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On the Relative Degree of Simultaneously Stabilizing Controllers

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ABSTRACT

In this brief paper [1] we present new necessary and sufficient conditions on the controller for the existence of a single controller to stabilize a set of n SISO plants: $P_1,\ P_2,\ ...,\ P_n$. As

[1] This result was first published in (Bredemann, 1995).

is well known this is equivalent to the existence of a single stable controller that stabilizes $n-1$ plants (strong stabilization). It was shown in (Blondel, 1994) that the simultaneous stabilization problem is transcendental and cannot be solved using algebraic functions. Our only hope in approaching the general solution to the simultaneous stabilization problem using algebraic functions is either to enlarge the class of controllers for which sufficient conditions exist, or to restrict the class of controllers from which a controller must exist. This paper restricts the search for existence of simultaneously stabilizing controllers to the class of exactly proper controllers.

Key Words. Simultaneous stabilization, linear systems, improper controllers.

1 Introduction

The problem of stabilizing n different plants is a longstanding problem in the robust control literature. The problem is relevant in applications where the plant is only known to belong to a set of n different plants, or where the failure of sensors or actuators will drastically change the plant from its current description. The problem has been studied in conjunction with the problem of stabilizing a nonlinear plant, which is linearized about n operating points.

It has been shown (Vidyasagar, 1985) that simultaneously stabilizing n plants with any controller is equivalent to simultaneously stabilizing $n-1$ plants with a stable controller $C(s)$, i.e. the strong simultaneous stabilization of $n-1$ plants. Unfortunately, this latter problem is yet unsolved except in the case where $n = 1$ (Vidyasagar, 1985). There exists necessary and sufficient conditions and a synthesis procedure for solving the strong stabilization of one plant as described in (Youla et al., 1974). Namely, a plant is strongly stabilizable if and only if it satisfies the Parity-Interlacing-Property (PIP).

For the case, where $n > 2$ few results have appeared in the direction of a general solution to the simultaneous stabilization problem. In (Ghosh, 1986) for example, the simultaneous stabilization of 3 different plants are shown to be equivalent to the the partial pole placement of a single plant with a stable minimum phase controller. In (Blondel, 1994), Blondel extended Ghosh's result to more than 3 plants. When one of the difference plants is minimum phase and exactly proper, then stabilizing $k \geq 3$ SISO plants with any controller is equivalent to stabilizing $(k-2)$ SISO plants with a *bistable* controller, a stable controller, whose inverse is also stable. Unfortunately, stabilizing even one plant with a bistable controller remains an open problem.

Recently, Yao, Schaefers and Darouach restated the necessary and sufficient conditions for the simultaneous stabilization of 3 or more plants using observers with state feedback in (Yao et al., 1994). They showed that simultaneously observing n plants is equivalent to strongly simultaneously observing $n-1$ plants, i.e. to simultaneously observing $n-1$ plants with a stable functional observer. This is analogous to the results in (Vidyasagar, 1985) for which general solutions for more than 2 plants do not exist.

There are several results with sufficient conditions for simultaneous stabilization. In (Barmish and Wei, 1985), the case, where n minimum-phase plants have the same sign in their high-frequency gains, is shown to be sufficient for simultaneous stabilization. A similar sufficient condition was treated in (Chapellat and Bhattacharyya, 1988). Emre arrived at sufficient conditions in (Emre, 1983) to stabilize n plants with the same closed loop characteristic polynomial. The sufficient conditions in (Emre, 1983) are very restrictive. Debowski and Kurylowicz showed in (Debowski and Kurylowicz, 1986) that if there are three minimum phase plants such that the two difference plants, formed from the difference of one of the plants with the other two, are minimum phase and exactly proper, then the three plants can be simultaneously stabilized. Blondel, Champion and Gevers extended these results in (Blondel et al., 1993). They showed that if there exists one plant, such that its differences formed with all other plants, are minimum phase and exactly proper, then all plants can be simultaneously stabilized.

Efforts toward the general solution of the problem found necessary and sufficient conditions for a plant to be stabilized by a stable controller with no real unstable zeros in (Wei, 1990). In the spirit of (Wei, 1990), Blondel et al. have presented necessary conditions in (Blondel et al., 1991) to simultaneously stabilize more than 2 plants.

Unfortunately, the necessary and sufficient conditions to simultaneously stabilize more than two plants are not computable. These conditions effectively translate the problem into another unsolved problem. Blondel showed in (Blondel, 1994) that the existence of a compensator which strongly simultaneously stabilizes two second order plants is "rationally undecidable". There are an infinite number of steps of elementary operations, such as addition, subtraction, multiplication, division, logical AND, logical OR, etc., required to determine existence of the solution. Therefore, the solution to the simultaneous stabilization problem for three or more plants, which is equivalent to the strong simultaneous stabilization of two or more plants, is also in general rationally undecidable. Our only hope in approaching the general solution to the simultaneous stabilization problem using algebraic functions is either to enlarge the class of systems for which sufficient conditions for simultaneous stabilization exist, or to restrict the class of controllers from which a simultaneously stabilizing controller must exist. This paper does the latter.

Several papers have addressed the requirements on the relative degree of the controller, which stabilizes one or more plants. In (Vidyasagar et al., 1982), Vidyasagar, Schneider and Francis showed that a strictly proper plant can be stabilized by a proper controller and that every controller that stabilizes a strictly proper plant must be proper. Toker and

Ozcaldiran (Toker and Ozcaldiran,) showed that if a plant can be strongly stabilized with an improper controller, then it can be strongly stabilized with a proper controller. Blondel showed in (Blondel, 1994) that if $k \geq 3$ plants can be stabilized by an improper controller, then the plants can be stabilized by a proper controller.

In this paper, one more non-tractable necessary and sufficient condition is presented. This new condition restricts the class of controllers, from which the question of existence may be addressed. If any controller exists, which simultaneously stabilizes a collection of plants, then there must exist an exactly proper controller, a controller with equal numerator and denominator order, which simultaneously stabilizes these plants.

This paper is organized in the following manner. Section 2 defines the problem and presents modifications to a lemma initially published by Barmish and Wei in (Barmish and Wei, 1985), which is used in the proof of the main result in section 3. Finally, our conclusions are presented in section 4.

2 Problem Statement and Useful Lemmas

The problem addressed in this paper is the following: Given n single-input-single-output (SISO) plants $P_1(s);\ P_2(s);\ \cdots\ ; P_n(s)$, does there exist a *single stable* compensator $C(s)$ such that the closed-loop (unity feedback) system is internally stable for any of the given plants. As is well known, see for example (Vidyasagar, 1985), the closed-loop systems are internally stable if and only if each of the three transfer functions

$$\frac{1}{1+P_i(s)C(s)},\ \frac{P_i(s)}{1+P_i(s)C(s)}, \tag{1}$$

are bounded-input-bounded-output (BIBO) stable. The strong simultaneous stabilizing compensator $C(s)$ must then make all of the above transfer functions stable.

Let us first recall an available result to be used in the sequel. The new results in this paper are based upon the lemma proved by Barmish and Wei in (Barmish and Wei, 1985).

Lemma 1 (Barmish and Wei (Barmish and Wei, 1985)) *Given two polynomials, $g(s)$ and $h(s)$, of finite degree, $o(g)$ and $o(h)$ respectively, with fixed real coefficients, where*

1. *$h(s)$ is strictly Hurwitz with positive coefficients,*
2. *$g(s)$ is monic,*
3. *$o(g) \leq o(h) + 1$,*

then there exists $\epsilon_{max} > 0$ such that $\forall \epsilon :\ 0 < \epsilon < \epsilon_{max}$, the polynomial $f(s) = h(s) + \epsilon g(s)$ is strictly Hurwitz with positive coefficients.

A minor variation, which allows subtraction of the two functions, is given in the following lemma.

Lemma 2 *Given two polynomials, $g(s)$ and $h(s)$, of finite degree, $o(g)$ and $o(h)$ respectively, with fixed real coefficients, where*

1. *$h(s)$ is strictly Hurwitz with positive coefficients,*

2. $g(s)$ *is monic,*

3. $o(g) \leq o(h)$,

then there exists $\epsilon_{max} > 0$ *such that* $\forall \epsilon$, $f(s) = h(s) - \epsilon g(s)$ *is strictly Hurwitz with positive coefficients.*

Proof of Lemma 2:

Hurwitz testing matrices H_ϵ^-, H, and $H^"$ are generated for $f(s)$, $h(s)$, and $g(s)$ respectively, as in Case 1 of the proof given by Barmish and Wei, but using $H_\epsilon^- = H - \epsilon H^"$ rather than $H_\epsilon^+ = H + \epsilon H^"$.

The norm of a matrix is understood to be the square root of the maximum eigenvalue of the product of the matrix multiplied by its conjugate transpose. Observing that $\|H_\epsilon^-\| = \|H - \epsilon H^"\| \geq \|H\| - \epsilon \|H^"\|$, and $\|H_\epsilon^+\| = \|H - \epsilon H^"\| \geq \|H\| - \epsilon \|H^"\|$ the remainder of the proof is identical. ■

A useful corollary, which minimizes the complexity of theorem proofs that follow, relaxes the monic requirements on $g(s)$ and the sign of the coefficients of $h(s)$.

Corollary 1 *Given two polynomials,* $g(s)$ *and* $h(s)$, *of finite degree,* $o(g)$ *and* $o(h)$ *respectively, with fixed real coefficients, where*

1. $h(s)$ *is strictly Hurwitz,*

2. $o(g) \leq o(h)$,

then there exists $\epsilon_{max} > 0$ *such that* $\forall \epsilon$ $f(s) = h(s) + \epsilon g(s)$ *is strictly Hurwitz and the sign of all of the coefficients of* $f(s)$ *are the same as the sign of all of the coefficients of* $h(s)$.

Proof of Corollary 1:

Let g_0 represent the highest order coefficient of $g(s)$. Define $q(s)$ and $r(s)$ as

$$q(s) = \frac{1}{g_0} g(s), \; r(s) = \frac{1}{g_0} h(s)$$

Then $q(s)$ is monic, $r(s)$ is strictly Hurwitz, and $d_q \leq d_r$, where d_q and d_r represent the degree of $q(s)$ and $r(s)$ respectively. If the sign of g_0 is the same as the sign of the coefficients of $h(s)$, then from Lemma 1, there exists $\epsilon_{max} > 0$, such that $p(s) = q(s) + \epsilon r(s)$ is strictly Hurwitz with positive coefficients $\forall \epsilon : \; 0 < \epsilon < \epsilon_{max}$. If the sign of g_0 is the opposite of the sign of all the coefficients of $h(s)$, then from Lemma 2, there exists $\epsilon_{max} > 0$, such that $p(s) = q(s) - \epsilon[-r(s)] = q(s) + \epsilon r(s)$ is strictly Hurwitz with positive coefficients $\forall \epsilon : \; 0 < \epsilon < \epsilon_{max}$. Therefore, $f(s) = g_0 \cdot p(s)$ is also strictly Hurwitz and the sign of all of the coefficients of $f(s)$ are the same as the sign of all of the coefficients of $h(s)$. This completes the proof. ■

3 Main Results

The theorems in this section show that a necessary and sufficient condition for simultaneous stabilization is that there must exist an exactly proper simultaneously stabilizing controller.

Theorem 1 *If the n proper plants: $P_i = \frac{n_i}{d_i}$, are stabilized by an improper controller, C_{np}, then the n plants are simultaneously stabilized by an exactly proper controller, C_{ep}.*

Proof of Theorem 1:

Assume there exists an improper controller, $C_{np} = \frac{n_c}{d_c}$, of relative degree $r_c = o(d_c) - o(n_c) < 0$, which simultaneously stabilizes the n plants. Then P_iC_{np} have no RHP pole-zero cancellations and each of the following three closed loop transfer functions are proper with a common strictly Hurwitz denominator polynomial, $n_in_c + d_id_c$, for each plant, P_i.

$$
\begin{aligned}
CLTF_{1i} &= \frac{P_iC_{np}}{1 + P_iC_{np}} = \frac{n_in_c}{(n_in_c + d_id_c)} \\
CLTF_{2i} &= \frac{C_{np}}{1 + P_iC_{np}} = \frac{d_in_c}{(n_in_c + d_id_c)} \\
CLTF_{3i} &= \frac{P_i}{1 + P_iC_{np}} = \frac{n_id_c}{(n_in_c + d_id_c)}
\end{aligned}
$$

In order for $CLTF_{2i}$ to be proper, P_i must be exactly proper or improper $\forall i = 1, 2, \ldots, n$. Otherwise, if P_i is strictly proper, the degree of the numerator exceeds the degree of either term in the denominator and this closed loop transfer function is improper. Since P_i is assumed to be proper, it must be exactly proper $\forall i = 1, 2, \ldots, n$.

Let $r_{1i}, \; r_{2i}, \;$ and r_{3i} represent the relative degree of the closed loop transfer functions $CLTF_{1i}, \; CLTF_{2i}, \;$ and $CLTF_{3i}$ respectively. Then

$$
r_{1i} = 0, \; r_{2i} = 0, \; r_{3i} = -r_c
$$

Consider the modified controller,

$$
C_{np}^1 = C_{np} \cdot \frac{1}{(\epsilon_1 s + 1)} = \frac{n_c}{d_c(\epsilon_1 s + 1)}
$$

The relative degree of C_{np}^1, $r_c^1 = r_c + 1$, is one degree closer to being exactly proper than C_{np}. There are uncountably many choices of ϵ_1 to prevent any pole-zero cancellations with any of the plant numerators. Therefore, in any continuous interval, ϵ_1 can be chosen to avoid such cancellations.

The new common denominator polynomial of the closed loop transfer functions is

$$h_i^1 = (n_i n_c + d_i d_c) + \epsilon_1 d_i d_c s = h_i + \epsilon_1 d_i d_c s$$

where

$$h_i = (n_i n_c + d_i d_c)$$

The degree of h_i is greater than or equal to the degree of $d_i d_c s$. Therefore, from Corollary 1, ϵ_1 can be chosen sufficiently small so that h_i^1 is strictly Hurwitz $\forall i = 1, 2, \ldots, n$.

It will next be shown that each of the closed loop transfer functions remain proper. The closed loop transfer functions formed with the modified controller, C_{np}^1 are

$$\begin{aligned} CLTF_{1i}^1 &= \frac{P_i C_{np}^1}{1 + P_i C_{np}^1} = \frac{n_i n_c}{h_i^1} \\ CLTF_{2i}^1 &= \frac{C_{np}^1}{1 + P_i C_{np}^1} = \frac{d_i n_c}{h_i^1} \\ CLTF_{3i}^1 &= \frac{P_i}{1 + P_i C_{np}^1} = \frac{n_i d_c (\epsilon_1 s + 1)}{h_i^1} \end{aligned}$$

The degree of h_i^1 remains the same as the degree of h_i. Therefore, $CLTF_{1i}^1$ and $CLTF_{2i}^1$ remain exactly proper. The relative degree of $CLTF_{3i}^1$, r_{3i}^1, is one less than the relative degree of $CLTF_{3i}$.

$$r_{3i}^1 = r_{3i} - 1 = -r_c - 1 \geq 0$$

Therefore, C_{np}^1 simultaneously stabilizes the n plants.

If C_{np}^1 is exactly proper, $r_c^1 = 0$, then the proof is complete. If C_{np}^1 is still improper, $r_c^1 < 0$, then this procedure is repeated until an exactly proper compensator, $C_{np}^{-r_c}$, is reached. With this compensator, all three closed loop transfer functions are exactly proper and the denominator polynomials, $h_i^{-r_c}$, are strictly Hurwitz. The exactly proper simultaneously stabilizing controller is of the form

$$C_{ep} = C_{np} \cdot \prod_{k=1}^{-r_c} \frac{1}{(\epsilon_k s + 1)}$$

where ϵ_k is chosen as described above $\forall k = 1, 2, \ldots, -r_c$. This completes the proof. ■

Theorem 2 *If the n proper plants: $P_i = \frac{n_i}{d_i}$, are stabilized by a strictly proper controller, C_{sp}, then the n plants are simultaneously stabilized by an exactly proper controller, C_{ep}.*

Proof of Theorem 2:

Assume there exists a strictly proper controller, $C_{sp} = \frac{n_c}{d_c}$, of relative degree $r_c = o(d_c) - o(n_c) > 0$, which simultaneously stabilizes the n plants. Then $P_i C_{sp}$ have no RHP pole-zero cancellations and each of the following three closed loop transfer functions are proper with a common strictly Hurwitz denominator polynomial, $n_i n_c + d_i d_c$, for each plant, P_i.

$$CLTF_{1i} = \frac{P_i C_{sp}}{1 + P_i C_{sp}} = \frac{n_i n_c}{(n_i n_c + d_i d_c)}$$
$$CLTF_{2i} = \frac{C_{sp}}{1 + P_i C_{sp}} = \frac{d_i n_c}{(n_i n_c + d_i d_c)}$$
$$CLTF_{3i} = \frac{P_i}{1 + P_i C_{sp}} = \frac{n_i d_c}{(n_i n_c + d_i d_c)}$$

It is interesting to note that in order for $CLTF_{3i}$ to be proper, P_i must be proper $\forall i = 1, 2, \ldots, n$. Otherwise, if P_i is improper, the degree of the numerator exceeds the degree of either term in the denominator and this closed loop transfer function is improper. P_i is assumed to be proper $\forall i$.

Let $r_{1i},\ r_{2i},$ and r_{3i} represent the relative degree of the closed loop transfer functions $CLTF_{1i},\ CLTF_{2i},$ and $CLTF_{3i}$ respectively. Let r_i represent the relative degree of the plant P_i, $\forall i = 1, 2, \ldots, n$. Then

$$r_{1i} = r_i + r_c,\ r_{2i} = r_c,\ r_{3i} = r_i$$

Consider the modified controller,

$$C_{sp}^1 = C_{sp} \cdot (\epsilon_1 s + 1) = \frac{n_c(\epsilon_1 s + 1)}{d_c}$$

The relative degree of C_{sp}^1, $r_c^1 = r_c - 1$, is one degree closer to being exactly proper than C_{sp}. There are uncountably many choices of ϵ_1 to prevent any pole-zero cancellations with any of the plant numerators. Therefore, in any continuous interval, ϵ_1 can be chosen to avoid such cancellations.

The new common denominator polynomial of the closed loop transfer functions is

$$h_i^1 = (n_i n_c + d_i d_c) + \epsilon_1 n_i n_c s = h_i + \epsilon_1 n_i n_c s$$

where

$$h_i = (n_i n_c + d_i d_c)$$

The degree of h_i is greater than or equal to the degree of $n_i n_c s$. Therefore, from Corollary 1, ϵ_1 can be chosen sufficiently small so that h_i^1 is strictly Hurwitz $\forall i = 1, 2, \ldots, n$.

It will next be shown that each of the closed loop transfer functions remain proper. The closed loop transfer functions formed with the modified controller, C_{sp}^1 are

$$\begin{aligned} CLTF_{1i}^1 &= \frac{P_i C_{sp}^1}{1 + P_i C_{sp}^1} = \frac{n_i n_c(\epsilon_1 s + 1)}{h_i^1} \\ CLTF_{2i}^1 &= \frac{C_{sp}^1}{1 + P_i C_{sp}^1} = \frac{d_i n_c(\epsilon_1 s + 1)}{h_i^1} \\ CLTF_{3i}^1 &= \frac{P_i}{1 + P_i C_{sp}^1} = \frac{n_i d_c}{h_i^1} \end{aligned}$$

The degree of h_i^1 remains the same as the degree of h_i. Therefore, the relative degree of $CLTF_{3i}^1$, r_{3i}^1, remains the same as the relative degree of $CLTF_{3i}$.

$$r_{3i}^1 = r_{3i} = r_i$$

The relative degree of $CLTF^1_{1i}$ and $CLTF^1_{2i}$, r^1_{1i} and r^1_{2i} respectively, are each one less than the relative degree of $CLTF_{1i}$ and $CLTF_{2i}$ respectively.

$$r^1_{1i} = r_{1i} - 1 = r_i + r_c - 1 \geq 0$$

$$r^1_{2i} = r_{2i} - 1 = r_c - 1 \geq 0$$

Therefore, C^1_{sp} simultaneously stabilizes the n plants.

If C^1_{sp} is exactly proper, $r^1_c = 0$, then the proof is complete. If C^1_{sp} is still strictly proper, $r^1_c > 0$, then this procedure is repeated until an exactly proper compensator, $C^{r_c}_{sp}$, is reached. With this compensator, the relative degree of $CLTF^{r_c}_{1i}$ and $CLTF^{r_c}_{3i}$ are equal to the relative degree of P_i, $\forall i$, the relative degree of $CLTF^{r_c}_{2i}$ is exactly proper, and the common denominator polynomials, $h^{r_c}_i$, are strictly Hurwitz $\forall i = 1, 2, \ldots, n$. The exactly proper simultaneously stabilizing controller is of the form

$$C_{ep} = C_{sp} \cdot \prod_{k=1}^{r_c} (\epsilon_k s + 1)$$

where ϵ_k is chosen as described above $\forall k = 1, 2, \ldots, r_c$. This completes the proof. ■

Theorem 3 *The n proper plants: $P_i = \frac{n_i}{d_i}$, if and only if the n plants are simultaneously stabilizable with an exactly proper controller.*

Proof of Theorem 3:

The proof of sufficiency is obvious. For the proof of necessity, assume there exists a controller that simultaneously stabilizes the n plants. If the controller is exactly proper, the proof is complete. If the controller is improper, then from Theorem 1, there exists an exactly proper controller, which simultaneously stabilizes the n plants. If the controller is strictly proper, then from Theorem 2, there exists an exactly proper controller, which simultaneously stabilizes the n plants. This completes the proof. ■

Theorem 3 also holds for improper plants.

4 Conclusions

In this paper, we have established that if there exists any controller, which simultaneously stabilizes 2 or more plants, then there must exist an exactly proper simultaneously stabilizing controller. This restricts the class of controllers from which the question of existence may be addressed and indicates that simultaneous stabilization with strictly proper controllers is a more difficult task.

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