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Distance in Matrices and Their Applications to Fuzzy Models and Neutrosophic Models

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PREFACE

In this book authors for the first time introduce the notion of distance between any two $m \times n$ matrices. If the distance is 0 or $m \times n$ there is nothing interesting. When the distance happens to be a value t; $0 < t < m \times n$ the study is both innovating and interesting. The three cases of study which is carried out in this book are

1. If the difference between two square matrices is large, will it imply the eigen values and eigen vectors of those matrices are distinct? Several open conjectures in this direction are given.
2. The difference between parity check matrix and the generator matrix for the same C(n, k) code is studied. This will help in detecting errors in storage systems as well as in cryptography.

3. Finally this concept of difference in matrices associated with fuzzy models can be implemented in the study of the deviance or closeness of the experts on any common problem.

Thus in this book the notion of difference in matrices is applied to different sets of problems which is both innovative and interesting. Several open conjecture makes this book a boon to any researcher.

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W.B.VASANTHA KANDASAMY
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Chapter One

INTRODUCTION TO SOME FUZZY MODELS AND NEUTROSOPHIC FUZZY MODELS

In this chapter we just give the reference books for these concepts which are used in the later chapters.

For the Fuzzy Cognitive Maps (FCMs) model please refer Kosko the father of FCMs. For more refer [44-5]. The concept of neutrosophy (or indeterminancy) was introduced by Florentin Smarandache in [66-9].

Here we use I to be the usual notation for interminancy. Further fuzzy neutrosophic numbers are denoted by

$$\{a + bI \mid a, b \in [0, 1], I^2 = I\}$$

Please refer [66-9] for more information.

Finally the construction of Neutrosophic Cognitive Maps (NCMs) was introduced in 2003 [79]. These concepts were defined and used in the analysis of real world problem [75-6, 88, 92].

Next the generalization of Fuzzy Cognitive Maps was made in 2003 [93]. These newly constructed models were termed as Fuzzy Relational Maps. This concept was introduced in [79].

These new models has a domain space and a range space and function in a similar way like FCMs except the hidden pattern happens to be a fixed pair or a limit cycle pair one state vector from the domain space and another from the range space.

So FRMs are advantageous for it saves time but can be applied only when the collection of attributes can be divided into two disjoint classes. These models have been appropriately adopted in several real world problems [93-4]. For more about FRMs refer [79].

Now this concept of fuzzy relational maps model have been developed to the case of Neutrosophic Relational Maps model in 2003 [79]. NRMs model function similar to FRMs.

Next the notion of fuzzy relational equations models [43, 90] have been utilized in different ways in the study [86, 103].

These models as the name suggests works on the equation of fuzzy matrices. For more about the functioning of these models refer [43, 90]. Now as in cae of FCMs we can for FRMs also build NRMs.

Study of NRMs inseated of FRMs by some experts is mainly due to the fact that some of the relations cannot be determined that is they remain indeterminate as far as that expert is concerned.

Thus we need the concept of NRMs also for when the expert feels there is indeterminancy he / she can use NRMs [90, 103].

Next we consider the fuzzy relational equations (FRE) models. As the name suggests FRE models function on the matrix equation P o Q = R. Further they work on predicted solutions and also we are not guaranteed of a solution for the equation P o Q = R may give a solution or may not yield a solution.

It is important to note when a solution does not exist we seek the method of layered neural networks. For the functioning of FRE in working a real world problem please refer [43, 90].

Now sometimes in the fuzzy relational equations formed certain experts may not be in a position to say the weightage they may give an indeterminate weightage. In such situations the notion of Neutrosophic Relational Equations (NRE) plays a vital role.

For more about their properties and functioning please refer [43, 90]. All these three models and their neutrosophic analogue have been defined and used in many real world problems.

Now the notion of Bidirectional Associative Memories (BAM) have been introduced in [45].

It works in an interval [–a, a] using the synaptic connection matrix relating the domain space and the range space.

Now we define neutrosophic Bidirectional Associative Memories (NBAM). We just briefly recall the working and construction of them from [45].

Let n neutrons in a field F_X synaptically connect with p neutrons in a field F_Y.

Imagine an axon from the ith neuron in F_X that terminates in a synapse m_{ij} that abuts the jth neuron in F_Y. We assume that the real number m_{ij} summarizes the synapse and the m_{ij} changes so slowly relative to activation fluctuation that it is constant.

Thus we assume no learning when $m_{ij} = 0$. The synaptic value m_{ij} might represent the average rate of release of a neurotransmitter such as norepinephrine. So as a rate m_{ij} can be +ve, –ve or zero.

The synaptic matrix or connection matrix M is an n by p matrix of real numbers whose entries are the synaptic efficacies m_{ij}. The ij^{th} synapse is excitatory if $m_{ij} > 0$, inhibitory if $m_{ij} < 0$.

Thus a BAM system (F_X, F_Y, M) is bidirectionally stable if all inputs converage to fixed point equilibria. Bidirectional stability provides one example of global or absolute stability.

However in the study of BAM the concept of Hamming metric is used but we in this book are not interested in recalling them or using them.

We assume the working of a BAM in a bandwidth interval $[-B, B]$, $B \in Z^+$.

Here we do not bring in the logic and the derivation for construction of this model, we request the reader to refer to a great book [45].

However we just indicate the functioning of a BAM by an example and this example is by no means a data pertaining to a real world problem.

Let M be a 7 × 4 synaptic connection matrix associated with a BAM model.

$$M = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 2 & 0 \\ -2 & 3 & 4 & 3 & 2 & 3 & 2 \\ 3 & -1 & -2 & 1 & -1 & -2 & -2 \\ -2 & 0 & 2 & 0 & 1 & 3 & 1 \end{bmatrix}.$$

Let $Y_K = (-4, -3, 1, 2, 5, -3, -1)$ be the initial input vector.

$$\begin{aligned} S(Y_K) &= (0\ 0\ 1\ 1\ 1\ 0\ 0) \\ S(Y_K)M^t &= (0\ 9\ -2,\ 3) && = X_{K+1} \\ S(X_{K+1}) &= (0\ 1\ 0\ 1) \\ S(X_{K+1})\ M &= (-4, 3, 6, 3, 3, 6, 3) && = Y_{K+2} \\ S(Y_{K+2}) &= (0\ 1\ 1\ 1\ 1\ 1\ 1) \end{aligned}$$

$$S(Y_{K+2})\,M^t = (2\ 17\ {-7}\ 7) = X_{K+3}$$

$$S(X_{K+3}) = (1\ 1\ 0\ 1)$$

$$S(X_{K+3})\,M = (0\ 3\ 6\ 3\ 3\ 6\ 3) = Y_{K+4}$$

$$S(Y_{K+4}) = (0\ 1\ 1\ 1\ 1\ 1\ 1);$$

leading to a fixed binary pair;

$$\{(0\ 1\ 1\ 1\ 1\ 1\ 1), (1\ 1\ 0\ 1)\}.$$

We see equilibrium of the model is achieved at the time K+4 when the starting time was K.

Now by using an experts opinion get the synaptic connection neutrosophic matrix M_1 if that particular experts wishes to work with indeterminancy. M_1 is given in the following:

$$M_1 = \begin{bmatrix} 4 & I & 1+I & 0 & 0 & 3 & I \\ -2I+3 & 3 & 4 & 2 & 3 & 2+I & 2 \\ 3 & -1-I & -2I & 0 & I & 0 & -1 \\ -2 & 2I & 2 & 0 & -I+1 & 3 & 1 \end{bmatrix}.$$

Now one can use M_1 and obtain the fixed pair. Since only data can deal with on or of state we work with them.

If $X_K = (3+I, -4, -2I, 4)$ then $S(X_K) = (1\ 0\ 0\ 1)$.

Note if $x = a + bI$ occurs and $a < b$ then replace it by zero. If $x = a + bI$, $a > b$ then replace by 1.

If $x = cI$ replace by zero.

If $x = -a$ $(a > 0)$ replace by zero.

This is the way working is carried out using neutrosophic synaptic connection matrix. However interested reader can refer [45] for more information.

Finally for the working of the Fuzzy Associative Memories (FAM) refer [45].

In this book authors are mainly interested not in finding solutions or making predictions using experts. Authors are more interested in finding the relation between the experts opinion when more than one expert deals with the problem.

More so; analyse the result and the dynamical system provided by the expert. Such analysis will certainly throw more light on the problem.

Further in case of FCMs, NCMs, FRMs and NRMs using the experts opinion we get the directed graphs we can analyse the directed graphs using the newly constructed Kosko-Hamming weight.

As most of the fuzzy models have been described defined or developed by Kosko, authors to keep on record his research contributions name the distance functions used to study or analyse the model as Kosko-Hamming distance.

Clearly this distance is not a Hamming distance for in the first place it cannot find distance between any two row vectors of the same order. They need to work on the initial state of vector and be associated with some fuzzy model.

Likewise even the Kosko-Hamming weight cannot be found for any row vector is infact the distance between the resultant and the initial vector.

In this book such techniques are developed in chapter II. In chapter three we develop the notion of special distance between any two matrices of some order. This study is new and innovative and gives more inside information about the very problem and the model.

Chapter Two

DISTANCE IN ROW MATRICES

In this chapter we for the first time define the notion of distances in matrices. We know if A and B are two row matrices of order 1 × n then the distance d(A, B) is defined as the Hamming distance and study in this direction is made in a systematic way. But however if the matrices are not row matrices we do not have any such concept of distance. Now in this book we define the notion of distance between two matrices provided they are of the same order. To this end we recall the following definition.

DEFINITION 2.1: *Let A and B any two 1 × n row matrices. The distance or the well known Hamming distance between A and B is defined as the number of places in which A differs from B.*

We will illustrate this situation by some examples.

Example 2.1: Let A = (3, 2, –1, 0, 7, 4, 5) and B = (3, 2, 5, 0 4, 2, 5) be any two 1 × 7 row vectors. The Hamming distance between A and B is 3.

Example 2.2: Let X = (1 0 1 1 1 1 0 0 0 1) and Y = (0 1 1 0 0 1 0 1 1 1) vectors. The Hamming distance between X and Y is 6.

The concept of Hamming distance finds its applications in coding theory and best approximations.

However the authors have defined in [87] the concept of Kosko-Hamming distance in case of FCMs and NCMs models. For more about these please refer [87].

Just for the sake of completion we recall the definition of the Kosko-Hamming distance in the following.

DEFINITION 2.2: *Let M_1 and M_2 be any two FCMs related with $n \times n$ matrices working on the same set of concepts C_1, C_2, …, C_n. We consider a state vector $X = (a_1, …, a_n)$ where $a_i \in \{0, 1\}$.*

Let the resultant of X on M_1 and M_2 be given by Y_1 and Y_2 respectively.

The Hamming distance between Y_1 and Y_2 is defined as the Kosko-Hamming distance denoted by $d_k(Y_1, Y_2) \leq n$.

This is a special type of distance between two vectors. We call this as a special type for any two vectors of same length we cannot define Kosko-Hamming distance, for the Kosko-Hamming distance to be defined

- *(i) We need two FCMs models on the same problem with same number of attributes.*
- *(ii) We need to work with the same initial state vector X.*
- *(iii) We get the corresponding resultant vectors say Y_1 and Y_2 for the same X.*
- *(iv) The Hamming distance between Y_1 and Y_2 is defined as the Kosko-Hamming distance and it is denoted by $d_k(Y_1, Y_2)$.*

That is Kosko-Hamming distance measures how far two experts agree or disagree upon a given resultant state vector or to be more non abstract how far two experts agree or disagree upon the on state of the nodes in the resultant for the same initial state vector.

Thus from the study we can get the deviation or the maximum deviated state vectors of a given node can be specially analysed and the causes found out so that the researcher finds the cause of such deviation and further analyses the property of those special nodes.

Thus the notion of Kosko-Hamming distance helps one to analyse the dynamical system as well as the nature of the experts who give their opinion regarding the social issues. Those nodes which have largest deviation may be termed as eccentric or unusual or uncommon node.

Such study is new and this technique analysis the very system and the expert.

We give one or two examples of Kosko-Hamming distance.

Example 2.3 : Let two experts work on the same problem using FCM with the same set of attributes say $C_1, \ldots, C_7$.

Let the matrices associated with the first expert be denoted by M_1 which is as follows:

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

and

$$M_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \end{array} \\ \begin{bmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

and let M_2 be the matrix associated with the second expert which is as follows:

Now it is pertinent to keep on record that these are only illustrative examples and do not form any part of any real problem.

Let X = (0 1 0 0 0 0 0) be the given initial vector. To find the effect of X on M_1 and M_2.

$$\begin{aligned} XM_1 &= (1\ 0\ 0\ 0\ 0\ 0\ 0) \\ &\to (1\ 1\ 0\ 0\ 0\ 0\ 0) \quad \text{(after updating).} \\ &= Y_1 \text{ (say)} \end{aligned}$$

$$\begin{aligned} Y_1M_1 &\to (1\ 1\ 1\ 0\ 0\ 0\ 0) = Y_2 \text{ (say)} \\ Y_2M_1 &\to (1\ 1\ 1\ 0\ 1\ 0\ 0) = Y_3 \text{ (say)} \\ Y_3M_1 &\to (1\ 1\ 1\ 0\ 1\ 0\ 0) = Y_4 \text{ (say).} \end{aligned}$$

Since $Y_4 = Y_3$ the hidden pattern of the dynamical system is a fixed point given by Y_3 = (1 1 1 0 1 0 0).

Now let us find the effect of the same X on the dynamical system M_2.

$$\begin{aligned} XM_2 &\to (0\ 1\ 0\ 0\ 0\ 0\ 1) = Z_1 \text{ (say)} \\ Z_1M_2 &\to (0\ 1\ 0\ 0\ 0\ 1\ 1) = Z_2 \text{ (say)} \end{aligned}$$

$Z_2M_2 \rightarrow (0\ 1\ 0\ 0\ 0\ 1\ 1) = Z_3$ (say).

Clearly $Z_2 = Z_3$ so the hidden pattern of the dynamical system M_2 is a fixed point given by $Z_2 = (0\ 1\ 0\ 0\ 0\ 1\ 1)$.

Now we find the Kosko-Hamming distance between Y_3 and Z_2.

$$d_k(Y_3, Z_2) = 5.$$

We see both the experts vary very much from each other on the node C_2. They agree upon only on the node C_4. Hence the researcher should take a special study on the two experts opinion for cause of the very large deviation. The researcher should note such nodes and their resultant.

Let X = (0 0 0 1 0 0 0) be the initial state vector for which the experts wishes to find the hidden pattern using the matrices M_1 and M_2.

$$XM_1 \rightarrow (0\ 0\ 0\ 1\ 0\ 0\ 1) = Y_1 \text{ (say)}$$
$$Y_1M_1 \rightarrow (0\ 0\ 0\ 1\ 0\ 1\ 1) = Y_2 \text{ (say)}$$
$$Y_2M_1 \rightarrow (0\ 0\ 0\ 1\ 0\ 1\ 1) = Y_3 \text{ (say).}$$

Clearly $Y_2 = Y_3$; thus the hidden pattern of X is the fixed point given by $Y_2 = (0\ 0\ 0\ 1\ 0\ 1\ 1)$.

Now we find the hidden pattern of X on the dynamical system M_2, $XM_2 \rightarrow (1\ 0\ 0\ 1\ 0\ 0\ 0) = Z_1$ (say)

$$Z_1M_2 \rightarrow (1\ 0\ 0\ 1\ 1\ 0\ 0) = Z_2 \text{ (say)}$$
$$Z_2M_2 \rightarrow (1\ 0\ 1\ 1\ 1\ 0\ 0) = Z_3 \text{ (say)}$$
$$Z_3M_2 \rightarrow (1\ 0\ 1\ 1\ 1\ 0\ 0) = Z_4 \text{ (say).}$$

$Z_4 = Z_3$, hence the hidden pattern is a fixed point. We see the Kosko-Hamming distance between Z_3 and Y_2 is given by $d_H(Y_2, Z_3) = d_H((0\ 0\ 0\ 1\ 0\ 1\ 1), (1\ 0\ 1\ 1\ 1\ 0\ 0)) = 5$.

We see the on state of the node C_4 also makes both the experts differ drastically. So on the nodes C_2 and C_4 they have a very varied opinion.

Let X = (0 0 0 0 0 0 1) be the initial state vector for which we find the hidden pattern using M_1 and M_2.

$$XM_1 \to (0\ 0\ 0\ 0\ 0\ 1\ 1) = Y_1 \text{ (say)}$$
$$Y_1M_1 \to (0\ 0\ 0\ 1\ 0\ 1\ 1) = Y_2 \text{ (say)}$$
$$Y_2M_1 \to (0\ 0\ 0\ 1\ 0\ 1\ 1) = Y_3 \text{ (say)}.$$

Clearly $Y_3 = Y_2$. Hence the hidden pattern is a fixed point.

Consider the effect of X on the dynamical system M_2.

$$XM_2 \to (0\ 0\ 0\ 0\ 0\ 1\ 1) = Z_1$$
$$Z_1M_1 \to (0\ 0\ 0\ 0\ 0\ 1\ 1) = Z_2$$
$$Z_2 = Z_4.$$

The Kosko-Hamming distance $d_H(Y_2, Z_2) = 1$. The experts are close for the node C_7.

Finally consider the on state of the node C_5.
X = (0 0 0 0 1 0 0); we find the effect of X on the dynamical system M_2 is as follows:

$$XM_2 \to (0\ 1\ 1\ 0\ 1\ 0\ 0) = Y_1 \text{ (say)}$$
$$Y_1M_2 \to (0\ 1\ 1\ 1\ 1\ 0\ 1) = Y_2 \text{ (say)}$$
$$Y_2M_2 \to (1\ 0\ 1\ 1\ 1\ 1\ 1) = Y_3 \text{ (say)}$$
$$Y_3M_2 \to (1\ 0\ 1\ 1\ 1\ 1\ 1) = Y_4 \text{ (say)}.$$

Clearly $Y_3 = Y_4$ is a fixed point.

Now we study the effect of X on M_1.

$$XM_1 \to (0\ 1\ 0\ 0\ 1\ 0\ 0) = Z_1 \text{ (say)}$$
$$Z_1M_1 \to (1\ 1\ 0\ 0\ 1\ 0\ 0) = Z_2 \text{ (say)}$$
$$Z_2M_1 \to (1\ 1\ 1\ 0\ 1\ 0\ 0) = Z_3 \text{ (say)}$$
$$Z_3M_1 \to (1\ 1\ 1\ 0\ 1\ 0\ 0) = Z_4 \text{ (say)}.$$

But $Z_4 = Z_3$ hence the hidden pattern is a fixed point $d_k(Y_3, Z_3) = 4$. The deviation is fairly large.

Thus the experts have diverse opinion of the same problem for the same node.

This is clearly evident from the Kosko-Hamming distance between the resultant vectors.

Now we can from the study of Kosko-Hamming distance determine how far two experts agree on the on state of a node that is the effect of the on state of that node on the dynamical system by finding the Hamming distance between the initial state vector X and the resultant state vector X_t which is termed as the Kosko-Hamming weight.

So Kosko-Hamming weight is different from the Hamming weight for Kosko-Hamming weight determines the Hamming distance between the state vector given by the hidden pattern and the initial state vector.

Such study is new and the greater the Kosko-Hamming weight the greater the impact that on state node has on the dynamical system.

We will illustrate this by some examples.

Example 2.4: Let three experts say E_1, E_2 and E_3 work on the same problem with the same set of attributes or concepts say $C_1, \ldots, C_8$.

The connection matrices related with experts E_1, E_2 and E_3 is respectively M_1, M_2 and M_3.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \left[\begin{array}{cccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

be the connection matrix associated with the first expert.

Let

$$M_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

be the connection matrix associated with the second expert.

Let

$$M_3 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \end{array}$$

be the connection matrix given by the third expert.

Now we find the effect of X = (0 0 1 0 0 0 0 0) on all the three dynamical systems and calculate the Kosko-Hamming distances and the Kosko-Hamming weight of the vector X relative to the three experts.

The effect of X on the dynamical system M_1 is given in the following

$$\begin{aligned}
XM_1 &\to (0\ 0\ 1\ 0\ 1\ 0\ 0\ 0) = Y_1 \text{ (say)} \\
Y_1M_1 &\to (0\ 0\ 1\ 1\ 1\ 0\ 0\ 0) = Y_2 \text{ (say)} \\
Y_2M_1 &\to (1\ 0\ 1\ 1\ 1\ 0\ 0\ 1) = Y_3 \text{ (say)} \\
Y_3M_1 &\to (1\ 1\ 1\ 1\ 1\ 0\ 1\ 1) = Y_4 \text{ (say)} \\
Y_4M_1 &\to (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = Y_5 \text{ (say)} \\
Y_5M_1 &\to (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = Y_6 \text{ (say).}
\end{aligned}$$

$Y_5 = Y_6$ is a fixed point.

We see the Kosko-Hamming weight of X is $d_k(X, Y_5) = 7$.

This clearly shows that this node is very powerful node for it makes every other node to come to on state. But however whether it an influential node is to be analysed.

The effect of X on the dynamical system M_2 is given in the following.

$$
\begin{aligned}
XM_2 &\to (1\ 0\ 1\ 0\ 0\ 0\ 0\ 0) = Z_1 \text{ (say)}\\
Z_1M_2 &\to (1\ 0\ 1\ 0\ 1\ 0\ 0\ 0) = Z_2 \text{ (say)}\\
Z_2M_2 &\to (1\ 1\ 1\ 0\ 1\ 0\ 0\ 0) = Z_3 \text{ (say)}\\
Z_3M_2 &\to (1\ 1\ 1\ 0\ 1\ 0\ 0\ 1) = Z_4 \text{ (say)}\\
Z_4M_2 &\to (1\ 1\ 1\ 0\ 1\ 0\ 1\ 1) = Z_5 \text{ (say)}\\
Z_5M_2 &\to (1\ 1\ 1\ 0\ 1\ 1\ 1\ 1) = Z_6 \text{ (say)}\\
Z_6M_2 &\to (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = Z_7 \text{ (say)}\\
Z_7M_2 &\to (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = Z_8 \text{ (say)}.
\end{aligned}
$$

Clearly $Z_7 = Z_8$ so hidden pattern is a fixed point. Hence the Kosko - Hamming weight of X relative to M_2 is also $d_k(X, Z_7) = 7$.

So this X is also a powerful node as all other nodes come to on state in the resultant according to the second expert also.

Now we find the effect of X = (0 0 1 0 0 0 0 0) on the dynamical system M_3.

$$
\begin{aligned}
XM_3 &\to (1\ 0\ 1\ 0\ 0\ 0\ 0\ 0) = S_1 \text{ (say)}\\
S_1M_3 &\to (1\ 0\ 1\ 0\ 0\ 0\ 0\ 1) = S_2 \text{ (say)}\\
S_2M_3 &\to (1\ 0\ 1\ 0\ 1\ 0\ 0\ 1) = S_3 \text{ (say)}\\
S_3M_3 &\to (1\ 1\ 1\ 0\ 1\ 0\ 0\ 1) = S_4 \text{ (say)}\\
S_4M_3 &\to (1\ 1\ 1\ 1\ 1\ 0\ 0\ 1) = S_5 \text{ (say)}\\
S_5M_3 &\to (1\ 1\ 1\ 1\ 1\ 0\ 1\ 1) = S_6 \text{ (say)}\\
S_6M_3 &\to (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = S_7 \text{ (say)}\\
S_7M_3 &\to (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = S_8 \text{ (say)}.
\end{aligned}
$$

The hidden pattern is a fixed point so $S_7 = S_8$.

Further the Kosko-Hamming weight $d_k(X, S_7) = 7$. This node also is a powerful node so all nodes come to on state.

Now we find the Kosko-Hamming distance for the resultant vectors by the three experts with respect to the initial state vector X.

$d_k(Y_5, Z_7) = 7$
$d_k(Y_5, S_7) = 7$ and
$d_k(Z_7, S_7) = 7$.

Thus the Kosko-Hamming distance of all the three pairs of resultant vectors by the three experts whose initial vector is X is the same in this case.

Let us consider the initial state vector B = (0 0 0 0 0 1 0 0); with only the node C_6 in the on state and all other nodes. The effect of B on the matrix M_1 is

$BM_1 \rightarrow (0\ 0\ 0\ 0\ 1\ 1\ 0\ 0) = Y_1$ (say)
$Y_1M_1 \rightarrow (0\ 0\ 0\ 1\ 1\ 1\ 0\ 0) = Y_2$ (say)
$Y_2M_1 \rightarrow (1\ 0\ 0\ 1\ 1\ 1\ 0\ 1) = Y_3$ (say)
$Y_3M_1 \rightarrow (1\ 1\ 0\ 1\ 1\ 1\ 1\ 1) = Y_4$ (say)
$Y_4M_1 \rightarrow (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = Y_5 = Y_4$ (say)
$Y_5M_1 \rightarrow (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = Y_6 = Y_5$ (say).

Thus the hidden pattern in this case is a fixed point given by $Y_5 = Y_4$.

Consider $XM_2 \rightarrow (0\ 0\ 0\ 1\ 0\ 1\ 0\ 0) = S_1$ (say)

$S_1M_2 \rightarrow (0\ 0\ 0\ 1\ 0\ 1\ 0\ 0) = S_2$ (say),

$S_2 = S_1$ a fixed point.

Consider the effect of X on the dynamical system M_3.

$XM_3 \rightarrow (0\ 0\ 1\ 0\ 0\ 1\ 0\ 0) = Z_1$ (say)
$Z_1M_3 \rightarrow (1\ 0\ 1\ 0\ 0\ 1\ 0\ 0) = Z_2$ (say)
$Z_2M_3 \rightarrow (1\ 0\ 1\ 0\ 0\ 1\ 0\ 1) = Z_3$ (say)
$Z_3M_3 \rightarrow (1\ 0\ 1\ 0\ 1\ 1\ 0\ 1) = Z_4$ (say)
$Z_4M_3 \rightarrow (1\ 1\ 1\ 0\ 1\ 1\ 0\ 1) = Z_5$ (say)
$Z_5M_3 \rightarrow (1\ 1\ 1\ 1\ 1\ 1\ 0\ 1) = Z_6$ (say)
$Z_6M_3 \rightarrow (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = Z_7$ (say)
$Z_7M_3 \rightarrow (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = Z_8$ (say).

Thus the hidden pattern is a fixed point given by $Z_8 = Z_7$.

Now the Kosko-Hamming weight of X is

$w_k(Y_5) = d_k(X, Y_5) = (7)$.
$w_k(S_2) = d_k(X, S_2) = 1$ and
$w_k(Z_7) = d_k(X, Z_7) = 7$.

Thus the Kosko-Hamming weight is the maximum in case of the dynamical systems associated with experts one and three which clearly shows the on state of C_6 makes on all other nodes their by making C_6 a powerful node. However according to the second expert the node C_6 is not powerful node as the weight is only one. So the two experts vary from the second expert.

Thus the Kosko-Hamming weight denoted by w_k is the weight of the resultant Y of X if t of the nodes are on in X then weight of the resultant Y denoted by
$w_k(Y)$ = {number of differences between X and Y}.

Clearly Kosko-Hamming weight is different from the usual Hamming weight.

Likewise the Kosko-Hamming distance is different from the usual Hamming distance. Both Kosko-Hamming distance and Kosko-Hamming weight are dependent on the initial state vector X associated with a fuzzy model.

THEOREM 2.1: *Let M be the connection matrix of the FCM with n attributes. For any initial state vector X we see the Kosko-Hamming weight of the resultant state vector of Y of X denoted by $w_k(Y) = d_k(X, Y)$ measures the influence of those on states in X over the system.*

(i) *$w_k(Y) = 0$ if and only if X has no impact on the dynamical system.*
(ii) *If $w_k(Y) = n - r$; where r is the number of on state vectors in X shows the collection of those nodes in*

X in the on state are powerful set of nodes as all nodes come to on state in Y.

(iii) $w_k(Y) = d_k(X, Y) = t$, $0 \leq t \leq n-r$ *and that number shows the power of the r nodes in the on state in X over the dynamical system.*

The proof is direct and hence left as an exercise to the reader.

However we are not in a position to conclude they are the most influential nodes of the system.

That is why we only use the term powerful node.

We will illustrate these situations by some examples.

Example 2.5: Let M be the matrix associated with the FCM model given by an expert.

$$M = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

be the connection matrix.

Let X = (0 1 0 0 0 1) be the given initial state vector. To find the effect of X on the dynamical system M is as follows.

$XM \to (0\ 1\ 0\ 0\ 0\ 1) = Y_1$
$Y_1M \to (0\ 1\ 0\ 0\ 0\ 1) = Y_2$.

$Y_1 = Y_2$ is a fixed point.

We see $d_k(X, Y_1) = (0)$ that is the Kosko-Hamming distance is zero. Further Kosko-Hamming weight is also zero.

$$w_k(X) = d_k(X, Y_1) = 0.$$

Thus the on state of these two nodes has nil effect on the system.

Let X = (1 1 0 0 0 1) be the given initial state vector.

To find the hidden pattern of X on the dynamical system

$$XM \to (1\ 1\ 0\ 1\ 0\ 1) = Y_1$$
$$Y_1 M \to (1\ 1\ 0\ 1\ 1\ 1) = Y_2$$
$$Y_2 M \to (1\ 1\ 0\ 1\ 1\ 1) = Y_3$$

$$Y_2 = Y_3.$$

The Kosko-Hamming distance is $d_k(X, Y_2) = 2 = w_k(x)$ the Kosko-Hamming weight.

Let M_1 be the another matrix of the same problem given by the second expert.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

Let X = (0 1 0 0 0 1) be the initial state vector.

$$XM \to (0\ 1\ 0\ 1\ 0\ 1) = Z_1$$

$Z_1M \rightarrow (0\ 1\ 0\ 1\ 0\ 1) = Z_2 = Z_1$.

The hidden pattern is a fixed point.

The Kosko-Hamming distance $d_k(Z_1, Y_2) = 2$.

The two experts do not vary much vary in one or two nodes of the given three nodes.

Example 2.6: Let us consider the two experts connection matrices of the FCMs of the problem with the same set of attributes. Let

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

and

$$M_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

be any two connection matrices.

To find the effect of the state vector;

X = (1 0 0 1 0 0 0 0) on the dynamical system M_1 and M_2.

$$XM_1 \rightarrow (1\ 0\ 0\ 1\ 0\ 0\ 1\ 0) = Y_1$$
$$Y_1M_1 \rightarrow (1\ 0\ 0\ 1\ 0\ 0\ 1\ 1) = Y_2$$
$$Y_2M_1 \rightarrow (1\ 0\ 0\ 1\ 0\ 0\ 1\ 1) = Y_3$$

The hidden pattern of X is a fixed point given by $Y_3 = Y_2$. Let us find the effect of the initial state vector X on $XM_2 \rightarrow (1\ 0\ 0\ 1\ 0\ 1\ 0\ 0) = Z_1$.

$$Z_1M_2 \rightarrow (1\ 0\ 0\ 1\ 0\ 1\ 0\ 0) = Z_2 \text{ (say)}$$
$$Z_2M_2 \rightarrow (1\ 0\ 0\ 1\ 0\ 1\ 0\ 0) = Z_3 \text{ (say)}.$$

We see $Z_3 = Z_2$ is the fixed point.

Now the Kosko-Hamming distance $d(Y_2, Z_2) = 3$.

Thus the two experts differ in three nodes. The Kosko-Hamming weight of X given by the first expert

$$w_k(X) = d_k(X, Y_3) = 2.$$

The Kosko-Hamming weight associated with the second expert is

$$d_k(X, Z_2) = w_k(X) = 1$$

which shows the influence of the on state of these two nodes is very limited. The effect is only on one more node.

Such analysis of the hidden patterns and the relation between experts are possible only by using the Kosko-Hamming distance and Kosko-Hamming weight.

Next we study the Kosko-Hamming distance in case of Fuzzy Relational Maps (FRMs).

We know in case of FRMs we get for any initial state vector a pair of resultant vectors. However we can find only effect of them in case of Kosko-Hamming weight as well as Kosko-Hamming distance.

We shall first illustrate this situation by an example or two.

Example 2.7: Let two experts work on a problem using FRMs with the same number of domain and range attributes.

$$\text{Let } M_1 = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

be the dynamical system associated with the first expert.

Let

$$M_2 = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

be dynamical system associated with the second expert.

Let us now find the hidden pattern

X = (1 0 0 0 0 0) an initial vector from the domain space of attributes.

$$XM_1 \rightarrow (1\ 0\ 0\ 0\ 1) = Y \text{ (say)}$$
$$Y M_1^t \rightarrow (1\ 0\ 0\ 0\ 1\ 0) = X_1 \text{ (say)}$$
$$X_1\ M \rightarrow (1\ 0\ 0\ 0\ 1) = Y_1 \text{ (say)}$$
$$Y_1 M_1^t \rightarrow (1\ 0\ 0\ 0\ 1\ 0) = X_2.$$

But $X_2 = X_1$, thus the hidden pattern is a fixed pair given by {(1 0 0 0 1 0), (1 0 0 0 1)}.

Now we find the effect of X on the dynamical system M_2.

$$XM_2 \rightarrow (0\ 1\ 0\ 0\ 1) = Z_1$$
$$Z_1 M_2^t \rightarrow (1\ 0\ 0\ 0\ 0\ 1) = Y_1$$
$$Y_1M_2 \rightarrow (1\ 1\ 0\ 0\ 1) = Z_2$$
$$Z_2\ M_2^t \rightarrow (1\ 0\ 0\ 1\ 0\ 1) = Y_2$$
$$Y_2M_2 \rightarrow (1\ 1\ 0\ 0\ 1) = Z_3.$$

We see $Z_3 = Z_2$. Thus the hidden pattern of the dynamical system is a fixed pair given by {(1 0 0 1 0 1), (1 1 0 0 1)}.

Now the Kosko-Hamming distance is given by

$$d_k((1\ 0\ 0\ 0\ 1\ 0), (1\ 0\ 0\ 1\ 0\ 1)) = 3 \text{ and}$$

$$d_k((1\ 0\ 0\ 0\ 1), (1\ 1\ 0\ 0\ 1)) = 1.$$

Thus we see the two experts vary in three nodes in the domain space where as they differ only in one node in the range space.

Now we work using the initial vector from the range space.

Let B = (0 1 0 0 0) be the initial state vector given by the expert from the range space.

$$B M_1^t \rightarrow (0\ 1\ 0\ 0\ 0\ 1) = Y_1 \text{ (say)}$$
$$Y_1M_1 \rightarrow (0\ 1\ 0\ 0\ 0) = X = B.$$

Thus the hidden pattern is a fixed pair given by
{(0 1 0 0 0), (0 1 0 0 0 1)}.

Now we find the hidden pattern of B on dynamical system given by the matrix M_2

$$B M_2^t \to (1\ 0\ 0\ 0\ 0\ 1) = Z \text{ (say)}$$
$$ZM_2 \to (0\ 1\ 0\ 0\ 1) = B_1$$
$$B_1\ M_2^t \to (1\ 0\ 0\ 0\ 0\ 1) = Z_1.$$

$Z_1 = Z$. Thus the resultant is a fixed point given by the pair
{(0 1 0 0 1), (1 0 0 0 0 1)}.

The Kosko-Hamming distance given by

$$d_k((0\ 1\ 0\ 0\ 0), (0\ 1\ 0\ 0\ 1)) = 1 \text{ and}$$

$$d_k((0\ 1\ 0\ 0\ 0\ 1), (1\ 0\ 0\ 0\ 0\ 1)) = 2.$$

Thus the two experts mostly agree on the effect of the state vector X.

Example 2.8: Let M_1 and M_2 be the two connection matrices of the FRM given by two experts working on the same set of attributes on the same problem.

Let the connection matrices of both experts be as follows:

$$M_1 = \begin{array}{c} \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \text{ and}$$

$$M_2 = \begin{array}{c} \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

be the two matrices related with the FRM of the two experts.

Now we find the effect of the initial state vector $X = (0\ 0\ 0\ 1\ 0\ 0)$ from the domain space. Only the node d_4 is in the on state and all other nodes are in the off state.

$$XM_1 \to (0\ 1\ 0\ 0\ 0\ 1\ 0\ 0) = Y_1 \text{ (say)}$$
$$Y_1 M_1^t \to (0\ 0\ 0\ 1\ 0\ 1) = X_1 \text{ (say)}$$
$$X_1 M_1 \to (0\ 1\ 0\ 1\ 0\ 1\ 0\ 0) = Y_2 \text{ (say)}$$
$$Y_2 M_1^t \to (0\ 0\ 0\ 1\ 0\ 1) = X_2.$$

But $X_2 = X_1$. Thus the hidden pattern is the fixed point pair given by

$$B = \{(0\ 0\ 0\ 1\ 0\ 1), (0\ 1\ 0\ 1\ 0\ 1\ 0\ 0)\}.$$

Now we find the effect of X on the system M_2.

$$XM_2 \to (1\ 0\ 0\ 0\ 0\ 1\ 0\ 0) = Z_1 \text{ (say)}$$
$$Z_1 M_2^t \to (0\ 0\ 0\ 1\ 0\ 0) = X_2 = X_1.$$

Thus the hidden pattern is the fixed point pair given by

$$B_1 = \{(0\ 0\ 0\ 1\ 0\ 0), (1\ 0\ 0\ 0\ 0\ 1\ 0\ 0)\}.$$

Now the Kosko-Hamming distance pair is (1, 3).

We see the difference is not very large.

Let us consider the on state of the node in the range space Y = (0 0 0 0 1 0 0 0).

$$Y M_1^t \to (0\ 0\ 0\ 0\ 0\ 0) = Z_1 \text{ say}$$
$$Z_1M_1 \to (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0).$$

The hidden pattern is a fixed point pair given by {(0 0 0 0 0 0), (0 0 0 0 1 0 0 0)}

Now we find the effect of the initial state vector Y on the dynamical system M_2.

$$Y M_2^t \to (0\ 0\ 0\ 0\ 1\ 0) = Z_1 \text{ (say)}$$
$$Z_1M_2 \to (0\ 0\ 0\ 0\ 1\ 0\ 0\ 1) = Y_1$$
$$Y_1\ M_2^t \to (0\ 0\ 0\ 0\ 1\ 1) = Z_2 \text{ (say)}$$
$$Z_2\ M_2 \to (0\ 0\ 0\ \ 1\ 1\ 0\ 0\ 1) = Y_2 \text{ (say)}$$
$$Y_2\,M_2^t \to (0\ 1\ 0\ 0\ 1\ 1) = Z_3 \text{ (say)}$$
$$Z_3M_2 \to (0\ 0\ 0\ 1\ \ 1\ 0\ 0\ 1) = Y_3.$$

Thus the hidden pattern is a fixed pair given by

{(0 1 0 0 1 1), (0 0 0 1 1 0 0 1)}.

The Kosko - Hamming distance pair is given by (3, 2).

According to the first expert no effect on the system is possible. But according to the second expert there is impact on the system.

Let us consider the on state of the nodes C_1 and C_3 in the range space; that is X = (1 0 1 0 0 0 0 0).

The effect of X on M_1 is given by

$$X M_1^t \to (1\ 0\ 1\ 0\ 1\ 0) = Z_1$$
$$Z_1M_1 \to (1\ 0\ 1\ 0\ 0\ 0\ 1\ 0) = X_1$$
$$X_1\,M_1^t \to (1\ 0\ 1\ 0\ 1\ 0\ ) = Z_2$$
$$Z_2M_1 \to (1\ 0\ 1\ 0\ 0\ 0\ 1\ 0) = X_2.$$

But $X_2 = X_1$.

Thus the hidden pattern is a fixed pair given by {(1 0 1 0 1 0), (1 0 1 0 0 0 1 0)}.

Now we study the effect of X on M_2.

$$X M_2^t \to (0\ 0\ 0\ 1\ 0\ 0) = Y_1 \text{ (say)}$$
$$Y_1 M_2 \to (1\ 0\ 1\ 0\ 0\ 1\ 0\ 0) = X_1 \text{ (say)}$$
$$X_1 M_2^t \to (0\ 0\ 0\ 1\ 0\ 0) = Y_2.$$

But $Y_2 = Y_1$. Thus the hidden pattern is a fixed pair given by {(0 0 0 1 0 0), (1 0 1 0 0 1 0 0)}.

Now the Kosko-Hamming distance pair is {(4, 2)}.

This is the way the Kosko-Hamming distance for a pair of hidden patterns is determined.

We can also define the Kosko-Hamming distance in case of Fuzzy Relational Equations (FRE) model, Bidirectional Associative Memories model (BAM) model and the Fuzzy Associative Memories (FAMs) model.

But the way they analyse these models is different from FRMs and FCMs.

We will first illustrate how the notion of Kosko-Hamming distance of two vectors in the case of FRE.

We have already recalled the working of the FREs in chapter I.

Suppose we use a FRE where the domain or the row variables are n in number and the number of column variables are m in number and we work with a $n \times m$ FRE model. Suppose two experts work with the same order $n \times m$ matrices.

We have a pre determined limit set say $S_1, S_2, \ldots, S_n$ with the fixed values $(s_1, s_2, \ldots, s_n)$ and each s_i lies between 0 and 1, $1 \leq i \leq n$.

Now we adopt the notion of Kosko-Hamming distance in two ways.

If after working with a set of values $(a_1, \ldots, a_n)$ we find $d_H((s_1, \ldots, s_n), (b_1, \ldots, b_n))$ where $(b_1, \ldots, b_n)$ is the resultant given by $(a_1, \ldots, a_n)$. If the number of differences is say t.

We say t constraints differ from the fixed values.

However this does not give the positive or negative deviance but the amount of deviance from the fixed value.

So we define the new notion of Kosko-Hamming deviance function;

$$f : (s_1, \ldots, s_n) \to (b_1, \ldots, b_n) \text{ by } f(s_1, \ldots, s_n)$$
$$= (s_1 - b_1, s_2 - b_1, \ldots, s_n - b_n).$$

We know $s_i - b_i$ is + ve or –ve or 0; $1 \leq i \leq n$.

If zero we say there is no deviance.

If $s_i - b_i < 0$ we say negative deviance and if $s_i - b_i > 0$ we say the deviance is positive.

We know or can assess that negative deviance is always accepted and the positive deviance is not to that extent accepted.

Since all our working is based on approximations we can get a value for $s_i - b_i > 0$ to be negligible and for some value the resultant to be rejected.

Further if $s_i - b_i < 0$ then if s_i is the acceptance value then b_i is clearly more acceptable.

Finally we find for two resultant vectors for the same input the Kosko-Hamming distance between two experts are defined in two ways.

(1) $d_k(x, y)$ = difference between the positions.
(2) $d_k(x, y) = (|x_1 - y_1|\ |x_2 - y_2|, \ldots, |x_n - y_n|)$

where we find $|x - y| = (|x_1 - y_1|\ |x_2 - y_2|, \ldots, |x_n - y_n|)$.

where $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$.

We will fix a pre-assigned value α; $|\alpha| < 1$ to accept or reject the solution or inform the deviation is small or large.

We will illustrate this situation by some examples.

Example 2.9: Let two experts work with the same problem having the same number of domain concepts and range concepts. They work with same set of limit sets.

The limit set is as follows:

$s_1 \geq 0.6$ it is good $s_1 < 0.6$ not so good.

$s_2 \geq 0.5$ it is acceptable if $s_2 < 0.5$ not acceptable.

$s_3 \geq 0.6$ the value of tolerable if $s_3 < 0.6$ is not tolerable.

$s_4 \geq 0.5$ the exception is best $s_4 < 0.5$ the exception not at its best.

$s_5 \geq 0.6$ the acceptance level is good if $s_5 < 0.6$ cannot be accepted.

$s_6 \geq 0.4$ the quality is fair $s_6 < 0.4$ is not that fair.

$s_7 \geq 0.6$ the quality is preferable if $s_7 < 0.6$ the quality is not preferable.

We give the membership matrix given by two experts.

$$M_1 = \begin{bmatrix} 0 & 0.6 & 0 & 0.7 & 0.8 & 0.6 & 0.8 \\ 0.3 & 0.1 & 0.6 & 0.6 & 0 & 0.5 & 0.7 \\ 0.6 & 0.8 & 0 & 0.7 & 0.7 & 0.5 & 0.6 \\ 0 & 0 & 0.7 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0.3 & 0 & 0.2 & 0 \\ 0.1 & 0 & 0 & 0.2 & 0 & 0.1 & 0 \\ 0.2 & 0 & 0.7 & 0.6 & 0 & 0.5 & 0.7 \end{bmatrix}$$

is the membership matrix given by the first expert.

The membership matrix M_2 is given by the second expert.

$$M_2 = \begin{bmatrix} 0.1 & 0.7 & 0.1 & 0.8 & 0.9 & 0.7 & 0.8 \\ 0.4 & 0.2 & 0.7 & 0.7 & 0.1 & 0.4 & 0.6 \\ 0.7 & 0.8 & 0.1 & 0.7 & 0.1 & 0.6 & 0.6 \\ 0.2 & 0.1 & 0.8 & 0.6 & 0 & 0.1 & 0 \\ 0.5 & 0 & 0.6 & 0.4 & 0 & 0.2 & 0.3 \\ 0.1 & 0.1 & 0.2 & 0.3 & 0.1 & 0.1 & 0.1 \\ 0.3 & 0 & 0.8 & 0.4 & 0 & 0.4 & 0.8 \end{bmatrix}.$$

Now we find the solutions for
$P = (0.7, 0.5, 0.7, 0.4, 0.8, 0.6, 0.3)$ using M_1 and M_2.

$P o M_1 = R_1$ and
$P o M_2 = R_2$.
$P o M_1 = (0.8, 0.6, 0.7, 0.7, 0.6, 0.2, 0.7)$.
$P o M_2 = (0.8, 0.7, 0.7, 0.7, 0.6, 0.3, 0.7)$.

We find the Kosko-Hamming distance of R_1 and R_2 for the value P is given in the following.

$d_k(R_1, R_2) = (2)$. This shows the two experts almost agree with the values. The distance between components of R_1 and R_2 is

$$d_k(R_1, R_2) = (0, 0.1, 0, 0, 0, 0.1, 0).$$

The difference is very small.

The deviance function $f : (s_1, \ldots, s_7) \to R_1$ is as follows.

$$f\{(0.6, 0.5, 0.6, 0.5, 0.6, 0.4, 0.6\}$$

$$= (0.2, 0.1, 0.1, 0.2, 0, 0.2, 0.1).$$

$$f : (s_1, \ldots, s_7) \to R_2 \text{ is as follows.}$$

$$f\{(0.6, 0.5, 0.6, 0.5, 0.6, 0.4, 0.6\}$$

$$= (0.2, 0.2, 0.1, 0.2, 0, 0.1, 0.1).$$

If we take $\alpha = 0.3$ then certainly the deviation function is negligible.

This is the way different types of Kosko-Hamming distance functions are defined.

They help in analyzing the resultant vector using the three types of Kosko-Hamming distance / functions.

Interested reader can give more examples.

Next we show how this concept of Kosko-Hamming distance is used in the FAM (Fuzzy Associative Memories) model.

The functioning of FAM's is recalled in chapter I of this book.

We know a FAM model functions on the collection of attributes in the domain space and a collection of attributes from the range space.

We have a map assigning membership function. Using these we get the fuzzy matrix given by the experts.

Using the same fit vector say A for the same problem and using the two experts whose associated matrices are M_1 and M_2 we find resultant say B_1 and B_2 respectively.

Now we find the Kosko-Hamming distance.

$$d_k(B_1, B_2) = n \geq 0.$$

This measures the deviation between the two experts.

We will illustrate this situation by an example or two.

Example 2.10: Let us consider the three matrices given by three experts associated with the FAM model in the analysis of the problems related with the HIV / AIDs patients. [75-6]

On the domain side bad habits leading to HIV / AIDs is taken and consequences suffered by HIV / AIDs patients is taken as the range space.

Let M_1 be the matrix given by the first expert

$$M_1 = \begin{bmatrix} 0.6 & 0.7 & 0.8 & 0.6 \\ 0.4 & 0.3 & 0.6 & 0.2 \\ 0.5 & 0.7 & 1 & 0.8 \\ 0.7 & 0.3 & 0.5 & 0.6 \\ 0.4 & 0.7 & 0.5 & 0.6 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}.$$

Let M_2 be the matrix given by the second expert

$$M_2 = \begin{bmatrix} 0.5 & 0.6 & 0.7 & 0.6 \\ 0.5 & 0.4 & 0.5 & 0.3 \\ 0.4 & 0.6 & 1 & 0.7 \\ 0.6 & 0.2 & 0.5 & 0.5 \\ 0.2 & 0.5 & 0.4 & 0.5 \\ 0.4 & 0.6 & 0.6 & 0.5 \end{bmatrix}.$$

Let M_3 be the matrix given by the third expert

$$M_3 = \begin{bmatrix} 0.7 & 0.7 & 0.7 & 0.7 \\ 0.5 & 0.2 & 0.5 & 0.3 \\ 0.4 & 0.8 & 1 & 0.6 \\ 0.5 & 0.4 & 0.5 & 0.5 \\ 0.3 & 0.5 & 0.5 & 0.5 \\ 0.3 & 0.6 & 0.6 & 0.5 \end{bmatrix}.$$

Let A = (0.7, 0.5, 0.8, 0.4, 0.3, 0.6) be the input vector.

We get the following output vectors for M_1, M_2 and M_3 as
B_1 = (0.6, 0.7, 0.8, 0.8),
B_2 = (0.5, 0.6, 0.8, 0.7) and
B_3 = (0.7, 0.8, 0.8, 0.7) respectively.

The Kosko-Hamming distances are as follows:

$d_k(B_1, B_2) = 3$,
$d_k(B_1, B_3) = 3$ and
$d_k(B_2, B_3) = 2$.

We see the experts two and three are more close than the experts one and two and one and three.

Thus the notion of Kosko-Hamming distance helps one to find the closeness of the expert for the output vectors for the given input vector.

This is the way we find the closeness in the prediction of the experts working on the same problem relative with the same input vector.

It is left as an exercise to the reader to find the examples of Kosko-Hamming distance in case of FAM.

It is interesting we can also mention that we can find the average FAM. We take say n experts working on the same problem with or domain attributes and s range attributes, so that each expert has a r × s, FAM matrix associated with them.

Let $M_1, M_2, \ldots, M_n$ be the n, r × s matrices of the FAM. The average FAM matrix is $1/n \sum M_i = M$.

Now if we work with A an initial state vector. If B_i is the resultant output state vector using the matrix M_i and B is the resultant of the output vector for the initial state vector A using M.

We find the Kosko-Hamming distance $d_k(B, B_i) = t$, if t is small we say the difference between the average and the expert one is not very large.

So here also the notion of Kosko-Hamming distance is used. We will illustrate this situation by an example.

Consider the above examples we find

$$M = 1/3\,(M_1 + M_2 + M_3)$$

$$= \begin{bmatrix} 0.6 & 0.67 & 0.73 & 0.63 \\ 0.46 & 0.3 & 0.53 & 0.26 \\ 0.43 & 0.7 & 1 & 0.7 \\ 0.6 & 0.3 & 0.5 & 0.53 \\ 0.3 & 0.56 & 0.46 & 0.53 \\ 0.4 & 0.56 & 0.56 & 0.5 \end{bmatrix}.$$

For the same A = (0.7, 0.5, 0.8, 0.4, 0.3, 0.6) we find B, B = (0.6, 0.7, 0.8, 0.7);

$$d_k(B, B_1) = 1,$$
$$d_k(B, B_2) = 2 \text{ and } d_k(B, B_3) = 2.$$

Thus from the average the deviation is small.

Hence we can also find the average and get the Kosko-Hamming distance.

If we fix a min or value of acceptance if $d_k(B, B_i) = t$, t a pre determined value by the researcher expert.

This is also one of the mode of analysis and acceptance of a experts opinions.

Likewise we in case of FRM and FCMs also find the average value of the experts who work with the problem and then find the Kosko-Hamming distance between the resultant state vector of an initial vector using the average matrix and the resultant state vector of each of the experts.

This will be illustrated by an example or two.

Example 2.11: Let us consider the following FREs whose limit sets are already given in example.

$$\text{Let } P_1 = \begin{bmatrix} 0 & 0.6 & 0 & 0.7 & 0.8 & 0.6 & 0.8 \\ 0.3 & 0.1 & 0.6 & 0.6 & 0 & 0.5 & 0.7 \\ 0.6 & 0.8 & 0 & 0.7 & 0.7 & 0.5 & 0.6 \\ 0 & 0 & 0.7 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0.3 & 0 & 0.2 & 0 \\ 0.1 & 0 & 0 & 0.2 & 0 & 0.1 & 0 \\ 0.2 & 0 & 0.7 & 0.6 & 0 & 0.5 & 0.7 \end{bmatrix}$$

be the membership matrix of the FRE given by the first expert.

$$P_2 = \begin{bmatrix} 0.1 & 0.7 & 0.1 & 0.8 & 0.9 & 0.7 & 0.8 \\ 0.4 & 0.2 & 0.7 & 0.7 & 0.1 & 0.4 & 0.6 \\ 0.7 & 0.8 & 0.1 & 0.7 & 0.1 & 0.6 & 0.6 \\ 0.2 & 0.1 & 0.8 & 0.8 & 0 & 0.1 & 0 \\ 0.5 & 0 & 0.6 & 0.4 & 0 & 0.2 & 0.3 \\ 0.1 & 0.1 & 0.2 & 0.3 & 0.1 & 0.1 & 0.1 \\ 0.3 & 0 & 0.8 & 0.4 & 0 & 0.4 & 0.8 \end{bmatrix}$$

be the membership matrix given by the second expert.

$$P_3 = \begin{bmatrix} 0.1 & 0.6 & 0 & 0.6 & 0.8 & 0.7 & 0.8 \\ 0.2 & 0.1 & 0.5 & 0.6 & 0.1 & 0.5 & 0.7 \\ 0.6 & 0.9 & 0.2 & 0.8 & 0.7 & 0.8 & 0.7 \\ 0.4 & 0.5 & 0.7 & 0.6 & 0.1 & 0.2 & 0.1 \\ 0.6 & 0.5 & 0.6 & 0.5 & 0.1 & 0.2 & 0.5 \\ 0.5 & 0.4 & 0.1 & 0.4 & 0.1 & 0.2 & 0.4 \\ 0.2 & 0.6 & 0.4 & 0.6 & 0.1 & 0.6 & 0.7 \end{bmatrix}$$

be the membership matrix of the FRE given by the third expert.

$$P_4 = \begin{bmatrix} 0.1 & 0.7 & 0 & 0.7 & 0.8 & 0.8 & 0.2 \\ 0.5 & 0.5 & 0.6 & 0.7 & 0.4 & 0.6 & 0.7 \\ 0.7 & 0.7 & 0.5 & 0.4 & 0.7 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.5 & 0.6 & 0.1 & 0.2 & 0.4 \\ 0.6 & 0.4 & 0.6 & 0.7 & 0.4 & 0.3 & 0.5 \\ 0.4 & 0.2 & 0.4 & 0.5 & 0.2 & 0.1 & 0 \\ 0.2 & 0.1 & 0.7 & 0.5 & 0 & 0.5 & 0.5 \end{bmatrix}$$

be the membership matrix of the FRE given by the fourth expert.

Let

$$P_5 = \begin{bmatrix} 0 & 0.6 & 0 & 0.6 & 0.7 & 0.7 & 0.8 \\ 0.3 & 0.1 & 0.5 & 0.5 & 0 & 0.5 & 0.7 \\ 0.5 & 0.7 & 0.1 & 0.6 & 0.6 & 0.5 & 0.6 \\ 0.1 & 0 & 0.7 & 0.6 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0 & 0.6 & 0.3 & 0 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.2 & 0.1 & 0.1 & 0.2 \\ 0.3 & 0.1 & 0.7 & 0.6 & 0.1 & 0.5 & 0.6 \end{bmatrix}$$

be the membership matrix of the FRE given by the fifth expert.

$$P = \frac{1}{5}(P_1 + P_2 + P_3 + P_4 + P_5)$$

$$= \begin{bmatrix} 0.06 & 0.64 & 0.02 & 0.68 & 0.8 & 0.7 & 0.68 \\ 0.34 & 0.2 & 0.58 & 0.62 & 0.12 & 0.5 & 0.68 \\ 0.62 & 0.78 & 0.18 & 0.64 & 0.56 & 0.6 & 0.6 \\ 0.26 & 0.22 & 0.68 & 0.64 & 0.06 & 0.12 & 0.12 \\ 0.36 & 0.18 & 0.6 & 0.44 & 0.1 & 0.22 & 0.28 \\ 0.24 & 0.16 & 0.16 & 0.32 & 0.1 & 0.12 & 0.14 \\ 0.2 & 0.08 & 0.66 & 0.54 & 0.06 & 0.5 & 0.66 \end{bmatrix}.$$

Let X = (0.7, 0.5, 0.7, 0.4, 0.8, 0.6, 0.3) be the given vector for which we solve the equation.

P_1 o X = (0.8, 0.6, 0.7, 0.7, 0.6, 0.2, 0.7)
= R_1.

P_2 o X = (0.8, 0.7, 0.7, 0.7, 0.6, 0.3, 0.7)
= R_2.

P_3 o X = (0.8, 0.5, 0.7, 0.7, 0.6, 0.5, 0.6)
= R_3.

$$P_4 \text{ o } X = (0.8, 0.6, 0.7, 0.6, 0.6, 0.4, 0.7) = R_4.$$

$$P_5 \text{ o } X = (0.7, 0.5, 0.6, 0.7, 0.6, 0.2, 0.7) = R_5.$$

and

$$P \text{ o } X = (0.8, 0.58, 0.62, 0.68, 0.6, 0.32, 0.66) = R \cong (0.8, 0.6, 0.6, 0.7, 0.6, 0.3, 0.7).$$

Now we find the Kosko-Hamming distance $d_k(R_i, R)$, $1 \leq i \leq 5$ and $d_k(R_i, R_j)$ $i \neq j$, $1 \leq i, j \leq 5$.

By this study of Kosko-Hamming distance we learn how much the opinion of two experts are close or otherwise.

By finding $d_k(R_i, R)$ we find how close an expert is form the average FRE.

$d_k(R_1, R) = 2$,
$d_k(R_2, R) = 2$,
$d_k(R_3, R) = 4$,
$d_k(R_4, R) = 3$ and
$d_k(R_5, R) = 3$.

We see only the third expert varies from the average in 4 co ordinates. Experts 4 and 5 vary from the average in three coordinates and so on.

$d_k(R_1, R_2) = 2$,
$d_k(R_1, R_3) = 6$,
$d_k(R_1, R_4) = 2$,
$d_k(R_1, R_5) = 3$,
$d_k(R_2, R_3) = 6$,
$d_k(R_2, R_4) = 3$,
$d_k(R_2, R_5) = 4$,
$d_k(R_3, R_4) = 4$,

$$d_k(R_3, R_5) = 4 \text{ and}$$
$$d_k(R_4, R_5) = 5.$$

We see when even the expert three is compared with the other experts give a bigger deviation.

However experts 4 and 5 differ, very highly deviant. Thus we think of other functions to find the differences.

We fix a predetermined parameter α; $0 \leq \alpha \leq 1$ and using that $\propto$ to find the closeness or deviation.

We define a notion called Kosko-Hamming distance deviation function $f_k^d(R_i, R_j) = |x_i - x_j|$ thus

$$f_k^d : (R_i, R_j) \to (x_1, \ldots, x_7) \in [0, 1]^7$$

We will illustrate this situation by some examples.

$$f_k^d(R_1, R_2) = (0, 0.1, 0, 0, 0, 0.1, 0).$$

Suppose the parameter α is taken as 0.2 we get how far the Kosko-Hamming deviation function is away from the parameter.

$$f_k^d(R_1, R_2) = (0, 0.1, 0, 0, 0, 0.1, 0).$$

So the variation in the expert one and two is negligible

$$f_k^d(R_1, R_3) = (0, 0.1, 0, 0, 0, 0.3, 0.1).$$

Some how as far as the 6^{th} attributes is concerned it is deviant apart from that the variation is negligible.

$$f_k^d(R_1, R_4) = (0, 0, 0, 0.1, 0, 0.2, 0).$$

Experts one and four are not deviant from each other.

$$f_k^d(R_1, R_5) = (0.1, 0.1, 0.1, 0, 0, 0, 0).$$

Experts one and five do not vary.

f_k^d (R_2 R_3) = (0.1, 0.2, 0, 0.3, 0.2, 0.3, 0.4).

Clearly experts two and three are very much deviant in their opinion.

f_k^d (R_2, R_4) = (0, 0.1, 0, 0.1, 0, 0.1, 0).

Experts two and four agree upon their results so not deviant.

f_k^d (R_2, R_5) = (0.1, 0.2, 0.1, 0, 0, 0.1, 0).

So the two experts mostly agree on their solution.

f_k^d (R_3, R_4) = (0, 0.1, 0, 0.1, 0, 0.1, 0.1).

The experts R_3 and R_4 also agree on their solution.

f_k^d (R_3, R_5) = (0.1, 0, 0.1, 0, 0, 0.3, 0.1).

There is some deviation between experts 3 and 5.

f_k^d (R_4, R_5) = (0.1, 0.1, 0.1, 0.1, 0, 0.2, 0).

Experts four and five are almost agreeable on this resultant.

We find

f_k^d (R, R_1) = (0, 0, 0.1, 0, 0, 0, 0)

thus the opinion of the first expert and the average opinion are one and the same.

f_k^d (R, R_2) = (0, 0.1, 0.1, 0, 0, 0, 0).

The second expert opinion and the average opinion are the same.

$$f_k^d\ (R_1, R_3) = (0, 0.1, 0.1, 0, 0, 0.2, 0.1).$$

The third experts opinion and the average opinion are the same

$$f_k^d\ (R_1, R_4) = (0, 0.1, 0.1, 0, 0, 0.1, 0).$$

The deviation of the fourth expert and the average is almost the same.

$$f_k^d\ (R, R_5) = (0.1, 0.1, 0, 0, 0, 0.1, 0).$$

The opinion of the fifth expert and the average resultant are nearly the same.

This is the way the model's results are analysed.

Finally we find

$f_k^d\ (S, R_i)$ where $S = (s_1, \ldots, s_7)$,
the assumed limit set of the FRE.

$$S = (0.6, 0.5, 0.6, 0.5, 0.6, 0.4, 0.6)$$

$$f_k^d (S, R_1) = (0.2, 0.1, 0.1, 0.2, 0, 0.2, 0.1)$$

$$f_k^d (S, R_2) = (0.2, 0.2, 0.1, 0.2, 0, 0.1, 0.1)$$

$$f_k^d (S, R_3) = (0.2, 0, 0.1, 0.2, 0, 0.1, 0)$$

$$f_k^d (S, R_4) = (0.2, 0.1, 0.1, 0.2, 0, 0, 0.1)$$

$$f_k^d (S, R_5) = (0.1, 0, 0, 0.2, 0, 0.2, 0.1)$$

and

$$f_k^d(S, R) = (0.2, 0.1, 0, 0.2, 0, 0, 0.1).$$

The deviation from the limit set to all the resultant vectors including the average happens to be acceptable with the pre assigned parameter.

We have already introduced in [103], the concept of new average FRM and the new average FCMs. Fore more about these concepts please refer [103].

Now we see how the resultant varies from the average resultant using the new technique of Kosko-Hamming distance deviance function.

First we will illustrate this situation by an example of FCM.

For the FCM given in example 2.4 of this book we find the average of the three connection matrices M_1, M_2 and M_3.

Let $M = \frac{1}{3}[M_1 + M_2 + M_3]$.

Using this M we can find the hidden pattern for any state vector and find for this resultant the Kosko-Hamming distance and also the Kosko-Hamming distance deviation function.

This measures how much two experts agree upon any arbitrary initial state vector.

Also the study helps in the analysis of how much each expert varies from the average connection matrix. Such study is innovative and shows a lot of analysis can be done also about each and every expert in particular and the system in general.

On similar lines we can for any collection of say some n experts opinion using FRM with same number of row attributes and same number of column attributes find the average, say if

$N_1, N_2, \ldots, N_n$ are the n connection matrices given by the n-experts then let $N = \frac{1}{n}(N_1 + \ldots + N_n)$

Now we can find for any initial state vector X the resultant hidden pattern pair say (P_1, M_1) using the collection matrix N_1;

(P_2, M_2) related to the connection matrix N_2 and so on (P_n, M_n) be the hidden pattern pair of X using the connection matrix N_n.

Let (P, M) be the hidden pattern pair of X using the average connection matrix N.

We find the Kosko-Hamming distance pair given by

$$\{(P_i, P_j), (M_i, M_j)\};\ i \neq j,\ 1 \leq i, j \leq n.$$

Using this pair of integers we can conclude the closeness or non-closeness in their opinion regarding the initial state vector. Thus this way of analysis is new and Kosko-Hamming distance and Kosko-Hamming distance deviation function helps one to study these concepts. This method helps in analysis experts closeness as well as the systems intricacies.

Next we proceed onto work or define Kosko-Hamming distance in case of Bidirectional Associative Memories (BAMs). The concept of BAM is recalled in chapter I from [Kosko]. Here we only indicate the functioning on BAMs.

We use the synaptic connection matrix of the BAM model.

$$\text{Let } M = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ a_{21} & \ldots & a_{2n} \\ \vdots & & \vdots \\ a_{m1} & \ldots & a_{mn} \end{bmatrix}$$

be m × n matrix.

For any initial input vector

$X_k = (a_1, \ldots, a_n)$ we find $S(X_k)M = Y_{k+1}$.

$S(Y_{k+1}) = X_{k+2}$ and so on.

We set say $S(X_{k+t})$ as a fixed point.

Then the binary pair (A, B) represents a fixed point of a BAM model or the dynamic system.

Suppose M_1 be the synaptic connection matrix of the BAM model for the same problem by another expert.

For same X_k we find the fixed point binary pair say (C, D).

We find $d_k((A, B), (C, D))$

$= (d_k (A, C), d_k(B, D))$
$= (r, s)$ gives the Kosko-Hamming distance pair.

We will illustrate this situation by some examples.

Example 2.12: Let 3 experts work on a problem using BAM model.

Let M_1, M_2 and M_3 be the synaptic connection matrices given by the three experts.

$$M_1 = \begin{bmatrix} -3 & -2 & 2 & 0 & -1 \\ -4 & 3 & 0 & -1 & 2 \\ 3 & 2 & -2 & -3 & 3 \\ -2 & -1 & 1 & 2 & -3 \\ 4 & 3 & 2 & 0 & 3 \\ 4 & 3 & -2 & -1 & 3 \\ 2 & 0 & -2 & -2 & 0 \end{bmatrix}$$

be the synaptic matrix given by the who works on the scale [–5, 5].

$$\text{Let } M_2 = \begin{bmatrix} -2 & -3 & 3 & 0 & -2 \\ -4 & 3 & 0 & -2 & 3 \\ 3 & 2 & -2 & -2 & 4 \\ -2 & -2 & 2 & 3 & 4 \\ 4 & 3 & 3 & 0 & 3 \\ 3 & 3 & -2 & -1 & 4 \\ 2 & 0 & -3 & -3 & 0 \end{bmatrix}$$

be the synaptic matrix given by the second expert who also works only with the same scale [–5, 5].

$$\text{Let } M_3 = \begin{bmatrix} -4 & -1 & 4 & 0 & -3 \\ -4 & 3 & 0 & -3 & 1 \\ 3 & 2 & -2 & -1 & 2 \\ -2 & -3 & 3 & 1 & 2 \\ 4 & 3 & 1 & 0 & 3 \\ 2 & 3 & -2 & -1 & 2 \\ 2 & 0 & -1 & -1 & 0 \end{bmatrix}$$

be the synaptic matrix given by the expert on the interval [–5, 5]

$$\text{Let } X_k = (-4, -3, 1, 2, 5, -3, -1)$$

be the initial input vector.

Consider $S(X_k) = (0\ 0\ 1\ 1\ 1\ 0\ 0)$;

$$S(X_k)M_1 = (5, 4, 1, -1, 3)$$

$$= Y_{k+1}$$

$$S(Y_{k+1})\ M_1^t = (-4, 1, 6, -5, 5, 8, 0)$$

$$= X_{k+2}$$

$$S(X_{k+2}) = (0\ 1\ 1\ 0\ 1\ 1\ 0)$$

$$S(X_{k+2})\ M_1 = (7, 11, –2, –5, 11)$$

$$= Y_{k+3}$$

$$S(Y_{K+3}) = (1\ 1\ 0\ 0\ 1)$$

$$S(Y_{K+3})\ M_1^t = (–6, 1, 8, –6, 10, 10, 2)$$

$$= (X_{K+4})$$

$$S(Y_{K+4}) = (0\ 1\ 1\ 0\ 1\ 1\ 1)$$

$$S(X_{k+4})\ M_1 = (5, 11, –4, –4, 11)$$

$$= Y_{K+5}$$

$$S(Y_{K+5}) = (1\ 1\ 0\ 0\ 1).$$

Thus the binary pair {(0 1 1 0 1 1 1), (1, 1, 0, 0, 1)} = A_1 represents the fixed point of the dynamical system M_1.

Now we find the effect of

$$X_K = (–4, –3, 1, 2, 5, –3, –1)$$

as the initial output vector.

$$S(X_K) = (0\ 0\ 1\ 1\ 1\ 0\ 0)$$

$$S(X_K)M_2 = (5, 3, 3, 1, 11) = Y_{K+1}$$

$$S(Y_{K+1}) = (1\ 1\ 1\ 1\ 1)$$

$$S(Y_{K+1})\ M_2^t = (–4, 0, 5, 5, 1, 3, 7, –4)$$

$$= X_{K+2}.$$

$S(X_{K+2}) = (0\ 0\ 1\ 1\ 1\ 1\ 0)$

$S(X_{K+2})M_2 = (8\ 6\ 5\ 0\ 15) = Y_{K+3}$

$S(Y_{K+3}) = (1\ 1\ 1\ 0\ 1)$

$S(Y_{K+3})\ M_2^t = (-4, 2, 7, 2, 13, 8, -1)$

$= X_{K+4}$

$S(X_{K+4}) = (0\ 1\ 1\ 1\ 1\ 1\ 0)$

$S(X_{K+4})\ M_2 = (4, 9, 1, -2, 18)$

$= Y_{K+5}$

$S(Y_{K+5}) = (1\ 1\ 1\ 0\ 1)$.

The fixed binary pair is $\{(0\ 1\ 1\ 1\ 1\ 1\ 0), (1\ 1\ 1\ 0\ 1)\} = A_2$

$$d(A_1, A_2) = (2, 1).$$

Consider the same initial input vector.

$Y_k = (-4, -3, 1, 2, 5, -3, -1)$

$S(Y_K) = (0\ 0\ 1\ 1\ 1\ 0\ 0)$

We find $S(X_K)\ M_3 = (5, 2, 2, 0, 7)$

$= Y_{K+1}$

$S(Y_{K+1}) = (1\ 1\ 1\ 0\ 1)$

$S(Y_{K+1})\ M_3^t = (-4, 0, 5, 0, 11, 5, 1)$
$= X_{K+2}$

$S(X_{K+2}) = (0\ 0\ 1\ 0\ 1\ 1\ 1)$.

$$S(X_{K+2})\,M_3 = (11, 8, -4, -3, 7)$$
$$= Y_{K+3}$$

$$S(Y_{K+3}) = (1\ 1\ 0\ 0\ 1)$$

$$S(Y_{K+3})\,M_3^t = (-8, 0, 7, -3, 10, 7, 2)$$
$$= X_{K+4}$$

$$S(X_{K+4}) = (0\ 0\ 1\ 0\ 1\ 1\ 1)$$

$$S(X_{K+4})M_3 = (11, 8, -4, -3,7)$$
$$= Y_{K+5}$$

$$S(Y_{K+5}) = (1\ 1\ 0\ 0\ 1)$$

$$S(Y_{k+6}) = (0\ 0\ 1\ 0\ 1\ 1\ 1)$$
$$= S(X_{K+4})$$

Thus the fixed binary pair
$A_3 = \{(0\ 0\ 1\ 0\ 1\ 1\ 1), (1\ 1\ 0\ 0\ 1)\}$.

$$d_k(A_1, A_3) = \{(2, 0)\}.$$

$$d_k(A_2, A_3) = \{(2, 1)\}.$$

Thus A_1 is more agreeable with A_3. More or less the three experts are not very deviant in their opinion for this X_k.

Likewise we can find the closeness or the deviation from expert to expert.

Thus the new notion of Kosko-Hamming distance pair will help one to analyse the deviation or the closeness of two experts.

We can also find the Kosko-Hamming distance pair between each of the experts and the average value.

Let

$$M = 1/3\ (M_1 + M_2 + M_3)$$

$$= \begin{bmatrix} -3 & -2 & 3 & 0 & -2 \\ -4 & 3 & 0 & -2 & 2 \\ 3 & 2 & -2 & -2 & 3 \\ -2 & -2 & 2 & 2 & 3 \\ 4 & 3 & 2 & 0 & 3 \\ 3 & 3 & 2 & -1 & 3 \\ 2 & 0 & -2 & -2 & 0 \end{bmatrix}.$$

$X_K = (-4, -3, 1, 2, 5, -3, -1)$

$S(X_K) = (0\ 0\ 1\ 1\ 1\ 0\ 0)$

$S(X_K)M = (5\ 3\ 2\ 0\ 9) = Y_{K+1}$

$S(Y_{K+1}) = (1\ 1\ 1\ 0\ 1)$

$S(Y_{K+1})\ M^t = (-4, 1, 6, 1, 12, 11, 0) = X_{K+2}$

$S(X_{K+2}) = (0\ 1\ 1\ 1\ 1\ 1\ 0)$

$S(X_{K+2})M = (4\ 9\ 4\ -3\ 14) = Y_{K+3}$

$S(Y_{K+3}) = (1\ 1\ 1\ 0\ 1).$

Thus the fixed binary pair of the dynamical system M is a

$$A = \{(0\ 1\ 1\ 1\ 1\ 1\ 0), (1\ 1\ 1\ 0\ 1)\}.$$

Now we find the Kosko-Hamming distance

$d_k(A, A_1) = \{(2, 1)\}$ and
$d_k(A, A_2) = \{(0, 0)\}$.
$d_k(A, A_3) = \{(2, 1)\}$.

Thus the Kosko-Hamming distance from the average dynamical system is very close to the systems M_1, M_2 and M_3 and infact the value of A_2 and A are the same.

Thus for the first time we have introduced the new notion of New average BAM by an example.

Infact if n-experts work on a problem using BAM and if M_1, M_2, …, M_n are the associated synaptic matrices all working in the interval [–s, s].

We construct the new synaptic matrix as

$$M = \frac{1}{n}\sum_{i=1}^{n} M_i = (m_{ij}),$$

if m_{ij} is not an integer say $m_{ij} = p.q$ where q is the decimal part and if $q > 0.5$ make $p.q = p + 1$ if $q \leq 0.5$ make $p.q = p$.

This is the way by which M has all its values to be integers.

This was illustrated by an example with $n = 3$.

Now we find Kosko-Hamming distance $d_k(A_i, A_j)$ where A_i is the fixed point pair associated with the initial state vector X_k of the ith expert who has given the opinion using the synaptic matrix M_i. The same is true for A_j, $i \neq j$, $1 \leq i, j \leq n$.

Thus $d_k(A_i, A_j)$ gives a pair which show the closeness or deviation of the expert A_i with A_j; $1 \leq i, j \leq n$.

Now $d_k(A, A_i)$, the Kosko-Hamming distance gives a pair of the ith expert opinion on the initial state vector X_k with the average synaptic matrix A for the same initial state vector X_k, $i = 1, 2, …, n$.

This is the way the deviation or closeness of each expert with the average for any initial state vector is analysed.

Now for the given initial state vector X_k using the n × m synaptic connection matrix M if $S(X_{k+r})$ is a fixed point then we find the Kosko-Hamming distance $d_k(S(X_{k+r}), X_k) = t$; t a number lying in $0 \leq t \leq (n-1)$.

We define X_k to be the powerful initial state vector if t is the largest value when compared with all the appropriate X_k and $S(X_{k+r})$ using Kosko-Hamming distance we call that initial state vector as the most powerful state vector.

If another X_{K_1} gives $S(X_{K_1+s})$ and if the Kosko-Hamming distance between X_{K_1} and $S(X_{K_1+s})$ is less than t but is the next largest value then we call X_{K_1} as the more powerful initial state vector and so on.

Thus the Kosko-Hamming distance measures the power of each initial state vector; as most powerful, more powerful, just powerful, least powerful, not powerful and so on.

Next one can proceed onto study this for NCMs, NRMs, NREs and NBAMs the concept of Kosko-Hamming distance and Kosko-Hamming distance deviation function. We feel such study is a matter of routine and can be easily carried out with some simple and appropriate modifications.

We will illustrate these situations by an example or two.

Let us consider n experts working on the same problem with the same number of concepts using the Neutrosophic Cognitive Maps Model.

Let suppose $S_1, S_2, \ldots, S_n$ be the n connection neutrosophic matrices associated with the n-experts of the NCMs model.

For any fixed initial state vector X we find the hidden patterns using S_1, S_2, …, S_n and let them be denoted by Y_1, Y_2, …, Y_n.

We find the Kosko-Hamming distance $d_k(Y_i, Y_j) = m_t$, $i \neq j$, $1 \leq i, j \leq n$. This number m_t shows whether two experts opinion are close are very deviated or deviated.

Further we can find the New Average NCM of these n NCMs as

$$S = 1/n\ (S_1 + S_2 + \ldots + S_n).$$

For that X if Y is the hidden pattern then we find the n-Kosko-Hamming distance for Y and Y_i, i = 1, 2, …, n that is we find $d_k(Y, Y_i) = P_t$ for each i. If P_t is small we conclude the deviation from the average NCM model of each of the expert is negligible, otherwise it is large.

Such study can be done only using Kosko-Hamming distance. Finally $d_k(X, Y_i)$ gives the most influential node ($1 \leq i \leq n$) or more influential node or so on. We can also say which expert represents the average dynamical system.

Interested reader can construct one such model.

Now we can in case of NRM (Neutrosophic Relational Maps) also consider or use Kosko-Hamming distance on a pair of fixed pair of points.

Thus for instance if n experts work on a problem using Neutrosophic relational Maps model. All the experts use the same set of domain attributes and range attributes.

Here we denote the n matrices by P_1, P_2, …, P_n. If X is the initial state vector from the domain space whose effect we want to study on the n dynamical systems given by the experts.

Let $Y_1, Y_2, \ldots, Y_n$ be the hidden pattern of the n experts using the dynamical systems $P_1, P_2, \ldots, P_n$ respectively.

We find

$$d_H(Y_i, Y_j) = t_{ij} \text{ (say) } i \neq j;\ t_{ij} \geq (0, 0).$$

If t_{ij} is a small pair of numbers we assume the experts more or less agree on the outcome of the initial state vector X. This is important to note that any two experts need not totally agree with each other on every initial state vector. Sometimes on certain initial state vectors the two experts may disagree, those state vectors can be specially analysed.

It is at the juncture the Kosko-Hamming distance plays a vital role.

Further when we find the Kosko-Hamming weight $d_H(Y_i, X)$ for i=1, 2, …, n, we can find the nature of the nodes. The Kosko-Hamming weight will make one know the most influential node and so on.

This is the way the notion of Kosko-Hamming distance and Kosko-Hamming weight play a vital role in the NCMs model.

It can be carried as a matter of routine so it is left as an exercise to the reader to give examples of this situation.

Next we can as in case of FCMs define the notion of New Simple Average NCMs (NASNCMs) in the case of NRMs also.

Let

$$N = 1/n\,(N_1 + \ldots + N_n) = (n_{ij}).$$

If $n_{ij} = a + bI$ and both a and b less than $\lfloor n/2 \rfloor$ then $n_{ij} = 0$

If $n_{ij} = a + bI$ and both a and b greater than equal to $\lfloor n/2 \rfloor$ then $n_{ij} = 1 + I$.

If $n_{ij} = a$ and $a < \lfloor n/2 \rfloor$ then $n_{ij} = 0$.

If $n_{ij} = a$ and $a \geq \lfloor n/2 \rfloor$ then $n_{ij} = 1$.

If $n_{ij} = bI$ and $b < \lfloor n/2 \rfloor$ then $n_{ij} = 0$.

If $n_{ij} = bI$ and $b \geq \lfloor n/2 \rfloor$ then $n_{ij} = I$.

If $n_{ij} = a + bI$ and $a \geq \lfloor n/2 \rfloor$ $b < \lfloor n/2 \rfloor$ then $n_{ij} = 1$.

If $n_{ij} = a + bI$ and $a < \lfloor n/2 \rfloor$ and $b \geq \lfloor n/2 \rfloor$ then $n_{ij} = I$.

This is the way the NSANCM matrix is constructed.

Now if Y is the hidden pattern associated with the dynamical system N.

We find $d_k(Y, Y_i)$; i = 1, 2, …, n. If $d_k(Y, Y_i)$ is small we see the deviation of the experts opinion from the expert is acceptable if other way we see that expert has a different view on the problem and we start to study that expert more closely and also analyse his / her closeness with other experts same procedure is done using Kosko-Hamming distance and Kosko-Hamming weight.

Next we can define for Neutrosophic Relational Maps (NRMs) model the concept and Kosko-Hamming distance and Kosko-Hamming weight.

In this case it will be Kosko-Hamming distance pair and Kosko-Hamming weight pair.

We will first describe this situation.

Let m experts work on a problem all of them use only Neutrosophic Relational Maps (NRMs) model.

Let $S_1, S_2, \ldots, S_m$ be then m $t \times s$ neutrosophic matrices given by the m experts. Expert 1, expert 2, …, expert m respectively.

Now let us work with the domain space nodes which is of t dimension that is $D = \{(a_1, \ldots, a_t) \mid a_i \in \{0, 1\}, 1 \leq i \leq t\}$ and the range space $R = \{(b_1, \ldots, b_s) \mid b_j \in \{0, 1\}, 1 \leq j \leq s\}$ of the nodes associated with the neutrosophic system dynamical system.

Let X be a $1 \times t$ row matrix associated with the domain space with some one node in the on state which is taken as the initial state vector.

We find the hidden pattern pair which may be a fixed pair or a limit cycle pair.

Let XS_i gives the hidden pattern of the dynamical system given by (P_i, M_i) where P_i is a $1 \times t$ resultant state vector and M_i is a $1 \times s$ resultant state vector from the domain and range space respectively. The same procedure is repeated for every dynamical system $i = 1, 2, \ldots, m$.

Now we find the Kosko-Hamming distance pair between any two experts i and j.

Let (P_i, M_i) and (P_j, M_j) be the fixed point pair given by the experts i and j.

The Kosko-Hamming distance pair is $d_k((P_i, M_i), (P_j, M_j))$

$= d_k((P_i, P_j), (M_i, M_j))$

$= (a_{ij}, b_{ij})$ (both a_{ij} and b_{ij} are values greater than or equal 0, integers).

If (a_{ij}, b_{ij}) is very small the experts agree upon the resultant on the specific vector.

If the deviation is very large we see the experts differ and a study is made on those initial state vectors over which experts differ largely.

Another method of this is if we find Y to be a 1 × s initial state vector from the range space. Now using the neutrosophic dynamical system S_i we find YS_i^t.

If the hidden pattern is a fixed point pair say (A_i, B_i), and if the using S_j we get (A_j, B_j) to be the fixed point pair we find the Kosko-Hamming distance $d_k((A_i, B_i), (A_j, B_j)) = d_k((A_i, A_j), (B_i, B_j)) = (a_{ij}, b_{ij})$ where both a_{ij} and b_{ij} are positive integers or zero.

If (a_{ij}, b_{ij}) is a small pair of integers then we say the experts i and j agree upon that specific initial state vector. If the pair is large we see they differ and study the situation for that initial state vector and about those two experts.

Thus i and j can vary from 1 to m ($i \neq j$). This is the way the Kosko-Hamming distance pair plays a role in the analysis of the NRMs.

Now we can also find Kosko-Hamming weight pair in case of the initial state vector with the hidden pattern. For instance if for a initial state vector X from the domain space the resultant is (A, B), where A is a 1 × t row vector and B is a 1 × s row vector from the domain and range space respectively.

Now we find then Kosko-Hamming weight pair which is as follows:

$$d_k((A, B), (X, (0))$$

$$= (m, n) \text{ where } n = \text{weight of B.}$$

If both m and n are very large then we say the initial state vector X for which if C_p is the associated node then the pth node of the domain space to be the most influential node.

If B also happens to be so we say this pair is the most influential node pair.

Likewise we can define more influential node, just influential node, influential node, least influential node and so on. We see some time the pair will be so at other time only one of them will be so.

Now we can as in case of NASNCMs define the notion of NANRMs also, which we will describe in a line or two.

Let $S_1, S_2, \ldots, S_m$ be the m, NRMs matrices given by the m experts.

We find $S = 1/m\ (S_1 + S_2 + \ldots + S_m)$

$= (s_{ij})$

where s_{ij}'s have been remade as in case of NASNCMs described earlier.

Now we using the NANRMs dynamical system find for initial vector X from the domain space D.

Let XS lead to the hidden pattern pair (V, W) where V is in the domain space and W is in the range space.

We find the Kosko-Hamming distance pair

$d_k((A_i, B_i), (V, W))$

$= d_k((A_i, V), (B_i, W))$

$= (e, f)$ (e and f positive integers including zero).

Now seeing the pair (e, f) we can predict whether for this initial state vector X the i^{th} expert is close to the average value or not.

We can for the NANRMs find the most influential node, more influential node, and so on using the notion of Kosko-Hamming weight pair.

This study of going from FRMs to NRMs is a matter of routine and hence left as an exercise to the reader.

The reader is advised to adopt Kosko-Hamming distance pair method of analysis for atleast one real world problem.

Next we give a brief description of NRE (Neutrosophic Relational Equations) model in the following.

For more about NREs model refer [90].

Let some n experts work on the same problem using the Neutrosophic Relational Equations (NREs) model.

Let all them agree upon to work with the same set of limit set fixed value.

All the n experts work with same set of domain attributes and range attributes. Let $N_1, N_2, \ldots, N_n$ denote the set of n matrices associated with each of the n experts.

Here for any given matrix by the expert will be fuzzy neutrosophic matrix.

Any state vector X be taken we find min $\{X, N_i\} = P_i$, $i = 1, 2, \ldots, n$. Here we find as in case of FRE use the Kosko-Hamming distance function and find the value or distance between two experts P_i and P_j.

If the difference is negligible we accept that both the experts mostly agree upon their views regarding the vector X as in case of FREs.

Also we find the NANRE is given by and if
$N = 1/n (N_1 + N_2 + \ldots + N_n)$.

We find using N the effect of the initial state vector X. If P is the resultant we find use the Kosko-Hamming distance function d_k and find the distance between P and any experts opinion P_i.

If the deviation is negligible we accept otherwise analyse the cause for such large deviations.

It is left as an exercise for the reader to find suitable real valued problem and use NREs with some experts, construct the models use the Kosko-Hamming distance function to analyse the resultants given by the model.

Next we proceed to find the use of Kosko-Hamming distance in Neutrosophic Bidirectional Associative Memories (NBAMSs).

In the BAMs if we instead of an interval [–n, n] if use the interval [– (a + bI), (a + bI)], a and b equal integers that is [–(a + aI), a + aI] and if the entries of the neutrosophic synaptic matrix B takes its entries as integers values or a combination of both we then call the model as NBAM model. NBAM model function in a similar way to as that of a BAM model.

Let n experts work on the same problem in the same neutrosophic interval using same set of domain and range attributes.

Let $L_1, L_2, \ldots, L_n$ be the n Neutrosophic synaptic connection matrices of the n experts.

Suppose X_k be the initial state vector. Using X_k on L_i gives the pair of fixed points say ({S(X_i), S(Y_i)}.

Suppose for the same initial vector X_k, (S(X_j), S(Y_j)) be the fixed pair using the synaptic connection neutrosophic matrix L_j, $1 \leq i, j \leq n$.

Now we can find the Kosko-Hamming distance $d_k((S(X_i), S(Y_i)), (S(X_j), S(Y_j)))$.

Let $d_k((S(X_i), S(Y_i)), (S(X_j), S(Y_j))$

$= (d_k(S(X_i), S(X_j), d_k(S(Y_i), S(Y_j)))$

= (a, b), a and b be positive integers greater than or equal to zero.

If a and b are negligibly small we assume that both the experts more or less agree upon the resultant for that initial vector X_k.

Such comparative study can be made only by using the new notion of Kosko-Hamming distance.

We can also as in case of BAMs find the New Average Neutrosophic Bidirectional Associative Memories (NANBAM) model. Using this we can find Kosko-Hamming distance.

The working is also a matter of routine and hence left as an exercise to the reader. This model certainly saves time and money.

However we briefly describe how the New Average Neutrosophic Bidirection Associative Memories (NANBAM) model.

$$\text{Let } L = 1/n\,(L_1 + \ldots + L_n)$$

$$= (l_{ij}).$$

We see if $l_{ij} = a + bI$ and both $a = p.q$ and $b = s.t$ then

$a = p$ if $q \leq 0.5$

$a = p + 1$ if $q > 0.5$.

Similar procedure is carried out for $b = s.t$.

(Here p is the integer part and q is the decimal part in a = p.q).

Now using that L we find for X_k; $S(X_k)L$.

If (S(X), S(Y)) is the fixed pair of hidden pattern then we can find using Hamming Kosko distance, how far the i^{th} expert varies from the average synaptic connection matrix. Such study can reveal how far NANBAM average differs from the experts opinion on any initial vector X.

Now it is pertinent to keep on record that we can also find mixed special average NFCM, mixed special average NFRM, mixed special average NFRE and mixed special average NBAM.

We will just describe these in a line or two.

Suppose m + n experts want to work on a problem with m of them working using FCMs and n of them work using NCMs, all of them work only with the same set of concepts.

Now we can find the Special mixed Average NFCMs (SANFCMs) of these m + n dynamical system.

Let $S = \frac{1}{m+n}(P_1 + \ldots + P_m + R_1 + \ldots + R_n)$ where P_i's are the connection matrices of the m experts who have used FCMs and R_j's are the connection matrices of the n experts who have used the NCMs, $1 \le i \le m$, $1 \le j \le n$.

Let $S = (s_{ij})$ the value of s_{ij} is determined as usual.

If $s_{ij} = a + bI$, $a < \left\lfloor \frac{m+n}{2} \right\rfloor$ and $b < \left\lfloor \frac{m+n}{2} \right\rfloor$ then $s_{ij} = 0$.

If $s_{ij} = a + bI$, $a < \left\lfloor \frac{m+n}{2} \right\rfloor$ and $b \ge \left\lfloor \frac{m+n}{2} \right\rfloor$ replace by I.

If $s_{ij} = a + bI$, $a \geq \left\lfloor \frac{m+n}{2} \right\rfloor$ and $b \geq \left\lfloor \frac{m+n}{2} \right\rfloor$ then replace by 1+I.

If $a \geq \left\lfloor \frac{m+n}{2} \right\rfloor$ and $b < \left\lfloor \frac{m+n}{2} \right\rfloor$ then replace by 1.

So $S = (a_{ij})$; $a_{ij} \in \{0, 1, I, 1 + I\}$.

S is the dynamical system associated with the mixed New Average Neutrosophic Fuzzy Cognitive Maps (NANFCMs) model.

Likewise we calculate for the m + n FRMs and NRMs, where m experts work with FRMs and n experts work using NRMs. In the same way for m FREs and n NREs.

Suppose are see m experts work with BAMs on the same problem with same domain and range attributes and n experts work with NBAM on the same problem with the same set of attributes, we find the mixed special new average NBAM in the same way provided the BAMs experts works on the scale [–a, a] then NBAMs experts work on the scale [– (a + aI), a + aI].

We suggest a few problems for the reader in the following.

Problems:

1. Distinguish between Hamming distance and Kosko-Hamming distance.

2. What are the benefits of using Kosko-Hamming distance in FCMs model?

3. Construct a FCMs model for a real world problem with some t number of experts and use Kosko-Hamming distance to analyse the problem.

4. Prove Kosko-Hamming distance can be used to find the influential nodes of the directed graph of FCMs.

5. What are the advantages of using the Kosko-Hamming weight?

6. Show (Kosko-Hamming weight) of any resultant vector can be used to find the most influential node, more influential node and so on.

7. Prove Kosko-Hamming distance not only analyses the closeness or deviation two experts opinion but also has other functions on the dynamical system.

8. Prove by adopting Kosko-Hamming distance on FRMs which give a pair of values and using this notion analyse the model.

9. What are the advantages of using Kosko-Hamming distance on FRMs?

10. Study or introduce any other interesting properties derived using Kosko-Hamming distance on FRMs.

11. Show by examples Kosko-Hamming distance on FREs can be beneficial to analyse the experts opinion and the model.

12. Work with a set of experts using FRE and their comments using the Kosko-Hamming distance and Kosko-Hamming distance deviance functions.

13. Show Kosko-Hamming distance plays a vital role in studying the New Average FCMs and the various experts opinion using FCMs whose NASFCMs is formed.

14. Study the problem 13 in case of New Average FRMs.

15. Study the problem 13 in case of NAFREs.

16. Describe NCMs and the use of Kosko-Hamming distance and find influential nodes by implementing NCMs in a real world problem.

17. Distinguish between NASFCMs and NASNFCMs.

18. Describe the functioning of NANCMs.

19. Compare NANFCMs and NANCMs.

20. What are the advantages of using special NASNFCMs?

21. Use NRMs in the real world problem using a set of t experts.
    (i) Use Kosko-Hamming distance to study and analyse the model.
    (ii) Find NANRMs model. Using NANRMs model study its closeness with respect to experts.
    (iii) What are the advantages of using NANRMs model?

22. Find the special features associated with special NANFRMs in studying a problem.

23. What are the advantages of using NAFRMs in the place of special NANFRMs?

24. Distinguish between NANRMs and NANFRMs.

25. Show using the concept of Kosko-Hamming function / distance / weight in the BAMs model helps one to analyse the effect of every initial vector.

26. Define and describe the notion of NABAMs and the use of Kosko-Hamming distance.

27. Define, describe and develop a NBAMs model using a real world problem.

28. Using Kosko-Hamming distance for the NBAMs model, analyse the problem.

29. What are the advantages of using NABAMs?

30. What are the advantages of using NANBAMs?

31. Show the use and advantage of the special mixed NANBAMs.

32. Show using Kosko-Hamming distance and weight we can find the most influential node, more influential node, just influential node, influential node, less influential node, least influential node and so on.

33. Study and analyse the concepts described in problem 32 by constructing a real world problem using FCMs.

34. Study problem 32 and 33 in case of NCMs.

35. Show the problem 32 may be made use of in FRMs, NCMs and NRMs.

36. Illustrate each of the situations by some real world problems.

37. Show by example that influential nodes in general are not powerful nodes and vice versa.

38. Use the Kosko-Hamming distance function to analyse the FRE model.

39. Describe some of the special features associated with the Kosko-Hamming distance function.

40. Show Kosko-Hamming distance is different from usual Hamming distance.

41. What are the advantages of using Kosko-Hamming distance function in FREs?

42. Using the notion of Kosko-Hamming distance function in NREs to analyse a real world problem.

43. What are the merits of using NAFREs?

44. What are the advantages of using NANREs?

45. Define, describe and develop the notion of special NANFREs?

46. Compare special NANFRE with NAFRE?

47. Compare special NANFRE with NANRE?

48. Obtain any special and striking features enjoyed by Kosko-Hamming weight?

49. Obtain any special feature attached in the use of Kosko-Hamming distance in NRMs, FRMs and mixed special average NANFRMs.

50. Show the Kosko-Hamming weight in the above cases can predict the nature of the nodes.

51. Prove or disprove the most influential node related to one expert in general need not be the most influential node related to every other expert.

52. Can the same in problem 51 be said about powerful nodes?

53. Prove use of NASFCMs more powerful than combined FCMs in case of several experts.

54. Prove use of NASNCMs is better than combined NCMs in case of several experts.

55. Prove or disprove that it is always possible to get the connection matrix with entries from the set {0, 1} instead of the set {0, 1, –1} by redefining the nodes without changing the underlying principle.

56. Give any other method of analyzing the NASFCMs model.

57. Can we say NASFCMs model is a needed one in case of certain problems?

58. Prove the concept of indeterminacy is a powerful tool in generalizing most of the fuzzy models.

59. Enumerate by an example that use of Neutrosophic fuzzy models in case of certain problems.

60. Give an example of a problem in which neutrosophic concepts plays a vital role in the prediction of solutions.

61. Show use of NAFREs saves time.

62. Prove use of mixed NANFREs can both save time and economy.

63. Show by an example that the fluctuations of the resultant with predicted values can be best studied using the Kosko-Hamming distance function.

64. Obtain any other innovating results about this type of analysis of these models.

Chapter Three

DISTANCE IN MATRICES

In this chapter we define the concept of distance in general matrices of same order. We see as applications whether the distance in matrices affect the characteristic equations and characteristic values.

How the difference in matrices affect the code space is also analysed. Finally we study the effect of difference in matrices over the distinctness of the linear transformation and linear operations.

Also distance in matrices play a role in the study of experts opinion and the related fuzzy model. Finally the distance and its effect on the graph for graphs are represented by matrices.

Let us before we define this new concept justify our study to be a natural or generalization of Hamming distance.

Recall Hamming distance measures the distance between two row vectors of same order. Now the column matrix is nothing but the transpose of the row vector. When the Hamming distance is defined for row vectors then why not for the transpose of the row vector which is a column vector. Hence distance between two $m \times 1$ column matrices A and B is defined in a similar way.

Now we just define it.

DEFINITION 3.1: *Let A be a n × 1 column matrix and B be a n × 1 column matrix. The special distance between A and B defined by*

d(A, B) = (number of places in which A and B differ)

d(A, B) ≥ 0 and

d(A, B) = 0 if A and B are identical 0 ≤ d(A, B) ≤ n and d(A, B) = n if A and B are not the same on any of the coordinates.

We will illustrate this by some examples.

Example 3.1: Let

$$A = \begin{bmatrix} 0 \\ 7 \\ 2 \\ -1 \\ 3.7 \\ 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 7 \\ 2 \\ 0 \\ 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

be two 8 × 1 column matrices. The distance between A and B is 4 denoted by d(A, B) = 4.

Thus we see $d(A^t, B^t) = 4$ where $A^t = (0, 7, 2, -1, 3.7, 0, 0, 1)$ and $B^t = (0, 7, 2, 0, 1, 2, 3, 1)$.

Thus we can find the Hamming distance of a column matrix also.

Example 3.2: Let

$$M = \begin{bmatrix} 0.3 \\ 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 5 \end{bmatrix} \text{ and } N = \begin{bmatrix} 0.5 \\ 1 \\ 2 \\ 4 \\ 5 \\ 3 \\ 1 \end{bmatrix}$$

be two 7 × 1 column matrices d(M, N) = 5.

The authors see that Hamming distance in case of column matrices is nothing but the Hamming distance of transpose of them. So such definition is in keeping with the classical definition.

Next we proceed onto define special distance between two m × n matrices m ≠ 1 and n ≠ 1 (m can be equal to n).

DEFINITION 3.2: *Let M and N be two m × n matrices we say d(M, N) = number of places in which the m × n matrices M and N are different. Clearly the special distance d(M, N) ≥ 0 and 0 ≤ d(M, N) ≤ m × n.*

We will illustrate this by an example or two and then give the applications of them.

Example 3.3: Let

$$A = \begin{bmatrix} 0 & 5 & 2.1 & 7 \\ 8 & 1 & 0 & 3 \\ 1 & 2 & 5 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 5 & 1 & 7 \\ 8 & 1 & 0 & 8 \\ 1 & 2 & 3 & 0 \end{bmatrix}$$

be two 3 × 4 matrices d(A, B) = 3.

Clearly 0 ≤ d(A, B) ≤ 12.

We see d(A, A) = 0 and d(B, B) = 0.

Example 3.4: Let

$$A = \begin{bmatrix} 0.7 & 0.9 & 0.16 \\ 0 & 9.1 & 11 \\ 2 & 3 & 4 \\ 5 & 0 & 6 \\ 7 & 8 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1.2 & 0.9 & 0.16 \\ 0 & 0 & 11 \\ 2 & 3 & 4 \\ 5 & 0 & 0 \\ 7 & 8 & 1 \end{bmatrix}$$

be any two 5 × 3 matrices.

d(A, B) = 3.

So the distance between A and B is 3; that is A and B differ from each other in 3 places.

Example 3.5: Let

$$A = \begin{bmatrix} 0 & 2 & 3 & 4 & 5.1 & 6 & 7 \\ 8 & 9 & 0 & 1 & 2 & 3 & 0 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 0 & 2 & 3 & 4 & 5 & 6 & 0 \\ 8 & 9 & 0 & 1 & 2 & 4 & 2 \end{bmatrix}$$

be any two 2 × 7 matrices d(A, B) = 4, 0 ≤ d(A, B) ≤ 14.

Thus we see A and B differ in four places.

Example 3.6: Let

$$M = \begin{bmatrix} 0 & 3 & 4 \\ 5 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } N = \begin{bmatrix} 2 & 1 & 5 \\ 5 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

be two 3 × 3 matrices. The distance between them is defined as d(M, N) = 3. That is M and N differ in 3 places.

Now we study the eigen values the associated characteristic equations when two matrices have distance one, two, three and so on.

Next we study the codes associated with a k × n matrix (or a n–k × n parity check matrix) when the distance between them is one, two and so on.

Such study can later be used in storage systems as when difference is 1 we assume one error and see the impact of the error on the system. Such study is not only innovative but lead to several types of new methods in retrieving the stored data and so on.

Further we can use the generator matrix and parity check matrix. When in the matrix a change is made we study the codes when a single change is made. Study in the difference, that can make into two distinct codes or sometimes the same type of code.

Finally we see the applications of the difference in the expert's dynamical systems on the same problem and study the difference of these matrices. Such study will help in further analysis of the problem. Also in case these codes sometimes the hidden small changes can guarantee security.

First we study the case of eigen values and eigen vectors.

Example 3.7: Let

$$A = \begin{bmatrix} 3 & 0 \\ 7 & 2 \end{bmatrix}$$

be a 2 × 2 matrix. The eigen values and the characteristic equation of A is as follows.

$$|A - \lambda| = \left\| \begin{bmatrix} 3 & 0 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right\|$$

$$= \left\| \begin{bmatrix} 3-\lambda & 0 \\ 7 & 2-\lambda \end{bmatrix} \right\|$$

$$= (3 - \lambda)(2 - \lambda) = 0$$

$$= \lambda^2 - 5\lambda + 6.$$

$\lambda = 3$ and $\lambda = 2$ are characteristic values.

Let $A_1 = \begin{vmatrix} 3 & 1 \\ 7 & 2 \end{vmatrix}$ be any matrix.

$d(A_1, A) = 1.$

Eigen values of A_1 are

$$|A_1 - \lambda| = \left\| \begin{bmatrix} 3-\lambda & 1 \\ 7 & 2-\lambda \end{bmatrix} \right\|$$

$$= (3 - \lambda)\,(2 - \lambda) - 7$$

$$= 6 - 5\lambda + \lambda^2 - 7$$

$$= \lambda^2 - 5\lambda - 1 = 0$$

$$\lambda = \frac{+5 \pm \sqrt{25+4}}{2} = \frac{+5 \pm \sqrt{29}}{2}.$$

$A_2 = \begin{vmatrix} 3 & 0 \\ 5 & 2 \end{vmatrix}$ be another 2×2 matrix $d(A, A_2) = 1$

$$|A_2 - \lambda| = \begin{bmatrix} 3-\lambda & 0 \\ 5 & 2-\lambda \end{bmatrix}$$

$= (3 - \lambda) (2 - \lambda)$

$= 6 - 5\lambda + \lambda^2$

$$\lambda = \frac{5 \pm \sqrt{25-4\times 6}}{2} = \frac{5 \pm 1}{2} = 3,2$$

We see all the characteristic values are same for A and A_2.

Let $A_3 = \begin{bmatrix} 1 & 0 \\ 7 & 2 \end{bmatrix}$ be the square matrix. $|A_3 - \lambda|$

$$= \begin{bmatrix} 1-\lambda & 0 \\ 7 & 2-\lambda \end{bmatrix} = 0; (1 - \lambda) (2 - \lambda) = 0$$

$\lambda = 1$ and $\lambda = 2$. We see $d(A, A_3) = 1$.

Let $A_4 = \begin{vmatrix} 3 & 0 \\ 7 & 5 \end{vmatrix}$; $A_4 - \lambda \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$$= \begin{vmatrix} 3-\lambda & 0 \\ 7 & 5-\lambda \end{vmatrix} = 0$$

gives $\lambda = 3$ and 5. $d(A_4, A) = 1$. That is one of the eigen values is different and other is the same.

We see if $A_5 = \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix}$ be matrix such that $d(A_5, A) = 2$.

$$|A_5 - \lambda| = \begin{vmatrix} 3-\lambda & 1 \\ 5 & 2-\lambda \end{vmatrix} = 0$$

$$(3 - \lambda)(2 - \lambda) - 5 = 0.$$

$$6 - 5\lambda + \lambda^2 - 5 = 0$$

$$\lambda^2 - 5\lambda + 1 = 0$$

$$\lambda = \frac{+5 \pm \sqrt{25-4}}{2} = \frac{+5 \pm \sqrt{21}}{2}.$$

We see $d(A_5, A) = 2$ and both the eigen values are different.

Let $A_6 = \begin{vmatrix} 3 & 0 \\ 2 & 4 \end{vmatrix}$, $d(A, A_6) = 2$

$$|A_6 - \lambda| = \begin{vmatrix} 3-\lambda & 0 \\ 2 & 4-\lambda \end{vmatrix}$$

$$= (3 - \lambda)(4 - \lambda) = 0$$

$$\lambda = 3 \text{ and } \lambda = 4.$$

We see only one eigen value is common between A and A_6.

Let $A_7 = \begin{vmatrix} 3 & 1 \\ 6 & 5 \end{vmatrix}$; $A = \begin{vmatrix} 3 & 0 \\ 7 & 2 \end{vmatrix}$

We see $d(A_7, A) = 3$.

Eigen values of A_7 is $|A_7 - \lambda| = 0$

$$\begin{vmatrix} 3-\lambda & 1 \\ 6 & 5-\lambda \end{vmatrix} = 0 \text{ implies } (3 - \lambda)(5 - \lambda) - 6 = 0$$

$$15 - 5\lambda - 3\lambda + \lambda^2 - 6 = 0$$

$$\lambda^2 - 8\lambda + 9 = 0$$

$$\lambda = \frac{8 \pm \sqrt{64-36}}{2} = \frac{8 \pm \sqrt{28}}{2} = 4 \pm \sqrt{7}.$$

We see both the eigen values are different.

If $d(A_t, A) = 4$ we do not work with it for it is not in keeping to study them as they are treated as different matrices.

Thus we observe a change or distance between matrices A and A_1 may change the eigen values or in some cases may not affect the eigen values. In some cases the eigen values are drastically affected.

It is left as an open problem.

Problem 1: Find in a $n \times n$ square matrices A, B with $d(A, B) = 1$, the maximum changes in the eigen values of A from B for n a sufficient large n.

Problem 2: Study problem 1 if $d(A, B) = 2$.

Problem 3: Compare results in problems (1) and (2).

Problem 4: Study problem (2) if $d(A, B) = 3$.

Problem 5: Let $d(A, B) = r$ and $d(A, B) = r + t$; when will the eigen values of A and B be closer, for r or $r + t$ $(t \geq 1)$.

Problem 6: Is it ever possible to have the characteristic equation of A and B to be same where both the entries above and below diagonals are non zero and
$d(A, B) = r \ (r \geq 1)$.

Problem 7: Let $\alpha_1, \ldots, \alpha_n$ and $\beta_1, \ldots, \beta_n$ be the n-eigen values of the $n \times n$ matrices A and B respectively.

(1) If $d(A, B) = 1$ can we say $|\alpha_i - \beta_i| = \delta_1$ where δ_1 will be small; $i = 1, 2, \ldots, n$.
(2) If $d(A,B) = 2$ can we say $|\alpha_i - \beta_i| = \delta_2$ with $\delta_2 > \delta_1$.
(3) If $d(A,B) = t$ can we say $|\alpha_i - \beta_i| = \delta_t$ with $\delta_t > \delta_{t-1}$ and so on $(1 \leq i \leq n)$.

Problem 8: If $d(A, B) = n - 1$ where A and B are $n \times n$ matrices; Is it possible for A and B to have atleast one common eigen value?

Problem 9: Boils down to; can we say atleast the characteristic equations have a common root?

So study in this direction is dormant and left as a collection of open problems for the reader.

Example 3.8: Let

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be the given 3×3 matrices $d(A, B) = 1$.

We see eigen values A and B are identical and the characteristic equation of A and B is the same given by $(\lambda - 1)(\lambda - 2)(\lambda - 3)$.

Thus for every B_1 which is different from A not on the diagonal elements have identical eigen values and characteristic equation.

Prove or disprove if A is diagonal matrix and all other entries are zero and if B is a matrix with d(A, B) = 1 with no difference in the diagonal terms. The characteristic values of A and B are the same and the characteristic equation is the same for A and B.

In example 3.2 let

$$B_1 = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

$d(A, B_1) = 2$.

Eigen values of A are 3, 2, 1.

Eigen values of B_1 are given by
$(3 - \lambda)(2 - \lambda)(1 - \lambda) + 4(2 - \lambda) = 0$

$$(2 - \lambda)[(3 - 4\lambda + \lambda^2) + 4]$$

$= (2 - \lambda)[\lambda^2 - 4\lambda + 7)$.

So the characteristic values are 2,

$$\frac{+4 \pm \sqrt{16 - 4 \times 7 \times 1}}{2}$$

which is imaginary.

So if A takes its entries from reals B_1 does not have the characteristic values as a pair of them is imaginary.

So for $d(A, B_1) = 2$ we see we have only one of the eigen values to be the same other two does not exist.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } B_2 = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$d(A, B_2) = 2.$

Eigen values of A are 3, 2, 1.
The characteristic equation for B_2 ;

$$|B_2 - \lambda| = \left\| \begin{matrix} 3-\lambda & 1 & -1 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{matrix} \right\|$$

$$= (3 - \lambda)\ [(2 - \lambda)\ (1 - \lambda)] - 1.0 + (-1).0$$

We see A and B_2 have the same characteristic equation and hence the same set of eigen values though

$d(A, B_2) = 2.$

$$\text{Consider } B_3 = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$d(A, B_3) = 3.$ The eigen values of A are 3, 2 and 1.

Characteristic equation of B_3 is

$$\left\| \begin{matrix} 3-\lambda & 1 & 2 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{matrix} \right\|$$

$$= (3 - \lambda)\ (2 - \lambda)\ (1 - \lambda) - 1 \times 0 + 2 \times 0.$$

The characteristic values of B_3 are the same as that of A.

In view of this we have following theorem.

THEOREM 3.1: *Let A be a diagonal matrix. All matrices B with same diagonal elements as A but which is a upper triangular (or a lower triangular) matrix with* $d(A, B) \leq (n-1) + (n-2) + (n-3) + \ldots + 1$ *have same set of eigen values and the same characteristic equation.*

The proof is direct and hence left as an exercise to the reader.

We will however illustrate this situation by some examples.

Example 3.9: Let

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

be the diagonal matrix.

$$B_1 = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

is a matrix such that $d(A, B_1) = 1$ and it is easily verified both A and B_1 have same characteristic values and characteristic equation.

$$\text{Let } B_2 = \begin{bmatrix} 2 & 4 & 1 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

be a matrix such that the special distance from A and B_2 is 2; that is $d(A, B_2) = 2$.

Characteristic equation of B_2 is

$$\begin{vmatrix} 2-\lambda & 4 & 1 & 0 \\ 0 & 5-\lambda & 0 & 0 \\ 0 & 0 & 1-\lambda & 0 \\ 0 & 0 & 0 & 7-\lambda \end{vmatrix}$$

$$= 2-\lambda \begin{vmatrix} 5-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 7-\lambda \end{vmatrix} - 4 \begin{vmatrix} 0 & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 7-\lambda \end{vmatrix} +$$

$$1 \begin{vmatrix} 0 & 5-\lambda & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7-\lambda \end{vmatrix} - 0 \begin{vmatrix} 0 & 5-\lambda & 0 \\ 0 & 0 & 1-\lambda \\ 0 & 0 & 0 \end{vmatrix}$$

$= (2 - \lambda)\,(5 - \lambda)\,(1 - \lambda)\,(7 - \lambda).$

The same characteristic equation as that of A.

Let $B_3 = \begin{bmatrix} 2 & 4 & 1 & 2 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$ be the matrix with same diagonal values $d(A, B_3) = 3$.

It is easily verified the characteristic equation of B_3 is the same as that of A.

Let $B_4 = \begin{bmatrix} 2 & 4 & 1 & 2 \\ 0 & 5 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$ be the matrix with same set of diagonal elements as that of A with special distance $d(A, B_4) = 4$.

Clearly the characteristic equation of B_4 is the same as that of A.

Now let $B_5 = \begin{bmatrix} 2 & 4 & 1 & 2 \\ 0 & 5 & 7 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$ be the matrix with same set of diagonal elements as that of A with special distance $d(A, B_5) = 5$.

We see both B_5 and A have the same set of eigen values.

Finally let $B_6 = \begin{bmatrix} 2 & 4 & 1 & 2 \\ 0 & 5 & 7 & 3 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 7 \end{bmatrix}$ be the matrix whose eigen values are the same as that of A.

Clearly $d(B_6, A) = 6$ but eigen values of B_6 and A are the same.

Now having seen this example which is the case when n = 4 in the theorem; the result is true.

Note we cannot say up to d(A, B) = 6 we can have the result to be true only when the different elements are from the upper diagonal (or lower diagonal) or used in the mutually exclusive sense. However if

d(A, B) = 2 with C given in the form

$$C = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 5 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$ we see the characteristic equations of

A and C are not the same though d(A,C) = 2.

$$|C - \lambda| = \begin{vmatrix} 2-\lambda & 0 & 1 & 0 \\ 0 & 5-\lambda & 0 & 0 \\ 1 & 0 & 1-\lambda & 0 \\ 0 & 0 & 0 & 7-\lambda \end{vmatrix}$$

$$= 2 - \lambda \begin{vmatrix} 5-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 7-\lambda \end{vmatrix} - 0 |\quad| +$$

$$1\begin{vmatrix} 0 & 5-\lambda & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 7-\lambda \end{vmatrix} - 0|\ |$$

$= (2 - \lambda)(5 - \lambda)(1 - \lambda)(7 - \lambda) + (-1)(5 - \lambda)(7 - \lambda)$

$\neq (2 - \lambda)(5 - \lambda)(1 - \lambda)(7 - \lambda)$.

Hence the claim.

Since eigen values play a major role and if during transmission some errors have occurred in the matrix and if they are of the same form we can find the distance and the related deviation in the eigen values.

Next we proceed onto analyse the problem when the special distance is different in the generator matrix of a code or the parity check matrix of a code.

We just recall the definition of a code word but for more about them refer [49].

We first make an assumption that the elements of a finite field represent the underlying alphabets for coding. Let $G(F_q)$ be the Galois field with q elements q a prime or a power of a prime.

Code words are written in the form $x = (x_1, \ldots, x_n)$ where the first k symbols $x_1, x_2, \ldots, x_k$ are message symbols and the $n - k$ elements $x_{k+1}, \ldots, x_n$ are the check symbols (or control symbols). All these $x_i \in G(Fq)$, $1 \leq i \leq n$.

A code word x satisfies the system of linear equations $Hx^T = (0)$ where H is the $(n - k) \times n$ matrix with elements from $G(F_q)$ we can always derive the standard form of H as (A, I_{n-k}) where A is an $n - k \times k$ matrix and I_{n-k} the $n-k \times n - k$ identity matrix.

We will first illustrate this by an example.

Example 3.10: Let q = 3, n = 7 and k = 4. The message a_1, a_2, a_3, a_4 is encoded as the code word x = a_1, a_2 a_3 a_4 x_5 x_6 x_7.

Here x_5, x_6 and x_7 are such that for the given matrix

$$H = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = (A, I_3).$$

We have $Hx^T = (0)$ that is

$$a_3 + x_5 = 0$$
$$a_1 + a_4 + x_6 = 0$$
$$a_2 + x_7 = 0$$

gives $x_5 = a_3$, $x_6 = a_1 + a_4$ and $x_7 = a_2$.

Now we find the code words using these codes as follows;

0 0 0 0 0 0 0	1 0 0 1 0 0 0
1 0 0 0 0 1 0	0 1 1 0 1 0 1
0 1 0 0 0 0 1	0 1 0 1 0 1 1
0 0 1 0 1 0 0	0 0 1 1 1 1 0
1 0 1 0 1 1 0	1 1 1 0 1 1 1
1 1 0 0 0 1 1	1 1 0 1 0 0 1
1 0 1 1 1 0 0	0 1 1 1 1 1 1
0 0 0 1 0 1 0	1 1 1 1 1 0 1

$Hx^T = (0)$ are also known as check equations.

The set of all n dimensional vectors x satisfying $Hx^T = (0)$ over $G(F_q)$ is called a linear code over $G(F_q)$ of block length n.

The matrix H is called the parity check matrix of the code C. C is also called the linear (n, k) code.

The set C of solutions of $Hx^t = (0)$; the solution space of this system of equations form a subspace of F_q^n over $G(F_q)$ of dimension k.

For the given parity check matrix in the standard form $H = (A, I_{n-k})$, the matrix $G = (I_k, -A^t)$ is called the canonical generator matrix or encoding matrix of a (n, k) code. In this case we have $GH^t = (0)$.

We will illustrate this situation by an example.

Example 3.11: Let

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

be the canonical generator matrix. We find for any message a the code is obtained from x = aG.

We shall get the set of code words using G

0 0 0 0 0 0 0	0 1 1 1 0 1 0
1 0 0 1 1 0 1	1 0 1 1 1 1 1
0 1 0 1 1 0 0	1 1 0 0 1 0 1
0 0 1 0 1 1 0	1 1 1 0 0 1 1

For more about algebraic codes refer [ ].

Now we show if we have a parity check matrix H and if we have another parity check matrix H_1 which is such that the special distance $d(H, H_1) = t$ where both H and H_1 are over the same field $G(F_q)$.

We study the code words of H and H_1.

$$\text{Let } H = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \text{ and}$$

$$H_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

be any two parity check matrices $d(H, H_1) = 3$.

Now the code words C using the parity check matrix H is as follows:

0 0 0 0 0 0 0
1 0 0 0 1 0 0
0 1 0 0 0 1 0
0 0 1 0 1 0 0
0 0 0 1 0 0 1
0 0 1 1 1 1 1
1 1 0 0 0 1 0
1 0 1 0 0 0 0
0 1 1 0 0 1 0
0 1 0 1 0 0 1
1 0 0 1 1 1 1
1 1 1 0 1 1 0
1 1 0 1 0 0 1
1 0 1 1 0 1 1
0 1 1 1 0 0 1
1 1 1 1 1 0 1

Now the code words C_1 associated with the parity check matrix H_1 is as follows:

0 0 0 0 0 0 0
1 0 0 0 1 0 0
0 1 0 0 1 1 0
1 0 1 0 0 0 0
1 0 0 1 1 1 1
0 1 1 0 0 1 0
0 1 0 1 1 0 1
1 1 0 1 0 0 1
1 1 1 1 1 0 1
0 0 1 0 0 0 0
0 0 0 1 0 1 1
1 1 0 0 0 1 0
0 0 1 1 1 1 1
1 1 1 0 1 1 0
0 1 1 1 0 0 1
1 0 1 1 0 1 1

However number of code words common between C and C_1 is as follows:

{(0 0 0 0 0 0 0), (1 0 0 0 1 0 0), (1 1 0 0 0 1 0),(0 0 1 1 1 1 1), (1 0 1 0 0 0 0), (1 1 1 1 1 0 1), (0 1 1 1 0 0 1), (1 0 0 1 1 1 1), (1 1 1 0 1 1 0), (1 1 0 1 0 0 1), (1 0 1 1 0 1 1), (1 1 0 1 0 0 1)}

Only four code words are different if some deviation has taken place.

The special Hamming distance between the code words of H and H_1 is as follows:

$$d_k((0\ 1\ 0\ 0\ 0\ 1\ 0)\ (0\ 1\ 0\ 0\ 1\ 1\ 0)) = 1$$
$$d_k((0\ 0\ 1\ 0\ 1\ 0\ 0)\ (0\ 0\ 1\ 0\ 0\ 0\ 0)) = 1$$
$$d_k((0\ 0\ 0\ 1\ 0\ 0\ 1)\ (0\ 0\ 0\ 1\ 0\ 1\ 1)) = 1$$
and $d_k((0\ 1\ 0\ 1\ 0\ 0\ 1)\ (0\ 1\ 0\ 1\ 1\ 0\ 1)) = 1$.

Further if $d(H, H_1) = 3$ we see out of 16 code words 12 are the same only four code words are different and the distance between them is negligible; in all cases the Hamming distance is one.

Example 3.12 : Let G be the generator matrix of the code C

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$ and G_1 be generator matrix

of the code C_1.

$$G_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

We see the special distance between G and G_1 is 3, that is $d_k(G, G_1) = 3$.

It is easily verified that only four out of the 16 code words are wrong and the rest is correct inspite of the special distance between the generator matrices G and G_1 being 3.

We give one more example before we proceed onto study more about the special distances of matrices of same order.

Example 3.13: Let

$$H = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

be a parity check matrix and

$$H' = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

be the parity check matrix wrongly stored as H.

The special distance between H and H′ is $d(H, H') = 9$.

Now we see the differences got using the parity check matrices H and H′.

The code words C associated with H are C =

$$\left\{\begin{array}{ccccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & & & \\
1 & 0 & 0 & 0 & 1 & 1 & 0 & & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 1 & & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 1 & & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 & & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & & & & & & & &
\end{array}\right\}$$

Let C′ denote the code words associated with H′

$$C' = \left\{\begin{array}{ccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 0
\end{array}\right\}.$$

Only out of 16 code words four are the same rest are different.

Now we find the distance between the corresponding 12 code words in C and C_1.

$$d_k((0\ 1\ 0\ 0\ 0\ 1\ 1), (0\ 1\ 0\ 0\ 1\ 0\ 0)) = 3$$

$$d_k((0\ 0\ 1\ 0\ 1\ 1\ 1), (0\ 0\ 1\ 0\ 1\ 1\ 0)) = 1$$

$$d_k((0\ 0\ 0\ 1\ 1\ 0\ 1), (0\ 0\ 0\ 1\ 0\ 1\ 0)) = 3$$

$$d_k((1\ 1\ 0\ 0\ 1\ 0\ 1), (1\ 1\ 0\ 0\ 0\ 1\ 0)) = 3$$

$$d_k((1\ 0\ 1\ 0\ 0\ 0\ 1), (1\ 0\ 1\ 0\ 0\ 0\ 0)) = 1$$

$$d_k((0\ 1\ 1\ 0\ 1\ 0\ 0), (0\ 1\ 1\ 0\ 0\ 1\ 0)) = 2$$

$$d_k((0\ 1\ 1\ 1\ 0\ 0\ 1), (0\ 1\ 1\ 1\ 0\ 0\ 0)) = 1$$

$$d_k((1\ 1\ 1\ 0\ 0\ 1\ 0), (1\ 1\ 1\ 0\ 1\ 0\ 0)) = 2$$

$$d_k((1\ 0\ 0\ 1\ 0\ 1\ 1), (1\ 0\ 0\ 1\ 1\ 0\ 0)) = 3$$

$$d_k((0\ 0\ 1\ 1\ 0\ 1\ 0), (0\ 0\ 1\ 1\ 1\ 1\ 0)) = 1$$

$$d_k((1\ 1\ 1\ 1\ 1\ 1\ 1), (1\ 1\ 1\ 1\ 1\ 1\ 0)) = 1$$

$$d_k((1\ 0\ 1\ 1\ 1\ 0\ 0), (1\ 0\ 1\ 1\ 0\ 1\ 0)) = 2.$$

We see the deviations are also large and the number of code words which are different are also large.

Thus we see from these two examples if the special distance between the parity check matrices is large then the number of non identical code words associated with them is also large.

From this we only conjecture the following.

Problem 10: Let H be a m × n parity check matrix; H′ a m × n parity check matrix distorted from H.

(i) If the special distance $d_k(H, H')$ is large; does it imply several code words in H and H′ are different and the distance (Hamming) of these appropriate code words are also large.

(ii) Can we say if the special distance $d_k(H, H')$ is considerably small then the number of code words different in H and H′ will also be small and the Hamming distance between the respective code words will also be small?

Next we proceed onto give the special distance between the generator matrices G and G′ got from G with errors.

Let

$$G = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

be the generated matrix of a C(7, 3) code.

Let G′ be the distorted version of G given by

$$G' = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Now the code words C generated by G is as follows:

$$C = \left\{\begin{matrix} 0&0&0&0&0&0&0 & & 1&0&1&0&0&1&0 \\ 1&0&0&0&1&0&1 & & 1&1&1&0&0&1&0 \\ 0&1&0&0&0&0&0 & & 1&1&0&1&0&1&1 \\ 0&0&1&0&1&1&1 & & 1&0&1&1&1&0&0 \\ 0&0&0&1&1&1&0 & & 1&1&1&1&1&0&0 \\ 0&0&1&1&0&0&1 & & 0&1&0&1&0&0&1 \\ 0&1&1&0&0&0&1 & & 1&0&0&1&0&1&1 \\ 1&1&0&0&1&0&1 & & 0&1&1&1&1&1&0 \end{matrix}\right\}.$$

The code words C′ generated by G′ is as follows:

$$C' = \left\{\begin{matrix} 0&0&0&0&0&0&0 & & 0&1&0&1&1&0&0 \\ 0&1&1&0&1&1&0 & & 1&1&1&1&0&0&1 \\ 1&1&0&0&0&1&1 & & 0&0&0&1&0&0&0 \\ 1&0&0&1&1&1&1 & & 1&1&1&0&0&0&1 \\ 1&0&1&0&1&0&1 & & 0&0&1&1&0&1&0 \\ 0&1&1&1&1&1&0 & & 0&1&0&0&1&0&0 \\ 0&0&1&0&0&1&0 & & 1&0&0&0&1&1&1 \\ 1&1&0&1&0&1&1 & & 1&0&1&1&1&0&1 \end{matrix}\right\}$$

The special distance between G and G′ given by

$$d_3(G, G') = 12.$$

Now we see out of 16 code words only three are identical.

All the other 13 code words are different.

$$d_k((1\ 0\ 0\ 0\ 1\ 0\ 1), (1\ 0\ 0\ 0\ 1\ 1\ 1)) = 1$$

$$d_k((0\ 1\ 0\ 0\ 0\ 0\ 0), (0\ 1\ 0\ 0\ 1\ 0\ 0)) = 1$$

$d_k((0\ 0\ 1\ 0\ 1\ 1\ 1), (0\ 0\ 1\ 0\ 0\ 1\ 0)) = 2$

$d_k((0\ 0\ 0\ 1\ 1\ 1\ 0), (0\ 0\ 0\ 1\ 0\ 0\ 0)) = 2$

$d_k((0\ 0\ 1\ 1\ 0\ 0\ 1), (0\ 0\ 1\ 1\ 0\ 1\ 0)) = 2$

$d_k((0\ 1\ 1\ 0\ 0\ 0\ 1), (0\ 1\ 1\ 0\ 1\ 1\ 0)) = 3$

$d_k((1\ 1\ 0\ 0\ 1\ 0\ 1), (1\ 1\ 0\ 0\ 0\ 1\ 1)) = 2$

$d_k((1\ 0\ 1\ 0\ 0\ 1\ 0), (1\ 0\ 1\ 0\ 1\ 0\ 1)) = 3$

$d_k((1\ 1\ 1\ 0\ 0\ 1\ 0), (1\ 1\ 1\ 0\ 0\ 0\ 1)) = 2$

$d_k((1\ 0\ 1\ 1\ 1\ 0\ 0), (1\ 0\ 1\ 1\ 1\ 0\ 1)) = 1$

$d_k((1\ 1\ 1\ 1\ 1\ 0\ 0), (1\ 1\ 1\ 1\ 0\ 0\ 1)) = 2$

$d_k((0\ 1\ 0\ 1\ 0\ 0\ 1), (0\ 1\ 0\ 1\ 1\ 0\ 0)) = 2$

$d_k((1\ 0\ 0\ 1\ 0\ 0\ 1), (1\ 0\ 0\ 1\ 1\ 1\ 1)) = 2$

It is pertinent to keep on record the maximum value of d_k for this code C(7, 4) can only be 3 so in some places the deviation is very high and only in three code words the distance is 1.

So if $d_k(G, G')$ is large so is the deviation among code words generated by G and G′.

Thus this study of (comparison) distance between the generator matrices and that of the code words can be analysed.

If G and G′ be the generator matrices of a C(n, k) code; then the maximum distance between the code words can be (n – k) only.

So if n–k is the special Hamming distance function then we cannot say the distance is small infact the distance is the largest.

Now we propose the following open conjecture.

Conjecture 3.1:

(i) If G and G′ are associated with two C(n, k) codes what is maximum $d_k(G, G')$ so that none of the code words other than (0, 0, 0, 0, 0, 0, …, 0) are different?

(ii) What is the bound value of $d_k(G, G')$ so that all code words are identical?

(iii) What is the value of $d_k(G, G')$ so that all the distance between code words is less than n–k?

Next we proceed on to study the implications of the distance between two fuzzy models of same type having the same type having the same order of the matrices.

We first define the notion of special distance between connection matrices FCMs for the same problem.

DEFINITION 3.3: *Let two experts work on the same problem with same number of concepts / attributes using FCMs model. Suppose let M_1 and M_2 be the connection matrices of the dynamical system given by the two experts. We define $d_k(M_1, M_2)$ to be the special distance between the dynamical systems/connection matrices.*

Then study the Kosko-Hamming distance between the resultant vectors (hidden pattern) for the same initial state vector X.

This will help one to analyse the models and the experts opinion.

We will first illustrate this situation by an example.

Example 3.14: Let us study this notion in case of the FCMs model given by four experts.

Let E_1, E_2, E_3 and E_4 be the four connection matrices given by them.

$$E_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

is the connection matrix given by the first expert.

$$E_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

associated with the second expert.

$$\text{Let } E_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

be the connection matrix given by the third expert.

$$\text{Let } E_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

be the connection matrix given by the fourth expert.

New $d_k(E_1, E_2) = 7$ that is difference between the matrices E_1 and E_2 is 7.

We now give the Kosko-Hamming distance function between the hidden pattern given by the two experts for the C_i's where $C_1 = (1\ 0\ \ldots\ 0)$, $C_2 = (0\ 1\ 0\ 0\ \ldots\ 0)$, $C_3 = (0\ 0\ 1\ 0\ \ldots\ 0)$ and so on $C_7 = (0\ 0\ \ldots\ 0\ 1)$.

We tabulate them in the following as table 1.

Table 1

C_i's	Hidden pattern given by E_1	Hidden pattern given by E_2	$d(E_1, E_2)$
(100000000)	(100001111)	(110000111)	2
(010000000)	(110111111)	(110101111)	1
(001000000)	(101111111)	(111101111)	2
(000100000)	(100111111)	(110101111)	2
(000010000)	(100111111)	(110111111)	1
(000001000)	(100001111)	(110101111)	2
(000000100)	(100001111)	(110101111)	2
(000000010)	(100001111)	(110101111)	2
(000000001)	(100001111)	(110101111)	2

We see the two experts have more or less the same opinion as the deviation is two or less deviation is two or less than two.

Now table 2 gives the difference between expert 1 and 3.

Table 2

The initial vectors C_i's	Hidden patterns given by E_1	Hidden pattern given by E_3	$d(E_1, E_3)$
(100000000)	(100001111)	(100001110)	1
(010000000)	(110111111)	(110111111)	0
(001000000)	(101111111)	(011110001)	5
(000100000)	(100111111)	(010110001)	5
(000010000)	(100111111)	(010110001)	5
(000001000)	(100011111)	(100001110)	1
(000000100)	(100001111)	(100001110)	1
(000000010)	(100001111)	(100001110)	1
(000000001)	(100001111)	(110111111)	3

We observe the experts (1) and 3 vary drastically in case of the initial state vectors on the nodes C_3, C_4 and C_5. But otherwise they largely agree upon other nodes.

Now table 3 gives the Kosko-Hamming distance of the hidden pattern gives for C_1, C_2, …, C_9 as initial state vectors given by experts 1 and 4.

C_i's	Hidden pattern of expert 1	Hidden pattern expert 4	$d(E_1, E_4)$
(100000000)	(100001111)	(101000111)	2
(010000000)	(110111111)	(010111000)	4
(001000000)	(101111111)	(101001111)	2
(000100000)	(100111111)	(111111111)	2
(000010000)	(100111111)	(111111111)	2
(000001000)	(100111111)	(101001111)	1
(000000100)	(100001111)	(101001111)	1
(000000010)	(100001111)	(101000111)	2
(000000001)	(100001111)	(101000111)	2

We see except for the node C_2 the two experts agree upon all other nodes.

Table 4 gives the hidden pattern and Kosko-Hamming distance of the experts 2 and 3.

Table 4

C_i's	Hidden pattern of expert 2	Hidden pattern expert 3	$d_K(2, 3)$
(100000000)	(110000111)	(100001110)	3
(010000000)	(110101111)	(110111111)	1
(001000000)	(111101111)	(011110001)	5
(000100000)	(111101111)	(010110001)	6
(000010000)	(110111111)	(010110001)	4

(000001000)	(110101111)	(100001110)	3
(000000100)	(110101111)	(100001110)	3
(000000010)	(110101111)	(100001110)	3
(000000001)	(110101111)	(110111111)	1

Clearly from the table we see the experts do not agree upon the resultants the deviations are large.

Now table 5 gives the Kosko-Hamming distance of hidden pattern of the experts 2 and 4.

Table 5

C_i's	**Hidden pattern of expert 2**	**Hidden pattern expert 4**	**$d_K(E_2, E_4)$**
(100000000)	(110000111)	(101000111)	2
(010000000)	(110101111)	(010111000)	5
(001000000)	(111101111)	(101001111)	2
(000100000)	(110101111)	(111111111)	2
(000010000)	(110111111)	(111111111)	1
(000001000)	(110101111)	(101001111)	2
(000000100)	(110101111)	(101001111)	3
(000000010)	(110101111)	(101000111)	3
(000000001)	(110101111)	(101000111)	4

We see experts 2 and 4 do not agree upon the opinion.

Now we give in table 6 the Kosko-Hamming distance of the hidden pattern given by the experts 3 and 4.

Table 6

C_i's	Hidden pattern of expert 3	Hidden pattern expert 4	$d_K(E_3, E_4)$
(100000000)	(100001110)	(101000111)	3
(010000000)	(110111111)	(010111000)	4
(001000000)	(011110001)	(101001111)	7
(000100000)	(010110001)	(111111111)	5
(000010000)	(010110001)	(111111111)	5
(000001000)	(100001110)	(101001111)	2
(000000100)	(100001110)	(101001111)	2
(000000010)	(100001110)	(101000111)	3
(000000001)	(110111111)	(101000111)	5

However expert 3 and expert 4 do not agree upon for the same initial state vector the resultant hidden pattern. They are deviant. Further expert three happens to hold views entirely different from the three experts.

Only the introduction of the Kosko-Hamming distance function can give such nice results and yield of such comparisons.

Next we proceed on to study the Kosko-Hamming weight function in case of FCMs and the concept of influential or vital nodes of the FCMs. We see the Kosko-Hamming weight will give the most influential node of the dynamical system and the expert opinion.

Let us consider the first experts opinion for all the hidden patterns the Kosko-Hamming weight of the hidden patterns.

$$d_k((1\ 0\ 0\ 0\ \ldots\ 0),(1\ 0\ 0\ 0\ 0\ \ 1\ 1\ 1\ 1)) = 4$$

$$= w_k(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1).$$

$$d_k((0\ 1\ 0\ 0\ \ldots\ 0),(1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1))$$

$= w_k(1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1) = 7.$

$d_k((0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0),(1\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1))$

$= w_k(1\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = 7.$

$d_k((0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0),(1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1))$

$= w_k(1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1) = 6.$

$d_k((0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0),(1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1))$

$= w_k(1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1) = 6.$

$d_k((0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0),(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1) = 4.$

$d_k((0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0),(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1) = 4.$

$d_k((0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0),(1\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1))$

$= w_k(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1) = 4.$

$d_k((0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1),(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1) = 4.$

Thus according to the first expert C_2 and C_3 the nodes 1 and 2 are the most influential nodes as the Kosko-Hamming distance is 7.

Likewise the nodes C_4 and C_5 are the more influential node as the Kosko-Hamming distance is 6.

The rest of the five nodes have Kosko-Hamming weight to be four.

Next we study the Kosko-Hamming weight of C_1, C_2, …, C_9 of the second expert.

$d_k((1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0),(1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1))$

$= w_k(1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1) = 4.$

$d_k((0\ 1\ 0\ 0\ \ldots\ 0),(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1) = 6.$

$d_k((0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0),(1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1) = 7.$

$d_k((0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0),(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1) = 6.$

$d_k((0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0),(1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1) = 7.$

$d_k((0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0),(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1) = 6.$

$d_k((0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0),(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1) = 6.$

$d_k((0\ 0\ 0\ 0\ \ \ldots\ 01\ 0),(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1) = 6.$

$d_k((0 \ldots 0\ 1),(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1) = 6.$

The nodes C_3 and C_5 are the most influential nodes and the Kosko-Hamming weight of C_3 and C_5 is 7.

The nodes C_4, C_6, C_7, C_8, C_9 are the more influential nodes as the Kosko-Hamming weight of them is 6.

The rest of the nodes of Kosko-Hamming weight is 4.

Now we study the Kosko-Hamming weights of C_1, C_2, C_3, …, C_9 given by the third expert.

$d_k((1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0),(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0))$

$= w_k(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0) = 3.$

$d_k((0\ 1\ 0\ 0 \ldots 0),(1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1) = 7$

$d_k((0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0),(0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1))$

$= w_k(0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1) = 4$

$d_k((0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0),(0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1))$

$= w_k(0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1) = 3$

$d_k((0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0),(0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1))$

$= w_k(0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1) = 3$

$d_k((0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0),(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0))$

$= w_k(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0) = 3$

$d_k((0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0), (1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0))$

$= w_k(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0) = 3$

$d_k((0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0), (1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0))$

$= w_k(1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0) = 3$

$d_k((0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1), (1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 1) = 7.$

The weight of C_2 and C_9 have the heighest weight and they are the most influential nodes by the expert 3.

The more influential node by the expert 3 has weight C_3.

The Kosko-Hamming weight of the nodes $C_1, C_2, \ldots, C_9$ by the expert 4 is as follows:

$d_k((1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0),(1\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 1))$

$= w_k(1\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 1) = 4$

$d_k((0\ 1\ \ldots\ 0),(0\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 0))$

$= w_k(0\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 0) = 3$

$d_k((0\ 0\ 1\ 0\ \ldots\ 0),(1\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 1))$

$= w_k(1\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 1) = 5$

$d_k((0\ 0\ 0\ \ 1\ 0 \ldots 0),(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = 8$

$d_k((0\ 0\ 0\ 0\ 1\ \ 0\ \ 0\ \ 0\ \ 0),(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1))$

$= w_k(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = 8$

$d_k((0 \ldots 0\ 1\ 0\ 0\ 0),(1\ 0\ 1\ 0\ 0\ 1\ 1\ 1))$

$= w_k(1\ 0\ 1\ 0\ 0\ 1\ 1\ 1) = 5$

$d_k((0\ 0 \ldots 0\ 1\ 0\ 0),(1\ 0\ 1\ 0\ 0\ 1\ 1\ 1))$

$= w_k(1\ 0\ 1\ 0\ 0\ 1\ 1\ 1) = 5$

$d_k((0\ 0 \ldots 0\ 1\ 0),(1\ 0\ 1\ 0\ 0\ 0\ 1\ 1))$

$= w_k(1\ 0\ 1\ 0\ 0\ 0\ 1\ 1) = 4$

$d_k((0 \ldots 0\ 1),(1\ 0\ 1\ 0\ 0\ 0\ 1\ 1))$

$= w_k(1\ 0\ 1\ 0\ 0\ 0\ 1\ 1) = 4.$

Thus the nodes C_4 and C_5 are the most influential nodes of expert 4 and the Kosko-Hamming weight is 8.

Thus we see the most influential node also depends on the expert and varies from expert to expert.

Next we proceed onto find the Kosko-Hamming distance function pair in case of FRMs and the special distance between two FRMs given by two experts.

Example 3.15: Let M_1 and M_2 be the connection matrices associated with FRM given by two experts.

$$\text{Let } M_1 = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 1 \end{bmatrix} \text{ and}$$

$$M_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

be the 2 expert FRMs. The special distance between M_1 and M_2 denoted by

$$d_K(M_1, M_2) = 10.$$

$C_1 = (1\ 0\ 0\ 0\ 0\ 0)$ on M_1 gives
$C_1M_1 \to (1\ 1\ 0\ 1\ 0) = X_1$ say
$X_1 M_1^t \to (1\ 1\ 1\ 1\ 1\ 0) = Y_1$ say

$Y_1M_1 \to (1\ 1\ 1\ 1\ 1) = X_2$ say
$X_2 M_1^t \to (1\ 1\ 1\ 1\ 1\ 0)$.

Thus we get a fixed pair as the hidden pattern
$((1\ 1\ 1\ 1\ 1\ 0), (1\ 1\ 1\ 1\ 1)) = A.$

Now we find the hidden pattern pair of C_1 using M_2.

$C_1M_2 \rightarrow (1\ 1\ 1\ 1\ 0) = X_1$
$X_1\, M_2^t \ \rightarrow (1\ 1\ 1\ 1\ 1\ 0) = Y_1$
$Y_1M_2 \rightarrow (1\ 1\ 1\ 1\ 0) = X_2$.

Thus the hidden pattern is a fixed pair
$((1\ 1\ 1\ 1\ 1\ 0), (1\ 1\ 1\ 1\ 0)) = B$.

Now the Kosko-Hamming distance between the pair A and B is $d_K(A, B) = (0, 1)$.

Thus the two experts agree on the node C_1 or they are approximately very close.

Now the Kosko-Hamming weight relative to C_1 is
$d_K((1\ 1\ 1\ 1\ 1\ 0), (1\ 0\ 0\ 0\ 0\ 0)) = 4$.

Next let us consider

$R_1 = (1\ 0\ 0\ 0\ 0)$ from the range space
$R_1\, M_1^t \rightarrow (1\ 1\ 0\ 1\ 0\ 0) = X_1$ say
X_1 M1 $\rightarrow (1\ 1\ 1\ 1\ 0) = Y_1$ say
$Y_1\ M_1^t \rightarrow (1\ 1\ 1\ 1\ 1\ 0) = X_2$
$X_2\ M_1^t \rightarrow (1\ 1\ 1\ 1\ 1) = Y_2$ say
$Y_2\ M_1^t \rightarrow (1\ 1\ 1\ 1\ 1\ 0) = X_3\ (= X_2)$.

Thus the hidden pattern is a fixed point pair given by
$C = ((1\ 1\ 1\ 1\ 1), (1\ 1\ 1\ 1\ 1\ 0))$

Now we find the hidden pattern pair of the dynamical system using M_2

$R_1\, M_2^t \rightarrow (1\ 0\ 0\ 0\ 0\ 0) = X_1$ say
$X_1M_2 \rightarrow (1\ 1\ 1\ 1\ 0) = Y_1$
$Y_1\ M_2^t \rightarrow (1\ 1\ 1\ 1\ 1\ 0) = X_2$
$X_2\ M_2^t \rightarrow (1\ 1\ 1\ 1\ 0) = Y_2\ (= Y_1)$.

Thus the hidden pattern pair is a fixed point pair given by

$$D = ((1\ 1\ 1\ 1\ 0), (1\ 1\ 1\ 1\ 1\ 0))$$

Now the Kosko-Hamming distance pair of C, D is

$$d_K(C, D) = (1, 0).$$

Kosko-Hamming weight is

$$d_k = ((1\ 1\ 1\ 1\ 0), (1\ 0\ 0\ 0\ 0))$$

$= w_k\ (1\ 1\ 1\ 1\ 0) = 3$ this is case with expert two.

Incase of expert one;

$$d_k((1\ 1\ 1\ 1\ 1), (1\ 0\ 0\ 0\ 0))$$

$$= w_k\ (1\ 1\ 1\ 1\ 1) = 4.$$

Thus this node according to expert one is the most influential node.

The study in this direction can be carried out as in case of FCMs.

Next we show how the special distance between the membership matrices associated with FRE works and the relation between the experts.

Example 3.16: Let us consider the matrices of the FRE's given by 3 experts.

Let S_1, S_2 and S_3 be the three membership matrices given by the three experts

$$S_1 = \begin{bmatrix} 0 & 0.6 & 0 & 0.7 & 0.8 & 0.6 & 0.8 \\ 0.3 & 0.1 & 0.6 & 0.6 & 0 & 0.5 & 0.7 \\ 0.6 & 0.8 & 0 & 0.7 & 0.7 & 0.5 & 0.6 \\ 0 & 0 & 0.7 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0.3 & 0 & 0.2 & 0 \\ 0.1 & 0 & 0 & 0.2 & 0 & 0.1 & 0 \\ 0.2 & 0 & 0.7 & 0.6 & 0 & 0.5 & 0.7 \end{bmatrix}$$

be the membership matrix given by the first expert.

$$S_2 = \begin{bmatrix} 0.1 & 0.7 & 0.1 & 0.8 & 0.9 & 0.7 & 0.8 \\ 0.4 & 0.2 & 0.7 & 0.7 & 0.1 & 0.4 & 0.6 \\ 0.7 & 0.8 & 0.1 & 0.7 & 0.1 & 0.6 & 0.6 \\ 0.2 & 0.1 & 0.8 & 0.8 & 0 & 0.1 & 0 \\ 0.5 & 0 & 0.6 & 0.4 & 0 & 0.2 & 0.3 \\ 0.1 & 0.1 & 0.2 & 0.3 & 0.1 & 0.1 & 0.1 \\ 0.3 & 0 & 0.8 & 0.4 & 0 & 0.4 & 0.8 \end{bmatrix}$$

be the membership matrix given by the second expert.

Let

$$S_3 = \begin{bmatrix} 0.1 & 0.6 & 0 & 0.6 & 0.8 & 0.7 & 0.8 \\ 0.2 & 0.1 & 0.5 & 0.6 & 0.1 & 0.5 & 0.7 \\ 0.6 & 0.9 & 0.2 & 0.8 & 0.7 & 0.8 & 0.7 \\ 0.4 & 0.5 & 0.7 & 0.6 & 0.7 & 0.2 & 0.1 \\ 0.6 & 0.5 & 0.6 & 0.5 & 0.1 & 0.2 & 0.5 \\ 0.5 & 0.4 & 0.1 & 0.4 & 0.1 & 0.2 & 0.4 \\ 0.2 & 0.6 & 0.4 & 0.6 & 0.1 & 0.6 & 0.7 \end{bmatrix}$$

be the membership matrix of the FRE given by the third expert.

The special distance between S_1 and S_2 is $d(S_1, S_2) = 0$ for we measure it a very different way.

If $M =(m_{ij})$ and $n = (n_{ij})$ be two FRE membership matrices.

$d(M, N)$ = Number of elements $|(m_{ij} - n_{ij})|$ which are greater than 0.5.

Thus in case of S_1 and S_2
$d(S_1, S_2) = 0$.

Consider $d(S_1, S_3) = 0$ and $d(S_2, S_3) = 0$.

Now we find the effect of
$X = (0.7, 0.5, 0.7, 0.4, 0.8, 0.6, 0.3)$
On the dynamical systems S_1, S_2 and S_3 are

$$x \text{ o } S_1 = (0.8, 0.6, 0.7, 0.7, 0.6, 0.2, 0.7) = R_1$$

$$x \text{ o } S_2 = (0.8, 0.7, 0.7, 0.7, 0.6, 0.3, 0.7) = R_2$$

$$x \text{ o } S_3 = (0.7, 0.5, 0.7, 0.7, 0.6, 0.5, 0.6) = R_3$$

respectively.

$$d(R_1, R_2) = (0, 0.1, 0, 0, 0, 0, 0)$$
$$d(R_1, R_3) = (0.1, 0.1, 0, 0, 0, 0.2, 0.1)$$
$$d(R_2, R_3) = (0.1, 0.2, 0, 0, 0, 0.2, 0.1).$$

Thus experts one and two agree or have identical opinion.

Experts 1 and 3 and experts one and two do not deviate much from each other. In conclusion all the three experts holds the same set of opinion in this regard.

Next we can find the special distance between neutrosophic matrices as well as between a neutrosophic matrix and the usual matrix.

We will illustrate these two situations by examples.

Example 3.17: Let

$$A = \begin{bmatrix} I & 2I+1 & 0 & 0 \\ 3 & 2 & 1 & I \\ 4 & 0 & 2I & 0 \\ 7 & 0 & 3I & 1 \\ 5I & 7I & 0 & 0 \\ 2I & 4 & 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} I & 2 & 0 & 0 \\ 3 & 2 & 2I & I \\ 4 & 1 & 2I+1 & 0 \\ 7 & 2 & 1 & 1 \\ 5I & 7I & 0 & 0 \\ 2I & 4I & 2 & 3 \end{bmatrix}$$

be any two neutrosophic matrices.

The special distance between them is 7. This is the way the special distance is measured.

Example 3.18: Let

$$A = \begin{pmatrix} 3I & 0 & 1 & 2 & 3I \\ 2 & 4I & 0 & I+1 & 2 \end{pmatrix} \text{ and}$$

$$B = \begin{pmatrix} 3I & 1 & 1 & 2 & 3I+1 \\ 2 & 4I & 0 & I+1 & 2 \end{pmatrix}$$

be any two neutrosophic matrices. The special distance between A and B is 2.

We can also get the distance between usual or real matrices and neutrosophic matrices. We can find the distance between the neutrosophic matrix and the real matrix of same order.

We will illustrate this situation by an example.

Example 3.19: Let

$$A = \begin{bmatrix} 3 & 2 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 4 & -2 & 7 & -1 \\ 2 & 5 & 6 & 8 \\ -3 & 4 & 3 & 1 \\ 8 & 1 & 0 & 1 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 3I & 2 & 4 & I+3 \\ 0 & 1 & 3+I & 0 \\ 4 & -2+I & 7 & -1 \\ 2 & 5 & 6 & 8 \\ -3 & 4 & 3 & I \\ 8 & I+3 & 0 & 1 \end{bmatrix}$$

be the real and neutrosophic matrices respectively.

The special distance between A and B is 6.

Example 3.20: Let

$$M_1 = \begin{bmatrix} 0.3 & 2I+1 & 3+5I & 7 & 0 \\ 8 & 4 & 0 & 8I+1 & 3 \\ 7I & 0 & 5-4I & 0 & 2 \\ 1+3I & 2 & 8I & 3 & 4I \\ 8I & 0 & 0 & 6I & 0 \\ 0 & 2I & 4I & 2 & 7 \end{bmatrix}$$

be a neutrosophic matrix.

Let

$$M_2 = \begin{bmatrix} 0.3 & 1 & 2 & 7 & 6 \\ 8 & 4 & 0 & 6 & 3 \\ 6 & 0 & 4 & 0 & 2 \\ 0 & 2 & 2 & 4 & 1 \\ 8 & 7 & 3 & 2 & 0 \\ 7 & 0 & 5 & 2 & 7 \end{bmatrix}$$

be the real matrix.

The special distance between M_1 and M_2 is $d(M_1, M_2) = 17$.

Now we use these two concepts in case of NCMs we take two NCMs working on the same problem with same number of attributes.

We will first illustrate by some examples.

Example 3.21: Let us consider the two experts opinion using NCMs on the same problem. Let N_1 and N_2 be the connection neutrosophic matrices given by the two experts.

$$N_1 = \begin{bmatrix} 0 & I & -1 & 1 & 1 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 \\ -1 & I & 0 & 0 & I & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

be the NCMs connection matrix given by the first expert.

$$N_2 = \begin{bmatrix} 0 & 1 & -1 & 1 & I & 0 & -1 \\ 0 & 0 & 0 & 0 & I & 0 & I \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ I & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

be the connection neutrosophic matrix given by the second expert.

The special distance between N_1 and N_2 is 15.

Let $X_1 = (1\ 0\ 0\ 0\ 0\ 0\ 0)$ be the initial state vector.

$X_1N_1 \rightarrow (1\ I\ 0\ 1\ 1\ 0\ 0) = X_2$
$X_2N_1 \rightarrow (1\ I\ 0\ 1\ 1\ 0\ 0) = X_3\ (=X_2)$.
$X_1N_2 \rightarrow (1\ 1\ 0\ 1\ I\ 0\ 0) = X_2$
$X_2N_2 \rightarrow (1\ 1\ 0\ 1\ I\ 0\ 0) = X_3$
$X_3N_2 \rightarrow (1\ 1\ 0\ 1\ I\ 0\ 0) = X_4\ (= X_3)$.

Thus the Kosko-Hamming distance is 2.

So both the experts agree closely. This is the way Kosko-Hamming distance for the hidden patterns for the same initial state vector is determined.

By this method we can find the experts closeness or distance. Also using Kosko-Hamming weight we can predict the most influential node or just influential node and so on. The working is just like that of the usual FCMs model.

We can also find the special distance between the NCMs and FCMs model using the same number of nodes. This situation will be represented by some examples.

Example 3.22: Let

$$M_1 = \begin{bmatrix} 0 & 1 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

be the first experts connection using FCM.

$$\text{Let } N_1 = \begin{bmatrix} 0 & 1 & -1 & 1 & I & 0 & -1 \\ 0 & 0 & 0 & 0 & I & 0 & I \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ I & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

be the neutrosophic connection matrix using the NCMs model for the same problem.

The special distance between M_1 and N_1 is 9.

Let X = (1 0 0 0 0 0 0) be the initial vector.

$XM_1 \to (1\ 1\ 0\ 1\ 0\ 0\ 0) = X_1$
$X_1M_1 \to (1\ 1\ 0\ 1\ 0\ 1\ 0) = X_2$

$X_2M_1 \rightarrow (1\ 1\ 0\ 1\ 0\ 1\ 0) = X_3\ (= X_2)$
is the fixed point.

Now $XN_1 \rightarrow (1\ 1\ 0\ 1\ I\ 0\ 0) = X_1'$
$X_1N_1 \rightarrow (1\ 1\ 0\ 1\ I\ 0\ 0) = X_2'$

$X_2' N_1 \rightarrow (1\ 1\ 0\ 1\ I\ 0\ 0) = X_3'\ (= X_2')$
a fixed point.

The Kosko – Hamming distance of X_2 and X_2'
$d_K((1\ 1\ 0\ 1\ 0\ 1\ 0), (1\ 1\ 0\ 1\ I\ 0\ 0)) = 2$.

Thus the deviation is not very large.

We can study all the properties and develop and describe the analysis as in case of FCMs even if we use NCMs or FCMs and NCM.

The only criteria which we have to follow is that the experts should work on the same problem with same set of attributes or nodes.

Next we can use the theory of working with NRMs. Just like FRMs we can study several experts NRMs or one NRM and a FRM comparison and analysis is also possible.

These two situations will be described by examples.

Example 3.23: Let us suppose two experts work on a problem using same set of domain and range attributes. Both chooses to work with NRMs.

Let

$$N_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & I & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & I & I & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & I \end{bmatrix} \text{ and}$$

$$N_2 = \begin{bmatrix} I & 0 & 0 & 1 & 1 \\ 0 & I & 1 & 0 & 0 \\ I & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & I & I \\ 0 & 0 & 1 & I & 0 \\ 0 & 0 & 1 & 0 & I \\ 1 & 0 & I & 0 & 1 \\ 0 & I & 0 & 1 & 1 \end{bmatrix}$$

be the two connection matrices of the FRM given by the two experts.

Clearly the special distance between N_1 and N_2 is 18.

$$d(N_1, N_2) = 18.$$

Now let X= (1 0 0 0 0 0 0 0)

be the given initial state vector.

To find the effect of X on N_1 and N_2.

$$XN_1 \rightarrow (0\ 0\ 0\ 0\ 1) = Y_1$$

$Y_1 N_1^t \rightarrow (1\ 0\ 0\ 0\ 0\ 1\ 0\ I) = X_2$

$X_2 N_1 \rightarrow (\ 0\ 0\ 0\ I\ 1) = Y_2$

$Y_2\ N_1^t \rightarrow (1\ 0\ 0\ 0\ 1\ 0\ I) = X_3\ (=X_2).$

Let us denote the fixed point pair by

((1 0 0 0 0 1 0 I), (0 0 0 I 1)).

Now we find the effect of X on N_2.

$XN_2 \rightarrow (I\ 0\ 0\ 1\ 1) = Y_1$

$Y_1\ N_2^t \rightarrow (1\ 0\ I\ I\ I\ I\ 1\ 1) = X_1$

$X_1N_2 \rightarrow (I\ I\ I\ 1\ 1\ ) = Y_2$

$Y_2\ N_2^t \rightarrow (1\ I\ I\ I\ I\ I\ 1\ 1) = X_2$ say

$X_2\ N_2 \rightarrow (I\ I\ I\ 1\ 1) = Y_3\ (=Y_2).$

Thus hidden pattern is a fixed point pair given by

B = ((1 I I I I I 1 1), (I I I 1 1)).

The Kosko-Hamming distance pair is given by

$d_K(A, B) = (7, 4).$

Thus this pair proves beyond doubt that for the initial state vector (1 0 0 0 0 0 0 0) the experts do not agree upon they hold diverse opinion.

Likewise we can work with more number of initial state vectors.

Such type of analysis is not only new but it gives analysis with in the experts and the models they use.

Now we can also compare the FRM models with NRM models on the same problem by different experts.

This will be illustrated by the following example.

Example 3.24: Let two experts work on the same problem. One of them use the FRM model and the connection matrix given by the expert is as follows:

$$M_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

Let the second expert work with the same problem using the NRMs. Let M_2 be the neutrosophic connection matrix given by the second expert.

$$M_2 = \begin{bmatrix} I & 0 & 0 & 1 & 1 \\ 0 & I & 1 & 0 & 0 \\ I & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & I & I \\ 0 & 0 & 1 & I & 0 \\ 0 & 0 & 1 & 0 & I \\ 0 & 0 & I & 0 & 1 \\ 0 & I & 0 & 1 & 1 \end{bmatrix}.$$

We the special distance between M_1 and M_2 is 10.

Now we find the Kosko-Hamming distance between the experts on the node X = (1 0 0 0 0 0 0 0)

$$XM_1 \to (0\ 0\ 0\ 1\ 1) = Y_1$$
$$Y_1 M_1^t \to (1\ 0\ 0\ 0\ 0\ 1\ 1\ 1) = X_1$$
$$X_1M_1 \to (0\ 0\ 0\ 1\ 1) = Y_2$$

$Y_2\ M_1^t \rightarrow (1\ 0\ 0\ 0\ 0\ 1\ 1\ 1) = X_2\ (=X_1)$.

Let A = ((1 0 0 0 0 1 1 1) (0 0 0 1 1))
be the fixed point pair of the initial state vector.

X = (1 0 0 0 0 0 0 0) be the initial state vector.

$XM_2 \rightarrow (I\ 0\ 0\ 1\ 1) = Y_1$
$Y_1\ M_2^t \rightarrow (1\ 0\ I\ I\ I\ I\ 1\ 1) = X_1$
$X_1M_2 \rightarrow (I\ I\ I\ 1\ 1\ ) = Y_2$
$Y_2\ M_2^t \rightarrow (1\ I\ I\ I\ I\ I\ 1\ 1) = X_2$
$X_2M_2 \rightarrow (I\ I\ I\ 1\ 1) = Y_3\ (= Y_1)$.

Thus the hidden pattern is a fixed pair given by
B = {(1, I, I, I, I, I, 1, 1), (I, I, I, 1, 1)}.

The Kosko-Hamming distance pair between A, B is
$d_k(A, B) = (5, 3)$.

The deviation infact is large.

Thus we can in case of two nodes one a neutrosophic model and other real fuzzy model we can compare them using these newly introduced tools.

We will just give how these distance functions work on the Neutrosophic Relational Equations (NRE) models.

We will illustrate this situation by an example or two.

Example 3.25: Let two experts work using NREs on the same problem.

Let M_1 and M_2 be the neutrosophic fuzzy matrix given by them.

$$\text{Let } M_1 = \begin{bmatrix} 0.2I & 0.6 & 0.3I & 0.7 & 0.8 & 0.6 & 0.8 \\ 0.3 & 0.7I & 0.6 & 0.6 & 0.3I & 0.5 & 0.7 \\ 0.6 & 0.8 & 0.7I & 0.7 & 0.7 & 0.5 & 0.7 \\ 0.5I & 0.2 & 0.7 & 0.6 & 0.3 & 0.2 & 0.7 \\ 0.7 & 0.4I & 0.6 & 0.3 & 0.2I & 0.7 & 0.5 \\ 0.1 & 0.5 & 0.3 & 0.2 & 0.7 & 0.1 & 0.5I \\ 0.2 & 0 & 0.7 & 0.6 & 0 & 0.5 & 0.7 \end{bmatrix}$$

the neutrosophic membership matrix of the NRE given by the first expert.

Let M_2 be the neutrosophic membership matrix of the NRE given by the second expert which is as follows:

$$M_2 = \begin{bmatrix} 0.I & 0.6 & 0 & 0.4 & 0.7 & 0.8 & 0.9 \\ 0.3I & 0.I & 0.4 & 0.6I & 0.I & 0.5 & 0.7 \\ 0.5 & 0.9 & 0.2I & 0.5 & 0.6 & 0.5 & 0.6 \\ 0.3 & 0.4I & 0.6 & 0.4 & 0.2 & 0.I & 0.2 \\ 0.5 & 0.5 & 0.6 & 0.5 & 0.I & 0.2 & 0.5 \\ 0.3 & 0.2 & 0.2 & 0.3 & 0.2 & 0.I & 0.3 \\ 0.I & 0.5 & 0.2 & 0.4 & 0.I & 0.4 & 0.5 \end{bmatrix}.$$

Now let both the expert work with

$X = (0.7, 0.5, 0.7, 0.4, 0.8, 0.6, 0.3)$

Using M_1

$M_1 \text{ o } X^t = (0.8, 0.6, 0.7I, 0.7, 0.7, 0.7, 0.7) = R_1$.

Let the second expert work with the same X.

$M_2 \text{ o } X^t = (0.7, 0.5, 0.6, 0.6, 0.6, 0.3, 0.5) = R_2$.

Now the distance function between R_1 and R_2 is

(0.1, 0.1, | 0.7I, –0.6| 0.1, 0.1, 0.4, 0.2)

We see except for the third node the deviation is not very large for it is less than 0.5.

Likewise for any initial state vector X we can find the Kosko-Hamming distance between the two experts opinion by find the Kosko-Hamming distance of the resultant for the initial state vector X.

Such study is considered as a matter of routine and is left as an exercise to the reader.

Now we give an example in which we find the special distance between the membership matrix of an FRE and the membership matrix of a NRE.

This is illustrated by the following example.

Example 3.26: Let two experts work on the same problem with same set of attributes one works using NRE and the other uses FRE. How we compare them is illustrated in the following.

Let the expert working with FRE has the following membership matrix.

$$S_1 = \begin{bmatrix} 0.1 & 0.7 & 0 & 0.7 & 0.8 & 0.8 & 0.2 \\ 0.5 & 0.5 & 0.6 & 0.7 & 0.4 & 0.6 & 0.7 \\ 0.7 & 0.7 & 0.5 & 0.4 & 0.7 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.5 & 0.6 & 0.1 & 0.2 & 0.4 \\ 0.6 & 0.4 & 0.6 & 0.7 & 0.4 & 0.3 & 0.5 \\ 0.4 & 0.2 & 0.4 & 0.5 & 0.2 & 0.1 & 0 \\ 0.2 & 0.1 & 0.7 & 0.5 & 0 & 0.5 & 0.5 \end{bmatrix}.$$

Let the second expert work with the same set of variables and initial limit points but with the NRE membership matrix S_2 which is as follows:

$$S_2 = \begin{bmatrix} 0.I & 0.5 & 0.2 & 0.7 & 0.6 & 0.9 & 0.5 \\ 0.6 & 0.8 & 0.8 & 0.9 & 0.2 & 0.6 & 0.5 \\ 0.4 & 0.2 & 0.5 & 0.4 & 0.7 & 0.6 & 0.3I \\ 0.5 & 0.4 & 0.2 & 0.5 & 0.2 & 0.2 & 0.3 \\ 0.5 & 0.4 & 0.5 & 0.8 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.2 & 0.6 & 0.5 & 0.2 & 0.I & I \\ 0.2 & 0.3 & 0.6I & 0.4 & 0 & 0.2 & 0.2 \end{bmatrix}.$$

Now we see the special distance between S_1 and S_2 is 6.

Now we find the effect of the initial state vector

$X = (0.7, 0.5, 0.7, 0.4, 0.8, 0.6, 0.3)$.

Using the membership matrix S_1.

$S_1 \text{ o } X = (0.8, 0.6, 0.7, 0.7, 0.6, 0.2, 0.7) = R_1$.

Similarly the effect of X on the membership matrix S_2 of the NRE is as follows:

$S_2 \text{ o } X = (0.6, 0.7, 0.7, 0.5, 0.6, 0.6, 0.6I) = R_2$.

The Kosko-Hamming distance function $d_K(R_1, R_2) = 1$ using $|x_i - x_j| \leq 5$ is replaced as zero difference

$d_K(R_1, R_2) = (|0.8 - 0.6|, |0.6 - 0.7|, |0.7 - 0.7|, |0.7 - 0.5|, |0.6 - 0.6|, |0.2 - 0.6|, |0.7 - 0.6I|)$

$= (0.2, 0.1, 0, 0.2, 0, 0.4, | 0.7 - 0.6I|)$

$= 1$

using $|x_i - x_j| \leq 0.5$ is 0 when both x_i and x_j are real and > 0.5 is 1 if one of x_i or x_j is neutrosophic then 1 or I according as the coefficient value of them and if both x_i and x_j are neutrosophic I or 0 as usual and

$|x_i - x_j| = a + bI$ if $a \leq 0.5$ and $b \leq 0.5$ it is zero.
if even one of $a > 0.5$ or $b > 0.5$ it is 1.

Only by this rule we calculate the Kosko-Hamming distance function provided they are the resultant of the same initial state vector X. (However expert has the liberty to work differently by assigning different values).

Next we indicate how in the first place super Hamming distance can be defined for two super row vectors.

DEFINITION 3.4: *Let $X = (a_1 \mid \ldots \mid \ldots \mid \ldots a_n)$ and $Y = (a_1' \mid \ldots \mid \ldots \mid a_n')$ be two super row vectors. The super Hamming distance between X and Y can be defined if and only if*

(i) Both X and Y are of same order.
(ii) Both X and Y enjoy the same type of partition.

Then the super Hamming distance $d^s(X, Y)$ is the number of places in which X and Y differ.

We will first illustrate this situation by an example or two.

Example 3.27: Let X = (3 7 | 2 1 0 | 5 2 0 | 7) and Y = (3 7 | 3 1 0 | 2 2 0 | 3) be two super row matrices of same type.

The super Hamming distance $d^S(X, Y) = 3$.

Example 3.28: Let X = (1 | 2 3 4 | 5 7 | 0 2 3) and Y = (1 2 | 3 4 5 | 7 0 | 2 3) be two super row vector. They are not of same type but we see entries in both X and Y are identical, still we cannot define the super Hamming distance for them.

Example 3.29: Let X = (3 1 | 5 6 2 | 1 | 3 0 1 1 | 7 | 2 1) and Y = (3 1 | 5 6 4 | 5| 3 0 1 1| 7 | 3 2) be any two super row vectors of same type; the super Hamming distance $d^S(X, Y) = 4$.

We can as in case of usual matrices, define the notion of special super distance. Here also the concept of super special distance is possible provided the super matrices of same natural order and they are of same type.

Unless these two criteria's are satisfied it is not possible to define super special distance between them.

We will illustrate these situations by some examples.

Example 3.30: Let

$$X = \left[\begin{array}{c} 3 \\ \hline 7 \\ \hline 8 \\ 9 \\ \hline 4 \\ 3 \\ 2 \\ \hline 1 \\ 0 \\ 3 \\ -1 \end{array}\right] \text{ and } Y = \left[\begin{array}{c} 3 \\ \hline 7 \\ \hline 1 \\ 9 \\ \hline 4 \\ 3 \\ 0 \\ \hline 1 \\ 0 \\ 2 \\ -7 \end{array}\right]$$

be two super column vectors.

The special super Hamming distance $d^S(X, Y) = 4$.

Now we see both X and Y have same natural order and they are of same type.

Example 3.31: Let

$$X = \begin{bmatrix} 3 \\ 2 \\ \hline 1 \\ 1 \\ 1 \\ \hline 1 \\ 4 \\ \hline 2 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 3 \\ 2 \\ 1 \\ \hline 1 \\ 1 \\ \hline 1 \\ 4 \\ 2 \end{bmatrix}$$

be two super column vector both these are same entries and same natural order but are of different types so we are not in a position to define super special distance between them.

Example 3.32: Let

$$X = \left(\begin{array}{ccc|c|cc|cc} 3 & 4 & 0 & 7 & 10 & 1 & 0 & 2 \\ 2 & 5 & 0 & 8 & 9 & 0 & 3 & 1 \\ 1 & 0 & 1 & 9 & -1 & 2 & 1 & 1 \end{array}\right) \text{ and}$$

$$Y = \left(\begin{array}{ccc|c|cc|cc} 3 & 4 & 1 & 7 & 10 & 1 & 0 & 2 \\ 2 & 5 & 0 & 8 & 8 & 0 & 3 & 1 \\ 1 & 2 & 1 & 0 & -1 & 2 & 1 & 4 \end{array}\right)$$

be two super row vectors of same type.

The special super distance $d^S(X, Y) = 5$.

Example 3.33: Let

$$Y = \left[\begin{array}{cccc} 0 & 1 & 1 & 1 \\ \hline 6 & 5 & 0 & 2 \\ \hline 7 & 1 & 1 & 0 \\ -1 & 2 & 3 & 6 \\ -3 & 1 & 2 & 4 \\ 1 & 2 & 3 & 0 \\ \hline 3 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \text{ and } X = \left[\begin{array}{cccc} 0 & 1 & 1 & 1 \\ \hline 6 & 5 & 2 & 2 \\ \hline 7 & 1 & 1 & 0 \\ -1 & 2 & 3 & 6 \\ -3 & 1 & 2 & 5 \\ 1 & 2 & 3 & 6 \\ \hline 3 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

be any two super column matrices of same type d (Y, X) = 3.

Example 3.34: Let

$$X = \left[\begin{array}{ccc} 2 & 3 & 1 \\ \hline 4 & 5 & 7 \\ 8 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 1 & 2 \\ 0 & 3 & 3 \\ \hline 1 & 8 & 0 \\ 3 & 1 & 7 \\ 8 & 1 & 8 \end{array}\right] \text{ and } Y = \left[\begin{array}{ccc} 2 & 3 & 1 \\ \hline 4 & 5 & 7 \\ 8 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline 1 & 1 & 2 \\ 0 & 3 & 3 \\ 1 & 8 & 0 \\ 3 & 1 & 7 \\ 8 & 1 & 8 \end{array}\right]$$

be any two super column matrices.

We see they are of same natural order. But they are not same type so special distance cannot be found.

Example 3.35: Let

$$M = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 7 & 8 & 9 & 1 & 2 \\ 1 & 3 & 0 & 1 & 0 & 0 \\ \hline 3 & 0 & 1 & 1 & 4 & -1 \\ 0 & 0 & 1 & 0 & 3 & 1 \\ 6 & 0 & 2 & 0 & 0 & 1 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 \end{array}\right] \text{ and}$$

$$N = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 7 & 8 & 9 & 1 & 2 \\ 1 & 3 & 6 & 1 & 0 & 0 \\ \hline 3 & 0 & 1 & 1 & 4 & 2 \\ 0 & 0 & 1 & 0 & 3 & 1 \\ 6 & 0 & 2 & 0 & 0 & 5 \\ \hline 1 & 2 & 3 & 4 & 5 & 7 \end{array}\right]$$

be any two super matrices of same natural order and same type.

The special distance between M and N is $d^S(M, N) = 4$.

Example 3.36: Let $X = \left[\begin{array}{cc|ccc|c} 3 & 1 & 1 & 3 & 8 & 4 \\ 5 & 6 & 8 & 9 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 & 1 & 3 \\ 2 & 2 & 4 & 8 & 0 & 4 \\ 1 & 0 & 2 & 1 & 0 & 1 \\ \hline 5 & 6 & 3 & 4 & 5 & 4 \\ 9 & 6 & 2 & 0 & 0 & 0 \\ \hline 1 & 0 & 2 & 0 & 1 & 5 \end{array}\right]$

$$\text{and } Y = \left[\begin{array}{cc|cc|cc} 3 & 1 & 1 & 3 & 8 & 4 \\ 5 & 6 & 8 & 9 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 & 1 & 3 \\ 2 & 2 & 4 & 8 & 0 & 4 \\ \hline 1 & 0 & 2 & 1 & 0 & 1 \\ 5 & 6 & 3 & 4 & 5 & 4 \\ \hline 9 & 6 & 2 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 5 \end{array}\right]$$

be any two super matrices.

We see they are of same natural order but we cannot find the special distance between them.

Example 3.37: Let

$$S = \left[\begin{array}{cc|c} 9 & 2 & 7 \\ 1 & 4 & 8 \\ \hline 9 & 1 & 0 \end{array}\right] \text{ and } R = \left[\begin{array}{cc|c} 9 & 2 & 0 \\ 1 & 4 & 8 \\ \hline 0 & 1 & 1 \end{array}\right]$$

be any two super matrices.

We see the special distance $d^S(S, R) = 3$.

Example 3.38: Let

$$R = \left[\begin{array}{c|cc} 3 & 1 & 0 \\ \hline 8 & 4 & 5 \\ 0 & 7 & 1 \end{array}\right] \text{ and } S = \left[\begin{array}{cc|c} 0 & 1 & 6 \\ \hline 7 & 8 & 1 \\ \hline 1 & 4 & 4 \end{array}\right]$$

be any two super matrices. They are of same natural order but of different type.

So the special distance between R and S cannot be determined.

Example 3.39: Let A = $\left[\begin{array}{cc|cccc|c} 9 & 0 & 1 & 2 & 1 & 2 & 0 \\ 6 & 1 & 0 & 0 & 0 & 1 & -1 \\ \hline 7 & 2 & 1 & 0 & 6 & 0 & 3 \\ 8 & 1 & 0 & 6 & 0 & 3 & 1 \\ 0 & 2 & 6 & 7 & 2 & 1 & 6 \\ 1 & 0 & 3 & 0 & 6 & 1 & 2 \\ \hline 8 & 1 & 1 & 0 & 1 & 0 & 3 \end{array}\right]$

and B = $\left[\begin{array}{cc|cccc|c} 9 & 0 & 1 & 2 & 1 & 2 & 7 \\ 6 & 1 & 0 & 0 & 0 & 6 & -1 \\ \hline 7 & 2 & 1 & 0 & 6 & 0 & 3 \\ 8 & 1 & 0 & 6 & 0 & 3 & 1 \\ 0 & 2 & 6 & 7 & 2 & 1 & 6 \\ 1 & 0 & 3 & 0 & 6 & 1 & 2 \\ \hline 8 & 1 & 1 & 6 & 1 & 2 & 4 \end{array}\right]$

be any two super matrices of same type.

The special distance between A and B $d^S(A, B) = 4$.

Now we can as in case of usual fuzzy models FCMs, FRMs, FREs, NCMs, NRMs and NREs apply it to super fuzzy models also.

This task is left as in case of usual matrices for the reader to find Kosko-Hamming distance between two initial super vectors.

We give example of them.

Example 3.40: Let us consider the fuzzy super FCMs model given by the following super matrix M_1 given by the first expert.

$$M_1 = \left[\begin{array}{cccccc|ccccc} 0 & 1 & -1 & 0 & 0 & 1 & & & & & \\ 1 & 0 & 0 & 1 & 1 & 0 & & & & & \\ -1 & 0 & 0 & 1 & 0 & 0 & & & (0) & & \\ 0 & 0 & 1 & 0 & -1 & 0 & & & & & \\ 1 & 0 & 0 & 0 & 0 & 1 & & & & & \\ 0 & 1 & -1 & 0 & 0 & 0 & & & & & \\ \hline & & & & & & 0 & 1 & 0 & 1 & 0 \\ & & & & & & 1 & 0 & 1 & 0 & 1 \\ & & (0) & & & & 0 & 1 & 0 & -1 & 0 \\ & & & & & & 1 & 0 & -1 & 0 & 1 \\ & & & & & & 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

Let M_2 be the super matrix given by the second expert.

$$M_2 = \left[\begin{array}{cccccc|ccccc} 0 & 1 & 0 & 0 & 0 & 1 & & & & & \\ 1 & 0 & 0 & 1 & 1 & 0 & & & & & \\ 0 & 0 & 0 & 1 & 0 & 0 & & & (0) & & \\ 0 & 0 & 0 & 0 & -1 & 0 & & & & & \\ 1 & 0 & 0 & 0 & 0 & 1 & & & & & \\ 0 & 1 & 0 & 0 & 0 & 0 & & & & & \\ \hline & & & & & & 0 & 1 & 0 & 1 & 0 \\ & & & & & & 1 & 0 & 1 & 0 & 1 \\ & & (0) & & & & 0 & 1 & 0 & 0 & 0 \\ & & & & & & 1 & 0 & 0 & 0 & 1 \\ & & & & & & 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

We see both the experts work on the same set of attributes. The super matrices of the super FCM are of same type.

The special distance $d(M_1, M_2) = 6$.

Let us consider the super initial row vector

$X = [1\ 0\ 0\ 0\ 0\ 1 \mid 0\ 1\ 0\ 0\ 0]$

$X \text{ o } M_1 \to [1\ 1\ 0\ 0\ 0\ 1 \mid 1\ 1\ 1\ 0\ 1] = Y_1$

$Y_1 \text{ o } M_1 \to [1\ 1\ 0\ 1\ 1\ 1 \mid 1\ 1\ 1\ 0\ 1] = Y_2$

$Y_2 \text{ o } M_1 \to [1\ 1\ 0\ 1\ 1\ 1 \mid 1\ 1\ 1\ 0\ 1] = Y_3\ (= Y_2)$.

Let us find the effect of X on the super FCM M_2.

$X\ M_2 \to [1\ 1\ 0\ 0\ 0\ 1 \mid 1\ 1\ 1\ 0\ 1] = Z_1$

$Z_1\ M_2 \to [1\ 1\ 0\ 1\ 1\ 1 \mid 1\ 1\ 1\ 1\ 1] = Z_2$
$Z_2\ M_2 \to [1\ 1\ 0\ 1\ 0\ 1 \mid 1\ 1\ 1\ 1\ 1] = Z_3$
$Z_3 M_2 \to (1\ 1\ 0\ 1\ 1\ 1 \mid 1\ 1\ 1\ 1\ 1] = Z_4 = Z_3$

The hidden pattern is a limit cycle.

The Kosko-Hamming distance between $d_K(Y_2, Z_2) = (0, 1)$

and $d_K(Y_2, Z_3) = (1, 1)$.

This is the way the Kosko-Hamming distance for the initial vector X is formed. Likewise we can find the super Kosko-Hamming distance.

Similarly we can find the Kosko-Hamming distance of two super FRMs. This study is a matter of routine and is left as an exercise to the reader.

We will find super Kosko-Hamming distance for any set of experts opinions we can also find the super Kosko-Hamming weight of the hidden pattern for any initial vector X.

$$d^S ((1\ 1\ 0\ 1\ 1\ 1 \mid 1\ 1\ 1\ 0\ 1), (1\ 0\ 0\ 0\ 0\ 1 \mid 0\ 1\ 0\ 0\ 0)) = w^S (1\ 1\ 0\ 1\ 1\ 1 \mid 1\ 1\ 1\ 0\ 1) = (3, 3)$$

Likewise we find the super Kosko-Hamming weight $d^S((1\ 1\ 0\ 1\ 1\ 1 \mid 1\ 1\ 1\ 1\ 1), (1\ 0\ 0\ 0\ 0\ 1 \mid 0\ 1\ 0\ 0\ 0) = (3, 4)$

$$= w^S ((1\ 1\ 0\ 1\ 1\ 1 \mid 1\ 1\ 1\ 1\ 1)).$$

Interested reader can work for different initial state super vectors.

Now we proceed onto give some problems for the reader.

Problems

1. Find any other nice applications and uses of the special distance between two matrices of same order.

2. Show the distance between two square matrices changes in general the eigen values associated with it.

3. Let $A = \begin{pmatrix} 3 & 4 & 2 & 1 \\ 0 & 1 & 4 & 0 \\ 2 & -4 & 5 & 6 \\ 0 & 1 & 0 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 & 2 & 1 \\ 7 & 1 & 4 & 2 \\ 2 & 0 & 5 & 6 \\ 0 & 1 & 0 & 7 \end{pmatrix}$

   be any two 4×4 matrices.

   (i) Find how many eigen values of A and B are identical?

   (ii) Find the special distance between A and B.

(iii) If $C = \begin{pmatrix} 3 & 4 & 2 & 1 \\ 0 & 1 & 4 & 1 \\ 2 & -4 & 0 & 2 \\ 0 & 1 & 0 & 7 \end{pmatrix}$ find the special distance

between A and C and B and C.

(iv) How many eigen values of A and C are identical?

(v) Find the number of eigen values which are common between B and C.

4. If instead of a usual square matrix a upper or a lower triangular matrix is used study the maximum number of acceptance changes that can occur without changing the eigen values.

5. Let $A = \begin{bmatrix} 3 & 7 & 2 & 1 & 0 & 5 \\ 0 & 1 & 0 & 5 & 2 & 1 \\ 0 & 0 & 9 & 0 & 7 & 9 \\ 0 & 0 & 0 & 8 & 1 & 2 \\ 0 & 0 & 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & 0 & 0 & 11 \end{bmatrix}$ and

$$B = \begin{bmatrix} 3 & 0 & 3 & 4 & 6 & 7 \\ 0 & 1 & 5 & 2 & 1 & 8 \\ 0 & 0 & 9 & 1 & 9 & 9 \\ 0 & 0 & 0 & 8 & 3 & 4 \\ 0 & 0 & 0 & 0 & 9 & 7 \\ 0 & 0 & 0 & 0 & 0 & 11 \end{bmatrix}$$

be two upper triangular matrices.

(i) Find d(A, B).
(ii) Are the eigen vectors and values of A different from B?

6. Obtain any special feature about eigen values of two square matrices which are at a maximum distance.

7. Obtain a minimum distance between any two square matrices so that they have the same set of eigen values.

8. Prove or disprove if some of the diagonal values vary in two square matrices then they have different eigen values.

9. Describe the role of special distance of matrices in FCMs.

10. Let $A_1 = \begin{bmatrix} 9 & 2 & 0 & 0 \\ 1 & 8 & 0 & 7 \\ 4 & 0 & 9 & 8 \\ 1 & 2 & 3 & 5 \end{bmatrix}$ and $A_2 = \begin{bmatrix} 7 & 2 & 0 & 0 \\ 1 & 2 & 0 & 7 \\ 4 & 0 & 9 & 8 \\ 1 & 2 & 3 & 5 \end{bmatrix}$

be any two square matrices.

(i) Find the special distance between A_1 and A_2.

11. Let $M_1 = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$,

$$M_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$M_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

be the 3 connection matrices associated with FCMs given by three experts.

(i) Find the special distance between the matrices M_1 and M_2, M_2 and M_3 and M_1 and M_3.

(ii) Find for the six on state of nodes.

$X_1 = (1\ 0\ 0\ 0\ 0\ 0)$,
$X_2 = (0\ 1\ 0\ 0\ 0\ 0)$,
$X_3 = (0\ 0\ 1\ 0\ 0\ 0)$,
$X_4 = (0\ 0\ 0\ 1\ 0\ 0)$,
$X_5 =(0\ 0\ 0\ 0\ 1\ 0)$ and
$X_6 = (0\ 0\ 0\ 0\ 0\ 1)$ the Kosko-Hamming distance of the hidden patterns between the experts.

(iii) Find the Kosko-Hamming weight of the hidden patterns.

(iv) Find the most influential nodes and more influential nodes of the 3 experts.

12. Let $S_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

be the connection matrix given by the first expert.

Let S_2 be the connection matrix given by the second expert which is as follows:

$$S_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Study questions (i) to (iv) of problem 11 for this pair.

13. What are the special features helpful in the study of special distance in FRM models?

14. Describe using a FRM model the most influential node, more influential node and so on.

15. Let $M_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$ and

$$M_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

be two FRM connection matrices given by two experts on the same problem with same number of range and domain nodes.

(i) Find the special distance between M_1 and M_2.

(ii) Find on how many domain nodes the experts agree upon the resultant.

(iii) In how many range nodes experts do not agree upon the resultant vectors?

(iv) Find the influential nodes of the domain and range spaces.

16. Using several experts and using FRMs to analyse a real world problem, find the closeness or deviation of experts.

    Show in general the most influential node is not the same for all experts.

17. Show in Fuzzy Relational Equations the notion of special distance and Kosko-Hamming distance plays a very vital role in the study of these FRE models of different experts.

18. Let $M_1 = \begin{bmatrix} 0.8 & 0.4 & 1 & 0 & 0 & 0.7 \\ 0.1 & 0.7 & 0 & 0.5 & 0 & 0.6 \\ 0 & 0 & 0.8 & 0.4 & 0.7 & 0 \\ 0 & 0.2 & 0 & 0 & 0 & 0.5 \\ 0.3 & 0.9 & 0 & 1 & 0.2 & 0 \\ 1 & 0 & 0 & 0 & 0.7 & 0 \\ 0 & 0.2 & 0.5 & 0.1 & 0.8 & 0 \end{bmatrix}$ and

$$M_2 = \begin{bmatrix} 1 & 0.3 & 0.8 & 0 & 0.1 & 0.8 \\ 0 & 0.8 & 0.1 & 0.7 & 0 & 0.7 \\ 0.2 & 0.1 & 0.9 & 0.6 & 0.5 & 0.7 \\ 0.8 & 0.3 & 0 & 0.1 & 0.2 & 0.6 \\ 0.5 & 0.8 & 0.1 & 0.7 & 0.7 & 0.7 \\ 0.8 & 0 & 0 & 0.1 & 0.6 & 0 \\ 0 & 0.2 & 0.6 & 0.2 & 0.8 & 0.3 \end{bmatrix}$$

be the membership matrices of FRE given by two experts.

(i) Find the special distance between M_1 and M_2.

(ii) Using same initial state vector X study the effect of these matrices and find the Kosko-Hamming distance between them.

(iii) Can we speak of Kosko-Hamming weight in this case?

19. Study for two NCMs of two experts the notion of Kosko-Hamming distance function and Kosko-Hamming weight function.

20. Let $S_1 = \begin{bmatrix} 0 & I & 1 & 0 & 1 \\ 0 & 0 & 0 & I & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \end{bmatrix}$ and $S_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & I & 1 \\ 1 & 0 & 0 & 1 & I \\ 0 & 1 & I & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

be two connection neutrosophic matrices given by two different experts.

(i) Find the special distance between S_1 and S_2.

(ii) For the set of initial state vectors X_1 = (1 0 0 0 0), X_2 = (0 1 0 0 0), X_3 = (0 0 1 0 0), X_4 = (0 0 0 10) and X_5 = (0 0 0 0 1) find for the resultant vectors Kosko-Hamming distances.

(iii) Using Kosko-Hamming weight determine the most influential node and the least influential node.

21. Study the notion of Kosko-Hamming distance function pair and Kosko-Hamming weight in case of NRMs given by two experts.

22. Let $N_1 = \begin{bmatrix} I & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & I & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & I & 0 & 0 & I \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$ and

$$N_2 = \begin{bmatrix} I & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & I & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & I \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

be the two relational matrix of NRMs given by two experts on a problem.

(i) Find the special distance between N_1 and N_2.

(ii) Find for the initial state vector X_1 = (1 0 … 0) X_3 = (0 0 1 0 0 0 0 0), X_4 = (00 0 10 0 0 0), X_5 = (0 0 0 0 1 0 0 0), X_6 =(0 0 0 0 0 1 0 0) and X_8 = (0 0 0 0 0 0 0 1) in the domain space, the corresponding hidden pattern pairs and study using Kosko-Hamming distance function, the closeness or distance of the two experts opinion.

(iii) For the set of initial vector Y_1 = (1 0 0 0 0), Y_3 = (0 0 1 0 0) and Y_5 = (0 0 0 0 1) in the range space find the hidden pattern pair and study the relation between the experts using Kosko-Hamming distance functions.

(iv) Find also the Kosko-Hamming weights of the resultant vector pairs of these initial vectors given in (ii) and (iii) and find the most influential node and just influential nodes.

23. Show by appropriately modifying the problem one can use the same and study questions (i) to (iv) of problem 22 for a FCM given by one expert and for a NCM given by another expert who work with the same problem and the same set of attributes.

24. Show if two experts work on the problem with same set of domain and range attributes using FRM and NRM then one can study questions (i) to (iv) of problem 22 for this FRM and NRM.

25. Show for membership matrix of NREs given experts, we can use the concept of Kosko-Hamming distance function to analyse the models.

26. Let two experts work on the same problem using the same set of attributes with neutrosophic matrices S_1 and S_2 where

$$S_1 = \begin{bmatrix} 0.I & 0.2 & 0 & 0.7I & 1 & 0 & 1 \\ 1 & 0 & I & 0 & 0.7 & 0.2 & 0 \\ 0.3 & I & 1 & 0.2I & 0 & 0 & 0.7 \\ 0.7I & 0 & 0.3I & 0 & 0.2I & 1 & 0 \\ 0 & 0.8I & 0 & 1 & 0.8 & 0 & 0.6I \\ 0.3I & 0 & 0 & 0 & 0 & 0 & 0.7 \\ 0.I & 1 & 0.2I & 0.7 & 0.2I & I & 0 \\ 0.8I & I & 0.5I & 0 & 0 & 0 & 0.2I \end{bmatrix} \text{ and}$$

$$S_2 = \begin{bmatrix} 0.3 & 1 & 0 & 0.2I & 1 & 0 & I \\ 0 & I & 0 & 0.4I & 0 & 0.2 & 0.3 \\ 0.5I & 0 & 0.7 & 0 & 0.8 & 0.4 & 0 \\ 0 & 0.1 & 0.8I & 0.4 & 0 & 0.6 & 0.7 \\ 1 & 0.3 & I & 0 & 0.3 & I & 0.2 \\ 0.8 & 0.3I & 0 & 0.7 & 0.6I & 0 & 0.4I \\ 0.7I & 0.6 & 1 & 0.4 & 0 & I & 0.2 \\ 0.2 & 0.4I & 0.3I & 0 & 0.8I & 0 & 0.6I \end{bmatrix}$$

be the neutrosophic membership matrices of the NREs given by two experts.

(i) Find the special distance between the matrices S_1 and S_2.

(ii) For 3 different essential initial points find the Kosko-Hamming distances.

(iii) Can we get the most influential node? Justify your answer.

27. Show these concepts can be applied to super fuzzy models also.

28. Let us consider two super diagonal matrices who work on the same problem given by 3 sets of experts.

$S_1 =$

$$\left[\begin{array}{c|c|c}
\begin{matrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{matrix} & (0) & (0) \\
\hline
(0) & \begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix} & (0) \\
\hline
(0) & (0) & \begin{matrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{matrix}
\end{array}\right]$$

and S_2 =

$$\left[\begin{array}{c|c|c}
\begin{array}{ccccc} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array} & (0) & (0) \\ \hline
(0) & \begin{array}{ccc} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{array} & (0) \\ \hline
(0) & (0) & \begin{array}{ccccccc} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}
\end{array}\right]$$

be the expert opinions.

(i) Find for the initial state super vector $X = (1\ 0\ 0\ 0\ 0 \mid 0\ 0\ 1 \mid 0\ 0\ 1\ 0\ 0\ 0\ 0)$ the resultant Kosko-Hamming distance function using S_1 and S_2.

(ii) Find the special distance between S_1 and S_2.

(iii) Find the most influential nodes for S_1 and S_2.

29. Study the other super fuzzy models and find the influential nodes.

30. Obtain any other interesting notions while studying super fuzzy models.

FURTHER READING

1. Adamopoulos, G.I., and Pappis, C.P., Some Results on the Resolution of Fuzzy Relation Equations, Fuzzy Sets and Systems, 60 (1993) 83-88.

2. Adams, E.S., and D.A. Farber. Beyond the Formalism Debate: Expert Reasoning, Fuzzy Logic and Complex Statutes, Vanderbilt Law Review, 52 (1999), 1243-1340. http://law.vanderbilt.edu/lawreview/vol525/adams.pdf

3. Adlassnig, K.P., Fuzzy Set Theory in Medical Diagnosis, IEEE Trans. Systems, Man, Cybernetics, 16 (1986) 260-265.

4. Ashbacher, C. Introduction to Neutrosophic Logic, American Research Press, Rehoboth, 2002. http://www.gallup.unm.edu/~smarandache/IntrodNeutLogic.pdf

5. Axelord, R. (ed.) Structure of Decision: The Cognitive Maps of Political Elites, Princeton Univ. Press, New Jersey, 1976.

6. Banini, G.A., and R. A. Bearman. Application of Fuzzy Cognitive Maps to Factors Affecting Slurry Rheology, Int. J. of Mineral Processing, 52 (1998) 233-244.

7. Bechtel, J.H. An Innovative Knowledge Based System using Fuzzy Cognitive Maps for Command and Control, Storming Media, Nov 1997. http://www.stormingmedia.us/cgi-bin/32/3271/A327183.php

8. Birkhoff, G., Lattice Theory, American Mathematical Society, 1979.

9. Blanco, A., Delgado, M., and Requena, I., Solving Fuzzy Relational Equations by Max-min Neural Network, Proc. 3rd IEEE Internet Conf. On Fuzzy Systems, Orlando (1994) 1737-1742.

10. Brannback, M., L. Alback, T. Finne and R. Rantanen. Cognitive Maps: An Attempt to Trace Mind and Attention in Decision Making, in C. Carlsson ed. Cognitive Maps and Strategic Thinking, Meddelanden Fran Ekonomisk Statsvetenskapliga Fakulteten vid Abo Akademi Ser. A 442 (1995) 5-25.

11. Brubaker, D. Fuzzy Cognitive Maps, EDN ACCESS, 11 April 1996. http://www.e-insite.net/ednmag/archives/1996/041196/08column.htm

12. Brubaker, D. More on Fuzzy Cognitive Maps, EDN ACCESS, 25 April 1996. http://www.e-insite.net/ednmag/archives/1996/042596/09column.htm

13. Caudill, M. Using Neural Nets: Fuzzy Cognitive Maps, Artificial Intelligence Expert, 6 (1990) 49-53.

14. Cechiarova, K., Unique Solvability of Max-Min Fuzzy Equations and Strong Regularity of Matrices over Fuzzy Algebra, Fuzzy Sets and Systems, 75 (1995) 165-177.

15. Cheng, L., and Peng, B., The Fuzzy Relation Equation with Union or Intersection Preserving Operator, Fuzzy Sets and Systems, 25 (1988) 191-204.

16. Chung, F., and Lee, T., A New Look at Solving a System of Fuzzy Relational Equations, Fuzzy Sets and Systems, 99 (1997) 343-353.

17. Craiger, J.P. Causal Structure, Model Inferences and Fuzzy Cognitive Maps: Help for the Behavioral Scientist, International Neural Network Society, Annual Meeting World Congress Neural Networks, June 1994.

18. Craiger, J.P., and M.D. Coovert. Modeling Dynamic Social and Psychological Processes with Fuzzy Cognitive Maps. In Proc. of the 3rd IEEE Conference on Fuzzy Systems, 3 (1994) 1873-1877.

19. Craiger, J.P., R.J. Weiss, D.F. Goodman, and A.A. Butler. Simulating Organizational Behaviour with Fuzzy Cognitive Maps, Int. J. of Computational Intelligence and Organization, 1 (1996) 120-123.

20. Di Nola, A., and Sessa, S., On the Set of Composite Fuzzy Relation Equations, Fuzzy Sets and Systems, 9 (1983) 275-285.

21. Di Nola, A., On Solving Relational Equations in Brouwerian Lattices, Fuzzy Sets and Systems, 34 (1994) 365-376.

22. Di Nola, A., Pedrycz, W., and Sessa, S., Some Theoretical Aspects of Fuzzy Relation Equations Describing Fuzzy System, Inform Sci., 34 (1984) 261-264.

23. Di Nola, A., Pedrycz, W., Sessa, S., and Sanchez, E., Fuzzy Relation Equations Theory as a Basis of Fuzzy Modeling: An Overview, Fuzzy Sets and Systems, 40 (1991) 415-429.

24. Di Nola, A., Relational Equations in Totally Ordered Lattices and their Complete Resolution, J. Math. Appl., 107 (1985) 148-155.

25. Di Nola, A., Sessa, S., Pedrycz, W., and Sanchez, E., Fuzzy Relational Equations and their Application in Knowledge Engineering, Kluwer Academic Publishers, Dordrecht, 1989.

26. Dickerson, J.A., and B. Kosko. Virtual Worlds as Fuzzy Cognitive Maps, Presence, 3 (1994) 173-189.

27. Dickerson, J.A., Z. Cox, E.S. Wurtele and A.W. Fulmer. Creating Metabolic and Regulatory Network Models using Fuzzy Cognitive Maps. http://www.botany.iastate.edu/~mash/metnetex/NAFIPS01v3a.pdf

28. Drewniak, J., Equations in Classes of Fuzzy Relations, Fuzzy Sets and Systems, 75 (1995) 215-228.

29. Fang, S.C., and Li, G., Solving Fuzzy Relation Equations with a Linear Objective Function, Fuzzy Sets and Systems, 103 (1999) 107-113.

30. Fuzzy Thought Amplifier. The Fuzzy Cognitive Map Program, Fuzzy Systems Engineering, USA. http://www.fuzzysys.com/ftaprod.html

31. Gavalec, M., Solvability and Unique Solvability of Max-min Fuzzy Equations. Fuzzy Sets and Systems, 124 (2001) 385-393.

32. Gottwald, S., Approximate Solutions of Fuzzy Relational Equations and a Characterization of t-norms that Define Matrices for Fuzzy Sets, Fuzzy Sets and Systems, 75 (1995) 189-201.

33. Gottwald, S., Approximately Solving Fuzzy Relation Equations: Some Mathematical Results and Some Heuristic Proposals, Fuzzy Sets and Systems, 66 (1994) 175-193.

34. Guo, S.Z., Wang, P.Z., Di Nola, A., and Sessa, S., Further Contributions to the Study of Finite Fuzzy Relation Equations, Fuzzy Sets and Systems, 26 (1988) 93-104.

35. Hafner, V.V. Cognitive Maps for Navigation in Open Environments, http://citeseer.nj.nec.com/hafner00cognitive.html

36. Hagiwara, M. Extended Fuzzy Cognitive Maps, Proc. IEEE International Conference on Fuzzy Systems, (1992) 795-801.

37. Harary, F. Graph Theory, Narosa Publications (reprint, Indian edition), New Delhi, 1969.

38. Hirota, K., and Pedrycz, W., Specificity Shift in Solving Fuzzy Relational Equations, Fuzzy Sets and Systems, 106 (1999) 211-220.

39. Kardaras, D., and B. Karakostas. The Use of Fuzzy Cognitive maps to Stimulate the Information Systems Strategic Planning Process, Information and Software Technology, 41 (1999) 197-210.

40. Kardaras, D., and G. Mentzas. Using fuzzy cognitive maps to model and analyze business performance assessment, In Prof. of Int. Conf. on Advances in Industrial Engineering – Applications and Practice II, Jacob Chen and Anil Milal (eds.), (1997) 63-68.

41. Khan, M.S., M. Quaddus, A. Intrapairot, and A. Chong, Modelling Data Warehouse Diffusion using Fuzzy Cognitive Maps – A Comparison with the System Dynamics Approach. http://wawisr01.uwa.edu.au/2000/Track%204/gModelling.PDF

42. Kim, H.S., and K. C. Lee. Fuzzy Implications of Fuzzy Cognitive Maps with Emphasis on Fuzzy Causal Relations

and Fuzzy Partially Causal Relationship, Fuzzy Sets and Systems, 97 (1998) 303-313.

43. Klir, G.J., and Yuan, B., Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice-Hall, Englewood Cliffs NJ, 1995.

44. Kosko, B. Fuzzy Cognitive Maps, Int. J. of Man-Machine Studies, 24 (1986) 65-75.

45. Kosko, B., Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence, Prentice Hall of India, 1997.

46. Kurano, M., Yasuda, M., Nakatami, J., and Yoshida, Y., A Fuzzy Relational Equation in Dynamic Fuzzy Systems, Fuzzy Sets and Systems, 101 (1999) 439-443.

47. Lee, K.C., J.S. Kim, N.H. Chang and S.J. Kwon. Fuzzy Cognitive Map Approach to Web-mining Inference Amplification, Expert Systems with Applications, 22 (2002) 197-211.

48. Lee, K.C., W.J. Lee, O.B. Kwon, J.H. Han, P.I. Yu. A Strategic Planning Simulation Based on Fuzzy Cognitive Map Knowledge and Differential Game, Simulation, 71 (1998) 316-327.

49. Lidl, R., and Pliz, G., Applied abstract algebra', Springer Verlag, USA, 1984.

50. Liu, F., and F. Smarandache. Intentionally and Unintentionally. On Both, A and Non-A, in Neutrosophy. http://lanl.arxiv.org/ftp/math/papers/0201/0201009.pdf

51. Liu, F., and F. Smarandache. Logic: A Misleading Concept. A Contradiction Study toward Agent's Logic, in Proceedings of the First International Conference on Neutrosophy, Neutrosophic Logic, Neutrosophic Set, Neutrosophic Probability and Statistics, Florentin

Smarandache editor, Xiquan, Phoenix, ISBN: 1-931233-55-1, 147 p., 2002, also published in "Libertas Mathematica", University of Texas at Arlington, 22 (2002) 175-187. http://lanl.arxiv.org/ftp/math/papers/0211/0211465.pdf

52. Liu, F., and Smarandache, F., Intentionally and Unintentionally. On Both, A and Non-A, in Neutrosophy. http://lanl.arxiv.org/ftp/math/papers/0201/0201009.pdf

53. Loetamonphing, J., and Fang, S.C., Optimization of Fuzzy Relation Equations with Max-product Composition, Fuzzy Sets and Systems, 118 (2001) 509-517.

54. Luo, C.Z., Reachable Solution Set of a Fuzzy Relation Equation, J. of Math. Anal. Appl., 103 (1984) 524-532.

55. Luoh, L., Wang, W.J., Liaw, Y.K., New Algorithms for Solving Fuzzy Relation Equations, Mathematics and Computers in Simulation, 59 (2002) 329-333.

56. Pedrycz, W., Inverse Problem in Fuzzy Relational Equations, Fuzzy Sets and Systems, 36 (1990) 277-291.

57. Pedrycz, W., Processing in Relational Structures: Fuzzy Relational Equations, Fuzzy Sets and Systems, 25 (1991) 77-106.

58. Pelaez, C.E., and J.B. Bowles. Using Fuzzy Cognitive Maps as a System Model for Failure Modes and Effects Analysis, Information Sciences, 88 (1996) 177-199.

59. Praseetha, V.R. A New Class of Fuzzy Relation Equation and its Application to a Transportation Problem, Masters Dissertation, Guide: Dr. W. B. Vasantha Kandasamy, Department of Mathematics, Indian Institute of Technology, April 2000.

60. Praseetha, V.R., A New Class of Fuzzy Relation Equation and its Application to a Transportation Problem, Masters Dissertation, Guide: Dr. W. B. Vasantha Kandasamy,

Department of Mathematics, Indian Institute of Technology, April 2000.

61. Ram Kishore, M. Symptom disease model in children, Masters Dissertation, Guide: Dr. W. B. Vasantha Kandasamy, Department of Mathematics, Indian Institute of Technology, Chennai, April 1999.

62. Ramathilagam, S. Mathematical Approach to the Cement Industry problems using Fuzzy Theory, Ph.D. Dissertation, Guide: Dr. W. B. Vasantha Kandasamy, Department of Mathematics, Indian Institute of Technology, Madras, November 2002.

63. Ramathilagam, S., Mathematical Approach to the Cement Industry problems using Fuzzy Theory, Ph.D. Dissertation, Guide: Dr. W. B. Vasantha Kandasamy, Department of Mathematics, Indian Institute of Technology, Madras, November 2002.

64. Silva, P.C. Fuzzy Cognitive Maps over Possible Worlds, Proc. of the 1995 IEEE International Conference on Fuzzy Systems, 2 (1995) 555-560.

65. Siraj, A., S.M. Bridges, and R.B. Vaughn. Fuzzy cognitive maps for decision support in an intelligent intrusion detection systems, www.cs.msstate.edu/~bridges/papers/nafips2001.pdf

66. Smarandache, F. (editor), Proceedings of the First International Conference on Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics, Univ. of New Mexico – Gallup, 2001. http://www.gallup.unm.edu/~smarandache/NeutrosophicProceedings.pdf

67. Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic, Preface by Charles Le, American Research Press, Rehoboth, 1999, 2000. Second edition of the Proceedings of the First International Conference on Neutrosophy,

Neutrosophic Logic, Neutrosophic Set, Neutrosophic Probability and Statistics, University of New Mexico, Gallup, 1-3 December 2001. http://www.gallup.unm.edu/~smarandache/eBook-Neutrosophics2.pdf

68. Smarandache, F. Collected Papers III, Editura Abaddaba, Oradea, 2000. http://www.gallup.unm.edu/~smarandache/CP3.pdf

69. Smarandache, F. Neutrosophic Logic - Generalization of the Intuitionistic Fuzzy Logic, To be presented at the Special Session on Intuitionistic Fuzzy Sets and Related Concepts, of International EUSFLAT Conference, Zittau, Germany, 10-12 September 2003. http://lanl.arxiv.org/ftp/math/papers/0303/0303009.pdf

70. Styblinski, M.A., and B.D. Meyer. Fuzzy Cognitive Maps, Signal Flow Graphs, and Qualitative Circuit Analysis, in Proc. of the 2nd IEEE International Conference on Neural Networks (ICNN – 87), San Diego, California (1988) 549-556.

71. Styblinski, M.A., and B.D. Meyer. Signal Flow Graphs versus Fuzzy Cognitive Maps in Applications to Qualitative Circuit Analysis, Int. J. of Man-machine Studies, 18 (1991) 175-186.

72. Stylios, C.D., and P.P. Groumpos. Fuzzy Cognitive Maps: a Soft Computing Technique for Intelligent Control, in Proc. of the 2000 IEEE International Symposium on Intelligent Control held in Patras, Greece, July 2000, 97-102.

73. Taber W. R. Fuzzy Cognitive Maps Model Social Systems, Artificial Intelligence Expert, 9 (1994) 18-23.

74. Uma, S. Estimation of Expert Weights using Fuzzy Cognitive Maps, Masters Dissertation, Guide: Dr. W.B.Vasantha Kandasamy, Department of Mathematics, Indian Institute of Technology, Chennai, March 1997.

75. Vasantha Kandasamy and Smarandache Florentin, Analysis of social aspects of migrant labourers living with HIV/AIDS using Fuzzy Theory and Neutrosophic Cognitive Maps, Xiquan, Phoenix, 2004.

76. Vasantha Kandasamy, W.B. and Florentin Smarandache, Introduction to n-adaptive Fuzzy Models to Analyze Public opinion in AIDS, Hexis, Arizona, 2006.

77. Vasantha Kandasamy, W.B., and A. Minor. Estimation of Production and Loss or Gain to Industries Using Matrices, Proc. of the National Conf. on Challenges of the 21st century in Mathematics and its allied topics, Feb. 3-4, 2001, Univ. of Mysore, 211-218.

78. Vasantha Kandasamy, W.B., and Balu, M. S., Use of Weighted Multi-Expert Neural Network System to Study the Indian Politics, Varahimir J. of Math. Sci., 2 (2002) 44-53.

79. Vasantha Kandasamy, W.B., and Florentin Smarandache, Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps, Xiquan, Phoenix, 2003.

80. Vasantha Kandasamy, W.B., and Indra, V., Maximizing the passengers comfort in the madras transport corporation using fuzzy programming, *Progress of Mat.*, Banaras Hindu Univ., **32** (1998) 91-134.

81. Vasantha Kandasamy, W.B., and Mary John, M., Fuzzy Analysis to Study the Pollution and the Disease Caused by Hazardous Waste From Textile Industries, Ultra Sci, 14 (2002) 248-251.

82. Vasantha Kandasamy, W.B., and Ram Kishore, M., Symptom-Disease Model in Children using FCM, Ultra Sci., 11 (1999) 318-324.

83. Vasantha Kandasamy, W.B., and Balu, M.S., Use of Weighted Multi-Expert Neural Network System to Study the Indian Politics, Sandipani Academy, 2 (2002) 44-53.

84. Vasantha Kandasamy, W.B., and Smarandache, F., Neutrosophic Lattices, 2 Neutrosophic Sets and Systems, (2014) 42-47.

85. Vasantha Kandasamy, W.B., and Pramod, P., Parent Children Model using FCM to Study Dropouts in Primary Education, Ultra Sci., 13, (2000) 174-183.

86. Vasantha Kandasamy, W.B., and Praseetha, R., New Fuzzy Relation Equations to Estimate the Peak Hours of the Day for Transport Systems, J. of Bihar Math. Soc., 20 (2000) 1-14.

87. Vasantha Kandasamy, W.B., Vasuki, R., and Thulukkanam, K., Kosko Hamming Distance in the analysis of FCMs to study the problems of locals due to dumping of solid waste in Kodungaiyur, Ultra Scientist, 26 (2014) 55-62.

88. Vasantha Kandasamy, W.B., and Uma, S., Combined Fuzzy Cognitive Map of Socio-Economic Model, Appl. Sci. Periodical, 2 (2000) 25-27.

89. Vasantha Kandasamy, W.B., and Uma, S., Fuzzy Cognitive Map of Socio-Economic Model, Appl. Sci. Periodical, 1 (1999) 129-136.

90. Vasantha Kandasamy, W.B., and Smarandache, F., Fuzzy Relational Equations and Neutrosophic Relational Equations, Hexis (Church Rock, USA), 2004.

91. Vasantha Kandasamy, W.B., Smarandache, F., and Unnisa, I., Supermodular Lattices, Educational Publisher Inc., Ohio, 2012.

92. Vasantha Kandasamy, W.B., and V. Indra. Applications of Fuzzy Cognitive Maps to Determine the Maximum Utility

of a Route, J. of Fuzzy Maths, publ. by the Int. fuzzy Mat. Inst., 8 (2000) 65-77.

93. Vasantha Kandasamy, W.B., and Yasmin Sultana, FRM to Analyse the Employee-Employer Relationship Model, J. Bihar Math. Soc., 21 (2001) 25-34.

94. Vasantha Kandasamy, W.B., and Yasmin Sultana, Knowledge Processing Using Fuzzy Relational Maps, Ultra Sci., 12 (2000) 242-245.

95. Vasantha Kandasamy, W.B., Florentin Smarandache and K. Ilanthenral, Elementary Fuzzy Matrix Theory and Fuzzy Models for Socio Scientists, Automaton, 2007.

96. Vasantha Kandasamy, W.B., Mary John, M., and T. Kanagamuthu. Study of Social Interaction and Woman Empowerment Relative to HIV/AIDS, Maths Tiger, 1(4) (2002) 4-7.

97. Vasantha Kandasamy, W.B., Neelakantan, N.R., and S. Ramathilagam. Maximize the Production of Cement Industries by the Maximum Satisfaction of Employees using Fuzzy Matrix, Ultra Science, 15 (2003) 45-56.

98. Vasantha Kandasamy, W.B., Neelakantan, N.R., and Kannan, S.R. Replacement of Algebraic Linear Equations by Fuzzy Relation Equations in Chemical Engineering, In Recent trends in Mathematical Sciences, Proc. of Int. Conf. on Recent Advances in Mathematical Sciences held at IIT Kharagpur on Dec. 20-22, 2001, published by Narosa Publishing House, (2001) 161-168.

99. Vasantha Kandasamy, W.B., Neelakantan, N.R., and Kannan, S.R., Operability Study on Decision Tables in a Chemical Plant using Hierarchical Genetic Fuzzy Control Algorithms, Vikram Mathematical Journal, 19 (1999) 48-59.

100. Vasantha Kandasamy, W.B., Smarandache, F., and Amal, K., Super Fuzzy Matrices and Super Fuzzy Models for Social Scientist, Infolearnquest, 2008.

101. Vasantha Kandasamy, W.B., Smarandache, F., and Ilanthenral, K., Pseudo Lattice Graphs and their Applications to Fuzzy and Neutrosophic Models, EuropaNova., 2014.

102. Vasantha Kandasamy, W.B., Smarandache, F., and Ilanthenral, K., New Techniques to analyse the prediction of Fuzzy models and Neturosophic models, EuropaNova., 2014.

103. Vasantha Kandasamy, W.B., Sekar, P., and Vidhyalakshmi, New type of Fuzzy Relational Equations and Neutrosophic Relational Equations – to analyse Customer Preference to Street shops, Neutrosophic Sets and Systems, 3(2014), 70-78.

104. Yasmin Sultana, Construction of Employee-Employee Relationship Model using Fuzzy Relational Maps, Masters Dissertation, Guide: Dr. W. B. Vasantha Kandasamy, Department of Mathematics, Indian Institute of Technology, April 2000.

105. Yen, J., Langari, R., and Zadeh, L.A., Industrial Applications of Fuzzy Logic and Intelligent Systems, IEEE Press, New York 1995.

106. Yuan, Miao and Zhi-Qiang Liu. On Causal Inference in Fuzzy Cognitive Maps, IEEE Transactions on Fuzzy Systems, 81 (2000) 107-119.

107. Zadeh, L.A., A Theory of Approximate Reasoning, Machine Intelligence, 9 (1979) 149- 194.

108. Zimmermann, H.J., Fuzzy Set Theory and its Applications, Kluwer, Boston, 1988.

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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The distance between two row matrices was defined by Hamming. However the distance between two column matrices or for that matter any two matrices of same order are not defined systematically. Here in this book this concept is defined and it can help in the analysis of storage of data, deviation in eigen values and in finding difference in opinion of experts using some fuzzy models. Some open problems are also suggested.

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