Two Applications of High Order Methods: Wave Propagation and Accelerator Physics

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Abstract

Numerical simulations of partial differential equations (PDE) are used to predict the behavior of complex physics phenomena when the real life experiments are expensive. Discretization of a PDE is the representation of the continuous problem as a discrete problem that can be solved on a computer. The discretization always introduces a certain inaccuracy caused by the numerical approximation. By increasing the computational cost of the numerical algorithm the solution can be computed more accurately. In the theory of numerical analysis this fact is called the convergence of the numerical algorithm. The idea behind high order methods is to improve the rate of convergence so a slight increase of numerical work will lead to a significant increase of accuracy.

We consider the wave equation on the domain around the body of a complex shape. This problem is challenging since because of the geometrical complexity. In this work we propose the Hybrid Hermite-Discontinuous Galerkin method of the overset grids for the wave equation. It combines highly efficient Hermite methods acting on Cartesian grids with the Discontinuous Galerkin method acting on a curvilinear grid around the body. The results of long time simulation will be presented to demonstrate the high order accuracy and stability of the proposed method.

High energy particle physicists would like polarized electron beams in the next generation of circular particle accelerators. The so-called DK formulas predict well the level of polarization at current energies but they may not be valid at the higher energies of the next generation machines. We aim the check the validity of the formulas and extend them if needed by numerically integrating the so-called full Bloch equation. This is a system of PDEs which is more fundamental than the DK formulas and is believed to contain all the relevant physics. The full Bloch equations can be classified as an advection-diffusion problem in 3 degrees of freedom which is an enormous problem to solve on the computer. Our idea is to use the fast Fourier transform for the angular dimensions and introduce the parallel in Fourier modes numerical algorithm. The results of numerical simulation for a one degree of freedom and a two degrees of freedom model circular accelerator will be presented.