teach Mathematics variable (RTTrc) after cross tabulation. The percentages in Table 19 were lower than in Table 18 considering the sum of moderate and large extent percentages in Table 19.

Table 19

Percentage of Participants Who Indicated Readiness to Teach Subjects and Those Who indicated Readiness to Teach Mathematics

Mathematics Subjects	Prepared Percent
General Mathematics	54.5%
Algebra I	53.8%
Algebra II	46.2%
Geometry	43.2%
Coordinate Geometry	38.5%
AP Mathematics	23.1%
Precalculus	26.9%
Calculus	19.2%

Note: The cross-tabulation results between readiness to teach each mathematics subject and readiness to teach mathematics considering all subjects.

The percentages of respondents who stated that they were well-prepared to teach mathematics subjects and those who were well-prepared to teach mathematics were lower in all eight mathematics subjects when compared to their responses in Table 18.

Secondary mathematics teachers reported how mathematics methods instructors, colleagues, independent learning, and professional development courses influenced them to acquire MKT for effective mathematics. The researcher calculated the percentage of participants who were not at all influenced, influenced to a small extent, influenced moderately, and influenced to a large extent. Table 19 summarizes the responses of participants about the extent of preparation teach mathematics subjects in secondary schools.

Table 20

Distribution of Secondary School Teachers' Responses on How They are Influenced to Acquire MKT (n = 27)

Variables	Not at all Assisted	Small Extent	Moderate Extent	Large Extent	Missing Percent
Mathematics Methods Instructors	3.8	23.1	19.2	38.5	7.7
Colleagues	0	3.9	34.6	61.5	0
Independent Learning	3.8	7.7	42.3	46.2	0
Professional Development	11.5	23.1	38.5	26.9	0

Note: How mathematics methods instructors, colleagues, independent learning, and professional development influenced the teacher to acquire MKT.

The data, on how mathematics methods instructors, colleagues, independent learning, and professional development influenced respondents to acquire MKT suggested that more than 50 percent of the respondents surveyed considered themselves to be influenced moderate or large extent to acquire MKT in each of the four variables; however, additional analysis was necessary. The researcher examined the relationship between the four variables influenced to acquire MKT and readiness to teach mathematics. The researcher conducted a cross-tabulation analysis between each type of the four variables and readiness to teach mathematics to check how the respondents were prepared.

The percentages of respondents who stated that they were influenced to acquire MKT through mathematics methods instructors, colleagues, independent learning, and professional development and readiness to teach mathematics were lower in all four categories when compared to their responses in Table 20. The differences in the percentages are well explained by the participants of the semi-structured interviews.

Table 21

The Distribution of Percentage of Participants Influenced to Acquire MKT and Those Who indicated Readiness to Teach Mathematics (N = 27)

Variables	Prepared Percent
Mathematics methods instructors	46.2%
Colleagues	50.0%
Independent learning	46.2%
Professional development	46.2%

Note: The cross-tabulation results show the preparedness of teachers to teach mathematics

Professional Development of Teachers

In this study, the respondents were asked to respond to questions about their professional development that took place in secondary schools. Some of the professional development skills were training in the content, training in methods, training in improving student critical learning, training in mathematics assessment, training in diversity and inclusion in mathematics. There were also other variables like attending a conference, giving a presentation at a conference, and taking in an innovative program in mathematics education.

The variables mentioned in the survey were organized so that participants with insights into the professional development that takes place in secondary schools should be able to understand. As it is seen in Table 22, the variables registered very high percentages of participation. The highest percentage of participation were recorded in training in mathematics content, training in mathematics methods, training in mathematics assessment, attending conference in mathematics, and training in critical thinking in mathematics.

Table 22

Distribution of Percentage of Secondary Teachers Who Said Yes to Each Type of Activity

Type of Training/Activity	Affirmative Percent
Training in mathematics content	88.5%
Training in mathematics methods	84.6%
Training in mathematics assessment	84.6%
Training in improving critical thinking in students	84.6%
Training in diversity and inclusion in mathematics	69.2%
Attended a conference in mathematics	84.6%
Presented at a conference in mathematics	34.6%
An innovative project in mathematics education	42.3%

Note: The percentages of mathematics teachers who participated in the professional development activities.

The percentages of the respondents who agreed participating in the professional development activities are indicated in Table 22. The results needed more analysis because in Table 20, there is 26.9 percent of the respondents that indicated that professional development influenced MKT, and 38.5 percent indicated that they were influenced to a moderate extent. As such, the researcher conducted a cross tabulation analysis between each of the variables and readiness to teach mathematics. The researcher conducted across tabulation analysis between each of the variables and readiness to teach mathematics to check the influence of professional development in acquiring MKT. The least results were recorded in doing innovative projects in mathematics education 23.1 percent. Table 23 summarizes the results of the analysis of the professional development activities.

Table 23

A Cross Tabulation Table for Training in Each Activity and Readiness to Teach Mathematics (N = 27)

Type of Training/Activity	Percent prepared
Training in mathematics content	53.8
Training in mathematics methods	53.8
Training in mathematics assessment	53.8
Training in improving critical thinking in students	53.8
Training in diversity and inclusion in mathematics	53.8
Attended a conference in mathematics	53.8
Presented at a conference in mathematics	26.9
An innovative project in mathematics education	23.1

Note: The cross-tabulation results between the professional development activities and readiness to teach mathematics

Contingency Tables and Chi-Square Analyses

After collecting the respondents' responses, the researcher constructed contingency tables and calculated Chi-Square analyses on how mathematics methods courses influenced the SMTs to acquire MKT to teach mathematics in secondary schools. The researcher decided to create five new variables that would allow him to conduct Chi-Square analyses further. The variables created represented the major concepts and constructs of the study. The variables created were teachers' perceptions about their readiness to teach mathematics (RTTrc), the importance of mathematics methods instructors to teach mathematics (IMMIrc), the importance of mathematics methods courses (MMCrc), the importance of mathematics preparation programs (MPPrcc), and the importance of appropriate methods of teaching (IAMTrc). Table 24 summarizes, by construct, the respondents' responses to the items.

Table 24

Frequencies and Percentages of the Participants Who Agreed and Disagreed with the New Categorical Variables

Constructed Categorical Variable	Frequency Disagree	Percent Disagree	Frequency Agree	Percent Agree
Readiness to Teach Mathematics	12	44.4	15	55.6
Mathematics Methods Courses	14	51.9	13	48.1
Mathematics Preparation Programs	15	55.6	12	44.4
Appropriate Methods of Teaching Math	2	7.4	25	92.6
Mathematics Methods Instructors	16	59.3	11	40.7

Note: The five categorical variables that were constructed to find out how the variables influenced readiness to teach mathematics (Agree=1, Disagree=0)

The variable Mathematics methods courses (MMCrc) that the researcher created meant current mathematics teachers' perceptions about mathematics methods courses' effectiveness during teacher preparation. The sum of each participant's responses to the two mathematics methods courses' items recorded as "helpful=1" or "not helpful=0" using the process outlined in Chapter Three. While 48.1% agreed that mathematics methods courses are helpful, 51.9% did not agree as shown in Table 24.

In Table 25 below, the researcher conducted a contingency table analysis to examine the relationship between the composite categorical variable "mathematics methods courses" (MMCrc) and "readiness to teach mathematics categorical variable " (RTTrc). It was noted that there was no relationship between MMCrc score and RTTrc score because the difference between the observed and expected counts is not minimal, calculated as 0.8. In Table 25, there are four sets of observed and expected data separated in values by 0.8.

Table 25

Cross-Tabulation Between Mathematics Methods Instructors (MMCrc) and Readiness to Teach (RTTrc)

			RTTrc Not Helpful	Helpful	Total
Mathematics methods courses	Not Helpful	Observed Count	7	7	14
		Expected Count	6.2	7.8	14.0
	Helpful	Observed Count	5	8	13
		Expected Count	5.8	7.2	13.0
Total		Observed Count	12	15	27
		Expected Count	12.0	15.0	27.0

Note: A contingency table analysis of mathematics methods courses and readiness to teach mathematics

The researcher computed a Chi-Square statistic to examine the relationship between mathematics methods courses and current teachers' readiness to teach mathematics. This resulted in a Chi-square statistic value of 0.363. The p-value was p = 0.547, which is greater than 0.05. Based on this sample, there was no statistically significant relationship between mathematics methods courses and readiness to teach mathematics ($\chi^2 = 0.363,\ p = 0.547 > 0.05$). This means that mathematics methods courses did not have a greater influence on MKT of the respondents.

The researcher defined and created the variable importance of mathematics methods instructors (IMMCrc) meant how mathematics methods instructors helped to prepare teachers to teach mathematics. It was the sum of each participant's responses to the two mathematics methods instructor items recorded as "helpful=1" or "not helpful=0" using the process outlined in Chapter Three. While 40.7% agreed that mathematics methods instructors were helpful, 59.3 % disagreed, as shown in Table 24.

Table 26

Cross-Tabulation Between Mathematics Methods Instructors (IMMIrc) and Readiness to Teach Mathematics (RTTrc)

			RTTrc Not Helpful	Helpful	Total
Mathematics methods instructors	Not Helpful	Observed Count	4	4	8
		Expected Count	3.4	4.4	8.0
	Helpful	Observed Count	9	11	19
		Expected Count	8.6	10.6	19.0
Total		Observed Count	12	15	27
		Expected Count	12.0	15.0	27.0

Note: A contingency table analysis of the importance of mathematics methods instructors and readiness to teach mathematics

In Table 26, the researcher conducted a contingency table analysis to examine the relationship between the composite categorical variable importance of mathematics methods instructors (IMMIrc) and readiness to teach mathematics categorical variable (RTTrc). The results showed that there was no relationship between (IMMIrc) score and (RTTrc) score because the difference between the observed and expected counts was not minimal. The four sets of observed and predicted data, separated in value by 0.4, showed this result.

The researcher also computed the Chi-Square statistic, which resulted in a value of 2.217. The p-value was p = 0.137 which is greater than 0.05. Based on this sample, this suggests that there was no significant relationship between the importance of mathematics methods instructors and readiness to teach mathematics ($\chi^2 = 2.217,\ p = 0.137 > 0.05$). This means that mathematics methods instructors did not have a greater influence in preparing the mathematics teachers to teach mathematics in secondary schools.

The researcher defined and created the variable importance of mathematics preparation programs (IMPPrc) meant how mathematics preparation programs helped to prepare teachers to teach mathematics. It was the sum of each participant's responses to the two mathematics preparation program items recorded as "helpful=1" or "not helpful=0" using the process outlined in Chapter Three. While 44.4% agreed that mathematics preparation programs were helpful, 55.6 % disagreed, as shown in Table 24.

In Table 27, the researcher conducted a contingency table analysis to examine the relationship between the composite categorical variable Importance of mathematics preparation programs (IMPPrc) and readiness to teach mathematics categorical variable (RTTrc). The results show there is no relationship between (IMPPrc) score and (RTTrc) score because the difference between the observed and expected counts was not minimal. The four sets of observed and expected data, separated in value by 0.6, showed this result.

The researcher then computed a Chi-square statistical analysis to examine the relationship between importance of mathematical preparation programs (IMPPrc) and current mathematics teachers' readiness to teach mathematics. This resulted in a Chi-square statistic value of 0.142. The p-value was 0.708 greater than 0.05. Based on this sample, that there was no statistically significant relationship between the importance of mathematics preparation programs and readiness to teach mathematics ($\chi^2 = 0.142$, $p = 0.708 > 0.05$). This means mathematics preparation programs did not have a greater influence on MKT preparing the mathematics teachers to teach mathematics their MKT influenced the readiness of teaching of mathematics during teacher preparation. It was the sum of each participant's responses to the two MKT items recorded as "important=1" or "not important=0" using the process outlined in Chapter Three. While 44.4% agreed that (IMPPrc) was important, 55.6% did not agree as it is shown in Table 24.

Table 27

Cross-tabulation Between Importance of (IMPPrc) and Readiness to Teach Mathematics (RTTrc)

			RTTrc		To.tal
			Not important	Important	
Mathematics	Not	Count	0	1	1
Preparation Programs	important	Expected Count	4	6	10
		Count	12	14	26
	Important	Expected Count	11.6	14.4	26.0
Total		Count	12	15	27
		Expected Count	12.0	15.0	27.0

Note: A contingency table analysis of the importance of mathematics preparation programs and readiness to teach mathematics

In Table 28, the researcher conducted a contingency table analysis to examine the relationship between the composite categorical variable appropriate methods of teaching

(AMTrc) and readiness to teach mathematics categorical variable (RTTrc). The results showed that there was no relationship between (AMTrc) score and (RTTrc) score because the difference between the observed and expected counts was not minimal. The four sets of observed and predicted data, separated in value by 1.1, show this result.

The researcher then computed a Chi-Square statistic to examine the relationship between importance of appropriate methods of teaching (IAMTrc) and current mathematics teachers' readiness to teach mathematics. This resulted in a Chi-square statistic value of 0.831**.** The p-value was 0.362 greater than 0.05. Hence, there was no statistically significant relationship between the importance of appropriate methods of teaching and readiness to teach mathematics ($\chi^2 = 0.831\ \ p = 0.362 > 0.05$). This means appropriate methods of teaching did not have a greater influence on MKT preparing the mathematics teachers.

Table 28

Cross-tabulation Between Importance of Appropriate Methods of Teaching (IAMTrc) and Readiness to Teach Mathematics (RTTrc)

			(RTTrc) Disagree	Agree	Total
Appropriate Methods	Disagree	Count	2	0	2
of Teaching		Expected Count	9	1.1	2.0
	Agree	Count	10	15	25
		Expected Count	11.1	13.9	25.0
Total		Count	12	15	27
		Expected Count	12.0	15.0	27.0

Note: This table shows a contingency table analysis of appropriate methods of teaching and readiness to teach mathematics

Performance of the Instrument

The instrument the researcher developed for this study had not been validated independently. The researcher estimated Cronbach's alpha reliability coefficient, a "correlational measure of the reliability or internal consistency of the items in a scale" to ensure that the four constructs (mathematics methods courses, importance of mathematics methods instructors, importance of mathematics preparation programs, and importance of appropriate methods of teaching mathematics) were measuring aspects of the same thing and that it was appropriate to add up items for an overall rating scale (Vogt, 2007, p. 90). The Cronbach's alpha reliability coefficients for each of the constructs were the following: mathematics methods courses (0.783), the importance of mathematics methods instructors (IMMIrc) (0.712), the importance of mathematics preparation programs (0.683), and importance of appropriate methods of teaching mathematics (0.919).

The performance of the instrument might have been improved if the length of the instrument was reduced. Although individual items were clustered, the number of responses that each participant was asked to respond to have been too great. Survey fatigue might have been a factor in some of the participants' responses. The inclusion of a gift card option at the end of the instrument seemed practical at the onset of the study; however, there were problems in accessing the second survey if a participant used his or her phone. This may have resulted in fewer cards being distributed because of participants having to add a more cumbersome step to access the gift card. The terminology in the instrument created several areas of confusion. For example, the terminology "mathematical knowledge for teaching" could not be easily understood and responded to accordingly. "Effective teaching of mathematics" could have caused multiple meanings in some respondents.

Summary of the Quantitative Findings

Descriptive analysis of the respondents' raw data revealed that mathematics methods courses did not influence MKT in the teachers for effective mathematics teaching. Inferential analysis showed that the MKT levels of the respondents did not change much during their teacher preparation. The data revealed that the content and instruction of mathematics methods courses did not contribute to the effective teaching of mathematics. A more detailed discussion of the results will be presented in Chapter Five.

Findings from the Qualitative Study

The qualitative phase of the study focused on explaining the information gained from the survey to help answer the second research question: In what ways did current secondary mathematics teachers perceive that the content and instruction in their mathematics methods courses contributed to their effective teaching of mathematics?

Results and Analysis of Semi-Structured Interviews

Four current secondary mathematics teachers teaching mathematics in public schools volunteered to participate in semi-structured interviews to share their perceptions of how mathematics methods courses contributed to their effective mathematics teaching. Two of the participants, Mbachie and Eneka, did not take any mathematics methods course during their teacher preparation. Yochie and Mackie took one mathematics methods course during their teacher education preparation. Some of the secondary mathematics teachers did not participate in the interviews due to the restrictions of COVID-19. This affected data collection, and thusly, the data collected was not as robust as initially anticipated. Next, demographics of the secondary mathematics teachers who participated in the semi-structured interviews are presented.

Demographics of the Interview Participants

Mbachie was one of the secondary mathematics teachers who participated in the interviews. He had a bachelor's degree in chemical engineering from the State University 1, with a mathematics component. He had a master's degree in business administration. He was teaching Algebra I and Geometry in spring 2021. He had not done any mathematics methods courses. He got a teaching license to teach mathematics from the State after doing education courses and general methods courses such as class management. At present, he is teaching mathematics at one of the high schools in the southwestern state, and he had two years of teaching experience.

Eneka was another secondary mathematics teacher who participated in the semi-structured interviews. He had a bachelor’s degree in electrical engineering from State University 1. He had a Master of Arts in Education Leadership from State University 2. He had 12 years of teaching experience, and he had not taken any mathematics methods courses. He was teaching Algebra I, Geometry, and Algebra II in spring 2021. After writing State University 1 mathematics certification examination from the Public Education Department in the southwestern state, he got his teaching licensure. Unlike Mbachie who did teaching licensure, Eneka had never gone to a college for teacher preparation. He is now teaching mathematics at one of the high schools in the southwestern state of the United States.

Yochie was another mathematics teacher who participated in the semi-structured interviews. She had a Bachelor of Science with a major in mathematics from State University 2. She has 18 years of teaching experience, and she was teaching Algebra I and Geometry. Mathematics and the teacher preparation program led her to get the teaching licensure to teach mathematics in the southwestern state.

Mackie was another secondary mathematics teacher that the researcher interviewed. He had a Bachelor of Education with a major in mathematics from another university. During his teacher education preparation, the mathematics methods course and teacher reciprocity licensure enabled him to teach in the southwestern state. He had six years of teaching experience, and he was teaching Algebra I and Algebra II in spring 2021. He is also teaching mathematics at one of the high schools in the southwestern state of the United States.

Responses to the Semi-Structured Questions

The four secondary mathematics teachers were asked ways in which mathematics methods courses developed their mathematics understanding of mathematics topics. Mbachie and Eneka were quick to answer that they could not remember how mathematics methods courses helped them understand mathematics topics since they had not done any mathematics methods course. In contrast, Yochie remembered that she learned writing course outlines in the mathematics methods course. She also learned how to teach mathematics topics in the class, but it was not enough. Mackie said he could not understand much of what was happening in the class because what the instructor was teaching was not very clear. Mostly, the methods of how to teach mathematics were not very clear. Mackie also said that there was no connection between the topics done in mathematics methods class and high school mathematics. He was confused and not very ready to teach mathematics after completion of the course.

The four secondary mathematics teachers were asked the components of the teacher education preparation that helped select and sequence teaching strategies. Mbachie said that he valued professional development (PD) activities that were done at the school level. Eneka noted that he appreciated observations more than anything else. He said that one could learn how to teach mathematics by observing expert teachers teaching mathematics. He continued that this was what he had been doing to gain experience in teaching mathematics. Yochie and Mackie said that they loved the practical teaching part of the teacher preparation especially when a student teacher taught fellow students in the class on a mathematics topic from high school mathematics. Mackie said this helped students to acquire teaching techniques faster than just going through material during teaching.

The teachers were asked about the connections that existed between college mathematics and secondary school mathematics. Mbachie and Eneka said that they saw no real connections between college mathematics and high school mathematics because they did not do methods courses. Eneka saw some relationships with content that he did in college algebra and calculus. Yochie pointed out that there were no connections mathematics methods courses and high mathematics because it was not discussed in class. Mackie said that he noted some connections between high school mathematics and mathematics methods courses. He said that there was some geometry involved in the math methods courses though not telling how to teach geometry.

The interviewee teachers were also asked how the mathematics methods courses prepared them to teach mathematics in secondary school. Mbachie and Eneka emphatically said mathematics methods courses were not required to teach mathematics. They pointed out that what matters most was content courses that one could do during teacher preparation. Eneka added that the best way to learn to teach mathematics was observing experts who were teaching mathematics in high school. I suggested this means field experience that student teachers do as part of their training. Yochie was not full y prepared to teach mathematics when he came out of the teacher preparation. He said, “I have learned most of the techniques of how to teach mathematics while on the job." Mackie concurred with Yochie that he was not fully prepared to teach mathematics from the teacher preparation program.

The last question that they were asked the teachers to comment on the teacher education preparation programs that took place in the universities, and how they could be improved. All the teachers said that the teacher preparation programs were not necessary because many teachers were teaching mathematics without going through the traditional preparation programs. Eneka pointed out that it would be better to do field experience than to do mathematics methods courses. He stressed that observation was more important than doing the mathematics methods courses. Yochie stated that there was a need for more work in the mathematics methods courses for the courses to make sense in teacher preparation. When probed to explain more, he said the content in the mathematics methods should be more focused on how to teach mathematics than what was being done doing.

Insights from the Semi-Structured Interviews

Analysis of the interview transcripts revealed four insights related to the current secondary mathematics teacher's perceptions of their preparation to teach mathematics. The participants indicated that the mathematics methods course had a minimal contribution to the effective teaching of mathematics. Yochie and Mackie expressed that MKT was addressed in their mathematics methods class, but it was not enough to make them fully ready to teach mathematics. After completing the mathematics methods course, Yochie and Mackie conveyed a sense of unpreparedness to teach mathematics because they were not fully trained how to teach mathematics topics from the mathematics curriculum. In the next section, the researcher presents the insights that were inferred during qualitative data analysis.

Insight 1: Current secondary school mathematics teachers perceived that the content and instruction of the mathematics methods course had minimal contribution on their effective teaching of mathematics in secondary schools.

The mathematics teachers expressed that the content and instruction in the mathematics methods course did not directly address MKT for teaching mathematics effectively. When initially asked about their experiences in the mathematics methods course. The two participants Yochie and Mackie expressed that the course did not focus on teaching the topics in secondary school mathematics. Therefore, the course was not very beneficial on the teaching of mathematics. Eneka expressed that he learned how to teach mathematics by observing experienced teachers because he had never done a methods course in mathematics, and he thought it was not necessary for teachers to take mathematics methods courses as he was teaching without it. Yochie stated:

> I don't know how to say this without being super negative. It's been one of the worst classes I've ever had… I dreaded going every week, and I was not sure if it was just because our professor was her first-year teaching at a college level.

Mackie shared similar feelings when asked if his experience in the mathematics methods course was positive, "No, not really, the professor is nice. Um, we, I mean, I haven't learned a whole lot of actual strategies in it." Mbachie said, "I do not have a good experience of it since I had never taken one." Eneka said, "I have not done any mathematics methods course, but it makes no difference to me." Overall, mathematics methods course did not have any meaningful help to the teachers for effective teaching of mathematics.

The four participants were asked to explain any content that represented mathematical ideas, exploring errors in student thinking, examining unusual approaches students might use or providing mathematical explanations to students to investigate the influence the course had on MKT. The mathematics strategies course seemed to lack activities that devoted MKT.

Yochie said, “We were introduced to different strategies but, when to use those strategies was not really expressed." Even with a lack of evidence in the mathematics strategies course, participants were able to identify content from another course that they did positively influence their MKT development. For example, Yochie shared, "I felt like my, my Mathematics for Teachers course taught me more about methods than this one." The Mathematics for Teachers course was taught within the mathematics department with multiple instructors.

Insight 2: Current secondary mathematics teachers perceived that the content and instruction of a Mathematics for Teachers course had a positive influence on their development of MKT.

Before investigating how participants perceived the mathematics methods course contributed to their effective teaching, each interviewee was asked to share their other mathematics experiences while in college. All of the four interview participants expressed that a Mathematics for Teachers course was a transformative experience for them as future teachers. Mbachie and Yochie revealed that the efforts of the professor were instrumental in growing their confidence to teach mathematics. Mbachie stated:

> In the Mathematics for Teachers class like by far, that was one of my favorite classes. Yeah, it was because it was Professor X teaching and she is, I mean, phenomenal. Like I've never enjoyed, I mean, it took me all the way to get to college, until I enjoyed math class and it was because of her. She made the incredibly enjoyable; I don't even know how to explain it, she was wonderful.

Yochie shared:

> My Mathematics for Teachers course was one of the best mathematics classes I've ever taken. And the professor was awesome because I wouldn't consider myself amazing at mathematics, and she really gave her students that confidence that they knew what they were doing, and she knew what she was teaching in order for them to be successful.

The comments of interviewees Mbachie and Yochie suggested that the actions of their instructor had a positive influence on their confidence to effectively teach mathematics. When asked to expand on how the Mathematics for Teachers course prepared them to teach, Mackie explained that "In the Mathematics for Teachers course, they did model how we could teach mathematical concepts to students, which was very beneficial." Mbachie asserted that the course helped to develop a deeper understanding of content by engaging them in solving mathematical problems while considering the thoughts of secondary-aged students. Mbachie's remarks suggested that the content and instruction of the Mathematics for Teachers course included activities that aligned with MKT as they were exposed to common errors, student thinking, and how to use mathematical strategies.

Silverman and Thompson (2008) suggested that teachers should practice decentering to develop an understanding of how their learners may approach mathematics. However, the interviewee mathematics teachers recognized that the efforts of the instructors in the Mathematics for Teachers course. The interviewee mathematics teachers recommended this course because the course was able to model strategies to teach mathematics and investigate student thinking were related to the MKT components. This was what the dissertation research aimed at studying: how mathematics methods courses influenced the development of MKT for effective teaching.

Insight 3: Current secondary mathematics teachers express unpreparedness to teach mathematics and expressed a desire for additional training.

The interviewee mathematics teachers shared varying degrees of unpreparedness when it came to the teaching of mathematics. Mbachie shared how mathematics had always been a source of struggle when teaching higher mathematics like Algebra II, Precalculus, AP Mathematics, and Calculus. "I'm not too concerned with my math knowledge because I know I'm willing to learn and I'm willing to put the thought and the work into planning and looking for all those questions students are going to ask." Similarly, Yochie and Mackie shared a desire to help their future students to build confidence.

Regarding ways in which they could improve their teaching effectiveness, the interviewees expressed a concern that they were unprepared to teach mathematics and suggested that the education curriculum included more mathematics methods courses. For example, Yochie stated, "I wish that we actually learned more strategies from our professor and how we could incorporate them into the classroom." Mackie said, "I don't feel like I learned enough of what I needed to learn, to be in the classroom."

Mbachie said that he did not take any mathematics methods course that could have helped him to learn how to teach mathematics from teacher preparation colleges. He learned how to teach mathematics mainly from colleagues at the schools where he has been teaching. Eneka strongly emphasized that observing experienced mathematics teachers was the best way to learn how to teach mathematics. He observed the expert teachers when he was on field experience. He also observed fellow teachers at the schools where he was teaching.

With the comments, the content and instruction in the mathematics methods courses did not contribute to the effective teaching of mathematics.

Insight 4: Current secondary mathematics teachers expressed that there were few connections between secondary mathematics and college mathematics for effective teaching of mathematics.

The interviewees expressed varying degrees about the connections of secondary mathematics and colleges mathematics. Mbachie stated that he saw very few connections between college mathematics and secondary mathematics because he could not see any connections when he was taking the courses in a university. Since he took no methods course, it was far for him to see some connections. Eneka expressed that he saw some connections between calculus and high school mathematics. He mentioned topics such as rates of change and sequences. He also mentioned some topics sequences and series, polynomials, and functions. These topics are found in Algebra II in high school and college algebra in college mathematics.

Yochie and Mackie also expressed to see some connections just as Eneka had seen connections between the secondary mathematics and college mathematics, but they did not see any connections between secondary mathematics and mathematics methods courses. Mackie elaborated that he never saw any strategies that were targeting how to teach secondary mathematics (6-12) in the mathematics methods courses. Yochie explained, "I thought I would be taught some secondary mathematics so that I am familiar with what I would be teaching." In general, the mathematics teachers should be familiar with secondary mathematics curriculum to teach mathematics effectively. Secondary mathematics content should be weaved together with mathematics methods courses for mathematics to make sense to the teachers. The interviewee secondary mathematics teachers explained that there were little connections between secondary mathematics and college mathematics.

Integration of Quantitative and Qualitative Findings

Inferential analysis of the mathematics methods course showed that MKT levels of current secondary mathematics teachers did not change over the course of the teacher preparation. Descriptive statistics conveyed the message that the mathematics methods course did not contribute to the effectiveness of teaching mathematics components that would support their MKT.

Analysis of responses of the interviewee teachers revealed that the interviewees expressed that the mathematics methods course did not contribute to their effective teaching of mathematics since the course content did not align with the MKT components. The interviewees shared that the course incorporated little to no opportunities to interact with student thinking or how to sequence learning experiences to advance student learning. Based on the interviews, it seems that lack of content focused on the MKT related to students and teaching may have contributed to the lack of long-lasting impact of mathematics teaching.

The quantitative analysis showed the frequencies and percentages of the teachers who were teaching lower-level secondary mathematics. The analysis of interviews revealed that lack of adequate preparation made the teachers to opt for lower-level mathematics teaching. The secondary school teachers expressed the need for further preparation to make them well prepared to teach all levels of secondary school mathematics.

Improving the interviews

The interview questions allowed the researcher to elicit information that helped to better understand how to improve the questionnaire as well as understand the possible reasons behind the questionnaire findings. The interviews would have garnered additional qualitative data if I had been able to conduct the interviews in person. I found out that it took several minutes to relax and read the questions in a natural way. The participants may also have been relaxed if the interview was done in person.

Although the number of participants who expressed interest in the interviews was 10, I ended up interviewing four participants. This was due to COVID-19 related issues of quarantine. The timing of the interview request might have contributed to the low response rate as I sent the request to the teachers who just finished the school year and were on vacation. This was a time of the year for anyone involved in education to rest before school started again.

Summary of the Findings

Data analysis from this study clearly indicated that current mathematics teachers rated MKT and appropriate methods of teaching as very important. The current secondary mathematics teachers also lowly rated the performance of mathematics methods courses in the preparation of mathematics teachers. The results could be influenced by the time since the teachers took the mathematics methods courses. This would make the teachers to forget what they learned in the mathematics methods courses. Some of the teachers did not take mathematics methods courses; hence, it was possible for the mathematics teachers to lowly rate the performance of the mathematics methods courses.

The research questions were (a) To what extent did current mathematics teachers perceive that mathematics methods courses influenced their MKT? (b) In what ways did current secondary mathematics teachers perceive that content and instruction in the mathematics methods courses contributed to their effective teaching of mathematics? For the first question the study showed that the mathematics methods course did not influence the current secondary mathematics teachers' MKT.

For the second question, the findings indicated that the content and instruction in the mathematics methods course did not contribute to the mathematics teachers' effectiveness of teaching. The results presented above indicated that the current mathematics teachers in the study were teaching mathematics with less coursework than what they would have taken in the teacher preparation. A more detailed summary and a discussion of the findings are presented in the Chapter Five.

CHAPTER 5

Summary and Discussion

This chapter of the dissertation is composed of two sections. The first section reviews the problem statement, explains the methodology used in the study, and summarizes the results of Chapter Four. The second section presents the discussion of the results. It discusses the interpretation of the results, implications of the study to educators, and the recommendations for further research. Finally, this section presents the limitations of the study and summarizes the results of Chapter Five.

Summary of the Problem Statement

As outlined in Chapter One, the purpose of this dissertation was to examine the perceptions of current secondary mathematics teachers about how their mathematics methods courses influenced their MKT for effective teaching of mathematics. The study also investigated how content and instruction of mathematics methods courses contributed to the MKT for effective mathematics teaching. Ball et al. (2008) pointed out that the use of MKT described secondary mathematics teachers' ability to select appropriate tasks, to anticipate errors students make, and to design instruction to advance teaching and learning of mathematics. This could explain mathematics for easy understanding, hence the need for MKT in teacher preparation.

The researcher also examined areas of teacher preparation that could influence the secondary mathematics (6-12) teachers' MKT for effective teaching of mathematics. This was done to accrue evidence that would support decisions in secondary mathematics teacher preparation and professional development. The study came at a time of rising importance for the preparation of secondary mathematics teachers, and at a time when there were high shortages of qualified mathematics teachers in the USA (NRC, 2012; Lai, 2019).

While deep content knowledge helped to teach mathematics, it was easy to find exceptions to have strong teachers who lacked profound content knowledge or those strong in content who were not strong in content presentation (Davis & Brown, 2009). Wasserman and Ham (2013) stated that understanding MKT was about knowledge of practical ways of teaching, investigating, and presenting mathematics. This study was meant to examine the perceptions of secondary mathematics teachers about how mathematics methods courses influenced the development of their MKT.

Review of the Methodology

The study employed a sequential explanatory mixed-methods study for data collection and analysis. The study used a survey instrument to collect data during the quantitative phase of the study. The study used frequencies, percentages, contingency tables, and Chi-Square statistics for quantitative data analysis. For all the quantitative data analyses, the researcher used SPSS version 28.

The study used semi-structured interviews to collect qualitative data, which had a challenge due to COVID-19, and only four mathematics teachers were interviewed. The qualitative analyses used categorization, coding, and conceptualization as in Miles and Haberman (1994), which was the same as thematic analysis described in Leavy (2017). The mixed-methods study was meant to provide more comprehensive evidence for studying the research problem than either a quantitative or qualitative method alone (Creswell, 2014). Creswell (2014) argued that a mixed-methods study was desirable to explain information that numbers could not explain; hence, the qualitative part of the study explained the findings of the quantitative part of the dissertation.

A Summary of the Results

As discussed in Chapter Four, the total sample included in the study was 27 valid responses after excluding incomplete surveys and ineligible participants. The sample was diverse relative to years of teaching experience, licensure type, license pathways, degree majors, and academic qualifications. The research questions guided a summary of the results. The overview starts with the first research question about the quantitative part of the study.

Research Question 1: To what extent did current secondary mathematics teachers perceive that mathematics methods course(s) influenced their MKT?

The extant literature (Ball et al., 2008; Hine, 2019; Shulman, 1986, 1987; Wasserman et al., 2018; Lai, 2019) indicated that most mathematics teachers started teaching mathematics with inadequate preparation and inadequate MKT. Mathematics methods courses were integrated with mathematics teaching preparation to assist in the development of MKT. This research examined how the secondary mathematics (6-12) teachers perceived their preparation for teaching mathematics about how mathematics methods courses influenced MKT during teacher preparation.

With reference to Table 15, 7.4 percent of the secondary mathematics teachers were prepared to teach mathematics to a large extent. The findings suggested that even when some of the mathematics teachers were confident in their abilities to teach mathematics, they were not consistently confident in their abilities to support students learning mathematics. These findings suggested that the mathematics teachers would need support to maximize student learning (Murray et al., 2013**).**

As noted in the demographic information in Table 16, the distribution of frequencies and percentages showed that it was essential for secondary mathematics teachers to have a sound knowledge of MKT with 96.3 percent. About 85.2 percent of the mathematics teachers indicated that appropriate methods of teaching were very important or extremely important. In Table 17, the finding indicated that 29.6 percent of the secondary mathematics teachers were not prepared at all or prepared to a small extent though they were teaching mathematics. The NRC (2010) pointed out a deep concern about the perceived shortages of highly qualified mathematics teachers in the United States.

The findings suggested that while secondary mathematics teachers appreciated the importance to have MKT and appropriate methods of teaching mathematics, they did not acquire much of the needed MKT and proper methods of teaching during their teacher preparation programs. Some secondary mathematics teachers were licensed through alternative pathways where there were no structured mathematics methods courses to help them to learn how to teach mathematics. In other circumstances, such as the other two interviewee secondary mathematics teachers, mathematics methods courses did not focus on the development of MKT (Masingila, 2012). Hence, there was need for improved standards of teaching mathematics.

The results from Table 18 indicate that 74.1 percent of the secondary mathematics teachers were prepared to teach general mathematics, 69.2 percent of the secondary mathematics teachers were prepared to teach Algebra I, 61.5 percent of the secondary mathematics teachers were prepared to teach Algebra II, 53.8 percent of the secondary mathematics teachers were ready to teach Geometry, and the percentages decreased for other higher-level mathematics such as AP Mathematics, Precalculus, and Calculus. The findings indicated that secondary mathematics teachers were well-prepared to teach lower-level mathematics as opposed to higher-level mathematics. Hine (2018) found a similar results where he interviewed 20 pre-service teachers. He noted that 85 percent of the mathematics teachers were comfortable teaching lower-level mathematics in a junior secondary school, unlike a senior secondary school.

Table 18 shows that most of the secondary mathematics teachers were competent to teach lower-level mathematics subjects. The table shows higher percentages of the teachers that were not prepared to teacher higher level mathematics subjects such that AP Mathematics and Calculus. In Table 19, a cross-tabulation of the responses on readiness to teach mathematics subjects and readiness to teach mathematics showed that the percentages decreased. This agreed with the results of the extant literature that secondary mathematics started teaching with insufficient knowledge of MKT to teach mathematics (Ball et al., 2008; CBMS, 2012, Morris, 2009; Lai, 2019). This implied that the secondary mathematics teachers might need more training or professional development to maximize student learning.

The results in Table 20 show that mathematics methods instructors influenced teachers' MKT with 57.7 percent. Colleagues influenced MKT with 96.1 percent. Independent learning influenced MKT with 88.5 percent while professional development influenced MKT with 65.4 percent. A cross-tabulation of the results with readiness to teach mathematics variable yielded similar results with mathematics education courses with mathematics methods courses having the lowest percentage of influence on MKT. Two of the secondary mathematics teachers, who took mathematics methods courses, explained the results when they were interviewed. They expressed dissatisfaction with the mathematics methods instructors that taught the mathematics methods courses. They seemed to know the mathematics methods content, but they expressed some doubts about teaching some topics in the mathematics methods course. Masingila (2012) pointed out that expert instructors should be teaching mathematics methods courses to improve the quality of mathematics teacher preparation.

Professional development was also examined to check how it influenced MKT when the mathematics teachers taught in their respective schools. Table 23 shows very high percentages of secondary mathematics teachers participating in professional development activities. A cross-tabulation analysis with the variable readiness to teach mathematics shows lower percentages of secondary mathematics teachers who participated in the activities. These results showed that secondary mathematics teachers needed additional professional development that could focus on the development of MKT.

Because of the results from descriptive analysis and cross-tabulation, the researcher conducted more analyses to understand better how mathematics methods courses influence the development of MKT for effective mathematics teaching. Contingency tables were created, and Chi-Square analysis were conducted to better understand how mathematics methods courses affected MKT. Some new categorical variables were created to assist in creating contingency tables and Chi-Square analyses. The new categorical variables were readiness to teach mathematics RTTrc, the importance of mathematics methods courses (IMMCrc), importance of mathematics preparation programs (IMPPrc), importance of mathematics methods instructors (IMMIrc), and importance of appropriate methods of teaching mathematics (IAMTMrc). The variables were created as explained in Chapter Three. These variables assisted in the computations of the Chi-Square statistical analysis because they increased the number of variables.

When asked about readiness to teach mathematics subjects, the categorical variable (RTTrc) indicated varying degrees of preparedness to teach mathematics subjects from 29.1% for Calculus to 74.1% for General Mathematics. When the researcher examined the sum of their readiness to teach mathematics scores, just over half or 55.6% reported that they were well-prepared to teach mathematics, as indicated in Table 24. However, when examining the relationship between teachers' readiness to teach mathematics responses subjects and teaching mathematics, a more complex story emerged that needed further analysis. After conducting a cross-tabulation analysis between readiness to teach mathematics by subject and readiness to teach mathematics, the researcher noted that the readiness to teach mathematics responses decreased in all the mathematics subjects. The researcher estimated a Chi-Square statistical analysis to test the relationship between total readiness to teach by mathematics subject responses and the readiness to teach mathematics responses. The Chi-Square value was $\chi^2 = 0.983$ and the corresponding p-value was 0.357 The p-value was more than 0.05, which suggested that, for this sample, there was no significant relationship between the mathematics teachers' readiness to teach mathematics subjects and the readiness to teach mathematics.

The data and supporting literature suggested that even if some mathematics teachers were confident in their abilities to teach mathematics, they were not consistently confident in teaching all the mathematics subjects to best support students learning (Hine, 2019; Chai et al., 2017). The finding suggested that mathematics teachers would need support to effectively teach mathematics to maximize student learning (Cuhadar, 2018). The interviewee secondary mathematics teachers acknowledged their lack of exposure to training in some professional development activities. There was always support from colleagues as indicated in Table 21 that 96.1% of the colleagues supported the teachers that needed assistance. In the next section, the researcher summarizes the findings of the second question dealing with qualitative phase of the study.

Research Question 2. In what ways did current secondary mathematics teachers perceive that content and instruction in the mathematics methods course(s) contributed to the effective teaching of mathematics?

In the qualitative phase of the study, the researcher interviewed four secondary mathematics teachers who volunteered during the survey that took place in the quantitative part of the study. Of the four secondary mathematics teachers, Mbachie and Eneka did not take a mathematics methods course during their teacher preparation while Yochie and Mackie took one mathematics methods course each during their teacher preparation. Mbachie, one of the interviewee secondary mathematics teachers, said that content and instruction in mathematics methods course did not significantly contribute MKT to him because he did not take any methods course during teacher preparation. He mentioned that the Mathematics for Teachers course that he took with the Department of Mathematics contributed MKT for effective teaching of mathematics. The other three interviewee secondary mathematics teachers also took the Mathematics for Teachers course at their universities contributing MKT more than the mathematics methods course that Yochi and Mackie took. Sullivan (2018) found out that special mathematics courses usually called capstone courses contribute to the development of MKT for teaching. These courses were offered in the final year of their teacher preparation.

While researchers have found out that mathematics teachers require preparation before they could start teaching (Darling-Hammond, 2000; Cochran-Smith et al., 2015; Shulman, 1986), Eneka, one of the interviewee secondary mathematics teachers, stated that mathematics methods were not required for effective teaching of mathematics. Hine (2019) and Masingila (2012) found out that MKT was required to teach mathematics effectively. Mathematics educators should teach mathematics methods courses to focus on the development of MKT. Alnord (2021) also argued that mathematics teachers should be specialized instructors in mathematics teacher education with a robust understanding of MKT development. Lai (2019) supported the idea that instructors teaching mathematics methods courses could have a well-developed knowledge of MKT to focus on MKT during their teacher preparation.

The secondary mathematics teachers indicated that MKT was essential for effective teaching of mathematics. This study found out that mathematics methods courses did not influence the development of MKT. The findings indicated that the content and instruction of mathematics methods courses did not significantly contribute to the effective teaching of mathematics. Therefore, teacher preparation programs should strive to prepare secondary mathematics teachers with a strong knowledge of MKT for effective teaching of mathematics.

Discussion of the Results

This sequential explanatory case study aimed to examine how mathematics methods courses influenced MKT for effective teaching. This study was conducted to bring an understanding of how mathematics teachers develop MKT when they were enrolled in mathematics methods courses. The development of MKT is essential in the teaching and learning of mathematics since mathematics become knowledge with the content of mathematics. In this section, the researcher presents a discussion of the results of the study.

Interpretations of the Results

The theoretical framework of the study was Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008; Hill et al., 2008; Shulman, 1986; Silverman & Thompson, 2008). This model comprised two main domains: subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Subject matter knowledge comprised teachers' knowledge of content within the course, how this content was connected to previous and future topics, and how the content was interpreted to assist the teaching of mathematics. Pedagogical content knowledge comprised teachers' knowledge of the content in relation to teaching practices, student learning, and the curriculum. Silverman and Thompson (2008) asserted that MKT developed when teachers connected content knowledge with pedagogical knowledge to create a new understanding of mathematics that supported students' teaching and learning of mathematics.

The framework had been widely used in mathematics education. Researchers argued that content knowledge without pedagogical content knowledge failed to support quality learning (Ball & Bass, 2011; Baki & Arslan, 2016; Bartell et al., 2013). The theoretical framework was used in this study because extensive evidence existed that there was positive influence of teachers' MKT on student achievement (Baki & Arslam, 2016; Shirvani, 2015). Baki and Arslan (2016) revealed that lack of MKT negatively affected classroom practices of the teacher. Ojose (2014) and Tajudin (2014) showed that if teachers had a deficit of mathematical knowledge, they were more likely to rely on teaching mathematics through routine procedures and could fail to develop a conceptual understanding of mathematics in their students. Hence, it is worthwhile for mathematics teachers to have the required mathematical knowledge for teaching mathematics.

While the quantitative part of the study showed that mathematics methods courses did not influence MKT when the mathematics teachers were enrolled in the courses, the mathematics teachers overwhelmingly showed that a solid knowledge of MKT was needed for effective teaching of mathematics with 20 respondents saying that it was very important as shown in Table 17. The researcher showed that students from teachers with higher MKT levels and stronger content and pedagogical knowledge outperformed students in classrooms with teachers having lower levels of MKT (Akbar & Sehrich, 2018; Baki & Arslan, 2016; Hehlol et al., 2018; Strand & Mills, 2014). This highlighted the reason for conducting this study—to examine and understand how mathematics methods courses influenced the development of MKT.

The results of the contingency tables and Chi-Square statistical analysis showed that there was no significant relationship between mathematics methods courses and readiness to teach mathematics RTTrc. MKT was crucial in developing the knowledge of content and teaching (KCT) responsible for using instructional strategies. Teachers who lacked MKT might struggle to plan effective mathematics lessons (Linder & Simpson, 2017). They might not have the ability to adjust content and instruction to meet the needs of their students (Lui & Bonner, 2016). Mathematics methods must influence teachers MKT because the ability to understand mathematics content, student thinking, and make appropriate instructional changes related directly to the knowledge of curriculum and students and knowledge of content and teaching in the MKT model (Ball et al., 2008; Hill and Chin, 2018).

In the qualitative phase of the study, there were four insights that emerged out after analysis of the interviews. One of the insights was that current secondary mathematics teachers perceived that content and instruction in mathematics methods courses had a minimal contribution to the teachers' effective teaching. The finding was essential to improve content and instruction in mathematics methods courses for effective teaching of mathematics. Ball et al. (2008) stated that mathematics teachers must have unique knowledge of MKT to identify, to design, and to sequence learning tasks that allowed students to engage in mathematical knowledge actively. Olson (2013) stated that teachers' perceptions of the teaching and learning process influenced how they engaged students in mathematical learning. Research had shown that MKT influenced the teaching of mathematics (Sullivan, 2019; Ball & Bass, 2005; Superfine & Li, 2014). Shulman (1987) stated that pedagogical knowledge was one of the knowledge bases that must be attained in the mathematics methods courses during teacher preparation. Franke et al. (2015) identified the challenges that students struggled with when engaging in mathematical ideas. Research had shown that MKT was instrumental when teaching mathematics; hence, a teacher's depth in MKT might limit their ability to engage students actively when teaching mathematics.

Another insight from the semi-structured interviews was that current mathematics teachers perceived that the content and instruction of the mathematics for teachers' course positively influenced MKT. All the four-interviewee secondary mathematic teachers spoke about the Mathematics for Teachers, which was offered in the Department of Mathematics. The insight highlighted that mathematics teacher preparation required mathematics courses specifically designed for teachers. NRC (2010) stated that "mathematics teachers need specific preparation for the challenge of teaching mathematics in ways that engage all students" (p. 104). Steele and Hillen (2012) pointed out that there is need for thoughtful integration of mathematics subject matter in mathematics methods courses to develop effective classroom procedures. Murray et al. (2013) stated that secondary mathematics teachers need deep knowledge of mathematics to be effective in their teaching. Mathematics content courses designed specifically for mathematics teachers provide an avenue to explore mathematics content to address the development of MKT. Holm and Kajander (2012) stated that special content courses have shown to raise MKT as indicated by the four mathematics teachers in semi-structured interviews that a course ‘mathematics for teachers’ stimulated their MKT. Holm et al. (2016) strongly advocated for the special content courses to support MKT development to best maximize the teaching and learning of mathematics.

The third insight was that current mathematics teachers expressed unpreparedness to teach higher-level mathematics subjects such as AP Mathematics, Precalculus, and Calculus, and then they expressed a desire for additional training to teach higher level mathematics. The results in Table 16 in the quantitative part of the study indicated that 81.5 percent of the mathematics teachers were prepared to a smaller extent or not prepared at all. The results in the contingency table and Chi-Square analyses also showed that there was no significant relationship the mathematics methods courses and readiness to teach mathematics RTTrc. The Chi-Square values was 0.983, and the p-value was 0.137. This implied that mathematics methods courses did not influence MKT for the secondary mathematics teachers. Hine and Thai (2019) found a similar result when studied 20 mathematics teachers who showed unpreparedness to teach upper-level mathematics courses, but they expressed readiness to teach lower-level mathematics.

The last insight was that the current secondary mathematics teachers found out that there are no connections among college mathematics, mathematics methods courses, and secondary school mathematics. That was what the interviewee secondary mathematics teachers said. A report by CBMS (2012) emphasized the importance of making connections between the mathematics undergraduates were learning and the school mathematics they would teach in secondary schools. Wasserman (2018) stated that the content of secondary mathematics ought to inform how undergraduate mathematics courses would be taught. That is, undergraduate mathematics for teacher preparation should focus on mathematics that secondary mathematics teachers would use to teach in secondary school. Speer et al. (2015) found that secondary mathematics teachers did not have sufficiently deep understanding of the mathematics in the secondary school curriculum. This showed that there was need to focus on secondary school mathematics during teacher preparation.

The results in this study showed that secondary mathematics teachers did not develop the required MKT, and therefore, they were not well-prepared to teach mathematics as indicated in Table 16. This is because most often mathematics teachers did not learn mathematics that they need in teacher preparation programs or on the job training (Kessel, 2009). There was need for strong connections between mathematics methods courses and secondary mathematics to produce secondary mathematics teachers with a strong background of MKT.

CBMS (2012) pointed out that “all teachers of mathematics need to be able to detect flaws in students’ arguments and help students the nature of those errors” (p.1). Wasserman (2017) also commented that mathematics methods courses formed a bridge between college mathematics and secondary mathematics. Hence, mathematics methods courses must be well-developed to support secondary mathematics teaching.

The results of the qualitative part of the study also show that the content and instruction in mathematics methods courses did not contribute to the effective teaching of mathematics because of the disconnections that existed between college mathematics and secondary mathematics. These disconnections of secondary mathematics and college mathematics hindered the development of MKT. Coffland and Xie (2015) stated the disconnections were in three ways (a) school mathematics was disconnected from real life (b) each mathematics course was disconnected from other courses in the mathematics curriculum (c) school mathematics was divorced from other subjects in the secondary curriculum. As such, the NCTM (2000) noted that students needed to understand mathematical ideas to apply them to topics in science and technology. The results in this study showed that the content and instruction in mathematics methods courses did not contribute to the development of MKT for effective teaching of mathematics. In the next section, the researcher presents the implications of the study to mathematics educators.

Recommendations for Teacher Educators

Based on the findings in this study, secondary mathematics methods courses did not influence MKT on the secondary mathematics teachers. Research had shown that teacher preparation fell short of developing mathematical pedagogical knowledge (NCTM, 2014). Therefore, teacher educators must seek to prepare secondary mathematics teachers to teach mathematics through the development of MKT. The integration of mathematics content and mathematics pedagogy into all forms of mathematics was essential for teacher preparation program.

Research has indicated that integration of mathematics methods courses and content mathematics influenced the development of MKT (Depaepe et al., (2015). This was supported because there was an exploration that blending the content and instructional practice improved MKT (Hoover et al., 2016; Auslander et al., 2016). Fernandez (2014) pointed out that mathematics teacher programs must have a clear plan on how to build MKT. As such, the best method through which mathematics teacher preparation programs could address MKT was through a combination of content, special content, and mathematics methods courses.

The researcher suggested a proposal for a mathematics education teacher preparation program in view of what research has pointed and the findings of this study. A teacher preparation program should consist of three domains: The first would be the undergraduate mathematics content courses just as they would be offered in many undergraduate mathematics programs. Wasserman et al. (2018) called them advanced mathematics courses for the upper-level mathematics courses. The second would be the mathematics special content courses that would be specifically for teachers (Stockton & Wasserman, 2018). These courses must combine high school mathematics and undergraduate mathematics to bridge the gap that existed between high school mathematics and college mathematics. The third would be mathematics methods courses dealing with how to teach mathematics in secondary schools.

The set of the courses should have connections from one domain to the other. The special mathematics courses should be offered starting from second year of their study as soon as the pre-service teachers have started doing the undergraduate mathematics. The mathematics teachers should be doing one course from each domain every semester or every other semester. The secondary mathematics teachers must provide students with a reformulation of mathematics of the mathematics that students would encounter later in the study (Heid et al., 2016). Table 31 summarizes the information about the mathematics domains listing the courses in each domain that would be used to develop MKT during teacher preparation.

Table 29

Framework of Mathematics Courses in the Teacher Preparation Programs

Mathematics Content Courses	Special Mathematics Content Courses	Mathematics Methods Courses
Undergraduate Mathematics	Mathematics for Secondary Teachers I	Teaching Secondary Mathematics I
Undergraduate Mathematics	Mathematics for Secondary Teachers II	Teaching Secondary Mathematics II
Upper-level Mathematics	Mathematics for Secondary Teachers III	Teaching Secondary Mathematics III

Note: Layout of math courses for secondary mathematics teachers during teacher preparation

Course Outcomes for the Courses of the Framework

The course outcomes would be grouped according to the domains of the courses. The undergraduate mathematics would have their own course outcomes. The special content courses would also have their own course outcomes. Finally, the mathematics methods courses would have their course outcomes.

The undergraduate mathematics courses were laid out to form the background knowledge of college mathematics. The courses were arranged in such a way that one course was built from the previous course for continuity of the teaching and learning process of mathematics. The teaching of undergraduate mathematics would serve the following outcomes (a) to get familiarized with properties of the integers such as prime factorization, divisibility, and congruence (b) to reason abstractly about mathematical structures (c) to recognize and comprehend correct proofs of formal statements and be able to formulate proofs clearly and concisely (d) to gain a working knowledge of important mathematical concepts in mathematics (e) to get introduced to and have knowledge of many mathematical concepts studied in mathematics (f) to comprehend and understand the connection and transition between previously studied mathematics and more advanced mathematics (g) to gain experience and confidence in proving theorems in mathematics (Rudin, 1976).

The special content courses would link undergraduate mathematics and mathematics methods courses. They would provide an avenue for the discussions of secondary school content mathematics for secondary mathematics teachers to understand what they would be teaching. The outcomes of the special mathematics content courses would be as follows: (a) to improve understanding of some of the mathematical concepts which are important in secondary school mathematics.(b) to improve understanding of the nature of mathematics: what is important, how it is practiced, how mathematical validity is determined (c) to improve understanding of the historical development of selected topics from secondary school mathematics (d) to develop a vision of good school mathematics (f) to increase ability to specify subject matter involved in a specific mathematics topic and make distinctions among them (g) to develop proficiency in the secondary mathematics content that would be taught in the secondary mathematics (6-12) (Bremigan et al., 2011).

The mathematics methods courses would be responsible for assisting the teachers to learn how to teach secondary mathematics. This would be made easy with the introduction of the special content courses, which would link undergraduate mathematics and mathematics methods courses. These courses would be made easy since the teachers would be able to know and understand the content from special mathematics content courses. The outcomes of the mathematics methods courses would be as follows: (a) to provide secondary mathematics (6-12) teachers with an understanding and knowledge of mathematics content that is needed for teaching secondary mathematics (b) to provide secondary mathematics teachers with additional content and knowledge to help them become more effective teachers (c) to cover advanced mathematical topics and research on teaching and learning of secondary mathematics (d) to deepen their comprehension of mathematics by studying advanced topics not covered in undergraduate curriculum (e) to develop the dispositions of life-long learners of mathematics (f) to develop an understanding and connections between undergraduate mathematics and secondary mathematics. (g) to improve understanding of various teaching strategies and their strengths and weaknesses (h) to increase ability to choose among lessons and curriculum materials based on the intended mathematical subject matter and the current understandings of the students (i) to understand why people learn mathematics and how it could be taught effectively (Brumbaugh & Rock, 2013).

The naming of special mathematics content courses could be varied. It could take the form of Algebra for Teachers, Geometry for Teachers, and Advanced Algebra for Teachers. These courses should bridge the gaps that existed between secondary mathematics and undergraduate mathematics. This set of mathematics courses was being done in most elementary mathematics programs (Ball et al., 2005). In fact, there was more mathematics in secondary school (6-12) than elementary school. Also, there was more advanced mathematics in secondary school than elementary school. Therefore, secondary mathematics preparation programs needed more mathematics courses than did elementary schools.

The set-up of mathematics courses would be done like that because many mathematics teachers found their mathematics preparation disconnected from their mathematics teaching

(Zazkis & Leikjin, 2010). Many mathematics teachers did not see connections between advanced courses and secondary mathematics (Ticknor, 2012; Wasserman et al., 2018). Mathematics courses were not applicable to the teaching of mathematics and that MKT was not represented much in mathematics courses intended for teachers (Lai, 2019).

The set-up of the mathematics courses was meant for the teachers to understand secondary school mathematics content, to understand methods of how to teach secondary school mathematics, and to understand the relationship between mathematics content and the methods applied to teach the content. Teachers must have a perspective on trajectory and growth of mathematical ideas beyond secondary school algebra (Mc Grory et al., 2012). The set-up of the courses would develop books that would be used for teaching mathematics because the current books weakly support teaching for development of MKT in secondary teacher education and most tasks focused on pure mathematics (Lai & Patterson, 2017).

The researcher noted that secondary mathematics, mathematics methods courses, and undergraduate mathematics were disconnected (CBMS, 2012; Lain, 2019). The course framework has been designed to connect all the mathematics that are needed for teacher preparation. The framework has also been designed to make the courses easily understood because of the connections. Wasserman et al. (2017) supported the idea of a bridge that such a layout creates a bring from undergraduate mathematics through special content courses to mathematics methods courses. Mathematics courses must be connected to improve the quality of teacher preparation. Mathematics should be connected to recognize and apply mathematics in contexts outside of mathematics (NCTM, 2000). Connections clear out misconceptions that students might have when they come to school. If this started in teacher preparation, teachers would recognize and use the connections among mathematical ideas to understand the concepts and use them when teaching secondary mathematics (NRC, 2010). So, connected coursework is powerful in the teaching of mathematics.

Recommendations for Further Research

This research found out that mathematics methods courses did not influence the development of MKT. Of particular interest were secondary mathematics teachers without a mathematics methods course preparation who made up a small percentage of the respondents in this study. They were unique in many ways from the larger population of secondary mathematics teachers. Future studies could specifically seek out these individuals to better understand their professional strengths and needs in the context of mathematical knowledge for teaching (MKT). Further research must be conducted to find out how teaching mathematics would be done without mathematics methods courses.

In the semi-structured interviews, one of the mathematics teachers pointed out that the best way to learn how to teach mathematics was observing expert teachers teaching. In this context, further research could be conducted into the role of field experiences in the preparation of mathematics teachers. Jackson et al. (2018) stated that prolonged field experiences had a positive experience on MKT, confidence, and understanding the perceptions of struggling students. More research must be conducted to fin d out how field experience how field experience would be done without the Mathematics Methods Courses.

It would also be important to gather demographic information on the secondary schools and universities to find out how they could develop MKT to the teachers. In this way, research could be done to gain a deeper understanding of MKT at school level to support learning of mathematics. As the results have shown, secondary mathematics teachers overwhelmingly agreed that MKT was a significant factor in mathematics teacher preparation. Therefore, further research to examine mathematics teacher preparation programs on the development MKT was suggested.

Limitations of the Study

This study had limitations that minimized the generalizability of the findings. First, the interviewee secondary mathematics teachers participated in the semi-structured interviews voluntarily. The factors that influenced voluntary participation are unknown. Hence, it might be possible that those who chose to participate in the interviews were secondary mathematics teachers with stronger dispositions toward sharing professional knowledge. As such, this might have affected their responses to the interview questions.

A second limitation in this study was that the secondary mathematics teachers self-reported the responses to the questions on the questionnaire. While it was assumed that respondents were truthful and spent the time to think deeply about each item on the questionnaire, these conditions could not be confirmed. As such, it might be one way that could have altered the responses.

A third limitation to this research was the fact that the study drew participants from a population of secondary mathematics teachers in the southwestern state of the United States. As a result, the findings reflected to the secondary mathematics teachers in a southwestern state. Therefore, the findings may not be applicable in other regional contexts.

A Summary of the Results

The dissertation examined current SMTs perceptions about how mathematics methods courses influenced the teachers' MKT. The study also investigated the ways in which content and instruction in mathematics methods courses contributed to effective teaching of mathematics. The researcher illustrated the power of MKT in the teaching and learning of mathematics. The results of the study revealed that current SMTs perceived that mathematics methods courses did not influence their MKT. The dissertation research also revealed that the content and instruction in mathematics methods courses had minimal contribution to the effective teaching of mathematics. The dissertation research revealed that mathematics teachers required special content courses that could have a positive influence on their mathematical knowledge for teaching. The dissertation research revealed that mathematics teachers required more mathematics methods courses to help in the teaching mathematics methods courses. The secondary mathematics teachers were supposed to understand the content that they would teach in the secondary school, and then get inducted on how to teach mathematics. This was very important because content knowledge alone was not sufficient to prepare teachers for teaching mathematics. There was need for coursework in mathematics pedagogy for mathematics teachers to know how to teach mathematics. CBMS (2012) stated that "coursework in mathematics pedagogy assumed to be part of a preparation program, but it not discussed in detail" (p. 39). There was need for a combination of mathematics content and pedagogy called mathematical knowledge for teaching for mathematics teachers.

The dissertation research revealed that secondary mathematics teachers might need more attention during teacher preparations. The researcher provided a perspective in MKT capabilities for mathematics to make sound decisions in the teaching and learning of mathematics. Finally, the research suggested a course framework for teacher preparation to influence MKT for effective teaching of mathematics. The coursework was purposefully arranged to create connections between college mathematics, mathematics for teachers and mathematics methods. This aimed at the development of MKT during teacher preparation to improve the quality of mathematics teacher preparation.

APPENDICES

APPENDIX A In-service Mathematics Survey Recruitment Email

Subject Line: Opportunity for SMTs to Participate in Survey Research on the Teacher Preparation of Mathematics Teachers

Dear Mathematics Teacher,

My name is Peterson C. Moyo. I am a doctoral candidate in the College of Education and Human Sciences (COEHS) at the State University 1. I am conducting a research study on the perceptions of in-service mathematics teachers about their teacher preparation to teach mathematics. You are receiving this email because you are a current in-service middle/high school mathematics teacher in public schools in the southwestern state. You are therefore asked to participate in the study.

The purpose of this research is to gain a better understanding of in-service mathematics teachers' perceptions about their teacher preparation to teach mathematics in middle and high schools. This research has the potential to inform the future teacher preparation and professional development programs of mathematics teachers. While much research has been done on effective teaching of mathematics, preparation of mathematics teacher settings remains an area in need of further study.

Your participation offers the opportunity to share your professional knowledge as a mathematics teacher and your knowledge in mathematics teacher preparation. Participation is open to any current middle and high school mathematics teacher who is teaching mathematics in public schools regardless of teaching experience.

If you agree to participate, the study is a 20-30 minute online survey that asks you to rate the importance of specific teaching practices and teacher preparation. The study has minimal risks for participants and can benefit the future preparation and professional development of mathematics teachers. Survey responses will be kept confidential. Survey completers will have the opportunity to enter a drawing of 20 $25 gift cards drawn every week from the start of the survey until all the gift cards are done.

Your participation in this study is entirely voluntary. If you feel you understand the study and would like to participate, click on the link at the bottom of this email, and you will be directed to the survey website. **(SURVEY LINK HERE)**

If you have questions before participating in the study, please contact Student X or the dissertation chair.

Thank you very much for your time to participate in the study.
Student X, Doctoral Candidate,
Teacher Education, Education Leadership & Policy, State University 1

APPENDIX B

Informed Consent for Online Survey
Perceptions about Current (SMTs) on their Teacher Preparation for Effective Teaching of Mathematics

My name is Peterson C. Moyo, a doctoral candidate in the College of Education and Human Sciences (COEHS) at the State University 1. I am conducting a research study on the perceptions of current mathematics teachers about their teacher preparation for effective teaching of mathematics. You are being asked to participate in this study because you are a current mathematics teacher in the public schools in the southwestern state.

The purpose of this research is to seek a better understanding of in-service mathematics teachers' perceptions of their teacher preparation. This research has the potential to inform the future teacher preparation programs and professional development programs of mathematics teachers. While much research has been done on effective teaching of mathematics, preparation of mathematics teacher settings remains an area in need of further study.

Your participation offers the opportunity to share your professional knowledge as a mathematics teacher and your knowledge of teacher preparation. Participation is open to any current middle and high school mathematics teacher teaching at least part-time in mathematics in public schools.

If you agree to participate, the study is a 20-30 minute online survey that asks you to rate the importance of specific teaching practices. The study has minimal risks for participants and has the potential to benefit the future preparation and professional development of mathematics teachers. Survey responses will be kept confidential. Survey completers will have the opportunity to enter a drawing of 20 $25 gift cards drawn every week from the start of the survey until all the gift cards are done. To enter a drawing, participants will fill out a separate questionnaire asking them for name and email address. Data collected from this study will be collected online and maintained in password-protected software. Once a survey has been submitted, the data will belong to the researcher.

Your participation in this study is entirely voluntary. If you feel you understand the study and would like to participate, go ahead, and start the survey. After you are done with the survey, please submit it.

If you have questions before participating in the study please contact Student X or the dissertation chair. If you have any questions about your rights as a research subject, or about what can do in case of any harm to you, or if you want to obtain information or offer input to the study, you may call the State University 1 Office of the IRB (OIRB) at (505) 277-2644 or irb.unm.edu.

By clicking *Yes* below you will be agreeing to participate in the above-described research study. Thank you for spending your time on this study.

APPENDIX C

SURVEY INSTRUMENT

In this part of the survey questionnaire, you are asked some basic information about where and what you are teaching in public schools in the southwestern state of the United States.

Demographic information

Instructions. Type your answer in the box that you choose to be the correct answer for each of the questions in this section

1. In which year did you graduate from college/university? ☐

2. How many years have you been teaching mathematics in middle or high school?

   In Middle School………………………….. ☐

   In High School…………………………… ☐

3. What grade levels are you teaching at your school? ☐

4. What subjects are you teaching in middle or high school? Check all that apply.

   Algebra I………………………………….. ☐

   Algebra II…………………………………. ☐

   Geometry………………………………….. ☐

   AP Mathematics…………………………… ☐

   Pre-calculus………………………………… ☐

   Calculus …………………………………… ☐

   General mathematics………………………. ☐

   Other (Specify)……………………………. ☐

5. What type of the state teaching licensure do you have?

Secondary Vocational-Technical (7-12)........ ☐

Secondary (7-12)…………………………… ☐

Middle-level (5-9) …………………………… ☐

Elementary (K-8)………………………….. ☐

Special Education (PreK-12)……….……… ☐

Other (Specify)…………………………….. ☐

6. What was your pathway to the state teaching licensure?

Four-year bachelor's degree…………... ☐

Master's degree program……………… ☐

Doctorate ……………………… ☐

Alternative licensure program………... ☐

Other (Specify)……………………….. ☐

7. What is your state licensure level?

Level One Alternative .... ☐

Level One……………........................... ☐

Level One Provisional…………..…... ☐

Level Two……………………...……. ☐

Level Two Professional…………….... ☐

Level Three-A……………………….. ☐

Level Three Instructional Leader……. ☐

Level Three-B………………………… ☐

Other (Specify)…………………..….. ☐

8. What is the highest level of education that you have completed?

   Bachelor's Degree……………………..… ☐

   Master's Degree…….......................... ☐

   Doctorate Degree…………………….. ☐

   Other (Specify) ..…….. …………..…. ☐

9. At what university did you earn your highest degree mentioned in 8?

   ☐

10. What degree did you earn at the end of your four-year studies? (Bachelor of Science in …, Bachelor of Education in …., Bachelor of Arts in ….. or Master of Science in ……)

   ☐

11. What was your major when you were in college/university?

   Applied Mathematics…….. ☐

   Pure Mathematics………… ☐

   Statistics…………………... ☐

   Physics……………………. ☐

   Mathematics Education…... ☐

   General Education……….. ☐

   Other (Specify)…..……….. ☐

Teaching and Learning of Mathematics

In this section, there are several questions about mathematics methods courses. For this study, mathematics methods courses refer to mathematics methods courses that are taught to pre-service mathematics teachers so that they can learn how to teach mathematics usually at the end of their undergraduate study (Lai, 2019). These courses are intended to familiarize teachers with the mathematics that they will teach in middle and high schools. They are intended to promote connections between university mathematics and secondary school mathematics and to strengthen understanding of secondary school mathematics (CBMS, 2012).

12. To what extent do you do the following in the teaching of mathematics to your students?
    Choose one number in each line of the lines

	Not at all	**Small Extent**	**Moderate Extent**	**Large Extent**
Relate lesson to students' daily lives	1	2	3	4
Ask students to explain their answers.	1	2	3	4
Encourage classroom discussions among students.	1	2	3	4
Link new content to students' prior knowledge..…	1	2	3	4

13. How often did you do the following activities as students with your instructor in your mathematics methods course at your university?

Choose one number in each of the lines below

	Rarely	**Often**	**Moderate often**	**Very often**
Explain the reasoning behind an idea	1	2	3	4
Represent and analyze relationships using tables, charts, or graphs…	1	2	3	4
Work on problems for which there is no immediately obvious method of the solution.................................	1	2	3	4
Use word problems to write equations to represent relationships	1	2	3	4

14. Based on your experience as a mathematics teacher in middle or high school, how important is it for mathematics teachers to

Choose one number in each of the lines below

	Not important	**Less important**	**Moderate Important**	**Very important**
Have mathematical knowledge for teaching.	1	2	3	4
Identify appropriate methods for teaching mathematics……….	1	2	3	4

15. To what extent did the mathematics methods courses influence your mathematical knowledge for teaching mathematics in middle or high school?

Not very much…………………..……1
Small Extent……...……………….…2
Moderate Extent…………………..3
Large Extent………………………4

Teacher Preparation for Teaching of Mathematics

16. To what extent did the mathematics methods courses prepare you to teach the following courses?

Choose one number in each of the lines below

	Not at all prepared	**Small Extent**	**Moderate Extent**	**Large Extent**
Algebra I………………..	1	2	3	4
Algebra II……………..…	1	2	3	4
Coordinate Geometry…...	1	2	3	4
AP Mathematics………...	1	2	3	4
Geometry………………..	1	2	3	4
Pre-calculus……………..	1	2	3	4
Calculus…………………	1	2	3	4
General Mathematics……	1	2	3	4

17. To what extent did the mathematics methods courses prepare you to teach mathematics in secondary school mathematics?

    Not at all prepared……….…..1
    Somewhat prepared…………..2
    Well prepared………….…..3
    Very well prepared…………..4

18. In the teaching of mathematics to your students in middle/high school, how would you characterize your confidence in doing the following?

	Very high	**High**	**Medium**	**Low**
Showing students a variety of problem-solving strategies.	4	3	2	1
Helping students appreciate the value of learning mathematics	4	3	2	1
Assessing students' comprehension of mathematics.........	4	3	2	1
Improving the understanding of struggling students……..	4	3	2	1
Making mathematics relevant to students………………..	4	3	2	1
Developing student's higher-order thinking skills……….	4	3	2	1

Effective Teaching of Mathematics

19. To what extent did each of the following help you to acquire mathematical knowledge for teaching mathematics effectively?

Circle one number in each line

	Large Extent	**Moderate Extent**	**Small Extent**	**Not at all**
Methodology course instructors…….	4	3	2	1
Colleagues…………………………...	4	3	2	1
Independent learning……………......	4	3	2	1
Professional development course…...	4	3	2	1

20. How helpful were the content and instruction of the mathematics methods courses in the organization and understanding of mathematics for effective teaching in middle and high schools?

    Not helpful at all………………..1
    Somewhat helpful………………2
    Very helpful…………………….3
    Extremely helpful………………4

21. How well did the teacher education mathematics preparation programs prepare you for effective of mathematics in middle and high school?

    Not at all prepared……….…......1
    Somewhat prepared…………..…2
    Well prepared………………......3
    Very well prepared…………..…4

Professional Development

22. In the past two years, have you participated in professional development in any of the following?

	YES	NO
Mathematics content……………………………….…..	1	0
Mathematics pedagogy……………………….………..	1	0
Improving student critical thinking…….……...........	1	0
Mathematics assessment…………………………....	1	0
Addressing diversity and inclusion in mathematics...	1	0

23. In the past two years, have you taken part in any of the following activities in Mathematics?

	YES	NO
I attended a workshop/conference……………………….	1	0
I gave a presentation at a workshop/conference…………	1	0
I took part in an innovative project for math pedagogy…	1	0

24. How often do you have the following types of interactions with other mathematics teachers at your middle or high school?

	Very often	**Often**	**Sometimes**	**Almost Never**
Discuss how to teach a particular topic…...	4	3	2	1
Collaborate in planning and preparing instructional materials…………….…….....	4	3	2	1
Work together to try out new ideas……….	4	3	2	1
Visit another classroom to learn more about teaching…………………………….	4	3	2	1
Share what you have learned about teaching experiences……………………..	4	3	2	1

25. Would like to be interviewed after you have completed this survey for further information about the teaching and teacher preparation of mathematics teachers in middle and high schools?

YES ☐ **NO**

Thank you very much for completing the survey.

APPENDIX D

Semi-structured Interview Protocol for Current Mathematics Teachers in a southwestern state of the United States

Thank you very much for agreeing to take part in this individual interview. The information gathered from this interview will be used to develop an understanding of how the mathematics methods courses develop mathematical knowledge for teaching that is used to teach secondary mathematics. This interview will be recorded to ensure accuracy. I assure you that all forms of identification will be removed from the data to protect your identity and privacy. At any time in the interview if you do not wish to answer a question or want to discontinue the conversation, feel free to do so. Before the interview is started, do the following:

1. Please review this consent form and keep this form for your records.
2. Do you have any questions before we start recording and begin the interview?

To begin with, I will ask you some general questions about your education, your mathematics background, and your teaching experiences.

1. How long have you been teaching mathematics at your school?
2. What degree did you receive at the end of your university study? Bachelor of Science in …?, Bachelor of Arts in …? etc
3. At what University did you earn your degree mentioned in 2?
4. At what university did you do your teaching licensure?
5. What grade levels are you teaching now?
6. How many mathematics methods courses did you take at the university? Explain.
7. What was the major focus of the mathematics methods course(s) that you took?
8. In what ways do you think the mathematics teacher preparation developed your understanding of mathematical topics to teach mathematics in secondary school?
9. Describe any components of the mathematics methods courses that helped you develop a sense of understanding of how to teach mathematics in public schools.
10. In what ways do you think your mathematics teacher preparation helped you to develop the skills to teach mathematics in secondary school?
11. What components of your mathematics teacher preparation helped you to develop the ability to identify, select, and sequence different teaching strategies?
12. In the teaching of mathematics methods courses, what connections did you see between college mathematics and secondary school mathematics?
13. What areas of the mathematics teacher preparation programs can be improved for effective mathematics teacher preparation for secondary school teachers?

APPENDIX E—CODEBOOK

Dataset	Current secondary school mathematics public teachers' perceptions about their mathematics teacher preparation to acquire MKT for effective teaching of secondary school mathematics
Overview	A study of current secondary mathematics public teachers' perceptions of mathematics public teachers who are teaching mathematics in secondary schools in the southwestern state of the United States.
Source	The data source for this study included the responses to the survey form current secondary mathematics teachers who were teaching mathematics in the southwestern state of the United States.
Sample Size	This survey involved 27 participants drawn from public schools in the southwestern state of the United States.

Col #	**Variable Name**	**Variable Description**	**Variable Metrics/Labels**
1	ID	Respondent ID Code	Integers
2	GRADYR	Graduation Year	1=1971-1980 2=1981-1990 3=1991-2000 4=2001-2010 5=2011-2020
3	TEACHEXP	Teaching Experience	1= 1-5 years 2= 6-10 years 3=11-15 years 4=16-20 years 5=21-25 years 6=25+ years

4	TEACHG6	Teaching Grade 6	$1=6^{th}$ Grade $2=7^{th}$ Grade $3=8^{th}$ Grade $4=9^{th}$ Grade $5=10^{th}$ Grade $6=11^{th}$ Grade $7=12^{th}$ Grade
5	TEACHG7	Teaching Grade 7	$1=6^{th}$ Grade $2=7^{th}$ Grade $3=8^{th}$ Grade $4=9^{th}$ Grade $5=10^{th}$ Grade $6=11^{th}$ Grade $7=12^{th}$ Grade
6	TEACHG8	Teaching Grade 8	$1=6^{th}$ Grade $2=7^{th}$ Grade $3=8^{th}$ Grade $4=9^{th}$ Grade $5=10^{th}$ Grade $6=11^{th}$ Grade $7=12^{th}$ Grade
7	TEACHG9	Teaching Grade 9	$1=6^{th}$ Grade $2=7^{th}$ Grade $3=8^{th}$ Grade $4=9^{th}$ Grade $5=10^{th}$ Grade $6=11^{th}$ Grade $7=12^{th}$ Grade
8	TEACHG10	Teaching Grade 10	$1=6^{th}$ Grade $2=7^{th}$ Grade $3=8^{th}$ Grade $4=9^{th}$ Grade $5=10^{th}$ Grade $6=11^{th}$ Grade $7=12^{th}$ Grade

9	TEACHG11	Teaching Grade 11	1=6th Grade 2=7th Grade 3=8th Grade 4=9th Grade 5=10th Grade 6=11th Grade 7=12th Grade
10	TEACHG12	Teaching Grade 12	1=6th Grade 2=7th Grade 3=8th Grade 4=9th Grade 5=10th Grade 6=11th Grade 7=12th Grade
11	TEACHGM	Teaching General Mathematics	1=General Mathematics 2=Algebra 1 3=Algebra 2 4=Geometry 5=AP Mathematics 6=Precalculus 7=Calculus
12	TEACHALG1	Teaching Algebra 1	1=General Mathematics 2=Algebra 1 3=Algebra 2 4=Geometry 5=AP Mathematics 6=Precalculus 7=Calculus
13	TEACHALG2	Teaching Algebra 2	1=General Mathematics 2=Algebra 1 3=Algebra 2 4=Geometry 5=AP Mathematics 6=Precalculus 7=Calculus

14	TEACHALGEO	Teaching Geometry	1=General Mathematics 2=Algebra 1 3=Algebra 2 4=Geometry 5=AP Mathematics 6=Precalculus 7=Calculus
15	TEACHAPM	Teaching AP Mathematics	1=General Mathematics 2=Algebra 1 3=Algebra 2 4=Geometry 5=AP Mathematics 6=Precalculus 7=Calculus
16	TEACHPRECALC	Teaching Precalculus	1=General Mathematics 2=Algebra 1 3=Algebra 2 4=Geometry 5=AP Mathematics 6=Precalculus 7=Calculus
17	TEACHCALC	Teaching Calculus	1=General Mathematics 2=Algebra 1 3=Algebra 2 4=Geometry 5=AP Mathematics 6=Precalculus 7=Calculus
18	LICTYPE	Type of teaching license	1=Sec Voc Tech (7-12) 2=Secondary (6-12) 3=Middle level (5-9) 4=Elementary (K-8) 5=Special Ed (PreK-12)

19	LICPATH	License Pathway	1=Alternative Route 2=Four-Year Degree 3=Master's Degree 4=Doctorate
19	LICLEVEL	Teaching license level	1=Level 1 2=Level 2 3=Level 3 4=Alternative License
20	HIGHQUAL	Highest Academic Qualification	1=Bachelor's 2=Master's 3Doctorate
20	UNIVDEG	Where university degree earned	1=UNM 2=NMSU 3=Other
21	DEGTYPE	Degree type earned	1=BS Math Education 2=BS Education 3=BA Education 4=BA Math Education 5=MS Mathematics 6=MS Math Education 7=PhD Math Education 8=Other
22	DEGMAJOR	Degree major in college	1=Applied Mathematics 2=General Mathematics 3=Mathematics 4=Math Education 5=Pure Mathematics 6=Physics 7=Statistics 8=Other
23	RELATELTDLIVES	Relate lesson to daily lives	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
24	ENCODIS	Encourage classroom discussions	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent

25	EXPLNANS	Ask students to explain their answers	1=Not at all Done 2=Small Exten 3=Moderate Extent 4=Large Extent
26	LICPRIORKNOW	Link new content to prior knowledge	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
27	MAUSEGRAPHS	Math methods course using charts, tables, or graphs as students	1=Rarely 2=Often 3=Moderate Often 4=Very often
28	IMPOHAVEMKT	Importance of having MKT in mathematics teachers	1=Not important 2=Important 3=Moderate important 4=Very important
29	IMPOAPROPMT	Importance of using appropriate methods of teaching	1=Not important 2=Important 3=Moderate important 4=Very important
30	MMINFMKT	Math methods courses influence MKT	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
31	MMHPTALGI	Math methods prepare teachers to teach Algebra I	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
32	MMHPTALGII	Math methods prepare teachers to teach Algebra II	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
33	MMHPTCGEOM	Math methods prepare teachers to teach coordinate geometry	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
34	MMHPTAPMATH	Math methods prepare teachers to teach AP mathematics	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent

35	MMHPTGEOM	Math methods prepare teachers to teach geometry	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
36	MMHPTCALC	Math methods prepare teachers to teach Precalculus	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
37	MMHPTMATH	Math methods prepare teachers to teach Calculus	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
38	MMHPTGENMATH	Math methods course helps to teach general mathematics	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
39	MMHPTMATH	Math methods course prepare teachers to teach mathematics	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
40	PSHACQMKT	Showing students a variety of problem-solving strategies help teachers to acquire MKT	1=Low 2=Medium 3=High 4=Very high
41	CMHAMKT	Assessing students comprehension of mathematics helps to acquire MKT	1=Low 2=Medium 3=High 4=Very high
42	IUSSIHELPACQMKT	Improving the understanding of struggling students in math helps acquire MKT	1=Low 2=Medium 3=High 4=Very high
43	HOTHELPACQMKT	Developing higher order thinking levels help acquire MKT	1=Low 2=Medium 3=High 4=Very high
44	MMIHPACQMKT	Math methods course instructors help to acquire MKT	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent

45	CHACQMKT	Colleagues help acquire MKT	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
46	ILHACQMKT	Independent learning helps acquire MKT	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
47	PDCHELPACQMKT	Professional development courses help acquire MKT	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
48	CIOMMHTMAT	Helpfulness of content and instruction of math methods courses in developing effective teaching	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
49	TPREPHETEACH	Helpfulness of math preparation programs for effective teaching of mathematics	1=Not at all Done 2=Small Extent 3=Moderate Extent 4=Large Extent
50	TPARTMATHCON	Taken part in activities in mathematics content	0=No 1=Yes
51	TPARTMATHMET	Taken part in activities in mathematics methods	0=No 1=Yes
52	TPARTIMPMCRT	Taken part in activities in improving critical thinking	0=No 1=Yes
53	TPARTMATHASS	Taken part in activities in mathematics assessment	0=No 1=Yes
54	TPARTMATHDIV	Taken part in activities in addressing diversity and inclusion	0=No 1=Yes
55	ATTCONFMATH	Attended a conference in mathematics	0=No 1=Yes
56	GAVEPRES	Gave a presentation in mathematics at a conference	0=No 1=Yes

57	INNOVPROJ	Taken part in innovative project in mathematics education	0=No 1=Yes
58	COLLABPLAN	Collaboration in planning and preparing instructional materials with other math teachers at school	1=Almost never 2=sometimes 3=Often 4=Very often
59	RTTrc	Readiness to teach mathematics	0=Not ready 1=Ready
60	IMMCrc	Importance of math methods courses to influence MKT	0=Disagree 1=Agree
61	IMPPrc	Importance of Math Preparation Programs to influence MKT	0=Disagree 1=Agree
62	IAMTMrc	Importance of appropriate methods of teaching mathematics	0=Disagree 1=Agree
63	IMMIrc	Importance of mathematics methods instructors	0=Disagree 1=Agree

APPENDIX F

Timeline for the Dissertation Study

Steps	**Activities**	**Stakeholders**	**Timeline**
Problem Identification	Research Questions	Researcher and Committee Chair	Summer 202
Literature Review	Inductive and Elaborative	Researcher and Committee	Fall 2020
Methodology	Population Selection 220 Mathematics Teachers Sample 27 math teachers	Researcher and Committee	Fall 2020
Data Collection	Survey Administration	Researcher and Teachers	Spring 2021
Data Analysis	Quantitative Data Analysis	Researcher and Chair	Spring 2021
Data Collection	Semi-structured Interviews	Researcher and Teachers	Spring 202
Data Analysis	Conceptualization, coding, categorization, thematic analysis	Researcher	Summer 2021
Reporting and closure	Dissertation write-up	Researcher and chair	Summer 2021
	Dissertation Defense	Researcher and Committee	Fall 2021
	Graduation	Researcher	Fall 2021

References

Almalki, S. (2016). Integrating quantitative and qualitative data in mixed methods research: Challenges and benefits. *Journal of Education and Learning*, *5*(3), 288-296.

Arnold, E.G., Burroughs, E. A., Fulton, E.W., & Mendoza Alvarez, J. A. (2021). *Applications of teaching secondary mathematics in undergraduate mathematics courses.* A paper for TS33 at the 14th International Congress on Mathematics Education. Shanghai, China.

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7*(2), 145-172.

Andreas, J. S., & Gabriel, J. S. (2014). Viewing mathematics for teaching as a form of applied mathematics: Implications for the mathematical preparation of teachers, *NOTICES of the AMS*, *61*, 266-276.

Auslander, S. S., Smith, S. Z., Smith, M. E., Hart, L. C., & Carothers, J. (2016). A case study of two groups of elementary prospective teachers' experiences in distinct mathematics content courses. *Southeastern Regional Association of Teacher Educators*, *25*(2), 1-15.

Baki, M., & Arslan, S. (2016). Reflections from pre-service teachers mathematics teaching process. *Hacettepe University Journal of Education*, *31*(4), 736-749. Doi:10.16986/HUJE.2015014664

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *Elementary School Journal*, *90*(4), 449-466. http://www.jstor.org/stable/1001941.

Ball, D. L. (2003). *What mathematical knowledge is needed for teaching mathematics? A paper presented at the Secretary's Summit on Mathematics, U.S. Department of Education*. http://www.ed.gov/rschstat/research/progs/mathscience/ball.html.

Ball, D. L., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher. *Journal of Teacher Education, 60*(5), 497-511.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *America Educator*, *59*(5), 389-407.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 81-104). Ablex.

Ball, D. L. &McDiarmid, G. W. (2003). The subject matter preparation of teachers. In W.R. Houston (Ed.), *Handbook of research on teacher education* (pp. 437-449). Macmillan.

Ball, D., & Wilson, S. (1990). *Knowing the subject and learning to it: Examining assumptions about becoming a mathematics teacher* (Research Report 90-7). National Center for Research on Teacher Learning. http://ncrtl.msu.edu/http/rreports/html/pdf/rr907.pdf

Banner, J., & Cannon, H. (1997). *The elements of teaching*. Yale University Press.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Jordan, A., Klusmann, U., Krauss, S. Neubrand, M., & Tsai, Y. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133-180.

Benedict, A., Holdheide, L., Brownell, M., & Foley, A. M. (2016). *Learning to teach: Practice-based preparation in teacher education. Special issues brief*. Center on Great Teachers & Leaders.

Beswick, K., & Goos, M. (2018). Mathematics teacher educator knowledge: What do we know and where to go from here? *Journal of Mathematics Teacher Education, 21*, 417-427. DOI: 10.1007/s10857-018-9416-4

Borko, H., & Putnam, R. T. (1996). Learning to teach. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology.* (pp. 673-708). Macmillan.

Bosse, M. J., Ries, H., & Chandler, K. (2013). Putting the modern in algebra. *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, *23*(1), 1-12.

Boyd, D., Grossman, P., Lankford, H., Loeb, S., Wycknoff, J., & Urban, I. (2009). Teacher preparation and student achievement. *Education, Evaluation, and Policy Analysis*, *31*(4), 416-440. DOI: 10.3102/0162373709353129

Brumbaugh, D. K., & Rock, D. R. (2013). *Teaching secondary mathematics*. (4th ed.), Routledge.

Brelias, A. (2015). Mathematics for what? High school students reflect on mathematics as a tool for social inquiry. *Democracy & Education*, *23*(1), 1-11.

Bremigan, E. G., Bremigan, R.D., & Lorch, J. D. (2011). *Mathematics for secondary school teachers*. Mathematical Association of America.

Bryman, A. (2006). Integrating quantitative and qualitative research: How is it done? *Qualitative Research*, *6*, 97-113.

Burghes, D., & Geach, R. (2011). *International comparative study in mathematics training: Recommendations for initial teacher training in England*. CfBT Education Trust. https://www.educationdevelopmenttrust.com/EducationDevelopmentTrust/files/b8/b899a6a5-5285-4563-846d-dc88e095b24f.pdf

California Space Education and Workforce Institute. (2008). *Recommendations to Improve Science, Technology, Engineering, and Mathematics (STEM)*. Education in California.

Cardetti, F., & Truxaw, M. P. (2014). Toward improving the mathematics preparation of elementary preservice teachers. *School Science and Mathematics, 114,* 1-9.

Chick, H. (2012). Pedagogical content knowledge. Paper presented at CEMENT Conference integrative model for teacher preparation. *Journal of Teacher Education*, *44*, 263-272.

Cochran, K. F., DeRuiter, J. A., & King, R. A. (1993). Pedagogical content knowledge: An integrative model for teacher preparation. *Journal of Teacher Education*, *44*, 263-272.

Cochran-Smith, M., Villegas, A. M., Abrams, I., Chavez-Moreno, L., Mills, T., & Stern, R. (2015). Critiquing teacher preparation research: An overview of the field, part II. *Journal of Teacher Education, 66*(2), 109-121. DOI: 10.1177.0022487114558268

Cochran-Smith, M. (2004). The problem of teacher education. *Journal of Teacher Education, 55*(4), 295-299. DOI: 10.1177/00

Cohen, L., Mannon, L., & Morrison, K. (2011). *Research methods in education*. (7th ed.). Routledge.

Conference Board of the Mathematical Sciences [CBMS], (2001). *The mathematical education of teachers I.* American Mathematical Society.

Conference Board of the Mathematical Sciences [CBMS], (2012). *The mathematical education of teachers II*. American Mathematical Society.

Cox, D. C., Chesler, J., Reisiegel, M., Kennedy, R., Newton, J., & Stone, J. (2013). *The status of capstone courses for preservice secondary mathematics teachers. Issues in the undergraduate preparation of school teachers*. http://www.k-12prep.math.ttu.edu/journal/4.curriculum/beisiegel01/article.pdf

Creswell, J. W. (2008). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (3rd.). Pearson Education, Inc.

Creswell, J. W. (2013). *Qualitative inquiry & research design: Choosing among five approaches* (4th ed.). SAGE.

Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). SAGE.

Creswell, J. (2015). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. Pearson

Creswell, J., & Plano Clark, V. (2007). *Designing and conducting mixed methods research* (3rd ed.). SAGE.

Creswell, J., & Plano Clark, V. (2011). *Designing and conducting mixed methods research* (2nd ed.). SAGE.

Crotty, M. (2003). The foundations of social research: *Meaning and perspectives in the research process.* (3rd.). SAGE.

Cuhadar, C. (2018). Investigation of pre-service teachers' level of readiness to technology integration in education. *Contemporary Educational Technology*, *9*(1), 61-75.

Darling-Hammond, L. (2000). How teacher education matters. *Journal of Teacher Education*, *51*(3), 166-173.

Darling-Hammond, L. & Sykes, G. (2003). A national teacher supply policy for education: The right way to meet. *Education Policy Analysis Archives*. *11*(33), 1-55.

Darling-Hammond L. (2006). Constructing 21st century teacher education. *Journal of Teacher Education*, *57*(3), 300-314. DOI:10.1177/0022487105285962

Darling-Hammond, L. (2008). Teacher learning that supports student learning. In B.Z. Presseisen (Ed.), *Teaching for Intelligence* (3rd ed.), (pp. 91-100). Corwin Press.

Darling-Hammond, L., & Bransford, J. (2005). *Preparing teachers for a changing world: What teachers should learn and be able to do.* John Wiley & Sons.

Darling-Hammond, L., & Oakes, J. (2019). *Preparing teachers for deeper learning*. Harvard Education Press.

Davis, R. (1967). Mathematics teaching with special reference to epistemological problems. *Journal of Research and Development in Education*, Monograph 1.

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education*, *34*, 12-25.

Dillman, D. A., Smith, J. D., & Christian, L. M. (2009). *Internet, mail, and mixed mode surveys: The tailored design method* (3rd ed.). John Wiley & Sons, Inc.

Dooly, M., Moore, E., & Vallejo, C. (2017). Research ethics. In E. Moore & M. Dooly (Eds.), *Qualitative approaches to research on plurilingual education* (pp. 351- 362). Research-publishing.net. DOI:10.14705/rpnet.2017.emmd2016.634

Dossey, J. A. McCrone, S. S., & Halvorsen, K. T. (2016). *Mathematics education in the United States, 2016: A capsule summary fact book.* National Council of Teachers of Mathematics.

Drew, C. (2011, November 4). Why science majors change their minds (It's just so darn hard). *The New York Times*. https://nyti.ms/sZ5v5J

Eggen, P., & Kauchek, D. (1994). *The teaching of education psychology: A research agenda*. Paper presented at the Annual Meeting at the America Education Research Association.

Erickson, K. A. (2014). *The road to excellence: The acquisition of expert performance in the arts and sciences, sports, and games*. Psychology Press.

Eriksson, K., Helenius, O., & Ryve, A. (2019). Using TIMSS items to evaluate the effectiveness of different instructional practices. *Instructional Science*, *47,* 1-18. https://nces.ed.gov/timss/faq.asp

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council Teachers Mathematics* (pp. 147-164). Macmillan.

Ferrini-Mundy, J., & Findell, B. (2001). The mathematics education of prospective teachers of secondary school mathematics: Old assumptions, new challenges. In T. Rishel (Ed.), *Mathematics and the mathematical sciences in 2010: What should students know?* (pp.31-41)). Mathematical Association of America.

Fetters, M. D., Curry, L. A., & Creswell, J. W. (2013). Achieving integration in mixed methods designs—principles and practices. *Health Services Research, 48*(6), 2134-2156.

Feuer, M. J., Floden, R. E., Chudowsky, N., & Ahn, J. (2013). *Evaluation of teacher preparation programs: Purposes, methods, and policy options*. National Academy of Education.

Fleischmann, H. L., Hopstock, P. J., Pelezar, M. P., & Shelley, B. E. (2010). *Highlights from PISA 2009*. *Performance of U. S. 15 year-old students in reading, mathematics, and science literacy in an international context:* (NCES 2017-048). National Center for Education Statistics, U. S. Department of Education.

Franke, M. L., Turrou, A, C., Webb, N.M., Ing, M., Wong, J., Shin, N., & Fernandez, C. (2017). Student engagement with other mathematical ideas: The role of teacher invitation and support moves. *The Elementary School Journal*, 116, 126-148.

Gardner, D. P. (1983). *A nation at risk.* U.S. Department of Education.

Gay, L. R., Mills, G. E., & Airasian, P. (2009). *Educational research: Competencies for analysis and applications.* (9th ed.). Pearson.

George, A. L., & Bennett, A. (2005*). Case studies and theory development in the social sciences.* MIT Press.

Glaser, B. G., & Strauss, A. (1967). *The discovery of grounded theory. Strategies for qualitative research*. Aldine.

Goodman, M. J., Sands, A. M., & Coley, R. J. (2015). *America's skills challenge: Millennials and the future*. Education Testing Service.

Goulding, M., Hatch, G., & Rodd, M. (2003). Undergraduate mathematics experience: Its significance in secondary mathematics teacher preparation. *Journal of Teacher Education*, *51*(3), 361-393.

Greenberg, J., Mckee, A., & Walsh, K. (2013). *Teacher prep review: A review of nation's teacher preparation programs*. National Council on Teacher Quality. http://www.nctq.org/teacherPrep.doi.org/10.2139/ssrn.2353894

Greenberg, J., & Walsh, K. (2008). *No common denominator: The preparation of elementary teacher in mathematics by American education schools*. National Council on Teacher Quality.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education*. Teachers College Press.

Hanushek, E. A., Peterson, P. E., & Woessman, L. (2010). U. S. math performance in global perspective: How well does the state do at producing high-achieving students? *Program on Education Policy & Governance*, 1-44.

Hill, H. C. (2011). The nature and effects of middle school mathematics teacher learning experiences. *Teachers College Record*, *113*(1), 205-234.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Education Research Journal*, *42*(2), 371-406.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematical for teaching. *Elementary School Journal*, *105*(1), 11–30.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific of students. *Journal for Research in Mathematics Education, 39*, 372-400.

Hill, H.C., & Chin, M.J. (2018). Connections between teachers' knowledge of studentsy, instruction, and achievements outcomes . American Educational Research Journal, *55*(5), 1076-1112.doi: 10.3102/0002831218769614

Hine, G. & Thai, T. (2019). Pre-service mathematics teachers' self-perceptions of readiness to teach secondary school mathematics. *Mathematics Teacher Education and Development*, *21*(2), 64-86.

Hine, G. S. C. (2015). Self-perceptions of pre-service mathematics teachers completing a graduate diploma of secondary education. *Issues in Education Research*, *25*(4), 480-500.

Holm, J., & Kajander, A. (2012). 'I finally get it!' Developing mathematical understanding during teacher education. *International Journal of Mathematical Education in Science and Technology, 43*(5), 563-574. DOI:10.1080/0020739X.2011.622804.

Holm, J., Kajander, A., & Avoseh, J. (2016). Factors relating to the development of mathematics for teaching. In Wood, M. B., Turner, E. E., Civil, J. A., & Eli, J. A. (Eds.), *Proceedings of the 38th annual meeting of the North American Chapter of* 91 *the International Group for the Psychology of Mathematics Education.* Tucson, AZ: The University of Arizona.

Hoover, M. H. & Ball, D. L. (2014). Making progress in U. S. mathematics education: Lessons learned—past, present, and future. In K. R. Leathom (Ed.), *Vital directions for mathematics education research* (pp. 15-44). Springer.

Hoover, M., Mosvold, R., Ball, D. L., & Lai, Y. (2016). Making progress on mathematical knowledge for teaching. *Mathematics Enthusiast. 13*(1&2). 3-34.

Hurrell, D. P. (2013). What teachers need to know to teach mathematics: An argument for a reconceptualized model. *Australian Journal of Teacher Education*, *38*(11), 53-64. http://dx.doi.org/10.14221/ajte.2013v38n11.3

Imig, D., Wiseman, D., & Imig, S. (2011). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, *105*(1), 11-30.

Johnson, L. R. (2017). *Community-based qualitative research: Approaches for education and the social sciences.* SAGE.

Johnson, J. A., Dupuis, V. L., Musial, D., Hall, G. E., & Gollnick, D. M. (2002). *Introduction to the foundations of American education* (12th ed.). Allyn and Bacon.

Kajander, A., & Holm, J. (2016). What math matters? Types of mathematics knowledge and relationships to methods course performance. *Canadian Journal of Science, Mathematics and Technology Education*, *16*(3), 273-283, DOI: 10.1080/14926156.2016.1183837.

Kastberg, D., Chan, J. Y., & Murray, G. (2016). *Performance of U. S. 15-year-old students in science, reading, and mathematics literacy in an international context: First look at PISA 2015* (NCES 2017-048). National Center for Education Statistics, U. S. Department of Education.

Kavanagh, J., & Rich, M. D. (2018). *Truth decay: An initial exploration of the diminishing role of facts and analysis in American public life*. RAND.

Kilic, H. (2011). Preservice secondary mathematics teachers' knowledge of students. *Turkish Online Journal of Qualitative Inquiry*, *2*(2), 17-35.

Kilpatrick, J. (1992). A history of research in mathematics education. In D. Grouws (Ed.), *Handbook research on mathematics research and teaching* (pp. 3-39). Macmillan.

Klein. D. (2003). *A brief history of K-12 mathematics education in the 20th century.* https://www.csum.edu/~vcmth00m/**A**History.html

Klein, F. (1932). *Elementary mathematics from an advanced from an advanced standpoint. Arithmetic, algebra* (Hedrick, E.R., & Noble, C. A. trans.). Macmillan.

Lai, Y. (2019). Accounting for mathematicians' priorities in mathematics courses for secondary teachers. *Journal of Mathematical Behavior*, *53*, 164-178.

Lai, Y. & Patterson, C. (2017). Opportunities presented by mathematics textbooks for prospective teachers to learn to use mathematics in teaching. In W. M. Smith, B. R. Lawler, J. Bowers, & L. Augustyn (Eds.), *Proceedings of the sixth annual mathematics teacher education partnership conference* (pp. 142-147). Association of Public and Land-Grant Universities.

Leavy, P. (2017). *Research design: Quantitative, qualitative, mixed methods, arts-based, and community-based participatory research approaches*. Guilford Press.

Lester, W. (2005). Poll shows America's love-hate relationship with math. *Lovelace Journal-World*. http://www2.ijworld.com//news/2005/aug/17/poll_shows_americas_lovehate_relationship_math/

Linder, S. M., & Simpson, A. (2017). Towards an understanding of early childhood mathematics education: A systematic review of the literature focusing on practicing and prospective teachers. *Contemporary Issues in Early Childhood*, *19*(3), 1-23. Doi: 10.1177/14639491177119553

Lovat, T. J. (2003). The role of the teacher: Coming of age? *Australia Council of Deans of Education*, 1-33.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Lawrence Erlbaum Associates.

Mahajan, S., Marciniak, Z., & Schmidt, B. (2016). *PISA mathematics in 2021: An analysis of the center for curriculum redesign*. Center for Curriculum Redesign

Mahajan, S. (2014). *Mathematics for the 21st century: What should students learn?* Center for Curriculum Redesign.

Malikow, M. (2006). Are teachers born or made? The necessity of teacher training programs. *National Forum of Teacher Education Journal*, *16*(3), 1-3.

Masingila, J. O., Olanoff, D., & Kimani, P. M. (2018). Mathematical knowledge for teaching teachers: Knowledge used and developed by mathematics teacher educators in learning to teach via problem solving. *Journal of Mathematics Teacher Education, 21*, 429-450.

Masingila, J. O., Olanoff, D. E., & Kwaka, D. K. (2012). Who teaches mathematics content courses for prospective elementary teachers in the United States? Results of a national survey. *Journal of Mathematics Teacher Education, 15,* 347-358. DOI: 10.10857-012-9215-2

Macmillan, J. H., & Schumacher, (2010). *Research in education: Evidence-based inquiry*, (7th ed.). Pearson.

Masters, G. (2009). *A shared challenge: Improving literacy, numeracy, and science learning in Queensland primary schools*. Australian Council for Educational Research.

Menon, R. (2009). Preservice teachers' subject matter knowledge of mathematics. *International Journal for Mathematics Teaching and Learning*. https://www.cimt.org.uk/journal/menon.pdf

Merriam, S. B. & Tisdell, E. J. (2016). *Qualitative research: A guide to design and Implementation.* (4th ed.). Josser-Bass.

Miles, M. B., Huberman, A. M., & Saldana, J. (1994). *Qualitative data analysis: A methods sourcebook.* (3rd ed.). SAGE.

Miller, M. K. & Davidson, D. M. (2006). What makes secondary school science and/or mathematics teacher highly qualified? *Science Educator*, *15*(1), 56-59.

Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review*, *13*, 125-145.

Morris, A. K., Hiebert, J., & Spitzer, S. M. (2009). Mathematical knowledge for teaching in planning and evaluating instruction: What can pre-service teachers learn? *Journal for Research in Mathematics Education*, *40*(5), 491-529.

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International Results in Mathematics and Science.* Retrieved from Boston College, TIMSS & PIRLS International Study Center https://timssandpirls.bc.edu/timss2019/international-results/

Mullis, I.V.S., Martin, M.O., Goh, S., & Cotter, K. (Eds.). (2016). *TIMSS 2015 encyclopedia: Education policy and curriculum in mathematics and science*. TIMSS & PIRLS International Study Center http://timssandpirls.bc.edu/timss2015/encyclopedia/

Murray, E., Baldinger, E., & Wasserman, N. (2017). Connecting advanced school mathematics. *IUMPST: The Journal*, *1*, 1-10.

National Center for Education Statistics, (2015). *The nation's report card*. Institute for Education Services, U. S. Department of Education.

National Center for Education Statistics [NCES], (2016). *2015: Mathematics & reading, grade 12.* Department of Education, Institute for Education Sciences.

National Commission on Excellence in Education (1983). *A nation at risk*. Government Printing Office.

National Council of Teachers of Mathematics [NCTM], (2000). *Curriculum and evaluation standards for school mathematics*. National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics [NCTM], (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. National Council of Teachers of Mathematics.

National Mathematics Advisory Panel [NMAP], (2008). Foundations for Success: The Final Report of the National Mathematics Advisory Panel, U.S. Department of Education.

National Research Council [NRC], (2001). *Adding it up: Helping children learn mathematics*. The National Academy Press. https://doi.org/10.17226/9822

National Research Council. (2010). Exploring the intersection of science education and 21st century skills: A workshop summary. M. Hilton (Ed.), *Board on science education, center for education, division of behavioral and social sciences and education.* The National Academies Press

National Research Council, (2012). Education for life and work: Developing transferable knowledge and skills in the 21st century. Committee on defining deeper learning and 21st century Skills. In J.W. Pellegrino and M.L. Hilton (Eds.), *Board on testing and assessment and board on science education, division of behavioral and social sciences and education*. The National Academies Press. https://doi.org/10.17226/13398.

Norton, S. (2010). How deeply and how well? How ready to teach mathematics after a one year program? *Mathematics Education and Special Education*, *12*(1), 65-84.

Ojose, B. (2014). Teaching certain mathematical axioms: The role played by conceptual knowledge of algebra. *National Teacher Educational Journal*, *7*(1), 37-44.

Olson, J. C., & Knott, L. (2013). When a problem is more than a teacher's question. *Educational Studies in Mathematics, 83,* 27-36.

Organization for Economic Cooperation and Development [OECD]. (2016). *PISA 2015 results (volume 1): Excellence and Equity in Education.* Organization for Economic Cooperation and Development.

Park, S., & Oliver, J. S. (2008). Revisiting the conceptualization of pedagogical content knowledge (PCK): PCK as a conceptual tool to understand teachers as professionals. *Research in Science Education, 38*(3), 261-284.

Paterson, B. (2020). Connections between real analysis and secondary mathematics. *The Journal*, *1*, 1-6.

Patton, M. Q. (2015). *Qualitative research & evaluation methods*. (4th ed.). SAGE.

Phelps, G. (2009). Just knowing how to read isn't enough! Assessing knowledge for teaching reading. *Educational Assessment, Evaluation, and Accountability*, *21*(2), 137-154.

Phillips, G.W. (2007). *Chance favors the prepared mind: Mathematics and science indicators for comparing states and nations.* American Institutes for Research.

Piaget, J. (1954). *The construction of reality in the child*. Ballantine Books.

Prescott, A. & Cavanagh, M. (2008). A situated perspective on learning to teach secondary mathematics. In M. Goos, R. Brown, & K. Makar (Eds.), *Proceedings of the 31st annual conference of the mathematics education research group of Australia*. MERGA, Inc.

Provasnik, S., Malley, L., Stephens, M,, M., Landeros, K., Prelims, R., & Tang, J. H. (2016). *Highlights from TIMSS and TIMSS advanced 2015: Mathematics and science achievement of U. S. students in grade 4 and 8 and in advanced courses at the end of high school in an international context (NCES 2017-002)*. U. S. Department of Education, National Center for Education Statistics. http://nces.ed.gov/pubsearch

Qian, H., & Youngs, P. (2016). The effect of teacher education programs on future elementary mathematics teachers' knowledge: A five-country analysis using TEDS-M data. *Journal of Mathematics Teacher Education, 19*, 371-396.

RAND and Ball, D. L. (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education*. RAND Corporation. https://www.rand.org/pubs/monograph_reports/MR1643.html

Rudin, W. (1976). *Principles of mathematical analysis*. (3rd ed.), McGraw-Hill.

Ravitch, S. M., & Carl, N. M. (2016). *Qualitative research: Bridging the conceptual, theoretical, and methodological*. SAGE

Ronfeldt, M., & Reininger, M. (2012). More or better student teaching? *Teaching and Teacher Education*, *28*, 1091–1106. DOI:10.1016/j.tate.2012.06.003

Rowland, T, (2014). The knowledge quartet: The genesis and application of a framework for analyzing mathematics teaching and deepening teachers' mathematics knowledge. *Sisyphus Journal of Education*, *1*(3), 15-43.

Russ-Eft, D. (2002). A topology of training design and work environmental factors affecting work learning place and transfer. *Human Resource Development Review*, *1*(1), 45-65.

Saldaña, J. (2016). *The coding manual for qualitative researchers* (3rd ed.). SAGE.

Schmidt, W.H., Burroughs, N. A., Cogan, L. S., & Houang, R. T. (2017). The role of subject matter content in teacher preparation: An international perspective for mathematics. *Journal of Curriculum Studies*, *49*(2), 111-131.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform, *Harvard Educational Review, 57*(1), 1-23.

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education, 11*, 499-511. DOI: 10.1007/s10857-008-9089-5

Speer, N. M., King, K. D., & Howell, H. (2014) Definitions of mathematical knowledge for teaching: Using these constructs in research on secondary and college mathematics teachers. *Journal of Mathematics Teacher Education*, *18,* 105-122. DOI: 10.1007/s10857-014-9277-4

Stacey, K. (2003). The need to increase attention to mathematical reasoning. In H. Hollingsworth, J. Lokan, & B. McCrae(Eds.), *Teaching mathematics in Australia: Results from the TIMSS 1999 Video Study.* Australian Council for Educational Research.

Stake, R. E. (1995). *The art of case study research*. SAGE.

Stewart, A. (2014). Case study. In J. Mills & M. Birks (Eds.). *Qualitative methodology: A practical guide* (pp. 145-159). SAGE.

Stockton, J. C., & Wasserman, N. H. (2017). Forms of knowledge of advanced mathematics for teaching. *The Mathematics Enthusiast*, *14*(1), 575-606.

Su, F. (2017). *Mathematics for human flourishing*. Presidential address, American Mathematical Society-Mathematics Association of America.

Sullivan, P. (2018). Supporting teachers in improving their knowledge of mathematics. *Journal of Mathematical Behavior*, *51*, 161-166.

Superfine, A. C., & Li, W. (2014). Developing mathematical knowledge for teaching teachers: A model for the professional development of teacher educators. *Issues in Teacher Education*, *23*(1), 113-132.

Tajudin, N.M. (2014). Exploring prospective mathematics teachers' mathematical knowledge for teaching algebraic problem-solving. *Proceedings of the Social Sciences Research ICSSR 2014,* Sabah.

Tatto, M., Ingerson, I., Schwille, J., Peck, R., Senk, S., & Rowley, G. (2008). *Teacher education and development study in mathematics (TEDS-M): Policy, practice, readiness to teach primary and secondary mathematics. Conceptual framework*. Teacher Education and Development International Study Center.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal of Research in Mathematics Education, 31*(1), 5-25. DOI:10.2307/949817

U. S. Department of Education. (2008). *Foundations for success*: *The final report of the National Mathematics Advisory Panel.* http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Vogt, W. P. (2007). *Quantitative research methods for professionals in education and other fields*. Pearson.

Vygotsky, L. (1978). *Mind in society: The development of the higher psychological process*. Harvard University Press.

Waggener, J. F. (1996). *A brief history of mathematics education in America* [unpublished Manuscript]. The University of Georgia. http://jwilson.coe.uga.edu/EMAT7050/HistoryWeggener.html

Wasserman, N.H. &Weber, K., Fukawa-Connelly, & McGuffey, W. (2019). Designing advanced mathematics to influence secondary mathematics teaching: fostering mathematics teachers' "attention to scope" *Journal of Mathematics Teacher Education*, *22*, 379-406.

Wasserman, N. H. (2018). Knowledge of nonlocal mathematics for teaching. *Journal of Mathematics Behavior, 49*, 116-128.

Wasserman, N. H., Weber, K., Villanueva, M., & Mejia-Ramos, J. P. (2018). Mathematics teachers' views of the limited utility of real analysis: A transport model hypothesis. *The Journal of Mathematics Behavior, 50,* 74-89.

Wasserman, N. H., Fukawa-Connelly, T., Villanueva, M., Mejia-Ramos, J. P., & Weber, K. (2017). Making real analysis relevant to secondary teachers: Building up from and stepping down to practice. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 27*(6), 559-578. DOI: 10.1080/10511970.2016.1225874

Whitehead, A. N. (1948). *An Introduction to mathematics*. Oxford University Press.

Yin, R. K. (2009). *Case study research: Design and methods* (4th ed.). SAGE.

Yin, R. K. (2014). *Case study research design and methods,* (5th ed.). SAGE.

Zazkis, R., & Leikin, R. (2010). Advanced mathematical knowledge in teaching practice: Perceptions of secondary mathematics. *Mathematics Thinking and Learning*, *12,* 263-281.