Neutrosophic Sets and Systems

Volume 24 Article 12

1-1-2019

Dombi Interval Valued Neutrosophic Graph and its Role in Traffic Control Management

D. Nagarajan

M. Lathamaheswari

S. Broumi

J. Kavikumar

University of New Mexico

Dombi Interval Valued Neutrosophic Graph and its Role in Traffic Control Management

D. Nagarajan[1], M.Lathamaheswari[2], S. Broumi[3] J. Kavikumar[4]

[1,2]Department of Mathematics, Hindustan Institute of Technology &Science, Chennai-603 103, India,

E-mail: dnrmsu2002@yahoo.com, E-mail:lathamax@gmail.com

[3]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955,Sidi Othman, Casablanca, Morocco,

E-mail: broumisaid78@gmail.com

[4]Department of Mathematics and Statistics, Faculty of Applied Science and Technology,

Universiti Tun Hussein Onn, Malaysia, E-mail:kavi@uthm.edu.my

Abstract. An advantage of dealing indeterminacy is possible only with Neutrosophic Sets. Graph theory plays a vital role in the field of networking. If uncertainty exist in the set of vertices and edge then that can be dealt by fuzzy graphs in any application and using Neutrosophic Graph uncertainty of the problems can be completely dealt with the concept of indeterminacy. In this paper, Dombi Interval Valued Neutrosophic Graph has been proposed and Cartesian product and composition of the proposed graphs have been derived. The validity of the derived results have been proved with the numerical example. This paper expose the use of Dombi triangular norms in the area of Neutrosophic graph theory. Advantages and limitations has been discussed for Crisp, Fuzzy, Type-2 Fuzzy, Neutrosophic Set, Interval Neutrosophic Set, Neutrosophic Graph and Interval Neutrosophic Graph.

Keywords: Dombi Triangular Norms, Fuzzy Graphs, Interval valued Neutrosophic Graph, Dombi Interval Valued Neutrosophic Graph, Cartesian product, Composition, Traffic Control Management.

1. Introduction

The generalized Dombi operator family was introduced by Dombi and applied to speech Recognition Task [1, 2]. Graph theory plays a vital role in different fields namely computer science, engineering, physics and biology to deal with complex networks [3, 4]. It is also used to solve various optimization problems in transportation where network is nothing but the logical sequence of the method and visualization possibility, permits surveys to be afforded. Also it responses to two equations simultaneously for the purpose and the procedure [5]. Permanent growth of the population is one of the main face of modern cities and it is the reason for building new roads and highways to avoid traffic problems and for stress free life of the people [6, 13].

A fuzzy set can be described mathematically by assigning a value, a grade of membership to each possible individual in the universe of discourse. This grade of membership associates a degree to which that individual either is similar or appropriate with the concept performed by the fuzzy set. A fuzzy subset of a set X is a mapping from membership to non-membership and is defined by $\eta : X \to [0,1]$ continuous rather than unexpected. Fuzzy relations are popular and important in the fields of computer networks, decision making, neural network, expert systems etc. [7, 16].Direct relationship and also indirect relationship also will be considered in graph theory [8].

Model of relation is nothing but a graph and it is a comfortable way of describing information involving connection between objects [9]. In graph, vertices are represented by vertices and relations by edges. While there is an impreciseness in the statement of the objects or it its communication or in both, fuzzy graph model can be designed for getting an optimized output. Maximizing the Utility of the application is always done by the researchers during the constructing of a model with a key characteristics reliability, complexity and impreciseness. Among these impreciseness plays an important role in maximizing the utility of the model. This situation can be described by fuzzy sets, introduced by Lotfi. A. Zadeh. Fuzzy graphs will be very useful, since for the real world problems, one gets the partial information [10, 11].

Performance evaluation can be defined as modeling and application and should be done before using them using graph theory[12]. When the system is huge and complex, it is challenging to extract the information about the system using classical graph theory and at this junction fuzzy graph can be used to examine the system [14]. A situation in which goods is shifted from one location to another can be dealt by graphs. For example, one can consider water supply, where water users and pipe join etc. are vertices and pipelines are edges [15]. The concept of graph theory was introduced by Euler in 1736 and it is a branch of combinatorics [16]. Fuzzy graph was introduced by Rosenfeld who has defined the fuzzy correlation of various graph theoretic notions such as cycles, paths, trees and connectedness and set some of their properties [18].

Zadeh formulated the term degree of membership and describe the notion of fuzzy set in order to deal with an impreciseness. Atanassov introduced intuitionistic fuzzy set by including the degree of non-membership in the concept of fuzzy set as an independent component. Samarandache introduced Neutrosophic set(NS) by finding the term degree of indeterminacy from the logical point of view as an independent component to handle with imprecise, indeterminate and unpredictable information which are exist in the real world problems The NSs are defined by truth, indeterminacy and false membership functions which are taking the values in the real standard interval. Wang et al. proposed the concept of single-valued Neutrosophic sets (SVNS) and Interval valued Neutrosophic Sets (IVNSs) as well, where the three membership functions are independent and takes value in the unit interval [0,1] [19, 20, 30, 32].

If uncertainty exists in the set of vertices or edges or both then the model becomes a fuzzy graph. Fuzzy graphs can be established by considering the vertex and edge sets as fuzzy, in the same way one can model interval valued fuzzy graphs, intuitionistic fuzzy graphs, interval valued fuzzy graphs, Neutrosophic graphs, single valued Neutrosophic graphs and interval valued Neutrosophic graphs [23]. Network of the brain is a Neutrosophic graph especially strong Neutrosophic graph[25]. Intelligent transport systems is a universal aspect gets the attention of worldwide interest from professionals in transportation, political decision makers and computerized industry. It is developed by understanding the progress of the road traffic in the interval of time and communication between the participants and structural elements available in the situation [26, 31].

Graph theory defines the relationship between various individuals and has got many number of applications in different fields namely database theory, modern sciences and technology, neural networks, data mining cluster analysis, expert systems image capturing and control theory [27]. The strength of the relationship in social networks can be analyzed by fuzzy graph theory and has got important potential [28]. While the network is large, analysis and evaluation of traffic will be very challenging one for the network managers and it can be done using dynamic Bandwidth [29]. Indeterminacy of the object or edge or both cannot be handled by fuzzy, intuitionistic fuzzy, bipolar fuzzy or interval valued fuzzy graphs and hence Neutrosophic graphs have been introduced [33].

Menger proposed triangular norms in the structure of probabilistic metric spaces and discussed by Schweizer Sklar. Also intersection and union of fuzzy sets have been proved by Alsina et al. Triangular norms play an important role application of fuzzy logic namely fuzzy graph and decision making process [34]. Some of the real world applications can be modelled in a better way with triangular norms especially t-norm than using minimum operations. Using this concept awareness of tracking in person for networks is possible [36]

Since intervals plays an essential role in graph theory and useful in the study of properties of fuzzy graphs which applied to surveying the land based on the concept of the distance between the vertices, using the concepts of Interval Valued Neutrosophic Graphs and Dombi fuzzy graphs, Dombi Single Valued and Dombi Interval Valued Neutrosophic Graphs have been proposed. Also Cartesian product and composition of Dombi Interval Valued Neutrosophic Graph have been derived. Numerical example also has been given for the validity of the results. The main goal of this paper is to emphasis that the minimum and maximum operators are not the only applicant for the logical reasoning of the classical graphs to Neutrosophic graphs.

2. Review of Literature

The authors of [1] introduced the generalized Dombi operator family and the multiplicative utility function. [2] applied the Generalized Dombi Operator Family to the Speech Recognition Task. [3] explained about Neutrosophic model and Control. [4] examined biological networks using graph theory. [5] established a computerized technique to solve network problems using graph theory. [6] analyzed a traffic control problem using cut-set of a graph and applied in arbitrary intersection to reduce the waiting time of the people.[7] proposed operation on a complement of a fuzzy graph. [8] proposed functional consistency of an input Gene Network. [9] proposed a program for coloring the vertex of a fuzzy graph. [10] applied vertex coloring function of a fuzzy graph to the traffic light problem.

[11] reviewed about Fuzzy Graph Theory.[12] evaluated an impreciseness produced in performance measures during the procedure of performance evaluation using graph theory. [13] introduced graph for the problem and circular arcs and applied in traffic management. [15] applied the concept of graph in traffic control management in city and airport.[16] described certain types of Neutrosophic graphs.[17] proposed a new dimension to graph theory. [18] proposed strong domination number using membership values of strong arcs in fuzzy graphs. [19] gave introduction to bipolar single valued Neutrosophic Graph theory. (Broumi et al. 2016) proposed an isolated Interval valued Neutrosophic graph.

[20] applied interval valued Neutrosophic in decision making problem to invest the money in the best company. [21] proved the necessary and sufficient condition for a Neutrosophic graph to be an isolated single valued Neutrosophic graph. [22] examined the properties of different types of degrees size and order of SVNGs and proposed the definition of regular SVNG. [25] proposed strong NGs and Sub graph Topological Subspaces. [26] presented a complete study of all existing Intelligent Transport systems namely research models and open systems.[27] applied graph theory concepts to SVNGs and examine a new type of graph model and concluded the result to crisp graphs, fuzzy graphs and intuitionistic fuzzy graphs and characterized their properties.

[28] examined asymmetrical partnership using fuzzy graph and detect hidden connections in Facebook. [29] proposed an optimized algorithm using the approach of rating of web pages and it assigns a minimum approved bandwidth to every connected user.[30] represented a graph model based on IVN sets.[31] proposed dimensional modeling of traffic in urban road using graph theory. [32] proposed uniform SVNGs. [33] proposed some of the results on the graph theory for complex NSs. [34] proposed Dombi fuzzy graphs and proved the standard operations on Dombi fuzzy graphs. [35] proposed fuzzy graph of semigroup.[36] proposed t-norm fuzzy graphs and discussed the importance of t-norm in network system.

3. Basic Concepts

Some basic concepts needed for proposing and deriving the results are listed below.

3.1 Graph (Ashraf et al. 2018)

A mathematical system $G=(V,E)$ is called a graph, where $V=V(G)$, a vertex set and $E=E(G)$ is anedge set. In this paper, undirected graph has been considered and hence every edge is considered as an unordered pair of different vertices.

3.2 Fuzzy Graph (Marapureddy 2018)

Let V be a non-empty finite set, λ be a fuzzy subsets on V and δbe a fuzzy subsets on $\mathrm{V}\times\mathrm{V}$. The pair $\mathrm{G}=(\lambda,\delta)$ is a fuzzy graph over the set V if $\delta(x,y)\le\min\{\lambda(x),\lambda(y)\}$ for all $(x,y)\in\mathrm{V}\times\mathrm{V}$ where λ is a fuzzy vertex and $\delta$is a fuzzy edge. Where:

1. A mapping $\lambda:\mathrm{V}\to[0,1]$ is called a **fuzzy subset** of V, where V is the non-empty set.
2. A mapping $\delta:\mathrm{V}\times\mathrm{V}\to[0,1]$ is a **fuzzy relation** on λ of V if $\delta(x,y)\le\min\{\lambda(x),\lambda(y)\}$
3. If $\delta(x,y)=\min\{\lambda(x),\lambda(y)\}$ then G is a **strong fuzzy graph**.

3.3 Dombi Fuzzy Graph (Ashraf et al. 2018)

A pair $G=(\lambda,\delta)$ is a Dombi fuzzy graph if $\delta(xy)\le\frac{\lambda(x)\lambda(y)}{\lambda(x)+\lambda(x)-\lambda(x)\lambda(y)}$, for all $x,y\in V$, where the Dombi fuzzy vertex set, $\lambda:V\to[0,1]$ is a fuzzy subset in V and the Dombi fuzzy edge set, $\delta:V\times V\to[0,1]$ is a symmetric fuzzy relation on λ.

3.4 Single Valued Neutrosophic Graph (SVNG) (Broumi et al. 2016)

A pair $G_N=(P,Q)$ is SVNG with elemental set v . Where:

1. Degree of truth membership, indeterminacy membership and falsity membership of the element $x_i\in \mathbf{V}$ are defined by $T_P:V\to[0,1]$, $I_P:V\to[0,1]$ and $F_P:V\to[0,1]$ respectively and $0\le T_P(x_i)+I_P(x_i)+F_P(x_i)\le 3,\forall x_i\in \mathbf{V}, i=1,2,3,...,n$
2. Degree of truth membership, indeterminacy membership and falsity membership of the edge $(x_i,y_j)\in \mathbf{E}$ are denoted by $T_Q:E\subseteq V\times V\to[0,1]$, $I_Q:E\subseteq V\times V\to[0,1]$ and $F_Q:E\subseteq V\times V\to[0,1]$ respectively and are defined by
   - $T_Q(\{x_i,y_j\})\le\min[T_P(x_i),T_P(y_j)]$
   - $I_Q(\{x_i,y_j\})\ge\max[I_P(x_i),I_P(y_j)]$
   - $F_Q(\{x_i,y_j\})\ge\max[F_P(x_i)F_P(y_j)]$

where $0\le T_Q(\{x_i,y_j\})+I_Q(\{x_i,y_j\})+F_Q(\{x_i,y_j\})\le 3,\forall\{x_i,y_j\}\in \mathbf{E}(i,j=1,2,...,n)$.

Also P is a single valued Neutrosophic vertex of V and Q is a single valued Neutrosophic edge set of $\mathbf{E}$. Q is a symmetric single valued Neutrosophic relation on P.

3.5Interval Valued Neutrosophic Graph (IVNG) (Broumi et al. 2016)

A pair $G_N=(P,Q)$ is IVNG, where $P=\langle[T_P^L,T_P^U],[I_P^L,I_P^U],[F_P^L,F_P^U]\rangle$, an IVN is set on V and $Q=\langle[T_Q^L,T_Q^U],[I_Q^L,I_Q^U],[F_Q^L,F_Q^U]\rangle$ is an IVN edge set on $\mathbf{E}$ satisfying the following conditions:

1. Degree of truth membership, indeterminacy membership and falsity membership of the element $x_i\in\mathbf{V}$ are defined by $T_P^L:V\to[0,1]$, $T_P^U:V\to[0,1]$, $I_P^L:V\to[0,1]$, $I_P^L:V\to[0,1]$, $I_P^U:V\to[0,1]$ and $F_P^L:V\to[0,1], F_P^U:V\to[0,1]$ respectively and $0\le T_P(x_i)+I_P(x_i)+F_P(x_i)\le 3,\forall x_i\in\mathbf{V}, i=1,2,3,...,n$
2. Degree of truth membership, indeterminacy membership and falsity membership of the edge $(x_i,y_j)\in\mathbf{E}$ are denoted by $T_Q^L:V\times V\to[0,1]$, $T_Q^U:V\times V\to[0,1]$ $I_Q^L:V\times V\to[0,1]$, $I_Q^U:V\times V\to[0,1]$ and $F_Q^L:V\times V\to[0,1]$, $F_Q^U:V\times V\to[0,1]$ respectively and are defined by
   - $T_Q^L(\{x_i,y_j\})\le\min[T_P^L(x_i),T_P^L(y_j)]$
   - $T_Q^U(\{x_i,y_j\})\le\min[T_P^U(x_i),T_P^U(y_j)]$
   - $I_Q^L(\{x_i,y_j\})\ge\max[T_P^L(x_i),T_P^L(y_j)]$
   - $I_Q^U(\{x_i,y_j\})\ge\max[I_P^U(x_i),I_P^U(y_j)]$
   - $F_Q^L(\{x_i,y_j\})\ge\max[F_P^L(x_i),F_P^L(y_j)]$
   - $F_Q^U(\{x_i,y_j\})\ge\max[F_P^U(x_i),F_P^U(y_j)]$

where $0\le T_Q(\{x_i,y_j\})+I_Q(\{x_i,y_j\})+F_Q(\{x_i,y_j\})\le 3,\forall\{x_i,y_j\}\in\mathbf{E}(i,j=1,2,...,n)$.

3.6 Triangular Norms (Ashraf et al. 2018)

A mapping $T:[0,1]^2 \to [0,1]$ is a binary operation and is called a triangular norm or T-Norm if $\forall x,y,z \in [0,1]$ it satisfies the following:

(1). $T(1,x)=x$ (Boundary condition)
(2). $T(x,y)=T(y,x)$ (Commutativity)
(3). $T(x,T(y,z))=T(T(x,y),z)$ (Associativity)
(4). $T(x,y)\le T(x,z)$, if $y \le z$ (Monotonicity)

- Triangular conorm or T-Conorm is also a binary operation $TC:[0,1]^2 \to [0,1]$ and is defined by $TC(x,y)=1-T(1-x,1-y)$

3.7 Dombi Triangular Norms (Dombi 2009, Dombi and Kocsor 2009, Ashraf et al. 2018)

Dombi product or T-Norm and T-Conorm are denoted by $\otimes$ and $\oplus$ respectively and defined by

$$TN(x,y)=x\underset{D}{\otimes}y=\frac{1}{1+\left[\left(\frac{1-x}{y}\right)^{\xi}+\left(\frac{1-y}{y}\right)^{\xi}\right]^{1/\xi}},\xi>0$$

$$TCN(x,y)=x\underset{D}{\oplus}y=\frac{1}{1+\left[\left(\frac{1-x}{y}\right)^{-\xi}+\left(\frac{1-y}{y}\right)^{-\xi}\right]^{1/-\xi}},\xi>0$$

This triangular norm contains the product, Hamacher operators, and Einstein operators and as the limiting case, minimum and maximum operators can be obtained. Multivariable case can be dealt easily by the new form of Hamacher family. Dombi operators have flexible parameters and hence the success rate will be greater one.

3.8 Hamacher Triangular Norms (Ashraf et al. 2018)

Hamacher product or T-Norm and T-Conorm are denoted by $\otimes$ and $\oplus$ respectively and defined by

$$T(x,y)=x\underset{H}{\otimes}y=\frac{xy}{\xi+(1-\xi)(x+y-xy)},\xi>0$$

$$TC(x,y)=x\underset{H}{\otimes}y=\frac{x+y+(\xi-2)xy}{1+(\xi-1)xy},\xi>0$$

3.9 Special Cases of Dombi and Hamacher Triangular Norms (Ashraf et al. 2018)

If we replace $\xi = 0$ in Hamacher family of triangular norms and $\xi = 1$ in Dombi family of triangular norms then

$T(x,y)=x\otimes y=\dfrac{xy}{(x+y-xy)}$ and $TC(x,y)=x\oplus y=\dfrac{x+y-2xy}{(1-xy)}$.

3.10 Standard Products of graphs (Ashraf et al. 2018)

Consider two $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ then the following products are defined by

1. Direct product : $E(G_1 \square G_2)=\{(x_1,x_2)(y_1,y_2)|x_1y_1 \in E_1 \ \&\ x_2y_2 \in E_1\}$
2. Cartesian Product: $E(G_1\times G_2)=\{(x_1,x_2)(y_1,y_2)|x_1=y_1 \ \&\ x_2y_2 \in E_2, or\ x_1y_1 \in E_1 \ \&\ x_2=y_2\}$
3. Strong Product: $E(G_1\times G_2)=\{(x_1,x_2)(y_1,y_2)|x_1=y_1 \ \&\ x_2y_2 \in E_2, or\ x_1y_1 \in E_1 \ \&\ x_2y_2 \in E_2\}$
4. Dombi fuzzy Cartesian product:
$$(\lambda_1\times\lambda_2)(x_1,x_2)=\frac{\lambda_1(x_1)\lambda_2(x_2)}{\lambda_1(x_1)+\lambda_2(x_2)-\lambda_1(x_1)\lambda_2(x_2)},\forall(x_1,x_2)\in \mathbf{V}_1\times\mathbf{V}_2$$

$$(\delta_1 \times \delta_2)((x,x_2)(x,y_2)) = \frac{\lambda_1(x)\delta_2(x_2y_2)}{\lambda_1(x)+\delta_2(x_2y_2)-\lambda_1(x_1)\delta_2(x_2y_2)}, \forall x \in \mathbf{V}_1, x_2y_2 \in \mathbf{E}_2$$

$$(\delta_1 \times \delta_2)((x_1,z)(y_1,z)) = \frac{\lambda_2(z)\delta_1(x_1y_1)}{\lambda_2(z)+\delta_1(x_1y_1)-\lambda_2(z)\delta_1(x_1y_1)}, \forall z \in \mathbf{V}_2, x_1y_1 \in \mathbf{E}_1$$

3.11 Neutrosophic Controllers (Aggarwal et al. 2010)

In the field of logic, fuzzy logic is a powerful one due its capacity of extracting the information from imprecise data. Human interpretations and computer simulation are the active and interested area among the researchers where the computers are managing only precise assessments. But the Neutrosophic logic has the capacity of recovering all sorts of logics. It is an appropriate choice to imitate the action of human brain which is assembled with handling with uncertainties and impreciseness.

Neutrosophic Controllers are giving an optimized results than the fuzzy supplement since they are more hypothesized and indeterminacy tolerant. These controllers would modify considerably according to the nature of the problem.

Modeling a proper mathematical structure of a control problem that would simulate the behavior of the system is very difficult. Due to impreciseness and indeterminacy in the data, unpredictable environmental disturbances, corrupt sensor getting a complete and determinate date is not possible in the real world problems. These situations can be handled by considering Neutrosophic linguistic terms and can be dealt effectively by considering interval valued Neutrosophic environment as it handles more impreciseness using lower and upper membership functions.

4. Proposed Dombi Interval Valued Neutrosophic Graph

Using the above special triangular norms Dombi Interval Valued Neutrosophic Graph (DIVNG) has been proposed and derived the Cartesian and composite products of DIVNG had been derived along with the numerical example.

4.1 Dombi Single Valued Neutrosophic Graph (DSVNG)

A single Valued Neutrosophic Graph is a DSVNG on V is defined to be a pair $G_N = (P,Q)$, where:

1. The functions $T_P : V \to [0,1]$, $I_P : V \to [0,1]$ and $F_P : V \to [0,1]$ are the degree of truth membership. Indeterminacy membership and falsity membership of the element $x_i \in V$ respectively and $0 \le T_P(x_i) + I_P(x_i) + F_P(x_i) \le 3, \forall x_i \in \mathbf{V}, i = 1,2,3,...,n$.
2. The functions $T_Q : E \subseteq V \times V \to [0,1]$, $I_Q : E \subseteq V \times V \to [0,1]$ and $F_Q : E \subseteq V \times V \to [0,1]$ are defined by

$$T_Q(\{x_i, y_j\}) \le \frac{T_P(x_i)T_P(y_j)}{T_P(x_i)+T_P(y_j)-T_P(x_i)T_P(y_j)}$$

$$I_Q(\{x_i, y_j\}) \ge \frac{I_P(x_i)+I_P(y_j)-2I_P(x_i)I_P(y_j)}{1-I_P(x_i)I_P(y_j)}$$

$$F_Q(\{x_i, y_j\}) \ge \frac{F_P(x_i)+F_P(y_j)-2F_P(x_i)F_P(y_j)}{1-F_P(x_i)F_P(y_j)}$$

Numerical Example:

Figure 1 is an example of DSVNG $G_N = (P,Q)$ of the graph $G = (V,E)$ such that the vertex set is $P = \{\langle i,(0.5,0.1,0.4)\rangle, \langle j,(0.6,0.3,0.2)\rangle, \langle k,(0.2,0.3,0.4)\rangle, \langle l,(0.4,0.2,0.5)\rangle\}$

and the edge set is given by

$Q = \{\langle ij,(0.4,0.3,0.5)\rangle, \langle jk,(0.2,0.4,0.5)\rangle\}, \langle kl,(0.2,0.4,0.6)\rangle, \langle il,(0.3,0.4,0.6)\rangle$

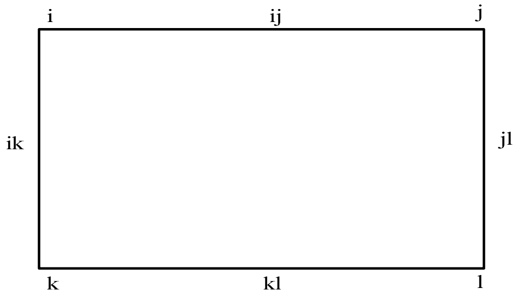

Fig.1.Dombi Single Valued Neutrosophic Graph

Numerical Computation:

To find $T_Q(ij)$:

$$T_Q(ij) \le \frac{(0.5)(0.6)}{(0.5)+(0.6)-(0.5)(0.6)} \le 0.4 \text{ , hence } T_Q(ij)=0.4$$

To find $I_Q(ij)$:

$$I_Q(ij) \ge \frac{(0.1)+(0.3)-2(0.1)(0.3)}{1-(0.1)(0.3)} \ge 0.3 \text{ , hence } I_Q(ij)=0.3$$

To find $F_Q(ij)$:

$$F_Q(ij) \ge \frac{(0.4)+(0.2)-2(0.4)(0.2)}{1-(0.4)(0.2)} \ge 0.5 \text{ , hence } I_Q(ij)=0.5$$

Similarly other values can be found.

- While the membership values of truth, indeterminacy and falsity are not in the interval form then the DSVNG can be used to get the solution or output of the system for the linguistic terms. Hence DSVNG is a special case of interval based Neutrosophic graph which is proposed below.

4.2 Dombi Interval Valued Neutrosophic Graph (DIVNG)

A pair $G_N=(P,Q)$ is DIVNG, where $P=\left\langle\left[T_P^L,T_P^U\right],\left[I_P^L,I_P^U\right],\left[F_P^L,F_P^U\right]\right\rangle$, is an IVN set on $\mathbf{V}$ and $Q=\left\langle\left[T_Q^L,T_Q^U\right],\left[I_Q^L,I_Q^U\right],\left[F_Q^L,F_Q^U\right]\right\rangle$ is an IVN edge set on $\mathbf{E}$ satisfying the following conditions:

1. Degree of truth membership, indeterminacy membership and falsity membership of the element $x_i\in\mathbf{V}$ are defined by $T_P^L:V\to[0,1]$, $T_P^U:V\to[0,1]$, $I_P^L:V\to[0,1]$, $I_P^L:V\to[0,1]$, $I_P^U:V\to[0,1]$ and $F_P^L:V\to[0,1]$, $F_P^U:V\to[0,1]$ respectively and $0\le T_P(x_i)+I_P(x_i)+F_P(x_i)\le 3,\forall x_i\in\mathbf{V}, i=1,2,3,...,n$
2. Degree of truth membership, indeterminacy membership and falsity membership of the edge $(x_i,y_j)\in\mathbf{E}$ are denoted by the functions, $T_Q^L:V\times V\to[0,1]$, $T_Q^U:V\times V\to[0,1]$ $I_Q^L:V\times V\to[0,1]$, $I_Q^U:V\times V\to[0,1]$ and $F_Q^L:V\times V\to[0,1]$, $F_Q^U:V\times V\to[0,1]$ respectively and are defined by

- $T_Q^L\left(\{x_i,y_j\}\right)\le\dfrac{T_P^L(x_i)T_P^L(y_j)}{T_P^L(x_i)+T_P^L(y_j)-T_P^L(x_i)T_P^L(y_j)}$
- $T_Q^U\left(\{x_i,y_j\}\right)\le\dfrac{T_P^U(x_i)T_P^U(y_j)}{T_P^U(x_i)+T_P^U(y_j)-T_P^U(x_i)T_P^U(y_j)}$
- $I_Q^L\left(\{x_i,y_j\}\right)\ge\dfrac{I_P^L(x_i)+I_P^L(y_j)-2I_P^L(x_i)I_P^L(y_j)}{1-I_P^L(x_i)I_P^L(y_j)}$
- $I_Q^U\left(\{x_i,y_j\}\right)\ge\dfrac{I_P^U(x_i)+I_P^U(y_j)-2I_P^U(x_i)I_P^U(y_j)}{1-I_P^U(x_i)I_P^U(y_j)}$
- $F_Q^L\left(\{x_i,y_j\}\right)\ge\dfrac{F_P^L(x_i)+F_P^L(y_j)-2F_P^L(x_i)F_P^L(y_j)}{1-F_P^L(x_i)F_P^L(y_j)}$

- $F_Q^U\left(\{x_i, y_j\}\right) \geq \dfrac{F_P^U(x_i) + F_P^U(y_j) - 2F_P^U(x_i)F_P^U(y_j)}{1 - F_P^U(x_i)F_P^U(y_j)}$

Where $0 \leq T_Q\left(\{x_i, y_j\}\right) + I_Q\left(\{x_i, y_j\}\right) + F_Q\left(\{x_i, y_j\}\right) \leq 3, \forall \{x_i y_j\} \in \mathbf{E}(i, j = 1, 2, ..., n)$.

Numerical Example: Fig. 1. is an example of Dombi Interval Valued Neutrosophic Graph

$$P = \{\langle i,[0.5,0.7],[0.2,0.3],[0.1,0.3]\rangle, \langle j,[0.6,0.7],[0.2,0.4],[0.1,0.3]\rangle$$
$$\langle k,[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle\}$$
$$Q = \{\langle ij,[0.4,0.5],[0.4,0.5],[0.3,0.5]\rangle, \langle jk,[0.3,0.5],[0.3,0.5],[0.3,0.5]\rangle$$
$$\langle ik,[0.3,0.5],[0.3,0.5],[0.3,0.5]\rangle\}$$

Numerical Computation:

To find $T_Q^L(ij)$:

$T_Q^L(ij) \leq \dfrac{(0.5)(0.4)}{(0.5)+(0.4)-(0.5)(0.4)} \leq 0.4$, hence $T_Q^L(ij) = 0.4$

To find $I_Q^L(ij)$:

$I_Q^L(ij) \geq \dfrac{(0.2)+(0.2)-2(0.2)(0.2)}{1-(0.2)(0.2)} \geq 0.3$, hence $I_Q^L(ij) = 0.4$

To find $F_Q^L(ij)$:

$F_Q^L(ij) \geq \dfrac{(0.1)+(0.1)-2(0.1)(0.1)}{1-(0.1)(0.1)} \geq 0.2$, hence $I_Q^L(ij) = 0.3$

Similarly other values can be found.

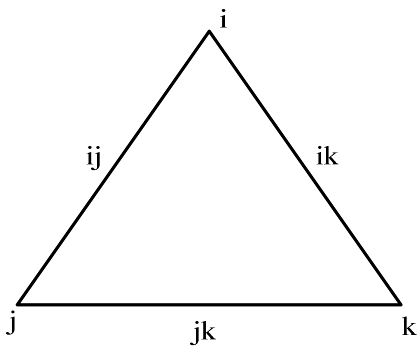

Fig.2. Dombi Interval Valued Neutrosophic Graph

- From the definition and numerical examples of DSVNG and DIVNG, it is found that Dombi Fuzzy Graph is a special case of DSVNG and DIVNG.

4.3Definition: Cartesian product of Dombi Interval Valued Neutrosophic Graphs

Consider λ_i, a Neutrosophic fuzzy subset of $\mathbf{V}_i$ and δ_i, a fuzzy subset of $\mathbf{E}_i, i = 1, 2$. Let $\mathbf{G}_{\mathbf{N}1}(\lambda_1, \delta_1)$ and $\mathbf{G}_{\mathbf{N}2}(\lambda_2, \delta_2)$ be two Dombi Neutrosophic Fuzzy Graphs of the crisp graphs $G_1^*(V_1, E_1)$ and $G_2^*(V_2, E_2)$ respectively and are defined by

$\left(\lambda_1^L \times \lambda_2^L\right)(x_1, x_2) = \dfrac{\lambda_1^L(x_1)\lambda_2^L(x_2)}{\lambda_1^L(x_1)+\lambda_2^L(x_2)-\lambda_1^L(x_1)\lambda_2^L(x_2)}$, for all $(x_1, x_2) \in V_1 \times V_2$ and

$\left(\delta_1^L \times \delta_2^L\right)((x, x_2)(x, y_2)) = \dfrac{\lambda_1^L(x)\lambda_2^L(x_2 y_2)}{\lambda_1^L(x)+\lambda_2^L(x_2 y_2)-\lambda_1^L(x)\lambda_2^L(x_2 y_2)}$, for all $x \in V_1$, $x_2 y_2 \in \mathbf{E}_2$

$\left(\delta_1^L \times \delta_2^L\right)((x_1, z)(y_1, z)) = \dfrac{\lambda_2^L(z)\delta_1^L(x_1 y_1)}{\lambda_2^L(z)+\delta_1^L(x_1 y_1)-\lambda_2^L(z)\delta_1^L(x_1 y_1)}$, for all $z \in \mathbf{V}_2$, $x_1 y_1 \in E_1$

Similarly for Indeterminacy and Falsity memberships with upper and lower membership values.

4.3.1Proposition

Let $\mathbf{G_{N1}}$ and $\mathbf{G_{N2}}$ be the Dombi IVN edge graphs of G_1 and G_2 respectively. Then Cartesian product of two Dombi IVN edge graphs is the Dombi Interval Valued Neutrosophic edge graph.

Proof:

Let $\mathbf{E} = \{(x, x_2)(x, y_2) / x \in V_1, x_2 y_2 \in \mathbf{E}_2\} \cup \{(x_1, z)(y_1, z) / z \in V_2, x_1 y_1 \in \mathbf{E}_1\}$

Consider $x \in V_1$, $x_2 y_2 \in E_2$

By the definition of Cartesian product of IVNG **(Broumi et al. 2016)**

$\left(T_{Q_1}^L \times T_{Q_2}^L\right)((x, x_2)(x, y_2))$

$= \min\left(T_{P_1}^L(x), T_{Q_2}^L(x_2 y_2)\right) = TN\left(1, T_{Q_2}^L(x_2 y_2)\right) \le \dfrac{T_{P_2}^L(x_2) T_{P_2}^L(y_2)}{T_{P_2}^L(x_2)+T_{P_2}^L(y_2)-T_{P_2}^L(x_2) T_{P_2}^L(y_2)}$

$\le \min\left(T_{P_1}^L(x), \min\left(T_{P_2}^L(x_2), T_{P_2}^L(y_2)\right)\right) = \min\left(\min\left(T_{P_1}^L(x), T_{P_2}^L(x_2)\right), \min\left(T_{P_1}^L(x), T_{A_2}^L(y_2)\right)\right)$

$= \min\left(\left(T_{P_1}^L \times T_{P_2}^L\right)(x, x_2), \left(T_{P_1}^L \times T_{P_2}^L\right)(x, y_2)\right)$

$\le \dfrac{\left(T_{P_1}^L \times T_{P_2}^L\right)((x, x_2))\left(T_{P_1}^L \times T_{P_2}^L\right)((x, y_2))}{\left(T_{P_1}^L \times T_{P_2}^L\right)((x, x_2))+\left(T_{P_1}^L \times T_{P_2}^L\right)((x, y_2))-\left(T_{P_1}^L \times T_{P_2}^L\right)((x, x_2))\left(T_{P_1}^L \times T_{P_2}^L\right)((x, y_2))}$

$\left(T_{Q_1}^U \times T_{Q_2}^U\right)((x, x_2)(x, y_2))$

$= \min\left(T_{P_1}^U(x), T_{Q_2}^U(x_2 y_2)\right) = TN\left(1, T_{Q_2}^U(x_2 y_2)\right) \le \dfrac{T_{P_2}^U(x_2) T_{P_2}^U(y_2)}{T_{P_2}^U(x_2)+T_{P_2}^U(y_2)-T_{P_2}^U(x_2) T_{P_2}^U(y_2)}$

$\le \min\left(T_{P_1}^U(x), \min\left(T_{P_2}^U(x_2), T_{P_2}^U(y_2)\right)\right) = \min\left(\min\left(T_{P_1}^U(x), T_{P_2}^U(x_2)\right), \min\left(T_{P_1}^U(x), T_{A_2}^U(y_2)\right)\right)$

$= \min\left(\left(T_{P_1}^U \times T_{P_2}^U\right)(x, x_2), \left(T_{P_1}^U \times T_{P_2}^U\right)(x, y_2)\right)$

$= \dfrac{\left(T_{P_1}^U \times T_{P_2}^U\right)((x, x_2))\left(T_{P_1}^U \times T_{P_2}^U\right)((x, y_2))}{\left(T_{P_1}^U \times T_{P_2}^U\right)((x, x_2))+\left(T_{P_1}^U \times T_{P_2}^U\right)((x, y_2))-\left(T_{P_1}^U \times T_{P_2}^U\right)((x, x_2))\left(T_{P_1}^U \times T_{P_2}^U\right)((x, y_2))}$

$\left(I_{Q_1}^L \times I_{Q_2}^L\right)((x, x_2)(x, y_2)) = \max\left(I_{P_1}^L(x), I_{Q_2}^L(x_2 y_2)\right)$

$\ge TCN\left(I_{P_1}^L(x), I_{Q_2}^L(x_2 y_2)\right) = \max\left(\max\left(I_{P_1}^L(x), I_{P_2}^L(x_2)\right), \max\left(I_{P_1}^L(x), T_{P_2}^L(y_2)\right)\right) = \max\left(\left(I_{P_1}^L \times I_{P_2}^L\right)((x, x_2)), \left(I_{P_1}^L \times I_{P_2}^L\right)((x, y_2))\right)$

$= \dfrac{\left(I_{P_1}^L \times I_{P_2}^L\right)((x, x_2))+\left(I_{P_1}^L \times I_{P_2}^L\right)((x, y_2))-2\left(I_{P_1}^L \times I_{P_2}^L\right)((x, x_2))\left(I_{P_1}^L \times I_{P_2}^L\right)((x, y_2))}{1-\left(I_{P_1}^L \times I_{P_2}^L\right)((x, x_2)).\left(I_{P_1}^L \times I_{P_2}^L\right)((x, y_2))}$

$\left(I_{Q_1}^U \times I_{Q_2}^U\right)((x, x_2)(x, y_2)) = \max\left(I_{P_1}^U(x), I_{Q_2}^U(x_2 y_2)\right)$

$\ge TCN\left(I_{P_1}^U(x), I_{Q_2}^U(x_2 y_2)\right) = \max\left(\max\left(I_{P_1}^U(x), I_{P_2}^U(x_2)\right), \max\left(I_{P_1}^U(x), T_{P_2}^U(y_2)\right)\right) = \max\left(\left(I_{P_1}^U \times I_{P_2}^U\right)((x, x_2)), \left(I_{P_1}^U \times I_{P_2}^U\right)((x, y_2))\right)$

$$=\frac{\left(I_{P_1}^U\times I_{P_2}^U\right)((x,x_2))+\left(I_{P_1}^U\times I_{P_2}^U\right)((x,y_2))-2\left(I_{P_1}^U\times I_{P_2}^U\right)((x,x_2))\left(I_{P_1}^U\times I_{P_2}^U\right)((x,y_2))}{1-\left(I_{P_1}^U\times I_{P_2}^U\right)((x,x_2)).\left(I_{P_1}^U\times I_{P_2}^U\right)((x,y_2))}$$

$$\left(F_{Q_1}^L\times F_{Q_2}^L\right)((x,x_2)(x,y_2))=\max\left(F_{P_1}^L(x),F_{Q_2}^L(x_2y_2)\right)$$
$$\geq TCN\left(F_{P_1}^L(x),F_{Q_2}^L(x_2y_2)\right)=\max\left(\max\left(F_{P_1}^L(x),F_{P_2}^L(x_2)\right),\max\left(F_{P_1}^L(x),F_{P_2}^L(y_2)\right)\right)=\max\left(\left(F_{P_1}^L\times F_{P_2}^L\right)((x,x_2)),\left(F_{P_1}^L\times F_{P_2}^L\right)((x,y_2))\right)$$
$$\geq\frac{\left(F_{P_1}^L\times F_{P_2}^L\right)((x,x_2))+\left(F_{P_1}^L\times F_{P_2}^L\right)((x,y_2))-2\left(F_{P_1}^L\times F_{P_2}^L\right)((x,x_2))\left(F_{P_1}^L\times F_{P_2}^L\right)((x,y_2))}{1-\left(F_{P_1}^L\times F_{P_2}^L\right)((x,x_2)).\left(F_{P_1}^L\times F_{P_2}^L\right)((x,y_2))}$$

$$\left(F_{Q_1}^U\times F_{Q_2}^U\right)((x,x_2)(x,y_2))=\max\left(F_{P_1}^U(x),F_{Q_2}^U(x_2y_2)\right)$$
$$\geq TCN\left(F_{P_1}^U(x),F_{Q_2}^U(x_2y_2)\right)=\max\left(\max\left(F_{P_1}^U(x),F_{P_2}^U(x_2)\right),\max\left(F_{P_1}^U(x),F_{P_2}^U(y_2)\right)\right)=\max\left(\left(F_{P_1}^U\times F_{P_2}^U\right)((x,x_2)),\left(F_{P_1}^U\times F_{P_2}^U\right)((x,y_2))\right)$$
$$\geq\frac{\left(F_{P_1}^U\times F_{P_2}^U\right)((x,x_2))+\left(F_{P_1}^U\times F_{P_2}^U\right)((x,y_2))-2\left(F_{P_1}^U\times F_{P_2}^U\right)((x,x_2))\left(F_{P_1}^U\times F_{P_2}^U\right)((x,y_2))}{1-\left(F_{P_1}^U\times F_{P_2}^U\right)((x,x_2)).\left(F_{P_1}^U\times F_{P_2}^U\right)((x,y_2))}$$

Consider $z\in V_2$, $x_1y_1\in E_1$

$$\left(T_{Q_1}^L\times T_{Q_2}^L\right)((x_1,z)(y_1,z))$$
$$=\min\left(T_{Q_1}^L(x_1y_1),T_{P_2}^L(z)\right)=TN\left(T_{Q_1}^L(x_1y_1),1\right)=T_{Q_1}^L(x_1y_1)\leq\frac{T_{P_1}^L(x_1)T_{P_1}^L(y_1)}{T_{P_1}^L(x_1)+T_{P_1}^L(y_1)-T_{P_1}^L(x_1).T_{P_1}^L(y_1)}$$
$$\leq\min\left(\min\left(T_{P_1}^L(x_1),T_{P_1}^L(y_1)\right),T_{P_2}^L(z)\right)=\min\left(\min\left(T_{P_1}^L(x_1),T_{P_2}^L(z)\right),\min\left(T_{P_1}^L(y_1),T_{P_2}^L(z)\right)\right)$$
$$=\min\left(\left(T_{P_1}^L\times T_{P_2}^L\right)(x_1,z),\left(T_{P_1}^L\times T_{P_2}^L\right)(y_1,z)\right)$$
$$\leq\frac{\left(T_{P_1}^L\times T_{P_2}^L\right)(x_1,z)\left(T_{P_1}^L\times T_{P_2}^L\right)(y_1,z)}{\left(T_{P_1}^L\times T_{P_2}^L\right)(x_1,z)+\left(T_{P_1}^L\times T_{P_2}^L\right)(y_1,z)-\left(T_{P_1}^L\times T_{P_2}^L\right)(x_1,z)\left(T_{P_1}^L\times T_{P_2}^L\right)(y_1,z)}$$

$$\left(T_{Q_1}^U\times T_{Q_2}^U\right)((x_1,z)(y_1,z))$$
$$=\min\left(T_{Q_1}^U(x_1y_1),T_{P_2}^U(z)\right)=TN\left(T_{Q_1}^U(x_1y_1),1\right)=T_{Q_1}^U(x_1y_1)\leq\frac{T_{P_1}^U(x_1)T_{P_1}^U(y_1)}{T_{P_1}^U(x_1)+T_{P_1}^U(y_1)-T_{P_1}^U(x_1).T_{P_1}^U(y_1)}$$
$$\leq\min\left(\min\left(T_{P_1}^U(x_1),T_{P_1}^U(y_1)\right),T_{P_2}^U(z)\right)=\min\left(\min\left(T_{P_1}^U(x_1),T_{P_2}^U(z)\right),\min\left(T_{P_1}^U(y_1),T_{P_2}^U(z)\right)\right)$$
$$=\min\left(\left(T_{P_1}^U\times T_{P_2}^U\right)(x_1,z),\left(T_{P_1}^U\times T_{P_2}^U\right)(y_1,z)\right)$$
$$\leq\frac{\left(T_{P_1}^U\times T_{P_2}^U\right)(x_1,z)\left(T_{P_1}^U\times T_{P_2}^U\right)(y_1,z)}{\left(T_{P_1}^U\times T_{P_2}^U\right)(x_1,z)+\left(T_{P_1}^U\times T_{P_2}^U\right)(y_1,z)-\left(T_{P_1}^U\times T_{P_2}^U\right)(x_1,z)\left(T_{P_1}^U\times T_{P_2}^U\right)(y_1,z)}$$

$$\left(I_{Q_1}^L\times I_{Q_2}^L\right)((x_1,z)(y_1,z))$$
$$=\max\left(T_{Q_1}^L(x_1y_1),T_{P_2}^L(z)\right)\geq\max\left(\max\left(I_{P_1}^L(x_1),I_{P_1}^L(y_1)\right),I_{P_2}^L(z)\right)=\max\left(\max\left(I_{P_1}^L(x_1),I_{P_2}^L(z)\right),\max\left(T_{P_1}^L(y_1),T_{P_2}^L(z)\right)\right)$$
$$=\max\left(\left(I_{P_1}^L\times I_{P_2}^L\right)(x_1,z),\left(I_{P_1}^L\times I_{P_2}^L\right)(y_1,z)\right)$$
$$\geq\frac{\left(I_{P_1}^L\times I_{P_2}^L\right)(x_1,z)+\left(I_{P_1}^L\times I_{P_2}^L\right)(y_1,z)-2\left(I_{P_1}^L\times I_{P_2}^L\right)(x_1,z)\left(I_{P_1}^L\times I_{P_2}^L\right)(y_1,z)}{1-\left(I_{P_1}^L\times I_{P_2}^L\right)(x_1,z)\left(I_{P_1}^L\times I_{P_2}^L\right)(y_1,z)}$$

$$\left(I_{Q_1}^U \times I_{Q_2}^U\right)((x_1,z)(y_1,z))$$
$$=\max\left(T_{Q_1}^U(x_1y_1),T_{P_2}^U(z)\right) \geq \max\left(\max\left(I_{P_1}^U(x_1),I_{P_1}^U(y_1)\right),I_{P_2}^U(z)\right)$$
$$=\max\left(\max\left(I_{P_1}^U(x_1),I_{P_2}^U(z)\right),\max\left(T_{P_1}^U(y_1),T_{P_2}^U(z)\right)\right)=\max\left(\left(I_{P_1}^U\times I_{P_2}^U\right)(x_1,z),\left(I_{P_1}^U\times I_{P_2}^U\right)(y_1,z)\right)$$
$$\geq\frac{\left(I_{P_1}^U\times I_{P_2}^U\right)(x_1,z)+\left(I_{P_1}^U\times I_{P_2}^U\right)(y_1,z)-2\left(I_{P_1}^U\times I_{P_2}^U\right)(x_1,z)\left(I_{P_1}^U\times I_{P_2}^U\right)(y_1,z)}{1-\left(I_{P_1}^U\times I_{P_2}^U\right)(x_1,z)\left(I_{P_1}^U\times I_{P_2}^U\right)(y_1,z)}$$

$$\left(F_{Q_1}^L \times F_{Q_2}^L\right)((x_1,z)(y_1,z))$$
$$=\max\left(F_{Q_1}^L(x_1y_1),F_{P_2}^L(z)\right) \geq \max\left(\max\left(F_{P_1}^L(x_1),F_{P_1}^L(y_1)\right),F_{P_2}^L(z)\right)$$
$$=\max\left(\max\left(F_{P_1}^L(x_1),F_{P_2}^L(z)\right),\max\left(F_{P_1}^L(y_1),F_{P_2}^L(z)\right)\right)=\max\left(\left(F_{P_1}^L\times F_{P_2}^L\right)(x_1,z),\left(F_{P_1}^L\times F_{P_2}^L\right)(y_1,z)\right)$$
$$\geq\frac{\left(F_{P_1}^L\times F_{P_2}^L\right)(x_1,z)+\left(F_{P_1}^L\times F_{P_2}^L\right)(y_1,z)-2\left(F_{P_1}^L\times F_{P_2}^L\right)(x_1,z)\left(F_{P_1}^L\times F_{P_2}^L\right)(y_1,z)}{1-\left(F_{P_1}^L\times F_{P_2}^L\right)(x_1,z)\left(F_{P_1}^L\times F_{P_2}^L\right)(y_1,z)}$$

$$\left(F_{Q_1}^U \times F_{Q_2}^U\right)((x_1,z)(y_1,z))=\max\left(F_{Q_1}^U(x_1y_1),F_{P_2}^U(z)\right) \geq \max\left(\max\left(F_{P_1}^U(x_1),F_{P_1}^U(y_1)\right),F_{P_2}^U(z)\right)$$
$$=\max\left(\max\left(F_{P_1}^U(x_1),F_{P_2}^U(z)\right),\max\left(F_{P_1}^U(y_1),F_{P_2}^U(z)\right)\right)=\max\left(\left(F_{P_1}^U\times F_{P_2}^U\right)(x_1,z),\left(F_{P_1}^U\times F_{P_2}^U\right)(y_1,z)\right)$$
$$\geq\frac{\left(F_{P_1}^U\times F_{P_2}^U\right)(x_1,z)+\left(F_{P_1}^U\times F_{P_2}^U\right)(y_1,z)-2\left(F_{P_1}^U\times F_{P_2}^U\right)(x_1,z)\left(F_{P_1}^U\times F_{P_2}^U\right)(y_1,z)}{1-\left(F_{P_1}^U\times F_{P_2}^U\right)(x_1,z)\left(F_{P_1}^U\times F_{P_2}^U\right)(y_1,z)}$$

Note: Cartesian product of two IVNEGs need not be IVNEG.

Numerical Example:

Consider two crisp graphs $G_1=(A_1,B_1)$ and $G_2=(A_2,B_2)$, where $V_1=\{a,b\}$, $V_2=\{c,d\}$, $E_1=\{a,b\}$ and $E_2=\{c,d\}$. Consider two Interval Valued Neutrosophic Graphs $\mathbf{G_{N1}}=(P_1,Q_1)$ and $\mathbf{G_{N2}}=(P_2,Q_2)$

$P_1=\{\langle i,[0.5,0.7],[0.2,0.3],[0.1,0.3]\rangle,\langle j,[0.6,0.7],[0.2,0.4],[0.1,0.3]\rangle\}$

$Q_1=\{\langle ij,[0.3,0.6],[0.2,0.4],[0.2,0.4]\rangle\}$ **(Broumi et al.2016)**

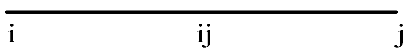

Fig. 3. 1 IVNG $\mathbf{G_{N1}}$

$P_2=\{\langle k,[0.4,0.6],[0.2,0.3],[0.1,0.3]\rangle,\langle l,[0.4,0.7],[0.2,0.4],[0.1,0.3]\rangle\}$

$Q_2=\{\langle kl,[0.3,0.5],[0.4,0.5],[0.3,0.5]\rangle\}$

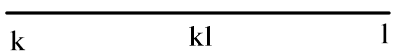

Fig. 4. IVNG $\mathbf{G_{N2}}$

Cartesian product of $\mathbf{G_{N1}}$ and $\mathbf{G_{N2}}$ $(\mathbf{G_{N1}}\times\mathbf{G_{N2}})$

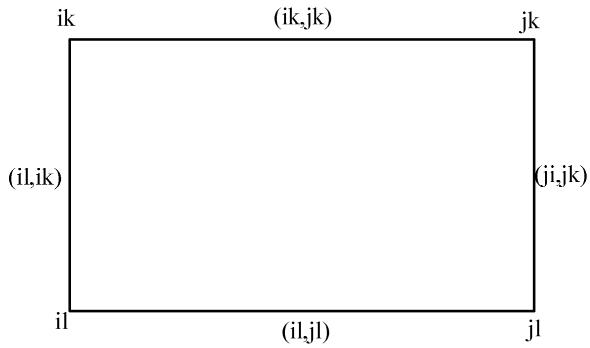

Fig. 5. Cartesian product of DIVNGs $(\mathbf{G_{N1}} \times \mathbf{G_{N2}})$

Interval Neutrosophic Vertices and Edges

$ik = \langle [0.4, 0.6], [0.2, 0.3], [0.1, 0.3] \rangle$, $jk = \langle [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] \rangle$

$il = \langle [0.4, 0.7], [0.2, 0.4], [0.1, 0.3] \rangle$, $jl = \langle [0.4, 0.7], [0.2, 0.4], [0.1, 0.3] \rangle$

$(ik, jk) = \langle [0.3, 0.6], [0.2, 0.4], [0.2, 0.4] \rangle$, $(jl, jk) = \langle [0.3, 0.5], [0.4, 0.5], [0.3, 0.5] \rangle$

$(il, jl) = \langle [0.3, 0.6], [0.2, 0.4], [0.2, 0.4] \rangle$, $(il, ik) = \langle [0.3, 0.5], [0.4, 0.5], [0.3, 0.5] \rangle$

To find ik:

$$\left(T_{P_1}^L \times T_{P_2}^L\right)(i,k) = \min\left(T_{P_1}^L(i), T_{P_2}^L(k)\right) = \min(0.5, 0.4) = 0.4$$

$$\left(I_{P_1}^L \times I_{P_2}^L\right)(i,k) = \max\left(I_{P_1}^L(i), I_{P_2}^L(k)\right) = \max(0.2, 0.2) = 0.2$$

$$\left(F_{P_1}^L \times F_{P_2}^L\right)(i,k) = \min\left(F_{P_1}^L(i), F_{P_2}^L(k)\right) = \max(0.1, 0.1) = 0.1$$

Similarly for other values.

Interval Neutrosophic Edges:

To find (ik, jk):

$$\left(T_{Q_1}^L \times T_{Q_2}^L\right)\left((i,k)(j,k)\right) = \min\left(T_{Q_1}^L(ij), T_{P_2}^L(k)\right) = \min(0.3, 0.4) = 0.3$$

$$\left(I_{Q_1}^L \times I_{Q_2}^L\right)\left((i,k)(j,k)\right) = \max\left(I_{Q_1}^L(ij), I_{P_2}^L(k)\right) = \max(0.2, 0.2) = 0.2$$

$$\left(F_{Q_1}^L \times F_{Q_2}^L\right)\left((i,k)(j,k)\right) = \max\left(F_{Q_1}^L(ij), F_{P_2}^L(k)\right) = \max(0.2, 0.1) = 0.2$$

To check the condition of Cartesian product of Dombi Interval Neutrosophic Edge Graph

$$\left(T_{Q_1}^L \times T_{Q_2}^L\right)\left((i,k)(j,k)\right) \le \frac{\left(T_{P_1}^L \times T_{P_2}^L\right)\left((i,k)\right)\left(T_{P_1}^L \times T_{P_2}^L\right)\left((j,k)\right)}{\left(T_{P_1}^L \times T_{P_2}^L\right)\left((i,k)\right) + \left(T_{P_1}^L \times T_{P_2}^L\right)\left((j,k)\right) - \left(T_{P_1}^L \times T_{P_2}^L\right)\left((i,k)\right)\left(T_{P_1}^L \times T_{P_2}^L\right)\left((j,k)\right)}$$

$0.3 \le \dfrac{(0.4)(0.4)}{(0.4)+(0.4)-(0.4)(0.4)} = 0.3$, hence satisfied.

$$\left(I_{Q_1}^L \times I_{Q_2}^L\right)((i,k)(j,k)) \geq \frac{\left(I_{P_1}^L \times I_{P_2}^L\right)((i,k)) + \left(I_{P_1}^L \times I_{P_2}^L\right)((j,k)) - 2\left(I_{P_1}^L \times I_{P_2}^L\right)((i,k))\left(I_{P_1}^L \times I_{P_2}^L\right)((j,k))}{1-\left(I_{P_1}^L \times I_{P_2}^L\right)((i,k))\left(I_{P_1}^L \times I_{P_2}^L\right)((j,k))}$$

$$0.2 \geq \frac{(0.2)+(0.2)-2(0.2)(0.2)}{1-(0.2)(0.2)} = 0.33$$, hence not satisfied

$$\left(F_{Q_1}^L \times F_{Q_2}^L\right)((i,k)(j,k)) \geq \frac{\left(F_{P_1}^L \times F_{P_2}^L\right)((i,k)) + \left(F_{P_1}^L \times F_{P_2}^L\right)((j,k)) - 2\left(F_{P_1}^L \times F_{P_2}^L\right)((i,k))\left(F_{P_1}^L \times F_{P_2}^L\right)((j,k))}{1-\left(F_{P_1}^L \times F_{P_2}^L\right)((i,k))\left(F_{P_1}^L \times F_{P_2}^L\right)((j,k))}$$

$$0.2 \geq \frac{(0.1)+(0.1)-2(0.1)(0.1)}{1-(0.1)(0.1)} = 0.2$$, hence satisfied.

Similarly for other edges.
Hence Cartesian product of two Dombi Interval Valued Neutrosophic Edge graphs need not be a DIVNEG.

4.4Definition: Composite product of Dombi Interval Valued Neutrosophic Graphs

Consider λ_i , a Neutrosophic fuzzy subset of $\mathbf{V}_i$ and δ_i , a fuzzy subset of $\mathbf{E}_i, i = 1,2$. Let $\mathbf{G_{N1}}(\lambda_1,\delta_1)$ and $\mathbf{G_{N2}}(\lambda_2,\delta_2)$ be two Dombi Interval Valued Neutrosophic Graphs of the crisp graphs $G_1^*(V_1,E_1)$ and $G_2^*(V_2,E_2)$ respectively and are defined by

$$\left(\lambda_1^L \circ \lambda_2^L\right)(x_1,x_2) = \frac{\lambda_1^L(x_1)\lambda_2^L(x_2)}{\lambda_1^L(x_1)+\lambda_2^L(x_2)-\lambda_1^L(x_1)\lambda_2^L(x_2)}$$, for all $(x_1,x_2) \in \mathrm{V}_1 \times \mathrm{V}_2$ and

$$\left(\delta_1^L \circ \delta_2^L\right)((x,x_2)(x,y_2)) = \frac{\lambda_1^L(x)\lambda_2^L(x_2y_2)}{\lambda_1^L(x)+\lambda_2^L(x_2y_2)-\lambda_1^L(x)\lambda_2^L(x_2y_2)}$$, for all $x \in \mathrm{V}_1$, $x_2y_2 \in \mathbf{E}_2$

$$\left(\delta_1^L \circ \delta_2^L\right)((x_1,z)(y_1,z)) = \frac{\lambda_2^L(z)\delta_1^L(x_1y_1)}{\lambda_2^L(z)+\delta_1^L(x_1y_1)-\lambda_2^L(z)\delta_1^L(x_1y_1)}$$, for all $z \in \mathbf{V}_2$, $x_1y_1 \in \mathrm{E}_1$

$$\left(\delta_1^L \circ \delta_2^L\right)((x_1,x_2)(y_1,y_2))$$

$$= \frac{\lambda_2^L(x_2)\lambda_2^L(y_2)\delta_1^L(x_1y_1)}{\lambda_2^L(x_2)\lambda_2^L(y_2)+\lambda_2^L(y_2)\delta_1^L(x_1y_1)+\lambda_2^L(x_2)\delta_1^L(x_1y_1)-2\lambda_2^L(x_2)\lambda_2^L(y_2)\delta_1^L(x_1y_1)},$$

for all $x_1y_1 \in \mathrm{E}_1$ and $x_2 \neq y_2$
Similarly for Indeterminacy and Falsity memberships with upper and lower membership values.

4.4.1 Proposition

The composite product of two Dombi Interval Valued Neutrosophic Edge graphs (DIVNEGs) of G_1 and G_2 is the DIVNEG.

Proof:

From the proof of 4.1.1

$$\left(T_{Q_1}^L \times T_{Q_2}^L\right)((x,x_2)(x,y_2)) \leq \frac{\left(T_{P_1}^L \times T_{P_2}^L\right)((x,x_2))\left(T_{P_1}^L \times T_{P_2}^L\right)((x,y_2))}{\left(T_{P_1}^L \times T_{P_2}^L\right)((x,x_2))+\left(T_{P_1}^L \times T_{P_2}^L\right)((x,y_2))-\left(T_{P_1}^L \times T_{P_2}^L\right)((x,x_2))\left(T_{P_1}^L \times T_{P_2}^L\right)((x,y_2))}$$

$$\left(T_{Q_1}^U \times T_{Q_2}^U\right)((x,x_2)(x,y_2)) \leq \frac{\left(T_{P_1}^U \times T_{P_2}^U\right)((x,x_2))\left(T_{P_1}^U \times T_{P_2}^U\right)((x,y_2))}{\left(T_{P_1}^U \times T_{P_2}^U\right)((x,x_2))+\left(T_{P_1}^U \times T_{P_2}^U\right)((x,y_2))-\left(T_{P_1}^U \times T_{P_2}^U\right)((x,x_2))\left(T_{P_1}^U \times T_{P_2}^U\right)((x,y_2))}$$

$$\left(I_{Q_1}^L \times I_{Q_2}^L\right)((x,x_2)(x,y_2))$$

$$\geq \frac{\left(I_{P_1}^L \times I_{P_2}^L\right)((x,x_2))+\left(I_{P_1}^L \times I_{P_2}^L\right)((x,y_2))-2\left(I_{P_1}^L \times I_{P_2}^L\right)((x,x_2))\left(I_{P_1}^L \times I_{P_2}^L\right)((x,y_2))}{1-\left(I_{P_1}^L \times I_{P_2}^L\right)((x,x_2)).\left(I_{P_1}^L \times I_{P_2}^L\right)((x,y_2))}$$

$$\left(I_{Q_1}^U \times I_{Q_2}^U\right)((x,x_2)(x,y_2))$$

$$\geq \frac{\left(I_{P_1}^U \times I_{P_2}^U\right)((x,x_2)) + \left(I_{P_1}^U \times I_{P_2}^U\right)((x,y_2)) - 2\left(I_{P_1}^U \times I_{P_2}^U\right)((x,x_2))\left(I_{P_1}^U \times I_{P_2}^U\right)((x,y_2))}{1-\left(I_{P_1}^U \times I_{P_2}^U\right)((x,x_2)).\left(I_{P_1}^U \times I_{P_2}^U\right)((x,y_2))}$$

$$\left(F_{Q_1}^L \times F_{Q_2}^L\right)((x,x_2)(x,y_2))$$

$$\geq \frac{\left(F_{P_1}^L \times F_{P_2}^L\right)((x,x_2)) + \left(F_{P_1}^L \times F_{P_2}^L\right)((x,y_2)) - 2\left(F_{P_1}^L \times F_{P_2}^L\right)((x,x_2))\left(F_{P_1}^L \times F_{P_2}^L\right)((x,y_2))}{1-\left(F_{P_1}^L \times F_{P_2}^L\right)((x,x_2)).\left(F_{P_1}^L \times F_{P_2}^L\right)((x,y_2))}$$

$$\left(F_{Q_1}^U \times F_{Q_2}^U\right)((x,x_2)(x,y_2))$$

$$\geq \frac{\left(F_{P_1}^U \times F_{P_2}^U\right)((x,x_2)) + \left(F_{P_1}^U \times F_{P_2}^U\right)((x,y_2)) - 2\left(F_{P_1}^U \times F_{P_2}^U\right)((x,x_2))\left(F_{P_1}^U \times F_{P_2}^U\right)((x,y_2))}{1-\left(F_{P_1}^U \times F_{P_2}^U\right)((x,x_2)).\left(F_{P_1}^U \times F_{P_2}^U\right)((x,y_2))}$$

Similarly for $z \in V_2$, $x_1y_1 \in E_1$.

Now consider $x_1y_1 \in E_1$, $x_2 \neq y_2$

$\left(T_{Q_1}^L \circ T_{Q_2}^L\right)((x_1,x_2)(y_1,y_2))$ **(Broumi et al. 2016)**

$$\leq \min\left(T_{P_2}^L(x_2), T_{P_2}^L(y_2), T_{Q_1}^L(x_1y_1)\right)$$

$$\leq \min\left(T_{P_2}^L(x_2), T_{P_2}^L(y_2), \min\left(T_{P_1}^L(x_1), T_{P_1}^L(y_1)\right)\right) = \min\left(\min\left(T_{P_1}^L(x_1), T_{P_2}^L(x_2)\right), \min\left(T_{P_1}^L(y_1), T_{P_1}^L(y_2)\right)\right)$$

$$= \min\left(\left(T_{P_1}^L \circ T_{P_2}^L\right)(x_1,x_2), \left(T_{P_1}^L \circ T_{P_2}^L\right)(y_1,y_2)\right)$$

$$= TN\left(\left(T_{P_1}^L \circ T_{P_2}^L\right)((x_1,x_2)), \left(T_{P_1}^L \circ T_{P_2}^L\right)((y_1,y_2))\right)$$

$$\leq \frac{\left(T_{P_1}^L \circ T_{P_2}^L\right)((x_1,x_2))\left(T_{P_1}^L \circ T_{P_2}^L\right)((y_1,y_2))}{\left(T_{P_1}^L \circ T_{P_2}^L\right)((x_1,x_2)) + \left(T_{P_1}^L \circ T_{P_2}^L\right)((y_1,y_2)) - \left(T_{P_1}^L \circ T_{P_2}^L\right)((x_1,x_2))\left(T_{P_1}^L \circ T_{P_2}^L\right)((y_1,y_2))}$$

$$\left(T_{Q_1}^U \circ T_{Q_2}^U\right)((x_1,x_2)(y_1,y_2))$$

$$= \min\left(T_{P_2}^U(x_2), T_{P_2}^U(y_2), T_{Q_1}^U(x_1y_1)\right)$$

$$\leq \min\left(T_{P_2}^U(x_2), T_{P_2}^U(y_2), \min\left(T_{P_1}^U(x_1), T_{P_1}^U(y_1)\right)\right) = \min\left(\min\left(T_{P_1}^U(x_1), T_{P_2}^U(x_2)\right), \min\left(T_{P_1}^U(y_1), T_{P_1}^U(y_2)\right)\right)$$

$$= \min\left(\left(T_{P_1}^U \circ T_{P_2}^U\right)(x_1,x_2), \left(T_{P_1}^U \circ T_{P_2}^U\right)(y_1,y_2)\right)$$

$$= TN\left(\left(T_{P_1}^U \circ T_{P_2}^U\right)((x_1,x_2)), \left(T_{P_1}^U \circ T_{P_2}^U\right)((y_1,y_2))\right)$$

$$\leq \frac{\left(T_{P_1}^U \circ T_{P_2}^U\right)((x_1,x_2))\left(T_{P_1}^U \circ T_{P_2}^U\right)((y_1,y_2))}{\left(T_{P_1}^U \circ T_{P_2}^U\right)((x_1,x_2)) + \left(T_{P_1}^U \circ T_{P_2}^U\right)((y_1,y_2)) - \left(T_{P_1}^U \circ T_{P_2}^U\right)((x_1,x_2))\left(T_{P_1}^U \circ T_{P_2}^U\right)((y_1,y_2))}$$

$$\left(I_{Q_1}^L \circ I_{Q_2}^L\right)((x_1,x_2)(y_1,y_2))$$

$$= \max\left(I_{P_2}^L(x_2), I_{P_2}^L(y_2), I_{Q_1}^L(x_1y_1)\right)$$

$$\geq \max\left(I_{P_2}^L(x_2), I_{P_2}^L(y_2), \max\left(I_{P_1}^L(x_1), I_{P_1}^L(y_1)\right)\right) = \max\left(\max\left(I_{P_1}^L(x_1), I_{P_2}^L(x_2)\right), \max\left(I_{P_1}^L(y_1), I_{P_1}^L(y_2)\right)\right)$$

$$= \max\left(\left(I_{P_1}^L \circ I_{P_2}^L\right)(x_1,x_2), \left(I_{P_1}^L \circ I_{P_2}^L\right)(y_1,y_2)\right)$$

$$= TCN\left(\left(I_{P_1}^L \circ I_{P_2}^L\right)((x_1,x_2)), \left(I_{P_1}^L \circ I_{P_2}^L\right)((y_1,y_2))\right)$$

$$\geq \frac{\left(I_{P_1}^L \circ I_{P_2}^L\right)((x_1,x_2)) + \left(I_{P_1}^L \circ I_{P_2}^L\right)((y_1,y_2)) - 2\left(I_{P_1}^L \circ I_{P_2}^L\right)((x_1,x_2)).\left(I_{P_1}^L \circ I_{P_2}^L\right)((y_1,y_2))}{1-\left(I_{P_1}^L \circ I_{P_2}^L\right)((x_1,x_2)).\left(I_{P_1}^L \circ I_{P_2}^L\right)((y_1,y_2))}$$

$$\left(I_{Q_1}^U \circ I_{Q_2}^U\right)((x_1,x_2)(y_1,y_2))$$

$$=\frac{I_{P_2}^U(x_2)I_{P_2}^U(y_2)I_{Q_1}^U(x_1y_1)}{I_{P_2}^U(x_2)I_{P_2}^U(y_2)+I_{P_2}^U(y_2)I_{Q_1}^U(x_1y_1)+I_{P_2}^U(x_2)I_{Q_1}^U(x_1y_1)-2I_{P_2}^U(x_2)I_{P_2}^U(y_2)I_{Q_1}^U(x_1y_1)},$$

$$=\max\left(I_{P_2}^U(x_2),I_{P_2}^U(y_2),I_{Q_1}^U(x_1y_1)\right)$$

$$=TCN\left(\left(I_{P_1}^U\circ I_{P_2}^U\right)((x_1,x_2)),\left(I_{P_1}^U\circ I_{P_2}^U\right)((y_1,y_2))\right)$$

$$\geq\frac{\left(I_{P_1}^U\circ I_{P_2}^U\right)((x_1,x_2))+\left(I_{P_1}^U\circ I_{P_2}^U\right)((y_1,y_2))-2\left(I_{P_1}^U\circ I_{P_2}^U\right)((x_1,x_2)).\left(I_{P_1}^U\circ I_{P_2}^U\right)((y_1,y_2))}{1-\left(I_{P_1}^U\circ I_{P_2}^U\right)((x_1,x_2)).\left(I_{P_1}^U\circ I_{P_2}^U\right)((y_1,y_2))}$$

$$\left(F_{Q_1}^L\circ F_{Q_2}^L\right)((x_1,x_2)(y_1,y_2))$$

$$=\max\left(F_{P_2}^L(x_2),F_{P_2}^L(y_2),F_{Q_1}^L(x_1y_1)\right)$$

$$\geq\max\left(F_{P_2}^L(x_2),F_{P_2}^L(y_2),\max\left(F_{P_1}^L(x_1),F_{P_1}^L(y_1)\right)\right)=\max\left(\max\left(F_{P_1}^L(x_1),F_{P_2}^L(x_2)\right),\max\left(F_{P_1}^L(y_1),F_{P_1}^L(y_2)\right)\right)$$

$$=\max\left(\left(F_{P_1}^L\circ F_{P_2}^L\right)(x_1,x_2),\left(F_{P_1}^L\circ F_{P_2}^L\right)(y_1,y_2)\right)$$

$$=TCN\left(\left(F_{P_1}^L\circ F_{P_2}^L\right)((x_1,x_2)),\left(F_{P_1}^L\circ F_{P_2}^L\right)((y_1,y_2))\right)$$

$$\geq\frac{\left(F_{P_1}^L\circ F_{P_2}^L\right)((x_1,x_2))+\left(F_{P_1}^L\circ F_{P_2}^L\right)((y_1,y_2))-2\left(F_{P_1}^L\circ F_{P_2}^L\right)((x_1,x_2)).\left(F_{P_1}^L\circ F_{P_2}^L\right)((y_1,y_2))}{1-\left(F_{P_1}^L\circ F_{P_2}^L\right)((x_1,x_2)).\left(F_{P_1}^L\circ F_{P_2}^L\right)((y_1,y_2))}$$

$$\left(F_{Q_1}^U\circ F_{Q_2}^U\right)((x_1,x_2)(y_1,y_2))$$

$$=\max\left(F_{P_2}^U(x_2),F_{P_2}^U(y_2),F_{Q_1}^U(x_1y_1)\right)$$

$$=TCN\left(\left(F_{P_1}^U\circ F_{P_2}^U\right)((x_1,x_2)),\left(F_{P_1}^U\circ F_{P_2}^U\right)((y_1,y_2))\right)$$

$$\geq\frac{\left(F_{P_1}^U\circ F_{P_2}^U\right)((x_1,x_2))+\left(F_{P_1}^U\circ F_{P_2}^U\right)((y_1,y_2))-2\left(F_{P_1}^U\circ F_{P_2}^U\right)((x_1,x_2)).\left(F_{P_1}^U\circ F_{P_2}^U\right)((y_1,y_2))}{1-\left(F_{P_1}^U\circ F_{P_2}^U\right)((x_1,x_2)).\left(F_{P_1}^U\circ F_{P_2}^U\right)((y_1,y_2))}$$

Hence the proposition.

Numerical Example:

Consider the same example as in numerical example for Cartesian product.

Composition of two IVNGs (Broumi et al. 2016)

$$P_1=\left\{\left\langle i,[0.5,0.7],[0.2,0.5],[0.1,0.3]\right\rangle,\left\langle j,[0.6,0.7],[0.2,0.4],[0.1,0.3]\right\rangle\right\}$$

$$Q_1=\left\{\left\langle ij,[0.3,0.6],[0.2,0.4],[0.2,0.4]\right\rangle\right\}$$

$$P_2=\left\{\left\langle k,[0.4,0.6],[0.3,0.4],[0.1,0.3]\right\rangle,\left\langle l,[0.4,0.7],[0.2,0.4],[0.1,0.3]\right\rangle\right\}$$

$$Q_2=\left\{\left\langle kl,[0.3,0.5],[0.4,0.5],[0.3,0.5]\right\rangle\right\}$$

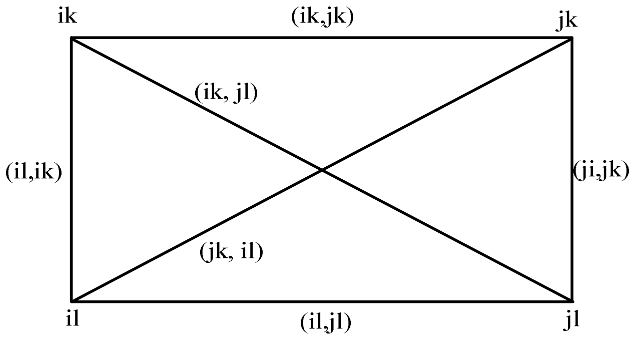

Fig. 6. Composition of G_{N1} and G_{N2} $(G_{N1} \circ G_{N2})$

$(ik, jk) = \langle [0.3,0.6],[0.3,0.4],[0.2,0.4] \rangle$, $(jl, jk) = \langle [0.3,0.5],[0.4,0.5],[0.3,0.5] \rangle$

$(il, jl) = \langle [0.3,0.6],[0.2,0.4],[0.2,0.4] \rangle$, $(il, ik) = \langle [0.3,0.5],[0.4,0.5],[0.3,0.5] \rangle$

$(ik, jl) = \langle [0.3,0.6],[0.3,0.4],[0.2,0.4] \rangle$, $(jk, il) = \langle [0.3,0.6],[0.3,0.4],[0.2,0.4] \rangle$

Other four edges has been given in Numerical Example 1.

To check for Dombi Composition of two IVNEGs is an IVNEG.
Consider $ij \in E_1$,

$$\left(T_{Q_1}^L \circ T_{Q_2}^L\right)((i,k)(j,l)) = \min\left[T_{P_2}^L(k), T_{P_2}^L(l), T_{Q_1}^L(ij)\right] = \min[0.4,0.4,0.3] = 0.3$$

$$\le \frac{\left(T_{P_1}^L \circ T_{P_1}^L\right)((i,k))\left(T_{P_1}^L \circ T_{P_1}^L\right)((j,l))}{\left(T_{P_1}^L \circ T_{P_1}^L\right)((i,k)) + \left(T_{P_1}^L \circ T_{P_1}^L\right)((j,l)) - \left(T_{P_1}^L \circ T_{P_1}^L\right)((i,k))\left(T_{P_1}^L \circ T_{P_1}^L\right)((j,l))}$$

$= \frac{(0.4)(0.4)}{(0.4)+(0.4)-(0.4)(0.4)} = 0.3$, hence satisfied

$$\left(I_{Q_1}^L \circ I_{Q_2}^L\right)((i,k)(j,l)) = \max\left[I_{P_2}^L(k), I_{P_2}^L(l), I_{Q_1}^L(ij)\right] = \max[0.3,0.2,0.2] = 0.3$$

$$\ge \frac{\left(I_{P_1}^L \circ I_{P_1}^L\right)((i,k)) + \left(I_{P_1}^L \circ I_{P_1}^L\right)((j,l)) - 2\left(I_{P_1}^L \circ I_{P_1}^L\right)((i,k))\left(I_{P_1}^L \circ I_{P_1}^L\right)((j,l))}{1-\left(I_{P_1}^L \circ I_{P_1}^L\right)((i,k))\left(I_{P_1}^L \circ I_{P_1}^L\right)((j,l))}$$

$= \frac{(0.2)+(0.2)-2(0.2)(0.2)}{1-(0.2)(0.2)} = 0.3$, hence satisfied

$$\left(F_{Q_1}^L \circ F_{Q_2}^L\right)((i,k)(j,l)) = \max\left[F_{P_2}^L(k), F_{P_2}^L(l), F_{Q_1}^L(ij)\right] = \max[0.1,0.1,0.2] = 0.2$$

$$\ge \frac{\left(F_{P_1}^L \circ F_{P_1}^L\right)((i,k)) + \left(F_{P_1}^L \circ F_{P_1}^L\right)((j,l)) - 2\left(F_{P_1}^L \circ F_{P_1}^L\right)((i,k))\left(F_{P_1}^L \circ F_{P_1}^L\right)((j,l))}{1-\left(F_{P_1}^L \circ F_{P_1}^L\right)((i,k))\left(F_{P_1}^L \circ F_{P_1}^L\right)((j,l))}$$

$= \frac{(0.1)+(0.1)-2(0.1)(0.1)}{1-(0.1)(0.1)} = 0.2$, hence satisfied

Similarly for other edges.
Hence composition two Dombi Interval Valued Neutrosophic Edge graphs is a DIVNEG.

5. Comparison of Traffic Control Management using different types of set and Graph theory

The below table expresses the advantage and limitations of crisp sets, Fuzzy sets (Type-1 Fuzzy Sets), Type-2 Fuzzy sets, Neutrosophic Sets (Single Valued Neutrosophic Graphs), Interval Valued Neutrosophic Sets, Dombi Fuzzy Graphs, Dombi Neutrosophic Graphs and Dombi Interval Valued Neutrosophic Graphs in traffic control management. This table also describe the role of triangular norms in control theory. In traffic control system, detection of latency of the vehicles in a roadway, estimation of the density will be done by sensors and it will send an interrupt signal to the control unit. After that using the controllers with logical operations, the roadway will be decided for giving the service first. At this junction, the logical operations of Dombi Interval Valued Neutrosophic Graphs can be used.

Traffic Control Management	Advantages	Limitations
Using Crisp Sets	• Fixed time period for all the traffic density • Achieved to characterize the real situation	• Cannot act while there is a fluctuation in traffic density • Unable to react immediately to unpredictable changes like driver's behavior. • Unable to handle with rapid momentous changes which disturb the continuity of the traffic.
Using Fuzzy Sets	• Different time duration can be considered according to the traffic density • Follow rule based approach which accepts uncertainties • Able to model the reasoning of an experienced human being • Adaptive and intelligent • Able to apply and handle real life rules identical to human thinking • Admits fuzzy terms and conditions • Performs the best security • It makes simpler to convert knowledge beyond the domain	• Adaptiveness is missing to compute the connectedness of the interval based input • Cannot be used to show uncertainty as it apply crisp and accurate functions • Cannot handle the uncertainties such as stability, flexibility and on-line planning completely since consequents can be uncertain
Using Type-2 Fuzzy Sets	• Rule based approach which accepts uncertainties completely • Adaptiveness (Fixed Type-1 fuzzy sets are used to calculate the bounds of the type reduced interval change as input changes) • Novelty (the upper and lower membership functions may be used concurrently in calculating every bound of the type reduced interval)	• Computational complexity is high as the membership functions themselves fuzzy.
Neutrosophic Sets	• Deals not only uncertainty but also indeterminacy due to unpredictable environmental disturbances	• Unable to rounding up and down errors of calculations
Interval Valued Neutrosophic Sets	• Deals with more uncertainties and indeterminacy • Flexible and adaptability • Able to address issues with a set of numbers in the real unit interval, not just a particular number. • Able to rounding up and down errors of calculations	• Unable to deal criterion incomplete weight information.
Neutrosophic Graphs	• When the terminal points and the paths are uncertain, optimized output is possible	• Unable to handle more uncertainties.
Interval Valued Neutrosophic Graphs	• Able to handle more uncertainties exist in the terminal points (vertices) and paths (edges)	• Unable to deal criterion incomplete weight information.
Dombi Fuzzy Graphs	• The Dombi Fuzzy Graph can portray the impreciseness strongly for all types of networks like traffic control.	• Indeterminacy cannot be dealt by Dombi Fuzzy Graph.
Dombi Neutrosophic Graphs	• Indeterminacy can be dealt by Dombi Neutrosophic Graph.	• Unable to handle uncertainty for interval values provided by the expert's

Dombi Interval Valued Neutrosophic Graphs	• Able to handle uncertainty properly for interval values provided by the expert's	• Unable to deal criterion incomplete weight information.

6. Conclusion

Dealing indeterminacy is an essential work to get an optimized output in any problem and control system as well. It is possible only with Neutrosophic logic and that too effectively by interval valued Neutrosophic setting as it has lower and upper membership function for three independent membership functions namely truth, indeterminacy and falsity. In this paper, Dombi Single valued Neutrosophic Graph and Dombi Interval valued Neutrosophic Graph have been proposed. Also it has been proved that Cartesian product and composition of two DIVNGs are DIVNG with numerical example. Also an importance of the Neutrosophic Controllers has been given theoretically and its use in traffic control management. It has been pointed out that instead of using minimum and maximum operations, triangular norms namely T Norm and T-Conorm can be used in control system such as traffic control management. Advantage and limitations has been discussed for crisp sets, fuzzy sets, and type-2 fuzzy sets, Neutrosophic Sets, Interval Valued Neutrosophic Sets, Neutrosophic Graphs, Interval Valued Neutrosophic Graphs, Dombi Fuzzy Graphs, Dombi Neutrosophic Graphs and Dombi Interval Valued Neutrosophic Graphs.

References

[1] J. Dombi. The Generalized Dombi Operator Family and the Multiplicative Utility Function. Soft Computing Based Modeling in Intelligent Systems, (2009), 115-131.

[2] G. Gosztolya, J. Dombi, and A. Kocsor. Applying the Generalized Dombi Operator Family to the Speech Recognition Task. Journal of Computing and Information Technology, (2009), 285-293.

[3] S. Aggarwal, R. Biswas, A. Q. Ansari. Neutrosophic Modeling and Control, International conference on Computer and Communication Technology, (2010), 718-723.

[4] G. A. Pavlopoulos, M. Secrier, C. N. Moschopoulos, T. G. Soldatos, S. Kossida, J. Aerts, R. Schneider, and P.G. Bagos. Using graph Theory to analyze biological networks, BioData Mining, 4(10)(2011), 1-27.

[5] S. Stoilova, and K. Nedelchev.An Automated Optimization using Graph Theory, Machines, Technologies, Materials, (2012),17-22.

[6] N. Baruah, and A.K. Baruah. On a Traffic Control Problem Using Cut-Set of Graph, International Journal of Advanced Networking and Applications, 3(4) (2012), 1240-1244.

[7] A. Dey, D. Ghosh and A. Pal, Operations on Complement of a Fuzzy Graph. Proceedings of the National Seminar on Recent Advances in Mathematics and its Applications in Engineerings Sciences, (2012), 43-50.

[8] F. Gomes-Vela, N. Diaz-Diaz, J. A. Lagares, J. A. Sanchez, and J. S. Aguilar-Ruiz, Using graph Theory to analyze gene network coherence. EMBnet.Journal 18 B, (2012), 32-34.

[9] A. Dey, D. Ghosh, and A. Pal, Vertex Coloring of a Fuzzy Graph, Proceedings of the National Seminar on Recent Advances in Mathematics and its Applications in Engineerings Sciences, (2012), 51-59.

[10] A. Dey, D. Ghosh, and A. Pal. An Application of Fuzzy Graph in Traffic Light Control, International Conference on Mathematics, (2013), 1-8.

[11] M. S. Sunitha, and S. Mathew. Fuzzy Graph Theory: A Survey, Annals of Pure and Applied Mathematics, 4(1) (2013), 92-110.

[12] I. S. Lopes, S. D. Sousa, and E. Nunes. Quantification of Uncertainty of Performance Measures Using Graph Theory, Congreso Galego de Estatistica e Investigacion de Operacions A Coruna, Project: Uncertainty in Performance Measurement, (2013), 1-9.

[13] D. Dave, and N. Jhala. Application of Graph Theory in Traffic Management. International Journal of Engineering and Innovative Technology, 3(12) (2014), 124-126.

[14] B. Prabha, V. M. Chandrasekaran, and G. Deepa, Energy of an Intuitionistic Fuzzy Graph. Italian Journal of Pure and Applied Mathematics, 32 (2014), 431-444.

[15] S. K. Bisen. Application of Graph Theory in Transportation Networks, International Journal of Scientific research and Management, 5(7) (2017), 6197-6201.

[16] R. Dhavaseelan, R. Vikramaprasad, and V. Krishnaraj. Certain Types of Neutrosophic Graphs, International Journal of Mathematical Sciences & Applications, 5(2) (2015), 333-339.

[17] W. B. V. Kandasamy, K. Ilanthendral, and F. Smarandache. Neutrosophic Graphs: A New Dimension to Graph Theory. (2015), DOI:10.5281/zenodo.31820.

[18] O. T. Manjusha, and M. S. Sunitha. Strong Domination in Fuzzy Graphs, Fuzzy Information and Engineering, 7(2015), 369-377.

[19] S. Broumi, F. Samarandache, M. Talea, and A. Bakali. An Introduction to Bipolar Single Valued Neutrosophic Graph Theory, Applied Mechanics and Materials,841 (2016), 184-191.

[20] S. Broumi, A. Bakali, M. Talea, and F. Samarandache. An Isolated Interval Valued Neutrosophic Graph, Uncertainty, XIII (2016), 67-80.

[21] S. Broumi, M. Talea, F. Samarandache, and A. Bakali. Decision-Making Method based on the Interval Valued Neutrosophic Graph, Future Technologies Conference, San Francisco, United States. (2016), 1-7

[22] S. Broumi, A. Bakali, M. Talea, and F. Samarandache. Isolated Single Valued Neutrosophic Graphs, Neutrosophic Sets and Systems, 11 (2016), 74-78.

[23] S. Broumi, F. Samarandache, M. Talea, and A. Bakali. Operations on Interval Valued Neutrosophic Graphs, New Trends in Neutrosophic Theory and Applications, (2016), 231-254.

[24] S. Broumi, M. Talea, F. Samarandache, and A. Bakali. Single Valued Neutrosophic Graphs: Degree, Order and Size, International Conference on Fuzzy Systems. (2016), 2444-2451.

[25] W. B. V. Kandasamy, K. Ilanthendral, and F. Smarandache. Strong Neutrosophic Graphs and Subgraph Topological Subspaces, (2016), ISBN-13: 978-1-59973-486-6, EuropaNova, Belgium, 2016, 9-18.

[26] N. K. Bhagat, and R. K. Patel. Traffic Management using Intelligence in Transportation Systems, International Journal for Scientific Research & Development, 3(12) (2016), 484-487.

[27] R. Sahin R. An approach to Neutrosophic graph theory with applications, Soft Computing. (2017), DOI: 10.107/s00500-017-2875-1.

[28] A. Jain, S. Rai, and A. Manaktala. Analyzing Asymmetrical Associations using Fuzzy Graph and Discovering Hidden Connections in Facebook, Global Journal of Enterprise Information System, 9(1) (2017), 1-12.

[29] P. C. Sethi P, and P. K. Behera. Network Traffic Management Using Dynamic Bandwidth on Demand, International Journal of Computer Science and Information Security, 15(6) (2017), 369-375.

[30] M. Akram, and M. Sitara. Representation of Graph Structure Based on I-V Neutrosophic Sets, International Journal of Algebra and Statistics, 6(1-2) (2017), 56-80.

[31] K. S. Oberoi, G. D. Mondo, Y. Dupuis, and P. Vasseur. Spatial Modeling of Urban Road Traffic Using Graph Theory, Spatial Analysis and GEOmatics,(2017), 1-14.

[32] S. Broumi, A. Dey, A. Bakali, M. Talea, F. Smarandache, L.H. Son, and D. Koley. Uniform Single Value Neutrosophic Graphs, Neutrosophic Sets and Systems, 17 (2017), 42-49.

[33] S. G. Quek, S. Broumi, G. Selvachandran, A. Bakali, M. Talea, and F. Smarandache. Some Results on the Graph Theory for Complex Neutrosophic Sets, Symmetry, 10 (190) (2018), 1-30.

[34] S. Ashraf, S. Naz, and E. E. Kerre. Dombi Fuzzy Graphs, Fuzzy Information and Engineering, Doi.org/10.1080/16168658.2018.1509520. (2018), 1-22.

[35] M. K. R. Marapureddy. Fuzzy Graph of Semigroup, Bulletin of the International Mathematical Virtual Institute. 8(2018) (2018), 439-448.

[36] J. N. Mordeson, and S. Mathew. t-Norm Fuzzy Graphs, New Mathematics and Natural Computation, 14(01) (2018), 129-143.

[37] M.Abdel-Baset, ,et al. An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field, Computers in Industry 106,(2019)94-110.

[38] M.Abdel-Basset, et al, An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number." Applied Soft Computing. 77, (2019) 438-452.

[39] M Abdel-Basset et al, A Group Decision Making Framework Based on Neutrosophic TOPSIS Approach for Smart Medical Device Selection." Journal of Medical Systems 43(2),(2019). 38.

Received: October 18, 2018 Accepted: January 15, 2019