Randomized Algorithms for Analysis and Control of Uncertain Systems-[Book review; R. Tempo, G. Calafiore, and F. Dabbene]

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emphases of this chapter are on the parts of the system that distinguished the Cornell team from its competitors and led to its success in competition.

III. A Very Useful Compendium

This *Handbook* is an oasis of information, offering a mix of basic and advanced topics, new solutions and technologies arising from the most recent research efforts, and emerging trends to help the reader stay current in this fast-changing field of NECS. It gives the reader an opportunity to quickly and thoroughly scan the field in sufficient depth to provide both specialized knowledge and a broad background of specific information.

As mentioned earlier, characterizing NECS precisely and uniquely is a challenge itself. The book, the first of its kind, could be improved in a number of ways.

- A chapter is needed to highlight what is NECS and what are the challenges. A summary section in the beginning of every part is also necessary to guide the reader.
- Since the stochastic/random phenomena are ubiquitous in NECS, a chapter that covers basic probability theory and stochastic processes is desirable.
- The topics of some chapters overlap heavily, e.g., there are three chapters on Bluetooth in control.
- Certain chapters are not tailored appropriately to the intended audience of the book. In *Finite Automata* of Part I (Fundamentals), for instance, the author should focus more on fundamental concepts in finite automata theory instead of spending a lot of space on such advanced notions as finite transducers.

Note that it is not a course book for students who want to learn control theory or embedded systems (no exercises or questions). While software is covered there is virtually no source code in this *Handbook*. It is not a programmer's book.

Overall, the *Handbook* is a very useful compendium. For additional information, readers may also want to refer to two related handbooks [1] and [2].

References

[1] W. S. Levine, *The Control Handbook*. Boca Raton, FL: CRC Press, 1996.

[2] R. Zurawski, *Embedded Systems Handbook*. Boca Raton, FL: CRC Press, 2005.

Randomized Algorithms for Analysis and Control of Uncertain Systems—R. Tempo, G. Calafiore, and F. Dabbene (London, U.K.: Springer-Verlag, 2005). *Reviewed by Peter Dorato and Chaouki T. Abdallah*

I. Introduction

Randomized algorithms have their origins in the Monte Carlo methods developed by Metropolis and Ulam [1] at the Los Alamos Scientific Laboratory during World War II. These methods were based on solving complex problems by converting difficult deterministic problems into statistical estimation problems. In [2, p. 199], Ulam describes the origins of the Monte Carlo method, in a conversation with Von Neumann, as they are driving from Los Alamos to Lamy in New Mexico, in 1946:

> "Johnny saw at once its great scope even though in the first hour of our discussion he evinced a certain skepticism. But when I became more persuasive, quoting statistical estimates of how many computations were needed to obtain rough results with this or that probability, he agreed, eventually becoming quite inventive in finding marvelous technical tricks to facilitate or speed up these computations." To quote Ulam further, "The one thing about Monte Carlo is that it never gives an exact answer, rather its conclusions indicate that the answer is so and so, within such and such error, with such and such probability-that is, with probability differing from one by such and such a small amount." This is what we now refer to as *probably approximately correct* (PAC) results.

Random algorithms have been applied to the solution of complex problems in many fields including physics, engineering, economics, etc. This book focuses on the application of random algorithms to the analysis and design of robust feedback control systems. The authors reduce the control problem of uncertain systems (robust control) to the problem of satisfying the following condition:

$$J(\mathbf{\Delta}, \theta) \leq \gamma, \quad \text{for all} \quad \Delta \in D$$

where $\mathbf{\Delta}$ is a vector on uncertain plant parameters, θ is a vector of design parameters, J is a performance function, γ is a given level of acceptable performance, and D is a set of possible plant parameter vectors. If a design vector is given, say $\theta = \theta_0$, and the above condition is to be verified for all $\mathbf{\Delta} \in D$, the control is referred to as a *robust analysis* problem. If the problem is to find a θ from a set of possible design values Θ, that meets the above condition, the control problem is referred to as a *robust synthesis* problem.

It is well known (see, for example, [3]) that many practical control problems can be reduced to conditions like the above. In general, solving robust analysis or synthesis problems is computationally very difficult (the existence problem may even be NP-hard [4]). With randomized algorithms, the analysis problem is solved by drawing N random samples of the uncertain-parameter vector $\mathbf{\Delta}$, and estimating the probability that the above condition is met. The robust synthesis problem is solved by drawing M *independent identically distributed* (i.i.d) samples of the design-vector θ, estimating the probability of meeting the above condition with each design sample (from N i.i.d. samples of $\mathbf{\Delta}$), and then selecting the design sample which minimizes

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the estimated probability. Monte Carlo theory specifies the number of samples, N and M, required to obtain results to given levels of accuracy and probabilistic confidence. The beauty of this Monte Carlo approach, also referred to as *statistical learning*, is that the number of samples required is finite and independent of the dimension of the real-valued vectors $\boldsymbol{\Delta}$ and θ. Thus, two serious problems are overcome: *the curse of the continuum* (having to deal with the infinity of real numbers), and *the curse of dimensionality* (having to deal with the high dimension of design and parameter vectors). Also, in the direct Monte Carlo approach, the number of samples M and N are independent of the complexity of the control problem, plant, and controller. However, the number of plant samples is coupled to the number of controller samples. When the concept of *uniform convergence of empirical mean* (UCEM) is introduced (Chapter 10), the number of samples M and N are decoupled. The Theory of UCEM is based on the so-called *Vapnik-Chervonenkis* (VC) dimension, which relates to the complexity of plant, controller, and performance measure. Unfortunately there is a serious price to be paid for this decoupling, i.e., the number of samples for many problems is unacceptably high (See, for example, [5]), and the discussions on pages 143 and 146 of the book under review.

One of the early applications of random algorithms to robust control is found in [6]. In this paper, only the robust analysis problem is considered. In [7], the theory of statistical learning is presented in some detail, but the application to robust control is only briefly discussed. In [8], the application of random methods to control problems is discussed in more detail, and in [5], Vidyasagar provides a comprehensive study of randomized algorithms for robust control synthesis. However, the text under review is the first book, and at the present time the only book, that covers both randomized-algorithms and robust control design fully.

II. Book Contents

The major topics covered in the book under review include the following.

- *Review of Probability Theory*-Chapter 2.
- *The Robust Control Problem*-Chapters 3 and 4.
- *Computational Complexity*-Chapter 5 and 10.
- *Probabilistic Methods for Robust Analysis*-Chapter 6.
- *Review of Monte Carlo and Quasi-Monte Carlo Methods*-Chapter 7
- *Randomized Algorithms for Robust Analysis and Synthesis*-Chapters 8 and 13.
- *Sample Size Bounds for Estimation and Optimization*-Chapter 9.
- *Sequential Algorithms*-Chapters 11 and 12.
- *Random Number Generation*-Chapters 14, 15, 16, 17, and 18.
- *Applications of Randomized Algorithms to Some Control Problems*-Chapter 19.

The material in this text is most appropriate for a graduate-level "special topics" control systems course. Although there are no homework problems given in the text, the material is explained in enough detail, and enough numerical examples are included, to make this a good textbook. Especially valuable in this text is the detailed discussion on random sample generation for vectors and matrices. The generation of sampled points using quasi-Monte Carlo methods is also briefly discussed (Chapter 7). However quasi-Monte Carlo algorithms are not random algorithms, and some deterministic properties of the performance measure must be known. Chapters 11 and 12, which deal with what the authors call, *sequential algorithms*, are based on the special assumption that the performance-specification function $J(\boldsymbol{\Delta}, \theta)$ is convex in θ for fixed $\boldsymbol{\Delta}$.

Prerequisites for the material in this book are, some knowledge of probability theory and some knowledge of control theory. However reviews of these topics are included in Chapters 2 and 3. Thus, the text is fairly well self contained.

III. Concluding Comments

As previously noted, this unique book could be used as a text for a graduate level special-topics course on random algorithms for robust control. It is also valuable as a reference book for research and industrial applications of random algorithms.

Until recently, articles and books on random algorithms have stressed the VC-dimension decoupling of sample estimates M and N. See, for example, [7] and [8]. However, the VC dimension is difficult to estimate, except for performance measures that are *multivariate polynomial functions*, and in Vidyasagar's article of 2001 [5], it was finally noted that the VC-dimension based number of samples is generally much too high for practical computations. Tempo *et al.* do acknowledge this problem also, but only after a much discussion of VC-dimension based theory. It might have been better for the reader, if the VC-dimension theory had been relegated to an Appendix. There is another small disconnect between theory and examples in the text. In discussing an application of quasi-Monte Carlo methods on page 298, a problem is considered which does not have a bounded variation, thus the number of sample points required cannot be justified theoretically.

Finally, it should be noted that some people remain nervous about the concept of *probably approximately correct* results. Even the noted probabilist De Finetti, expressed doubt about the reality of probability theory when he started his Volume on the subject [9] with the sentence: **PROBABILITY DOES NOT EXIST**. However, when problems reach a certain level of computational complexity, there are few other choices. Thus, the material in this book fills an important niche in the design of robust control systems.

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