2016

MATHEMATICS. POSSIBLE SUBJECTS FOR THE HIGH SCHOOL ENTRANCE EXAMINATION AND THE CAPACITY EXAMINATION IN ROMANIA

Florentin Smarandache
University of New Mexico, smarand@unm.edu

Constantin Coanda

Ionuț Ivanescu

MATHEMATICS

Sami Drăghici
(coordinator)

POSSIBLE SUBJECTS

Constantin Coandă
Ionuţ Ivănescu
Gheorghe Duţă
Florentin Smarandache
Gheorghe Drăgan
Lucian Tuţescu
Jean Ilie

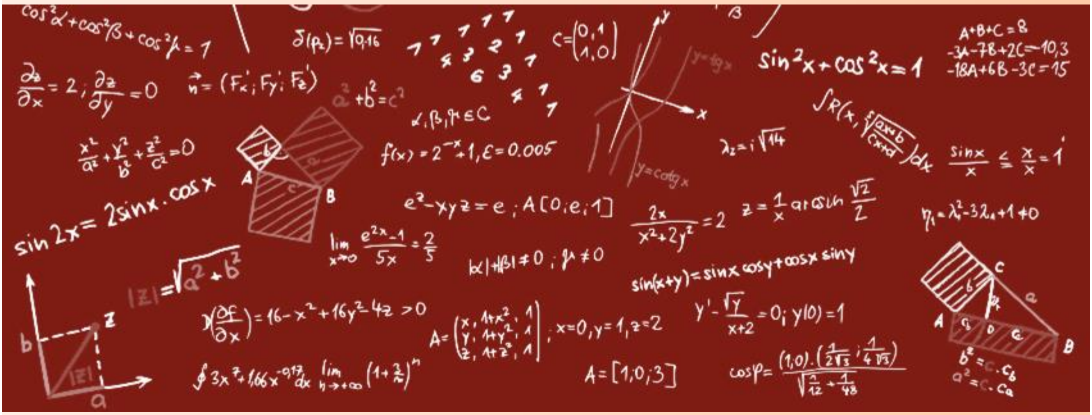
FOR THE HIGH SCHOOL ENTRANCE EXAMINATION

AND THE CAPACITY EXAMINATION IN ROMANIA

Grades V-VIII

High Schools Admission

Translated into English by Florentin Smarandache



pons

Constantin Coandă, Ionuț Ivănescu, Gheorghe Duță, Florentin Smarandache, Gheorghe Drăgan, Lucian Tuțescu, Jean Ilie, Sami Drăghici *(coordinator)*

MATHEMATICS. POSSIBLE SUBJECTS FOR THE HIGH SCHOOL ENTRANCE EXAMINATION AND THE CAPACITY EXAMINATION IN ROMANIA

Grades V-VIII
High Schools Admission

Translated into English by Florentin Smarandache

This book benefited from collaboration of:
Prof. Daniela Taclit (Olt)
Prof. Dumitru Cotoi (Craiova)
Prof. Elena Roşu (Işalniţa)
Prof. Elena Savin (Craiova)
Prof. Elena Tăbîrţă (Balş)
Prof. Florin Nicolaescu (Balş)
Prof. George Chiriţoiu (Balş)
Prof. Ionela Popa (Balş)
Prof. Marian Firicel (Calafat)
Prof. Mariana & Florin Benea (Craiova)
Prof. Mariana Pătraşcu (Craiova)
Prof. Valerica Pometescu (Craiova)
Prof. Valeriu Văduva (Craiova)
Prof. Sanda Zanfir (Craiova)

First issued (in Romanian):

0300, Piteşti. str. Fraţii Goleşti, 29
I.S.B.N.: 973 - 9291 - 42 – 2

Constantin Coandă, Ionuț Ivănescu, Gheorghe Duță,
Florentin Smarandache, Gheorghe Drăgan, Lucian Tuțescu,
Jean Ilie, Sami Drăghici *(coordinator)*

POSSIBLE SUBJECTS

for

THE HIGH SCHOOL ENTRANCE EXAMINATION AND THE CAPACITY EXAMINATION

IN ROMANIA

Grades V-VIII

High Schools Admission

Tests and Answers

Translated into English by Florentin Smarandache

Brussels, 2016

Peer reviewers:

VALERI KROUMOV
Okayama University of Science,
Okayama City, Japan

OCTAVIA CIRA
Universitatea "Aurel Vlaicu"
Arad, Romania

Editor:

Pons asbl
Quai du Batelage no. 5
1000, Bruxelles, Belgium

ISBN 978-1-59973-385-2

Cuprins

Foreword

The present book tries to offer students and teachers knowledge evaluation tools for all the chapters from the current Romanian mathematics syllabus.

In the evolution of teenagers, the phase of admission in high schools mobilizes particular efforts and emotions. The present workbook aims to be a permanent advisor in the agitated period starting with the capacity examination and leading to the admittance to high school.

The tests included in this workbook have a complementary character as opposed to the many materials written with the purpose to support all those who prepare for such examinations and they refer to the entire subject matter included in the analytical mathematics syllabus of arithmetic in Romania, algebra and geometry from the lower secondary grades.

These tests have been elaborated with the intention to offer proper support to those who use the workbook, assuring them the success and extra preparation for future exams.

For all the suggested subjects options we present grading scales, solutions, problems solving suggestions and complete answers, depending on the difficulty and the specificity of each problem.

We also inform you that each test has been organized according to the models of the subjects given at recent high school entrance exams and are thus structured so that the entire examination syllabus is covered and the school workbooks are thus rhythmically covered.

At the end of the workbook, we have included the subjects given at the mathematics examination in the years 1991-1997.

We would like to thank all those who have shown a special receptivity for the emergence of this workbook and have made its publication possible.

The authors

Tests Enunciations

Test no.1

I. 1. Determine the natural numbers a, b, c, knowing that:

$$8\cdot(2^a + \overline{bb}_{(10)}) + 4^c = 609$$

2. Determine the natural numbers a, b, c, d with the properties:

$$\frac{a}{b}=\frac{2}{3};\ \frac{b}{c}=\frac{3}{5};\ \frac{c}{d}=\frac{5}{7};\ \text{and } abcd = 17010.$$

3. Determine the set:

$$A=\left\{n\in\mathbf{N} \mid \sqrt{17-\sqrt{n}}\in\mathbf{N}\right\}.$$

II. 1. Let $a=\sqrt{45}-2\sqrt{2^2-1^2}$ and $b=\sqrt{25^{n+1}:5^{2n+1}}+\sqrt{12}$, $n\in\mathbb{N}$. Determine the smallest value of p, natural number for which $\left(b-a-\sqrt{12}\right)^p\in\mathbb{N}$.

2. Show that:

$$\overline{ab}\ \vdots\ 6 \text{ where } \sqrt{62+20\sqrt{6}}=a\sqrt{2}+b\sqrt{3}.$$

3. Let:

$$f:\mathbf{R}\to\mathbf{R},\ f(x)=\frac{a}{\sqrt{3}+1}x+\frac{b}{\sqrt{3}-1},\ a,\ b\in\mathbf{Q},$$

the graphic of which contains the point $M(\sqrt{3}-1;3)$. Determine the point with both coordinates as natural numbers, that belongs to the graphic of the function.

III. Solve the following equation:

$$\frac{\left(x^2-6\sqrt{3}x+27\right)\left(x-4\right)\left(x^2-10x+25\right)}{x^2\cdot\sqrt{x^2+2x+1}}\le 0.$$

2. The triangle ABC has $AB = 2cm$, $BC = 4cm$ and $m(\sphericalangle ABC) = 30°$. The projection of this triangle on a plane α is a triangle $A'B'C'$ with $m(\sphericalangle B'A'C') = 90°$ and the area of $3cm^2$. Calculate:

a) The area of the triangle ABC.

b) The sides of the triangle $A'B'C'$.

c) The dihedral angle from situated between planes (ABC) and α.

3. Let there be the triangle ABC and G its' center of mass. A random straight line traced through G intersects AB in M and AC in N. Let $BE \parallel AI \parallel CF$ so that the points C, G, E are collinear, B, G, F collinear and $BE, AI. CF$ are situated in the same semiplane with A determined by BC, I being the center of BC. Show that:

a) $\frac{MB}{MA} + \frac{NC}{NA} = 1$

b) $BE + CF \geq 4. GI$; when does the equality take place?

Grading scale:		1 point *ex officio*
	I.	1) 1p 2) 1p 3) 1p
	II.	1) 1p 2) 2p 3) 1p
	III.	1) 2p 2) 2p 3) 1p

Test no.2

I. 1. Calculate:

$$(2{,}5 \cdot 10^3 + 32 : 10^{-2}) \cdot 10 + \sqrt{(-2)^2}$$

2. Show that:

$$\sqrt{27+10\sqrt{2}} + \sqrt{27-10\sqrt{2}} \in \mathbf{N}.$$

II. 1. Determine the set A, if:

$$A = \left\{ x \in \mathbf{Z} \mid x = \frac{6n-7}{2n+1}, n \in \mathbf{Z} \right\}.$$

2. In the triangle ABC we trace the height $AD(D \in BC)$. Calculate BD in relation to the sides of the triangle $BC = a$; $AC = b$; $AB = c$.

III. 1. The base of a straight parallelepiped is a rhombus with the side measuring 8 and an angle of $60°$. The lateral area of the parallelepiped is 512. Determine the volume of the parallelepiped.

2. Determine at which distance from the top of a cone with the radius of the base being 4 and the height being 5, we have to trace a parallel plane with the base so that this cone will be divided in two parts having the same volume.

Grading scale: 1 point *ex officio*

I.	1) 1p 2) 1p
II.	1) 1p 2) 2p
III.	1) 2p 2) 2p

Test no.3

I. 1. a) Calculate:

$$[2^{-2}+(-1)^5+1{,}25]:\sqrt{\frac{1}{9}-\frac{1}{12}}$$

b) Divide number 6 in parts directly proportional with the numbers $0,(3)$ and $0{,}1(6)$.

2. Solve in $\mathbb{R}$ the system:

$$\begin{cases}31x+23y=54\\62x-y=61\end{cases}$$

II. Determine:

$$\frac{4x^2}{x^4+x^3+x+1}:\left(\frac{1}{x^2+2x+1}-\frac{2}{x+1}\cdot\frac{1}{1-x}+\frac{1}{x^2-2x+1}\right)$$

and write down the result as an irreducible fraction.

2. By dividing a polynomial $P(X)$ by X^3+3, we obtain a remainder equal to the quotients' square. Determine this remainder knowing that $P(-1)+P(1)+5=0$.

III. 1. The regular square pyramid $VABCD$ has the base side $AB=10\ cm$ and the height $VO=5\ cm$.

a) Calculate the lateral area and the volume of the pyramid

b) Determine the measure, in degrees, of the angles of the planes $(VBC)$and (ABC)

c) Considering that on VO there is the point P, equally positioned in relation to the 5 points of the pyramid, determine PV.

2. In a straight circular cylinder with the radius $\sqrt{2}$ and the height of $4\ cm$, $[AB]$ is the diameter and $[AA']$ and $[BB']$ are the generators:

a) Determine the total area and the volume of the cylinder

b) Let E be on the circle of diameter $[AB]$, so that $m=\widehat{(BE)}=60°$. State the measure in degrees of the angle formed by the straight lines AE and $A'B$.

Grading scale:		1 point *ex officio*
	I.	1) a) 0,5p b) 0,5p 2) 1p
	II.	1) 1p 2) 1p
	III.	1) a) 1p b) 1p c) 1p
		2) a) 1p b) 1p

Test no.4

I. 1. Solve:

a) $\left[0{,}25-4\left(3-\frac{5}{2}\right)\right]:(-1{,}4)$

b) $\left(\sqrt{0{,}25}-\sqrt{1{,}96}\right):\left(2\frac{1}{3}-3\frac{3}{5}\right)$

c) $\left(\sqrt{2}-2\sqrt{3}\right)^2-\left(2\sqrt{3}+\sqrt{2}\right)\left(\sqrt{2}-\sqrt{3}\right)+5\sqrt{6}$

2. Given the sets:

$A=\left\{n\in \mathbf{N}\mid \frac{6}{n-1}\in \mathbf{N}\right\}$ şi $B=\left\{n\in \mathbf{N}\mid \frac{8}{n+2}\in \mathbf{N}\right\}$.

Calculate: $A\cup B$; $A\cap B$; $A)$; $A\backslash B$.

II. 1. Solve the equation: $m(x+1)-m^2x=1$. Discussion.

2. Show that, if $a,b,c,x,y\in\mathbb{R}$, $a.b\neq 0$ so that $ax+by=c$, then: $x^2+y^2\geq\frac{c^2}{a^2+b^2}$.

III. 1. Determine the area of a triangle, knowing that, if its length is decreased by 10% and its height is augmented by 10%, its area will decrease by $20\ cm^2$.

2. The isosceles right triangle ABC is given, with $m(\sphericalangle B)=90°$, $AB=Ac=a$. In point O, the middle of the line segment AC, we raise a perpendicular on the plane of the triangle on which we designate a point D so that $DO=b$. Let $AM\perp BD$, $M\in(BD)$.

a) Show that $CM\perp DB$

b) Calculate the area of the triangle AMC.

Grading scale:		1 point *ex officio*
	I.	1) a) 0,5p b) 0,5p c) 0,5p 2) 1p
	II.	1) 1,5p 2) 1p
	III.	1) 2p 2) a) 1p b) 1p

Test no.5

I. 1. Calculate the integer part of the number a, where:

$$a=[2{,}5-0{,}4\cdot 3,(3)]^{-1}-\left(2\sqrt{2}+\sqrt{3}\right)\left(2\sqrt{2}-\sqrt{3}\right)\cdot\left(-\frac{7}{2^3}\right)^{-1}$$

2. Let p be a two-digit natural number, prime, that has equal digits. Determine the set A:

$$A=\left\{n \mid n\in \mathbf{N}^*, n \mid \overline{bb}, p+\overline{bb}=77\right\}$$

3. The natural numbers a, b, c, not equal to zero, with $0 < a < b < c$ are directly proportional to $4, 5, d$, $d \in \mathbb{N}^*$. If $c^2 \leq (2b-a)^2$ what percentage of $(a+c)$ does b represent?

II. Calculate:

$$\left(\sqrt{11+6\sqrt{2}}+\sqrt{11-6\sqrt{2}}\right)^{1998(a+b)}$$

where $a, b \in \mathbb{Q}$, so that:

$$\frac{a\sqrt{3}+b}{\sqrt{3}-1}\in \mathbf{Q}.$$

2. Determine A, knowing that:

$$A=\left\{a\in \mathbf{R}^*\setminus\{1\} \mid (a,a^2)\cap \mathbf{N}=\{2,3\}\right\}$$

3. Represent graphically the function $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=ax+b$, where a and b are prime numbers, $c\in\mathbb{Z}$, so that:

$$7\sqrt{a}+2\sqrt{b}=c\sqrt{3}.$$

III. 1. The trapezoid $ABCD$ has $AB \parallel CD$, $AB=12\ cm$, $CD=4\ cm$, $AB\subset\alpha$ ad $C\notin\alpha$, $D\notin\alpha$. Let:

$$AC\cap BD=\{O\},\ MN\parallel AB,\ O\in MN,\ M\in AD,\ N\in CB.$$

If T and R belong to the plane α, so that $MT \parallel NR$, calculate the length of TR.

2. On the plane of the triangle ABC, with $m(\sphericalangle A)=90°$, $m(\sphericalangle B)=15°$, $BC=16\ cm$, the perpendicular $AV=4\ cm$ is raised.

Calculate:

a) the distance from point A to the plane (VBC);

b) the measure of the dihedral angle formed by the planes (ABC) and (VBC).

Test no.6

I. 1. Determine the following set:

$$A = \left\{ x \in \mathbf{Z} \middle| x = \frac{6n-7}{2n+1}, n \in \mathbf{Z} \right\}.$$

2. Calculate:

$$\left(2{,}5 \cdot 10^3 + 32 : 10^{-2}\right) : 10^{-1} + \sqrt{(-2)^2}$$

3. Decompose in factors:

a) $(x-1)^2 - y^2$

b) $9x^2 + 6x + 1$

II. 1. Represent graphically the following function:

$f: \mathbf{R} \to \mathbf{R}, f(x) = \min\{2x - 1; 2x + 1\}.$

2. Determine the area of an equilateral triangle knowing the sum of the distances from a point M that belongs to the interior of the triangle, to its sides, in relation to the length of the triangle's side.

3. A cube with the side measuring 20 is given. Determine the distance between one of the cube's diagonals and a lateral edge it does not intersect.

III. The base of a pyramid is the right triangle ABC with $AB + AC + 8\ cm$ and the edge (SA) perpendicular on the base, $SA = 6\ cm$. Calculate:

a) The total area of the pyramid

b) The measure of the dihedral angle formed by the faces (SBC) and $[ABC]$

c) The volume of the pyramid $[SABC]$.

Grading scale:		1 point *ex officio*
	I.	1) 1p 2) 1p 3) 1p
	II.	1) 1p 2) 1p 3) 1p
	III.	a) 1p b) 1p c) 1p

Test no. 7

I. 1. Calculate:

$$\left(1,(3)+1,(6)\right)\cdot 0,(3)+\sqrt{6,25}:\frac{5}{2}.$$

2. Explain: $|x-1|+2|-3|$.

II. 1. Let ABC be an isosceles triangle $([AB]=[AC])$ and AD the bisector of the angle A, $D \in (BC)$. Let:

$DF \perp AB,\ F \in (AB)$ and $DE \perp AC.\ E \in (AC)$.

Show that $BFEC$ is an isosceles trapezoid.

2. Determine the set:

$$\left\{x \in \mathbf{N} \middle| n \in \mathbf{N},\ x=\frac{2n+7}{n+1}\right\}$$

III. 1. The right triangle ABC $(m\sphericalangle A)=90°$ rotates around the cathetus AC. Knowing that $AB=4$, $AC=3$, calculate the area and the volume of the body that is obtained.

2. Let $ABCD$be a square with the side measuring $3\ cm$. In A and B the perpendiculars $AA'=4\ cm$ and $BB'=6\ cm$ are raised on the plane $ABCD$. If $E \in A'B'$ so that $(A'E)\equiv(EB')$ and $F \in AB$ so that $(AF)\equiv(FD)$, calculate the volume of the pyramid $EABCD$.

Grading scale:		1 point *ex officio*	
	I.	1) 1p	2) 1p
	II.	1) 1,5p	2) 1,5p
	III.	1) 1,5p	2) 1,5p

Test no. 8

I. 1. Calculate:

$$3-5^{-2}\cdot 75+10^{0}+\sqrt{0{,}64}$$

2. Enumerate the elements of the sets:

$$A=\{x\in \mathbf{N} \mid 1+\sqrt{2}<x\leq 1+\sqrt{25}\}$$ and

$$B=\{x\in \mathbf{Z} \mid -5-\sqrt{2}<x<1+\sqrt{3}\}.$$

3. Determine the quotient and the remainder of the division of the polynomial:

$$P(X)=X^3-2X^2+2X+1 \text{ la } X-1$$

II. 1. Prove that in any right triangle the sum of the diameters of the inscribed and circumscribed circles of that triangle is equal to the sums of the catheti.

2. Solve the system:

$$\begin{cases} \dfrac{x}{3}=\dfrac{y}{4}=\dfrac{z}{5} \\ x+y+z=60 \end{cases}$$

3. Bring the following expression to its simplest form:

$$E(x,y)=\frac{2x+y}{x+y}+\frac{x^2+y^2}{x^2-y^2}-\frac{x-2y}{x-y}$$

III. In the regular tetrahedron $[OABC]$ where $OA\perp OB\perp OA$, $OA=OB=OC=8\ cm$ is given.

a) Calculate the area of the triangle ABC

b) Calculate the total area of the tetrahedron $[OABC]$

c) Calculate the volume of the tetrahedron $[OABC]$

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1p	3) 1p
	II.	1) 1p	2) 1p	3) 1p
	III.	a) 1p	b) 1p	c) 1p

Test no. 9

I. 1. Determine the elements of the sets X and Y knowing that the following conditions are simultaneously met:

a) $X \cap Y = \{3,4\}$

b) $X \cup Y = \{3, 4, 4, 6, 7\}$

c) The sum of Y's elements is an even number

2. a) Show that $E(X) = X^4 - 4X^3 + 12X - 9$ is divisible by $X - 1$ and $X - 3$

b) Decompose in indivisible factors $E(X)$

3. Solve the equation: $|2x - 3| + |4x - 6| = 0$

II. 1. Let ABC be a triangle and D the middle of (BC). Show that $AD < \frac{AB+AC}{2}$

2. Solve the inequation: $\frac{4-x}{x^2+4x+5} < 0$

3. Determine the real numbers x and y that verify the relation: $x^2 + y^2 + 2x + 2y + 2 = 0$.

III. A rectangular parallelepiped $ABCDA'B'C'D'$ has the dimensions: $AB = 4\ cm$, $AA' = 4\sqrt{3}\ cm$ and $BC = \sqrt{33} cm$. Calculate:

a) the measure of the parallelepiped's diagonal

b) the measure of the angle between the planes $(B'C'D')$ and (BAC)

c) the volume of the parallelepiped

d) the total area of the parallelepiped

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p	
II.	1) 1p	2) 1p	3) 1p	
III.	a) 0,75p	b) 0,75p	c) 0,75p	d) 0,75p

Test no.10

I. 1. Compare the numbers $\frac{777...75}{777...78}$ and $\frac{888...85}{888...89}$, if each number from the numerator and the denominator has n digits, $n \in \mathbb{N}, n > 2$.

2. Calculate : $1 - \frac{1}{2}.\left(0{,}1.10^2 - \sqrt{81}\right): 2^{-1}$

3. Solve the system:

$$\begin{cases} \frac{1}{x+3} - \frac{1}{y+1} = 0 \\ 3x + 2y = 9 \end{cases}$$

4. If $n \in \mathbb{N}^*$, prove that the number $\sqrt{n + \sqrt{n+1}}$ is irrational.

II. 1. Solve the equation:

$$\frac{\overline{xy}}{\overline{yx}} = 2 - \frac{y}{x}$$

2. If $x^2 + y^2 + z^2 = 1$, prove that:

$$|x + y + z| \leq \sqrt{3}$$

3. A parallel to the median (AD) of the triangle ABC cuts AB and AC in E and F, respectively. Show that $\frac{AE}{AF} = \frac{AB}{AC}$.

III. Given a right circular cone with the length of the base's diameter of $12\ cm$ and the length of the height being equal to $2/3$ of the diameter's length:

a) determine the volume of the cone

b) the lateral area of the cone

c) The lateral surface of the cone is displayed thus obtaining a sector of the circle. What is the measure of the angle of this sector of the circle?

Grading scale: 1 point *ex officio*

I.	1) 0,75p	2) 0,75p	3) 0,75p	4) 0,75p
II.	1) 1p	2) 1p	3) 1p	
III.	a) 1p	b) 1p	c) 1p	

Test no. 11

I. 1. Calculate:

a) $\left(0{,}001:\frac{1}{20}+\sqrt{0{,}0064}\right)\cdot\left(3^2+1^0\right)$

b) $(1+a)^2-(1-a)^2$

2. Let there be the function $f:\mathbb{R}\to\mathbb{R}$, described by $f(x)=2x+m$.

a) Determine the value of the parameter "m" so that the graphic passes through the point $A(2;5)$;

b) Represent graphically the determined function.

3. Solve the equation:

$\sqrt{x^2-2x+1}+\sqrt{1-2x+x^2}=2,\ x\in\mathbf{R}$..

II. 1. If $p>3$ is a prime number, show that p^2 divided by 24 will give the remainder 1.

2. If:

$P(X)=3X^2-3aX+a^2$

prove that $P(b)\geq 0$, irrespective of the what $b\in\mathbb{R}$ is.

3. Let O represent the point of intersection of a trapezoid. Prove that the straight line determined by the point O and by the intersection point of the un-parallel sides of the trapezoid goes through the middle of the trapezoid's bases.

III. In a cylinder whose axial section is a square with the side of $6\ cm$ a sphere is inscribed. Calculate:

a) The total area and the volume of the cylinder

b) The rapport between the volume of the sphere and that of the cylinder's.

Grading scale: 1 point *ex officio*

I.	1) a) 0,50p	b) 0, 50p	2) a) 0,50p b) 0,50p	3) 1p
II.	1) 1p	2) 1p	3) 1p	
III.	a) 1p	b) 1p		

Test no. 12

I. 1. Show that for $\forall n \in \mathbb{N}$, the expression:

$$E = 3 \cdot 5^{2n+1} + 2^{3n+1}$$

is divisible by 7.

2. Determine the digits $\underline{a}$ and $\underline{b}$ so that the fraction $\frac{\overline{52a}}{\overline{1b75}}$ can be simplified by 17.

3. Calculate:

$$[(-1)^n \cdot (-2^2)^n - 4^n] : (-4^n), \text{ where } n \in \mathbf{N}.$$

II. 1. Effectuate:

$$\min(-5;-2) + \frac{|-7|}{|-2|} - \sqrt{(-4)^2}$$

2. Justify why the equation:

$$\sqrt{3}\left(\sqrt{2} - x\right)^2 = \sqrt{2} - \sqrt{3}$$

doesn't have real solutions.

3. Simplify the fraction:

$$\frac{4+8+12+\ldots+400}{3+6+9+\ldots+300}.$$

III. 1. The measure of an angle is $7/8$ from its supplement. Show that $2/3$ from the measure of the bigger angle is $1°$ bigger than $3/4$ from the measure of the small angle.

2. Three parallel planes are given: α, β, γ and the points A, B in α and C, D in β. The straight lines AC, BC, BD and AD cut the plane γ in E, F, G and H. Prove that $EFGH$ is a parallelogram.

3. On the plane of the parallelogram $ABCD$ the perpendicular AE is raised. Calculate the distances from E to the straight lines BD, BC and CD, knowing that:

$$AB = AD = AE = a \text{ and } m(\sphericalangle BAD) = 60°.$$

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p
II.	1) 1p	2) 1p	3) 1p
III.	1) 1p	2) 1p	3) 1p

Test no. 13

I. 1. Find the natural number n, so that:

$A = 1^n + 2^n + 3^n + \ldots + 9^n$

is divisible by 5.

2. Prove that:

$$\frac{2^n \cdot 3^{n+1} \cdot 5^{n+2}}{30^{n+1}} = 2\frac{1}{2}, (\forall) n \in \mathbf{N}.$$

3. Knowing that $\frac{x}{y} = \frac{3}{4}$, find the value of the rapport $\frac{5y-7x}{6y-8x}$.

II. 1. Solve in $\mathbb{Z}$ the equation:

$|2x + 5 - |x - 2|| = 0.$

2. Show that the following number is a perfect square:

$n = 1991 + 2(1 + 2 + 3 + \ldots + 1990)$

3. Solve the equation system:

$$\begin{cases} \max\left[-3{,}1];-\sqrt{17};-|-3|\right] \cdot x - \{2{,}3\} \cdot y = 1{,}1(6) - \sqrt{576} \\ \min\left[2^{-3};\sqrt{6};1\right] \cdot x + 3^{-2} y = -72^{-1} \end{cases}$$

III. 1. Two adjacent angles have parallel bisectors. Determine the measure of each angle, knowing that the measure of one of them is five time bigger than the measure of the other.

2. Show that, if the diagonals of a rectangular trapezoid are perpendicular to one another, then the length of the perpendicular side on the bases is the geometric mean of the bases lengths.

3. Let A, B, C, D be four points that are not coplanar. We note E and F the projections of point A on the bisectors $\sphericalangle ABD$ and $\sphericalangle ACD$ respectively. Prove that $EF \parallel (BCD)$.

Grading scale:		1 point *ex officio*		
	I.	1) 0,75p	2) 1p	3) 1,25p
	II.	1) 1p	2) 0,75p	3) 1,25p
	III.	1) 0,75p	2) 1p	3) 1,25p

Test no. 14

I. 1. Determine the consecutive digits a, b, c, d in the base 10 knowing that $\overline{ab} = cd$.

2. Given $A = \sqrt{\frac{2x+5}{x-1}}$, determine $x \in \mathbb{Z}$, knowing that $A \in \mathbb{N}$.

3. Show that the following fraction is irreducible:

$$F = \frac{10n+53}{7n+37}$$

II. 1. Determine the set:

$$A = \left\{ x \in \mathbf{Z} \,\middle|\, \min\left(x+1; 4-\frac{x}{2} \right) \geq 1 \right\}$$

2. Represent graphically the following function:

$$f: \mathbf{R} \to \mathbf{R},\ f(x) = \sqrt{3^2 + 3^{-2} - \left(\frac{1}{3}\right)^2} \cdot x - |-2|.$$

3. Determine x from the proportion: $\frac{a}{x} = \frac{4^{993}}{0{,}25}$, where $a = 2^{1990} - 2^{1989} - 2^{1988}$.

III. 1. Show that in a right angle ABC with $m(\sphericalangle A) = 90°$, the following inequality takes place: $B.\sin C \leq \sin 30°$. When does the inequality take place?

2. Let A, B, C, D be for points that are not coplanar, so that:

$$BC^2 + AC^2 = AD^2 + BD^2 \text{ şi } AC^2 + AD^2 = BC^2 + BD^2.$$

Show that if M and N are the middles of the segments AB and CD respectively, then MN is the common perpendicular of the straight lines AB and CD.

3. The theorem of the three perpendiculars: enunciation and demonstration.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1,25p	3) 0,75p
II.	1) 1,25p	2) 0,75p	3) 1p
III.	1) 1,25p	2) 1,25p	3) 0,50p

Test no. 15

I. 1. Given the numbers:

$$a=\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\ldots+\frac{1992}{1993} \text{ and } b=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots+\frac{1}{1993}$$

Prove that $3|(a+b)$.

2. Find the form of the natural number $\underline{n}$ knowing that:

$$(-2)^5+(-1)^n+\left(-\frac{1}{2}\right)^2\cdot(-1)^{n+1}=-31\frac{1}{4}$$

3. Show that the following number is subunitary $(\forall)n\in\mathbb{N}^*$:

$$A=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\ldots+\frac{1}{n(n+1)}$$

II. 1. No matter what $n\in\mathbb{N}$ is, is the following number a perfect square?

$$a=(n^2+n-7)(n^2+n-3)+4$$

2. Determine the function $f:\mathbb{R}\to\mathbb{R}$, with the property:

$$f(x)+2f(1-x)=|x+1|$$

irrespective of what $x\in\mathbb{R}$ is and $f(x)$- linear function.

3. Given:

$$P(X)=X^{1993}+1993X^{1992}+1993X^{1991}+\ldots+1993X+1993$$

show that $P(-1992)=1$.

III. 1. In ΔABC we consider that $m(\sphericalangle A)=60°$ and $AB=2.AC$. Find the measures $\sphericalangle B$ and $\sphericalangle C$.

2. Prove that in any ΔABC the following relation takes place:

$$\frac{h_A}{\frac{1}{a}}=\frac{h_B}{\frac{1}{b}}=\frac{h_C}{\frac{1}{c}},$$

where a,b,c represent the lengths of the sides of ΔABC, and $h_A.h_B.h_C$ the heights corresponding to the tops A,B and C respectively.

3. A right regular pyramid has the side of the base of $6\sqrt{3}$ and the apothem of the pyramid equal to $5m$. Determine the lateral area and the volume of the pyramid.

Grading scale:		1 point *ex officio*		
	I.	1) 0,75p	2) 1p	3) 1,25p
	II.	1) 1p	2) 1p	3) 1p
	III.	1) 1,25p	2) 0,75p	3) 1p

Test no. 16

I. 1. Knowing that $a + b = 3$ and $a + c = 5$, calculate $a^2 + ac + 5b$.

2. If $\frac{x}{y} = \frac{4}{3}$, determine the rapport: $\frac{xy-y^2}{xy+y^2}$

3. Given the expression:

$E(x) = x^2 + x + p,$

where p is an integer number, show that if $E(2)$ or $E(3)$ are divisible by 6, then $E(5)$ is divisible by 6.

II. 1. Write as union of intervals the set:

$\{x \mid x \in \mathbf{R}, |x-1| > 2\}.$

2. Solve the equation:

$(x+1) + (x+2) + \ldots + (x+100) = 15050.$

3. For what values of m and n are the following equations equivalent?

$x^2+mx+1=0$ and $2x^2-nx+m=0$

III. 1. In a triangle ABC we know:

$m(\sphericalangle A) = 50°,\ m(\sphericalangle C) = 64°.$

Calculate the acute angle formed by the bisector of the angle B with the height raised from A.

2. In the cube $ABCDA'B'C'D'$, let Q be the projection of the top D on the diagonal AC'. Determine the value of the rapport $\frac{AQ}{AC'}$.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1p	3) 1p
	II.	1) 1p	2) 1,5p	3) 1,5p
	III.	1) 1p	2) 1p	

Test no. 17

I. 1. Show that:

$$\frac{2\cdot(-1)^k+2\cdot(-1)^{k+1}+np}{(n+p)\sqrt{np}}<\frac{1}{2},\ (\forall)k\in\mathbf{N},\ n,\ p>0,\ n\neq p.$$

2. If $\frac{a}{b}=\frac{13}{12}$, show that $\frac{3a-b}{4a+b}$ is a perfect cube.

3. Using:

$$E=\frac{1}{3}\left(\frac{1}{3a-2}-\frac{1}{3a+1}\right)$$

show that $S<1/3$ where:

$$S=\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\frac{1}{7\cdot10}+\ldots+\frac{1}{(3n-2)(3n+1)}$$

II. 1. Solve in $\mathbb{Z}\times\mathbb{Z}$ the equation:

$$8x^2-6xy+y^2-5=0$$

2. Given:

$$f:\ \mathbf{R}\to\mathbf{R},\ f(x)=\sqrt{x^2+6x+9}-\sqrt{14-6\sqrt{5}}.$$

Order in ascending order the numbers:

$$f(-3),\ f(1),\ f(-\sqrt{5}),\ f(1+\sqrt{5}).$$

3. Given the polynomial:

$$P(X)=a(1-b)(2X^2-3)^{2k+1}-b(5-6X^2)^{k-1}-a(4-5X^3)^{k+2}+1,$$

$k\in\mathbb{N}$ and $b\in\mathbb{Z}$, determine a and b so that the sum of the coefficients is 0.

III. 1. Show that if in a ΔABC, the side $\{AC\}$, the bisector $\sphericalangle B$ and the median of the side $[BC]$ are concurrent, then $m(\sphericalangle B)=2m(\sphericalangle C)$.

2. In ΔABC we know that $AB=9\ cm$, $BC=10\ cm$ and $m(\sphericalangle B)=60°$. Calculate the length of the segment AB.

3. A sphere, a circular right cylinder with a square axial section and a cube have the same area. Compare their volumes.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 0,75p	3) 1,25p
	II.	1) 1p	2) 1p	3) 1p
	III.	1) 0,75p	2) 1,25p	3)1p

Test no. 18

I. 1. Determine a so that:

$$|a\sqrt{3}-1| = \sqrt{3}-\sqrt{5}.$$

2. Show that the following numbers are smaller than 2:

$$\sqrt{2+\sqrt{2}} \text{ and } \sqrt{2+\sqrt{2+\sqrt{2}}}$$

3. Solve the system:

$$\begin{cases} \frac{1}{x}+\frac{1}{y}=0,7 \\ \frac{3}{x}-\frac{5}{y}=0,5 \end{cases}$$

II. 1. Determine x knowing that:

$$\frac{x}{3}=\frac{\overline{32a}}{15} \text{ and } 4 \mid \overline{32a}.$$

2. Given the real numbers x, y, z, show that:

a) $x^2+y^2+z^2 \geq xy+yz+xz$

b) if $x+y+z=1$, deduce that $x^2+y^2+z^2 \geq 1/3$.

III. 1. Let ABC be a triangle, with the sides' lengths $AB=a$, $AC=a\sqrt{2}$, $BC=a\sqrt{3}$. We note with P the foot of the height from A on BC and with M, N the projections of Pon the sides AB and AC respectively. Calculate the length of the segment MN in relation to a.

2. Let ABC be a right isosceles triangle. The hypotenuse BC is situated in a plane α and the plane of the triangle makes a $45°$ angle with the plane α. Determine the angles that the catheti AB and AC form with the plane α.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p		3) 1p
II.	1) 1p	2) a) 1p	b) 1,5p	
III.	1) 1,5p	2) 1p		

Test no. 19

I. 1. Given:

$$S = (-1)^{a_0} + (-1)^{a_1} + (-1)^{a_2} + \ldots + (-1)^{a_{100}}$$

and knowing that $a_0, a_1, \ldots, a_{100}$ are natural numbers, calculate $S_{min} + S_{max}$. Show that, generally, $S \neq 0$.

2. Determine the natural numbers $\underline{x}$ and $\underline{y}$ so that the geometric mean of the numbers 4^x and 8^y is 64.

3. Prove that the numbers $\overline{a4}.\overline{a6} + 1$ are perfect squares, irrespective of the number $\underline{a}$.

II. 1. Given:

$$A = \{x \mid x \in \mathbf{N} \text{ and } |3x - 2| \leq 7\} \text{ and}$$
$$B = \{x \mid x \in \mathbf{Z} \text{ and } x - 5 \in [-1, 2]\}$$

calculate $A \cap B$.

3. Given:

$$P(X) = 2X^2 - mX + n,\ P(X) \vdots (X-1)$$

and $P(X) : (X + 1)$ gives the remainder 2, determine m and n.

III. 1. In ΔABC, with $m(\sphericalangle C) = 60°$, the bisectors $(AK$ and $(BE$, where $K \in BC$, $E \in AC$ intersect in O. Show that $OK = OE$.

2. Given the square $ABCD$ and M a point in the interior of ΔABC, show that $MD > MB$.

3. A regular square pyramid has the side of the base of $16\ cm$ and the height of $6\ cm$. Calculate the lateral area and the lateral edge of the pyramid.

Grading scale:		1 point *ex officio*		
I.	1) 1p	2) 1p		3) 1p
II.	1) 1p	2) a) 1p	b) 1p	
III.	1) 1,25p	2) 1,25p		3) 0,50p

Test no. 20

I. 1. Given the sets:

$A = \{2^n, 4, 7\}$ and $B = \{12, 16, 125\}$

determine $n \in \mathbb{N}$ so that $A \cup B$ has five elements.

2. Knowing that $\frac{x}{y} = \frac{3}{4}$, calculate $\frac{2x-y}{3y-x}$.

II. 1. Calculate:

$$\frac{1}{2-\sqrt{2}} - \frac{1}{3-\sqrt{3}} - \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{6}.$$

2. In the triangle ABC we know that $AB = 6\ cm$, $AC = 14\ cm$ and $m(\sphericalangle B) = 60°$. Calculate BC and the lengths of the triangle's heights.

III. 1. Calculate the equations:

a) $x^2 + x - y^2 + y = 2;\ x, y \in \mathbf{N};$

b) $2x - (4x - 5) = 3x + 1;\ x \in \mathbf{R}.$

2. In a rights triangle ABC $(m(\sphericalangle A) = 90°)$ we know that $AB = 12\ cm$ and that the length of the median$Am = 10\ cm$ $(M \in BC)$. In the point M a perpendicular MN is raised on the plane of the triangle. If $MN = 8\ cm$, calculate:

a) the angle between the planes (NAB) and (ABC);

b) $\cos \sphericalangle[(NBC), (AMN)]$;

c) the distance from point A to the straight line NB;

d) the area and the volume of the pyramid $NABC$.

Grading scale: 1 point *ex officio*

I.	1) 0,50p	2) 0,50p
II.	1) 0,50p	2) 2p (4. 0,50p)
III.	1) 1p+ 0,50p	2) 0,75p + 1p + 1p + 1, 25p

Test no. 21

I. 1. Effectuate:

a) $2+2\cdot\{2+2\cdot[2+2\cdot(2-2:2)]\}\cdot 2^{-2}$

b) $\left(1+\frac{4}{3}\right)^2-\left(1+\frac{3}{4}\right)^2$

c) $\left(2-\sqrt{3}\right)^2-\left(8-4\sqrt{3}\right)$

2. a) Calculate the value of the rapport $\frac{3a+b}{3a-b}$ knowing that $\frac{2a}{3b}=\frac{1}{5}$.

b) After two consecutive price raises, the price of an object raised from $3500\ lei$ to $3969\ lei$. Knowing that the second raise was by 5%, determine the percent of the first raise.

II. 1. Solve in Q the equation:

$$5\left(y+\frac{2}{3}\right)-3=4\left(3y-\frac{1}{2}\right)$$

2. Solve in ℝ the inequation:

$$\frac{7x+1}{8}-\frac{5x-11}{11}<4$$

3. A vehicle has covered $150\ km$ in 3 hours. In the passing towns, the average speed was $40\ km/h$ and outside the towns, $70\ km/h$. How much time did he spend crossing the towns?

4. Determine the remainder of the division of the polynomial:

$$P(X)=X^3+mX^2+1$$

with $X-1$, if by dividing the polynomial $P(X)$ with $X+2$, the remainder is -3.

5. Represent graphically the function:

$f:\mathbf{R}\to\mathbf{R}$, where $f(x)=-\frac{1}{2}x+1$.

III. 1. Considering the right triangle ABC with $m(\sphericalangle A) = 90°$, O the middle of $[BC]$, $AD \perp BC$, $BC = 50\ cm$ and $OD = 7\ cm$. Calculate the measures of the segments $[AB]$ and $[AC]$.

2. Let $ABCDA'B'C'D'$ be a rectangular parallelepiped and M the middle of $[A'D']$. Knowing that $AB = 4a$, $BC = 2a$ and that the area of the triangle MBC is equal to $5a^2$, calculate:

a) the volume of the parallelepiped

b) the distance from B' to AM

c) the distance from A to (MBC)

3. In a cone with the dimensions $R = 8$ and $h = 6$ we cut a section with a parallel plane with the plane of the base, situated at a distance from the top of $1/3$ from the height of the cone. Determine:

a) the area and the volume of the cone

b) the area and the volume of the body of the obtained cone.

Grading scale: 1 point *ex officio*

I.	1) a) 0,50p b) 0,50p c) 0,50p
	2. a) 0,75p b) 0,75p
II.	1) 0,6p 2) 0,6p 3) 0,6p 4)0,6p 5) 0,6p
III.	1) 1p 2) 1p 3) 1p

Test no. 22

I. 1. The difference between two numbers is 100. If we divide the big number to the small number, we obtain the quotient 3 and the remainder 20. What are the two numbers?

2. Knowing that $\frac{a-13}{b} = \frac{a}{b+7}$, where $a, b \in \mathbb{N}^*$ determine $\frac{a}{b}$.

3. Calculate the value of the expression:

$$E = \frac{(-1)^1 + (-1)^2 + \ldots + (-1)^n}{(-1)^{n^1} + (-1)^{n^2} + (-1)^{n^3} + \ldots + (-1)^{n^n}}$$

where $n \in \mathbb{N}^*$.

II. 1. Solve the equation: $mx + 1 = x + m$, where m is a real parameter.

2. Given:

$$E(x) = \left[\left(\frac{x-1}{x+1}\right)^2 + 1 + \frac{2x-2}{x+1}\right] \cdot \frac{(x+1)^3}{16x^2}$$

Knowing that $E(x) \in \mathbb{Z}, x \in \mathbb{Z}$, determine the value of x.

3. Knowing that $x = \sqrt{11 - 6\sqrt{2}}$ and $y = 3 - \sqrt{2}$, calculate $a = (-1)^n . (x + y) =?$, where $n \in \mathbb{N}$.

III. 1. The measures of the angles A, B, C of the triangle ΔABC are proportional to $5, 6, 7$, respectively. Determine the height of the angle formed by the height of A and the bisector $\sphericalangle C$.

2. Let $ABCD$ be a orthodiagonal quadrilateral inscribed in the center circle O. If S is the projection of O on CD, then $OS = \frac{1}{2} AB$.

3. Let $ABCDA'B'C'D'$ be a cube with the edge $\underline{a}$ and M the middle of the segment AB and O the center of the square $BCC'B'$. Show that $D'O \perp (COM)$.

Grading scale:		1 point *ex officio*
	I.	1) 0,50p 2) 1,75p 3) 0,75p
	II.	1) 0,75p 2) 1,25p 3) 1p
	III.	1) 0,25p 2) 1,25p 3) 1,25p

Test no. 23

I. 1. Given:

$a = |2^n - 3| - |5 - 2^n|, n \in \mathbb{N}$

Prove that:

$|a| \in \{0, 2\}, (\forall) n \in \mathbb{N}$.

2. Determine the numbers $\overline{ab}$ in the 10 base, so that $\sqrt{\overline{ab} + \overline{ba}} \in \mathbb{N}$.

3. Determine the real numbers x, y and z for which:

$$\sqrt{x^2 + 6x + 34} + \sqrt{y^2 - 2y + 10} + \sqrt{z^2 - 6z + 25} = 12$$

II. 1. Given the polynomial:

$P(X) = X^{1992} - 1992X + 1991$

show that $P(X)$ is divided by $(X-1)^2$.

2. Solve in $\mathbb{R}$ the system of equations:

$$\begin{cases} [0,(3)x - 0,1(6)y] : \frac{1}{6} = 3^{101} : 3^{10^2} \\ [\max(-100; -12)]x + 10 \cdot \sqrt{0,36y} = \min(|-5|; -18) \end{cases}$$

3. Calculate:

$$E = \sqrt{7 + 2\sqrt{6}} - \sqrt{7 - 2\sqrt{6}}$$

III. 1. In ΔABC the parallels through B and C to the bisector $AD (D \in BC)$ intersect the straight lines AC and AB, respectively, in E, F. Prove that $[BC] \equiv [EF]$.

2. In ΔABC, let M be the middle of the side BC and P and Q the feet of the perpendiculars traced from M on AB and AC, respectively. Show that $\frac{MP}{MQ} = \frac{AC}{AB}$.

3. In the top A of a ΔABC, $AD \perp (ABC)$ is raised. Let $CE \perp AB$, $E \in AB$ and $EF \perp BD$, $F \in BD$. Show that $(BCD) \perp (CEF)$.

Grading scale: 1 point *ex officio*

I.	1) 0,75p	2) 0,50p	3) 1,75p
II.	1) 1,50p	2) 1p	3) 0,50p
III.	1) 1p	2) 1p	3) 1p

Test no. 24

I. 1. Let:

$A = \{x \in \mathbb{N} \mid 3 < 2x - 3 < 10\}$, $B = \{x \in \mathbb{N} \mid 5 < 3x - 2 < 11\}$.

Determine the sets:

$A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$.

2. Determine the natural numbers n for which $\frac{15}{2n+1} \in \mathbb{N}$.

3. If $x + y = s$ and $xy = p$ calculate $x^3 + y^3$ and $x^4 + y^4$ in relation to s and p.

II. 1. Determine the smallest common multiple of the polynomials:

$X^2 - (a+b)X + ab$ and $X^2 - (a+c)X + ac$

2. Determine $\frac{a}{b}$ knowing that $\frac{3a-2b}{5a-3b} = \frac{1}{5}$.

3. Show that:

$(a+b+c)^2 \leq 3(a^2+b^2+c^2)$.

III. 1. A rhombus has an angle of $120°$; the small diagonal is of $7\ cm$. Its perimeter is required.

2. Given the trapezoid $ABCD$, where $AD || BC$ and $AB = AD = DC$, $AB = a$ and $BC = 2a$. The diagonals AC and BD intersect in O. In O we raise the perpendicular on the plane of the trapezoid and we designate S so that: $SO = \frac{a}{2}$.

a) Show that the acute angles of the trapezoid have $60°$ each and that AC is perpendicular on AB, and that BD is perpendicular on DC.

b) Show that the triangle SAB is right triangle.

c) Determine the dihedral angle formed by the plane (SAD) with the plane of the trapezoid.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p	
II.	1) 1p	2) 1p	3) 1p	
III.	1) 0,5p	2) a) 1p	b) 0,5p	c) 1p

Test no. 25

I. 1. Calculate the sum:

$$S=(-1)^1\cdot 2+(-1)^2\cdot 4+(-1)^3\cdot 6+\ldots+(-1)^{997}\cdot 1994+998$$

2. We consider the integer number:

$$n = 1994^2 - 1994 - 1993.$$

Determine x from the proportion:

$$\frac{\frac{x}{1993}}{2}=\frac{1993}{\frac{n}{2}}.$$

3. Show that if:

$$a^2+b^2-2a\sqrt{2}-2b\sqrt{3}+5=0$$

then:

$$\left(\frac{2}{a}+\frac{3}{b}\right)(b-a)=-1.$$

II. 1. Determine $n \in \mathbb{N}^*$ so that the following polynomial is divisible by $Q(X) = 2X - 1$:

$$P(X) = 2^{n+1}\cdot X^n+2^n\cdot X^{n-1}+\ldots+2^2X-n^2$$

2. Solve the equation:

$$\sqrt{(x-4)^2}+\sqrt{(3-x)^2}=1$$

3. Solve in $\mathbb{N}$ the system of inequations:

$$\begin{cases} 6x+\frac{5}{7}>4x+7 \\ \frac{8x+3}{2}<2x+25 \end{cases}$$

III. 1. In ΔABC with $AB = \frac{2}{3}AC$ and $m(\sphericalangle BAC) = 60°$ we trace the median CM, where $M \in (AB)$. Show that $CM = BC$.

2. In ΔABC we have $AB = AC = 12\ cm$ and $BC = 8\ cm$. At what distance from the base BC do we have to trace the parallel BC so that the perimeter of the formed trapezoid measures $20\ cm$?

3. In the top A of ΔABC equilateral, we trace $AD \perp (ABC)$. Let $CE \perp AB$ and $EF \perp BD$. Prove that $DB \perp (CEF)$.

Grading scale:		1 point *ex officio*
	I.	1) 0,50p 2) 1p 3) 1,50p
	II.	1) 1,25p 2) 1,25p 3) 0,50p
	III.	1) 0,75p 2) 1,25p 3) 1p

Test no. 26

I. 1. Show that:

$18 \mid 27^{n+1} \cdot 5^{4n+1} - 3^{3n+2} \cdot 5^{4n+2}$, $(\forall) n \in \mathbf{N}$.

2. Calculate the sum:

$$\frac{1}{3 \cdot 10} + \frac{1}{10 \cdot 17} + \ldots + \frac{1}{830 \cdot 837}$$

3. If a and b are rational numbers, inversely proportional to c and d then:

$$\frac{cd}{ab} = \left(\frac{c+d}{a+b}\right)^2.$$

II. 1. For what values of m and n does the equation $x^2 + mx + n = 0$ admit the solutions 4 and -3?

2. Simplify:

$$\frac{X^2 - Y^2 - 2YZ - Z^2}{Y^2 - X^2 - 2XZ - Z^2}.$$

3. Is it true that:

$(-2) \cdot (-2)^{99} = 2^{100}$?

III. 1. A trapezoid $ABCD(AB||CD)$ is circumscribed to a circle having the center O. Show that: $CO \perp OB$ and $AO \perp DO$.

2. $ABCDA'B'C'D'$ is a rectangular parallelepiped with $AB = 9\ cm$, $AD = 15\ cm$ and $AA' - 20\ cm$. Determine the distance from B' to the diagonal AD'.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1,50p	3) 1,50p
	II.	1) 1p	2) 1p	3) 1p
	III.	1) 1p	2) 1p	

Test no. 27

I. 1. Determine $x, y, z \in \mathbb{N}$ that verify the equalities:

$$\frac{x}{8} = \frac{y}{6} = \frac{10}{z}.$$

2. The arithmetic mean of two numbers is 15 and their geometric mean is $10\sqrt{2}$. Determine the harmonic mean of the two numbers.

3. Show that the remainder of the division of a perfect square by 16 is also a perfect square.

II. 1. Solve the system:

$$\begin{cases} (m-1)x + 3y = 1 \\ x + (m+1)y = -1 \end{cases}, \text{ where } m \in \mathbf{R}.$$

2. Determine the rational numbers x and y that verify the relation:

$$x\left(\sqrt{2}+3\right) + y\left(3-\sqrt{2}\right) = 0$$

3. Simplify the fractions:

a) $$\frac{x^4+1}{x^2+x\sqrt{2}+1}$$

b) $$\frac{x^8-9}{\left(x^2-\sqrt{3}\right)\left(x^4+3\right)}.$$

III. 1. Having the drawing of a $13°$ angle, explain how we can obtain a $1°$ angle using only a ruler and a compass.

2. In ΔABC we have the medians AA', $A' \in (BC)$and BB', $B' \in (AC)$ and $AA' = 7{,}5\ cm$, $BB' = 6\ cm$ and $BC = 5\ cm$. Determine the perimeter of ΔABC and the length of the height from A of ΔABC.

3. If $ABCD$ is a regular tetrahedron and M is a point in the interior of the tetrahedron so that $MA = MB = MC = 3a$ and $MD = a\sqrt{3}$, determine AB.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 0,50p	3) 1,50p
	II.	1) 1,25p	2) 0,75p	3) 1p
	III.	1) 1p	2) 1p	3) 1p

Test no. 28

I. 1. A three-digit number has the sum of the digits equaling 7. Show that if the number is divided by 7, then the digit of the decimals is equal to the digit of unities.

2. What is the truth value of the proposition:

$$\{x \mid x \in \mathbb{Z}, |x - 1| = 5\} = \{-4, 6\}$$

3. The numbers a, b and c are distinct. Determine the real solutions of the equation:

$$(x-a)^2 + (x-b)^2 + (x-c)^2 = 0.$$

II. 1. Simplify:

$$\frac{(x^2-1)(x^2-3)+1}{(x^2-1)(x^2-4)+2}.$$

2. Let a, b, c, d be real positive numbers with $abcd = 1$. Then the following inequality takes place:

$$a^2 + b^2 + c^2 + d^2 + ab + ac + ad + cd + bc + bd \geq 10.$$

3. Calculate:

$$\sqrt{8} \cdot |-2| \cdot (-2)^2 \cdot \frac{1}{4\sqrt{2}}$$

III. 1. Let ABC be a random triangle with $AB > AC$. If CF and BE are the medians that correspond to the sides AB and AC respectively, $E \in (AC)$, $F \in (AB)$, prove that $BE > CF$.

2. $ABCA'B'C'$ is considered a regular triangular prism where $AB = AA' = a$. Calculate:

a) The distance from the top A' to the edge BC;

b) The distance from the top B' to the middle line of the base that is parallel to AB.

Grading scale: 1 point *ex officio*

I.	1) 1,50p	2) 1p	3) 0,50p
II.	1) 1p	2) 1,50p	3) 0,50p
III.	1) 1p	2) 2p	

Test no. 29

I. 1. Solve in $\mathbb{N}$ the equation: $xy - x + y = 18$.

2. Determine ab knowing that:

$$\frac{a}{1+\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{128}} = \frac{2^7}{b}.$$

3. State the value of truth of the proposition $\{x/x \in \mathbb{Z}, |x| < -10\}$ has one element.

II. 1. Show that for any $k \in \mathbb{N}$, k uneven, the number $N = 10^{3k} + 2^{3k}$ is divided by 7.

2. Decompose in factors:

$2a^2 - ab + 4ac - b^2 - bc + 2c^2$.

III. 1. Let ABC be a right triangle in A with $\sphericalangle B > \sphericalangle C$. We note with D the middle of the hypotenuse BC. The perpendicular in D on BC intersects the cathetus AC in E.

a) Prove that $m(\sphericalangle ADE) = m(\sphericalangle B) - m(\sphericalangle C)$

b) Determine the measures of the angles of the triangle ABC, knowing that $(AE) \equiv (ED)$.

2. A triangular pyramid with congruent lateral edges has a right triangle as base. Show that the plane of one of the lateral faces is perpendicular to the plane of the base.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1,50p	3) 0,50p
	II.	1) 1,50p	2) 1,50p	
	III.	1) 2p	2) 1p	

Test no. 30

I. 1. Calculate:

$$1998+\frac{3}{2}+\frac{5}{4}+\ldots+\frac{101}{100}-\left(\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{100}\right)$$

2. Determine the natural numbers n, for which:

$$\frac{3}{7}<\frac{n+1}{42}\leq\frac{1}{2}$$

3. Show that, if $P(1+x)=P(1-x), (\forall)x\in\mathbb{R}$, then the polynomial $P(X)=X^2+aX+1$ is the square of a binomial.

II. 1. Let there be a,b,c integer, uneven numbers. Show that there doesn't exist any $x\in\mathbb{Q}$ that verifies the relation:

$$ax^2+bx+c=0.$$

2. Let there be a,b,c three integer, positive not equal to zero numbers, so that $ab<c$. Show that $a+b\leq c$.

III. 1. In the triangle ABC $(AB\equiv AC)$ we note with M,N,P the middles of the sides AB, BC and AC respectively. If $\sphericalangle ABC=30°$ and $AM=4\ cm$, calculate the perimeter of the quadrilateral $AMNP$.

2. Let $ABCD$ be a random tetrahedron and M a point on the edge of AD. Show that the rapport AM/MD is equal to the rapport of the areas of the triangles ABC and DBC, if and only if M is equally distanced from the faces ABC and DBC.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p
II.	1) 1,50p	2) 1,50p	
III.	1) 1p	2) 2p	

Test no. 31

I. 1. Calculate:

$$\left(-\frac{1}{2}\right)^{1988} \cdot 2^{-1988} \cdot \left(\frac{1}{2}\right)^{-1988} \cdot (-2)^{1988}.$$

2. Determine three numbers, knowing that they are directly proportional to $2, 3, 4$ and that the difference between the biggest and the smallest one is 5.

3. Solve the equation:

$$\frac{|x-1|}{x-1} = 1.$$

II. 1. Let ABC be a random triangle and O the center of the circle circumscribed to the triangle. Let $AD \perp BC$, $D \in BC$. Calculate the measure of the angle $\sphericalangle DAO$ in relation to the angles of the triangle ABC.

2. Prove that in a trapezoid, the intersection point of the un-parallel sides with the middles of the bases are three collinear points.

III. 1. Let α_1 and $\alpha_2$be two perpendicular planes and d their intersection straight line. Let $A \in \alpha_1$ and $B \in d$ so that $AB \perp d$. Let $a \subset \alpha_2$ a random straight line. What can be said about the straight lines AB and a?

2. Let there be the equilateral triangle ABC, with the side a and $M \in (AC)$ so that $[AM] \equiv [BM]$ and $[AN] \equiv [NC]$. The triangle is bent along the line of MN until $(AMN) \perp (BMC)$. Calculate the measure of the dihedral angle of the planes (ABC) and (BMC).

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p
II.	1) 1,50p	2) 2p	
III.	1) 1p	2) 1,50p	

Test no. 32

I. 1. Calculate:

$$\frac{3+6+9+...+1995+1998}{2+4+6+...+1330+1332}.$$

2. Determine the natural numbers n for which:

$$\frac{2n-5}{n+1}<1.$$

3. We have four faucets. The first faucet fills a basin in one hour, the second in two hours, the third in three hours and the forth in four hours. In how many hours do the four faucets combined fill the basin?

II. 1. Show that the triangle with sides of the lengths a, b, c, where $2(a-b)(a+c)=(a-b+c)^2$, is a right-angled triangle.

2. Let there be the random triangle ABC and A', B', C' tangency points of the inscribed circle with the sides BC, CA and AB respectively. Calculate the lengths of the segments AC', BA' and CB'.

III. 1. On the plane of the square $ABCD$ $(AB=a)$ we raise the perpendicular $BB'=a$. Let N be the middle of $B'D$, M the middle of AB and P the middle of BC. Calculate the area of the triangle MNP and show that $B'D \perp (MNP)$.

2. The height of a right circular cone is 15 and the sum of the generating line and the radius is 25. Calculate the lateral area and the volume of the cone.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1,50p
II.	1) 1p	2) 1,50p	
III.	1) 2p	2) 1p	

Test no. 33

I. 1. Calculate:

$$\frac{\left(2^3\cdot 3^4\cdot 5^4\right):\left(3^3\cdot 5^3\cdot 2^5\right)-\frac{3}{4}+2,(3)+\frac{22}{33}}{\left(489\cdot 7+311\cdot 7-777\cdot 7-23\cdot 7\right)+2}.$$

2. The following equation is given:

$$\frac{3x+1}{x-5}=m,\ x\in\mathbf{R}\setminus\{5\}.$$

Solve and determine $m \in \mathbb{Z}$ so that the solution is an integer number.

3. The sum of three natural consecutive numbers is 1209. Determine these three numbers.

II. 1. Two adjacent angles have perpendicular bisectors. Determine the measure of each angle knowing that one measures five times as much as the other.

2. Let $ABCD$ be a rhombus and E a random point on one of the diagonals. If M, N, P, Q are the feet of the perpendiculars from E on the straight lines AB, BC, CD and DA respectively, show that the quadrilateral $MNPQ$ is unwritable.

III. 1. A straight line a contained in the plane α is parallel to a different plane β. Determine the truth value of the proposition: "The plane α is parallel to plane β."

2. Let A, B, C, D be four points that are not coplanar. Through a point M situated on the segment AB we trace a parallel plane to AC and BD. This plane intersects BC in Q, CD in P and AD in N. Prove that $MNPQ$ is a parallelogram.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1,50p	3) 1p
II.	1) 1,50p	2) 2p	
III.	1) 1p	2) 1p	

Test no. 34

I. 1. Determine the value of x from the equality:

$$10 \cdot \{x - 10 \cdot [362 + 10 \cdot (24 + 24 : 4)]\} = 100$$

2. In a road trip, Ioana spends $3/4$ of her savings, which represents: $240\ 000$ lei. What sum did Ioana have?

3. Calculate:

$$E(x) = x^3 + (a+1)x^2 + (a+b)x + b + 1$$

knowing that:

$$x^2+ax+b=0.$$

II. 1. Let f and g be two linear functions. Determine these functions knowing that:

$$2f(x+1) + g(x-1) = 2x + 14 \text{ and } f(x+1) - 2g(x-1) = 6x + 2$$

for any $x \in \mathbb{R}$.

2. Determine the measures of a triangle's angles, knowing that they are directly proportional to the numbers: $1/2, 1/3$ and $1/6$.

III. 1. Let $ABCD$ be a rectangle. In D we raise the perpendicular on the regular plane on which we take point M. Let P and Q be the projections of the points A and C, respectively, on MB. If $AB = 6\ cm$, $BC = 4cm$ and $PQ = 3cm$, calculate: MB.

2. Given a cube $ABCDA'B'C'D'$ with the edge a:

a) Calculate the distance from point the A to the diagonal BD' ;

b) Prove that $BD' \perp (AB'C)$

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p
II.	1) 1,50p	2) 1p	
III.	1) 1,50p	2) 2p	

Test no. 35

I. 1. Calculate:

$$2^3 + [(2^3)^7 - (2^7)^3 + 3 \cdot 3^4 - 81] : \{3^2 \cdot [301 - 10 \cdot (24 + 2^{1986} : 2^{1985} \cdot 3)]\}$$

2. In two boxes there are a total of 120 crayons. Determine the number of crayons from each box knowing that if we take 15 crayons from the first box and we put them in the second box, then the two boxes will contain the same number of crayons.

3. Show that:

$$\sqrt{x^2 - 2x + 1} + \sqrt{y^2 - 4y + 4} + \sqrt{z^2 - 6z + 9} = 0$$

if and only if $x = 1$, $y = 2$ and $z = 3$.

II. 1. Calculate the area of the isosceles trapezoid $ABCD$where $AB||CD$ knowing that $AB = 26$, $DC = 16$ and $AC \perp CB$.

2. Prove that the sum of the distances of a triangle to a straight line exterior to the triangle is equal to the sum of the distances of the middles of the triangle's sides to the same straight line.

III. 1. Given a regular quadrilateral prism $ABCDA'B'C'D'$. The side of the square's base has $2cm$, and the diagonal AC' has $4cm$.

a) Prove that the triangle ACC' is isosceles.

b) Calculate the total area of the prism.

2. A cube $ABCDA'B'C'D'$ is inscribed in a sphere with the radius of a cm. Determine the volume of the cube.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1p	3) 1,50p
	II.	1) 1p	2) 1,50p	
	III.	1) 1p	2) 2p	

Test no. 36

I. 1. The price of an object was 2500 lei at the beginning. After two consecutive price reductions with the same number of percentages, the price dropped to 2025 lei. With what per cent did the price of the object went down with each reduction?

2. Calculate the sum: $S = 1 + 3 + 5 + \cdots + 1995 + 1997 + 1999$.

3. Show that $x^6 + x^4 - 2x^3 - 2x^2 + 2 \geq 0$, indifferent of what $x \in \mathbb{R}$.

II. 1. Simplify the expression:

$$E(a,b) = \frac{a+b-\sqrt{a^2-b^2}}{a+b+\sqrt{a^2-b^2}} + \frac{a+b+\sqrt{a^2-b^2}}{a+b-\sqrt{a^2-b^2}}$$

2. Show that the parallel EF traced to the bases of trapezoid $ABCD$ through the point M of intersection of the diagonals is divided in two equal parts in relation to this point.

III. 1. In the body of a cone $R = 13cm$, $r = 5cm$, and the generating line is inclined on the base with an angle of $45°$. Determine the lateral area, the total area and the volume of the cone's body.

2. Determine the volume of a regular tetrahedron with the edge a.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1,50p	3) 1p
	II.	1) 1p	2) 1,50p	
	III.	1) 1p	2) 2p	

Test no. 37

I. 1. Solve:

$$\frac{30\cdot 4\frac{1}{4}-11\frac{1}{5}:9\frac{1}{3}}{14:2\frac{2}{9}+8\frac{2}{5}\cdot 14\frac{2}{7}}:\frac{1:6+12:5}{2\frac{1}{2}\cdot 15-4\frac{13}{15}\cdot 7\frac{3}{5}}.$$

2. Solve the equation:

$$\frac{1}{3}\left\{\frac{1}{3}\left[\frac{1}{3}\left(\frac{1}{3}x+2\right)+2\right]+2\right\}-1=0.$$

3. Let there be three integer numbers so that each represents the arithmetic mean of the other two. Show that the three numbers are equal.

II. 1. Determine the 1st grade function (linear) whose graphic passes through the points $A(3,1)$ and $B(1,3)$.

2. Compare the numbers: $\sqrt{7}-3$ and $\sqrt{11}-4$.

III. 1. In the isosceles triangle ABC ($[AB]\equiv[AC]$) we take the segments $[BM]\equiv[CM]$ (M between A and B, N on the extension of $[AC]$). Prove that the straight line BC passes through the middle of $[MN]$.

2. In the straight parallelepiped $ABCDA'B'C'D'$ we know that $AA'=a$, $AB=b$ and $AD=c$. The diagonal of the parallelepiped forms with the three faces where it is traced angles with the measures α,β,γ, Prove that:

$$\sin^2\alpha+\sin^2\beta+\sin^2\gamma=1.$$

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1p	3) 1,50p
	II.	1) 1p	2) 1p	
	III.	1) 1,50p	2) 2p	

Test no. 38

I. 1. Let:

$$A = \{ x \mid x \in \mathbf{R}, -4 < x < 8\} \text{ and } B = \{x \mid x \in \mathbf{R}, -5 < x \leq 3\}.$$

a) Write, using intervals, the sets A and B.

b) Calculate:

$$A \cup B,\ A \cap B,\ A \setminus B,\ B \setminus A \text{ and } (B \setminus A) \cap \mathbf{Z},$$

where $\mathbb{Z}$ represents the set of the integer numbers.

2. Let there be the functions:

$$f: \mathbf{R} \to \mathbf{R},\ f(x) = 2x + 1 \text{ and } g: \mathbf{R} \to \mathbf{R},\ g(x) = x + 3.$$

Determine the coordinates of the intersection point of the graphics of the two functions.

3. Solve the equation: $mx - 3 = 3m - x$, where m is a real parameter.

II. 1. Bring to a simpler form the equation:

$$E(x) = \left\{ \left[\frac{(8 + x^3)(x - 2)}{x^4 - 2x^3 + 4x^2} \right]^2 + \frac{2x^2 - 8}{x^2} + 1 \right\} : \frac{2x^2 - 4}{x^4}$$

2. Show that the middles of a random triangle's sides and the foot of one of the triangle's heights are the tops of an isosceles triangle.

III. 1. A pyramid has as base an equilateral triangle with the side a. Determine the volume of this pyramid knowing that its height is twice as big as the height of the base triangle.

2. Let there be the tetrahedron $VABC$, where $VB \equiv VC$. The edges VA, AB and AC are divided in three equal parts and are noted with M, N, P the points closest to A. Show that the triangle MNP is the ninth part from the area of the face VBC.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1,50p	3) 1,50p
II.	1) 1p	2) 1p	
III.	1) 1p	2) 2p	

Test no. 39

I. 1. Calculate:

$$-\sqrt{2}-\sqrt{(-2)^2}+|1-\sqrt{2}|+2|\sqrt{8}-3|+|-\sqrt{2}|^2.$$

2. Determine x, y, z knowing that:

$$x-y=\frac{y+z}{4}=\frac{z}{2} \text{ and } x+2z=14.$$

3. Show that:

$$(2-\sqrt{3})^2=7-4\sqrt{3}$$

and determine all the integer numbers x so that:

$$x^2<7-4\sqrt{3}.$$

II. Bring to a simpler form the expression:

$$E(x)=\left[\left(\frac{1-8x^3}{4x^4+2x^3+x^2}\cdot\frac{1+2x}{x^2}\right)^3+\frac{12x^2-48x^4}{x^{12}}\right]\cdot\frac{x^{12}}{1+4x^2+16x^4}.$$

2. Let there be the quadrilateral $ABCD$, where $m(\sphericalangle A)=m(\sphericalangle C)$. The bisector of the angle B cuts the sides DC and AD in the points E and F. Prove that the triangle DEF is isosceles.

III. 1. On the plane of the square $ABCD$with the side a we raise the perpendiculars $AM=\frac{a\sqrt{6}}{6}$ and $CP=\frac{a\sqrt{6}}{2}$. We unite P with M. Prove that the planes of the triangles MBD and PBD are perpendicular.

2. Given a right circular cone with the diameter of the base of 12 cm and the height equal to $2/3$ of the diameter, determine the lateral area, the total area and the volume of the cone.

Grading scale:	1 point *ex officio*		
I.	1) 1p	2) 1 p	3) 1,50p
II.	1) 1,50p	2) 1,50p	
III.	1) 1,50p	2) 1p	

Test no. 40

I. 1. Calculate:

a) $(-1)^{n+1} + (-1)^{n+2} + (-1)^{n+3}, n \in \mathbf{N}$

b) $3^n + (-3)^n, n \in \mathbf{N}$

2. At a show, 320 tickets are distributed, some costing 300 lei per ticket, others costing 400 lei a ticket. How many tickets from each category should be sold in order to obtain $100\ 000$ lei?

3. Solve the following system of inequations, in relation to the values of the real parameter m:

$$\begin{cases} 5x + 3 > -7 \\ mx \geq 0 \end{cases}$$

II. 1. Simplify the fraction:

$$\frac{2x^3 - 6x^2 + 6x - 4}{2x^2 - 2x + 2}.$$

2. Let there be the rectangular triangle $ABC(\sphericalangle = 90°)$ where we know the lengths of the catheti $AB = c$ si $AC = b$. Determine the areas of the triangles ABD and BDC obtained by tracing the bisector of the angle B $(D \in AC)$.

III. 1. We consider an angle $\sphericalangle AOB$ measuring $120°$. On the bisector of this angle we take a point P so that $OP = 6\ cm$. In P we raise a perpendicular on the plane of the angle on which we consider the point Q so that $PQ = 4\ cm$. Determine the distances from Q to the triangles' sides.

2. Given a regular tetrahedron with the edge a, determine the height, the apothem and the value of the cosine of the dihedral angle of two faces of the tetrahedron.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1,50 p	3) 1,50p
II.	1) 1p	2) 1,50p	
III.	1) 1p	2) 1,50p	

Test no. 41

I. 1. Determine $\frac{a}{b}$ knowing that:

$$\frac{7a-2b}{10a+8b}=\frac{4}{15}$$

2. Determine $n \in \mathbb{N}$ so that the following equality takes place:

$$3^n = 2^n.$$

3. Solve in $\mathbb{Z}$ the inequation:

$$\frac{7y-5}{y+4} \leq 3.$$

II. 1. Determine a and b so that by dividing the polynomial:

$$P(X) = X^4+aX^3-X^2+bX+3$$

to $X-2$ the remainder will be 5 and by dividing it to $X+3$ the remainder will be 1.

2. Solve the following system, where a, b are real parameters not equal to zero:

$$\begin{cases} \frac{x}{a}+\frac{y}{b}=c \\ \frac{x}{b}-\frac{y}{a}=0 \end{cases}$$

III. 1. In the parallelogram $ABCD$, $AB = 2.BC$ and $m(\sphericalangle A) = 60°$. Let M be the middle of the side CD. Let $\{E\} = AM \cap BD$.

a) Prove that the triangle BCM is equilateral.

b) Prove that $ABMD$ is a trapezoid inscribed in the circle with the diameter AB.

2. A circular sector of $120°$ with the area of $12\pi\ cm^2$ is wrapped in such a way as to form the lateral surface of a cone. It is required to:

a) Determine the total area of the cone and its volume.

b) determine the area of the sphere inscribed in the cone.

Grading scale:	1 point *ex officio*		
I.	1) 1p	2) 1 p	3) 1,50p
II.	1) 1p	2) 1,50p	
III.	1) 1,50p	2) 1,50p	

Test no. 42

I. 1. Solve the equation:

$$|x-2|+|2x-4|+|3x-6|=0.$$

2. Determine the geometric mean of the numbers 7 and 10 with two exact decimals.

3. Three numbers are inversely proportional to the numbers $4, 9$ and 16. Determine what percent does the second number represent from the arithmetic mean of the other two.

II. 1. Determine the quotient and the remainder of the division of the polynomial:

$$4X^4-3X^3+2X^2-X+1$$

to X^2+1.

2. Find the smallest natural number that divided consecutively to $3, 5, 7$ will always give the same remainder 1 and the quotient different from 0.

III. 1. Given a cube with the length of the diagonal a, determine its volume.

2. In a pyramid having the base a rectangular trapezoid $ABCD$ we have:

$$\sphericalangle A=90^\circ,\ \sphericalangle D=90^\circ,\ AB=36\text{ cm},\ AD=15\text{ cm}.$$

The big diagonal BD is divided by the small diagonal AC in the rapport: $\frac{3}{10}$. The height of the pyramid is $VO=40\ cm$, O representing the intersection point of the diagonals of the base. Determine the perimeter of the triangle VOD and the volume of the pyramid.

Grading scale:		1 point *ex officio*		
	I.	1) 1,50p	2) 1 p	3) 1,50p
	II.	1) 0,50p	2) 1,50p	
	III.	1) 1p	2) 2p	

Test no. 43

I. 1. Calculate the value of the expression $E = 2a - 2b + c - 2d$ for:

$$a = \frac{1}{\sqrt{5}-1};\ b = \frac{1}{\sqrt{3}+2};\ c = \frac{1}{\sqrt{2}+\sqrt{3}};\ d = \frac{1}{\sqrt{5}+1}.$$

2. What hour of the day is it if there is $1/7$ of the day left from what has already passed? (The day has 24 hours and it begins at 12 midnight).

3. Let $a, b \in \mathbb{R}$, $a < b$. Show that no matter what $t_1, t_2 \in (0,1)$ is, with the property $t_1 + \ t_2 = 1$, then $t_1.a + t_2.b \in (a, b)$.

II. 1. Prove that the triangle the sides of which verify the equality:

$$a^2 + b^2 + c^2 = ab + bc + ca$$

is equilateral.

2. Prove that the polynomial:

$$nX^n - (n-1)X^{n-1} + (n-2)X^{n-2} - n + 1$$

is divisible by $X - 1, n \in \mathbb{N}^*$.

III. The body of a right circular cone has the radiuses of the bases 9 and 15, and the generating line 10. Determine the lateral area and the volume of the cone that the body of the cone comes from.

Grading scale:	1 point *ex officio*		
I.	1) 1 p	2) 1,50 p	3) 2p
II.	1) 2p	2) 1p	
III.	1,50p		

Test no. 44

I. 1. Solve:

a) $(-2)^3+10\cdot[(-1)^{1000}+(-1)^{99}+2\cdot(-2+2\cdot 3)]$.

b) $(-2)^{101}:2^{99}-10\cdot\{-2-2\cdot[(-4)^5:4^4-2]\}$.

2. The sum of two prime numbers is 39. Determine the numbers.

3. Simplify the fraction:

$$\frac{x^2-4x+4}{2x^3+x^2-3x-14}.$$

II. 1. Determine the relation between the natural numbers m and n so that the following polynomials will divide by X^2-1:

$P(X)=m\cdot X^{2n+1}+nX+m+n$

2. Solve the equation $2mx=x+4m-2$ making a discussion around the values of the real parameter m.

III. 1. The angles of the triangle ABC are directly proportional to $14, 12$ and 10. The bisector $\sphericalangle ABC, [BN](N\in AC)$ intersects the segment $[AM]$, the symmetrical of $[AB]$ in relation to the height $[AD]$ in the point P. Determine the measures of the quadrilateral $PMCN$.

2. Determine the volume of a cube inscribed in a sphere with the radius r.

Grading scale: 1 point *ex officio*

I.	1) a) 0,50p b) 0,50 p	2) 1,50p	3) 1p
II.	1) 1,50p	2) 1,50p	
III.	1) 1p	2) 1,50p	

Test no. 45

I. 1. Solve:

a) $$\frac{\left[\left(-\frac{2}{3}\right)^2+\left(\frac{1}{-3}\right)^2-\left(-\frac{4}{9}\right)+(-2)^2\right]^3}{\left(-\frac{5}{6}\right)^3-(-1)^3-1}+\frac{\frac{3}{4}+\frac{1}{2}}{\frac{5}{2}}$$

b) $$\frac{\left[\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)^{200}+\frac{1}{2^{201}}\right]\cdot\left[\left(-\frac{1}{5}\right)^{99}:\frac{1}{5^{98}}\right]}{\left[\left(-\frac{1}{2}\right)^{100}:\frac{1}{2^{98}}\right]\cdot\left[\left(\frac{1}{5}\right)^{60}:\left(\frac{1}{5}\right)^{30}\right]}$$

2. Decompose in factors the polynomial:

$P(X)=X^4+64.$

3. The father is 28 years old today and the son is 8. In how many years the age of the father will be three times bigger than the son's?

II. 1. Solve the system:

$$\begin{cases}\sqrt{2}x+3y=1\\ 3\sqrt{2}x+7y=1\end{cases}$$

2. Let $a, b \in \mathbb{R}$. Prove the inequality:

$\sqrt{(1+a)(1+b)}\geq 1+\sqrt{ab}$.

III. 1. Let ABC be a random triangle and AD the bisector of the angle $\sphericalangle BAC$ $(D \in BC)$. On $[BA$ let there be the point E so that $A \in (BE)$ abd $[AE] \equiv [AC]$. Prove $AD||EC$.

2. Let $ABCD$ and $DCEF$ be two squares situated in perpendicular planes. We note with M, N, P the middles of the segments EF, AD, AB.

a) Show that the triangle MNP is a right triangle.

b) Calculate the tangent of the dihedral angle formed by the planes (MNP) and (ABC).

Grading scale: 1 point *ex officio*

I.	1) a) 0,50p	b) 0,50p	2) 1,50p	3) 1p
II.	1) 1p		2) 1,50p	
III.	1) 1p		2) 2p	

Test no. 46

I. 1. Calculate the arithmetic mean of the numbers a and b, where:

$$a = \frac{2\sqrt{1+\frac{1}{3}} : \sqrt{1-\frac{1}{5}}}{5\sqrt{1+\frac{1}{3}} \cdot \sqrt{1-\frac{1}{5}}}, \quad b = \left[\left(\frac{2}{3}\right)^4 + \left(\frac{5}{6}\right)^2 \cdot \left(\frac{3}{4}\right)^3\right] \cdot \frac{9}{2437}$$

2. Divide the number 49 in three equal parts so that if we add to the first part $1/3$ from the sum of the other two, to the second one $1/4$ from the sum of the other two and to the third $1/5$ from the sum of the other two, we obtain equal numbers.

3. Solve the system of inequations:

$$\begin{cases} \dfrac{1+x}{\frac{1}{2}} < 1 + \dfrac{x}{3} \\ \dfrac{3x+2}{2} \leq \dfrac{2x+1}{4} + \dfrac{3}{2} \end{cases}$$

II. 1. Determine the area of the triangle formed by the coordinates' axis and the graphic of the function $f: \mathbb{R} \to \mathbb{R}, f(x) = x - 5$.

2. Prove that, irrespective of what the real, non-equal to zero number x is:

$$\left|x + \frac{1}{x}\right| \geq 2.$$

3. Decompose in factors the polynomial:

$$X^4 + X^2 + 4.$$

III. 1. Let $ABCD$be a random triangle and D a point situated on the extension of AC so that $\sphericalangle BAC \equiv \sphericalangle CBD$. Prove that BD is a proportional mean between AD and CD.

2. A straight circular cylinder has the radius of the base of $1{,}5\ cm$ and the area of the axial section of $12\ cm$.

Determine:

a) The diagonal of the axial section.

b) The lateral area, the total area and the volume of the cylinder.

c) The diagonal of the deployment of the lateral face of the cylinder.

Grading scale: 1 point *ex officio*

I.	1) 1 p	2) 1 p	3) 1p
II.	1) 1,50p	2) 1,50p	3) 1p
III.	1) 1p	2) 1p	

Test no. 47

I. 1. Determine x from the equality:

$$\left\{\left[4\frac{3}{5}-\frac{6\frac{1}{14}}{13\frac{1}{3}:\left(5\frac{1}{3}-x\right)-3\frac{4}{7}}\right]\cdot\frac{9}{14}+1\frac{1}{3}\right\}:4\frac{1}{4}=\frac{2}{3}.$$

2. Show that the sum of the squares of three consecutive numbers raised by 1 is a number divisible by 3.

3. Determine all the pairs of natural numbers that have the geometric mean (proportional) equal to 11.

II. 1. Trace the graphic of the function:

$$f:\mathbf{R}\to\mathbf{R},\ f(x)=\begin{cases} x+1, & \text{for } x\in(-\infty,-1] \\ 1, & \text{for } x\in(-1,1) \\ x, & \text{for } x\in[1,\infty) \end{cases}$$

2. Show that if $a,b,c\in\mathbb{R}$ so that $(a+b+c)^2=3(a^2+b^2+c^2)$, then $a=b=c$.

III. 1. A triangle ABC is given, where $AB=AC=6cm$ and $\sphericalangle A=120°$. Let O be the center of the circle circumscribed to this triangle and D the point diametrically opposed to A.

a) Prove that the triangle BCD is equilateral.

b) Let d be a perpendicular straight line in O on the plane of the triangle BCD and M a random point situated on d. Prove that $MB=MC=MD$

c) Show that $MB\perp CD$.

d) Calculate the volume of the tetrahedron $MBCD$ knowing that $MB=6\sqrt{3}cm$.

Grading scale: 1 point *ex officio*

I.	1) 1 p	2) 1,50 p	3) 1,50p
II.	1) 1,50p	2) 1,50p	
III.	1) 2p		

Test no. 48

I. 1. Compare the numbers: a) $(0,1)^{10}$ and $(0,1)^{11}$; b) $(0,1)^5$ and $(0,01)^5$.

2. The difference between two natural numbers is 46. Determine the two numbers knowing that by dividing the biggest one to the smallest one we obtain the quotient 3 and the remainder 2.

3. Determine $x \in \mathbb{R}$ knowing that:

$$5-\frac{x-5}{5}<x<6-\frac{x-6}{6}.$$

II. 1. The measures of an quadrilateral's angles are directly proportional to $3, 6, 12$ and 15. Determine the measures of these angles.

2. The polynomial $P(X) = X^3 + aX^2 + bX + 1$ is divisible by $X^2 - 1$. Determine a and b then decompose in factors the polynomial for the determined a and b.

III. 1. Let D be a random point on the base BC of a triangle ABC. Through B and C we trace parallels to AD. These meet the extensions of AC in M and of AB in N. Prove that $MB \cdot DC = NC \cdot DB$.

2. Let there be the square $ABCD$ and M a point in space so that $MA \perp (ABC)$. Prove there exists a point in space equally distanced from A, B, D and M.

Grading scale:		1 point *ex officio*		
	I.	1) a) 0,50p b) 0,50p	2) 1p	3) 1p
	II.	1) 1p	2) 1,50p	
	III.	1) 1,50p	2) 2p	

Test no. 49

I. 1. Knowing that $a - \frac{1}{a} = 5$, where $a \in \mathbb{R}\backslash\{0\}$, calculate $a^2 + \frac{1}{a^2}$ and $a^3 + \frac{1}{a^3}$.

2. 20 m of fabric cost $140\ 000\ lei$. How much will $8\ m$ of the same fabric cost?

3. Determine x from the proportion $\frac{x}{2} = \frac{\overline{5a5}}{3}$, where a is a digit having the property that the number $\overline{5a5}$ is divisible by 9.

II. 1. Demonstrate that in any triangle the half of the perimeter is bigger than any side.

2. Solve in $\mathbb{Z}$ the system of inequations:

$$\begin{cases} 3x + 7 > 7x - 9 \\ x - 3 > -3x + 1 \end{cases}$$

III. 1. In the triangle ABC we trace a parallel to the median AD that cuts the sides AB, AC in E, F, respectively. Prove that:

$$\frac{AE}{AF} = \frac{AB}{AC}.$$

2. Let there be the plane α and the semi-straight line $OX (O \in \alpha)$ that makes with the plane α a $30°$ angle. A point A situated on the same of the plane α with OX projects on α in B and on OX in C. It is required:

a) To calculate BC if $OA = 17\ cm, OB = 8\ cm, OC = 12\ cm$.

b) Let C' be the projection of C on α and M the middle of OC'. Prove that $MA \perp OC'$.

Grading scale: 1 point *ex officio*

I.	1)1,50 p	2) 1p	3) 1p
II.	1) 1p	2) 1p	
III.	1) 1,50p	2) 2p	

Test no. 50

I. 1. Calculate the sum:

$$S = 2 - 4 + 6 - 8 + \dots + 1994 - 1996 + 1998.$$

2. 10 workers can finish a project in 10 days by working 10 hours a day. In how many days will 5 workers finish the project if they work 5 hours a day?

3. There exists $x, y \in \mathbb{R}$ so that:

$$\sqrt{4x - 5y + 1} + \sqrt{-5x - 4y + 9} = 0\ ?$$

II. 1. Determine $m \in \mathbb{R}$ so that 3 is the solution to the equation:

$$\frac{m}{3} - x = \frac{x}{3} - m.$$

2. The quotients of the division of a polynomial $P(X)$ through $X - a, X - b$ are:

$$X^2 - 4X + 5 \text{ and } X^2 - 6X + 3.$$

Determine a and b and the polynomial knowing that the free term of the polynomial is 1.

III. Let M be a point on the diagonal AC of a random quadrilateral $ABCD$. We trace $MP||AB$ and $MQ||CD$ where $P \in BC$ and $Q \in AD$. Prove that:

$$\frac{MP}{AB} + \frac{MQ}{CD} = 1.$$

2. Let A and B be two points situated in two planes α and β, perpendicular one to the other. We consider AA' and BB' the perpendiculars from A and B on the intersection of the planes. We note $AA' = a$, $BB' = b$, $A'B' = c$. Calculate the length of the segment AB and the tangents of the angles formed by AB with the planes α and β.

Grading scale: 1 point *ex officio*

I.	1)1,50 p	2) 1p	3) 1p
II.	1) 1p	2) 1,50p	
III.	1) 1,50p	2) 1,50p	

Test no. 51

I. 1. Calculate:

$$\frac{1}{1-\frac{1}{1-\frac{1}{1-\frac{1}{2}}}}.$$

2. Knowing that:

$$\frac{3}{a}=\frac{4}{b}=\frac{12}{c}=13$$

prove that:

$$a^2+b^2+c^2=1.$$

3. For what values of $n \in \mathbb{N}$ is the number $\sqrt{1000-25\sqrt{n}}$ a natural number?

II. 1. Solve the system:

$$\begin{cases} \frac{3-x}{2}<\frac{x}{5} \\ 4x<3x+1 \end{cases}$$

2. Let there be the function:

$$f: \mathbf{R} \to \mathbf{R},\ f(x)=2x+3.$$

Find a point situated on the graphic of the function that has equal coordinates.

III. 1. The side of an equilateral triangle measures $8\ cm$. Calculate the radius of a circle whose area is a quarter from the triangle's area.

2. A cone is sectioned by a parallel plane with the base at a distance equal to $1/3$ of the height in relation to the top of the cone. The body of the cone thus obtained has a volume of $52\ cm^2$. Determine the volume of the cone.

Grading scale:		1 point *ex officio*		
	I.	1)1,50p	2) 1p	3) 2p
	II.	1) 1p	2) 1,50p	
	III.	1) 1 p	2) 1,50p	

Test no. 52

I. 1. a) Determine a number knowing that $3/4$ of it is 30.

b) From $45\ kg$ of seawater $500\ g$ of salt are obtained. What quantity of seawater is required in order to obtain $20\ kg$ of salt?

2. The arithmetic mean of three natural numbers is 20. The first number is three times smaller than the second. The arithmetic mean of the second and the third is 25. determine the three numbers.

3. Divide the number 4200 in parts directly proportional to the numbers $5, 9, 12$ and 14.

II. 1. Write the expression $x^6 - 2x^3$ as a difference of squares.

2. Determine the area of a triangle with the sides measuring $6, 8$ and 10.

III. 1. Let M and N be the middles of the non-parallel sides of a trapezoid and P and Q the middles of its' diagonals. Show that the segments MN and PQ have the same middle.

2. Let ABC be a right triangle in A and let D be a point of the perpendicular traced in B on the plane of the triangle. We note with M and N the projections of the point A on the straight lines BC and CD and with P and Q the projections of the point B on the straight lines AD and CD. Prove that the planes (AMN) and (BQP) are parallel and that the triangles AMN and BPQ are also parallel.

Grading scale: 1 point *ex officio*

I.	1) a) 0,50p	b) 0,50p	2) 1p	3) 1p
II.	1) 1p		2) 1p	
III.	1) 2p		2) 2 p	

Test no. 53

I. 1. a) Determine $n \in \mathbb{N}$ so that:

$3n + 1 \mid 2n + 4$.

b) Determine the arithmetical mean of the numbers:

$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

2. Determine the integer values of the real parameter m so that the solution to the following equation is a strictly positive number:

$mx + 7 = m - x$

3. In a classroom there are 32 students. What percent of the girls' number do the boys represent, knowing that the number of girls is 16 times higher than that of the boys?

II. 1. Let there be the linear function:

$f: \mathbf{R} \to \mathbf{R}, f(x) = ax + b, a \neq 0$ and $x_1, x_2 \in \mathbf{R}, x_1 \neq x_2$.

Prove that:

$f(x_1) \neq f(x_2)$ and that $f\left(\frac{x_1+x_2}{2}\right) = \frac{f(x_1)+f(x_2)}{2}$.

2. Simplify the fraction:

$\frac{4x^3+3x^2-4x-3}{x^3+x^2-x-1}$.

III. 1. Prove that in an equilateral triangle the sum of the distances from one point in the interior of the triangle to the sides is equal to the height of the equilateral triangle.

2. Let $ABCD$ be a square with the center O and d_1, d_2 the perpendiculars traced from A and C on the plane of the square. Let $M \in d_1$ and $N \in d_2$ so that $MN \perp OM$. Prove that $MN \perp (BMD)$.

Grading scale: 1 point *ex officio*

I.	1) a) 1 p b) 1 p	2) 1p	3) 1p
II.	1) 1 p	2) 1p	
III.	1) 1,50 p	2) 1,50 p	

Test no. 54

I. 1. Compare the numbers:

$a = (27^{31} - 2 \cdot 9^{46} + 4^{102} : 2^{203} - 3^{92})^{213}$ and $b = (-2)^{142}$.

2. For what values of x does the equality $|x| = x + 0$ take place?

3. Does the equation:

$$(x - 1)^2 = x^2 + 2x - 3$$

and the inequation:

$$x^{1998} - \frac{1}{2} > 0$$

have common solutions?

II. 1. Let there be the linear function:

$$f: \mathbf{R} \to \mathbf{R},\ f(x) = ax + b,\ a \neq 0.$$

Prove that, irrespective of what real x is, :

$$f(x - 1) + 2f(x + 2) = 3f(x + 1).$$

2. Let there be the polynomials:

$P(X)=X^8 - X^5 - X^3 + 1$ and $Q(X)=X^4 + X^3 + X^2 + X + 1$.

Prove that the polynomial $Q(X)$ divides the polynomial $P(X)$.

III. 1. Prove that in a right triangle, the median and the height traced from the top of the right angle form between them an angle equal to the difference of the acute angles of the triangle.

2. Let there be the trapezoid $ABCD$ with $AB||CD, AD = AB + DC$. We consider $E \in BC$ so that $CE = BE$. In the point D we trace the perpendicular DF on the plane of the trapezoid so that $DF = AE$. Prove that $FE = AD$ and $FE \perp AE$.

Grading scale:		1 point *ex officio*		
	I.	1) 1 p	2) 1p	3) 1p
	II.	1) 1 p	2) 1p	
	III.	1) 2 p	2) 2 p	

Test no. 55

I. 1. Two numbers have a rapport of $1/4$ and their geometric mean is 15. Determine the numbers.

2. Calculate:

$$\left|\sqrt{5}-3\right|-\left|\sqrt{7}-5\right|+\left|\sqrt{11}-\sqrt{7}\right|.$$

3. Compare the numbers:

$\sqrt{13}-7$ and $\sqrt{15}-8$.

II. 1. One person deposits his savings at the C.E.C with a rate of 10% per year. What sum did the person deposit if after two year he has $121\ 000\ lei$?

2. Let there be the function:

$f\colon \mathbf{R} \to \mathbf{R},\ f(x) = mx + 4.$

Determine $m \in \mathbb{R}$, knowing that the point $A(3{,}19)$ belongs to the graphic of the function.

III. 1. Determine the sides of a right triangle, knowing that the difference of the catheti is $1$and the length of the hypotenuse is 5.

2. The lateral area of the body if a right circular cone is equal to 225π, the generating line 25 and the height 24. Calculate the volume of the body of the cone.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1,50p
II.	1) 1,50p	2) 1p	
III.	1) 1p	2) 2p	

Test no. 56

I. 1. Effectuate:

$$\left[\left(1:\frac{\frac{x^4+1}{x^2-x\sqrt{2}+1}-x\sqrt{2}-2}{x^6-1}\right)+x^6\right]:\left(x^4+1\right).$$

2. Three workers can finish a project in 4 days. The first, working alone, can finish it in 10 days and the second in 12 days. In how many days can the third one finish it?

3. Determine $m \in \mathbb{R}$ so that the following equation does not admit real solutions:

$$m^2x + 3 = 9x + m$$

II. 1. Solve the following inequation, where m is a random real parameter:

$$mx + 1 \leq x + m$$

2. Given the following polynomials, where $m, n \in \mathbb{R}$, determine m and n so that Q divides P:

$$P(X) = X^3 - 3X^2 + mX - 3 \text{ and } Q(X) = X^2 - nX + 1$$

III. 1. Let H be the intersection point of the heights traced on the sides AC and AB of the triangle ABC and M and N the intersections of the bisector of the angle A with these heights. Prove that the triangle HMN is an isosceles.

2. The side of a regular hexagonal pyramid measures $6\ cm$, and the lateral edge forms with the plane of the base a $45°$ angle. Determine the volume of the pyramid.

Grading scale:	1 point *ex officio*		
I.	1) 1p	2) 1,50p	3) 1,50p
II.	1) 1,50p	2) 1,50p	
III.	1) 1p	2) 1p	

Test no. 57

I. 1. Calculate:

$(a^{10} + a^{11} + ... + a^{20}) : (a^0 + a^1 + ... + a^{10})$.

2. Determine x from the equality:

$$\left\{\left[\left(\frac{2x-3}{5}-0,(3)\right)\cdot 3\frac{1}{2}-3-4,(3)\right]:\frac{1}{5}+5\right\}:0,(5)=27.$$

3. Let:

$A = \{x \mid x \in \mathbf{N}, x < 124\}$ and $B = \{x \mid x \in \mathbf{N}, 38 < x < 100\}$.

Calculate $A \cap B, A \cup B, A \backslash B$ and $B \backslash A$.

II. 1. Prove that any uneven number can be written as the difference of the squares of two natural numbers.

2. Determine the linear function $f: \mathbb{R} \to \mathbb{R}, f(x) = ax + b$ with the property that $f(x + 1) = 5x + 2$, no matter what real x is.

III. 1. Let ABC be a triangle inscribed in a circle, D the point where the perpendicular from A on BC cuts the circle and E the point diametrically opposed to the top A. Prove that the angles $\sphericalangle BAE$ and $\sphericalangle DAC$ are congruent.

2. A cone having the length of the radius of the base a has the generating line equal to the diameter of the base. Calculate the volume and the area of the sphere circumscribed to the cone.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1p	3) 1p
	II.	1) 1,50p	2) 1,50p	
	III.	1) 1,50p	2) 1,50p	

Test no. 58

I. 1. Prove that the product of any three consecutive natural numbers is divisible by 3.

2. The rapport between the sum and the difference of numbers x and y is $\frac{9}{5}$. Determine the value of the rapport $\frac{x}{y}$.

3. Solve the system of inequations:

$$\begin{cases} 5x - 3 > 1 + x \\ \frac{1}{2} - 3x \leq \frac{2}{3}x - 5 \end{cases}$$

II. 1. Show that in any right triangle the height lowered from the right top is equal at most to half of the hypotenuse.

2. Decompose in factors the polynomial:

$P(X) = X^4 - 2X^3 + 2X^2 - 2X + 1.$

III. 1. Let ABC be a triangle inscribed in a circle, BD and CE two heights of the triangle, H their intersection and G the point diametrically opposed to A. What kind of quadrilateral is $BGCH$?

2. We consider a sphere with the radius of $3\ cm$ that is nickelated by coating it with a thick layer of nickel of $1\ mm$. Determine the volume of the required quantity of nickel.

Grading scale:		1 point *ex officio*		
	I.	1) 1,50p	2) 1p	3) 1p
	II.	1) 1,50p	2) 1,50p	
	III.	1) 1,50p	2) 1,50p	

Test no. 59

I. 1. Prove the identity:

$$\frac{1-ax+(a+x)x}{2ax-a^2x^2-1}:\left[1+\frac{a^2+2ax+x^2}{(1-ax)^2}\right]=-\frac{1}{1+a^2}.$$

2. Let a, b be two natural numbers so that $a+b$ is an uneven number. Prove that $a \cdot b$ is an even number.

3. Show that the number $\sqrt{5}$ is an irrational number.

II. 1. Simplify the fraction:

$$\frac{x^3-x^2-3x-1}{x^4-4x^2-4x-1}.$$

2. Prove that the polynomial $Q(X) = X^2 + X$ divides the polynomial $(X+1)^n - X^n - 1$ for $n \in \mathbb{N}$, n uneven.

III. 1. Prove that the bisectors of two joined angles of a quadrilateral are perpendicular.

2. We consider a pyramid that has as base an isosceles right triangle and whose height congruent with a cathetus of the base $(= a)$ falls in the top of an acute angle of the base. Determine the lengths of the side edges and the volume of the pyramid.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1p	3) 2p
	II.	1) 1,50p	2) 1,50p	
	III.	1) 1p	2) 1p	

Test no. 60

I. 1. Determine all the natural numbers of the form $\overline{8x1y}$ divisible by 15.

2. A ball rises with $5/7$ from the heights from which it falls. From what height did it fall the first time if the third time it rose at $\frac{1250}{343}$ m?

3. The following sets are given:

$$A=\left\{n\in\mathbf{N}\,\middle|\,\frac{12}{n-2}\in\mathbf{N}\right\} \text{ and } B=\left\{n\in\mathbf{N}\,\middle|\,\frac{6}{n}\in\mathbf{N}\right\}.$$

Determine $A\cup B, A\cap B, A\backslash B, B\backslash A$ and $A\times B$.

II. 1. Solve the inequation:

$$\frac{|x-4|}{|x-2|+3}\le 0.$$

2. Prove that any two heights of a triangle are inversely proportional with the sides upon which they fall.

III. 1. Let $ABCD$ be a parallelogram with $AB=2a$, $AD=a$ and $m(\sphericalangle DAB)=60°$. On the same side with the plane (ABC) we raise perpendiculars on the plane in A, B, C and D. On the perpendicular B we take $BM=2a$, on the perpendicular D we take $DN=3a$ and on the perpendicular C we take P so that $MP=NP$.

a) Calculate CP.

b) The perpendicular in A on the plane $(ABCD)$ cuts the plane (MNP) in Q. Calculate the length of AQ.

c) Calculate the distances from P and Q to the straight line BD.

2. Determine the volume of a regular tetrahedron inscribed in a sphere with the radius r.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p
II.	1) 1,50p	2) 1p	
III.	1) 1,50p	2) 2p	

Test no. 61

I. 1. Determine the minimum $n \in \mathbb{N}$ so that:

$$\frac{27n+65}{8} \in \mathbf{N}.$$

2. State the value of truth of the proposition:

$$\{x \mid x \in \mathbf{Z}, |x+18|+|x-4|=0\} \neq \Phi.$$

3. Study the monotony of the function:

$$f: \mathbf{R} \to \mathbf{R},\ f(x)=\begin{cases} x+1, x \in (-\infty,0) \\ 1, x \in [0,4] \\ 2x-1, x \in (4,\infty) \end{cases}$$

II. 1. Prove that for any $n \in \mathbb{N}$ the following expression is divisible by 8:

$$3 \cdot 9^n + 5 \cdot 17^n$$

2. Determine $a, b, c \in \mathbb{R}$, so that the polynomial:

$$P(X)=X^4+(a+4)X^3+bX^2+6X+c$$

is divisible by the polynomial:

$$Q(X)=X^3+6X^2+11X+6.$$

3. Show that, if $a, b \in \mathbb{R}$ and $a+b=1$, then:

$$a^3+3ab+b^3=1.$$

III. 1. Let ABC be a random triangle. The straight line that passes through B and is parallel to the tangent in A at the circumscribed circle ΔABC intersects AC in D. Prove that:

$$AB^2 = AC \cdot AD.$$

2. A triangle ABC with the sides $AB = 8\ cm, BC = 5\ cm, AC = 7\ cm$ has the side BC included in a plane α. Determine the area of the triangle's projection on the plane knowing that the projection is a right triangle.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p
II.	1) 1p	2) 1p	3) 1p
III.	1) 1p	2) 2p	

Test no. 62

I. 1. If $a, b, c \in \mathbb{N}$ and $2a - 3b + 8c = 0$ then $6|b(a + c)$.

2. A bank gives an annual interest of 5%. What sum will a person who deposited $100\$$ in 3 years?

3. Determine $a, b, c \in \mathbb{N}^*$, so that:

$a^2 \leq b; b^2 \leq c; c \leq a^2$.

II. 1. Determine the natural numbers $\overline{abc}$ so that the sum of its' digits squares is the square of a prime number with the form:

$3k + 2, k \in \mathbb{N}$.

2. Show that the following number is divisible by 27:

$N = 15^{n+1} + 3^{n+1} \cdot 5^n + 3^{n+2} \cdot 5^n$

III. 1. In the right triangle ABC in A, $AB = 6\ cm, AC = 8\ cm$, D is the foot of A's height, and O is the center of the circle circumscribed to the triangle ABC. Calculate the length of the segment DO.

2. The volume of a cone is V. The cone is divided in three bodies through two parallel planes that pass through points of the height that divide it in three congruent segments. Calculate the volume of the middle body.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1p	3) 1p
	II.	1) 2p	2) 1p	
	III.	1) 1p	2) 2p	

Test no. 63

I. 1. Simplify:

$$\sqrt{2}\cdot\sqrt{2+\sqrt{2}}\cdot\sqrt{2+\sqrt{2+\sqrt{2}}}\cdot\sqrt{2-\sqrt{2+\sqrt{2}}}.$$

2. a) If x and y are positive numbers, show that:

$$xy \le \frac{(x+y)^2}{4}.$$

b) If $x+y=k$ then the heighest value of the product xy is $\frac{k^2}{4}$.

3. Determine a and b, real, so that the following polynomials will divide:

$$P(X)=X^4-3X^3+ax+b;\ Q(X)=X^2-5X+4.$$

II. 1. Let $x, y \in \mathbb{R}^*$ and $x+y=1$. Show that:

$$\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\ge 9.$$

2. Prove that the following number divides by 6, $(\forall)n \in \mathbb{N}^*$:

$$N = 1\cdot2\cdot3 + 2\cdot3\cdot4 + \dots + n(n+1)(n+2)$$

III. 1. Let ABC be a random triangle, $AA' \perp BC$ and R the radius of the circle circumscribed to the triangle. Prove that:

a) $AB^2 - AC^2 = A'B^2 - A'C^2$.

b) $AB \cdot AC = 2R \cdot AA'$.

2. The total area of a regular quadrilateral pyramid is S and the lateral face has the top angle equal to $60°$. Calculate the height of the pyramid.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p
II.	1) 2p	2) 1p	
III.	1) a) 1p b) 1p	2) 1p	

Test no. 64

I.1. Show that the following numbers are perfect squares:

$a = 3^{2n+3} \cdot 4^{2n+3} - 2^{2n+1} \cdot 6^{2n+3}, n \in \mathbf{N}$,

$b = 1998 + 2(1 + 2 + 3 + \dots + 1997)$.

2. Let there be the sequence of rapports $\frac{a}{3} = \frac{b}{5} = \frac{c}{7}$. Knowing that $abc = 840$, determine a, b, c.

II. 1. The sum of a three-digit number is 12. If we add 99 to this number, we obtain a number from the same digits inversely ordered. Knowing that the sum of the decimals is a prime number, determine the number.

2. The rapport between the area and the length of a circle is $1/2$. Calculate the lengths of the sides and the apothems of the equilateral triangle, the square and the regular hexagon inscribed in this circle.

III. 1. Let:

$P(X) = mX^3 + 2X^2 - 5X + n, m, n \in \mathbf{R}$.

Determine m and n so that the remainder of the division of $P(X)$ to $X^2 - 3X + 4$ will be $-5X + 4$.

2. The apothem of a regular quadrilateral pyramid makes with the plane of the base a $45°$ angle. Knowing that the length of this apothem is equal to $6\sqrt{2}\ cm$, calculate:

a) the lateral area and the total area of the pyramid;

b) the volume of the pyramid;

c) we cut a section with a plane parallel to the base at a distance equal to $2/3$ of the height in relation to the base. Calculate the area of the section;

d) calculate the rapport between the volume of the newly formed pyramid and the volume of the initial pyramid;

e) calculate, in two ways, the volume of the obtained pyramid body.

Grading scale: 1 point *ex officio*

I.	1) a) 0,75p b) 0,75p	2) 1,25p
II.	1) 0,75p	2) 1,25p
III.	1) 0,75p	2) 1p + 0,5p + 0,5p + 0,5p + 1p

Test no. 65

I. 1. Determine the numbers $\overline{x2y}$, knowing that $\overline{x2y} = 51y - 50x + 20$.

2. The prices of four merchandise assortments are inversely proportional to $\frac{2}{5}; \frac{3}{4}; \frac{1}{6}; \frac{6}{7}$. Knowing that the four assortments cost $1980\ lei$, determine the price of each assortment.

II. 1. Given the numbers:

$$a = \frac{2}{\sqrt{2} - \sqrt{3} + 1} + \frac{\sqrt{6} + \sqrt{2} - 4}{1 + \sqrt{2} - \sqrt{3}} - \sqrt{3}$$

$$b = \sqrt{4 - 2\sqrt{3}} + \left|2 - \sqrt{3}\right| - \left|-1 - \sqrt{2}\right| - \left|1 - \sqrt{2}\right| - 1$$

Calculate: $a^2 - b^2$.

2. In the triangle ABC we know: $AB = 5\ cm$, $BC = 10\ cm$ and $m(\sphericalangle C) = 30°$. Let M be the symmetrical of A in relation to BC. Show that $BM \perp MC$.

III. 1. Determine $x \in \mathbb{R}$ so that:

$$\text{a) } \frac{1}{x-2} \le \frac{2}{2x-1}; \text{ b) } \frac{\left(x^2+5\right)\left(x^2 - \frac{1}{2}x + \frac{1}{4}\right)(1-x)}{\left(x^2+x+1\right)(x-2)} \le 0.$$

2. In a regular quadrilateral pyramid with the height of $16\ cm$ and the side of the base of $24\ cm$, we make a section with a plane parallel to the base at $3/4$ from the height in relation to the base. Calculate:

a) the sine of the angle formed by a lateral face with the plane of the base;

b) the tangent of the angle formed by an edge with the pane of the base;

c) the lateral area of the pyramid's body obtained by sectioning the initial pyramid;

d) the area of the section made in the pyramid's body through a plane that pass through two parallel diagonals of the base.

Grading scale: 1 point *ex officio*

I.	1) 0,50p	2) 0,75p
II.	1) 1,50p	2) 0,75p
III.	1) 2p	2) 0,75p + 0,75p + 1p + 1p

Test no. 66

I. 1. Calculate:

a) $\sqrt{9+4\sqrt{5}}-\sqrt{9-4\sqrt{5}}$

b) $(0,2)^2-(-5)^{-2}:0,2(6)+2^{-2}$

2. The rapport of two numbers is $0,3$ and their difference is 35. Determine the numbers.

3. Solve the system:

$$\begin{cases} x-2y=-1 \\ 2x-3y=1 \end{cases}$$

II. 1. Show that the following is a perfect square:

$P(X)=(X^2+X+1)(X^2+X+3)+1$

2. Determine the area of a right triangle that has the sum of the catheti $14\ cm$, and their difference $2\ cm$.

3. The arithmetic mean of 3 numbers is 3. Determine the numbers knowing that the arithmetic mean of the other two is $11\frac{1}{2}$.

III. Let there be the isosceles triangle ABC, $AB = AC = 10\ cm$, $BC = 16\ cm$. On the plane of the triangle we raise BM, perpendicular to it, $BM = 6\ cm$.

a) Determine the area of the triangle MBA.

b) Determine the distance from C to the plane MBA.

c) Determine the volume of the tetrahedron $ABCM$.

Grading scale: 1 point *ex officio*

I.	1) a) 0,50p b) 0,50p	2) 1p	3) 1p
II.	1) 1 p	2) 1p	3) 1p
III.	a) 1p b) 1p c) 1p		

Test no. 67

I. 1. Determine what is the biggest number of the form $\overline{ab}$ that fulfills the condition:

$$\overline{ab} = 8a + 2b.$$

2. Determine x and y natural prime numbers so that:

$$\frac{x^2 - y^2}{2x^2 - 3y^2} = \frac{24}{23}.$$

II. 1. Show that:

a) $\frac{x}{y} + \frac{y}{x} \geq 2, (\forall)x, y \in \mathbf{R}_+$

b) $x^2 + y^2 + z^2 \geq xy + xz + yz, (\forall)x, y, z \in \mathbf{R}.$

2. The lengths of two sides of a triangle are of $3\ cm$ and $6\ cm$. Prove that the bisector of the angle between them in not bigger than $4\ cm$.

III. 1. Simplify the fraction:

$$\frac{(x^2 + x + 1)^2 - 1}{x^4 + 2x^3 + 5x^2 + 4x + 4}.$$

2. In the body of a regular quadrilateral pyramid with the volume of $5920\ cm^2$, the side of the big base is $20\ cm$, the side of the little base is $2\ cm$; determine:

a) the height, the apothem and the lateral edge of the pyramid's body;

b) the lateral area and the total area of the body;

c) the volume and the lateral area of the pyramid that generates the body;

d) the measure of the angle formed by the lateral edge with the plane of the big base of the pyramid's body.

Grading scale: 1 point *ex officio*

I.	1) 0,50p b) 0,75p	
II.	1) 1,75p	2) 1p
III.	1) 1p	2) 1p + 1p+ 1p + 1p

Test no. 68

I. 1. Effectuate:

a) $2 \cdot [(10-3)(85-16 \cdot 5)-2(1+100:25)]-5(1+90:30)$;

b) $10 \cdot (1{,}64 \cdot 0{,}5 + 0{,}0236 : 0{,}02)$;

c) $15^7 : (3^2)^3 : (5^2 \cdot 5^3)$.

2. During a vacation a student had to solve 240 problems, but he solved 15% more. How many problems did the student solve?

II. 1. Effectuate:

a) $\left[\left(7^{-1}+7^{-2}\right):\left(3 \cdot 7^{-3}\right)+7^{-2}:\left(-\frac{7}{2}\right)^{-3}\right]:\left(\frac{24}{427}\right)^{-1}$;

b) $\left[(-21)^{-1}\right]^{-2} : 21^{-1} - 21^{-1}$.

2. Prove that, in a trapezoid, the difference of the bases is bigger than the difference of the non-parallel sides.

III. 1. Determine the remainder of the division of the polynomial $P(X) = X^2 - aX + 3$ to $X - 2$, knowing that by dividing it to $X - 1$ the remainder is 3; $a \in \mathbb{R}$.

2. In a straight triangle ABC $(m(\sphericalangle A) = 90°)$ we know $AB = 9\ cm, AC = 12\ cm$. Let BP $(P \in AC)$ be the bisector of the angle B. In the point P er raise a perpendicular on the plane of the triangle, $MP = 10\ cm$. Calculate:

a) the tangent of the angle between the planes (MBC) and (ABC);

b) the distance from point A to the plane (MPB);

c) the sine of the angle between the straight line MB and the plane (MPD), where $PD \perp BC$ $(D \in BC)$;

d) the area and the volume of the prism that has as base the triangle PBC and the height MP.

Grading scale:	1 point *ex officio*	
I.	1) 0,25p + 0,25p +0,25p	2) 0,50p
II.	1) 0,50p + 0,25p	2) 1p
III.	1) 1p	2) 1p + 1p+ 1p+ 2p

Test no. 69

I. 1. Determine the smallest natural number that, once divided by 12 will give the remainder 7, divided by 8 will give the remainder 3 and divided by 18 will give the remainder 13.

2. Calculate:

$$\sqrt{\frac{\left(2-\frac{2}{75}\right)\left(13-\frac{2}{3}\right)}{\left(3+1\frac{1}{6}\right)\left(1+\frac{1}{2}\right)}:\frac{37}{75}}.$$

II. 1. Determine $x \in \mathbb{Z}$ for which the expression E is an integer number:

$$E=\frac{\sqrt{2}\cdot\sqrt{9+4\sqrt{2}}+\left|\sqrt{2}-\sqrt{3}\right|+\left|\sqrt{3}-5\right|}{x-2}$$

2. Determine the area of a parallelogram whose diagonals measure $10\ cm$ and $16\ cm$, and the angle between them has $30°$.

III. 1. Determine the remainder of the division of the polynomial:

$$P(X) = X^3 - 2aX^2 + bX + 3$$

to $aX + b$, knowing that it is divisible by $(X-1)(X+1)$.

2. Let ABC be an isosceles triangle with $AB + AC + 8\ cm, m(\sphericalangle A) = 120°$ and $AD \perp BC (D \in BC)$. In the point D we raise a perpendicular on the plane of the triangle, $DM = 4\ cm$. Calculate:

a) the distance from the point D to the plane (MAC);

b) the angle between the straight line MD and the plane (MAC);

c) the area and volume of the pyramid $MABC$.

Grading scale: 1 point *ex officio*

I.	1) 1,25p	2) 0,75p
II.	1) 1p	2) 1p
III.	1) 1,50p	2) 1p + 1p+ 1,50p

Test no. 70

I. 1. Knowing that:

$a = 6, b - c = 4, d = [2 \cdot 7 \cdot (7^2)^6 + 3 \cdot 7^3 \cdot (7^2)^5] : 7^{12}.$

Calculate:

a) $ab - ac$; b) $a^2 + ab - ac + d^2$

2. Three workers dig a ditch in 3 hours. How many workers dig the same ditch in half an hour?

II. 1. Show that the following numbers are natural:

$$a = \sqrt{11 - 6\sqrt{2}} + \sqrt{7 - 4\sqrt{3}} + \sqrt{5 + 2\sqrt{6}}$$

$$b = \frac{1}{\sqrt{2} + 1} + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{4} + \sqrt{3}}$$

2. Let ABC be a triangle having the lengths of the sides $AB = 3\ cm,\ AC = 3\sqrt{2}, BC = 3\sqrt{3}$. Let P be the foot of the height from A on the side BC, and M and N the projections of P on the sides AB, AC, respectively. Calculate the length of the segment MN.

III. 1. Let $P(X)$ be a polynomial of a degree bigger than 2. Knowing that divided at $X - 1$ it will give the remainder 1, divided at $X - 2$, it will give the remainder -1 and divided at $X - 3$ it will give the remainder 2, determine the remainder of dividing $P(X)$ at $(X - 1)(X - 2)(X - 3)$.

2. A trapezoid $ABCD$ $(AB||CD)$ has $AB = 30\ cm, DC = 10\ cm$ and the non-parallel sides $AD = 12\ cm, BC = 16\ cm$. In the point B we raise a perpendicular on the plane of the trapezoid, $BM = 10\ cm$. Calculate:

a) the distance from the point M to the straight line AD;

b) the value of one of the trigonometric functions of the angle formed by the straight line AD with the plane (MBD);

c) the angle between the planes (MAB) and (MDC);

d) the area and the volume of the pyramid $MABCD$.

Grading scale:	1 point *ex officio*	
I.	1) 0,75p	2) 0,25p
II.	1) 1p	2) 1,25p
III.	1) 1,50p	2) 1p + 1p+ 1,25p

Test no. 71

I. 1. Calculate:

$$\left\{\frac{\left[\left(\frac{16}{125}\right)^{5}\cdot(0,4)^{8}\right]^{3}}{\left(\frac{2}{5}\right)^{69}}-2^{15}+\left(\sqrt{0,125}\right)^{4}\cdot\frac{1}{16}\right\}:\frac{1}{2^{10}}-(-1).$$

2. Two children deposited at C. E. C sums in the ratio of $2/3$. What sum did every child deposit knowing that $3/5$ from the sum deposited by the second one, raised by 30% of the sum deposited by the first one is with $156\ 000$lei smaller than the sum of the two children combined?

3. Determine the values of $x \in \mathbb{R}$ for which the following expression is defined:

$$E(x)=\sqrt{\frac{x+1}{x-1}}.$$

II. 1. Solve the following equation, where m is a real parameter:

$$(m+5)\cdot(m-3)\cdot x=m+5.$$

2. Does there exist a linear function whose graphic contains the points $A(-1,-1), B(3,3)$ and $C(0,2)$?

III. 1. On the sides AB and AC of an equilateral triangle ABC we consider the points D and E respectively, so that $AD = CE$. Let M be the intersection of the straight lines BE and CD. Calculate $m(\sphericalangle BMC)$.

2. In a cylinder, the radius of the base has $4\ cm$ and the height $14\ cm$. Determine the radius of a sphere whose area is equivalent to the total area of the cylinder.

Grading scale:	1 point *ex officio*		
I.	1) 1p	2) 1,50p	3) 1,50p
II.	1) 1,50p	2) 1p	
III.	1) 1,50p	2) 1p	

Test no. 72

I. 1. Calculate:

$$\sqrt{10+\sqrt{4+\sqrt{6+\sqrt{9}}}}\cdot\sqrt{10-\sqrt{4+\sqrt{6+\sqrt{9}}}}.$$

2. At an auction the price of an object rose by 20% then by 1% of the new price. Thus, the object sold with $264\ 000\ lei$. What was the starting price of the object?

3. x being a random real number, determine:

$|-x|$ şi $\left|\left(2-\sqrt{3}\right)(x-1)\right|$.

II. 1. Determine the intersections with the coordinates' axis of the function $f:\mathbb{R}\to\mathbb{R}, f(x)=2x-3$, then trace the graphic of the function.

2. Solve the system of inequations:

$$\begin{cases}\dfrac{\left(\sqrt{2}-x\right)^2}{2}\geq x\left(0,5x-\sqrt{8}\right)\\(x-2)^2-1\geq(x-1)^2-5\end{cases}.$$

III. 1. Let ABC be an isosceles triangle ABC $(AB=AC)$ inscribed in a circle. In A and C we trace the tangents to the circle and we note with T their intersection. Prove that:

$AC^2=BC\cdot CT$.

2. The planes of two joined lateral faces of a regular quadrilateral pyramid form a $120°$ angle. Calculate the volume of the pyramid if the side of the base has the length a.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1,50p	3) 1p
	II.	1) 1p	2) 1p	
	III.	1) 1,50p	2) 2p	

Test no. 73

I. 1. Calculate:

a) $\frac{1}{\sqrt{7}}+\frac{1}{\sqrt{28}}+\frac{1}{\sqrt{63}}+\frac{1}{\sqrt{112}}$; b) $\frac{1}{\sqrt{2}-1}-\frac{1}{\sqrt{2}+1}$.

2. Prove that:

$$(a-3)^2+(a+2)^2+(a+3)^2+(a+4)^2=(a-1)^2+a^2+(a+1)^2+(a+6)^2,$$

irrespective of what a, real number is.

3. Bring to a simpler form the expression:

$$E(x)=\frac{(x^2-1)\sqrt{x^2-4}+x^3-3x-2}{(x^2-1)\sqrt{x^2-4}+x^3-3x+2}\cdot\sqrt{\frac{x+3}{x-3}}.$$

II. 1. Determine the positive real numbers x that satisfy the inequality:

$$1988x-1987>1987x-1988.$$

2. Determine $a, b, c \in \mathbb{R}$, so that the polynomial:

$$P(X)=6X^3-16X^2+26X-12$$

can be written as:

$$P(X)=(3X-2)(aX^2+bX+c).$$

III. 1. Let there be a circle with the center O and BO a radius perpendicular on a diameter. We unite B with a point A of the diameter and we note with P the intersection between BA and the circle. The tangent in P cuts the extension of the diameter in C. Prove that $CA = CP$.

2. Calculate the radius of the sphere circumscribed in a right circular cone with the generating line of $50\ cm$ and the radius of the base of $30\ cm$.

Grading scale: 1 point *ex officio*

I.	1) a) 0,5p b) 0,5p	2) 1p	3) 1p
II.	1) 1,50p	2) 2p	
III.	1) 2p	2) 1,50p	

Test no. 74

I. 1. What sign do the following differences have: $2\sqrt{11} - 6$; $3\sqrt{2} - 2\sqrt{3}$?

2. We know that $x + y = s$ and $xy = p$. Calculate $x^2 + y^2$ and $x^3 + y^3$.

3. Determine the total area of a rectangle knowing that if we raise the length $9/5$ times and we reduce the width $1/3$ times, the area of the rectangular surface will shrink with $120\ cm^2$.

II. 1. Solve the following system, where a and b are random real parameters, $a \neq b, a \neq -b$:

$$\begin{cases} \frac{x}{a+b} + \frac{y}{a-b} = \frac{1}{a-b} \\ \frac{x}{a+b} - \frac{y}{a-b} = \frac{1}{a+b} \end{cases}$$

2. Determine the quotient and the remainder of the division of the polynomial:

$$P(X) = 6X^5 - 28X^4 + 37X^3 - 16X^2 + 12X + 4$$

to the polynomial:

$$Q(X) = 3X^3 - 8X^2 + 4X - 4.$$

III. 1. The rapport of a right triangle's catheti is $3/4$ and the length of the hypotenuse is $175\ cm$. Determine the length of the two catheti and the difference between the two segments determined on the hypotenuse by the height that originates in top of the right angle.

2. The sides of a regular triangular pyramid body's bases are $6\ cm$ and $16\ cm$ long. The lateral face forms with the plane of the base an angle of $60°$. Calculate the volume and the lateral area of the pyramid's body.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1,50p	3) 1p
II.	1) 1,50p	2) 1p	
III.	1) 1,50p	2) 1,50p	

Test no. 75

I. 1. Calculate:

$$(-1)^{2m} + (-1)^{2n+1} + (-1)^{p}$$

where m, n, p are random natural numbers.

2. Determine what the price of a book was before two price reductions, knowing that the first reduction was 5%, the second $15\ \%$ and that the actual price is $12\ 920\ lei$.

3. Prove that irrespective of what the natural number $n \neq 1$ is, the following fraction isn't an integer number.

II. 1. Determine the linear functions f and g knowing that, irrespective of real x:

$$f(x+1) + 2g(x-1) = 3 \text{ and } 2f(x+1) - 2g(x-1) = 3x$$

2. Let there be the polynomial:

$$P(X) = X^4 - 3X^2 + 2X + 1.$$

Calculate $P(\sqrt{3} - \sqrt{2})$.

III. 1. In a circle we inscribe a random triangle ABC. The perpendicular taken from A on AC cuts the circle in M, and the perpendicular traced from A on AB cuts the circle in N. Prove that the straight lines MN and BC are parallel.

2. A regular triangular pyramid has the side of the base a and the lateral edge. Determine the lateral area and the volume of the pyramid.

Grading scale:	1 point *ex officio*		
I.	1) 1p	2) 1p	3) 1,50p
II.	1) 1,50p	2) 1p	
III.	1) 1,50p	2) 1,50p	

Test no. 76

I. 1. Let a, b, c be three random real numbers. If $a < b$ does it follow that: $a \cdot c < b \cdot c$?

2. Prove that the following number is divisible by 15, for any n , natural number:

$$72^n + 3^{2n+1} \cdot 2^{3n+1} + 8^{n+1} \cdot 9^n$$

3. Prove that, for any n, non-equal to zero, natural number:

$$\frac{3^n}{4^n} < \frac{3^n + 1}{4^n + 1}$$

II. 1. Let there be the polynomial:

$$P(X) = X^5 + aX^2 + bX + 1.$$

Determine the real constants a and b so that the sum of the polynomial's coefficients is 2 and that $P(X)$ is divisible by the polynomial $X + 1$.

2. Prove that, if a, b, c are the lengths of a triangle's sides, then:

$$(b^2 + c^2 - a^2) - 4b^2c^2 < 0.$$

III. 1. Let $ABCD$ be an isosceles trapezoid $(AB||CD)$, where $BC = CD = DA = a$, and $AB = 2a$. Calculate the area of the trapezoid.

2. We consider a quadrilateral pyramid where one of the lateral faces is an equilateral triangle and its plane is perpendicular with the plane of the base. Knowing that the side of the base has length a, calculate the lengths of the lateral edges, the volume of the pyramid and the angles formed by the lateral edges with the plane of the base.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1,50p	3) 1,50p
	II.	1) 1p	2) 1,50p	
	III.	1) 1p	2) 1,50p	

Test no. 77

I. 1. If a, b are two random real numbers, so that $a < b$, then: $a^2 < b^2$?

2. Prove that if between the pairs of numbers (a, b) and (c, d) there exists a direct proportionality, then the following equality takes place:

$$\frac{ab}{cd} = \left(\frac{a+b}{c+d}\right)^2.$$

3. Determine the smallest natural number that, divided, in a row by $3, 4, 5$ will give the same remainder.

II. 1. The following equation is given:

$$\frac{2x+1}{x-3} = m.$$

Solve and determine $m \in \mathbb{Z}$ so that the solution is an integer number.

2. Determine the sum of the following polynomial's coefficients:

$$P(X) = (5X^4 - 4)^{1986} + (9X^3 - 8)^{1997} + (1 - 2X)^{1998}$$

III. 1. Let $ABCD$ be a parallelogram and $N \in (CD), M \in (BC)$. Through C we trace $CP||AM$ and $CQ||AN$ $\big(P \in (AD), Q \in (AB)\big)$. Prove that the straight lines PM, QN and EF are concurrent $\{E\} = CP \cap AN, \{F\} = CQ \cap AM)$.

2. We consider a prism $ABCDA'B'C'D'$ that has a square as a base and where the projection of the top A' on the plane of the base is the top C. Knowing that $AB = a, AA' = 2a$, determine the volume of the prism, the angle formed by a lateral edge with the plane of the base and the angles of the lateral faces.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1p	3) 1,50p
	II.	1) 1p	2) 1,50p	
	III.	1) 1,50p	2) 1,50p	

Test no. 78

I. 1. a) If $x^2 = y^2$ does it follow that $x = y$?

b)If $x < 0$, calculate $\sqrt{x^2}$.

2. If from a number we subtract, one a time $2, 4, 8, 16$, we obtain four numbers that when added, give the double of the respective number. Determine that number.

3. Determine the real parameter m so that the following equation does not admit real solutions:

$$(m-2)x + 3x = m-5$$

II. 1. Determine the linear function $f: \mathbb{R} \to \mathbb{R}$ with the property that, for any real x:

$$f(x + 2) = -4x - f(3).$$

2. Prove that if a polynomial $P(X)$ is divisible by $X - a$ and $X - b$, then it is divisible with the polynomial $(X - a)(X - b)$ (we made the presupposition that $a \neq b$).

III. 1. Let ABC be a right triangle ($\sphericalangle A = 90°$), D is the foot of the height traced from A and $E, F$the projections of the point D on the catheti. Prove that:

$$AE^2 + AF^2 = AD^2.$$

2. The axial section of a cone is an equilateral triangle with the area $9\sqrt{3}cm^2$. Determine the total area and the volume of the cone.

Grading scale:		1 point *ex officio*	
	I.	1) a) 0,50p b) 0,50p 2) 1p 3) 1,50p	
	II.	1) 1,50p	2) 2p
	III.	1) 1p	2) 1p

Test no. 79

I. 1. Verify the equality:

$$\left(3\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3=\left[\left(3\frac{1}{2}\right)^2-3\frac{1}{2}\cdot\frac{1}{3}+\left(\frac{1}{3}\right)^2\right]\cdot\left(3\frac{1}{2}+\frac{1}{3}\right).$$

2. Determine x from the equality:

$$12\frac{1}{3}:\left\{\left[2\frac{3}{4}:\left(3\frac{1}{3}-1\frac{7}{8}\cdot x\right)\right]\cdot\frac{8}{11}+1\frac{2}{3}\right\}=5.$$

3. Prove that, by adding to a number 8 times the sum of its' digits, we obtain a number divisible by 3.

II. 1. Decompose in factors the polynomial:

$$P(X)=X^4-4X^3+6X^2-4X+1.$$

2. Solve the following system of inequations:

$$\begin{cases}2(x-3)-1<5\\ \frac{3x}{8}-7>\frac{x}{12}\end{cases}$$

III. 1. We consider a convex quadrilateral with parallel diagonals. Let O be their intersection point. Show that O 's projections on the quadrilateral's sides are the tops of a writable quadrilateral.

2. In a regular quadrilateral pyramid we inscribe a cube so that four of its' tops are situated on the lateral edges of the pyramid, and the other four tops, on the plane of the base. Determine the volume of the cube if the side of the pyramid's base is a and the height of the pyramid is h.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1,50p
II.	1) 1p	2) 1p	
III.	1) 2p	2) 1,50p	

Test no. 80

I. 1. Calculate:

$$[2^{48} : 2^{18} + (3^4)^5 + 6^{23} : 6^{13}] : [2^{10} \cdot 3^{10} + 2^{17} \cdot 2^{13} + (3^5)^4].$$

2. Solve the equation $|x - 4| = 5$.

3. In a cellar there are 7 full barrels, 7 half full barrels abd 7 empty barrels. Divide these barrels in three equal groups so that there is the same number of barrels in each group and the same quantity of liquid.

II. 1. Solve the following system, where a, b are real parameters:

$$\begin{cases} \dfrac{x}{a+b} + \dfrac{y}{a-b} = 2a \\ \dfrac{x-y}{2ab} = \dfrac{x+y}{a^2+b^2} \end{cases}$$

2. Let there be the function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = ax + b$. Determine a and b so that:

$$\frac{1}{4} f(x-1) + \frac{3}{4} f(x+2) = f(x+1),$$

for any real a.

III. 1. Show that the length of the tangent common to two exterior tangent circles is the proportional mean between the diameters of the two circles.

2. A pyramid $VABC$ has congruent and perpendicular lateral edges $(VA = VB = VC, VA \perp VB \perp VC \perp VA)$. Determine the total area and the volume of the pyramid, knowing that $VA = VB = VC = a$.

Grading scale:		1 point *ex officio*		
	I.	1) 1p	2) 1p	3) 2p
	II.	1) 1,50p	2) 1p	
	III.	1) 1p	2) 1,50p	

Test no. 81

I. 1. A set A has 11 elements and a set B has 10 elements. If $A \cup B$ has 13 elements, how many elements does $A \cap B$?

2. Solve in Q the equations:

a) $$\frac{x}{2+5:(-5)} = \frac{\frac{1}{2}-\frac{1}{2}\cdot 2}{\frac{3}{4}};$$

b) $mx + 3 = 3x + m.$

II. 1. Calculate the arithmetic, geometric and harmonic mean of the numbers a and b where:

$$a = \sqrt{252} - \sqrt{112} + \sqrt{448} \text{ and } b = \sqrt{112} + \sqrt{700} - \sqrt{28}.$$

2. The lengths of a triangle's sides are $AB = 13\,m,\ BC = 21\,m, CA = 20\,m$. Determine the heights of the triangle.

III. 1. Represent graphically the function $f: \mathbb{R} \to \mathbb{R}$, defined through:

$$f(x) = \begin{cases} -x, & x < -1 \\ x, & -1 \le x \le 1 \\ -2, & 1 < x \le 2 \\ 0, & 2 < x < 5 \\ x-6, & x \ge 5 \end{cases}$$

2. Let $ABCD$ be a rectangle with $BC = 5\,cm, m(\sphericalangle BOC) = 60°$ ($\{O\} = AC \cap BD$). In the point A we raise a perpendicular on the plane of the rectangle, $AM = 5\,cm$. Calculate:

a) the tangent formed by the planes $(MBD)$and (ABC);

b) the angle between the straight line MA and (MBD);

c) the distance from point A to the plane (MBC);

d) the angle between the planes (MAB) and (MDC);

e) the area and the volume of the pyramid $MABCD$.

Grading scale: 1 point *ex officio*

I.	1) 0,25p	2) 0,50p
II.	1) 0,75p	2) 1p
III.	1) 1,25p	2) 1p + 1p + 1p +1p + 1,25p

Test no. 82

I. 1. We consider the sets:

$A = \{3, 4\}, B = \{1, 2, 5\}, C = \{3, 6\}.$

Effectuate:

$(A \cup B) \cap C, A \cup (B \cap C), A - (B \cap C), C - (A \cup B).$

2. The number 132 is divided in parts, directly proportional with the numbers $3\frac{1}{2}, 2\frac{1}{3}, 5\frac{1}{6}$. What are the numbers?

II. 1. Solve the equations:

a) $3 \cdot \left\{x - \frac{3x-1}{4} - \left[1 - 2\left(x - \frac{x+3}{5}\right)\right]\right\} = 5x - 2$;

b) $\frac{1 - \frac{x}{2}}{3} - \frac{2 - \frac{x}{4}}{4} + 3x = 23$.

2. In a circle we trace the diameter AB. Let M be a point of the circle and N its' projection on AB. The following are given: $AN = 12\ cm, NB = 3\ cm$. Determine the lengths of the segment MN and the chords of AM and BM.

III. 1. Effectuate:

a) $[-2; 4) \cap \mathbf{N}^*$;

b) $(-2{,}3; 5] \cap \mathbf{R}^*$;

c) $\{-1; 0; 1; 3\} \cap \mathbf{N}$;

d) $[-3; 7] \setminus (-2; 9]$;

e) $[-3; 7] \cup (-2; 9]$.

2. An equilateral triangle ABC has the length of the middle line MN $(M \in AB, N \in AC)$ equal to $4\ cm$. In the point M we raise a perpendicular $MD = 2\ cm$ on the plane of the triangle. Calculate:

a) the distance from B to the plane (DAC);

b) the angle between the planes (DAC) and (ABC);

c) the angle between the straight line AC and the plane (DAB);

d) the area and the volume of the pyramid with the top in D and the base MNC.

Grading scale: 1 point *ex officio*

I.	1) 0,25p	2) 0, 50p
II.	1) 0,75p + 0,5p	2) 0, 75 p
III.	1) 1,25p	2) 1p + 1p + 1p + 1,25p

Test no. 83

I. 1. Calculate:

a) $2^{74} : (2^3 \cdot 2^{70}) + 103 : (2 + 2 \cdot 102)$;

b) $\left[\left(\frac{1}{4} - 0{,}1 : 2\right) \cdot \frac{5}{13} + 1 : \left(\frac{3}{4} + \frac{1}{3}\right)\right] \cdot \frac{3}{8}$.

2. The sum of three consecutive powers of the number 3 is 351. Determine these numbers.

3. At a sport race, a certain number of students had to participate. 15% more students registered, reaching a total of 69 children. How many students were due to participate at the race?

II. 1. Solve the system:

$$\begin{cases} 3{,}9 + 2(0{,}2x - 1) = 1{,}5y \\ -21{,}3 - 3\left(\frac{1}{x} - 2\right) = \frac{10 + 0{,}3y - 2x}{x} \end{cases}$$

2. In the triangle $ABC(m(\sphericalangle A) = 90°)$ we trace the height $AD(D \in BC)$, Prove that:

a) $AD \cdot AB = AC \cdot BD$;

b) $AD \cdot AC = AB \cdot CD$.

III. 1. Let:

$f: \mathbf{R} \to \mathbf{R}$, $f(2x - 5) = 2x - 3 - f(7)$.

a) Determine $f(x)$;

b) Does the point $A(0, 2)$ belong to the graphic of the function $f(x)$?

2. Let $ABCD$ be a parallelogram with $AB = 10\ cm, m(\sphericalangle ABD) = 60°$ and $BD = 16\ cm$. In the point B we raise a perpendicular on the parallelogram's plane, $BM = 10\ cm$. Calculate:

a) the tangent of the angle between the planes (MAC) and (ABC);

b) the distance from point A to the plane (MBD);

c) the angle between the straight line MC and the plane (MAD);

d) the area and the volume of the pyramid $MABCD$.

Grading scale:	1 point *ex officio*		
I.	1) 1p	2) 0,50p	3) 0,50p
II.	1) 1p	2) 1,25p	
III.	1) 1p	2) 1p + 1p + 1p + 1,25p	

Test no. 84

1. The number $A = 5^2 - [(42)^2]^0$ is equal to :

A: 60; B: 24; C: 29.

2. The solution of the inequation $2(x+3)+4=30$ is:

A: 5; B: 12; C: 10.

3. The area of a right triangle with the catheti of $8\ cm, 10\ cm$ is:

A: 80 cm^2; B: 40 cm^2; C: 18 cm^2.

4. The value of the fraction $F(x) = \frac{2x-1}{x+1}$ for $x = 1$ is:

A: $\frac{2}{5}$; B: $\frac{1}{3}$; C: 0,5.

5. The volume of a cube with a $10\ cm$ edge is:

A: 100 cm^3; B: 1000 cm^3; C: 10 000 cm^3.

6. The domain of existence of the expression $E(x) = \sqrt{x-4} - 7$ is:

A: $(4, +\infty)$; B: $[4, +\infty)$; C: $(-\infty, 4]$.

7. We consider $ABCD$ (right) with $AB = m, BC = n, m > n$. Let M be the middle of $[AB]$ and N the middle of $[BC]$. What relation is there between m and n so that $\Delta NDM$is right?

A: $m = n\sqrt{2}$; B: $m = 2n$; C: $m = n\sqrt{3}$.

8. The solution of the system:

$$\begin{cases} x = 2 \\ 2x + y = 10 \end{cases}$$

is:

A: (2, 2); B: (2, 6); C: (2, 1).

9. The simplest form of the expression:

$$E(x) = \left(\frac{x}{x^2 - x} + \frac{x+2}{2 + x - 2x^2 - x^3} + \frac{3x^2}{6x^2 + 6x}\right) \cdot \left(\frac{x}{2} - \frac{0,5}{x}\right)$$

is:

A: $\frac{x+3}{4}$; B: $\frac{x+3}{2}$; C: $\frac{x+6}{4}$.

10. In a right circular cone with $R = 5\ cm, h = 12\ cm$ we cut a section parallel to the base, having the area equal to 4π. Show that A_1 and V the issuing cone body is:

A: $\frac{273\pi}{3}cm^2, \frac{468\pi}{3}cm^3$;

B: $\frac{172\pi}{3}cm^2, \frac{368\pi}{3}cm^3$;

C: $\frac{162\pi}{3}cm^2, \frac{458\pi}{3}cm^3$.

Grading scale: 1 point *ex officio*

1) 0, 75p;
2) 0, 75p;
3) 0, 75p;
4) 0, 75p;
5) 1p;
6) 1p;
7) 1p;
8) 1p;
9) 1p;
10) 1p

Test no. 85

1. The value of the number:

$$8^{16} : (2 \cdot 2^2 \cdot 4^{10})^2 - 2^1 - 2^0$$

is:

A: 0; B: 2; C: 1.

2. Given the powers 2^{30} and 8^{10} we have:

A: $2^{30} > 8^{10}$; B: $2^{30} < 8^{10}$; C: $2^{30} = 8^{10}$.

3. If we effectuate:

$$\left(-\frac{5}{7}\right):\left(-\frac{3}{4}\right)\cdot\frac{2}{3}:\frac{5}{9}\cdot 0\cdot\frac{4}{7}$$

we have: A: 2; B: 3; C: 0.

4. If $\frac{x}{y} = \frac{4}{5}$ then the value of the rapport $\frac{2x-y}{x+3y}$ is:

A: $\frac{2}{5}$; B: $\frac{19}{3}$; C: $\frac{3}{19}$.

5. The solution to the system of inequations:

$$\begin{cases} x + y\sqrt{2} = 2\sqrt{2} \\ \frac{3x}{\sqrt{2}} - 2y = 1 \end{cases}$$

is:

A: $(\sqrt{2}, 1)$; B: $(2\sqrt{2}, \sqrt{2})$; C: $(0,3)$.

6. Effectuate the division:

$$(x^4 + 3x^2 - 2x + 3) : (x^2 - x + 1)$$

and obtain the remainder:

A: 3; B: −1; C: 0.

7. Two adjacent angles have their uncommon sides in extension. The angle of their bisector has the measure:

A: 50°; B: 100°; C: 90°.

8. If a rectangle has $l = 5\ cm, L = 5\sqrt{3}$, then the angle of the diagonals is:

A: 30°: B: 60°: C: 45°.

9. If $A = \{2, 3, 5\}, B = \{1, 4, 5\}$, then card. $A \cup B$ is:

A: 5; B: 3; C: 4.

10. In a regular quadrilateral pyramid's body $L = 100\ cm, l = 6\ cm, h = 2\ cm$. Determine the lateral area and the volume of the body and the pyramid where it comes from.

Grading scale: 1 point *ex officio*

1) 1p;
2) 1p;
3) 0,5p;
4) 1p;
5) 0,5p;
6) 0,75p;
7) 0,75p;
8) 1p;
9) 0,5p;
10) 0,5p+ 0,5p + 0,5p+ 0,5p

Test no. 86

1. $\frac{2}{5}$ from $2\ m$ is:

A: 8 cm; B: 8 dm; C: 0,8 m.

2. The value of the expression:

$(-1)\cdot(-2+3)+4:(5-6^0)$

is:

A: 2; B: –5; C: 0.

3. The number of elements of the subset:

$M = \{ x \mid x \in \mathbb{Z}^*, -2 \le x < 5 \}$

is:

A: 7; B: 5; C: 6.

4. The value of the expression $E = 3(x^2-1) + x(x-2) - 4x(x-1) - 2x + 2$ is:

A: x; B: 2; C: –1.

5. The solution of the system:

$$\begin{cases} \frac{1}{x} - \frac{1}{y} = 5 \\ \frac{3}{x} + \frac{2}{y} = 25 \end{cases}$$

is:

A: (7, 2); B: (3, 4); C: $\left(\frac{1}{7}, \frac{1}{2}\right)$.

6. $\frac{3}{x-2} > 0$ for:

A: x < 2; B: x = 2; C: x > 2.

7. The value of the polynomial $P(X) = X^4 - 3X^2 + 5X - 7$ for $X = -1$ is:

$A: 2; B: -4; C: -14.$

8. If two complementary angles have the rapport $\frac{2}{7}$, then their measures are:

$A: 10° \; and \; 35°$; B: $40° \; and \; 140°$; $C: 20° \; and \; 70°$.

9. If a cube has the edge of $10\ cm$, then its' diagonal is:

$A: 30\ cm; B: 10\sqrt{3}\ cm; C: 50\ cm.$

10. A cone has $G = 10\ cm$ and the angle between it and the plane of the base is $60\ °$. Determine: the height, the volume, the total area and the angle of the circular sector that comes from the extension.

Grading scale: 1 point *ex officio*
1) 1p;
2) 1p;
3) 0,75p;
4) 1p, 0,75p;
5) 0,75p;
6) 0,5p;
7) 1p;
8) 0,75p;
9) 1p;
10) 0,5p

Test no. 87

1. The value of the expression $E = \frac{2}{3} \cdot \left(\frac{5}{2} - \frac{5}{9}\right) : \frac{4}{3}$ is:
$A: 1; B: 10; C: \frac{35}{36}$.

2. The solution of the equation $\frac{5}{9} \cdot x = 8$ is:
$A: \frac{1}{2}; B: \frac{5}{72}; C: \frac{72}{5}$.

3. The value of the expression $|3\sqrt{2} - 2\sqrt{5}| - 2(\sqrt{5} + \sqrt{2})$ is $A: \sqrt{3}; B: \sqrt{2}; C: -5\sqrt{2}$.

4. How many natural values of x verify the relation $\frac{2}{3} < \frac{x+2}{5} \leq \frac{2}{7}$? $A: 3; B: 0; C: 2$.

5. The system $\begin{cases} x + y = 3 \\ 2x + 2y = 6 \end{cases}$ is:
A: compatible, determined; B: incompatible; C : compatible, undetermined.

6. Let $f: \mathbb{R} \to \mathbb{R}, f(x) = 2x + 1$. The function is:
A: constant; B: strictly increasing; C: strictly decreasing.

7. The area of the equilateral triangle inscribed in the circle with the radius of $5\ cm$ has the value:

A: $\frac{5\sqrt{3}}{2}$; B: $\frac{75\sqrt{3}}{2}$; C: $\frac{75\sqrt{3}}{4}$.

8. If in a rhomb a diagonal has the same length as its' side, then the acute angle of the rhomb is:

A: 50°; B: 60°; C: 120°.

9. If a regular quadrilateral pyramid has $h = 9\ cm$, the side of the base $6\ cm$, then its' total area is:

A: $72\sqrt{10}$; B: $36\sqrt{10}$; C: $36(1+\sqrt{10})$.

Grading scale: 1 point *ex officio*

1) 1p;
2) 0,5p;
3) 1p;
4) 1p;
5) 1p;
6) 1p;
7) 1p;
8) 1p;
9) 1,5p

Test no. 88

1. For what value of x the following equality verifies:

$[20 \cdot (x : 4 - 5) \cdot 5 : 100 - 3] : 11 = 2$

A: 100; B: 80; C: 120; D: 200.

2. How many three digit numbers divided by 12 will give the remainder 5?

A: 73; B: 74; C: 75; D: 76.

3. A student has the sum of 18 000 *lei* in an equal number of banknotes of 500 and 1000 *lei*. How many banknotes of each type does the student have?

A: 31; B: 35; C: 37; D: 40.

4. The number:

$a = (3^{2n} + 3^{2n+1} + 3^{2n+2} + 3^{2n+3}) \cdot 3^{-2n}$, $n \in \mathbf{N}$ is:

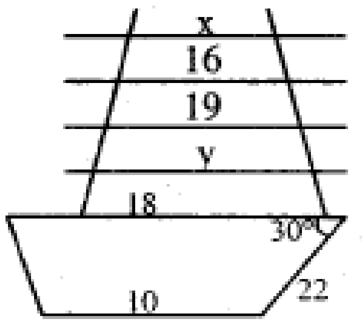
A: 11; B: 12; C: 13; D: 14.

5. If the angle formed by a rhomb's diagonal with one of its' sides has 36°, then an angle of the rhomb can have:

A: 100°; B: 90°; C: 105°; D: 108°.

6. In the graphic below, a, b, c, d are equidistant parallels. The measures x, y are:

A: 14, 21; C: 13, 22; D: 13, 21; E: 14, 20.



7. The trapezoid above has the area of:

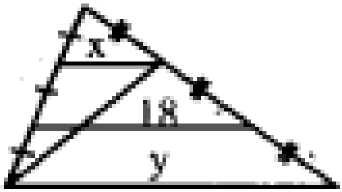
A: 154; B: 180; C: 156; D: 152.

8. The side AB of ΔABC with $AC = 7$, and $BC = 19$ can have the measure belonging to the interval:

A: [3, 17); B: (3, 18); C: (4, 16); D: (3, 17).

9. In the figure below, the pair (x, y) is:

A: (12, 34); B: (14, 36); C: (12, 36); D: (10, 34).



10. The equilateral triangle BCD and the isosceles triangle ABC with $m(\sphericalangle A) = 90°$ are situated in perpendicular planes. Let G be the center of mass of ΔABC and $MN||BC, M \in (AB), N \in (AC)$. If E is the middle of $[BC]$ and $BC = 12\ cm$, determine:

a) the area of ΔDMN;

b) $d\big(E, (DMN)\big)$;

c) $tg\ \theta, \theta = m(\sphericalangle(ABC), (DAB))$.

Grading scale: 1 point *ex officio*

1: 0,75p;

2: 0,75p;

3. 1p;

4: 0,75p;

5: 0,75p;

6: 0,75p;

7: 0,75p;

8: 0,75p;

9: 1p;

10: 0,75p.

Test no. 89

1. Which of the following inequalities is false:

A: $2 \cdot 3^{15} > 2^{16}$; B: $\left(7^4\right)^5 < 7^{5^2}$; C: $3^{92} > 2^{186}$; D: $5^{94} < 3^{141}$.

2. if the first number is 3 times smaller than the second one, the second is 3 times smaller than the sum of the other two and 5 times smaller than the third one, then the numbers are:

A: 47, 15, 20; B: 45, 20, 20; C: 45, 15, 30; D: 45, 15, 20.

3. A square shaped table has a perimeter of $48\ dm$. If $1\ m^2$ of melanin cost $6250\ lei$, then to cover the superior side of the table cost:

A: 9 200 lei; B: 9 100 lei; C: 9 000 lei; D: 7 800 lei.

4. After a price reduction of 12% and one of 10%, a suit costs $79200\ lei$. What is the initial price?

A: 79 000 lei; B: 82 200 lei; C: 80 200 lei; D: 79 200 lei.

5. Which sentence is true:

A. The straight line of equation $x = 3$ is parallel with the axes of the abscissae.

B. The straight line of equation $y - 2 = 0$ is parallel with the axes of the ordinates.

C. The straight lines $2x = 3$ and $2y = 3$ are parallel.

6. Let $ABCD$ be a square, ΔABE equilateral in its' interior and ΔBCF equilateral in the exterior of the square. If $AB = 5\ cm$, then the affirmation is false:

A. The points D, E, F collinear.

B. The straight lines EB, BFperpendicular.

C. $EF = 5\sqrt{2}\ cm$; D: $EC \leq 2,5\ cm$.

7. Let there be the expression:

$$E(x) = \frac{4x^2 - 15 + (2x-3)^2 + 4x}{1 + 4x + 4x^2}.$$

The following affirmation is wrong:

A: $E(-1)=10$; B: $E(x)=\frac{2(2x-3)}{2x+1}$; C: $E(2)=\frac{1}{5}$; D: $E(-3)=\frac{18}{5}$.

8. If:

$\sin x=\frac{3}{5}$, where $0<x<90°$ and $E=\frac{4\text{tg}\,x-3\text{ctg}\,x}{3\sin x+2\cos x}$,

then the following affirmation is true:

A: $E<-1$; B: $E>0$; C: $E=\frac{5}{17}$; D: $E=-\frac{5}{17}$.

9. If $ABCDA'B'C'D'$ is a cube and I, J, K the middles of the edges $[AB]$, $[AA']$, $[B'C']$, which affirmation is false:

A. ΔIJK isosceles

B. ΔKBI right

C. B, C', D are equidistant from I and J

D. $\Delta BKA'$ equilateral

10. In the interior of a square's plane, we build, in the exterior, equilateral triangles with sides of $10\ cm$. Determine:

a) if the 4 triangles and the square can form the faces of a pyramid;

b) the area and the volume of the pyramid obtained.

Grading scale: 1 point *ex officio*

1: 0,75 p;

2: 1 p;

3. 0,75 p;

4: 1 p;

5: 0,5 p;

6: 1 p;

7: 1 p;

8: 1 p;

9: 1 p;

10: 1 p.

Test no. 90

I. 1. Calculate:

a) $\frac{1}{6}+\frac{2}{6}+\frac{3}{6}$; b) $1\frac{1}{2}\cdot 0.25$; c) $3^3 : 3^2$; d) $(8+2^4\cdot 2):8-4$.

2. Is 2 a solution for the equation:

$2(3x-1):5-1=(x-1)\cdot 3-2$?

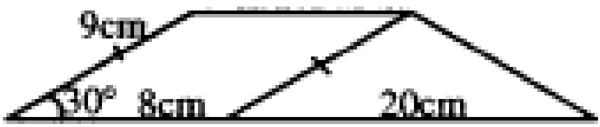
3. In what type of triangles are two middle lines congruent?

II. 1. Can the following numbers form a proportion: $4, 8, 9$ and 18.

2. Determine the parallelogram in which the diagonals can determine with the sides four congruent triangles.

3. Draw a rectangle that can be split into two squares. Can it be split into three isosceles triangles?

III. 1. Determine the area of the trapezoid here below:



2. An equilateral triangle forms with a plane a $60°$ angle. Determine its' side if the area of its' projection on the plane is $8\sqrt{3}cm^2$.

3. A right parallelepiped has the sides of the base of $4\ cm, 6\ cm$ and the height of $5\ cm$. Is it true that the sum of all the edges is $60\ cm$?

Grading scale:		1 point *ex officio*
	I.	1) a) 0, 2 p b) 0, 2 p c) 0, 2 p d) 0, 4 p
		2) 1 p 3) 1 p
	II.	1) 1 p 2) 1 p 3) 1p
	III.	1) 1 p 2) 1 p 3) 1 p

Test no. 91

1. The solution of the equation:

$$\frac{x+1}{2}-\frac{3x+1}{5}-4=x-15 \text{ is:}$$

A: 1; B: $\frac{113}{11}$; C: −2.

2. If:

$p=3+2\sqrt{2}$, $q=3-2\sqrt{2}$,

the geometric mean is:

A: 2; B: 1; C: 12.

3. The value of the expression:

$$0{,}03:10^{-2}+\sqrt{500}-\frac{50}{\sqrt{5}} \text{ is:}$$

A: 3; B: 1; C: 0,5.

4. If the side of an equilateral triangle is 12, then its' area is:

A: $36\sqrt{3}$; B: $12\sqrt{3}$; C: $108\sqrt{2}$.

5. If the length of a circle is 24π, its' area is:

A: 96π; B: 144π; C: 48π.

6. The sides of a triangle are $6, 8, 12$. A similar triangle has the perimeter 78. Its' sides are:

A: 18, 24, 36; B: 18, 36, 40; C: 10, 16, 36.

7. The length of a rectangle is three times bigger than its' width. If its area is $27\ cm^2$, then the perimeter will be:

A: 14; B: 24; C: 40.

8. A linear function that satisfies the relation $2f(x)+x=f(3-x)$, has the value $f(3)$ equal to:

A: −2; B: −1; C: 0; D: 1; E: 2.

9. Given the functions:

$A=\{x\in \mathbb{N} \mid x+5<8\}$ and $B=\{x\in \mathbb{N}^* \mid 2x\leq 10\}$

$A\cap B$ is: A: $\{0, 1, 2\}$; B: $\{1, 2\}$; C: $\{2\}$.

10. We consider $ABCD, A'B'C'D'$ situated in different planes, parallelograms. Let M, N, P, Q be the middles of $[AA'], [BB'], [C, C'], [DD']$. Prove that $MNPQ$ is a parallelogram.

Grading scale: 1 point *ex officio*
1: 0,5p;
2: 0,5p;
3: 0,75p;
4: 0,5p;
5: 0,5p;
6: 1p;
7: 1p;
8: 1p;
9: 1p;
10: 2,25p

Test no. 92

1. Making the calculations:

$$\frac{1}{2}:2+1:\frac{1}{2}-4:4^{-1}$$

we obtain:

A: -55; B: $-\frac{55}{4}$; C: $\frac{1}{4}$.

2. Effectuate:

$$3\sqrt{12}-\frac{3-\sqrt{3}}{\sqrt{3}}+\sqrt{49}-\sqrt{27}.$$

We obtain:

A: 8; B: $3\sqrt{3}$; C: $2\sqrt{3}+8$.

3. The arithmetic mean of the numbers $\frac{1}{2},\frac{1}{3},\frac{1}{6}$ is:

A: $\frac{1}{3}$; B: $\frac{2}{3}$; C: 1.

4. The graphic of $f:\mathbb{R}\to\mathbb{R}, f(x)=2x-1$ contains the point $A(a,7)$. Then a is:

A: 2; B: 3; C: 4.

4. Eight faucets with the same debit can fill a tank in 6 hours. In how many hours do 3 faucets fill the tank?

A: 16 $hours$; B: $2\frac{1}{4}$ $hours$; C: 3 $hours$.

6. Let:

$$P(X)=5X^4+3X^3-X^2-7X+2.$$

The remainder of the division to $X+3$ is:

A: 1; B: 338; C: 100.

7. The area of the equilateral triangle inscribed in the circle with the radius $R=2\ cm$ is:

A: $3\sqrt{3}$; B: $4\sqrt{3}$; C: 6.

8. The measures of the angles ΔABC are proportional to $2, 3, 7$. Then they can be:

A: 30°, 60°, 90°; B: 30°; 45°, 105°; C: 30°, 50°, 100°.

9. The volume of the regular tetrahedron with the side of $2\ cm$ is:

A: $\frac{2\sqrt{2}}{3}$; B: $\frac{2\sqrt{3}}{3}$; C: $\frac{4\sqrt{6}}{3}$.

10. The volume of a right circular cone whose section is a triangle with congruent sides of $5\ cm$, and the third side of $8\ cm$, is:

A: 16π; B: 20π; C: 3π.

Grading scale: 1 point *ex officio*
1: 0, 75 p;
2: 1 p;
3: 1 p;
4: 0, 75 p;
5: 1 p;
6: 0, 75 p;
7: 0, 75 p;
8: 1 p;
9: 1 p;
10: 1 p.

Test no. 93

1. Let $A = \{-2, -1, 0, 1, 2\}$. How many of its' subsets have the sum of the elements equal to zero?

A: 3; B: 5; C: 1

2. If $n \in \mathbb{N}^*$ then $9^{3n} \cdot 3^{4n}$ admits only one of the values:

A: 3^{10n}; B: 27^n; C: 3^{12n}.

3. The rapport between the price of a notebook and that of a book is $2/5$ and the double price of the book plus the triple price of the notebook is $6400\ lei$. The price of the book and the notebook is:

A: 2 400, 600; B: 1 200, 500; C: 2 000, 800.

4. Two circles are cut in A and B. If BD and BC are diameters, what measure does $\sphericalangle CAD$ has?

A: 120°; B: 90°; C: 180°.

5. A, B, C, D, three random non-collinear are given. The parallel through A at BC cuts BD in M, the parallel through B at AD cuts AC in N. The position of the straight lines MN and DC is:

A: perpendicular; B: parallel; C: different situation.

6. Let $ABCD$ be a square with O the center, and M, N the symmetricals of A in relation to B and of D in relation to A. The triangle MON is:

A: right; B: right isosceles; C: equilateral.

7. If $\frac{x}{y} = 0,4$, then $\frac{5x-2y}{3x-7y}$ is:

A: 5; B: 2; C: 0.

8. The solution of the system:

$$\begin{cases} \dfrac{7-x}{2} + 4 < \dfrac{3+4x}{5} + 3 \\ \dfrac{5}{3}x + 5(4-x) < 2(4-x) \end{cases}$$ is:

A: $(9, \infty)$; B: $[9, \infty)$; C: $(-\infty, 9)$

9. Let:

$$F(x) = \frac{x^4 - 25x^2 + 60x - 36}{x^2 - 3x + 2}.$$

Is not defined for:

A: $\{-1, -2\}$; B: $\{-1, 2\}$; C: $\{1, 2\}$.

10. Given the body of a regular triangular pyramid with $L = 12, l = 6$ and $V = 63\sqrt{3}$. Determine:

a) the height and the apothem;

b) the lateral area A_1;

c) the volume of the pyramid where it comes from.

Grading scale: 1 point *ex officio*

1: 0, 5 p;
2: 0, 5 p;
3: 1 p;
4: 0, 5 p;
5: 1 p;
6: 1 p;
7: 1 p;
8: 1 p;
9: 1 p;
10: 1, 5 p.

Test no. 94

1. The coordinates of the intersection point of the graphics of the functions $f, g: \mathbb{R} \to \mathbb{R}, f(x) = 2x - 3, g(x) = x + 2$ are:

A: (5, 6); B: (5, 2); C: (−5, 3); D: (5, 7); E: (7, 5).

2. The solution to the equation:

$3(2x - 1) - 5(x - 3) + 6(3x - 4) = 83$ is:

A: −1; B: 5; C: 1/2.

3. In the random ΔABC, $AB = 10\ cm, AC = 16\ cm$ and $m(\sphericalangle A) = 60°$. The area of ΔABC is:

A: 152 cm^2; B: 96 cm^2; C: $40\sqrt{3}$ cm^2.

4. The measure of two supplementary angles are in the rapport of $2/3$. The measures are:

A: 120°, 60°; B: 108°, 72°; C: 100°, 80°.

5. The catheti of a right triangle are $10\sqrt{3}$ and 24. The side of an equivalent equilateral triangle is:

A: 26; B: $15\sqrt{3}$; C: $4\sqrt{30}$.

6. Knowing that:

$$\frac{2a}{3} = \frac{3b}{5} = \frac{5c}{4} \text{ and } a + b + c = 119$$

show that a, b, c are:

A: (45, 50, 24); B: (12, 16, 40); C: (19, 39, 50).

7. The solution of the system:

$$\begin{cases} \frac{9x}{2} - \frac{5y}{3} = 13 \\ \frac{5x}{2} + \frac{2y}{3} = 12 \end{cases} \quad \text{is:}$$

A: (4, 5); B: (4, 3); C: (3, 4).

8. We consider the inequation:

$$0 > \frac{3x + 2}{18}$$

Its solution is:

A: $(2, \infty)$; B: $(-\infty, -2/3)$; C: $[1, \infty)$.

9. From a quantity of cotton 24% fiber are obtained. How much cotton is needed in order to obtain $240\ kg$ of fiber?

A: 2 316 kg; B: 108 kg; C: 1000 kg.

10. Let ΔABC be equilateral. On $[AB]$ and $[AC]$ we take E and F so that $AE = 2 \cdot BE$ and $CF = 2 \cdot AF$. The straight lines BF and CE intersect in D. On the perpendicular in A on (ABC) we take M so that $AM = MB = 30.$ Requirements:

a) show that $(EFM) \perp (FAM)$;

b) calculate $d(M, EF), d(M, CE)$.

Grading scale: 1 point *ex officio*
1: 1 p;
2: 0, 5 p;
3: 1 p;
4: 1 p;
5: 1 p;
6: 1 p;
7: 0, 75 p;
8: 0, 25 p;
9: 0, 75 p;
10: 1, 75 p

Test no. 95

I. 1. Calculate:

$$\left(17\frac{1}{2}-8\frac{1}{4}\cdot\frac{10}{11}\right):1\frac{2}{3}.$$

2. Calculate $\sqrt{13}$ with two exact decimals and test it.

3. Divide no. 1 in parts, inversely proportional to the numbers: $\frac{1}{2}; 0(3); 4$.

II. 1. Solve the system of inequations:

$$\begin{cases} -2x+1>7 \\ 2x-5\geq -1 \end{cases}.$$

2. Determine the function $f:\mathbb{R}\to\mathbb{R}$ whose graphic contains $A(2,-5), B(5,2)$.

3. Bring to the simplest form the expression:

$$F(x)=\left(\frac{1}{x-1}-\frac{1}{x+1}\right):\left(\frac{1}{x-1}+\frac{1}{x+1}-\frac{4}{x^2-1}\right)$$

III. 1. In a triangle ABC $(AB = AC = 10\ cm)$ and $m(\sphericalangle A) = 120°$. Determine the heights corresponding to the side AC.

2. On the plane of the rectangle $ABCD$ we raise the perpendicular MD so that $MA = 15\sqrt{2}\ cm, MB = 5\sqrt{34}\ cm$ and $MC = 25\ cm$. Determine its' sides and the distance from diagonal AC to the point M.

3. The axial section of a right circular cone is an equilateral triangle whose side is $8\ cm$. Calculate the total area and the volume of the cone.

Grading scale:	1 point *ex officio*			
	I.	1) 1 p	2) 1p	3) 1p
	II.	1) 1 p	2) 1p	3) 1p
	III.	1) 1 p	2) 1p	3) 1p

Test no. 96

I. Calculate:

1) $\left(\frac{1}{2}\right)^{-2}-\left(1-\frac{3}{2}\right)^{3}-\left(\frac{1}{0{,}125}\right)^{-1}$

2) $\frac{\sqrt{2}-1}{2\sqrt{2}}-\frac{\sqrt{2}+1}{2}+\frac{\left(3\sqrt{2}+\frac{1}{4}\right)^{2}}{4}$

3) $\frac{a}{b}$ knowing that $\frac{3a-2b}{a-b}=\frac{3}{2}$.

II. 1. Solve in $\mathbb{R}$:

a) $2(x+1)=3x+1$

b) $\frac{x-1}{3}<7+\frac{x+1}{5}$

2. Determine $a, b \in \mathbb{R}$ if $P(X)=aX^3+3X^2-bX$ is divisible to $X-1$ and $X+2$.

3. The following expression is given:

$$E(x)=\left(\frac{x^2-x}{x^2+1}+\frac{2x^2}{x^3-x^2+x-1}\right):\frac{x^2}{x^2-1}$$

a) Determine the set of existence of the expression

b) Write $E(x)$ as an irreducible fraction.

III. 1. Calculate the area of a right triangle with a cathetus of $8\ cm$ and an hypotenuse of $10\ cm$.

2. The area of the parallelogram $ABCD$ is $30\ cm^2, BC=5$. Determine the distance from B to AD.

3. In a regular quadrilateral pyramid the base apothem is $6\ cm$ and the pyramid apothem is $10\ cm$. Calculate:

a) the lateral edge

b) the lateral area, the total area and the volume

c) the angle between the plane of the base and a lateral face.

Grading scale: 1 point *ex officio*

I.	1) 1 p		2) 1p	3) 1p
II.	1) 0, 5 p + 0, 5 p		2) 1p	
		3) 0, 25p + 0, 75 p		
III.	1) 1 p		2) 0, 5 p	
		3) 0,5p + 0,5p + 0,5p		

Test no. 97

I. 1. Calculate:

a) $6^5 : (-6)^4 - (-3)^2 + 1^0 + (-3) \cdot (+2)$

b) $\sqrt{63} - 7\sqrt{3} + \sqrt{147} - \frac{7}{\sqrt{7}}$

c) $\frac{1}{8} + \left(\frac{5}{24} - \frac{1}{18}\right) : \frac{11}{6}$

2. Show that:

$(a-3)^2 + (a+2)^2 + (a+3)^2 + (a+4)^2 = (a-1)^2 + a^2 + (a+1)^2 + (a+6)^2$

3. Given $\frac{a}{b} = 0{,}8$, calculate: $\frac{2a+3b}{3b}$

II. 1. Solve in $\mathbb{R}$:

a) $2x - 5 = x - \frac{1}{2}$

b) $\begin{cases} \frac{1}{5}x - \frac{1}{2}y = -11 \\ \frac{1}{4}x - \frac{1}{15}y = 3 \end{cases}$

2. 7 workers finish a project in 18 days. In how many days do 9 workers finish the same project?

3. Calculate the area of ΔABC that has $AB = 5\ cm, AC = 12\ cm, BC = 13\ cm$.

III. Represent graphically in the same system of coordinates the graphics of the functions:

$f, g: \mathbf{R} \to \mathbf{R}$, $f(x) = x - 2$ and $g(x) = -2x + 1$.

2. A triangle ABC has BC contained in α. If $A \notin \alpha$ and $M \in AB, N \in AC$, establish the position of the straight line MN in relation to α, if:

a) $AM = 5cm, AN = 10cm, BM = 3cm, NC = 6cm$

b) $AM = 12cm, AN = 4cm, BM = 15cm, NC = 8cm$

Grading scale: 1 point *ex officio*

I.	1) 0, 5 p + 0, 5 p + 0, 5 p	2) 0, 75p	3) 0, 75p
II.	1) 0, 25 p + 0, 75 p	2) 1p	3) 1 p
III.	1) 1, 50 p	2) 0, 75 p + 0, 75 p	

Test no. 98

I. 1. Effectuate:

$$\frac{18}{5}\cdot\left[0,(6)+0,1(6)\right].$$

2. The sum of two consecutive natural numbers is 11. Determine the numbers.

3. Given $A=\{-2,0,2\}, B=\{-3,0,3\}$, calculate:

$A\cup B$, $A\cap B$, $A-B$, $B\setminus A$.

II. 1. Solve the system:

$$\begin{cases} 3y+x=1 \\ 2x+5y=3 \end{cases}$$

2. Determine the quotient and the remainder of the division of the polynomial:

$P(X)=X^4-2X^3-3X^2+4X+4$ to $X+1$.

3. Simplify the fraction:

$$\frac{4x+4}{12x+12}.$$

III. 1. Determine the area and the perimeter of an equilateral triangle with the side of $12\ cm$.

2. Let there be cube with the edge of $10\ cm$. Determine its diagonal, area and volume.

3. The lateral area of a right circular cone is $544\,\pi\ cm^2$ and $G=34\ cm$. Determine:

a) the area and the volume of the cone

b) the volume of the cone body obtained by sectioning the cone with a plane parallel to the base, taken through the middle of the cone's height.

Grading scale: 1 point *ex officio*

I.	1) 1 p	2) 1p	3) 1p
II.	1) 1 p	2) 1p	3) 1p
III.	1) 1 p	2) 1p	3) 0, 5 p + 0, 5 p

Test no. 99

I. 1. Given the sets:

$$A = \left\{ x \in \mathbf{R} \middle| -1 < \frac{4x+10}{2} < 13 \right\} \text{ and } B = \left\{ x \in \mathbf{R} \middle| 3 < \frac{2x+14}{2} < 10 \right\}$$

determine $A \cup B, A \cap B, A - B$.

2. Determine x from:

$$\frac{x}{\sqrt{10}} - \frac{4\sqrt{8}}{\sqrt{5}} = \frac{3\sqrt{20}}{5};$$

3. Trace the graphic of the function:

$\mathbf{f}: (1, 5) \to \mathbf{R}, f(x) = x - 3.$

II. 1. Find the solution $x \in \mathbb{Z}$ of the system:

$$\begin{cases} \frac{x+1}{2} - \frac{x+1}{3} \geq 0 \\ (x-1)(x+1) \leq (x-1)^2 \end{cases}.$$

2. A rhomb $ABCD$has $AB = 4\ cm$ and $m(\sphericalangle A) = 60°$. In D we raise a perpendicular on the plane so that $MD = 4\ cm$. Let P be the middle of MD and Q the middle of MB. Determine:

a) the distance from M to the straight line BC

b) the distance from M to the straight line AC

c) the angle formed by (APQ) and (ABC)

3. Determine $x \in \mathbb{Z}$, so that $x^2 = a$, where:

$$a = 8,4 : 0,8 + 3 \cdot \left(3\frac{1}{2} - 1,25 \cdot 2\right) - \left(\sqrt{5} - \frac{2,5}{\sqrt{5}}\right) \cdot \sqrt{5} - 2$$

III. 1. Determine the height and the area of ΔABC equilateral with $AB = 10$.

2. An angle is equal to $2/3$ from its' complement. Determine the angles.

3. Determine the rapport between the lowest common denominator and the least common multiple of the numbers 144 and 720.

Grading scale: 1 point *ex officio*

I.	1) 1 p	2) 1p	3) 0, 50 p
II.	1) 0, 50 p	2) 0, 75p	3) 1p
III.	1) 1 p	2) 0, 50 p	3) 0, 75 p

Test no. 100

I. 1. Calculate:

$$[(5\cdot36+5\cdot44):5+(12\cdot81+19\cdot12):100]:92$$

2. A number is $12,56$ times smaller than another. Determine the numbers knowing that their sum is 1356.

3. Solve and discuss the equation in relation to the parameter $m \in \mathbb{R}$:

$$m^2x-1=3mx+4x$$

II. 1. In ΔABC, the bisectors of the angles formed by the median $[AM]$ with the side $[BC]$ intersects the other two sides on P and Q. Prove that $PQ||BC$.

2. Determine the function:

$$f: \mathbf{R} \rightarrow \mathbf{R},\ f(5-2x)=4x+a$$

and then trace the graphic of the function if it passes through $M(3,3)$.

3. Let A, B, C, D be four non coplanar points, with $AB = AC = AD = BC = CD = BD$. On the segments $[AB], [BC], [CD]$ and $[AD]$ we consider M, N, P, Q so that $AM = BN = CP = DQ$. If the point O is the middle of $[NQ]$, show the $NQ \perp (MOP)$.

III. 1. Calculate the arithmetic and geometric mean of the numbers:

$$p=\left(2-\sqrt{3}\right)^2 \text{ and } q=\left(2+\sqrt{3}\right)^2.$$

2. Determine the rapport between the area of a square with the side a and the area of the square that has the first's square diagonal as its side.

3. Calculate the area of a trapezoid where the middle line has $12\ cm$ and the height of $6\ cm$.

Grading scale: 1 point *ex officio*

I.	1) 0, 75 p	2) 0, 75 p	3) 0, 75 p
II.	1) 1, 25 p	2) 1, 25 p	3) 1, 25 p
III.	1) 1 p	2) 1 p	3) 1 p

Test no. 101

1. If:

$$\frac{5x-7}{2}-\frac{2x+7}{3}=3x-4,$$

the solution of the equation is:

A: $-1/2$; B: 7; C: 1.

2. If:

$$m=17+12\sqrt{2},\ n=17-12\sqrt{2},$$

the geometric mean of the numbers is:

A: 12; B: 1/5; C: 1.

3. If:

$$x=0{,}03:10^{-2}+\sqrt{500}-\frac{50}{\sqrt{5}},\ y=2^{-1}:\left(-\frac{1}{2}\right)^2-0{,}5,$$

$(x+y)^2$ is:

A: $\frac{1}{4}$; B: $\frac{13}{2}$; C: $\frac{81}{4}$.

4. In ΔABC acute angled, AD the height and $\frac{DC}{DB}=\frac{1}{2}$. If the parallel through C at AB intersects AD in E. Then the parallel through E passes through:

A: *the middle of* B , $B: N \in (BC)$ where $CN=\frac{1}{4}BC$; $C: P \in (BC)$ so that $\frac{BP}{BC}=\frac{1}{5}$.

5.On the plane of ΔABC equilateral we raise the perpendicular $DA=12$. We trace $CE \perp AB, E \in (AB)$ and $EF \perp DB$, $F \in (DB)$. If $AB=12$, then the area ΔCEF is:

A: $9\sqrt{6}$; B: $9\sqrt{3}$; C: $6\sqrt{3}$ cm^2.

6. The graphic of:

$f: [-2, 2] \setminus \{0\} \to \mathbf{R},\ f(x)=\sqrt{3}x-5$ is:

A: finite set of points; B: semiright; C: a reunion of segments.

7. Let $ABCD$ regular $(AB = 10, BC = 8)$ and $OA \perp (ABC)$ with $OA = 6$. We note with E the middle of OC. Then the distance from O to (ABE) is:

A: $\frac{39}{5}$; B: 15; C: $\frac{24}{5}$.

8. A set X has 10 elements, Y has 8 elements. If $X \cup Y$ has 12 elements, then $X \cap Y$ has:

A: 10 elements; B: 6 elements; C: 8 elements.

9. The biggest common denominator of the numbers 1450, 144, 6006 is:

A: 2; B: 15; C: 6.

10. The bisector of supplement of the $45°$ angle determines with its sides angles of:

A: 105°; B: 10°3′; C: 67°30′.

Grading scale: 1 point *ex officio*
1) 1 p
2) 1 p
3) 1 p
4) 1 p
5) 1p
6) 1p
7) 1 p
8) 1 p
9) 0, 5 p
10) 1, 5 p

Test no. 102

1. If $m = 0,4$ and $n = 2,5$ then their arithmetic mean is:

A: $1\frac{9}{20}$; B: $\frac{2}{5}$ C: $\frac{2}{29}$.

2. The solution of the equation:

$7\left(2x - \frac{2}{3}\right) - 2(3x - 3{,}5) = 9$ is:

A: 2; B: $\frac{5}{6}$; C: 4,2.

3. Determine the value of $m \in \mathbb{R}$ if $M(2, 3 - m) \in G_f$ where:

f: $\mathbf{R} \to \mathbf{R}$, $f(x) = (m-1)x - m + 3$.
A: -2; B: -1; C: 0; D: 1.

4. If the hypotenuse of a right triangle is 13 and one of the cathetus has $5\ cm$, then the projection of the other cathetus on the hypotenuse is:

A: 8 cm; B: $\frac{144}{13}$ cm; C: 7,5 cm.

5. Let $ABCD$ be right ($AB = 8, BC = 6$) and $MC \perp (ABC), MC = \frac{24}{5}$. The distance from M to BD is:

A: 10; B: $\frac{24\sqrt{2}}{5}$; C: 8,5.

6. Let $ABCD$ be a rhombus, $A' \notin (ABC)$. If the angles formed by AA' with AD and AB are congruent and $A'P \perp (ABC)$, then the points A, P, C are:

A:colinear; B: non-colinear.

7. The solution of the system:

$$\begin{cases}(x+1)(y-1) = (x+5)(y-5) \\ (x+2)(y+3) = (x+4)(y-1)\end{cases}$$ is:

A: (3, 2); B: (−4, 2); C: (−1, 0).

8. Compare $m = 9^{17}$ and $n = 27^{11}$. Then:

A: m = n; B: m > n; C: m < n.

9. If the angles of a triangle are proportional to $4, 5, 9$, then the triangle is:

A: equilateral; B: isosceles; C: right.

10. A set A has 12 elements, another set B has 10 elements and $A \cap B$ has 5 elements. Then $A \cup B$ has:

A: 13; B: 15; C: 22.

Grading scale: 1 point *ex officio*

1) 0, 5 p
2) 1 p
3) 1 p
4) 1 p
5) 1p
6) 1p
7) 1 p
8) 1 p
9) 1 p
10) 0, 5 p

Test no. 103

1. The product of all the integer numbers between -100 and $+100$ is:

A: a very large number that cannot be calculated; B: 0; C: 129 560.

2. $\sqrt{2}$ with an approximation of a thousand is:

A: 1,414; B: 1,420; C: 1,415.

3. The decomposition in factors of the polynomial:

$x^2 + 2xy + y^2 - 4$ is:

A: $(x + y - 2)(x + y + 2)$; B: $(x + y - 2)^2$; C: $(x + y - 4)(x + y + 1)$.

4. The smallest natural number, not equal to zero, that, once divided to $2, 3, 5$ gives the same remainder, different to zero is:

A: 30; B: 31; C: 61.

5. The graphic of the function: $f: \mathbb{R} \to \mathbb{R}, f(x) = 2x + 3$ intersects the axis OX in the abscissa point:

A: -3; B: $-\frac{3}{2}$; C: $\frac{3}{2}$.

6. If: $\frac{2a+3b}{5a+2b} = \frac{3}{4}$ then $\frac{a}{b}$ is:

A: $\frac{1}{7}$; B: $\frac{7}{6}$; C: $\frac{6}{7}$.

7. If a right circular cone has $V = 100\pi\ cm^3$ and $h = 12\ cm$ then the total area is:

A: 90π; B: 95π; C: 65π.

8. The area of the triangle with the sides $2\sqrt{3}\ cm, 2\ cm$ and $4\ cm$ is:

A: 4; B: $2\sqrt{3}$; C: $4\sqrt{3}$.

9. The quadrilateral that has the tops in the middles of the sides of a rhombus is:

A: right; B: square; C: parallelogram.

10. Let there be the cube $ABCDA'B'C'D'$ with the side $AB = 10\ cm$. Calculate:

a) the total area and the volume of the cube

b) the area and the volume of the pyramid $B'A'BC'$

c) one of the trigonometric function of the angle formed by BB' with the plane $A'BC'$.

Grading scale: 1 point *ex officio*

1) 0, 75 p
2) 0, 75 p
3) 1 p
4) 1 p
5) 0, 5 p
6) 1p
7) 0, 75 p
8) 0, 75 p
9) 0, 5 p
10) a) 0, 5 p b) 1 p c) 0, 5 p

Test no. 104

1. Making the calculations:

$-10+10:(-2)-\sqrt{1024}$

we obtain:

A: -32; B: 0; C: 47.

2. In the interval $(-4,2]$ we find:

A: 6 integer numbers; B: an infinity of integer numbers; C: 5 integer numbers.

3. The equation:

$X \cap \{5, 7, 8\} = \{5, 7\}$ has:

A: a single solution; B: two solutions; C: an infinity of solutions.

4. $\sqrt{(\sqrt{7}-\sqrt{5})^2}$ is:

A: $\sqrt{7}-\sqrt{5}$; B: $\sqrt{5}-\sqrt{7}$; C: $\sqrt{2}$.

5. In a right triangle the cathetus that opposes the $30°$ angle is:

A: half of the hypotenuse; B: a quarter of the hypotenuse; C: congruent with the other cathetus.

6. The area of the equilateral triangle with the side of $3\ cm$ is:

A: $9\sqrt{3}$; B: $\frac{9\sqrt{3}}{4}$; C: $\frac{9\sqrt{3}}{2}$.

7: The system:

$$\begin{cases} 3x+y=9 \\ x+2y=8 \end{cases}$$

has the solution:

A: $(2, 4)$; B: $(2, 3)$; C: $(3, 2)$.

8. The remainder of the division of the polynomial:

$P(X)=2X^2\sqrt{3}+X-7$ to $X-1$ is:

A: $2\sqrt{3}$; B: $2\sqrt{3}-7$; C: $2\sqrt{3}-6$.

9. If $A = \{1, 2, 3\}, B = \{0, 1, 2, 3, 4\}$ then card $(A \times B)$ is:

A: 15; B: 3; C: 4.

10. Let $VABC$ be a regular triangular pyramid with $AB = 6\ cm, VA = 5cm$. Calculate:

a) the total area and the volume of the pyramid;

b) the area ΔVAM where M is the middle of BC

c) a trigonometric function of the angle between the lateral edge and the plane of the base.

Grading scale: 1 point *ex officio*

1) 0,5p
2) 0,5p
3) 0,75p
4) 0,75p
5) 0,5p
6) 1p
7) 1p
8) 1 p
9) 1p
10) a) 1p b) 0,5p c) 0,5p

Test no. 105

I. 1. Calculate:

$$\left(-1\frac{1}{5}\right)\cdot\left(-2\frac{1}{2}\right)+\left(-\frac{1}{5}\right)^{98}:\frac{1}{5^{29}}.$$

2. Divide the no. 180 in three parts so that: the second number is half of the first and $2/3$ of the third.

3. Rationalize:

$$\frac{1}{\sqrt{2}-3};\frac{2}{2\sqrt{5}-\sqrt{19}}.$$

II. 1. Given the fraction:

$$F(x)=\frac{4x^3-32}{x^3+(x+2)^3}:$$

a) Show that $F9x)=2-\frac{6}{x+1}$;

b) For what real values x, $F(x)$ is defined;

c) For what integer values of $x, F(x)$is integer.

2. Let there be the function $f:\mathbb{R}\to\mathbb{R}$, given by $f(x)=ax+b$, whose graphic passes through the points $A(1,0)$ and $B(0,1)$. Determine the form of the function and represent it graphically.

III. 1. In a right triangle ABC with $m(\sphericalangle A)=90°$ we know the catheti $AB=6\ cm, AC=8\ cm$. Calculate the height corresponding to the hypotenuse, the area of the triangle and the radius of the circumscribed circle.

2. The body of a regular quadrilateral pyramid has the diagonal of $9\ m$ and the sides of the bases of $7\ m$ and $5\ m$. Calculate the lateral area, the total area and the volume of the body.

Grading scale: 1 point *ex officio*

I.	1) 1 p	2) 1p	3) 1p
II.	1) a) 1 p; b) 0, 50 p; c) 0, 40 p	2) 1 p	
III.	1) 1, 50 p	2) 1, 50 p	

Test no. 106

I. 1. Calculate:

a) $\left(\frac{1}{324}-\frac{7}{150}-\frac{17}{240}\right)\cdot\left(0{,}5-\frac{1}{2}\right)$

b) $(-0{,}1)^3\cdot(-1{,}2)^2\cdot(-10)^6$.

2. Determine the numbers x, y, z, knowing that:

$x-y=\frac{y+z}{4}=\frac{z}{3}$ and that: $x+2z=24$

3. Let there be the numbers:

$a=\sqrt{3}+\sqrt{2}$ and $b=\sqrt{3}-\sqrt{2}$.

Calculate the arithmetic, geometric and harmonic means.

II. 1. We consider the following number:

$a=\frac{1}{\sqrt{3+\sqrt{5}}}+\frac{1}{\sqrt{3-\sqrt{5}}}$.

a) Show that $a^2 \in \mathbb{Q}$;

b) For what values of the real parameter m, the polynomial:

$P(X)=16X^4+8X^2+m$ divides by: $X-m$.

2. Let $ABCD$ be a parallelogram. We consider M, N, P, Q the middles of the sides AB, BC, CD, DA.

The straight lines AN, BP, CQ, DM determine a parallelogram.

What is the rapport between the area of this parallelogram and the area of the initial one?

III. 1. On the plane of the right triangle ABC, with the catheti $AB = 3\ cm, AC = 4m$ we raise a perpendicular $AM = 24\ m$. Calculate the distance from M to BC and the measure of the angle corresponding to the dihedral angle formed by the planes ABC and MBC.

2. In a right circular cone with the diameter of the base of $12\sqrt{2}\ cm$ and the height equal to $6\ cm$, we inscribe a cube, in such

a way that one of its' bases will be in the interior of the cone's plane and the tops of the other base will be situated on the conic layer:

a) Calculate the lateral area and the volume of the cone

b) Calculate the volume of the cube.

Grading scale: 1 point *ex officio*

I.	1) a) 0, 50 p b) 0, 50 p	2) 1p	3) 1p
II.	1) a) 0, 75 p; b) 0, 75 p; c) 0, 40 p	2) 1, 50 p	
III.	1) 1, 50 p	2) 1, 50 p	

Test no. 107

1. Let:

$$a = \frac{8 - 4{,}7 : \left(5 - 0{,}8 : 2\frac{4}{6}\right) + \frac{4}{5} \cdot 1{,}25}{\left(5\frac{3}{9} - 3\frac{3}{4}\right) : 1\frac{7}{12} + 2} \cdot \frac{3}{8}.$$

Specify if:

A: $a < 1$; B: $a = 1$; C: $a > 2$.

2. Determine card $(X \cup Y)$ if:

$$X = \left\{ n \in \mathbf{Z} \middle| \frac{12}{2n+3} \in \mathbf{Z} \right\} \text{ and } Y = \left\{ n \in \mathbf{Z} \middle| \frac{6}{n+3} \in \mathbf{Z} \right\}$$

The result is: $A: 9; B: 8; C: 4.$

3. The harmonic mean of the numbers:

$E_1 = 5\sqrt{6} - 6\sqrt{24} + 7\sqrt{54} - 2\sqrt{244} + 3$ and

$E_2 = \sqrt{\left(2 - \sqrt{5}\right)^2} - \left|3 - \sqrt{5}\right| - 2\sqrt{5} + 6$ is:

$A: 2,5; B: 4; C: 2.$

4. If $\frac{x}{2} = \frac{y}{3}$ the value of the rapport $\frac{6x-4y+3z}{3x-2y+z}$ is:

$A: \frac{3}{2}; B: 8; C: 3.$

5. The solution of the equation $2(x-1) = \frac{7x-3}{2}$ is:

$A: \frac{1}{3}; B: -\frac{1}{3}; C: 3.$

6. What term should be added to the expression $x^2 + \frac{1}{25}$ to obtain the square of a binomial?

$A: \frac{2x}{5}; B: \frac{x^2}{5}; C: \frac{1}{5}.$

7. The lengths of a triangle's sides are: $10, 24, 26\ cm$. Its area is:

A: $1224\ cm^2$; B: $181\ cm^2$; C: $1500\ cm^2$.

8. An angle is $28°$ smaller than its complement. How much do they measure?

A: $30°, 60°$; B: $31°, 59°$; C: $28°, 62°$.

9. In a n equilateral triangle with the side $10\ cm$, the height corresponding to a side is:

A: 5 cm; B: $3\sqrt{5}$ cm; C: $5\sqrt{3}$ cm.

10. A body is shaped like a $40\ cm$ long cylinder, continued with two hemispheres at each ending, of the same radius as the cylinder. If $R = 30\ cm$, the total area is:

A: $6000\pi\ cm^2$; B: $5400\pi\ cm^2$; C: $600\pi\ cm^2$.

Grading scale: 1 point *ex officio*
1) 1p
2) 1p
3) 1p
4) 1p
5) 0,75p
6) 0,75p
7) 0,75 p
8) 0, 5p
9) 1p
10) 1p

Test no. 108

1. Calculate:

$$\left\{[0,2+0,(2)]\cdot\frac{9}{19}+\frac{4}{5}\right\}^3+1.$$

The right answer is:

A: $\frac{1}{19}$; B: 2; C: 9.

2. Calculate:

$$\sqrt{\left(2-\sqrt{3}\right)^2}+\sqrt{\left(\sqrt{2}-\sqrt{3}\right)^2}+\sqrt{\left(4-\sqrt{17}\right)^2}-\sqrt{\left(4-\sqrt{17}\right)^2}$$

We obtain:

A: $2-\sqrt{2}$; B: $2-\sqrt{3}$; C: 2.

3. The system:

$$\begin{cases}5(3x+y)-8(x-6y)=200\\200(2x-3y)-13(x-y)=520\end{cases}$$

has the solution:

A: (20, 3); B: (7, 2/3); C: (21, 1).

4. From $20\ kg$ of grapes $12\ l$ of wine are produced. How much wine can be produced from $5100\ kg$?

A: 3 000 l; B: 3 060 l; C: 3 100 l.

5. If:

$$P(X) = X^4 - X^3 + 2X^2 - nX + 4,$$

$n \in \mathbb{R}$ is divisible by $X+1$, then n is:

A: –8; B: 8; C: 10.

6. Let:

$$f: \mathbf{R} \to \mathbf{R},\ f(x) = (m-1)x + 5.$$

Knowing that its graphic passes through $A(1,3)$, then m is:

A: –2; B: –1; C: 2.

7. An isosceles trapezoid has the bases of 12 and $6\ cm$, the acute angle measuring $60°$. Its perimeter is:

A: 32; B: 24; C: 30.

8. The triangle ABC has the area $168\, cm^2$. If AM is a median, the area ΔAMB is:

A: 42 cm^2; B: 84 cm^2; C: 112 cm^2.

9. The total area of a right circular cylinder is $132\pi\ cm^2$, and the lateral area $96\pi\ cm^2$. Its volume is:

A: $144\sqrt{3}$ cm^3; B: 132π cm^3; C: $144\sqrt{2}$ cm^3.

10. The diagonal of a right parallelepiped with the dimensions $12, 10$ and $6\ cm$ is:

A: $2\sqrt{70}$; B: 28; C: $4\sqrt{7}$.

Grading scale: 1 point *ex officio*
1) 1p
2) 0,75p
3) 1p
4) 1p
5) 1p
6) 0,75p
7) 1p
8) 0,75p
9) 1p
10) 0,75p

Test no. 109

1. If $A - B$ has 5 elements, $B - A$ has 7 elements and $A \cup B$ has 14 elements, how many elements does $A \cap B$ have?

A: 5; B: 2; C: 10.

2. In a classroom there are 40 students out of which 55% are girls. How many boys are there in the classroom?

A: 22; B: 18; C: 20.

3. An angle is $28°$ smaller than its complement. How much do they measure?

A: 30° ,60°; B: 31°, 59°; C: 30°, 58°.

4. The solution of the equation:

$$\frac{1}{2} - \frac{5x-4}{4} = \frac{x}{2} - 1 - \frac{x+1}{3} \text{ is:}$$

A: 2; B: −2; C: 0.

5. In a triangle ABC:

$BC = 9$ cm, $AA' = 8$ cm $(AA' \perp BC)$ and

$BB' = 10$ cm $(BB' \perp AC)$.

The side AC measures:

A: 6 cm; B: 12 cm; C: 7,2 cm.

6. Knowing that the difference between the radius of the circumscribed circle and the radius of the inscribed circle in the equilateral triangle is $15\ cm$, its side is:

A: $15\sqrt{3}$; B: $30\sqrt{3}$; C: $15\sqrt{2}$.

7. The elements of the set:

$$A = \left\{ x \in Z \middle| x - 1 \le \frac{2x-1}{3} < x + 1 \right\} \text{ are:}$$

A: $\{-3, -1, -2, 0, 1, 2\}$; B: $\{-2, -1, 0, 1\}$; C: $\{-3, -2, -1, 0, 1\}$.

8. Show that the polynomials $P(X), Q(X)$ that verify the relation:

$$(X^2 - X + 1)\,P(X) + (X^2 + X - 1)\,Q(X) = 2$$

and are of first degree are:

A: $P(X) = X + 2$, $Q(X) = -X + 2$; B: $P(X) = -X + 2$, $Q(X) = X + 2$;
C: $P(X) = X - 1$, $Q(X) = X + 1$.

9. The weighted arithmetic mean of the numbers $25, 36, 52, 48$ with the weights $3, 6, 2$ and 4 is:

A: 41; B: $39\frac{2}{15}$; C: $38\frac{2}{15}$.

10. A regular triangular pyramid with the height $4\sqrt{3}\ cm$ and the angle formed by a lateral face and the plane of the base of $60°$, has the lateral area and the volume:
A: $100\ cm^2$, $200\ cm^3$; B: $95\sqrt{3}\ cm^2$, $196\ cm^3$; C: $96\sqrt{3}\ cm^2$, $192\ cm^3$.

Grading scale: 1 point *ex officio*
1) 0,75p
2) 0,75p
3) 0,75p
4) 0,75p
5) 1p
6) 1p
7) 1p
8) 1p
9) 1p
10) 1p

Test no. 110

1. The simple form of the fraction:

$$F(x)=\frac{x^2-25}{x^2-10x+25} \text{ is:}$$

A: $\frac{x+5}{5}$; B: $\frac{x+5}{x-5}$; C: $\frac{x-5}{x+5}$.

2. The triangle ABC has the area $168\, cm^2$ and AM the median. What is the area of ΔAMB?

A: 48 cm^2; B: 84 cm^2; C: 112 cm^2.

3. By calculating:

$$\sqrt{2}+\sqrt{8}+\sqrt{18}+...+\sqrt{200}$$

we obtain:

A: $10\sqrt{2}$; B: $60\sqrt{2}$; C: $55\sqrt{2}$

4. We mix water of $70°$ with water of $20°$. What is the rapport of the quantities used so that the mixture has $40°$?

A: 1; B: 2/3; C: 1/2.

5. A parallelogram $ABCD$ has the area $120\sqrt{3}\, cm^2$ and the sides AB, AD have the lengths 15 and $16\, cm$. What is $m(\sphericalangle A)$?

A: 60°; B: 45°; C: 120°.

6. The result of the division:

$$\frac{x^2-y^2}{4xy}:\frac{x-y}{2y} \text{ is:}$$

A: $\frac{2x}{x+y}$; B: $\frac{2xy}{x+y}$; C: $\frac{x+y}{2x}$.

7. The graphic of the function:

$$f: \mathbf{R}\to\mathbf{R},\ f(x)=\frac{3x+7}{3} \text{ is:}$$

A: a segment; B: a straight line; C: a semi-straight line.

8. A right circular cylinder has $R=5\, cm, h=8\, cm$. The area of the axial section is:

A: 80 cm^2; B: 90 cm^2; C: 40 cm^2.

9. The sum of the digits of the number:

$10^{n+1} + 2 \cdot 10^n + 4$ is:

A: 14; B: 7; C: 2n.

10. A right circular cone has $R = 15\ cm, h = 36\ cm$.

a) Determine the total area of the cone

b) At what distance from the top do we section with a parallel plane to the base so that the generating line of the body has $26\ cm$?

c) Determine the rapport of the volumes of the two bodies.

Grading scale: 1 point *ex officio*

1) 0, 5 p
2) 0, 5 p
3) 1 p
4) 1, 5 p
5) 1 p
6) 1 p
7) 1 p
8) 1 p
9) 0, 5 p
10) 1, 5 p

Test no. 111

I. Calculate:

1) $25 : 5 + 13 : 13 - 24 \cdot 2 : 8$

2) $2\sqrt{6} \cdot \left(2\sqrt{50} - 3\sqrt{48}\right)$

3) $8 \cdot (-1)^3 \cdot \left[(-2)^5 - (-3)^3\right]^2$

II. 1. Determine the numbers $x \in \mathbb{Z}$ for which $\frac{6}{x-2} \in \mathbb{Z}$.

2. Determine $A = \{x \in \mathbb{R} \mid |x| < 3\}$.

3. Let:

$f: \mathbf{R} \to \mathbf{R}, f(x) = (m-3)x + \frac{m-1}{5}$.

If $(91, -2) \in G_f$, determine $m \in \mathbb{R}$.

III. 1. If the sides of a triangle are directly proportional to $3, 4, 5$ and its perimeter is $48\ cm$, determine the area of the triangle.

2. Calculate $2/3$ out of 540.

3. A right parallelepiped has $L = 10\ cm$, $l = 8\ cm, h = 12\ cm$. Determine the volume, the total area and its diagonal.

Grading scale: 1 point *ex officio*

I.	1) 0, 50 p	2) 1p	3) 1p
II.	1) 1 p;	2) 1 p	3) 1 p
III.	1) 1 p	2) 0, 50 p	3) 0, 50 p; 1 p; 0, 50 p

Test no. 112

I. Effectuate:

1) $\left[0,(5):0,1(5)-3\frac{1}{2}\right]:\frac{1}{7}-\frac{1}{3}$

2) $\left[2^{-2}+(-2)^{-2}\right]\cdot\left[(-1)^{10}+(-1)^{-10}\right]$

3) $(x^2-2x+1)+(2x^2+x-3)-(3x^2-x+4)$

II. 1. Let there be the sets:

$A=\{2x-3;x+1;3x-1\}$ and $B=\{2x-1;2x+1;4x-7\}$.

Determine $x \in \mathbb{N}$ so that $A = B$.

2. Determine the remainder of the division of:

$P(X)=X^2-2X+3$ to $X-1$.

3. Show that, for any $x \in \mathbb{R}$:

$x^2-4x+5>0$.

III. 1. In ΔABC isosceles, $AB = AC = 30\ cm$ and $BC = 12\sqrt{5}\ cm$. Determine the lengths of the heights.

2. Determine three numbers directly proportional to $4, 5, 6$ that have the arithmetic mean 25.

3. A regular triangular pyramid has the height $12\ cm$ and the angle between the lateral edge and the plane of the base is $30°$. Determine: the volume, the total area, the sine of the angle between a lateral face and the base, the distance from one top of the base to the opposite face.

Grading scale: 1 point *ex officio*

I.	1) 1 p	2) 1p	3) 0, 50 p
II.	1) 1 p;	2) 0, 50 p	3) 0, 50 p
III.	1) 1 p	2) 1 p	3) 0, 5 p + 1 p+0, 5 p + 0, 5 p

Test no. 113

I. Effectuate:

1) $3 \cdot [5 + (2 + 3 \cdot 8) : 13]$

2) $0,5 \cdot 0,75 \cdot 5\frac{1}{3}$

3) $\frac{11}{14} + \frac{2}{7} \cdot \frac{21}{20}$

II. Determine the unknown number:

1) $\frac{x}{2} = \frac{10}{12}$; 2) $5(x-1) = 2x - 3$; 3) $x - 2 < 3x + 5$.

III. 1. Effectuate:

$(x+1)^2 - 2(x-1)(x+1) + (x-1)^2$.

2. In a right triangle, a cathetus has $8\ cm$ and the hypotenuse $10\ cm$. Determine the height corresponding to the hypotenuse and the area of the triangle.

3. Determine the volume and the total area for a regular triangular prism knowing that it has the height of $10\ cm$ and the side of the base of $4\ cm$.

Grading scale: 1 point *ex officio*

I.	1) 0, 50 p	2) 0, 50 p	3) 1 p
II.	1) 0, 50 p;	2) 0, 50 p	3) 1 p
III.	1) 1 p	2) 2 p	3) 2 p

Test no. 114

I. 1. Calculate:

$-9 + 9 \cdot 3^{-1} + \sqrt{1{,}21}$.

2. Let there be the sets:

$A = \{x \in \mathbf{N} \mid 3(x + 1) \leq 2x + 7\}$, $B = (-\infty, 2]$.

Calculate $A \cup B$ and $A \cap B$.

3. If $\frac{a}{b} = \frac{2}{3}$, calculate $\frac{2a+3b}{5a-b}$.

II. 1. Determine a, b prime numbers, so that $2a + 3b = 21$.

2. Let there be the polynomial:

$P(X) = 5X^4 - 3X^2 + mX - 1$.

a) Find $m \in \mathbb{R}$, knowing that the remainder of the division of $P(X)$ to $X + 2$ is 7;

b) For m determined, determine the sum of the coefficients.

3. Given the function:

$f: \mathbf{R} \to \mathbf{R}$, $f(x) = 2x + 4$.

a) Represent the function graphically;

b) Determine $a \in \mathbb{R}$, so that $A(-1, a)$ belongs to the graphic.

III. 1. We consider $ABCD$ a rhombus and M, N, P, Q the middles of its' sides.

a) Establish the nature of $MNPQ$;

b) Determine the rapport of the areas of $ABCD$ and $MNPQ$.

2. Let $ABCDA'B'C'D'$ be a cube with the side of $6\ cm$ and M, N the middles of the segments AA' and BD'.

a) Calculate the total area and the volume of the cube;

b) Where is MN situated in relation to BD?

3. Given a right circular cone with $R = 3\ cm$ and $H = 4\ cm$, determine:

a) The lateral area;

b) The volume of the cone.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p
II.	1) 1p;	2) 0,5p + 0,5p	3) 0,5p + 0,5p
III.	1) 0,5p + 0,5p	2) 0,5p + 0,5p	3) 0,5p + 0,5p

Test no. 115

1. By calculating:

$$-\frac{3}{5}+\frac{3}{5}:\frac{9}{10}$$

we obtain:

A: 0; B: $\frac{3}{50}$; C: $\frac{1}{15}$.

2. $\sqrt{(\sqrt{2}-\sqrt{7})^2}$ is equal to:

A: $\sqrt{2}-\sqrt{7}$; B: $\sqrt{7}-\sqrt{2}$; C: $\sqrt{5}$.

3. The number $2^{100}-2^{99}-2^{98}$ is equal to:

A: 2^{98}; B: 2^{-97}; C: cannot be calculated.

4. If card $A=3$, card $B=5$ and $A\cap B=\emptyset$, then card $A\cup B$ is:

A: 8; B: 5; C: 3.

5. If $\frac{a}{b}=\frac{5}{8}$ then $\frac{2a+3b}{7a+b}$ is:

A: $\frac{34}{40}$; B: $\frac{34}{43}$; C: $\frac{17}{20}$.

6. We consider the polynomial:

$P(X)=5X^3-7X^2+3X+8$.

Then, $P(\sqrt{2})$ is:

A: $10\sqrt{2}+3$; B: $13\sqrt{2}-6$; C: $3\sqrt{2}$.

7. A right circular cone has $A_t=90\pi, A_l=65\pi$. Its height is:

A: 12 cm; B: 13 cm; C: 10 cm.

8. The area of a parallelogram $ABCD$ with $AB=5\ cm, AD=3\ cm$ and $m(\sphericalangle A)=45°$ is:

A: $\frac{15\sqrt{2}}{4}$; B: $\frac{15\sqrt{2}}{2}$; C: $\frac{15\sqrt{3}}{2}$.

9. The perimeter of an isosceles trapezoid $ABCD$ with the bases $AB=10\ cm, CD=4\ cm$ and $m(\sphericalangle A)=60°$ is:

A: 38 cm; B: 26 cm; C: 20 cm.

10. The total area of a right parallelepiped with the sides $3\ cm, 5\ cm, 7\ cm$ is:

A: 142 cm^2; B: 144 cm^2; C: 71 cm^2.

Grading scale: 1 point *ex officio*

1) 1 p
2) 1 p
3) 1 p
4) 0, 75 p;
5) 1 p
6) 1 p
7) 0, 75 p
8) 0, 75 p
9) 1 p
10) 0, 75 p

Test no. 116

1. The set of solutions of the equation $3(x-2) = 2x+7$ is:

A: ϕ; B: 1; C: 13.

2. Effectuating the product $(x+y)(x^2 - xy + y^2)$ we obtain:

A: $x^3 + y^3$; B: $x^3 - y^3$; C: grade II polynomial.

3. $\sqrt{144+25}$ is:

A: 13; B: 17; C: 15.

4. Given the sets :

$A = \{x \in N \mid x < 10\}$, $B = \{x \in N \mid x \text{ divide } 12\}$, $A \cap B$ is:

A: ϕ; B: $\{1, 2, 3, 4, 6\}$; C: $\{2, 3, 4, 6\}$.

5. The system:

$\begin{cases} x+2y=3 \\ 2x+4y=6 \end{cases}$ has:

A: one solution; B: no solution; C: an infinity of solutions.

6. A right parallelepiped with the lengths of the sides of $2, 3, 4\ cm$ has the volume:

A: 24 cm^3; B: 8 cm^3; C: 26 cm^3.

7. $VABCD$ regular quadrilateral pyramid with the side of the base $4\ cm$ and the apothem $4\ cm$. The measure of the dihedral angle of two opposing lateral faces is:

A: $90°$; B: $60°$; C: another solution.

8. We consider the triangle ABC and AD the height. If $BD = 4\ cm, CD = 2\ cm, AD = 3cm$, then the distance from C to AD is:

A: $\frac{18}{6}$; B: $\frac{18}{5}$; C: $\frac{12}{5}$.

9. Let there be the rhombus $ABCD$ with $AB = 5\ cm$ and $m(\sphericalangle A) = 60°$. Then the length of the diagonal AC is:

A: $5\sqrt{3}$; B: 4; C: $6\sqrt{3}$.

10. Consider the polynomial:

$$P(X) = (X - 2)^{100} - 7X + 15.$$

The sum of its coefficients is:

A: a number that cannot be counted; $B: 0$; $C: 8$.

Grading scale: 1 point *ex officio*

1) 1 p
2) 0, 75 p
3) 0, 75 p
4) 1 p;
5) 0, 5 p
6) 1 p
7) 1 p
8) 1 p
9) 1 p
10) 1p

Test no. 117

I. 1. Effectuate:

$$[0{,}6 - 0{,}(3) - 0{,}6 \cdot \left(\frac{1}{2^3} : 0{,}25\right) + 1/3] : 0{,}1 - 0{,}2 \cdot 5.$$

2. Determine the numbers a, b, c knowing that they are directly proportional to the numbers $4, 6, 12$ and that their product is 360.

3. Rationalize the denominator:

$$\frac{5}{\sqrt{2}}; \frac{\sqrt{8}}{2\sqrt{5}}; \frac{3}{\sqrt{5} - \sqrt{7}}.$$

II. 1. Show that the following inequality takes place:

$$x^2 + y^2 + z^2 \geq xy + yz + xz;\ x, y, z \geq 0.$$

2. Let there be the polynomial:

$$P(X) = X^2 - 2X - 8.$$

a) Determine the remainder of the division of the polynomial through $X + 2$ and $X - 4$ without making the divisions

b) Simplify:

$$\frac{X^3 + 8}{X^2 - 2X - 8}$$

III. 1. The un-parallel sides BC and AD of the trapezoid $ABCD$ intersect in the point M. Calculate the lengths of the segments MA, MB, MC and MD knowing that $AB = 20\ cm, BC = 6\ cm, CD = 15\ cm$ and $DA = 8\ cm$.

2. In a regular triangular pyramid, we know that the side of the base is $5\sqrt{3}\ cm$ and the height is $6\ cm$. Calculate:

a) the lateral area, the total area and the volume of the pyramid

b) the area and the volume of the cone circumscribed to this pyramid.

Grading scale: 1 point *ex officio*

I.	1) 1 p	2) 1 p	3) 1 p
II.	1) 1 p	2) a) 1 p b) 1 p	
III.	1) 1 p	2) a) 1 p b) 1 p	

Test no. 118

I. 1. Effectuate:

$$\left[\frac{1}{1,2(27)}+\frac{(0,5)^2}{0,(6)}-\frac{22}{27}-0,375+1\right]^{1996}.$$

2. Knowing that three numbers are directly proportional to $5, 7$ and 11 and that the difference between the biggest and the smallest number is 42, determine the three numbers.

3. Calculate:

$$\left(\sqrt{2}+\frac{3}{\sqrt{2}}\right)\cdot\sqrt{2}:\sqrt{3}\left(\frac{2}{\sqrt{3}}-\sqrt{3}\right)$$

II. 1. Show that the following inequality takes place:

$$\frac{a^4+b^4}{a^2b^2}\geq 2;\ a,b\geq 0.$$

2. If the polynomial:

$$X^4+4aX^3+6bX^2+4cX+d$$

is divisible with:

$$X^3+3aX^2+3bX+c,$$

show that the first is the square and the second the cube of a binomial.

3. Given the function:

$$f:\mathbf{R}\to\mathbf{R},\ f(x)=mx+n,$$

determine m and n knowing that f passes through the points $A(1,2), B(-1,0)$ and make the graphic representation.

III. 1. The diagonals of a trapezoid $ABCD(AB||CD)$ intersect in the point N. Determine the lengths of the segments NA, NB, NC and ND knowing that $AB = 20\ cm, CD = 10\ cm, AC = 21\ cm$ and $BD = 12\ cm$.

2. The total area of a regular quadrilateral pyramid is $14,76\ cm^2$ and the total area is $18\ cm^2$. Calculate the volume of the pyramid.

3. Calculate the area and the volume of the sphere that has area of the big circle $9\,\pi\ cm^2$.

Grading scale: 1 point *ex officio*

I.	1) 1 p	2) 1 p	3) 1 p
II.	1) 1 p	2) 1 p	3) 1 p
III.	1) 1 p	2) 1 p	3) 1 p

Test no. 119

I. 1. Effectuate:

$$\left(\frac{1}{2}\right)^3 \cdot \left[3{,}331-(1+0{,}1)^3\right]^2.$$

2. The arithmetic mean of three numbers is 20. The first number is three times smaller than the second and the arithmetic mean between the second and the third is 25. Determine the numbers.

3. Calculate:

$$\left(\frac{2}{5\sqrt{2}}-\frac{1}{\sqrt{22}}+\frac{3}{\sqrt{75}}\right):\left(2\sqrt{3}\right)^{-1}.$$

II. 1. Verify if the following inequality takes place for $xy = 1$:

$$\left(x^2+\frac{y^2}{x^2}\right)\left(y^2+\frac{x^2}{y^2}\right)\geq 4$$

2. a) Show that the polynomial:

$$P(X) = X^4 - X^3 - 6X^2 + 4X + 8$$

is divisible by the polynomials $Q(X) = X - 2$ and $R(X) = X + 2$.

b) Simplify the function:

$$F(x) = \frac{x^4 - x^3 - 6x^2 + 4x + 8}{x^4 - 16}.$$

III. 1. In a right triangle with the hypotenuse of $2\ dm$, the measure of the angle btween the height and the median, traced from the top of the right angle is $30°$. Determine the length of the height traced from the top of the right angle.

2. A pyramid with the base $ABCD$ rectangle, with $AB = 2a, BC = a$, the height $SD = 2a$. On the edge SBwe take the point P at the middle.

a) Show that the triangle APC is isosceles and calculate its area.

b) Calculate the lateral area of the pyramid and its volume.

Grading scale: 1 point *ex officio*

I.	1) 1 p	2) 1 p		3) 1 p
II.	1) 1 p	2) a) 1 p	b) 1 p	
III.	1) 1 p	2) a) 1 p	b) 1 p	

Test no. 120

I. 1. Effectuate:

$$\left\{\frac{111}{500}:\left(\frac{1}{5}+\frac{1}{50}+\frac{1}{500}\right)+[0,(14)+0,(27)]\cdot\frac{198}{41}\right\}:0{,}06.$$

2. Determine three numbers that are directly proportional to $7, 4, 12$ and knowing that the sum of the two biggest ones is 380.

3. Calculate:

$$\left(\frac{2}{3\sqrt{3}}+\frac{1}{\sqrt{3}}-\frac{8\sqrt{3}}{6}\right):\left(-\frac{9}{7\sqrt{3}}\right)$$

II. 1. Solve the system of inequations:

$$\begin{cases} 3(x-2)\le 7(x-1)-5 \\ 3x-4\le 9(x-2)+3 \end{cases}.$$

2. Given the polynomial:

$$P(X)=X^4+mX^3-7X^2+nX+p.$$

Determine m, n, p, knowing that $P(X)$ divides by $X-1, X-2$ and $X-3$, then solve the equation $P(X)=0$.

3. In a right triangle the projections of the catheti on the hypotenuse have $7\ cm$ and $63\ cm$. Determine the length of the height traced from the top of the right angle.

III. 1. Calculate the lateral area, the total area and the volume of the cube whose diagonal has $10\sqrt{3}\ cm$.

2. A pyramid has as base a right trapezoid $ABCD\ (AB||BC, \sphericalangle A = 90° = \sphericalangle B), AD = a, BC = 2a, AB = 2a$. The height VO falls in O the middle of AB, and $VO = a$. Calculate:

a) the areas of the triangles VAB, VAD and VBC.

b) the volume of the pyramid.

Grading scale: 1 point *ex officio*

I.	1) 1 p	2) 1 p	3) 1 p
II.	1) 1 p	2) 1 p	3) 1 p
II.	1) 1 p	2) a) 1 p b) 1 p	

Test no. 121

I. 1. Effectuate:

$$\left\{\left[\frac{4}{3}-0,(3)\right]^2 : 0.25\right\} : \left(\frac{37}{100} : 0,0925\right) + 12,5 + 0,64.$$

2. A jacket costs $576.000\ lei$. After two consecutive sales the price of the jacket got to $400.000\ lei$. Knowing that the sales have been proportional to the new prices, calculate the price of the jacket after the first sale.

3. Calculate:

$$3\cdot\frac{2}{5}-\left(\frac{4}{\sqrt{9}-1}+\frac{\sqrt{27}}{\sqrt{3}}-\frac{4}{9}\right)$$

II. 1. Solve the system of inequations:

$$\begin{cases} 9+\dfrac{4x-11}{7}<\dfrac{x-3}{5} \\ 1-\dfrac{1-x}{-1}>\dfrac{-x-1}{-1} \end{cases}.$$

2. Determine the set of values of the real parameter a for which the following polynomials are prime one to the other:

$P(X) = X^3 - 2X^2 - X + 2$ and $Q(X) = X^3 - X^2 + 4X + a$

3. Determine the length of the height from the top of the right angle in the right triangle with the hypotenuse of $13\ cm$ and the rapport of the catheti $4/9$.

III. 1. On the plane of the rhombus $ABCD$, with the side equal to $18\ cm$ and $m(\sphericalangle D) = 120°$ we raise the perpendicular $AP = 9\ cm$. Determine the distances from the point P to BC, BD and CD.

2. A pyramid has congruent lateral edges and forms with the plane of the base $45°$ angles. The base is an isosceles trapezoid with the acute angles of $60°$ and the bases 6 and $8\ cm$. Calculate:

a) the radius of the circle circumscribed to the trapezoid

b) the volume of the pyramid.

Grading scale: 1 point *ex officio*

I.	1) 1 p 2) 1 p 3) 1 p
II.	1) 1 p 2) 1 p 3) 1 p
III.	1) 1 p 2) a) 1 p b) 1 p

Test no. 122

I. 1. Effectuate:

$$\left[0{,}25+\left(\frac{7}{30}-\frac{1}{24}\right):\left(\frac{2}{3}+0{,}5\right)\right]:0{,}1(6)-1:\frac{7}{2^4}.$$

2. If $\frac{3a}{5b}=\frac{2}{7}$, calculate $\frac{7a-2b}{7a+2b}$.

3. Calculate:

a) $\sqrt{\left(1-\sqrt{5}\right)^2}+\sqrt{\left(\sqrt{5}-3\right)^2}$;

b) $\sqrt{18+2\sqrt{17}}+\sqrt{18-2\sqrt{17}}$.

II. 1. Determine the values of x that simultaneously simplify the inequations:

$3-\frac{3-7x}{10}+\frac{x+1}{2}>4-\frac{7-3x}{5}$ and

$7(3x-6)+4(17-x)>11-5(x-3)$

2. a) Given the polynomial:

$P(X)=2X^2-X-3$.

Calculate $P(-1)$.

b) Simplify the fraction:

$$\frac{(x-1)^2-(x-2)^2}{(x+1)(2x-3)}.$$

III. 1. On the plane of the rectangle $ABCD$ with $AB=16\ cm, BC=12\ cm$ we raise the perpendicular $MD=12\ cm$. Determine:

a) The distance from the point M to the straight line AB and BC

b) The distance from D to the plane (MBC)

2. The axial section of a cone is an isosceles triangle with a $18\ cm$ perimeter and the lengths of the segment that unites the

middles of the un-parallel sides of $4\ cm$. In it we make a section parallel to the plane of the base at $2/3$ from the height in relation to the top. Calculate:

a) the total area and the volume of the cone

b) the total area and the volume of the cone body

Grading scale: 1 point *ex officio*

I. 1) 1 p 2) 1 p 3) a) 0, 50 p b) 0, 50 p

II. 1) 1 p 2) a) 1 p b) 1 p

III. 1) a) 0, 75 p b) 0, 75 p 2 a) 0, 75 p b) 0, 75 p

Test no. 123

I. 1. Effectuate:

$$\left(1+\frac{13}{25}\right):\left(0{,}24+\frac{1}{75}\right)+\frac{1}{3}\cdot\left(\frac{1{,}17}{3}-0.03\right)$$

2. Knowing that $\frac{x}{y}=0,(3)$, calculate the value of the expression:

$$\frac{3x-7y}{5x+4y}$$

3. Calculate:

$$\frac{\sqrt{6}+\sqrt{2}}{\sqrt{3}+1}+\frac{\sqrt{27}+\sqrt{45}}{\sqrt{3}+\sqrt{5}}+\frac{\sqrt{75}-\sqrt{54}}{5-3\sqrt{2}}.$$

II. 1. Determine the elements of the set:

$$X=\{x\in\mathbf{Z}\mid x-2\le\frac{2x-3}{3}<x\}.$$

2. Show that $(X-1)^2$ divides $P(X)=X^4-3X+3$.

3. Solve the system:

$$\begin{cases}2\sqrt{2}x+y\sqrt{3}=7\\3\sqrt{2}x-4\sqrt{3}y=-6\end{cases}$$

III. 1. A parallelepiped $ABCDA'B'C'D'$ has the dimensions $AB=3\ cm, BC=4\ cm$ and $AA'=12\ cm$. Calculate:

a) the length of the diagonal of the parallelepiped, its area and its volume

b) the distance from C to AC'

2. The body of a cone has the generating line of the base of $26\ cm$, the radius of the big base of $15\ cm$ and the height of $24\ cm$.

a) Determine the lateral area and the volume of the cone body

b) Calculate the volume of the cone where the body issues from

Grading scale: 1 point *ex officio*

I.	1) 1 p 2) 1 p 3) 1 p
II.	1) 1 p 2) 1 p
III.	1) a) 0, 75 p b) 0, 75 p 2 a) 0, 75 p b) 0, 75 p

Test no. 124

1. The numbers $a = 2^{46}, b = 3^{31} - 9^{15}$ have the relation:

A: a < b; B: a = b; C: a > b.

2. Which of the equalities is false?

A: $\frac{1-\sqrt{3}}{2+\sqrt{3}} = \frac{(1-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$; B: $\frac{1-\sqrt{3}}{2+\sqrt{3}} = \frac{5-3\sqrt{3}}{2^2-\sqrt{3}^2}$;

C: $\frac{1-\sqrt{3}}{2+\sqrt{3}} = 5-3\sqrt{3}$; D: $\frac{1-\sqrt{3}}{2+\sqrt{3}} = -5+3\sqrt{3}$.

3. Which of the inequalities is false?

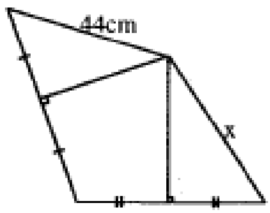
A: $a^3 \le a^2 < a$; B: $a \le \sqrt{a} \le \frac{1}{a}$; C: $a^2 \le \sqrt{a} \le \frac{1}{a^2}$; D: $\frac{1}{a} < \sqrt{a} < a$.

4. Knowing that $x, x-2, x+2$ can be the sides of a right triangle, then its hypotenuse can be:

A: 9; B: 8; C: 10; D: 6.

5. In the figure below, x measures:

A: 40 cm; B: 42 cm; C: 43 cm; D: 44 cm.



6. Given the polynomial $P(X) = 4X^2 - aX + b$, if $P(1) = P(2) = 1$, then the equation $\frac{P(X)}{2X-3} = 1$ has the solution:

A: {3}; B: {2}; C: {4}; D: {−2}.

7. Let:

$$E(x) = \frac{3x-2}{9x^2-12x+4} - \frac{2x}{3x^2+2x} + \frac{4}{4-9x^2}.$$

The values a for which $E(a) \in \mathbb{Z}$ are:

A:{−1/3, −2}; B: {−1/2, −1}; C: {−1/3, −1}; D: {0, 1}

8. On the plane of ΔABC equilateral we raise MA, NC, perpendicular, with $MA = 2NC$. The following affirmation is false:

A: ΔMNC right; B: $MA \perp BC$; C: $[MB] \equiv [MC]$; D: $NA = NB$.

9. In a conic glass with the top at the bottom, with the radius $6\ cm$ and the height $8\ cm$, we introduce a ball with the radius $1\ cm$ and we pour water until the ball is covered and no more. The height of the water is:

A: 7/3 cm; B: 8/3 cm; C: 3 cm; D: 4 cm.

10. If $ABCD$ is a regular tetrahedron, and I, J, K, L the middles of the edges $[AC], [AD], [BC], [BD]$, the following affirmation is false:

A: $IJLK$ rhombus; B: $CD \subset$ in the mediating plane $[AB]$; C: $BC \perp AD$; D: $KI||BD$.

Grading scale: 1 point *ex officio*

1) 1 p
2) 1 p
3) 1 p
4) 1 p;
5) 0, 5 p
6) 1 p
7) 1 p
8) 0, 5 p
9) 1 p
10) 1p

Test no. 125

1. The number:

$$n = (3^{n+2} \cdot 4^{n+3} + 3^{n+3} \cdot 2^{2n+4}) : 12^{n+2}$$

has the value:

A: 6; B: 7; C: 8; D: 12.

2. If:

$$\frac{3}{x} = \frac{4}{y} = \frac{5}{8} = \frac{6}{z},$$

then (x, y, z) is:

A: (4,8; 6,4; 9,6); B: (40; 60; 90);

C: (30; 40; 60); D: (2,4; 3,2; 4,8).

3. What is the probability that by randomly replacing n with a number, the number $\frac{8}{n}$ is a natural number?

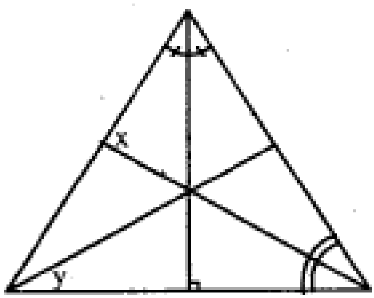
A: 1/3; B: 4/10; C: 3/10; D: 4/9.

4. The number $-\sqrt{2}$ belongs to the interval:

A: $(-\sqrt{2}, 5]$; B: $[-1{,}41; 2)$; C: $(-\sqrt{3}; -1{,}41]$; D: $(-6, -\frac{2}{\sqrt{2}}]$

5. In the triangle below, the measures of the angles x and y are:

A: 122°, 41°; B: 121°, 42°; C: 123°, 41°; D: 124°, 42°.



6. The following proposition is false:

A: The equation straight line $x + 5 = 0$ is perpendicular on Ox;

B: The equation straight line $y = -1$is perpendicular on Oy;

C: The straight lines $2x = 3$ and $3y - 1 = 0$ are perpendicular;

D: The straight lines $y = 2x + 3$ and $y = 2x - 3$ are perpendicular.

7. If $\Delta ABC, \Delta ACD, \Delta ADB$ right and isosceles (A, B, C, D non-coplanar), and I is the middle of $[BC]$, then DI has:

A: $2\sqrt{6}$; B: $\sqrt{6}$; C: 6; D: $3\sqrt{6}$.

8. In three identical cones there is liquid up to the heights $h, 2h, 3h$ (starting from the top). Their volumes can be:

A: V, 2V, 3V; B: V, 4V, 9V; C: V, 6V, 9V; D: V, 8V, 27V.

9. Let $ABCDEFGH$ be a cube with $I, J, K, L, M, N, O, P, R, S, T$ the middles of $AB, BC, CD, DA, AE, BF, CG, DH, EF, FG, GH, HE$. The following straight lines are not perpendicular:

A: AE and NO; B: EF and OJ; C: AH and IF; D: MN and SI.

10. In $ABCDEFGH$ parallelepiped, $ABCD$ a square with $AB = 10$ and the height $AE = x$. If $m(\sphericalangle EBO) = 60°$, x will be:

A: 9; B: 10; C: 11; D: $10\sqrt{2}$.

Grading scale: 1 point *ex officio*

1) 0, 75 p
2) 0, 75 p
3) 1 p
4) 0, 50 p;
5) 1 p
6) 1 p
7) 1 p
8) 0, 5 p
9) 1 p
10) 1p

Test no. 126

I. 1. Let :

$$A=\left\{-5;\frac{4}{2};-\frac{6}{3};-\frac{8}{\sqrt{4}};-\frac{7}{7};\frac{-12}{-2};\sqrt{3};\frac{1}{5};-\sqrt{2};\frac{0}{3};\sqrt{100}\right\}.$$

Calculate :

a) $A\cap\mathbf{N}$; b) $A\cap\mathbf{Z}$; c) $A-\mathbf{Q}$.

2. Solve :

a) $2x=7,\ x\in\mathbf{Q}$; b) $-3x+1=4,\ x\in\mathbf{Z}$;

c) $\begin{cases}x+y=5\\ x-y=11\end{cases},\ x, y\in\mathbf{R}$; d) $|x-1|\leq 2$.

3. Determine $x\in\mathbb{Z}$ so that:

a) $\frac{7}{x}\in\mathbf{N}$; b) $\frac{3}{x-3}\in\mathbf{Z}$; c) $\frac{x+5}{x-2}\in\mathbf{N}$.

II. 1. A student reaches school in 15 minutes if he walks, in 6 minutes with the bicycle and 2 minutes with the taxi. If, on foot, the speed of the student is $6\ km/h$, determine the average distance with the bicycle and the taxi.

2. In $ABCD$ right trapezoid $(m(\sphericalangle A)=m(\sphericalangle D)=90°)$, $\mathrm{AC}=10\ \mathrm{cm}$, $\mathrm{BD}=8\sqrt{2}\ \mathrm{cm}$. If AC and BD intersect in o, determine the distance from O to AD when $AB=8\ cm$.

3. Let there be the sets:

$$A=\left\{(x,y)\middle|y>\frac{1}{2}x+1\right\}\text{ and }B=\left\{(x,y)\middle|y<\frac{1}{2}x+1\right\}.$$

a) Represent graphically:

$f:\mathbf{R}\to\mathbf{R},\ f(x)=\frac{1}{2}x+1$;

b) Cross out B and establish which of the following points belong to B:

$M_1(2,\ 3)$, $M_2(0,\ 0)$, $M_3(0,\ 1)$, $M_4(0,\ 2)$, $M_5(-2,\ 0)$,

$M_6(-3,\ 0), M_7\left(\sqrt{2}, \sqrt{2}\right),\ M_8\left(\frac{\sqrt{2}}{2}, 2\right)$

c) Verify that:

$\mathbf{R} \times \mathbf{R} \setminus A \cup B = G_f$.

III. 1. In $\Delta ABC, m(\sphericalangle B) = 45°, m(\sphericalangle C) = 60°, AD \perp BC, D \in (BC), CE \perp AB, E \in (AB)$. If $BD = 6\ cm$, determine $\sin A \cos A$.

2. On the plane ΔABC we raise AA', BB' perpendiculars. If $AB = 6\ cm, AA' = 8\ cm,$ determine $BB' = x$, so that $\Delta A'B'C'$ right in A'.

3. Let $ABCDEFGH$ be a right parallelepiped with the base $ABCD$a square, $AB = 10\ cm/$

a) If $AC \cap BD = \{0\}$, then $OE \perp BD$;

b) If $AE = 10\ cm$, calculate $\sin \sphericalangle AOE$ and $\cos \sphericalangle EBO$.

c) If $m(\sphericalangle AOE) = 60°$, determine AE.

d) If $AE = 8\ cm$, determine the sum of all the edges, the area and the volume of the parallelepiped.

Grading scale: 1 point *ex officio*

I.	1) 1p	2) 1p	3) 1p
II.	1) 1p	2) 1p	3) 1p
III.	1) 1p	2) 1p	3) 1p

Entry examination in the 9th grade, high school

Session: July 1991

I. 1. Calculate:

$$(-0{,}5)^3 + (-1)^4 + \frac{5}{2} \cdot \sqrt{0{,}0025} - \left(\frac{1}{\sqrt{5}} + \sqrt{5}\right) \cdot \sqrt{5}.$$

2. Given the sets:

$A = \{-1, 0, 1\}$ and $B = \{x \mid x \in \mathbf{R}$ şi $|x| \geq 1\}$

Write the set B as a reunion of intervals and then effectuate $A \cap B$.

3. Solve the inequation:

$$\frac{\sqrt{2}x - 1}{2x^2 + 2 - 2\sqrt{2}x} > 0.$$

4. a) What is the condition that a sum of non-negative numbers is zero?

b) Solve the equation:

$|x^2 - 9| + |2x - 6| = 0.$

II. 1. Let there be the functions:

$f: [-5, 2] \to \mathbf{R}$, $f(x) = -x - 3$ and $g: [2, 5] \to \mathbf{R}$, $g(x) = x + 1$.

a) Decide if the point $A(-2, -1)$ belongs to one of the graphics of the functions f or g;

b) Represent graphically, in the same system of coordinate axes the functions f or g;

c) For $x \in [-5, 2) \cap [2,5]$, solve the system of equations:

$$\begin{cases} x - y = -1 \\ x + y = -3 \end{cases}$$

2. Simplify the fraction:

$$E(x) = \frac{(x^2 + 2x + 3)^2 - 5(x^2 + 2x + 3) + 6}{x^3 + 3x^2 + 2x}.$$

III. 1. Let $ABCD$ be a rhombus with the side equal to a and the acute angle $A = 60°$. In the point A we raise the perpendicular $AM = \frac{3a}{2}$ on the plane (ABC). Determine:

a) the volume of the right prism with the rhombus $ABCD$ as base and the height equal to AM;

b) the plane angle of the dihedral formed by the planes (ABC) and (BMD);

c) the distance from the point M to the straight line BC;

d) the volume of the pyramid with the top in B and the base AMD.

Entry examination in the 9^{th} grade, high school

Session: July 1992

I. Calculate:

1) $\frac{3}{5}+\frac{3}{5}:\frac{6}{5}+\frac{1}{2}$;

2) $2^{-1}:\left(-\frac{1}{2}\right)^2-0{,}5+\frac{1}{\sqrt{3}-\sqrt{2}}-\left(\sqrt{3}+\sqrt{2}\right)$.

II. 1. Solve the equation:

$2x+3=-5,\ x\in\mathbf{R}$.

2. Given $\frac{a}{b}=0{,}6$, calculate $\frac{2a+3b}{3b}$.

3. Solve in $\mathbb{R}$ the system of equations:

$$\begin{cases} x+2[y-3(x-1)]=25 \\ y-3[x-2(y+1)]=0 \end{cases}$$

III. Given the following sets, determine $A\cap B$ and $A\cup B$:

$A=\left\{x\in\mathbf{R}\left|\frac{1-3x}{-2}<3\right.\right\}$ and $B=\{x\in\mathbf{N}^*|x \text{ divides } 12\}$.

IV. We consider the linear function:

$f:\mathbf{R}\to\mathbf{R},\ f(x)=x+1$ and $x_1\neq x_2$.

Show that:

a) $f(x_1)\neq f(x_2)$; b) $\frac{f(x_1)+f(x_2)}{2}=f\left(\frac{x_1+x_2}{2}\right)$.

V. Show that for any $a\in\mathbb{R}$, the following number is a perfect square:

$(a^3+3a^2+a)\cdot(a^3+3a^2+a+2)+1$

VI. Effectuate:

$\left(\frac{3}{x-2}-\frac{2}{x+2}-\frac{10}{x^2-4}\right)\cdot\frac{x^2-4x+4}{x}$

VII. In the rectangle $ABCD$ with $AB = 4\ cm$ and $BC = 3cm$ we trace the diagonal AC. Determine the height DI of the triangle ADC and the cosine of the angle $\sphericalangle CDI$.

VIII. The base of the pyramid $VABCD$ is the rhombus $ABCD$ with $AB = a\sqrt{3}, BD = 2a$ and the height $VD = \frac{2a\sqrt{2}}{3}$. Calculate:

1) the total area and the volume of the pyramid;
2) the angle formed by the planes (VBC) and (ABC);
3) the distance from the top D to the face (VBC).

Entry examination in the 9th grade, high school

Session: July 1993

I. Calculate:

a) $(-4)^2-(-2)^3-4^2$;

b) $\frac{1}{5}-\frac{1}{5}:\frac{2}{3}$;

c) $(x+2)(2-x)-(x-1)^2$;

d) the biggest divisor of the polynomials:

$9-X^2$; X^2-4X+3; X^2-6X+9;

e) the area of the equilateral triangle ABC, knowing that $AB=8\sqrt{3}\ cm$.

2. Solve:

a) $3(x-2)-(-6+x)(-2)=-18$;

b) $3x^2-2x-1=0$;

c) $-4x+8\le 0$.

3. Given:

$$E(x)=\left(\frac{1}{x-3}+\frac{9-x^2}{x^2-x+1}\cdot\frac{1}{x-3}\right):\frac{10-x}{x^3+1}-2$$

a) Verify if $E(x)=\frac{7-x}{x-3}$;

b) Write the real numbers for which $E(x)$ hasn't a defined value.

4. The cube $ABCDA'B'C'D'$ has the side measuring $20\ cm$ and M is the middle of $[CC']$.

a) Calculate the total area and the volume of the cube;

b) Calculate the volume of the pyramid $D'ABD$;

c) Calculate the area of the triangle $D'MA$;

d) Calculate the distance from the point M to the plane determined by the point D' and the straight line BD.

Grading scale: 1 point *ex officio*

1. a) 0,50p b) 0,50p c) 0,50p d) 0,75p e) 0,75p
2. a) 0,50p b) 0,50p c) 0,50p
3. a) 0,75p b) 0,75p
4. a) 0,50p b) 0,50p c) 0,50p d) 0,50p

Entry examination in the 9th grade, high school

Special Session: June 1994

I. Calculate:

a) $(5-15)^2-(5^2-15^2)+\sqrt{25^2-15^2}$;

b) $\left[\frac{1}{5}+0,(3)\right]\cdot\left\{\left(\frac{1}{2}\right)^2-\frac{1}{2}\cdot 0,(3)+[0,(3)]^2\right\}$;

c) $P(1-\sqrt{2})$, where $P(X)=X^2-2X-1$.

II. 1. A right circular cone and a right circular cylinder have equal radiuses and heights. Determine the value of the rapport of the volumes of the two figures.

2. The square $ABCD$ is given, with $AB = 4\ cm$. Let there be the points M and N on the segments $[AB]$ and $[DC]$ respectively, so that $AM = CN = 3\ cm$. Calculate: a) the area of the quadrilateral $MBND$; b) the distance from point B to the straight line MD.

III. 1. Given the function:

$f: \mathbf{R} \to \mathbf{R}$, $f(x) = 2x - 3$.

a) Represent the given function graphically;

b) Determine the numbers $m \in \mathbb{R}$, for which $f(m) \leq \frac{1-m}{-2}$.

2. The polynomial $P(X)$ is divided by the polynomial $X^2 - 4$ and we obtain the quotient $(X^3 = X + 1)$. Determine the polynomial $P(X)$ knowing that $P(2) = 6$ and $P(-2) = 2$.

IV. 1. A regular quadrilateral pyramid has the side of the base of $8\ cm$ and the height $3\ cm$.

Calculate:

a) the volume of the pyramid;

b) the lateral area of the pyramid.

2. A regular triangular prism has the lateral edge of $9\ cm$ and the edge of the base is $2/3$ of the height.

Calculate:

a) the lateral area of the prism;

b) the volume of the prism;

c) the distances from the center of one of the lateral faces to the other faces of the prism.

Grading scale: 1 point *ex officio*

I. a) 1p b) 0,50p c) 0,5p;

II. 1. For expressing the volumes (0,5p), end result (0,5p);

2. a) 0, 75 p b) 0, 25 p ;

III. 1. a) 1 p b) 1 p;

2. For the schema of the theorem of division (0,25p), end result (0,25p);

IV. 1. a) The figure (0, 25 p), end result (0, 5 p)

b) 0, 50 p

2. a) 0,50p b) 0,15p c)4-0,15p=0,60p

Entry examination in the 9th grade, high school

Session: July 1994

I. 1. Calculate:

a) $\frac{16}{81}-\frac{16}{81}\cdot\frac{9}{4}:\frac{4}{9}$;

b) $0{,}006\cdot(-10)^3+\frac{33}{0{,}5}\cdot10^{-1}$.

2. In a plane reported to a system of orthogonal axes xOy, represent graphically the function:

$f: \mathbf{R}\to\mathbf{R},\ f(x)=\frac{1}{3}x-0{,}5$.

3. Given the following sets, calculate $A\cap B$:

$A=\{x \mid x\in\mathbf{R} \text{ and } 4x-3(x+1)\geq 0\}$ and

$B=\{x \mid x\in\mathbf{R} \text{ and } 2(x+1)-8\leq 0\}$.

4. In the right triangle ABC with the right angle in $A, AB = 6\ cm, AC = 8\ cm$. a) Calculate the lengths of the hypotenuses $[BC]$, of the height $[AD]$ and of the segment $[BD]$; b) If the parallel through D to the straight line AC intersects the cathetus $[AB]$ in E, determine the rapport of the areas of the triangle BED and BAC.

II. 1. Show that:

a) the following number is integer:

$n=\sqrt{63}-7\sqrt{3}+\sqrt{147}-2\sqrt{7}-\frac{7}{\sqrt{7}}$

b) the following numbers have the sum $2\sqrt{3}$ and the product 2:

$|1-\sqrt{3}|$ and $|\sqrt{3}+1|$

2. The arithmetic mean of three numbers is 7. Determine one of these numbers knowing that the arithmetic mean of the other two is $5{,}50$.

3. Determine the radius, the height, the volume and the lateral area of a right circular cone, for which the deployment of the lateral surface is a disk sector with a $120°$ angle and a radius of $9\ cm$.

III. 1. Let there be the polynomial:

$$P(X) = a^2X^4 - 2abX^3 + b^2X^2 + a^2X - 2a + 1.$$

Determine $a, b \in \mathbb{Z}$ so that $P(X)$ is divisible by $X - 1$.

2. Simplify:

$$E(x) = \frac{2x^2 - 4x - 6}{x^3 - x^2 - 5x - 3}.$$

3. Calculate the volume of a regular quadrilateral pyramid knowing that its height is $h = 6\ cm$ and the rapport between the lateral area and the area of the base is $k = 3$.

Grading scale: 1 point *ex officio*

$I.$ 1) $0,50\ p$ 2) $0,75\ p$ 3) $0,75\ p$ 4) $1p$

$II.$ 1) $1,25\ p$ 2) $1\ p$ 3) $0,75p$

$III.$ 1) $0,50\ p$ 2) $0,75\ p$ 3) $1,75p$

Grading and notation scale :

I. 1. $a)$ $0,25\ p$ $b)$ $0,25\ p$

2. The table of values of the Ist order function $(0,50p)$;

Graphic representation $(0,25p)$.

3. Writing the set A as a real numbers interval $(0,25p)$;

Writing the set B as a real numbers interval $(0,25p)$;

$A \cap B(0,25p)$

4. a) BC 0,10p, AD 0,15p, BD 0,25p;

b) $\Delta BED \sim \Delta BAC$ and the rapport of similarity $(0,25p)$;

the rapport of the areas of the two triangles $(0,25p)$;

II. 1. a) $n = 0(0,75p)$ b) the sum $(0,25p)$; c) the product $(0,25p)$;

2. The formula of the arithmetic mean of three numbers $(0,50p)$;

The sum of two numbers is the double of the arithmetic mean and replacing the three numbers in the arithmetic mean $(0,25p)$;

Solving the equation $(0,25p)$;

3. The radius of the cone $(0,25p)$, the height $(0,20p)$, the volume $(0,20p)$, the lateral area $(0,10p)$;

III. 1. Calculating $P(1)(0,15p)$, writing the sum of the squares equal to zero $(0,25p)$, determining a, b $(0,20p)$;

2. Decomposing the nominator in irreducible factors $(0,25p)$, of the denominator $(0,25p)$, simplification $(0,25p)$.

3. $A_{lat} = \dfrac{P_b \cdot A_p}{2}$ (0,25p); A_b (0,25p);

the rapport of the two areas $(0,25p)$.

$\left(\dfrac{h_{pir}}{a_b}\right)^2$ (0,50p);

determining the apothem and the volume of the pyramid $(0,50p)$.

Entry examination in the 9th grade, high school

Session: July 1995

I. 1. Calculate:

a) $4-4\cdot(-5)+2\cdot(-8)$;

b) $\left[3\cdot\left(\frac{1}{3}\right)^{-1}-1:\frac{1}{3}+(0{,}5)^2\right]:2\frac{1}{2}$;

c) $\left[\frac{7}{2\sqrt{6}}-\frac{6}{\sqrt{24}}+\frac{5}{\sqrt{54}}\right]\cdot\left(\frac{13}{\sqrt{216}}\right)^{-1}$.

2. A rectangle with the perimeter of $153{,}6\ cm$ has the dimensions proportional to the numbers 7 and 9. Determine the dimensions of the rectangle.

3. We consider the right triangle $BAC(m(\sphericalangle A)=90°)$, having $AB=6\ cm$ and $AC=8\ cm$. Through the middle M of the cathetus $[AB]$ we trace a parallel to AC that intersects the hypotenuse in N. Determine:

a) the height of the triangle $ABCD$ corresponding to the hypotenuse;

b) what per cent of the ABC triangle's area represents the area of the triangle MBN.

4. Determine the volume of a right circular cone that has the total area of $96\pi\ cm^2$ and the lateral area $60\pi\ cm^2$.

II. 1. Determine the irreducible fraction that it can be brought to:

$$E(x)=\left(\frac{x^2-x}{x^2-1}-\frac{2x^2}{1-x+x^2-x^3}\right):\frac{x^2}{x^2-1}$$

2. Consider the function:

$f\colon \mathbf{R}\to\mathbf{R},\ f(x)=mx+2m,\ m\in(-3,+\infty)$.

a) Determine m so that the point $A(m, 3)$ belongs to the graphic of the function f;

b) Represent graphically the function $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x + 2$.

3. Determine the elements of the set:

$$A = \left\{ x \in \mathbb{Z} \,\middle|\, \sqrt{\left(\sqrt{2}+1\right)^2}\, x = 6 - \sqrt{6 - 4\sqrt{2}}\, x \right\}$$

III. 1. The remainder of the division of the polynomial $P(X)$ by $X^3 - 2$ is equal to the square of the quotient. Determine this remainder knowing that:

$$\mathbf{P(-2) + P(2) + 34 = 0}$$

2. We consider the regular triangular pyramid $SABC$ having the height of $\sqrt{2}\ cm$, the edge of the base of $2\sqrt{3}\ cm$ and the points M, N, P respectively, the middles of the edges $[AB], [BC], [CA]$ of the base.

a) Calculate the lateral area and the volume of the pyramid;

b) If $SM \perp SN$ show that $SP \perp (SMN)$.

Grading and notation scale:

I. 1. a) 0, 50 p b) 0, 50 p c) 0, 50 p

2. The formula of the rectangle's perimeter and the proportionality $(0, 25p)$; end result $(0, 25p)$;

3. a) The hypotenuse $(0, 25p)$, the height corresponding to the hypotenuse $(0, 25p)$;

b) $(0, 25p)$;

4. The radius of the base $(0, 10p)$, the generating line $(0, 15p)$, the height $(0, 25p)$, the volume $(0, 25p)$;

II. 1. Decomposing the nominator in factors and the denominator of the first fraction $(0, 25p)$, the smallest common multiple and the amplification $(0, 25p)$, the calculations of the denominator of the obtained fraction from the brakets $(0,25p)$;, end result $(0,25p)$;

2. a) $f(m) = m^2 + 2m = 3$, $m^2 + 2m - 3 = 0$ (0,25p),

$(m-1)(m+3) = 0$ $(0,25p)$, solving the equation $(0,25p)$. b) the pair of points $(0,25p)$, the graphic $(0,25p)$.

3. Eliminating the first radical and the square $(0,25p)$, writing the number under the second radical as a square $(0,25p)$, eliminating the radical and the square, reducing the similar terms $(0,25p)$, the solution of the equation and the end result $(0,25p)$.

III. 1. $P(X) = (X^3 - 2)\cdot Q(X) + Q^2(X)$ (0,20p),

grad $Q^2 < 3 \Rightarrow$ grad $Q = 1$ and $Q(X) = aX + b$, $a, b \in R$ (0,10p),

$P(X) = (X^3 - 2)(aX + b) + (aX + b)^2$ (0,15p),

$P(-2) + P(2) + 34 = 0 \Leftrightarrow (-2a + b - 5)^2 +$
$+ (2a + b + 3)^2 = 0$ (0,20p),

the system $\begin{cases} -2a + b - 5 = 0 \\ 2a + b + 3 = 0 \end{cases}$ and the solution (0,25p),

the expression of $Q^2(X)(0,15p)$

2. a) The aphotem of the pyramid (0,15p), A_l (0,15p), V(0,15p);

b) $OM \perp PN$ (0,20p), $PN \perp (SOM)$ (0,15p),

$SM \perp (SPN)$ (0,15p) end result (0,25p).

Entry examination in the 9th grade, high school

Session: August-September 1995

I. 1. Calculate:

a) $2^3 - 3\frac{1}{2} \cdot \frac{2}{7}$;

b) $-2 : \left(-\frac{1}{5}\right) - 0{,}001 \cdot 1000$.

2. Solve in $\mathbb{R}$ the equation:

$\frac{2}{3}(3x - 12) = x - 4$

3. Determine a natural number knowing that 7% of it represents 28.

4. A right circular cylinder with the radius of $6\ cm$ has the lateral area of $156\pi\ cm^2$. Determine the generating line of the cylinder.

II. 1. Calculate:

a) $(3 - \sqrt{6})(3 + \sqrt{6})$; b) $\left(\frac{2}{\sqrt{3}} - \sqrt{3}\right) \cdot \sqrt{3}$.

2. We consider the sets:

$A = \{x \mid x \in \mathbb{Z} \text{ and} -2 \leq x < 3\}$ and $B = \{x \mid x \in \mathbb{Z} \text{ and } x \text{ divides } 15\}$.

Determine the sets $A \cup B$ and $A \cap B$.

3. Effectuate and simplify, as much as possible, the result:

$E(x) = \left(\frac{4}{x+3} - \frac{5-3x}{9-x^2}\right) : \frac{x-7}{x-3}$.

4. In the right triangle ABC, ($m(\sphericalangle A) = 90°$), with $AB = 6cm, BC = 10\ cm$, we note with D the foot of the height from A on the hypotenuse. The parallel through D to AB intersects $[AC]$ in E. Determine BD and EC.

III. 1. Determine $m, m \in \mathbb{R}$ knowing that the polynomial:

$$P(X) = X^2 + mX + n$$

gives the remainder 2 by division to $X - 1$ and the remainder 5 by division to $X - 3$.

2. Simplify the fraction:

$$E(x) = \frac{4x^2 - 4x - 15}{4x^2 - 12x + 5}$$

3. A regular triangular pyramid $VABC$ has the height of $6\ cm$ and the apothem of $4\sqrt{3}$. Calculate:

a) the lateral area and the volume of the pyramid;

b) what distance from the top is point P located, on the height of the pyramid so that the triangle VPB is isosceles with the base $[VB]$.

Grading scale: 1 point *ex officio*

$I.$ 1) $a)$ $0,50\ \ p$ $b)$ $0,75\ p$; 2) $1\ p$; 3) $0,75\ p$ 4) $1p$

$II.$ 1) $a)$ $0,50$ $b)$ $0,50\ p$; p 2) $0,75\ p$; 3) $0,50p$; 4) $0,75\ p$

$III.$ 1) $0,50\ p$ 2) $0,50\ p$ 3) $a)$ $0,50p$ $b)$ $0,50\ p$

Entry examination in the 9^{th} grade, high school

Session: August-September 1995

I. 1. Calculate:

a) $6+6:2\cdot 3-3\cdot 5$; b) $\frac{6}{12}+\frac{5}{15}+\frac{4}{24}$;

c) $\frac{\sqrt{18}-3}{\sqrt{2}-1}-\left(\frac{1}{\sqrt{3}}-\frac{1}{3}\right):\frac{\sqrt{3}-1}{3}-\frac{\sqrt{80}}{2\sqrt{5}}$.

2. In the classroom there are 18 girls, which represents 60% of the students from the class. How many students are in the class?

3. Determine the smallest natural number that divided, in turn by $12, 18, 40$, gives the same remainder 7 every time.

II. 1. Solve in the naturals numbers set:

a) $2(x+8)=5x+1$; b) $x-1\geq\frac{3x-5}{2}$.

2. Let there be the function:

$f: \mathbf{R}\to\mathbf{R}$, $f(x)=2x-1$.

a) Trace the graphic of the function f;

b) Determine the points of the graphic of function f that have the abscissa equal to the ordinate.

3. Decompose in irreducible factors:

$(x^2-4)^2-(x^2-5x+6)^2$.

4. Let there be the polynomial:

$P(X)=(2X-3)^{1996}+X+1$.

a) Calculate $P(1)$ and $P(2)$;

b) Determine the remainder of the division of the polynomial $P(X)$ to $(X-1)(X-2)$.

III. 1. In ΔABC, we have $m(\sphericalangle ABC)=90°, m(\sphericalangle ACB)=30°)$ and $AD\perp BC, D\in BC$. Prove that the area of the triangle ABD is equal to a third of the area of the triangle ACD.

2. The cube $ABCDA'B'C'D'$ has the edge of $\sqrt{2}\ cm$. Determine:

a) the total area, the volume and the diagonal of the cube;

b) the angle of the straight lines AB' and BD;

c) the angle of the straight line AB' with the plane (BDD').

3. The body of a right circular cone has the lateral area equal to $100\pi\ cm^2$, the height of $8\ cm$ and the generating line of $10\ cm$. Determine:

a) the volume of the cone body;

b) the volume of the cone where the body comes from.

Grading and notation scale: 1p *ex officio*

$I.$ 1) $a)$ $0,75\ p$ $b)$ $0,50\ p$; $c)$ $0,25\ p$

2) $0,75\ p$;

3) for $x-7$ is the least common multiple of the numbers $12, 18$ and 40 is granted $(0,25\ p)$, end result $(0,25\ p)$

$II.$ 1) $a)$ $0,75$ b) for $x \leq 3$ are granted $(0,40\ p)$, end result $(0,10\ p)$

2) a) 0,50p; b) f(x) = x (0,10p), end result (0,15p).

3) $0,50p$;

4) a) (0,15+0,15p);

b) for the theorem of the division with remainder are granted $(0,20\ p)$, end result $(0,25\ p)$.

III. 1) figure (0,25p), AB = BC/2 (0,25p), end result (0,25p)

2) a) figure (0,25p), area (0,20p), volume (0,20p), diagonal (0,10p); b) 0,25p; c) 0,25p

3) a) figure (0,25p), R + r = 10 cm (0,25p); R − r = 6 cm (0,10p), end result (0,15p); b) 0,25p.

Entry examination in the 9th grade, high school

Session: July 1997

I. 1. Calculate:

$$8 - 8 : 2 + 10^0 + 0,2 - \frac{1}{5}.$$

2. Effectuate:

$$\frac{1}{2}\sqrt{32} + \sqrt{50} - \sqrt{18}$$

and approximate the result with a decimal by adding.

3. After having spent $1/3$ of the sum he had one day, and the second day $1500\ lei$, a student is left with 25% of the total sum. What sum did the student initially have?

II. 1. We consider the sets:

$A = \{x \in \mathbf{N} \mid 2x - 10 \leq 6(1 - x)\}$ and

$$B = \left\{x \in \mathbf{Z} \middle| \frac{3x+5}{x+1} \in \mathbf{Z}\right\}$$

Determine $A \cap B, B \setminus A$.

III. 1. Let M be a point inside an equilateral triangle with the side $2\sqrt{3}$. Calculate the sum of the distances from M to the sides of the triangle.

2. We consider the regular quadrilateral pyramid $VMNPQ$ with the side of the base $MN = 10\ cm$ and the height $VO = 5\sqrt{2}$. Determine:

a) the lateral area and the volume of the pyramid;

b) the position of the point T on the edge VN, for whom $MT + PT$ is minimal and the value of this minimum.

3. The axial section of a right circular cone's body is an isosceles trapezoid with the bases of $6\ cm$ and $18\ cm$ and with perpendicular diagonals. Determine:

a) the lateral area and the volume of the body;
b) the volume of the cone where the body comes from.

Grading and notation scale: 1 p *ex officio*

I. 1. 1. $8:2=4$ (0,25p), $10^0=1$ (0,25p), end result (0,50p).

2. $\frac{1}{2}\sqrt{32}=2\sqrt{2}$ (0,25p), $\sqrt{50}=5\sqrt{2}$ (0,20p),

$\sqrt{18}=3\sqrt{2}$ (0,20p), $2\sqrt{2}+5\sqrt{2}-3\sqrt{2}=4\sqrt{2}$ (0,15p),

$4\sqrt{2}\approx 5{,}7$ (0,20p).

3. $x-\frac{1}{3}x-1500=25\%\cdot x$ (0,50p), $x=3600$ lei (0,50p).

II. 1. $A=\{0,1,2\}$ (0,25p), $\frac{3x+5}{x+1}=3+\frac{2}{x+1}$ (0,15p),

$x+1\in\{\pm1,\pm2\}$ (0,15p), $B=\{-3,-2,0,1\}$ (0,15p),

$A\cap B=\{0,1\}$ (0,15p), $B\setminus A=\{-3,-2\}$ (0,15p).

2. a) $\cap Ox$ (0,20p), $\cap Oy$ (0,20p),
the drawing of the straight line (0,10p).

b) $\operatorname{tg}\alpha=\frac{2}{\frac{2\sqrt{3}}{3}}$ (0,25p), $\alpha=60^\circ$ (0,25p).

3. $X^3-X=X(X-1)(X+1)$ (0,40p),
according to the theory of Bezout $-1,0$ or 1 are roots of the denominator $(0,30p)$, end result $(0,30p)$

III. 1. The drawing (0,15p), $S_{MAB}+S_{MBC}+S_{MCA}=$

$=S_{ABC}$ (0,20p), end result (0,40p).

2. a) The drawing (0,15p), $ap=5\sqrt{3}$ cm (0,20p),

A_{lat} (0,20p), Vol (0,20p)

b) $VN\perp(MTP)$ (0,15p), VMN equilateral (0,15p),

$VT=NT$ (0,10p), the minimum $=10\sqrt{3}$ cm (0,10p).

3. a) The drawing (0,15p), $h_{tr} = 12$ cm (0,10p), $g = 6\sqrt{3}$ cm (0,15p), A_{lat} (0,10p), V_{tr} (0,10p).

b) $h_c = 18$ cm (0,30p), $V_c = 486\pi$ cm^3 (0,10p).

Entry examination in the 9th grade, high school

Session: August 1997

I. 1. Effectuate:

a) $-2^2+|-2|-2^0+\sqrt{5^2-3^2}$;

b) $\frac{1}{3}-\frac{3}{5}:1\frac{1}{5}$;

c) $\left[\left(\sqrt{3}-2\right)\left(2+\sqrt{3}\right)\right]^{1997}$.

2. Solve:

a) $3x-\left(\frac{1}{2}+x\right)=0$;

b) $\begin{cases}3x-2y=1\\2x-3y=-1\end{cases}$;

c) $|(x-2)(x-3)|+|2x-4|\leq 0$.

II. 1. a) Effectuate the function:

$f\colon \mathbf{R}\to\mathbf{R}$, $f(x)=mx+n$, $m,n\in\mathbf{R}$,

whose graphic intersects the coordinate axes in the points $A(0,2)$ and $B(2,0)$, respectively.

b) Represent graphically the following function, then determine the real values of a for which $|f(a)\leq 5$:

$f\colon \mathbf{R}\to\mathbf{R}$, $f(x)=-x+2$

2. The following expression is given:

$$E(x)=\left(\frac{1-2x}{1-x}+\frac{2-x^2}{x^2-1}\right):\frac{x^3-1}{x+1}.$$

a) Bring it to the simplest form;

b) Specify the real values of x for which $E(x)$ makes sense;

c) Determine the set:

$M=\{x\in\mathbf{Z}\mid E(x)\in\mathbf{Z}\}$.

III. 1. Given an isosceles trapezoid with the big base of $5\ cm$, the little base of $3\ cm$ and the acute sides $2\ cm$ each determine:

a) the area of the trapezoid;

b) the radius circumscribed to this trapezoid.

2. The axial section of a cone is an equilateral triangle with the side of $6\ cm$. Determine:

a) the total area and the volume of the cone;

b) the measure of the circle's sector angle, obtained by deploying the lateral surface of the cone.

3. The body of a regular quadrilateral pyramid has the areas of the bases of $100\ \ cm^2$ and $36\ cm^2$, and the lengths of the lateral edges of $10\sqrt{2}\ cm$ each. Determine:

a) the lateral area and the volume of the body;

b) the height of the pyramid where the body comes from;

c) the distance from the center of the big base to the plane of a lateral face of the body.

Grading and notation scale: $1\ p$ ex officio

I. 1. a) The calculation of the terms $(0,40\ p)$, end result $(0,10\ p)$;

b) The division $(0,25\ p)$, the subtraction $(0,25\ p)$;

c) The calculation from the straight brackets $(0,30\ p)$, end result $(0,20\ p)$.

2. a) Finding the value of $x(0,50\ p)$;

b) Determining the solution of the system $(0,50\ p)$;

c) Determining the result correctly (with justification) $(0,50\ p)$;

II. 1. a) Determining the function $(0,50\ p)$;

b) Graphic representation $(0,50\ p), |-a+2| \leq 5$ and the end result $(0,50\ p)$.

2. a) Making the calculations from the equation $(0,40\ p)$, simplification and end result $(0,35\ p)$;

b) Specifying the values of x $(0,25\ p)$;

c) Determining the set M $(0,25\ p)$.

III. 1. a) The calculation of the height of the trapezoid $(0,25\ p)$, determining the area of the trapezoid $(0,25\ p)$;

b) Determining the radius $(0,50\ p)$.

2. The total area $(0,25\ p)$, the volume $(0,25\ p)$;

b) Determining the angle of the circular sector $(0,25\ p)$.

3. a) Determining the apothem of the body $(0,15\ p)$, determining the height of the body $(0,15\ p)$, the lateral area $(0,20\ p)$, the volume $(0,20\ p)$;

b) Locating the height of the pyramid where the body comes from $(0,30\ p)$;

c) Determining the distance from the center of the big base to the plane of a lateral face $(0,50\ p)$.

Tests Solutions

Test no. 1

I. 1. 4^c is an uneven number $\Rightarrow c = 0 \Rightarrow 8(2^a + 11b) + 1 =$
$= 609 \Rightarrow 2^a + 11b = 76 \Rightarrow 2^a < 76 \Rightarrow a \le 6$; the only acceptable option is $a = 5 \Rightarrow 32 + 11b = 76 \Rightarrow b = 4$.

2. $\frac{a}{2}=\frac{b}{3}$; $\frac{b}{3}=\frac{c}{5}$; $\frac{c}{5}=\frac{d}{7} \Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{5}=\frac{d}{7}=k$,

$k>0 \Rightarrow \begin{cases} a=2k \\ b=3k \\ c=5k \\ d=7k \end{cases}$. We see that $b-a=k$; $\left.\begin{matrix} b-a\in \mathbf{Z} \\ k>0 \end{matrix}\right\} \Rightarrow$

$\Rightarrow k \in \mathbf{N}^*$. $abcd = 17\,010 \Rightarrow 2k \cdot 3k \cdot 5k \cdot 7k = 17\,010 \Rightarrow k^4 = 81 \Rightarrow$

$\Rightarrow k = 3 \Rightarrow \begin{cases} a=6, b=9 \\ c=15, d=21 \end{cases}$

3. $\sqrt{17-\sqrt{n}} = a$, $a \in \mathbf{N} \Leftrightarrow 17-\sqrt{n} = a^2 \Rightarrow 17 = a^2 + \sqrt{n} \Rightarrow$
$\Rightarrow a^2 < 17 \Rightarrow a^2 \in \{16, 9, 4, 1\}$; $a^2 = 16 \Rightarrow \sqrt{n} = 1 \Rightarrow n = 1$; $a^2 = 9 \Rightarrow$
$\Rightarrow \sqrt{n} = 8 \Rightarrow n = 64$, $a^2 = 4 \Rightarrow \sqrt{n} = 13 \Rightarrow n = 169$; $a^2 = 1 \Rightarrow$
$\Rightarrow \sqrt{n} = 16 \Rightarrow n = 256 \Rightarrow A = \{1, 64, 169, 256\}$.

II. 1. $a = 3\sqrt{5} - 2\sqrt{3}$; $b = 3\sqrt{5^{2n+2} : 5^{2n+1}} + 2\sqrt{3} = 3\sqrt{5} + 2\sqrt{3}$;
$b-a = 4\sqrt{3}$, $(b-a-\sqrt{12})^p \in \mathbf{N} \Leftrightarrow (4\sqrt{3}-2\sqrt{3})^p \in \mathbf{N} \Leftrightarrow$

$\Leftrightarrow \left.\begin{matrix} (2\sqrt{3})^p \in \mathbf{N} \\ p \text{ minimum}, p \ne 0 \end{matrix}\right| \Rightarrow p = 2$.

2. $\sqrt{62+20\sqrt{6}} = \sqrt{62+\sqrt{2400}} = \sqrt{50}+\sqrt{12} = 5\sqrt{2}+2\sqrt{3}$,
$a = 62$; $b = 2400$; $c^2 = 62^2 - 2400 = 3844 - 2400 = 1444 \Rightarrow c = 38$.
$5\sqrt{2}+2\sqrt{3} = a\sqrt{2}+b\sqrt{3}$, $a\in\mathbf{N}^*$, $b\in\mathbf{N} \Rightarrow (5-a)\sqrt{2} = \sqrt{3}(b-2) | \sqrt{3} \Leftrightarrow$

$$\Leftrightarrow (5-a)\sqrt{6} = 2(b-2)\text{ ; if } 5-a \neq 0 \Rightarrow \left.\begin{array}{l} \sqrt{6} = \dfrac{3(b-2)}{5-a} \\ \sqrt{6} \in \mathbf{R}\setminus\mathbf{Q} \\ \dfrac{3(b-2)}{5-a} \in \mathbf{Q} \end{array}\right| (F) \Rightarrow$$

$52 \neq M_6.$

$5-a=0$ şi $b-2=0 \Rightarrow a=5;\ b=2 \Rightarrow \overline{ab}=52$; $52 = M_2$, $52 \neq M_3$.

3. $f(x) = \dfrac{a(\sqrt{3}-1)}{2}x + \dfrac{b(\sqrt{3}+1)}{2}$; $M(\sqrt{3}-1;3) \in G_f \Leftrightarrow$

$f(\sqrt{3}-1) = 3 \Leftrightarrow a(3-2\sqrt{3}+1) + b(\sqrt{3}+1) = 6 \Leftrightarrow \sqrt{3}(b-2a) = 6-4a-b;$

$$\text{if } b-2a \neq 0\ \ \left.\begin{array}{l} \sqrt{3} = \dfrac{6-4a-b}{b-2a} \\ \sqrt{3} \in \mathbf{R}\setminus\mathbf{Q} \\ \dfrac{6-4a-b}{b-2a} \in \mathbf{Q} \end{array}\right| (F) \Rightarrow b-2a=0 \Rightarrow 6-4a-b=0.$$

$$\begin{cases} b-2a=0 \\ -b-4a=-6 \end{cases} \Rightarrow a=1 \text{ şi } b=2 \Rightarrow f(x) = \frac{\sqrt{3}-1}{2}x + \frac{2(\sqrt{3}+1)}{2};$$

let $P(x_0, y_0) \in G_f$, $x_0, y_0 \in \mathbf{Z} \Rightarrow f(x_0) = y_0 \Rightarrow \dfrac{\sqrt{3}-1}{2} \cdot x_0 + \dfrac{2(\sqrt{3}+1)}{2} = y_0 \Rightarrow$

$\Rightarrow \sqrt{3}x_0 - x_0 + 2\sqrt{3} + 2 = 2y_0 \Rightarrow \sqrt{3}(x_0+2) = 2y_0 + x_0 - 2;$

$$\text{if } x_0+2 \neq 0 \Rightarrow \left.\begin{array}{l} \sqrt{3} = \dfrac{2y_0+x_0-2}{x_0+2} \\ \sqrt{3} \in \mathbf{R}\setminus\mathbf{Q} \\ \dfrac{2y_0+x_0-2}{x_0+2} \in \mathbf{Q} \end{array}\right| (F) \Rightarrow x_0+2=0 \text{ şi } 2y_0+x_0-2=0 \Rightarrow$$

$$\Rightarrow \begin{cases} x_0 = -2 \\ y_0 = 2 \end{cases} \Rightarrow P(-2, 2).$$

III. 1. $\dfrac{(x-3\sqrt{3})^2(x-4)(x-5)^2}{x^2\sqrt{(x+1)^2}} \leq 0$. $x \neq 0$; $x+1 \neq 0 \Rightarrow x \neq -1$;

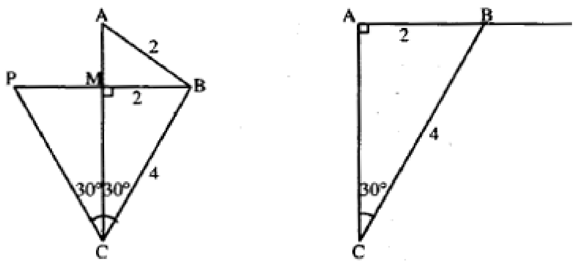
$x^2 > 0, (\forall)x \in \mathbf{R}^*$; $|x+1| > 0, (\forall)x \in \mathbf{R}\setminus\{-1\}$. $\left(x-3\sqrt{3}\right)^2 \geq 0$;

$(x-5)^2 \geq 0 \Rightarrow x-4 \leq 0 \Rightarrow x \leq 4 \Rightarrow x \in (-\infty, 4]$.

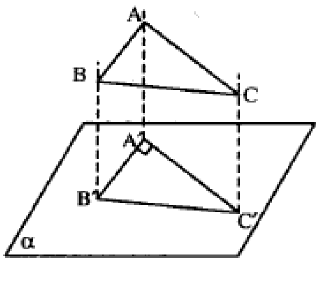
$x \in ((-\infty, 4] \setminus \{-1, 0\}) \cup \{3\sqrt{3}, 5\}$

2. a) Using the method of *reductio ad absurdum* we prove that ΔABC is right in A. We assume that $m(\sphericalangle CAB) \neq 90°$. Let P be the symmetric of B in relation to AC. ΔPCB isosceles with $m(\sphericalangle PCB) = 60° \Rightarrow \Delta PCB\ equilateral \Rightarrow PB = 4\ cm \Rightarrow MB = 2\ cm$. In ΔAMB; $MB = AB(F)$

$$AC = 2\sqrt{3} \Rightarrow S_{ABC} = \frac{2 \cdot 2\sqrt{3}}{2} = 2\sqrt{3}\ cm^2.$$



b) The right angle $\sphericalangle BAC$ is projected after the right angle $\sphericalangle B'A'C'$. Then we have two cases:



1) $AB \parallel \alpha \Rightarrow AB = A'B'$ şi $AB \parallel A'B' \Rightarrow A'B' = 2$ cm $\Rightarrow \dfrac{A'B' \cdot A'C'}{2} = 3 \Rightarrow$

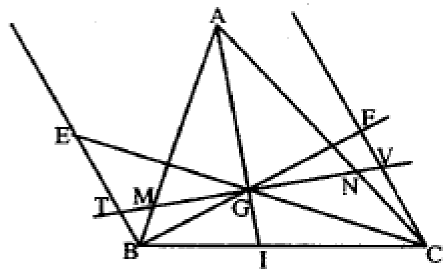
$\Rightarrow 2 \cdot A'C' = 6 \Rightarrow A'C' = 3 \Rightarrow B'C' = \sqrt{4+9} = \sqrt{13}$.

$S' = S \cdot \cos u \Leftrightarrow \cos u = \dfrac{S'}{S} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2} \Rightarrow u = 30° \Rightarrow \sphericalangle((ABC); \alpha) = 30°$.

2) If $AC \parallel \alpha \Rightarrow A'C' = AC = 2\sqrt{3} \Rightarrow A'B' = \sqrt{3} \Rightarrow B'C' = \sqrt{15}$.

III. 3. Let $MN \cap (BE = \{T\}$ şi $MN \cap (CF = \{V\}$

$\dfrac{MB}{MA} = \dfrac{BT}{AG}$; $\dfrac{NC}{NA} = \dfrac{CV}{AG}$



In the trapezoid TBCV: GI is a middle line $\Rightarrow BT + CV = 2 \cdot GI$

$\dfrac{MB}{MA} + \dfrac{NC}{NA} = \dfrac{BT}{AG} + \dfrac{CV}{AG} = \dfrac{2GI}{AG} = 1$

b) We easily prove that:

$\dfrac{1}{GI} = \dfrac{1}{BE} + \dfrac{1}{CF}$.

We use the inequality:

$(a+b)\left(\dfrac{1}{a} + \dfrac{1}{b}\right) \geq 4 \Rightarrow \left(\dfrac{1}{BE} + \dfrac{1}{CF}\right)(BE + CF) \geq 4 \Rightarrow BE + CF \geq$

$\geq \dfrac{4}{\dfrac{1}{BE} + \dfrac{1}{CF}} \Leftrightarrow BE + CF \geq \dfrac{4}{\dfrac{1}{GI}} = 4GI$

The equality would take place if $a = b \Leftrightarrow \frac{1}{BE} = \frac{1}{CF} \Rightarrow BE = CF \Rightarrow BCFE$ parallelogram with the center G and EF is a parallel traced to BC throught the middle of AG.

Test no. 2

I. **1.** 57 002;

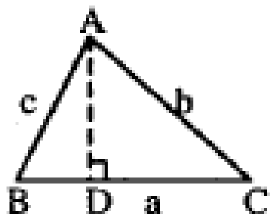
2. $27+10\sqrt{2}=(5+\sqrt{2})^2$; $27-10\sqrt{2}=(5-\sqrt{2})^2$, so the number:

$\sqrt{27+10\sqrt{2}}+\sqrt{27-10\sqrt{2}}=5+\sqrt{2}+5-\sqrt{2}=10$

II. **1.** $x=3-\dfrac{10}{2n+1}\in \mathbf{Z} \Rightarrow 2n+1\in\{\pm1;\pm2;\pm5;\pm10\}\Rightarrow$

$\Rightarrow n\in\{0,2,-3,-1\} \Rightarrow A=\{-7,1,5,13\}$.

2. We distinguish three cases:



a) If $m(\sphericalangle B)<90^\circ$ then $BD=\dfrac{a^2-b^2+c^2}{2a}$

b) If $m(\sphericalangle B)=90^\circ$ then $BD=0$

c) If $m(\sphericalangle B)>90^\circ$ then $BD=\dfrac{b^2-a^2-c^2}{2a}$.

III. **1.** $V=512\sqrt{3}$

2. Noting with d the required distance it follows that:

$\left(\dfrac{d}{5}\right)^3=\dfrac{1}{2}\Rightarrow d^3=\dfrac{125}{2}\Rightarrow d=\dfrac{5}{\sqrt[3]{2}}$.

Test no. 3

I. 1. a) 3; b) $a + b = 6$, $\frac{a}{\frac{1}{3}} = \frac{b}{\frac{15}{90}} = \frac{a+b}{\frac{1}{3}+\frac{1}{6}} = \frac{a+b}{\frac{1}{2}} = 12 \Rightarrow$

$\Rightarrow a = 4;\ b = 2.$

2. $x = y = 1.$

II. 1. $\frac{(x-1)^2}{x^2 - x + 1}$; 2. $P(X) = (X^3 + 3)C(X) + C^2(X).$

Because $\text{grad}\ C^2(X) < \text{grad}\ (X^3 + 3) = 3$ it follows that $C(X) = aX + b$.
We obtain $2(-a + b) + (-a + b)^2 + 4(a + b) + (a + b)^2 + 5 = 0$ or

$$\left(a + \frac{1}{2}\right)^2 + \left(b + \frac{3}{2}\right)^2 = 0$$

From this $a = -1/2;\ b = -3/2$, and $R(X) = \frac{1}{4}X^2 + \frac{3}{2}X + \frac{9}{4}$.

III. 1. a) $A_{lat} = 100\sqrt{2}\ cm^2;\ V = \frac{500}{3}\ cm^3.$

b) $\sphericalangle VMO$ is the angle associated plane and $(\sphericalangle VMO) = 45°$

c) $PV = 7{,}5\ cm$

2. a) $A_t = 2\pi\sqrt{2}\left(\sqrt{2} + 4\right)cm^2;\ V = 8\pi\ cm^3.$

b) the required angle has $60°$.

Test no. 4

I. 1. a) 5; b) $\frac{27}{19}$; c) 18.

2. $A = \{0, 2, 3, 4, 7\}$, $B = \{0, 2, 6\}$; $A \cup B = \{0, 2, 3, 4, 6, 7\}$, $A \cap B = \{0, 2\}$, $A \setminus B = \{3, 4, 7\}$.

II. 1. The equation is written:

$m(1 - m)x = 1 - m.$

If $m \neq 0$ and $m \neq 1$ the equation admits the solution $x = 1/m$.

If $m = 0$ the equation doesn't admit solutions.

If $m = 1$ the equation has solutions any real number.

2. The inequality is written:

$(a^2 + b^2)(x^2 + y^2) \geq (ax + by)^2$

and after the calculations and reducing the similar terms it becomes: $(ay - bx)^2 \geq 0$.

III. 1. The required area is $2000\ cm^2$

a) $\Delta ADB = \Delta CDB$ (being isosceles triangles even)

b) From ΔADB, using the areas we calculate:

$$AM = a\sqrt{\frac{a^2 + 4b^2}{2(a^2 + 2b^2)}}$$

As $AM = MC$ from the isosceles triangle AMC we can calculate:

$$MO = \frac{ab}{\sqrt{a^2 + 2b^2}}$$

Because MO is a height in ΔAMC, we obtain:

$$S_{\Delta AMC} = \frac{a^2 b}{2\sqrt{a^2 + 2b^2}}.$$

Test no. 5

I. 1. $a=\left(\frac{5}{2}-\frac{4}{10}\cdot\frac{10}{3}\right)^{-1}-(8-3)\cdot\left(-\frac{8}{7}\right)=\left(\frac{5}{2}-\frac{4}{3}\right)^{-1}+$

$+\frac{40}{7}=\left(\frac{7}{6}\right)^{-1}+\frac{40}{7}=\frac{6}{7}+\frac{40}{7}=\frac{46}{7}=6\frac{4}{7}\ [a]=6$

2. $p=11;\ \overline{bb}=66,\ A=\{1,2,3,6,11,66,22,33\}$

3. $\frac{a}{4}=\frac{b}{5}=\frac{c}{d}=k,\ k>0,\ a>0,\ a<b<c,$

$$\left.\begin{cases} a=4k & b-a=k \\ b=5k \Rightarrow & b-a\in\mathbb{Z} \\ c=dk & b>a \end{cases}\right\}\Rightarrow k\in\mathbb{N}^*\Rightarrow\begin{cases} a\geq 4 \\ b\geq 5 \\ c\geq d \end{cases}$$

$a<b<c\Rightarrow 4k<5k<dk\Rightarrow d>5\Rightarrow d\geq 6\ (1)$

$c^2\leq(2b-a)^2\Leftrightarrow d^2k^2\leq(6k)^2\Leftrightarrow d^2k^2\leq 36k^2\Rightarrow d\leq 6\ (2)$

From (1) şi $(2)\Rightarrow d=6\Rightarrow\begin{cases} a=4k \\ b=5k \\ c=6k \end{cases},\ a+c=10k\Rightarrow$

$10k$ ----------- 100%

$5k$ ---------------- $x\quad\Rightarrow x=50\%.$

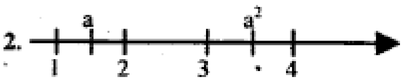
II. 1. $\frac{a\sqrt{3}+b}{\sqrt{3}-1}=p,\ p\in\mathbb{Q}\Leftrightarrow\frac{(a\sqrt{3}+b)(\sqrt{3}+1)}{2}=p\Leftrightarrow$

$\Leftrightarrow 3a+a\sqrt{3}+b\sqrt{3}+b=2p\Rightarrow\sqrt{3}(a+b)=2p-3a-b.$

$$\text{If }\ a+b\neq 0\Rightarrow\left.\begin{array}{l}\sqrt{3}=\frac{2p-3a-b}{a+b}\\ \sqrt{3}\in\mathbb{R}\setminus\mathbb{Q}\\ \frac{2p-3a-b}{a+b}\in\mathbb{Q}\end{array}\right|(F)\Rightarrow a+b=0\Rightarrow$$

$\Rightarrow\left(\sqrt{11+6\sqrt{2}}+\sqrt{11-6\sqrt{2}}\right)^{19980}=1.$

2.

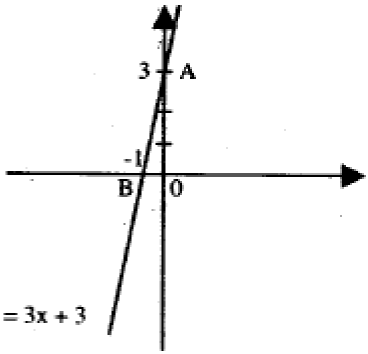


$1<a<2<3<a^2<4\Rightarrow a\in(\sqrt{3};2)\Rightarrow A=(\sqrt{3},2)$

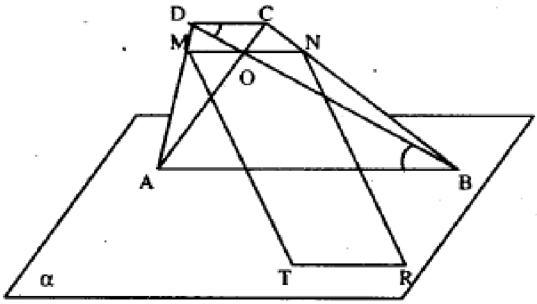
3. $7\sqrt{a}+2\sqrt{b}=c\sqrt{3} \Rightarrow 49a+4b+28\sqrt{ab}=3c^2 \Rightarrow \sqrt{ab}\in Q \Rightarrow$

$\Rightarrow a=b \Rightarrow 49a+4a+28a=3c^2 \Rightarrow 81a=3c^2 \Leftrightarrow 27a=c^2 \Rightarrow$

$\Rightarrow \left.\begin{array}{l} 9\cdot 3a=c^2 \\ a\in M_3 \end{array}\right| \Rightarrow a=3,\ b=3 \Rightarrow f(x)=3x+3.$



III. 1. $MT \parallel NR \Rightarrow M, N, T, R$ coplanar



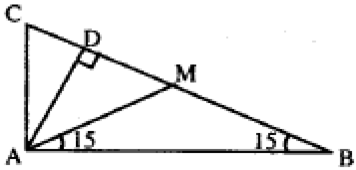
Let $\beta=(M, N, T)$

$\left.\begin{array}{l} MN \parallel \alpha \\ MN\subset\beta \\ \alpha\cap\beta=TR \end{array}\right\} \Rightarrow MN \parallel TR$ and $MT \parallel NR \Rightarrow MNRT$ parallelogram $\Rightarrow$

$\Rightarrow\ [MN]\equiv[TR]\ \ MN=\dfrac{2AB\cdot CD}{AB+CD}=\dfrac{2\cdot 12\cdot 4}{16}=6$ cm.

2. a) Let there be the median $(AM) \Rightarrow AM=MB=MC \Rightarrow$

$\Rightarrow \Delta AMB$ isoscel $\Rightarrow m(\sphericalangle AMB)=150° \Rightarrow m(\sphericalangle AMD)=30°$.



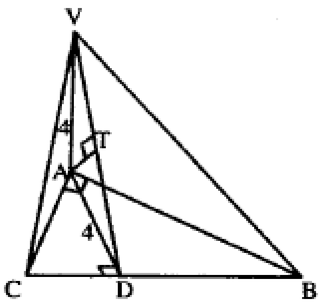
In ΔADM: $AD = \frac{AM}{2} = \frac{8}{2} = 4$ cm

$$\left.\begin{array}{l} VA\perp(ABC)\\ AD\perp BC\\ BC\subset(ABC)\end{array}\right\} \overset{T3\perp}{\Rightarrow} AD\perp BC \Rightarrow \left.\begin{array}{l} \left.\begin{array}{l}BC\perp(VAD)\\ BC\subset(VBC)\end{array}\right\}\Rightarrow (VBC)\perp(VAD)\\ \text{But } (VBC)\cap(VAD)=AD\\ \text{Let } AT\perp AD\end{array}\right\}\Rightarrow$$

$\Rightarrow AT \perp (VBC) \Rightarrow d(A; (VBC)) = AT$

ΔVAD: $VD = 4\sqrt{2} \Rightarrow AT = \frac{4\cdot 4}{4\sqrt{2}} = 2\sqrt{2}$.

b) $\sphericalangle((VBC);(ABC)) = \sphericalangle VDA$ şi $m(\sphericalangle VDA) = 45°$.



Test no. 6

I. 1. $x = 3 - \dfrac{10}{2n+1}$, as $x \in \mathbb{Z}$, it follows that

$2n + 1 \mid 10 \Rightarrow (2n + 1) \in \{-5, -1, 1, 5\}$,

so $n \in \{-6, -1, 0, 2\} \Rightarrow A = \{-7, 1, 5, 13\}$

2. $(2{,}5 \cdot 10^3 + 32 : 10^{-2}) : 10^{-1} + \sqrt{(-2)^2} =$

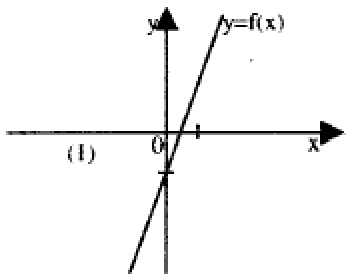
$= \left(2500 + 32 : \dfrac{1}{100}\right) : \dfrac{1}{10} + \sqrt{4} = (2500 + 3200) \cdot 10 + 2 = 57000 + 2 = 57002.$

3. a) $(x-1)^2 - y^2 = (x - y - 1)(x + y - 1)$;

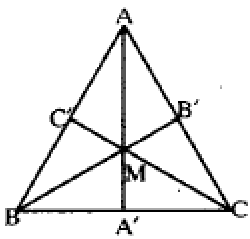
b) $9x^2 + 6x + 1 = (3x+1)^2$

II. 1. $f(x) = \begin{cases} 2x - 1, & 2x - 1 \le 2x + 1 \\ 2x + 1, & 2x - 1 > 2x + 1 \end{cases}$,

so $f(x) = 2x - 1\ (\forall)x \in \mathbb{R}$



2. $A_{[ABC]} = A_{[MBC]} + A_{[MCA]} + A_{[AMB]}$ (1)



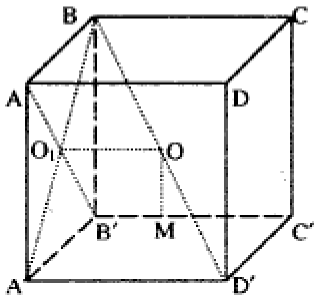
We note h- the height of the triangle with "a" the triangle side, then from (1) it follows that:

$$\frac{a \cdot h}{2} = \frac{a \cdot MA'}{2} + \frac{a \cdot MB'}{2} + \frac{a \cdot MC'}{2} \Rightarrow h = MA' + MB' + MC',$$

so the sum of the distances from the interior of ΔABC equilateral at the sides is equal to the height of the equilateral triangle. But:

$$h = \frac{a\sqrt{3}}{2} \Rightarrow a = \frac{2h}{\sqrt{3}} \text{ so } A_{[ABC]} = \frac{1}{2} \cdot \frac{2h}{\sqrt{3}} \cdot h = \frac{h^2}{\sqrt{3}}.$$

3. Let BD' be the considered diagonal and $B'C'$ indicated in the problem.



Because $B'C' \parallel (BA'D')$ because $B'C' \parallel A'D' \Rightarrow d(B'C', (BA'D')) =$ the length of the perpendicular lowered from a random point of the straight line $B'C'$ on the plane $(BA'D')$.

Let $B' \in (B'C')$ from $B'A' \perp A'D'$, $BA' \perp A'D'$ and $BO_1 \perp A'D'$ and $BO_1 \perp A'B$ ($ABB'A'$ = square) $\Rightarrow B'O_1 \perp (BA'D')$.

We build $O_1O \parallel A'D' \Rightarrow O_1O \parallel B'C'$ ($O \in (BD')$).

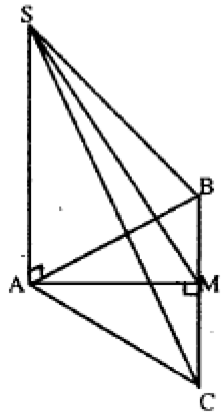
We build $OM \parallel O_1B'$ ($M \in (B'C')$) $\Rightarrow OM \perp B'C'$, $OM \perp (BA'D')$.

From $OM \perp BD'$ and $OM \perp B'C' \Rightarrow OM$ represents the distance from the straight line $B'C'$ to the straight line BD'.

But O_1OMB' parallelogram $\Rightarrow OM = BO_1 = \frac{m\sqrt{2}}{2} = 10\sqrt{2}$.

III. a) $A_{[SAB]} = A_{[SAC]} = 24 \text{ cm}^2$ (1)

$A_{[BAC]} = 32 \text{ cm}^2$ (2)



Let $AM \perp BC$, then $AM = \dfrac{AB \cdot AC}{BC} = 4\sqrt{2}$ cm

From $SA \perp (ABC)$ and $AM \perp BC \Rightarrow SM \perp BC$.

From ΔSAM right $\Rightarrow SM^2 = SA^2 + AM^2$, so $SM = 2\sqrt{17}$.

so $A_{[SBC]} = \dfrac{BC \cdot SM}{2} = 16\sqrt{34}$ (3)

From (1), (2) şi (3) we obtain $A_t = 80 + 16\sqrt{34} = 16\left(5 + \sqrt{34}\right)\text{cm}^2$.

b) $\text{tg} \sphericalangle AMS = \dfrac{AS}{AM} = \dfrac{6}{4\sqrt{2}} = \dfrac{3\sqrt{2}}{4}$

c) $V = \dfrac{1}{3} A[ABC] \cdot SA = \dfrac{1}{3} \cdot 32 \cdot 6 = 64 \text{ cm}^3$.

Test no. 7

I. 1. $(1,(3)+1,(6))\cdot 0,(3)+\sqrt{6,25}:\frac{5}{2}=\left(1\frac{3}{9}+1\frac{6}{9}\right)\cdot\frac{3}{9}+2,5\cdot\frac{2}{5}=$

$=\left(\frac{12}{9}+\frac{15}{9}\right)\cdot\frac{1}{3}+1=\frac{27}{9}\cdot\frac{1}{3}+1=1+1=2.$

2. $||x-1|+2|-3|=\begin{cases}||x+1|-3|, x\geq 1\\ ||-x+3|-3|, x<1\end{cases}=$

$=\begin{cases}|x-2|, x\geq 1, x\geq -1\\ |-x-4|, x\geq 1, x<-1\\ |-x|, x<1, x\leq 3\\ |x-6|, x<1, x>3\end{cases}=\begin{cases}|x-2|, x\geq 1\\ |-x|, x<1\end{cases}=\begin{cases}x-2, x\geq 1, x\geq 2\\ -x+2, x\geq 1, x<2\\ -x, x\leq 0, x\leq 1\\ x, x>0, x\leq 1\end{cases};$

$|x-1|=\begin{cases}x-1, x\geq 1\\ 1-x, x<1\end{cases}$ and $|x-2|=\begin{cases}x-2, x\geq 2\\ 2-x, x<2\end{cases}$

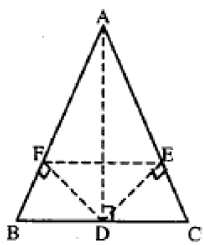
so $||x-1|+2|-3|=\begin{cases}x-2, x\geq 2\\ 2-x, x\in(2,1]\\ -x, x\leq 0\\ x, x\in(0,1)\end{cases}$

II. 1. $\Delta ABC=\Delta$ isosceles $\Rightarrow AD\perp BC$

From:

$\Delta ADF\equiv\Delta ADE\Rightarrow[AF]\equiv[AE]\Rightarrow[BF]\equiv[CE]$

and applying the reciprocal T. Thales $\Rightarrow EF||BC$. So $BFEC=$ isosceles trapezoid.



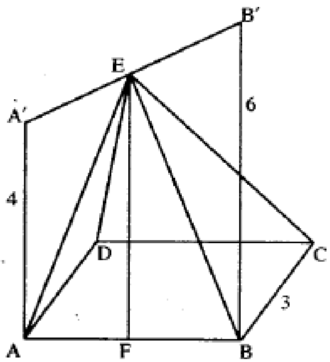
2. $x = \frac{2n+7}{n+1} = \frac{2(n+1)+5}{n+1} = 2 + \frac{5}{n+1} \Rightarrow n+1 \in \{1, 5\} \Rightarrow \{3, 7\}$

III. 1. $A_t = 36\pi$; $V = 16\pi$.

2. $A'ABB'$ = trapezoid $\Rightarrow$ EF = middle line $\Rightarrow$

$\Rightarrow EF = \frac{AA' + BB'}{2} \Rightarrow EF = 5.$

$(ABB'A') \perp (ABCD) \Rightarrow EF \perp (ABCD) \Rightarrow EF$ the height of the pyamid EABCD. $V_{EABCD} = \frac{1}{3} A_{ABCD} \cdot EF = \frac{1}{3} \cdot 9 \cdot 5 = 15.$



Test no. 8

I. 1. $3 - 5^{-2} \cdot 75 + 10^0 + \sqrt{0,64} = 3 - \frac{1}{25} \cdot 75 + 1 + 0,8 = 1,8.$

2. $A = \{3, 4, 5, 6\}$; $B = \{-6, -5, -4, -3, -2, -1, 0, 1, 2\}$.

3. $P(X) = (X - 1) \cdot Q(X) + R$, where $Q(X) = X^2 - X + 1$, $R = 2$.

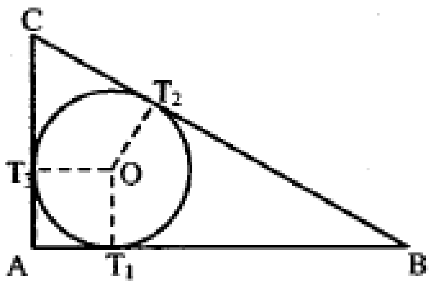
II. 1. it is known that the tangents traced from an exterior point to a circle are congruent, so:

$(BT_1) \equiv (BT_2)$, $(CT_2) \equiv (CT_3)$ şi $(AT_1) \equiv (AT_3)$ (1)

Let O be the center of the circle inscribed in ΔABC right angled and $T_1, T_2, T_3$the tangent points, then the quadrilateral OT_1AT_3 is a square.

$OT_1 = OT_3 = r$ şi $m(\sphericalangle OT_1A) = m(\sphericalangle T_1AT_3) = m(\sphericalangle AT_3O) = 90°$,

so $AT_1 + AT_3 = 2r$. (2)



The diameter of the circle circumscribed to the right triangle is $[BC]$, so:

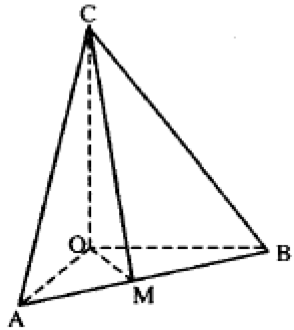
$BC = BT_2 + CT_2 = 2R$ (3)

From (1), (2) and (3) $\Rightarrow AB + AC = 2R + 2r$.

2. $\frac{x}{3} = \frac{y}{4} = \frac{z}{5} = \frac{x+y+z}{3+4+5} \Rightarrow x = 15, y = 20, z = 25.$

3. $E(x, y) = 2(x^2 + y^2)$ with the conditions $x \neq \pm y$.

III. a) From the isosceles right triangle AOB, with $(OA) \equiv (OB)$ we obtain $AB = 8\sqrt{2}\ cm$.



From $\left.\begin{array}{l} CO \perp OA \\ CO \perp OB \end{array}\right\} \Rightarrow CO \perp (OAB)$, let $OM \perp AB$,

then $CM \perp AB$ (according to T.3⊥).

From ΔAOB right isosceles $\Rightarrow OM = \dfrac{OA \cdot OB}{AB} = 4\sqrt{2}$.

From $\Delta COM \Rightarrow CM = 4\sqrt{6}$.

$A_{\Delta ABC} = \dfrac{CM \cdot AB}{2} = \dfrac{4\sqrt{6} \cdot 8\sqrt{2}}{2} = 32\sqrt{3}\ cm^2$.

b) $A_{[AOB]} = A_{[AOC]} = A_{[BOC]} = 32\ cm^2$.

$A_t = 3 \cdot 32 + 32\sqrt{3} = 32\left(3 + \sqrt{3}\right) cm^2$.

c) $V = \dfrac{1}{3} \cdot A_b \cdot h = \dfrac{1}{3} \cdot \dfrac{8 \cdot 8}{2} \cdot 8 = \dfrac{256}{3}\ cm^3$.

Test no. 9

I. 1. The problem has the following solutions:

a) $X = \{3, 4, 6, 7\}$; $Y = \{3, 4, 5\}$

b) $X = \{3, 4, 5, 6\}$; $Y = \{3, 4, 7\}$

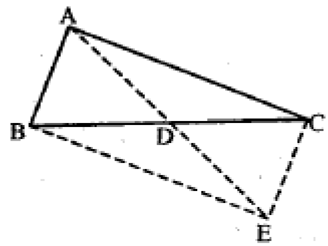
c) $X = \{3, 4\}$; $Y = \{3, 4, 5, 6, 7\}$

2. $E(X) = (X-1)(X-3)(X-\sqrt{3})(X+\sqrt{3})$

3. $|2x-3| + |4x-6| = 0 \Leftrightarrow 2x-3=0$ and $4x-6=0$

$\Rightarrow x = 3/2$ the solution of the equation.

II. 1. We extend $(AD$ with the segment $(DE) \equiv (AD)$. From the fact that $(BD) \equiv (DC)$ and $(AD) \equiv (DE)$ it follows that $ABEC$ = parallelogram $\Rightarrow (AB) = (EC)$.



From ΔAEC, it follows that $AE < AC + EC$, but $AE = 2AD$, $CE = AB$, so $AD < \dfrac{AB+AC}{2}$.

2. $\dfrac{4-x}{x^2+4x+5} < 0 \Leftrightarrow \dfrac{4-x}{(x+2)^2+1} < 0 \Leftrightarrow 4-x<0 \Leftrightarrow x>4 \Rightarrow$
$\Rightarrow x \in (4;+\infty)$

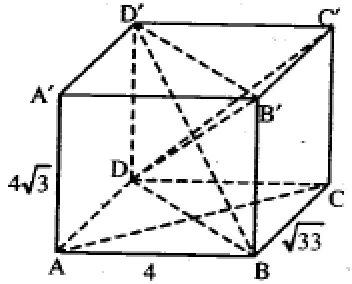
3. $X^2+Y^2+2X+2Y+2=0 \Leftrightarrow (X^2+2X+1)+(Y^2+2Y+1)=$
$=0 \Leftrightarrow (X+1)^2+(Y+1)^2=0 \Leftrightarrow X+1=0$ and $Y+1=0 \Leftrightarrow X=-1$ and $Y=-1$.

III. a) $d = BD' = \sqrt{AB^2+BC^2+AA'^2} = \sqrt{16+33+48} = \sqrt{97}$ cm.

c) $V = 4 \cdot 4\sqrt{3} \cdot \sqrt{33} = 48\sqrt{11}$ cm^3.

d) $A_t = 2(4 \cdot 4\sqrt{3} + 4 \cdot \sqrt{33} + 4\sqrt{3} \cdot \sqrt{33}) =$

$= 8\left(4\sqrt{3} + \sqrt{33} + 3\sqrt{11}\right) \text{cm}^2.$



Test no. 10

I. 1. We note with $a = \underbrace{11111\ldots1}_{n\ times}$ and we compare the numbers $\frac{7a-2}{7a+1}$ and $\frac{8a-3}{8a+1}$ making their difference.

$$\frac{7a-2}{7a+1} - \frac{8a-3}{8a+1} = \frac{4a+1}{(7a+1)(8a+1)} > 0 \Rightarrow \frac{7777...75}{7777...78} > \frac{8888...85}{8888...89}$$

2. $1 - \frac{1}{2} \cdot \left(0.1 \cdot 10^2 - \sqrt{81}\right) : 2^{-1} = 1 - \frac{1}{2}(10-9) : \frac{1}{2} = 1 - 1 = 0$

3. For $x \neq 3$ and $y \neq -1$ the system is:

$\Leftrightarrow \begin{cases} y+1 = x+3 \\ 3x+2y = 9 \end{cases} \Leftrightarrow \begin{cases} x-y=-2 \\ 3x+2y=9 \end{cases}$ with the solution $(1; 3)$

4. We assume the number is rational, so there exists:

$x \in N^*$ so that $x^2 = n + \sqrt{n+1}$.

It follows that there exists $y \in N^*$ so that $y^2 = n + 1$ so $n = y^2 - 1$.

And we obtain $x^2 = y^2 - 1 + y$. From $y^2 < y^2 + y - 1 < y^2 + 2y + 1 = (y+1)^2 \Rightarrow y^2 < x^2 < (y+1)^2$ which is false. So the assumption is false, it follows that $\sqrt{n+\sqrt{n+1}} \notin Q$.

II. 1. We obtain the equation:

$\frac{10x+y}{10y+x} = 2 - \frac{y}{x}$ or $(x-y)\cdot(4x-5y) = 0$. The solutions follow $(x; x)$ or $x = 5$ and $y = 4$.

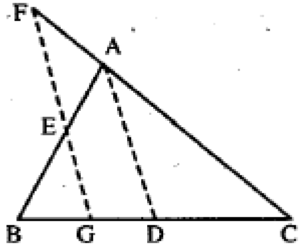
2. We know that: $x^2 + y^2 + z^2 \geq xy + yz + xz \Rightarrow 2(x^2 + y^2 + z^2) \geq$ $\geq 2(xy + yz + xz)$ and adding in both terms $x^2 + y^2 + z^2$ we get:

$3(x^2 + y^2 + z^2) \geq (x+y+z)^2 \Leftrightarrow (x+y+z)^2 \leq 3$, i.e. $|x+y+z| \leq \sqrt{3}$.

3. AD =median, $GF||AD$

In ΔABD; $GE \parallel AD \Rightarrow$ according to T. Thales that $\frac{AE}{AB} = \frac{GD}{BD}$ (1)

In ΔGCF; $GF \parallel AD \Rightarrow$ according to T. Thales that $\frac{AF}{AC} = \frac{GD}{DC}$ (2)



$$(1):(2) \Rightarrow \frac{AE}{AF} \cdot \frac{AC}{AB} = 1 \Rightarrow \frac{AE}{AF} = \frac{AB}{AC}.$$

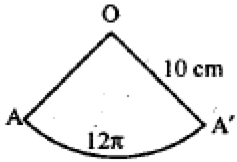
III. a) From the problem it follows that $R = 6; h = 8; G = 10$ the elements of the cone.

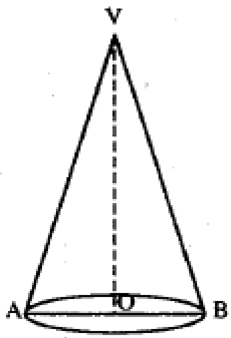
$$V = \frac{1}{3}\pi R^2 h = 96\pi \text{ cm}^3.$$

b) $A_l = \pi RG$, $A_l = 60\pi \text{ cm}^2$.

c) By deploying the lateral surface of the cone we obtain a sector of circle with the radius of $10\ cm$ and the length of the arch (AA') is equal to $2\pi R = 12\pi$. Using cross-multiplication we obtain:

$$\begin{array}{l} 360^\circ \text{ ———————— } 20\pi \\ \underline{x^\circ \text{ ———————— } 12\pi} \\ x^\circ = \dfrac{360 \cdot 12\pi}{20\pi} = 216^\circ. \end{array}$$





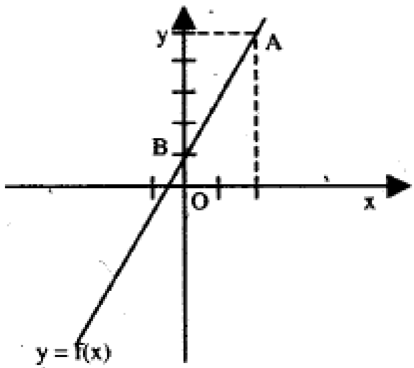
Test no. 11

I. 1. a) $\left(0{,}001:\frac{1}{20}+\sqrt{0{,}0064}\right)\cdot\left(3^2+1^0\right)=\left(\frac{1}{1000}\cdot\frac{20}{1}+\sqrt{\frac{64}{10000}}\right)\cdot$
$\cdot(9+1)=\left(\frac{1}{50}+\frac{4}{50}\right)\cdot 10=1$

b) $(1+a)^2-(1-a)^2=4a$

2. $f(x)=2x+m$, from the condition $A(2,5)\in G_f\Rightarrow 5=4+m\Rightarrow$
$\Rightarrow m=1$.

$f(x)=2x+1$, $B(0;1)$, $A(2;5)\in G_f$



3. $\sqrt{x^2-2x+1}+\sqrt{1-2x+x^2}=2$, $x\in\mathbf{R}\Leftrightarrow|x-1|+|x-1|=$
$=2\Leftrightarrow 2\cdot|x-1|=2\Leftrightarrow|x-1|=4\Leftrightarrow x-1=\pm1$, $x\in\{0,2\}$.

II. 1. From $p>3$, prime, it follows that $p=2k+1, k>1$. It follows that:

$p^2=(2k+1)^2=4k(k+1)+1=8f+1$,

so the remainder of the division of p^2 to 8 is 1. From p prime it follows that p can be of the form:

$p=3k+1$ or $p=3k+2$, $k\in\mathbf{N}$ so $p^2=3(3k^2+2k)+1$ or
$p^2=3(3k^2+4k+1)+1$ and so $p^2-1\,\vdots\,3$. So $p^2-1\,\vdots\,24$

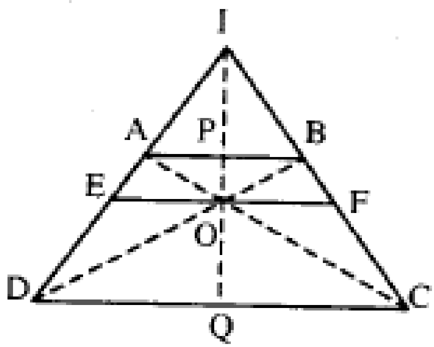
so the remainder of the division of p^2 to 24 is 1.

$$2.\ P(b) = 3b^2 - 3ab + a^2 = 3(b^2 - ab) + a^2 = 3\left(b^2 - 2\cdot\frac{a}{2}\cdot b + \frac{a^2}{4}\right) +$$

$$+ a^2 - \frac{3a^2}{4} = 3\left(b - \frac{a}{2}\right)^2 + \frac{a^2}{4} \geq 0.$$

The equality takes place for $a = 0$ and $b = 0$.

3. Let I be the intersection point of the nonparallel sides of the trapezoid, P and Q the intersection of the straight line OI to the bases.



We trace through O the parallel EF to the bases of the trapezoid. We have the equalities:

$\frac{EO}{AB} = \frac{DE}{DA} = \frac{CF}{CB} = \frac{OF}{AB}$ so $EO = OF$.

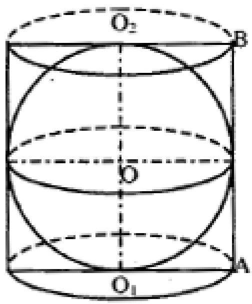
We also have the equalities:

$\frac{AP}{EO} = \frac{IP}{IO} = \frac{PB}{OF}$ from where it follows that: $AP = PB$ (because $EO = OF$),

i.e. P is the middle of (AB). Then we have: $\frac{EO}{DQ} = \frac{IO}{IQ} = \frac{OF}{QC}$ so

$DQ = QC$, i.e. Q is the middle of (CD).

III. a) The length of the cylinder radius is $3\ cm$ and the length G of the cylinder's generating line is $6\ cm$. The length of the inscribed sphere is $3\ cm$.



$A_t = 2\pi R(R + G)$, $A_t = 54\pi\ cm^2$; $V_{cil} = \pi R^2 h$, $V = 54\pi\ cm^2$

b) $V_{sf} = \frac{4\pi R^3}{3}$, $V_{sf} = 36\pi\ cm^3$, $\frac{V_{sf}}{V_{cil}} = \frac{2}{3}$.

Test no. 12

I. 1. $E = 3\cdot 5\cdot 5^{2n} + 2\cdot 2^{3n} = 15\cdot 5^{2n} + 2\cdot 2^{3n} = (17-2)\cdot 5^{2n} + 2\cdot 3^{3n} =$
$= 17\cdot 5^{2n} - 2(5^{2n} - 2^{3n}) = 17\cdot 5^{2n} - 2(25^n - 8^n) = 17\cdot 5^{2n} -$
$- 2(25-8)(25^{n-1} + \ldots + 8^{n-1}) = 17[5^{2n} - 2(25^{n-1} - \ldots + 8^{n-1})]$

2. The given fraction is simplified by 17 if and only if $17|\overline{52a}$ and $17|\overline{1b75}$, which leads to:

: $\overline{52a} = 17k$, $\overline{1b75} = 17l$ and $k, l \in \mathbb{N}^*$.

$520 \le \overline{52a} \le 529 \Rightarrow 520 \le 17k \le 529 \Rightarrow 30{,}5\ldots \le k \le 31{,}1\ldots$ and as $k \in \mathbb{N}^*$ we have $\underline{k = 31} \Rightarrow \overline{52a} = 17\cdot 31 = 527 \Rightarrow a = 7$.

$1075 \le \overline{1b75} \le 1975 \Rightarrow 1075 \le 17l \le 1975 \Rightarrow 63{,}2\ldots \le l \le 116{,}1\ldots$

and as $l \in \mathbb{N}^* \Rightarrow l \in \{64, \ldots, 116\}$. From $\left.\begin{array}{l} 25 \mid \overline{1b75} \\ 2 \nmid \overline{1b75} \end{array}\right\} \Rightarrow l = 75$.

So $\overline{1b75} = 17\cdot 75 = 1275 \Rightarrow b = 2$.

3. $[(-1)^n \cdot (-2^2)^n - 4^n] : (-4^n) = \{[(-1)\cdot(-4)]^n - 4^n\} : (-4^n) =$
$= (4^n - 4^n) : (-4^n) = 0 : (-4^n) = 0$.

II. 1. $\min(-5; -2) = -5$; $\dfrac{|-7|}{|-2|} = \dfrac{7}{2}$; $\sqrt{(-4)^2} = |-4| = 4$.

So $\min(-5;-2) + \dfrac{|-7|}{|-2|} - \sqrt{(-4)^2} = -5 + \dfrac{7}{2} - 4 = -\dfrac{11}{2}$

2. $\left.\begin{array}{r} \sqrt{3}\left(\sqrt{2} - x\right)^2 \ge 0 \\ \sqrt{2} - \sqrt{3} < 0 \end{array}\right| \Rightarrow$ the equation can't have a real solution.

3. $\dfrac{4+8+12+\ldots+400}{3+6+9+\ldots+300} = \dfrac{4(1+2+3+\ldots+100)}{3(1+2+3+\ldots+100)} = \dfrac{4}{3}$

III. 1. Let x be the measure of the angle's supplement. So:

$x + \dfrac{7}{8}x = 180° \Rightarrow x = 96°$.

$180° - 96° = 84°$; $\dfrac{2}{3}\cdot 96° = 64°$; $\dfrac{3}{4}\cdot 84° = 63° \Rightarrow 64° - 63° = 1°$.

2. We apply the theorem: Two parallel planes are intersected by a second plane after two straight parallel lines. In this way we show that the opposite sides of the quadrilateral $EFGH$ are parallel and so it is a parallelogram.

3. Because $AB = AD = ABCD -$rbombus and the diagonals are perpendicular between themselves ⇒

$\Rightarrow d(E, BD) = EO = \frac{a\sqrt{7}}{2}$, where $AC \cap BD = \{O\}$.

Let $AP \perp BC$, $d(E, BC) = AP = d(E, CD) = \frac{a\sqrt{7}}{2}$.

Test no. 13

I. 1. Establishing the last digit of each term we notice that for any uneven number n we obtain $A \vdots 5$.

2. So $2^n \cdot 3^{n+1} \cdot 5^{n+2} = 30^{n+1} \cdot \frac{5}{2} \Rightarrow 2^n \cdot 3^n \cdot 3 \cdot 5^n \cdot 5^2 = 30^n \cdot 30 \cdot \frac{5}{2} \Rightarrow$

$(2 \cdot 3 \cdot 5)^n \cdot 75 = 30^n \cdot 75 \Rightarrow 30^n \cdot 75 = 30^n \cdot 75$.

3. $x = \frac{3}{4} y \Rightarrow \frac{5y-7x}{6y-8x} = \frac{y\left(5-7\cdot\frac{3}{4}\right)}{y\left(6-8\cdot\frac{3}{4}\right)} = \frac{-\frac{1}{4}}{0}$ = imposibile.

So, if $\frac{x}{y} = \frac{3}{4}$ the rapport $\frac{5y-7x}{6y-8x}$ doesn't exist.

II. 1. $|2x+5-|x-2|| = 0 \Rightarrow 2x+5 = |x-2|$.

a) $x \geq 2 \Rightarrow 2x+5 = x-2 \Leftrightarrow x = -7$ not acceptable;

b) $x < 2 \Rightarrow 2x+5 = 2-x \Leftrightarrow x = -1$ acceptable.

2. $n = 1991 + 2\cdot(1+2+3+\ldots+1990) = 1991 +$

$+ 2 \cdot \frac{(1+1990)\cdot 1990}{2} = 1991 + 1991 \cdot 1990 = 1991 \cdot 1991 = 1991^2$.

3. We know that $[x]$ – the integer part of x, $|x|$ – the module of x, $\{x\}$ – the decimal part of x. The system becomes:

$$\begin{cases} \max\left(-4;-\sqrt{17};-3\right)\cdot x - 0{,}3\cdot y = \frac{7}{6} - 24 \\ \min\left(\frac{1}{8};\sqrt{6};1\right)\cdot x + \frac{1}{9}y = -\frac{1}{72} \end{cases} \Leftrightarrow \begin{cases} -3x - \frac{3}{10}y = \frac{7}{6} - 24 \\ \frac{1}{8}x + \frac{1}{9}y = -\frac{1}{72} \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$$

III. 1. They are angles adjacent to the bisectors perpendicular between them $\Rightarrow$ they are supplementary angles. Let their measures be:

x and $5x \Rightarrow x + 5x = 180° \Rightarrow x = 30°$.

2. Let there be the trapezoid ABCD with $AB \parallel DC$, $AB > DC$, $AD \perp AB$ and $AC \perp BD$. From D we build $DD' \parallel AC$, $D' \in AB \Rightarrow$ $\Rightarrow D'D \perp DB$ and in $\Delta D'DB$ according to the theory of height we have: $AD^2 = D'A \cdot AB$. But $D'A = DC \Rightarrow AD^2 = DC \cdot AB$ – q.e.d.

3. The plane AEF intersects the straight lines BD and CD respectively in the points P and Q. In ΔABP, $[BE]$ is bisector and height $\Rightarrow \Delta ABP$ is isosceles $\Rightarrow [BE]$ and median $\Rightarrow AE = EP$. Analogously $AF = FQ$. In ΔAPQ, according to the reciprocal of Thales' theorem:

$$\left.\begin{array}{l} EF \parallel PQ \\ \text{But } PQ \subset (BCD) \end{array}\right| \Rightarrow EF \parallel (BCD) - \text{q.e.d.}$$

Test no. 14

I. 1. $\overline{ab} = cd \Rightarrow 10a + b = cd$ (1)

But $a = x - 1;\ b = x;\ c = x + 1;\ d = x + 2$ (2)

From (1)and (2) $\Rightarrow 10(x-1) + x = (x+1)(x+2) \Leftrightarrow x^2 - 8x + 12 = 0 \Leftrightarrow$
$\Leftrightarrow (x-6)(x-2) = 0 \Rightarrow x_1 = 6$ and $x_2 = 2$.

i) If $x = 6 \Rightarrow a = 5;\ b = 6;\ c = 7;\ d = 8;$

ii) If $x = 2 \Rightarrow a = 1;\ b = 2;\ c = 3;\ d = 4.$

2. Obviously $\frac{2x+5}{x-1}$ has to be a perfect square $\Rightarrow$

$$\Rightarrow \frac{2x+5}{x-1} = \frac{2x-2+7}{x-1} = 2 + \frac{7}{x-1} \in \mathbf{N} \Rightarrow \frac{7}{x-1} \in \mathbf{N} \Rightarrow x - 1 \mid 7 \Rightarrow$$

$x - 1 \in \{\pm 1, \pm 7\} \Rightarrow x \in \{-6, 0, 2, 8\}$.

But $\frac{2x+5}{x-1}$ is a perfect square only for $x \in \{-6, 2\}$.

3. Let $d \mid 10n + 53$ and $d \mid 7n + 37 \Rightarrow d \mid 7(10n + 53) -$
$- 10(7n + 37) \Leftrightarrow d \mid 1.$

II. 1. We know $\min(a, b) = \begin{cases} a \text{ for } a \leq b \\ b \text{ for } a > b \end{cases}$. So:

$$\min\left(x + 1; 4 - \frac{x}{2}\right) = \begin{cases} x + 1 & \text{for } \leq 2 \\ 4 - \frac{x}{2} & \text{for } > 2 \end{cases}$$

If $x \leq 2$ we have $\min\left(x + 1; 4 - \frac{x}{2}\right) = x + 1 \Leftrightarrow x + 1 \geq 1 \Rightarrow x \geq 0 \Rightarrow$

$\Rightarrow x \geq 0 \Rightarrow x \in \{0, 1, 2\}$ (1)

If $x > 2$ we have $\min\left(x + 1; 4 - \frac{x}{2}\right) = 4 - \frac{x}{2} \Rightarrow 4 - \frac{x}{2} \geq 1 \Rightarrow$

$\Rightarrow x \leq 6 \Rightarrow x \in \{3, 4, 5, 6\}$ (2)

From (1) and (2) $\Rightarrow A = \{0, 1, 2, 3, 4, 5, 6\}$.

2. $f(x) = \sqrt{3^2 + 3^{-2} - \left(\frac{1}{3}\right)^2} \cdot x - \left|-2\right| \Rightarrow f: \mathbf{R} \to \mathbf{R},\ f(x) = 3x - 2.$

We assign real values to x and we obtain the two points necessary for the graphical interpretation.

3. Obviously:

$a = 2^{1988}(2^2 - 2 - 1) = 2^{1988} \cdot 1 = 2^{1988}$.

So:

$$\frac{2^{1988}}{x} = \frac{2^{1996}}{\frac{1}{2^2}} \Rightarrow x = \frac{2^{1998}}{2^{1996}} \cdot \frac{1}{2^2} \Rightarrow x = \frac{2^{1998}}{2^{1998}} \Rightarrow x = 1 .$$

III. 1. $\left.\begin{array}{l} \sin B = \frac{AC}{BC} \\ \sin C = \frac{AB}{BC} \end{array}\right| \Rightarrow \sin B \cdot \sin C = \frac{AC \cdot AB}{BC^2} = \frac{AC \cdot AB}{AC^2 + AB^2} \Rightarrow$

$$\Rightarrow \frac{1}{\sin B \cdot \sin C} = \frac{AC^2}{AC \cdot AB} + \frac{AB^2}{AC \cdot AB} = \frac{AC}{AB} + \frac{AB}{AC} \qquad (1).$$

Observation:

$AC > 0$ şi $AB > 0 \Rightarrow (AC - AB)^2 \geq 0 \Rightarrow AC^2 + AB^2 \geq 2AB \cdot AC \mid$

$\mid :(AC \cdot AB) \Rightarrow \frac{AC}{AB} + \frac{AB}{AC} \geq 2 \qquad (2).$

From (1) and (2) $\Rightarrow \frac{1}{\sin B \cdot \sin C} \geq 2 \Rightarrow \sin B \cdot \sin C \leq \frac{1}{2} \Rightarrow$

$\Rightarrow \sin B \cdot \sin C \leq \sin 30°$ q.e.d.

Obviously, if $AC = AB \Rightarrow \Delta ABC$ right and isosceles $\Leftrightarrow$ $\sin B \, . \sin C = \sin 30°$ q.e.d.

2. We apply the theory of the median in ΔABC:

$$CM^2 = \frac{2(BC^2 + AC^2) - AB^2}{4} \qquad (1)$$

We apply the theory of the median in ΔDAB:

$$DM^2 = \frac{2(BD^2 + AD^2) - AB^2}{4} \qquad (2)$$

From (1) and $(2) \Rightarrow \Delta CMD$ isosceles, but $CN = ND \Rightarrow$ $MN \perp CD$ (I). We apply the theory of the median in ΔACD and ΔBCD and we obtain $AN = BN \Rightarrow \Delta ABN$ isosceles, but $BM = MA \Rightarrow MN \perp AB$ (II). From (1) and $(2) \Rightarrow MN$ is the common nperpendicular of the straight lines AB and CD q.e.d

Test no. 15

I. It is obvious that:

$$a+b=\underbrace{\left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{2}{3}+\frac{1}{3}\right)+\left(\frac{3}{4}+\frac{1}{4}\right)+\ldots+\left(\frac{1992}{1993}+\frac{1}{1993}\right)}_{1992\text{ brackets}}=\underbrace{1+1+1+\ldots+1}_{1992\text{ brackets}}=1992$$

and so:

$3\mid 1992\Rightarrow 3\mid(a+b)$ q.e.d.

2. Making the calculations for $n=2k$ and $n=2k+1$ where $n\in\mathbb{N}$ we obtain that the equality takes place for $n=2k$.

3. It is obvious that:

$$\frac{1}{1\cdot 2}=\frac{1}{1}-\frac{1}{2};\ \frac{1}{2\cdot 3}=\frac{1}{2}-\frac{1}{3}\ \text{ and so }\ \frac{1}{n\cdot(n+1)}=\frac{1}{n}-\frac{1}{n+1}$$

as (obs.) $\frac{1}{n+1}>0$ and we will have:

$$A=\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots+\frac{1}{n}-\frac{1}{n+1}=1-\frac{1}{n+1}\Rightarrow A<1\quad \text{q.e.d.}$$

II. 1. We note:

$$n^2+n=t\Rightarrow (t-7)(t-3)+4=t^2-10t+25=(t-5)^2$$

so:

$$a=(n^2+n-7)(n^2+n-3)+4=(n^2+n-5)^2.$$

so $\underline{a}$ is a perfect square.

2. We give to $\underline{x}$ the values 0 and 1 and we obtain the system:

$$\begin{cases}f(0)+2f(1)=1\\ f(1)+2f(0)=2\end{cases}\Leftrightarrow\begin{cases}f(0)+2f(1)=1\\ -4f(0)-2f(1)=-4\end{cases}\Leftrightarrow\begin{cases}f(0)=1\\ f(1)=0\end{cases}\qquad(1)$$

As $f(x)$ is linear $\Rightarrow f(x)=ax+b.\qquad(2)$

From (1) and $(2)\Rightarrow f:\mathbb{R}\to\mathbb{R}, f(x)=x-2.$

3. $P(X)=(X^{1993}+1992X^{1992})+(X^{1992}+1992X^{1991})+\ldots+$

$+X^2+1992X+X+1993=(X+1992)(X^{1992}+X^{1991}+X^{1990}+\ldots+$

$+X^2+X)+X+1993.$

It is obvious that:

$P(-1992)=0\cdot A-1992+1993=1\Rightarrow P(-1992)=1.$

III. 1. We extend the side AC with $[CA'] \equiv [AC]$ (C between A and A'). We obtain $\Delta AA'B$ where $[AA'] \equiv [AB]$ and $m(\sphericalangle A) = 60^\circ \Rightarrow$ $\Rightarrow \Delta AA'B$ equilateral. CB median in $\Delta AA'B$ equilateral $\Rightarrow BC \perp AA'$ so $m(\sphericalangle C) = 90^\circ$ and $m(\sphericalangle B) = 30^\circ$.

2. $A_{\Delta ABC} = \frac{1}{2} a \cdot h_A = \frac{1}{2} a \cdot h_B = \frac{1}{2} a \cdot h_C \Rightarrow a \cdot h_A = a \cdot h_B = a \cdot h_C \Rightarrow$

$\Rightarrow \frac{h_A}{\frac{1}{a}} = \frac{h_B}{\frac{1}{b}} = \frac{h_C}{\frac{1}{c}}$ q.e.d.

3. Let there be the pyramid VABC, where ΔABC equilateral $AB = 6\sqrt{3} \Rightarrow R\sqrt{3} = 6\sqrt{3} \Rightarrow R = 6m$. Let OM be the aphotem of the base $\Rightarrow OM = \frac{1}{2} R = 3m$. From ΔVOM $(\sphericalangle O = 90^\circ) \Rightarrow VO =$

$= \sqrt{VM^2 - OM^2} = \sqrt{25-9} = \sqrt{16} = 4$ m. $A_l = \frac{1}{2} P_b \cdot A_p = \frac{1}{2} \cdot AB \cdot VM = \frac{1}{2} \cdot$

$\cdot 18\sqrt{3} \cdot 5 = 45\sqrt{3}$ m^2

$V = \frac{1}{3} A_b \cdot h_p = \frac{1}{3} \frac{(6\sqrt{3})^2 \sqrt{3}}{4} \cdot 4 = \frac{36 \cdot 3 \cdot \sqrt{3}}{3} = 36\sqrt{3}$ m^3.

Test no. 16

I. 1. $a^2 + ac + 5b = a(a + c) + (a + c)b = (a + b)(a + c) = 15.$

2. $\frac{x}{y} = \frac{4}{3} \Leftrightarrow \frac{x - y}{x + y} = \frac{4 - 3}{4 + 3} \Leftrightarrow \frac{x - y}{x + y} = \frac{1}{7} \Rightarrow \frac{xy - y^2}{xy + y^2} = \frac{x - y}{x + y} = \frac{1}{7}.$

3. $E(2) = 6 + p$, $E(3) = 12 + p \Rightarrow 6 \mid p$; as $E(5) = 30 + p \Rightarrow 6 \mid E(5)$.

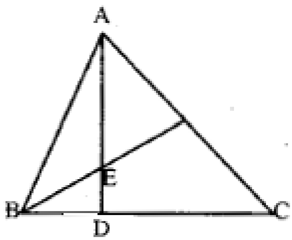
II. 1. $|x - 1| > 2 \Leftrightarrow 2 < x - 1 < -2 \Leftrightarrow 3 < x < -1 \Leftrightarrow x \in (-\infty, -1) \cup$
$\cup (3, \infty)$.

2. $(x + 1) + (x + 2) + \ldots + (x + 100) = 15050 \Leftrightarrow 100x + (1 + 2 + \ldots +$
$+ 100) = 15050 \Leftrightarrow 100x + 50 \cdot 101 = 15050 \Leftrightarrow 100x = 10000 \Leftrightarrow x = 100.$

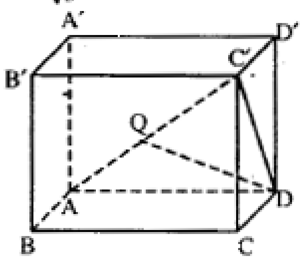
3. We must have:

$+ 100) = 15050 \Leftrightarrow 100x + 50 \cdot 101 = 15050 \Leftrightarrow 100x = 10000 \Leftrightarrow x = 100.$

III. 1. Let D be the foot of the height and $\sphericalangle BED$ the required angle. We calculate the measures of the angles ABD and $EBD \Rightarrow m(\sphericalangle BED) = 57°$.



2. $C'C \perp (ABCD)$, $CD \perp AD \Rightarrow T.3\perp \Rightarrow C'D \perp AD$.
So $\Delta ADC'$ is right $(m(\sphericalangle ADC') = 90°)$. $AC' = a\sqrt{3}$, $C'D = a\sqrt{2}$, $AD = a$,
$AD^2 = AQ \cdot AC' \Rightarrow AQ = \frac{a}{\sqrt{3}} \Rightarrow \frac{AQ}{AC'} = \frac{1}{3}$.



Test no. 17

I. 1. We notice that:

$$2\cdot(-1)^k + 2\cdot(-1)^{k+1} + np = 2\cdot(-1)^k - 2\cdot(-1)^k + np = np.$$

So $\dfrac{np}{(n+p)\sqrt{np}} < \dfrac{1}{2} \Leftrightarrow \left(\sqrt{n} - \sqrt{p}\right)^2 > 0$ so true.

2. $\dfrac{a}{b} = \dfrac{13}{12} \Rightarrow b = \dfrac{12}{13}a$.

So $\dfrac{3a-b}{4a+b} = \dfrac{3a - \dfrac{12}{13}a}{4a + \dfrac{12}{13}a} = \dfrac{a\cdot\dfrac{27}{13}}{a\cdot\dfrac{64}{13}} = \dfrac{27}{64} = \left(\dfrac{3}{4}\right)^3$.

3. Making the calculations we obtain:

$$E = \frac{1}{(3a-2)(3a+1)}.$$

Using this, we will obtain:

$$S = \frac{1}{3}\left[\left(1 - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) + \ldots + \left(\frac{1}{3n-2} - \frac{1}{3n+1}\right)\right] = \frac{n}{3n+1} <$$

$$< \frac{n}{3n} = \frac{1}{3}.$$

So $S < 1/3$ q.e.d.

II. 1. $8x^2 - 6xy + y^2 - 5 = 0 \Rightarrow 8x^2 - 4xy - 2xy + y^2 - 5 = 0 \Rightarrow$ $\Rightarrow 4x(2x - y) - y(2x - y) = 5 \Rightarrow (2x - y)(4x - y) = 5 \Rightarrow (x, y) \in \{(2, 3);$ $(-2, -9); (-2, -3); (2, 9)\}$.

2. It is obvious that:

$$f(x) = \sqrt{(x+3)^2} + \sqrt{\left(3 - \sqrt{5}\right)^2} \Rightarrow f(x) = |x+3| + \left|3 + \sqrt{5}\right| \Rightarrow f(x) = |x+3| +$$

$$+\left(3 - \sqrt{5}\right).$$

We immediately have:

$$f(-3) = 3 - \sqrt{5};\ f(1) = 7 - \sqrt{5};\ f\left(-\sqrt{5}\right) = 2\left(3 - \sqrt{5}\right);\ f\left(1 + \sqrt{5}\right) = 7$$

and from here the immediate ordering.

3. It is obvious that:

$$P(1) = 0 \Rightarrow a(1-b)(-1)^{2k+1} - b(-1)^{k+2} - a(-1)^{k+2} + 1 = 0.$$

For $\underline{k}$ even $\Rightarrow (b-2)(a-1)=1 \Rightarrow a=2$ and $b=3$.

For $\underline{k}$ uneven $\Rightarrow b(a+1)=-1 \Rightarrow a=-2$ and $b=1$.

III. 1. Let $(BF$ bisector $\sphericalangle B$, $F \in (AC)$. F is on the line segment bisector of $[BC]$, so $[FC] \equiv [FB] \Leftrightarrow \sphericalangle FCB \equiv \sphericalangle FBC \Leftrightarrow m(\sphericalangle B) = 2m(\sphericalangle C)$. q.e.d.

2. In ΔABC we construct the height CD, so $CD \perp AB$. Using the two right triangles thus obtained BCD and ACD we will determine the lengths of the segments AD and BD and so:

$AB = (5+\sqrt{6})$cm when $\sphericalangle A$ is acute and $AB = (5-\sqrt{6})$cm when $\sphericalangle A$ is obtuse.

3. We note with $R -$ the radius of the sphere, $r -$the radius of the cylinder and $a -$ the edge of the cube. We have:

$$4\pi R^2 = 6\pi r^2 = 6a^2 \Rightarrow R = \frac{\sqrt{3}}{\sqrt{2\pi}}a;\ r = \frac{a}{\sqrt{\pi}}.$$

$$V_{sf} = 2\cdot\frac{\sqrt{3}}{\sqrt{2\pi}}\cdot a^3,\ V_{cil} = \frac{2a^3}{\sqrt{\pi}},\ V_{cub} = a^3.\ \text{Deci } V_{cub} < V_{cil} < V_{sf}.$$

Test no. 18

I. 1. $\left|a\sqrt{3}-1\right| \geq 0$ and $\sqrt{3}-\sqrt{5}<0 \Rightarrow a \in \Phi$

2. $\sqrt{2+\sqrt{2}}<2 \Leftrightarrow 2+\sqrt{2}<4 \Leftrightarrow \sqrt{2}<2$. $\sqrt{2+\sqrt{2+\sqrt{2}}}<$ $<\sqrt{2+\sqrt{2}}<2$.

3. We note $\frac{1}{x}=a, \frac{1}{y}=b$; the system becomes:

$$\begin{cases} a+b=0.7 \\ 3a-5b=0.5 \end{cases} \Leftrightarrow \begin{cases} 3a+3b=2.1 \\ 3a-5b=0.5 \end{cases} \Leftrightarrow \begin{cases} a+b=0.7 \\ 8b=1.6 \end{cases} \Leftrightarrow \begin{cases} a=0.5 \\ b=0.2 \end{cases}.$$

II. 1. $4 \mid \overline{32a} \Rightarrow a \in \{0, 4, 8\}$. For $a=0 \Rightarrow \frac{x}{3}=\frac{320}{5} \Rightarrow x=192$.

For $a=4 \Rightarrow \frac{x}{3}=\frac{324}{5} \Rightarrow x=194.4$.

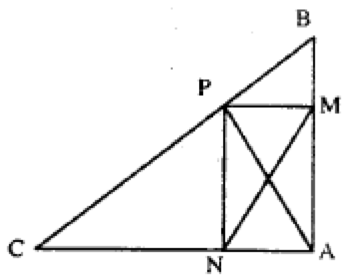
For $a=8 \Rightarrow \frac{x}{3}=\frac{328}{5} \Rightarrow x=196.8$.

2. a) $(x-y)^2+(x-z)^2+(y-z)^2 \geq 0 \Leftrightarrow 2x^2+2y^2+2z^2 \geq 2xy+$ $+2xz+2yz \Leftrightarrow x^2+y^2+z^2 \geq xy+xz+yz$.

b) From $x+y+z=1 \Rightarrow x^2+y^2+z^2+2(xy+xz+yz)=1$ and as: $xy+xz+yz \leq x^2+y^2+z^2 \Rightarrow 3(x^2+y^2+z^2) \geq 1 \Leftrightarrow x^2+y^2+z^2 \geq 1/3$.

III. 1. We observe that:

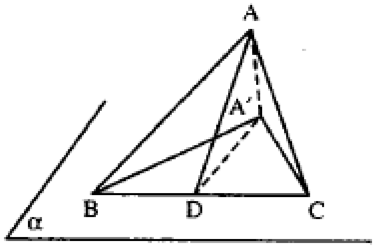
$AB^2+AC^2=3a^2=BC^2$.



So the triangle ABC is right in A. We immediately deduce that $ANMP$ is right and so:

$MN = AP = \frac{AB \cdot AC}{BC} = \frac{a\sqrt{6}}{3}$.

2. Let D be the foot of the perpendicular traced from A on BC; $BD = DC$.



$\Delta ABA' \equiv \Delta ACA'$ (ic) so $A'D \perp BC$; the plane angle corresponding to the dihedral is ADA'; $m(\sphericalangle ADA') = 45°$. Let $AB = a \Rightarrow AC = a$, $BC = a\sqrt{2}$.

$AD = \frac{BC}{2} = \frac{a\sqrt{2}}{2}$; $AA' = AD \cdot \sin 45° = \frac{a}{2}$; $\sin ACA' = \frac{AA'}{AC} = \frac{1}{2} \Rightarrow$

$\Rightarrow m(\sphericalangle ACA') = 30°$; the same for $m(\sphericalangle ABA') = 30°$.

Test no. 19

I. 1. For all the odd exponents:

$$\Rightarrow S = \underbrace{-1-1-1-\ldots-1}_{101 \text{ terms}} = -101 = S_{min}.$$

For all the even exponents:

$$\Rightarrow S = \underbrace{1+1+1+\ldots+1}_{101 \text{ terms}} = 101 = S_{max}.$$

So:

$S_{min} + S_{max} = -101 + 101 = 0.$

In general, $-101 \le S \le 101$. As they are an odd number of terms $\Rightarrow S \neq 0$.

2. From the geometric mean $\Rightarrow$

$\Rightarrow \sqrt{4^x \cdot 8^y} = 64 \Leftrightarrow 2^{2x} \cdot 2^{3y} = \left(2^6\right)^2 \Leftrightarrow 2^{2x+3y} = 2^{12} \Leftrightarrow 2x + 3y = 12 \Leftrightarrow 2x =$
$= 12 - 3y$ and $2x \in \mathbf{N} \Rightarrow 12 - 3y \in \mathbf{N}$. So: $12 - 3y \ge 0 \Rightarrow 12 \ge 3y \Leftrightarrow y \le 4$.
Let $y = 4 \Rightarrow 2x = 0 \Rightarrow x = 0 \Rightarrow (0,4)$ a solution and so on ... $y = 3 \Rightarrow$
$\Rightarrow 2x = 3 \Rightarrow x \notin \mathbf{N}$ ...

3. $\overline{a4} \cdot \overline{a6} + 1 = \overline{a4} \cdot \left(\overline{a4} + 2\right) + 1 = \left(\overline{a4}\right)^2 + 2\overline{a4} + 1 = \left(\overline{a4} + 1\right)^2$ q.e.d.

II. 1. $|3x - 2| \le 7 \Leftrightarrow \begin{cases} 3x - 2 \le 7 \\ 3x - 2 \ge -7 \end{cases} \Leftrightarrow \begin{cases} x \le 3 \\ x \ge -\frac{5}{3} \end{cases} \Leftrightarrow \left.\begin{matrix} x \in \left[-\frac{5}{3}, 3\right] \\ x \in \mathbf{N} \end{matrix}\right| \Rightarrow$

$\Rightarrow A = \{0,1,2,3\}$.

$x - 5 \in [-1, 2] \Leftrightarrow -1 \le x - 5 \le 2 \Leftrightarrow 4 \le x \le 7 \Leftrightarrow \left.\begin{matrix} x \in [4,7] \\ x \in \mathbf{Z} \end{matrix}\right| \Rightarrow B = \{4,5,6,7\}$.

So $A \cap B = \emptyset$.

2. $\begin{cases} x + 2^n y = 4^n \\ 2x - y = 2^{2n+1} \end{cases} \Leftrightarrow \begin{cases} x + 2^n y = 4^n \\ 2^{n+1} x - 2^n y = 2^{3n+1} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x\left(1 + 2^{n+1}\right) = 2^{2n} + 2^{3n+1} \\ x + 2^n y = 4^n \end{cases} \Leftrightarrow \begin{cases} x = 2^{2n} \\ y = 0 \end{cases}$

3. $\left.\begin{array}{l}\text{From } P(X) \vdots (X-1) \\ \text{From } P(X) \vdots (X+1) \text{ remainder } 2\end{array}\right\} \Rightarrow \begin{cases} P(1)=0 \\ P(-1)=2 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} -m+n=-2 \\ m+n=0 \end{cases} \Leftrightarrow \begin{cases} n=-1 \\ m=1 \end{cases}.$

III. 1. From V we build:

$OL \perp AB,\ OM \perp BC,\ ON \perp AC \Rightarrow OL = OM = ON$

$m(\sphericalangle OEN) = m(\sphericalangle C) + m(\sphericalangle OBC) = 60^\circ + m(\sphericalangle OBC)$ (1)

$m(\sphericalangle OKM) = 180^\circ - m(\sphericalangle AKB) = 180^\circ - [180^\circ - (m(\sphericalangle OAB) +$
$+ m(\sphericalangle B))] = m(\sphericalangle OAB) + m(\sphericalangle B) = m(\sphericalangle A/2) + m(\sphericalangle B) = 180^\circ -$
$- m(\sphericalangle OAC + 60^\circ) = 120^\circ - m(\sphericalangle A/2) = 60^\circ + m(\sphericalangle ABO) = 60^\circ +$
$+ m(\sphericalangle OBC)$ (2)

From (1) and (2) $\left.\begin{array}{l} m(\angle OEN) = m(\angle OKM) \\ \Rightarrow ON = OM \\ m(\hat{N}) = m(\hat{M}) = 90^\circ \end{array}\right| \Rightarrow \Delta ONE \equiv \Delta OMK \Rightarrow \Rightarrow OE = OK$ q.e.d.

2. We build $AM \Rightarrow \Delta MAB$ and ΔMAD. In ΔMAD we have: $m(\sphericalangle MAD) > m(\sphericalangle DAC) \Rightarrow m(\sphericalangle MAD) > 45^\circ$. Analogously in ΔBAM we have $m(\sphericalangle MAB) > 45^\circ$. Compare : ΔMAB and ΔMAD where:

$\left.\begin{array}{l} AB = AD \\ AM = AM \\ m(\sphericalangle MAD) > m(\sphericalangle MAB) \end{array}\right| \Rightarrow \Delta MAB \not\equiv \Delta MAD \Rightarrow MD > MB$

3. From the calculations we will find the apothem of the pyramid equal to $10\ cm$ and so:

$A_l = \frac{1}{2} \cdot P_b \cdot A_p = \frac{1}{2} \cdot 4 \cdot 16 \cdot 10 = 320\ cm^2$, and the lateral edge is equal to $2\sqrt{41}$ cm.

Test no. 20

I. 1. $n = 4$; 2. $\frac{2}{9}$.

II. 1. $\frac{1}{2}$; 2. $BC = 16$ cm, $h_a = 3\sqrt{3}$ cm, $h_b = \frac{24\sqrt{3}}{7}$ cm, $h_c = 8\sqrt{3}$ cm.

III. 1. a) $S = \{(1, 0); (1, 1)\}$; b) $x = \frac{4}{5}$.

2. a) $45°$; b) $AM \perp MN$ and $BM \perp MN$, $[\sphericalangle(NBC), (AMN)] =$
$= \sphericalangle AMB$; $\cos \sphericalangle AMB = \frac{7}{25}$

c) We use the equality of the products between the bases and the heights in the triangle ABN or using the theorem of the three perpendiculars by building:
$AD \perp (NBC)$ $(D \in BC)$; $DE \perp NB$ $(E \in NB) \Rightarrow AE \perp NB$,
$AE = \frac{48\sqrt{82}}{41}$ cm.

d) $A = 16(16 + 3\sqrt{2})$ cm^2; $V = 256$ cm^3.

Test no. 22

I. 1. Let a and $a+100$ be the two numbers; obviously $a+100=3a+20$. So $a=40$ small number and 140 the large number.

2. From $\frac{a-13}{b}=\frac{a}{b+7} \Leftrightarrow (a-13)(b+7)=ab \Leftrightarrow 7a=13b+91$.

We note $\frac{a}{b}=k \Rightarrow a=bk \Rightarrow b=\frac{91}{7k-13}$. But $b \in \mathbf{N}^* \Rightarrow \frac{91}{7k-13} \in \mathbf{N}^* \Rightarrow$

$\Rightarrow 7k-13 \in D_{91} \Leftrightarrow 7k-13 \in \{1, 7, 13, 91\} \Rightarrow k=\frac{a}{b} \in \left\{2, \frac{20}{7}, \frac{26}{7}, \frac{104}{7}\right\}$.

3. For n – even number $\Rightarrow E=0$.

For n – uneven number $\Rightarrow E=\frac{1}{n}$.

II. 1. $mx+1=x+m \Leftrightarrow m-1=x(m-1)$

a) If $m-1\neq 0 \Rightarrow m\neq 1 \Rightarrow S=\{1\}$

b) If $m-1=0 \Rightarrow m=1 \Rightarrow S=\mathbf{R}$.

2. By making the calculations we have:

$$\left.\begin{array}{l} E(x)=\frac{x+1}{4} \\ E(x)\in \mathbf{Z} \end{array}\right| \Rightarrow \frac{x+1}{4} \in \mathbf{Z} \Leftrightarrow x+1=M4 \Leftrightarrow x+1=4k \text{, where:}$$

$k \in \mathbf{Z} \Rightarrow x=4k-1, k\in \mathbf{Z}$.

3. $x=\sqrt{11-6\sqrt{2}} \Leftrightarrow x=\sqrt{\left(3-\sqrt{2}\right)^2} \Leftrightarrow x=|3-\sqrt{2}| \Leftrightarrow x=3-\sqrt{2}$.

So:

$a=(-1)^n\cdot(x+y)=(-1)^n(3-\sqrt{2}+3-\sqrt{2})=(-1)^n(6-2\sqrt{2})$.

We study the two cases n – even number and n – uneven number and we obtain two values of the repetitive expression:

n - even $\Rightarrow a=6-2\sqrt{2}$

n - uneven $\Rightarrow a=2\sqrt{2}-6$

III. 1. From the data in the problem we have:

$$\frac{m(\hat{A})}{5}=\frac{m(\hat{B})}{6}=\frac{m(\hat{C})}{7}=\frac{180^\circ}{18^\circ}=10^\circ \Rightarrow m(\sphericalangle A)=50^\circ;\ m(\sphericalangle B)=60^\circ,$$

$m(\sphericalangle C)=70°$. Let $AA' \perp BC$ and CC' the bisector $\sphericalangle C$. If $CC' \cap AA'=$ $=\{P\} \Rightarrow m(\sphericalangle C'PA) = 55°$.

2. Let $ABCD$ be an orthodiagonal quadrilateral inscribed in the circle with the center O. If S is the projection of O on CD, then $OS = AB/2$. Let E be a point, diametrically opposed to D. $[SO]$ is a middle line in ΔDCE and so:

$SO = CE / 2$ (1); $m(\sphericalangle CDE) = 90° - m(\sphericalangle DEC) = 90° - m(\sphericalangle DAC) =$ $= m(\sphericalangle ADB) \Rightarrow \widehat{AB} \equiv \widehat{CE} \Rightarrow AB = CE$ (2). From (1) and (2) $) \Rightarrow$ $\Rightarrow SO = AB / 2$ q.e.d.

3. Using Pythagoras' theorem in the right triangles formed we obtain:

$D'M = \frac{3}{2}a$; $MO = \frac{\sqrt{3}}{2}a$; $D'O = \frac{\sqrt{6}}{2}a$. As $OC = \frac{\sqrt{2}}{2}a$ and $D'C =$ $= a\sqrt{2}$ according to the reciprocal of the Phytagorean theorem $\Rightarrow$ $\Rightarrow \Delta OMD'$ and $\Delta OCD'$ are right in $O \Rightarrow D'O \perp OM$, $D'O \perp OC$ and $OM \nparallel OC \Rightarrow D'O \perp (COM)$ q.e.d.

Test no. 23

I. 1. For $n=0 \Rightarrow a=|2^0-3|-|5-2^0|=|-2|-|4|=2-4=$
$=-2 \Rightarrow a=-2$.

For $n=1 \Rightarrow a=|2^1-3|-|5-2^1|=|-1|-|3| \Rightarrow a=-2$.

For $n=2 \Rightarrow a=|2^2-3|-|5-2^2|=|1|-|1|=0 \Rightarrow a=0$.

For $n \geq 3 \Rightarrow a=2^n-3+5-2^n=2$. So $|a| \in \{0, 2\}$ q.e.d.

2. It is obvious that:

$\overline{ab}=10a+b \Rightarrow \sqrt{\overline{ab}+\overline{ba}}=\sqrt{11(a+b)} \in N \Rightarrow a+b=11$ and a, b digits $\Rightarrow$
$\Rightarrow$ the numbers: 92; 38; 83; 47; 74; 56 and 65.

3. We observe that:

$$\sqrt{x^2+6x+34}=\sqrt{x^2+6x+9+25}=\sqrt{(x+3)^2+25} \geq \sqrt{25}=5$$
$$\sqrt{y^2-2y+10}=\sqrt{y^2-2y+1+9}=\sqrt{(y-1)^2+9} \geq \sqrt{9}=3$$
$$\sqrt{z^2-6z+25}=\sqrt{z^2-6z+9+16}=\sqrt{(z-3)^2+16} \geq \sqrt{16}=4$$

So we have a sum of positive numbers ≥ 12, the minimum value 12 will be taken when each square will take the minimum value i.e. zero. So:

$(x+3)^2=0 \Rightarrow x=-3$; $(y-1)^2=0 \Rightarrow y=1$; $(z-3)^2=0 \Rightarrow z=3$.

II. 1. $P(X) = X(X^{1991}-X)-1991(X-1) \Rightarrow P(X) =$
$=(X-1)[X^{1991}+X^{1990}+ \ldots +X-(1+1+\ldots+1)] \Rightarrow P(X)=(X-1)^2(X^{1990}+$
$+2X^{1989}+3X^{1988}+\ldots+1991X+1991) \Rightarrow P(X) \vdots (X-1)^2$.

2. The given system is written:

$$\begin{cases} \left(\frac{1}{3}x-\frac{1}{6}y\right)\cdot 6=3 \\ -12x+10\cdot 0.6y=-18 \end{cases} \Leftrightarrow \begin{cases} 2x-y=3 \\ -12x+6y=-18 \end{cases} \Leftrightarrow \begin{cases} 12x-6y=18 \\ -12x+6y=-18 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 0x+0y=0 \\ x\in \mathbf{R};\ y \in \mathbf{R} \end{cases} \Rightarrow \begin{cases} \forall x \in \mathbf{R} \\ y=2x-3 \end{cases} \text{ or } \begin{cases} \forall y \in \mathbf{R} \\ x=\dfrac{3+y}{2} \end{cases}.$$

3. $E=\sqrt{7+2\sqrt{6}}-\sqrt{7-2\sqrt{6}}=\sqrt{\left(\sqrt{6}+1\right)^2}-\sqrt{\left(\sqrt{6}-1\right)^2}=$

$$=\left|\sqrt{6}+1\right|-\left|\sqrt{6}-1\right| = \sqrt{6}+1-\left(\sqrt{6}-1\right)= \sqrt{6}+1-\sqrt{6}+1=2$$

or we use the formula of the double radicals.

III. 1. Obviously:

$\sphericalangle AFC \equiv \sphericalangle BAD$ ($\sphericalangle$ correspondent) and $\sphericalangle ACF \equiv \sphericalangle CAD$ ($\sphericalangle$ alternate internal) $\Rightarrow \Delta AFC$ isosceles . Analogously we prove that also ΔABF is isosceles. $\Delta EAF \equiv \Delta CAB$ (L.U.L.) $\Rightarrow [EF] \equiv [BC]$ q.e.d.

2. $BM = MC \Rightarrow A_{\Delta ABM} = A_{\Delta AMC} \Rightarrow \frac{1}{2} \cdot AB \cdot PM = \frac{1}{2} \cdot AC \cdot QM \Rightarrow$

$$\Rightarrow AB \cdot PM = AC \cdot QM \Leftrightarrow \frac{PM}{MQ} = \frac{AC}{AB} \quad \text{q.e.d.}$$

3. Because $DE \perp CE$, by using the theorem of the three perpendiculars $\Rightarrow CE \perp (ABD)$. As $EF \perp BD \Rightarrow CF \perp BD$. As EF and CF are not parallel $\Rightarrow BD \perp (ECF)$, but $BD \subset (BCD) \Longrightarrow (BCD) \perp (CEF)$.

Test no. 24

I. 1. $3 < 2x - 3 < 10 \Leftrightarrow 3 < x < \frac{13}{2}$; but $x \in \mathbf{N} \Rightarrow x \in \{4, 5, 6\}$.

$5 < 3x - 2 < 11 \Leftrightarrow \frac{7}{3} < x < \frac{13}{3}$; but $x \in \mathbf{N} \Rightarrow x \in \{3, 4\}$.

So $A=\{4, 5, 6\}$, $B=\{3, 4\} \Rightarrow A \cup B=\{3, 4, 5, 6\}$, $A \cap B=\{4\}$, $A \setminus B=\{5, 6\}$, $B \setminus A=\{3\}$.

2. $\frac{15}{2n+1} \in \mathbf{N} \Rightarrow 2n+1 \in \{1,3,5,15\} \Rightarrow n \in \{0,1,2,7\}$

3. $x^3 + y^3 = (s^2 - 3p)s$; $x^4 + y^4 = s^4 - 4s^2p + 2p^2$.

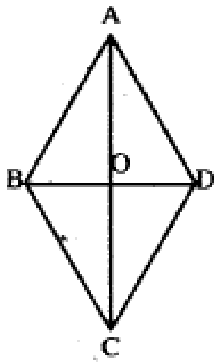
II. 1. $x^2 - (a+b)x + ab = (x-a)(x-b)$; $x^2 - (a+c)x + ac = (x-a)(x-c)$. c.m.m.m.c. $= (x-a)(x-b)(x-c)$.

2. $\frac{3a-2b}{5a-3b} = \frac{1}{5} \Leftrightarrow \frac{3\frac{a}{b}-2}{5\frac{a}{b}-3} = \frac{1}{5} \Leftrightarrow 15\frac{a}{b} - 10 = 5\frac{a}{b} - 3 \Leftrightarrow$

$\Leftrightarrow 10\frac{a}{b} = 7 \Leftrightarrow \frac{a}{b} = \frac{7}{10}$.

3. $(a+b+c)^2 \le 3(a^2+b^2+c^2) \Leftrightarrow 2ab+2ac+2bc \le 2a^2+2b^2+2c^2 \Leftrightarrow$
$\Leftrightarrow 0 \le (a-b)^2 + (a-c)^2 + (b-c)^2$; equality for $a=b=c$

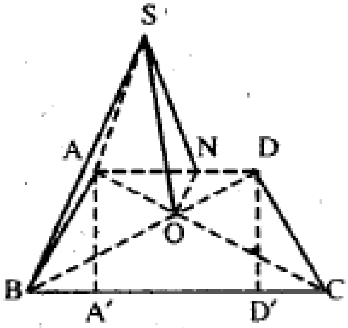
III. 1. $m(\sphericalangle ABC) = 120° \Rightarrow \Delta ABD$ equilateral $\Rightarrow$
$\Rightarrow AB = BD = AD = 7 \Rightarrow P(ABCD) = 28$ cm.



2. a) let $D' = pr_{BC}D$, $A' = pr_{BC}A$, $A'D' = AD = a$ $BA' = D'C = \frac{a}{2}$;

in $\Delta AA'B$: $\cos \sphericalangle ABA' = \frac{1}{2} \Rightarrow m(\sphericalangle ABA') = 60°$ the same for $m(\sphericalangle DCD') = 60°$.

ΔABD isosceles $AB = AD = a \Rightarrow m(\sphericalangle DBC) = m(\sphericalangle ADB) = m(\sphericalangle ABD)$, so $m(\sphericalangle DBC) = 30° \Rightarrow m(\sphericalangle BDC) = 90°$ the same for $m(\sphericalangle BAC) = 90°$.



b) $SO \perp (ABCD)$, $OA \perp AB \Rightarrow SA \perp AB$.

c) Let $ON \perp AD \Rightarrow SN \perp AD$: the plane angle corresponding to the dihedral angle formed by the planes (SAD) and $(ABCD)$ is $\sphericalangle SNO$.

$$tg \sphericalangle SNO = \frac{SO}{ON};\ ON = \frac{a}{2\sqrt{3}} \Rightarrow tg \sphericalangle SNO = \frac{\frac{a}{2}}{\frac{a}{2\sqrt{3}}} = \sqrt{3} \Rightarrow m(\sphericalangle SNO) = 60°.$$

Test no. 25

I. 1. $S = -2+4-6+8-\ldots-1994+998 = \underbrace{2+2+2+\ldots+2}_{498 \text{ terms}} -$

$-1994+998=0.$

2. So: $n = 1994(1994-1)-1993 = 1994\cdot1993-1993 = 1993^2$.

From $\dfrac{\frac{x}{1993}}{2} = \dfrac{1993}{\frac{n}{2}} \Rightarrow \dfrac{\frac{x}{1993}}{2} = \dfrac{1993}{\frac{1993^2}{2}} \Rightarrow \dfrac{x}{2\cdot1993} = \dfrac{1993\cdot2}{1993^2} \Leftrightarrow x=4$.

3. We have: $a^2+b^2-2a\sqrt{2}-2b\sqrt{3}+5=0 \Leftrightarrow (a-\sqrt{2})^2+$

$+(b-\sqrt{3})^2=0 \Leftrightarrow \begin{array}{l} a-\sqrt{2}=0 \Rightarrow a=\sqrt{2} \\ b-\sqrt{3}=0 \Rightarrow b=\sqrt{3} \end{array}$ because $\begin{array}{l} (a-\sqrt{2})^2 \geq 0 \\ (b-\sqrt{3})^2 \geq 0 \end{array}$

So: $\left(\dfrac{2}{a}+\dfrac{3}{b}\right)(a-b) = \left(\dfrac{2}{\sqrt{2}}+\dfrac{3}{\sqrt{3}}\right)(\sqrt{2}-\sqrt{3}) = \dfrac{2\sqrt{3}+3\sqrt{2}}{\sqrt{6}}\cdot(\sqrt{2}-\sqrt{3}) =$

$= \dfrac{2\sqrt{6}-6+6-3\sqrt{6}}{\sqrt{6}} = \dfrac{-\sqrt{6}}{\sqrt{6}} = -1$.

II. 1. So $P(X) \,\vdots\, (2X-1) \Leftrightarrow P(1/2)=0 \Leftrightarrow 2^{n+1}\cdot\dfrac{1}{2^n}+2^n\cdot\dfrac{1}{2^{n-1}}+$

$+\ldots+2^2\cdot\dfrac{1}{2}-n^2=0 \Leftrightarrow \underbrace{2+2+2+\ldots+2}_{n\text{-terms}}-n^2=0 \Leftrightarrow \left.\begin{array}{l} 2n-n^2=0 \\ n\in N^* \end{array}\right| \Rightarrow n=2$.

2. $\sqrt{(x-4)^2}+\sqrt{(3-x)^2}=1 \Leftrightarrow |x-4|+|3-x|=1$

x	$-\infty$		3		4		$+\infty$
x – 4	– – – – – –	– – – –	– – – – –	– – –	0	+ + + + + + + +	
3 - x	+ + + + + + + + +		0	– – – – – – – – – – – – – – – –			

For: a) $x\in(-\infty,3) \Leftrightarrow -x+4+3-x=1 \to S_1$ (1)

b) $x\in[3,4) \Leftrightarrow -x+4-3+x=1 \to S_2$ (2)

c) $x \in [4, +\infty) \Leftrightarrow x - 4 - 3 + x = 1 \rightarrow S_3$ (3)

From (1), (2) and (3) $\Rightarrow S = S_1 \cup S_2 \cup S_3 = [3, 4]$. So $x \in [3, 4]$.

3. Immediate solution through calculation:

$\Rightarrow x \in \{4, 5, 6, 7, 8, 9, 10, 11\}$.

III. 1. Let $D \in [AB$ where $[BD] \equiv [BM]$, $B \in [MD] \Rightarrow \Delta ADC$ equilateral $\Delta MAC \equiv \Delta BDC$ (LUL) $\Rightarrow [MC] \equiv [BC]$.

2. Let $MN \parallel BC \Rightarrow MN + 2BM = 12$ (1)

$\Delta AMN \sim \Delta ABC \Rightarrow \frac{MN}{8} = \frac{12 - BM}{12} \Leftrightarrow 12MN + 13MB = 96$ (2)

From (1) and (2) $\Rightarrow MB = 3$ cm and $MN = 6$ cm. Let $ED = x$. So $\frac{AE}{AD} = \frac{MN}{BC}$; $\frac{8\sqrt{2} - x}{8\sqrt{2}} = \frac{3}{4} \Leftrightarrow x = 2\sqrt{2}$. So $ED = 2\sqrt{2}$, where $AD \perp BC$ and $AD \cap MN = \{E\}$.

3. From $DA \perp (ABC)$ and $EC \subset (ABC) \Rightarrow DA \perp EC \Rightarrow$

$$\Rightarrow \left.\begin{array}{l} EC \perp AD \\ EC \perp AB \\ AD \not\parallel AB \end{array}\right| \Rightarrow \left.\begin{array}{l} EC \perp (ABD) \\ \text{Dar } BD \subset (ABD) \end{array}\right| \Rightarrow \left.\begin{array}{r} EC \perp BD \Rightarrow BD \perp EC \\ \text{But } EF \perp BD \Rightarrow BD \perp FE \\ EC \not\parallel FE \end{array}\right| \Rightarrow$$

$\Rightarrow BD \perp (EFC)$ q.e.d.

Test no. 26

I. 1. $27^{n+1} \cdot 5^{4n+1} - 3^{3n+2} \cdot 5^{4n+2} = 27 \cdot 27^n \cdot 5^{4n+1} - 9 \cdot 27^n \cdot 5 \cdot 5^{4n+1} =$
$= 9 \cdot 27^n \cdot 5^{4n+1}(3-5) = -18 \cdot 27^n \cdot 5^{4n+1} \ \vdots\ 18.$

2. $\frac{1}{3 \cdot 10} + \frac{1}{10 \cdot 17} + \ldots + \frac{1}{830 \cdot 837} = \frac{1}{7}\left(\frac{1}{3} - \frac{1}{10} + \frac{1}{10} - \frac{1}{17} + \ldots + \right.$
$\left. + \frac{1}{830} - \frac{1}{837}\right) = \frac{1}{7}\left(\frac{1}{3} - \frac{1}{837}\right) = \frac{1}{7} \cdot \frac{834}{3 \cdot 837} = \frac{276}{7 \cdot 837} = \frac{276}{5859}$

3. $ac = bd \Rightarrow \frac{c}{b} = \frac{d}{a} = \frac{c+d}{a+b} = k \Rightarrow \frac{cd}{ab} = k^2 = \left(\frac{c+d}{a+b}\right)^2$

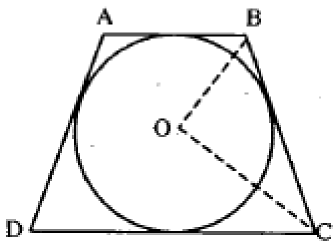
II. 1. 4 and -3 being soultions of the equation $x^2 + mx + n = 0$ we have:

$$\begin{cases} 16 + 4m + n = 0 \\ 9 - 3m + n = 0 \end{cases} \Leftrightarrow \begin{cases} 7m + 7 = 0 \\ 9 - 3m + n = 0 \end{cases} \Leftrightarrow \begin{cases} m = -1 \\ n = -12 \end{cases}$$

2. $\frac{x^2 - y^2 - 2yz - z^2}{y^2 - x^2 - 2xz - z^2} = \frac{x^2 - (y+z)^2}{y^2 - (x+z)^2} = \frac{(x+y+z)(x-y-z)}{(x+y+z)(y-x-z)} =$
$= \frac{x-y-z}{y-x-z}.$

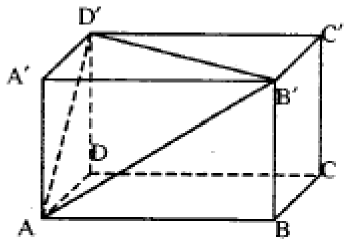
3. Yes.

III. 1. OB and OC are the bisectors of the supplementary angles $\sphericalangle ABC$ and $\sphericalangle BCD$, it follows that:
$m(\sphericalangle OBC) + m(\sphericalangle OCB) = 90°$ analogously $m(\sphericalangle OAD) + m(\sphericalangle ODA) = 90°$ so $CO \perp OB$ and $AO \perp DO$.



2. $d(B', AD') = \frac{2A(AB'D')}{AD'}.$

We calculate AB', $B'D'$ and AD' from the right triangles $\Delta ADD'$ with Pythagoras's theorem and then $A(AB'D')$ with the Heron formula.



Test no. 27

I. 1. $\frac{x}{8}=\frac{y}{6}=\frac{10}{z}=k \Rightarrow \left.\begin{array}{l} x=8k\in\mathbf{N} \\ y=6k\in\mathbf{N} \\ z=\frac{10}{k}\in\mathbf{N}\end{array}\right| \Rightarrow k\in\left\{\frac{1}{2},2,\frac{5}{2},5,10\right\}$

We obtain the triplets:

(4, 3, 20); (8, 6, 10); (16, 12, 5); (20, 15, 4); (40, 30, 2); (80, 60, 1).

2. Let a and b be the two numbers; we have:

$$\left.\begin{array}{l} a+b-30 \\ ab=200\end{array}\right| \Rightarrow \text{Harmonic mean} = \frac{2ab}{a+b}=\frac{2\cdot 200}{30}=\frac{40}{3};$$

3. The perfect squares have one of the forms:

a) 4k or

b) 8k+1

From the theorem of the division with remainder we have:

a) $4k=16p+r \Rightarrow r=4k-16p \Leftrightarrow r=4(k-4p) \Rightarrow r=4s \Rightarrow r$

r is a perfect square.

b) $8k+1=16p+3 \Rightarrow r=8(k-2p)+1 \Rightarrow r=8s+1 \Rightarrow r$

r is a perfect square.

II. 1. Solving the system will yield:

$$\begin{cases} x=\dfrac{m+4}{m^2-4} \\ y=\dfrac{m}{4-m^2}\end{cases} \quad (1)$$

Discussion:

1) if $m \neq \pm 2$ – system compatible and determined with the solution (1)

2) if $m=\pm 2$ – incompatible system

2. The given relation $\Leftrightarrow$

$$\Leftrightarrow x\sqrt{2}+3x+3y-\sqrt{2}y=0 \Leftrightarrow (x-y)\sqrt{2}+3(x+y)=0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x-y=0 \\ x+y=0 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ y=0 \end{cases}$$

3. a) $$\frac{x^4+1}{x^2+x\sqrt{2}+1}=\frac{x^4+2x^2+1-2x^2}{x^2+x\sqrt{2}+1}=\frac{\left(x^2+1\right)^2-\left(x\sqrt{2}\right)^2}{x^2+x\sqrt{2}+1}=$$

$$=\frac{\left(x^2+x\sqrt{2}+1\right)\left(x^2-x\sqrt{2}+1\right)}{x^2+x\sqrt{2}+1}=x^2-x\sqrt{2}+1.$$

b) $$\frac{x^8-9}{\left(x^2-\sqrt{3}\right)\left(x^4+3\right)}=\frac{\left(x^4\right)^2-3^2}{\left(x^2-\sqrt{3}\right)\left(x^4+3\right)}=\frac{\left(x^4-3\right)\left(x^4+3\right)}{\left(x^2-\sqrt{3}\right)\left(x^4+3\right)}=$$

$$=\frac{\left(x^2\right)^2-\left(\sqrt{3}\right)^2}{x^2-\sqrt{3}}=\frac{\left(x^2+\sqrt{3}\right)\left(x^2-\sqrt{3}\right)}{x^2-\sqrt{3}}=x^2+\sqrt{3}.$$

III. 1. Let $m[\sphericalangle(x0y)] = 13°$. We build:

$[\sphericalangle(zOy)] = 7 \cdot m[\sphericalangle(xOy)] = 7 \cdot 13° = 91°,$

and in the point O we raise:

$OA \perp Oy \Rightarrow m[\sphericalangle(AOy)] = 90° \Rightarrow m[\sphericalangle(Aoz)] = 1°$

2. We use the formula:

$$m_a^2 = \frac{2\left(b^2+c^2\right)-a^2}{4}$$

$m_a$is the length of the median corresponding to the side BC, and a, b, c are the lengths of the sides ΔABC(see the geometry manual, grade VII, the 1993 edition, page 37).

Because $MA = MB = MC \Rightarrow M$ is situated on the perpendicular from the center of the circumscribed circle ΔABC on its plane, so $M \in (DO)$ where $DO \perp (ABC); DO$- the height of the tetrahedron $ABCD$. Expressing AO and OM according to AB by using Pythagoras's theorem in the right triangle OAM we will obtain $AB = 3\sqrt{2}.a$.

Test no. 28

I. 1. Let $N = \overline{xyz}$, $x \neq 0$; we have $x + y + z = 7$ and $7 \mid N = 100x + 10y + z$.

We deduce: $\left.\begin{matrix} y + z = 7 - x \\ x \geq 1 \Rightarrow 7 - x \leq 6 \end{matrix}\right\} \Rightarrow y + z \leq 6$

$7 \mid (14 \cdot 7x + 7y) + (2x + 3y + z) \Rightarrow 7 \mid 2x + 3y + z = (x + y + z) + x + 2y \Rightarrow$
$\Rightarrow 7 \mid 7 + (x + 2y) \Rightarrow 7 \mid x + 2y$; but $x = 7 - y - z \Rightarrow 7 \mid 7 + y - z \Rightarrow 7 \mid y - z$;
but $-9 \leq y - z \leq 9$ and $|y - z| \leq y + z \leq 6 \Rightarrow y = z$.

2. True because $|x - 1| = 5 \Leftrightarrow x - 1 = 5$ or $x - 1 = -5 \Leftrightarrow$
$\Leftrightarrow x = 6$ or $x = -4$.

3. $(x - a)^2 + (x - b)^2 + (x - c)^2 > 0$

because only takes place for $x = a = b = c$, the equation doesn't have real solutions.

II. 1. Let $x^2 - 1 = a$; the expression becomes:

$$\frac{a(a-2)+1}{a(a-3)+2} = \frac{a^2 - 2a + 1}{a^2 - 3a + 2} = \frac{(a-1)^2}{(a-1)(a-2)} = \frac{a-1}{a-2} = \frac{x^2 - 2}{x^2 - 3}.$$

2. We have:

$$\left.\begin{matrix} a^2 + b^2 \geq 2ab \\ c^2 + d^2 \geq 2cd \end{matrix}\right\} \Rightarrow a^2 + b^2 + c^2 + d^2 \geq 2(ab + cd) \geq 4\sqrt{abcd} =$$
$$= 4 \Rightarrow a^2 + b^2 + c^2 + d^2 \geq 4$$

$$ab + cd \geq 2\sqrt{abcd} = 2;\ bc + da \geq 2\sqrt{abcd} = 2;\ ac + bd \geq 2\sqrt{abcd} = 2$$

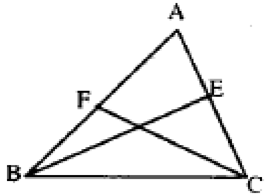
Adding these four inequalities member by member, we obtain:

$$a^2 + b^2 + c^2 + d^2 + ab + cd + bc + da + ac + bd \geq 4 + 2 + 2 + 2 = 10$$

3. $\sqrt{8} \cdot |-2| \cdot (-2)^2 \cdot \frac{1}{4\sqrt{2}} = 2\sqrt{2} \cdot 2 \cdot 4 \cdot \frac{1}{4\sqrt{2}} = 4$.

III. 1. With the theorem of the median we have:

$$m_c < m_b \Leftrightarrow m_c^2 < m_b^2 \Leftrightarrow \frac{2(a^2 + b^2) - c^2}{4} < \frac{2(a^2 + c^2) - b^2}{4} \Leftrightarrow$$
$$\Leftrightarrow 2a^2 + 2b^2 - c^2 < 2a^2 + 2c^2 - b^2 \Leftrightarrow b^2 < c^2 \Leftrightarrow b < c.$$



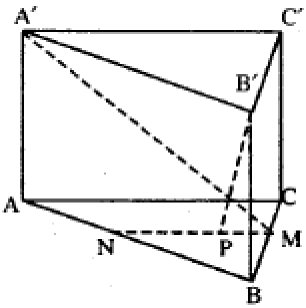
2. a) In the plane $A'BC$ let $A'M \perp BC$, $A'A \perp (ABC)$.

$$A'M \perp BC \Rightarrow T3\perp \Rightarrow AM \perp BC \Rightarrow AM = \frac{a\sqrt{3}}{2} \Rightarrow A'M = \frac{a\sqrt{7}}{2}.$$

A′

b) Let MN be a middle line and P the foot of the perpendicular lowered from B' on MN. So $B'B \perp (NBM)$.

$$B'P \perp MN \Rightarrow T3\perp \Rightarrow BP \perp MN \Rightarrow B'P = \frac{a\sqrt{19}}{4}.$$



Test no. 29

I. 1. $xy - x + y = 18 \Leftrightarrow xy - x + y - 1 + 1 = 18 \Leftrightarrow x(y-1) + y - 1 = 17 \Leftrightarrow (x+1)(y-1) = 17 \Leftrightarrow \begin{cases} x+1=1 \\ y-1=17 \end{cases}$ or $\begin{cases} x+1=17 \\ y-1=1 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x=0 \\ y=18 \end{cases}$ or $\begin{cases} x=16 \\ y=2 \end{cases}$

2. $ab = 2^7(1 + 2^{-1} + 2^{-2} + \ldots + 2^{-7}) = 1 + 2 + \ldots + 2^7$.

Let $S = 1+2+\ldots+2^7 \Rightarrow (2-1)S = (2-1)(1+2+\ldots+2^7) = 2-1+2^2-2+\ldots+$

$+2^8-2^7 = 2^8 - 1$. So $ab = 2^8 - 1$.

3. Because $|x| \geq 0 \Rightarrow$ the truth value is false.

II. 1. $N = 10^{3k}+2^{3k} = 1000^k+8^k = 1000^k+1^k+8^k-1^k =$
$= (1000+1)(1000^{k-1}-1000^{k-2}+\ldots-1000+1) + (8-1)(8^{k-1}+8^{k-2}+\ldots+8+1) = M_7$
$+ M_7 = M_7$ because $1001 = M_7$.

The following formulas have been used:

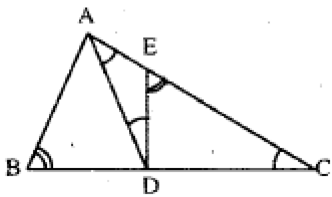
$a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + \ldots + b^{n-1}), (\forall) n \in \mathbb{N}$, uneven

$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \ldots + b^{n-1}), (\forall) n \in \mathbb{N}^*$

2. $2a^2 + 4ac + 2c^2 - b(a+c) - b^2 = 2(a+c)^2 - b(a+c) - b^2 =$
$= (a+c)^2 - b(a+c) + (a+c)^2 - b^2 = (a+c)(a+c-b) + (a+c-b)(a+$
$+c+b) = (a+c-b)(2a+b+2c)$.

III. 1. a) Because AM is a median relative to the hypotenuse in the triangle ABC, the triangles ADC and ADB are isosceles. So:

$\sphericalangle DAC \equiv \sphericalangle ACD$ (1)



The angle $\sphericalangle DEC$ being exterior to the the triangle ADE, we have:

$\sphericalangle DEC = \sphericalangle DAC + \sphericalangle ADE$. (2)

In ΔABC and DEC, the angles $\sphericalangle ABC$ and $\sphericalangle DEC$ have the same complement ($\sphericalangle ACB$), so:

$\sphericalangle ABC \equiv \sphericalangle DEC$ (3)

From (1), (2) and (3) $\Rightarrow$

$\Rightarrow \sphericalangle ABC = \sphericalangle DEC = \sphericalangle DAC + \sphericalangle ADE = \sphericalangle ACD + \sphericalangle ADE \Rightarrow$

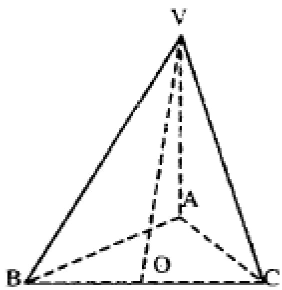
$\Rightarrow \sphericalangle ADE = \sphericalangle ABC - \sphericalangle ACD = \sphericalangle B - \sphericalangle C.$

b) $AE = ED \Rightarrow \Delta ADE$ isosceles, so $\sphericalangle EAD \equiv \sphericalangle ADE$.

Using (1) $\Rightarrow \sphericalangle C \equiv \sphericalangle ADE$ and from $\sphericalangle ADE = \sphericalangle B - \sphericalangle C \Rightarrow \sphericalangle B = 2\sphericalangle C$.

As $\sphericalangle B$ and $\sphericalangle C$ are complementary $\Rightarrow m(\sphericalangle B) = 60°$ and $m(\sphericalangle C) = 30°$.

2. Let ABC ($\sphericalangle A = 90°$) be the base of the pyramid $VABC$ and as $(VA) \equiv (VB) \equiv (VC)$, it follows that O the foot of the pyramid's height is the center of the circumscribed circle ΔABC. So O is the middle of the hypotenuse BC and $VO \perp (ABC)$, $VO \subset (VBC) \Rightarrow (VBC) \perp (ABC)$.



Test no. 30

$$\text{I. 1. } 1998+\frac{3}{2}-\frac{1}{2}+\frac{5}{4}-\frac{1}{4}+\ldots+\frac{101}{100}-\frac{1}{100}=1998+(1+1+\ldots+1)$$

The number of terms in brackets is equal to the number of terms of the sum $3+5+\cdots+101$ which is 50. So the sum is $1998+50=2048$.

$$\text{2. } \frac{3}{7}<\frac{n+1}{42}\le\frac{1}{2}\Leftrightarrow 18<n+1\le 21\Leftrightarrow 17<n\le 20\Rightarrow S=\{18,19,20\}$$

$$\text{3. } P(1+x)=P(1-x)\Leftrightarrow(1+x)^2+a(1+x)+1=(1-x)^2+$$
$$a(1-x)+1\Leftrightarrow 2ax=4x\ (\forall)x\in\mathbf{R}\Rightarrow a=2\Rightarrow P(x)=x^2+2x+1=(x+1)^2$$

II. 1. We assume that:

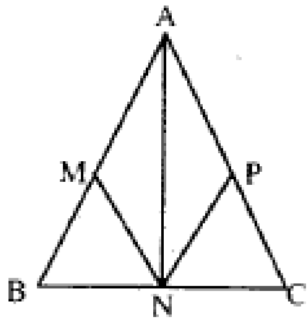
$$(\exists)x\in\mathbf{Q},\ x=\frac{m}{n};\ m,\ n\in\mathbf{Z},\ n\neq 0,\ (m,\ n)=1,$$

which verifies the relation: $ax^2+bx+c=0$, a,b,c uneven. By replacing, we obtain $am^2+bmn+cn^2=0$ which is impossible because the left number is odd. In conclusion, there is no $x\in\mathbb{Q}$ that verifies the relation from the enunciation.

2. The natural numbers a and b, being non equal to zero $\Rightarrow$

$$\Rightarrow a\ge 1 \text{ şi } b\ge 1\Leftrightarrow a-1\ge 0 \text{ şi } b-1\ge 0\Rightarrow(a-1)(b-1)\ge 0\Rightarrow$$
$$\Rightarrow ab-a-b+1\ge 0\Rightarrow ab+1\ge a+b.$$

According to the hypothesis $ab<c$ and so: $a+b<c+1$. As a,b,c are natural $\Rightarrow a+b\le c$.



$$\text{III. 1. } (AB)\equiv(AC),\ (BN)\equiv(NC)\Rightarrow AN\perp BC.$$

In ΔANB ($\sphericalangle ANB = 90°$), $(BM) \equiv (MA) \Rightarrow NM = \frac{1}{2} AB = AM = 4$ cm.

The same for $NP = AP = \frac{1}{2} AC = AM = 4$ cm.

2. Writing in two ways the volume of the tetrahedron $MABC$ and $MBCD$ we have:

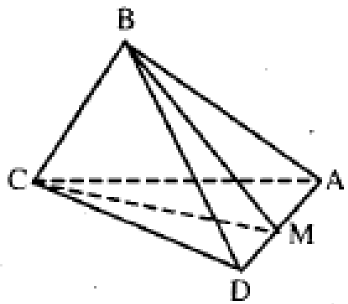
$$\frac{V[MABC]}{V[MBCD]} = \frac{S[ABC] \cdot d(M,(ABC))}{S[BCD] \cdot d(M,(BCD))} \qquad (1)$$

$$\frac{V[MABC]}{V[MBCD]} = \frac{V[BCAM]}{V[BCDM]} = \frac{S[CAM] \cdot d(B,(CAB))}{S[CDM] \cdot d(B,(CDM))} = \frac{S[CAM]}{S[CDM]} =$$

$$= \frac{AM \cdot d(C, AM)}{DM \cdot d(C, DM)} = \frac{AM}{DM} \qquad (2)$$

From (1) and (2) $\Rightarrow \frac{AM}{DM} = \frac{S[ABC] \cdot d(M,(ABC))}{S[BCD] \cdot d(M,(BCD))} \qquad (3)$

It follows from (3) $d(M,(BCD)) = d(M,(ABC)) \Leftrightarrow \frac{MA}{MD} = \frac{S[ABC]}{S[BCD]}$



Test no. 31

I. **1.** 1; **2.** 5; 7; 5 and 10; **3.**

We analyze the cases $x < 1$ and $x > 1$. If $x < 1$ the equation doesn't have real solutions. If $x > 1$, then, any number from the interval $(1, \infty)$ is a solution to the equation.

II. 1. Let E be the point diametrically opposed to A.

$\sphericalangle EAC = 90° - \sphericalangle AEC$. But $\sphericalangle AEC \equiv \sphericalangle ABC \Rightarrow \sphericalangle EAC = 90° - \sphericalangle ABC = \sphericalangle BAD$. So $\sphericalangle EAC \equiv \sphericalangle BAD = 90° - \sphericalangle ABC$. $\sphericalangle DAO = \sphericalangle BAC - (\sphericalangle BAD + \sphericalangle CAE) = \sphericalangle BAC - 2 \cdot \sphericalangle BAD = \sphericalangle BAC - 2 \cdot (90° - \sphericalangle ABC) \Rightarrow \sphericalangle DAO = \sphericalangle BAC + 2 \cdot \sphericalangle ABC - 180°$.

2. Given the trapezoid BCD ($AB \parallel CD$) and M the middle of AB, N the middle of DC and $\{P\} = AD \cap BC$. We assume that $PM \cap DC = \{N'\}$ and we will show that

$N' = N$. $DN' \parallel AM \Rightarrow \Delta PDN' \sim \Delta PAM \Rightarrow \dfrac{PN}{PM} = \dfrac{DN'}{AM}$ (1).

Similarly $\dfrac{PN}{PM} = \dfrac{CN'}{BM}$ (2). From (1) and (2) $\Rightarrow \dfrac{DN'}{AM} = \dfrac{CN'}{BM}$. But $AM \equiv BM \Rightarrow DN' \equiv CN' \Rightarrow N' = N$ (the middle of DC)

III. 1. $AB \perp a$.

2. $45°$ because the triangle AQP is right isosceles (we have noted with Q the middle of MN and with P the middle of BC).

Test no. 32

I. 1. $\frac{3+6+9+...+1995+1998}{2+4+6+...+1330+1332}=\frac{3(1+2+3+...+665+666)}{2(1+2+3+...+665+666)}=\frac{3}{2}$

2. $\frac{2n-5}{n+1}<1 \Leftrightarrow 2n-5<n+1 \Leftrightarrow n<6, n\in \mathbb{N} \Rightarrow n\in \{0, 1, 2, 3, 4, 5\}$.

3. Let x represent the capacity of the tank and t the time it takes for the 4 faucets to fill the tank (t measured in minutes). In a minute the first faucet fills the $60th$ part of the tank, the second the $120th$ part, the third, the $180th$ part and the fourth, the $240th$ part.

$$t\cdot\frac{x}{60}+t\cdot\frac{x}{120}+t\cdot\frac{x}{180}+t\cdot\frac{x}{240}=x \Rightarrow tx\left(\frac{1}{60}+\frac{1}{120}+\frac{1}{180}+\frac{1}{240}\right)=x \Rightarrow$$

$$\Rightarrow t\left(\frac{1}{60}+\frac{1}{120}+\frac{1}{180}+\frac{1}{240}\right)=1 \Rightarrow t\cdot\frac{25}{720}=1 \Rightarrow t=\frac{720}{25} \Rightarrow t=\frac{144}{5} \Rightarrow$$

$$\Rightarrow t=28\frac{4}{5}\text{ min}.$$

II. 1. $2(a-b)(a+c)=(a-b+c)^2 \Leftrightarrow 2a^2+2ac-2ab-2bc=$
$=a^2+b^2+c^2-2ab-2bc+2ac \Leftrightarrow a^2=b^2+c^2$.

2. $AC'=p-a$, $BA'=p-b$, $CB'=p-c$,

where a, b, c are the sides of the triangle and $p=\frac{a+b+c}{2}$.

III. 1. From the congruence of the triangles $B'BM$ and $DAM \Rightarrow$
$\Rightarrow B'M\equiv DM \Rightarrow \Delta B'MD$ is isosceles $\Rightarrow MN\perp B'D$. Similarly $NP\perp B'D$. So $B'D\perp MNP$.

2. $h=15$; $G+R=25 \Rightarrow h^2=G^2-R^2=225 \Rightarrow$
$\Rightarrow (G-R)(G+R)=225 \Rightarrow (G-R)\cdot 25=225 \Rightarrow G-R=9$.

As $G+R=25 \Rightarrow G=17$ and $R=8$.

$A=\pi RG=\pi\cdot 17\cdot 8=136\pi$. $V=\frac{1}{3}\pi R^2h=\frac{1}{3}\cdot 64\cdot 15=320\pi$.

Test no. 33

I. 1. If $m \neq 3$, the equation has a unique solution, $x = \frac{5m+1}{m-3}$. If $m = 3$, the equation has no solutions. $x \in \mathbb{Z} \Rightarrow m - 315m + 1$. But $m - 315m - 15 \Rightarrow m - 31(5m + 1) - (5m - 15) \Rightarrow m - 3116 \Rightarrow m - 3 \in \{-16, -8, -4, -2, -1, 1, 2, 4, 8, 16\} \Rightarrow m \in [-13, -5, -1, 1, 2, 4, 5, 7, 11, 17]$.

3. 402, 403 and 404.

II. 1. It should be proven that the two angles are additional. One finds the measures: 30^0 and 150^0.

2. Using the fact that a point situated on the bisector of an angle is equally situated from the sides of the angle $\Rightarrow EM = EQ$ and $EP = PN$. $\sphericalangle PEQ = \sphericalangle MEN$ (opposite) $\Leftrightarrow \Delta MEN = \Delta QEP \Rightarrow MN = PQ$. ΔAQM and ΔCPN are isosceles $\Rightarrow AC \perp PN$ and $AC \perp MQ \Rightarrow PN \parallel MQ$. Therefore, $MNPQ$ is an isosceles trapezoid $\Rightarrow$ $MNPQ$ is an inscriptible quadrilateral.

III. 1. The proposition is false.

2. One uses the fact that the intersection of a plane parallel to a plane containing the line is another line that is parallel to the given line.

Test no. 34

I. **1.** $x = 6630$; **2.** 32 000 lei; **3.** $E(x) = x^3 + (a+1)x^2 + (a+b)x + b + 1 = x^3 + ax^2 + x^2 + ax + bx + b + 1 = x^3 + x^2 + ax + b + ax^2 + bx + 1 = x^3 + 0 + ax^2 + bx + 1 = x(x^2 + ax + b) + 1 = x \cdot 0 + 1 = 1.$

II. 1. Forming with the two equalities a system with the unknowns $f(x+1)$ and $g(x-1)$, we get: $f(x+1) = 2x + 6$ and $g(x-1) = -2x + 2$. Let $f(x) = ax + b, a, b \in \mathbb{R} \Rightarrow f(x+1) = ax + a + b.$ Therefore $ax + a + h = 2x + 6$, irrespective of $x \in \mathbb{R} \Rightarrow a = 2$ and $a + b = 6 \Rightarrow a = 2, b = 4$. Analogously, one determines $g(x) = -2x, 2, 90^0, 60^0$ and 30^0.

$$\textbf{III. 1. } \left.\begin{array}{l} DM \perp (ABCD) \\ DA \perp AB \\ AB \subset (ABCD) \end{array}\right\} \Rightarrow MA \perp AB$$

applying cathetus theorem in the triangle $MAB \Rightarrow AB^2 = PB \cdot MB$. Therefore,

$$\frac{AB^2}{BC^2} = \frac{PB \cdot MB}{QB \cdot MB} \Rightarrow \left(\frac{6}{4}\right)^2 = \frac{PB}{QB} \Rightarrow$$

$$\Rightarrow \frac{9}{4} = \frac{PB}{QB} \Rightarrow \frac{9-4}{4} = \frac{PB - QB}{QB} \Rightarrow \frac{5}{4} = \frac{3}{QB} \Rightarrow QB = \frac{12}{5} \text{ cm.}$$

$$BC^2 = QB \cdot MB \Rightarrow 16 = \frac{12}{5} \cdot MB \Rightarrow MB = \frac{5 \cdot 16}{12} = \frac{20}{3} \text{ cm.}$$

2. a) $\frac{a\sqrt{6}}{3}$; b) $BA = BB' = BC = a \Rightarrow B$ is situated on the perpendicular taken in the middle of the circle circumscribed to $\Delta AB'C$ on the plane of this triangle. $DA = DB' = DC = a\sqrt{2}cm \Rightarrow D'$ is also situated on the perpendicular taken in the middle of the circle circumscribed to the triangle $\Delta AB'C$ on the plane of this triangle. Therefore, $BD' \perp (AB'C)$.

Test no. 35

I. 1. 26; 2. 45 pencils and 75 pencils.

3. $\sqrt{(x-1)^2}+\sqrt{(y-2)^2}+\sqrt{(z-3)^2}=0 \Leftrightarrow |x-1|+$

$+|y-2|+|z-3|=0 \Leftrightarrow x=1, y=2, z=3.$

II. 1. We take $DD' \perp AB$ and $CC' \perp AB$.

$D' \in AB,\ C' \in AB,\ AD' = C'B = 5$ cm $\Rightarrow C'A = 21$ cm,

Height theorem applied in the rectangular triangle ACB ($\sphericalangle C = 90^0$), one gets:

$CC'^2 = AC' \cdot C'B \Rightarrow CC'^2 = 21 \cdot 5$ cm $\Rightarrow CC'^2 = 105$ cm $\Rightarrow CC' = \sqrt{105}$ cm.

$$A_{ABCD} = \frac{CC'(AB+CD)}{2} \Rightarrow A_{ABCD} = \sqrt{105} \cdot 21 \text{ cm}.$$

2. One uses the fact that in a trapezoid the middle line has equal length with half sum of the bases.

III. 1. a) In the right triangle ACB $(AB = BC = 2\text{cm})$ one calculates $AC = 2\sqrt{2}\text{cm}$. Therefore $\Delta ACC'$ is isosceles $(AC = CC')$. b) $8 + 16\sqrt{2}$ cm.

2. The diagonal of the cube is the sphere diameter. Denoting by x the cube side, one gets:

$$x^2 + 2x^2 = 4a^2 \Rightarrow 3x^2 = 4a^2 \Rightarrow x = \frac{2a\sqrt{3}}{3} \Rightarrow V = \frac{8a^3\sqrt{3}}{9}.$$

Test no. 36

I. **1.** 10%; **2.** $S = 1000 \cdot 2000 = 2\,000\,000$; **3.** $x^6 + x^4 - 2x^3 - 2x^2 + 2 = x^6 - 2x^3 + 1 + x^4 - 2x^2 + 1 = (x^3 - 1)^2 + (x^2 - 1)^2 \geq 0$.

II. 1. $E(a,b) = \frac{2a}{b}$.

2. Let AB and CD the trapezoid bases. $E \in AD, F \in BC$. From the similarity of triangles DEM and DAB, it follows $\frac{EM}{AB} = \frac{DM}{DB}$. Analogously $\frac{MF}{AB} = \frac{CM}{AC}$. But $\frac{DM}{DB} = \frac{CM}{AC}$ (obtained from the similarity of triangles DMC and BMA, then using derivatives proportions.

Therefore $\frac{EM}{AB} = \frac{DM}{DB} \Rightarrow EM = FM$.

III. 1. $A_l = 144\pi\sqrt{2}$ cm^2; $A_t = \left(194 + 144\sqrt{2}\right)\pi$ cm^2; $V = \frac{2072}{3}\pi$ cm^3 .

2. $\frac{a^3\sqrt{11}}{24}$.

Test no. 37

I. $\frac{1}{5}$. 26; 2. $x = 3$; 3. Let a, b, c the three numbers $a = \frac{b+c}{2}$, $b = \frac{a+c}{2}$, $c = \frac{a+b}{2}$. Replacing $a = \frac{b+c}{2}$ in the last two equalities and effectuating the calculus, we get $b = c \Rightarrow a = b \Leftrightarrow a = b = c$.

II. 1. Let $f: \mathbf{R} \to \mathbf{R}, f(x) = ax + b, a,b \in \mathbf{R}$.
From hypothesis, we get the relations $3a + b = 1, a = b - 3 \Rightarrow a = -1, b = 4$.

2. We suppose that

$$\sqrt{7} - 3 < \sqrt{11} - 4 \Leftrightarrow 4 - 3 < \sqrt{11} - \sqrt{7} \Leftrightarrow 1 < \sqrt{11} - \sqrt{7} \Leftrightarrow$$

$$\Leftrightarrow 1^2 < \left(\sqrt{11} - \sqrt{7}\right)^2 \Leftrightarrow 1 < 11 - 2\sqrt{77} + 7 \Leftrightarrow 2\sqrt{77} < 17 \Leftrightarrow 4 \cdot 77 < 289 \Leftrightarrow 308 < 289$$

(false). Therefore:

$$\sqrt{7} - 3 > \sqrt{11} - 4.$$

III. 1. Applying Menelaus theorem in the triangle AMN cut by the secant BC.

2. Let α the angle formed by the diagonal AC' with the facet $ABB'A'$, β the angle formed with the facet $ABCD$, and γ the angle formed with the facet $ADD'A'$:

$$\sin\alpha = \frac{B'C'}{AC'};\ \sin\beta = \frac{CC'}{AC'};\ \sin\gamma = \frac{D'C'}{AC'};$$

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = \frac{B'C'^2}{AC'^2} + \frac{CC'^2}{AC'^2} + \frac{D'C'^2}{AC'^2} = \frac{B'C'^2 + CC'^2 + D'C'^2}{AC'^2} = \frac{AC'^2}{AC'^2} = 1$$

Test no. 38

I. **1.a)** $A = (-4, 8); B = (-5, 3]$. b) $A \cup B = (-5, 8)$; $A \cap B = (-4, 3]$; $A \setminus B = (3, 8)$; $B \setminus A = (-5, -4]$; $(B \setminus A) \cap \mathbb{Z} = \{-4\}$.

2. Solving the equation system, we obtain $P(2,5)$.

$$\begin{cases} 2x + 1 = y \\ x + 3 = y \end{cases}$$

3. If:

$$m \in \mathbb{R} \setminus \{-1\} \Rightarrow x = 3$$

it is the only solution of the equation.

If $m = -1 \Rightarrow$ any real number is a solution of the equation.

II. **1.** $E(x) = 2x^2 - 4$.

2. $A'B'C'D'$ is an isosceles trapezoid.

III. **1.** $\frac{a^3}{4}$. **2.** $A_{VBC} = \frac{VB \cdot VC \cdot \sin B\hat{V}C}{2} = \frac{VB^2 \cdot \sin B\hat{V}C}{2}$.

$$A_{MNP} = \frac{MN \cdot MP \cdot \sin N\hat{M}P}{2} = \frac{MN^2 \cdot \sin N\hat{M}P}{2}.$$

Test no. 39

I. **1.** $5-2\sqrt{8}$; **2.** $x=6$, $y=4$, $z=4$; **3.** $x^2<7-4\sqrt{3} \Leftrightarrow x^2<\left(2-\sqrt{3}\right)^2 \Leftrightarrow$
$\Leftrightarrow x \in \left(-2+\sqrt{3}, 2-\sqrt{3}\right)$. $\quad x \in \mathbf{Z} \Rightarrow x \in \{0\}$.

II. **1.** $E(x)=1-4x^2$. **2.** $\sphericalangle DEF \equiv \sphericalangle BEC = 180^\circ - (\sphericalangle BCD + \frac{1}{2}\sphericalangle ABC)$; $\sphericalangle DFE = 180^\circ -$

$$-(\sphericalangle BAD + \frac{1}{2}\sphericalangle ABC).$$

But

$$\sphericalangle BAD \equiv \sphericalangle BCD.$$

Therefore

$$\sphericalangle DEF \equiv \sphericalangle DFE \Rightarrow \Delta DEF$$

is isosceles.

III. 1. Let O the middle of BD. The triangles MBD and PBD being isosceles $\Rightarrow MO \perp BD$ and $PO \perp BD$.Therefore, the dihedral angle of planes (MBD) and (PBD) is MOP. Calculate MO and PO in the rectangular triangles MAO and PAO. Then, calculate MP in trapezoid $ACPM$. According to the reciprocal of Pythagoras Theorem, $\sphericalangle MOP = 90^0$.

2. $A_l = 60\pi \text{ cm}^2$; $A_t = 96\pi \text{ cm}^2$; $V = 96\pi \text{ cm}^3$.

Test no. 40

I. 1. a) If n is an even number: -1; if n is an odd number: 1.

b) 0 if n is odd, and $2 \cdot 3^n$ if n is even.

2. 280 tickets costing 300 lei and 40 tickets costing 400 lei.

3. If $m \geq 0$, the solution is $x \in [0, \infty)$. If $m < 0$, the solution is $x \in (-2, 0]$.

II. 1. We get $x - 2$.

2. We take:

$$DE \perp BC,\ E \in BC.\ \Delta ABD \equiv \Delta BDE \Rightarrow BE = AB = a = \sqrt{b^2 + c^2}$$

$$\Delta ABC \sim \Delta DEC \Rightarrow \frac{DE}{AB} = \frac{CE}{AC} \Rightarrow \frac{DE}{c} = \frac{a-c}{b} \Rightarrow DE = \frac{c(a-c)}{b}$$

$$A_{BDC} = \frac{BC \cdot DE}{2} = \frac{ac(a-c)}{2b}.\ A_{ABD} = A_{ABC} - A_{BDC} = \frac{bc}{2} - \frac{ac(a-c)}{2b} = \frac{c(b^2 - a^2 + ac)}{2b}.$$

III. 1. 5 cm. **2.** $\frac{a\sqrt{6}}{3}, \frac{a\sqrt{3}}{2}, \frac{1}{3}$.

Test no. 41

I. **1.** $\frac{a}{b}=\frac{62}{85}$. **2.** $n=0$. **3.** $y\in\left(-4,\frac{17}{4}\right]$.

II. **1.** $\left.\begin{matrix} P(2)=5\Rightarrow 4a+b=-5 \\ P(-3)=1\Rightarrow 27a+3b=-74 \end{matrix}\right\}\Rightarrow$

$a=-\frac{59}{15}, b=\frac{161}{15}$. **2.** $x=c\cdot\frac{ab^2}{a^2+b^2}$, $y=c\cdot\frac{a^2b}{a^2+b^2}$.

III. 1. a) ΔBCM is an isosceles triangle having an angle of 60^0, so it is an equilateral triangle.

b) It is obvious that $ABMD$ is an isosceles trapezoid. Let N be the middle of AB. $MNBC$ is a parallelogram. Therefore:

$$MN\equiv BC=\frac{1}{2}AB\Rightarrow MN\equiv AN\equiv NB\Rightarrow \sphericalangle MAN\equiv\sphericalangle AMN=u.$$

$$\sphericalangle MBN\equiv\sphericalangle BMN=v.$$

From the fact that the angles' sum in triangle AMB is 180^0,

$$\Rightarrow 2(u+v)=180^\circ\Rightarrow u+v=90^\circ,$$

therefore $\sphericalangle AMB=90^0$.

2. a) Sector's area is equal to the third part of the area of the circle from which it is derived. Therefore, circle's area is $36\pi\ \text{cm}^2\Rightarrow$ the radius of the circle from which the sector was derived is 6 cm. This radius will be the generator of the cone. The length of the part from the circle situated into the sector is the third part from the length of the arc from which the sector was derived, so it is equal to 4μ cm. But this length is also the length of the basic arc of the cone, so the radius of the basic circle of the cone is 2 cm. Therefore, $R=2$ cm and $G=6$ cm $\Rightarrow$ the height of the cone $h=\sqrt{32}=4\sqrt{2}$ cm. We calculate $A_1=16\pi$ cm^2. $V=\frac{16\sqrt{2}}{3}\pi$ cm^3.

b) We obtain the radius of the sphere, of length $\sqrt{2}$ cm. $A=8\pi$ cm^2.

Test no. 42

I. **1.** $x = 2$; **2.** 8,36; **3.** 70,(1)%.

II. 1. The quotient is $4x^2 - 3x - 2$, and the rest is $2x + 3$.

2. 106.

III. 1. $\frac{a\sqrt{3a}}{9}$. **2.** $\frac{DO}{OB} = \frac{DC}{AB} \Rightarrow \frac{3^{\prime}}{10} = \frac{DC}{36} \Rightarrow DC = \frac{108}{10} = \frac{54}{5}$. $DB^2 = DA^2 + AB^2 \Rightarrow$

$\Rightarrow DB^2 = 225 + 1296 = 1521 \Rightarrow DB = 39$.

$\frac{DO}{OB} = \frac{3}{10} \Rightarrow \frac{DO}{DO + OB} = \frac{3}{3 + 10} \Rightarrow \frac{DO}{DB} = \frac{3}{13} \Rightarrow \frac{DO}{39} = \frac{3}{13} \Rightarrow DO = 9$.

$VD^2 = VO^2 + DO^2 \Rightarrow VD^2 = 1600 + 81 = 1681 \Rightarrow VD = 41$. $P_{VOD} = VO + OD + VD = 90$ cm.

$A_{ABCD} = \frac{DA(AB + DC)}{2} = 351\text{ cm}^2$. $V_{VABCD} = \frac{1}{3} VO \cdot A_{ABCD} = 468\text{ cm}^3$.

Test no. 43

I. 1. $3(\sqrt{3}-1)-\sqrt{2}$.

2. Let's suppose it is x o'clock. Therefore, x hours passed by. There are $(24-x)$ hours left. So: $24-x=\frac{1}{7}x \Rightarrow x=21.$ The result is 9 pm.

3. We have to prove that:

$$a < t_1a + t_2b < b.\ t_1a + t_2b > a \Leftrightarrow t_1a + (1-t_1)b > a \Leftrightarrow (t_1-1)a + (1-t_1)b > 0 \Leftrightarrow$$

$$\Leftrightarrow (1-t_1)b - (1-t_1)a > 0 \Leftrightarrow (1-t_1)(b-a) > 0$$

True, because: $1-t_1>0$ and $b-a>0$.

II. 1. The enunciated equality is equivalent with:

$$(a-b)^2+(b-c)^2+(c-a)^2=0 \Leftrightarrow a=b=c \Rightarrow$$

The triangle is equilateral.

2. $P(1)=0 \Rightarrow X-1 \mid P(X)$.

III. We consider a axial section VAB in the cone from which the truncated cone $ABB'A'$ was derived.

$A' \in (VA),\ VO \perp AB,\ O \in (AB),\ VO \cap A'B' = \{O'\},$

$$O' \in A'B'.\quad \Delta VO'B' \sim \Delta VOB;\ \frac{O'B'}{OB}=\frac{VB'}{VB} \Leftrightarrow \frac{9}{15}=\frac{VB'}{VB'+10} \Rightarrow VB'=15;\ VB=25.$$

$$VO^2=VB^2-OB^2=625-225=400 \Rightarrow VO=20.\ V_{con}=\frac{1}{3}\pi\cdot 15^2\cdot 20=1500\pi.$$

$$A_{lat}=\pi\cdot 25\cdot 15=375\pi.$$

Test no. 44

I. 1. a) 72.

b) -104

2. 37 and 2.

3. $\frac{x-2}{2x^2} + 5x + 7$.

II. 1. $P(1) = 0 \Rightarrow m + n = 0$. $P(-1) = 0 \Rightarrow -m - n + m + n = 0 \Rightarrow 0 = 0$.

So $m = -n$.

2. If $m \neq \frac{1}{2} \Rightarrow x = 2$ is the unique solution. If $m = \frac{1}{2} \Rightarrow$ any natural number is a solution to the equation.

III. 1. $\sphericalangle BAC = 70°$, $\sphericalangle ABC = 60°$, $\sphericalangle ACB = 50°$. $\sphericalangle BAD = 30° \Rightarrow \sphericalangle BAM = 60°$.
$\sphericalangle ABN = ½ \cdot \sphericalangle ABC \Rightarrow \sphericalangle ABN = 30°$. $\sphericalangle APB = 180° - \sphericalangle BAP - \sphericalangle ABP \Rightarrow$
$\Rightarrow \sphericalangle APB = 90° \Rightarrow \sphericalangle NPM = 90°$. $\sphericalangle PNC = \sphericalangle BAN + \sphericalangle ABN = 70° + 30° = 100°$.
$\sphericalangle PMC = \sphericalangle BAM + \sphericalangle ABM = 60° + 60° = 120°$.

2. The cube's diagonal is the sphere's diameter. Let x the edge length of the cube.

$$3x^2 = 4r^2 \Rightarrow$$

$$\Rightarrow x = \frac{2r\sqrt{3}}{3}.\ V = \left(\frac{2r\sqrt{3}}{3}\right)^3 = \frac{8r^3\sqrt{3}}{9}.$$

Test no. 45

I. 1. a) $-\frac{431}{2}$; b) 0. **2.** $P(X) = X^4 + 64 = (X^2)^2 + 8^2 = (X^2)^2 + 2\cdot 8\cdot X^2 + 8^2 - 2\cdot 8\cdot X^2 =$
$= (X^2 + 8)^2 - 16X^2 = (X^2 + 8)^2 - (4X)^2 = (X^2 - 4X + 8)\cdot(X^2 + 4X + 8).$

3. Let x be the number of years when the father's age will be three times bigger than son's age. $3(8 + x) = 28 + x \Rightarrow x = 2.$

II. 1. $x = -\sqrt{2}$, $y = 1$.

2. $\sqrt{(1+a)(1+b)} \geq 1+\sqrt{ab} \Leftrightarrow (1+a)(1+b) \geq \left(1+\sqrt{ab}\right)^2 \Leftrightarrow 1+a+b+ab \geq 1+2\sqrt{ab}+ab \Leftrightarrow$
$\Leftrightarrow a+b \geq 2\sqrt{ab} \Leftrightarrow a-2\sqrt{ab}+b \geq 0 \Leftrightarrow \left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$

True.

III. 1. The triangle AEC is isosceles.

$\Rightarrow \sphericalangle AEC \equiv \sphericalangle ACE.\ \sphericalangle BAC = \sphericalangle AEC + \sphericalangle ACE = 2\cdot\sphericalangle ACE.$

But:

$\sphericalangle BAC = 2\cdot\sphericalangle DAC \Rightarrow 2\cdot\sphericalangle ACE = 2\cdot\sphericalangle DAC \Rightarrow \sphericalangle ACE \equiv \sphericalangle DAC \Rightarrow AD \parallel CE.$

2. a) We denote by a the length of the squares' side. Let Q the middle of DC. The triangle MQP is an isosceles and rectangular triangle.

$\Rightarrow MP = a\sqrt{2}$, $MQ \perp (ABC) \Rightarrow MQ \perp NQ$. $MN^2 = MQ^2 + NQ^2 \Rightarrow$

$\Rightarrow MN^2 = a^2 + \frac{a^2}{2} = \frac{3a^2}{2}$, $NP^2 = \frac{a^2}{2}$.

We observe that $NP^2 + NM^2 = MP^2$. It follows that the triangle PNM is rectangular in N.

b) $NQ \parallel AC, NP \parallel BD$, but $BD \perp AC \Rightarrow NQ \perp NP$, and because $NM \parallel NP \Rightarrow$ the required angle is $\sphericalangle MNQ$. Therefore:

$$\operatorname{tg} \sphericalangle MNQ = \frac{MQ}{NQ} = \frac{a}{\frac{a\sqrt{2}}{2}} = \sqrt{2}.$$

Test no. 46

I. 1. $a=\frac{1}{2}, b=\frac{1}{576} \Rightarrow \frac{a+b}{2}=\frac{\frac{1}{2}+\frac{1}{576}}{2}=\frac{289}{1152}$. **2.** 13, 17, 19. **3.** $x \in \left(-\infty, -\frac{3}{5}\right)$

II. 1. 12,5. **2.** $\left|x+\frac{1}{x}\right| \geq 2 \Leftrightarrow \left(x+\frac{1}{x}\right)^2 \geq 4 \Leftrightarrow x^2+2+\frac{1}{x^2} \geq 4 \Leftrightarrow x^2+\frac{1}{x^2} \geq 2 \,\Big|\cdot x^2 \Leftrightarrow$

$\Leftrightarrow x^4+1 \geq 2x^2 \Leftrightarrow x^4-2x^2+1 \geq 0 \Leftrightarrow (x^2-1)^2 \geq 0$.

true.

3. $X^4+X^2+4=X^4+2X^2+4-2X^2=(X^2+2)^2-(\sqrt{2}X)^2=(X^2-\sqrt{2}X+2)(X^2+\sqrt{2}X+2)$

III. 1. $\Delta BDC - \Delta ADB$ (because they have two congruent angles)

$\Rightarrow \frac{BD}{AD}=\frac{CD}{BD} \Rightarrow BD^2=AD \cdot CD$

$\Rightarrow BD=\sqrt{AD \cdot CD}$. 2.a) 5; b) $A_l = 12\pi$, $A_t = 16{,}50\,\pi$, $V = 13{,}50\,\pi$; c) $\sqrt{9\pi^2+16}$.

Test no. 47

I. 1. $x = 3\frac{1}{5}$.

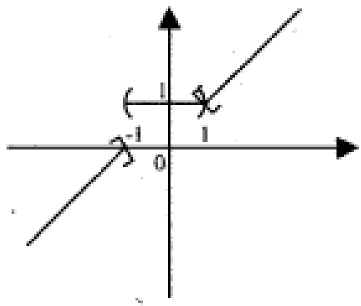
2. Let $n-1$ and $n+1$ the three numbers:

$$(n-1)^2 + n^2 + (n+1)^2 + 1 = n^2 - 2n + 1 +$$

$$+ n^2 + 2n + 1 + n^2 + 1 = 3n^2 + 3 = 3(n^2+1) \vdots 3.$$

3. Let $m, n \in \mathbb{N}$ such that $\sqrt{mn} = 11 \Rightarrow mn = 121$ or $m = 1$ and $n = 121$, or $m = 11$ and $n = 111$, or $m = 121$ and $n = 1$.

II. 1.



2. $(a+b+c)^2 = 3(a^2+b^2+c^2) \Rightarrow a^2+b^2+c^2 = ab+bc+ca \Rightarrow$

$$\Rightarrow \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] = 0 \Rightarrow a = b = c.$$

III. a) AO is the mediator of $[BC]$. But ABC is an isosceles triangle, so it follows that AO is the bisector of the angle $\sphericalangle BAC \Rightarrow \sphericalangle BAD \equiv \sphericalangle CAD = 60^0$. But $\sphericalangle DBC \equiv \sphericalangle CAD$ and $\sphericalangle DCB \equiv \sphericalangle BAD$ because the quadrilateral $ABCD$ is inscriptible. Therefore $\sphericalangle DBC \equiv \sphericalangle DCB = 60^0 \Rightarrow \sphericalangle BDC = 60^0$. Therefore the triangle BDC is equilateral.

b) $\Delta MOB \equiv \Delta MOC \equiv \Delta MOD \Rightarrow MB \equiv MC \equiv MD$.

c) $CD \perp OM$, but $CO \perp OB$ (OB is the mediator of the segment CD because O is the center of the circle circumscribed to the triangle BCD). Therefore $CD \perp (BOM) \Rightarrow CD \perp BM$.

d) $\frac{81\sqrt{13}}{2}$ cm^3.

Test no. 48

I. **1.** $(0,1)^{10} > (0,1)^{11}$; $(0,1)^5 > (0,01)^5$. **2.** 22 şi 68. **3.** $x \in (5,6)$.

II. **1.** 30, 60, 120, 150. **2.** $\left.\begin{array}{l} P(1)=0 \Rightarrow a+b+2=0 \\ P(-1)=0 \Rightarrow a-b=0 \end{array}\right\} \Rightarrow a=b=-1$

$P(X) = X^3 - X^2 - X + 1 = X^2(X-1) - (X-1) = (X-1)(X^2-1) = (X-1)^2(X+1)$

III. **1.** $\Delta CAD \sim \Delta CMB \Rightarrow \frac{AD}{MB} = \frac{DC}{BC}$; $\Delta BAD \sim \Delta BNC \Rightarrow \frac{AD}{NC} = \frac{BD}{DC}$.

2. Let O be the middle of the segment MC. We show that $OA \equiv OB \equiv OD \equiv OM$.

$$\left.\begin{array}{l} MA \perp (ABC) \\ AD \perp DC \\ DC \subset (ABC) \end{array}\right\} \Rightarrow MD \perp DC$$

Analogously, $MB \perp BC$.

Because the triangle MDC is rectangular ($\sphericalangle MDC = 90^0$) and DO is the median corresponding to hypotenuse, it follows that $DO = \frac{MC}{2}$. Analogously, $OB = \frac{MC}{2}$.

$$\left.\begin{array}{l} MA \perp (ABC) \\ AC \subset (ABC) \end{array}\right\} \Rightarrow MA \perp AC$$

Because Ao is the median corresponding to hypotenuse, it follows that $AO = \frac{MC}{2}$. Therefore:

$$OA \equiv OB \equiv OC \equiv OD \equiv OM = \frac{MC}{2}.$$

Test no. 49

I. **1.** $a-\frac{1}{a}=5\Rightarrow a^2-2+\frac{1}{a^2}=25\Rightarrow a^2+\frac{1}{a^2}=27$

$$a-\frac{1}{a}=5\Rightarrow\left(a-\frac{1}{a}\right)^3=125\Rightarrow a^3-3a^2\cdot\frac{1}{a}+3a\cdot\frac{1}{a^2}-\frac{1}{a^3}=125\Rightarrow$$

$$\Rightarrow a^3-3a+3\cdot\frac{1}{a}-\frac{1}{a^3}=125\Rightarrow a^3-\frac{1}{a^3}-3\left(a-\frac{1}{a}\right)=125\Rightarrow a^3-\frac{1}{a^3}-15=125\Rightarrow$$

$\Rightarrow a^3-\frac{1}{a^3}=140$. **2.** 56 000 lei. **3.** $\overline{5a5}\vdots 9\Rightarrow 10+a\vdots 9\Rightarrow a=8$; $\frac{x}{2}=\frac{585}{3}\Rightarrow x=390$.

II. **1.** $\frac{a+b+c}{2}<a\,\Big|\cdot 2\Leftrightarrow a+b+c<2a\Leftrightarrow b+c<a$

- true.

Analogously,

$\frac{a+b+c}{2}<b$ şi $\frac{a+b+c}{2}<c$. **2.** $x\in(1,4), x\in\mathbb{Z}\Rightarrow x\in\{2,3\}$.

III. 1. Let

$\{G\}=EF\cap BC$, $AD\parallel FG\Rightarrow\frac{AC}{AF}=\frac{CD}{DG}$ şi $\frac{AE}{BE}=\frac{GD}{BG}$.

Multiplying member by member the two equalities, we obtain:

$\frac{AC}{AF}\cdot\frac{AE}{BE}=\frac{CD}{BG}$. But: $\frac{BE}{AB}=\frac{BG}{BD}$.

Multiplying this two new equalities, we get:

$\frac{AC}{AF}\cdot\frac{AE}{AB}=1\Rightarrow\frac{AE}{AF}=\frac{AB}{AC}$.

2. a) In:

ΔACO, $\sphericalangle ACO=90°$, $AC=\sqrt{AO^2-OC^2}=\sqrt{145}$

ΔABO, $\sphericalangle ABO=90°$, $AB=\sqrt{OA^2-OB^2}=15$

$\Delta CC'O$, $\sphericalangle CC'O=90°$, şi $\sphericalangle COC'=30°\Rightarrow CC'=\frac{OC}{2}=6$

b) $\Delta OBC'$ is isosceles, because $OB=BC'=8$; BM median $\Rightarrow$

$$\left.\begin{array}{l}AB\perp\alpha\\ BM\perp OC'\\ OC'\subset\alpha\end{array}\right\}\Rightarrow AM\perp OC'.$$

Test no. 50

I. **1.** $S = 2 - 4 + 6 - 8 + ... - 1996 + 1998 = 2(1 - 2 + 3 - 4 + ... - 998 + 999) =$

$= 2\left(\underbrace{-1-1-1-...-1}+999\right) = 2(-499 + 999) = 2 \cdot 500 = 1000$. **2.** 40 zile. **3.** $x = 1, y = 1$.

II. **1.** $m = 3$. **2.** $P(X) = (X - a)(X^2 - 4X + 5) + R_1$; $P(X) = (X - b)(X^2 - 6X + 3) + R_2$.

$P(X) = X^3 - (4 + a)X^2 + (5 + 4a)X - 5a + R_1$

$P(X) = X^3 - (6 + b)X^2 + (3 + 6b)X - 3b + R_2$. $R_1, R_2 \in \mathbb{R}$.

Rezultă că $4 + a = 6 + b$, $5 + 4a = 3 + 6b$, $-5a + R_1 = -3b + R_2 = 1$.

Deci $a = 7$, $b = 5$, $R_1 = 36 \Rightarrow P(X) = X^3 - 11X^2 + 33X + 1$.

III. **1.** $\Delta CPM \sim \Delta CBA \Rightarrow \frac{MP}{AB} = \frac{CM}{AC}$. $\Delta AQM \sim \Delta ADC \Rightarrow \frac{MQ}{CD} = \frac{AM}{AC}$.

2. Let AA' and BB' the perpendiculars on the planes intersection, In the rectagular triangle $BB'A$, we have:

$AB^2 = BB'^2 + AB'^2$. $AB'^2 = AA'^2 + A'B'^2$.

From rectangular triangle $BB'A$, we get:

$\text{tg } u = \frac{b}{a^2 + c^2}$. $v = m(\sphericalangle ABB')$. $\text{tg } v = \frac{a}{b^2 + c^2}$.

Test no. 51

I. 1. $\frac{1}{2}$; 2. $a = \frac{3}{13}, b = \frac{4}{13}, c = \frac{12}{13}$ and they are replaced in relation $a^2 + b^2 + c^2 = 1$.

3. $\sqrt{1000 - 25\sqrt{n}} = \sqrt{25(40 - \sqrt{n})} = 5\sqrt{40 - \sqrt{n}} \in \mathbb{N} \Rightarrow 40 - \sqrt{n}$

- it is a perfect square, and n is also a perfect square:

$n \in \{16, 225, 576, 961, 1296, 1521, 1600\}$.

II. 1. $x \in \varphi$. (The system has no solution.)

2. $P(-3, -3)$.

III. **1.** $r = 2\sqrt{\frac{\sqrt{3}}{\pi}}$ cm.

2. v = 2 cm^3, V = 54 cm^3.

Test no. 52

I. 1. a) 40; b) 400 g. 2. 10, 30 şi 20. 3. 525, 945, 1260, 1470.
II. 1. $x^6 - 2x^3 = x^6 - 2x^3 + 1 - 1 = (x^3 - 1)^2 - 1^2$.

2. Notice that the triangle is rectangular, $A = \frac{6 \cdot 8}{2} = 24$.

III. 1. Let O the middle of PQ. $MP \equiv QN = \frac{1}{2}DC$, DC being a trapezoid base. Prove that the points M, P, Q and N are collinear.
$OM = OP + PM$ and $ON = OQ + QN$. As $OP \equiv OQ$ and $PM \equiv QN \Rightarrow OM \equiv ON \Rightarrow O$ is the middle of MN.

2. From $BD \perp (ABC)$ and $AB \perp AC \Rightarrow AD \perp AC$.

$$\left.\begin{array}{l} AC \perp AD \\ AC \perp AB \end{array}\right\} \Rightarrow AC \perp (ABD) \quad (ABD) \perp (ABC),$$

because $AD \subset (ABD)$ and $AD \perp (ABC)$.
$AM \perp BC \Rightarrow AM \perp (BCD)$.
Analogously,
$BP \perp (ACD)$. $CD \perp AM$
$CD \perp AN \Rightarrow CD \perp (AMN)$.
From planes' parallelism, it follows that $AN \parallel PQ$ and $MN \parallel QB$, therefore $\sphericalangle ANM' \equiv \sphericalangle BQP$. So, the rectangular triangles are similar.

Test no. 53

I. **1.** $3n+1 \mid 2n+4 \Rightarrow 3n+1 \mid 3(2n+4) \Rightarrow 3n+1 \mid 6n+12$ (1)

$3n+1 \mid 3n+1 \Rightarrow 3n+1 \mid 2(3n+1) \Rightarrow 3n+1 \mid 6n+2$ (2)

From (1) and (2)

$\Rightarrow 3n+1 \mid 6n+12-(6n+2) \Rightarrow 3n+1 \mid 10 \Rightarrow 3n+1 \in \{1, 2, 5, 10\} \Rightarrow$

$\Rightarrow 3n \in \{0, 1, 4, 9\}$.

Because $n \in \mathbb{N}$,

$\Rightarrow n \in \{0, 3\}$. 2. 5. 3. 33,(3) %.

II. 1. Check by direct calculation.

2. $\dfrac{4x+3}{x+1}$.

III. 1. Let ABC an equilateral triangle, and M a point inside the triangle, A', B', C' being its projections on BC, AC, and respectively AB. D is the height feet from A.

$$A_{ABC} = \frac{AD \cdot BC}{2}.$$

$$A_{ABC} = A_{MAB} + A_{MBC} + A_{MAC} = \frac{MC' \cdot AB}{2} + \frac{MB' \cdot AB}{2} + \frac{MA' \cdot AB}{2}.$$

$$\frac{AD \cdot AB}{2} = \frac{(MC' + MA' + MB') \cdot AB}{2} \Rightarrow AD = MA' + MB' + MC'.$$

2. $\left.\begin{array}{l} BD \perp AC \\ BD \perp AM \end{array}\right\} \Rightarrow BD \perp (MACN) \Rightarrow BD \perp MN$. Deci $\left.\begin{array}{l} MN \perp BD \\ MN \perp OM \end{array}\right\} \Rightarrow MN \perp (BMD)$.

Test no. 54

I. 1. $a > b$.

2. $x \in (-\infty,0]$.

3. $x = 1$ is a common solution.

II. 1. Check by direct calculation.

2. Effectuate the division; the rest is 0.

III. 1. We draw a rectangular triangle ABC ($\sphericalangle A = 90^0$), where $AD \perp BC, D \in BC$ and E is the middle of $[BC]$.

$\sphericalangle EAC \equiv \sphericalangle ECA$. $\sphericalangle DAE = 90^\circ - \sphericalangle AED = 90^\circ - (\sphericalangle EAC + \sphericalangle ECA) = 90^\circ - (\sphericalangle C + \sphericalangle C) =$
$= 90^\circ - \sphericalangle C - \sphericalangle C = \sphericalangle B - \sphericalangle C$.

2. Let G be the middle of AD.

$$GE = \frac{DC + AB}{2} \Rightarrow GE = \frac{1}{2}AD.$$

It is easy to prove that $m(\sphericalangle AED) = 90^0$.

In the rectangular triangle:

AED: $AD^2 = AE^2 + ED^2$.

Because:

$AE \equiv FD \Rightarrow FE \equiv AD$.

$DF \perp (ABCD)$ (1); $DE \perp AE$ (2); $DE, AE \subset (ABCD)$ (3) $\Rightarrow$

from (1), (2) and (3) $FE \perp AE$.

Test no. 55

I. 1. 6 and 24.

2. $\sqrt{11}-\sqrt{5}-2$. 3. $\sqrt{13}-7<\sqrt{15}-8$.

II. 1. 100 000 lei. 2. $m = 2$.

III. 1. 3, 4, 5. 2. $A_l = \pi G(R + r) = \pi \cdot 25(R + r) = 225\pi \Rightarrow R + r = 9$.

$G^2 = h^2 + (R - r)^2 \Rightarrow$

$\Rightarrow 625 = 576 + (R - r)^2 \Rightarrow R - r = 7$.

Therefore, $R = 8, r = 1$. The volume is

$$\frac{\pi \cdot 24}{3}(64 + 1 + 8) = 584\pi.$$

Test no. 56

I. 1. $x^2 + 1$.

2. 15 days.

3. $m^2x + 3 = 9x + m \Leftrightarrow x(m^2 - 9) = m - 3 \Leftrightarrow (m - 3)(m + 3)\cdot x = m - 3$.

For $m = -3$ the equation has no real solutions.

II. 1. $mx + 1 \le x + m \Leftrightarrow x(m - 1) \le m - 1$.

2. $m = 0, m = 1$.

III. 1. Let

$\{B'\} = BH \cap AC, \{C'\} = CH \cap AB$.

$90° - \sphericalangle MAB' = \sphericalangle AMB' \Rightarrow$

$\sphericalangle AMB' = 90° - \frac{1}{2} \cdot \sphericalangle BAC \Rightarrow$

$\sphericalangle HMN = 90° - \frac{1}{2} \cdot \sphericalangle BAC$.

$\sphericalangle HNM = \sphericalangle C'NA = 90° - \sphericalangle C'AN = 90° - \frac{1}{2} \sphericalangle BAC$.

Therefore, $\sphericalangle HNM \equiv \sphericalangle HMN \Rightarrow \Delta HMN$ is isosceles.

2. $V = 162$ cm^3.

Test no. 57

I. **1.** $(a^{10} + a^{11} + .. + a^{20}) : (a^0 + a^1 + ... + a^{10}) =$

$a^{10}(a^0 + a^1 + ... + a^{10}) : (a^0 + a^1 + ... + a^{10}) = a^{10}$.

2. $x = 9$. **3.** $A \cap B = \{39, 40, ..., 98, 99\}$; $A \cup B = \{0, 1, 2, ..., 122, 123\}$.

$A \setminus B = \{0, 1, ..., 38,$

$100, 101, ... 123\}$; $B \setminus A = \phi$.

II. **1.** $2n + 1 = (n + 1)^2 - n^2$,

2. $a = 5, b = -3$.

III. Let

$AD \cap BC = F$. $\sphericalangle BAF = 90° - \sphericalangle ABC$. $\sphericalangle ACE = 90°$

because *AE* is a diameter.

$\sphericalangle CAE = 90° - \sphericalangle AEC = 90° - \sphericalangle ABC$

Therefore:

$\sphericalangle CAE \equiv \sphericalangle BAF$ (1); $\sphericalangle BAE = \sphericalangle BAF + \sphericalangle DAF$ (2); $\sphericalangle CAD = \sphericalangle CAE + \sphericalangle DAF$ (3).

$\Rightarrow \sphericalangle BAE \equiv \sphericalangle CAD$.

2. $A = \dfrac{16a^2\pi^5}{4\pi^2 - 1}$; $V = \dfrac{32a^3\pi^6\sqrt{4\pi^2 - 1}}{9(4\pi^2 - 1)^2}$.

Test no. 58

I. 1. Let $n-1, n$ and $n+1$ the three numbers. We have to prove that $(n-1)\cdot n\cdot(n+1) \vdots 3.$ (1) Dividing the natural number n by 3, we can have the rests 0, 1 or 2. Therefore, n can be $3k, 3k+1, 3k+2$, where $k \in \mathbb{N}$. Replacing n with one of those, one can easily get the conclusion.

$$2.\ \frac{x}{y}=\frac{14}{4}.\ 3.\ x \in [\frac{3}{2};\infty).$$

II. Let a, b the catheti's length, h the length of the height taken from the right vertex, and c the length of hypotenuse. We have to prove that the following is true:

$$h \le \frac{c}{2} \Leftrightarrow \frac{a\cdot b}{c} \le \frac{c}{2} \Leftrightarrow 2ab \le c^2 \Leftrightarrow 2ab \le a^2+b^2 \Leftrightarrow$$

$$\Leftrightarrow a^2 - 2ab + b^2 \ge 0 \Leftrightarrow (a-b)^2 \ge 0$$

We have equality in the case of a right isosceles triangle.

$$2.\ X^4-2X^3+2X^2-2X+1 = X^4+2X^2+1-2X^3-2X = (X^2+1)^2-2X(X^2+1) =$$
$$=(X^2+1)(X^2-2X+1)=(X^2+1)(X-1)^2.$$

III. 1. Because AEHD is an inscriptible quadrilateral,

$$\Rightarrow \sphericalangle EAD + \sphericalangle EHD = 180° \ (1).$$

Because ABGC is also an inscriptible quadrilateral,

$$\Rightarrow \sphericalangle EAD + \sphericalangle BGC = 180° \ (2).$$

From (1) and (2),

$$\Rightarrow \sphericalangle EHD \equiv \sphericalangle BGC.$$

But

$$\sphericalangle EHD \equiv \sphericalangle BHC \ (3).$$

and because

$$\sphericalangle ABD \equiv \sphericalangle ACE \Rightarrow \sphericalangle HBG \equiv \sphericalangle HCG \ (4).$$

it follows that $BHCG$ is a quadrilateral with congruent opposite angles, therefore it is a parallelogram.

$$2.\ \frac{4}{3}\pi\left(31^3-30^3\right)mm^3.$$

Test no. 59

I. 1. Direct calculation.

2. $a+b$ – odd $\Rightarrow a$ – even and b – odd or a – odd and b – even. Both cases, $a \cdot b$ is an even number.

3. We assume by absurd that $\sqrt{5} \in Q \Rightarrow$ there exist $m,n \in \mathbb{Z}$, m,n prime numbers such that $\frac{m}{n} = \sqrt{5} \Rightarrow \frac{m^2}{n^2} = 5 \Rightarrow$

$$\Rightarrow m^2 = 5n^2 \Rightarrow 5 \mid m^2 \Rightarrow 5 \mid m \Rightarrow m = 5k,\ k \in \mathbf{N}.$$

$$(5k)^2 = 5n^2 \Rightarrow 25k^2 = 5n^2 \Rightarrow 5k^2 = n^2 \Rightarrow 5 \mid n^2 \Rightarrow 5 \mid n.$$

But $5|m$ – is in contradiction with $(m,n) = 1$. Therefore $\sqrt{5} \notin Q$.

II. 1. $\frac{1}{x+1}$. **2.** $P(-1) = P(0) = 0$.

III. 1. Let E be the intersection of bisectors taken from vertices A and B of the parallelogram $ABCD$.

$$\sphericalangle AEB = 180° - (\sphericalangle EAB + \sphericalangle EBA) = 180° - (\frac{1}{2} \cdot \sphericalangle DAB + \frac{1}{2} \cdot \sphericalangle CBA) =$$

$$= 180° - \frac{1}{2}(\sphericalangle DAB + \sphericalangle CBA) = 180° - \frac{1}{2} \cdot 180° = 90°.$$

Therefore, $AE \perp BE$.

2. We have the pyramid $VABC$ with the base ABC ($\sphericalangle A = 90^0, AB \equiv AC$).

$$VA = a\sqrt{2},\ VB = a\sqrt{3},\ VC = a.\ V_{ABCD} = \frac{1}{3} \cdot a \cdot \frac{a^2}{2} = \frac{a^3}{6}.$$

Test no. 60

I. **1.** $\overline{8x1y} \vdots 15 \Rightarrow \overline{8x1y} \vdots 5 \quad \overline{8x1y} \vdots 3$

$\overline{8x1y} \vdots 5 \Rightarrow y \in \{0,5\}$. $\overline{8x1y} \vdots 3 \Rightarrow 8+x+1+y \vdots 3 \Rightarrow x+y \in \{0, 3, 6, 9, 12, 15, 18\}$.

We find the numbers:

8010, 8310, 8610, 8910, 8115, 8415, 8715.

2. 10 m. 3. $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 14\}$; $A \cap B = \{3, 6\}$.

$A \setminus B = \{4, 5, 8, 14\}$. $B \setminus A = \{1, 2\}$

$A \times B = \{(3, 1), (3, 2), (3, 3), (3, 6), (4, 1), (4, 2), (4, 3),$
$(4, 6), (5, 1), (5, 2), (5, 3), (5, 6),$
$(6, 1), (6, 2), (6, 3), (6, 6), (8, 1), (8, 2), (8, 3), (8, 6),$
$(14, 1), (14, 2), (14, 3), (14, 6)\}$.

II. 1. The inequations has no solutions.

2. The area of a triangle is the demi-product between the height and the side on which the height falls.

III. 1. a) Ne denote CP $-x$. From the rectangular trapezoids BCPM and CDNP we get:

$$MP^2 = a^2 + (x - 2a)^2$$
$$NP^2 = 4a^2 + (3a - x)^2.$$

Because $MP \equiv NP$,

$$a^2 + (x - 2a)^2 =$$
$$= 4a^2 + (3a - x)^2 \Rightarrow x = 4a \Rightarrow MP \equiv NP = a\sqrt{5}.$$

$$\text{b)}\ \left.\begin{array}{l}(ADNQ) \parallel (BCPM)\\ (ADNQ) \cap (MNP) = NQ\\ (BCPM) \cap (MNP) = MP\end{array}\right\} \Rightarrow NQ \parallel MP.$$

c) Let

$AR \perp BD$, $R \in BD$. $BD^2 = AB^2 + AD^2 - 2AB \cdot AD \cdot \cos 60° = 3a^2$.

$AB^2 = AD^2 + DB^2 \Rightarrow AD \perp DB \Rightarrow R = D$.

$$\left.\begin{array}{l} QA \perp (ABCD) \\ AD \perp DB \\ DB \subset (ABCD) \end{array}\right\} \Rightarrow QD \perp DB.\ d(Q, DB) = QD = a\sqrt{2}.$$

Analogously,

$$PB \perp DB.\ d(P, DB) = \sqrt{a^2 + 16a^2} = a\sqrt{17}.$$

2. The side of the tetrahedron has the value of

$$\frac{2\sqrt{6}}{3}r.\ V = \frac{8R^3\sqrt{3}}{27}.$$

Test no. 61

I. 1. $\frac{27n+65}{8} = 3n+8+\frac{3n+1}{8} \in N \Leftrightarrow 3n+1 \in \{8, 16, 24, ...\}$. $3n+1=16 \Rightarrow n=5$.

2. False, because:

$$|x+18|+|x-4|>0$$

3. $f(x)$ is increasing on $\mathbb{R}$.

II. 1. $3 \cdot 9^n + 5 \cdot 17^n = 3(8+1)^n + 5(2 \cdot 8 + 1)^n =$

$3(M8+1)+5(M8+1) = M8+3+5 = M8.$

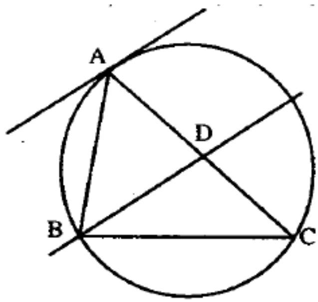
2. $X^3 + 6X^2 + 11X + 6 = (X+1)(X+2)(X+3)$

$a = 2, b = 11, c = 0.$

3. $a^3 + 3ab + b^3 = (a+b)(a^2 - ab + b^2) + 3ab =$

$= a^2 + 2ab + b^2 = (a+b)^2 = 1.$

III. 1. $\Delta ABD \sim \Delta ABC \Rightarrow \frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AB^2 = AC \cdot AD.$



2. Let A' the projection of A on the plane α. In rectangular triangles $AA'B$ and $AA'C$, we have $A'B^2 = 8^2 - AA'^2$ and $A'C^2 = 7^2 - AA'^2$, from where we get:

$$A'B^2 - A'C^2 = 64 - 49 = 15.$$

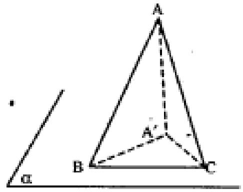
Also, in triangle $BA'C$, we have

$$A'B^2 + A'C^2 = 25.$$

From the two relations, we get:

$$A'B^2 = 20 \quad A'C^2 = 5 \Rightarrow A'B \cdot A'C = 10.$$

$A(BA'C) = \dfrac{A'B \cdot A'C}{2} = 5\ cm^2.$



Test no. 62

I. **1.** $2a - 3b + 8c = 0 \Leftrightarrow 2a + 8c = 3b \Leftrightarrow 2(a + 4c) = 3b \Leftrightarrow 2 \mid b$

$3 \mid (a + 4c) \Leftrightarrow$

$\Leftrightarrow 2 \mid b$

$3 \mid (a + c) + 3c \Leftrightarrow 2 \mid b$

$3 \mid (a + c) \Leftrightarrow 6 \mid b(a + c).$

2. The sum after every year represents 105% = 1,05 form previous year. After 3 years, the sum becomes:

$1{,}05^3 \cdot 100\$ = 115{,}7625\$.$

3. $a^2 \leq b \Rightarrow a^4 \leq b^2 \leq c;$

$c \leq a^2 \Rightarrow a^4 \leq b^2 \leq c \leq a^2 \Rightarrow a^4 \leq a^2 \Rightarrow a^2(a^2 - 1) \leq 0 \Rightarrow$

$\Rightarrow a \in [-1, 1] \cap N^* \Rightarrow a = 1, b = 1, c = 1.$

II. Let $A^2 = a^2 + b^2 + c^2$; because a, b, c are numbers, it follows that:

$A^2 \leq 9^2 + 9^2 + 9^2 = 243 < 256;$

Consequently,

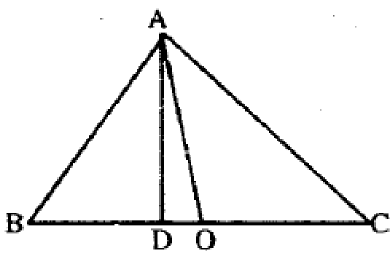
$A \in \{2, 3, 5, 7, 11, 13\}$

However, A being of form 3k + 2, we still have the possibilities

$A \in \{2, 5, 11\}.$

Considering each case, we find the solutions: 200, 269, 296, 304, 340, 403, 430, 500, 629, 667, 676, 766, 792, 926, 962.

III. 1. BC = 10 cm.



The center of the circle circumscribed to the triangle is at the middle of BC, and then OA = OB = OC = 5. Expressing triangle's area in two ways, we get the relation:

$$AD \cdot BC = AB \cdot AC \Rightarrow AD = \frac{AB \cdot AC}{BC} = \frac{24}{5}$$

From ΔADO, we get:

$$DO = \sqrt{AO^2 - AD^2} = \frac{7}{5}.$$

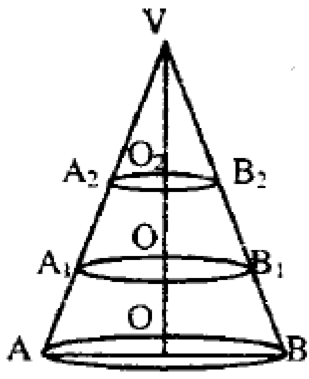
2. We denote:

$$V_1 = V_{con}(VA_1B_1);\ V_2 = V_{con}(VA_2B_2);$$

$$V_3 = V_{trcon}(A_1B_1A_2B_2).$$

Obviously,

$$V_3 = V_1 - V_2.$$



But

$$\frac{V_1}{V} = \left(\frac{VO_1}{VO}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27};\quad \frac{V_2}{V} = \left(\frac{VO_2}{VO}\right)^3 = \frac{1}{27}.$$

$$V_3 = V_1 - V_2 \Rightarrow \frac{V_3}{V} = \frac{V_1}{V} - \frac{V_2}{V} = \frac{8}{27} - \frac{1}{27} = \frac{7}{27} \Rightarrow V_3 = \frac{7V}{27}$$

Test no. 63

I. 1. $\sqrt{2}\cdot\sqrt{2+\sqrt{2}}\cdot\sqrt{2+\sqrt{2+\sqrt{2}}}\cdot\sqrt{2-\sqrt{2+\sqrt{2}}}=\sqrt{2}\cdot\sqrt{2+\sqrt{2}}\cdot\sqrt{2-\sqrt{2}}=\sqrt{2}\cdot\sqrt{2}=2.$

2. a) $xy\le\frac{(x+y)^2}{4}\Leftrightarrow 4xy\le x^2+2xy+y^2\Leftrightarrow 0\le(x-y)^2,$

- true.

b) $x+y=k$

$$\Rightarrow xy\le\frac{(x+y)^2}{4}=\frac{k^2}{4}.$$

3. $Q(X)=(X-1)(X-4);$

P(X) divides by Q(X) if P(1)=0, P(4)=0, from where we have the system:

$$\begin{cases}a+b=2\\4a+b=-64\end{cases}\Rightarrow\begin{cases}a=-22\\b=24\end{cases}$$

II. 1. $x>0,\ y>0,\ x+y=1$

The inequality transcribes equivalently:

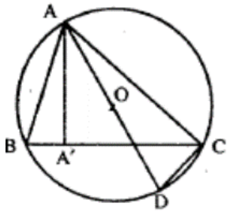
$$\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\ge 9\Leftrightarrow 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{xy}\ge 9\Leftrightarrow\frac{x+y+1}{xy}\ge 8\Leftrightarrow\frac{2}{xy}\ge 8\Leftrightarrow xy\le\frac{1}{4}$$

Inequality is easily obtained from the means' inequality:

$$\sqrt{xy}\le\frac{x+y}{2}=\frac{1}{2}\Rightarrow xy\le\frac{1}{4}.$$

2. The product $k(k+1)(k+2)$ is divided by 6, as a product of a three consecutive natural numbers. Then, N being the sum of n divisible products by 6, it follows $N|6$.

III. 1. a) $A'B^2=AB^2-AA'^2;\ A'C^2=AC^2-AA'^2\Rightarrow$
$\Rightarrow A'B^2-A'C^2=(AB^2-AA'^2)-(AC^2-AA'^2)=$
$=AB^2-AC^2.$



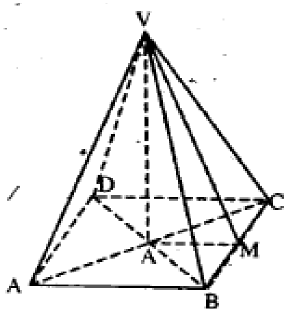
b) The triangles ABA’ and ADC are similar, and then:

$$\frac{AB}{AD}=\frac{AA'}{AC}\Rightarrow$$

$$\Rightarrow AB\cdot AC = AD\cdot AA' \Rightarrow AB\cdot AC = 2R\cdot AA'.$$

2. Let VABCD a regular quadrilateral pyramid. VO is the height of the pyramid and $OM \perp BC$. From the theorem of the three perpendiculars, it follows that $VM \perp BC$. We denote by a the base, and we have:

$$S = A_l + A(ABCD) = \frac{4a\cdot VM}{2}+a^2 = 2a\cdot VM + a^2.$$



The triangle VBC is equilateral, and then $VM=\frac{a\sqrt{3}}{2}$. Replacing in the above relation, we get

$$a=\sqrt{\frac{S}{\sqrt{3}+1}};$$

Calculate in the triangle VOM:

$$VO=\sqrt{\frac{S}{\sqrt{6}+\sqrt{2}}}.$$

Test no. 64

I. **1.** $a = \left(2^2 \cdot 3^2 \cdot 2^{2n} \cdot 3^n\right)^2;$

$b = 1998 + 2 \cdot \frac{1998 \cdot 1997}{2} = 1998^2$

2. $a = 6$; $b = 10$; $c = 14$.

II. **1.** 273, 354, 435, 516.

2. $l_3 = 1$; $a_3 = \frac{\sqrt{3}}{2}$; $l_4 = \sqrt{2}$; $a_4 = \frac{\sqrt{2}}{2}$; $l_6 = \sqrt{3}$; $a_6 = \frac{1}{2}$.

III. **1.** $m = -\frac{6}{5}$; $n = -\frac{12}{5}$. **2.** a) $A_l = 144\sqrt{2}\ \text{cm}^2$;

$A_t = 144\left(\sqrt{2} + 1\right)\text{cm}^2$. b) $V_1 = 288\ \text{cm}^3$.

c) We denote by l_4' the square side, and we have:

$\frac{l_4}{12} = \frac{1}{3}$; $l_4 = 4$ cm; section area $= 16$ cm^2.

d) $V_2 = \frac{32}{3}\text{cm}^3$; $\frac{V_2}{V_1} = \frac{1}{27}$.

e) $V = 277\frac{1}{3}$ cm^3.

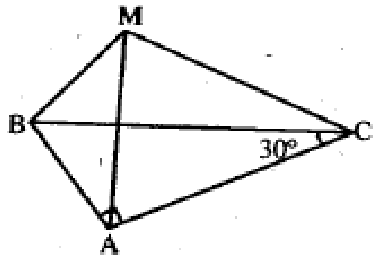
Test no. 65

I. 1. 123 and 329.

2. 450, 240, 1080, 210.

II. 1. $a = 1, b = -2\sqrt{2};\ a^2 - b^2 = -7.$

2. Observe that the triangle ABC is rectangular ($m\sphericalangle A = 90^0$), and that the triangles ABC and MBC are congruent.



III. 1. a) $x \in \left(\frac{1}{2}, 2\right)$. b) $x \in \mathbb{R} - (1, 2)$.

2. a) Build $OM \perp BC$ and, according to the theorem of the three perpendiculars, $VM \perp BC$, therefore the dihedral angle can be measured through the angle OMV, so: $\sin \sphericalangle OMV = \frac{VO}{VM}$, and in triangle VOM ($VO \perp VM$), $VM^2 = VO^2 + OM^2 \Rightarrow VM = 20$ cm, consequently $\sin \sphericalangle OMV = \frac{4}{5}$.

b) The angle formed by an edge with the base plane is e.g. $\sphericalangle VAO$, and

$$\text{tg} \sphericalangle VAO = \frac{VO}{AO} = \frac{2\sqrt{2}}{3}.$$

c) Still have to find the small base side. VO and VM determine a plane in the triangle VOM ($m\sphericalangle O = 90^0$).

$$OM \parallel ON \Rightarrow \frac{VO_1}{VO} = \frac{ON}{OM} = \frac{VN}{VM} \Rightarrow \frac{1}{4} = \frac{ON}{12} = \frac{VN}{20} \Rightarrow ON = 3 \text{ cm};$$

$$VN = 5 \text{ cm} \Rightarrow A'B' = 6 \text{ cm}. \ NM = 15 \text{ cm}.$$

The lateral sides are trapezoids, having the bases of 24 cm and 6 cm, and the height NM = 15 cm.

d) The section is the trapeze ACC'A', with bases:

$AC = 24\sqrt{2}$ cm, $A'C' = 6\sqrt{2}$ cm

and the height:

$OO_1 = VO - VO_1 = 12$ cm.

Test no. 66

I. 1.a) $\sqrt{3 \pm 4\sqrt{5}} = \sqrt{5 \pm 2 \cdot 2\sqrt{5} + 4} = \sqrt{\left(\sqrt{5} \pm 2\right)^2} = \sqrt{5} \pm 2$; b) 0,14.

2. $a = 0{,}3 \cdot b$; $b - a = 35$;

$b = 50$; $a = 15$. 3. $x = 5$; $y = 3$.

II. 1. $x^2 + x = t \Rightarrow P(x) = (t+1)(t+3) + 1 = t^2 + 4t + 4 = (t+2)^2 = (x^2 + x + 2)^2$.

2. $b + c = 14$; $b - c = 2 \Rightarrow b = 8$; $c = 6$

3. $a + b + c = 9$; $b + c = 23$; $a = -14$.

III. 1. The area of MBA is $\frac{BM \cdot BA}{2} = 30$ cm².

2. $MB \perp (ABC) \Rightarrow (MBA) \perp (ABC) \Longrightarrow CP \perp$ the right of intersection of the two planes. $AB \perp (MBA)$. Calculate CP by writing the area of triangle ABC in two ways $\Longrightarrow CP = 9.6$.

3. $\text{Vol} = \frac{S_{(MBA)} \cdot CP}{3} = 96 \text{ cm}^3$

$V_{(ABCM)} = \frac{S_{(ABC)} \cdot MB}{3}$.

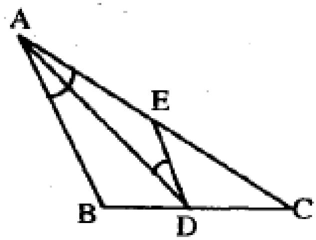
Test no. 67

I. 1. 48. 2. $x = 7, y = 5$.

II. 1. a) Start from $(x - y)^2 \geq 0$.

b) $(x - y)^2 \geq 0$; $(x - z)^2 \geq 0$; $(y - z)^2 \geq 0$.

2. Build $DE \parallel AB$. The triangle ADE is isosceles. Observe that the triangles CED and CAB are similar, and it follows that - from the fundamental theorem of similarity: $a = 2$. Then use inequalities between the sides of triangle ADE.

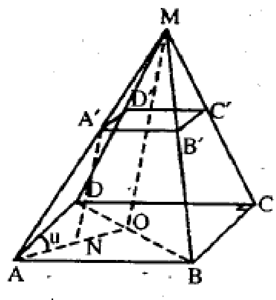


III. 1. $\frac{(x^2+x+1)^2-1}{x^4+2x^3+5x^2+4x+4} = \frac{(x^2+x)(x^2+x+2)}{(x^2+x+2)^2} = \frac{x^2+x}{x^2+x+2}$.

2. a) $i = 40$, $a_p = 41$; $m = \sqrt{1762}$.

b) $A_{lat} = 1804\ cm^2$; $A_{tot} = 2208\ cm^2$.

c) $V = 5914{,}08\ cm^3$, $A_{lat} = 1822{,}2\ cm^2$. d) $\sin u = 0{,}95$.

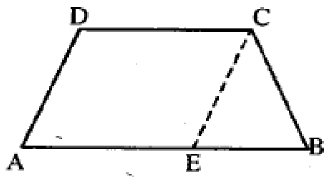


Test no. 68

I. 1.a) 30; b) 20; c) 75. 2. 276.

II. 1. a) 1; b) 0.

2. Build $CE \parallel AD$ $(E \in AB)$ and use inequalities between the sides of triangle CEB.



III. 1. 5. 2. Build $PD \perp BC \Rightarrow MD \perp BC$.

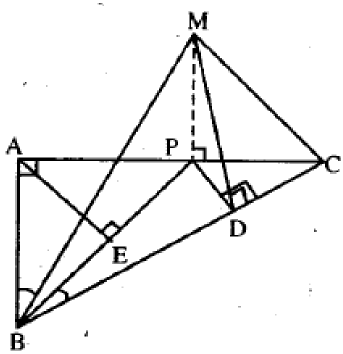
$\sphericalangle[(MBC),(ABC)] = \sphericalangle MDP$. $tg \sphericalangle MDP = \frac{20}{9}$.

b) $(MPB) \perp (ABC)$, $d[A, (MPB)] = AE = \frac{9\sqrt{5}}{5}$ $(E \in BP)$.

c) $MP \perp (ABC)$; $MP \subset (MPD) \Rightarrow (MPD) \perp (ABC)$
$\sphericalangle[MB,(MPD)] = \sphericalangle(MB, MD) = \sphericalangle BMD$.

$\sin \sphericalangle BMD = \frac{18\sqrt{805}}{805}$.

d) $A = \frac{45}{2}\left(2\sqrt{5}+13\right)cm^2$; $V = \frac{675}{2} cm^3$.

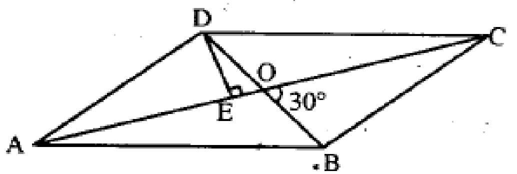


Test no. 69

I. **1.** 67. 2. 4.

II. **1.** $E = \frac{9}{x-2}$; $x \in \{-7, -1, 1, 3, 5, 11\}$.

2. $S_{ABCD} = 2 \cdot S_{ADC} = DE \cdot AC = 40\ cm^2$.



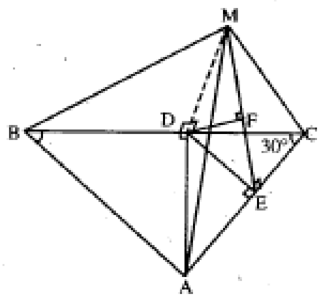
III. **1.** $-\frac{35}{27}$. **2.** a) $d(D, (MAC)) = \frac{4\sqrt{21}}{7}$;

b) Let

$DF \perp (MAC)$, $\sphericalangle[MD, (MAC)] = \sphericalangle DMF$,

$\sin \sphericalangle DMF = \frac{\sqrt{21}}{7}$;

c) $A_{tot} = 16\left(2\sqrt{3} + \sqrt{7}\right) cm^2$; $V = \frac{64\sqrt{3}}{3}\ cm^3$.



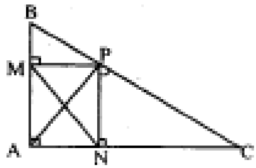
Test no. 70

I. 1. a) 24; b) 1285.

2. 18 workers.

II. 1. $a = 5, b = 1.$

2. Observe that the triangle ABC is rectangular, and ANPM is a rectangle, so $MN = AP = \sqrt{6}$ cm.



III. 1. $\frac{5}{2}x^2 - \frac{19}{2}x + 8.$

2. a) Let

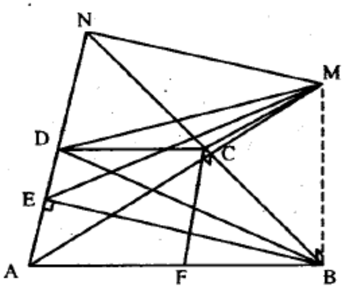
$AD \cap BC = \{N\}$, $\sphericalangle BNA = 90°$,

$d(M, AD) = MN = 26$

b) $(MBD) \perp (ABD)$.

$AA' \perp BD$, $AA' = \frac{48\sqrt{17}}{17}$;

$\sin \sphericalangle ADA' = \frac{AA'}{AD} = \frac{4\sqrt{17}}{17}$.



c) $\text{tg} \sphericalangle [(MAB), (MDC)] = \frac{24}{25}$.

d) $A = 647 \text{ cm}^2$; $V = 640 \text{ cm}^3$.

Test no. 71

I. 1. -1.

2. 120.000 lei and 180.000 lei.

3. $x \in (-\infty, -1] \cup (1, \infty)$.

II. 1. If $m = -5$, the equation admits as solution any real number. If $m = 3$, the equation does not have real solutions. If $m \in \mathbb{R} \setminus \{-5, 3\}$, the equation has as unique solution $x = \frac{1}{m-3}$.

2. No.

III. 1. $\Delta ADE \equiv \Delta CEB$,

because:

$$\begin{cases} AD \equiv CE \\ \widehat{DAC} \equiv \widehat{BCE} \\ AC \equiv BC \end{cases}. \text{ Deci } \sphericalangle EBC \equiv \sphericalangle ACD.$$

$\sphericalangle ACD + \sphericalangle MCB = 60° \Rightarrow \sphericalangle MBC + \sphericalangle MCB = 60° \Rightarrow \sphericalangle BMC = 120°$.

2. $\sqrt{2\left(4+\sqrt{202}\right)}$.

Test no. 72

I. **1.** $\sqrt{96-\sqrt{6+\sqrt{9}}}=\sqrt{93}$. **2.** 200 000 lei. **3.** $\left|\left(2-\sqrt{3}\right)(x-1)\right|=\begin{cases}\left(2-\sqrt{3}\right)(x-1)\\ \left(\sqrt{3}-2\right)(x-1)\end{cases}$

$$|-x|=|x|=\begin{cases}x, x\geq 0\\ -x, x<0\end{cases}.$$

II **1.** A(0, –3) şi B(3 / 2, 0). **2.** $x\in\left[-\frac{\sqrt{2}}{2},2\right]$.

III. **1.** $\Delta ABC\sim\Delta TAC\Rightarrow\frac{TC}{AC}=\frac{AC}{BC}\Rightarrow AC^2=TC\cdot BC.$

2. Let $VABCD$ be the pyramid having its vertex in V and the base $AB=a$. We draw $AF\perp VB\Rightarrow CF\perp VB.$ The triangle AFC is isosceles; also:

$$\sphericalangle AFC=120°.\ OA=\frac{a\sqrt{2}}{2}.$$

$$\frac{OA}{AF}=\sin 60^\circ\Rightarrow AF=\frac{a\sqrt{6}}{3}.$$

Let G the middle of AB. $\Delta VGB\sim\Delta AFB$

$$\Rightarrow\frac{VG}{AF}=\frac{GB}{FB}\Rightarrow VG=AF\cdot\frac{GB}{FB}.$$

$$FB=\sqrt{AB^2-AF^2}=\frac{a\sqrt{3}}{3};\quad VO=\sqrt{VG^2-GO^2}=\frac{a}{2}.\quad V_{VABCD}=\frac{1}{3}VO\cdot S_{ABCD}=\frac{a}{6}.$$

Test no. 73

I. **1.** a) $\frac{25\sqrt{7}}{84}$;

b) Direct calculation.

3. $\sqrt{\frac{x+3}{x-3}}$.

II. **1.** $x \in (0, \infty)$. **2.** $a = 2; b = -4; c = 6$.

III. 1. Prove using $\sphericalangle APC \equiv \sphericalangle CAP$.

2. $\frac{45}{2}$ cm.

Test no. 74

I. **1.** $2\sqrt{11} - 6 > 0; 3\sqrt{2} - 2\sqrt{3} > 0$.

2. $x^2 + y^2 = s^2 - 2p;\ x^3 + y^3 = s^3 - 3sp$.

3. $A = \frac{300}{11}$ cm^2.

II. 1. We take $\frac{x}{a+b}$ and $\frac{y}{a-b}$, and we get: $x = \frac{a}{a-b}$, $y = \frac{b}{a+b}$.

2. The quotient is $2x^2 - 4x - 1$, and the rest is 0.

III. 1. 140, 105 and 49 cm.

2. $V = \frac{485\sqrt{3}}{3}$ cm^3. $A_l = 110\sqrt{3}$ cm^2.

Test no. 75

I. 1. -1 if p is odd and 1 if p is even.

2. 16.000 lei.

3. Suppose there is $n \in \mathbb{N}, n \neq 1$ such that the fraction to be an integer number.

$$\Rightarrow 2n+3 \mid 3n+2 \Rightarrow 2n+3 \mid 2(3n+2) \Rightarrow 2n+3 \mid 6n+4$$

$$2n+3 \mid 2n+3 \Rightarrow 2n+3 \mid 3(2n+3) \Rightarrow 2n+3 \mid 6n+9.$$

$$2n+3 \mid (6n+9)-(6n+4) \Rightarrow 2n+3 \mid 5 \Rightarrow$$

$$\Rightarrow 2n+3 \in \{1, 5\},\ 2n+3 \neq 1.$$

Therefore, any $n \in \mathbb{N}, n \neq 1$, the fraction

$$\frac{3n+2}{2n+3} \notin \mathbb{Z}.$$

II. 1. $f(x) = x$, for any $x \in \mathbb{R}$ and $g(x) = -\frac{1}{2}x + \frac{1}{2}$, for any $x \in \mathbb{R}$.

2. $P\left(\sqrt{3}-\sqrt{2}\right) = 35 - 14\sqrt{6} + 2\left(\sqrt{3}-\sqrt{2}\right).$

III. 1. Unite M with C and N with B.

$\sphericalangle MAB \equiv \sphericalangle NAC \quad (1)$

$\sphericalangle NAC \equiv \sphericalangle NMC \equiv \sphericalangle NBC \quad (2)$

$\sphericalangle MAB \equiv \sphericalangle MCB \quad (3).$

$(1), (2)$ şi $(3) \Rightarrow \sphericalangle NMC \equiv \sphericalangle MCB \Rightarrow MN \parallel BC.$

2. $A_l = a^2\sqrt{15}; \quad V = \frac{a^3\sqrt{14}}{6}.$

Test no. 76

I. 1. Not always.

2. $72^n + 3^{2n+1} \cdot 2^{3n+1} + 8^{n+1} \cdot 9^n =$

$3^{2n} \cdot 2^{3n} + 3^{2n+1} \cdot 2^{3n+1} + 2^{3n+3} \cdot 3^{2n} = 3^{2n} \cdot 2^{3n} \cdot (1 + 3 \cdot 2 + 2^3) =$

$= 15 \cdot 2^{3n} \cdot 3^{2n} \ \vdots\ 15.$

3. $\dfrac{3^n}{4^n} \leq \dfrac{3^n+1}{4^n+1} \Leftrightarrow 3^n \cdot (4^n + 1) \leq 4^n \cdot (3^n + 1) \Leftrightarrow$

$3^n \cdot 4^n + 3^n \leq 3^n \cdot 4^n + 4^n \Leftrightarrow 3^n \leq 4^n$

- true for any $n \in \mathbb{N}$.

II. 1. $a = b = 0.$

2. $(b^2 + c^2 - a^2)^2 - 4b^2c^2 < 0 \Leftrightarrow (b^2 + c^2 - b^2 - c^2 + 2bc \cdot \cos A)^2 - 4b^2c^2 < 0$

$\Leftrightarrow 4b^2c^2 \cdot \cos^2 A < 4b^2c^2 \mid :4b^2c^2 \Leftrightarrow \cos^2 A < 1$

- true.

III. 1. $\dfrac{3a^2\sqrt{3}}{4}$. **2.** $a, a, a\sqrt{2}, a\sqrt{2}$. $V = \dfrac{a^3\sqrt{3}}{6}$.

Test no. 77

I. 1. Not always. For example, $-3 < -1$, but $(-3)^2 > (-1)^2$.

2. $\frac{a}{c} = \frac{b}{d} = k$. $\frac{a \cdot b}{c \cdot d} = \frac{a}{c} \cdot \frac{b}{d} = k^2$. $\left(\frac{a+b}{c+d}\right)^2 = \left(\frac{kc+kd}{c+d}\right)^2 = \left[\frac{k(c+d)}{c+d}\right]^2 = k^2$.

3. 60.

II. 1. $x(m-2) = 3m+1$.

If $m = 2 \Rightarrow$ the equation has no real solutions. If $m \neq 2 \Rightarrow$ the equation has the solution:

$$x = \frac{3m+1}{m-2},\ x \in \mathbb{Z} \Leftrightarrow m-2 \mid 3m+1 \Leftrightarrow m \in \{-5, 1, 3, 9\}.$$

2. The sum of polynomial coefficients

$$P(X) = a_n \cdot X^n + a_{n-1}X^{n-1} + \ldots + a_1 X + a_0$$

is:

$$P(1) = a_n + a_{n-1} + \ldots + a_1 + a_0.$$

In our case, $P(1) = 3$.

III. 1. Use the fact that in a parallelogram the diagonals are halved.

2. $V = a^3\sqrt{2}$. The angle created by an edge with the base plane is 45^0. The facets angels have the value of 60^0 and 120^0.

Test no. 78

I. 1. a) Not necessarily. E.g., $(-5)^2 = 5^2$, but $-5 \neq 5$.

b) $\sqrt{x^2} = |x|$.

2. 15.

3. $m = -1$.

II. 1. $f: \mathbb{R} \to \mathbb{R},\ f(x) = -4x + 10.$

2. Apply the theorem of dividing with rest to polynomials,

$$P(X) = (X-a)(X-b)\cdot C(X) + \alpha X + \beta.$$

$$X - a \mid P(X) \Rightarrow P(a) = 0 \Rightarrow \alpha a + \beta = 0 \qquad (1)$$

$$X - b \mid P(X) \Rightarrow P(b) = 0 \Rightarrow \alpha b + \beta = 0 \qquad (2)$$

From (1) and (2), it follows that

$$\Rightarrow \alpha(a-b) = 0.$$

But

$$a \neq b \Rightarrow \alpha = 0.$$

$$\alpha a + \beta = 0 \Rightarrow \beta = 0.$$

Therefore,

$$(X-a)(X-b) \mid P(X).$$

II. 1. $AE^2 + AF^2 = FE^2 = AD^2$

2. $A_l = \pi 3(3+6) = 27\pi \text{ cm}^2$. $V = \dfrac{1}{3}\cdot\pi\cdot 9\cdot 5 = 15\pi \text{ cm}^3$.

Test no. 79

I. 1. Calculate every member of the equality.

2. $x = 4/9$.

3. Let

$$\overline{a_n a_{n-1} a_{n-2} \ldots a_2 a_1}$$

The number

$$\overline{a_n a_{n-1} a_{n-2} \ldots a_2 a_1} + 8(a_n + a_{n-1} + \ldots + a_2 + a_1) =$$

$$= 10^{n-1} \cdot a_n + 8a_n + 10^{n-2} \cdot a_{n-1} + 8a_{n-1} + \ldots + 10a_2 + 8a_2 + a_1 + 8a_1 = (10^{n-1} + 8) \cdot a_n +$$
$$+ (10^{n-2} + 8) \cdot a_{n-1} + \ldots + (10 + 8) \cdot a_2 + 9 \cdot a_1 \ \vdots\ 9$$

II. **1.** $P(X) = X^4 - 4X^3 + 6X^2 - 4X + 1 = (X^2 - 2X + 1)^2 = (X - 1)^4$.

2. The system has no real solutions.

III. 1. Let M, N, P, Q be the projections of O on the sides AB, BC, CD, and respectively DA.

$$m(\sphericalangle ONP) = m(\sphericalangle OCP)$$
$$m(\sphericalangle ONM) = m(\sphericalangle OBA)$$
$$m(\sphericalangle OQM) = m(\sphericalangle OAM)$$
$$m(\sphericalangle OQP) = m(\sphericalangle ODP)$$

Taking into account that:

$$m(\sphericalangle OBA) + m(\sphericalangle OAM) = 90^\circ$$
$$m(\sphericalangle OCP) + m(\sphericalangle ODP) = 90^\circ$$

It follows that

$$m(\sphericalangle MNP) + m(\sphericalangle MQP) = 180^\circ \Rightarrow$$

- the quadrilateral is inscriptible.

2. $V = \left(\dfrac{a \cdot h}{a + h} \right)^3$.

Test no. 80

I. 1. 1.

2. The solutions are 9 and -1.

3. A group with two full barrels, three barrels half-full and two empty barrels; another group with three full barrels, one only half full and three empty barrels; and one more group with two full barrels, three half-full barrels and two empty barrels.

II. **1.** $x=(a+b)^2$; $y=(a-b)^2$.

2. $a=0$, and $b\in\mathbb{R}$. Therefore, we obtain the function $f:\mathbb{R}\to\mathbb{R}, f(x)=b$ for any $x\in\mathbb{R}$.

III. 1. Let O_1 and O_2 be the centers of the two circles, r_1 and r_2 be their radii, and A, B the contact points of circles O_1 and O_2 with their common tangent. In the rectangular triangle O_1CO_2, we have:

$$O_2C^2=O_1O_2^2-O_1C^2\Rightarrow O_2C^2=(r_1+r_2)^2-(r_1-r_2)^2\Rightarrow$$

$$\Rightarrow O_2C^2=4r_1r_2\Rightarrow O_2C=\sqrt{2r_1\cdot 2r_2}\ .$$

2. The base length is 6 cm.

$$A_t=9\sqrt{3}\left(\sqrt{3}+1\right)\text{cm}^2;\quad V=\frac{VA\cdot VB\cdot VC}{6}=9\sqrt{2}\ \text{cm}^3.$$

Test no. 81

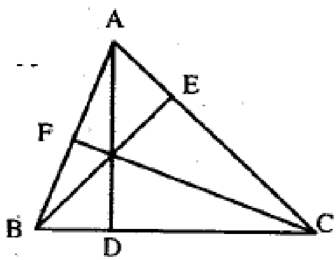
I. 1. 8 elements.

2. a) $-2/3$. b) If $m = 3, x \in Q$; if $m \neq 3, x = 1$.

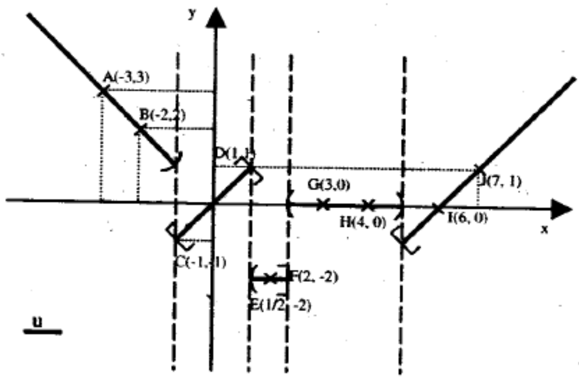
II. 1. $a = 10\sqrt{7}$; $b = 12\sqrt{7}$; $m_a = 11\sqrt{7}$;

$$m_g = 2\sqrt{210};\ m_{arm} = \frac{120\sqrt{7}}{11}.$$

2. Applying the Pythagorean theorem in triangles ADB and ADC, we get $BD = 5$ cm. In this way, we can calculate $AD = 12$ cm, $BE = \frac{63}{5}$ cm, $CF = \frac{252}{13}$ cm.



III. 1.



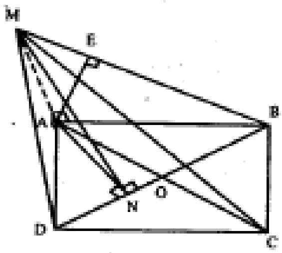
2. a) $\text{tg} \sphericalangle MNA = \frac{2\sqrt{3}}{3}$,

where

$AN \perp BD \ (N \in BD)$.

b) $\sphericalangle[AM, (MBD)] = \sphericalangle AMN, (AMN) \perp (MBD)$

$\text{tg} \sphericalangle AMN = \frac{\sqrt{3}}{2}$.



c) Fie $AE \perp MB$; $d(A, (MBC)) = \frac{5\sqrt{3}}{2}$.

d) $\sphericalangle[(MAB), (MDC)] = \sphericalangle AMD = 45°$.

e) $A_{tot} = \frac{25}{2} \cdot \left(3 + 3\sqrt{3} + \sqrt{6}\right) cm^2$. $V = \frac{125\sqrt{3}}{3} cm^3$.

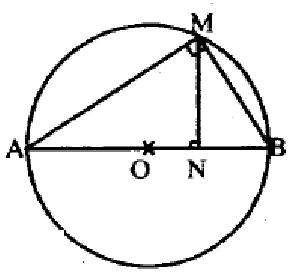
Test no. 82

I. **1.** $\{3\}$, $\{3, 4\}$, $\{3, 4\}$, $\{6\}$. **2.** 42, 28, 62.

II. **1.** a) 7; b) 8.

2. In the rectangular triangle AMN, one applies the height theorem and the cathetus theorem, and finds out:

$$MN = 6 \text{ cm}, AM = 6\sqrt{5} \text{ cm și } BM = 3\sqrt{5} \text{ cm.}$$



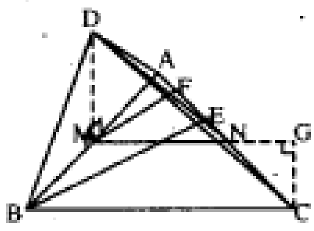
III. **1.** $\{1, 2, 3\}$; $(-2{,}3; 0)\cup(0; 5]$; $\{0, 1, 3\}$; $[-3; -2]$; $[-3; 9]$.

2. a) $V_{DABC} = 32 \cdot \frac{\sqrt{3}}{3} \text{ cm}^3$; $A_{DAC} = 16 \text{ cm}^2$; $d(B,(DAC)) = 2\sqrt{3}$ cm.

b) $\sphericalangle DFM = 30°$

c) $(DAB) \perp (ABC)$; $\sphericalangle[AC, (DAB)] = \sphericalangle CAB = 60°$.

d) $A = 4(3 + 2\sqrt{3})\text{cm}^2$; $V = \frac{8\sqrt{3}}{3} \text{cm}^3$.



Test no. 83

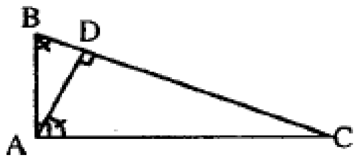
I. 1. a) $\frac{5}{2}$; b) $\frac{3}{8}$.

2. 27; 81; 243.

3. 60 pupils.

II. 1. S(-1, 1).

2. Observe that the triangles DBA and DAC are similar and apply the fundamental theorem of similarity.

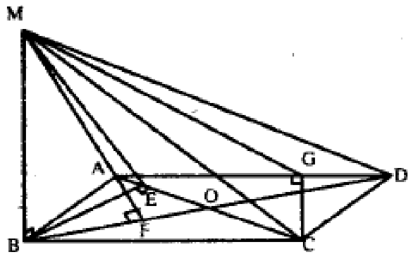


III. 1. $f(x) = x - \frac{5}{2}$; $A(0, 2) \notin G_f$.

2. a) $tg \sphericalangle [(MAC), (ABC)] = \frac{\sqrt{7}}{2}$.

b) $d[A, (MBD)] = 5\sqrt{3}$.

c) Express the volume of pyramid MADC in two ways. Let $CC' \perp (MAD); sin\sphericalangle CMC' = 0{,}339$.



d) $A_{tot} = 10(12 + \sqrt{97} + \sqrt{73} + 8\sqrt{3})cm^2$. $V = \frac{800\sqrt{3}}{3} cm^3$.

Test no. 84

1B, 2C, 3B, 4C, 5B, 6B, 7A, 8B, 9A, 10A.

Test no. 85

1C, 2C, 3C, 4C, 5A, 6C, 7C, 8B, 9A.

10: $A_{lt} = 64\sqrt{2}$, $V_t = \frac{392}{3}$, $A_{lp} = 100\sqrt{2}$, $V_p = \frac{500}{3}$.

Test no. 86

1BC, 2C, 3C, 4C, 5C, 6C, 7C, 8C, 9B.

10: $h = 5\sqrt{3}$, $V = \frac{125\pi\sqrt{3}}{3}$, $A_l = 75\pi$, $\sphericalangle = 180^\circ$.

Test no. 87

1C, 2C, 3C, 4B, 5C, 6B, 7C, 8B, 9C.

Test no. 88

1C, 2C, 3B, 4D, 5D, 6B, 7A, 8D, 9C, 10:a) $16\sqrt{7}$ cm^2; b) $\frac{3\sqrt{21}}{7}$; c) $\sqrt{6}$.

Test no. 89

1C, 2D, 3C, 4D, 5D, 6D, 7C, 8D, 9D, 10: 1) Da; 2) $A_l = 100(\sqrt{3}+1)$ $V = \frac{500\sqrt{2}}{3}$.

Test no. 90

I. 1. a) 1; b) 3/8; c) 3; d) 1.   2. Yes.   3. Isosceles.

II. 1 Yes. 2. Rhombus, square.

3.    

III. 1. $8 \cdot 9 \sin 30° + 20 \cdot 9 \cdot \frac{1}{2} \sin 30° = 81$.

2. $A_\Delta = A_{pr} \cos\theta \Rightarrow l = 4$.

3. $4 \cdot (4+6+5) = 60$.

Test no. 91

1B, 2B, 3A, 4A, 5B, 6A, 7B, 8C, 9B.

Test no. 92

1B, 2C, 3A, 4C, 5A, 6B, 7A, 8B, 9A, 10A.

Test no. 93

1A, 2A, 3C, 4C, 5B, 6B, 7C, 8A, 9A.

10 a) $h = 3$, $a_l = 2\sqrt{3}$, b) $A_l = 54\sqrt{3}$; c) $V = 72\sqrt{3}$.

Test no. 94

1D, 2B, 3C, 4B, 5C, 6A, 7B, 8B 9C, 10) $\frac{30\sqrt{70}}{7}$.

Test no. 95

I. **1.** 6. **2.** 3, 60. **3.** $\frac{8}{21}, \frac{1}{21}, \frac{4}{7}$. **II.** **1.** ϕ. **2.** $\frac{7}{3}x - \frac{29}{3}$. 3. $\frac{1}{x-2}$.

III. **1.** $5\sqrt{3}$. **2.** 20 şi 15. **3.** 48π cm^2; $\frac{69\pi\sqrt{3}}{3}$ cm^3.

Test no. 96

I. **1.** 4. **2.** $4\frac{9}{16}$. **3.** $\frac{a}{b} = \frac{1}{3}$.

II. **1.** a) $x = 1$; b) $x \in (-\infty, 1\,13/2)$. **2.** $a = 3, b = 6$. **3.** a) $x \in \mathbf{R} - \{0, \pm 1\}$; b) $\frac{x+1}{x}$.

III. **1.** 24 cm^2. **2.** 6 cm. **3.** $2\sqrt{34}$, 240 cm^2; 384 cm^2, 384 cm^3, $tg\ \alpha = \frac{4}{3}$.

Test no. 97

I. **1.** 8, $2\sqrt{7}$, $\frac{5}{24}$.

2. Formulas.

3. $\frac{23}{15}$. **II.** **1.** $x \in \left\{\frac{9}{2}\right\}$, $x = 20, y = 30$.

2. 14 days. 3. 30 cm^2.

Test no. 98

I. **1.** 3. **2.** 5 şi 6. **3.** $\{-3, -2, 0, 2, 3\}$, $\{0\}$, $\{-2, 2\}$, $\{-3\}$.
II. **1.** $(4, -1)$. **2.** $C(x) = x^3 + 4 - 3x^2$, $R = 0$. 3. 1/3.
III. **1.** 36 cm, $36\sqrt{3}$ cm^2. **2.** $10\sqrt{3}$, 600, 1000. **3.** 800π, 2560π, 2240π.

Test no. 99

I. **1.** $6\sqrt{2}+16$.
II. **1.** $[-1, 1]$. **2.** $2\sqrt{7}$, $2\sqrt{5}$, 30°. **3.** ± 3.
III. **1.** $5\sqrt{3}$. **2.** 54°, 36°. **3.** $\frac{1}{5}$.

Test no. 100

I. 1, 2, 100 and 1256.

III. **1.** $m_a = 7$, $m_g = 1$. **2** $\frac{1}{2}$. 3. 72.

Test no. 101

1B, 2C, 3C, 4A, 5A, 6C, 7C, 8B, 9A, 10C.

Test no. 102

1A, 2B, 3D, 4B, 5B, 6A, 7B, 8B, 9C, 10B.

Test no. 103

1B, 2A, 3A, 4B, 5B, 6C, 7A, 8B, 9A.

10. a) 600; 1000. b) $25\left(\sqrt{3}+4\right);\frac{500}{3}$. c) $\operatorname{tg} \alpha = \frac{\sqrt{2}}{2}$.

Test no. 104

1C, 2A, 3C, 4A, 5A, 6B, 7B, 8C, 9A.

10. a) $9\left(\sqrt{3}+4\right);3\sqrt{39}$; b) $\frac{3\sqrt{39}}{2}$; c) $\cos u = \frac{2\sqrt{3}}{5}$.

Test no. 105

I. 1. -2; 2. We denote the parts with x, y, z.

$$2y + y + \frac{3y}{2} = 180 \Rightarrow y = 40;\ x = 80;\ z = 60.$$

3. $-\sqrt{2}+\sqrt{3};\ 4\sqrt{5}-2\sqrt{19}$.

II. 1. a) $F(x) = \frac{4(x-2)(x^2+2x+4)}{2(x+1)(x^2+2x+4)} = \frac{2(x-2)}{x+1} = 2 - \frac{6}{x+1}$;

b) $f : \mathbb{R} - \{-1\} \to \mathbb{R}$; c) $x \in \{-7, -4, -3, -2, 0, 1, 2, 5\}$.

2. $a = -1;\ b = 1 \Rightarrow f(x) = -x + 1$.

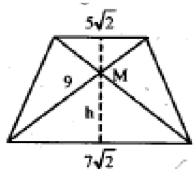
III. 1. $h = \frac{6 \cdot 8}{10} = 4{,}8\ \text{cm};\ S_{\Delta} = \frac{6 \cdot 8}{2} = 24\ \text{cm}^2;\ R = \frac{i_p}{2} = \frac{10}{2} = 5\ \text{cm}$.

2. The section formed by diagonals is an isosceles trapezoid with the bases congruent with the diagonals.

$h = 3$,

$V = 109$

$A_s = 24\sqrt{10}$

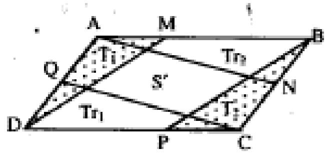


Test no. 106

I. 1. a) 0; b) −1440. 2. $\frac{x-y+y+z+z}{1+4+3}=\frac{24}{8}=3 \Rightarrow \frac{z}{3}=3 \Rightarrow z=9.$

3. a) $m_a=\sqrt{3}$; $m_g=1$; $m_{arm}=\frac{\sqrt{3}}{3}$.

II. 1. a) $a^2=\frac{5}{2}\notin Q$; b) $m=-120$.



$$S_{\Delta AMD}=S_{\Delta CPB}=\frac{S}{4}$$

$$S_{Tr_1}=S_{Tr_2}=\frac{3S'}{4}.$$

$$S=2\cdot\frac{S}{4}+2\cdot\frac{3S'}{4}+S' \Rightarrow \frac{S'}{S}=\frac{1}{5}.$$

Test no. 107

1B, 2A, 3C, 4C, 5B, 6A, 7B, 8B, 9C, 10A.

Test no. 108

1B, 2A, 3C, 4B, 5A, 6B, 7C, 8B, 9C, 10A.

Test no. 109

1C, 2B, 3B, 4A, 5C, 6B, 7A, 8A, 9B, 10C.

Test no. 110

1B, 2B, 3C, 4B, 5A, 6C, 7B, 8A, 9B.

10. a) $A_t = 810\pi$ cm^2; b) $d = 12$ cm; c) $k = \frac{1}{27}$.

Test no. 111

I. **1.** 0. **2.** $8\left(5\sqrt{3} - 9\sqrt{2}\right)$ 3. -200. **II.** **1.** $x \in \{3, 1, 4, 0, 5, -1, 8, -4\}$. **2.** $A = (-3, 3)$. **3.** $m = 1$.
III. **1.** 96 cm^2. **2.** 360. **3.** $V = 960$ cm^3, $A_t = 592$ cm^2, $d = 2\sqrt{7}$ cm.

Test no. 112

I. **1.** 1/6. **2.** 1. 3. -6. **II.** **1.** 2. **2.** 2. **3.** $(x-2)^2 + 1 > 0$. **III.** **1.** $12\sqrt{5}$;24;24. **2.** 20, 25, 30.
3. $A_t = 324\left(\sqrt{3} + \sqrt{7}\right)$; $V = 1296\sqrt{3}$; $\sin \sphericalangle x = \frac{2\sqrt{7}}{7}$; $d = \frac{36\sqrt{21}}{7}$.

Test no. 113

I. **1.** 21. **2.** 2. **3.** $\frac{38}{25}$.

II. **1.** $\frac{5}{3}$. **2.** $\frac{2}{3}$. **3.** $x \in \left(-\frac{7}{2}, +\infty\right)$ **III.** **1.** 4. **2.** $\frac{24}{5}$, 24. **3.** $40\sqrt{3}$, $8\left(\sqrt{3} + 15\right)$

Test no. 114

I. 1. 4,79. 2. $A = \{0, 1, 2, 3, 4\}$. 3. $\frac{13}{9}$.

II. 1. 3; 5. 2. $m = 18$. 3. $a = 2$.

III. 1. a) rectangle

; b) 2. 2. $\bar{2}16{,}216$. 3. 15π, 12π.

Test no. 115

1C, 2B, 3A, 4A, 5B, 6B, 7B, 8B, 9B, 10A.

Test no. 116

1C, 2A, 3A, 4B, 5C, 6A, 7B, 8B, 9A, 10C.

Test no. 117

I. 1. 2. 2. $\frac{a}{4} = \frac{b}{6} = \frac{c}{12}$ $abc = 36\,000 \Rightarrow a = 20;\ b = 30;\ c = 60$. 3. $\frac{5\sqrt{2}}{2}$; $\frac{\sqrt{10}}{5}$; $\frac{3(\sqrt{5}+\sqrt{7})}{-2}$.

II. 1. $x + y \ge 2\sqrt{xy}\ (x, y \ge 0)$.

$$x^2 + y^2 \ge 2\sqrt{x^2y^2} \ge 2xy; \qquad y^2 + z^2 \ge 2\sqrt{y^2z^2} \ge 2yz; \quad x^2 + z^2 \ge 2\sqrt{x^2z^2} \ge 2xz$$

$$2x^2 + 2y^2 + 2z^2 \ge 2xy + 2yz + 2xz \mid :2 \Rightarrow x^2 + y^2 + z^2 \ge xy + yz + xz.$$

2. $P(-2) = 0;\ P(4) = 0$. 3. $F(x) = \frac{x^3 + 8}{x^2 - 2x - 8} = \frac{(x+2)(x^2 - 2x + 4)}{(x+2)(x-4)} = \frac{x^2 - 2x + 4}{x - 4}$

III. 1. $MD = 24;\ MA = 32;\ MC = 18;\ MB = 24$.

2. a) $A_p = 6.5;\ A_l = \frac{195\sqrt{3}}{4};\ A_t = \frac{135\sqrt{3}}{4};\ V = \frac{75\sqrt{3}}{2};\ m = \sqrt{61}$.

Test no. 118

I. **1.** 1. **2.** $\frac{x}{5}=\frac{y}{7}=\frac{z}{11} \Rightarrow z-x=42 \Rightarrow x=35; y=49; z=77$. **3.** 5; -1.

II. **1.** $\frac{a^4+b^4}{a^2b^2}=\frac{a^4}{a^2b^2}+\frac{b^4}{a^2b^2}=\frac{a^2}{b^2}+\frac{b^2}{a^2}\geq 2\sqrt{\frac{a^2}{b^2}\cdot\frac{b^2}{a^2}}\geq 2$.

2. $a=b=c=d=1 \Rightarrow x^4+4x^3+6x^2+4x+1=(x^2+2x+1)^2$; $x^3+3x^2+3x+1=(x+1)^3$

3. $m=1; n=1 \Rightarrow f(x)=x+1$.

III. **1.** $\Delta ANB \sim \Delta CND \Rightarrow NA=14; NC=7; ND=4; NB=8$.

2. $A_p=4,1; h=4; V=4,32\ cm^3$. **3.** $A_{sf}=36\pi\ cm^2; V=36\pi\ cm^3$.

Test no. 119

I. **1.** $\frac{1}{2}$. **2.** $\frac{x+y+z}{3}=20; y=3x; \frac{y+z}{2}=25 \Rightarrow x=10; y=30; z=20$.

II. **1.** $\left(x^2+\frac{y^2}{x^2}\right)\left(y^2+\frac{x^2}{y^2}\right)\geq 2\sqrt{x^2\cdot\frac{y^2}{x^2}}\cdot 2\sqrt{y^2\cdot\frac{x^2}{y^2}}\geq 4xy\geq 4$.

2. a) $P(2)=0; P(-2)=0$. b) $F(x)=\frac{(x-2)^2(x+2)(x-1)}{(x-2)(x+2)}=(x-2)(x-1)$

III. **1.** $h=5\sqrt{3}$.

Test no. 120

I. **1.** 50. **2.** $\frac{x}{4}=\frac{y}{7}=\frac{z}{12}$ şi $y+z=380 \Rightarrow x=80; y=140; z=240$.

II. **1.** $n\in[3/2,\infty)$.

2. $m=-3; n=27$ şi $p=-18$. $P(X)=0$

III. **1.** $d=l\sqrt{3}\Rightarrow \ell=10; A_t=600\ cm^2; V=1000\ cm^3$.

2. a) $S_{\Delta VAB}=a^2, S_{\Delta VAD}=\frac{a^2\sqrt{2}}{2}; S_{\Delta VBC}=a^2\sqrt{2}$. b) a^3.

Test no. 121

I. 1. 14, 14. 2. We denote by x the cost of the jacket after the first sale. The jacket costs 400.000 after the new sale:

$\frac{576000-x}{x}=\frac{x-400000}{400000} \Rightarrow \frac{576000}{x}=\frac{x}{400000} \Rightarrow x=480000$ lei. **3.** $-\frac{47}{3}$.

II. 1. $x \in (-\infty, -281/13)$. **2.** $a \in \mathbf{R}-\{-6, -4, 4\}$. 3. 4, 9.

III. 1. The three distances are equal: 18 cm.

2. a) $R=\frac{2\sqrt{39}}{3}$ cm; b) $V=\frac{14\sqrt{13}}{3}$ cm^3.

Test no. 122

I. 3. a) 2; b) $\sqrt{18+2\sqrt{17}}=\sqrt{(\sqrt{17}+1)^2}=\sqrt{17}+1$. $\sqrt{18-2\sqrt{17}}=\sqrt{(\sqrt{17}-1)^2}=\sqrt{17}-1$

II. 2. a) 0; b) $\frac{2x-3}{(x+1)(2x-3)}=\frac{1}{x+1}$.

III. 1. a) T3⊥ $\Rightarrow d(M, AB)=MA=12\sqrt{2}$; $d(M, BC)=MC=20$.

b) $d(D, MBC)=DP=\frac{48}{5}$ $(DP \perp MC)$.

2. a) $R=4$; $G=5$; $h=3$. b) $\frac{100\pi}{9}$ cm^2; $\frac{304\pi}{27}$ cm^3.

Test no. 123

I. 1. 6,12; 2. $-18/17$. 3. $\frac{\sqrt{2}(\sqrt{2}+1)}{\sqrt{2}+1}+\frac{3(\sqrt{3}+\sqrt{5})}{\sqrt{3}+\sqrt{5}}+\frac{\sqrt{3}(5-3\sqrt{2})}{5-3\sqrt{2}}=\sqrt{2}+3+\sqrt{3}$.

II. 3. $x=\sqrt{2}$; $y=\sqrt{3}$.

III. 1. $d=13$; $A_l=192$ cm^2; $V=144$ cm^3.

2. a) $r=5$; $A_t=520\pi$ cm^2; $V=2600\pi$ cm^3. b) $h_{con}=36$; $V_{con}=2700\pi$ cm^3.

Test no. 124

1A, 2D, 3D, 4C, 5D, 6B, 7C, 8A, 9B, 10A.

Test no. 125

1B, 2A, 3D, 4D, 5C, 6D, 7A, 8D, 9C, 10B.

Test no. 126

I. **1.** a) $\{0, 4, 6, 10\}$, b) $\{-5, -4, -2, -1, 0, 4, 6, 10\}$; c) $\{\sqrt{3}, -\sqrt{2}\}$
2. a) $\{7/2\}$; b) $\{-1\}$; c) $\{(8, -3)\}$; d) $[-1, 3]$. **3.** a) $\{1, 7\}$; b) $\{0, 2, 4, 6\}$; c) $\{-5, 1, 2, 9\}$.

II. **1.** 15 km/h, 45 km/h. **2.** $3\frac{3}{7}$ cm. **3.** $M_3, M_7 \in B$.

III. **1.** $\sin \sphericalangle A = \sin 75^\circ = \frac{CE}{AC} = \frac{\sqrt{6}+\sqrt{2}}{4}$, $\cos \sphericalangle A = \cos 75^\circ = \frac{AE}{AC} = \frac{\sqrt{6}-\sqrt{2}}{4}$.

2. $BB' = x = \frac{41}{4}$ cm. **3.** a) T3⊥; b) $\sin \sphericalangle AOE = \frac{\sqrt{6}}{3}$; $\cos \sphericalangle EBO = \cos 60^\circ = \frac{1}{2}$;
c) $AE = 5\sqrt{6}$; d) $S = 112$ cm; $A_l = 160\ cm^2$, $V = 128\ cm^3$.

The present book tries to offer students and teachers knowledge evaluation tools for all the chapters from the current Romanian mathematics syllabus.

In the evolution of teenagers, the phase of admission in high schools mobilizes particular efforts and emotions. The present workbook aims to be a permanent advisor in the agitated period starting with the capacity examination and leading to the admittance to high school.

The tests included in this workbook have a complementary character as opposed to the many materials written with the purpose to support all those who prepare for such examinations and they refer to the entire subject matter included in the analytical mathematics syllabus of arithmetic in Romania, algebra and geometry from the lower secondary grades.

These tests have been elaborated with the intention to offer proper support to those who use the workbook, assuring them the success and extra preparation for future exams.

ISBN 978-1-59973-385-2

9 781599 733852 >