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Abstract. In this paper, we presented another concept of neutrosophic open sets called neutrosophic semi-α-open sets and studied their fundamental properties in neutrosophic topological spaces. We also present neutrosophic semi-α-interior and neutrosophic semi-α-closure and study some of their fundamental properties.
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1. Introduction

In 2000, G.B. Navalagi [4] presented the idea of semi-α-open sets in topological spaces. The concept of "neutrosophic set" was first given by F. Smarandache [2,3]. A.A. Salama and S.A. Alblowi [1] presented the concept of neutrosophic topological space (briefly NTS). The objective of this paper is to present the concept of neutrosophic semi-α-open sets and study their fundamental properties in neutrosophic topological spaces. We also present neutrosophic semi-α-interior and neutrosophic semi-α-closure and obtain some of its properties.

2. Preliminaries

Throughout this paper, $(\mathcal{U},T)$ (or simply $\mathcal{U}$) always mean a neutrosophic topological space. The complement of a neutrosophic open set (briefly N-OS) is called a neutrosophic closed set (briefly N-CS) in $(\mathcal{U},T)$. For a neutrosophic set $\mathcal{A}$ in a neutrosophic topological space $(\mathcal{U},T)$, $Ncl(\mathcal{A})$, $Nint(\mathcal{A})$ and $\mathcal{A}^c$ denote the neutrosophic closure of $\mathcal{A}$, the neutrosophic interior of $\mathcal{A}$ and the neutrosophic complement of $\mathcal{A}$ respectively.

Definition 2.1:
A neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U},T)$ is said to be:
(i) A neutrosophic pre-open set (briefly NP-OS) [7] if $\mathcal{A} \subseteq Nint(Ncl(\mathcal{A}))$. The complement of a NP-OS is called a neutrosophic pre-closed set (briefly NP-CS) in $(\mathcal{U},T)$. The family of all NP-OS (resp. NP-CS) of $\mathcal{U}$ is denoted by NPO$(\mathcal{U})$ (resp. NPC$(\mathcal{U})$).
(ii) A neutrosophic semi-open set (briefly NS-OS) [6] if $\mathcal{A} \subseteq Ncl(Nint(\mathcal{A}))$. The complement of a NS-OS is called a neutrosophic semi-closed set (briefly NS-CS) in $(\mathcal{U},T)$. The family of all NS-OS (resp. NS-CS) of $\mathcal{U}$ is denoted by NSO$(\mathcal{U})$ (resp. NSC$(\mathcal{U})$).
(iii) A neutrosophic α-open set (briefly Nα-OS) [5] if $\mathcal{A} \subseteq Nint(Ncl(Nint(\mathcal{A})))$. The complement of a Nα-OS is called a neutrosophic α-closed set (briefly Nα-CS) in $(\mathcal{U},T)$. The family of all Nα-OS (resp. Nα-CS) of $\mathcal{U}$ is denoted by NαO$(\mathcal{U})$ (resp. NαC$(\mathcal{U})$).

Definition 2.2:
(i) The neutrosophic pre-interior of a neutrosophic set $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U},T)$ is the union of all NP-OS contained in $\mathcal{A}$ and is denoted by $PNint(\mathcal{A})$[7].
(ii) The neutrosophic semi-interior of a neutrosophic set $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U},T)$ is the union of all NS-OS contained in $\mathcal{A}$ and is denoted by $SNint(\mathcal{A})$[6].
(iii) The neutrosophic α-interior of a neutrosophic set $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U},T)$ is the union of all Nα-OS contained in $\mathcal{A}$ and is denoted by $\alpha Nint(\mathcal{A})$[5].

Definition 2.3:
(i) The neutrosophic pre-closure of a neutrosophic set $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U},T)$ is the intersection of all NP-CS that contain $\mathcal{A}$ and is denoted by $PNcl(\mathcal{A})$[7].
(ii) The neutrosophic semi-closure of a neutrosophic set $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U},T)$ is the

intersection of all NS-CS that contain $\mathcal{A}$ and is denoted by $SNcl(\mathcal{A})$[6].

(iii) The neutrosophic α-closure of a neutrosophic set $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, T)$ is the intersection of all Nα-CS that contain $\mathcal{A}$ and is denoted by $\alpha Ncl(\mathcal{A})$[5].

Proposition 2.4 [5]:
In a neutrosophic topological space $(\mathcal{U}, T)$, then the following statements hold, and the equality of each statement are not true:
(i) Every N-OS (resp. N-CS) is a Nα-OS (resp. Nα-CS).
(ii) Every Nα-OS (resp. Nα-CS) is a NS-OS (resp. NS-CS).
(iii) Every Nα-OS (resp. Nα-CS) is a NP-OS (resp. NP-CS).

Proposition 2.5 [5]:
A neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, T)$ is a Nα-OS iff $\mathcal{A}$ is a NS-OS and NP-OS.

Lemma 2.6:
(i) If $\mathcal{K}$ is a N-OS, then $SNcl(\mathcal{K}) = Nint(Ncl(\mathcal{K}))$.
(ii) If $\mathcal{A}$ is a neutrosophic subset of a neutrosophic topological space $(\mathcal{U}, T)$, then $SNint(Ncl(\mathcal{A})) = Ncl(Nint(Ncl(\mathcal{A})))$.
Proof: This follows directly from the definition)2.1) and proposition (2.4).

3. Neutrosophic Semi-α-Open Sets

In this section, we present and study the neutrosophic semi-α-open sets and some of its properties.

Definition 3.1:
A neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, T)$ is called neutrosophic semi-α-open set (briefly NSα-OS) if there exists a Nα-OS $\mathcal{H}$ in $\mathcal{U}$ such that $\mathcal{H} \subseteq \mathcal{A} \subseteq Ncl(\mathcal{H})$ or
equivalently if $\mathcal{A} \subseteq Ncl(\alpha Nint(\mathcal{A}))$. The family of all NSα-OS of $\mathcal{U}$ is denoted by NSαO$(\mathcal{U})$.

Definition 3.2:
The complement of NSα-OS is called a neutrosophic semi-α-closed set (briefly NSα-CS). The family of all NSα-CS of $\mathcal{U}$ is denoted by NSαC$(\mathcal{U})$.

Proposition 3.3:
It is evident by definitions that in a neutrosophic topological space $(\mathcal{U}, T)$, the following hold:
(i) Every N-OS (resp. N-CS) is a NSα-OS (resp. NSα-CS).
(ii) Every Nα-OS (resp. Nα-CS) is a NSα-OS (resp. NSα-CS).

The converse of the above proposition need not be true as seen from the following example.

Example 3.4:
Let $\mathcal{U} = \{u\}$, $\mathcal{A} = \{\langle u, 0.5, 0.5, 0.4\rangle : u \in \mathcal{U}\}$, $\mathcal{B} = \{\langle u, 0.4, 0.5, 0.8\rangle : u \in \mathcal{U}\}$, $\mathcal{C} = \{\langle u, 0.5, 0.6, 0.4\rangle : u \in \mathcal{U}\}$, $\mathcal{D} = \{\langle u, 0.4, 0.6, 0.8\rangle : u \in \mathcal{U}\}$.
Then $T = \{0_N, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, 1_N\}$ is a neutrosophic topology on $\mathcal{U}$.
(i) Let $\mathcal{H} = \{\langle u, 0.5, 0.1, 0.3\rangle : u \in \mathcal{U}\}$, $\mathcal{A} \subseteq \mathcal{H} \subseteq Ncl(\mathcal{A}) = \langle u, 0.6, 0.4, 0.2\rangle$, the neutrosophic set $\mathcal{H}$ is a NSα-OS but is not N-OS. It is clear that $\mathcal{H}^c = \{\langle u, 0.5, 0.9, 0.7\rangle : u \in \mathcal{U}\}$ is a NSα-CS but is not N-CS.
(ii) Let $\mathcal{K} = \{\langle u, 0.5, 0.1, 0.2\rangle : u \in \mathcal{U}\}$, $\mathcal{A} \subseteq \mathcal{K} \subseteq Ncl(\mathcal{A}) = \langle u, 0.6, 0.4, 0.2\rangle$, the neutrosophic set $\mathcal{K}$ is a NSα-OS, $\mathcal{K} \nsubseteq Nint(Ncl(Nint(\mathcal{K})) = Nint(Ncl(\langle u, 0.5, 0.5, 0.4\rangle)) = Nint(\langle u, 0.6, 0.4, 0.2\rangle) = \langle u, 0.5, 0.5, 0.4\rangle$, the neutrosophic set $\mathcal{K}$ is not Nα-OS. It is clear that $\mathcal{K}^c = \{\langle u, 0.5, 0.9, 0.8\rangle : u \in \mathcal{U}\}$ is a NSα-CS but is not Nα-CS.

Remark 3.5:
The concepts of NSα-OS and NP-OS are independent, as the following examples shows.

Example 3.6:
In example (3.4), then the neutrosophic set $\mathcal{H} = \{\langle u, 0.5, 0.1, 0.3\rangle : u \in \mathcal{U}\}$ is a NSα-OS but is not NP-OS, because $\mathcal{H} \nsubseteq Nint(Ncl(\mathcal{H})) = Nint(\langle u, 0.6, 0.4, 0.2\rangle) = \langle u, 0.5, 0.5, 0.4\rangle$.

Example 3.7:
Let $\mathcal{U} = \{a, b\}$, $\mathcal{A} = \{\langle 0.4, 0.8, 0.9\rangle, \langle 0.7, 0.5, 0.3\rangle\}$, $\mathcal{B} = \{\langle 0.5, 0.8, 0.6\rangle, \langle 0.8, 0.4, 0.3\rangle\}$, $\mathcal{C} = \{\langle 0.4, 0.7, 0.9\rangle, \langle 0.6, 0.4, 0.4\rangle\}$, $\mathcal{D} = \{\langle 0.5, 0.7, 0.5\rangle, \langle 0.8, 0.4, 0.6\rangle\}$.
Then $T = \{0_N, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, 1_N\}$ is a neutrosophic topology on $\mathcal{U}$.
Then the neutrosophic set $\mathcal{K} = \{\langle 1, 1, 0.3\rangle, \langle 0.7, 0.3, 0.6\rangle\}$ is a NP-OS but is not NSα-OS.

Remark 3.8:
(i) If every N-OS is a N-CS and every nowhere neutrosophic dense set is N-CS in any neutrosophic topological space $(\mathcal{U}, T)$, then every NSα-OS is a N-OS.
(ii) If every N-OS is a N-CS in any neutrosophic topological space $(\mathcal{U}, T)$, then every NSα-OS is a Nα-OS.

Remark 3.9:
(i) It is clear that every NS-OS and NP-OS of any neutrosophic topological space $(\mathcal{U}, T)$ is a NSα-OS (by proposition (2.5) and proposition (3.3) (ii)).
(ii) A NSα-OS in any neutrosophic topological space $(\mathcal{U}, T)$ is a NP-OS if every N-OS of $\mathcal{U}$ is a N-CS (from proposition (2.4) (iii) and remark (3.8) (ii)).

Theorem 3.10:
For any neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, T)$, $\mathcal{A} \in$ NαO$(\mathcal{U})$ iff there exists a N-OS $\mathcal{H}$ such that $\mathcal{H} \subseteq \mathcal{A} \subseteq Nint(Ncl(\mathcal{H}))$.

Proof: Let $\mathcal{A}$ be a Nα-OS. Hence $\mathcal{A} \subseteq Nint(Ncl(Nint(\mathcal{A})))$, so let $\mathcal{H} = Nint(\mathcal{A})$, we get $Nint(\mathcal{A}) \subseteq \mathcal{A} \subseteq Nint(Ncl(Nint(\mathcal{A})))$. Then there exists a N-OS $Nint(\mathcal{A})$ such that $\mathcal{H} \subseteq \mathcal{A} \subseteq Nint(Ncl(\mathcal{H}))$, where $\mathcal{H} = Nint(\mathcal{A})$.
Conversely, suppose that there is a N-OS $\mathcal{H}$ such that $\mathcal{H} \subseteq \mathcal{A} \subseteq Nint(Ncl(\mathcal{H}))$.
To prove $\mathcal{A} \in$ NαO$(\mathcal{U})$.
$\mathcal{H} \subseteq Nint(\mathcal{A})$ (since $Nint(\mathcal{A})$ is the largest N-OS contained in $\mathcal{A}$).
Hence $Ncl(\mathcal{H}) \subseteq Nint(Ncl(\mathcal{A}))$, then $Nint(Ncl(\mathcal{H})) \subseteq Nint(Ncl(Nint(\mathcal{A})))$.
But $\mathcal{H} \subseteq \mathcal{A} \subseteq Nint(Ncl(\mathcal{H}))$ (by hypothesis). Then $\mathcal{A} \subseteq Nint(Ncl(Nint(\mathcal{A})))$.
Therefore, $\mathcal{A} \in$ NαO$(\mathcal{U})$.

Theorem 3.11:
For any neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, T)$. The following properties are equivalent:
(i) $\mathcal{A} \in$ NSαO$(\mathcal{U})$.
(ii) There exists a N-OS say $\mathcal{H}$ such that $\mathcal{H} \subseteq \mathcal{A} \subseteq Ncl(Nint(Ncl(\mathcal{H})))$.
(iii) $\mathcal{A} \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$.
Proof:
$(i) \Longrightarrow (ii)$ Let $\mathcal{A} \in$ NSαO$(\mathcal{U})$. Then there exists $\mathcal{K} \in$ NαO$(\mathcal{U})$, such that $\mathcal{K} \subseteq \mathcal{A} \subseteq Ncl(\mathcal{K})$. Hence there exists $\mathcal{H}$ N-OS such that $\mathcal{H} \subseteq \mathcal{K} \subseteq Nint(Ncl(\mathcal{H}))$ (by theorem (3.10)). Therefore, $Ncl(\mathcal{H}) \subseteq Ncl(\mathcal{K}) \subseteq Ncl(Nint(Ncl(\mathcal{H})))$, implies that $Ncl(\mathcal{K}) \subseteq Ncl(Nint(Ncl(\mathcal{H})))$. Then $\mathcal{H} \subseteq \mathcal{K} \subseteq \mathcal{A} \subseteq Ncl(\mathcal{K}) \subseteq Ncl(Nint(Ncl(\mathcal{H})))$. Therefore, $\mathcal{H} \subseteq \mathcal{A} \subseteq Ncl(Nint(Ncl(\mathcal{H})))$, for some $\mathcal{H}$ N-OS.
$(ii) \Longrightarrow (iii)$ Suppose that there exists a N-OS $\mathcal{H}$ such that $\mathcal{H} \subseteq \mathcal{A} \subseteq Ncl(Nint(Ncl(\mathcal{H})))$. We know that $Nint(\mathcal{A}) \subseteq \mathcal{A}$. On the other hand, $\mathcal{H} \subseteq Nint(\mathcal{A})$ (since $Nint(\mathcal{A})$ is the largest N-OS contained in $\mathcal{A}$). Hence $Ncl(\mathcal{H}) \subseteq Ncl(Nint(\mathcal{A}))$, then $Nint(Ncl(\mathcal{H})) \subseteq Nint(Ncl(Nint(\mathcal{A})))$, therefore $Ncl(Nint(Ncl(\mathcal{H}))) \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$.
But $\mathcal{A} \subseteq Ncl(Nint(Ncl(\mathcal{H})))$ (by hypothesis). Hence $\mathcal{A} \subseteq Ncl(Nint(Ncl(\mathcal{H}))) \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$, then $\mathcal{A} \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$.
$(iii) \Longrightarrow (i)$ Let $\mathcal{A} \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$.
To prove $\mathcal{A} \in$ NSαO$(\mathcal{U})$. Let $\mathcal{K} = Nint(\mathcal{A})$; we know that $Nint(\mathcal{A}) \subseteq \mathcal{A}$. To prove $\mathcal{A} \subseteq Ncl(Nint(\mathcal{A}))$.
Since $Nint(Ncl(Nint(\mathcal{A}))) \subseteq Ncl(Nint(\mathcal{A}))$. Hence, $Ncl(Nint(Ncl(Nint(\mathcal{A})))) \subseteq Ncl(Ncl(Nint(\mathcal{A}))) = Ncl(Nint(\mathcal{A}))$. But $\mathcal{A} \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ (by hypothesis). Hence, $\mathcal{A} \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ $\subseteq Ncl(Nint(\mathcal{A})) \Longrightarrow \mathcal{A} \subseteq Ncl(Nint(\mathcal{A}))$. Hence, there exists a N-OS say $\mathcal{K}$, such that $\mathcal{K} \subseteq \mathcal{A} \subseteq Ncl(\mathcal{A})$. On the other hand, $\mathcal{K}$ is a Nα-OS (since $\mathcal{K}$ is a N-OS). Hence $\mathcal{A} \in$ NSαO$(\mathcal{U})$.

Corollary 3.12:
For any neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, T)$, the following properties are equivalent:
(i) $\mathcal{A} \in$ NSαC$(\mathcal{U})$.
(ii) There exists a N-CS $\mathcal{F}$ such that $Nint(Ncl(Nint(\mathcal{F}))) \subseteq \mathcal{A} \subseteq \mathcal{F}$.
(iii) $Nint(Ncl(Nint(Ncl(\mathcal{A})))) \subseteq \mathcal{A}$.
Proof:
$(i) \Longrightarrow (ii)$ Let $\mathcal{A} \in$ NSαC$(\mathcal{U})$, then $\mathcal{A}^c \in$ NSαO$(\mathcal{U})$. Hence there is $\mathcal{H}$ N-OS such that $\mathcal{H} \subseteq \mathcal{A}^c \subseteq Ncl(Nint(Ncl(\mathcal{H})))$ (by theorem (3.11)). Hence $(Ncl(Nint(Ncl(\mathcal{H}))))^c \subseteq \mathcal{A}^{c^c} \subseteq \mathcal{H}^c$, i.e., $Nint(Ncl(Nint(\mathcal{H}^c))) \subseteq \mathcal{A} \subseteq \mathcal{H}^c$. Let $\mathcal{H}^c = \mathcal{F}$, where $\mathcal{F}$ is a N-CS in $\mathcal{U}$. Then $Nint(Ncl(Nint(\mathcal{F}))) \subseteq \mathcal{A} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS.
$(ii) \Longrightarrow (iii)$ Suppose that there exists $\mathcal{F}$ N-CS such that $Nint\left(Ncl\left(Nint(\mathcal{F})\right)\right) \subseteq \mathcal{A} \subseteq \mathcal{F}$, but $Ncl(\mathcal{A})$ is the smallest N-CS containing $\mathcal{A}$. Then $Ncl(\mathcal{A}) \subseteq \mathcal{F}$, and therefore: $Nint\left(Ncl(\mathcal{A})\right) \subseteq Nint(\mathcal{F}) \Longrightarrow$ $Ncl\left(Nint\left(Ncl(\mathcal{A})\right)\right) \subseteq Ncl\left(Nint(\mathcal{F})\right) \Longrightarrow$ $Nint(Ncl(Nint(Ncl(\mathcal{A})))) \subseteq Nint(Ncl(Nint(\mathcal{F}))) \subseteq \mathcal{A} \Longrightarrow Nint(Ncl(Nint(Ncl(\mathcal{A})))) \subseteq \mathcal{A}$.
$(iii) \Longrightarrow (i)$ Let $Nint(Ncl(Nint(Ncl(\mathcal{A})))) \subseteq \mathcal{A}$.
To prove $\mathcal{A} \in$ NSαC$(\mathcal{U})$, i.e., to prove $\mathcal{A}^c \in$ NSαO$(\mathcal{U})$.
Then $\mathcal{A}^c \subseteq (Nint(Ncl(Nint(Ncl(\mathcal{A})))))^c = Ncl(Nint(Ncl(Nint(\mathcal{A}^c))))$, but $(Nint(Ncl(Nint(Ncl(\mathcal{A})))))^c = Ncl(Nint(Ncl(Nint(\mathcal{A}^c))))$.
Hence $\mathcal{A}^c \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}^c))))$, and therefore $\mathcal{A}^c \in$ NSαO$(\mathcal{U})$, i.e., $\mathcal{A} \in$ NSαC$(\mathcal{U})$.

Proposition 3.13:
The union of any family of Nα-OS is a Nα-OS.
Proof: Let $\{\mathcal{A}_i\}_{i\in\Lambda}$ be a family of Nα-OS of $\mathcal{U}$.
To prove $\bigcup_{i\in\Lambda}\mathcal{A}_i$ is a Nα-OS,
i.e., $\bigcup_{i\in\Lambda}\mathcal{A}_i \subseteq Nint(Ncl(Nint(\bigcup_{i\in\Lambda}\mathcal{A}_i)))$.
Then $\mathcal{A}_i \subseteq Nint(Ncl(Nint(\mathcal{A}_i)))$, $\forall i \in \Lambda$.
Since $\bigcup_{i\in\Lambda} Nint(\mathcal{A}_i) \subseteq Nint(\bigcup_{i\in\Lambda}\mathcal{A}_i)$ and $\bigcup_{i\in\Lambda} Ncl(\mathcal{A}_i) \subseteq Ncl(\bigcup_{i\in\Lambda}\mathcal{A}_i)$ hold for any neutrosophic topology.
We have
$$\begin{aligned}\bigcup_{i\in\Lambda}\mathcal{A}_i &\subseteq \bigcup_{i\in\Lambda} Nint(Ncl(Nint(\mathcal{A}_i)))\\ &\subseteq Nint(\bigcup_{i\in\Lambda} Ncl(Nint(\mathcal{A}_i)))\\ &\subseteq Nint(Ncl(\bigcup_{i\in\Lambda}(Nint(\mathcal{A}_i)))\\ &\subseteq Nint(Ncl(Nint(\bigcup_{i\in\Lambda}\mathcal{A}_i))).\end{aligned}$$
Hence $\bigcup_{i\in\Lambda}\mathcal{A}_i$ is a Nα-OS.

Theorem 3.14:
The union of any family of NSα-OS is a NSα-OS.
Proof: Let $\{\mathcal{A}_i\}_{i\in\Lambda}$ be a family of NSα-OS. To prove $\bigcup_{i\in\Lambda}\mathcal{A}_i$ is a NSα-OS. Since $\mathcal{A}_i \in$ NSαO$(\mathcal{U})$. Then there is a Nα-OS $\mathcal{B}_i$ such that $\mathcal{B}_i \subseteq \mathcal{A}_i \subseteq Ncl(\mathcal{B}_i)$, $\forall i \in \Lambda$. Hence $\bigcup_{i\in\Lambda}\mathcal{B}_i \subseteq \bigcup_{i\in\Lambda}\mathcal{A}_i \subseteq \bigcup_{i\in\Lambda} Ncl(\mathcal{B}_i) \subseteq Ncl(\bigcup_{i\in\Lambda}\mathcal{B}_i)$.
But $\bigcup_{i\in\Lambda}\mathcal{B}_i \in$ NαO$(\mathcal{U})$ (by proposition (3.13)).
Hence $\bigcup_{i\in\Lambda}\mathcal{A}_i \in$ NSαO$(\mathcal{U})$.

Corollary 3.15:

The intersection of any family of NSα-CS is a NSα-CS.
Proof: This follows directly from the theorem (3.14).

Remark 3.16:
The following diagram shows the relations among the different types of weakly neutrosophic open sets that were studied in this section:

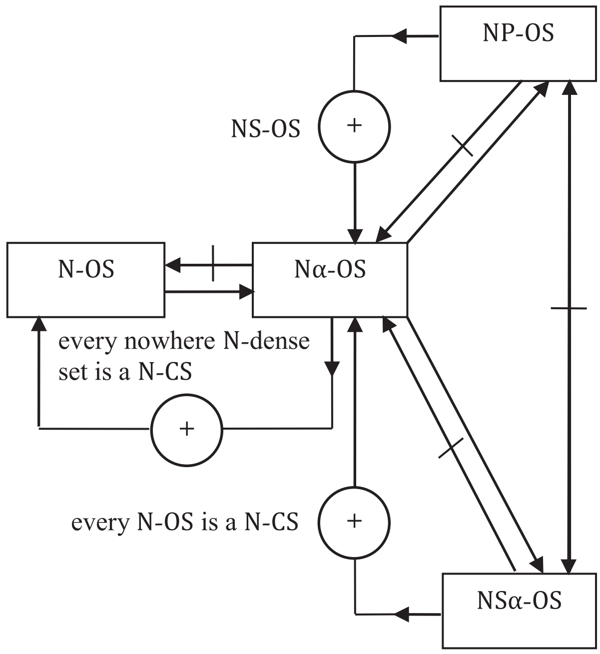

Diagram (3.1)

4. Neutrosophic Semi-α-Interior and Neutrosophic Semi-α-Closure

We present neutrosophic semi- α -interior and neutrosophic semi- α -closure and obtain some of its properties in this section.

Definition 4.1:
The union of all NSα-OS in a neutrosophic topological space $(\mathcal{U}, T)$ contained in $\mathcal{A}$ is called neutrosophic semi-α -interior of $\mathcal{A}$ and is denoted by $S\alpha Nint(\mathcal{A})$, $S\alpha Nint(\mathcal{A}) = \bigcup\{\mathcal{B}: \mathcal{B} \subseteq \mathcal{A}, \mathcal{B}$ is a NSα-OS$\}$.

Definition 4.2:
The intersection of all NSα - CS in a neutrosophic topological space $(\mathcal{U}, T)$ containing $\mathcal{A}$ is called neutrosophic semi- α -closure of $\mathcal{A}$ and is denoted by $S\alpha Ncl(\mathcal{A})$, $S\alpha Ncl(\mathcal{A}) = \bigcap\{\mathcal{B}: \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a NSα-CS$\}$.

Proposition 4.3:
Let $\mathcal{A}$ be any neutrosophic set in a neutrosophic topological space $(\mathcal{U}, T)$, the following properties are true:
(i) $S\alpha Nint(\mathcal{A}) = \mathcal{A}$ iff $\mathcal{A}$ is a NSα-OS.
(ii) $S\alpha Ncl(\mathcal{A}) = \mathcal{A}$ iff $\mathcal{A}$ is a NSα-CS.
(iii) $S\alpha Nint(\mathcal{A})$ is the largest NSα-OS contained in $\mathcal{A}$.
(iv) $S\alpha Ncl(\mathcal{A})$ is the smallest NSα-CS containing $\mathcal{A}$.
Proof: (i), (ii), (iii) and (iv) are obvious.

Proposition 4.4:
Let $\mathcal{A}$ be any neutrosophic set in a neutrosophic topological space $(\mathcal{U}, T)$, the following properties are true:
(i) $S\alpha Nint(1_N - \mathcal{A}) = 1_N - (S\alpha Ncl(\mathcal{A}))$,
(ii) $S\alpha Ncl(1_N - \mathcal{A}) = 1_N - (S\alpha Nint(\mathcal{A}))$.
Proof: (i) By definition, $S\alpha Ncl(\mathcal{A}) = \bigcap\{\mathcal{B}: \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a NSα-CS$\}$
$1_N - (S\alpha Ncl(\mathcal{A})) = 1_N - \bigcap\{\mathcal{B}: \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a NSα-CS$\}$
$= \bigcup\{1_N - \mathcal{B}: \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a NSα-CS$\}$
$= \bigcup\{\mathcal{H}: \mathcal{H} \subseteq 1_N - \mathcal{A}, \mathcal{H}$ is a NSα-OS$\}$
$= S\alpha Nint(1_N - \mathcal{A})$.
(ii) The proof is similar to (i).

Theorem 4.5:
Let $\mathcal{A}$ and $\mathcal{B}$ be two neutrosophic sets in a neutrosophic topological space $(\mathcal{U}, T)$. The following properties hold:
(i) $S\alpha Nint(0_N) = 0_N$, $S\alpha Nint(1_N) = 1_N$.
(ii) $S\alpha Nint(\mathcal{A}) \subseteq \mathcal{A}$.
(iii) $\mathcal{A} \subseteq \mathcal{B} \Longrightarrow S\alpha Nint(\mathcal{A}) \subseteq S\alpha Nint(\mathcal{B})$.
(iv) $S\alpha Nint(\mathcal{A} \cap \mathcal{B}) \subseteq S\alpha Nint(\mathcal{A}) \cap S\alpha Nint(\mathcal{B})$.
(v) $S\alpha Nint(\mathcal{A}) \cup S\alpha Nint(\mathcal{B}) \subseteq S\alpha Nint(\mathcal{A} \cup \mathcal{B})$.
(vi) $S\alpha Nint(S\alpha Nint(\mathcal{A})) = S\alpha Nint(\mathcal{A})$.
Proof: (i), (ii), (iii), (iv), (v) and (vi) are obvious.

Theorem 4.6:
Let $\mathcal{A}$ and $\mathcal{B}$ be two neutrosophic sets in a neutrosophic topological space $(\mathcal{U}, T)$. The following properties hold:
(i) $S\alpha Ncl(0_N) = 0_N$, $S\alpha Ncl(1_N) = 1_N$.
(ii) $\mathcal{A} \subseteq S\alpha Ncl(\mathcal{A})$.
(iii) $\mathcal{A} \subseteq \mathcal{B} \Longrightarrow S\alpha Ncl(\mathcal{A}) \subseteq S\alpha Ncl(\mathcal{B})$.
(iv) $S\alpha Ncl(\mathcal{A} \cap \mathcal{B}) \subseteq S\alpha Ncl(\mathcal{A}) \cap S\alpha Ncl(\mathcal{B})$.
(v) $S\alpha Ncl(\mathcal{A}) \cup S\alpha Ncl(\mathcal{B}) \subseteq S\alpha Ncl(\mathcal{A} \cup \mathcal{B})$.
(vi) $S\alpha Ncl(S\alpha Ncl(\mathcal{A})) = S\alpha Ncl(\mathcal{A})$.
Proof: (i) and (ii) are evident.
(iii) By part (ii), $\mathcal{B} \subseteq S\alpha Ncl(\mathcal{B})$. Since $\mathcal{A} \subseteq \mathcal{B}$, we have $\mathcal{A} \subseteq S\alpha Ncl(\mathcal{B})$. But $S\alpha Ncl(\mathcal{B})$ is a NSα - CS . Thus $S\alpha Ncl(\mathcal{B})$ is a NSα-CS containing $\mathcal{A}$. Since $S\alpha Ncl(\mathcal{A})$ is the smallest NSα-CS containing $\mathcal{A}$, we have $S\alpha Ncl(\mathcal{A}) \subseteq S\alpha Ncl(\mathcal{B})$. Hence, $\mathcal{A} \subseteq \mathcal{B} \Longrightarrow S\alpha Ncl(\mathcal{A}) \subseteq S\alpha Ncl(\mathcal{B})$.
(iv) We know that $\mathcal{A} \cap \mathcal{B} \subseteq \mathcal{A}$ and $\mathcal{A} \cap \mathcal{B} \subseteq \mathcal{B}$.
Therefore, by part (iii), $S\alpha Ncl(\mathcal{A} \cap \mathcal{B}) \subseteq S\alpha Ncl(\mathcal{A})$ and $S\alpha Ncl(\mathcal{A} \cap \mathcal{B}) \subseteq S\alpha Ncl(\mathcal{B})$.
Hence $S\alpha Ncl(\mathcal{A} \cap \mathcal{B}) \subseteq S\alpha Ncl(\mathcal{A}) \cap S\alpha Ncl(\mathcal{B})$.
(v) Since $\mathcal{A} \subseteq \mathcal{A} \cup \mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{A} \cup \mathcal{B}$, it follows from part (iii) that $S\alpha Ncl(\mathcal{A}) \subseteq S\alpha Ncl(\mathcal{A} \cup \mathcal{B})$ and $S\alpha Ncl(\mathcal{B}) \subseteq S\alpha Ncl(\mathcal{A} \cup \mathcal{B})$.
Hence $S\alpha Ncl(\mathcal{A}) \cup S\alpha Ncl(\mathcal{B}) \subseteq S\alpha Ncl(\mathcal{A} \cup \mathcal{B})$.
(vi) Since $S\alpha Ncl(\mathcal{A})$ is a NSα-CS, we have by proposition (4.3) part (ii), $S\alpha Ncl(S\alpha Ncl(\mathcal{A})) = S\alpha Ncl(\mathcal{A})$.

Proposition 4.7:
For any neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, T)$, then:

(i) $Nint(\mathcal{A}) \subseteq \alpha Nint(\mathcal{A}) \subseteq S\alpha Nint(\mathcal{A}) \subseteq S\alpha Ncl(\mathcal{A}) \subseteq \alpha Ncl(\mathcal{A}) \subseteq Ncl(\mathcal{A})$.
(ii) $Nint(S\alpha Nint(\mathcal{A})) = S\alpha Nint(Nint(\mathcal{A})) = Nint(\mathcal{A})$.
(iii) $\alpha Nint(S\alpha Nint(\mathcal{A})) = S\alpha Nint(\alpha Nint(\mathcal{A})) = \alpha Nint(\mathcal{A})$.
(iv) $Ncl(S\alpha Ncl(\mathcal{A})) = S\alpha Ncl(Ncl(\mathcal{A})) = Ncl(\mathcal{A})$.
(v) $\alpha Ncl(S\alpha Ncl(\mathcal{A})) = S\alpha Ncl(\alpha Ncl(\mathcal{A})) = \alpha Ncl(\mathcal{A})$.
(vi) $S\alpha Ncl(\mathcal{A}) = \mathcal{A} \cup Nint(Ncl(Nint(Ncl(\mathcal{A}))))$.
(vii) $S\alpha Nint(\mathcal{A}) = \mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A}))))$.
(viii) $Nint(Ncl(\mathcal{A})) \subseteq S\alpha Nint(S\alpha Ncl(\mathcal{A}))$.

Proof: We shall prove only (ii), (iii), (iv), (vii) and (viii).

(ii) To prove $Nint(S\alpha Nint(\mathcal{A})) = S\alpha Nint(Nint(\mathcal{A})) = Nint(\mathcal{A})$. Since $Nint(\mathcal{A})$ is a N-OS, then $Nint(\mathcal{A})$ is a NSα-OS. Hence $Nint(\mathcal{A}) = S\alpha Nint(Nint(\mathcal{A}))$ (by proposition (4.3)). Therefore:
$Nint(\mathcal{A}) = S\alpha Nint(Nint(\mathcal{A}))$......................(1)
Since $Nint(\mathcal{A}) \subseteq S\alpha Nint(\mathcal{A}) \Rightarrow Nint(Nint(\mathcal{A})) \subseteq Nint(S\alpha Nint(\mathcal{A})) \Rightarrow Nint(\mathcal{A}) \subseteq Nint(S\alpha Nint(\mathcal{A}))$.
Also, $S\alpha Nint(\mathcal{A}) \subseteq \mathcal{A} \Rightarrow Nint(S\alpha Nint(\mathcal{A})) \subseteq Nint(\mathcal{A})$. Hence:
$Nint(\mathcal{A}) = Nint(S\alpha Nint(\mathcal{A}))$......................(2)
Therefore by (1) and (2), we get $Nint(S\alpha Nint(\mathcal{A})) = S\alpha Nint(Nint(\mathcal{A})) = Nint(\mathcal{A})$.

(iii)To prove $\alpha Nint(S\alpha Nint(\mathcal{A})) = S\alpha Nint(\alpha Nint(\mathcal{A})) = \alpha Nint(\mathcal{A})$. Since $\alpha Nint(\mathcal{A})$ is Nα-OS, therefore $\alpha Nint(\mathcal{A})$ is NSα-OS. Therefore by proposition (4.3):
$\alpha Nint(\mathcal{A}) = S\alpha Nint(\alpha Nint(\mathcal{A}))$......................(1)
Now, to prove $\alpha Nint(\mathcal{A}) = \alpha Nint(S\alpha Nint(\mathcal{A}))$. Since $\alpha Nint(\mathcal{A}) \subseteq S\alpha Nint(\mathcal{A}) \Rightarrow \alpha Nint(\alpha Nint(\mathcal{A})) \subseteq \alpha Nint(S\alpha Nint(\mathcal{A})) \Rightarrow$
$\alpha Nint(\mathcal{A}) \subseteq \alpha Nint(S\alpha Nint(\mathcal{A}))$.
Also, $S\alpha Nint(\mathcal{A}) \subseteq \mathcal{A} \Rightarrow \alpha Nint(S\alpha Nint(\mathcal{A})) \subseteq \alpha Nint(\mathcal{A})$. Hence:
$\alpha Nint(\mathcal{A}) = \alpha Nint(S\alpha Nint(\mathcal{A}))$......................(2)
Therefore by (1) and (2), we get $\alpha Nint(S\alpha Nint(\mathcal{A})) = S\alpha Nint(\alpha Nint(\mathcal{A})) = \alpha Nint(\mathcal{A})$.

(iv) To prove $Ncl(S\alpha Ncl(\mathcal{A})) = S\alpha Ncl(Ncl(\mathcal{A})) = Ncl(\mathcal{A})$. We know that $Ncl(\mathcal{A})$ is a N-CS, so it is NSα-CS. Hence by proposition (4.3), we have:
$Ncl(\mathcal{A}) = S\alpha Ncl(Ncl(\mathcal{A}))$......................(1)
To prove $\boldsymbol{Ncl(\mathcal{A}) = Ncl(S\alpha Ncl(\mathcal{A}))}$.
Since $\boldsymbol{S\alpha Ncl(\mathcal{A}) \subseteq Ncl(\mathcal{A})}$ (by part (i)).
Then $\boldsymbol{Ncl(S\alpha Ncl(\mathcal{A})) \subseteq Ncl(Ncl(\mathcal{A})) = Ncl(\mathcal{A}) \Rightarrow Ncl(S\alpha Ncl(\mathcal{A})) \subseteq Ncl(\mathcal{A})}$. Since $\boldsymbol{\mathcal{A} \subseteq S\alpha Ncl(\mathcal{A}) \subseteq Ncl(S\alpha Ncl(\mathcal{A}))}$, then $\boldsymbol{\mathcal{A} \subseteq Ncl(S\alpha Ncl(\mathcal{A}))}$. Hence $\boldsymbol{Ncl(\mathcal{A}) \subseteq Ncl\left(Ncl(S\alpha Ncl(\mathcal{A}))\right) = Ncl(S\alpha Ncl(\mathcal{A}))}$
$\boldsymbol{\Rightarrow Ncl(\mathcal{A}) \subseteq Ncl(S\alpha Ncl(\mathcal{A}))}$ and therefore: $\boldsymbol{Ncl(\mathcal{A}) = Ncl(S\alpha Ncl(\mathcal{A}))}$...................................(2)
Now, by (1) and (2), we get that $Ncl(S\alpha Ncl(\mathcal{A})) = S\alpha Ncl(Ncl(\mathcal{A}))$.
Hence $Ncl(S\alpha Ncl(\mathcal{A})) = S\alpha Ncl(Ncl(\mathcal{A})) = Ncl(\mathcal{A})$.

(vii) To prove $S\alpha Nint(\mathcal{A}) = \mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A}))))$.
Since $S\alpha Nint(\mathcal{A}) \in NS\alpha O(\mathcal{U}) \Rightarrow S\alpha Nint(\mathcal{A}) \subseteq Ncl(Nint(Ncl(Nint(S\alpha Nint(\mathcal{A})))))$
$= Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ (by part (ii)).
Hence $S\alpha Nint(\mathcal{A}) \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$, also $S\alpha Nint(\mathcal{A}) \subseteq \mathcal{A}$. Then:
$S\alpha Nint(\mathcal{A}) \subseteq \mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A}))))$.............(1)
To prove $\mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ is a NSα-OS contained in $\mathcal{A}$.
It is clear that $\mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A})))) \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ and also it is clear that $Nint(\mathcal{A}) \subseteq Ncl(Nint(\mathcal{A})) \Rightarrow Nint(Nint(\mathcal{A})) \subseteq Nint(Ncl(Nint(\mathcal{A}))) \Rightarrow Nint(\mathcal{A}) \subseteq Nint(Ncl(Nint(\mathcal{A}))) \Rightarrow Ncl(Nint(\mathcal{A})) \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ and $Nint(\mathcal{A}) \subseteq Ncl(Nint(\mathcal{A}))$ $\Rightarrow Nint(\mathcal{A}) \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ and $Nint(\mathcal{A}) \subseteq \mathcal{A} \Rightarrow Nint(\mathcal{A}) \subseteq \mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A}))))$.
We get $Nint(\mathcal{A}) \subseteq \mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A})))) \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$.
Hence $\mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ is a NSα-OS (by proposition (4.3)). Also, $\mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ is contained in $\mathcal{A}$. Then $\mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A})))) \subseteq S\alpha Nint(\mathcal{A})$ (since $S\alpha Nint(\mathcal{A})$ is the largest NSα-OS contained in $\mathcal{A}$). Hence:
$\mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A})))) \subseteq S\alpha Nint(\mathcal{A})$.............(2)
By (1) and (2), $S\alpha Nint(\mathcal{A}) = \mathcal{A} \cap Ncl(Nint(Ncl(Nint(\mathcal{A}))))$.

(viii) To prove that $Nint(Ncl(\mathcal{A})) \subseteq S\alpha Nint(S\alpha Ncl(\mathcal{A}))$.
Since $S\alpha Ncl(\mathcal{A})$ is a NSα-CS, therefore $Nint(Ncl(Nint(Ncl(S\alpha Ncl(\mathcal{A}))))) \subseteq S\alpha Ncl(\mathcal{A})$ (by corollary (3.12)). Hence $Nint(Ncl(\mathcal{A})) \subseteq Nint(Ncl(Nint(Ncl(\mathcal{A})))) \subseteq S\alpha Ncl(\mathcal{A})$ (by part (iv)).
Therefore, $S\alpha Nint\left(Nint(Ncl(\mathcal{A}))\right) \subseteq S\alpha Nint(S\alpha Ncl(\mathcal{A})) \Rightarrow$
$Nint(Ncl(\mathcal{A})) \subseteq S\alpha Nint(S\alpha Ncl(\mathcal{A}))$ (by part (ii)).

Theorem 4.8:
For any neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, T)$. The following properties are equivalent:
(i) $\mathcal{A} \in NS\alpha O(\mathcal{U})$.
(ii) $\mathcal{H} \subseteq \mathcal{A} \subseteq Ncl(Nint(Ncl(\mathcal{H})))$, for some N-OS $\mathcal{H}$.
(iii) $\mathcal{H} \subseteq \mathcal{A} \subseteq SNint(Ncl(\mathcal{H}))$, for some N-OS $\mathcal{H}$.
(iv) $\mathcal{A} \subseteq SNint(Ncl(Nint(\mathcal{A})))$.
Proof:
$(i) \Rightarrow (ii)$ Let $\mathcal{A} \in NS\alpha O(\mathcal{U})$, then $\mathcal{A} \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ and $Nint(\mathcal{A}) \subseteq \mathcal{A}$. Hence $\mathcal{H} \subseteq \mathcal{A} \subseteq Ncl(Nint(Ncl(\mathcal{H})))$, where $\mathcal{H} = Nint(\mathcal{A})$.
$(ii) \Rightarrow (iii)$ Suppose $\mathcal{H} \subseteq \mathcal{A} \subseteq Ncl(Nint(Ncl(\mathcal{H})))$, for some N-OS $\mathcal{H}$.

But $SNint(Ncl(\mathcal{H})) = Ncl(Nint(Ncl(\mathcal{H})))$ (by lemma (2.6)).
Then $\mathcal{H} \subseteq \mathcal{A} \subseteq SNint(Ncl(\mathcal{H}))$, for some N-OS $\mathcal{H}$.
$(iii) \Longrightarrow (iv)$ Suppose that $\mathcal{H} \subseteq \mathcal{A} \subseteq SNint(Ncl(\mathcal{H}))$, for some N-OS $\mathcal{H}$. Since $\mathcal{H}$ is a N-OS contained in $\mathcal{A}$. Then $\mathcal{H} \subseteq Nint(\mathcal{A}) \Longrightarrow Ncl(\mathcal{H}) \subseteq Ncl(Nint(\mathcal{A}))$
$\Longrightarrow SNint(Ncl(\mathcal{H})) \subseteq SNint(Ncl(Nint(\mathcal{A})))$.
But $\mathcal{A} \subseteq SNint(Ncl(\mathcal{H}))$ (by hypothesis), then
$\mathcal{A} \subseteq SNint(Ncl(Nint(\mathcal{A})))$.
$(iv) \Longrightarrow (i)$ Let $\mathcal{A} \subseteq SNint(Ncl(Nint(\mathcal{A})))$. But
$SNint(Ncl(Nint(\mathcal{A}))) = Ncl(Nint(Ncl(Nint(\mathcal{A}))))$
(by lemma (2.6)). Hence $\mathcal{A} \subseteq Ncl(Nint(Ncl(Nint(\mathcal{A}))))$
$\Longrightarrow \mathcal{A} \in$ NSαO$(\mathcal{U})$.

Corollary 4.9:
For any neutrosophic subset $\mathcal{B}$ of a neutrosophic topological space $(\mathcal{U}, T)$, the following properties are equivalent:
(i) $\mathcal{B} \in$ NSαC$(\mathcal{U})$.
(ii) $Nint(Ncl(Nint(\mathcal{F}))) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS.
(iii) $SNcl(Nint(\mathcal{F})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS.
(iv) $SNcl(Nint(Ncl(\mathcal{B}))) \subseteq \mathcal{B}$.
Proof:
$(i) \Longrightarrow (ii)$ Let $\mathcal{B} \in$ NSαC$(\mathcal{U}) \Longrightarrow$
$Nint(Ncl(Nint(Ncl(\mathcal{B})))) \subseteq \mathcal{B}$ (by corollary (3.12))
and $\mathcal{B} \subseteq Ncl(\mathcal{B})$. Hence we get
$Nint(Ncl(Nint(Ncl(\mathcal{B})))) \subseteq \mathcal{B} \subseteq Ncl(\mathcal{B})$.
Therefore $Nint(Ncl(Nint(\mathcal{F}))) \subseteq \mathcal{B} \subseteq \mathcal{F}$, where $\mathcal{F} = Ncl(\mathcal{B})$.
$(ii) \Longrightarrow (iii)$ Let $Nint(Ncl(Nint(\mathcal{F}))) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS. But $Nint(Ncl(Nint(\mathcal{F}))) = SNcl(Nint(\mathcal{F}))$ (by lemma (2.6)). Hence $SNcl(Nint(\mathcal{F})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS.
$(iii) \Longrightarrow (iv)$ Let $SNcl(Nint(\mathcal{F})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS. Since $\mathcal{B} \subseteq \mathcal{F}$ (by hypothesis), hence $Ncl(\mathcal{B}) \subseteq \mathcal{F}$
$\Longrightarrow Nint(Ncl(\mathcal{B}) \subseteq Nint(\mathcal{F}) \Longrightarrow SNcl(Nint(Ncl(\mathcal{B})))$
$\subseteq SNcl(Nint(\mathcal{F})) \subseteq \mathcal{B} \Longrightarrow SNcl(Nint(Ncl(\mathcal{B}))) \subseteq \mathcal{B}$.
$(iv) \Longrightarrow (i)$ Let $SNcl(Nint(Ncl(\mathcal{B}))) \subseteq \mathcal{B}$.
But $SNcl(Nint(Ncl(\mathcal{B}))) = Nint(Ncl(Nint(Ncl(\mathcal{B}))))$ (by lemma (2.6)). Hence $Nint(Ncl(Nint(Ncl(\mathcal{B})))) \subseteq \mathcal{B} \Longrightarrow \mathcal{B} \in$ NSαC$(\mathcal{U})$.

5. Conclusion

In this work, we have defined new class of neutrosophic open sets called neutrosophic semi-α-open sets and studied their fundamental properties in neutrosophic topological spaces. The neutrosophic semi-α-open sets can be used to derive a new decomposition of neutrosophic continuity, neutrosophic compactness, and neutrosophic connectedness.

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