Support-Neutrosophic Set: A New Concept in Soft Computing

Nguyen Xuan Thao
Florentin Smarandache
Nguyen Van Dinh

Follow this and additional works at: https://digitalrepository.unm.edu/nss_journal

Recommended Citation
https://digitalrepository.unm.edu/nss_journal/vol16/iss1/16

This Article is brought to you for free and open access by UNM Digital Repository. It has been accepted for inclusion in Neutrosophic Sets and Systems by an authorized editor of UNM Digital Repository. For more information, please contact amywinter@unm.edu.
Support-Neutrosophic Set: A New Concept in Soft Computing

Nguyen Xuan Thao,1 Florentin Smarandache,2 Nguyen Van Dinh1

1Faculty of Information Technology, Vietnam National University of Agriculture (VNUA)
   Email: nxthao2000@gmail.com, nvdinh2000@gmail.com
2Department of Mathematics University of New Mexico Gallup, NM, USA.
   E-mail: smarand@unm.edu

Abstract. Today, soft computing is a field that is used a lot in solving real-world problems, such as problems in economics, finance, banking... With the aim to serve for solving the real problem, many new theories and/or tools which were proposed, improved to help soft computing used more efficiently. We can mention some theories as fuzzy sets theory (L. Zadeh, 1965), intuitionistic fuzzy set (K Atanasov, 1986), neutrosophic set (F. Smarandache 1999). In this paper, we introduce a new notion of support-neutrosophic set (SNS), which is the combination a neutrosophic set with a fuzzy set. So, SNS set is a direct extension of fuzzy set and neutrosophic sets (F. Smarandache). Then, we define some operators on the support-neutrosophic sets, and investigate some properties of these operators.

Keywords: support-neutrosophic sets, support-neutrosophic fuzzy relations, support-neutrosophic similarity relations

1 Introduction

In 1998, Prof. Smarandache gave the concept of the neutrosophic set (NS) [3] which generalized fuzzy set [10] and intuitionistic fuzzy set [1]. It is characterized by a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F). Over time, the sub-class of the neutrosophic set were proposed to capture more advantageous in practical applications. Wang et al. [5] proposed the interval neutrosophic set and its operators. Wang et al. [6] proposed a single-valued neutrosophic set as an instance of the neutrosophic set accompanied with various set theoretic operators and properties. Ye [8] defined the concept of simplified neutrosophic set whose elements of the universe have a degree of truth, indeterminacy and falsity respectively that lie between [0, 1]. Some operational laws for the simplified neutrosophic set and two aggregation operators, including a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator were presented.


Practically, let’ consider the following case: a customer is interested in two products A and B. The customer has one rating of good (i), indeterminacy (ii) or not good (iii) for each of the products. These ratings (i),(ii) and (iii) (known as neutrosophic ratings) will affect the customer's decision of which product to buy. However, the customer's financial capacity will also affect her decision. This factor is called the support factor, with the value is between 0 and 1. Thus, the decision of which product to buy are determined by truth factors (i), indeterminacy factors (ii), falsity factors (iii) and support factor (iv). If a product is considered good and affordable, it is the best situation for a buying decision. The most unfavorable situation is when a product is considered bad and not affordable (support factor is bad), in this case, it would be easy to refuse to buy the product.

Another example, the business and purchase of cars in the Vietnam market. For customers, they will care about the quality of the car (good, bad and indeterminacy, they are neutrosophic) and prize, which are considered as supporting factors for car buyers. For car dealers, they are also interested in the quality of the car, the price and the government's policy on importing cars such as import duties on cars. Price and government policies can be viewed as supporting components of the car business.

In this paper, we combine a neutrosophic set with a fuzzy set. This raise a new concept called support-neutrosophic set (SNS). In which, there are four...
membership functions of an element in a given set. The
remaining of this paper was structured as follows: In
section 2, we introduce the concept of support-
neutrosophic set and study some properties of SNS. In
section 3, we give some distances between two SNS sets.
Finally, we construct the distance of two support-
neutrosophic sets.

2 Support-Neutrosophic set

Throughout this paper, U will be a nonempty set called
the universe of discourse. First, we recall some the concept
about fuzzy set and neutrosophic set. Here, we use
mathematical operations on real numbers. Let \( S_1 \) and \( S_2 \) be
two real standard or non-standard subsets, then

\[
S_1 + S_2 = \{ x | x = s_1 + s_2, s_1 \leq S_2, s_2 \in S_2 \}
\]

\[
S_1 - S_2 = \{ x | x = s_2 - s_1, s_1 \leq S_2, s_2 \in S_2 \}
\]

\[
S_2 = \{ 1^+ - S_2 = \{ x | x = 1^+ - s_2, s_2 \in S_2 \}
\]

\[
S_1 \times S_2 = \{ x | x = s_1 \times s_2, s_1 \in S_1, s_2 \in S_2 \}
\]

\[
S_1 \lor S_2 = [\max(\inf S_1, \inf S_2), \max(\sup S_1, \sup S_2)]
\]

\[
S_1 \land S_2 = \{ \min(\inf S_1, \inf S_2), \min(\sup S_1, \sup S_2) \}
\]

\[
d(S_1, S_2) = \inf_{s_1 \in S_1, s_2 \in S_2} d(s_1, s_2)
\]

Remark: \( S_1 \land S_2 = S_1 \lor S_2 \) and \( S_1 \lor S_2 = S_1 \land S_2 \). Indeed, we
consider two cases:

+ if \( \inf S_1 \leq \inf S_2 \) and \( \sup S_1 \leq \sup S_2 \), then \( 1 - \inf S_2 \leq 1 - \inf S_1 \) and \( \sup S_1 \leq \sup S_2 \) and \( S_1 \lor S_2 = S_1 \land S_2 \).

+ if \( \inf S_1 < \inf S_2 \) \leq \( \sup S_2 \) \leq \( \sup S_1 \). Then \( S_1 \land S_2 = [\inf S_1, \sup S_2] \) and \( S_1 \lor S_2 = [1 - \sup S_2, 1 - \inf S_1] \). Hence \( S_1 \land S_2 = S_1 \lor S_2 \). Similarly, we have \( S_1 \lor S_2 = S_1 \land S_2 \).

Definition 1. A fuzzy set \( A \) on the universe \( U \) is an object of the form

\[
A = \{(x, \mu_A(x)) | x \in U \}
\]

where \( \mu_A(x) \in [0,1] \) is called the degree of membership of \( x \) in \( A \).

Definition 2. A neutrosophic set \( A \) on the universe \( U \) is an object of the form

\[
A = \{(x, T_A(x), I_A(x), F_A(x)) | x \in U \}
\]

where \( T_A \) is a truth – membership function, \( I_A \) is an indeterminacy–membership function, and \( F_A \) is falsity – membership function of \( A \). \( T_A(x), I_A(x) \) and \( F_A(x) \) are real standard or non-standard subsets of \( [0^-, 1^+], \) that is

\[
T_A: U \rightarrow [0^-, 1^+], \quad I_A: U \rightarrow [0^-, 1^+], \quad F_A: U \rightarrow [0^-, 1^+]
\]

In real applications, we usually use

\[
T_A: U \rightarrow [0,1], \quad I_A: U \rightarrow [0,1], \quad F_A: U \rightarrow [0,1]
\]

Now, we combine a neutrosophic set with a fuzzy set. That leads to a new concept called support-
neutrosophic set (SNS). In which, there are four membership functions of each element in a given set. This
new concept is stated as follows:

Definition 3. A support – neutrosophic set (SNS) \( A \) on the
universe \( U \) is characterized by a truth –membership function \( T_A \), an indeterminacy–membership function \( I_A \), a falsity – membership function \( F_A \) and support–membership function \( s_A \). For each \( x \in U \) we have \( T_A(x), I_A(x), F_A(x) \) and \( s_A(x) \) are real standard or non-standard subsets of \( [0^-, 1^+], \) that is

\[
T_A: U \rightarrow [0^-, 1^+], \quad I_A: U \rightarrow [0^-, 1^+], \quad F_A: U \rightarrow [0^-, 1^+], \quad s_A: U \rightarrow [0^-, 1^+]
\]

We denote support – neutrosophic set (SNS)

\[
A = \{(x, T_A(x), I_A(x), F_A(x), s_A(x)) | x \in U \}
\]

There is no restriction on the sum of \( T_A(x), I_A(x), F_A(x) \), so \( 0^- \leq \sum T_A(x) + \sum I_A(x) + \sum F_A(x) \leq 3^+ \), and \( 0^- \leq s_A(x) \leq 1^+ \).

When \( U \) is continuous, a SNS can be written as

\[
A = \int_U < T_A(x), I_A(x), F_A(x), s_A(x) > dx
\]

When \( U = \{x_1, x_2, \ldots, x_n\} \) is discrete, a SNS can be written as

\[
A = \sum_{i=1}^n < T_A(x_i), I_A(x_i), F_A(x_i), s_A(x_i) > x_i
\]

We denote SNS(\( U \)) is the family of SNS sets on \( U \).

Remarks:

+ The element \( x_0 \in U \) is called “worst element” in \( A \) if

\[
T_A(x_0) = 0, I_A(x_0) = 0, F_A(x_0) = 1, s_A(x_0) = 0 \quad . \quad . \quad \text{The element } x^+ \in U \text{ is called “best element” in } A \text{ if}
\]

\[
T_A(x^+) = 1, I_A(x^+) = 1, F_A(x^+) = 0, s_A(x^+) = 1
\]
(if there is restriction \( \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 1 \) then the element \( x^* \in U \) is called “best element” in \( A \) if 
\[
T_A(x^*) = 1, I_A(x^*) = 0, F_A(x^*) = 0, s_A(x^*) = 1.
\]
+ the support – neutrosophic set \( A \) reduces an neutrosophic set if \( s_A(x) = c \in [0,1] \), \( \forall x \in U \). 
+ the support – neutrosophic set \( A \) is called a support-standard neutrosophic set if 
\[
T_A(x), I_A(x), F_A(x) \in [0,1] \text{ and } T_A(x) + I_A(x) + F_A(x) \leq 1 
\]
for all \( x \in U \). 
+ the support – neutrosophic set \( A \) is a support-intuitionistic fuzzy set if \( T_A(x), F_A(x) \in [0,1] \), \( I_A(x) = 0 \) and \( T_A(x) + F_A(x) \leq 1 \) for all \( x \in U \). 
+ A constant SNS set 
\[(a, \alpha, \beta, \theta, \gamma) = \{(x, \alpha, \beta, \theta, \gamma) | x \in U \}\]
where \( 0 \leq \alpha, \beta, \theta, \gamma \leq 1 \). 
+ the SNS universe set is 
\[
U = 1_U = (1,1,0,0,1) = \{(x, 1,1,0,1) | x \in U \}
\]
+ the SNS empty set is 
\[
U = 0_U = (0,0,1,0) = \{(x, 0,0,1,0) | x \in U \}
\]

**Definition 4.** The complement of a SNS \( A \) is denoted by \( c(A) \) and is defined by 
\[
\begin{align*}
T_{c(A)}(x) &= F_A(x) \\
I_{c(A)}(x) &= \{1\} - I_A(x) \\
F_{c(A)}(x) &= T_A(x) \\
s_{c(A)}(x) &= \{1\} - s_A(x)
\end{align*}
\]
for all \( x \in U \).

**Definition 5.** A SNS \( A \) is contained in the other SNS \( B \), denote \( A \subseteq B \), if and only if 
\[
\begin{align*}
\inf T_A(x) &\leq \inf T_B(x), & \sup T_A(x) &\leq \sup T_B(x) \\
\inf F_A(x) &\geq \inf F_B(x), & \sup F_A(x) &\geq \sup F_B(x) \\
\inf s_A(x) &\leq \inf s_B(x), & \sup s_A(x) &\leq \sup s_B(x)
\end{align*}
\]
for all \( x \in U \).

**Definition 6.** The union of two SNS \( A \) and \( B \) is a SNS \( C = A \cup B \), that is defined by 
\[
\begin{align*}
T_C &= T_A \lor T_B \\
I_C &= I_A \lor I_B \\
F_C &= F_B \land F_B \\
s_C &= s_A \lor s_B
\end{align*}
\]

**Definition 7.** The intersection of two SNS \( A \) and \( B \) is a SNS \( D = A \cap B \), that is defined by 
\[
\begin{align*}
T_D &= T_A \land T_B \\
I_D &= I_A \land I_B \\
F_D &= F_B \lor F_B \\
s_D &= s_A \land s_B
\end{align*}
\]

**Example 1.** Let \( U = \{x_1, x_2, x_3, x_4\} \) be the universe. Suppose that 
\[
A = \left\{ \left[ 0.5, 0.8 \right], \left[ 0.4, 0.6 \right], \left[ 0.2, 0.7 \right], \left[ 0.7, 0.9 \right] \right\}
\]
\[
x_1
\]
\[
+ \left\{ \left[ 0.4, 0.5 \right], \left[ 0.45, 0.6 \right], \left[ 0.3, 0.6 \right], \left[ 0.5, 0.8 \right] \right\}
\]
\[
x_2
\]
\[
+ \left\{ \left[ 0.5, 0.9 \right], \left[ 0.4, 0.5 \right], \left[ 0.6, 0.7 \right], \left[ 0.2, 0.6 \right] \right\}
\]
\[
x_3
\]
\[
+ \left\{ \left[ 0.5, 0.9 \right], \left[ 0.3, 0.6 \right], \left[ 0.4, 0.8 \right], \left[ 0.1, 0.6 \right] \right\}
\]
\[
x_4
\]
and 
\[
B = \left\{ \left[ 0.2, 0.6 \right], \left[ 0.3, 0.5 \right], \left[ 0.3, 0.6 \right], \left[ 0.6, 0.9 \right] \right\}
\]
\[
x_1
\]
\[
+ \left\{ \left[ 0.45, 0.7 \right], \left[ 0.4, 0.8 \right], \left[ 0.9, 1 \right], \left[ 0.4, 0.9 \right] \right\}
\]
\[
x_2
\]
\[
+ \left\{ \left[ 0.1, 0.7 \right], \left[ 0.4, 0.8 \right], \left[ 0.6, 0.9 \right], \left[ 0.2, 0.7 \right] \right\}
\]
\[
x_3
\]
\[
+ \left\{ \left[ 0.5, 1 \right], \left[ 0.2, 0.9 \right], \left[ 0.3, 0.7 \right], \left[ 0.1, 0.5 \right] \right\}
\]
\[
x_4
\]
are two support – neutrosophic set on \( U \).

We have 
+ complement of \( A \), denote \( c(A) \) or \( \sim A \), defined by
\[ c(A) = \left\langle \begin{array}{c}
0.2,0.7 \\
0.4,0.6 \\
0.5,0.8 \\
0.1,0.3
\end{array} \right\rangle x_1 + \left\langle \begin{array}{c}
0.3,0.6 \\
0.4,0.55 \\
0.2,0.5
\end{array} \right\rangle x_2 + \left\langle \begin{array}{c}
0.6,0.7 \\
0.5,0.6 \\
0.5,0.9
\end{array} \right\rangle x_3 + \left\langle \begin{array}{c}
0.4,0.8 \\
0.4,0.7 \\
0.5,0.9
\end{array} \right\rangle x_4 + \text{Union } C = A \cup B: \]

\[ C = \left\langle \begin{array}{c}
0.5,0.8 \\
0.4,0.6 \\
0.2,0.6 \\
0.7,0.9
\end{array} \right\rangle x_1 + \left\langle \begin{array}{c}
0.45,0.7 \\
0.45,0.8 \\
0.3,0.6 \\
0.4,0.9
\end{array} \right\rangle x_2 + \left\langle \begin{array}{c}
0.5,0.9 \\
0.4,0.8 \\
0.6,0.7 \\
0.2,0.7
\end{array} \right\rangle x_3 + \left\langle \begin{array}{c}
0.5,1 \\
0.3,0.9 \\
0.3,0.7 \\
0.1,0.6
\end{array} \right\rangle x_4 + \text{the intersection } D = A \cap B: \]

\[ D = \left\langle \begin{array}{c}
0.2,0.6 \\
0.3,0.5 \\
0.3,0.7 \\
0.6,0.9
\end{array} \right\rangle x_1 + \left\langle \begin{array}{c}
0.4,0.5 \\
0.4,0.6 \\
0.9,1 \\
0.4,0.8
\end{array} \right\rangle x_2 + \left\langle \begin{array}{c}
0.1,0.7 \\
0.4,0.5 \\
0.6,0.9 \\
0.2,0.6
\end{array} \right\rangle x_3 + \left\langle \begin{array}{c}
0.5,0.9 \\
0.2,0.6 \\
0.4,0.8 \\
0.1,0.5
\end{array} \right\rangle x_4 \]

**Proposition 1.** For all \( A, B, C \in \text{SNS}(U) \), we have

(a) If \( A \subseteq B \) and \( B \subseteq C \), then \( A \subseteq C \),
(b) \( c(c(A)) = A \),
(c) Operators \( \cap \) and \( \cup \) are commutative, associative, and distributive,
(d) Operators \( \cap, \sim \) and \( \cup \) satisfy the law of De Morgan. It means that \( \bar{A \cap B} = \bar{A} \cup \bar{B} \) and \( \bar{A \cup B} = \bar{A} \cap \bar{B} \).

**Proof.**

It is easy to verify that (a), (b), (c) is truth.

We show that (d) is correct. Indeed, for each

\[ T_{-(A \cap B)} = F_{A \cap B} = F_A \lor F_B = T_A \lor T_B \]

\[ I_{-(A \cap B)} = (1) - I(A \cap B) = I(A) \land I(B) \]

\[ F_{-(A \cap B)} = T_{A \cap B} \land T_B = F_A \land F_B \]

\[ s_{-(A \cap B)} = (1) - s(A \cap B) = s(A) \land s(B) \]

So that \( \bar{A \cap B} = \bar{A} \cup \bar{B} \). By same way, we have \( \bar{A \cup B} = \bar{A} \cap \bar{B} \). \( \bigcirc \)

**3 The Cartesian product of two SNS**

Let \( U, V \) be two universe sets.

**Definition 8.** Let \( A, B \) two SNS on \( U, V \), respectively. We define the Cartesian product of these two SNS sets:

(a) \[ A \times B = \left\{ (x,y), T_{A \times B}(x,y), I_{A \times B}(x,y), F_{A \times B}(x,y) \mid x \in U, y \in V \right\} \]

where

\[ T_{A \times B}(x,y) = T_A(x)T_B(y), \]

\[ I_{A \times B}(x,y) = I_A(x)I_B(y), \]

\[ F_{A \times B}(x,y) = F_A(x)F_B(y) \]

and

\[ s_{A \times B}(x,y) = s_A(x)s_B(y), \forall x \in U, y \in V. \]

(b) \[ A \otimes B = \left\{ (x,y), T_{A \otimes B}(x,y), I_{A \otimes B}(x,y), F_{A \otimes B}(x,y) \mid x \in U, y \in V \right\} \]

Where

\[ T_{A \otimes B}(x,y) = T_A(x)T_B(y), \]

\[ I_{A \otimes B}(x,y) = I_A(x)I_B(y), \]

\[ F_{A \otimes B}(x,y) = F_A(x)F_B(y) \]

and

\[ s_{A \otimes B}(x,y) = s_A(x)s_B(y), \forall x \in U, y \in V. \]
Example 2. Let \( U = \{x_1, x_2\} \) be the universe set. Suppose that

\[
A = \begin{bmatrix}
[0.5,0.8],[0.4,0.6],[0.2,0.7],[0.7,0.9] \\
x_1 \\
[0.4,0.5],[0.45,0.6],[0.3,0.6],[0.5,0.8] \\
x_2
\end{bmatrix}
\]

and

\[
B = \begin{bmatrix}
[0.2,0.6],[0.3,0.5],[0.3,0.6],[0.6,0.9] \\
x_1 \\
[0.45,0.7],[0.4,0.8],[0.9,1],[0.4,0.9] \\
x_2
\end{bmatrix}
\]

are two SNS on \( U \). Then we have

\[
A \times B = \begin{bmatrix}
[0.25,0.72],[0.16,0.3],[0.12,.49],[0.14,0.54] \\
(x_1, x_2) \\
[0.225,0.56],[0.16,0.48],[0.18,0.7],[0.28,1] \\
(x_1, x_2) \\
[0.2,0.45],[0.18,0.3],[0.18,0.42],[0.1,0.48] \\
(x_1, x_2) \\
[0.2,0.45],[0.135,0.36],[0.12,0.48],[0.05,0.48] \\
(x_1, x_2) \\
[0.5,0.7],[0.4,0.6],[0.2,0.7],[0.7,0.9] \\
(x_1) \\
[0.4,0.5],[0.45,0.6],[0.3,0.6],[0.5,0.8] \\
(x_2)
\end{bmatrix}
\]

and

\[
A \otimes B = \begin{bmatrix}
[0.5,0.8],[0.4,0.5],[0.6,0.7],[0.2,0.6] \\
(x_1, x_2) \\
[0.5,0.8],[0.3,0.6],[0.4,0.8],[0.1,0.6] \\
(x_1, x_2) \\
[0.4,0.5],[0.4,0.5],[0.6,0.7],[0.2,0.6] \\
(x_1, x_2) \\
[0.4,0.5],[0.3,0.6],[0.4,0.8],[0.1,0.6] \\
(x_2, x_2)
\end{bmatrix}
\]

Proposition 2. For every three universes \( U, V, W \) and three universe sets \( A \) on \( U, B \) on \( V, C \) on \( W \). We have

a) \( A \times B = B \times A \) and \( A \otimes B = B \otimes A \)

b) \( (A \times B) \times C = A \times (B \times C) \)

and \( (A \otimes B) \otimes C = A \otimes (B \otimes C) \)

Proof. It is obvious.

4 Distance between support-neutrosophic sets

In this section, we define the distance between two support-neutrosophic sets in the sense of Szmidt and Kacprzyk are presented:

Definition 9. Let \( U = \{x_1, x_2, ..., x_n\} \) be the universe set. Given \( A, B \in SNS(U) \), we define

a) The Hamming distance

\[
d_{SNS}(A, B) = \frac{1}{n} \sum_{i=1}^{n} d(T_A(x_i), T_B(x_i)) + d(I_A(x_i), I_B(x_i)) + d(F_A(x_i), F_B(x_i)) + d(s_A(x_i), s_B(x_i))
\]

b) The Euclidean distance

\[
e_{SNS}(A, B) = \left( \frac{1}{n} \sum_{i=1}^{n} [d^2(T_A(x_i), T_B(x_i))] + d^2(I_A(x_i), I_B(x_i)) + d^2(F_A(x_i), F_B(x_i)) + d^2(s_A(x_i), s_B(x_i))] \right)^{\frac{1}{2}}
\]

Example 3. Let \( U = \{x_1, x_2\} \) be the universe set. Two SNS \( A, B \in SNS(U) \) as in example 2 we have \( d_{SNS}(A, B) = 0.15; e_{SNS}(A, B) = 0.15 \).

If

\[
C = \begin{bmatrix}
[0.5,0.7],[0.4,0.6],[0.2,0.7],[0.7,0.9] \\
x_1 \\
[0.4,0.5],[0.45,0.6],[0.3,0.6],[0.5,0.8] \\
x_2
\end{bmatrix}
\]

and

\[
D = \begin{bmatrix}
[0.2,0.4],[0.3,0.5],[0.3,0.6],[0.6,0.9] \\
x_1 \\
[0.6,0.7],[0.4,0.8],[0.9,1],[0.4,0.9] \\
x_2
\end{bmatrix}
\]

then \( d_{SNS}(C, D) = 0.25 \) and \( e_{SNS}(C, D) = 0.2081 \).
Conclusion

In this paper, we introduce a new concept: support-neutrosophic set. We also study operators on the support-neutrosophic set and their initial properties. We have given the distance and the Cartesian product of two support – neutrosophic sets. In the future, we will study more results on the support-neutrosophic set and their applications.

References


Received: June 14, 2017. Accepted: June 28, 2017.