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# Multi Attribute Decision Making Strategy on Projection and Bidirectional Projection Measures of Interval Rough Neutrosophic Sets

Surapati Pramanik<sup>1</sup>, Rumi Roy<sup>2</sup>, Tapan Kumar Roy<sup>3</sup>, Florentin Smarandache<sup>4</sup>

<sup>1</sup> Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O.-Narayanpur, District –North 24 Parganas, Pin code-743126, West Bengal, India. \*E-mail: [sura\\_pati@yahoo.co.in](mailto:sura_pati@yahoo.co.in)

<sup>2</sup> Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, P.O.-Botanic Garden, Howrah-711103, West Bengal, India, E-mail: [roy.rumi.r@gmail.com](mailto:roy.rumi.r@gmail.com)

<sup>3</sup> Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, P.O.-Botanic Garden, Howrah-711103, West Bengal, India, E-mail: [roy\\_t\\_k@gmail.com](mailto:roy_t_k@gmail.com)

**Abstract.** In this paper, we define projection and bidirectional projection measures between interval rough neutrosophic sets and prove their basic properties. Then two new multi attribute decision making strategies are proposed based on interval rough neutrosophic projection

and bidirectional projection measures respectively. Then the proposed methods are applied for solving multi attribute decision making problems. Finally, a numerical example is solved to show the feasibility, applicability and effectiveness of the proposed strategies.

**Keywords:** Projection measure, Bidirectional projection measure, Interval rough neutrosophic set, MADM problem.

## 1 Introduction

The concept of neutrosophic set [1, 2, 3, 4, 5] introduced by Smarandache is a generalization of crisp set [6], fuzzy set [7] and intuitionistic fuzzy set [8]. To use neutrosophic set in real fields, Wang et al. extended it to single valued neutrosophic set [9].

Broumi et al. introduced rough neutrosophic set [10, 11] by combining the concept of rough set [12] and neutrosophic set.

Broumi and Smarandache defined interval rough neutrosophic set [13] by combining the concept of rough set and interval neutrosophic set theory [14].

Projection measure is a very useful for solving decision making problems because it takes into account the distance as well as the included angle between points. Yue [15] studied projection based MADM problem in crisp environment. Yue also [16] presented a projection method to obtain weights of the experts in a group decision making problem. Xu and Da [17] and Xu [18] studied projection method for decision making in uncertain environment with preference information. Yang et al. [19] develop projection method for material selection in fuzzy environment. Xu and Hu [20] developed two projection based models for MADM in intuitionistic fuzzy and interval valued intuitionistic fuzzy environment. Zeng et al. [21] provided weighted projection algorithm for intuitionistic fuzzy

MADM problems and interval-valued intuitionistic fuzzy MADM problems. Chen and Ye [22] developed the projection based model for solving MADM problem and applied it to select clay-bricks in construction field.

To overcome the shortcomings of the general projection measure Ye [23] introduced a bidirectional projection measure between single valued neutrosophic numbers and developed MADM method for selecting problems of mechanical design schemes under a single valued neutrosophic environment. Ye [24] also presented the bidirectional projection method for multiple attribute group decision making with neutrosophic numbers. Dey et al. [25] defined weighted projection measure with interval neutrosophic environment and applied it to solve MADM problems with interval valued neutrosophic information. Yue [26] proposed a projection based approach for partner selection in a group decision making problem with linguistic value and intuitionistic fuzzy information. Dey et al. [27] defined projection, bidirectional projection and hybrid projection measures between bipolar neutrosophic sets and presented bipolar neutrosophic projection based models for MADM problems. Pramanik et al. [28] defined projection and bidirectional projection measure between rough neutrosophic sets and proposed the decision making methods based on them.

**Research gap** MADM strategy using projection and bidirectional projection measures under interval rough neutrosophic environment.

**Research questions**

- (i) Is it possible to define two new projection and bidirectional projection measure between interval rough neutrosophic sets?
- (ii) Is it possible to develop two new MADM strategies based on the proposed measures in interval rough neutrosophic environment?

The objectives of the paper are

- (i) To define two new projection and bidirectional projection measure between interval rough neutrosophic sets.
- (ii) To develop two new MADM strategies based on the proposed measures in interval rough neutrosophic environment.

**Contributions**

- (i) In this paper, we propose projection and bidirectional projection measures under interval rough neutrosophic environment.
- (ii) In this paper, we develop two new MADM strategies based on the proposed measures in interval rough neutrosophic environment.
- (iii) We also present numerical example to show the effectiveness and applicability of the proposed measures.

Rest of the paper is organized as follows: Section 2 describes preliminaries of neutrosophic number, SVNS, RNS and IRNS. Section 3 presents definitions and properties of proposed projection and bidirectional projection measure between IRNSs. Section 4 describes the MADM methods based on projection and bidirectional projection measures of IRNSs. In section 5 we describe a numerical example. Finally, section 6 presents the conclusion.

**2 Preliminaries**

In this Section, we provide some basic definitions regarding SVNSs, IRNSs which are useful in the paper.

**2.1 Neutrosophic set:**

In 1999, Smarandache gave the following definition of neutrosophic set(NS) [1].

Definition 2.1.1. Let X be a space of points (objects) with generic element in X denoted by x. A NS A in X is

characterized by a truth-membership function  $T_A$ , an indeterminacy membership function  $I_A$  and a falsity membership function  $F_A$ . The functions  $T_A$ ,  $I_A$  and  $F_A$  are real standard or non-standard subsets of  $(-0, 1^+)$  that is  $T_A: X \rightarrow (-0, 1^+)$ ,  $I_A: X \rightarrow (-0, 1^+)$  and  $F_A: X \rightarrow (-0, 1^+)$ . It should be noted that there is no restriction on the sum of  $T_A(x)$ ,  $I_A(x)$  and  $F_A(x)$  i.e.  $-0 \leq T_A(X) + I_A(X) + F_A(X) \leq 3^+$

**Definition 2.1.2:** (complement)

The complement of a neutrosophic set A is denoted by  $C(A)$  and is defined by  $T_{c(A)}(x) = \{1^+\} - T_A(x)$ ,  $I_{c(A)}(x) = \{1^+\} - I_A(x)$ ,  $F_{c(A)}(x) = \{1^+\} - F_A(x)$ .

**Definition 2.1.3:** (Containment)

A neutrosophic set A is contained in the other neutrosophic set B, denoted by  $A \subseteq B$  iff

$$\inf T_A(x) \leq \inf T_B(x), \sup T_A(x) \leq \sup T_B(x), \\ \inf I_A(x) \geq \inf I_B(x), \sup I_A(x) \geq \sup I_B(x), \\ \inf F_A(x) \geq \inf F_B(x), \sup F_A(x) \geq \sup F_B(x) \quad \forall x \in X$$

**Definition 2.1.4:** (Single-valued neutrosophic set).

Let X be a universal space of points (objects) with a generic element of X denoted by x. A single valued neutrosophic set A is characterized by a truth membership function  $T_A(x)$ , a falsity membership function  $F_A(x)$  and indeterminacy function  $I_A(x)$  with

$$T_A(x), I_A(x) \text{ and } F_A(x) \in [0, 1] \quad \forall x \text{ in } X$$

When X is continuous, a SNVS S can be written as follows

$$A = \int_x \langle T_A(x), F_A(x), I_A(x) \rangle / x \quad \forall x \in X$$

and when X is discrete, a SVNS S can be written as follows

$$A = \sum \langle T_A(x), F_A(x), I_A(x) \rangle / x \quad \forall x \in X$$

For a SVNS S,  $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$ .

**Definition 2.1.5:**

The complement of a single valued neutrosophic set A is denoted by  $c(A)$  and is defined by  $T_{c(A)}(x) = F_A(x)$ ,  $I_{c(A)}(x) = 1 - I_A(x)$ ,  $F_{c(A)}(x) = T_A(x)$ .

Definition 2.1.6: A SVNS A is contained in the other SVNS B, denoted as  $A \subseteq B$  iff,

$$T_A(x) \leq T_B(x), I_A(x) \geq I_B(x)$$

$$\text{and } F_A(x) \geq F_B(x) \quad \forall x \in X.$$

**2.2 Rough neutrosophic set**

Rough neutrosophic sets [10, 11] are the generalization of rough fuzzy sets [29, 30] and rough intuitionistic fuzzy sets [31].

**Definition 2.2.1:**

Let Y be a non-null set and R be an equivalence relation on Y. Let P be neutrosophic set in Y with the membership function  $T_P$ , indeterminacy function  $I_P$  and non-membership function  $F_P$ . The lower and the upper

approximations of P in the approximation (Y, R) denoted by are respectively defined as:

$$\underline{N(P)} = \langle \langle x, T_{\underline{N(P)}}(x), I_{\underline{N(P)}}(x), F_{\underline{N(P)}}(x) \rangle / y \in [x]_R, x \in Y \rangle$$

and

$$\overline{N(P)} = \langle \langle x, T_{\overline{N(P)}}(x), I_{\overline{N(P)}}(x), F_{\overline{N(P)}}(x) \rangle / y \in [x]_R, x \in Y \rangle$$

where,

$$T_{\underline{N(P)}}(x) = \wedge z \in [x]_R T_P(Y),$$

$$I_{\underline{N(P)}}(x) = \wedge z \in [x]_R I_P(Y),$$

$$F_{\underline{N(P)}}(x) = \wedge z \in [x]_R F_P(Y)$$

and

$$T_{\overline{N(P)}}(x) = \vee z \in [x]_R T_P(Y),$$

$$I_{\overline{N(P)}}(x) = \vee z \in [x]_R I_P(Y),$$

$$F_{\overline{N(P)}}(x) = \vee z \in [x]_R F_P(Y)$$

So,

$$0 \leq T_{\underline{N(P)}}(x) + I_{\underline{N(P)}}(x) + F_{\underline{N(P)}}(x) \leq 3$$

and

$$0 \leq T_{\overline{N(P)}}(x) + I_{\overline{N(P)}}(x) + F_{\overline{N(P)}}(x) \leq 3$$

Here  $\vee$  and  $\wedge$  denote “max” and “min” operators respectively,  $T_P(y)$ ,  $I_P(y)$  and  $F_P(y)$  are the membership, indeterminacy and non-membership of Y with respect to P.

Thus NS mapping,

$\underline{N}, \overline{N}: N(Y) \rightarrow N(Y)$  are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair  $(\underline{N(P)}, \overline{N(P)})$  is called the rough neutrosophic set in (Y, R).

Definition 2.2.2 If  $N(P) = (\underline{N(P)}, \overline{N(P)})$

is a rough neutrosophic set in (Y, R), the rough complement of N(P) is the rough neutrosophic set denoted by

$$\sim N(P) = ((\overline{N(P)})^c, (\underline{N(P)})^c)$$

,where

$$(\underline{N(P)})^c \text{ and } (\overline{N(P)})^c$$

are the complements of neutrosophic sets  $\underline{N(P)}$  and  $\overline{N(P)}$  respectively.

### 2.3 Interval rough neutrosophic set

Interval neutrosophic rough set is the hybrid structure of rough sets and interval neutrosophic sets. According to Broumi and Smarandache interval neutrosophic rough set is the generalizations of interval valued intuitionistic fuzzy rough set.

#### Definition 2.3.1

Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by  $[x]_R$  for  $X \in U$ , the lower and upper approximation of X with respect to (U, R) are denoted by respectively,

$\underline{RX}$  and  $\overline{RX}$  and are defined by

$$\underline{RX} = \{x \in U : [x]_R \subseteq X\},$$

$$\overline{RX} = \{x \in U : [x]_R \cap X \neq \emptyset\}.$$

Now if  $\underline{RX} = \overline{RX}$ , then X is called definable; otherwise X is called a rough set.

#### Definition 2.3.2

Let U be a universe and X, a rough set in U. An intuitionistic fuzzy rough set A in U is characterized by a membership function  $\mu_A: U \rightarrow [0, 1]$  and non-membership function  $\nu_A: U \rightarrow [0, 1]$  such that  $\mu_A(\underline{RX})=1$  and  $\nu_A(\overline{RX})=0$  ie,  $[\mu_A(x), \nu_A(x)] = [1, 0]$  if  $x \in (\underline{RX})$  and  $\mu_A(U - \overline{RX}) = 0$ ,  $\nu_A(U - \overline{RX}) = 1$  ie,

$$0 \leq \mu_A(\overline{RX} - \underline{RX}) + \nu_A(\overline{RX} - \underline{RX}) \leq 1$$

#### Definition 2.3.3

Assume that, (U, R) be a Pawlak approximation space, for an interval neutrosophic set

$$A = \{ \langle x, [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] \rangle : x \in U \}$$

The lower approximation  $\underline{A}_R$  and the upper approximation  $\overline{A}_R$  of A in the Pawlak approximation space (U, R) are expressed as follows:

$$\begin{aligned} \underline{A}_R &= \{ \langle x, [\wedge_{y \in [x]_R} \{T_A^L(y)\}, \vee_{y \in [x]_R} \{T_A^U(y)\}], \\ &[\vee_{y \in [x]_R} \{I_A^L(y)\}, \wedge_{y \in [x]_R} \{I_A^U(y)\}], \\ &[\vee_{y \in [x]_R} \{F_A^L(y)\}, \wedge_{y \in [x]_R} \{F_A^U(y)\}] \rangle : x \in U \} \\ \overline{A}_R &= \{ \langle x, [\vee_{y \in [x]_R} \{T_A^L(y)\}, \wedge_{y \in [x]_R} \{T_A^U(y)\}], \\ &[\wedge_{y \in [x]_R} \{I_A^L(y)\}, \vee_{y \in [x]_R} \{I_A^U(y)\}], \\ &[\wedge_{y \in [x]_R} \{F_A^L(y)\}, \vee_{y \in [x]_R} \{F_A^U(y)\}] \rangle : x \in U \} \end{aligned}$$

The symbols  $\wedge$  and  $\vee$  indicate “min” and “max” operators respectively. R denotes an equivalence relation for interval neutrosophic set A. Here  $[x]_R$  is the equivalence class of the element x. It is obvious that

$$[\wedge_{y \in [x]_R} \{T_A^L(y)\}, \wedge_{y \in [x]_R} \{T_A^U(y)\}] \subseteq [0, 1],$$

$$[\vee_{y \in [x]_R} \{I_A^L(y)\}, \vee_{y \in [x]_R} \{I_A^U(y)\}] \subseteq [0, 1],$$

$$[\vee_{y \in [x]_R} \{F_A^L(y)\}, \vee_{y \in [x]_R} \{F_A^U(y)\}] \subseteq [0, 1].$$

$$\text{and } 0 \leq \wedge_{y \in [x]_R} \{T_A^U(y)\} + \vee_{y \in [x]_R} \{I_A^U(y)\} + \vee_{y \in [x]_R} \{F_A^U(y)\} \leq 3$$

Then  $\underline{A}_R$  is an interval neutrosophic set (INS)

Similarly, we have

$$\begin{aligned}
 & [\vee_{y \in [x]_R} \{T_A^L(y)\}, \vee_{y \in [x]_R} \{T_A^U(y)\}] \subset [0,1], \\
 & [\wedge_{y \in [x]_R} \{T_A^L(y)\}, \wedge_{y \in [x]_R} \{T_A^U(y)\}] \subset [0,1], \\
 & [\wedge_{y \in [x]_R} \{F_A^L(y)\}, \wedge_{y \in [x]_R} \{F_A^U(y)\}] \subset [0,1]
 \end{aligned}$$

and

$$\begin{aligned}
 0 \leq & \vee_{y \in [x]_R} \{T_A^U(y)\} + \wedge_{y \in [x]_R} \{I_A^U(y)\} + \\
 & \wedge_{y \in [x]_R} \{F_A^U(y)\} \leq 3
 \end{aligned}$$

Then  $\underline{A}_R$  is an interval neutrosophic set.

If  $\underline{A}_R = \overline{A}_R$  then A is a definable set, otherwise A is an interval valued neutrosophic rough set. Here,  $\underline{A}_R$  and  $\overline{A}_R$  are called the lower and upper approximations of interval neutrosophic set with respect to approximation space (U,R) respectively.  $\underline{A}_R$  and  $\overline{A}_R$  are simply denoted by  $\underline{A}$  and  $\overline{A}$  respectively.

**3 Projection and Bidirectional projection measure of interval rough neutrosophic sets :**

Existing projection and bidirectional projection measure does not deal with interval rough neutrosophic set(IRNS)s. Therefore, a new projection and bidirectional projection measure between IRNSs is proposed.

Assume that there are two IRNSs

$$\begin{aligned}
 M = \{ & \langle x_i, ([T_{iM}^-, T_{iM}^+], [I_{iM}^-, I_{iM}^+], [F_{iM}^-, F_{iM}^+], \\
 & [T_{iM}^-, T_{iM}^+], [I_{iM}^-, I_{iM}^+], [F_{iM}^-, F_{iM}^+]) \rangle : i = 1, 2, \dots, n\}
 \end{aligned}$$

and

$$\begin{aligned}
 N = \{ & \langle x_i, ([T_{iN}^-, T_{iN}^+], [I_{iN}^-, I_{iN}^+], [F_{iN}^-, F_{iN}^+], \\
 & [T_{iN}^-, T_{iN}^+], [I_{iN}^-, I_{iN}^+], [F_{iN}^-, F_{iN}^+]) \rangle : i = 1, 2, \dots, n\}
 \end{aligned}$$

Then the inner product of M and N denoted by M.N can be defined as

$$\begin{aligned}
 M.N = & \sum_{i=1}^n [ \frac{T_{iM}^- \cdot T_{iN}^- + T_{iM}^+ \cdot T_{iN}^+ + I_{iM}^- \cdot I_{iN}^- + I_{iM}^+ \cdot I_{iN}^+}{2} \\
 & + \frac{F_{iM}^- \cdot F_{iN}^- + F_{iM}^+ \cdot F_{iN}^+ + T_{iM}^- \cdot T_{iN}^- + T_{iM}^+ \cdot T_{iN}^+}{2} \\
 & + \frac{I_{iM}^- \cdot I_{iN}^- + I_{iM}^+ \cdot I_{iN}^+ + F_{iM}^- \cdot F_{iN}^- + F_{iM}^+ \cdot F_{iN}^+}{2} ]
 \end{aligned}$$

The modulus of M can be defined as

$$\|M\| = \sqrt{\sum_{i=1}^n \left[ \begin{aligned} & (\frac{T_{iM}^-}{2})^2 + (\frac{T_{iM}^+}{2})^2 + (\frac{I_{iM}^-}{2})^2 + (\frac{I_{iM}^+}{2})^2 \\ & + (\frac{F_{iM}^-}{2})^2 + (\frac{F_{iM}^+}{2})^2 + (\frac{T_{iM}^-}{2})^2 + (\frac{T_{iM}^+}{2})^2 \\ & + (\frac{I_{iM}^-}{2})^2 + (\frac{I_{iM}^+}{2})^2 + (\frac{F_{iM}^-}{2})^2 + (\frac{F_{iM}^+}{2})^2 \end{aligned} \right]}$$

and the modulus of N can be defined as

$$\|N\| = \sqrt{\sum_{i=1}^n \left[ \begin{aligned} & (\frac{T_{iN}^-}{2})^2 + (\frac{T_{iN}^+}{2})^2 + (\frac{I_{iN}^-}{2})^2 + (\frac{I_{iN}^+}{2})^2 \\ & + (\frac{F_{iN}^-}{2})^2 + (\frac{F_{iN}^+}{2})^2 + (\frac{T_{iN}^-}{2})^2 + (\frac{T_{iN}^+}{2})^2 \\ & + (\frac{I_{iN}^-}{2})^2 + (\frac{I_{iN}^+}{2})^2 + (\frac{F_{iN}^-}{2})^2 + (\frac{F_{iN}^+}{2})^2 \end{aligned} \right]}$$

Definition4.1. The projection of M on N can be defined as

$$Proj(M)_N = \frac{1}{\|N\|} M.N.$$

Definition4.2. The bidirectional projection measure between the RNSs M and N is defined as

$$\begin{aligned}
 BProj(M, N) &= \frac{1}{1 + \|\|M\| - \|N\|\| M.N} \\
 &= \frac{\|M\| \|N\|}{\|M\| \|N\| + \|\|M\| - \|N\|\| M.N}
 \end{aligned}$$

Here also the bidirectional projection measure satisfies the following properties :

- (1) BProj(M,N) = BProj(N,M);
- (2) 0 ≤ BProj(M,N) ≤ 1;
- (3) BProj(M,N) = 1, iff M = N.

**Proof:**

$$\begin{aligned}
 (i) & BProj(M, N) \\
 &= \frac{1}{1 + \|\|M\| - \|N\|\| M.N} \\
 &= \frac{1}{1 + \|\|N\| - \|M\|\| N.M} \\
 &= BProj(N, M)
 \end{aligned}$$

(ii)As

$$\frac{1}{1 + \|\|M\| - \|N\|\| M.N} \geq 0$$

and

$$\frac{1}{1 + \|\|M\| - \|N\|\| M.N} \leq 1$$

so, 0 ≤ BProj(M, N) ≤ 1;

(iii)If M=N then

$$\begin{aligned}
 BProj(M, N) &= BProj(M, M) \\
 &= \frac{1}{1 + \|\|M\| - \|M\|\| M.M} \\
 &= 1
 \end{aligned}$$

**4. Projection And Bidirectional Projection Based Decision Making Methods For MADM Problems With Interval Rough Neutrosophic Information**

In this section, we develop projection and bidirectional projection based decision making models to solve MADM problems with interval rough neutrosophic information. Consider C={C<sub>1</sub>, ..., C<sub>m</sub>} be the set of attributes and A={A<sub>1</sub>, ..., A<sub>n</sub>} be a set of alternatives. Now we provide two algorithms for MADM problems involving interval rough neutrosophic information.

**4.1. Algorithm 1.(see Fig 1)**

Step 1. The value of alternative A<sub>i</sub>(i=1,.....,n) for the attribute C<sub>j</sub>(j=1,.....,m) is evaluated by the decision maker

in terms of IRNSs and the interval rough neutrosophic decision matrix is constructed as:

$$D = \langle z_{ij} \rangle_{n \times m} = \begin{pmatrix} z_{11} & z_{12} & \dots & z_{1m} \\ z_{21} & z_{22} & \dots & z_{2m} \\ \dots & \dots & \dots & \dots \\ z_{n1} & z_{n2} & \dots & z_{nm} \end{pmatrix}$$

where

$$z_{ij} = \langle ([T_{iM}^-, T_{iM}^+], [I_{iM}^-, I_{iM}^+], [F_{iM}^-, F_{iM}^+], [T_{iM}^-, T_{iM}^+], [I_{iM}^-, I_{iM}^+], [F_{iM}^-, F_{iM}^+]) \rangle$$

with

$$0 \leq \bigvee_{y \in [X]_R} \{T_A^U(y)\} + \bigwedge_{y \in [X]_R} \{I_A^U(y)\} + \bigwedge_{y \in [X]_R} \{F_A^U(y)\} \leq 3$$

Step 2. Calculate the weighted alternative decision matrix  
For the attribute  $C_j$  ( $j=1, \dots, m$ ) the weight vector of attribute is considered as :  $W = (w_1, w_2, \dots, w_m)$  with

$$w_j \geq 0 \quad \text{and} \quad \sum_{j=1}^m w_j = 1$$

On calculating

$$s_{ij} = \langle ([w_j T_{iM}^-, w_j T_{iM}^+], [w_j I_{iM}^-, w_j I_{iM}^+], [w_j F_{iM}^-, w_j F_{iM}^+], [w_j T_{iM}^-, w_j T_{iM}^+], [w_j I_{iM}^-, w_j I_{iM}^+], [w_j F_{iM}^-, w_j F_{iM}^+]) \rangle$$

for  $i=1, 2, \dots, n$  and  $j=1, 2, \dots, m$ , we obtain the weighted alternative decision matrix

$$S = \langle s_{ij} \rangle_{n \times m} = \begin{pmatrix} s_{11} & s_{12} & \dots & s_{1m} \\ s_{21} & s_{22} & \dots & s_{2m} \\ \dots & \dots & \dots & \dots \\ s_{n1} & s_{n2} & \dots & s_{nm} \end{pmatrix}$$

Step 3. Determine the ideal solution  $S^*$ .

For benefit type attribute,

$$S^* = \{(\min_i \underline{T}_{ij}, \max_i \underline{I}_{ij}, \max_i \underline{F}_{ij}), (\max_i \overline{T}_{ij}, \min_i \overline{I}_{ij}, \min_i \overline{F}_{ij})\}$$

For cost type attribute,

$$S^* = \{(\max_i \underline{T}_{ij}, \min_i \underline{I}_{ij}, \min_i \underline{F}_{ij}), (\min_i \overline{T}_{ij}, \max_i \overline{I}_{ij}, \max_i \overline{F}_{ij})\}$$

Step 4. Compute the projection measure between  $S^*$  and  $Z_i = \langle Z_{ij} \rangle_{n \times m}$  for all  $i = 1, \dots, n$  and  $j = 1, \dots, m$ .

Step 5. Ranking of alternatives is prepared based on the values of projection measure. The highest value reflects the best alternatives.

Step 6. End.

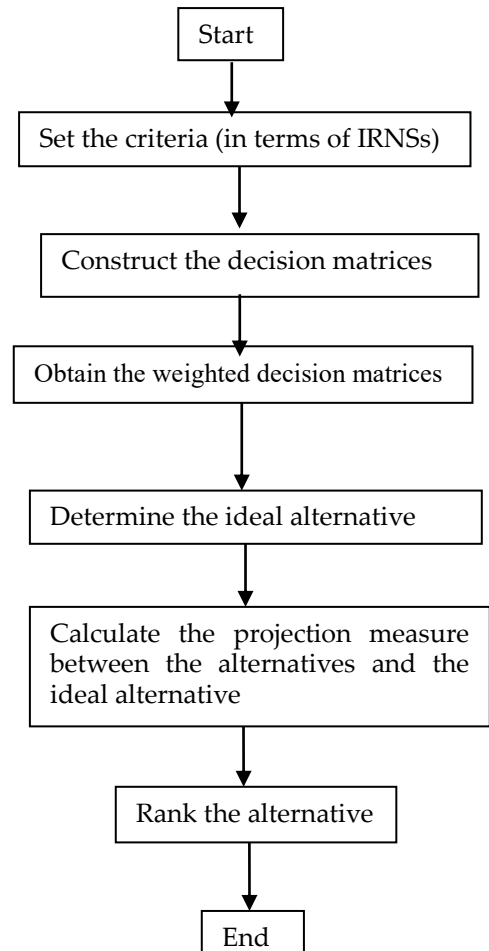


Fig 1. A flowchart of the proposed decision making method

**4.2. Algorithm 2.**(see Fig 2)

Step 1. The value of alternative  $A_i$  ( $i=1, \dots, n$ ) for the attribute  $C_j$  ( $j=1, \dots, m$ ) is evaluated by the decision maker in terms of IRNSs and the interval rough neutrosophic decision matrix is constructed as:

$$D = \langle z_{ij} \rangle_{n \times m} = \begin{pmatrix} z_{11} & z_{12} & \dots & z_{1m} \\ z_{21} & z_{22} & \dots & z_{2m} \\ \dots & \dots & \dots & \dots \\ z_{n1} & z_{n2} & \dots & z_{nm} \end{pmatrix}$$

where

$$z_{ij} = \langle ([T_{iM}^-, T_{iM}^+], [I_{iM}^-, I_{iM}^+], [F_{iM}^-, F_{iM}^+], [T_{iM}^-, T_{iM}^+], [I_{iM}^-, I_{iM}^+], [F_{iM}^-, F_{iM}^+]) \rangle$$

with

$$0 \leq \bigvee_{y \in [x]_R} \{T_A^U(y)\} + \bigwedge_{y \in [x]_R} \{I_A^U(y)\} + \bigwedge_{y \in [x]_R} \{F_A^U(y)\} \leq 3$$

Step 2. Calculate the weighted alternative decision matrix  
 For the attribute  $C_j$  ( $j=1, \dots, m$ ) the weight vector of attribute is considered as :  $W = (w_1, w_2, \dots, w_m)$  with

$$w_j \geq 0 \quad \text{and} \quad \sum_{i=1}^n w_j = 1$$

On calculating

$$s_{ij} = \langle ([w_j T_{ij}^-, w_j T_{ij}^+], [w_j I_{ij}^-, w_j I_{ij}^+], [w_j F_{ij}^-, w_j F_{ij}^+], [w_j T_{ij}^-, w_j T_{ij}^+], [w_j I_{ij}^-, w_j I_{ij}^+], [w_j F_{ij}^-, w_j F_{ij}^+]) \rangle$$

for  $i=1, 2, \dots, n$  and  $j=1, 2, \dots, m$ , we obtain the weighted alternative decision matrix

$$S = \langle s_{ij} \rangle_{n \times m} = \begin{pmatrix} s_{11} & s_{12} & \dots & s_{1m} \\ s_{21} & s_{22} & \dots & s_{2m} \\ \dots & \dots & \dots & \dots \\ s_{n1} & s_{n2} & \dots & s_{nm} \end{pmatrix}$$

Step 3. Determine the ideal solution  $S^*$ .

For benefit type attribute,

$$S^* = \{(\min_i T_{ij}, \max_i I_{ij}, \max_i F_{ij}), (\max_i \bar{T}_{ij}, \min_i \bar{I}_{ij}, \min_i \bar{F}_{ij})\}$$

For cost type attribute,

$$S^* = \{(\max_i T_{ij}, \min_i I_{ij}, \min_i F_{ij}), (\min_i \bar{T}_{ij}, \max_i \bar{I}_{ij}, \max_i \bar{F}_{ij})\}$$

Step 4. Compute the bidirectional projection measure between  $S^*$  and  $Z_i = \langle Z_{ij} \rangle_{n \times m}$  for all  $i = 1, \dots, n$  and  $j = 1, \dots, m$ .

Step 5. Ranking of alternatives is prepared based on the values of bidirectional projection measure. The highest value reflects the best alternatives.

Step 6. End.

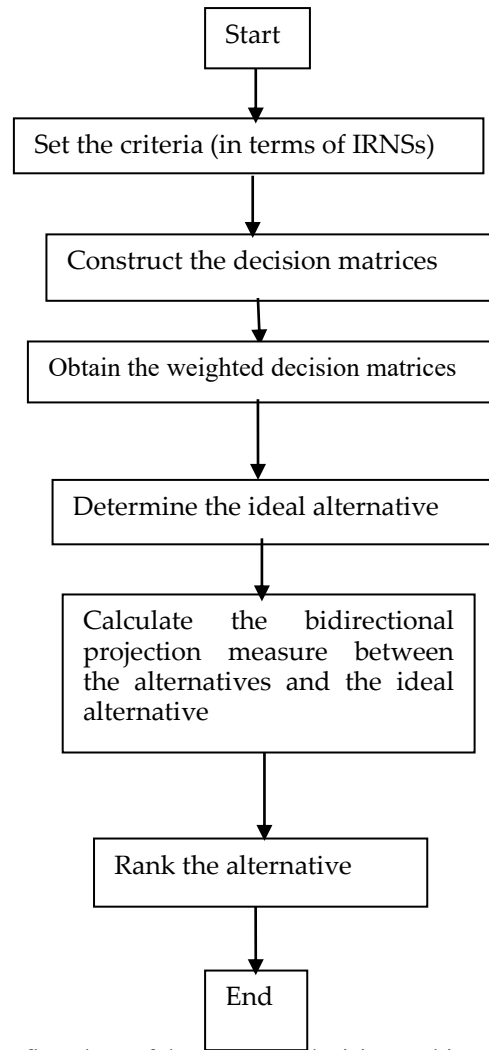


Fig 2. A flowchart of the proposed decision making method

**5. A Numerical Example:**

Assume that a decision maker intends to select the most suitable laptop for random use from the three initially chosen laptops ( $A_1, A_2, A_3$ ) by considering four attributes namely: features  $C_1$ , reasonable price  $C_2$ , customer care  $C_3$ , risk factor  $C_4$ . Based on the proposed approach discussed in section 5, the considered problem is solved by the following steps:

Step1: Construct the decision matrix with interval rough neutrosophic number

The decision maker construct the decision matrix with respect to the three alternatives and four attributes in terms of interval rough neutrosophic number.

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
A <sub>1</sub>	<([.6, .7], [.3, .5], [.3, .4]), ([.8, .9], [.1, .3], [.1, .2])>	<([.5, .7], [.3, .4], [.1, .2]), ([.7, .9], [.3, .5], [.3, .4])>	<([.5, .6], [.4, .5], [.4, .6]), ([.7, .8], [.2, .4], [.3, .4])>	<([.8, .9], [.3, .4], [.5, .6]), ([.7, .8], [.3, .5], [.3, .5])>
A <sub>2</sub>	<([.7, .8], [.2, .3], [.0, .2]), ([.7, .9], [.1, .2], [.1, .2])>	<([.6, .7], [.1, .2], [.0, .2]), ([.6, .7], [.1, .3], [.1, .3])>	<([.5, .7], [.2, .3], [.1, .2]), ([.6, .9], [.3, .5], [.2, .4])>	<([.7, .8], [.3, .5], [.1, .3]), ([.5, .7], [.5, .6], [.2, .3])>
A <sub>3</sub>	<([.6, .7], [.3, .4], [.0, .3]), ([.6, .9], [.1, .2], [.1, .2])>	<([.5, .7], [.2, .4], [.2, .4]), ([.6, .8], [.1, .3], [.1, .2])>	<([.6, .8], [.2, .4], [.3, .4]), ([.6, .8], [.2, .5], [.3, .5])>	<([.4, .7], [.2, .4], [.4, .5]), ([.5, .8], [.2, .5], [.0, .2])>

Step 2: The weight vectors considered by the decision maker are 0.35, 0.25, 0.25 and 0.15 respectively. The weighted decision matrix is:

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
S <sub>1</sub>	<([0.21,0.245], [0.105,0.175], [0.105,0.14]), ([0.28,0.315], [0.035,0.105], [0.035,0.07])>	<([0.125,0.175], [0.075,0.1], [0.025,0.05]), ([0.175,0.225], [0.075,0.125], [0.075,0.1])>	<([0.125,0.15], [0.1,0.125], [0.1,0.15]), ([0.175,0.2], [0.05,0.1], [0.075,0.1])>	<([0.12,0.135], [0.045,0.06], [0.075,0.09]), ([0.105,0.12], [0.045,0.075], [0.045,0.75])>
S <sub>2</sub>	<([0.245,0.28], [0.07,0.105], [0.0,0.07]), ([0.245,0.315], [0.035,0.07], [0.035,0.07])>	<([0.15,0.175], [0.025,0.05], [0.0,0.05]), ([0.15,0.175], [0.025,0.075], [0.025,0.075])>	<([0.125,0.175], [0.05,0.075], [0.025,0.05]), ([0.15,0.225], [0.075,0.125], [0.05,0.1])>	<([0.105,0.12], [0.045,0.75], [0.015,0.045]), ([0.075,0.105], [0.075,0.09], [0.03,0.045])>
S <sub>3</sub>	<([0.21,0.245], [0.105,0.14], [0.0,0.105]), ([0.21,0.315], [0.035,0.7], [0.035,0.7])>	<([0.125,0.175], [0.05,0.1], [0.05,0.1]), ([0.15,0.2], [0.025,0.075], [0.025,0.05])>	<([0.15,0.2], [0.05,0.1], [0.075,0.1]), ([0.15,0.2], [0.05,0.125], [0.075,0.125])>	<([0.06,0.105], [0.03,0.06], [0.06,0.075]), ([0.075,0.12], [0.03,0.075], [0.0,0.03])>

Step3: Determine the benefit type attribute and cost type attribute

Here three benefit type attributes C<sub>1</sub>, C<sub>2</sub>, C<sub>3</sub> and one cost type attribute C<sub>4</sub>. We calculate the ideal alternative as follows:

$$S^* = \{ < ([.21, .245], [.07, .175], [.105, .14]), ([.28, .315], [.035, .07], [.035, .07]) >, < ([.15, .175], [.075, .1], [.05, .1]), ([.175, .225], [.025, .075], [.025, .05]) >, < ([.15, .15], [.1, .1], [.1, .1]), ([.175, .225], [.075, .125], [.075, .125]) >, < ([.12, .135], [.03, .06], [.015, .045]), ([.075, .105], [.075, .09], [.045, .075]) > > \}$$

Step4: Calculate the projection and bidirectional projection measure of the alternatives

$$\|S_1\| = 0.918273, \|S_2\| = 0.829533,$$

$$\|S_3\| = 0.832331, \|S^*\| = 0.818175.$$

$$S_1.S^* = 0.815425, S_2.S^* = 0.563137, S_3.S^* = 0.7337.$$

$$\begin{aligned} \text{Proj}(S_1)_{S^*} &= 0.99663886, \\ \text{Proj}(S_2)_{S^*} &= 0.68828490, \\ \text{Proj}(S_3)_{S^*} &= 0.89675192. \\ \Rightarrow \text{Proj}(S_1)_{S^*} &> \text{Proj}(S_3)_{S^*} > \text{Proj}(S_2)_{S^*}. \\ \text{BProj}(S_1, S^*) &= 0.92453705, \\ \text{BProj}(S_2, S^*) &= 0.99364454, \\ \text{BProj}(S_3, S^*) &= 0.98972051. \\ \Rightarrow \text{BProj}(S_2, S^*) &> \text{BProj}(S_3, S^*) > \text{BProj}(S_1, S^*). \end{aligned}$$

Step5: Rank the alternatives

Ranking of alternatives is prepared based on the descending order of projection and bidirectional measures. The highest value reflects the best alternatives.

Hence, according to the projection measure, the laptop A<sub>1</sub> is the best alternative and according to the bidirectional



projection measure, the laptop  $A_2$  is the best alternative. As bidirectional projection measure gives better result than projection measure, so  $A_2$  is the best laptop for random use.

### 6. Comparative study and discussions:

Mondal and Pramanik study the MADM method in interval rough neutrosophic environment using cosine, dice and Jaccard similarity measure [32]. We take the same problem and solve the problem using projection and bidirectional projection measure based decision making method. In the existing methods,  $S_2$  is the best alternatives. But in new method  $S_1$  is the best alternative.

### 7. Conclusion:

In this paper, we have defined projection measure, weighted projection measure, bidirectional projection measure, weighted bidirectional projection measure between interval rough neutrosophic sets. We have also proved their basic properties. We have developed two new MADM strategies based on the proposed projection and bidirectional projection measures respectively. Finally, we have solved a numerical example to demonstrate the feasibility, applicability and effectiveness of the proposed strategies. The proposed strategies can be applied to solve different MADM problems such as teacher selection [33, 34, 35], school selection [36], weaver selection [37, 38, 39], brick field selection [40, 41], logistics center location selection [42, 43], data mining [44] etc. The proposed strategies can also be extended for MAGDM in interval rough neutrosophic environment.

### References

- [1] F. Smarandache. A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth, (1998).
- [2] F. Smarandache. Linguistic paradoxes and tautologies. *Libertas Mathematica*, University of Texas at Arlington, IX (1999), 143-154.
- [3] F. Smarandache. A unifying field in logics: neutrosophic logics. *Multiple Valued Logic*, 8(3) (2002), 385-438.
- [4] F. Smarandache. Neutrosophic set- a generalization of intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics*, 24(3) (2005), 287-297.
- [5] F. Smarandache. Neutrosophic set-a generalization of intuitionistic fuzzy set. *Journal of Defense Resources Management*, 1(1) (2010), 107-116.
- [6] G. Cantor. Über unendliche, lineare Punktmannigfaltigkeiten V [On infinite, linear point-manifolds(sets)]. *Mathematische Annalen*, 21 (1883), 545-591.
- [7] L. A. Zadeh. Fuzzy sets. *Information and Control*, 8(3) (1965), 338-353.
- [8] K. T. Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1) (1986), 87-96.
- [9] H. Wang, F. Smarandache, Y. Q. Zhang and R. Sunderraman. Single valued neutrosophic sets. *Multispace and Multistructure*, 4 (2010), 410-413.
- [10] S. Broumi, F. Smarandache and M. Dhar. Rough neutrosophic sets. *Italian Journal of Pure and Applied Mathematics*, 32 (2014), 493-502.
- [11] S. Broumi, F. Smarandache and M. Dhar. Rough neutrosophic sets. *Neutrosophic Sets and Systems*, 3 (2014), 60-66.
- [12] Z. Pawlak. Rough sets. *International Journal of Information and Computer Sciences*, 11(5) (1982), 341-356.
- [13] S. Broumi and F. Smarandache. Interval neutrosophic rough sets. *Neutrosophic Sets and Systems*, 7 (2015), 23-31.
- [14] H. Wang, F. Smarandache, Y. Q. Zhang and R. Sunderraman. *Interval neutrosophic sets and logic: theory and applications in computing*. Hexis, Phoenix, 2005.
- [15] Z. L. Yue. Approach to group decision making based on determining the weights of experts by using projection method. *Applied Mathematical Modelling*, 36(7) (2012), 2900-2910.
- [16] Z. L. Yue. Application of the projection method to determine weights of decision makers for group decision making, *Scientia Iranica*, 19(3) (2012), 872-878.
- [17] Z. S. Xu and Q. L. Da. Projection method for uncertain multi-attribute decision making with preference information on alternatives. *International Journal of Information Technology & Decision Making*, 3 (2004), 429-434.
- [18] Z. Xu. On methods for uncertain multiple attribute group decision making with uncertain multiplicative preference information on alternatives, *Fuzzy Optimization and Decision Making*, 4 (2005), 131-139.
- [19] L. Yang, S. Xiao, S. Yuan. Projection method for material selection problem with triangular fuzzy numbers. *Advanced Materials Research*, 1046 (2014), 375-379.
- [20] Z. Xu, and H. Hu. Projection models for intuitionistic fuzzy multiple attribute decision making. *International Journal of Information Technology & Decision Making*, 9(2) (2010), 267-280.
- [21] S. Zeng, T. Balezentis, J. Chen, and G. Luo. A projection method for multiple attribute group decision making with intuitionistic fuzzy information, *Informatica*, 24(3) (2013), 485-503.
- [22] J. Chen and J. Ye. A projection model of neutrosophic numbers for multiple attribute decision making of clay-brick selection, *Neutrosophic Sets and Systems*, 12 (2016), 139-142.
- [23] J. Ye. Projection and bidirectional projection measures of single valued neutrosophic sets and their decision – making method for mechanical design scheme, *Journal of Experimental & Theoretical Artificial Intelligence*, 2016, DOI:10.1080/0952813X.2016.1259263.
- [24] J. Ye. Bidirectional projection method for multiple attribute group decision making with neutrosophic number. *Neural Computing and Application*, 2015, Doi: 10.1007/s00521-015-2123-5.
- [25] P. P. Dey, S. Pramanik, and B. C. Giri. Extended projection based models for solving multiple attribute decision making problems with interval valued neutrosophic information, *New Trends in Neutrosophic Theory and Applications*. Pons asbl, Brussels, 2016, 127-140.

- [26] Z. L. Yue. An intuitionistic fuzzy projection-based approach for partner selection, *Applied Mathematical Modelling*, 37 (2013), 9538-9551.
- [27] P. P. Dey, S. Pramanik, and B. C. Giri. Bipolar neutrosophic projection based models for multi-attribute decision making problems. Retrieved March 30, 2017 from <http://vixra.org/pdf/1702.0140>.
- [28] S. Pramanik, R. Roy and T. K. Roy. Multi criteria decision making based on projection and bidirectional projection measures of rough neutrosophic sets. In *New Trends in Neutrosophic Theory and Applications*; Smarandache, F., Pramanik, S., Eds.; Pons Editions: Brussels, Belgium; Volume II, In Press.
- [29] D. Dubios and H. Prade. Rough fuzzy sets and fuzzy rough sets, *International Journal of General System*, 17 (1990), 191-208.
- [30] S. Nanda, and S. Majumdar. Fuzzy rough sets. *Fuzzy Sets and Systems*, 45 (1992), 157–160.
- [31] K. V. Thomas and L. S. Nair. Rough intuitionistic fuzzy sets in a lattice, *International Mathematics Forum*, 6 (27) (2011), 1327–1335.
- [32] K. Mondal and S. Pramanik. Decision making based on some similarity measures under interval rough neutrosophic environment. *Neutrosophic Sets and Systems*, 10 (2015), 46-57. doi.org/10.5281/zenodo.571358.
- [33] S. Pramanik, and D. Mukhopadhyaya. Grey relational analysis based intuitionistic fuzzy multi-criteria group decision-making approach for teacher selection in higher education. *International Journal of Computer Applications*, 34 (2011), 21-29.
- [34] K. Mondal and S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education under simplified Neutrosophic environment. *Neutrosophic Sets and Systems*, 6 (2014), 28-34.
- [35] S. Pramanik, R. Roy, and T. K. Roy. Teacher selection strategy based on bidirectional projection measure in neutrosophic number environment. In F. Smarandache, M. Abdel – Basset & V. Chang (Eds.), *Neutrosophic Operational Research*, Vol. II. Brussels: Pons Editions. 2017.
- [36] K. Mondal and S. Pramanik. Neutrosophic decision making model of school choice. *Neutrosophic Sets and Systems*, 7 (2015), 62-68.
- [37] P. P. Dey, S. Pramanik and B. C. Giri. Multi-criteria group decision making in intuitionistic fuzzy environment based on grey relational analysis for weaver selection in Khadi institution. *Journal of Applied and Quantitative Methods*, 10 (4) (2015), 1-14.
- [38] P. P. Dey, S. Pramanik and B. C. Giri. An extended grey relational analysis based interval neutrosophic multi-attribute decision making for weaver selection. *Journal of New Theory*, 9 (2015), 82-93.
- [39] P. P. Dey, S. Pramanik and B. C. Giri. Extended projection based models for solving multiple attribute decision making problems with interval valued neutrosophic information. In F. Smarandache, & S. Pramanik (Eds.), *New Trends in Neutrosophic Theory and Applications*, Brussels: Pons Edition, 2016, 127-140.
- [40] K. Mondal and S. Pramanik. Intuitionistic fuzzy multi criteria group decision making approach to quality-brick selection problem. *Journal of Applied Quantitative Methods* 9 (2) (2014), 35-50.
- [41] K. Mondal and S. Pramanik. Neutrosophic decision making model for clay-brick selection in construction field based on grey relational analysis. *Neutrosophic Sets and Systems*, 9 (2015), 72-79.
- [42] S. Pramanik, S. Dalapati, and T. K. Roy. Logistics center location selection approach based on neutrosophic multi-criteria decision making. *New Trends in Neutrosophic Theories and Applications*, Pons-Editions, Brussels, 2016, 161-174.
- [43] S. Pramanik, S. Dalapati, and T. K. Roy. Neutrosophic multi-attribute group decision making strategy for logistics center location selection. In F. Smarandache, M. A. Basset, & V. Chang (Eds), *Neutrosophic Operational Research Volume III*, Pons asbl, Brussels, 2016, 13-32.
- [44] K. Mondal, S. Pramanik, and B. C. Giri. Role of Neutrosophic Logic in Data Mining. *New Trends in Neutrosophic Theory and Application*, Pons-Editions, Brussels, 2016, 15-23.
- [45] Abdel-Basset, M., Mohamed, M., Smarandache, F., & Chang, V. (2018). Neutrosophic Association Rule Mining Algorithm for Big Data Analysis. *Symmetry*, 10(4), 106.
- [46] Abdel-Basset, M., & Mohamed, M. (2018). The Role of Single Valued Neutrosophic Sets and Rough Sets in Smart City: Imperfect and Incomplete Information Systems. *Measurement*. Volume 124, August 2018, Pages 47-55
- [47] Abdel-Basset, M., Gunasekaran, M., Mohamed, M., & Smarandache, F. A novel method for solving the fully neutrosophic linear programming problems. *Neural Computing and Applications*, 1-11.
- [48] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2018). A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*, 1-22.
- [49] Abdel-Basset, M., Mohamed, M., & Chang, V. (2018). NMCDA: A framework for evaluating cloud computing services. *Future Generation Computer Systems*, 86, 12-29.
- [50] Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. *Journal of Intelligent & Fuzzy Systems*, 33(6), 4055-4066.
- [51] Abdel-Basset, M.; Mohamed, M.; Smarandache, F. An Extension of Neutrosophic AHP–SWOT Analysis for Strategic Planning and Decision-Making. *Symmetry* 2018, 10, 116.

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