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A Study of the Application of Digital Computers for Controlling Vehicle Traffic at a signalized Intersection

John V. Wait

APPLICATION OF DIGITAL COMPUTERS - WAIT

A STUDY OF THE APPLICATION OF DIGITAL COMPUTERS FOR CONTROLLING VEHICLE TRAFFIC AT A SIGNALIZED INTERSECTION

By

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A B S T R A C T

This study outlines a method whereby a small, specialized "on-line" digital computer could be used to control urban vehicle traffic at a signalized intersection. In the basic system proposed, the computer uses as input data a "history" of the instantaneous vehicle flow, in terms of number of cars arriving and arrival time. The data are stored for a period of 240 seconds in a magnetic drum memory. Once each 1/2 second, the computer examines the arrival pattern of cars and compares the delay being imposed on the waiting cars versus the delay which would be imposed on the cars on the moving phase if the lights were changed for a sufficient time to clear the waiting queue. The computer then decides when to change the signal lights and what the timing interval for the next phase should be.

The results of a Monte Carlo simulation of the basic system are included. A greatly simplified two-phase intersection model is postulated, consisting of two unidirectional flows of Poisson distributed traffic with empirically

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intervals of [illegible]

average delay of [illegible]

are compared to [illegible]

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fixed-time control [illegible]

system appeared [illegible]

and consequently [illegible]

particularly [illegible]

deviated crudely [illegible]

response to [illegible]

To extend [illegible]

tion situations [illegible]

signal, multi-lane and multi-leg [illegible]

conflicting manoeuvres [illegible]

to be examined. [illegible]

extended to consider [illegible]

A study of [illegible]

amount of field [illegible]

basic system [illegible]

and field studies have yielded a system capable of handling traffic at a single intersection, the work should be continued toward the development of a system that provides optimum control of a network of intersections. It is felt that this approach offers a means for significantly improving the traffic handling capability of an urban street network.

ACKNOWLEDGEMENTS

The studies leading to the writing of this paper were initiated under the guidance of Prof. R. K. Moore, Chairman of the University of New Mexico Department of Electrical Engineering. The Monte Carlo simulation of the basic system was performed with the assistance of personnel of the Sandia Corporation Laboratories of the Atomic Energy Commission, and the author expresses especial gratitude to the members of Divisions 5242, 5243 and 5126 for their generous cooperation in this part of the study. There were many others whose advice and assistance contributed to the completion of the thesis; among these are Prof. Moore, Prof. M. C. May of the University Civil Engineering Department, Maj. R. E. Wright, of the Research Directorate, Kirtland AFB, and Mr. Frank C. Burton, Traffic Engineer for the City of Albuquerque. The author is also grateful to Mrs. Patricia K. Franklin and Mr. James P. Farrar for the typing and reproduction of the final copies.

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CHAPTER I

Introduction

The need for improved methods for controlling urban traffic is well recognized. It is an accepted fact that the efficient accommodation of urban vehicle traffic is presently a major problem to traffic engineers and city planning agencies. In general, the traffic demands created by the growth of cities and city populations increase so rapidly that the traffic engineer cannot revise and expand the capacity of the street networks and other traffic handling facilities to meet the demand adequately. Although in itself rather costly, good signal control equipment is in many respects a relatively economical means of utilizing existing street capacities most efficiently. Consequently, there has been much work done in recent years toward the development of electronic control devices which provide more efficient control of signalized intersections and intersection networks.

The material presented here is intended as a preliminary approach to a technique for providing an integrated system

of signalized intersections controlled by a digital computer or group of computers. This system would use flow information from a large number of traffic detectors as input data and would perform "on-line" calculations to determine the optimum timing pattern for the lights in the street network. It should be noted that most existing systems designed to perform this type of function are not actually computers, but are devices which select predetermined timing patterns on the basis of the average volume being detected on major arteries or at key intersections. In present systems, the timing patterns, and the volume levels at which the pattern changes are to be made are all determined by the traffic engineer, who usually must use empirical signal timing techniques and his experience to arrive at the settings to be made on the control system. Various remote control systems are in use which provide means for assimilating traffic flow information, and changing timing patterns in some semi-automatic fashion. However, it appears that although digital computers have been used for some time in the analysis of traffic flow problems,[1] little consideration has been given to actually using a computer to continuously calculate and control signal timing.

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[1] See References 9, 12, 18

This study is only intended to cover a preliminary problem which seems fundamentally related to the overall system problem; specifically, that of controlling a single intersection with a small digital computer. While an effort has been made to consider the economic factors associated with the design of such a computer, no conclusive statements will be made as to whether the estimated cost of such a device would be warranted. It is felt that a fair evaluation can be made only after a prototype unit has been constructed and field tested. Also no discussion of the warrants (criteria) for the use of this type of signal controller at a particular intersection is included.

The rest of this chapter includes a review of traffic characteristics as related to signal control. Chapter II outlines the basic operation of the proposed system for controlling two intersecting traffic flows, the computer sequence, and the results obtained from a Monte Carlo simulation of the system operation. Chapter III deals with means for extending the basic system to include the consideration of more complicated situations. In Chapter IV the conclusions made about the practicality and usefulness of the system are discussed and an outline is presented of the

areas which should next be examined, if study of this problem were to be continued.

Basic Functions of an Intersection Controller - Existing Types

The purpose of a traffic signal is to break the traffic flow into two or more phases of flow or movement, in order to handle high volumes with the greatest safety and a minimum of delay. In this paper, the term phase applies to a selection of traffic movements which are simultaneously given a common signal aspect (usually a green or red light). A complete sequence of phases will be termed a cycle. In addition to the green and red intervals assigned to each phase, the signal cycle may contain clearance intervals (normally a yellow light) in order to facilitate the transfer of movement and to assist pedestrians in crossing an intersection. Before discussion the factors affecting optimum timing of traffic signals, it is perhaps of value to list the existing types of signal controllers commonly in use.

a. Fixed-Time Operation. The most common type of intersection controller is known as the fixed-time controller. Average time intervals for the various phases are set into the controller on the basis of anticipated traffic demands. Provision may be made for remotely changing the timing pattern

or for coordinating the timing with that of adjancent controllers, but the basic distinction of timing the lights on the basis of average or anticipated demand applies.

b. Traffic-actuated Operation. The principle of traffic-actuated signal control has been used for many years. The so-called semi-actuated controller is basically a fixed time controller modified so that the green light will be assigned either permanently or for a relatively long interval to the major phase unless one or more vehicles actuate a detector, thereby transferring the green light to the minor phase for a relatively short interval, to allow movement of the cars on the minor phase. Several more complex controllers which can be more accurately called traffic-actuated or automatic controllers are also in use. The most efficient of the presently available types is the "volume-density" controller. This is an electronic device which contains a number of RC-timed thyaratron switching circuits and various auxiliary relays and stepping switches. The device is designed to adjust the green intervals on the basis of the number of cars waiting, the time that the first waiting car has been delayed, and the instantaneous flow (density) of traffic sensed on the moving phase.

In all probability, a "volume-density" controller will perform fairly efficiently over a wide range of traffic demands, provided that the many necessary settings are made properly. However, while it does tend to operate on the basis of the actual demand, it fails to consider several aspects of the traffic flow, which, if taken into consideration, should improve the efficiency of the controller.

For example:

a. Present controllers do not consider the detailed pattern of traffic arrivals on the waiting phase; i.e. exactly how long each car has been waiting.

b. Present controllers are not able to examine future traffic demands on all phases, for example, they do not appear to be able to predict the arrival of a platoon of cars far enough in advance to allow optimization of signal settings so as to eliminate delay to large bunches of cars.

c. Present controllers have many settings which must be correctly made for proper operation and changes in traffic demands throughout the day may be great enough to cause these settings to be considerably in error.

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c) [illegible]

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Although the system proposed in the later sections of this paper is undoubtedly subject in some degree to the same limitations, it is hopefully better able to examine traffic in a more detailed sense, and to adjust the signal operation to meet wide variations in demand more efficiently than can present devices.

Characteristics of Traffic

In proposing a system for controlling the signals at a traffic intersection, one must, of course, first consider the characteristics of traffic, in order to assess the requirements which the controller must fulfill. Considerable work has been done toward describing the characteristics of vehicle traffic, both in free flow conditions and at intersections. Only those performance characteristics which appear to be pertinent to the basis for system operation are considered here. It is felt that description of the characteristics given are accepted in the Traffic Engineering literature as being valid; whenever these descriptions are modified to fit the requirements of the system, the reasons for modifications will be discussed.

Experimental data indicate that under conditions of free flow, the distribution of vehicle arrivals can be described with reasonable accuracy by a Poisson distribution.

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are weighted [illegible]

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Thus:

$$P(X) = e^{-m} \frac{m^x}{x!}$$

Where P(X) is the probability of a certain number of cars, X, arriving in a given interval, and m is the average number arriving in that same interval.

It can be shown that this distribution leads to an exponential distribution of arrival times, i.e.

$$P(t) = e^{-mT} - e^{-m(T+1)}$$

Where P(t) is the probability of two successive vehicles being separated by a spacing t such that $T < t < (T + 1)$

This description of traffic flow has been found to be reasonably accurate, even for relatively high traffic volumes with one notable exception: P(t) is very small for $0 < t < 1$ sec.[1] That is, the minimum time spacing between vehicles is about one second. This limitation causes the actual pattern of vehicle arrivals to be somewhat modified, particularly at high volumes.

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[1] Reference 10, p 77

A modified description of traffic flow was used for developing the proposed system and for studying its performance, wherein real time is quantized into 1/2-second intervals under the assumption that $P(X) = 0$ for $0 \quad t \quad 1/2$. The 1/2-second sampling interval appears adequate, and this is one of the basic parameters of the proposed system. In simulating the operation of the system on an IBM 704 computer, the 1/2-second sampling interval was also assumed, and a random number series was used to generate the presence or absence of a car in each 1/2 second real-time interval in such a manner that the average number of units generated corresponds to the volume of traffic being simulated.

Queuing Behavior

Another important characteristic of traffic which relates to intersection operation is the behavior of queues of vehicles. Again, the results of previous field studies have been relied upon heavily. According to the work of Greenshields, et al,[1] the time required for a queue of waiting cars to enter the intersection, after being

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[1] Reference 10, p 27

given a "go" signal is:

Vehicle No. N	Entrance Time Sec	Difference
1	3.8	
2	6.9	3.1
3	9.6	2.7
4	12.0	2.4
5	14.2	2.2
6	16.3	2.1
over 6	2.1N + 3.7	2.1

These values can be used not only to determine the blocking period of the waiting cars, but also to calculate the approximate amount of queuing delay incurred by each waiting car, in addition to the delay presented by the red light itself. In addition to the queue holding times tabulated above, it is generally assumed that some additional delay time should be included for the time required for the queue to accelerate to a reasonable running speed. Greenshields recommends 2 sec. for the first car, and 1 sec. for the second and third cars. Thus the additional delay experienced by each car in a waiting queue

after the "go" light is given approximately:

N number of vehicles	Queue Delay of Nth vehicle-sec.	Total Delay for N vehicles vehicle seconds
1	2	2
2	4.8	6.8
3	10.6	17.4
4	12.1	29.4
5	14.2	43.6
6	16.3	59.9
7	18.4	78.3
8	20.5	98.8
9	22.6	121.4

Note that for more than 6 vehicles, the additional queuing delay time per car is (4.2N + 7.4) 1/2-seconds or approximately (4N + 7) 1/2-seconds. The total delay is seen to be closely approximated by the sum of an arithmetic series:

$$\text{Total Queuing delay} = \sum_{1}^{N} (4i + 7) \quad -11$$

$$= 2N^2 + 9N - 11 \text{ vehicle-1/2 sec.}$$

The delay as calculated by this expression compares to the delay derived from the empirical data as follows:

Number of Vehicles N	Calculated Delay Vehicle-1/2-sec.	Empirical Delay Vehicle-1/2-sec.	Difference Percent
1	0	4	-
2	15	13.6	7
3	34	34.8	3
4	57	58.8	3
5	84	87.2	3
6	115	119.8	4
7	150	156.6	4.2
8	189	197.6	4.47
9	232	242.8	4.57
10	279	292.2	5.0
N	2N + 9N-11		<5

Spot-Speeds

Since it is assumed that the controller will be handling urban vehicle traffic, the spot speeds of the vehicles in the flow are assumed to be distributed fairly closely around an average lane speed, which is related to the zoned speed for the intersection approaches. Several studies have shown that proper speed zoning can

be used to achieve a close distribution of speeds.[1] Although it is well known that the average speed of traffic in a given stream is a function of the stream volume, use is made of the above assumption in determining the passage time from the point of vehicle detection upstream of the controller to the intersection. If the passage time is somewhat in error, the overall effect on the system performance is not considered to be too serious, because although the error will affect the information about the traffic supplied to the computer in a detailed sense, the character of information over a period of several seconds will not be greatly changed.

Of course, many other characteristics of traffic have been considered, but those mentioned above are listed primarily because they relate to several of the basic assumptions to follow which might otherwise be considered somewhat questionable.

<u>Present Signal Timing Methods</u>

a. <u>Fixed Time Controllers</u>

The utilization of the above traffic characteristics

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[1]Reference 14, p 60

in determing an optimum solution to the signal timing problem is somewhat difficult. An approximate determination of the signal timing required for a given volume on each phase of movement can be made by any one of a number of empirical formulas. For purposes of comparison, the average green light intervals found from the Monte Carlo simulation were compared to the theoretical values presented by Matson[1] for a fixed-time system, wherein a random arrival of cars is assumed, and green times are determined so as to allow passage of all waiting cars 95% of the time. These theoretical values appear in Figures II-5, 6, 7, and 8. Calculation of delay by a simplified average arrival method is used for comparison purposes[2], although the values obtained by this method are only approximate.

b. Traffic Actuated Controllers

If the traffic volumes change considerably throughout the day the fixed-time controller does not provide effective control. An efficient traffic-actuated controller may be made to respond to changes in traffic demands in

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[1] Reference 14, p 340

[2] Ibid p 334

several ways. One common way is to provide an initial green interval plus an added interval which is extended on the basis of the arrival rate of cars on the waiting and moving phases. As more cars wait longer, the amount of green interval "bought" by each passing car is decreased so that the light is changed unless the demand on the green phase remains high. Certain other boundary conditions on the controller operation are also provided so that practical minimum and maximum green and red times are maintained. Most existing equipment uses RC-timed vacuum tube circuits some of which are appropriately reset by the passage of vehicles across detectors, to provide timing of the various cycle intervals. While they do provide a fairly efficient and flexible means of meeting changing traffic demands they do not consider many of the factors which appear to affect an accurate determination of signal timing. First of all, these controllers, if they can be called computers in any sense, are basically analog devices. The counting of cars is done with capacitor charge and discharge circuits, and the timing circuits are considerably non-linear, so that

the overall result is a system which modifies the signal timing in a manner that in general should improve the operation, but which also is basically dependent upon choices of settings made by the traffic engineer, for maximum effectiveness. On most controllers, there are many settings which must be made. These settings, if properly made, will provide efficient operation over a fairly wide range of volumes, but they must be made on primarily an "educated guess" basis, depending upon predicted traffic volumes.

In view of the many approximations which are an essential part of the modus operandi of existing traffic-actuated systems, it appears that a more sophisticated system might be devised. It is hoped that a system developed from the approach discussed in this paper would determine signal timing more efficiently on the basis of detailed traffic flow information, using only a few essential assumptions about traffic characteristics and actually calculating the delay to be incurred by vehicles on the various phases, and alloting green time in a more equitable manner. No detailed comparison could be made

between the performance of the proposed system and that of existing traffic-actuated controllers since little data was available on the operation of these devices when set up for optimum performance at various traffic volumes.

Coordination

Still another complex problem which will be considered here in only a general manner is the functional relationship of an intersection controller to others in a network of intersections. The overall problem of timing and coordination will not be discussed, however, the capability of the proposed system to respond to platooning of vehicle flow by neighboring signals will be outlined.

CHAPTER II

Basic Operation of the Proposed System

It is emphasized that the discussion to follow deals with a system whose operation was developed from a greatly simplified intersection model. It is agreed at the outset that many of the assumptions made in constructing the basic model would tend to make accurate estimates of system performance in a physical situation somewhat difficult. However, the use of a simplified model made it possible for the operation of the system to be simulated by Monte Carlo methods in a straight-forward manner, and made the basic system functions more easily identified and studied. Later sections will discuss the extension of the system to allow consideration of other factors.

The model assumes two intersecting unidirectional flows. Each flow consists of vehicle units which arrive randomly with a mean arrival rate corresponding to the volume on that phase. The intersection is assumed to be isolated from other intersections, i.e., it has free-flow approaches. Initially, no consideration is given to turning

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cells [illegible]

results [illegible]

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allow consideration of [illegible] features.

Any [illegible]

Glass [illegible]

randomly with [illegible]

volume [illegible]

problem [illegible]

representing [illegible]

movements, and the controller system is formulated so as to assign a "green" light alternately to each flow or phase, so as to reduce the average car-second delay to a minimum. A three second "yellow" or clearance interval is provided between transfers.

If a moving phase loses the green interval, it is assumed that any car arriving after the first 2 seconds of the yellow interval will be blocked until the green light is returned. A waiting line of cars is assumed to clear in accordance with the characteristics described above, in Chapter I. If N is the number of waiting cars, then the initial green interval is made (4N + 7) 1/2-seconds.

The discussion of the system for controlling the model intersection will be divided into the following subjects:

a. Origin and Nature of the Information Supplied to the System

   1. Sampling of traffic flow.

   2. Prediction of traffic arrivals.

b. Reception and Storage of Vehicle Flow Data in the Computer Memory

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It [illegible]

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light is reduced, [illegible]
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than the initial [illegible]

seconds.

The [illegible]

model [illegible]

subjects:

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1) [illegible]

1. [illegible]

2. [illegible]

b. [illegible]

in the [illegible]

c. Outline of System Computing Sequence.

d. Discussion of the Physical equipment required to perform the Various System Functions.

Following this, the results of a Monte Carlo simulation of the system will be summarized.

Origin and Nature of the Information Supplied to the System

Sampling

Under the assumption that vehicle headways are always greater than one half-second, the information on traffic flow can be quantized into 1/2-second intervals. Detectors placed in the path of arriving vehicles are assumed to provide a switch closure denoting the passage of each vehicle. A sampling circuit (see Appendix A) provides a train of synchronized pulses to the computer input circuitry. These pulses occur with a 1/2-second period. Presence of a car in a given 1/2-second interval can be denoted by a positive pulse and the absence of a car by a zero (or negative) pulse. A typical pulse train might appear as shown in Fig. II-1.

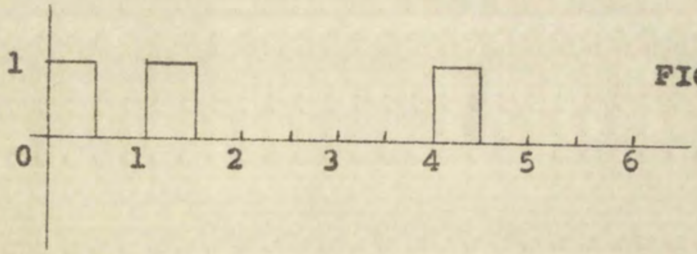

FIGURE II-1

e. Outline of System Computing Sequence.

b. Discussion of the Physical and Logical [illegible]

to perform the Various System Functions.

Following this, the results of a Monte Carlo simulation of the system will be summarized.

<u>Origin and Nature of the Information Supplied to the System Sampling</u>

[illegible] the [illegible] of [illegible] vehicle [illegible] are always greater than one half-second, the information on traffic flow can be quantized into 1/2-second intervals. Detectors placed in the path of arriving vehicles are designed to provide a switch closure denoting the passage of each vehicle. A sampling circuit (see Appendix B) provides a train of synchronized pulses to the computer input circuitry. These pulses occur once in a 1/2-second period. Presence of a car in a given 1/2-second interval can be denoted by a positive pulse and the absence of a car by a zero (or negative) pulse. A typical pulse train might appear as shown in Fig. II-1.

Prediction of Traffic Arrivals

It is felt that the principal advantages which might be achieved by the system are based on the assumption that prediction of future traffic arrivals can be made, and that the prediction information can be effectively utilized to improve the signal timing. Since we are initially characterizing the traffic arrivals by a random time series, we cannot expect to make any detailed prediction of future traffic arrivals from the information provided by detectors located close to the intersection. Instead, it appears better to place the detectors "upstream" as far as is practical (See Fig. II-2). By doing this, the detector information actually is prediction information about future arrivals. There are several obvious limitations on the accuracy of this technique, which must be considered.

Let D be the distance from the detector to the intersection. A signal from the detector then denotes the arrival of a car at t_a seconds in the future, where

$$t_a = \frac{D}{V_{av}}$$

[illegible]

[illegible]

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[illegible]

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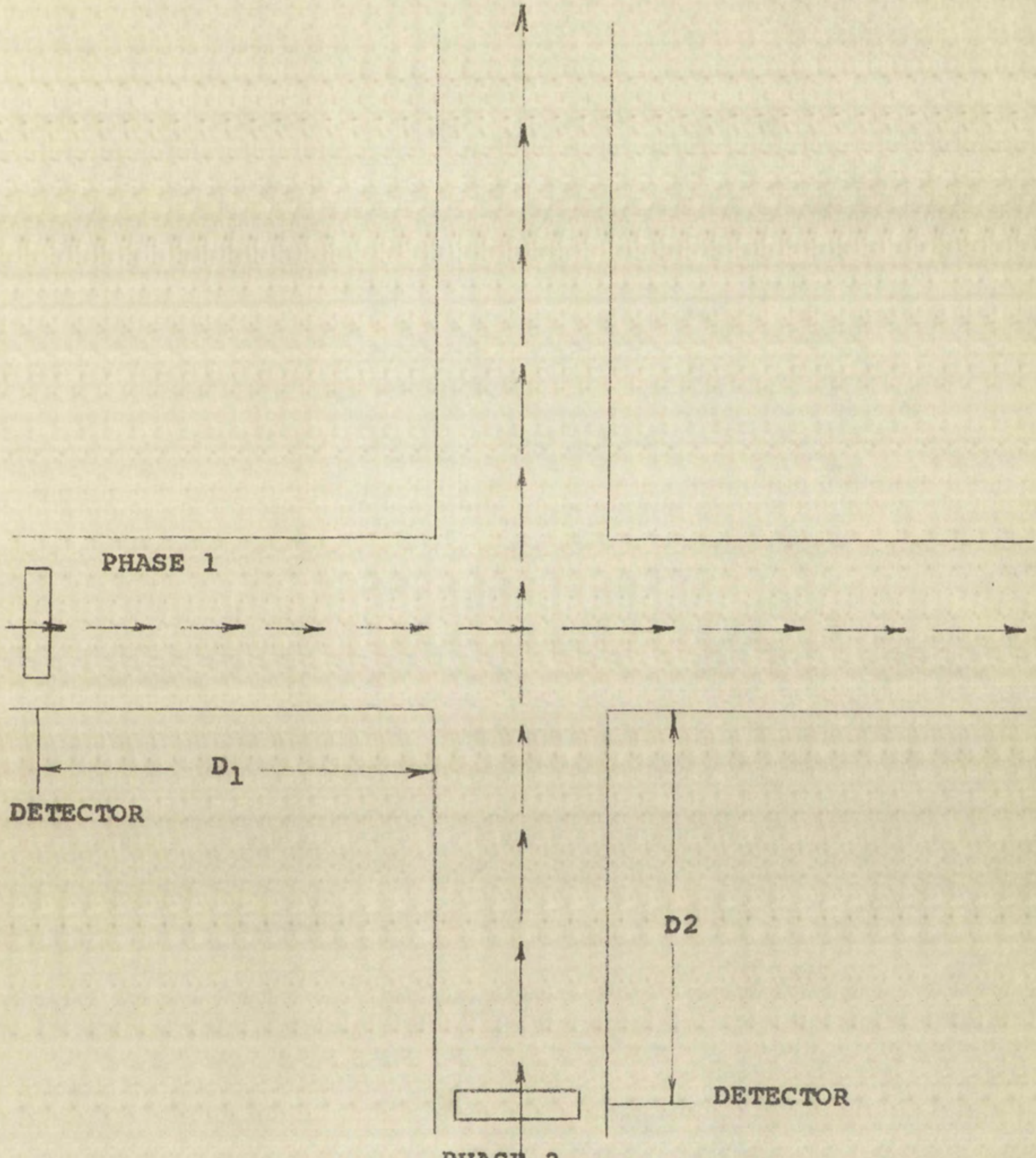

FIGURE II-2 SIMPLIFIED INTERSECTION, DETECTOR LOCATION

and V_{av} is the average velocity of the particular vehicle detected. One major problem, then, is knowing what the average velocity of the vehicle will be after it is detected. If the traffic flow were not too dense, say less than 400 vph, and if the intersection approaches were zoned for a specific speed, then probably the distribution of vehicle velocities would be fairly close to some constant, measurable mean value. Under these conditions, a fairly accurate arrival time could be derived from an upstream detector. However, as the traffic volume increased on a particular phase, the average vehicle velocities would probably decrease. This would mean that the predicted arrival time should be increased for high volumes. For a given detector distance, the exact relationship between arrival time and volume of traffic would probably be difficult to determine. There is considerable information already available which could be used to estimate the effect of volume on average speed,[1] however, for the simplified model, it is assumed that the intersection

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[1]Matson, Ref. 14 p 58, indicates that urban spot speeds average about 25-30 mph (35-45 ft/sec) and that zoning to the 10-90 percentile speed tends to produce optimum control of speed.

approaches are zoned to produce an average vehicle velocity of about 40 ft per sec. The detectors are placed 1200 feet upstream, so that, on the average cars should arrive about 30 seconds after detection. This 30 second interval is assumed to be constant for all traffic volumes. Of course, for high volumes, the actual interval will be somewhat longer. However, a vehicle arriving later than expected probably will still be included in a given blocking interval, and the difference between the predicted time, based on free flow, and the actual arrival time will be properly counted as intersection delay by the system. In general, the error in arrival time which may occur at high volumes will somewhat blur the description of traffic arrivals, but the information provided will still be quite useful in describing the average characteristics of the traffic flow.

Reception and Storage of Vehicle Flow Data in the Computer Memory

The information provided by the vehicle detectors must be stored in the memory unit at the computer in a usable fashion. It is proposed that this be accomplished

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[illegible]

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by storing the pulses representing the arrivals of vehicles on a magnetic drum. For a two-flow intersection, this would require at least two tracks on the drum, each track representing 480 1-bit words. The address of each word would correspond to one 1/2 second interval of real time. Information is stored for the interval from two minutes in the past to two minutes in the future. Address zero would correspond to the present. Thus a "1" in address 120 would mean that a car arrived 60 seconds ago (this sign convention is chosen for computing convenience.)

Since it is necessary to shift the stored data only one address at a time, the displacement of the reading and writing heads would be only a small fraction of an inch. So that the information can be easily read and transferred ahead, two bit positions are provided for each phase. At a given time, only one of the two positions for a given phase will actually contain the information desired, the other position having been erased during the transfer process. Thus, the total drum space alloted for storage of flow information would consist of four tracks. One or

by [illegible]

[illegible]

[illegible] would [illegible]

Each card [illegible]

of each word, would [illegible]

of [illegible]

from two [illegible]

Address cards would [illegible]

In addition [illegible]

and [illegible]

[illegible]

and address of a [illegible]

and [illegible]

[illegible]

transferral [illegible]

[illegible]

[illegible]

for which [illegible] have been [illegible] during the [illegible]

process. [illegible]

of [illegible]

the other of the first two tracks would contain vehicle arrivals on Phase A, and one of the other two would contain the phase B data.

The 1's and 0's would be inserted in the various addresses by circuitry which was operated from the vehicle detectors. If the prediction of arrivals can be made assuming a constant arrival time after a car crosses the detector, say 30 seconds later, then the writing heads would be placed at some angle from the reading heads, corresponding to a space of 60 words "upstream". Beyond an address of -60, the future predicted arrivals would actually be "echos" from past arrivals, and would represent the "demand" of future arrivals only in an average sense, not a detailed sense. However, arrivals this far in the future will not affect the total delay calculation for that phase as much as will near arrivals, (i.e., less than 30 sec.) which are recorded accurately.

It appears that the assumption of a fixed average arrival time from detector to intersection may be reasonable, because, first of all, the detector must be placed well

[illegible]

arrivals on [illegible]

tain the phenom[illegible]

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[illegible]

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It appears that [illegible]

arrival time from detector [illegible]

because, first of all [illegible]

upstream to get adequate prediction information. If it is placed upstream far enough, the shifting of car spacings and the variations in car speeds after detection may cause sufficient variation in arrival time to make any predictions on the basis of speed at detection scarcely more accurate than those based on an average velocity (and therefore average arrival time). Perhaps as an added refinement, the value of average arrival time could be varied with the volume on each phase, in accordance with some predetermined schedule. However, this feature has the disadvantage of requiring that the prediction information be entered into the drum at a different address as the predicted arrival time changes.

Outline of System Computing Sequence

After the information about vehicle arrivals has been stored in the computer memory, several operations must be performed, all within a 1/2-second interval, so that a decision about whether or not to change the lights can be made. The principal criterion selected as a basis for this decision, is the equalization of total delay in

vehicle-seconds on each phase. Certain other constraints are used to assist in making the decision and to satisfy various practical considerations.

First, it is assumed that if a green light is assigned to a new phase, the initial interval will be long enough to allow clearance of all vehicles that are expected to be blocked by either the preceding red light or by the queue clearance time. The number of 1/2-seconds, T_g, required to clear N cars is taken to be

$$T_g = (4N + 7) \text{ 1/2-seconds}$$

If, after the initial green interval has elapsed, it appears that the time should be extended, this is done up to a maximum of 236 1/2-seconds (including last 2 seconds of the preceding yellow interval, this creates a maximum blocking time of two minutes).

As mentioned previously, a yellow light clearance interval of three seconds is used between each change. The system assumes that all cars scheduled to arrive after the first second of the yellow interval are to be included in calculating the number waiting during the subsequent red interval. With the exception of these

boundaries, the signal timing is based upon the sequence of calculations to be outlined below.

a) For the waiting phase, the following calculations are made:

(1) $T_B = T_{gm} + 4$

T_B = time when cars began to be blocked on the waiting phase, including four 1/2-seconds of the preceding yellow interval.

T_{gm} = green interval already elapsed on moving phase. T_{gm} is the initial condition carried over from the preceding calculation.

(2) $$N_w = \sum_{-6}^{T_B} K_t$$

Where: N_w = number of cars blocked by the red light up to the present time.

t = arrival time in 1/2-seconds

$$K_t = \begin{cases} 1 \text{ if car arrived at time } t \\ 0 \text{ if no car arrived at time } t \end{cases}$$

The summation begins at t = (-6), the earliest time when the green light can be presented, following a 3-second yellow interval.

(3) $T_{gw} = (4N_w + 7)$

= green time required to clear N_w cars

(4) At this point, an iterative process is begun. Note that the time, T_{gw}, represents an added potential blocking interval to cars on the now waiting phase. It is necessary to check for additional cars blocked by this interval. If more cars are found to be blocked, then N_w should be increased accordingly. This causes a further extension of T_{gw}, etc.; thus an iterative calculation is performed until N_w does not change as T_{gw} is appropriately extended. There are three steps to the process:

a. N_w', a new value for the number blocked, including queue blocking is found

$$N_w' = \sum_{-(6 + T_{gw})}^{T_B} K_t$$

b. If $N_w' > N_w$, a new value for T_{gw} is found, with $N_w = N_w'$

c. $T_{gw} = (4N_w' + 7)$

The above process is repeated using revised estimates for N_W and T_{gW} until $N_W' = N_W$ in step b. It is estimated that the number of iterations required at high traffic volumes is about seven (see Appendix C).

(5) Once it is known how many cars are waiting, N_W, and how much green time is required to clear the cars, T_{gW}, it is possible to estimate the total delay incurred by the waiting cars:

$$D_W = \sum_{-(6+T_{gW})}^{T_B} K_t\,(6+t) + \sum_{i=1}^{N_W} (4i + 7) - 11$$

$$= \underbrace{\sum K_t\,(6+t)}_{\text{Term A}} + \underbrace{2N_W^2 + 9N_W - 11}_{\text{Term B}}$$

D_W is the total delay in vehicle 1/2-seconds incurred by N_W cars, if the lights are changed after the present sequence of calculations. The terms in this equation include delay due both to the red light blocking interval and the queue clearance time. Note that T_B is the time when cars began to be blocked. The queue will have cleared at time $-(6 + T_{gW})$, actually, cars arriving more than six 1/2-seconds in the future will not see a red

light. However, these cars have a negative arrival time, which will correct the summation of term A for the fact that they are blocked only by the queue.

Term B provides the additional queuing delay based on all cars blocked (see Chap. I, page 10). D_w is stored for later comparison to the delay calculated for the other phase.

(6) $$T_c = T_{gw} + 12$$

where $(-T_c)$ is the earliest time at which the green light can be returned to the now moving phase. Note that

$T_c = 6 + 6 + T_{gw}$ = two yellow intervals plus the clearance time for the waiting queue.

(b) Similar steps are carried out to find D_m, the total delay that would be incurred by the cars on the now moving phase. This calculation is similar to above sequence, except that the initial blocking interval is from -2 to $-T_c$. Again, the number of cars blocked by this interval and the resulting queue is found by iteration.

(c) Next D_w, the total delay on the waiting phase is compared to D_m, the total delay that would occur to cars on the moving phase if the lights were changed. If

[illegible]

[illegible]

[illegible]

[illegible]

for all [illegible]

[illegible]

phase.

(c) [illegible]

where (r) is the [illegible]

[illegible]

[illegible]

(b) Similar [illegible]

total decay [illegible]

moving phase [illegible]

[illegible]

[illegible]

[illegible]

(d) [illegible]

[illegible]

data on the [illegible]

$D_w > D_m$, the lights are changed, to provide an initial green interval to the now waiting phase of T_{gw} 1/2-seconds. If $D_w < D_m$, the computer waits until one 1/2-second has elapsed since the start of the previous calculation, shifts the 1's and 0's representing vehicle flow data, and then repeats the sequence outlined above, with T_{gm} increased by 1.

Physical Equipment Required for Basic System

Since the manipulations to be performed on the digitized vehicle information are straightforward arithmetic operations and logical decisions, it has thus far been tacitly assumed that digital circuits can be devised to do the work. A detailed design of the circuitry required is not included here. However, it would be naive to assert that there would be no problems involved in developing a simple, reliable package of units to perform the required functions. Therefore, some consideration is next given to the overall system organization and equipment requirements visualized for the actual device.

The major units in the system include:

a) A magnetic drum and associated reading,

[illegible]

[illegible]

[illegible]

accordance [illegible]

[illegible]

flow [illegible]

when [illegible]

[illegible]

Since the [illegible]

discussed [illegible]

[illegible]

been partly [illegible]

to do the work [illegible]

required in [illegible]

be [illegible]

developing a simple [illegible]

form the results [illegible]

is next given to the [illegible]

equipment requirements [illegible]

The [illegible]

a) A supply of [illegible]

writing and synchronizing equipment, for storage of vehicle information.

b) A computer programming and control unit

c) Specialized arithmetic units

d) Temporary Storage Registers

e) Signal light control circuitry

f) An enclosure designed to provide adequate environmental protection for critical system components.

Magnetic Drum Memory Unit

For the elementary two-flow case, the computer memory should be capable of storing vehicle arrival data for both phases (designated $\emptyset_1$ and $\emptyset_2$) and for a time interval of 480 1/2-seconds. The 480 1/2-seconds correspond to the interval from 2 minutes in the past to 2 minutes in the future. In order to allow some time for calculations between each reading of flow data, it is proposed that a bit spacing corresponding to about 600 bits per revolution be used. Thus about 80% of a revolution is used to read flow data and 20% for other operations. A small magnetic drum operating at 3600 rpm would

provide this capability, with a reading rate of about 36 KC. This frequency is taken to be the basic clock rate for the computer.

The associated drum circuitry is shown in Fig. II-3. It contains the elements required to:

a) Generate timing pulses for the computer.

b) Generate a number representing the instantaneous address at which the data reading heads are positioned.

c) Sample the traffic detector signals and store the arrival data for each phase in the proper position in the drum.

d) Advance the flow data one address each 1/2-second, and create an "echo" of past flow in the future arrival addresses.

e) Provide a series of 1's and 0's corresponding to the flow data on a specific phase, and for a specific range of addresses.

These functions are performed as follows:

a) Tracks CP and TP provide 36 KC and 60 cps synchronizing pulses respectfully. CP contains

600 permanent machined bits and TP one bit. The Timing and Synchronizing Generator uses these pulses to create the basic timing for the computer. It produces at least three time references:

1) A 36 KC clock pulse for individual sequencing of computing steps, change of drum address, etc.

2) A 60 cps pulse, denoting the beginning of a drum revolution, i.e., some specified drum position.

3) A 2 cps pulse, indicating the 1/2-second intervals at which sampling and data shifting should be accomplished.

b) Along with the Timing generator, the CP and TP tracks operate an Address Counter. This register contains a binary number between - 100101101 and + 100101100 (decimal -300 to +300), denoting the relative position of the drum with respect to the reading heads. The sense of drum rotation is from address +300 down through 0 to address -300. The address

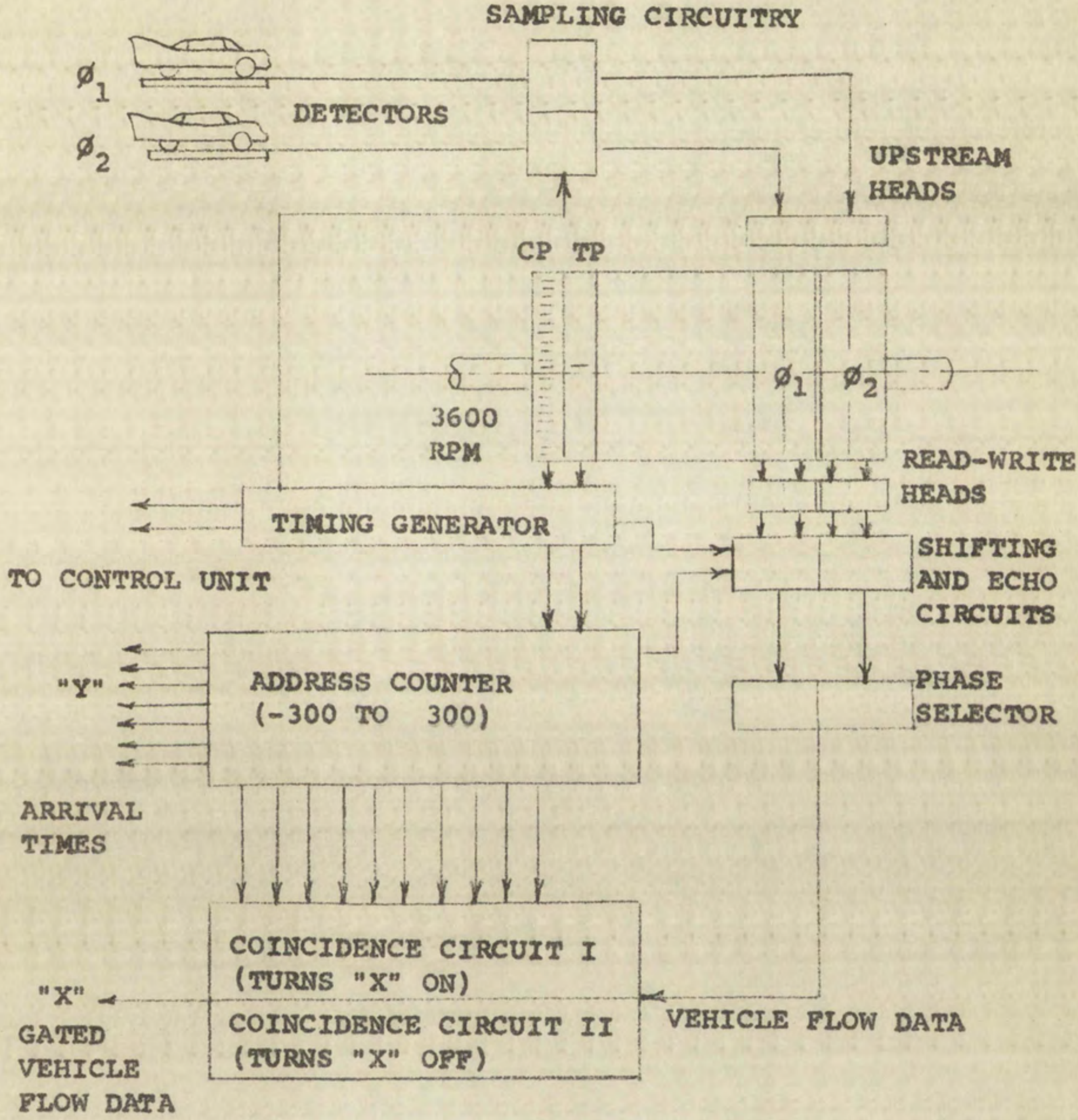


FIGURE II-3 MEMORY DRUM CIRCUITRY

would appear on output lines from the counter, so that it could be used (in parallel form) for various computing operations. Address 0 would correspond to the present in real time, addresses from +1 to +240, past time, and addresses -1 to -240, future time. The remaining addresses can be used for indexing computer operations, but no vehicle arrival data will be stored in these positions.

c) Sampling circuitry, similar to that in Appendix A, receives the signals from the vehicle detectors and produces properly timed pulses to the upstream writing heads. If the detectors were, say, 30 seconds of travel time from the intersection, the signals would be stored in address -60. The upstream writing heads would be physically displaced from the other heads, and the functions described in this paragraph would be performed more or less independently, without any direct relationship to the main computing sequence, except that pulses from the timing generator would insure insertion of the sampled data in the proper address.

d) Once each 1/2-second, the 2 cps timing pulse also initiates operation of the shifting circuitry. This causes each bit of flow data to be read out and inserted

would appear on output lines from the counter [illegible]. It could be used in parallel with the [illegible] recording operations. Address 0 would correspond to the present in real time, addresses from +1 to +120, past time, and addresses -1 to -240, future time. The remaining addresses can be used for iterative computer operations. Data on vehicle arrival data will be stored in these positions.

c) Sampling circuitry, similar to that in Appendix A, receives the signals from the vehicle detectors and produces properly timed pulses to the upstream writing heads. If the detectors were, say, 30 seconds of travel time from the intersection, the signals would be stored in address -60. The upstream writing heads would be physically displaced from the other heads, and the functions described in this paragraph would be performed more or less independently, without any direct relationship to the main computing sequence, except that pulses from the timing generator would insure insertion of the sampled data in the proper address.

d) Once each 1/2-second, the 2 cps timing pulse also initiates operation of the shifting circuitry. This causes each bit of flow data to be read out and inserted

in the next highest address. Thus at one instant track $\emptyset_{11}$ might be the active track for Phase 1 traffic. When a shift is initiated, the data on $\emptyset_{11}$ are read and re-written on track $\emptyset_{12}$ with a + 1 change in each address or location. The concept of two tracks per phase is used to allow for a reasonable physical spacing between the reading and writing heads used in the transfer process. A special delay circuit or another set of writing heads is also required to insert the data from address +240 into adress -240, so that the "echo" of old flow data can be used for prediction information, beyond the interval wherein accurate detector data is available (i.e. between addresses -61 and -240).

e) The address scalar works in conjunction with coincidence circuits I and II and the phase selector, to allow the computer to view the vehicle flow information on a particular phase, and for a given range of addresses. For example, suppose the computer wants the number of cars arriving from t = +30 to t = -6 on a desired phase; then +30 is put in coincidence circuit I, -6 in circuit II, and a signal is sent to the phase selector designating the phase

[illegible]

[illegible]

[illegible]

[illegible]

location. [illegible]

below [illegible]

[illegible]

special delay [illegible]

[illegible]

[illegible]

[illegible]

in advance [illegible]

[illegible]

[illegible]

[illegible]

allow the [illegible]

a distribution [illegible]

For example, [illegible]

[illegible]

[illegible]

[illegible]

(ϕ_1 or ϕ_2) to be read. This produces a train of 1's and 0's at point "X" representing the vehicle arrivals in this interval. As each 1 appears at point "X", the corresponding arrival time appears as a binary number on lines "Y" from the address counter. The signals on lines "X" and "Y" provide the basic data used by the computer. Their appearance on these lines is, of course, directed and timed by the computer control unit.

Computer Programming and Control Unit

Another band of tracks on the same magnetic drum can be used to store control statements for sequencing the computer operations. The location of these statements probably can be such that they would provide instructions to the computer in between periods when the memory reading heads are providing flow information to the arithmetic units. For example, during 72° of drum rotation the computer receives instructions from the control portion of the drum, telling it to count the vehicle arrivals on a given phase between two addresses. Then during the next 288° of rotation, the outputs of the memory heads are gated to the arithmetic units for the desired interval. Along with the stored program on the drum, one or more special instruction counters will probably

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

competition. [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

be needed to assist in instruction sequencing. The instructions can be used to operate a control matrix, which actually performs the interconnections of system elements.

Specialized Arithmetic Units

In discussing the special arithmetic units required, it is assumed that the memory circuitry, operating under control of the programming and control unit, is capable of providing the following basic information:

a) A train of pulses, representing the number of cars arriving (or blocked) on a specific phase, between two time intervals T_a and T_b. This will be called the vehicle counting signal.

b) The D_q accumulator; used to calculate queuing delay.

c) The $\emptyset_w$ and $\emptyset_m$ delay accumulators, working in conjunction with the D_q accumulator and the D_r coincidence circuit; used to calculate total delay. The functions of these units are outlined below.

The 9 bit N-accumulator receives the vehicle counting signal, and counts the number of cars arriving in a given interval. These pulses are actually inserted in the

third digit of the accumulator. For example, suppose ten pulses were received from the memory. These would be counted, and appear in the accumulator from point B to the left:

```
0001010|00 |
       B   A
```

Note that reading from point A, the accumulator reads 4N. Thus, by adding a 7 (binary 111), the number (4n + 7) is formed. In this example, the accumulator might next receive the number 111 (probably in parallel form) starting from point A. It then reads:

```
0001011|11 |
       B   A
```

which is a binary 47. This is the green time in 1/2-seconds required to clear 10 vehicles. By adding the vehicle counting signal at point B, various other number of the form (4n + K) could be easily formed by adding the number K from a special source in the machine program. For example, the number (4N + 19) is the time at which a green light can be returned to a phase, if it is removed for a time sufficient to clear N cars on another phase. The N accumulator also probably will be used to find sums and differences in certain special calculations during the computing sequence.

The D_q accumulator is a special digital circuit that receives the vehicle counting signal and produces the number

$$D_q = 2N^2 + 9N - 11 = \sum_{i=1}^{N} (4N + 7) \quad -11$$ [1]

Logically, the number might most easily be calculated using the second form. D_q, as discussed previously,

---

[1]See Chap. I, p 10

which is a [illegible]

technique required [illegible]

[illegible]

of the [illegible]

the number X [illegible]

terms. For example, [illegible]

which a given [illegible]

is removed for [illegible]

another phase. [illegible]

used to find [illegible]

calculation [illegible]

The λ_1 [illegible]

that receives [illegible]

the number

$n_2 = 2N^2$ [illegible]

[illegible]

lated using the second [illegible]

See Chap. [illegible]

represents the total queuing delay in vehicle 1/2-seconds incurred by N cars.

The $\emptyset_w$ and $\emptyset_m$ accumulators store the total delay incurred by the cars arriving in a given time interval. This is done in two steps. First, the address signals and the vehicle counting signal are fed to a coincidence circuit, called the D_r circuit. The purpose of this circuit is to allow passage of the address only when there is a pulse in the vehicle counting signal representing an actual vehicle arrival. If the waiting phase is being worked on, the $\emptyset_w$ accumulator forms the sum of those addresses passed through the D_r circuit, and thus contains

$$D_r = \sum_{i=1}^{N} K_i(t) \cdot t$$

Where $K_i(t) = \begin{cases} 1 \text{ if a car arrived at time } t \\ 0 \text{ if no car arrived} \end{cases}$

t = arrival time (address)

D_r = delay incurred by cars blocked by the interval T_a to T_b.

The number D_q is added into $\emptyset_w$, to form the total delay $(D_r + D_q)$ associated with the N cars arriving in

represents the total [illegible]

seconds, required by N data.

The C_i and D_i [illegible]

induced by the data [illegible]

This is done in two steps [illegible]

and the vehicle [illegible]

circuit, called the [illegible]

circuit is to allow passage of the [illegible]

there is a pulse in the [illegible]

and an actual vehicle arrival [illegible]

being worked on, the D_i [illegible]

these addresses passed through the [illegible] circuit [illegible]

contains

$$S_D = \sum^{N} X_i(T) \quad \text{[illegible]}$$

where $X_i(t) = \{$ 1 if [illegible]
0 if [illegible]

t = [illegible]

D_i = delay [illegible]
the [illegible]

The number S_D is [illegible]

delay [illegible] associated with [illegible]

the specified interval on the waiting phase. Similarly, the ϕ_m accumulator contains the delay calculated on the moving phase.

Temporary Storage Registers

During the 1/2-second when the computer is calculating a decision about signal timing, there is a requirement for storage of several numbers used in the computation. It is estimated that no more than ten such numbers need to be stored for part or all of the calculating sequence. It is likely that some of them could easily be stored in between instructions on the program section of the drum. However, others should be stored in separate registers so that they can be rapidly retrieved at the required time in the computing sequence.

Signal Light Control Circuitry

Of course, the values of signal times calculated by the computer have to be used to control the lights. To do this, control circuitry must be provided to convert the binary numbers generated by the system into the desired time interval. After a new initial green interval has been calculated, the computer is held in stand-by condition, the green interval is stored in a simple counter,

and appropriate relays, supplying power to the signal lights, are actuated. At the end of each 1/2-second interval, a "1" is subtracted from the counter. When the counter is cleared, the computing sequence is again started. The signals are held in the same state until the computer determines that a transfer should be made. Then the light control circuitry is so instructed, and the timing counter is filled with a new green interval for the alternate phase.

Environmental Control of System Components

It is noted in passing that if the type of system proposed is intended to operate in the out-of-doors, the equipment will be subjected to rather severe environmental conditions. In particular, the magnetic drum could be seriously affected by temperature variations and dust. Since it is desirable to have a device capable of operating for a maximum length of time without maintenance or adjustment, it appears that careful consideration must be given to environmental factors in the design of the system components. The use of encapsulated circuits and components might be required to achieve stable and reliable long-term operation.

and appropriate relays connecting [illegible]

lights are actuated. At the end of each timing

interval, a "1" is subtracted from the [illegible]

the counter is cleared, the [illegible]

started. The signals are held in the [illegible]

the computer [illegible]

When the light control signals [illegible]

the timing counter is filled with a new [illegible]

for the alternate phase.

Environmental Control of Remote Equipment

It is noted in passing that the [illegible] system proposed is intended to operate [illegible] equipment will be subjected to [illegible] conditions. In particular, the magnetic [illegible] seriously affected by temperature [illegible] Since it is desirable to have a [illegible] for a maximum length of time [illegible] adjustment, it appears that careful [illegible] given to environmental factors in the design of the [illegible] components. The use of encapsulated [illegible] might be required to achieve stable and reliable [illegible] operation.

System Block Diagram

Fig. II-4 shows an overall block diagram of the system discussed. The major paths of signal flow are shown, but a complete presentation of all sub-units and signal paths is not intended. Of course, a small regulated power supply for operation of the computer circuitry, is required.

Results of the Monte Carlo Simulation of the Elementary Two-Phase Controller

Using the IBM 704 program given in Appendix B, the operation of the system was simulated for various combinations of phase volumes. For each pair of volumes, ten cycles were calculated. Table II-A shows a typical set of values for the case of 400 vph versus 800 vph. T_{g1} and T_{g2} are the green-light intervals given to each phase. D_1 and D_2 are the associated values of total delay calculated for the waiting phase.

The values obtained for the other sets of volumes are presented, in Appendix D. In every case, there was an extreme variation of individual intervals around the mean value, which indicated that the system was responding

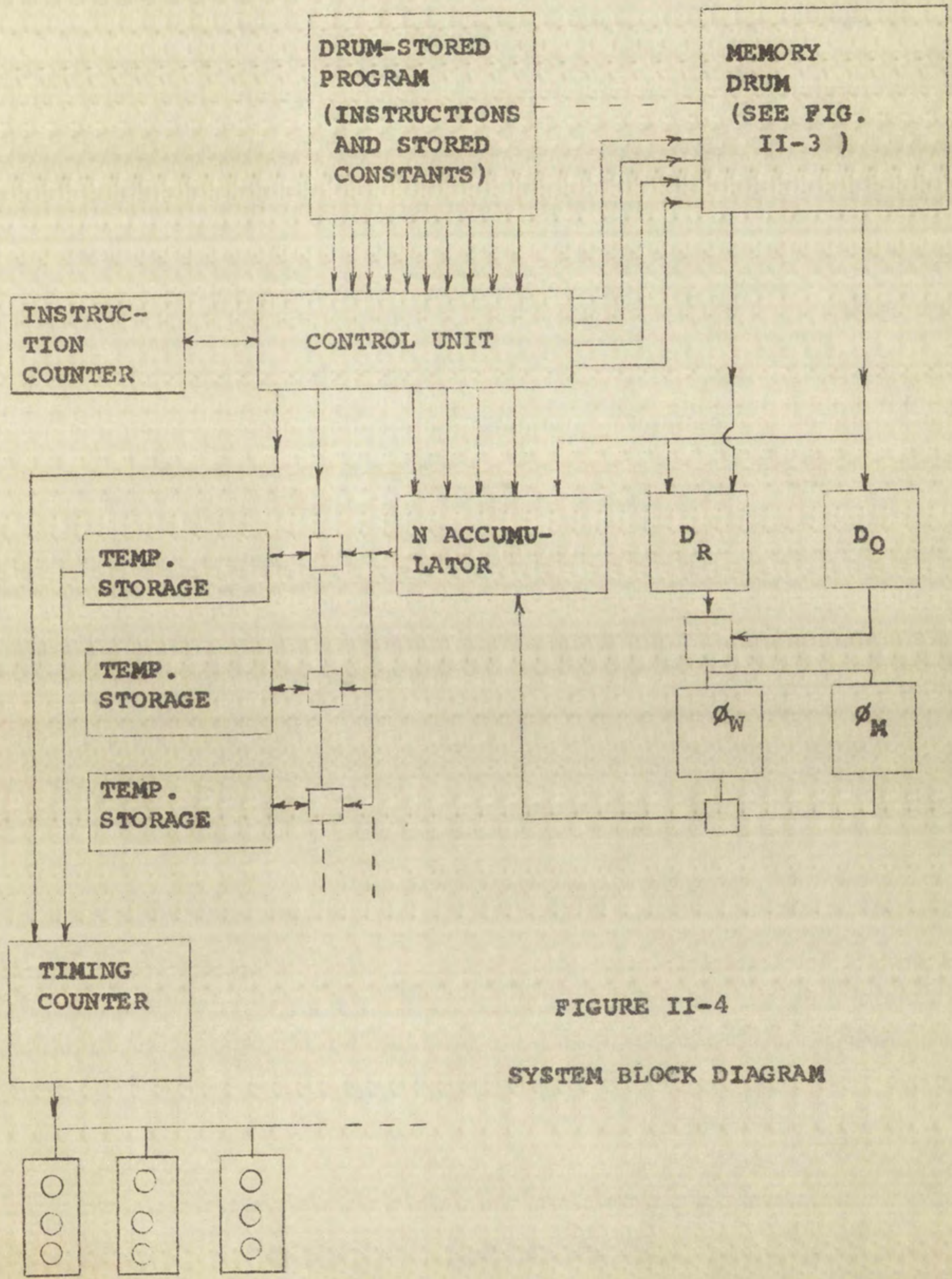


FIGURE II-4

SYSTEM BLOCK DIAGRAM

to short-time variations in demand as anticipated. In the case of 400 vph versus 800 vph, two sample comparisons can be made between the computer-controlled intersection and the fixed time controlled intersection.

Table II-A

400 vph		800 vph	
D_1	T_{g1}	T_{q2}	D_2
vehicle-seconds	seconds	seconds	vehicle-seconds
158	23	33.5	153.5
31.5	11.5	42.5	76
76	11.5	17.5	83
75.5	9.5	48.5	165
136.5	19.5	23.5	198.5
110	11.5	41.5	142
33.5	7.5	23.5	24
41	9.5	9.5	44.5
109	22.5	44.0	149.5
167.5	19.5	19.5	105.5
93.9 av	14.5 av	30 av	112 av

a) Green-light intervals.

The average green times were 14.5 and 30 seconds for the 400 vph and 800 vph flows respectively. The accuracy of these mean values is, of course, poor, since there were only ten different values calculated and the individual variations were large. In this particular case, the variance of the sample values around the means were found to be

$$s_1{}^2 = \frac{\sum \Delta_1^2}{10} = 31.1 \text{ sec}^2\text{, for the 400 vph flow}$$

$$s_2{}^2 = \frac{\sum \Delta_2^2}{10} = 161 \text{ sec}^2\text{, for the 800 vph flow}$$

The sample standard deviation of the values are then:

$$s_1 = \sqrt{31.1} = 5.6 \text{ sec.}$$

$$s_2 = \sqrt{161} = 12.7 \text{ sec.}$$

In order to test the significance of the mean value of the small number of samples available, the "student" t distribution was used.[1]

---

[1]See Reference 11

[illegible]

[illegible]

[illegible] accuracy [illegible]

[illegible]

[illegible]

[illegible] such, [illegible]

with regard [illegible]

[illegible]

[illegible]

[illegible]

Then [illegible]

where

[illegible]

[illegible]

value of the [illegible]

[illegible]

[1] See [illegible]

If it is desired to know with 90% confidence that the mean green time, G, lies between (G-t) and (G+t) then for 10 trials

$$t = \frac{1.8S}{\sqrt{N-1}} = \frac{1.8S}{\sqrt{9}} = 0.6S$$

In this case then

$$t_1 = 3.6 \text{ sec}$$
$$t_2 = 6.6 \text{ sec}$$

so that

$$T_{g_1} = 14.5 \pm 3.6 \text{ sec for 400 vph}$$

$$T_{g_2} = 30.0 \pm 6.6 \text{ sec for 800 vph}$$

The theoretical green times for a fixed-time controller would be about 28 and 43 seconds.[1]

b) Delays

Using the theoretical fixed-time controller green intervals, an estimate of the average delay for that type of system can be made. Several standard techniques are used. The average arrival and uniform spacing methods discussed by Greenshields[2] are rather involved, particularly

---

[1]These values are given by Matson (Ref. 14, p 338) as those intervals that will pass as may vehicles as arrive in one cycle 95% of the time.

[2]Reference 10, p 100

for high volumes. Matson gives a simplified method yielding an answer which is probably optimistic.[1] This method assumes that T_d, the average delay per vehicle on a given phase is

$$T_d \gtrapprox \frac{R + 4.75}{2}$$

where R is the blocking interval in seconds. The other standard methods yield figures for delay that are greater than the above, approaching one-half of the total cycle time for high volumes. Consequently, favorable comparisons to the above formula should be valid.

If a three-second yellow interval is assumed

$$R = T_g + 3$$

where T_g is the green time on the opposite phase.

Then the delay for the fixed-time case is

$$T_d \gtrsim \frac{T_g + 7.75}{2} > \frac{T_g}{2}$$

Regardless of the criteria used for comparison, it is readily apparent that a system with lower average green times should have less delay.

---

[1]Reference 14, p 334

Thus in the present example, for the 400 vph phase

$$T_{D_1} \geqq \frac{46 + 4.5}{2} = 25.3 \text{ sec/vehicle}$$

For the 800 vph phase

$$T_{D_2} \geqq \frac{31 + 4.5}{2} = 17.7 \text{ sec/vehicle}$$

The average delay is

$$\frac{T_{D_1} \cdot V_1 + T_{D_2} \cdot V_2}{V_1 + V_2} \geqq 20.2 \text{ sec/vehicle}$$

for the fixed time controller.

Using the values of total waiting phase delay produced in the Monte Carlo simulation, the average delay in the computer-controlled system was found to be 12.9 seconds. Similar comparisons could be made for each case simulated.

In general, the results of the Monte Carlo simulation indicated that the computer-controlled system should yield less delay. Table II-B summarizes the results obtained. The average green times assigned to the two phases increased with increasing volume in each phase and the total cycle time increased with total volume. The case of 200 vph versus 200 vph did not follow this trend; however, the variation of individual values was so large that the true mean values could be

This is the average delay for the 500 vph phase:

$$D_1 = \frac{\Sigma \ldots}{\ldots} = 18.07 \text{ sec/vehicle}$$

for the 250 vph phase

$$D_2 = \frac{\Sigma \ldots}{\ldots} = 19.17 \text{ sec/vehicle}$$

The average delay is

$$\frac{D_1 V_1 + D_2 V_2}{V_1 + V_2} = 18.2 \text{ sec/veh}$$

for the fixed time controller.

Using the values of total waiting time as given by the Monte Carlo simulation, the average delay in the computer-controlled system was found to be 17.7 sec/veh. Similar comparisons could be made for each of the other levels.

In general, the results of the Monte Carlo simulation indicated that the computer-controlled system should yield less delay. Table IV-B shows the results obtained. The average green times assigned to the two phases increased with increasing volume in each phase and the total cycle time increased with total volume. The case of 250 vph versus 250 vph did not fit this trend; however, the variation of individual values was so large that the true mean values could be

considerably different. In general, the mean green times are not felt to be accurate to within less than 5 seconds.

As expected, the system tended to saturate as the total volume approached 1800 vph. This is due to the assumption that a green time of at least 2 seconds per car is required for clearance. Since 1800 vph represents an average arrival headway of 2 seconds, this is the upper limit of the intersection capacity. Dead times, due to yellow light times and queue starting times make the actual intersection maximum capacity less than the theoretical 1800 vph.

Figures II-5, 6, 7 and 8 compare the theoretical values of fixed green intervals to the mean values for the computer controlled system. Note that in general, the average green times are less in the case of the computer controlled system than the fixed-time case. The curves tend to cross at low volumes, since the theoretical curves are calculated without consideration of yellow light intervals. Since the above equations indicate that delays are proportional to the green times, it is concluded that at high volumes, the delay per vehicle should be considerably less for the computer-controlled system.

Table II-B

Summary of Results - Monte Carlo Simulation of a Two-Phase Intersection Controller*

V_1 vph	G_1 sec	t_1 sec	V_2 vph	G_2 sec	t_2 sec	T_D sec/veh	V_t vph	C sec
200	33.0	8.8	200	33.0	8.8	12.6	400	72.0
	9.5	3.3	400	26.5	15.4	10.0	600	42.0
	13.0	4.1	600	27.5	6.7	9.1	800	46.5
	12.0	2.6	1000	51.0	10.6	12.2	1200	70.0
	21.5	4.8	1200	85.5	17.2	13.4	1400	113.0
	16.5	4.3	1400	107.0	20.0	24.8	1600	123.5
400	26.5	5.4	200	9.5	3.3	10.0	600	42.0
	15.5	4.2	400	15.5	4.2	10.3	800	37.0
	17.5	3.9	600	24.0	7.3	15.5	1000	47.5
	14.5	3.3	800	30.0	7.6	12.9	1200	50.5
	16.1	4.4	1000	56.2	12.2	15.4	1400	78.3
	32.1	9.0	1200	94.0	18.0	22.8	1600	132.1
600	27.5	6.9	200	13.0	4.1	9.1	800	46.5
	24.0	7.3	400	17.5	3.9	15.5	1000	47.5
	24.0	2.2	600	24.0	2.2	16.0	1200	54.0
	20.5	5.1	800	34.0	5.1	15.6	1400	60.5
	41.0	11.9	1000	57.0	11.9	22.4	1600	104.0
800	40.0	16.5	200	12.0	3.1	10.0	1000	58.0
	30.0	7.6	400	14.5	3.3	12.9	1200	50.5
	34.0	5.1	600	20.5	5.1	15.6	1400	60.5
	76.0	7.4	800	76.0	7.4	22.1	1600	158.0

*G_1 and G_2 are average green intervals $\pm t_1$ and $\pm t_2$ are the 90% confidence ranges of G_1 and G_2.

V_1 and V_2 are the respective phase volumes. T_D is the average vehicle delay for both phases. V_t is the total intersection volume, and C is the total cycle time.

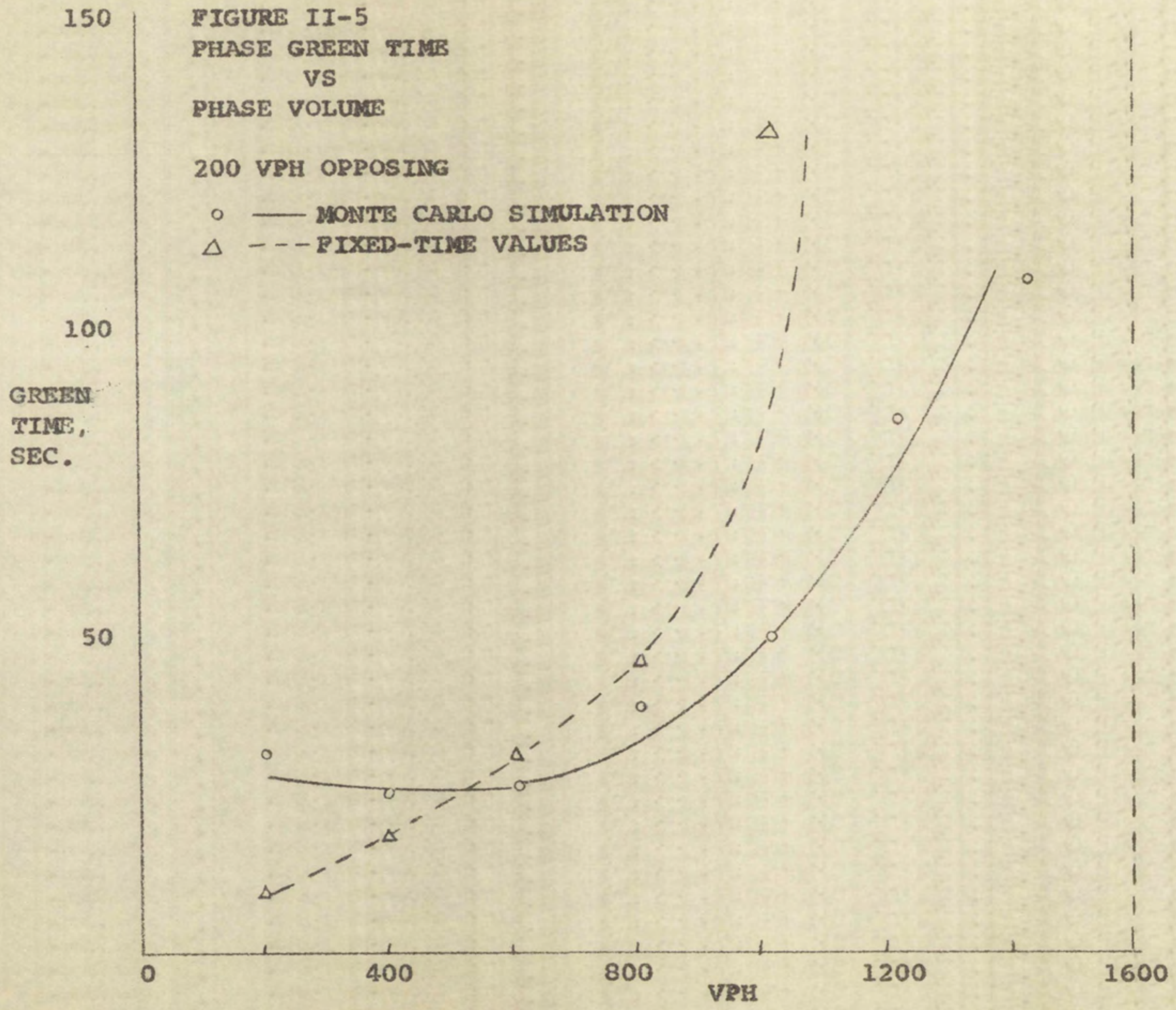


FIGURE II-5
PHASE GREEN TIME
VS
PHASE VOLUME
200 VPH OPPOSING
○ —— MONTE CARLO SIMULATION
△ - - - FIXED-TIME VALUES
150
100
50
GREEN TIME, SEC.
0
400
800
1200
1600
VPH

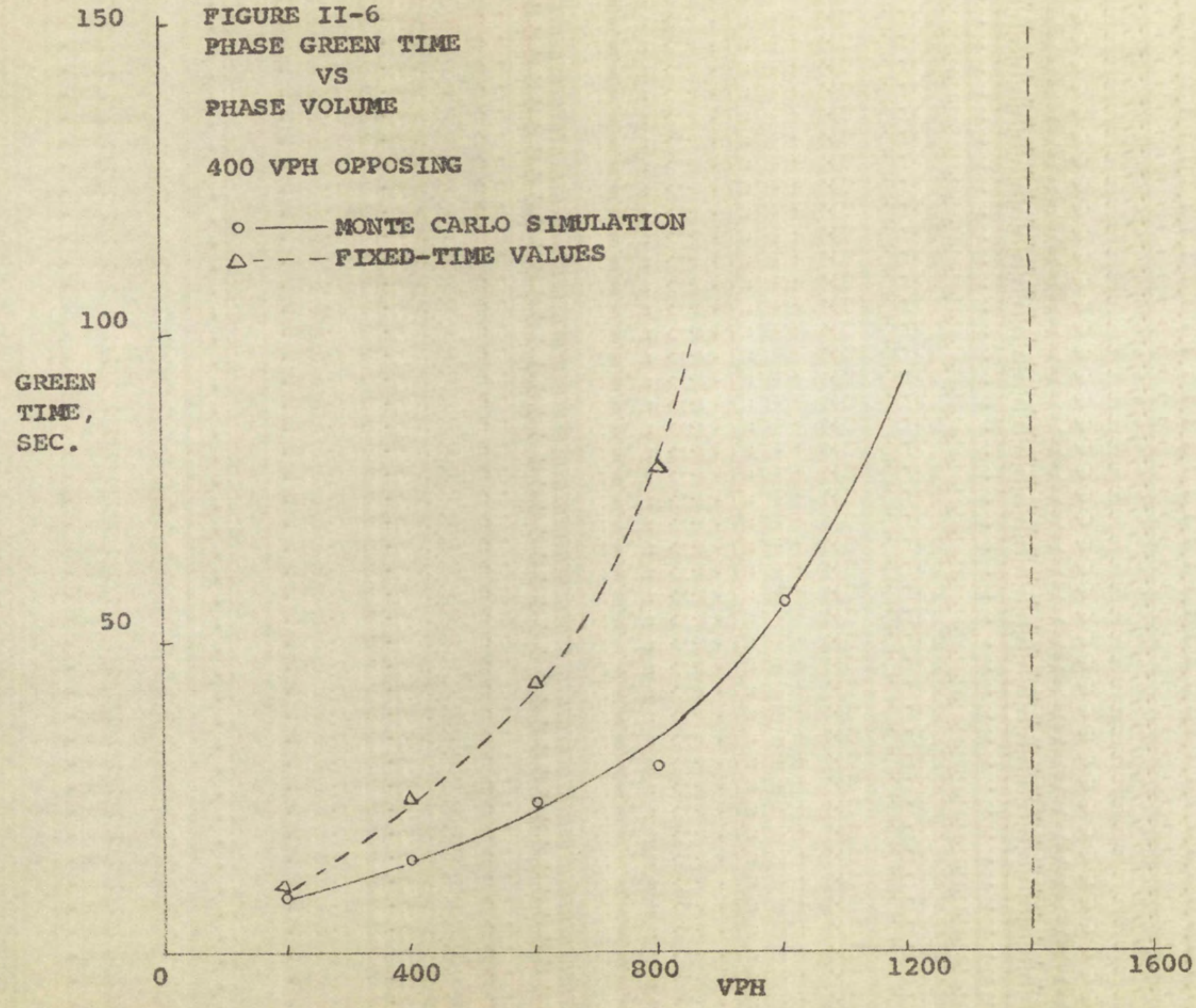


FIGURE II-6
PHASE GREEN TIME
VS
PHASE VOLUME
400 VPH OPPOSING
MONTE CARLO SIMULATION
FIXED-TIME VALUES
GREEN TIME, SEC.
150
100
50
0
400
800
1200
1600
VPH

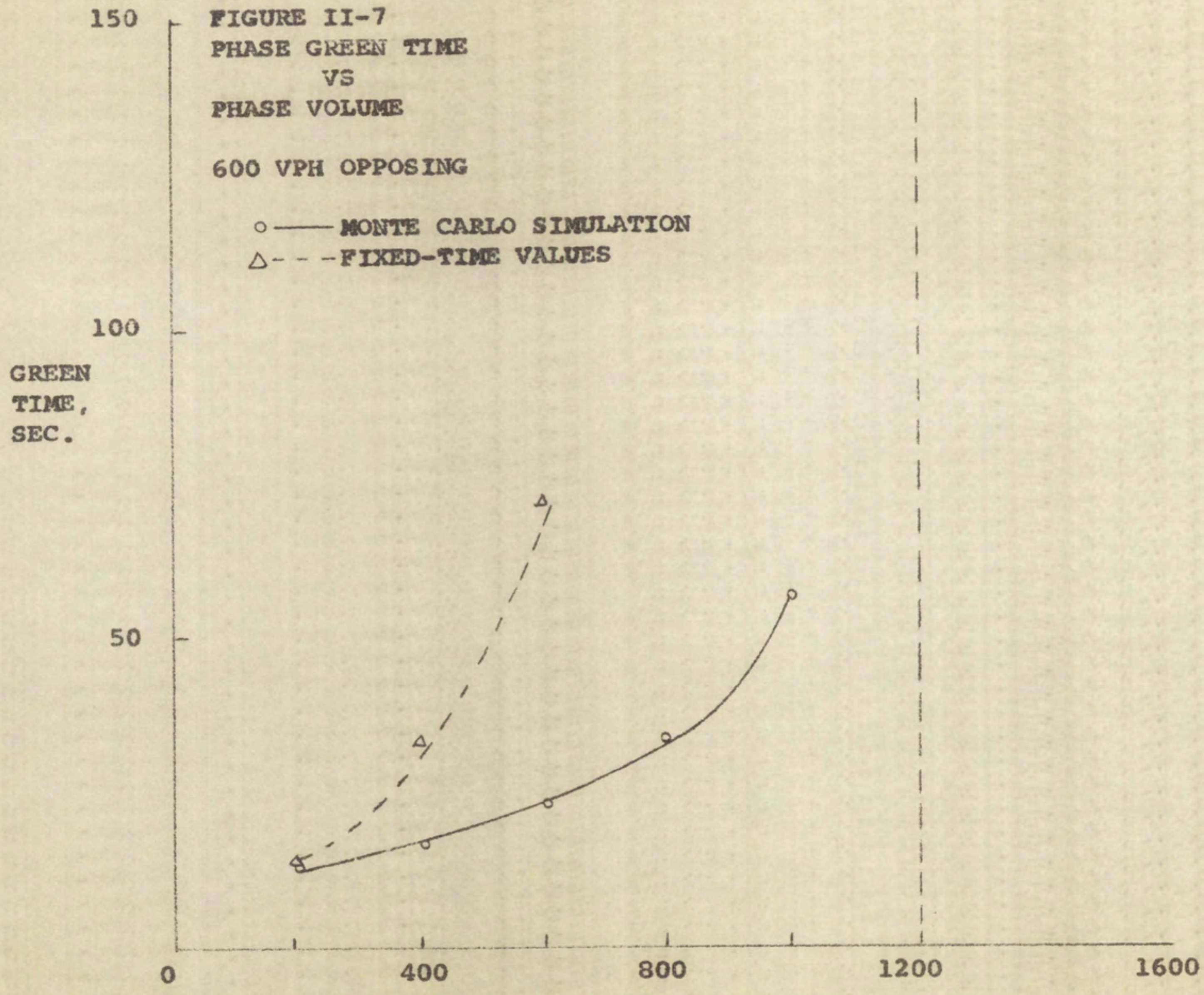
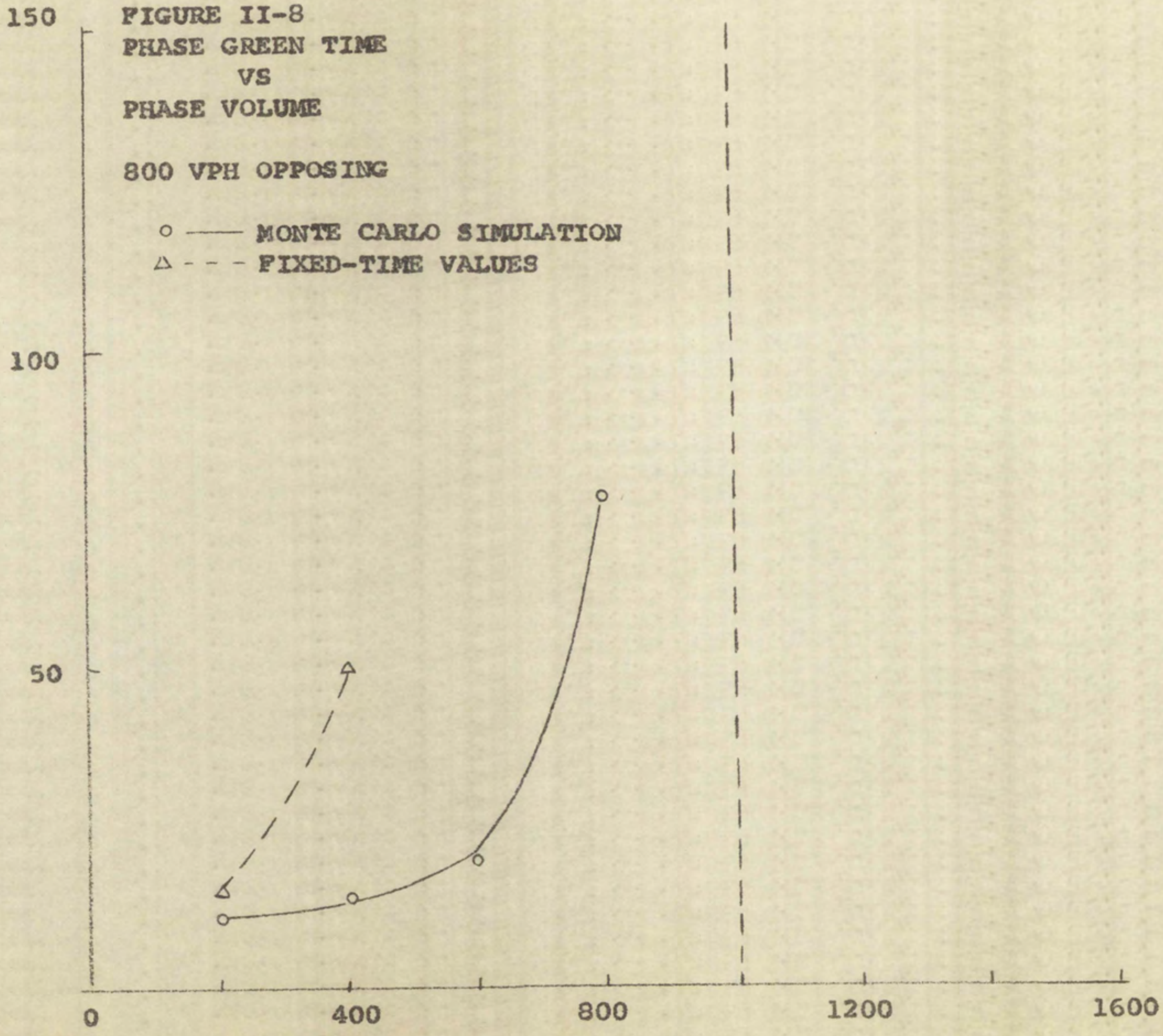


FIGURE II-7
PHASE GREEN TIME
VS
PHASE VOLUME


150
100
50
0
400
800
1200
1600
FIGURE II-8
PHASE GREEN TIME
VS
PHASE VOLUME
800 VPH OPPOSING
MONTE CARLO SIMULATION
FIXED-TIME VALUES

CHAPTER III

Extension of Basic System to More Complicated Situations

As indicated in Chapter II, the intersection model used in formulating the system is considerably simplified. It is the purpose of this chapter to suggest means by which the system could be extended to include consideration of other factors and situations. However, detailed quantitative analyses of the problems created by these extensions are not taken to be within the scope of this basic study. In all cases, it is assumed that the basic approach of the original system would still apply. Discrete vehicle flow information would be used to calculate the number of cars blocked, the green times required and the delays experienced on each phase.

Consideration of Two-directional Flows

One necessary extension of the system is the consideration of the two-directional nature of real traffic flows. This would necessitate doubling the number of tracks in the flow data storage portion of the drum, so that for a two phase intersection, 4 tracks would be required to store

the arrival information on one phase. Thus for a north-south movement, two tracks would alternately store the data for the northbound traffic, and two more tracks, for the southbound traffic. In the calculating sequence, the total number of cars in a given blocking interval would be found as before. However, the number blocked on each direction would have to be found separately, so that the green times (T_{gw} and T_{gm}) could be calculated for all four directions. Then for each phase the larger of the two values would be used in completing calculations about signal timing. Total waiting and moving phase delays would still be compared as the criterion for the decision, but again, the four directions would be considered separately in finding queuing delays, etc.

The added complications created by the above considerations, would mainly serve to increase the size of the temporary storage registers and the arithmetic units in the computer. However, the time required to perform the calculations should not be greatly increased, if the calculations on one phase are performed simultaneously.

Consideration of Multi-Lane Traffic

Another consideration that would probably be necessary is that of multi-lane traffic flows. This problem cannot be solved simply by adding detectors for each lane, and feeding all information into a common track on the memory drum, because of the possibility of simultaneous arrivals. Moreover, the time required to clear a given total number of vehicles waiting in two lanes is obviously not the same as for one lane, but is closer to the time required to clear only the longest of the two queues. Again, the handling of this more complex situation is felt to be possible. Additional pairs of storage tracks for each lane would be needed. Simultaneous calculations for each phase are possible, so that the decision could be made in the same length of time as in the case of the elementary model.

Multi-Phase Operation

It appears that the basic computer system discussed above would be adaptable to multi-phase intersection control. Once a sequence of phases for the intersection signal

cycle is determined, the computer could be made to examine the flow information and calculate the delay being incurred on the several waiting phases, as well as the predicted delay on the moving phase. The computer would have to make a more refined decision as to when to change the lights, and how to do so. In some cases, it might be desirable to delete the next regular movement, in order to better meet the demand of following phases. From a human factors standpoint this probably is allowable, provided that the movements are deleted, but not re-ordered.

Note that the system may have many more calculations to perform in the 1/2-second computing time. Delay figures will have to be calculated on all phases, even though some are not scheduled to receive the green light next. If several high-volume phases are involved, the prediction information required to do the calculations may have to be available for a considerable time in the future, in order to predict the delays involved in changing through several phases before returning to the present moving

phase. However, it is quite possible that effective results still could be obtained using the "echo" of flow information from past demands for predicting more than 30-40 seconds in the future.

Consideration of Left-Turn Maneuvers

Considerable success in the handling of heavy left-turn movements has been achieved with present traffic-actuated controllers by considering the conflicting movement as a separate and sometimes optional phase.

With this in mind, it appears that the basic approach in the consideration of left-turn demand would be to use left-turn corridors with separate detectors to provide demand information for a left-turn phase, just as the other detectors would serve the normal through phases. In this case, it appears that the left-turn detectors should be placed fairly close to the intersection. Prior to entering the left-turn corridor, the cars would have been counted by the upstream detectors, and the data used in predicting phase delay. Then, if any of these units were also counted as desiring to make a left turn, this data would be used to generate a left-turn interval to

follow the normal straight-through signal on that phase. If only 10 or 20 percent of the units were expected to desire a left-turn, it is felt that a simplified treatment of the demand information could be used. Instead of again using drum storage of exact arrival times, the number of detected units would merely counted, so as to allow calculation of required clearance time and queuing delay time. In this case, the additional blocking interval and delay would be assigned to the parent phase from which the left-turning cars originated. Although a Monte Carlo simulation of this situation was not attempted, it is felt that it could be done without too much difficulty.

Pedestrian Movement

A means of discretely counting pedestrian traffic and incorporating the information into the system in an equitable manner is not readily apparent. If pedestrians are to be allowed to move with the vehicles on each phase, then perhaps the best solution is to incorporate pedestrian-actuated detectors that do not actually count pedestrians, but that indicate to the system that at least one pedestrian desires to cross. Then when the given light

is assigned to that phase, it is automatically given a minimum interval sufficient to allow pedestrians to cross. In present timing methods, a 15-second minimum is often advised.[1] Note that the results of the simulation discussed in Chapter II indicate that the green intervals would be at least 12-15 seconds under most conditions, so that perhaps a 15 second minimum interval could be incorporated into the system boundary conditions, without seriously affecting the performance. A further refinement would be an auxiliary unit that presented a "walk" signal for only part of the assigned green interval. Thus if a 50 second interval was to be assigned, a 35 second "walk" light would be given, followed by a 15-second "don't walk" interval. This would help insure that stragglers would not block the intersection during the terminal portion of the green interval, when cars are completing turning maneuvers.

Consideration of Platooning a Coordination

The basic system as already outlined inherently should be capable of responding to platooning effects automatically, since instantaneous vehicle demand is used as the basis for

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[1]Reference 14, p 341

determining the signal timing. Therefore, a degree of automatic coordination with other signal controllers seems to be inherent in the system performance.

If a combined system of intersection controllers were to be used some form of interconnection would undoubtedly improve system performance. Information about the release of cars from upstream intersections could be fed into a particular controller for use in predicting future arrivals (see Fig. III-1).

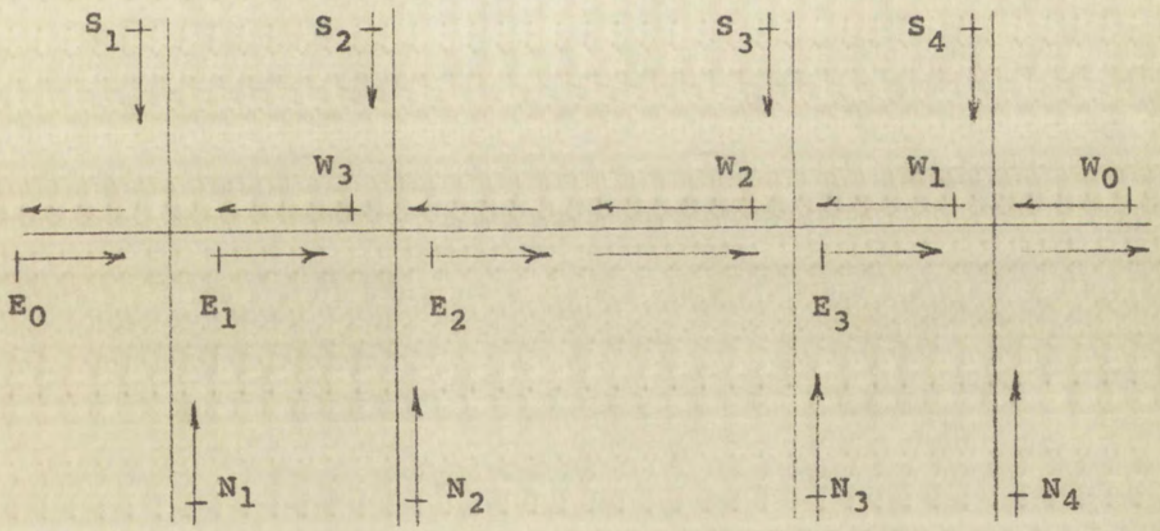

Figure III-1 Network of Intersections

In this case, coordination of signal timing is desired between the controllers at the four intersections shown. Detectors E_o, W_o, s_1 ------ s_4, and N_1 ------- N_4 would be used as upstream detectors as previously discussed. Detectors W_1 ------ W_3 and E_1 -------- E_3, would be placed at the outlet of each intersection, so as to provide prediction information for the downstream controllers. Thus, the controller at point 1 would get arrival information not only from detectors E_o, N_1, S_1 and W_3, but if desired, also from W_2, W_1 and W_o as well. Therefore, it would be possible to gain some prediction of arrivals at point 1 for a considerable time in the future, at least for the westbound flow, for which coordination is desired. Successive corrections for turn-off and turn-on maneuvers would be made by the intermediate detectors so that good coordination appears possible.

Controlling a Network of Intersections with a Single Computer

Although more will be said in Chapter IV about the economic feasibility of using a digital computer to control traffic signals, it is obvious that such a system is

much more worthy of consideration, if it were able to control a network of intersections, with a correspondingly lower per-intersection cost.

Further study in this area appears hopeful. In the above example, it would seem feasible to build one larger computer capable of controlling all four intersections. In this case, a common memory drum, with sets of writing and reading heads placed around the periphery could be used. Also, the use of common computer timing, control and power units would help reduce the per-intersection cost of the system. However, using a 50-100 KC computing speed it is doubtful if there would be a great deal of saving in the size of the arithmetic units and storage registers involved, since probably the sets of calculations required for timing each intersection would have to be performed simultaneously. However, since the sizes of the numbers to be handled are small, and the type of arithmetic operations are simple and can be efficiently timed, it appears quite possible that a relatively small computer with a single set of very fast arithmetic units (say with 1 microsecond operation times) could handle several intersections simultaneously.

CHAPTER IV

Conclusions

The work outlined here is intended to detail a solution to only one specific problem of somewhat limited scope. That is, a digital computing sequence for controlling the flow of two single-lane, unidirectional, intersecting flows of idealized vehicles. Admittedly, this limits the usefulness of the results. The extensions of the system discussed in Chapter III have been made qualitative in nature primarily because more specific discussions do not appear to be of value, until some of the concepts already developed have been tested experimentally and until considerably more theoretical work has been done. This chapter estimates the usefulness and practicality of using a computer for purposes of signal control and outlines the areas that should next be examined, if study of this problem is to be continued.

Single Intersection Control

Assuming that the system, as formulated can actually be used in the design of a device for controlling a single signalized intersection, the resulting equipment will undoubtedly be expensive. Present traffic-actuated controllers are priced in the range of $2000-3000. It is estimated that in relatively large quantities, a computer-controller, including the memory drum and all other components, could be made for perhaps $10,000-25,000 exclusive of development costs. Obviously, this makes the use of such a device difficult to justify from an economic standpoint. However, if a slightly more complex unit, costing in the same price range, could be made to handle the demands on a complicated multi-phase intersection, then the cost becomes more justifiable. The results of the limited amount of Monte Carlo simulation thus far performed indicate that such a device should respond effectively to instantaneous changes in traffic demand, as evidenced by the wide variation of signal times assigned to individual segments of traffic flows having constant mean volumes. Although a quantitative evaluation of the degree of reduction in vehicle delay

achieved is difficult, it seems apparent that such a system could come close to reducing this delay to a minimum. The value of this reduction is a subjective measurement. Therefore, the justification for using the device would be greatly dependent upon its capability for improving the traffic handling capacity of a specific intersection. As expected, the system appears to be most useful in cases where the volumes of traffic flow are high and change radically throughout the day. The usefulness of the system, cannot really be evaluated until it has been used in a field situation, so that its capabilities can be observed under actual operating conditions. This cannot be done until the basic system has been extended to include consideration of the several factors discussed in Chapter III.

Multiple Intersection Control

If a somewhat larger Computer were built that had the capability of observing traffic flow data from many detectors and then controlling a large network of intersections, this type of control becomes more feasible. Using a multiple-track high-speed drum for data storage, and a computing rate in the order of 1 MC, it seems

possible that even the detailed calculations discussed in the single intersection system could be performed for 10 or 20 intersections, still in the 1/2-second sampling interval. An even more fruitful avenue of approach might be followed through studies of a system using a less detailed type of calculation, with longer sampling intervals and somewhat averaged data.

If this could be done with a unit costing on the order of $25,000, then the per-intersection cost is competitive with present traffic-actuated controllers. It potentially has the added advantages of increasing traffic handling capacities and reducing the overall traffic delay, particularly if means for getting optimum inter-intersection coordination are used. In view of the advantages visualized and the problems involved in achieving these advantages, the following additional phases of study are outlined.

Phase II. Extension of the Single Intersection Problem

The next portion of the study should include the extension of the single intersection control system to include detailed consideration of turning maneuvers, bi-directional flows, multi-lane flows and eventually,

multi-phase situations. It is felt that this work can best be done through further Monte Carlo simulations. Including the development of more complex programs and the computer time involved in perfecting these programs and producing answers, this phase, in itself, could become a rather extensive program. The amount of computer time required might run into the tens-of-hours, so that the cost involved could be considerable.

Phase III. Field Trials with Mock-up Equipment

Obviously, a point of diminishing returns would be reached if theoretical problems were used as the only basis for continuing the study. At some point, field experiments should be made.

For example, fairly simple detecting and recording equipment could be used to verify the feasibility of predicting traffic with upstream detectors. In particular, it is important to study the effect of changes in volume on the arrival time. Through these studies, it would be possible to find the maximum prediction times that can be obtained with acceptable accuracy.

Another set of experiments could be used to verify the accuracy of some of the empirical equations discussed

in preceding chapters. For example, it was proposed that a queue of N cars be given a green time of

$$T = KN + C \quad \text{1/2-seconds}$$

where $K = 4$; $C = 7$

Field studies would aid in assuring that this time is correct, or would tend to indicate that some other relationship might be better.

As an advanced step in this phase, it would be advantageous to mock up a computer, using temporarily assembled plug-in units and a modified magnetic drum. This device could be used to control an intersection, using temporarily installed detectors to supply flow data. Here the basic system could be studied under dynamic conditions, and various modifications tested.

<u>Phase IV. Multi-intersection Control</u>

After additional theoretical and field studies have yielded a system capable of handling traffic effectively at a single intersection, the work should be continued toward the development of a system that provides optimum control of a network of intersections. Again, Monte Carlo

system simulation would be advisable. However, it is not felt that the multi-intersection problem can be effectively studied until it has been demonstrated experimentally that some of the basic concepts previously discussed are valid.

Phase V. Optimum System Design and Evaluation

Once an effective multi-intersection system has been developed a prototype computer should be constructed and evaluated, in order to gain detailed knowledge of its advantages and disadvantages. Also, the Traffic Engineering Agencies of major municipalities should be consulted to determine what system features might be most desirable, so that a flexible system could be developed with a minimum of unnecessary refinements.

General Conclusions

Thus far, only a few elementary concepts have been formulated. These must be field tested and additional techniques must be developed. Some of the basic concepts that have resulted from the present study are:

a) It should be possible to supply detailed information about traffic flow to a digital computer. If vehicle detector information is sampled at approximately a 1/2-sec rate, the flow data can be quantized with little or no loss of information.

b) The criterion of equalizing total delay in vehicle-seconds appears to be a valid means of determining signal timing at an intersection. If information is available about individual vehicle arrival times the total delay created by a given blocking interval can be found by a summation

$$D = \sum_{T_A}^{T_B} K_t \cdot t \quad + \quad \sum_{1}^{N} (4N + 7) \qquad -11$$

c) It appears that the amount of green time required to clear a given number of cars can be calculated with reasonable accuracy by a simple linear equation

$$T = 4N + 7$$

d) Prediction of vehicle arrivals, using upstream detectors, while it may yield less accurate information about arrival times, allows a more effective determination of signal timing. The validity of this statement is actually a very fundamental assumption here. Before much additional work is done, it is felt that field experiments to verify this concept should be performed.

In general, the approach taken to the problem of

traffic control has been that each traffic unit can and should be treated as a discrete entity, and consideration should be given to its position, its queuing behavior, and the delay it experiences. This is certainly not a unique idea. However, at first glance, it makes the amount of data that must be handled, in order to solve even a simple intersection control problem appear to be unnecessarily great. Certainly, high speed digital circuits would be needed, in order to handle the calculations, and probably a system operating on this basis would not be economically practical, unless considerable improvement in the traffic handling capacity of a network of urban streets and intersections is obtainable. However, it is felt that this approach offers a possible means by which such a significant improvement in traffic control can be achieved.

[illegible]

should be treated in a [illegible]

[illegible]

and the [illegible]

[illegible]

amount of [illegible]

[illegible]

unnecessarily [illegible]

[illegible]

and probably a [illegible]

[illegible]

[illegible]

urban [illegible]

[illegible]

[illegible]

can be [illegible]

APPENDIX A

Detector Sampling Circuit

In order to store the information provided by a vehicle detector into the magnetic drum of a computer, a simple sampling circuit is required. Such a circuit is shown in Fig. A-1.

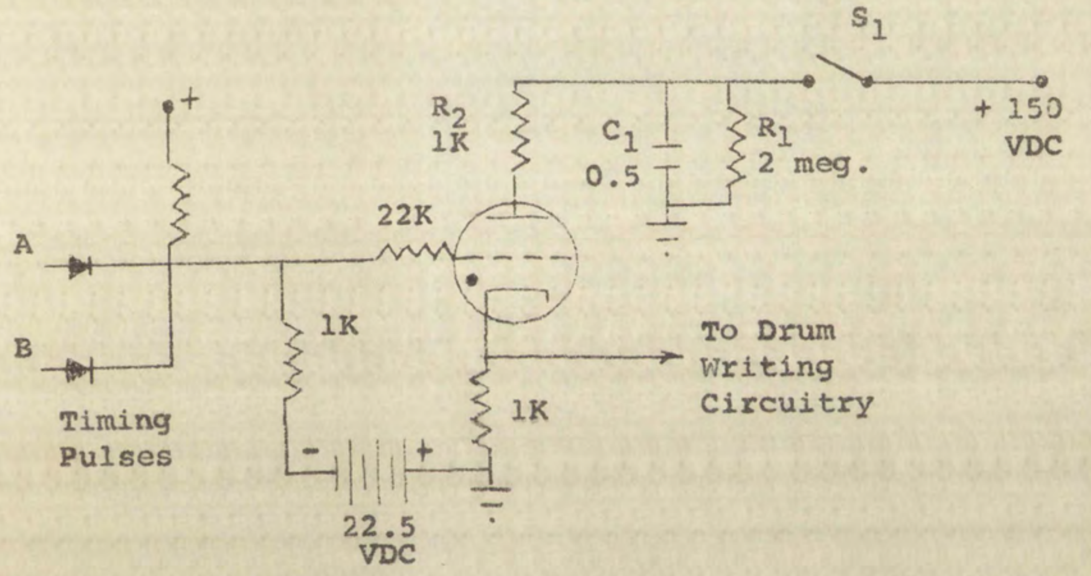

Fig. A-1

The thyratron is biased so that it is normally non-conducting, unless a positive pulse is applied to the grid. It is assumed that the detector contains some form of mechanism which closes a switch, s_1 once for the passage of one car. Closure of this switch charges capacitor C_1.

[illegible] de[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The [illegible]

[illegible]

It is assumed that [illegible]

mechanism which [illegible]

of one cat. [illegible]

thus sending a pulse on the drum writing circuitry.

Note that a writing pulse is generated only if a car has actuated s_1 and stored a charge on the capacitor within the last sampling interval.

Input A may be used to insert the writing pulse in the proper address, and input B may be used to determine the sampling rate. Assuming a 1/2 second sampling rate, time constant R_1C_1 will maintain sufficient voltage on the thyratron plate for 1/2 second. Resistor R_1 is not absolutely necessary, but, its use assures that any residual charge on C_1 after generation of a writing pulse will not cause generation of another. A sketch of the output pulse obtained at the cathode of the thyratron is shown in Fig. A-2.

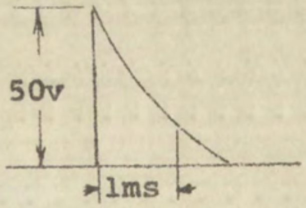

Fig. A-2

APPENDIX B

Fortran Program for Simulating System Operation

A listing of the Fortran statements used to program the IBM 704 computer for Monte Carlo simulation of the system is shown below:

```
      DIMENSION NV(2), K(1240,2), KK(60), KI(1240,2)

  1.  READ 5, NC, JG,
  2.  READ 5, (NV(M), M = 1,2)
  3.  PRINT 5, (NV(M), M = 1,2)
  5.  FORMAT (2I4)
  6.  FORMAT (I5)
  8.  FORMAT (30I2)
 14.  DO 50 M = 1,2
 15.  DO 40 I = 760, 1240
 16.  A = NV(M)
 17.  IF(RANDOMF(0) - (A/7200.)) 27, 27, 21
 21.  K(I,M) = 0
 25.  GO TO 40
 27.  K(I,M) = 1
 40.  CONTINUE
 50.  CONTINUE
 53.  MW = 1
 54.  MM = 2
 59.  DO 350 ND = 1, NC
 60.  DO 100 KS = 1, JG
 61.  DO 90 M = 1,2
 62.  DO 68 I = 760, 1240
 68.  KI(I,M) = K(I,M)
 69.  DO 70 I = 760, 1239
 70.  K(I+1,M) = KI(I,M)
 71.  K(760,M) = KI(1240,M)
 72.  A = NV(M)
 73.  IF(RANDOMF(0) - (A/7200.)) 85, 85, 80
```

```
 80.   K(940,M) = 0
 82.   GO TO 90
 85.   K(940,M) = 0
 90.   CONTINUE
100.   CONTINUE
110.   JR = JG + 1004
111.   M = MW
112.   NB = 0
113.   NBI = 0
114.   LDW = 0
115.   DO 120 I = 994, JR
120.   NB = K(I,M) + NB
123.   JCL = 987 - 4*NB
124.   IF (JCL - 760) 125, 126, 126
125.   JCL = 760
128.   DO 130 I = JCL,JR
130.   NBI = K(I,M) + NBI
134.   IF (NBI-NB) 136, 147, 140
136.   SENSE LIGHT 1
140.   NB = NBI
141.   NBI = 0
142.   GO TO 123
147.   IF (NB) 148, 162, 150
148.   SENSE LIGHT 4
149.   PAUSE
150.   DO 160 I = JCL, JR
160.   LDW = LDW + (K(I,M))*(I-994)
161.   LDW = LDW + 2*NB**2 + 9 * NB - 11
162.   JX = 981 - 4*NB
       M = MM
       NBI = 0
       NB = 0
       LDM = 0
164.   IF (JX - 760) 165, 166, 167
165.   JX = 760
166.   JCL = 760
       GO TO 185
167.   DO 170 I = JX, 998
170.   NB = K(I,M) + NB
180.   JCL = JX - 4*NB - 13
183.   IF (JCL-760) 184, 185, 185
184.   JCL = 760
185.   DO 190 I = JCL, 998
```

```
190.    NBI = K(I,M)+NBI
195.    IF (NBI - NB) 196, 206, 200
196.    SENSE LIGHT 2
200.    NB = NBI
201.    NBI = 0
202.    GO TO 180
206.    IF (NB) 207,224,210
207.    SENSE LIGHT 4
208.    PAUSE
210.    DO 220 I = JCK, 998
220.    LDM = LDM + K(I,M) *(I-JX)
221.    LDM = LDM + 2*NB**2+9*NB-11
223.    IF (LDW) 224, 225, 225
224.    LDW = 0
225.    IF (LDM) 226, 228, 228
226.    LDM = 0
228.    IF (LDW-LDM) 230, 230, 300
230.    JG = JG+1
240.    IF (JG-236) 250, 300, 295
250.    DO 280   M = 1,2
251.    DO 252 I = 760, 1240
252.    KI(I,M) = K(I,M)
253.    DO 254 I = 760, 1239
254.    K(I+1,M) = KI(I,M)
255.    K(760,M) = KI(1240, M)
256.    A = NV(M)
257.    IF(RANDOMF(0) - (A/7200.)) 270, 270, 260
260.    K(940,M) = 0
265.    GO TO 280
270.    K(940,M) = 1
280.    CONTINUE
285.    GO TO 110
295.    JG = 236
300.    MW = MM
305.    IF (MM - 1) 306, 310, 320
306.    SENSE LIGHT 3
310.    MM = 2
315.    GO TO 325
320.    MM = 1
325.    PRINT 6, LDW
326.    PRINT 6, LDM
327.    PRINT 340, MW, JG
```

```
330.   JG = 988 - JX
335.   PRINT 340, MM, JG
340.   FORMAT (I5, I10)
350.   CONTINUE
360.   GO TO 2
```

The symbols used in the program stand for the following quantities (All values are fixed point unless otherwise noted):

Symbol	Quantity	Range of Values
M	Number of traffic phase	1 or 2
NV(M)	Volume of traffic on phase M in vph	100 to 1600
NC	Number of light changes to be simulated for each pair NV(1) and NV(2)	up to 1000
JG	Green time in 1/2 seconds	1 to 236
I	Real time variable in 1/2 sec I 1000 represents future arrivals I 1000 represents past arrivals	760 to 1240
K(I,M)	Traffic variable, representing arrival of a car at time I on phase M	0 or 1
MW	Number of waiting phase	1 or 2
MM	Number of moving phase	1 or 2 (MM $\neq$ MW)

---

[1] 1000 is taken as the zero time reference, since I is to be used as a subscript for other variables, and Fortran language forbids negative subscripts.

Symbol	Quantity	Range of Values
ND, KS	Dummy variables for DO loop indices	
A	Floating Point Dummy variable for Random Function	A = NV
JR	Absolute time in 1/2 seconds when red light was assigned to waiting phase	1008 to 1240
JCL	Absolute time in 1/2 seconds when last car in waiting queue will clear	760 to 983
JX	Absolute time when light would be returned to moving phase if present number of waiting cars is cleared	760 to 971
NB	Number of cars blocked	1 to 240
NBI	Intermediate value of NB	
LDW	Total delay in vehicle-1/2 seconds to cars in waiting phase	
LDM	Total delay to cars on moving phase if lights are changed	

The functions of the principal statements are as follows:

a. Statements 1 - 8 These are input statements to provide input data to the machine. NC is the number of trials to be made for each pair of flow volumes. JG is an initial value of Green light interval to be given phase 2.

NV(1) and NV(2) are the mean volumes on phases 1 and 2 in vehicles per hour.

b. <u>Statements 14 - 50</u> This set of statements generates K(I,M), the string of 1's and 0's representing the arrival of cars at time I on phase M. The 1's and 0's are generated using a pseudo-random function generated by the machine (RANDOMF(0)). The average number of 1's in a given time interval will correspond to NV, the volume on a particular phase. K(I,M) represents the history of traffic arrivals which in the actual system would be supplied by vehicle detector information.

c. <u>Statements 53 and 54</u> Initially, phase 1 is made the waiting phase and phase 2 the moving phase.

d. <u>Statement 59</u> This instructs the machine to go through NC changes of the intersection signals for the particular pair of values for NV(1) and NV(2) provided above.

e. <u>Statement 60 - 100</u> After each signal transfer, this set of statements shifts the traffic flow information JG 1/2 seconds to simulate the waiting period of the initial green interval. K(I,M) is shifted one 1/2 second at a time

and new simulated flow information is inserted at I = 940. This assumes the detectors are placed 60 1/2 seconds upstream from the intersection.

For time I < 940, the "echo" of past flow information is used to describe the traffic flow in an average sense. The "echo" is generated by statement 71.

f. Statement 110 The absolute time at which the waiting phase was blocked is calculated to be: JR = JG + 1004. This includes 4-1/2-seconds of blocking time due to the last 2 seconds of the yellow interval.

g. Statements 111 - 114 These statements initialize M, NB, NBT, and LDW prior to the calculation of the number of cars waiting and the delay on the waiting phase.

h. Statements 115 and 120 The number of cars blocked by the red interval is calculated by adding up the number of 1's appearing in K(I,MW) between I = 994 and I = JR. I = 994 is the earliest time at which a green light can be given to the waiting phase, assuming a 3 second (6 - 1/2-second) clearance yellow light for the moving phase.

i. <u>Statement 123</u> This statement calculates the time (JCL) at which the waiting queue of NB cars will no longer block cars on that phase, if the lights are changed.

JCL = 1000 - (4NB + 7) - 6 = 987 - 4NB

where 1000 represents zero time at the instant of calculations, (4NB + 7) is the green time required to clear NB cars, and the number 6 is the yellow interval in 1/2-seconds which must be provided to the moving phase prior to re-assigning the green light.

j. <u>Statements 124 and 125</u> These statements check to see that JCL is not less than 760. In the actual system this time should not be less than 760, since this would represent a blocking time in excess of the 2 - minute maximum.

k. <u>Statements 128 - 142</u> This group of statements recalculates NB using the new blocking interval JCL to JR. This intermediate value of NB (called NBI) is compared to the first value and if NBI is greater than NB, NB is replaced by the new value. Statements 123 to 142 form an iterative loop which is used to find the total number of cars blocked, including the blocking caused by the waiting

queue. An iterative process is required since the discovery of additional blocked cars may extend the blocking interval to create interference to even more cars (See Appendix C).

l. Statements 147 to 161 Once a final value of NB has been found for the waiting phase, these statements calculate the total delay, LDW, in vehicle 1/2-seconds to the NB cars. If NB = 0, i.e. no cars have been blocked, LDW = 0

If NB = 1, then LDW is found in two steps (1) The red light blocking delay is found by statements 150 and 160 which yield

$$LDW_1 = \sum_{I = JCL}^{JR} K(I,MW)\ (I - 994)$$

where (I - 994) is the waiting time for each blocked car. Note the Ith term has a value only when K(I,MW) = 1, i.e. if a car has arrived in that time segment.

(2) An added delay due to queue clearance time is found by statement 161, then

$$LDW_2 = LDW_1 + 2NB^2 + 9NB - 11$$

(See Chapter I for origin of this equation)

m. Statements 162 to 221 This group of statements performs the same set of calculations for the moving phase.

Initially the blocking interval is taken from JX, the time at which the green light could be re-assigned to the presently moving phase if the waiting phase was given enough green time to clear all blocked cars calculated above. The blocking interval is assumed to run up to I = 998, which is one second after the yellow light would appear if the lights are changed. Once a value for NB, now the number potentially blocked on the moving phase, is found, a corrected value for the beginning of the blocking interval JCL, is found which again includes queue clearance time. By an iterative process, a final value for NB is found and then LDM, the potential delay on the moving phase is calculated.

n. Statements 223 - 226 These statements verify that neither LDW or LDM are negative, if they are, they are set equal to zero.

o. Statement 228 This compares LDW and LDM. If and only if LDW, the waiting phase delay is greater than LDM' the moving phase delay, control is transferred to Statement 300 and the lights are changed. If LDW = LDM control is transferred to Statement 230 and the necessary operations

are performed to shift the traffic flow information so that a new calculation can be made one 1/2-second later.

p. Statements 230, 240 and 295 JG is increased by one 1/2 second and checked to see that it is not greater than 236,[1] the assumed upper bound for a green interval. If JG > 236 control is transferred to statement 295, wherein JG is set equal to 236. If JG = 236, control is transferred to statement 300 and the lights are changed.

q. Statements 250 to 285 The traffic flow information is shifted one 1/2-second to simulate the waiting period allowed before the delay situation is again examined. New information is generated for I = 940. Control then transfers to statement 110 for a new calculation of LDW and LDM.

r. Statements 300 - 320 If control reaches statement 300, this means that the lights are to be changed. These statements interchange the values for MW and MM, thus indicating the transfer.

---

[1] In the actual system, the upper bound is 240.

appendix [illegible]

are [illegible]

[illegible]

by the 1/2 [illegible]

greater than [illegible]

increase [illegible]

past 25% [illegible]

control is [illegible]

changed.

[illegible]

market is [illegible]

period allowed [illegible]

New information [illegible]

transfers to [illegible]

but [illegible]

[illegible]

past [illegible]

These statements [illegible]

thus indicating [illegible]

[illegible]

s. Statements 325 - 327 The machine is instructed to print out LDW, LDM and the total green time, JG, which was used by the previous moving phase.

t. Statements 330 and 335 An initial value for the green interval to be given to the new moving phase is calculated and printed. This calculation uses the fact that

JX = 981 - 4 NB

Since we want

JG = 4NB + 7 (See Chapter I)

Then JG = 988 - JX

u. Statement 350 This statement completes the DO loop starting with statement 59. When this loop has been completed NC times control is transferred to Statement 360.

v. Statement 360 This calls for a new pair of values for NV(1) and NV(2) from the card reader and the above process is repeated.

The above program is probably somewhat inefficient. It is translated by the Fortran routine into 769 share assembly program statements. The computer requires about 15 seconds to calculate one signal transfer. In many cases, of course, this may include several intermediate calculations of LDW and LDM for each 1/2-second of green interval extension until LDW $>$ LDM.

APPENDIX C

Iterations Required To Find the Total Number of Cars Blocked

Problem. Given a red light interval of T 1/2-seconds, blocking a lane of Poisson distributed traffic with a mean spacing of A 1/2-seconds, how many iterative calculations must be made to determine the total number of cars blocked by both the interval R and the resulting queue?

The probability of K cars arriving in a given time interval, T, is

$$P(K) = \frac{e^{-m} m^K}{K!}$$

where: $m = \frac{T}{A}$ = the average number arriving in the interval

Thus in an interval T

$$P(K) = \frac{e^{-\left(\frac{T}{A}\right)} \left(\frac{T}{A}\right)^K}{K!}$$

The time required to clear a queue of K cars is taken as

$\Delta T_o = 4K + 7$ 1/2-seconds

thus the expected value of ΔT_o is

$$\Delta T_o = \sum_{1}^{\infty} P(K)\,(4K + 7)$$

$$= \sum_{1}^{\infty} \frac{e^{-m} m^K}{K!}\,(4K + 7) \quad ; K = T/A$$

But $\sum_{0}^{\infty} P(K) = 1$

Therefore $\sum_{1}^{\infty} P(K) = 1 - P(0) = (1 - e^{-m})$

And
$$\Delta T_o = 4\sum_{1}^{\infty} K\,P(K) + 7\sum_{1}^{\infty} P(K)$$

$$= 4\sum_{1}^{\infty} \frac{e^{-m}m^{K}}{(K-1)!} + 7(1 - e^{-m})$$

$$= 4\,M \sum_{1}^{\infty} P(K-1) + 7\,(1 - e^{-m})$$

$$= 4\,M \sum_{j=0}^{\infty} P(j) + 7\,(1 - e^{-m})$$

where $j = k-1$

$$\Delta T_o = 4\,M + 7(1-e^{-m})$$

The sum of T and ΔT_o represents a revised value of total blocking time. If additional cars are blocked by this interval, then additional clearance time ΔT_1 is required. The time $(T+\Delta T_o + \Delta T_1)$ may again block more cars, and so forth. The total blocking time is $(T + \Delta T_o + \Delta T_1 + \cdot\cdot\cdot \Delta T\ )$. Since all times are quantized to the nearest 1/2-second, it will be assumed that a sufficient number of iterations will have been performed when $\Delta T_i < 1/2$. Each additional blocked car is assumed to create a ΔT of 4.

Therefore $\Delta T_i = 4 \, \Delta K_i$

where ΔK_i is the ith incremental number of blocked cars over and above the number blocked by T.

$$E(\Delta K_1) = \sum_{0}^{\infty} P_1(K) K_1 \quad ; \text{ where } P_1(K) = P(K) \text{ with } m = m_1 = \frac{\Delta T_o}{A}$$

$$E(\Delta T_1) = 4 \sum_{0}^{\infty} P_1(K) K_1 = 4 \sum_{0}^{\infty} \frac{e^{-m_1} m_1^{K}}{K!} \; K = 4 m_1 = \frac{4 \Delta T_o}{A}$$

Similarly $E(\Delta T_2) = 4 m_2 = \dfrac{4 \Delta T_1}{A} = \left(\dfrac{4}{A}\right)^2 \Delta T_o$

And $E(\Delta T_i) = \left(\dfrac{4}{A}\right)^i \Delta T_o = \left(\dfrac{4}{A}\right)^i \; 4m + 7(1 - e^{-m})$

$$m = \frac{T}{A}$$

As a typical example, consider a highly saturated two-flow intersection with 800 vph on each phase. Thus A = 7200/800 = 9 - 1/2-seconds.

Let $T = 240$ - 1/2 seconds

Then = $M = 240/9 = 26$

And $(1 - e^{-m}) = 1$

$$\Delta T_i = \left(\frac{4}{9}\right)^i \left[4 \cdot 26 + 7\right] = \left(\frac{4}{9}\right)^i \; 111$$

For $\Delta T_i < 1/2$

Then $\left(\dfrac{4}{9}\right)^i < 1/2 \cdot \dfrac{1}{111}$

Or $\left(\dfrac{4}{9}\right)^i < \dfrac{1}{222} = 0.0045$

And $i > 6.6$ iterations

Thus, under the severest conditions, the total number of iterations should not be more than 7, after the initial number of blocked cars is found. The conclusion that is drawn from the above is that a total of 14 drum revolutions would be adequate to get a sufficiently converged value of the number of cars potentially blocked by a given red-light interval, T. If 30 revolutions are available in each 1/2-second calculating period, then ample time is provided to perform this calculation, as well as the other operations required to determine delay, etc.

APPENDIX D

Tabulation of Results from the Monte Carlo Simulation

Table II-B summarizes the data given below. The raw data was calculated using the 1/2-second as the unit of time. Conversion to seconds is the only modification performed on the data listed here. The symbols used are:

ϕ_1 = volume on phase one in vehicles per hour

ϕ_2 = volume on phase two in vehicles per hour

G_1 = green interval assigned to phase one in seconds

G_2 = green interval assigned to phase two in seconds

D_1 = total delay incurred by phase one vehicles in vehicle-seconds

D_2 = total delay incurred by phase two vehicles in vehicle-seconds

ϕ_1 = 200 vph ϕ_2 = 200 vph

G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
54.0	107.5	75.5	86.5
38.5	61.5	47.5	55.5
25.5	122.0	75.5	6.5
30.5	20.0	24.0	21.5
25.0	33.5	31.0	8.0
10.0	34.5	5.5	0.5
15.5	29.0	42.0	0.5
7.5	28.0	27.5	75.0
74.5	27.0	16.5	250.5
7.5	6.0	28.0	37.0

ϕ_1 = 200 vph ϕ_2 = 400 vph

G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
5.5	12.5	12.5	32.0
25.5	19.0	29.0	49.5
8.0	13.0	29.0	17.5
9.5	68.0	43.5	81.0
9.5	46.0	30.5	61.5
7.5	29.5	19.5	29.5
7.5	23.5	14.0	23.5
9.5	24.0	22.5	96.5
7.0	15.0	29.5	20.5
5.5	17.5	34.5	27.5

ϕ_1 = 200 vph ϕ_2 = 600 vph

G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
11.5	73.0	21.0	107.5
18.5	26.0	23.5	27.5
6.5	125.0	31.5	47.5
16.5	0.5	20.0	24.5
29.5	6.5	21.0	77.5
10.0	24.5	30.0	63.5
10.5	16.5	11.5	12.0
9.5	118.5	56.0	123.5
9.5	58.0	26.5	59.5
5.5	28.0	35.5	29.5

ϕ_1 = 200 vph		ϕ_2 = 800 vph	
G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
13.5	66.0	34.0	84.5
5.5	11.0	13.5	4.0
5.5	15.5	23.5	98.0
7.5	27.5	39.5	34.0
14.5	28.5	24.5	52.5
18.0	118.5	27.0	179.0
7.5	29.0	43.5	54.0
11.5	59.0	17.0	69.5
15.5	254.5	65.0	176.0
21.5	203.0	111.0	50.5

ϕ_1 = 200 vph		ϕ_2 = 1000 vph	
G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
20.0	207.5	55.5	154.5
12.0	30.5	23.5	70.5
13.5	184.0	59.0	155.0
13.5	107.5	35.0	55.0
21.5	419.5	80.5	513.0
13.5	195.0	63.5	52.5
7.5	42.0	31.5	12.0
7.5	72.0	46.5	24.0
11.5	233.5	74.5	138.0
12.0	75.5	49.5	104.5

ϕ_1 = 200 vph		ϕ_2 = 1200 vph	
G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
16.5	63.5	53.5	208.5
5.5	420.5	118.0*	537.5
13.5	359.5	97.5	309.0
17.5	384.0	95.5	265.5
15.5	226.0	70.5	300.5
5.5	38.5	43.5	16.0
11.5	84.0	44.5	140.5
97.0	323.0	118.0*	357.0
15.5	301.5	97.0	807.5
15.5	264.0	118.0*	493.0

*Saturated

ϕ_1 = 200 vph ϕ_2 = 1400 vph

G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
7.5	191.5	118.0*	523.0
11.5	162.5	112.0	317.0
14.0	97.5	62.5	398.5
17.5	385.0	118.0*	915.0
17.5	392.5	71.5	367.0
15.5	351.0	118.0*	1018.5
17.5	459.0	118.0*	1971.5
23.5	507.0	118.0*	2391.5
19.5	620.0	118.0*	1629.5
19.5	477.5	114.0	445.5

ϕ_1 = 400 vph ϕ_2 = 400 vph

G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
28.5	197.5	55.5	19.5
11.5	44.0	9.5	26.0
9.5	26.0	17.5	20.5
7.5	15.0	18.0	67.0
10.5	12.0	11.5	6.5
9.5	14.0	11.5	14.5
14.0	40.5	5.5	13.0
7.5	47.0	18.5	85.5
17.5	38.0	17.5	3.5
11.5	78.0	18.5	85.0

ϕ_1 = 400 vph ϕ_2 = 600 vph

G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
13.5	53.0	13.5	72.5
24.5	57.0	16.0	29.0
7.5	35.0	11.5	39.5
15.5	107.0	26.5	112.5
18.5	46.0	19.5	68.0
13.5	107.5	19.5	92.5
20.0	236.5	41.0	304.5
11.5	63.0	51.5	61.5
31.0	41.5	15.5	73.0
19.0	168.0	26.5	275.0

ϕ_1 = 400 vph		ϕ_2 = 800 vph	
G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
23.0	158.0	33.5	153.5
11.5	31.5	42.5	76.0
11.5	76.0	17.5	84.0
9.5	75.5	48.5	165.0
19.5	136.5	23.5	198.5
11.5	110.0	41.5	142.0
7.5	33.5	23.5	24.0
9.5	41.0	9.5	44.5
22.5	109.0	44.0	149.5
19.5	167.5	19.5	185.5

ϕ_1 = 400 vph		ϕ_2 = 1000 vph	
G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
11.5	63.0	33.5	44.5
9.5	152.0	61.5	175.5
15.5	219.0	66.5	168.5
31.5	649.0	100.0	474.0
23.5	375.5	63.5	396.5
23.5	496.0	72.5	454.0
9.5	20.5	49.5	142.0
9.5	94.0	33.5	142.0
9.5	56.0	30.0	85.0
17.5	228.0	51.5	264.5

ϕ_1 = 400 vph		ϕ_2 = 1200 vph	
G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
37.5	772.0	118.0*	813.5
38.0	780.0	116.5	793.5
41.5	1013.5	118.0*	800.0
43.5	1017.5	114.0	952.0
56.5	790.0	114.0	1087.0
21.5	649.0	129.5	491.0
13.5	195.0	46.5	297.0
17.5	280.0	51.0	596.5
19.5	282.0	88.5	452.0
**	**	**	**

*Saturated

**Only 18 trial solutions were calculated at this point due to termination of scheduled computer time.

ϕ_1 = 600 vph ϕ_2 = 600 vph

G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
32.0	192.0	27.5	141.0
16.5	126.5	23.5	81.0
19.5	124.0	23.0	141.0
29.5	173.5	17.5	110.0
33.0	177.5	42.5	383.0
29.5	375.5	35.5	253.5
25.5	182.5	27.5	78.0
11.5	43.5	13.5	9.0
24.5	46.5	18.0	77.5
19.5	119.0	11.5	59.5

ϕ_1 = 600 vph ϕ_2 = 800 vph

G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
29.5	164.0	33.5	218.0
14.0	51.0	29.5	16.5
17.5	196.5	43.5	169.5
19.5	139.5	23.5	56.5
19.5	88.5	33.0	165.5
31.0	174.5	23.5	115.5
37.5	487.5	38.5	225.5
29.5	480.0	51.0	489.5
15.5	113.5	37.0	112.5
11.5	84.5	25.0	122.0

ϕ_1 = 600 vph ϕ_2 = 1000 vph

G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
18.0	106.5	27.5	118.5
15.5	147.5	26.5	216.5
23.5	345.0	63.0	461.5
25.5	250.0	49.0	493.0
75.5	576.0	81.0	885.5
59.5	915.5	79.5	844.5
37.5	495.0	48.0	472.0
53.5	857.5	54.5	713.0
43.5	310.5	56.0	656.5
57.5	885.0	84.0	568.0

ϕ_1 = 800 vph		ϕ_2 = 800 vph	
G_1 sec	D_1 vehicle-sec	G_2 sec	D_2 vehicle-sec
71.5	1007.0	93.0	859.0
77.5	777.5	118.0*	908.0
81.5	781.0	89.5	616.0
91.5	848.5	87.5	576.0
99.5	1092.5	81.5	690.0
70.5	1237.5	89.5	799.5
73.0	760.0	75.5	578.0
49.0	636.5	49.5	520.5
63.5	802.5	43.5	751.5
68.5	708.0	49.5	687.0

*Saturated

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