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Abstract: In this paper we introduce the concept of a new class of an ordered neutrosophic bitopological spaces. Besides giving some interesting properties of these spaces. We also prove analogues of Uryshon's lemma and Tietze extension theorem in an ordered neutrosophic bitopological spaces.

Keywords: Ordered neutrosophic bitopological space; lower(resp.upper) pairwise neutrosophic G_δ-α-locally T_1-ordered space; pairwise neutrosophic G_δ-α-locally T_1-ordered space; pairwise neutrosophic G_δ-α-locally T_2-ordered space; weakly pairwise neutrosophic G_δ-α-locally T_2-ordered space; almost pairwise neutrosophic G_δ-α-locally T_2-ordered space and strongly pairwise neutrosophic G_δ-α-locally normally ordered space.

1 Introduction and Preliminaries

The concept of fuzzy sets was introduced by Zadeh [17]. Fuzzy sets have applications in many fields such as information theory [15] and control theory [16]. The theory of fuzzy topological spaces was introduced and developed by Chang [7]. Atanassov [2] introduced and studied intuitionistic fuzzy sets. On the other hand, Coker [8] introduced the notions of an intuitionistic fuzzy topological space and some other related concepts. The concept of an intuitionistic fuzzy α-closed set was introduced by B. Krsteshka and E. Ekici [5]. G. Balasubramanian [3] was introduced the concept of fuzzy G_δ set. Ganster and Reilly used locally closed sets [10] to define LC-continuity and LC-irresoluteness. The concept of an ordered fuzzy topological space was introduced and developed by A. K. Katsaras [11]. Later G. Balasubmanian [4] introduced and studied the concepts of an ordered L-fuzzy bitopological spaces. F. Smarandache [[13], [14] introduced the concepts of neutrosophy and neutrosophic set. The concepts of neutrosophic crisp set and neutrosophic crisp topological space were introduced by A. A. Salama and S. A. Alblowi [12].

In this paper, we introduce the concepts of pairwise neutrosophic G_δ-α-locally T_1-ordered space, pairwise neutrosophic G_δ-α-locally T_2-ordered space, weakly pairwise neutrosophic G_δ-α-locally T_2-ordered space, almost pairwise neutrosophic G_δ-α-locally T_2-ordered space and strongly pairwise neutrosophic G_δ-α-locally normally ordered space. Some interesting propositions are discussed. Urysohn's lemma and Tietze extension theorem of an strongly pairwise neutrosophic G_δ-α-locally normally ordered space are studied and established.

Definition 1.1. [7] Let X be a nonempty set and $A \subset X$. The characteristic function of A is denoted and defined by $\chi_A(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \notin A \end{cases}$

Definition 1.2. [13, 14] Let T,I,F be real standard or non standard subsets of $]0^-, 1^+[$, with $sup_T = t_{sup}, inf_T = t_{inf}$
$sup_I = i_{sup}, inf_I = i_{inf}$
$sup_F = f_{sup}, inf_F = f_{inf}$
$n - sup = t_{sup} + i_{sup} + f_{sup}$
$n - inf = t_{inf} + i_{inf} + f_{inf}$. T,I,F are neutrosophic components.

Definition 1.3. [13, 14] Let X be a nonempty fixed set. A neutrosophic set [briefly NS] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A.

Remark 1.1. [13, 14]

(1) A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A\rangle$ in $]0^-, 1^+[$ on X.

(2) For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.4. [12] Let X be a nonempty set and the neutrosophic sets A and B in the form

$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\bar{A} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$; [Complement of A]

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

(f) $[\,]A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;

(g) $\langle\rangle A = \{\langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.5. [12] Let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X. Then

(a) $\bigcap A_i = \{\langle x, \wedge\mu_{A_i}(x), \wedge\sigma_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X\}$;

(b) $\bigcup A_i = \{\langle x, \vee\mu_{A_i}(x), \vee\sigma_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x \in X\}$.

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

Definition 1.6. [12] $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 1.7. [9] A neutrosophic topology (NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq T$.

In this case the ordered pair (X, T) or simply X is called a neutrosophic topological space (NTS) and each neutrosophic set in T is called a neutrosophic open set (NOS). The complement $\overline{A}$ of a NOS A in X is called a neutrosophic closed set (NCS) in X.

Definition 1.8. [9] Let A be a neutrosophic set in a neutrosophic topological space X. Then

$Nint(A) = \bigcup\{G \mid G$ is a neutrosophic open set in X and $G \subseteq A\}$ is called the neutrosophic interior of A;

$Ncl(A) = \bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$ is called the neutrosophic closure of A.

Corollary 1.1. [9] Let A,B,C be neutrosophic sets in X. Then the basic properties of inclusion and complementation:

(a) $A \subseteq B$ and $C \subseteq D \Rightarrow A \cup C \subseteq B \cup D$ and $A \cap C \subseteq B \cap D$,

(b) $A \subseteq B$ and $A \subseteq C \Rightarrow A \subseteq B \cap C$,

(c) $A \subseteq C$ and $B \subseteq C \Rightarrow A \cup B \subseteq C$,

(d) $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$,

(e) $\overline{A \cup B} = \overline{A} \cap \overline{B}$,

(f) $\overline{A \cap B} = \overline{A} \cup \overline{B}$,

(g) $A \subseteq B \Rightarrow \overline{B} \subseteq \overline{A}$,

(h) $\overline{\overline{(A)}} = A$,

(i) $\overline{1_N} = 0_N$,

(j) $\overline{0_N} = 1_N$.

Now we shall define the image and preimage of neutrosophic sets. Let X and Y be two nonempty sets and $f : X \to Y$ be a function.

Definition 1.9. [9]

(a) If $B = \{\langle y, \mu_B(y), \sigma_B(y), \gamma_B(y)\rangle : y \in Y\}$ is a neutrosophic set in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the neutrosophic set in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$.

(b) If $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ is a neutrosophic set in X,then the image of A under f, denoted by $f(A)$, is the neutrosophic set in Y defined by $f(A) = \{\langle y, f(\mu_A)(y), f(\sigma_A)(y), (1 - f(1 - \gamma_A))(y)\rangle : y \in Y\}$. where

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$f(\sigma_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \sigma_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$(1 - f(1 - \gamma_A))(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 1, & \text{otherwise,} \end{cases}$$

For the sake of simplicity, let us use the symbol $f_-(\gamma_A)$ for $1 - f(1 - \gamma_A)$.

Corollary 1.2. [9] Let A , $A_i (i \in J)$ be neutrosophic sets in X, B, $B_i (i \in K)$ be neutrosophic sets in Y and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(c) $A \subseteq f^{-1}(f(A))$ { If f is injective,then $A = f^{-1}(f(A))$ } ,

(d) $f(f^{-1}(B)) \subseteq B$ { If f is surjective,then $f(f^{-1}(B)) = B$ },

(e) $f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j)$,

(f) $f^{-1}(\bigcap B_j) = \bigcap f^{-1}(B_j)$,

(g) $f(\bigcup A_i) = \bigcup f(A_i)$,

(h) $f(\bigcap A_i) \subseteq \bigcap f(A_i)$ { If f is injective,then $f(\bigcap A_i) = \bigcap f(A_i)$},

(i) $f^{-1}(1_N) = 1_N$,

(j) $f^{-1}(0_N) = 0_N$,

(k) $f(1_N) = 1_N$, if f is surjective,

(l) $f(0_N) = 0_N$,

(m) $\overline{f(A)} \subseteq f(\overline{A})$, if f is surjective,

(n) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

Definition 1.10. [1] A neutrosophic set A in a neutrosophic topological space (X, T) is called a neutrosophic α-open set $(N\alpha OS)$ if $A \subseteq Nint(Ncl(Nint(A)))$.

2 Ordered neutrosophic G_δ-α-locally bitopological Spaces

In this section, the concepts of a neutrosophic G_δ set, neutrosophic α-closed set, neutrosophic G_δ-α-locally closed set, upper pairwise neutrosophic G_δ-α-locally T_1-ordered space, lower pairwise neutrosophic G_δ-α-locally T_1-ordered space, pairwise neutrosophic G_δ-α-locally T_1-ordered space, pairwise neutrosophic G_δ-α-locally T_2-ordered space, weakly pairwise neutrosophic G_δ-α-locally T_2-ordered space, almost pairwise neutrosophic G_δ-α-locally T_2-ordered space and strongly pairwise neutrosophic G_δ-α-locally normally ordered space are introduced. Some basic properties and characterizations are discussed. Urysohn's lemma and Tietze extension theorem of an strongly pairwise neutrosophic G_δ-α-locally normally ordered space are studied and established.

Definition 2.1. Let (X, T) be a neutrosophic topological space. Let $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ be a neutrosophic set of a neutrosophic topological space X. Then A is said to be a neutrosophic G_δ set (briefly NG_δS) if $A = \bigcap_{i=1}^{\infty} A_i$, where each $A_i \in T$ and $A_i = \langle x, \mu_{A_i}, \sigma_{A_i}, \gamma_{A_i} \rangle$.

The complement of neutrosophic G_δ set is said to be a neutrosophic F_σ set(briefly NF_σS).

Definition 2.2. Let (X, T) be a neutrosophic topological space. Let $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ be a neutrosophic set on a neutrosophic topological space (X, T). Then A is said be a neutrosophic G_δ-α-locally closed set (in short,NG_δ-α-lcs) if $A = B \cap C$, where B is a neutrosophic G_δ set and C is an neutrosophic α-closed set.

The complement of a neutrosophic G_δ-α-locally closed set is said to be a neutrosophic G_δ-α-locally open set (in short,NG_δ-α-los).

Definition 2.3. Let (X, T) be a neutrosophic topological space. Let $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ be a neutrosophic set in a neutrosophic topological space (X, T). The neutrosophic G_δ-α-locally closure of A is denoted and defined by

NG_δ-α-$lcl(A)$=$\bigcap\{B{:}B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ is a neutrosophic G_δ-α-locally closed set in X and $A \subseteq B\}$.

Definition 2.4. Let (X, T) be a neutrosophic topological space. Let $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ be a neutrosophic set in a neutrosophic topological space (X, T). The neutrosophic G_δ-α-locally interior of A is denoted and defined by

NG_δ-α-$lint(A)$=$\bigcup\{B{:}B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ is a neutrosophic G_δ-α-locally open set in X and $B \subseteq A\}$.

Definition 2.5. Let X be a nonempty set and $x \in X$ a fixed element in X. If $r, t \in I_0 = (0, 1]$ and $s \in I_1 = [0, 1)$ are fixed real numbers such that $0 < r + t + s < 3$, then $x_{r,t,s} = \langle x, r, t, s \rangle$ is called a neutrosophic point (briefly NP) in X, where r denotes the degree of membership of $x_{r,t,s}$, t denotes the degree of indeterminacy and s denotes the degree of nonmembership of $x_{r,t,s}$ and $x \in X$ the support of $x_{r,t,s}$.

The neutrosophic point $x_{r,t,s}$ is contained in the neutrosophic $A(x_{r,t,s} \in A)$ if and only if $r < \mu_A(x), t < \sigma_A(x), s > \gamma_A(x)$.

Definition 2.6. A neutrosophic set $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ in a neutrosophic topological space (X, T) is said to be a neutrosophic neighbourhood of a neotrosophic point $x_{r,t,s}, x \in X$, if there exists a neutrosophic open set $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ with $x_{r,t,s} \subseteq B \subseteq A$.

Definition 2.7. A neutrosophic set $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ in a neutrosophic topological space (X, T) is said to be a neutrosophic G_δ-α-locally neighbourhood of a neutrosophic point $x_{r,t,s}, x \in X$, if there exists a neutrosophic G_δ-α-locally open set $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ with $x_{r,t,s} \subseteq B \subseteq A$.

Notation 2.1. In what follows, we denote neutrosophic neighbourhood A of a in X by neutrosphic neighbourhood A of a neutrsophic point $a_{r,t,s}$ for $a \in X$.

Definition 2.8. A neutrosophic set $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ in a partially ordered set $(X, \leq)$ is said to be an

(i) increasing neutrosophic set if $x \leq y$ implies $A(x) \subseteq A(y)$.That is,
$\mu_A(x) \leq \mu_A(y)$, $\sigma_A(x) \leq \sigma_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$.

(ii) decreasing neutrosophic set if $x \leq y$ implies $A(x) \supseteq A(y)$.That is,
$\mu_A(x) \geq \mu_A(y)$,$\sigma_A(x) \geq \sigma_A(y)$and $\gamma_A(x) \leq \gamma_A(y)$.

Definition 2.9. An ordered neutrosophic bitopological space is a neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ (where τ_1 and τ_2 are neutrosophic topologies on X) equipped with a partial order $\leq$.

Definition 2.10. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an upper pairwise neutrosophic T_1-ordered space if $a, b \in X$ such that $a \nleq b$, there exists a decreasing τ_1 neutrosophic neighbourhood (or) an decreasing τ_2 neutrosophic neighbourhood A of b such that $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ is not a neutrosophic neighbourhood of a.

Definition 2.11. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a lower pairwise neutrosophic T_1-ordered space if $a, b \in X$ such that $a \nleq b$, there exists an increasing τ_1 neutrosophic neighbourhood (or) an increasing τ_2 neutrosophic neighbourhood A of a such that $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ is not a neutrosophic neighbourhood of b.

Example 2.1. Let $X = \{1, 2\}$ with a partial order relation $\leq$. Let $\tau_1 = \{0_N, 1_N, A\}$ and $\tau_2 = \{0_N, 1_N, B\}$ where $A = \langle (0.3, 0.3, 0.5), (0.7, 0.7, 0.4) \rangle$ and $B = \langle (0.5, 0.5, 0.5), (0.5, 0.5, 0.5) \rangle$ be any two topologies on X. Then $(X, \tau_1, \tau_2, \leq)$ is an ordered neutrosophic bitopological space. Let $1_{(0.25,0.3,0.5)}$ and $2_{(0.25,0.25,0.35)}$ be any two neutrosophic points on X. For $1_{(0.25,0.3,0.5)} \nleq 2_{(0.25,0.25,0.35)}$, there exists an increasing τ_1 neutrosphic neighbourhood A of $1_{(0.25,0.3,0.5)}$ such that A is not neutrosophic neighbourhood of $2_{(0.25,0.25,0.35)}$. Therefore $(X, \tau_1, \tau_2, \leq)$ is a lower pairwise neutrosophic T_1-ordered space.

Definition 2.12. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a pairwise neutrosophic T_1-ordered space if and only if it is both upper and lower pairwise neutrosophic T_1-ordered space.

Definition 2.13. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an upper pairwise neutrosophic G_δ-α-locally T_1-ordered space if $a, b \in X$ such that $a \nleq b$, there exists a decreasing τ_1 neutrosophic G_δ-α-locally neighbourhood (or) a decreasing τ_2 neutrosophic G_δ-α-locally neighbourhood $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ of b such that A is not a neutrosophic G_δ-α-locally neighbourhood of a.

Definition 2.14. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a lower pairwise neutrosophic G_δ-α-locally T_1-ordered space if $a, b \in X$ such that $a \nleq b$, there exists an increasing τ_1 neutrosophic G_δ-α-locally neighbourhood (or) an increasing τ_2 neutrosophic G_δ-α-locally neighbourhood $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ of a such that A is not a neutrosophic G_δ-α-locally neighbourhood of b.

Definition 2.15. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a pairwise neutrosophic G_δ-α-locally T_1-ordered space if and only if it is both upper and lower pairwise neutrosophic G_δ-α-locally T_1-ordered space.

Proposition 2.1. For an ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ the following are equivalent

(i) X is a lower (resp. upper) pairwise neutrosophic G_δ-α-locally T_1-ordered space.

(ii) For each $a, b \in X$ such that $a \nleq b$, there exists an increasing (resp. decreasing) τ_1 neutrosophic G_δ-α-locally open set(or) an increasing (resp.decreasing) τ_2 neutrosophic G_δ-α-locally open set $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ such that $A(a) > 0$ (resp. $A(b) > 0$) and A is not a neutrosophic G_δ-α-locally neighbourhood of b (resp.a).

Proof:
(i)⇒(ii) Let X be a lower pairwise neutrosophic G_δ-α-locally T_1-ordered space. Let $a, b \in X$ such that $a \nleq b$. There exists an increasing τ_1 neutrosophic G_δ-α-locally neighbourhood (or) an increasing τ_2 neutrosophic G_δ-α-locally neighbourhood A of a such that A is not a neutrosophic G_δ-α-locally neighbourhood of b. It follows that there exists a τ_i neutrosophic G_δ-α-locally open set ($i = 1(or)2$), $A_i = \langle x, \mu_{A_i}, \sigma_{A_i}, \gamma_{A_i} \rangle$ with $A_i \subseteq A$ and $A_i(a) = A(a) > 0$. As A is an increasing neutrosophic set, $A(a) > A(b)$ and since A is not a neutrosophic G_δ-α-locally neighbourhood of b, $A_i(b) < A(b)$ implies $A_i(a) = A(a) > A(b) \geq A_i(b)$. This shows that A_i is an increasing neutrosophic set and A_i is not a neutrosophic G_δ-α-locally neighbourhood of b, since A is not a neutrosophic G_δ-α-locally neighbourhood of b.
(ii)⇒(i) Since A_1 is an increasing τ_1 neutrosophic G_δ-α-locally open set (or) increasing τ_2 neutrosophic G_δ-α-locally open set. Now, A_1 is a neutrosophic G_δ-α-locally neighbourhood of a with $A_1(a) > 0$. By (ii), A_1 is not a neutrosophic G_δ-α-locally neighbourhood of b. This implies, X is a lower pairwise neutrosophic G_δ-α-locally T_1-ordered space.

Remark 2.1. Similar proof holds for upper pairwise neutrosophic G_δ-α-locally T_1-ordered space.

Proposition 2.2. If $(X, \tau_1, \tau_2, \leq)$ is a lower (resp. upper) pairwise neutrosophic G_δ-α-locally T_1-ordered space and $\tau_1 \subseteq \tau_1^*, \tau_2 \subseteq \tau_2^*$, then $(X, {\tau_1}^*, {\tau_2}^*, \leq)$ is a lower (resp. upper) pairwise neutrosophic G_δ-α-locally T_1-ordered space.
Proof:
Let $(X, \tau_1, \tau_2, \leq)$ be a lower pairwise neutrosophic G_δ-α-locally T_1-ordered space. Then if $a, b \in X$ such that $a \nleq b$, there exists an increasing τ_1 neutrosophic G_δ-α-locally neighbourhood (or) an increasing τ_2 neutrosophic G_δ-α-locally neighbourhood $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ of a such that A is not a neutrosophic G_δ-α-locally neighbourhood of b. Since $\tau_1 \subseteq \tau_1^*$ and $\tau_2 \subseteq \tau_2^*$. Therefore, if $a, b \in X$ such that $a \nleq b$, there exists an increasing ${\tau_1}^*$ neutrosophic G_δ-α-locally neighbourhood (or) an increasing ${\tau_2}^*$ neutrosophic G_δ-α-locally neighbourhood $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ of a such that A is not a neutrosophic G_δ-α-locally neighbourhood of b. Thus $(X, {\tau_1}^*, {\tau_2}^*, \leq)$ is a lower pairwise neutrosophic G_δ-α-locally T_1-ordered space.

Remark 2.2. Similar proof holds for upper pairwise neutrosophic G_δ-α-locally T_1-ordered space.

Definition 2.16. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a pairwise neutrosophic T_2-ordered space if for $a, b \in X$ with $a \nleq b$, there exist a neutrosophic open sets $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ such that

A is an increasing τ_i neutrosophic neighbourhood of a, B is a decreasing τ_j neutrosophic neighbourhood of b ($i, j = 1, 2$ and $i \neq j$) and $A \cap B = 0_N$.

Definition 2.17. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a pairwise neutrosophic G_δ-α-locally T_2-ordered space if for $a, b \in X$ with $a \nleq b$, there exist a neutrosophic G_δ-α-locally open sets $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ such that A is an increasing τ_i neutrosophic G_δ-α-locally neighbourhood of a, B is a decreasing τ_j neutrosophic G_δ-α-locally neighbourhood of b ($i, j = 1, 2$ and $i \neq j$) and $A \cap B = 0_N$.

Definition 2.18. Let $(X, \leq)$ be a partially ordered set. Let $G = \{(x, y) \in X \times X \mid$
$x \leq y, y = f(x)\}$. Then G is called a graph of the partially ordered $\leq$.

Definition 2.19. Let X be any nonempty set. Let $A \subseteq X$. Then we define a neutrosophic set χ^*_A is of the form $\langle x, \chi_A(x), \chi_A(x), 1 - \chi_A(x) \rangle$.

Definition 2.20. Let $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ be a neutrosophic set in an ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$. Then for $i = 1(or)2$, we define

I_{τ_i}-G_δ-α-$li(A)$ = increasing τ_i neutrosophic G_δ-α-locally interior of A
= the greatest increasing τ_i neutrosophic G_δ-α-locally open
set contained in A

D_{τ_i}-G_δ-α-$li(A)$ = decreasing τ_i neutrosophic G_δ-α-locally interior of A
= the greatest decreasing τ_i neutrosophic G_δ-α-locally open
set contained in A

I_{τ_i}-G_δ-α-$lc(A)$ = increasing τ_i neutrosophic G_δ-α-locally closure of A
= the smallest increasing τ_i neutrosophic G_δ-α-locally closed
set containing in A

D_{τ_i}-G_δ-α-$lc(A)$ = decreasing τ_i neutrosophic G_δ-α-locally closure of A
= the smallest decreasing τ_i neutrosophic G_δ-α-locally closed
set containing in A.

Notation 2.2. (i) The complement of a neutrosophic set $\chi_G{}^*$, where G is the graph of the partial order of X is denoted by $\chi^*_{\overline{G}}$.

(ii) I_{τ_i}-G_δ-α-$lc(A)$ is denoted by $I_i(A)$ and D_{τ_j}-G_δ-α-$lc(A)$ is denoted by $D_j(A)$, where $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ is a neutrosophic set in an ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$, for $i, j = 1, 2$ and $i \neq j$.

(iii) I_{τ_i}-G_δ-α-$li(A)$ is denoted by $I_i{}^\circ(A)$ and D_{τ_j}-G_δ-α-$li(A)$ is denoted by $D_j{}^\circ(A)$, where $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ is a neutrosophic set in an ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$, for $i, j = 1, 2$ and $i \neq j$.

Definition 2.21. Let A and B be any two neutrosophic sets of a nonempty set X. Then a neutrosophic set $A \times B$ on $X \times X$ is of the form $A \times B = \langle (x, y), \mu_{A \times B}, \sigma_{A \times B}, \gamma_{A \times B} \rangle$ where
$\mu_{A \times B}((x, y)) = \mu_A(x) \wedge \mu_B(y), \sigma_{A \times B}((x, y)) = \sigma_A(x) \wedge \sigma_B(y)$ and $\gamma_{A \times B}((x, y)) = \gamma_A(x) \vee \gamma_B(y)$, for every $(x, y) \in X \times X$

Proposition 2.3. For an ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ the following are equivalent

(i) X is a pairwise neutrosophic G_δ-α-locally T_2-ordered space.

(ii) For each pair $a, b \in X$ such that $a \nleq b$, there exist a τ_i neutrosophic G_δ-α-locally open set $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ and τ_j neutrosophic G_δ-α-locally open set $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ such that $A(a) > 0, B(b) > 0$ and $A(x) > 0, B(y) > 0$ together imply that $x \nleq y$.

(iii) The neutrosophic set χ^*_G, where G is the graph of the partial order of X is a τ^*-neutrosophic G_δ-α-locally closed set, where τ^* is either $\tau_1 \times \tau_2$ or $\tau_2 \times \tau_1$ in $X \times X$.

Proof:
(i)⇒(ii) Let X be a pairwise neutrosophic G_δ-α-locally T_2-ordered space.
Assume that suppose $A(x) > 0$, $B(y) > 0$ and $x \leq y$. Since A is an increasing τ_i neutrosophic G_δ-α-locally open set and B is a decreasing τ_j neutrosophic G_δ-α-locally open set, $A(x) \leq A(y)$ and $B(y) \leq B(x)$. Therefore $0 < A(x) \cap B(y) \leq A(y) \cap B(x)$, which is a contradiction to the fact that $A \cap B = 0_N$. Therefore $x \nleq y$.
(ii)⇒(i) Let $a, b \in X$ with $a \nleq b$, there exists a neutrosophic sets A and B satisfying the properties in (ii). Since $I_i{}^\circ(A)$ is an increasing τ_i neutrosophic G_δ-α-locally open set and $D_j{}^\circ(B)$is decreasing τ_j neutrosophic G_δ-α-locally open set, we have $I_i{}^\circ(A) \cap D_j{}^\circ(B)$=$0_N$. Suppose $z \in X$ is such that $I_i{}^\circ(A)(z)$ $\cap$ $D_j{}^\circ(B)(z)$ >0. Then $I_i{}^\circ(A) > 0$ and $D_j{}^\circ(B)(z) > 0$. If $x \leq z \leq y$, then $x \leq z$ implies that $D_j{}^\circ(B)(x) \geq D_j{}^\circ(B)(z)$ >0 and $z \leq y$ implies that $I_i{}^\circ(A)(y) \geq I_i{}^\circ(A)(z)$ >0 then $D_j{}^\circ(B)(x)$ >0 and $I_i{}^\circ(A)(y)$ >0. Hence by (ii), $x \nleq y$ but then $x \leq y$. This is a contradiction. This implies that X is pairwise neutrosophic G_δ-α-locally T_2-ordered space.
(i)⇒ (iii) We want to show that χ^*_G is a τ^* neutrosophic G_δ-α-locally closed set. That is to show that $\chi^*_{\overline{G}}$ is τ^* neutrosophic G_δ-α-locally open set. It is sufficient to prove that $\chi^*_{\overline{G}}$ is a neutrosophic G_δ-α-locally neighbourhood of a point $(x, y) \in X \times X$ such that $\chi^*_{\overline{G}}(x, y) > 0$. Suppose $(x, y) \in X \times X$ is such that $\chi^*_{\overline{G}}(x, y) > 0$. We have $\chi^*_G(x, y) < 1$. This means $\chi^*_G(x, y) = 0$. Thus $(x, y) \notin G$ and hence $x \nleq y$. Therefore by assumption (i), there exist neutrosophic G_δ-α-locally open sets A and B such that A is an increasing τ_i neutrosophic G_δ-α-locally neighbourhood of a , B is an decreasing τ_j neutrosophic G_δ-α-locally neighbourhood of b ($i, j = 1, 2$ and $i \neq j$) and $A \cap B = 0_N$. Clearly $A \times B$ is an $IF\tau^*$ G_δ-α-locally neighbourhood of (x, y). It is easy to verify that $A \times B \subseteq \chi_{\overline{G}}$. Thus we find that $\chi_{\overline{G}}$ is an τ^* NG_δ-α-locally open set. Hence (iii) is

established.

(iii)⇒(i) Suppose $x \nleq y$. Then $(x, y) \notin G$,where G is a graph of the partial order. Given that $\chi^*_{\overline{G}}$ is τ^* neutrosophic G_δ-α-locally closed set. That is $\chi^*_{\overline{G}}$ is an τ^* neutrosophic G_δ-α-locally open set. Now $(x, y) \notin G$ implies that $\chi^*_{\overline{G}}(x, y) > 0$. Therefore $\chi^*_{\overline{G}}$ is an τ^* neutrosophic G_δ-α-locally neighbourhood of $(x, y) \in X \times X$. Hence we can find that τ^* neutrosophic G_δ-α-locally open set $A \times B$ such that $A \times B \subseteq \chi^*_{\overline{G}}$ and A is τ_i neutrosophic G_δ-α-locally open set such that $A(x) > 0$ and B is an τ_j neutrosophic G_δ-α-locally open set such that $B(y) > 0$. We now claim that $I_i{}^\circ(A) \cap D_j{}^\circ(B)$=$0_N$. For if $z \in X$ is such that $(I_i{}^\circ(A) \cap D_j{}^\circ(B))(z)>$ 0, then $I_i{}^\circ(A)(z) \cap D_j{}^\circ(B)(z) > 0$. This means $I_i{}^\circ(A)(z)>$ 0 and $D_j{}^\circ(B)(z)>$ 0. And if $a \leq z \leq b$, then $z \leq b$ implies that $I_i{}^\circ(A)(b) \geq I_i{}^\circ(A)(z) > 0$ and $a \leq z$ implies that $D_j{}^\circ(B)(a) \geq D_j{}^\circ(B)(z) > 0$. Then $D_j{}^\circ(B)(a) > 0$ and $I_i{}^\circ(A)(b) > 0$ implies that $a \nleq b$ but then $a \leq b$. This is a contradiction. Hence (i) is established.

Definition 2.22. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a weakly pairwise neutrosophic T_2-ordered space if given $b < a$ (that is $b \leq a$ and $b \neq a$), there exist an τ_i neutrosophic open set $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ such that $A(a) > 0$ and τ_j neutrosophic open set $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ such that $B(b) > 0$ $(i, j = 1, 2$ and $i \neq j)$ such that if $x, y \in X$, $A(x) > 0$, $B(y) > 0$ together imply that $y < x$.

Definition 2.23. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a weakly pairwise neutrosophic G_δ-α-locally T_2-ordered space if given $b < a$ (that is $b \leq a$ and $b \neq a$), there exist an τ_i neutrosophic G_δ-α-locally open set $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ such that $A(a) > 0$ and τ_j neutrosophic G_δ-α-locally open set $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ such that $B(b) > 0$ $(i, j = 1, 2$ and $i \neq j)$ such that if $x, y \in X$, $A(x) > 0$, $B(y) > 0$ together imply that $y < x$.

Definition 2.24. The symbol $x \parallel y$ means that $x \leq y$ and $y \leq x$.

Definition 2.25. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an almost pairwise neutrosophic T_2-ordered space if given $a \parallel b$, there exist a τ_i neutrosophic open set $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ such that $A(a) > 0$ and τ_j neutrosophic open set $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ such that $B(b) > 0$ (i,j=1,2 and $i \neq j$) such that if $x, y \in X$, $A(x) > 0$ and $B(y) > 0$ together imply that $x \parallel y$.

Definition 2.26. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be an almost pairwise neutrosophic G_δ-α-locally T_2-ordered space if given $a \parallel b$, there exist a τ_i neutrosophic G_δ-α-locally open set $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ such that $A(a) > 0$ and τ_j neutrosophic G_δ-α-locally open set $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ such that $B(b) > 0$ $(i, j = 1, 2$ and $i \neq j)$ such that if $x, y \in X$, $A(x) > 0$ and $B(y) > 0$ together imply that $x \parallel y$.

Proposition 2.4. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is a pairwise neutrosophic G_δ-α-locally T_2-ordered space if and only if it is a weakly pairwise neutrosophic G_δ-α-locally T_2-ordered and almost pairwise neutrosophic G_δ-α-locally T_2-ordered space.

Proof:

Let $(X, \tau_1, \tau_2, \leq)$ be a pairwise neutrosophic G_δ-α-locally T_2-ordered space. Then by Proposition 3.3 and Definition 3.20, it is a weakly pairwise neutrosophic G_δ-α-locally T_2-ordered space. Let $a \parallel b$. Then $a \nleq b$ and $b \nleq a$.Since $a \nleq b$ and X is a pairwise neutrosophic G_δ-α-locally T_2-ordered space. We have τ_i neutrosophic G_δ-α-locally open set $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ and τ_j neutrosophic G_δ-α-locally open set $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ such that $A(a) > 0$, $B(b) > 0$ and $A(x) > 0$, $B(y) > 0$ together imply that $x \nleq y$. Also since $b \nleq a$,there exist τ_i neutrosophic G_δ-α-locally open set A^*=$\langle x, \mu_{A^*}, \gamma_{A^*} \rangle$ and τ_j neutrosophic G_δ-α-locally open set B^*=$\langle x, \mu_{B^*}, \gamma_{B^*} \rangle$ such that $A^*(a) > 0$, $B^*(b) > 0$ and $A^*(x) > 0$, $B^*(y) > 0$ together imply that $y \nleq x$. Thus $I_i{}^\circ(A \cap A^*)$ is an τ_i neutrosophic G_δ-α-locally open set such that $I_i{}^\circ(A \cap A^*)(a) > 0$ and $I_j{}^\circ(B \cap B^*)$ is a τ_j neutrosophic G_δ-α-locally open set such that $I_j{}^\circ(B \cap B^*)(b) > 0$.Also $I_i{}^\circ(A \cap A^*)(x) > 0$ and $I_j{}^\circ(B \cap B^*)(y) > 0$ togetherimply that $x \parallel y$. Hence X is an almost pairwise neutrosophic G_δ-α-locally T_2-ordered space.

Conservely, let X be a weakly pairwise neutrosophic G_δ-α-locally T_2-ordered and almost pairwise neutrosophic G_δ-α-locally T_2-ordered space. We want to show that X is a pairwise neutrosophic G_δ-α-locally T_2-ordered space. Let $a \nleq b$. Then either $b < a$ (or) $b \nleq a$. If $b < a$ then X being weakly pairwise neutrosophic G_δ-α-locally T_2-ordered space, there exist τ_i neutrosophic G_δ-α-locally open set A and τ_j neutrosophic G_δ-α-locally open set B such that $A(a) > 0$,$B(b) > 0$ and such that $A(x) > 0$,$B(y) > 0$ together imply that $y < x$. Thus $x \nleq y$. If $b \nleq a$, then $a \parallel b$ and the result follows easily since X is an almost pairwise neutrosophic G_δ-α-locally T_2-ordered space. Hence X is a pairwise neutrosophic G_δ-α-locally T_2-ordered space.

Definition 2.27. Let $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ be neutrosophic sets in an ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$. Then A is said to be a τ_i neutrosophic neighbourhood of B if $B \subseteq A$ and there exists τ_i neutrosophic open set C=$\langle x, \mu_C, \sigma_C, \gamma_C \rangle$ such that $B \subseteq C \subseteq A$,$(i = 1(or)2)$.

Definition 2.28. Let $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ and $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ be neutrosophic sets in an ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$. Then A is said to be a τ_i neutrosophic G_δ-α-locally neighbourhood of B if $B \subseteq A$ and there exists τ_i neutrosophic G_δ-α-locally open set $C = \langle x, \mu_C, \sigma_C, \gamma_C \rangle$ such that $B \subseteq C \subseteq A$,
$(i = 1(or)2)$.

Definition 2.29. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is said to be a strongly pairwise neutrosophic G_δ-α-locally normally ordered space if for every pair $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ is a decreasing τ_i neutrosophic G_δ-α-locally closed set and $B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle$ is an decreasing τ_j neutrosophic G_δ-α-locally open set such that $A \subseteq B$ then there exist decreasing τ_j neutrosophic G_δ-α-locally open set A_1=$\langle x, \mu_{A_1}, \gamma_{A_1}\rangle$ such that $A \subseteq A_1 \subseteq D_i(A_1) \subseteq B$,$(i, j = 1, 2$ and $i \neq j)$.

Proposition 2.5. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ the following are equivalent

(i) $(X, \tau_1, \tau_2, \leq)$ is a strongly pairwise neutrosophic G_δ-α-locally normally ordered space.

(ii) For each increasing τ_i neutrosophic G_δ-α-locally open set A=$\langle x, \mu_A, \sigma_A, \gamma_A\rangle$ and decreasing τ_j neutrosophic G_δ-α-locally open set B=$\langle x, \mu_B, \sigma_B, \gamma_B\rangle$ with $A \subseteq B$ there exists an decreasing τ_j neutrosophic G_δ-α-locally open set A_1 such that $A \subseteq A_1 \subseteq NG_\delta$-$\alpha$-$lcl_{\tau_i}(A_1) \subseteq B$,$(i, j = 1, 2$ and $i \neq j)$.

Proof: The Proof is simple.

Notation 2.3. (i) The collection of all neutrosophic set in nonempty set X is denoted by ζ^X.

(ii) Let X be any nonempty set and $A \in \zeta^X$. Then for $x \in X$, $\langle \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ is denoted by $A^\sim$.

Definition 2.30. A neutrosophic real line $\mathbb{R}_\mathbb{I}(I)$ is the set of all monotone decreasing neutrosophic $A \in \zeta^\mathbb{R}$ satisfying $\cup\{A(t) : t \in \mathbb{R}\} = 1^\sim$ and $\cap\{A(t) : t \in \mathbb{R}\} = 0^\sim$ after the identification of neutrosophic sets $A, B \in \mathbb{R}_\mathbb{I}(I)$ if and only if $A(t-) = B(t-)$ and $A(t+) = B(t+)$ for all $t \in \mathbb{R}$ where $A(t-) = \cap\{A(s) : s < t\}$ and $A(t+) = \cup\{A(s) : s > t\}$.

The neutrosophic unit interval $\mathbb{I}_\mathbb{I}(I)$ is a subset of $\mathbb{R}_\mathbb{I}(I)$ such that $[A] \in \mathbb{I}_\mathbb{I}(I)$ if the membership , indeterminancy and nonmembership of A are defined by

$$\mu_A(t) = \begin{cases} 1, & \text{t<0;} \\ 0, & \text{t>1.} \end{cases} \quad \sigma_A(t) = \begin{cases} 1, & \text{t<0;} \\ 0, & \text{t>1.} \end{cases} \quad \text{and } \gamma_A(t) = \begin{cases} 0, & \text{t<0;} \\ 1, & \text{t>1.} \end{cases}$$

respectively. The natural neutrosophic topology on $\mathbb{R}_\mathbb{I}(I)$ is generated from the subbasis $\{L^\mathbb{I}{}_t, R^\mathbb{I}{}_t : t \in \mathbb{R}\}$ where $L^\mathbb{I}_t, R^\mathbb{I}_t : \mathbb{R}_\mathbb{I}(I) \to \mathbb{I}_\mathbb{I}(I)$ are given by $L^\mathbb{I}_t[A] = \overline{A(t-)}$ and $R^\mathbb{I}_t[A] = A(t+)$, respectively.

Definition 2.31. Let $(X, \tau_1, \tau_2, \leq)$ be an ordered neutrosophic bitopological space. A function $f : X \to \mathbb{R}_\mathbb{I}(I)$ is said to be a τ_i $lower^*(resp.upper^*)$ neutrosophic G_δ-α-locally continuous function if $f^{-1}(R^\mathbb{I}{}_t)$ $(resp.f^{-1}(L^\mathbb{I}{}_t))$ is an increasing (or)an decreasing τ_i(resp.τ_j) neutrosophic G_δ-α-locally open set, for each $t \in \mathbb{R}$ $(i, j = 1, 2$ and $i \neq j)$.

Proposition 2.6. Let $(X, \tau_1, \tau_2, \leq)$ be an ordered neutrosophic bitopological space. Let $A = \langle x, \mu_A, \sigma_A, \gamma_A\rangle$ be a neutrosophic set in X and let $f : X \to \mathbb{R}_\mathbb{I}(I)$ be such that

$$f(x)(t) = \begin{cases} 1^\sim & if \quad t < 0 \\ A^\sim & if \quad 0 \leq t \leq 1 \\ 0^\sim & if \quad t > 1 \end{cases}$$

for all $x \in X$ and $t \in \mathbb{R}$. Then f is a τ_i $lower^*(resp.\tau_j upper^*)$ neutrosophic G_δ-α-locally continuous function if and only if A is an increasing (or) a decreasing τ_i (resp. τ_j) neutrosophic G_δ-α-locally open (resp. closed) set $(i, j = 1, 2$ and $i \neq j)$.

Proof:

$$f^{-1}(R^\mathbb{I}{}_t) = \begin{cases} 1^\sim & if \quad t < 0 \\ A^\sim & if \quad 0 \leq t \leq 1 \\ 0^\sim & if \quad t > 1 \end{cases}$$

implies that f is τ_i $lower^*$ neutrosophic G_δ-α-locally continuous function if and only if A is an increasing (or) a decreasing τ_i neutrosophic G_δ-α-locally open set in X.

$$f^{-1}(L^\mathbb{I}{}_t) = \begin{cases} 1^\sim & if \quad t < 0 \\ A^\sim & if \quad 0 \leq t \leq 1 \\ 0^\sim & if \quad t > 1 \end{cases}$$

implies that f is τ_j $upper^*$ neutrosophic G_δ-α-locally continuous function if and only if A is an increasing (or) a decreasing τ_j neutrosophic G_δ-α-locally closed set in X $(i, j = 1, 2$ and $i \neq j)$.

Uryshon's lemma

Proposition 2.7. An ordered neutrosophic bitopological space $(X, \tau_1, \tau_2, \leq)$ is a strongly pairwise neutrosophic G_δ-α-locally normally ordered space if and only if for every $A = \langle x, \mu_A, \sigma_A, \gamma_A\rangle$ is decreasing τ_i neutrosophic closed set and $B = \langle x, \mu_B, \sigma_B, \gamma_B\rangle$ is an increasing τ_j neutrosophic closed set with $A \subseteq B$, there exists increasing neutrosophic function $f : X \to \mathbb{I}_\mathbb{I}(I)$ such that $A \subseteq f^{-1}(\overline{L_1}) \subseteq f^{-1}(R_0) \subseteq B$ and f is a τ_i $upper^*$ neutrosophic G_δ-α-locally continuous function and τ_j $lower^*$ neutrosophic G_δ-α-locally continuous function $(i, j = 1, 2$ and $i \neq j)$.

Proof:

Suppose that there exists a function f satisfying the given conditions. Let $C = \langle x, \mu_C, \sigma_C, \gamma_C\rangle = f^{-1}(\overline{L^\mathbb{I}{}_t})$ and $D = \langle x, \mu_D, \sigma_D, \gamma_D\rangle$=$f^{-1}(R^\mathbb{I}{}_t)$ for some $0 \leq t \leq 1$. Then $\overline{C} \in \tau_i$ and $D \in \tau_j$ and such that $A \subseteq C \subseteq D \subseteq \overline{B}$. It is easy to verify that D is a decreasing τ_j neutrosophic G_δ-α-locally open set and C is an increasing τ_i neutrosophic G_δ-α-locally closed set. Then there exists decreasing τ_j neutrosophic G_δ-α-locally open set C_1 such that $C \subseteq C_1 \subseteq D_i(C_1) \subseteq D$, $(i, j = 1, 2$ and $i \neq j)$. This proves that X is a strongly pairwise neutrosophic G_δ-α-locally normally ordered space.

Conversely, let X be a strongly pairwise neutrosophic G_δ-α-locally normally ordered space. Let A be a decreasing τ_i neutrosophic G_δ-α-locally closed set and B be an increasing τ_j neutrosophic G_δ-α-locally closed set. By the Proposition 3.6, we can construct a collection $\{C_t \mid t \in \mathbb{I}\} \subseteq \tau_j$, where $C = \langle x, \mu_{C_t}, \gamma_{C_t}\rangle$, $t \in \mathbb{I}$ such that $A \subseteq C_t \subseteq B$, NG_δ-α-$lcl_{\tau_i}(C_s) \subseteq C_t$ whenever $s < t$, $A \subseteq C_0$ $C_1 = B$ and $C_t = 0_N$ for $t < 0$,$C_t = 1_N$ for $t > 1$. We define a function $f : X \to \mathbb{I}_\mathbb{I}(I)$ by $f(x)(t) = C_{1-t}(x)$. Clearly f is well defined. Since $A \subseteq C_{1-t} \subseteq B$, for $t \in \mathbb{I}$. We have $A \subseteq f^{-1}(\overline{L^\mathbb{I}{}_1}) \subseteq f^{-1}(R^\mathbb{I}{}_0) \subseteq B$. Furthermore $f^{-1}(R^\mathbb{I}{}_t) = \bigcup_{s<1-t} C_s$ is a τ_j neutrosophic G_δ-α-locally open set and $f^{-1}(\overline{L^\mathbb{I}{}_t}) = \bigcap_{s>1-t} C_s = \bigcap_{s>1-t} NG_\delta$-$\alpha$-$lcl_{\tau_i}(C_s)$ is an τ_i neutrosophic G_δ-α-locally closed set. Thus f is a τ_j $lower^*$ neutrosophic G_δ-α-locally continuous function and τ_i $upper^*$ neutrosophic G_δ-α-locally continuous function and is an increasing neutrosophic function.

Tietze extension theorem

Proposition 2.8. Let $(X, \tau_1, \tau_2, \leq)$ be an ordered neutrosophic bitopological space the following statements are equivalent.

(i) $(X, \tau_1, \tau_2, \leq)$ is a strongly pairwise neutrosophic G_δ-α-locally normally ordered space.

(ii) If $g, h : X \to \mathbb{R}_\mathbb{I}(I)$,$g$ is an τ_i $upper^*$ neutrosophic G_δ-α-locally continuous function, h is a τ_j $lower^*$ neutrosophic G_δ-α-locally continuous function and $g \subseteq h$, then there exists $f : X \to \mathbb{I}_\mathbb{I}(I)$ such that $g \subseteq f \subseteq h$ and f is a τ_i $upper^*$ neutrosophic G_δ-α-locally continuous function and τ_j $lower^*$ neutrosophic G_δ-α-locally continuous function $(i, j = 1, 2$ and $i \neq j)$.

Proof:
(ii)⇒(i) Let A=$\langle x, \mu_A, \sigma_A, \gamma_A\rangle$ and B=$\langle x, \mu_B, \sigma_B, \gamma_B\rangle$ be a neutrosophic G_δ-α-locally open sets such that $A \subseteq B$. Define $g, h : X \to \mathbb{R}_\mathbb{I}(I)$ by

$$g(x)(t) = \begin{cases} 1^\sim & if \quad t < 0 \\ A^\sim & if \quad 0 \leq t \leq 1 \\ 0^\sim & if \quad t > 1 \end{cases} \quad \text{and} \quad h(x)(t) = \begin{cases} 1^\sim & if \quad t < 0 \\ B^\sim & if \quad 0 \leq t \leq 1 \\ 0^\sim & if \quad t > 1 \end{cases}$$

for each $x \in X$. By Proposition 3.6, g is an τ_i $upper^*$ neutrosophic G_δ-α-locally continuous function and h is an τ_j $lower^*$ neutrosophic G_δ-α-locally continuous function. Clearly, $g \subseteq h$ holds,so that there exists $f : X \to \mathbb{R}_\mathbb{I}(I)$ such that $g \subseteq f \subseteq h$. Suppose $t \in (0, 1)$. Then $A = g^{-1}(R^\mathbb{I}{}_t) \subseteq f^{-1}(R^\mathbb{I}{}_t) \subseteq f^{-1}(\overline{L^\mathbb{I}{}_t}) \subseteq h^{-1}(\overline{L^\mathbb{I}{}_t}) = B$. By Proposition 3.7, X is a strongly pairwise neutrosophic G_δ-α-locally normal ordered space.
(i)⇒(ii) Define two mappings $A, B : Q \to I$ by $A(r) = A_r = h^{-1}(\overline{R^\mathbb{I}{}_r})$ and $B(r) = B_r = g^{-1}(L^\mathbb{I}{}_r)$, for all $r \in Q$ (Q is the set of all rationals). Clearly, A and B are monotone increasing families of a decreasing τ_i neutrosophic G_δ-α-locally closed sets and decreasing τ_j neutrosophic G_δ-α-locally open sets of X. Moreover $A_r \subset B_{r'}$ if $r < r'$. By Proposition 3.5, there exists an decreasing τ_j neutrosophic G_δ-α-locally open set $C = \langle x, \mu_C, \sigma_C, \gamma_C\rangle$ such that $A_r \subseteq NG_\delta\text{-}\alpha\text{-}lint_{\tau_i}(C_r)$, $NG_\delta\text{-}\alpha\text{-}lcl_{\tau_i}(C_r) \subseteq NG_\delta\text{-}\alpha\text{-}lint_{\tau_i}(C_{r'})$, $NG_\delta\text{-}\alpha\text{-}lcl_{\tau_i}(C_r) \subseteq B_{r'}$ whenever $r < r'$ $(r, r' \in Q)$. Letting $V_t = \bigcap_{r<t} \overline{C_r}$ for $t \in R$, we define a monotone decreasing family $\{V_t \mid t \in R\} \subseteq I$. Moreover we have $NG_\delta\text{-}\alpha\text{-}lcl_{\tau_i}(V_t) \subseteq NG_\delta\text{-}\alpha\text{-}lint_{\tau_i}(V_s)$ whenever $s < t$. We have,

$$\begin{aligned} \bigcup_{t\in R} V_t &= \bigcup_{t\in R} \bigcap_{r<t} \overline{C_r} \\ &\supseteq \bigcup_{t\in R} \bigcap_{r<t} \overline{B_r} \\ &= \bigcup_{t\in R} \bigcap_{r<t} g^{-1}(\overline{L^\mathbb{I}{}_r}) \\ &= \bigcup_{t\in R} g^{-1}(\overline{L^\mathbb{I}{}_r}) \\ &= g^{-1}(\bigcup_{t\in R} \overline{L^\mathbb{I}{}_t}) \\ &= 1_N \end{aligned}$$

Similarly, $\bigcap_{t\in R} V_t = 0_N$. Now define a function $f : (X, \tau_1, \tau_2, \leq) \to \mathbb{R}_\mathbb{I}(I)$ satisfying the required conditions. Let $f(x)(t) = V_t(x)$, for all $x \in X$ and $t \in R$. By the above discussion, it follows that f is well defined. To prove f is a τ_i $upper^*$ neutrosophic G_δ-α-locally continuous function and τ_j $lower^*$ neutrosophic G_δ-α-locally continuous function (i,j=1,2 and $i \neq j$). Observe that $\bigcup_{s>t} V_s = \bigcup_{s>t} NG_\delta\text{-}\alpha\text{-}lint_{\tau_i}(V_s)$ and $\bigcap_{s>t} V_s = \bigcap_{s>t} NG_\delta\text{-}\alpha\text{-}lcl_{\tau_i}(V_s)$. Then $f^{-1}(R_t) = \bigcup_{s>t} V_s = \bigcup_{s>t} NG_\delta\text{-}\alpha\text{-}lint_{\tau_i}(V_s)$ is an increasing τ_i neutrosophic G_δ-α-locally open set. Now $f^{-1}(\overline{L_t}) = \bigcap_{s>t} V_s = \bigcap_{s>t} NG_\delta\text{-}\alpha\text{-}lcl_{\tau_i}(V_s)$ is a decreasing τ_j neutrosophic G_δ-α-locally closed set. So that f is a τ_i $upper^*$ neutrosophic G_δ-α-locally continuous function and τ_j $lower^*$ neutrosophic G_δ-α-locally continuous function. To conclude the proof it remains to show that $g \subseteq f \subseteq h$. That is $g^{-1}(\overline{L^\mathbb{I}{}_t}) \subseteq f^{-1}(\overline{L^\mathbb{I}{}_t}) \subseteq h^{-1}(\overline{L^\mathbb{I}{}_t})$ and $g^{-1}(R^\mathbb{I}{}_t) \subseteq f^{-1}(R^\mathbb{I}{}_t) \subseteq h^{-1}(R^\mathbb{I}{}_t)$ for each $t \in R$. We have,

$$\begin{aligned} g^{-1}(\overline{L^\mathbb{I}{}_t}) &= \bigcap_{s<t} g^{-1}(\overline{L^\mathbb{I}{}_s}) \\ &= \bigcap_{s<t} \bigcap_{r<s} g^{-1}(\overline{L^\mathbb{I}{}_r}) \\ &= \bigcap_{s<t} \bigcap_{r<s} \overline{B_r} \\ &\subseteq \bigcap_{s<t} \bigcap_{r<s} \overline{C_r} \\ &= \bigcap_{s<t} V_s \\ &= f^{-1}(\overline{L^\mathbb{I}{}_t}) \end{aligned}$$

and

$$f^{-1}(\overline{L^{\mathbb{I}}_t}) = \bigcap_{s<t} V_s$$
$$= \bigcap_{s<t}\bigcap_{r<s} \overline{C_r}$$
$$\subseteq \bigcap_{s<t}\bigcap_{r<s} \overline{A_r}$$
$$= \bigcap_{s<t}\bigcap_{r<s} h^{-1}(\overline{R^{\mathbb{I}}_r})$$
$$= \bigcap_{s<t} h^{-1}(\overline{L^{\mathbb{I}}_s})$$
$$= h^{-1}(\overline{L_t})$$

Similarly, we obtain

$$g^{-1}(R^{\mathbb{I}}_t) = \bigcup_{s>t} g^{-1}(R^{\mathbb{I}}_s)$$
$$= \bigcup_{s>t}\bigcup_{r>s} g^{-1}(\overline{L^{\mathbb{I}}_r})$$
$$= \bigcup_{s>t}\bigcup_{r>s} \overline{B_r}$$
$$\subseteq \bigcup_{s>t}\bigcup_{r>s} \overline{C_r}$$
$$= \bigcup_{s>t} V_s$$
$$= f^{-1}(\overline{R^{\mathbb{I}}_t})$$

and

$$f^{-1}(\overline{R^{\mathbb{I}}_t}) = \bigcup_{s>t} V_s$$
$$= \bigcup_{s>t}\bigcup_{r>s} \overline{C_r}$$
$$\subseteq \bigcup_{s>t}\bigcup_{r>s} \overline{A_r}$$
$$= \bigcup_{s>t}\bigcup_{r>s} h^{-1}(\overline{R^{\mathbb{I}}_r})$$
$$= \bigcup_{s>t} h^{-1}(\overline{R^{\mathbb{I}}_s})$$
$$= h^{-1}(\overline{R^{\mathbb{I}}_t})$$

Hence the proof.

Proposition 2.9. Let $(X, \tau_1, \tau_2, \leq)$ be a strongly pairwise neutrosophic G_δ-α-locally normally ordered space. Let $\overline{A} \in \tau_1$ and $\overline{A} \in \tau_2$ be crisp and let $f : (A, \tau_1/A, \tau_2/A) \to \mathbb{I}_{\mathbb{I}}(I)$ be a τ_i $upper^*$ neutrosophic G_δ-α-locally continuous function and τ_j $lower^*$ neutrosophic G_δ-α-locally continuous function (i,j=1,2 and $i \neq j$). Then f has a neutrosophic extension over $(X, \tau_1, \tau_2, \leq)$ (that is, $F : (X, \tau_1, \tau_2, \leq) \to \mathbb{I}_{\mathbb{I}}(I)$.
Proof:

Define $g : X \to \mathbb{I}_{\mathbb{I}}(I)$ by

$$g(x) = f(x) \quad if \quad x \in A$$
$$= [A_0] \quad if \quad x \notin A$$

and also define $h : X \to \mathbb{I}_{\mathbb{I}}(I)$ by

$$h(x) = f(x) \quad if \quad x \in A$$
$$= [A_1] \quad if \quad x \notin A$$

where $[A_0]$ is the equivalence class determined by $A_0 : \mathbb{R}_{\mathbb{I}}(I) \to \mathbb{I}_{\mathbb{I}}(I)$ such that

$$A_0(t) = 1^\sim \quad if \quad t < 0$$
$$= 0^\sim \quad if \quad t > 0$$

and $[A_1]$ is the equivalence class determined by $A_1 : \mathbb{R}_{\mathbb{I}}(I) \to \mathbb{I}_{\mathbb{I}}(I)$ such that

$$A_1(t) = 1^\sim \quad if \quad t < 1$$
$$= 0^\sim \quad if \quad t > 1$$

g is a τ_i $upper^*$ neutrosophic G_δ-α-locally continuous function and h is a τ_j $lower^*$ neutrosophic G_δ-α-locally continuous function and $g \subseteq h$. Hence by Proposition 3.8, there exists a function $F : X \to \mathbb{I}_{\mathbb{I}}(I)$ such that F is a τ_i $upper^*$ neutrosophic G_δ-α-locally continuous function and τ_j $lower^*$ neutrosophic G_δ-α-locally continuous function and $g(x) \subseteq F(x) \subseteq h(x)$ for all $x \in X$. Hence for all $x \in A$, $f(x) \subseteq F(x) \subseteq f(x)$. So that F is a required extension of f over X.

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