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Abstract

In this paper, the polar controller is applied to the three-state, one-mode Moore-Greitzer Compressor model. A benchmark is first established with a backstepping controller. The polar control method is then explained, and compared to the backstepping controller. The polar controller is used successfully to control the surge and stall problem in the presence of both disturbances and uncertainties.

1. Introduction

The rotating stall and surge control problem in axial flow compressors has been studied in several recent papers [1]-[3], and others. The control motivation being that at the maximum efficiency operating condition, the compressor is at an unstable equilibrium, and therefore without sufficient control, the compressor will leave the axisymmetric design flow, and enter either a rotating stall, deep surge, or classic surge. Rotating stall is an inherently two-dimensional local compression system oscillation. It manifests itself as a region of severely reduced flow that rotates at a fraction of the rotor speed. Surge is a one-dimensional axisymmetric global compression system oscillation that can cause flameout and engine damage. Traditionally, designers have avoided the problems of rotating stall and surge by accepting a lower efficiency of the compression system.

One model to describe the instabilities of the compressor is the three-state, one-mode Moore-Greitzer model found in [4] and [5]. Several variations exist on the basic model, and in this paper one such variation is used to evaluate the control law required for stabilization. To achieve stabilization, several techniques have been applied by various authors: backstepping [5], nonlinear robust control [3], and nonlinear disturbance rejection [2]. These three control techniques will be discussed, and the recently introduced polar control method [6] will be applied, and compared to previous works. The recently published method of stabilizing nonlinear systems via polar control [6], provides an efficient, multistage logic controller with adjustable parameters. The design of the controller is based on a Lyapunov approach using backstepping, where the designer has the freedom to cancel unwanted nonlinearities during backstepping. However, in the standard backstepping approach, one cannot expect to cancel the nonlinearities exactly. Therefore in the polar control design, an error term is defined, and the system is then modified to reflect the error and its derivative, making the polar controller robust in the presence of uncertainties.

This paper is organized as follows. Section 2. presents an overview of a benchmark backstepping design for a surge and stall model. Section 3. presents the polar controller design. Section 4. deals with controller design for disturbance rejection while section 5. deals with the robust control problem. Finally, our conclusions are given in section 6.

2. Backstepping Design Overview

The backstepping technique developed in [5] is first reviewed to establish a benchmark for analyzing the control law. To ensure that the controllers are evaluated evenly, the initial conditions used in [3] are used throughout this paper.

The first model utilized is found in equations (2.7), (2.8), and (2.9) of [5], and is duplicated in equation (1) below. This model has already been translated to the origin such that the equilibrium state is at $\phi_e = \psi_e = R_e = 0$. The control law has also been defined to be $u = \frac{1}{\beta^2}(\phi + 1 - \Phi_T)$.

$$\begin{aligned} \dot{\phi} &= -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R \\ \dot{R} &= \sigma R(-2\phi - \phi^2 - R) \\ \dot{\psi} &= u \end{aligned} \tag{1}$$

Where R, the normalized stall cell squared amplitude, is restricted to being a nonnegative number. Note that ϕ is the mass flow translated to the origin, Φ_T is the mass flow through the throttle, and ψ is the pressure rise translated to the origin. ψ is allowed to be negative, but has a lower bound defined by $\psi = \Psi - \Psi_{c0} - 2$, where Ψ is the untranslated pressure rise, and is restricted to being a positive number due to physical limitations. Ψ_{c0}, β, and σ

are constant parameters and β and σ are required to be positive. Note that $R = 0$ is an invariant set, and defines a plane. Therefore if the system moves to a state where $R = 0$, it will stay on that plane, and the system could be represented as a two state system. This is what Krstić and Kokotović have noted and used to start their design process in the section "Design for No-Stall Model" in [5]. They have also noted that an upper bound exists for R as defined by equation (2.11) in [5], and have exploited these two properties to simplify their design. By careful examination of (1), it can be seen that once ϕ is forced to the zero state, R will follow, and therefore it is not necessary to control R directly. This process was started by Krstić and Kokotović and will be used throughout this paper. Following the procedure in [5] a control law was found to be

$$\begin{aligned} u &= (c_2+c_0)c_0\phi - (c_2+c_0)\psi - 3(c_2+c_0)R \\ &+ 3R\left[\left(\frac{\sigma(\sigma+1)}{2} - c_0\right)\phi - \frac{\sigma^2}{2c_0}\psi\right] \\ &- 3R\left[\sigma\left(\frac{3\sigma}{2c_0} - 1\right)R\right] \end{aligned} \tag{2}$$

Where $c_0 = c_1 + \frac{9}{8}$, $c_1 \geq 0$, and $c_2 > 0$ make the system globally asymptotically stable. Note that when $R = 0$, the control law reduces to a linear controller. The closed-loop system defined by (1) and (2) was simulated using initial conditions $\phi_0 = -0.25$, $R_0 = 0.5$, and $\psi_0 = 0$, and system parameters $\sigma = 3.6$ and $c_1 = c_2 = 1$. The results of the simulation are shown in Figure 1.

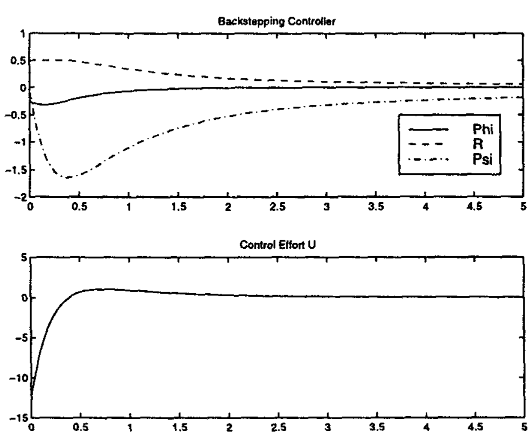

Figure 1: Krstić and Kokotović's Backstepping Controller, and the control effort u.

3. Polar Control

In order to implement the polar controller, three steps must be considered on the m-dimensional space R^m. The first step to consider is the simplex partition of the unit sphere in R^m into $m+1$ partition vectors [6]. Next, the polar partition is established to ensure that an overlapping partition exists on R^m for $0 \leq \omega \leq \frac{1}{m}$; where ω is a sector width parameter that defines the amount of overlap between partitions. Finally, the polar controller is established. In this paper, we will only consider a polar controller for a system with a strict feedback structure; which is in contrast to the embedded polar controller for strict feedback systems. The difference between the two structures is that the former has only one design step, and relies on the strict feedback nature of the system. The latter utilizes strict feedback as well, but develops a controller to directly influence each state of the plant. Effectively the system with one the one-step controller design is used to drive the plant to a stability manifold and once on the manifold the system will stabilize. The embedded polar controller works to directly stabilize each state of the system until the entire plant reaches the desired operating point.

The simplex partition is defined on the m-dimensional space, and is a collection of $m+1$ unit length vectors $\{\Pi_1, \Pi_2, ..., \Pi_{m+1}\}$ satisfying the following conditions:

$$\begin{aligned} \Pi_i^T\Pi_i &= 1, i = 1, 2, ..., m+1 \\ \Pi_i^T\Pi_j &= \eta, i \neq j \\ \|\Pi_i - \Pi_j\| &= d, i \neq j \\ \Pi_1 + \Pi_2 + ... + \Pi_{m+1} &= 0 \end{aligned} \tag{3}$$

where $\|\cdot\|$ is the 2-norm. The polar partition of R^m is defined by the open sets

$$C_i(\omega) = \left\{ S \in R^m : V_i^T\left(\frac{S}{\|S\|}\right) > \omega, \right\} \tag{4}$$

where $\omega > 0, V_i = -\Pi_i, i = 1, 2, ..., m+1$. The polar control is defined as

$$U = K(\|S\|)\left(\frac{1}{\sum_{i=1}^{m+1}\delta_i}\right)\sum_{i=1}^{m+1}\delta_i\Pi_i \tag{5}$$

where m is the dimension of the control input, and where

$$\begin{aligned} \delta_i &= max(\gamma_i, 0) && (6) \\ \gamma_i &= V_i^T\frac{S}{\|S\|} - \omega, 0 \leq \omega \leq \frac{1}{m} && (7) \\ V_i &= -\Pi_i;\ i = 1, 2, ..., m+1 && (8) \end{aligned}$$

The function $K(\|S\|)$ is a monotonically increasing scalar function of $\|S\|$, and the condition on ω is to ensure that an overlapping partition exits on $\Re^m$. While the backstepping controller made use of the knowledge of the nonlinearities of the system, the polar controller does not. The polar controller merely dominates the nonlinearities, and thus provides a degree of robustness. Since the backstepping controller can utilize the nonlinearities to the designer's advantage, the controller can be made to use less control effort than the polar controller, but overall the polar controller can be made more robust.

Using the model of (1) while following the approach used in [5], avoiding cancellations, using the fact that R has an upper bound, and that $\dot{R}$ is negative semidefinite, a polar controller was developed. Letting $\psi_c = -\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3$, and noting that ψ_c can be rewritten as $-\frac{1}{2}\left(\phi + \frac{3}{2}\right)^2 \phi + \frac{9}{8}\phi$, the first equation of model (1) can be transformed into

$$\dot{\phi} = -\psi_{des}(\phi, R) + \frac{9}{8}\phi - \frac{1}{2}\left(\phi + \frac{3}{2}\right)^2 \phi - 3R\phi - 3R$$

Now letting $\psi_{des}(\phi, R) = (c_1 + \frac{9}{8})\phi - 3R = c_0\phi - 3R$, and defining $S = \psi - \psi_{des}$, we obtain

$$\begin{aligned}
\dot{\phi} &= -S - c_1\phi - \frac{1}{2}\left(\phi + \frac{3}{2}\right)^2 \phi - 3R\phi \\
\dot{S} &= c_0\left[S + c_1\phi + \frac{1}{2}\left(\phi + \frac{3}{2}\right)^2 \phi + 3R\phi\right] \\
&\quad +3\sigma R\left(-2\phi - \phi^2 - R\right) + U \qquad (9) \\
\dot{R} &= \sigma R\left(-2\phi - \phi^2 - R\right)
\end{aligned}$$

Now applying the rules for polar control, and noting that for this particular case $m = 1$, V_i was chosen such that $V_1 = 1$ and $V_2 = -1$. The term $K(\|S\|)$ was chosen to be $K * \|S\|$, where K is a positive real constant that can be varied to increase the gain of the controller. By increasing the gain of the controller, the nonlinearities and any perturbations or disturbances can be dominated by the controller U, and thus global asymptotic stability is achieved. Using the same values as before for σ, c_0, and c_1, and choosing values of $K = 1$, and $\omega = 0.3$, the closed loop system was simulated. The initial conditions are the same as before, except that the initial condition for S was determined by evaluating $S = \psi - \psi_{des}$ at ψ_0 and $\psi_{des,0}$, and was found to be -1.3. The results of the simulation are shown in Figure 2. There are several comments worth noting about the polar controller. First, it can be shown using Lyapunov techniques that the system is asymptotically stable. Second, notice that the time scale in Figure 2 is twice that in Figure 1. This was only done to show the richness of the transient behavior with the polar controller; however, both systems show approximately the same levels of decay after 5 seconds of simulation. A third and most important result is that the initial magnitude of the control effort is about 4 times less with the polar controller than with the backstepping controller of [5]. This result is extremely important if one is to implement this controller in hardware since the magnitude of the control effort can determine the type of hardware used. Another important result to notice is that while ψ has a definite lower bound, S does not.

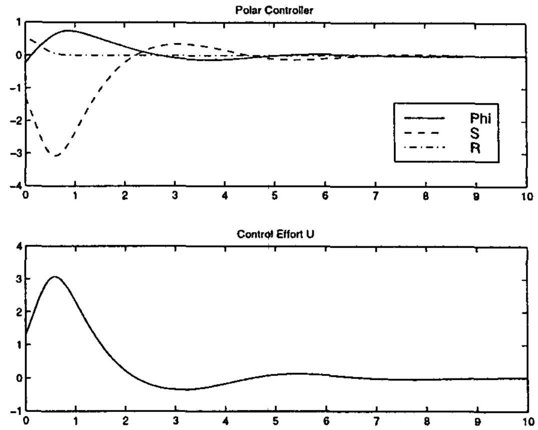

Figure 2: Polar Controller, and the control effort U

4. Disturbance Rejection

In [2], Haddad et al. have recognized that one of the main contributors for the system to enter either a rotating stall or surge, is the presence of disturbances. The model becomes in this case,

$$\begin{aligned}
\dot{\phi} &= -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3A^2\phi - 3A^2 + \beta_2\omega_2 \\
\dot{A} &= \frac{1}{2}\sigma A\left(-2\phi - \phi^2 - A^2\right) + \beta_1\omega_1 \qquad (10) \\
\dot{\psi} &= u
\end{aligned}$$

for the system of [2], and converting to polar form yields

$$\begin{aligned}
\dot{\phi} &= -S - c_1\phi - \frac{1}{2}\left(\phi + \frac{3}{2}\right)^2 \phi - 3A^2\phi + \beta_2\omega_2 \\
\dot{S} &= c_0\left[S + c_1\phi + \frac{1}{2}\left(\phi + \frac{3}{2}\right)^2 \phi + 3A^2\phi - \beta_2\omega_2\right] \\
&\quad +3\sigma A^2\left(-2\phi - \phi^2 - A^2\right) + 6A\beta_1\omega_1 + U
\end{aligned}$$

$$\dot{A} = \frac{1}{2}\sigma A\left(-2\phi - \phi^2 - A^2\right) + \beta_1\omega_1 \quad (11)$$

where β_1, β_2 are positive constants and ω_1, ω_2 are L_2 external disturbances as defined in [2]. To analyze the system, the same disturbances used in [2] were applied i.e, $\omega_1 = 0.1e^{-0.5t}\sin t$ and $\omega_2 = 0.5e^{-0.05t}\sin 0.7t$. Using the model of (1), the control law defined in [5], and adding the disturbance, the system was not able to yield valid results from the ODE integrator. By making the substitution $R = A^2$, and forming the system (10), the ODE integrator gave valid results, and after substituting $R = A^2$ into (2), the system of (10) was simulated as shown in Figure 3. Note that in [2], the control uses

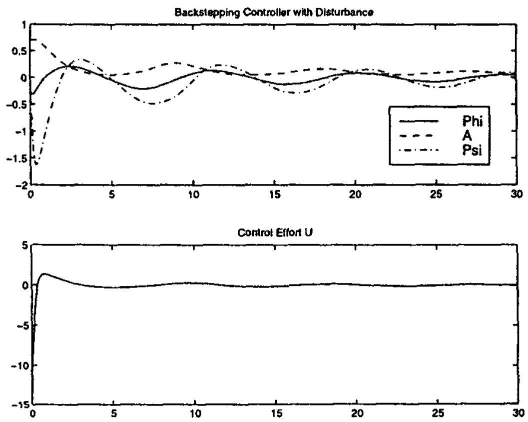

Figure 3: Backstepping controller with L_2 disturbance, and the control effort u.

a negative u, whereas in this paper the control uses a positive u. The polar controller of (11) was also able to stabilize the system as shown in Figure 4. Notice that with the disturbance added, the controller effort is larger. The control effort can be reduced and smoothed out by simply increasing the gain of the controller. The gain of the polar controller was increased from $K = 1$ to $K = 2$, as shown in Figure 5. Specific attention is drawn to this result since this is one of the main advantages of the polar controller. By simply increasing the gain of the control law, a level of robustness is achieved, and the disturbance is rejected much easier. While the difference is subtle for this example, it will be shown later that the increase in gain can yield significant reductions. One very important observation is that for the controller in [2], the compressor flow ψ is highly oscillatory, and takes a long time to die out. With the polar controller, the compressor flow ψ, has a smaller oscillation frequency and dies out quicker. This is true

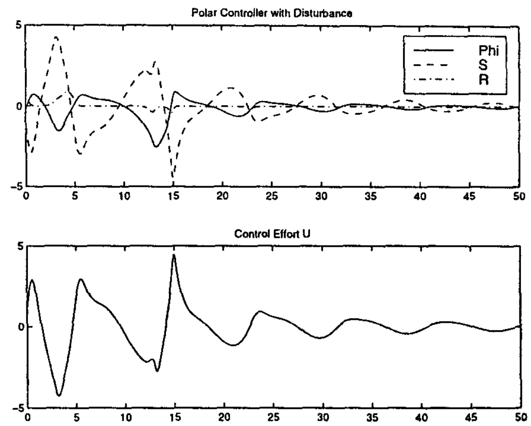

Figure 4: Polar Controller with L_2 Disturbance,and the control effort U. Gain $K = 1$.

for both values $K = 1$, and $K = 2$. It can be shown that with higher gains, the compressor flow ϕ will die out quicker with virtually no oscillation. The initial control effort does tend to increase, thus creating a practical upper bound on the size of the gain K. Nevertheless, the polar controller is able to reject the disturbances more efficiently than the controller of [2].

5. Robust Control

In this section, the system of (1) is again modified to study robustness, and the polar controller obtained earlier is applied. To obtain the uncertain model, we add the term $\Delta\Psi$ to the first modified equation where,

$$\begin{aligned}
\Delta\Psi &= k_1\Pi(\phi)\sin\omega\phi \\
\Pi(\phi) &= tan^{-1}[\mu(\phi - a)] - \tan^{-1}[\mu(\phi - b)]
\end{aligned}$$

so that,

$$\dot{\phi} = -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3A^2\phi - 3A^2 + \Delta\Psi \quad (12)$$

and after transforming to polar form yields the following equations:

$$\begin{aligned}
\dot{\phi} &= -S - c_1\phi - \frac{1}{2}\left(\phi + \frac{3}{2}\right)^2\phi - 3A^2\phi + \Delta\Psi \\
\dot{S} &= c_0\left[S + c_1\phi + \frac{1}{2}\left(\phi + \frac{3}{2}\right)^2\phi + 3A^2\phi - \Delta\Psi\right] \\
&\quad +3\sigma A^2\left[-2\phi - \phi^2 - A^2\right] + U \\
\dot{A} &= \frac{1}{2}\sigma A\left(-2\phi - \phi^2 - A^2\right) \quad (13)
\end{aligned}$$

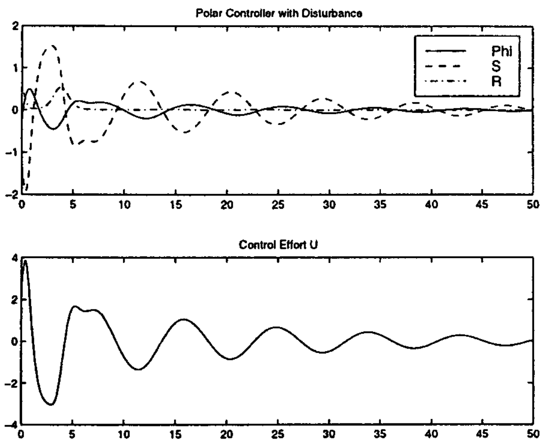

Figure 5: Polar Controller with L_2 Disturbance, and control effort U. Gain $K = 2$.

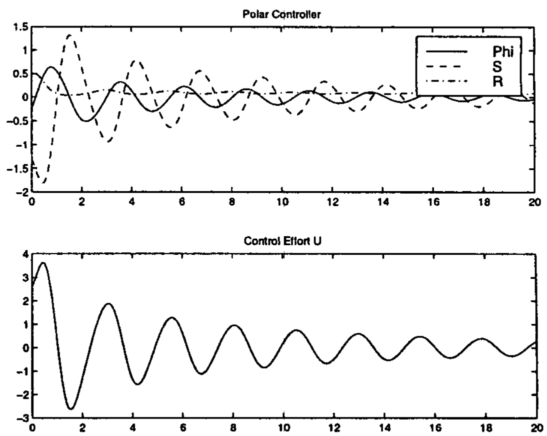

Figure 6: Polar Controller with Perturbation for Robust Control. Gain $K = 2$.

Using the values $k_1 = 0.375\pi$, $\omega = 2.0$, $a = 0$, $b = 1$, and $\mu = 1$, the system was simulated, and the results are shown in Figure 6 where $K = 2$. Haddad et al. [3] have stabilized the system with virtually no oscillation and a little overshoot; whereas the polar controller has a large overshoot. The advantage of the polar controller is that one simple controller has been developed, and stabilization was achieved by increasing the gain. The controller by Haddad et al. is a much more complicated controller, and their robust controller is significantly different then the controller found for disturbance rejection; whereas the polar controller used here is the same controller used throughout.

6. Conclusion

The polar controller has been shown to be an effective control method for the rotating stall and surge problem. While the controller tends to oscillate, it is a simple controller that has a lower peak amplitude for the control effort than the backstepping controller of [5]. The polar controller was also shown to suppress disturbances, and by simply increasing the gain of the controller the overall control effort could be reduced. The polar controller was also shown to have a degree of robustness by increasing the gain of the controller. The ability to increase the gain allows for one controller to be used for both the disturbance rejection, and robust control models. This is in contrast to the controllers developed in [3] and [2], where different and complicated controllers are developed for each case. In future research, the polar controller will be modified to control both rotating stall and surge. In future research, more work will be done to study the effects of different forms of $K(||S||)$, and varying other parameters such as ω and the degree of the polynomial in $||S||$.

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