Neutrosophic Sets and Systems

Volume 25 Article 6

1-1-2019

Interval Neutrosophic Vague Sets

Hazwani Hashim

Lazim Abdullah

Ashraf Al-Quran

Interval Neutrosophic Vague Sets

Hazwani Hashim[1], Lazim Abdullah[2] and Ashraf Al-Quran[3]

[1] Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara (UiTM) Kelantan, Campus Machang, Kelantan, 18500, Malaysia Email: hazwanihashim@kelantan.uitm.edu.my

[2] School of Informatics and Applied Mathematics, University Malaysia Terengganu, 21030 Kuala Terengganu, Terengganu, Malaysia. E-mail: lazim_m@umt.edu.my

[3] Department of Mathematics, Faculty of Science, Jerash University, Jerash, 26150 Jordan.
E-mail: a.quraan@jpu.edu.jo

Abstract.
Recently, vague sets and neutrosophic sets have received great attention among the scholars and have been applied in many applications. But, the actual theoretical impacts of the combination of these two sets in dealing uncertainties are still not fully explored until now. In this paper, a new generalized mathematical model called interval neutrosophic vague sets is proposed, which is a combination of vague sets and interval neutrosophic sets and a generalization of interval neutrosophic vague sets. Some definitions of interval neutrosophic vague set such as union, complement and intersection are presented. Furthermore, the basic operations, the derivation of its properties and related example are included.
Keywords: neutrosophic set, vague set, interval neutrsophic vague set.

1 Introduction

Many attempts that used classical mathematics to model uncertain data may not be successful. This is due to the concept of uncertainty which is too complicated and not clearly defined. Therefore, many different theories were developed to solve uncertainty and vagueness including the fuzzy set theory [1], intuitionistic fuzzy set [2], rough sets theory [3], soft set [4], vague sets [5], soft expert set [6] and some other mathematical tools. There are many real applications were solved using these theories related to the uncertainty of these applications [7], [8], [9], [10] . However, these theories cannot deals with indeterminacy and consistent information. Furthermore, all these theories have their inherent difficulties and weakness. Therefore, neutrosophic set is developed by Smarandache in 1998 which is generalization of probability set, fuzzy set and intuitionistic fuzzy set [11]. The neutrosophic set contains three independent membership functions. Unlike fuzzy and intuitionistic fuzzy sets, the memberships in neutrosophic sets are truth, indeterminacy and falsity. The neutrosophic set has received more and more attention since its appearance. Hybrid neutrosophic set were introduced by many researchers [12], [13], [14], [15], [16]. In line with these developments, these extensions have been used in multi criteria decision making problem such as ANP, VIKOR, TOPSIS and DEMATEL with different application [17]–[20].

Vague sets have been introduced by Gau and Buehrar in 1993 as an extension of fuzzy set theory [5]. Vague sets is considered as an effective tool to deal with uncertainty since it provides more information as compared to fuzzy sets [21]. Several studies have revealed that, many researchers have combined vague sets with others theories. Xu et al. proposed vague soft sets and examined its properties [22]. Later, Hassan [23] have combined vague set with soft expert set and its operations were introduced. In addition, others hybrid theories such as complex vague soft set [24], interval valued vague soft set [25], generalized interval valued vague set [26] and possibility vague soft set [27] were presented to solve uncertainty problem in decision making. Recently, Al- Quran and Hassan [28] proposed new hybrid of neutrosophic vague such as [29], [30], [31] and [32].

Until now, there has been no study on interval neutrosophic vague set (INVS) and its combination particularly with vague sets. Therefore, the objective of this paper is to develop a mathematical tool to solve uncertainty problem, namely INVS which is a combination of vague sets and interval neutrosophic set and as a generalization of interval neutrosophic vague set. This set theory provides an interval-based membership structure to handle the neutrosophic vague data. This feature allows users to record their hesitancy in assigning membership values which in turn better capture the vagueness and uncertainties of these data.

This paper is structured in the following manner. Section 2 presents some basic mathematical concepts to enhance the understanding of INVS. Section 3 describes definitions IVNS and its properties together with example. Finally, conclusion of INVS is stated in section 4.

2 Preliminaries

Some basic concepts associated to neutrosophic sets and interval neutrosophic set are presented in this section.

2.1 Vague Set

Definition 2.1 [5]

Let e be a vague value, $e=\left[\bar{t}_e, 1-\bar{f}_e\right]$ where $\bar{t}_e \in [0,1]$, $\bar{f}_e \in [0,1]$ and $0 \le \bar{t}_e \le 1-\bar{f}_e \le 1$. If $\bar{t}_e = 1$ and $\bar{f}_e = 0$ then e is named a unit vague value so as $e=[1,1]$. Meanwhile if $t_e = 0$ and $f_e = 1$, hence e is named a zero vague value such that $e=[0,0]$.

Definition 2.1.1 [5]

Let e and f be two vague values, where $e=\left[\bar{t}_e, 1-\bar{f}_e\right]$ and $f=\left[\bar{t}_f, 1-\bar{f}_f\right]$. If $\bar{t}_e=\bar{t}_f$ and $\bar{f}_e = \bar{f}_f$, then vague values e and f are named equal $\left(\text{i.e.} \left[\bar{t}_e, 1-\bar{f}_e\right] = \left[\bar{t}_f, 1-\bar{f}_f\right]\right)$.

Definition 2.1.2 [5]

Let p be a vague set of the universe E . If $\forall e_n \in E$, $t_p(e_n)=1$ and $f_p(e_n)=0$, then p is named a unit vague set where $1 \le n \le m$. If $t_p(e_n)=0$ and $t_p(e_n)=1$ hence p named a zero vague set where $1 \le n \le m$.

2.2 Neutrosophic Set

Definition 2.2 [11]
A neutrosophic set e in E is described by three functions: truth membership function $V_p(e)$, indeterminacy-membership function $W_p(e)$ and falsity-membership function $X_p(e)$ as $p=\{< e: V_p(e), W_p(e), X_p(e), e \in E\}$ where $V, W, X : E \to \ ]^{-}0, 1^{+}[$ and $^{-}0 \le \sup V_p(e) + \sup W_p(e) + \sup X_p(e) \le 3^{+}$

2.3 Interval Neutrosophic Set

Definition 2.3 [12]
Let E be a universe. An interval neutrosophic set denoted as (INS) can be defined as follows:
$p=\{(< [V_p^L(e), V_p^U(e)], [W_p^L(e), W_p^U(e)], X_p^L(e), X_p^U(e)) > | e \in E\}$ where for each point $e \in E$, we have $V_p(e) \in [0,1]$, $W_p(e) \in [0,1]$, $X_p(e) \in [0,1]$ and $^{-}0 \le \sup V_p(e) + \sup W_p(e) + \sup X_p(e) \le 3^{+}$.

2.4 Neutrosophic Vague Set

Definition 2.4 [28]
A neutrosophic vague set p in E denoted (NVS) as an object of the form
$p_{NV} = \{< e : V_{p_{NV}}(e), W_{p_{NV}}(e), X_{p_{NV}}(e) > | e \in E\}$ and
$V_{p_{NV}}(e) = [V^-, V^+]$, $W_{p_{NV}}(e) = [W^-, W^+]$, $X_{p_{NV}}(e) = [X^-, X^+]$
where
$V^+ = 1 - X^-$, $X^+ = 1 - V^-$ and $^{-}0 \le V^- + W^- + X^- \le 2^+$.

Definition 2.4.1 [28]
Let α be a NVS in E . Then α is called a unit NVS where $1 \le n \le m$
$V_{\alpha_{NV}}(e) = [1,1]$, $W_{\alpha_{NV}}(e) = [0,0]$, $X_{\alpha_{NV}}(e) = [0,0]$
.
Definition 2.4.2 [28]
Let β be a NVS in E . Then β is called a zero NVS where $1 \le n \le m$.
$V_{\beta_{NV}}(e) = [0,0]$, $W_{\beta_{NV}}(e) = [1,1]$, $X_{\beta_{NV}}(e) = [1,1]$

For two NVS

$p_{NV} = \{< e : V_{p_{NV}}(e), W_{p_{NV}}(e), X_{p_{NV}}(e) > | e \in E\}$ and

$q_{NV} = \{< e : V_{q_{NV}}(e), W_{q_{NV}}(e), X_{q_{NV}}(e) > | e \in E\}$

The relations of NVS is presented as follows:

(i) $p_{NV} = q_{NV}$ if and only if $V_{p_{NV}}(e) = V_{q_{NV}}(e)$, $W_{p_{NV}}(e) = W_{q_{NV}}(e)$ and $X_{p_{NV}}(e) = X_{q_{NV}}(e)$.

(ii) $p_{NV} \subseteq q_{NV}$ if and only if $V_{p_{NV}}(e) \le V_{q_{NV}}(e)$, $W_{p_{NV}}(e) \ge \bar{I}_{q_{NV}}(e)$ and $X_{p_{NV}}(e) \ge X_{q_{NV}}(e)$.

(iii) The union of p and q is denoted by $R_{NV} = p_{NV} \cup q_{NV}$ is defined by;

$$V_{R_{NV}}(e) = \left[\max\left(V^-_{p_{NVe}}, V^-_{q_{NVe}}\right), \max\left(V^+_{p_{NVe}}, V^+_{q_{NVe}}\right)\right]$$

$$W_{R_{NV}}(e) = \left[\min\left(W^-_{p_{NVe}}, W^-_{q_{NVe}}\right), \min\left(W^+_{p_{NVe}}, W^+_{q_{NVe}}\right)\right],$$

$$X_{R_{NV}}(e) = \left[\min\left(X^-_{p_{NVe}}, X^-_{q_{NVe}}\right), \min\left(X^+_{p_{NVe}}, X^+_{q_{NVe}}\right)\right]$$

(iv) The intersection of p and q is denoted by $S_{NV} = p_{NV} \cap q_{NV}$ is defined by;

$$V_{S_{NV}}(e) = \left[\min\left(V^-_{p_{NVe}}, V^-_{q_{NVe}}\right), \min\left(V^+_{p_{NVe}}, V^+_{q_{NVe}}\right)\right]$$

$$W_{S_{NV}}(e) = \left[\max\left(W^-_{p_{NVe}}, W^-_{q_{NVe}}\right), \max\left(W^+_{p_{NVe}}, W^+_{q_{NVe}}\right)\right],$$

$$X_{S_{NV}}(e) = \left[\max\left(X^-_{p_{NVe}}, X^-_{q_{NVe}}\right), \max\left(X^+_{p_{NVe}}, X^+_{q_{NVe}}\right)\right]$$

(v) The complement of a NVS p_{NV} is denoted by p^c and is defined by

$$V^c_{p_{NV}}(e) = \left[1 - V^+, 1 - V^-\right]$$

$$W^c_{p_{NV}}(e) = \left[1 - W^+, 1 - W^-\right]$$

$$X^c_{p_{NV}}(e) = \left[1 - X^+, 1 - X^-\right]$$

3 Interval Neutrosophic Vague Sets

The formal definition of an INVS and its basic operations of complement, union and intersection are introduced. Related properties and suitable examples are presented in this section.

Definition 3.1

An interval valued neutrosophic vague set A_{INV} also known as INVS in the universe of discourse E. An INVS is characterized by truth membership, indeterminacy membership and fasilty-membership functions is defined as:

$$A_{INV} = \{e, [\tilde{V}^L_A(e), \tilde{V}^U_A(e)], [\tilde{W}^L_A(e), \tilde{W}^U_A(e)], [\tilde{X}^L_A(e), \tilde{X}^U_A(e)] > | e \in E\}$$

$$\tilde{V}^L_A(e) = \left[V^{L^-}, V^{L^+}\right], \tilde{V}^U_A(e) = \left[T^{U^-}, T^{U^+}\right],$$

$$\tilde{W}^L_A(e) = \left[W^{L^-}, W^{L^+}\right], \tilde{W}^U_A(e) = \left[W^{U^-}, W^{U^+}\right],$$

$$\tilde{X}^L_A(e) = \left[X^{L^-}, X^{L^+}\right], \tilde{X}^U_A(e) = \left[X^{U^-}, X^{U^+}\right]$$

where

$V^{L^+} = 1 - X^{L^-}, X^{L^+} = 1 - V^{L^-}, V^{U^+} = 1 - X^{U^-}, X^{U^+} = 1 - V^{U^-}$ and

$^{-}0 \leq V^{L^{-}} + V^{U^{-}} + W^{L^{-}} + W^{U^{-}} + X^{L^{-}} + X^{U^{-}} \leq 4^{+}$

$^{-}0 \leq V^{L^{+}} + V^{U^{+}} + W^{L^{+}} + W^{U^{+}} + X^{L^{+}} + X^{U^{+}} \leq 4^{+}$

An INVS A_{INV} when E is continuous is presented as follows:

$$A_{INV} = \int_E \frac{\left\langle \left[\tilde{V}_A^L(e), \tilde{V}_A^U(e)\right], \left[\tilde{W}_A^L(e), \tilde{W}_A^U(e)\right], \left[\tilde{X}_A^L(e), \tilde{X}_A^U(e)\right] \right\rangle}{e} : e \in E$$

and when E is discrete an INVS A_{INV} can be presented as follows:

$$A_{INV} = \sum_{i=1}^{n} \frac{\left\langle \left[\tilde{V}_A^L(e), \tilde{V}_A^U(e)\right], \left[\tilde{W}_A^L(e), \tilde{W}_A^U(e)\right], \left[\tilde{X}_A^L(e), \tilde{X}_A^U(e)\right] \right\rangle}{e_i} : e \in E$$

$0 \leq \sup \tilde{V}_A(e) + \sup \tilde{W}_A(e) + \sup \tilde{X}_A(e) \leq 3$

Example 3.1

Let $E = \{e_1, e_2, e_3\}$. Then

$$A_{INV} = \left\{ \begin{array}{l} \dfrac{e_1}{\langle \{[0.2,0.5],[0.2,0.3]\}, \{[0.1,0.6],[0.3,0.6]\}, \{[0.5,0.8],[0.7,0.8]\} \rangle}, \\ \dfrac{e_2}{\langle \{[0.4,0.5],[0.1,0.7]\}, \{[0.5,0.5],[0.1,0.3]\}, \{[0.5,0.6],[0.3,0.9]\} \rangle} \\ \dfrac{e_3}{\langle \{[0.6,0.9],[0.2,0.5]\}, \{[0.3,0.7],[0.4,0.6]\}, \{[0.1,0.4],[0.5,0.8]\} \rangle} \end{array} \right\}$$

is an INVS subset of E .

Consider Example 3.1 .Then we check the INVS for e_1 by Definition 3.1 as follows:

$V^{L^{+}} = 1 - X^{L^{-}} = 0.5 + 0.5 = 1$, $X^{L^{+}} = 1 - V^{L^{-}} = 0.8 + 0.2 = 1$

$V^{U^{+}} = 1 - X^{U^{-}} = 0.3 + 0.7 = 1$, $X^{U^{+}} = 1 - V^{U^{-}} = 0.8 + 0.2 = 1$

Using condition $^{-}0 \leq V^{L^{-}} + V^{U^{-}} + W^{L^{-}} + W^{U^{-}} + X^{L^{-}} + X^{U^{-}} \leq 4^{+}$,

therefore we have $0.2 + 0.2 + 0.1 + 0.3 + 0.5 + 0.7 = 2$ and

$^{-}0 \leq V^{L^{+}} + V^{U^{+}} + W^{L^{+}} + W^{U^{+}} + X^{L^{+}} + X^{U^{+}} \leq 4^{+}$, therefore we have

$0.5 + 0.3 + 0.6 + 0.6 + 0.8 + 0.8 = 3.6$

The calculations for INVS in Example 3.2, Example 3.3 are calculated similarly.

Definition 3.2

Consider Φ_{INV} be an INVS of the universe E where $\forall e_n \in E$,

$\tilde{V}_{\Phi_{INV}}^L(e) = [1,1], \tilde{V}_{\Phi_{INV}}^U(e) = [1,1]$,

$\tilde{W}_{\Phi_{INV}}^L(e) = [0,0], \tilde{W}_{\Phi_{INV}}^U(e) = [0,0]$,

$\tilde{X}_{\Phi_{INV}}^L(e) = [0,0], \tilde{X}_{\Phi_{INV}}^U(e) = [0,0]$

Therefore, Φ_{INV} is defined a unit INVS where $1 \leq n \leq m$

Consider δ_{INV} be a INVS of the universe E where $\forall e_n \in E$

$\tilde{V}^L_{\delta_{INV}}(e)=[0,0], \tilde{V}^U_{\delta_{INV}}(e)=[0,0]$,

$\tilde{W}^L_{\delta_{INV}}(e)=[1,1], \tilde{W}^U_{\delta_{INV}}(e)=[1,1]$,

$\tilde{X}^L_{\delta_{INV}}(e)=[1,1], \tilde{X}^U_{\delta_{INV}}(e)=[1,1]$

Therefore, δ_{INV} is defined a zero INVS where $1 \le n \le m$

Definition 3.3

Let A^c_{INV} is defined as complemnet of INVS

$$\left(\tilde{V}^L_A\right)^c(e)=\left[1-V^{L^+},1-V^{L^-}\right],\ \left(\tilde{V}^U_A\right)^c(e)=\left[1-V^{U^+},1-V^{U^-}\right],$$

$$\left(\tilde{W}^L_A\right)^c(e)=\left[1-W^{L^+}1-,W^{L^-}\right],\ \left(\tilde{W}^U_A\right)^c(e)=\left[1-W^{U^+},1-W^{U^-}\right],$$

$$\left(\tilde{X}^L_A\right)^c(e)=\left[1-X^{L^+},1-X^{L^-}\right],\ \left(\tilde{X}^U_A\right)^c(e)=\left[1-X^{U^+},1-X^{U^-}\right]$$

Example 3.2

Considering **Example 3.1**, by using Definition 3.3, we have

$$A^c_{INV}=\left\{\begin{array}{c}\dfrac{e_1}{\langle\{[0.5,0.8],[0.7,0.8]\},\{[0.4,0.9],[0.4,0.7]\},\{[0.2,0.5],[0.2,0.3]\}\rangle},\\ \dfrac{e_2}{\langle\{[0.5,0.6],[0.3,0.9]\},\{[0.5,0.5],[0.7,0.9]\},\{[0.4,0.5],[0.1,0.7]\}\rangle}\\ \dfrac{e_3}{\langle\{[0.1,0.4],[0.3,0.8]\},\{[0.3,0.7],[0.4,0.6]\},\{[0.6,0.9],[0.2,0.7]\}\rangle}\end{array}\right\}$$

Definition 3.5

Let A_{INV} and B_{INV} be two INVS of the universe. If $\forall e_n \in E$,

$\tilde{V}^L_A(e_n)=\tilde{V}^L_B(e_n)$, $\tilde{V}^U_A(e_n)=\tilde{V}^U_B(e_n)$, $\tilde{W}^L_A(e_n)=\tilde{W}^L_B(e_n)$, $\tilde{W}^U_A(e_n)=\tilde{W}^U_B(e_n)$, $\tilde{X}^L_A(e_n)=\tilde{X}^L_B(e_n)$ and $\tilde{X}^U_A(e_n)=\tilde{X}^U_B(e_n)$

Then the INVS A_{INV} and B_{INV} are equal, where $1 \le n \le m$

Definition 3.6

Let A_{INV} and B_{INV} be two INVS of the universe. If $\forall e_n \in E$,

$\tilde{V}^L_A(e_n)\le\tilde{V}^L_B(e_n)$ and $\tilde{V}^U_A(e_n)\le\tilde{V}^U_B(e_n)$,

$\tilde{W}^L_A(e_n)\ge\tilde{W}^L_B(e_n)$ and $\tilde{W}^U_A(e_n)\ge\tilde{W}^U_B(e_n)$,

$\tilde{X}^L_A(e_n)\ge\tilde{X}^L_B(e_n)$ and $\tilde{X}^U_A(e_n)\ge\tilde{X}^U_B(e_n)$

Then the INVS A_{INV} are included by B_{INV} denoted by $A_{INV} \subseteq B_{INV}$, where $1 \le n \le m$.

Definition 3.7

The union of two INVS A_{INV} and B_{INV} is a INVS C_{INV}, written as $C_{INV}=A_{INV}\cup B_{INV}$ is defined as follows:

$$\tilde{V}^L_A(e)=\left[\max\left(V^{L^-}_A,V^{L^-}_B\right),\max\left(V^{L^+}_A,V^{L^+}_B\right)\right] \text{ and } \tilde{V}^U_A(e)=\left[\max\left(V^{U^-}_A,V^{U^-}_B\right),\max\left(V^{U^+}_A,V^{U^+}_B\right)\right],$$

$$\tilde{W}^L_A(e)=\left[\min\left(W^{L^-}_A,W^{L^-}_B\right),\min\left(W^{L^+}_A,W^{L^+}_B\right),\right] \text{ and } \tilde{W}^U_A(e)=\left[\min\left(W^{U^-}_A,W^{U^-}_B\right),\min\left(W^{U^+}_A,W^{U^+}_B\right),\right]$$

$$\tilde{X}^L_A(e)=\left[\min\left(X^{L^-}_A,X^{L^-}_B\right),\min\left(X^{L^+}_A,X^{L^+}_B\right)\right] \text{ and } \tilde{X}^U_A(e)=\left[\min\left(X^{U^-}_A,X^{U^-}_B\right),\min\left(X^{U^+}_A,X^{LU^+}_B\right)\right]$$

Definition 3.8

The intersection of two INVS A_{INV} and B_{INV} is a INVS C_{INV}, written as $D_{INV} = A_{INV} \cap B_{INV}$, is defined as below:

$$\tilde{V}_A^L(e) = \left[\min\left(V_A^{L^-}, V_B^{L^-}\right), \min\left(V_A^{L^+}, V_B^{L^+}\right) \right] \text{ and } \tilde{V}_A^U(e) = \left[\min\left(V_A^{U^-}, V_B^{U^-}\right), \min\left(V_A^{U^+}, V_B^{U^+}\right) \right],$$

$$\tilde{W}_A^L(e) = \left[\max\left(W_A^{L^-}, W_B^{L^-}\right), \max\left(W_A^{L^+}, W_B^{L^+}\right), \right] \text{and } \tilde{W}_A^U(e) = \left[\max\left(W_A^{U^-}, W_B^{U^-}\right), \max\left(W_A^{U^+}, W_B^{U^+}\right) \right],$$

$$\tilde{X}_A^L(e) = \left[\max\left(X_A^{L^-}, X_B^{L^-}\right), \max\left(X_A^{L^+}, X_B^{L^+}\right) \right] \text{ and } \tilde{X}_A^U(e) = \left[\max\left(X_A^{U^-}, X_B^{U^-}\right), \max\left(X_A^{U^+}, X_B^{LU+}\right) \right]$$

Example 3.3

Consider that there are two INVS A_{INV} and B_{INV} consist of $E = \{e_1, e_2, e_3\}$ defined as follows:

$$A_{INV} = \left\{ \begin{array}{l} \dfrac{e_1}{\langle\{[0.2,0.5],[0.2,0.3]\},\{[0.1,0.6],[0.3,0.6]\},\{[0.5,0.8],[0.7,0.8]\}\rangle}, \\ \dfrac{e_2}{\langle\{[0.4,0.5],[0.1,0.7]\},\{[0.5,0.5],[0.1,0.3]\},\{[0.5,0.6],[0.3,0.9]\}\rangle} \\ \dfrac{e_3}{\langle\{[0.6,0.9],[0.2,0.5]\},\{[0.3,0.7],[0.4,0.6]\},\{[0.1,0.4],[0.5,0.8]\}\rangle} \end{array} \right\}$$

$$B_{INV} = \left\{ \begin{array}{l} \dfrac{e_1}{\langle\{[0.2,0.6],[0.4,0.9]\},\{[0.5,0.5],[0.3,0.6]\},\{[0.4,0.8],[0.1,0.6]\}\rangle}, \\ \dfrac{e_2}{\langle\{[0.2,0.6],[0.2,0.3]\},\{[0.2,0.6],[0.5,0.5]\},\{[0.4,0.0.8],[0.7,0.8]\}\rangle} \\ \dfrac{e_3}{\langle\{[0.4,0.9],[0.2,0.5]\},\{[0.1,0.8],[0.2,0.6]\},\{[0.1,0.6],[0.5,0.8]\}\rangle} \end{array} \right\}$$

By using Definition 3.7, then we obtain INV union, $C_{INV} = A_{INV} \cup B_{INV}$ presented as follows:

$$C_{INV} = \left\{ \begin{array}{l} \dfrac{e_1}{\langle\{[0.2,0.6],[0.4,0.9]\},\{[0.1,0.5],[0.3,0.6]\},\{[0.4,0.8],[0.1,0.6]\}\rangle}, \\ \dfrac{e_2}{\langle\{[0.4,0.6],[0.2,0.7]\},\{[0.2,0.5],[0.1,0.3]\},\{[0.5,0.6],[0.3,0.8]\}\rangle} \\ \dfrac{e_3}{\langle\{[0.6,0.9],[0.2,0.5]\},\{[0.1,0.7],[0.2,0.6]\},\{[0.1,0.4],[0.5,0.8]\}\rangle} \end{array} \right\}$$

Moreover, by using Definition 3.8, we obtained INV intersection, $D_{INV} = A_{INV} \cap B_{INV}$ as follows:

$$D_{INV} = \left\{ \begin{array}{l} \dfrac{e_1}{\langle\{[0.2,0.5],[0.2,0.3]\},\{[0.5,0.6],[0.3,0.6]\},\{[0.5,0.8],[0.7,0.8]\}\rangle}, \\ \dfrac{e_2}{\langle\{[0.2,0.5],[0.1,0.3]\},\{[0.5,0.6],[0.5,0.5]\},\{[0.5,0.8],[0.7,0.9]\}\rangle} \\ \dfrac{e_3}{\langle\{[0.4,0.9],[0.2,0.5]\},\{[0.3,0.8],[0.4,0.6]\},\{[0.1,0.6],[0.5,0.8]\}\rangle} \end{array} \right\}$$

Proposition 3.1 Let A_{INV} and B_{INV} be two INVS in X. Then

(i) $A_{INV} \cup A_{INV} = A_{INV}$

(ii) $A_{INV} \cap A_{INV} = A_{INV}$

(iii) $A_{INV} \cup B_{INV} = B_{INV} \cup A_{INV}$

(iv) $A_{INV} \cap B_{INV} = B_{INV} \cap A_{INV}$

Proof (i):
If x is any arbitrary element in $A_{INV} \cup A_{INV} = A_{INV}$ by definition of union, we have $x \in A_{INV}$ or $x \in A_{INV}$. Hence $x \in A_{INV}$. Therefore $A_{INV} \cup A_{INV} \subseteq A_{INV}$. Conversely, If x is any arbitrary element in $A_{INV} \cup A_{INV} = A_{INV}$, then $x \in A_{INV}$ and $x \in A_{INV}$. Therefore,
$A_{INV} \cup A_{INV} \subseteq A_{INV}$ $\therefore A_{INV} \cup A_{INV} = A_{INV}$

Proof (ii): similar to the proof of (i).

Proof (iii):
Let x is any arbitrary element in $A_{INV} \cup B_{INV} = B_{INV} \cup A_{INV}$, then by definition of union, $x \in A_{INV}$ and $x \in B_{INV}$. But, if x is in A_{INV} and B_{INV}, then it is in B_{INV} or A_{INV}, and by definition of union, this means $x \in B_{INV} \cup A_{INV}$. Therefore, $A_{INV} \cup B_{INV} \subseteq B_{INV} \cup A_{INV}$.
The other inclusion is identical: if x is any element $B_{INV} \cup A_{INV}$. Therefore, then we know $x \in B_{INV}$ or $x \in A_{INV}$. But, $x \in B_{INV}$ or $x \in A_{INV}$ implies that x is in A_{INV} or B_{INV}; hence, $x \in A_{INV} \cup B_{INV}$. Therefore, $B_{INV} \cup A_{INV} \subseteq A_{INV} \cup B_{INV}$. Hence $A_{INV} \cup B_{INV} = B_{INV} \cup A_{INV}$.

Proof (iv): same to the proof (iii)

Proposition 3.2 Let A_{INV}, B_{INV} and C_{INV} be three INVS over the common universe X. Then,

(i) $A_{INV} \cup (B_{INV} \cup C_{INV}) = (A_{INV} \cup B_{INV}) \cup C_{INV}$

(ii) $A_{INV} \cap (B_{INV} \cap C_{INV}) = (A_{INV} \cap B_{INV}) \cap C_{INV}$

(iii) $A_{INV} \cup (B_{INV} \cap C_{INV}) = (A_{INV} \cup B_{INV}) \cap (A_{INV} \cup C_{INV})$

(iv) $A_{INV} \cap (B_{INV} \cup C_{INV}) = (A_{INV} \cap B_{INV}) \cup (A_{INV} \cap C_{INV})$

Proof (i):
First, let x be any element in $A_{INV} \cup (B_{INV} \cup C_{INV})$. This means that $x \in A_{INV}$ or $x \in (B_{INV} \cup C_{INV})$. If $x \in A_{INV}$ then $x \in (B_{INV} \cup C_{INV})$; hence, $x \in A_{INV} \cup (B_{INV} \cup C_{INV})$. On the other side, if $x \in A_{INV}$, then $x \in (B_{INV} \cup C_{INV})$. This means $x \in B_{INV}$ or $x \in C_{INV}$. If, $x \in B_{INV}$ then
$x \in A_{INV} \cup B_{INV} \Rightarrow x \in (A_{INV} \cup B_{INV}) \cup C_{INV}$. If $x \in C_{INV}$, Then, $x \in (A_{INV} \cup B_{INV}) \cup C_{INV}$ Hence,
$A_{INV} \cup (B_{INV} \cup C_{INV}) \subseteq (A_{INV} \cup B_{INV}) \cup C_{INV}$.
For the reverse inclusion, let x be any element of $(A_{INV} \cup B_{INV}) \cup C_{INV}$ .Then, $x \in (A_{INV} \cup B_{INV})$ or $x \in C_{INV}$. If $x \in (A_{INV} \cup B_{INV})$, we know $x \in A_{INV}$ or $x \in B_{INV}$. If $x \in A_{INV}$, then $x \in A_{INV} \cup (B_{INV} \cup C_{INV})$. If $x \in B_{INV}$,then $x \in (B_{INV} \cup C_{INV})$. Hence $x \in A_{INV} \cup (B_{INV} \cup C_{INV})$. On the other side, if $x \in C_{INV}$, hence $x \in (B_{INV} \cup C_{INV})$, and so, $A_{INV} \cup (B_{INV} \cup C_{INV})$.
Therefore, $A_{INV} \cup (B_{INV} \cup C_{INV}) \subseteq (A_{INV} \cup B_{INV}) \cup C_{INV}$.
Thus, $A_{INV} \cup (B_{INV} \cup C_{INV}) = (A_{INV} \cup B_{INV}) \cup C_{INV}$.
Proof (ii): associativity of intersections is similar to the proof (i)

Proof (iii): Distributive Laws are satisfied for INVS

Let $x \in A_{INV} \cup (B_{INV} \cap C_{INV})$. If $x \in A_{INV} \cup (B_{INV} \cap C_{INV})$ then x is either in A_{INV} or in $(B_{INV} \cap C_{INV})$.

This means that $x \in A_{INV}$ or $x \in (B_{INV} \cap C_{INV})$.

If $x \in A_{INV}$ or $\{x \in B_{INV}$ and $x \in C_{INV}\}$.

Then, $\{x \in A_{INV}$ or $x \in B_{INV}\}$ and $\{x \in A_{INV}$ or $x \in C_{INV}\}$.

So we have,

$x \in A_{INV}$ or B_{INV} and $x \in A_{INV}$ or C_{INV}

$x \in (A_{INV} \cup B_{INV})$ and $x \in (A_{INV} \cup B_{INV})$

$x \in (A_{INV} \cup B_{INV}) \cap (A_{INV} \cup B_{INV})$

Hence, $A_{INV} \cup (B_{INV} \cap C_{INV}) \Rightarrow (A_{INV} \cup B_{INV}) \cap (A_{INV} \cup C_{INV})$

Therefore,

$A_{INV} \cup (B_{INV} \cap C_{INV}) \subseteq (A_{INV} \cup B_{INV}) \cap (A_{INV} \cup C_{INV})$

Let $x \in (A_{INV} \cup B_{INV}) \cap (A_{INV} \cup C_{INV})$. If $x \in (A_{INV} \cup B_{INV}) \cap (A_{INV} \cup C_{INV})$ then x is in $(A_{INV}$ or $B_{INV})$ and x $(A_{INV}$ or $C_{INV})$.

So we have,

$x \in (A_{INV}$ or $B_{INV})$and $x \in (A_{INV}$ or $C_{INV})$

$\{x \in A_{INV}$ or $x \in B_{INV}\}$and $\{x \in A_{INV}$ or $x \in C_{INV}\}$

$x \in A_{INV}$ or $\{x \in B_{INV}$ and $x \in C_{INV}\}$

$x \in A_{INV} \cup \{x \in (B_{INV}$ and $C_{INV})\}$

$x \in A_{INV} \cup \{x \in (B_{INV} \cap C_{INV})\}$

$x \in A_{INV} \cup (B_{INV} \cap C_{INV})$

$x \in (A_{INV} \cup B_{INV}) \cap (A_{INV} \cup C_{INV}) \Rightarrow x \in A_{INV} \cup (B_{INV} \cap C_{INV})$

Therefore,

$(A_{INV} \cup B_{INV}) \cap (A_{INV} \cup C_{INV}) \subseteq A_{INV} \cup (B_{INV} \cap C_{INV})$

$\therefore A_{INV} \cup (B_{INV} \cap C_{INV}) = (A_{INV} \cup B_{INV}) \cap (A_{INV} \cup C_{INV})$

Proof (iv): similar to the prove of (iii)

Proposition 3.3:

(i) $(A_{INV} \cup B_{INV})^c = A^c_{INV} \cap B^c_{INV}$

(ii) $(A_{INV} \cap B_{INV})^c = A^c_{INV} \cup B^c_{INV}$

(iii) $\left(A^c_{INV} \cup B^c_{INV} \cup C^c_{INV}\right) = (A_{INV} \cap B_{INV} \cap C_{INV})^c$

(iv) $\left(A^c_{INV} \cap B^c_{INV} \cap C^c_{INV}\right) = (A_{INV} \cup B_{INV} \cup C_{INV})^c$

Proof (i):

Let $x \in (A_{INV} \cup B_{INV})^c$

$\Rightarrow x \notin A_{INV} \cup B_{INV}$

$\Rightarrow x \notin A_{INV}$ and $x \notin B_{INV}$

$\Rightarrow x \in A^c_{INV}$ and $x \in B^c_{INV}$

Since for all $x \in (A_{INV} \cup B_{INV})^c$ such that $x \in A^c_{INV} \cap B^c_{INV}$

Therefore, $(A_{INV} \cup B_{INV})^c \subseteq A^c_{INV} \cap B^c_{INV}$

Proof (ii): similar to the prove of (i)

Proof (ii):

Let $x \in \left(A_{INV}^{c} \cup B_{INV}^{c} \cup C_{INV}^{c}\right)$

$\Rightarrow x \in A_{INV}^{c} \cup x \in B_{INV}^{c} \cup x \in C_{INV}^{c}$

$\Rightarrow x \notin A_{INV} \cup x \notin B_{INV} \cup x \notin C_{INV}$

$\Rightarrow x \notin \left(A_{INV} \cap B_{INV}\right) \cap C_{INV}$

$\Rightarrow x \notin \left(A_{INV} \cap B_{INV} \cap C_{INV}\right)$

$\Rightarrow x \in \left(A_{INV} \cap B_{INV} \cap C_{INV}\right)^{c}$

Since for all $x \in \left(A_{INV}^{c} \cup B_{INV}^{c} \cup C_{INV}^{c}\right)$ such that $x \in \left(A_{INV} \cap B_{INV} \cap C_{INV}\right)^{c}$

Therefore, $\left(A_{INV}^{c} \cup B_{INV}^{c} \cup C_{INV}^{c}\right) \subseteq \left(A_{INV} \cap B_{INV} \cap C_{INV}\right)^{c}$

Proof (iv): similar to the prove of (iv)

4 Conclusion

In this paper, the concept of interval neutrosophic vague was successfully established. The idea of this concept was taken from the theory of vague sets and interval neutrosophic. Neutrosophic set theory is mainly concerned with indeterminate and inconsistent information. However, interval nutrosophic vague sets were developed to improvise results in decision making problem. Meanwhile, vague set capturing vagueness of data. It is clear that, interval neutrosophic vague sets, can be utilize in solving decision making problems that inherited uncertainties. The basic operations involving union, complement, intersection for interval neutrosophic vague set was well defined. Subsequently, the basic properties of these operations related to interval neutrosophic vague set were given and mathematically proven. Finally, some examples are presented. In future, this new extension will broaden the knowledge of existing set theories and subsequently, can be used in practical decision making problem.

References

[1] L. A. Zadeh, “Fuzzy sets,” Inf. Control, 8(3), (1965),338–353.

[2] K. T. Atanassov, “Intuitionistic fuzzy sets,” Fuzzy Sets Syst., 20(1),(1986), 87–96.

[3] Z. Pawlak, “Rough sets,” Int. J. Comput. Inf. Sci., 11(5), (1982), 341–356.

[4] P. K. Maji, R. Biswas, and A. R. Roy, “Soft set theory,” Comput. Math. with Appl., 45(4-5), (2003), 555–562.

[5] W.-L. Gau and D. J. Buehrer, “Vague sets,” IEEE Trans. Syst. Man. Cybern., 23(2) , no. 2, pp. 610–614, 1993.

[6] F. G. Lupiáñez, “On Neutrosophic Sets and Topology,” Procedia Comput. Sci., vol. 120, (2017), 975–982.

[7] Z. Xiao, S. Xia, K. Gong, and D. Li, “The trapezoidal fuzzy soft set and its application in MCDM,” Appl. Math. Model., 36(12), (2012), 5844–5855.

[8] M. Deveci, “Site selection for hydrogen underground storage using interval type-2 hesitant fuzzy sets,” Int. J. Hydrogen Energy, 43, (2018), 9353–9368.

[9] X. Geng, X. Chu, and Z. Zhang, “A new integrated design concept evaluation approach based on vague sets,” Expert Syst. Appl., 37, (2010), 6629–6638.

[10] H. Shidpour, C. Da Cunha, and A. Bernard, “Group multi-criteria design concept evaluation using combined rough set theory and fuzzy set theory,” Expert Syst. Appl., 64, (2016), 633–644.

[11] F. Smarandache, “Neutrosophy: neutrosophic probability, set, and logic,” RehobothAmerican Res. Press, (1998), 1–141.

[12] H. Wang, F. Smarandache, Y.-Q. Zhang, and R. Sunderraman, “Interval Neutrosophic Sets and Logic: Theory and Applications in Computing,” in Hexis, Phoenix, (2005).

[13] P. Kumar Maji, “Neutrosophic soft set,” Ann. Fuzzy Math. Informatics, 5(1), (2013), 157–168.

[14] M. Alkhazaleh and S. Uluçay, “Neutrosophic Soft Expert Sets,” Neutrosophic Soft Expert Sets. Appl. Math., 6, (2015), 116–127.

[15] S. Alias, D. Mohamad, and A. Shuib, “Rough Neutrosophic Multisets,” Neutrosophic Sets Syst.,16, (2017), 80–88.

[16] M. Ali and F. Smarandache, “Complex neutrosophic set,” Neural Comput. Appl., 28(7), (2017), 1817–1834.

[17] M. Abdel-Basset, G. Manogaran, A. Gamal, and F. Smarandache, “A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria,” Des. Autom. Embed. Syst., 22(3), (2018), 257–278.

[18] M. Abdel-Baset, V. Chang, A. Gamal, and F. Smarandache, “An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field,” Comput. Ind., 106, (2019), 94–110.

[19] M. Abdel-Basset, M. Saleh, A. Gamal, and F. Smarandache, “An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number,” Appl. Soft Comput., 77, (2019), 438–452.

[20] M. Abdel-Basset, G. Manogaran, A. Gamal, and F. Smarandache, "A Group Decision Making Framework Based on Neutrosophic TOPSIS Approach for Smart Medical Device Selection," J. Med. Syst., 43(2), (2019), 38.
[21] D. Zhang, J. Zhang, K.-K. Lai, and Y. Lu, "An novel approach to supplier selection based on vague sets group decision," Expert Syst. Appl., 36, (2009), 9557–9563.
[22] W. Xu, J. Ma, S. Wang, and G. Hao, "Vague soft sets and their properties," Comput. Math. with Appl., 59(2), (2010), 787–794.
[23] N. Hassan and K. Alhazaymeh, "Vague soft expert set theory," in AIP Conference Proceedings, (2013), 953–958.
[24] G. Selvachandran, P. K. Maji, I. E. Abed, and A. R. Salleh, "Complex vague soft sets and its distance measures," J. Intell. Fuzzy Syst., 31(1), (2016), 55–68.
[25] K. Alhazaymeh and N. Hassan, "Interval-Valued Vague Soft Sets and Its Application," Adv. Fuzzy Syst., 2012, (2012), 1–7.
[26] K. A. Alhazaymeh, N. Hassan, and K. Alhazaymeh, "Generalized Interval-Valued Vague Soft Set," Appl. Math. Sci., 7(140), (2013), 6983–6988.
[27] K. Alhazaymeh and N. Hassan, "Possibility Vague Soft Set and Its Application in Decision Making," Int. J. Pure Appl. Math., 77(4), (2012), 549–563.
[28] S. Alkhazaleh, "Neutrosophic Vague Set Theory," Crit. Rev., X(July), 29–39.
[29] Al-Quran and A. Hassan, "Neutrosophic Vague Soft Set and its Applications," Malaysian Journal of Mathematical Sciences., 11(2), (2017), 141-163.
[30] A. Al-Quran and N. Hassan, "Neutrosophic vague soft expert set theory," Artic. J. Intell. Fuzzy Syst., 30(6), (2016), 3691–3702, 2016.
[31] A. Al-Quran and N. Hassan, "Neutrosophic vague soft multiset for decision under uncertainty," Songklanakarin J. Sci. Technolgy, 40(2) , (2018), 290–305.
[32] N. Hassan and A. Al-Quran, "Possibility neutrosophic vague soft expert set for decision under uncertainty," in AIP Conference Proceedings, 1830, (2017), 070007.

Received: November 22, 2018. Accepted: March 22, 2019