5-23-1960

A Study of Minimum Element Syntheis of a Two-Pole RC Driving Point Impedance

William F. Carlson

MINIMUM ELEMENT SPREAD SYNTHESIS · CARLSON

MANUSCRIPT THESES

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A STUDY OF MINIMUM ELEMENT SPREAD SYNTHESIS OF A TWO-POLE RC DRIVING-POINT IMPEDANCE

By

William F. Carlson

A Thesis
Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Electrical Engineering

The University of New Mexico

1960

This thesis, directed and approved by the candidate's committee, has been accepted by the Graduate Committee of the University of New Mexico in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Dean

May 23, 1960

Date

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Guillemin [1] and Truxal [2] have noted the fact that given a driving-point impedance function of the complex frequency variable $s = \sigma + j\omega$, the element spread in the network realization of the function is dependent on the method of synthesis used. It appears attractive to consider a method of predicting, from the impedance function, the element spread which will be obtained by use of a particular method of synthesis.

The approach taken in this thesis is from the point of view of economy. A network containing resistors of values 1 ohm and 1 megohm may present problems of accuracy and element tolerance limits which are not aided by the fact that all elements may be increased or decreased by a constant multiplier without changing the shape of the network gain curve.

Furthermore, it is certainly more economical where several identical networks are to be built, to purchase elements of the same value or near the same value.

For these reasons the work of this thesis is pointed toward achieving networks with minimum element spread.

The RC driving point impedance is widely encountered in the design of filters and compensating networks for control systems. One common form for the impedance function of this type of network is that having two finite poles and two finite zeros. Its synthesis will illustrate all the methods used

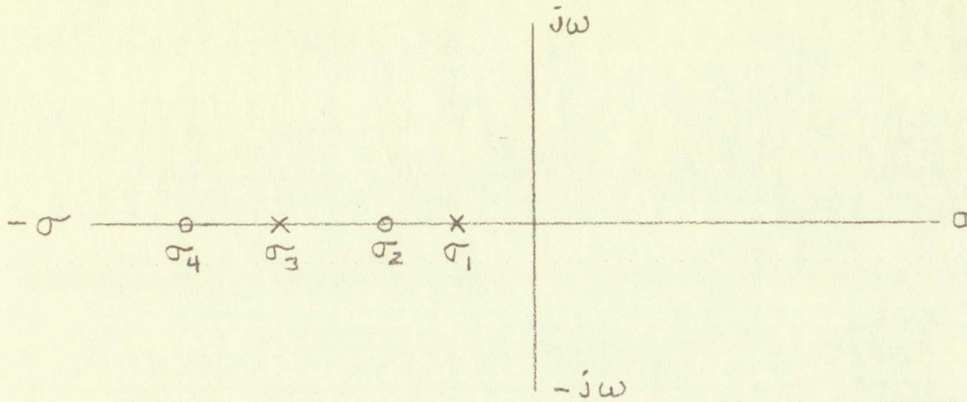
in this study of networks having minimum element spreads, and for this reason it has been chosen for investigation.

There is an infinite number of methods of synthesizing the network corresponding to the function described above, if the removal of portions of poles is considered. However, since economy is the prime consideration here, the allowable methods have been limited to the four so-called canonic forms (First Foster form, Second Foster form, First Cauer form, Second Cauer form) described by Guillemin [3] plus the partial removal of one of the impedance or admittance poles, followed by one of the four canonic forms of synthesis of the remainder. Only one partial pole is removed in all the following work, since each time part of a pole is removed, two additional elements are added to the final network. The four canonic forms of synthesis, of course, yield networks with the minimum number of elements attainable.

Consider the driving-point impedance given by

$$Z(s) = \frac{H(s+\sigma_2)(s+\sigma_4)}{(s+\sigma_1)(s+\sigma_3)}$$

If this function is to represent an RC driving-point impedance, the pole-zero configuration in the complex plane must have the form shown below: [4]

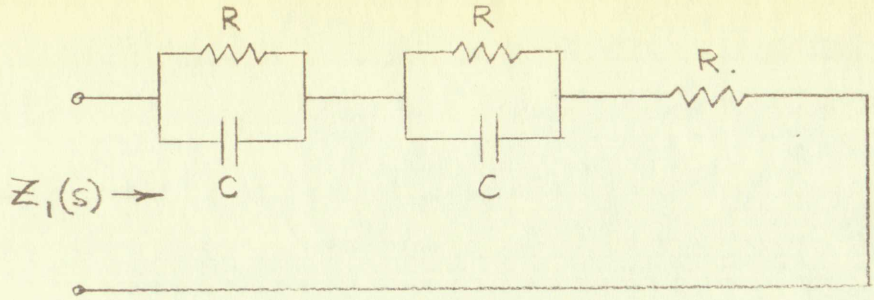


The first circumstance to be considered is the possibility of achieving synthesis of this function with equal valued resistors and equal valued capacitors. A network having this characteristic will be defined as an equi-element network.

The forms of the networks resulting from synthesis by the four canonic methods are given by Guillemin [3] . Since the form in a given case is known, equal values can be assigned to all resistors and to all capacitors, and the driving-point impedance, $Z_1(s)$, of the network determined. This function can then be compared with $Z(s)$, the given function, to see how the pole-zero locations are affected by the requirement of equal element values.

1. First Foster Form

The equi-element two pole-two zero RC network in this form is:

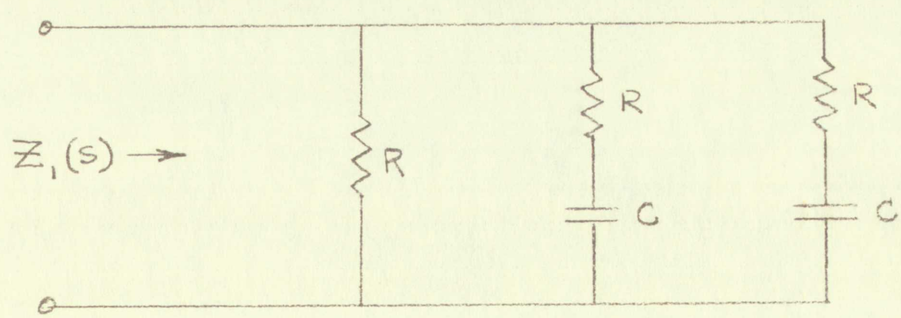


$$Z_1(s) = \frac{R\left(s + \frac{3}{RC}\right)}{\left(s + \frac{1}{RC}\right)}$$

which is now to be compared with $Z(s)$. $Z_1(s)$ contains only one finite pole and one finite zero, while $Z(s)$ contains two of each. It is apparent then, that the given $Z(s)$ cannot be realized as an equi-element network, using the First Foster method of synthesis.

2. Second Foster Form

The network form is:



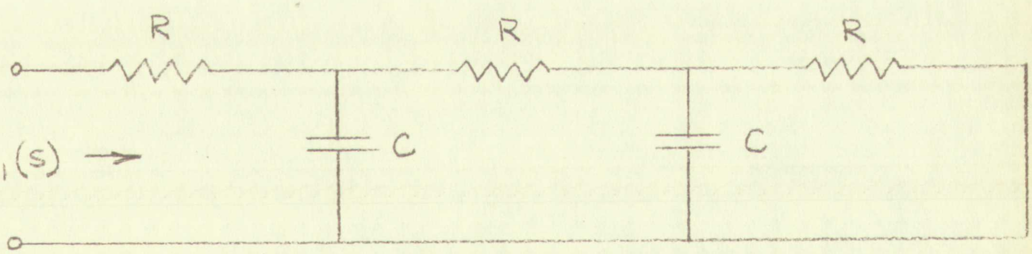
$$Z_1(s) = \cfrac{1}{\cfrac{1}{R} + \cfrac{2}{R + \cfrac{1}{Cs}}}$$

$$= \frac{R}{3} \frac{\left(s + \frac{1}{RC}\right)}{\left(s + \frac{1}{3RC}\right)}$$

which again has one less pole and one less zero than the given function. Synthesis by this method, therefore, will not yield an equi-element network.

3. First Cauer Form

The network form is:



$$Z_1(s) = R + \cfrac{1}{Cs + \cfrac{1}{R + \cfrac{1}{Cs + \cfrac{1}{R}}}}$$

$$= R \frac{s^2 + \frac{4}{RC} s + \frac{3}{R^2C^2}}{s^2 + \frac{3}{RC} s + \frac{1}{R^2C^2}}$$

AND $Z(s) = \dfrac{H\left(s^2 + [\sigma_2 + \sigma_4] s + \sigma_2 \sigma_4\right)}{s^2 + [\sigma_1 + \sigma_3] s + \sigma_1 \sigma_3}$

Comparing coefficients of $Z_1(s)$ and $Z(s)$,

(1) $\sigma_2 + \sigma_4 = \dfrac{4}{RC}$ (3) $\sigma_1 + \sigma_3 = \dfrac{3}{RC}$

(2) $\sigma_2 \sigma_4 = \dfrac{3}{R^2C^2}$ (4) $\sigma_1 \sigma_3 = \dfrac{1}{R^2C^2}$

(5) $H = R$

There are 5 equations and 7 unknowns ($\sigma_1, \sigma_2, \sigma_3, \sigma_4,$ R, C, H). Let σ_1 and H be independent variables. Then from (1) and (3), (6) $\sigma_1 + \sigma_3 = \frac{3}{4}(\sigma_2 + \sigma_4)$;

from (2) and (4), (7) $\sigma_1 \sigma_3 = \frac{1}{3}\sigma_2 \sigma_4$,

and from (1) and (2),

(8) $$\sigma_2 \sigma_4 = \frac{3}{16}(\sigma_2 + \sigma_4)^2$$

Combining (6) and (8),

$$\sigma_2 \sigma_4 = \frac{3}{16}\left(\sigma_2^2 + 2\sigma_2\sigma_4 + \sigma_4^2\right)$$

OR, $$\sigma_4^2 - \frac{10}{3}\sigma_2\sigma_4 + \sigma_2^2 = 0$$

$$\sigma_4 = \frac{5}{3}\sigma_2 \pm \sqrt{\frac{25}{9}\sigma_2^2 - \sigma_2^2}$$

$$= \frac{5}{3}\sigma_2 \pm \frac{4}{3}\sigma_2$$

and since the magnitude of σ_4 must be greater than that of σ_2 for this RC function, the plus sign is chosen.

Combining (1) and (4),

$$\sigma_1 \sigma_3 = \frac{1}{16}\left[\frac{4}{3}(\sigma_1 + \sigma_3)\right]^2$$

OR, $$\sigma_3^2 - 7\sigma_1\sigma_3 + \sigma_1^2 = 0$$

$$\sigma_3 = \frac{7}{2}\sigma_1 \pm \frac{\sqrt{45}}{2}\sigma_1$$

and choosing again the plus sign because the magnitude of σ_3 must be greater than that of σ_1 ,

(10) $$\sigma_3 = \frac{7 + \sqrt{45}}{2}\sigma_1$$

Using (9), (10) and (1),

(11) $$\sigma_2 = \frac{3 + \sqrt{5}}{2}\sigma_1$$

(12) $$\sigma_3 = \frac{7 + 3\sqrt{5}}{2}\sigma_1$$

(13) $$\sigma_4 = \frac{9 + 3\sqrt{5}}{}\sigma_1$$

(5) $$H = R$$

As an example of the results, let $\sigma_1 = 2$, $H = 1$.

Then $$Z(s) = \frac{(S + 5.24)(S + 15.72)}{(S + 2)(S + 13.70)}$$

Use of the first Cauer form of expansion results in the following element values:

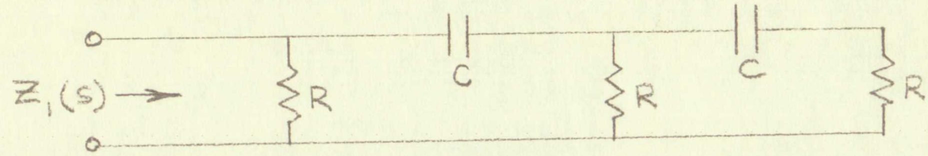
$$R_1 = R_2 = R_3 = 1 \text{ OHM}$$

$$C_1 = C_2 = 0.19 \text{ FARAD}$$

$$H = R$$

4. Second Cauer Form

The network is:



$$Z_1(s) = \frac{R}{3}\,\frac{S^2 + \frac{3}{RC}S + \frac{1}{R^2C^2}}{S^2 + \frac{4}{3RC}S + \frac{1}{3R^2C^2}}$$

Comparing as before with $Z(s)$ and solving the resulting equations with σ_1, and H as independent variables, it is found that an equi-element network will result if,

(14) $\sigma_2 = \frac{3}{2}(3-\sqrt{5})\sigma_1$

(15) $\sigma_3 = 3\sigma_1$

(16) $\sigma_4 = \frac{9+3\sqrt{5}}{2}\sigma_1$

(17) $H = \frac{R}{3}$

As an example, if $\sigma_1 = 2$ and $H = 1$,

$R_1 = R_2 = R_3 = 3$ OHMS

$C_1 = C_2 = 0.056$ FARAD

$H = \frac{R}{3}$

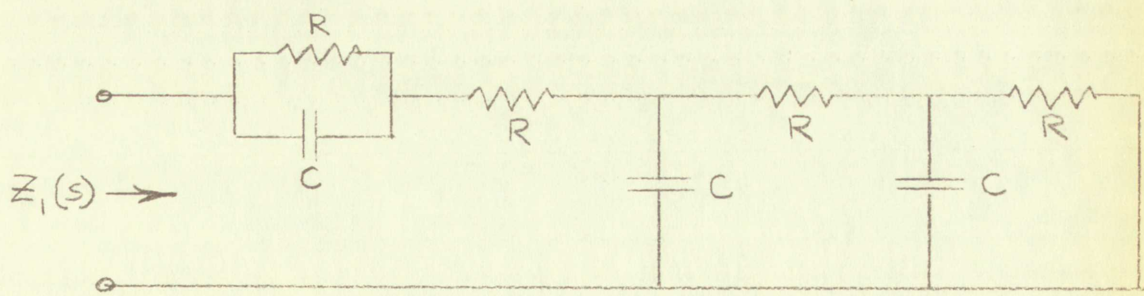
Thus, of the four canonic forms, only two - the First and Second Cauer forms - are capable of yielding equi-element networks.

It remains to consider the effect of partial pole removal on the synthesis. The removal of a portion of an impedance

or admittance pole leaves the form of the remainder unchanged; that is, the remainder will still contain two finite poles and two finite zeros. Also it has just been shown that a remainder function of this type cannot be developed into an equi-element network by the First or Second Foster forms. Therefore, these will not be considered in the discussion of partial pole removal.

The terms "higher" and "lower", when used to define relative pole locations, refer to the absolute value of the numerical pole location; i.e., if $|\sigma_3| > |\sigma_1|$ then σ_3 is the higher, and σ_1 the lower pole.

Consider the removal of part of an impedance pole followed by expansion of the remainder in the two Cauer forms. Expansion of the remainder in the first Cauer form will yield the following network:



where equal values have been assigned to like elements.

(18) $$Z_1(s) = \frac{R}{RCs+1} + \frac{R^3C^2s^2 + 4R^2Cs + 3R}{R^2C^2s^2 + 3RCs + 1}$$

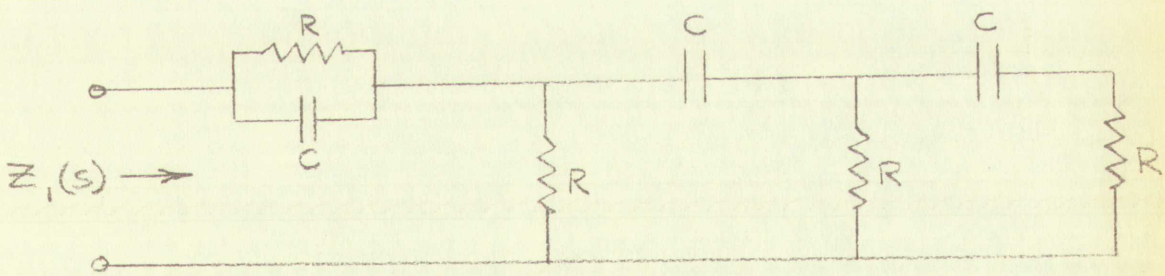
which will have a denominator in S^3 unless,

(1) The term $(RCS+1)$ is a factor of $(R^2C^2S^2+3RCS+1)$

or, (2) There is a common factor in the numerator and the denominator of the second term of $Z_1(s)$.

Straightforward division shows that neither condition (1) nor condition (2) is met. Therefore, since $Z_1(s)$ contains three finite poles, and $Z(s)$ only two, the conclusion is that the synthesis by this method will not yield an equi-element network.

Expansion of the remainder in the second Cauer form will yield a network as follows:



having the driving point impedance,

$$Z_1(s) = \frac{R}{RCS+1} + \frac{R^3C^2S^2+3R^2CS+R}{3R^2C^2S^2+4RCS+1}$$

which again will have a third order denominator unless conditions (1) and (2) above are met. Division as before shows that condition (1) is indeed met, and the factors in the denominator of $Z_1(s)$ are $(RCS+1)$ and $(3RCS+1)$, i.e.,

$$(19) \quad Z_1(s) = \frac{R^3C^2S^2 + 6R^2CS + 2R}{(3RCS+1)(RCS+1)}$$

$$= \frac{R^3C^2S^2 + 6R^2CS + 2R}{3R^2C^2S^2 + 4RCS + 1}$$

and there is now no common factor in numerator and denominator.

Comparing with the given $Z(s)$,

$$(20) \quad \sigma_2 + \sigma_4 = \frac{6}{RC}$$

$$(21) \quad \sigma_2 \sigma_4 = \frac{2}{R^2C^2}$$

$$(22) \quad \sigma_1 + \sigma_3 = \frac{4}{3RC}$$

$$(23) \quad \sigma_1 \sigma_3 = \frac{1}{3R^2C^2}$$

$$(24) \quad H = \frac{R}{3}$$

Before proceeding further, note that the parallel RC network formed by removal of part of an impedance pole has the impedance

$$Z'(s) = \frac{1/C}{S + \frac{1}{RC}}$$

Only the higher impedance pole has this form, the lower one having a denominator of $\left(S + \frac{1}{3RC}\right)$ instead of $\left(S + \frac{1}{RC}\right)$. Apparently only the removal of part of the higher impedance pole can be considered for our equi-element network.

The residue in the pole at $S = -\frac{1}{RC}$ is found to be $\frac{3}{2C}$. Removal of $\frac{1/C}{S + 1/RC}$ means the removal of $2/3$ of the pole at $S = -1/RC$, which then becomes one of the conditions for achieving the equi-element network.

Solution of equations (20) - (24) as before yields the relations between pole and zero locations which together with the required amount of partial pole removal will yield an equi-element network.

(25) $\sigma_2 = (9 - 3\sqrt{7})\sigma_1$

(26) $\sigma_3 = 3\sigma_1$

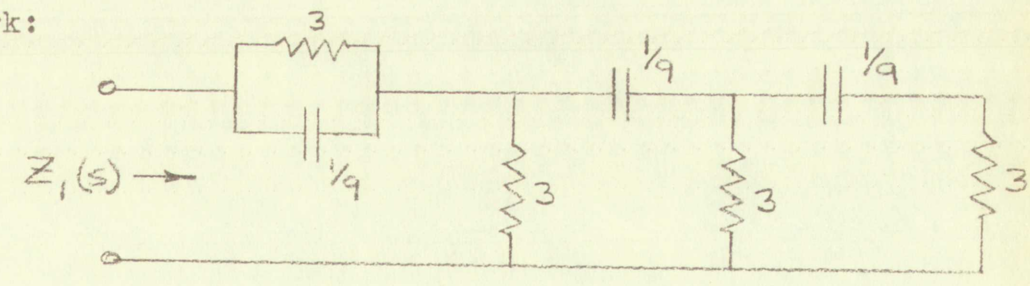
(27) $\sigma_4 = (9 + 3\sqrt{7})\sigma_1$

(28) $H = R/3$

For example, if $\sigma_1 = 1$ and $H = 1$,

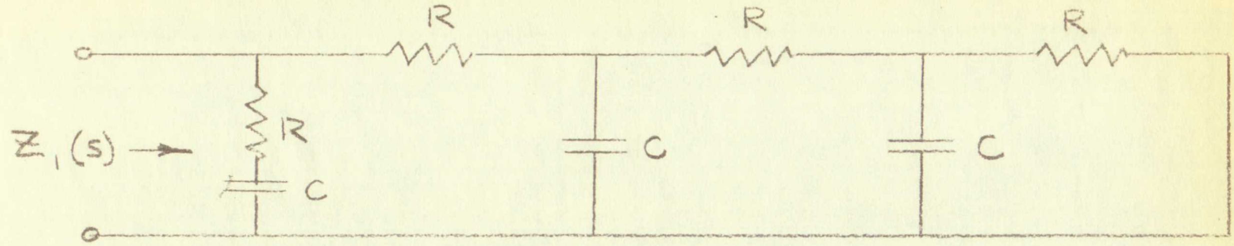
$$Z_1(s) = \frac{s^2 + 18s + 18}{(s+1)(s+3)}$$

Removal of $2/3$ of the pole at $s = -3$ and expansion of the remainder in the second Cauer form yields the following network:



and $H = R/3$ as required.

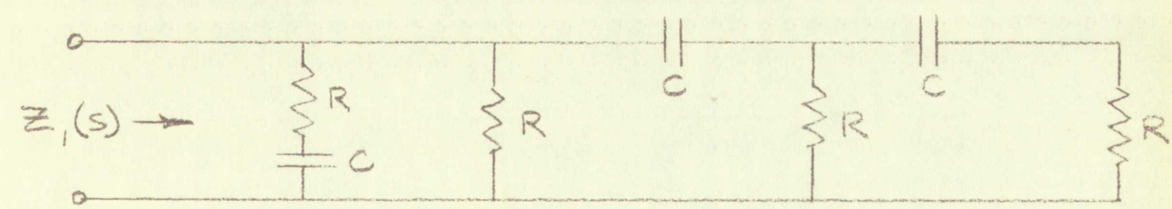
Consider now the removal of part of an admittance pole. The network for the first Cauer form of remainder expansion can be shown as,



$$Z_1(s) = \frac{1/C}{S + 1/RC} + R\,\frac{S^2 + \frac{4}{RC}S + \frac{3}{R^2C^2}}{S^2 + \frac{3}{RC}S + \frac{1}{R^2C^2}}$$

As was noted earlier, the denominator of the second term must contain a factor equal to the denominator of the first term in order that an equi-element network be attained. Such is not the case here, so this synthesis method will not yield the desired results.

Expansion of the remainder in the second Cauer form yields the network,



$$Z_1(s) = \frac{1/C}{S + 1/RC} + \frac{3}{R}\,\frac{S^2 + \frac{4}{3RC}S + \frac{1}{3R^2C^2}}{S^2 + \frac{3}{RC}S + \frac{1}{R^2C^2}}$$

and again there is no common factor in the denominators of the two terms.

In order to simplify the remainder of the work, let

(29) $\sigma_2 = K_2 \sigma_1$

(30) $\sigma_3 = K_3 \sigma_1$

(31) $\sigma_4 = K_4 \sigma_1$

(32) k' fraction of a pole removed in a partial pole removal.

The results of the preceding calculations can then be summarized.

Only three of the methods indicated on page 3 will synthesize the function given on page 4 and yield equi-element networks. Furthermore, certain relations among the locations of the poles and zeros of the given function must hold. These methods, and the corresponding relations between critical frequencies, are:

I. First Cauer Form

$K_2 = (3 + \sqrt{5})/2$

$K_3 = (7 + 3\sqrt{5})/2$

$K_4 = (9 + 3\sqrt{5})/2$

$(H = R)$

and again there is no common [illegible] for the [illegible] [illegible] the two terms.

In order to [illegible] the [illegible] for the [illegible]

(29) [illegible]

(30) [illegible]

(31) [illegible]

(32) [illegible] a gate [illegible] the [illegible]

network.

The results of the present [illegible] can be summarized

Only three of the pattern [illegible] of [illegible] [illegible] synthesize the function given on page [illegible] networks. [illegible], certain [illegible] on the [illegible] of the poles and zeros of the given function, [illegible] methods, and the [illegible] [illegible] [illegible] quencies, are:

1. Pierce Bridge Form

[illegible]

[illegible]

II. Second Cauer Form

$K_2 = 3(3-\sqrt{5})/2$

$K_3 = 3$

$K_4 = (9+3\sqrt{5})/2$

$(H = R/3)$

III. Remove part of the highest impedance pole and expand the remainder in the second Cauer form.

$k' = 2/3$

$K_2 = 9 - 3\sqrt{7}$

$K_3 = 3$

$K_4 = 9 + 3\sqrt{7}$

$(H = R/3)$

It appears, then, that only in an extremely small number of cases can a network of the form being studied be economically synthesized as an equi-element network. However, in most cases, if the element spread can be held within economically reasonable limits, some benefit will result. The remainder of this thesis is therefore concerned with a computer study of the element spread achieved, using the previously described methods of synthesis, for a wide range of

values of K_2, K_3, K_4 and k'. The results of the study have been presented as a table, entered with the given values of K_i, and giving all of the agreed upon methods of synthesis which will yield ratios of element values in the range

$$0.1 \leq \left\{ \frac{R_m}{R_n} \text{ AND } \frac{C_j}{C_K} \right\} \leq 10 \tag{33}$$

where $\frac{R_m}{R_n}$ and $\frac{C_j}{C_K}$ represent all possible ratios of the network resistors and capacitors.

In cases where this is not possible using any of the given methods, the range has been expanded to,

$$0.01 \leq \left\{ \frac{R_m}{R_n} \text{ AND } \frac{C_j}{C_K} \right\} \leq 100 \tag{34}$$

and these cases are distinguished by being to the right of a heavy black line in the table.

Using relations (29) - (31), $Z(s)$ can be written as,

$$Z(s) = \frac{(s + K_2\sigma_1)(s + K_4\sigma_1)}{(s+\sigma_1)(s + K_3\sigma_1)}$$

$$= \frac{s^2 + (K_2 + K_4)\sigma_1 s + K_2 K_4 \sigma_1^2}{s^2 + (1 + K_3)\sigma_1 s + K_3\sigma_1^2}$$

Starting with this $Z(s)$, the element values in terms of literal coefficients have been derived for the types of synthesis agreed upon earlier and repeated here for convenience.

1. First Foster Form
2. Second Foster Form
3. First Cauer Form
4. Second Cauer Form
5. Removal of portion of lower impedance pole followed by expansion of remainder in each of forms 1 - 4.
6. Removal of portion of higher impedance pole followed by 1 - 4.
7. Removal of portion of lower admittance pole followed by 1 - 4.
8. Removal of portion of higher admittance pole followed by 1 - 4.

Case 1 above will be used to illustrate the procedure, which is straightforward. H is assumed to be unity since all element values can be altered later to account for a value other than this.

$$Z(s) = \frac{s^2 + (K_2 + K_4)\sigma_1 s + K_2 K_4 \sigma_1^2}{s^2 + (1 + K_3)\sigma_1 s + K_3 \sigma_1^2}$$

so that,

$$Z(s) = 1 + \frac{(K_2 + K_4 - 1 - K_3)\sigma_1 s + (K_2 K_4 - K_3)\sigma_1^2}{(s + \sigma_1)(s + K_3 \sigma_1)}$$

and the residues A and B in the poles at $S = -\sigma_1$ and $S = -K_3\sigma_1$ are,

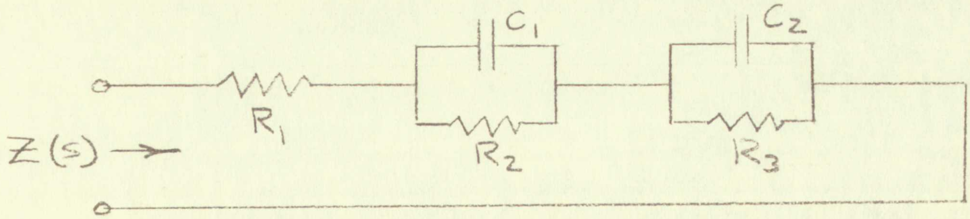
$$A = \frac{(K_2-1)(K_4-1)}{(K_3-1)}\sigma_1$$

$$B = \frac{(K_3-K_2)(K_4-K_3)}{(K_3-1)}\sigma_1$$

so that,

$$Z(s) = 1 + \frac{\frac{(K_2-1)(K_4-1)}{(K_3-1)}\sigma_1}{S+\sigma_1} + \frac{\frac{(K_3-K_2)(K_4-K_3)}{(K_3-1)}\sigma_1}{S+K_3\sigma_1}$$

and therefore, in the network shown below, which is the First Foster form of synthesis of $Z(s)$,



$R_1 = 1$

$$R_2 = \frac{(K_2-1)(K_4-1)}{(K_3-1)}$$

$$R_3 = \frac{(K_3-K_2)(K_4-K_3)}{K_3(K_3-1)}$$

$$C_1 = \frac{K_3-1}{(K_2-1)(K_4-1)\sigma_1}$$

$$C_2 = \frac{K_3-1}{(K_3-K_2)(K_4-K_3)\sigma_1}$$

so that the element ratios are,

$$R_2/R_1 = \frac{(K_2-1)(K_4-1)}{(K_3-1)}$$

$$R_3/R_1 = \frac{(K_3-K_2)(K_4-K_3)}{K_3(K_3-1)}$$

$$R_3/R_2 = \frac{R_3/R_1}{R_2/R_1}$$

$$C_2/C_1 = \frac{(K_2-1)(K_4-1)}{(K_3-K_2)(K_4-K_3)}$$

These relations, and the similar ones resulting from synthesis of $Z(s)$ by the methods of Cases 2 - 8, have been programmed for solution by an IBM-704 digital computer. Use has been made of the FORTRAN automatic assembly program available at The Los Alamos Scientific Laboratory to simplify the coding. The program for the solution of the equations pertaining to Cases 1 - 4 is given on page 27, and differs from that used in solving Cases 5 - 8 only in the equations representing the various element ratios.

A complete list of all equations solved for the various cases is lengthy and has not been included here. However, all of these equations are maintained in the author's file,

together with the complete programs for their solution, the binary cards for insertion in the computer and the resulting data.

The solutions have been obtained for values of K_i as follows:

K_2 = 2, 4, 6, 8, 10, 20, 40, 60, 80, 100

K_3 = 4, 6, 8, 10, 20, 40, 60, 80, 100, 200

K_4 = 6, 8, 10, 20, 40, 60, 80, 100, 200

and where partial pole removal is made, for values of k' as follows:

k' = 0.1, 0.3, 0.5, 0.7, 0.9

[illegible] the [illegible] series, [illegible]
[illegible] series for [illegible] and [illegible]
[illegible].

The following [illegible] have been [illegible] for [illegible] as follows:

K_1 = [illegible] 20, 30, 50, 100,

K_2 = 4, 6, [illegible] 25, 40, [illegible] 100, [illegible]

K_3 = 0, 5, 10, 20, 40, [illegible] 100, [illegible]

and where parallel pole removal [illegible], for values of [illegible] as follows:

[illegible] = 0.1, 0.2, 0.5, 0.7, [illegible]

RESULTS AND CONCLUSIONS

1. The results of the computer study are presented as a table, pp 29 through 46. The values of K_2, K_3 and K_4 are first computed for a given function. The table is entered with these, and the methods which will yield minimum element spread network (as defined by (33)) are noted. In some cases more than one method is possible, in which case, other circuit considerations will dictate the choice. Where use of a canonic form will give the desired results, only the canonic form has been listed, since resort to partial pole removal increases the number of circuit elements. A hand calculation has been made for each equation with a set of specific values of k', K_2, K_3 and K_4 in order to verify the correctness of computer solution.

2. In all cases, the values of element ratios are found to be independent of σ_1.

3. Inspection of the table shows, in general, that for pole-zero locations close together, minimum element-spread networks - as defined by (33) - may be achieved through use of the canonic forms. As the locations move out from each other, partial pole removal must be made to stay within this range. Finally as the locations move even farther apart, none of the agreed upon methods will yield element spreads within the desired range.

4. The table may readily be expanded to include values of K_i and k' closer together or to increase its range, merely by adding to the numbers read into the computer on data cards. The extent to which this may be done is limited only by computer time availability.

INSTRUCTIONS FOR USE OF TABLE, PP 29 to 46

$$Z(s) = \frac{(s + K_2\sigma_1)(s + K_4\sigma_1)}{(s+\sigma_1)(s + K_3\sigma_1)}$$

1. Compute K_2, K_3 and K_4

2. Enter the table with these numbers, choosing values of K_i in the table which are closest to the desired values. In most cases interpolation can be performed.

3. The letters and numbers in the boxes are interpreted as follows:

1 = First Foster form

2 = Second Foster form

3 = First Cauer form

4 = Second Cauer form

A = Lower impedance pole

B = Higher impedance pole

C = Lower admittance pole

D = Higher admittance pole

Thus, a sequence of letters and numbers such as (0.3D-1,2,3) means that 0.3 of the higher admittance pole should be removed and that the remainder should be expanded in the First or Second foster, or First Cauer form.

As an example, assume

$$Z(s) = \frac{(s+5)(s+40)}{(s+2)(s+9.2)}$$

Then,

$$K_2 = 2.5,\ K_3 = 4.6,\ K_4 = 20$$

The table is entered on page 29 using $K_2 = 2$, $K_3 = 4$, $K_4 = 20$, and the permissible methods are seen to be 1, 3 and 4. Thus for a minimum element spread synthesis the First Foster form, or the First or second Cauer forms, could be used. If K_4 were 80 or greater, the appropriate box would fall to the right of the heavy line, so that none of the methods would result in element ratios in the range defined by (33). However, the methods given here would assure ratios in the more extended range defined by (34); that is, in the range

$$0.01 \leq \left\{ \frac{R_m}{R_n} \text{ AND } \frac{C_j}{C_k} \right\} \leq 100$$

Then,

$$[illegible]$$

The tabulated values on page 59 ... [illegible]
and the permissible [illegible] are seen to be 1, 3 and 4, while
for a minimum element [illegible] in the first [illegible] form,
or the first or second lower forms, could be used. If [illegible]
were 60 or greater, the appropriate box would fall to the
right of the heavy line, so that none of the [illegible] would
result in element ratios in the range defined by [illegible]. How-
ever, the methods given here would ensure [illegible] in the
extended range defined by (56); that is, in the range

$$[illegible]$$

Acknowledgement is gratefully made to Mr. E. R. Ferdinand, who carefully prepared the tables for reproduction, and to Miss Judith Blevins, who did the meticulous typing of the final paper.

```
H       NET    1      W.F.CARLSON  N-4   7-5389
        DIMENSIONC2(10),C3(10),C4(10)
     10 READ1,N
        READ3,(C2(I),I=1,N),(C3(J),J=1,N),
       X(C4(K),K=1,N)
        DO20I=1,N
        DO30J=I,N
        DO40K=I,N
        R2R1A=(C2(I)-1.)*(C4(K)-1.)/(C3(J)-1.)
        R3R1A=(C3(J)-C2(I))*(C4(K)-C3(J))/(C3(J)*(C3(J)-1.))
        R3R2A=R3R1A/R2R1A
        C2C1A=(C2(I)-1.)*(C4(K)-1.)/((C3(J)-C2(I))*
       X(C4(K)-C3(J)))
        R2R1B=C3(J)*(C4(K)-C2(I))/(C4(K)*(C2(I)-1.)
       X*(C3(J)-C2(I)))
        R3R1B=C3(J)*(C4(K)-C2(I))/(C2(I)*(C4(K)-C3(J))
       X*(C4(K)-1.))
        R3R2B=R3R1B/R2R1B
        C2C1B=(((C2(I))**2)*(C4(K)-1.)*(C4(K)-C3(J)))
       X/(((C4(K))**2)*(C2(I)-1.)*(C3(J)-C2(I)))
        A=C2(I)+C4(K)
        B=C2(I)*C4(K)
        C=1.+C3(J)
        D=C3(J)
        R2R1C=((A-C)**2)/(C*(A-C)-(B-D))
        R3R1C=((A-C)*(B*C-A*D)-(B-D)**2)/
       X(D*(C*(A-C)-(B-D)))
        R3R2C=R3R1C/R2R1C
        C2C1C=(C*(A-C)-(B-D))**2/
       X((A-C)*(B*C-A*D)-(B-D)**2)
        R2R1D=(D*(A*(B*C-A*D)-B*(B-D)))/
       X((B*C-A*D)**2)
        R3R1D=(D*(A*(B*C-A*D)-B*(B-D)))/
       X((B-D)*(A*(B*C-A*D)-B*(B-D))-
       X(B*C-A*D)**2)
        R3R2D=R3R1D/R2R1D
        C2C1D=(B*(B-D)*(A*(B*C-A*D)-B*(B-D))
       X-B*(B*C-A*D)**2)/
       X((A*(B*C-A*D)-B*(B-D))**2)
        IF(SENSESWITCH1)14,15
     14 PRINT2,C2(I),C3(J),C4(K),R2R1A,
       XR2R1B,R2R1C,R2R1D,R3R1A,R3R1B,R3R1C,
       XR3R1D,R3R2A,R3R2B,R3R2C,R3R2D,
       XC2C1A,C2C1B,C2C1C,C2C1D
        GOTO40
     15 WRITEOUTPUTTAPE9,2,C2(I),C3(J),
       XC4(K),R2R1A,R2R1B,R2R1C,R2R1D,
       XR3R1A,R3R1B,R3R1C,R3R1D,R3R2A,R3R2B
       X,R3R2C,R3R2D,C2C1A,C2C1B,C2C1C,C2C1D
     40 CONTINUE
     30 CONTINUE
     20 CONTINUE
        PAUSE7
        GOTO10
      1 FORMAT(18I4)
      2 FORMAT(9H0      C2=F4.0,8H      C3=F4.0,
       X8H      C4=F4.0/(1H 1P4E13.2))
      3 FORMAT(18F4.0)
        END(0,1,0,1,1)
```

LIST OF REFERENCES

1 Guillemin, E. A., "Synthesis of Passive Networks", p. 80, John Wiley and Sons Inc., New York, 1957.

2 Truxal, J. G., "Control System Synthesis", p. 186, McGraw-Hill Book Co. Inc., New York, 1955.

3 Guillemin, E. A., "Synthesis of Passive Networks", pp. 107-126, John Wiley and Sons Inc., New York, 1957.

4 Guillemin, E. A., "Synthesis of Passive Networks", p. 64, John Wiley and Sons Inc., New York, 1957.

$K_2 = 2$

K_4 / K_3	6	8	10	20	40	60	80	100	200	400
4	1 2 3 4	1 2 3 4	1 2 3 4	1 3 4	0.3A-1 0.5A-1,3 0.7A-1,4 0.9A-4 0.7B-3,4 0.9B-4 0.7D-1	0.5A-1	1 2 3 4	1 2 3 4	1 3 4	1 3
6		1 2 3 4	1 2 3 4	1 2 3 4	1 3	0.5A-1 0.7A-4 0.9A-4 0.3B-3 0.5B-3 0.7B-3,4 0.9B-3,4 0.5D-4	0.5A-1 0.7B-3 0.9B-3,4	1 2 3 4	1 2 3 4	1 3
8			1 2 3	1 2 3 4	1 2 3 4	1 3	0.3B-3 0.5B-3 0.7B-3,4 0.9B-3,4	0.7B-3 0.9B-3,4	1 2 3 4	1 3 4
10				1 2 3 4	1 2 3 4	1 3 4	1 3	0.3B-3 0.5B-3 0.7B-3,4 0.9B-2,3,4	1 2 3 4	1 2 3 4

$K_2 = 2$

K_4 / K_3	6	8	10	20	40	60	80	100	200	400
20					1 2	2 4	2 4	2 4	0.5B-3 0.7B-4 0.9B-2	1 2 3 4
40						0.3B-3 0.5B-1,3 0.7B-3 0.9B-3	0.5B-1 0.5C-2	0.5C-2,4 0.7C-4	1 2 3 4	1 2 3 4
60							0.5B-1,3 0.7B-3	1 2 3 4	1 2 3 4	1 2 3 4
80								0.5B-3	1 2 3 4	1 2 3 4

$K_2 = 2$

K_4	6	8	10	20	40	60	80	100	200	400
K_3										
100									1 2 3 4	1 2 3 4
200										1 2 3

$K_2 = 4$

K_4 / K_3	8	10	20	40	60	80	100	200	400
6	2 3	2 3 4	3 4	0.3A-3 0.5A-4 0.7A-4 0.3D-3 0.5D-3 0.7D-3	1 2 3 4	1 2 3 4	1 2 3 4	1 3	1 3
8		2 3	1 2 3 4	0.3A-3,4 0.5A-3,4 0.7A-4 0.9A-4 0.7B-3 0.3D-3,4 0.5D-1,2,3,4 0.7D-1,3 0.9D-3	0.5D-3	1 2 3 4	1 2 3 4	1 2 3 4	1 3
10			1 2 3 4	3 4	0.5A-1,3 0.5D-3 0.7D-3	1 2 3 4	1 2 3 4	1 2 3 4	1 3
20				1 2 3 4	1 2 3 4	0.5A-1,3 0.9A-4 0.7B-2,4 0.9B-3 0.7C-1,3 0.9C-1 0.5D-2,4 0.9D-3	0.5A-1,3 0.7A-4 0.7B-4 0.9B-2 0.9C-1 0.5D-2,4	1 2 3 4	1 2 3 4

$K_2 = 4$

K_4 / K_3	8	10	20	40	60	80	100	200	400
40					1 3	1 3	1 3	0.7B-4 0.9B-2 0.7C-1,3	1 2 3 4
60						3	1 3	0.5B-1,3 0.7B-3 0.5C-2,3,4 0.7C-4	1 2 3 4
80							0.5C-1,3	0.5B-3	1 2 3 4
100								1 2 3 4	1 2 3 4

$K_2 = 4$

K_4	8	10	20	40	60	80	100	200	400
K_3									
200									1 2 3 4

$K_2 = 6$

K_4 / K_3	10	20	40	60	80	100	200	400
8	2 3	3	0.5A-4 0.7A-4 0.5D-3 0.7D-3	1 2 3 4	1 2 3 4	1 2 3 4	1 3	1 3
10		2 3 4	0.3A-4 0.5A-4 0.7A-4 0.3D-4 0.5D-3 0.7D-3	1 2 3 4	1 2 3 4	1 2 3 4	1 3	1 3
20			2 3 4	0.3A-3 0.5A-1,3 0.7A-2,4 0.9A-2,4 0.5C-3,4 0.7C-3,4 0.9C-4 0.3D-2 0.5D-2,3,4 0.7D-2,3 0.9D-3	0.5A-4 0.7A-4 0.5D-2,4	1 2 3 4	1 2 3 4	1 3
40				3	2 3	2 4	1 2 3 4	1 2 3 4

$K_2 = 6$

K_4	10	20	40	60	80	100	200	400
K_3								
60					0.5A-3 0.3C-3 0.5C-1 0.7C-1,3 0.9C-1	3	0.7C-3 0.9C-1	1 2 3 4
80						0.5A-3 0.3C-3 0.5C-1,3 0.7C-1,3	0.5C-1,2,3,4 0.7C-1,3	1 2 3 4
100							3	1 2 3 4
200								1 2 3 4

$K_2 = 8$

K_4	20	40	60	80	100	200	400
K_3							
10	3	0.5A-4 0.5D-3	1 2 3 4	1 2 3 4	1 2 3 4	1 3	1 3
20		2 4	0.5A-4 0.7A-4 0.3D-4 0.5D-2,4 0.7D-3	1 2 3 4	1 2 3 4	1 2 3 4	1 3
40			3	2 4	0.5A-1,3 0.7C-3 0.9C-3,4	1 2 3 4	1 2 3 4
60				0.5A-3 0.3C-3 0.5C-3 0.7C-1,3 0.9C-1,3	0.5A-1,3 0.3C-3 0.5C-1,2,3,4 0.7C-1,3 0.9C-1,3	1 2 3 4	1 2 3 4

$K_2 = 8$

K_4	20	40	60	80	100	200	400
K_3							
80					0.5C-3 0.7C-1,3 0.9C-1	0.7C-3 0.9C-1,3	1 2 3 4
100						0.5C-3 0.7C-1,3	1 2 3 4
200							1 2 3 4

$K_2 = 10$

K_4	40	60	80	100	200	400
K_3						
20	0.5A-2,3,4 0.7A-2 0.7C-3,4 0.9C-3 0.3D-2 0.5D-2,4 0.9D-3 0	0.5A-4 0.7A-4	1 2 3 4	1 2 3 4	1 2 3 4	1 3
40		0.3A-3 0.5A-3 0.1C-2 0.3C-2,3,4 0.5C-2,3,4 0.7C-1,2,3,4 0.9C-1,2,3,4 0.9D-3	0.5A-3 0.3C-4 0.5C-4 0.7C-3,4 0.9C-3,4	0.5D-2	1 2 3 4	1 2 3 4
60			0.3C-3 0.5C-2,3 0.7C-1,3 0.9C-1,3	0.5A-3 0.3C-2 0.5C-2,3,4 0.7C-1,2,3,4 0.9C-1,3,4	1 2 3 4	1 2 3 4
80				0.5C-3 0.7C-1,3 0.9C-1	0.9C-3	1 2 3 4

$K_2 = 10$

K_4	40	60	80	100	200	400
K_3						
100					0.5C-4 0.7C-3 0.9C-1	1 2 3 4
200						1 2 3 4

$K_2 = 20$

K_4	60	80	100	200	400
K_3					
40	0.9C-3	1 2 3 4	1 2 3 4	1 2 3 4	1 3
60		0.5C-2 0.9C-3	0.9C-3	1 2 3 4	1 3
80			0.5C-2 0.7C-3 0.9C-3	1 2 3 4	1 2 3 4
100				1 2 3 4	1 2 3 4

$K_2 = 20$

K_4	60	80	100	200	400
K_3					
200					1 2 3 4

$K_2 = 40$

K_4	80	100	200	400
K_3				
60	2 3 4	2 3 4	3 4	3
80		2 3 4	2 3 4	1 3
100			2 3 4	1 3
200				1 2 3 4

$K_2 = 60$

K_4	100	200	400
K_3			
80	2 3 4	3	3
100		2 3 4	3
200			2 3 4

$K_2 = 80$

K_4	200	400
K_3		
100	3	3
200		2 3 4

$K_2 = 100$

K_4	400
K_3	
200	3