The Development of a Novel Experiment on Confined, Flattened Brazilian Disks to Correlate Damage and Permeability in Brittle Geo-Materials

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GEO-MATERIALS

by

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ABSTRACT

This research develops a novel experiment using flattened Brazilian disks under confining pressure with concurrent permeability measurements and acoustic emission monitoring. The purpose of this work is to correlate damage and permeability in brittle geo-materials. Two series of tests are performed on concrete and one series on tuff, under a range of confining stresses between 2.76 and 13.79 MPa. Tests were continued beyond peak stress conditions, capturing data in the post-peak region. The acoustic emission data (cumulative energy and counts) are used to identify damage thresholds and track the progression of microscopic damage. This quantified damage is then correlated to pre-peak changes in permeability, generating unique data sets to aid in the development of hydromechanical models. These models are of particular interest in the fields of carbon sequestration and hydrofracking, as well as having national security interests concerning the underground storage of nuclear waste and the leakage of radioactive gases.
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I. INTRODUCTION

1.1 Motivation

Ever since our distant ancestors fashioned the first stone implements, the mechanics of brittle geo-materials have shaped the development of civilization. The pyramids, Roman aqueducts, and Mayan temples all show an innate understanding of the way rocks fracture. As the centuries progressed, intuition gave way to empirically formulated failure criteria, and society’s drive for innovation demanded an explanation for these empirical data. Over the past several decades, numerous theories have been developed to predict fracture initiation, propagation, and the coalescence of these processes that eventually lead to rock failure. However, the variety of geo-materials and plethora of both engineered and naturally occurring formations has made it difficult to deduce a comprehensive theory for the failure of brittle rock. Using both numerical and experimental methods, this work studies the failure of brittle geo-materials.

Increases in computational power, coupled with more advanced numerical methods have enhanced the capacity to simulate the complex behavior of rock mechanics. This capacity provides further opportunity to investigate not only the failure of geo-materials but also the impact such failures have on mechanical processes such as the enhancement of fluid transport.

Understanding rock failure alone has traditional applications in geotechnical, rock, earthquake, and mine engineering. This project aims to relate rock damage with permeability changes. The relationship is becoming increasingly important in the hydrofracking and carbon sequestration industries. Furthermore, issues relevant to national security, such as leakage of radioactive gas from the long-term storage of nuclear waste or underground nuclear explosive
tests, have prompted collaborative research efforts between the University of New Mexico and Los Alamos National Laboratories (LANL).

1.2 Background

To predict the behavior of brittle geo-materials, researchers at LANL have developed a computational mechanics tool, the Hybrid Optimization Software Suite (HOSS) (Rougier et al., 2014; Lei et al., 2014). HOSS utilizes parallelization of LANL’s supercomputers to simulate damage processes based on the combined finite-discrete element methodology. The combined finite-discrete element method (FDEM) merges finite element continuum mechanics (FEM) with key concepts of discrete element methods (DEM) (Munjiza, 2004; Munjiza et al., 2011; Munjiza et al., 2015).

While HOSS simulations have shown excellent agreement with benchmark problems (Rougier et al., 2014; Knight et al., 2015), there is always a question of the accuracy of such methods to reproduce fracture patterns and subsequent permeability changes. In order to capture damage and fracturing behavior, HOSS requires the implementation of detailed material models. Each geo-material, e.g., concrete, granite, tuff, limestone, etc., has unique characteristics that dictate the failure processes. These characteristics, which include material strength, fracture energies, and strain softening behavior, are obtained from experimental programs.

A collaborative effort between LANL and the University of New Mexico seeks to integrate LANL’s numerical simulations with the University’s experimental capabilities. This collaborative research has already yielded results on correlating damage with permeability using the Brazilian split tension test with a concurrent vacuum permeability system (Rusch,
2018; Leyba, 2018). The current research aims to expand on these results by modifying the conventional Brazilian test to incorporate a confining stress that allows material response to be measured under a wider range of stresses compared to the conventional, unconfined, Brazilian test.

### 1.3 Scope of Work

This project develops a novel experimental setup that incorporates a confined, flattened, Brazilian split tension test with concurrent permeability measurements and acoustic emissions sensing. This allows investigations into the relationships between stress, damage, and permeability. The tests will be performed on concrete as a representative sample of brittle geo-materials, and tuff, a geologic material formed from volcanic ash, but the experiment will be reproducible for various other materials. Numerical models will be used to analyze the complicated stress states in confined, flattened Brazilian tests. The confining stress is expected to increase the ductility of typically brittle geo-materials allowing permeability changes to be measured after peak stress, while also providing post-peak strain-softening data that is often difficult to capture experimentally.

This work develops an experimental approach to test confined, flattened Brazilian disks. In doing so, experiments under a range of confining stresses will be performed on concrete and tuff specimens. This allows for:

- Permeability changes and damage, as measured by acoustic emissions, to be correlated.
- The transition from tensile to compressive failure to be evaluated.
- Numerical simulations and experimental results to be compared and analyzed.
1.4 Outlining Thesis

The remainder of this Thesis is portioned into four major chapters. Chapter Two is a comprehensive review of literature relevant to this work. Fundamental fracture and damage theories for brittle geo-materials are discussed, along with basic concepts of fluid transport in geo-materials. A history of the Brazilian split tension test is provided as well as a review of the numerical methods, focused primarily on the FDEM. Chapter Three describes testing methods and is subdivided into numerical methods and the development of the experimental setup. Chapter Four describes the results from the numerical and experimental methods, with a discussion on relationships between the key aspects of the work. Finally, Chapter Five presents the conclusions and provides recommendations and revisions for further work. Supplemental material, such as calculations and theoretical analyses, are found in the Appendices.
II. LITERATURE REVIEW

2.1 Fracture

In the realm of rock engineering, the strength of rock determines the design of the engineering application. Unlike relatively ductile materials (e.g., steel) which have both yield and rupture limits, many brittle geo-materials experience sudden failure along a fracture plane and exhibit little to no plasticity. The stress when this occurs, designated as either tensile or compressive, is referred to as the strength of the material even though damage is accumulated as micro cracks propagate and coalesce prior to failure. Understanding the failure strength and mechanisms that control it, is crucial in rock mechanic fields.

2.1.1 Failure Criteria

A failure criterion for rock mechanics is usually phenomenological and a generalization of experimental data (Labuz et al., 2018). Two common examples are the Mohr-Coulomb and Hoek-Brown failure criteria. The Mohr-Coulomb criterion is an empirical failure envelope that predicts the linear increase in shear strength to a corresponding increase in confinement (Eqn. 2.1). The Hoek-Brown criteria is expressed using the ratio of maximum and minimum principal stresses (Eqn. 2.2).

\[
\text{Eqn. 2.1} \quad \tau_f = \sigma_f \tan(\varphi) + c \\
\text{Eqn. 2.2} \quad \sigma_{1f} = \sigma_{3f} + c(m \frac{\sigma_{3f}}{c} + s)^{0.5}
\]

Where, in Eqn. 2.1, \( \tau_f \) is the shear stress at failure, \( \sigma_f \) is the normal stress at failure, \( \varphi \) is the internal angle of friction, and \( c \) is the unconfined compressive strength. While in Eqn. 2.2, \( \sigma_{1f} \)
is the maximum principal stress at failure, $\sigma_{3f}$ is the minimum principal stress at failure, $c$ is the unconfined compressive strength, and $m$ and $s$ are dimensionless fitting parameters.

Both criteria match experimental data well in the confined compression regions (Fig. 2.1). However, when extrapolated into tensile regions, these criteria fail to capture the behavior between uniaxial compression and uniaxial tension, i.e., confined tension. (Patel and Martin, 2018; Hoek and Martin, 2014). Since, according to Hoek and Martin (2014), “the brittle failure process initiates and is, to a very large extent, controlled by the tensile strength of intact rock,” a better theoretical understanding of the confined tension region is required. Developing a comprehensive fracture theory for rock that can capture that behavior is vital for evaluating and preventing spalling (Ramsey and Chester, 2004). Spalling can be defined as in situ fracture initiation and propagation and is relevant to any applications that use underground excavations, such as the development nuclear waste repositories, underground mining, tunneling, and wellbore drilling.
Figure 2.1 a) Mohr-Coulomb failure envelope in normal/shear stress space ($\sigma$, $\tau$); b) Hoek-Brown failure envelope in principal stress space ($\sigma_3$, $\sigma_1$). Dashed lines represent the extrapolation of envelopes into tensile regions; compression is considered positive.
2.1.2 Griffith Theory for Brittle Geo-Materials

Griffith’s work on the flow, fracture, and rupture of solids has become fundamental in the field of fracture mechanics (Griffith, 1921; Griffith 1924). Griffith proposed that natural flaws, cracks, and discontinuities are distributed at the microscopic scale throughout a given material. Modeling these flaws as ellipses, or ‘penny’ shaped cracks, Griffith was able to show that the surface energy gained during the fracture is equal to the loss in strain energy of the system (Hoek, 1965). Originally applying Inglis’ stress concentration at crack tips under tension, Griffith extended his fracture theory to include localized tensile failure induced under uniaxial and biaxial compression. Under compression the crack growth tends towards the maximum compressive stress (Fig 2.2). In normal/shear stress space ($\sigma$, $\tau$), Griffith’s failure envelope takes the form of a parabola (Eqn. 2.3) (Brace, 1960).

\begin{equation}
\tau^2 + 4\sigma_1\sigma - 4\sigma_1^2 = 0
\end{equation}

Where $\tau$ is the shear stress, $\sigma$ is the normal stress, and $\sigma_1$ is the uniaxial tensile strength of the material.

Figure 2.2 Crack propagation from the tips of elliptical Griffith cracks under a) uniaxial tension b) uniaxial compression and c) biaxial compression (modified from Hoek and Martin, 2014).
The success of Griffith theory for biaxial compression indicated a possible application in geo-mechanics, since most rock, whether in-situ or engineered, is under compressive stress (Hoek and Martin, 2014). McClintock and Walsh (1962) recognized this and modified Griffith theory to account for closed cracks under compression to describe typical shear failure (Eqn. 2.4). Assuming that Griffith’s penny shaped cracks would initially close under bi-axial compression, it is first necessary to overcome the frictional forces along the crack surfaces before the initiation of tensile cracks can begin (McClintock and Walsh, 1962), where the shear stress at failure, \(\tau_f\), can be determined from the uniaxial tensile strength, \(\sigma_t\), the normal stress at failure, \(\sigma_f\), and the coefficient of friction, \(\mu\).

\[
\text{Eqn. 2.4} \quad \pm \tau_f = 2\sigma_t - \mu\sigma_f
\]

Hoek (1965) gave a detailed summary of the derivation of Griffith’s Theory and subsequent modifications. He also conducted experiments that confirmed tensile crack growth initiating at the void spaces and grain boundaries of rocks. Importantly, Hoek’s tests showed that the length of crack propagation was a function of the ratio of the minor and major principal compressive stresses, providing the basis for the Hoek-Brown failure criterion.

2.1.3 Fracture Propagation and Interaction

Many fracture theories for rock used Griffith’s fracture theory as a starting point. However, Patel and Martin (2018) note that an isolated Griffith crack in a plate is an inadequate model of a complex grain-boundary network. Even more importantly, “Griffith theory predicts that orientation of the most severely stressed crack, in a material with a variety of cracks of different length and orientation. It does not predict how this critical crack will propagate” (Brace, 1960). The apparent disparity between a theoretical description of microscopic fracture
initiation and macroscopic propagation and coalescence has led to the development of phenomenological models for crack propagation and interaction.

Brace and Bombolakis (1963) investigated Griffith cracks in photo elastic glass. Under uniaxial tension the critical orientation of an existing microcrack is perpendicular to the direction of loading. Once the critical stress is applied, stress concentration near the tip of the crack propagate unstably to failure, and this critical stress can be considered the tensile strength of the material (Brace and Bombolakis, 1963). Fracture behavior is more complicated under bi-axial compression. It was found that the critical orientation of existing microcracks is ~30 degrees from the maximum compressive stress, and that the maximum stress concentration of the microcrack is not precisely at the crack tip. At the critical stress, wing cracks form and gradually curve toward the maximum compressive stress. Unlike the crack growth under uniaxial tension, these microcracks have stable growth, and the wing cracks stop extending unless the critical stress is significantly increased (Brace and Bombolakis, 1963). However, other experiments by Brace (1960) show that the critical stress to induce crack growth is often the same as or very near the ultimate stress at failure. Brace concludes that one optimally orientated crack cannot cause failure without interacting with other preexisting microcracks.

Nasser and Horii (1983) proposed that “shear failure may be considered to be the result of unstable growth of a suitably orientated set of interacting, pre-existing cracks.” Brace and others had shown that if multiple cracks of the same size are within a few crack lengths of each other that the maximum tensile stress concentrations are greater than those of a single crack for the same loading condition (Brace and Bombolakis 1963).

An alternative concept was proposed by Reches and Lockner (1994). Known as the stepped-crack model, it postulates that a pair of optimal pre-existing cracks form a process
zone that induces damage as it advances, while the tail of the process zone becomes the fault nucleus, where a macroscopic failure forms from localized tensile stresses (Reches and Lockner, 1994). This concept was confirmed using acoustic emission sensors to monitor the location of microcrack growth. The authors explain, “a fault nucleated at one site [i.e., the process zone] and propagated from it without significant preceding damage. There are no indications that the fault surface formed by coalescence of a pre-existing, randomly distributed array of microcracks…Thus we infer that the advancing fracture itself induces organized damage restricted to its own plane” (Reches and Lockner, 1994). The geometry of the initial pair of interacting Griffith cracks is what determines fault inclination, more specifically the ratio of crack spacing to crack length. If the grain boundaries are pre-existing Griffith cracks, then standard estimates of crack density from the grain boundary network can limit the fault inclination to a range of 20-30 degrees – matching most empirical data for rock failure (Reches and Lockner, 1994). It is noteworthy that the model by Nasser and Horii also correctly predicts the ~20-30 degree inclination for shear fractures. This is likely because even though the two models describe crack interaction differently, the underlying assumption of both is that optimally orientated, pre-existing Griffith cracks exist and grow from tensile stress concentrations near the crack tips. It seems, therefore, that some variation of Griffith theory continues to be a valid explanation for the failure of brittle geo-materials.

2.1.4 Confined Tension Region

Brace and Bombolakis (1963), Nasser and Horii (1983), Paterson and Wong (2005), Zuo et al., (2008), and numerous others have investigated the failure of brittle geo-materials under either compressive or tensile loads. In a uniaxial tension test, microcracks extend unstably at the critical stress, perpendicular to the applied load, and the tensile failure is
commonly referred to as an extension fracture (Gudmundsson, 2011). On the other hand, a triaxial compression test fails as a result of interacting microcracks at an angle ~30 degrees off the axis of the major compressive loading direction; this is known as a shear fracture (Gudmundsson, 2011). It was long hypothesized that extension and shear fractures are the two extremes of a continuous spectrum of fracture types (Engelder, 1999; Ramsey and Chester, 2004), and that under a mixed-mode stress state with both compressive and tensile stresses, a corresponding mixed-mode or hybrid fracture would occur (Ramsey and Chester, 2004).

![Figure 2.3](image) Common failure envelopes and test conditions in Mohr-Coulomb stress space, highlighting the confined tension region.

To investigate mixed-mode fractures, Brace (1964) and Mogi (1967) ran a series of confined tension tests using a dog bone geometry. The dog bone geometry is a cylindrical sample with a reduced central region or ‘thin neck,’ which under a confining pressure induces tensile stresses along the length of the sample. Although often cited, these results were inconclusive, and Ramsey and Chester (2004), and Bobich (2005) conducted a modified
version of Brace’s experiment on Carrara marble and Berea sandstone, respectively, which proved the existence of hybrid fractures. Lan et al. (2019) also performed confined tension tests on dog-bone samples of Longmaxi shale and provided a summary of the limited results in the confined tension region.

Another test that generates a mixed-mode stress state is the confined Brazilian split tension test. In the standard Brazilian test, a thin disk is loaded in compression diametrically which induces compressive stress concentrations near the applied load, and a uniform tensile stress across the central part of the loaded diameter (ASTM, 2008). Under a hydrostatic confining pressure, the Brazilian test becomes a truly variable triaxial test, such that \( \sigma_1 > \sigma_2 > \sigma_3 \) as opposed to the dog bone tests, where \( \sigma_1 = \sigma_2 > \sigma_3 \). Jaeger and Hoskins (1966) performed experiments using the confined Brazilian disk test. Their results and the cumulative mixed-mode data from Brace (1964), Mogi (1967), Ramsey and Chester (2004), Bobich (2005), Lan et al. (2019) indicates that the failure envelope in the confined tension region is complicated and not well predicted by existing theories. The generalized Fairhurst equation (Fairhurst, 1964), Hoek-Brown tension cut-off (Hoek and Martin, 2014), and modified Griffith criterion (McClintock and Walsh, 1962) all attempt to provide a continuous transition from tensile to shear failure, but empirical data does not match to these criteria (Ramsey and Chester, 2004; Tang et al., 2016; Patel and Martin, 2018; Lan et al., 2019). One possibility is the influence of the intermediate principal stress, \( \sigma_2 \). Griffith-based theories rely on the ratio of the maximum and minimum principal stresses and often assume the effect of \( \sigma_2 \) is negligible.

2.1.5 Intermediate Principal Stress

One of the main objections to theories derived from Griffith is that the two-dimensional analysis oversimplifies the complexity of a true triaxial stress state (Fairhurst, 1964). Empirical
data from confined tension tests indicate this objection is valid and that the intermediate principal stress does affect the failure strength in the confined tension region (Patel and Martin, 2018). Because typical confined triaxial experiments are such that \( \sigma_2 = \sigma_3 \), strength criteria to explain these results ignore the influence of the intermediate principal stress and cannot account for the role of \( \sigma_2 \). Studies on the effect of the intermediate principal stress by Boker (1915) and von Karman (1911) show it has a much smaller effect than that of the minimum principal stress. Further research by Handin and Hager (1957), Murrell (1958), and Mogi (1967) consistently showed that the intermediate principal stress has a small but measurable influence on rock failure. This is in marked disagreement with the underlying assumptions of the Mohr-Coulomb, Griffith, and modified Griffith failure criteria – that the intermediate principal stress has no effect. Mogi’s explanation for this discrepancy is that when a fracture surface starts to develop, it does initiate at suitably orientated cracks to the major and minor principal stresses as assumed by Griffith. In a biaxial condition, this surface would be normal to the \( \sigma_1 - \sigma_3 \) plane, and parallel to the intermediate principal stress \( \sigma_2 \). However, if the initial microcracks orientation is slightly off from this \( \sigma_2 \) axis then the intermediate principal stress would exert a normal component on the cracks such that frictional resistance increases with the intermediate principal stress (Mogi, 1967).

Based on the review of literature, an understanding of fracture initiation and propagation in brittle geo-materials remains incomplete. While suitably orientated Griffith cracks representing grain boundaries or initial micro flaws are likely to initiate cracks from stress concentrations near their tips, the stress conditions necessary to cause the cracks to interact and propagate are not always accurately predicted by existing theories. It has been shown that under mixed-mode stress states with both tensile and compressive stresses,
corresponding hybrid fractures can occur (Bobich, 2005; Ramsey and Chester, 2004; Brace 1964). In the confined tension region, these hybrid fractures are the transition from extension fractures under tensile stresses to shear fractures under triaxial compressive stresses. Currently, there are no suitable failure criteria that provide failure envelopes which captures this transition. It was also shown by Mogi (1967) that current failure criteria incorrectly ignore the intermediate principal stress, and that a comprehensive fracture criterion needs to be a function of all three principal stresses.
2.2 Damage

2.2.1 Damage Theory in Brittle Geo Materials

In brittle geo-materials, damage mechanics has developed as a corresponding field to fracture mechanics to help explain the complicated processes between microscopic crack initiation and complete macroscopic failure. Damage is defined as the mechanics of an array of growing and interacting microcracks that describe local effects on gross mechanical properties (Ashby et al., 1986). As damage accumulates in brittle geo-materials, typical responses can include elastic stiffness degradation, induced anisotropy, and cohesion loss. Fracture and damage mechanics coincide when damage is equal to one, or when cohesive bonds are broken, and a full macroscopic fracture has formed (Mazars and Pijaudier-Cabot, 1996). Introducing this new state variable (D), ‘damage’ may be regarded as a continuous measure of the internal degradation of the material as microfractures break cohesive bonds (Eberhardt et al., 1999).

As the field of damage mechanics developed, work soon concentrated on identifying the different stages and thresholds of the failure process (Eberhardt et al., 1997). The key thresholds for rock under compressive stresses, proposed by Brace (1964), and Eberhardt et al. (1998) are as follows:

- crack closure
- crack initiation and stable crack growth
- crack damage and unstable crack growth
- crack coalescence resulting in failure and post-peak behavior
Eberhardt et al. (1998) pointed out that certain strength parameters, such as the uniaxial compressive strength are dependent on loading conditions and loading rates. The damage thresholds, however, specifically crack initiation and crack damage appear to be reliable material characteristics amongst different testing conditions. However, because damage is a scale dependent property, the interpretation of the common thresholds mentioned above remains ambiguous, and as a result, they well be redefined for this work in chapter four.

2.2.2 Damage Detection and Characterization Methods

Early methods to identify the damage thresholds used scanning electron microscopy (SEM) to observe crack propagation and coalescence (Chang and Lee, 2004). Other methods used analysis of stress-strain measurements. Eberhardt et al., (1998) applied the moving point regression technique on the axial stress-strain curve to produce a moving point average of the materials Young’s modulus and identified the damage thresholds at key points as the stiffness changed. More recently, a method using the maximum lateral volumetric strain as a reference has been developed by Nicksiar and Martin (2012). However, because SEM and stress-strain data are difficult to measure in situ, monitoring damage via acoustic emissions has become an important method for the damage assessment of rocks (Bruning, 2018).

2.2.3 Acoustic Emissions

Acoustic emissions are elastic waves produced by the sudden movement of a stressed material. When a material is stressed past a certain level, plastic deformation occurs, culminating in crack initiation, growth, and ultimately, failure. At every stage in this process, energy is released in the form of elastic waves. These waves radiate throughout the material and can be detected well before ultimate failure by acoustic emission systems. Acoustic
emission events can result from displacement events on the order of picometers. As a result, acoustic emission sensing has a wide range of applications from source location of existing flaws to structural health monitoring, or characterizing micro-damage for material studies (Pollock, 2013).

A typical acoustic emission system is shown in Fig. 2.4. A sensor is placed on the material of interest. A load is applied to said material, resulting in plastic deformation that generates elastic waves. The waves travel through the material to the sensor. The sensor records the signal and the pre-amplifier filters and amplifies the signal before passing it to the data acquisition system (DAQ). Other parametric information, such as time, or data about the load can be combined for analysis.

The most important component of an AE system is the sensor. All AE sensors rely on piezoelectric crystals which convert mechanical movement into voltage. Stress waves cause the piezoelectric crystal to vibrate, and these vibrations can then be transformed into an electrical signal. During permanent deformation the displacement of a given element
corresponds to a simple wave (Fig. 2.5) where the amount of energy released is proportional to the area under the stress wave (Birks et al., 1991).

![Diagram of Displacement and Velocity/Stress over Time](image)

**Figure 2.5** Primitive waveforms from an acoustic emission event (Pollock, 2013).

In reality, an AE sensor will never receive such a simple waveform for several reasons. As mentioned previously, the stress waves generated by permanent deformation radiate in all directions. These waves are only bound by the geometry of the structure itself, and as individual waves rebound around the structure, different waves from a single event interfere with each other. Furthermore, the material itself, and the AE sensors vibrate at their own natural frequencies, resulting in additional interference. The impact of this interference is that an AE sensor will see the typical waveform in Fig. 2.6 rather than the assumed primitive wave from Fig. 2.5. As a result of the complicated nature of recorded waveforms, full waveform analyses are not typically conducted. Instead, it is common to use statistical analysis of a series of acoustic emission events to show time history trends and triangulate the source of the events (Birks et al., 1991).
2.2.3.1 AE to identify damage thresholds

Rapid advances have been made using acoustic emission sensing to evaluate damage (Lockner, 1993; Eberhardt et al., 1998, 1999; Cai, et al., 2004; Chang and Lee, 2004; Zhao et al., 2013; Kim et al., 2015; Bruning et al., 2018). As a result, stages of crack development in rock are well established using AE methods. Eberhardt et al. (1998, 1999) identified crack initiation and crack damage thresholds using cumulative AE counts and amplitudes. Chang and Lee (2004) and Zhao et al. (2013) expanded on this work and showed that by using cumulative AE counts, crack initiation can be identified as the first significant increase in the cumulative AE curve, and the crack damage threshold can be identified when the cumulative AE curve diverges and increases abruptly.
2.3 Permeability

2.3.1 Fluid Movement in Geo Materials & Darcy’s Law

The permeability of geo-materials is of interest in many fields, including carbon sequestration, hydraulic fracturing, hydrocarbon production, and the design of nuclear waste repositories (Nertnieks, 1993). Fluids travel through void spaces which are distinguished as either pores or fractures (Bear, 1993). The flow of a single Newtonian fluid is well described by the Navier-Stokes equation. If continuity is applied, and the conservation of mass is combined with the Navier-Stokes equations, it can be reduced to the well-known Darcy’s Law, Eqn. 2.5 (Bear, 1993).

\[
\text{Eqn. 2.5} \quad q = -\frac{k}{\mu} \nabla P
\]

Where the flux, \(q\), is a function of the pressure gradient, \(\nabla P\), the intrinsic permeability, \(k\), and the dynamic viscosity, \(\mu\).

2.3.2 Corrections for Visco-Inertial Flow

In 1856 Henry Darcy described the flow of a fluid through porous sand beds after experimental observations. However, this form (Eqn. 2.5) makes several important assumptions which include the following:

- Steady-state conditions
- A non-compressible, viscous fluid
- Laminar flow
- One-dimensional, single-phase flow
If these assumptions are not valid, then certain corrections must be applied. For example, when flow velocities are sufficiently great, inertial effects render Darcy’s Law inaccurate and an alternative expression such as Forchheimers equation must be used to describe fluid flow (Zimmerman, R.W. et al., 2004). Additionally, gas flow through openings with a size comparable to the mean free path of the gas molecule causes interactions between the fluid particle and wall occur. This is known as the Klinkenberg effect and requires further modifications to Darcy’s Law (Rushing, J.A. et al., 2004).

In this work, the permeability of geo-materials is measured using nitrogen gas. Darcy’s Law for steady state flow of a compressible fluid takes the form shown in Eqn. 2.6, where $Q$ is the volumetric flowrate, $k$ is the intrinsic permeability, $A$ is the cross-sectional area, $P_1$ and $P_2$ are the upstream and downstream pressures, $\mu$ is the viscosity, $L$ is the length of the domain in the direction of flow, and $P_{\text{avg}}$ is the mean of pressures $P_1$ and $P_2$. This form of Darcy’s Law assumes a one-dimensional ideal gas, under laminar flow, with steady-state, and isothermal conditions.

\[
Q = -\frac{k A (P_1^2 - P_2^2)}{\mu L (2P_{\text{avg}})}
\]

2.3.3 Cubic Law

In compact rocks with low porosities, the permeability of a fracture network can be several orders of magnitude higher than the permeability of the pore network (Neretnieks, 1993). As such, it can be assumed that the permeability of the pore network is negligible and any macroscopic permeability is a result of the fracture network. Fractures are often idealized as a set of parallel plates separated by an aperture, $h$, to study flow (Neretnieks, 1993). When this assumption is made, the relation between flow and pressure gradient is governed by the
so-called cubic law (Witherspoon et al., 1980). When normalized by the cross-sectional area, the cubic law states that the permeability through parallel plates, $k$, is proportional to the square of the aperture between them, $h$, as shown in Eqn. 2.7.

$$\text{Eqn. 2.7} \quad k = \frac{h^2}{12}$$

Akhavan et al. (2012) showed that in brittle geo-materials, if mechanical aperture is used, the cubic law must be corrected to account for the roughness and tortuosity of fracture surfaces by introducing a correction factor $\alpha$ (Eqn. 2.8a). For small apertures (35 – 100 microns), this correction can affect permeability by several orders of magnitude. On plain and fiber-enforced mortar disks, fractured using the Brazilian split tension tests using factors accounting for roughness $R_r$, and tortuosity $\omega$, with a fitting parameter $b$, Eqn. 2.8a takes the form seen in Eqn. 2.8b (Akhavan et al., 2012).

$$\text{Eqn. 2.8a} \quad k = \frac{\alpha h^2}{12}$$

$$\text{Eqn. 2.8b} \quad k = \frac{\omega b^2}{12(1+8.8 R_r^{1.5})}$$

The fractures evaluated by Akhavan et al. were created using the Brazilian disk test. Water permeability was considered a function of crack geometry, but not a function of damage or stress. Wang et al. (1997) and Picandet et al. (2009) also tested permeability on Brazilian disk. However, these studies measured permeability after unloading and not while under load. Rastiello et al. (2014) did successfully conduct concurrent water permeability measurements on Brazilian disks during loading and showed that Brazilian disk tests provide further opportunities to study hydromechanical coupling, particularly concurrent air permeability enhancements of rock fractures under load.
2.4 Brazilian Test

2.4.1 Standard Brazilian Split Tension Test

The Brazilian disk test was developed simultaneously and independently by Carneiro and Akazawa in 1943 (Li and Wong, 2013). Since that time, it has been studied and applied extensively in rock mechanics, becoming a standard method of determining the tensile strength of brittle rock by the International Society of Rock Mechanics (ISRM) and the American Society for Testing and Materials (ASTM) (Gutierrez-Moizant et al., 2018). The test determines the tensile strength of brittle materials by diametrically loading a thin circular disk to failure (Fig. 2.8).

For point loads, a 2D solution was provided by Timoshenko (1934). However, if the load is applied over a small circumferential strip as recommended by Fairhurst (1964), then the stresses along the loaded diameter are solved for using Hondros’ analyses (1959).
Eqn. 2.9a \[ \sigma_{\theta y} = \frac{2P}{\pi} \left[ \frac{(1-r^2/R^2)\sin 2\alpha}{1-2r^2/R^2 \cos 2\alpha + r^4/R^4} - \tan^{-1}\left(\frac{1+r^2/R^2}{1-r^2/R^2}\right)\tan \alpha \right] \]

Eqn. 2.9b \[ \sigma_{r y} = -\frac{2P}{\pi} \left[ \frac{(1-r^2/R^2)\sin 2\alpha}{1-2r^2/R^2 \cos 2\alpha + r^4/R^4} + \tan^{-1}\left(\frac{1+r^2/R^2}{1-r^2/R^2}\right)\tan \alpha \right] \]

Where the radial and circumferential stresses \( \sigma_{\theta y} \) and \( \sigma_{r y} \) at any distance from the center of the disk, \( r \), can be given as a function of the diametrical load \( P \), the distributed loading arc, \( 2\alpha \), and the radius of the disk, \( R \). As seen in figure 2.8, the corresponding radial tensile stresses are constant across the central region of the diameter and simplifying Hondros’ solutions at the center of the disk yields the following (Jaeger and Hoskins, 1966):

\[ 2.4.1.1 \text{ Brazilian disk under confinement} \]

Jaeger and Hoskins (1966) suggested that if an additional confining pressure was applied to the Brazilian disk, then the transition from tensile to compressive failure could be studied. In the unconfined scenario, the tensile and compressive radial stresses (Eqn. 2.10) are equal to the minimum (\( \sigma_3 \)) and maximum (\( \sigma_1 \)) principal stresses at the center of the sample, with the intermediate principal stress, (\( \sigma_2 \)) equal to zero. If a constant hydrostatic pressure is applied to all surfaces of the Brazilian disk, then the principal stresses increase by a magnitude equal to the hydrostatic pressure (Fig 2.9). As a result, the principal stresses at the center of the disk are given by Eqn. 2.11 for confined Brazilian disks (Jaeger and Hoskins, 1966).
Eqn. 2.11a \[ \sigma_1 = \sigma_{yy} = \frac{6F}{\pi Dt} + P_c \]

Eqn. 2.11b \[ \sigma_2 = \sigma_{zz} = P_c \]

Eqn. 2.11c \[ \sigma_3 = \sigma_{xx} = \frac{-2F}{\pi Dt} + P_c \]

Where, \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are the principal stress, \( F \) is the diametrical load, \( P_c \) is the confining stress, \( D \) is the diameter of the disk, and \( t \) is the thickness of the disk.

![Confined Brazilian disk analysis](image)

**Figure 2.8** Confined Brazilian disk analysis.

### 2.4.2 Validity of the Brazilian Disk Test

Although a full review of the Brazilian disk test falls outside of the scope of this work, an extensive history is provided by Li & Wong (2012). There have been several noteworthy developments and critiques of the Brazilian test (Hondros, 1959; Fairhurst, 1964; Jaeger and Hoskins, 1966; Wijk, 1978; Yanagidani et al., 1978; Andreev, 1991; Wang et al., 2004; Yu et
al., 2006). The more recent focus, presented by some of these authors, deals with improvements and modifications to the standard testing procedure.

2.4.2.1 Crack Initiation & Stress Concentrations

As pointed out by Wijk (1978) the 2D analytical solution happens to be the idealized case for the Brazilian test, but in reality, the 2D theory of elasticity does not yield solutions that satisfy the 3D equations. This phenomenon is demonstrated effectively by Yu et al. (2006). A 3D FEM analysis showed that thicker Brazilian disks will have larger tensile stresses at the surface than in the center of the disk. Even samples that meet the ISRM recommended thickness-to-diameter ratio require corrections for the calculated tensile strengths.

Furthermore, according to the Griffith criterion, tensile cracks in the Brazilian disk should initiate at the center of the sample (Li & Wong, 2012). Some experimental results report cracks initiating away from the center (Fairhurst, 1964). However, it remains unclear if this is the result of stress concentrations near the loading points, or material heterogeneity. While both factors are important, several researchers have shown the importance of using curved loading arcs to distribute the diametrical load (Lu et al., 2018; Gutierrez-Moizant et al., 2018); yet implementing this in practice remains difficult.

2.4.2.2 Flattened Brazilian

To resolve the difficulties associated with the curved loading arcs and stress concentrations, the flattened Brazilian disk is recommended as an alternative (Patel & Martin, 2018; Wu et al., 2018; Yu et al., 2009). Adopting the method proposed by Wang and Xing (1999), two equal parallel planes are introduced to the Brazilian disk specimen (Fig 2.10). This configuration reduces stress concentrations near the specimen circumference, and cracks are
more likely to initiate from the center portion of the disk. While numerical methods have been used in recent years to analyze the flattened Brazilian disk – Wu et al., (2018) used a discrete element method (DEM), and Patel and Martin (2018) used a finite difference code, FLAC3D – these methods have their limitations. No evidence in literature exists that a confined, flattened Brazilian disk has been analyzed using a numerical code capable of capturing complete fracture and fragmentation processes.

![Diagram of Flattened Brazilian Test](image)

**Figure 2.9** Flattened Brazilian (modified from Wu et al., 2018).
2.5 Numerical Methods

There is a myriad of numerical methods for rock mechanic applications. Broadly, they can be categorized into continuum, discrete, and hybrid methods (Jing, 2003).

2.5.1 FEM

Continuum numerical methods treat the entire domain as a continuous body. Common forms are the boundary element method (BEM), and finite element method (FEM) (Lisjack, 2014). First developed in the 1960’s, the FEM became a popular tool for problems including structural analysis, heat transfer, fluid flow, and mass transfer (Logan, 1992). The FEM yields approximate values at discrete numbers of points in the continuum, providing solutions for complicated geometries where analytical solutions are impossible. However, a major limitation of continuum methods, like the FEM, is its lack of ability to capture fracture processes (Jing, 2003). Rock and granular materials are filled with discontinuities at the grain scale, that can be challenging to represent in a continuum model. Furthermore, complete detachment of neighboring elements due to the formation of a fracture network results in an ill-conditioned system of equations because the continuum mechanics require that fracture elements cannot be torn apart (Jing, 2003). While this problem can be addressed in more complex continuum models, discrete element methods were developed to more effectively solve the discontinuity problem (Lisjack, 2014).

2.5.2 DEM

The development of the discrete element method (DEM) for soil and granular materials can be primarily credited to Cundall and Strack (1979). According to their definitions, a DEM allows finite displacement, rotations, and complete detachments, and can also recognize new
contacts. It does so by modeling the domain of interest as an assembly of separate particles. This is ideal for granular assemblies and rock mechanics problems, but while effective, it may not provide the most efficient computational framework (Lisjack, 2014). More recently, hybrid models, that combine finite element continuum mechanics (FEM) with the key concepts of discrete element methods (DEM) such as contact detection and interaction, have emerged (Munjiza, 2004; Munjiza et al., 2015).

2.5.3 FDEM

The combined finite-discrete element method (FDEM) can capture both continuous and discontinuous mechanics if each discrete element is further discretized into a finite element mesh (Fig 2.11). First proposed by Munjiza (2004) the combined finite-discrete element methodology has since been the basis for several numerical codes (Rougier & Munjiza, 2010; Munjiza et al., 2012; Latham et al., 2012). One such implementation of the FDEM, is the Hybrid Optimization Software Suite (HOSS). Developed by researchers at Los Alamos National Laboratories in partnership with Munjiza, HOSS is a tool designed to capture high strain rate events, fracture and fragmentation processes, for a wide array of material models; making it applicable to a myriad of science and engineering problems.

In this work, HOSS was used to aid in the development of the novel experiment to test confined, flattened Brazilian disks. A brief description of the key attributes utilized by HOSS is given below. The model set-up is given in chapter three and simulation results are presented in Appendix A-6.
2.5.3.1 Governing Equation

The governing equation of the FDEM methodology developed by Munjiza (2004), is given below, where \( M \) is the lumped mass matrix, \( C \) is the damping matrix, \( \ddot{x} \) is the acceleration vector, \( \dot{x} \) is the velocity vector, and \( f \) is the force vector for each node.

\[
M \ddot{x} + C \dot{x} = f
\]  

2.5.3.2 Critical Time Step

HOSS implements the central difference explicit time integration scheme to solve Eqn. 2.12, which ensures stability during the evaluation of non-linear constitutive laws. As such, one important parameter is the critical time step (Eqn. 2.13). The critical time step is a function
of the characteristic length $h$, the Young’s modulus of the material $E$, and the density of the material $\rho$, and represents the largest possible time step that produces a stable time integration.

\[
\text{Eqn. 2.13} \quad \Delta t_{cr} = \frac{h}{\sqrt{E/\rho}}
\]

### 2.5.3.3 Damping

Damping is required to maintain the energy balance of the system. HOSS uses both a viscous damping context to represent the dissipation of energy due to the internal deformation of stressed elements, and a mass-proportional damping constant, Beta (Eqn. 2.14). Beta is a system property that depends on the variables described in Eqn. 2.13.

\[
\text{Eqn. 2.14} \quad \beta = 2 \frac{\pi \sqrt{E/\rho}}{h}
\]

### 2.5.3.4 Multiplicative Decomposition

The deformations of individual elements are decomposed into their respective translations, rotations, and stretches. These deformation modes are resolved using a multiplicative decomposition-based formulation which is incorporated naturally within FDEM using elastic-based Munjiza constants (Munjiza et al., 2015).

### 2.5.3.5 Contact Detection/Interaction

Contact detection in HOSS is performed using the Munjiza-Rougier-Carney-Knight (MRCK) algorithm (Munjiza et al., 2011; Lei et al., 2014). The MRCK contact detection algorithm decomposes the physical space rather than the solid domain, into identical squares. The search problem is then reduced to those pairs of contactor elements and target points that
share the same square (Fig 2.12). It is computationally efficient and applicable to systems containing many bodies of different shapes and sizes (Munjiza & Rougier, 2010).

![Diagram showing MRCK decomposition](image)

**Figure 2.11** Rendering of MRCK decomposition (adopted from Zei et al., 2014).

When contact is detected, a penalty function is used to calculate the forces between contacting elements. In the latest version of HOSS, contact interaction is represented by triangle to point kinematics (Fig. 2.13). The surfaces of target triangles are discretized into Gauss integration points, over which the interaction forces are integrated (Lei et al., 2014).
2.5.3.6 Material Model - Smeared Cracking Model & Strain

Softening Damage

The definition of material strain-softening behavior is the basis for the combined single and smeared crack approach to describe fracture (Munjiza, 1999). The curve is divided into two regions: a strain-hardening region, where no damage occurs, and the material behaves elastically until it nears yield, and a strain-softening region which is only reached when the stress surpasses the prescribed material strength (Fig. 2.14).
In the strain-softening region, stress is defined as a function of displacement, rather than strain. This displacement can be in either the normal or transverse direction (Fig 2.15). Once the strain-softening portion of the material model is reached, elements accumulate damage based on their local normal and tangential displacements (Eqn. 2.15). Maximum displacement thresholds are derived from both the mode I and II fracture energies ($G_I, G_{II}$), tensile and shear strengths ($\sigma_t, \tau$), and area under the strain-softening region ($A_0$). When the displacements exceed the maximum displacement thresholds, the cohesive bonds break and fully developed fractures form (Eqn. 2.16).
Figure 2.14 Normal and tangential displacements between elements (modified from Munjiza et al., 1999).

Eqn. 2.15a \[ \delta_T = \frac{G_I}{\sigma_T A_0} \]

Eqn. 2.15b \[ \delta_S = \frac{G_H}{\tau A_0} \]

Eqn. 2.16 \[ D = \begin{cases} 0 & \text{if } \delta \leq \delta_p \\ 1 & \text{if } \delta > \delta_c \\ \frac{\delta - \delta_p}{\delta_c - \delta_p} & \text{otherwise} \end{cases} \]

This process can result in the transformation of the continuum FEM mesh into a number of discrete bodies which interact with each other. As the simulation progresses, these interactions among newly formed discrete bodies are resolved with the help of the previously mentioned detection and interaction algorithms.

2.5.3.7 Parallelization

HOSS also benefits from the implementation of large-scale parallel processing (Lei et al., 2014). The entire computational domain is divided into Message Passing Interface (MPI)
domains (Fig 2.16). Each MPI domain is passed to its own processor, that all run in parallel. This increases the computational efficiency and significantly reduces the runtime of simulations, even those containing millions of elements.

Figure 2.15 Representation of MPI domains (red) with an average weight of 1000 elements per domain.

2.5.4 Motivation

The advantages of using the combined finite-discrete element method to simulate and study rock mechanic problems are well proven (Rougier et al., 2014; Tatone et al., 2010; Latham et al., 2008). In this paper, confined, flattened, Brazilian disk tests are simulated using HOSS. This provides a unique analysis of the fracture and fragmentation processes in the confined tension region. Analysis of the results allows insight into the development of experimental methods to test confined, flattened, Brazilian disks. Taken together with concurrent permeability measurements and acoustic emission sensing, this work introduces a novel approach to the study of hydromechanical processes.
III. METHODS

This project develops and implements a novel experiment on confined, flattened Brazilian disks, supplemented with numerical simulations. The experimental and numerical methodologies are distinct and divided into their corresponding sections below.

3.1 Experimental Methodology

3.1.1 Program

The goal of the experimental program is to develop a method to test flattened Brazilian disk samples of brittle geo-materials under confinement with concurrent permeability measurements and acoustic emission monitoring. Doing so allows researchers the opportunity to study hydromechanical coupling — the relationships between stress, damage, and permeability.

3.1.2 Testing Configuration

A diagram of the complete test setup is shown in Figure 3.1. A flattened Brazilian disk is cast in an impermeable polyurethane jacket, between two end caps. The end caps have ports that allow nitrogen gas to be pushed across the disk during the duration of the test, with outgoing flow vented to atmosphere and measured by flow meters. The jacketed sample is tested inside of a cylindrical steel pressure vessel rated to 34 MPa. A hydraulic pump is used to apply a hydrostatic confining pressure ranging from 0 – 20 MPa ± 0.2MPa. A hermetically sealed piston applies a diametrical load from a 433 kN Instron loading frame with a 2 kN resolution to the flattened Brazilian disk at a rate of 1e-6 m/s. Four acoustic emission sensors are attached to the outside of the pressure vessel and recorded acoustic emission events are processed by the Physical Acoustic corporation Mistras AEwin software.
Figure 3.1 Confined flattened Brazilian disk test configuration.
3.1.3 Materials

Flattened Brazilian disks were tested using tuff and concrete specimens. All specimens had a 50.80 mm diameter and 25.40 mm thickness to accommodate the size of the testing apparatus while maintaining ASTM D3967 requirements for a Brazilian disk test – $t/D \leq 0.5$. The disks were flattened 1.00 mm on the top and bottom of the loaded diameter using a Dayton table sander.

Tuff is an igneous rock formed from the consolidation of exploded volcanic materials. The cores used in this work were provided by Sandia National Laboratories and consisted of zeolitic, non-welded, unoriented tuff, with no observable fractures. The tuff was characterized as a light pink matrix of zeolitized ash with abundant lithic clasts ranging from 0.50 mm to 25.4 mm in size. The tuff was sealed in bees wax during drilling to preserve in situ moisture content and provided in limited quantities.

Figure 3.2 50.8 mm Tuff Brazilian disks. Note the variability in the size of lithic clasts (aggregates).
The concrete was mixed in the laboratory using Portland cement, water, sand, and aggregate with a maximum nominal size of 4.75 mm. The concrete specimens were created following the procedure in ASTM C192. The design mixture for the concrete is shown in Table 3.1.

![Concrete specimen](image)

**Figure 3.3** 50.8 mm Concrete Brazilian disk with 1.0 mm flattening on top and bottom.

**Table 3.1:** Concrete mixture.

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Cement</td>
<td>400 kg/m³</td>
</tr>
<tr>
<td>Water</td>
<td>180 kg/m³</td>
</tr>
<tr>
<td>Sand</td>
<td>790 kg/m³</td>
</tr>
<tr>
<td>Aggregate</td>
<td>990 kg/m³</td>
</tr>
<tr>
<td>Water/Cement Ratio</td>
<td>0.45</td>
</tr>
</tbody>
</table>
3.1.4 Pressure Vessel and Jacketing

The steel pressure vessel is rated to withstand a 34 MPa internal pressure. The inner dimensions measure 203.20 mm in height with a 127.00 mm diameter. A 31.75 mm diameter piston is hermetically sealed and can transfer the diametrical load while maintaining internal pressure (Fig 3.4).

Flattened Brazilian disks are placed between 50.80 mm diameter aluminum end caps. The Brazilian disks were flattened 1mm on top and bottom using a coarse-grain sanding belt. Perforated Teflon spacers are placed between the sample and the end caps in order to reduce friction between the sample and the end caps, while distributing flow evenly across the surface of the sample. The sample, spacers, and end caps are placed in a latex sleeve, then cast in a 6.35 mm thick layer of a deformable polyurethane rubber compound, PMC-744 from Smooth-On Inc. The latex sleeve ensures that there is no adhesive bonding between the PMC and the flattened Brazilian disk. The PMC is cast around the gas inlet and outlet ports (Fig. 3.5), which connect to the permeability system (Fig. 3.6)
Figure 3.5 a) Sample Jacket Diagram and b) Jacketed Sample.
Figure 3.6 Jacketed sample attached to permeability system and pressure vessel lid with loading configuration. This assembly is placed into the body of the pressure vessel prior to testing.
3.1.5 Permeability System

To measure permeability during loading nitrogen gas is pushed through the flattened Brazilian disk. Upstream pressure \((P_1)\) is controlled with a regulator that has a range of 0 – 344 kPa and recorded by an Omega pressure transducer of 0 – 207 kPa ± 0.1 kPa. Once the nitrogen gas passes through the sample it vents to atmosphere; therefore, downstream pressure, \((P_2) = (P_{atm})\). The volumetric flowrate of gas passing through the sample is monitored by a series of Alicat flowmeters with ranges of 0.000 – 1.000 L/min and 1.0 – 100.0 L/min respectively, with accuracies to 0.2% of the full ranges. Permeability is calculated using Eqn. 2.6, where \((L)\) is equal to the thickness of the sample, \((A)\) is the cross-sectional area of the flattened Brazilian disk, and \((\mu)\) is the dynamic viscosity of nitrogen, equal to 1.76e-05 Pa·s. It is assumed that the dynamic viscosity is constant as a function of pressure. The uncertainty in apparent permeability is ± 3%.

3.1.6 Acoustic Emission Setup

Acoustic emission sensors were placed on the outside of the pressure vessel to monitor stress-induced damage during loading. Four Micro30 acoustic emission transducers from Physical Acoustics Corporation (PAC) were used with an eight-channel AE signal processing system. AE signals were pre-amplified by a gain of 60 dB, and a digital threshold was set at 50 dB to filter out background noise. The sensors were attached to the exterior of the pressure vessel with elastic bands, and a layer of vacuum grease was applied to remove any air between interfaces, ensuring good acoustic coupling.

It is important to note that the AE sensors were not placed directly on the specimen and as such, acoustic emission signals had to pass through the sample, sample jacket, confining
fluid, and pressure vessel before reaching the sensors. Each media has different wave velocities making source location analysis via arrival times difficult to interpret. However, Zhao et al. (2013), found that placing sensors on the outside of the pressure vessel had a negligible impact on using the acoustic emission data to judge damage thresholds. Although AE waveforms are attenuated, it is still possible to analyze peak amplitude, rise time, event duration, and counts per hit (Fig. 3.6).

![Figure 3.7 Features of attenuated AE waveforms (Pollock, 2013).](image-url)
3.2 Numerical Methodology

3.2.1 Plan

Numerical simulations were conducted using HOSS to aid in the development of the experimental method. Firstly, a direct comparison of regular and flattened Brazilian disks was conducted. Next, the analytical solutions presented in Eqn. 2.11 were compared to the explicit stress state calculated in HOSS. Finally, confined flattened Brazilian disks were simulated under a range of confining pressures to gain insight into the role of the confining pressure. Figure 3.7 shows the model setup for a standard Brazilian disk with loading plates.

Figure 3.8 Model setup for a standard Brazilian disk test.
3.2.2 Geometry and Mesh Convergence

The simulated Brazilian disks matched the exact geometry of laboratory samples. The standard Brazilian disk, 50.8 mm in diameter and 25.4 mm thick, with two loading plates was represented by a mesh of tetrahedral elements generated by Cubit. Cubit is a pre-processing software developed at Sandia National Laboratories to generate two- and three-dimensional finite element meshes (cubit.sandia.gov). It allows for sets of tetrahedral elements to be grouped together as a ‘material block’ so that each material block can be assigned independent material properties. HOSS is also able to distinguish between material blocks that are cohesive or non-cohesive. This allows certain material blocks to be designated as FEM and simulated using pure continuum mechanics, greatly increasing computational efficiency. In this model, only the Brazilian disk is set as a cohesive material block, and the two steel plates are treated as FEM domains.

The mesh size of individual elements is an important aspect in numerical simulations. If the size is too large, the domain is not well represented, and accurate stresses cannot develop. If the size is too small, simulations become computationally expensive. To find the optimal mesh size for the Brazilian disk simulations, a mesh convergence study was performed (Appendix A-6), and 3 mm mesh size was selected.

3.2.3 Material Definitions

Both the concrete Brazilian disk and steel loading plates were modeled as isotropic, homogenous, fully 3D materials. Because the concrete is a cohesive material, it requires more material properties than the steel; this allows HOSS to capture the fracture propagation. The
material properties for each material were based on previous work by Rusch (2018) and Leyba (2018) and are shown in Table 3.2 and Table 3.3.

Table 3.2: Material properties of concrete.

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<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>E (GPa)</td>
<td>11</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>ν</td>
<td>0.17</td>
</tr>
<tr>
<td>Density</td>
<td>ρ (kg/m³)</td>
<td>2347</td>
</tr>
<tr>
<td>Normal and Shear Strengths</td>
<td>σ , τ (MPa)</td>
<td>5.5 , 9.2</td>
</tr>
<tr>
<td>Fracture Energies</td>
<td>G_I , G_{II} (J/m²)</td>
<td>100 , 128</td>
</tr>
<tr>
<td>Maximum Normal and Shear Displacements</td>
<td>δ_I , δ_{II} (mm)</td>
<td>0.036 , 0.028</td>
</tr>
</tbody>
</table>

Table 3.3: Material properties of steel.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>E (GPa)</td>
<td>200</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>ν</td>
<td>0.3</td>
</tr>
<tr>
<td>Density</td>
<td>ρ (kg/m³)</td>
<td>8050</td>
</tr>
</tbody>
</table>

3.2.4 Boundary Conditions and Speed Optimization

The boundary condition for Brazilian disk tests are straightforward. The disk is placed in between two steel plates. All nodes on the bottom plate are fixed, and the top plate is moved downward with a constant velocity. The material properties of the steel plates are appropriately defined, and their higher stiffness ensures they do not significantly deform. For confined tests, a uniform pressure is applied to all surfaces of the Brazilian disk. This pressure is applied instantaneously, resulting in an initial increase in the kinetic energy of the system. This kinetic energy reduces over time and must be allowed to equilibrate before the diametrical loading begins.
The diametrical load is produced by contact between the sample and the displacement of the top plate. As with experimental Brazilian disk tests, the displacement rate affects failure. Increasing the speed of the plate can help with computational efficiency by reducing runtime. However, if the displacement rate is too high, quasi-static conditions are no longer valid. To ensure quasi-static conditions, an optimization study was performed (Appendix A-6), and the optimal displacement rate 0.03 m/s was found.

3.2.5 Comparison of Experimental and Numerical Results

The numerical simulations used in this work are not quantitatively compared to the experimental results but are still useful in aiding the development of the experimental program. Although the model geometry and boundary conditions match the experimental conditions, the material properties do not. Because both the concrete and tuff samples have large aggregates relative to the dimensions of the Brazilian disk, material homogeneity cannot be consistently assumed. As a result, the numerical models use material properties based on previous work (Leyba, 2018; Rusch, 2018), and without precise descriptions of the material properties, a direct comparison of numerical and experimental is impossible. To avoid confusion, a full description of the numerical simulations and their results in presented in Appendix A-6.
IV. RESULTS AND DISCUSSION

This work includes the development of a novel experimental approach. To evaluate the approach, extensive analysis of the experimental setup was performed. This analysis was aided by, and includes, the numerical methods of section 3.2. The structure of the following chapter first describes the results used to analyze and develop the experimental methodology - section 4.1. The following section, 4.2, presents the experimental results of confined, flattened, Brazilian disk experiments with concurrent permeability and damage measurements on concrete and tuff.

4.1 Experimental Development

The experiments conducted in the test combined procedures and methods that are individually well established, including Brazilian split disk tests, acoustic emission monitoring, and gas permeability measurements of rock fractures. By combining these systems, a novel experimental methodology was fashioned that allows hydromechanical processes to be studied. In the process of combining these systems, certain modifications were used. These include using 1mm flattened Brazilian disks to reduce local crushing, and the application of an impermeable jacket with end caps to measure permeability. This section presents the analysis of the critical modifications and discusses their impact on the experimental results.
4.1.1 Classic vs. Flattened Brazilian Disks

An experimental comparison of standard and flattened Brazilian disks was performed. Four standard Brazilian disks with distributed arc loading and four 1 mm flattened Brazilian disks were tested according to ASTM standards. The force/displacement data for each test are averaged and displayed in Fig. 4.1.1. There is a noticeable difference in the peak displacement (Fig. 4.1.2), but this is caused by a combination of mechanical seating of the load frame and deviations in flattened surfaces of the flattened Brazilian disks. More importantly, the percent difference in mean peak force is less than 2.5%, indicating that a 1mm flattened depth is an acceptable modification to alleviate the common problem of localized crushing in the standard Brazilian disks without significantly impacting the failure strength of the material.
Figure 4.1.1 Statistical comparison of the peak strength for standard Brazilian disks (with distributed arc loading) and 1mm flattened Brazilian disks.

Figure 4.1.2 Comparison of standard (with distributed arc loading as recommended by the ASTM) and 1mm flattened Brazilian disks using force/displacement along the loaded diameter.
4.1.2 Analytical Stress Verification

The Brazilian disk test creates three unequal principal stresses at the center of the disk. The stress state of a differential element in the disk also greatly depends on the spatial location within the disk. Jaeger and Hoskins, (1966) made use of simplifying assumptions, such as plane stress, and material homogeneity to find analytical solutions at the critical location – the center of the disk (Eqn. 2.11). However, Eqn. 2.11 is derived by also assuming a 15-degree loading arc angle and that the confining pressure is evenly applied to all surfaces. The testing configuration in this work does not match these assumptions and corrections to Eqn. 2.11 are required.

These corrections account for the effective 32.1-degree loading arc angle \((2\alpha)\) on 1mm flattened Brazilian disk, and the fact that the confining pressure is not applied to the flattened surfaces; only the diametrical load \((F)\) is transmitted on those areas. By defining the flattened area as \((A_f)\), the analytical solution for the confined flattened Brazilian disk becomes Eqn. 4.1.

HOSS simulations were used to predict stresses that develop during a flattened confined Brazilian test. The stresses explicitly calculated by HOSS are compared to the analytical solutions (Eqn. 2.11 and Eqn. 4.1) in Table 4.1.1. The results in Table 4.1.1 show that for multiple loading conditions, the complicated stress states in the confined, flattened, Brazilian disk can be reasonably approximated by the analytical solution using Eqn. 4.1 when compared to the stresses explicitly calculated in HOSS. There error between the stresses calculated by HOSS and the analytical solution from Eqn. 4.1 is thought to the result of true 3D stresses analyzed under 2D plane stress conditions.
Figure 4.1.3 Loading configuration of the confined, flattened, Brazilian disk.

Eqn. 4.1a¹ \[ \sigma_1 = \sigma_{yy} = 0.9 \frac{6[F-(P_c A_f)]}{\pi D t} + P_c \]

Eqn. 4.1b¹ \[ \sigma_2 = \sigma_{zz} = P_c \]

Eqn. 4.1c¹ \[ \sigma_3 = \sigma_{xx} = 0.9 \frac{-2[F-(P_c A_f)]}{\pi D t} + P_c \]

¹The derivations for Eqn. 4.1 can be found in the Appendix (A-5).

Where, \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are the principal stress, \( F \) is the diametrical load, \( P_c \) is the confining stress, \( D \) is the diameter of the disk, \( t \) is the thickness of the disk, and \( A_f \) is the flattened area, which is loaded diametrically.
Table 4.1: Comparison of analytical and numerical solutions for maximum and minimum principal stresses at the center of a confined, flattened, Brazilian disk.

<table>
<thead>
<tr>
<th>$P_c$(MPa)</th>
<th>Numerical Solution from HOSS</th>
<th>Analytical Solution (Eqn. 2.11)</th>
<th>Corrected Analytical Solution (Eqn. 4.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\sigma_1$(MPa) 19.77</td>
<td>23.01</td>
<td>20.71</td>
</tr>
<tr>
<td></td>
<td>$\sigma_3$(MPa) -5.27</td>
<td>-7.67</td>
<td>-6.90</td>
</tr>
<tr>
<td>2.76</td>
<td>$\sigma_1$(MPa) 27.64</td>
<td>33.98</td>
<td>29.53</td>
</tr>
<tr>
<td></td>
<td>$\sigma_3$(MPa) -5.56</td>
<td>-7.65</td>
<td>-6.17</td>
</tr>
<tr>
<td>5.52</td>
<td>$\sigma_1$(MPa) 34.45</td>
<td>44.29</td>
<td>37.77</td>
</tr>
<tr>
<td></td>
<td>$\sigma_3$(MPa) -5.15</td>
<td>-7.40</td>
<td>-5.23</td>
</tr>
<tr>
<td>8.27</td>
<td>$\sigma_1$(MPa) 43.04</td>
<td>54.40</td>
<td>45.84</td>
</tr>
<tr>
<td></td>
<td>$\sigma_3$(MPa) -4.56</td>
<td>-7.11</td>
<td>-4.24</td>
</tr>
<tr>
<td>11.03</td>
<td>$\sigma_1$(MPa) 50.60</td>
<td>64.50</td>
<td>53.90</td>
</tr>
<tr>
<td></td>
<td>$\sigma_3$(MPa) -3.42</td>
<td>-6.80</td>
<td>-3.26</td>
</tr>
<tr>
<td>13.79</td>
<td>$\sigma_1$(MPa) 58.10</td>
<td>74.59</td>
<td>61.97</td>
</tr>
<tr>
<td></td>
<td>$\sigma_3$(MPa) -1.76</td>
<td>-6.49</td>
<td>-2.27</td>
</tr>
</tbody>
</table>

¹ Jaeger and Hoskins, 1966.

Note: The confining pressures ($P_c$) correspond to 0, 400, 800, 1200, 1600, and 2000 psi, respectively; the results in this work use SI units.
4.1.3 Jacket & End Cap Effects

During confined, flattened, Brazilian disk testing, a confining pressure is applied to the sample using hydraulic oil. The sample is placed between two end caps and cast in an impermeable polyurethane jacket to ensure that the confining pressure does not generate a pore pressure. The trade-off is that the jacket and end cap system affect the sample’s behavior in two ways. First, under confinement, the interfaces between the sample and end caps experience a frictional force proportional to the magnitude of the confining pressure. Second, the polyurethane jacket restricts motion and exerts an effective additional confining pressure on the sample, even if no hydrostatic confining pressure is applied.

This frictional force is reduced by placing smooth Teflon spacers between the sample and the end caps. This approach is similar to that used in conventional testing of concrete and rock samples in compression tests. The magnitude of this reduction is not evaluated, and any remaining frictional force is considered negligible.

The effect of the jacket on the sample was studied by running tests on unconfined, flattened, Brazilian disks with and without polyurethane jackets. Each test was performed five times, and the mean results show that the confinement from the polyurethane jacket increased the apparent strength of an unconfined sample by ~20% (Fig. 4.1.4). It is believed that this difference is independent of confining pressure, so the significance of the jacket effect diminishes relative to the increasing confining stress for confined samples. This is assumed because of the large difference in material properties, namely Young’s Modulus (Appendix A-7). Fig. 4.1.5 shows the force/displacement data for confined, flattened, Brazilian disk (with the jacket) at 2.76 MPa – the lowest confining pressure used in these experiments compared to the unconfined samples. The application of even this relatively low confining pressure
increases the sample’s apparent strength by 130%. Samples that are tested under higher hydrostatic confining pressures see even larger increases in apparent strength. Comparing the effect of the hydrostatic confining pressure (>130%), to the confinement from the polyurethane jacket (~20%), it is assumed that the jacket effect becomes negligible when a hydrostatic confining pressure of 2.76 MPa or greater is applied.
**Figure 4.1.4** Box-and-whisker plot of the peak strength of unconfined flattened Brazilian disks.

**Figure 4.1.5** Median force/displacement along the loaded diameter for flattened, unconfined, Brazilian disks with and without polyurethane jackets, and for a flattened, confined, Brazilian disk (with the PMC jacket), at the lowest confining pressure, 2.76 MPa.
4.1.4 Leakage Pathways

An additional area of concern with this experimental setup is the possibility of leakage pathways. The permeability measurements are made by pushing nitrogen gas across the confined, flattened, Brazilian disk, controlled with an upstream pressure regulator. A flexible latex sleeve keeps the polyurethane jacket from bonding to the circumferential surface of the sample, but it does allow for the possibility that nitrogen gas can travel around the sample, between the circumferential surface and the latex sleeve (Fig. 4.1.6).

![Diagram showing possible leakage pathways around the circumference of the sample between the sample surface and the latex sleeve.](image)

**Figure 4.1.6** Diagram showing possible leakage pathways around the circumference of the sample between the sample surface and the latex sleeve.

The possibility of leakage pathways was evaluated, and the results are presented in Fig. 4.1.7. At point A, there is a large, measurable permeability before any loading (either confining stress, or diametrical load) is applied. At point B, a hydrostatic confining pressure is applied. As the confining pressure increases the measurable permeability of the system decreases. At point C the confining pressure exceeds the maximum upstream pressure of the nitrogen gas, and the permeability of the system drops below the system limit, line \( f_g \), becoming
unmeasurable with the current system. It appears that the confining pressure forces the polyurethane jacket and latex membrane against the sample so tightly that any possible leakage pathways are closed. At time equal to zero, the hydrostatic pressure is maintained, and the confined, flattened Brazilian disk is loaded diametrically. As the diametrical load increases over the duration of the test, microfractures (measured by acoustic emissions) coalesce into a through-going macroscopic fracture network. At this point D, permeability is measurable and increases as fractures continue to coalesce. The nature of the Brazilian test tends to create a single extension fracture that dilates, and permeability becomes a function of the crack aperture. At point E the permeability starts to level off but never returns to pre-confinement levels, while staying well below the upper system limit denoted by line FG. From these results, it is concluded that the permeability of confined, flattened, Brazilian disk test is not the result of leakage pathways, and that any measured permeability flows through the sample.

![Diagram](image.png)

**Figure 4.1.7** Evaluation of possible leakage pathways for a confined, flattened, Brazilian disk test.
4.1.5 Permeability Corrections

Permeability measurements made during the confined, flattened, Brazilian disk test are believed to represent valid one-dimensional flow across the Brazilian disk. However, special consideration needs to be taken as these measurements represent only the apparent permeability of the sample. As described in section 2.3.2, the true intrinsic permeability may be misrepresented if non-linear or gas-slip conditions occur. Gas-slip, or the Klinkenberg effect, occurs when the aperture of a flow path is on the same order of magnitude as the free mean path of the fluid’s molecules. Under gas-slip conditions the apparent permeability depends on the pressure gradient across the sample, $\nabla P$. Non-linear flow due to visco-inertial conditions typically occurs when flowrates are sufficiently high to induce pressure losses due to inertial effects in addition to the pressure lost from viscous effects; visco-inertial flow also causes the apparent permeability to vary with the pressure gradient, $\nabla P$.

To identify gas-slip and/or non-linear flow, the apparent permeability of confined, flattened Brazilian disks of concrete ‘B’ was tested at three different stages of loading, pre-peak, peak, and post-peak strength. At each stage, the diametrical load was paused allowing for multiple permeability measurements. Each measurement was made at different values of mean pressure, $P_{\text{mean}}$, by controlling $P_1$, while downstream pressure, $P_2$, remained at atmospheric pressure. It is expected that a decrease in the mean pressure would cause a corresponding drop in the flowrate across the sample, and that apparent permeability would remain constant for a given loading condition (i.e., pre-peak, peak, post-peak).

The flow paths in these samples are fractures rather than connected pore structures. If the measured permeability is represented as an equivalent flow between two parallel plates, the hydraulic aperture is the distance separating those plates. In the case of a split tension test,
when a single extensional fracture is formed, the fracture size can be approximated by the hydraulic aperture. The hydraulic aperture is calculated from the apparent permeability using the cubic law (Eqn. 2.7). Analysis of the hydraulic apertures (Table 4.1.2) shows that even the smallest hydraulic apertures are orders of magnitude larger than the mean free path (MFP) of nitrogen. Thus, the gas molecules can travel through the fracture without significantly interacting with the fracture surfaces, providing evidence that no Klinkenberg corrections are necessary.

Table 4.1.2: Apparent permeability and hydraulic aperture at different regions of a confined, flattened Brazilian disk test on concrete ‘B’, compared to the mean free path (MFP) of nitrogen gas.

<table>
<thead>
<tr>
<th>Region</th>
<th>Apparent Permeability (m²)</th>
<th>Hydraulic Aperture (µm)</th>
<th>MFP nitrogen (µm)¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Peak Strength</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.71E-14</td>
<td>31.70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.37E-14</td>
<td>32.71</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.99E-14</td>
<td>32.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.89E-14</td>
<td>31.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.01E-14</td>
<td>32.17</td>
<td></td>
</tr>
<tr>
<td>Peak Strength</td>
<td></td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>1.86E-13</td>
<td>44.55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.79E-13</td>
<td>43.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.84E-13</td>
<td>44.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.88E-13</td>
<td>44.70</td>
<td></td>
</tr>
<tr>
<td>Post-Peak Strength</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.54E-13</td>
<td>59.96</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.81E-13</td>
<td>61.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.34E-13</td>
<td>63.29</td>
<td></td>
</tr>
</tbody>
</table>

¹The mean free path of nitrogen at standard temperature and atmospheric conditions.
Fig. 4.1.8 plots the apparent (or measured) permeability as a function of the inverse of the mean pressure, showing changes in permeability with respect to changes in the pressure gradient. In the pre-peak and peak regions, the apparent permeability stays nearly constant as mean pressure changes. This provides further evidence that no measurable gas-slip conditions are occurring even though the initial fracture networks developing during these regions likely have small hydraulic apertures.

![Figure 4.1.8](image)

**Figure 4.1.8** Evaluation of possible gas-slip and/or non-linear flow conditions for the confined, flattened Brazilian disk at different stages of loading. The nearly constant slopes in the pre-peak and peak regions suggest no correction are required for Klinkenberg effects. The change in apparent permeability as a function of the mean pressure in the post-peak regions indicates non-linear flow may be occurring.
In the post-peak region (Fig. 4.1.8), the change in pressure has a clear influence on the apparent permeability. By this time, the fracture aperture has dilated, and flowrates have increased; visco-inertial flow is likely occurring. Accurate identification and correction for visco-inertial flow requires multiple permeability measurements to be made while varying the mean pressure, at a single loading condition. In this test, as the load progresses, the fracture is constantly changing, and it is not possible to obtain sufficient data to develop the visco-inertial corrections. As a result, the Reynolds number is used to evaluate the possible visco-inertial flow.

The Reynolds number is a dimensionless ratio of viscous and inertial forces for fluid flow. The point at which flow transitions from viscous to inertial is marked by the critical Reynolds number. Hatambeigi et al. (2019) developed a relationship between the critical Reynolds number and the hydraulic aperture for visco-inertial flow in cement fractures. This expression is used to estimate the critical Reynolds numbers for the flow in Table 4.1.2. These critical Reynolds numbers are compared to the Reynolds numbers for flow in confined, flattened, concrete Brazilian disks in Table 4.1.3. Because the Reynolds numbers exceed the critical Reynolds number visco-inertial flow is likely occurring, even for the pre-peak and peak regions. As a result, all permeability measurements on confined, flattened Brazilian disk tests are corrected for visco-inertial flow. These corrections are detailed in appendix A-4.
Table 4.1.3: Hydraulic aperture and Reynolds numbers at different regions of a confined, flattened Brazilian disk test on concrete ‘B’, compared to critical Reynolds numbers to determine the possibility of visco-inertial flow.

<table>
<thead>
<tr>
<th>Region</th>
<th>Hydraulic Aperture (µm)</th>
<th>Reynolds Number</th>
<th>Critical Reynolds Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Peak Strength</td>
<td>31.70</td>
<td>54.29</td>
<td>11.68</td>
</tr>
<tr>
<td></td>
<td>32.71</td>
<td>52.12</td>
<td>12.17</td>
</tr>
<tr>
<td></td>
<td>32.14</td>
<td>42.35</td>
<td>11.89</td>
</tr>
<tr>
<td></td>
<td>31.98</td>
<td>34.75</td>
<td>11.81</td>
</tr>
<tr>
<td></td>
<td>32.17</td>
<td>28.23</td>
<td>11.91</td>
</tr>
<tr>
<td>Peak Strength</td>
<td>44.55</td>
<td>52.12</td>
<td>18.23</td>
</tr>
<tr>
<td></td>
<td>43.98</td>
<td>43.43</td>
<td>17.93</td>
</tr>
<tr>
<td></td>
<td>44.33</td>
<td>36.92</td>
<td>18.11</td>
</tr>
<tr>
<td></td>
<td>44.70</td>
<td>30.40</td>
<td>18.32</td>
</tr>
<tr>
<td>Post-Peak Strength</td>
<td>59.96</td>
<td>54.29</td>
<td>26.91</td>
</tr>
<tr>
<td></td>
<td>61.14</td>
<td>39.09</td>
<td>27.61</td>
</tr>
<tr>
<td></td>
<td>63.29</td>
<td>32.58</td>
<td>28.89</td>
</tr>
</tbody>
</table>
4.1.6 Summary

During the development of the experimental program for testing confined, flattened Brazilian disks, extensive analyses were performed. Experimental and numerical methods (see Appendix A-6) were used to show that flattening the 50.80 mm diameter Brazilian disks by 1 mm on top and bottom can still generate the unique stress states in a classic split tension test while minimizing local crushing effects on the diametrically loaded areas. HOSS – a state-of-the-art FDEM program – was used to verify the analytical solutions for the principal stresses at the center of the confined, flattened, Brazilian disk. An impermeable polyurethane jacket and end cap system is used to isolate the confined Brazilian disk allowing permeability to be measured concurrently during the experiments. The effects of the jacket and end caps on the apparent strength of the samples was minimized by trial and error, and any residual effect is assumed negligible for confined tests. The jacket system was thoroughly tested for leaks to ensure that only the permeability of the sample is measured. Finally, the assumptions used to calculate the apparent permeability were investigated. It was found that corrections for visco-inertial flow are required. Acoustic emission sensors were placed on the outside of the pressure vessel and passively monitor AE hits for the duration of the tests; they have no impact on the testing conditions. Altogether, these analyses indicate that confined, flattened Brazilian disks can be successfully tested.
4.2 Experimental Results

During the development of the novel experimental program, concrete specimens were used as cost-effective, representative samples of a brittle geo-material. Two series of confined, flattened Brazilian disk tests were performed on the concrete specimens, at confining pressures equal to 2.76, 5.52, 8.27, 11.03, and 13.79 MPa. These stresses are equal to 400, 800, 1200, 1600, and 2000 psi, respectively, and are presented hereafter in SI units. These confining stresses were chosen to approximate real-world conditions for deep wellbore excavations; corresponding to lithostatic stress states at depths up to 700 meters below ground surface depending on rock type. Once the testing procedure was determined, one series of experiments was performed on tuff specimens at 2.76, 8.27, and 13.79 MPa. The full testing matrix is shown in Table 4.2.1 with important differences noted.

Table 4.2.1: Testing matrix for confined, flattened Brazilian disks on brittle geo-materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Confining Pressure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Concrete A¹</td>
<td>5x</td>
</tr>
<tr>
<td>Concrete B²</td>
<td>5x</td>
</tr>
<tr>
<td>Tuff ³</td>
<td>1x</td>
</tr>
</tbody>
</table>

¹ Tests on Concrete A used a permeability system with a more limited range [E-15,E-11] m² vs. [E-17,E-10] m² for remaining tests. The load rate was also higher, 3E-6 m/s vs. 1E-6 m/s. Thus, permeability measurements were made by pausing the diametrical load every 60 seconds.

² Concrete B cured longer and exhibited higher unconfined strengths than Concrete A. Follows the procedure outlined in Chapter III.

³ The tuff sample were provided in limited quantities. Follows the procedure outlined in Chapter III.
4.2.1 Peak Stress Data

The confined, flattened Brazilian disk test is a true triaxial test that creates three unequal principal stresses such that, $\sigma_1 > \sigma_2 > \sigma_3$, with compression being positive. The unconfined case ($\sigma_2 = 0$) generates a tensile minimum principal stress that should result in failure by an extensional fracture along the loaded diameter. Even as the confining pressure increases the minimum principal stress continues, for a period, to be tensile. This type of failure is referred to as confined extension and spans the region between uniaxial tension and triaxial compression (see Section 2.1.4). Table 4.2.2 provides the peak force, maximum principal stress, $\sigma_1$, and the minimum principal stress, $\sigma_3$ at failure for each testing condition. For the unconfined tests, average values are used. Note that negative values indicate tension.
Table 4.2.2: Minimum and maximum principal stresses at failure (MPa) for confined, flattened Brazilian disks on brittle geo-materials. Calculated from the peak force using Eqn. 4.1.

<table>
<thead>
<tr>
<th>Confining Pressure (MPa)</th>
<th>Concrete A</th>
<th>Concrete B</th>
<th>Tuff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Force (kN)</td>
<td>4.95</td>
<td>8.66</td>
<td>8.46</td>
</tr>
<tr>
<td>$\sigma_1$ (MPa)</td>
<td>6.59</td>
<td>11.54</td>
<td>11.26</td>
</tr>
<tr>
<td>$\sigma_3$ (MPa)</td>
<td>-2.20</td>
<td>-3.85</td>
<td>-3.76</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>23.80</td>
<td>21.95</td>
</tr>
<tr>
<td>$\sigma_1$ (MPa)</td>
<td>31.03</td>
<td>33.15</td>
<td>30.68</td>
</tr>
<tr>
<td>$\sigma_3$ (MPa)</td>
<td>-6.67</td>
<td>-7.38</td>
<td>-6.55</td>
</tr>
<tr>
<td>2.76</td>
<td></td>
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<td></td>
</tr>
<tr>
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<td>35.10</td>
<td>-</td>
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<tr>
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<td>49.62</td>
<td>-</td>
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<tr>
<td>$\sigma_3$ (MPa)</td>
<td>-8.22</td>
<td>-9.19</td>
<td>-</td>
</tr>
<tr>
<td>5.52</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Force (kN)</td>
<td>35.90</td>
<td>46.70</td>
<td>35.40</td>
</tr>
<tr>
<td>$\sigma_1$ (MPa)</td>
<td>52.17</td>
<td>66.56</td>
<td>51.48</td>
</tr>
<tr>
<td>$\sigma_3$ (MPa)</td>
<td>-6.36</td>
<td>-11.15</td>
<td>-6.13</td>
</tr>
<tr>
<td>8.27</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Force (kN)</td>
<td>43.00</td>
<td>37.60</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_1$ (MPa)</td>
<td>63.08</td>
<td>55.90</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_3$ (MPa)</td>
<td>-6.32</td>
<td>-3.92</td>
<td>-</td>
</tr>
<tr>
<td>11.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Force (kN)</td>
<td>47.30</td>
<td>40.60</td>
<td>38.70</td>
</tr>
<tr>
<td>$\sigma_1$ (MPa)</td>
<td>70.25</td>
<td>61.28</td>
<td>58.72</td>
</tr>
<tr>
<td>$\sigma_3$ (MPa)</td>
<td>-5.03</td>
<td>-2.04</td>
<td>-1.19</td>
</tr>
</tbody>
</table>
Figures 4.2.1, 4.2.2, and 4.2.3 plot the results from Table 4.2.2. in principal stress space. It is difficult to draw definitive conclusions from these results because of the limited number of data points. Existing data for geo-materials under these conditions – having a combination of tensile and compressive stresses at failure – is limited but tends to show the same inconclusive behavior in terms of minimum principal stress (Brace, 1964; Ramsey and Chester, 2004; Lan et al., 2019). Seen in Fig. 4.2.1, increasing confining pressure causes an initial increase in tensile strength! This is not uncommon for geo-materials under these stress conditions, but the effect is pronounced.

Unfortunately, confining pressures were not high enough to cause the minimum principal stress to reach the compressive region, and the minimum principal stress remains tensile for these experimental conditions. However, $\sigma_3$ for both concrete A and concrete B is starting to tend towards compression at the higher confining pressures. Fig. 4.2.2 clearly shows the increase in the maximum principal stress (directly proportional to the strength) as the confining pressure increases. Fig. 4.2.3 shows the more common $\sigma_1, \sigma_3$ stress space. All test results fall in the region between uniaxial tension and uniaxial compression, and as the confining pressure increases the stress states appear to be moving towards the expected triaxial compression region.
Figure 4.2.1 Minimum principal stress, $\sigma_3$, as a function of confining pressure (the intermediate principal stress, $\sigma_2$).

Figure 4.2.2 Maximum principal stress, $\sigma_1$, as a function of confining pressure (the intermediate principal stress, $\sigma_2$).
Figure 4.2.3 Minimum-maximum stress state ($\sigma_1$, $\sigma_3$) for confined, flattened Brazilian disk tests at peak load.
4.2.2 Fracture Patterns

The results for the confined, flattened Brazilian disks produce no consistent fracture orientations with respect to the confining stress. In the confined tension region, under both compressive and tensile stresses, hybrid fractures are expected in homogenous materials (Ramsey and Chester, 2004; Bobich et al., 2005). Hybrid fractures can be described as primary tensile fractures inclined less than 20 degrees from the major principal stress axis.

Figure 4.2.43 shows the fractured Brazilian disks for the series of tests on concrete ‘B.’ While some fracture patterns meet the requirements for hybrid fractures – a single fracture formed inclined less than twenty degrees from the major principal stress axis – others often form ‘wedges’ near the contact points.

Figure 4.2.4 Fracture patterns for concrete ‘B’ confined, flattened Brazilian disks. Confining pressure increases from left to right: 2.76, 5.52, 8.27, 11.03, 13.79 MPa, respectively.
It is reasoned that in this work, the formation of wedged fractures and inconsistencies in fracture pattern orientations are due to variations in the stress field of the confined Brazilian disks caused by material heterogeneity. The large aggregate size relative to the sample radius, 4.75 mm vs. 25.40 mm in concrete, makes it difficult to assume material homogeneity. This is exhibited more clearly in the tuff samples, which have lithic clasts nearly as large as the radius of the Brazilian disks.

Figure 4.2.5 shows the fractures formed in confined, flattened, tuff Brazilian disks under increasing confining pressure. In all three cases, parallel fractures form in addition to the primary extensional fracture. They can be difficult to see at the lowest confining stress but become more pronounced as the confining stress increases; indicating that 1) the tuff is a weaker material than the concrete, and 2) that the tuff Brazilian disks do not produce uniform tensile stresses across the center of the sample – likely the result of material heterogeneity.

Figure 4.2.5a Flattened, tuff Brazilian disks, confined at 2.76 MPa.
Figure 4.2.5b Flattened, tuff Brazilian disks, confined at 8.27 MPa.

Figure 4.2.5c Flattened, tuff Brazilian disks, confined at 13.79 MPa.
4.2.3 Axial Force/Displacement Data

Figures 4.2.5 shows the force-displacement curves for the test series on concrete ‘B’. When compared to the unconfined test, the confined tests exhibit much higher peak strengths and peak displacements. The increase in peak strength as a function of confinement is consistent with literature and Fig. 4.2.2. However, the apparent increase in displacement is an artifact of both the mechanical seating of the loading frame, and the displacement of the impermeable PMC jacket before the piston makes contact with the sample! Future work that investigates the material response of the Brazilian disks should correct the toe region to remove the seating in accordance with ASTM practices (Fig. 4.2.6). However, for this work, the force/displacement curves are presented in their entirety. Figures 4.2.7 – 4.2.9 show the full force/displacement curves for the series of confined, flattened Brazilian disk tests on concrete ‘A’, concrete ‘B’, and tuff, respectively.

![Typical ASTM toe compensation correction](Figure 4.2.6)

*Figure 4.2.6* Typical ASTM toe compensation correction (ASTM, 2015).
Figure 4.2.7 Force/displacement along the loaded diameter for the series of confined, flattened Brazilian disk tests on concrete ‘A’. Note the different testing procedure (i.e. pausing) caused rebounding; seen here as drops in axial force.

Figure 4.2.8 Force/displacement along the loaded diameter for the series of confined, flattened Brazilian disk tests on concrete ‘B’.

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Although all the confined, flattened Brazilian disk tests in this work fall in the confined extension region, these results are too limited to provide definitive statistical conclusions. Other studies on brittle geo-materials that fail in the transition from tensile to compressive failure have shown that failure mechanism exhibit both shear and extensional characteristics (Brace, 1964; Ramsey and Chester, 2004; Bobich et al., 2005). The dominant failure mechanism is not fully understood, but further experiments in this region, using confined Brazilian disk, can provide key data for a better understanding of rock damage and fracture. While the respective series of confined, flattened Brazilian disk experiments in this work are too few in number to collectively analyze fracture mechanisms in the confined tension region, it is possible to establish relationships between damage and permeability for individual tests.

Figure 4.2.9 Force/displacement along the loaded diameter for the series of confined, flattened Brazilian disk tests on tuff.
4.2.4 Individual Data Sets

The experimental methodology for the confined, flattened Brazilian disk tests presented in Chapter III produces three distinct data sets:

1. Force vs. displacement for the diametrical load on the specimen
2. Permeability vs. time
3. Acoustic emissions vs. time

Additionally, the hydrostatic confining pressure can be controlled for the duration of the test. By considering the confining pressure and the diametrical load, the stress state at the center of the sample can be calculated at any time using Eqn. 4.1. The diametrical load is applied at a constant displacement rate. As such, the displacement can be translated into time and all three data sets can be synchronized. Results from individual tests are consistent and data from tests at 5.52 MPa for concrete and 8.27 MPa for tuff are shown below as a representation of typical results. Full results from the other tests are presented in the appendices.

Figure 4.2.10a shows the diametrical force and acoustic emission data synchronized as functions of time for the duration of a flattened Brazilian test confined at 5.52 MPa on a concrete B sample. Confining pressure is maintained with the manual release of a bleed valve. This results in the small, but regular, drops in diametrical force. The initial portion of the test – from time 0 to ~2000 sec – is primarily mechanical seating as the diametrical load is transferred through the polyurethane jacket before making contact with the sample. After that time (2000 sec) the load transitions and is carried increasingly by the sample. From there, the typical linear elastic range, peak force, and post-peak region can be identified. Because the test is ended when the polyurethane jacket fails (resulting in an immediate loss of confining
pressure), the full post-peak behavior is not captured in a typical test. The cumulative AE counts are consistent with expected results. They are limited before the linear elastic range, because the load is not high enough to induce plastic deformation, even at the microscopic scale. During the linear elastic range, microcracks are initiated and the AE counts grow steadily. The growth becomes exponential just before the post-peak region when microcracks become unstable and interact. Because the confining pressure holds the sample together in the post-peak region, unstable microcracks are able to continue interacting and the cumulative AE counts increases an order of magnitude between the peak time and the end of the test.

Fig. 4.2.10b shows the same diametrical force data synchronized with the discrete concurrent permeability measurements. The intrinsic permeability of intact concrete is believed to be on the order of 1E-18 m² (Rusch, 2018). This value below the resolution of the permeability system in these tests. As a result, the permeability is ‘measured’ at the lowest system limit until a microcrack fracture network develops that substantially increases the permeability of the sample. As seen in Fig. 4.2.10b (and other tests in this work) this is the case during the linear elastic region. The apparent permeability is corrected for visco-inertial flow (section 4.1.6) and the difference is noticeable, but minimal on the log scale. The permeability changes show a clear distinction before and after the peak strength, and this will be examined in more detail using the relationship shown in Fig. 4.2.10c.

Fig. 4.2.10c combines the AE data from 4.2.10a and the permeability data from 4.2.910. In the subsequent sections, the relationship between the two data sets is investigated by using the acoustic emission data to quantify damage. Fig. 4.2.11 shows that the results for individual confined, flattened Brazilian disk tests are consistent for different materials and at different confining pressures by presenting the results from a tuff sample and 8.27 MPa confinement.
Figure 4.2.10a Synchronized data for a flattened Brazilian disk test on concrete confined at 5.52 MPa. Axial force vs. time (using constant displacement rate loading) with cumulative acoustic emission counts.

Figure 4.2.10b Synchronized data for a flattened Brazilian disk test on concrete confined at 5.52 MPa. Axial force vs. time with discrete apparent permeability;
Figure 4.2.10 Synchronized data for a flattened Brazilian disk test on concrete confined at 5.52 MPa. Cumulative acoustic emission counts vs time with discrete apparent permeability. Note: The green dashed lines represent the measurable system limits for the permeability system, and the true intrinsic permeability of intact samples is below this line.
Figure 4.2.11a Synchronized data for a flattened Brazilian disk test on tuff confined at 8.27 MPa. Axial force vs. time (using constant displacement rate loading) with cumulative acoustic emission counts.

Figure 4.2.11b Synchronized data for a flattened Brazilian disk test on tuff confined at 8.27 MPa. Axial force vs time with discrete apparent permeability;
Figure 4.2.11c Synchronized data for a flattened Brazilian disk test on tuff at confined at 8.27 MPa. Cumulative acoustic emission counts vs. time with discrete apparent permeability. Note: The green dashed lines represent the measurable system limits for the permeability system, and the true intrinsic permeability of intact samples is below this line.
4.2.5 Damage and Permeability

In this work damage is quantified solely using acoustic emission measurements. Acoustic emission monitoring of rock failure has been well studied in recent years, and numerous techniques are found in literature that quantify damage evolution from acoustic emissions. Several of these techniques are applied in this study and can be categorized as:

a. The determination of damage thresholds.

b. The designation and measure of damage evolution using a damage parameter.

c. Statistical analysis of acoustic emission waveforms.

An omission from the above categories is the source location of acoustic emission hits. As described in section 3.1.2, the acoustic emission sensors are placed on the outer surface of the pressure vessel. This means that any acoustic emission event originating in the sample must pass through the sample itself, the polyurethane jacket, the confining hydraulic fluid, and the steel pressure vessel before reaching the AE sensor. As a result, AE waveforms travel through an effective anisotropic material that will vary the wave speed. Error induced from the variable wave speed means that triangulating hits based on arrival times invalidates attempts at source location. While further study may remedy this issue, it falls outside the scope of this work.

Acoustic emission sensors placed on the pressure vessel continuously monitor elastic stress waves generated by microscopic plastic deformation. This is the only data obtained during these experiments that can be used to interpret damage. No lateral or volumetric stress-strain data was recorded. Axial stress-strain data was recorded, but that captured the response of the entire experimental configuration, including the loading frame, rather than the direct
contact at the sample’s interface. This makes it difficult to use any digital or strain-based methods to quantify damage, such as digital imaging correlation (DIC) (Rusch, 2018), the lateral strain response method (LSR) (Nicksiar and Martin, 2012), the axial strain method (ASM) (Eberhardt, 1999), or the damage inelastic strain method (DISM) (Martin, 1993).

4.2.5.1 Determination of damage thresholds by acoustic emissions

Eberhardt (1998) showed that damage thresholds could be reliably determined from AE data. Further research over the past twenty years has shown that AE data is unique from test to test. Any changes in AE equipment or testing conditions provide additional variability and have made it challenging for a standardized method to emerge that can be used to identify damage thresholds from AE sources. There is also confusion regarding the names and descriptions of damage thresholds.

The common damage thresholds from literature, as described in section 2.2.1, are crack closure, crack initiation, crack damage, and crack coalescence. The problem is that the progression of damage is scale dependent and these thresholds don’t refer to the same ‘crack’. Crack closure describes the compression of pre-existing microscopic flaws, such as pore spaces. Crack initiation refers to the formation of new microscopic cracks originating from the tips of existing microscopic flaws. Crack damage represents the transition from stable to unstable crack growth – where the cracks themselves likely transition from microscopic to macroscopic. Finally, crack coalescence represents the formation of a full macroscopic failure plane. In this work, damage is studied at the microscopic scale using acoustic emissions and correlated to permeability enhancements. With this in mind, the author recommends that the common damage thresholds be redefined considering a ‘crack’ to represent the macroscopic scale. As a result, Eberhardt’s crack initiation will be referred to as damage initiation,
representing the initiation of microscopic damage originating from pre-existing stressed micro-flaws. Crack damage, or the transition from stable to unstable microscopic crack growth, becomes damage coalescence. Meanwhile, the culmination of unstable crack growth into a full macroscopic failure plane becomes crack formation. In this work the crack closure threshold is not considered because it elicits no measurable change in permeability. However, it is noted that for more porous materials, the compression of pre-existing micro-flaws would likely cause a drop in the apparent permeability. To summarize the common damage thresholds from literature identified in this work are redefined and listed below:

- Crack Initiation becomes **Damage Initiation (DI)**: represents the formation and stable growth of micro-cracks from pre-existing microscopic flaws.
- Crack Damage becomes **Damage Coalescence (DS)**: represents the transition from stable microscopic crack growth to unstable crack growth.
- Crack Coalescence becomes **Crack Formation (CI)**: represents the initiation of the process of a coalescence of a series of unstable macro-cracks into a complete macroscopic failure plane.

Current methods to determine damage thresholds plot pre-peak cumulative AE counts or energy as a function of the axial stress (Zhao, 2013; Chang and Lee, 2004). Inflection points on the resulting curve (such as points where the curve ‘increases significantly for the first time’) are then used to signify different damage thresholds. While generally consistent, it is commonly agreed that this method is subjective. In this work a more systematic approach was
used by applying Eberhardt’s moving point regression technique to the cumulative AE count/axial stress curve.

The moving point regression technique uses the first derivative of a given data set to highlight any slope changes (Eberhardt, 1998). Rather than finding the slope between two consecutive points, a linear line is fit to a ‘sliding window’ of $n$ data points. The ‘sliding window’ is moved through the x-y data providing a continuous derivative for successive windows. Eberhardt used this technique on axial stress-strain data to produce the moving point averages in the Young’s modulus. In this work the moving point regression is applied to the axial stress vs. pre-peak cumulative AE counts curve producing a linear regression that represents the change in cumulative AE counts with respect to an increase in axial stress. Fig. 4.2.12 shows the linear regression for the cumulative AE counts curve. This figure highlights the larger initial slope when loading begins (likely the result of seating and/or microtears in the polyurethane jacket), the constant increase in AE counts as the sample’s load increases (likely the region of stable crack growth), and finally the exponential increase in AE counts as the cracks become unstable and the sample approaches the peak strength.
Figure 4.2.12 Cumulative AE counts vs. Axial Stress with the first linear regression of AE Counts and Axial Stress.
The region of stable crack growth is defined by damage initiation that only increases with an increase in the applied load. It is hypothesized that the region of constant growth in cumulative AE counts with respect to axial stress represents this region (Fig. 4.2.12). The beginning of stable crack growth is marked by the damage initiation damage threshold (DI) and ended by the damage coalescence threshold (DS). At the damage coalescence threshold, crack growth is unstable, and a corresponding increase in cumulative AE counts is expected. When cumulative AE counts start to increase exponentially there is a marked increase in the slope of the linear regression curve that is used to signify the crack formation damage threshold (CI).

To calculate the damage initiation, damage coalescence, and crack formation thresholds a second-order linear regression is applied to the cumulative AE counts/axial stress curve. The second linear regression is equivalent to a second-order derivative and the region of constant slope in the first regression (representing the region of stable crack growth) can be identified in the second regression when set equal to zero. The initial and final points in that region mark the axial stress thresholds for damage initiation and damage coalescence. The crack formation threshold is identified as the point in the first linear regression curve when the slope increases an order of magnitude (i.e. from 10 to 100 counts/MPa). These thresholds can then be plotted on the axial stress-strain curve as seen in Fig. 4.2.13. Normalizing the damage thresholds to the peak stress, Table 4.2.3 shows that the damage thresholds identified with this method are consistent with expected ranges from literature.
Figure 4.2.13 Damage thresholds shown on the pre-peak diametric stress-strain curve for flattened, concrete ‘B’ Brazilian disk confined at 5.52 MPa.
Table 4.2.3: Damage Thresholds with respect to peak strength.

<table>
<thead>
<tr>
<th>Material</th>
<th>$P_c$ (MPa)</th>
<th>$\sigma_{dl}/\sigma_{peak}$</th>
<th>$\sigma_{ds}/\sigma_{peak}$</th>
<th>$\sigma_{cl}/\sigma_{peak}$</th>
</tr>
</thead>
<tbody>
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<td>Concrete A</td>
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<td>0.34</td>
<td>0.58</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>5.52</td>
<td>0.24</td>
<td>0.94</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>8.27</td>
<td>0.43</td>
<td>0.69</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>11.03</td>
<td>0.33</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>13.79</td>
<td>0.23</td>
<td>0.74</td>
<td>0.83</td>
</tr>
<tr>
<td>Concrete B</td>
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<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>5.52</td>
<td>0.34</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>8.27</td>
<td>0.36</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>11.03</td>
<td>0.59</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>13.79</td>
<td>0.43</td>
<td>0.84</td>
<td>0.88</td>
</tr>
<tr>
<td>Tuff</td>
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<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>8.27</td>
<td>0.38</td>
<td>0.88</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>13.79</td>
<td>0.32</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>Dolomite (Hatzor and Palchik, 1997)</td>
<td>-</td>
<td>0.6</td>
<td>1.0</td>
<td>N/A</td>
</tr>
<tr>
<td>Granite (Eberhardt et al., 1998)</td>
<td>-</td>
<td>0.39</td>
<td>0.76</td>
<td>N/A</td>
</tr>
<tr>
<td>Granite (Change and Lee, 2004)</td>
<td>-</td>
<td>0.40</td>
<td>0.84</td>
<td>N/A</td>
</tr>
<tr>
<td>Granite (Zhao et al., 2013)</td>
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<td>0.80</td>
<td>N/A</td>
</tr>
<tr>
<td>Diorite (Hidalgo and Nordlund, 2013)</td>
<td>-</td>
<td>0.51</td>
<td>0.90</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Fig. 4.2.14 combines the diametrical force vs. diametrical displacement data with the discretely measured permeability for the flattened concrete Brazilian disk test at 8.27 MPa confinement. It also denotes three damage thresholds as determined by acoustic emission counts. It can be seen in Fig. 4.2.14 that damage initiation (DI) begins early in the linear-elastic phase and elicits no measurable change in permeability. It is not until cracks density (or counts) increases sufficiently to form an interconnected microscopic fracture network that a measurable permeability can be detected with the current system. As is expected for a brittle geo-material, macroscopic crack formation (CI) closely follows unstable microscopic crack growth (DS) and the two thresholds consistently occur just before the peak strength of the specimen is reached. The post-peak region of the force/displacement curve also coincides with an abrupt change in permeability. The nature of the Brazilian disk test creates a single extension fracture, even under confining pressure. In the post-peak region, the single macroscopic extension fracture dilates as the sample experiences additional displacement-based loading. As a result, permeability likely becomes a function of the equivalent hydraulic aperture, and changes less severely with respect to diametrical displacement. This is in stark contrast to the pre-peak region, where as the microscopic fracture network is developing, permeability changes several orders of magnitude, initially starting well below the system limits.
Figure 4.2.14 Diametrical force vs. diametrical displacement with discrete apparent permeability measurements, and damage thresholds as determined by acoustic emissions for a flattened, concrete ‘B’ Brazilian disk confined at 5.52 MPa.
4.2.5.2 The designation and measure of damage evolution using a damage parameter

Previous studies have concluded that the energy released during AE events is the most accurate measure of microcracking (Kim, 2015; Bruning et al., 2018). Cumulative AE energy is used to measure damage evolution $D$, as shown in Eqn. 4.2, where $\Omega$ is the cumulative AE energy at a given load, and $\Omega_{\text{peak}}$ is the cumulative AE energy at the peak load (Bruning et al., 2018).

\[
\text{Eqn. 4.2} \quad D = \frac{\Omega}{\Omega_{\text{peak}}}
\]

If cumulative AE energy is limited to $0 < \Omega < \Omega_{\text{peak}}$, then damage ranges from 0 to 1 where 0 signifies the initial undamaged state of the sample and 1 represents the full macroscopic failure at the peak load. The key difference between Eqn. 4.2 and the relationship used by Bruning is that in Eqn. 4.2 the maximum cumulative AE energy is represented by the amount at the peak load, $\Omega_{\text{peak}}$, rather than the total cumulative AE energy at the end of the test, $\Omega_{\text{total}}$. This is done because damage should be considered a scale-dependent property. If $\Omega$ represents the energy released at the microscopic scale, then energy released at the macroscopic scale (i.e. frictional sliding of an existing macro fracture) should not be represented with the same damage parameter. All post-peak AE energy in these tests is considered the results of macroscopic frictional sliding. Therefore, the damage parameter, $D$, only considers cumulative AE energy up to the peak load, when cohesive bonds are failing and resulting in microcracks. However, to provide a continuous measure of damage that shows the transition from microscopic to macroscopic damage, Fig. 4.2.15 plots the permeability as a function of normalized cumulative AE energy.
The transition from microscopic to macroscopic damage is marked in Fig. 4.2.15 as the ‘peak strength.’ This line corresponds to the peak load and the apparent permeability transitions suddenly at this point. Before the peak strength, changes of permeability are several orders of magnitude and is believed to be a function of the fracture network developing as microcracks coalesce. After the peak strength, permeability likely becomes a function of the aperture of the single fracture, as the fully formed macroscopic fracture dilates under increasing diametrical displacement. Figures 4.2.16 to 4.2.18 illustrate this relationship is the pre-peak and post-peak regions respectively.

**Figure 4.2.15** Apparent permeability as a function of the normalized cumulative acoustic emission energy for a flattened concrete Brazilian disk test at 5.52 MPa confinement. The ‘peak strength’ denotes the normalized cumulative AE energy at the peak strength of the specimen, marking an abrupt change in slope.
Fig. 4.2.16 plots the permeability, measured before the peak strength of the sample is reached, as a function of the damage parameter described by Eqn. 4.2. The damage thresholds corresponding to damage initiation and stable micro-crack growth (DI), damage coalescence and unstable micro-crack growth (DS), and macroscopic crack formation (CI) are also shown. For materials with low intact permeability, the initiation of microcracks alone does not cause a measured change in permeability. As new micro-cracks continue to initiate (between DI and DS) the crack density increases, and cracks begin to interact. This interaction of stably growing microfractures (likely initiating at optimally orientated grain boundaries) forms a microfracture network, shown in Figure 4.2.17. It seems reasonable that this microfracture network controls the changes in measured permeability. Because this happens in the stable crack growth region (before the DS threshold), it is clear that this microfracture network can be developed, resulting in changes in permeability several orders of magnitude in size without compromising the integrity of the sample with respect to the peak strength.
Figure 4.2.16 Permeability as a function of the damage parameter as measure by AE energy with damage thresholds normalized to the peak strength for a confined, flattened Brazilian disk (concrete B, 5.52 MPa).

Figure 4.2.17 Schematic representation of the progression of pre-peak damage showing a) random pre-existing micro-flaws; b) damage initiation and the formation of new isolated micro-cracks; c) stable micro-crack growth, and the formation of an isolated through going fracture network causing changes in permeability; d) the continued development of a microscopic fracture network, increases in permeability, and damage coalescence.
Shortly after the damage coalescence threshold (DS), a macroscopic fracture plane forms, represented by crack initiation (CI) and the sample reaches its peak strength. It is hypothesized that the extensional fracture formed during a confined, flattened Brazilian disk test will dilate if the diametrical displacement continues post-peak. In this region, AE hits are primarily the result of frictional sliding as points along the extension fracture and no longer a measure of microscopic damage. As a result, it is more appropriate to express permeability as an equivalent hydraulic aperture. Fig. 4.2.18 plots the hydraulic aperture for a flattened, concrete Brazilian disk confined at 5.52 MPa as a function of the diametrical displacement, showing behavior consistent with a dilating extensional fracture in the post-peak region.

![Estimated hydraulic aperture as a function of diametrical displacement in the post-peak region of a confined, flattened Brazilian disk (concrete B, 5.52 MPa). Note that the hydraulic apertures are much larger than the mean free path of nitrogen gas; thus, Klinkenberg corrections are unnecessary.](image)

Figure 4.2.18
4.2.5.3 Statistical Analysis of AE Waveforms

The reason that the peak strength of the sample occurs at such a low relative value of the total cumulative AE energy (Fig. 4.2.14) is that the acoustic emission behavior of the test changes dramatically in the post-peak region. A closer examination of Fig. 4.2.9a reveals that the cumulative acoustic emission counts at the peak strength is on the order of several thousand counts. However, after the peak strength, the AE counts increase exponentially and by the end of the test, the cumulative counts are on the order of tens-of-thousands. The previously hypothesis states that the pre-peak AE counts are primarily capturing the failure of cohesive bonds and the development of new microfractures, while the post-peak AE counts are primarily the result of shear sliding at the interface of the newly formed macroscopic extension fracture. This extension fracture would normally result in catastrophic failure, but the confining pressure effectively holds the sample together allowing it to take post-peak load and a result, post-peak AE hits. Table 4.2.4 provides statistical analysis of the acoustic emission events before and after the peak strength for the flattened Brazilian disk test at 5.52 MPa confinement. The results from Table 4.2.4 show that the post-peak acoustic emission events exhibit significantly different characteristics than the pre-peak events. The reduction in average event amplitude, event duration, and event energy all support the hypothesis that the majority of post-peak AE events are the result of lower energy stress waves; the kind likely to be produced by shear sliding rather than cohesive bonds failing (Sun et al., 1991).

Table 4.2.4: Statistical Analysis of acoustic emission events for an 8.27 MPa confined, flattened Brazilian disk test.

<table>
<thead>
<tr>
<th></th>
<th>Total Counts</th>
<th>Mean Max Amplitude (dB)</th>
<th>Mean Event Duration (µs)</th>
<th>Mean Event Energy (dB s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Peak</td>
<td>2691</td>
<td>63.56</td>
<td>530</td>
<td>52.47</td>
</tr>
<tr>
<td>Post-Peak</td>
<td>19229</td>
<td>61.34</td>
<td>380</td>
<td>25.15</td>
</tr>
</tbody>
</table>
V. CONCLUSION

5.1 Summary

This project developed a novel experimental program to test brittle geo-materials using confined, flattened, Brazilian disks. The experiments quantify damage using acoustic emission sensors while making concurrent permeability measurements to correlate damage and permeability. A hydrostatic confining pressure allows for true triaxial stress states to develop that can represent *in situ* lithostatic conditions. Extensive analysis of the experimental program was performed to ensure the validity of the test results using both numerical and experimental methods. Concrete and tuff specimens were tested, and the experimental approach is applicable to other brittle geo-materials. Individual data sets for both concrete and tuff captured unique data that will be used to aid the development of hydromechanical models in the Hybrid Optimization Software Suite. In the era of supercomputers, these models will allow researchers to study the complex behavior of brittle rock fracture and subsequent enhancements in material properties, namely permeability and damage.

5.2 Findings and Applications

The combination of numerical and experimental methods to study the ‘flattened’ Brazilian disk showed that flattening is an acceptable modification to the standard Brazilian disk test which can help to alleviate the common problem of localized crushing due to stress concentration near the loading points. Reducing stress concentrations allows greater stresses to develop throughout the sample and causes the critical stress at the center of the sample to induce tensile failure even under confined conditions.
In this work, the experimental program always induced a (negative) tensile stress in the confined, flattened Brazilian disks even under moderate hydrostatic confining pressures. These so-called, confined extension tests create true triaxial conditions that fail the sample somewhere between uniaxial tension and uniaxial compression. The triaxial conditions may be relevant to \textit{in situ} stresses, particularly those found in deep wellbore excavations where rock spalling is a common problem. Previous researchers, using the dog-bone methodology (Brace, 1964; Ramsey and Chester 2004; Bobich, 2005; Lan et al., 2018) have shown that the behavior of rock fracture in confined extension regions is complicated. Hybrid fracture, which exhibit, both extensional and shear fracture characteristics, can occur in this region and the rock strength is not well predicted by existing failure criteria. Unlike the dog-bone tests, confined Brazilian disks allow the user to control and study the intermediate principal stress, which is known to affect rock strength. While the results from these experiments are too limited to draw meaningful conclusions, further studying the effects of the intermediate principal stress in the confined extension region may provide data crucial to developing accurate, comprehensive, failure criteria that capture the initial increases in tensile strengths under confinement.

In addition to studying the brittle failure of confined geo-materials, this work also quantified damage progression before and after the peak strength. Using cumulative acoustic emission counts combined with Eberhardt’s liner regression technique, damage thresholds described as damage initiation, damage coalescence, and crack formation could be identified. Cumulative acoustic emission energy has been shown to be the most accurate measure of damage and it was used to formulate a damage parameter that quantified pre-peak damage evolution. Statistical analysis of acoustic emission waveforms showed that the post-peak AE hits have lower energy but much higher volume. This indicates that a microscopic damage
parameter is unable to comprehensively track macroscopic damage in the post-peak region. More importantly, the pre-peak damage parameter and damage thresholds were compared to changes in permeability as damage progressed.

These tests were able to capture permeability enhancements several orders of magnitude in size well before the failure at peak strength induced a macroscopic fracture (Fig. 5.2.1; Fig. 5.2.3). This is in stark contrast to previous work using unconfined Brazilian disks by Rusch (2018) and Leyba (2018) who measured no permeability changes until after macroscopic fractures had formed. This indicates that the confined Brazilian disk can capture unique permeability changes as microscopic fracture networks grow in the pre-peak region. The quantification of damage provides further evidence for the validity of significant pre-peak permeability changes because damage initiation and stable micro-crack growth consistently occur well before the peak strength of the sample is reached. The damage initiation threshold is also consistently identified before measurable changes in permeability, from which it follows intuitively that microfractures have to form a connected, through going flow path before effecting the permeability. Furthermore, the transition from microscopic damage to macroscopic failure marks a clear and consistent transition in the behavior of permeability enhancement. Once a macroscopic fracture has formed, permeability likely becomes a function of that fracture dilation, as little to no new fracture networks are developing. The permeability data from these experiments captures the transition from microscopic to macroscopic damage. This data will allow researchers to study not just fracture mechanics, but also issues involving fluid transport; particularly those of interest to national security such as the migration of radioactive gases from underground nuclear explosions and nuclear waste repositories.
Figure 5.2.1 Permeability as a function of the damage parameter as measure by AE energy, with mean damage thresholds normalized to the peak strength for the series of confined, flattened Brazilin disks of concrete B. Note: the dashed lines are hypothesized changes from the initial intact permeability.

Figure 5.2.2 Permeability as a function damage, for the series of confined, flattened Brazilin disks on concrete B (rescaled to the area of interest).
Figure 5.2.3 Permeability as a function of the damage parameter as measure by AE energy, with mean damage thresholds normalized to the peak strength for the series of confined, flattened Brazilin disks of tuff. Note: permeability for tuff at 13.79 MPa was not steady-state.

Figure 5.2.4 Permeability as a function of damage, for the series of confined, flattened Brazilin disks on tuff (rescaled to the area of interest). Note: permeability for tuff at 13.79 MPa was not steady-state.
5.3 Future Work

The confined, flattened Brazilian disk test has been proven to provide unique data in a variety of fields. Fracture mechanics, damage mechanics, confining pressure, acoustic emissions waveforms, permeability, and visco-inertial flow can all be related from this test data, opening up further research into numerous topics. In addition to further refining the newly developed method, the confined, flattened, Brazilian disk test is recommended to study:

- Fracture mapping of microscopic fracture networks – This work indicates that a microscopic fracture network is developed before the peak strength of the sample is reached, and even before cracks grow unstably. Being able to map this fracture network would further aid the development of accurate hydromechanical models.

- Other brittle geo-materials – this test was applied to geo-materials under simplifying assumptions of isotropy and homogeneity. However, it is applicable to any suitable geo-material allowing researchers to develop extensive material models and study anisotropic and heterogenous conditions.

- Permeability as a function of stress – In this work, relationships were made between permeability and damage. However, permeability enhancements are also a function of the stress conditions. Cyclic loading tests with damage held constant, could reveal this duality.

- Visco-inertial permeability – Permeability was assumed to follow one-dimensional Darcy flow, but visco-inertial effects were shown to affect the apparent permeability. Future work could investigate the relationships between fracture apertures, fracture roughness, tortuosity, and visco-inertial flow in microscopic fractures.
5.4 Concluding Remarks

In conclusion, this work details the development of a novel experimental method to test confined, flattened Brazilian disks. These experiments were used to correlate damage (quantified by acoustic emissions) and permeability, and allow for future investigations into the confined tension region in concrete and tuff. In doing so, this project provides unique and difficult to capture data that can be used to compare and analyze the development of hydromechanical models. The novel experiment set forth in this thesis and the resulting numerical models will allow future researchers to study a wide variety of interconnected processes related to the fracture of brittle geo-materials.
VI. APPENDICES

6.1 A-1: Data for Experiments on Concrete A

6.2 A-2: Data for Experiments on Concrete B

6.3 A-3: Data for Experiments on Tuff

6.4 A-4: Permeability Corrections for Visco-Inertial Flow

6.5 A-5: Derivation of Analytical Solution for Confined, Flattened Brazilian Disk (Eqn. 4.1)

6.6 A-6: Numerical Results

6.1 A-1: Data for Experiments on Concrete A

The results for a concrete A, flattened Brazilian disk confined at 2.76 MPa.

Figure A-1.1a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-1.1b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-1.1c Cumulative AE counts vs. time with concurrent permeability.

Figure A-1.1d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-1.1e Pre-peak permeability as a function of damage.

Figure A-1.1f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
The results for a concrete A, flattened Brazilian disk confined at 5.52 MPa

Figure A-1.2a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-1.2b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-1.2c Cumulative AE counts vs. time with concurrent permeability.

Figure A-1.2d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A.1.2e Pre-peak permeability as a function of damage.

Figure A.1.2f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
The results for a concrete A, flattened Brazilian disk confined at 8.27 MPa

Figure A-1.3a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-1.3b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-1.3c Cumulative AE counts vs. time with concurrent permeability.

Figure A-1.3d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-1.3e Pre-peak permeability as a function of damage.

Figure A-1.3f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
The results for a concrete A, flattened Brazilian disk confined at 11.09 MPa

Figure A-1.4a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-1.4b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-1.4c Cumulative AE counts vs. time with concurrent permeability.

Figure A-1.4d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-1.4e Pre-peak permeability as a function of damage.

Figure A-1.4f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
The results for a concrete A, flattened Brazilian disk confined at 13.79 MPa

Figure A-1.5a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-1.5b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-1.5c Cumulative AE counts vs. time with concurrent permeability.

Figure A-1.5d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-1.5e Pre-peak permeability as a function of damage.

Figure A-1.5f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
6.2 A-2: Data for Experiments on Concrete B

The results for a concrete B, flattened Brazilian disk confined at 2.76 MPa

Figure A-2.1a Force vs. displacement along the loaded diameter with cumulative AE counts.
Figure A-2.1 Force vs. displacement along the loaded diameter with concurrent permeability.

Figure A-2.1c Cumulative AE counts vs. time with concurrent permeability.

Figure A-2.1 Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-2.1e Pre-peak permeability as a function of damage.

Figure A-2.1f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
The results for a concrete B, flattened Brazilian disk confined at 5.52 MPa

Figure A-2.2a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-2. Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-2.2c Cumulative AE counts vs. time with concurrent permeability.

Figure A-2.2d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-2.2e Pre-peak permeability as a function of damage.

Figure A-2.2f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
The results for a concrete B, flattened Brazilian disk confined at 8.27 MPa

Figure A-2.3a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-2.3b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-2.3c Cumulative AE counts vs. time with concurrent permeability.

Figure A-2.3d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-2.3e Pre-peak permeability as a function of damage.

Figure A-2.3f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
The results for a concrete, flattened Brazilian disk confined at 11.09 MPa

Figure A-2.4a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-2.4b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-2.4c Cumulative AE counts vs. time with concurrent permeability.

Figure A-2.4d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-2.4e Pre-peak permeability as a function of damage.

Figure A-2.4f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
The results for a concrete, flattened Brazilian disk confined at 13.79 MPa

Figure A-2.5a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-2.5b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-2.5c Cumulative AE counts vs. time with concurrent permeability.

Figure A-2.5d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-2.5e Pre-peak permeability as a function of damage.

Figure A-2.5f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
6.3 A-3: Data for Experiments on Tuff

The results for a tuff, flattened Brazilian disk confined at 2.76 MPa

![Graph](image)

Figure A-3.1a Force vs. displacement along the loaded diameter with cumulative AE counts.

![Graph](image)

Figure A-3.1b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-3.1c Cumulative AE counts vs. time with concurrent permeability.

Figure A-3.1d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-3.1e Pre-peak permeability as a function of damage.

Figure A-3.1f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
The results for a tuff, flattened Brazilian disk confined at 8.27 MPa

Figure A-3.2a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-3.2b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-3.2c Cumulative AE counts vs. time with concurrent permeability.

Figure A-3.2d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-3.2e Pre-peak permeability as a function of damage.

Figure A-3.2f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
The results for a tuff, flattened Brazilian disk confined at 13.79 MPa

Figure A-3.3a Force vs. displacement along the loaded diameter with cumulative AE counts.

Figure A-3.3b Force vs. displacement along the loaded diameter with concurrent permeability.
Figure A-3.3c Cumulative AE counts vs. time with concurrent permeability.

Figure A-3.3d Force vs. displacement along the loaded diameter with concurrent permeability and damage thresholds.
Figure A-3.3e Pre-peak permeability as a function of damage.

Figure A-3.3f Post-peak hydraulic aperture as a function of displacement along the loaded diameter.
6.4 A-4: Permeability Corrections for Visco-Inertial Flow

It was initially assumed that any permeability measurements made during these experiments would follow Darcy’s law for laminar flow such that the permeability maintained a constant relationship between the pressure gradient and the measured flowrate:

\[ -\nabla P = aQ \]  \hspace{1cm} \text{Eqn. A-4.1}

Where \(-\nabla P\), is the pressure gradient (the negative indicates flow travels from upstream to downstream pressure), \(Q\) is the volumetric flowrate and \(a\) is a constant expression including the permeability \((k)\), the cross-sectional area \((A)\), and the dynamic viscosity of the fluid \((\mu)\):

\[ a = \frac{\mu}{Ak} \]  \hspace{1cm} \text{Eqn. A-4.2}

Analysis of the effect of the pressure gradient of permeability and the critical Reynolds numbers in chapter four, section 4.1.6, indicated that visco-inertial flow was likely occurring; the result of high flowrates inducing turbulent flow. Visco-inertial flow requires a correction to Darcy’s law such that Eqn. A-4.1 becomes:

\[ -\nabla P = aQ + bQ^2 \]  \hspace{1cm} \text{Eqn. A-4.3}

Where \((b)\) is the inertial coefficient described by Forchheimer. In this equation the energy losses experienced by the pressure gradient are a non-linear function of both the viscous term \((a)\) and the inertial term \((b)\).

Hatambeigi et al., (2019) found that in cement fractures an inverse relationship exists between the inertial term \((b)\) and the hydraulic aperture \((h)\), where \((\lambda)\) and \((m)\) are constant regression coefficients.:

\[ b = \lambda h^{-m} \]  \hspace{1cm} \text{Eqn. A-4.4}
The viscous term \((a)\) can also be expressed as a function of the hydraulic aperture using the cubic law (Eqn. A-4.5) such that:

\[
\text{Eqn. A-4.5} \quad k = \frac{h^2}{12}
\]

\[
\text{Eqn. A-4.6} \quad a = \frac{12\mu}{h^3w}
\]

Where \((w)\) is the height of the fracture such that \(h \cdot w = A_f\) (area of the fracture).

With the inertial and viscous terms expressed as in terms of the hydraulic aperture Eqn.’s A-4.4 and A-4.6 can be substituted into A-4.3:

\[
\text{Eqn. A-4.7} \quad -\nabla P = \left(\frac{12\mu}{h^3w}\right)Q + (\lambda h^{-m})Q^2
\]

In this expression, the pressure gradient and flowrate are recorded directly, \((w)\) and \((\mu)\) are known constants and \((\lambda)\) and \((m)\) can be taken from the work by Hatambeigi et al. This leaves \((h)\) as the only unknown. Numerical solvers (i.e. Matlab) can be used to find this corrected hydraulic aperture \((h_c)\). If the flow is purely viscous this term would be the same as the apparent hydraulic aperture \((h)\) calculated from Eqn.’s A-4.1 and A-4.6. However, if flowrates are sufficiently high to induce inertial effects then one would expect the corrected hydraulic aperture \((h_c)\) from Eqn. A-4.7 to be larger than the apparent hydraulic aperture \((h)\) from Eqn.’s A-4.1 and A-4.6. This is indeed the case and the data from the experiments on confined, flattened Brazilian disks consistently shows the increase in the hydraulic aperture corrected for visco-inertial flow as a function of the Reynolds number (which is proportional to the flowrate). In the figures shown below, it is demonstrated that as the flowrate (represented by the Reynolds number) increases, the effect of the inertial coefficient \((b)\) requires a larger correction to the hydraulic aperture. Remember that the hydraulic aperture
represents the distance between two parallel plates with a flowrate equivalent to the permeability. Thus, the final step is using the cubic law in Eqn. A-4.5, with the corrected hydraulic aperture ($h_c$), to find the permeability ($k$) corrected for visco-inertial effects.

Note: Even though the corrections to the hydraulic apertures are 10 micrometers or less, this can have a significant effect on the corrected permeability for the very low permeabilities seen in these experiments, although this can be difficult to see on the log scale in appendices A-1 through A-3.
Figure A-4.1 Difference between corrected and apparent hydraulic aperture as a function of the Reynolds number for concrete B, at 5.52 MPa confinement.

Figure A-4.2 Difference between corrected and apparent hydraulic aperture as a function of the Reynolds number for concrete B, at 8.27 MPa confinement.
Figure A-4.3 Difference between corrected and apparent hydraulic aperture as a function of the Reynolds number for concrete B, at 11.09 MPa confinement.

Figure A-4.4 Difference between corrected and apparent hydraulic aperture as a function of the Reynolds number for concrete B, at 13.79 MPa confinement.
6.5 A-5: Derivation of Analytical Solution for Confined, Flattened Brazilian Disk (Eqn. 4.1)

Hondros (1959) derived the full stress field for a diametrically loaded Brazilian disk under plane stress conditions. The critical stresses at the center of the sample in polar coordinates are given by:

Eqn. A-5.1a \[ \sigma_\theta = -\frac{2F}{\pi Dt} \left( \frac{\sin 2\alpha}{\alpha} - 1 \right) \]

Eqn. A-5.1b \[ \sigma_r = \frac{6F}{\pi Dt} \left( \frac{\sin 2\alpha}{\alpha} - 1 \right) \]

Fairhurst (1964), Jaeger (1966) and others assumed the loaded angle, $2\alpha$, equal to 15 degrees, which simplified the term:

Eqn. A-5.2 \[ \frac{\sin 2\alpha}{\alpha} - 1 = 0.98 \approx 1 \]

If the stresses are translated into rectangular coordinates, assuming the 15-degree loading arc, the minimum and maximum principal stresses become the classic unconfined Brazilian disk solutions:

Eqn. A-5.3a \[ \sigma_3 = \sigma_{xx} = -\frac{2F}{\pi Dt} \]

Eqn. A-5.3c \[ \sigma_2 = \sigma_{zz} = 0 \]

Eqn. A-5.3b \[ \sigma_1 = \sigma_{yy} = \frac{6F}{\pi Dt} \]
Jaeger assumes that for confined Brazilian disks, a hydrostatic pressure is evenly applied to all surfaces of the disk, and by superposition the principal stresses become the combination of the classic solution and the confining pressure, $P_c$:

Eqn. A-5.4a  $\sigma_3 = \sigma_{xx} = \frac{-2F}{\pi Dt} + P_c$

Eqn. A-5.4c  $\sigma_2 = \sigma_{zz} = P_c$

Eqn. A-5.4b  $\sigma_1 = \sigma_{yy} = \frac{6F}{\pi Dt} + P_c$

However, as seen in Fig. 4.1.3 (shown here for convenience), on the flattened, confined Brazilian disk tests used in this work, no confining pressure is applied to the flattened surface. Additionally, the loaded angle, $2\alpha$, becomes 32.1 degrees for a 50.8 mm diameter disk, flattened by 1mm.

\[\text{Figure 4.1.3} \text{ Loading configuration of the confined, flattened, Brazilian disk. (Shown here for convenience)}\]
In this case, the term \( \frac{\sin 2\alpha}{a} - 1 = 0.9 \), which can no longer be approximated to 1.

Furthermore, force from the superimposed confining pressure, \( P_c \), must be subtracted from the flattened area, \( A_f \). These corrections applied to Eqn. A-5.4 provide a more accurate analytical solution for the stress state in the type of confined, flattened Brazilian disks used in the study. These solutions are used as Eqn. 4.1.

\[
\text{Eqn. A-5.5c} \quad \sigma_3 = \sigma_{xx} = 0.9 \frac{-2[F-(P_cA_f)]}{\pi D t} + P_c \\
\text{Eqn. A-5.5b} \quad \sigma_2 = \sigma_{zz} = P_c \\
\text{Eqn. A-5.5a} \quad \sigma_1 = \sigma_{yy} = 0.9 \frac{6[F-(P_cA_f)]}{\pi D t} + P_c
\]
6.6 A-6: Numerical Results

Mesh Convergence Study

A mesh convergence study was performed to find the optimal size of the tetrahedral meshed elements used in HOSS. Elements must be sufficiently small to accurately represent the developed stresses. However, reducing the size past a certain point increases the computational runtime, while only providing minimal stress refinements. In this work, a mesh convergence study was performed on standard Brazilian disks using mesh sizes of 2, 3, and 4 mm, respectively (Fig. A-6.1). The results from the mesh convergence study are shown in Figure A-6.2 and Table A-6.1, and a mesh size of 3 mm was selected. Notice the secondary peak in the force/displacement plot for the 4 mm mesh size.

![Mesh comparison of for sizes 2, 3, and 4 mm.](image)

**Figure A-6.1** Mesh comparison of for sizes 2, 3, and 4 mm.
Figure A-6.2 Force vs. displacement for unconfined Brazilian disks with different mesh sizes.

Table A-6.1: Results from the mesh convergence study.

<table>
<thead>
<tr>
<th>Mesh Size (mm)</th>
<th>Computational Processors</th>
<th>Simulation Runtime (hours)</th>
<th>Peak Force (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>163</td>
<td>48</td>
<td>11.8</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>40</td>
<td>11.4</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>16</td>
<td>8.3</td>
</tr>
</tbody>
</table>

**Speed Optimization**

Another convergence study was performed to find the optimal displacement loading rate. This is done on the standard Brazilian disk using the optimal mesh size of 3 mm, by running different simulations with increasingly slower displacement loading rates. As the loading rate slows, the peak force developed in the Brazilian disk converges and any further...
reductions have a minimal effect on the simulation results. This ensures quasi-static conditions while any further reductions in the loading rate only increase the computational runtime. The results for the speed optimization study are shown in Figure A-6.3 and Table A-6.2, and a displacement rate of 0.03 m/s was selected.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Mesh Size (mm)} & \text{Loading Rate (m/s)} & \text{Runtime (hours)} & \text{Peak Force (kN)} \\
\hline
3 & 0.01 & 128 & 10.8 \\
3 & 0.0165 & 80 & 10.9 \\
3 & 0.03 & 64 & 11.0 \\
3 & 0.3 & 48 & 14.8 \\
\hline
\end{array}
\]

\textbf{Figure A-6.3} Force vs. displacement for unconfined Brazilian disks loaded at different velocities.

\textit{Table A-6.2:} Results from the speed convergence study.
**Regular vs. Flattened Brazilian Disks**

Once the mesh convergence and speed optimization studies were completed, an analysis of the Brazilian disk and its modifications was possible. In this work, flattened Brazilian disks were used to alleviate the problem of localized crushing near the contact points (Fig. A-6.4). A numerical analysis of the effect of flattened Brazilian disks was performed.

![Figure A-6.4 Model Setup of Flattened Brazilian Disk.](image)

Figure A-6.5 shows the simulated force-displacement plots for standard or ‘classic’ Brazilian disks, standard Brazilian disks with loads distributed over increasing arc lengths, and 1mm flattened Brazilian disks. The classic Brazilian disk simulation creates a line load along the length of the Brazilian disk. As a result, stress concentrations at the loading points induce localized damage, greatly reducing the apparent stiffness of the simulated specimen. As the diametric load is distributed over larger areas, higher stresses are able to develop in the center of the sample and converge towards the tensile strength of the material. The 1mm flattened Brazilian disk has an effective loading arc angle of 32.1 degrees and is good approximation
with the 30-degree loading arc standard Brazilian disk. The error between these two is less than 10% and provides evidence that a 1mm flattened Brazilian disk is a good replacement for the standard Brazilian disk, while reducing the challenges associated with evenly distributing the diametric load over an arc length.

![Force vs. displacement graph showing different Brazilian disks](image)

**Figure A-6.5** Force vs. displacement for the following simulated Brazilian disks: 1) classic Brazilian; 2) classic Brazilian with a 15-degree loading arc; 3) classic Brazilian with a 30-degree loading arc; 4) 1mm flattened Brazilian with an effective 32.1-degree loading arc.

**Confined Brazilian Disks**

For confined, flattened, Brazilian disk tests, a uniform confining pressure was applied to every surface of the optimized flattened Brazilian disk except the flattened surfaces which are loaded diametrically by the steel plates (Fig. A-6.6.). A series of simulations applying confining pressures equal to 2.76, 5.52, 8.72, 11.09, and 13.79 MPa, respectively, was performed - matching the experimental conditions.
Figure A-6.6 Confined flattened Brazilian disk. Surfaces in red do not have an applied confining pressure.

Figure A-6.7 presents the force/displacement curves along the loaded diameter for the simulated series of confined, flattened, Brazilian disks. The peak forces from these simulations and the explicitly calculated stresses are used to create Table 4.1.1 and show the increase in apparent strength with confinement.

Figure A-6.7 Force vs. displacement for confined, flattened Brazilian disks simulations.
Future Numerical Work

As mentioned previously, the numerical results in this work were used to aid in the development of the experimental program by evaluating the effect of flattening on Brazilian disks and verifying the stress states developed in confined Brazilian disk tests. The simulations were not quantitatively compared to the experimental results! Verification and validation of the experimental program in HOSS will require further numerical work including accurate descriptions of material models, and accurate representations of the boundary conditions (e.g. the polyurethane jacket and end caps). This is a detailed process that falls outside the scope of this work. However, existing literature has shown the validity of HOSS to capture fracture processes (Rougier et al., 2014; Knight et al., 2015) and the author recommends that any further numerical work in HOSS focus on the ability of HOSS’s fracture network to accurately represent permeability enhancements by a direct comparison of the experimental results presented above (Appendix A-1; Appendix A-2; Appendix A-3).

Ugwu (2008) developed analytical solutions for the interface pressure of nested cylinders subjected to *in situ* lithostatic stresses. Those solutions are used in this work as an analogy to evaluate the effect of the PMC jacket on the confined Brazilian disks under hydrostatic confining pressures.

Ugwu considers a deep wellbore cement-formation interface (Fig. A-7.1), where the *in-situ* rock formation exerts a formation pressure \( P_f \), which generates an interface pressure \( P_i \) at the cement-formation boundary. Ugwu assumes plain strain conditions, axisymmetric deformations, no initial stresses in the cement, a perfect boundary, and treats the cement as a thick-walled pressure vessel.

![Diagram of cement-formation interface](image)

**Figure A-7.1** Cement-formation interface for deep wellbore (modified from Ugwu, 2008).
In the analogy to this work, the rock formation is considered the PMC jacket, which 
experiences the hydrostatic, rather than lithostatic, confining pressure, and the cement sheath 
can be considered the concrete Brazilian disk where \( b \) tends to zero. Thus, Ugwu’s solution 
depends on the formation geometry and the material properties; namely the Young’s Modulus 
and Poisson’s Ratio of concrete and PMC \( (E_c, v_c, E_{pme}, v_{pme}) \), and simplifies to:

\[
\text{Eqn. A-7.1} \quad P_i = \frac{FPf}{K}
\]

Where:

\[
K = \left( \frac{c}{E_{pme}} \right) \left[ \left( 1 - v_{pme}^2 \right) \left( \frac{d^2 + c^2}{d^2 - c^2} \right) + (\nu_{pme} + v_{pme}^2) \right] + \frac{c}{E_c} \left[ (1 - v_c^2) - (v_c - v_{pme}^2) \right]
\]

\[
F = \left( \frac{c}{E_{pme}} \right) \left( \frac{2d^2}{d^2 - c^2} \right) (1 - v_{pme}^2)
\]

Using Eqn. A-7.1, the change in interface pressure (between the PMC jacket and the 
concrete Brazilian disk) can be plotted as a function of increasing confining pressure for 
different material properties. Figure A-7.2 presents the results from Eqn. A-7.1 for PMC 
jackets with different values of Young’s Modulus. As the Young’s Modulus of the PMC jacket 
increases (i.e. becomes stiffer) the jacket is able to carry more of the load from the confining 
pressure and as a result, transmits only a portion of the confining pressure to the sample 
interface. For example, a PMC jacket with a Young’s Modulus of 15000 MPa (the light blue 
line) is sufficiently stiff that only 13.5 MPa of the original 15 MPa confining pressure is 
transmitted to the disk-jacket interface (seen as a negative 1.5 MPa interface pressure change). 
In this case, the Brazilian disk would be confined by both the hydrostatic pressure and the 
relatively ‘stiff’ PMC jacket before failing.
However, if the Young’s Modulus of the PMC is sufficiently low, (like the 0.72 MPa jackets used in this work) the confining pressure is perfectly transmitted to the concrete disk – which has a much higher Young’s Modulus (~11 GPa). As a result of the large difference in material properties, the author believes that the confinement from the PMC jacket does not increase as a function of confining pressure.

Figure A-7.2 Change in interface pressure between the concrete Brazilian disk and the PMC jacket as a function on increasing confining pressure for different values of the Young’s Modulus of PMC.
VII. REFERENCES


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