

1-1-2016

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Recommended Citation

Mondal, Kalyan; Surapati Pramanik; and Florentin Smarandache. "Multi-attribute Decision Making based on Rough Neutrosophic Variational Coefficient Similarity Measure." *Neutrosophic Sets and Systems* 13, 1 (2016). https://digitalrepository.unm.edu/nss_journal/vol13/iss1/2

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Multi-attribute Decision Making based on Rough Neutrosophic Variational Coefficient Similarity Measure

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Abstract: The purpose of this study is to propose new similarity measures namely rough variational coefficient similarity measure under the rough neutrosophic environment. The weighted rough variational coefficient similarity measure has been also defined. The weighted rough variational coefficient similarity measures between the rough ideal alternative and each alternative are

calculated to find the best alternative. The ranking order of all the alternatives can be determined by using the numerical values of similarity measures. Finally, an illustrative example has been provided to show the effectiveness and validity of the proposed approach. Comparisons of decision results of existing rough similarity measures have been provided.

Keywords: Neutrosophic set, Rough neutrosophic set; Rough variation coefficient similarity measure; Decision making.

1 Introduction

In 1965, L. A. Zadeh grounded the concept of degree of membership and defined fuzzy set [1] to represent/manipulate data with non-statistical uncertainty. In 1986, K. T. Atanassov [2] introduced the degree of non-membership as independent component and proposed intuitionistic fuzzy set (IFS). F. Smarandache introduced the degree of indeterminacy as independent component and defined the neutrosophic set [3, 4, 5]. For purpose of solving practical problems, Wang et al. [6] restricted the concept of neutrosophic set to single valued neutrosophic set (SVNS), since single value is an instance of set value. SVNS is a subclass of the neutrosophic set. SVNS consists of the three independent components namely, truth-membership, indeterminacy-membership and falsity-membership functions.

The concept of rough set theory proposed by Z. Pawlak [7] is an extension of the crisp set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. The hybridization of rough set theory and neutrosophic set theory produces the rough neutrosophic set theory [8, 9], which was proposed by Broumi, Dhar and Smarandache [8, 9]. Rough neutrosophic set theory is also a powerful mathematical tool to deal with incompleteness.

Literature review reflects that similarity measures play an important role in the analysis and research of clustering analysis, decision making, medical diagnosis, pattern recognition, etc. Various similarity measures [10, 11, 12, 13, 14, 15, 16, 17, 18] of SVNSs and hybrid SVNSs are

available in the literature. The concept of similarity measures in rough neutrosophic environment [19, 20, 21] has been recently proposed.

Pramanik and Mondal [19] proposed cotangent similarity measure of rough neutrosophic sets. In the same study [19], Pramanik and Mondal established its basic properties and provided its application to medical diagnosis. Pramanik and Mondal [20] also proposed cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. The same authors [21] also studied Jaccard similarity measure and Dice similarity measures in rough neutrosophic environment and provided their applications to multi attribute decision making. Mondal and Pramanik [22] presented tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. Together with F. Smarandache and S. Pramanik, K. Mondal [23] presented hypercomplex rough neutrosophic similarity measure and its application in multi-attribute decision making. Mondal, Pramanik, and Smarandache [24] presented several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in multi attribute decision making problems.

Different methods for multiattribute decision making (MADM) and multicriteria decision making (MCDM) problems are available in the literature in different environment such as crisp environment [25, 26, 27, 28, 29], fuzzy environment [30, 31], intuitionistic fuzzy environment [32, 33, 34, 35, 36, 37, 38, 39, 40], neutrosophic environment [41, 42, 43, 44, 45, 46, 47, 48,

49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62], interval neutrosophic environment [63, 65, 66, 67, 68], neutrosophic soft expert environment [69], neutrosophic bipolar environment [70, 71], neutrosophic soft environment [72, 73, 74, 75, 76], neutrosophic hesitant fuzzy environment [77, 78, 79], rough neutrosophic environment [80, 81], etc. In neutrosophic environment Biswas, Pramanik and Giri [82] studied hybrid vector similarity measure and its application in multi-attribute decision making. Getting motivation from the work of Biswas, Pramanik and Giri [82], for hybrid vector similarity measure in neutrosophic environment, we extend the concept in rough neutrosophic environment.

In this paper, a new similarity measurement is proposed, namely rough variational coefficient similarity measure under rough neutrosophic environment. A numerical example is also provided.

Rest of the paper is structured as follows. Section 2 presents neutrosophic and rough neutrosophic preliminaries. Section 3 discusses various similarity measures and variational coefficient similarity measure in crisp environment. Section 4 presents various similarity measures and variational similarity measure for single valued neutrosophic sets. Section 5 presents variational coefficient similarity measure and weighted variational coefficient similarity measure for rough neutrosophic sets and establishes their basic properties. Section 6 is devoted to present multi attribute decision making based on rough neutrosophic variational coefficient similarity measure. Section 7 demonstrates the application of rough variational coefficient similarity measures to investment problem. Finally, section 8 concludes the paper with stating the future scope of research.

2 Neutrosophic preliminaries

Definition 2.1 [3, 4, 5] Neutrosophic set

Let X be a space of points (objects) with generic element in X denoted by x . Then a neutrosophic set A in X is denoted by $A = \{x(T_A(x), I_A(x), F_A(x)) : x \in X\}$ where, $T_A(x)$ is the truth membership function, $I_A(x)$ is the indeterminacy membership function and $F_A(x)$ is the falsity membership function. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]^{-0}, 1^{+}[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ i.e. $^{-0} \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

Definition 2.2 [6] (Single-valued neutrosophic set).

Let X be a universal space of points (objects), with a generic element $x \in X$. A single-valued neutrosophic set (SVNS) $N \subset X$ is denoted by

$$N = \{ \langle T_N(x), I_N(x), F_N(x) \rangle / x, \forall x \in X, \text{ when } X \text{ is continuous;} \\ N = \sum_{i=1}^m \langle T_N(x), I_N(x), F_N(x) \rangle / x, \forall x \in X, \text{ when } X \text{ is discrete.}$$

SVNS is characterized by a true membership function $T_N(x)$, a falsity membership function $F_N(x)$ and an indeterminacy function $I_N(x)$ with $T_N(x), F_N(x), I_N(x) \in [0, 1]$ for all $x \in X$. For each $x \in X$, of a SVNS N $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$.

2.1 Some operational rules and properties of SVNSs

Let $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ be two SVNSs in X . Then the following operations are defined as follows:

- I. Complement: $N_A^c = \langle F_A, 1 - I_A, T_A \rangle \forall x \in X$.
- II. Addition: $N_A \oplus N_B = \langle T_A + T_B - T_A T_B, I_A I_B, F_A F_B \rangle$
- III. Multiplication: $N_A \otimes N_B = \langle T_A T_B, I_A + I_B - I_A I_B, F_A + F_B - F_A F_B \rangle$
- IV. Scalar Multiplication: $\lambda N_A = \langle 1 - (1 - T_A)^\lambda, I_A^\lambda, F_A^\lambda \rangle$ for $\lambda > 0$.
- V. $\langle N_A \rangle^\lambda = \langle (T_A)^\lambda, 1 - (1 - I_A)^\lambda, 1 - (1 - F_A)^\lambda \rangle$ for $\lambda > 0$.

Definition 2.3 [6]

Complement of a SVNS N is denoted by N^c and is defined by

$$T_{N^c}(x) = F_N(x); I_{N^c}(x) = 1 - I_N(x); F_{N^c}(x) = T_N(x)$$

Definition 2.4 [6]

A SVNS N_A is contained in the other SVNS N_B , denoted as $N_A \subseteq N_B$, if and only if

$$T_{N_A}(x) \leq T_{N_B}(x); I_{N_A}(x) \geq I_{N_B}(x); F_{N_A}(x) \geq F_{N_B}(x) \forall x \in X$$

Definition 2.5 [6]

Two SVNSs N_A and N_B are equal, i.e. $N_A = N_B$, if and only if $N_A \supseteq N_B$ and $N_A \subseteq N_B$

Definition 2.6 [6]

Union of two SVNSs N_A and N_B is a SVNS N_C , written as $N_C = N_A \cup N_B$. Its truth membership, indeterminacy-membership and falsity membership functions are related to those of N_A and N_B by

$$T_{N_C}(x) = \max(T_{N_A}(x), T_{N_B}(x)); I_{N_C}(x) = \min(I_{N_A}(x), I_{N_B}(x)); \\ F_{N_C}(x) = \min(F_{N_A}(x), F_{N_B}(x)) \text{ for all } x \text{ in } X.$$

Definition 2.7 [6] Intersection of two SVNSs N_A and N_B is a SVNS N_D , written as $N_D = N_A \cap N_B$, whose truth membership, indeterminacy-membership and falsity membership functions are related to those of N_A and N_B by

$$T_{N_C}(x) = \min(T_{N_A}(x), T_{N_B}(x)); I_{N_C}(x) = \max(I_{N_A}(x), I_{N_B}(x)); \\ F_{N_C}(x) = \max(F_{N_A}(x), F_{N_B}(x)) \text{ for all } x \text{ in } X.$$

Definition 2.8 Rough Neutrosophic Sets [8, 9]

Let Z be a non-null set and R be an equivalence relation on Z . Let P be neutrosophic set in Z with the

membership function T_P indeterminacy function I_P and non-membership function F_P . The lower and the upper approximations of P in the approximation (Z, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$\underline{N}(P) = \langle \langle x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) \rangle / z \in [x]_R, x \in Z \rangle,$$

$$\overline{N}(P) = \langle \langle x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) \rangle / z \in [x]_R, x \in Z \rangle,$$

Here, $T_{\underline{N}(P)}(x) = \wedge_z \in [x]_R T_P(z)$, $I_{\underline{N}(P)}(x) = \wedge_z \in [x]_R I_P(z)$,

$$F_{\underline{N}(P)}(x) = \wedge_z \in [x]_R F_P(z), T_{\overline{N}(P)}(x) = \vee_z \in [x]_R T_P(z),$$

$$I_{\overline{N}(P)}(x) = \vee_z \in [x]_R I_P(z), F_{\overline{N}(P)}(x) = \vee_z \in [x]_R F_P(z)$$

So, $0 \leq \sup T_{\underline{N}(P)}(x) + \sup I_{\underline{N}(P)}(x) + \sup F_{\underline{N}(P)}(x) \leq 3$

$$0 \leq \sup T_{\overline{N}(P)}(x) + \sup I_{\overline{N}(P)}(x) + \sup F_{\overline{N}(P)}(x) \leq 3$$

Here \vee and \wedge denote ‘‘max’’ and ‘‘min’’ operators respectively. $T_P(z)$, $I_P(z)$ and $F_P(z)$ denote respectively the membership, indeterminacy and non-membership function of z with respect to P . It is easy to see that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Z .

Thus NS mappings $\underline{N}, \overline{N} : N(Z) \rightarrow N(Z)$ are, respectively, referred to as the lower and the upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set [8, 9] in (Z, R) .

From the above definition, it is seen that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence classes of R . if $\underline{N}(P) = \overline{N}(P)$ i.e. $T_{\underline{N}(P)}(x) = T_{\overline{N}(P)}(x)$, $I_{\underline{N}(P)}(x) = I_{\overline{N}(P)}(x)$ and $F_{\underline{N}(P)}(x) = F_{\overline{N}(P)}(x), \forall x \in Z$.

P is said to be a definable neutrosophic set in the approximation (Z, R) . It can be easily proved that zero neutrosophic set $(0_N = (0, 1, 1))$ and unit neutrosophic sets $(1_N = (1, 0, 0))$ are definable neutrosophic sets.

Definition 2.9 [8, 9]

If $N(P) = (\underline{N}(P), \overline{N}(P))$ is a rough neutrosophic set in (Z, R) , the rough complement [8, 9] of $N(P)$ is the rough neutrosophic set denoted by $\sim N(P) = (\underline{N}(P)^c, \overline{N}(P)^c)$ where $\underline{N}(P)^c, \overline{N}(P)^c$ are the complements of neutrosophic sets of $\underline{N}(P), \overline{N}(P)$ respectively.

$$\underline{N}(P)^c = \langle \langle x, F_{\underline{N}(P)}(x), 1 - I_{\underline{N}(P)}(x), T_{\underline{N}(P)}(x) \rangle / x \in Z \rangle \text{ and}$$

$$\overline{N}(P)^c = \langle \langle x, F_{\overline{N}(P)}(x), 1 - I_{\overline{N}(P)}(x), T_{\overline{N}(P)}(x) \rangle / x \in Z \rangle$$

Definition 2.10 [8, 9]

If $N(P_1)$ and $N(P_2)$ are the two rough neutrosophic sets of the neutrosophic set P respectively in Z , then the following definitions [8, 9] hold:

$$N(P_1) = N(P_2) \Leftrightarrow \underline{N}(P_1) = \underline{N}(P_2) \wedge \overline{N}(P_1) = \overline{N}(P_2)$$

$$N(P_1) \subseteq N(P_2) \Leftrightarrow \underline{N}(P_1) \subseteq \underline{N}(P_2) \wedge \overline{N}(P_1) \subseteq \overline{N}(P_2)$$

$$N(P_1) \cup N(P_2) = \langle \underline{N}(P_1) \cup \underline{N}(P_2), \overline{N}(P_1) \cup \overline{N}(P_2) \rangle$$

$$N(P_1) \cap N(P_2) = \langle \underline{N}(P_1) \cap \underline{N}(P_2), \overline{N}(P_1) \cap \overline{N}(P_2) \rangle$$

$$N(P_1) + N(P_2) = \langle \underline{N}(P_1) + \underline{N}(P_2), \overline{N}(P_1) + \overline{N}(P_2) \rangle$$

$$N(P_1) \cdot N(P_2) = \langle \underline{N}(P_1) \cdot \underline{N}(P_2), \overline{N}(P_1) \cdot \overline{N}(P_2) \rangle$$

If N, M, L are the rough neutrosophic sets in (Z, R) , then the following proposition are stated from definitions [8, 9].

Proposition 1 [8, 9]

1. $\sim(\sim N) = N$
2. $N \cup M = M \cup N, N \cap M = M \cap N$
3. $(L \cup M) \cup N = L \cup (M \cup N),$
 $(L \cap M) \cap N = L \cap (M \cap N)$
4. $(L \cup M) \cap N = (L \cap M) \cap (L \cup N),$
 $(L \cap M) \cup N = (L \cap M) \cup (L \cap N)$

Proposition 2 [8, 9]

De Morgan’s Laws are satisfied for rough neutrosophic sets .

1. $\sim(N(P_1) \cup N(P_2)) = (\sim(N(P_1))) \cap (\sim(N(P_2)))$
2. $\sim(N(P_1) \cap N(P_2)) = (\sim(N(P_1))) \cup (\sim(N(P_2)))$

Proposition 3 [8, 9]

If P_1 and P_2 are two neutrosophic sets in U such that $P_1 \subseteq P_2$ then $N(P_1) \subseteq N(P_2)$

1. $N(P_1 \cap P_2) \subseteq N(P_1) \cap N(P_2)$
2. $N(P_1 \cup P_2) \supseteq N(P_1) \cup N(P_2)$

Proposition 4 [8, 9]

1. $\underline{N}(P) = \sim \overline{N}(\sim P)$
2. $\overline{N}(P) = \sim \underline{N}(\sim P)$
3. $\underline{N}(P) \subseteq \overline{N}(P)$

3 Similarity measures and variational coefficient similarity measure in crisp environment

The vector similarity measure is one of the important tools for the degree of similarity between objects. However, the Jaccard, Dice, and cosine similarity measures are often used for this purpose. Jaccard [83], Dice [84], and cosine [85] similarity measures between two vectors are stated below.

Let $X = (x_1, x_2, \dots, x_n)$ and $Y = (y_1, y_2, \dots, y_n)$ be two n -dimensional vectors with positive co-ordinates.

Definition 3.1 [83]

Jaccard index of two vectors (measuring the ‘‘similarity’’ of these vectors) can be defined as follows:

$$J(X, Y) = \frac{X \cdot Y}{\|X\|^2 + \|Y\|^2 - X \cdot Y} = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2 + \sum_{i=1}^n y_i^2 - \sum_{i=1}^n x_i y_i} \quad (1)$$

where $\|X\|^2 = \sum_{i=1}^n x_i^2$ and $\|Y\|^2 = \sum_{i=1}^n y_i^2$ are the Euclidean norm of X and Y , $X \cdot Y = \sum_{i=1}^n x_i y_i$ is the inner product of the vector X and Y .

Proposition 5 [83]

Jaccard index satisfies the following properties:

1. $0 \leq J(X, Y) \leq 1$

- 2. $J(X, Y) = J(Y, X)$
- 3. $J(X, Y) = 1$, for $X = Y$ i.e, $x_i = y_i (i = 1, 2, \dots, n)$ for every $x_i \in X$ and $y_i \in Y$

Definition 3.2 [84]

The Dice similarity measure can be defined as follows:

$$E(X, Y) = \frac{2XY}{\|X\|^2 + \|Y\|^2} = \frac{2\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2 + \sum_{i=1}^n y_i^2} \quad (2)$$

Proposition 6 [84]

The Dice similarity measure satisfies the following properties:

- 1. $0 \leq E(X, Y) \leq 1$
- 2. $E(X, Y) = E(Y, X)$
- 3. $J(X, Y) = 1$, for $X = Y$ i.e, $x_i = y_i (i = 1, 2, \dots, n)$ for every $x_i \in X$ and $y_i \in Y$.

Definition 3.3 [85]

The cosine similarity measure between two vectors X and Y is the inner product of these two vectors divided by the product of their lengths and can be defined as follows:

$$C(X, Y) = \frac{X.Y}{\|X\| \|Y\|} = \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2} \sqrt{\sum_{i=1}^n y_i^2}} \quad (3)$$

Proposition 7 [85]

The cosine similarity measure satisfies the following properties

- 1. $0 \leq C(X, Y) \leq 1$
- 2. $C(X, Y) = C(Y, X)$
- 3. $C(X, Y) = 1$, for $X = Y$ i.e, $x_i = y_i (i = 1, 2, \dots, n)$ for every $x_i \in X$ and $y_i \in Y$.

These three formulas are similar in the sense that they take values in the interval $[0, 1]$. Jaccard and Dice similarity measures are undefined when $x_i = 0$, and $y_i = 0$ for $i = 1, 2, \dots, n$ and cosine similarity measure is undefined when $x_i = 0$ or $y_i = 0$ for $i = 1, 2, \dots, n$.

Definition 3.4 [86]

Variational co-efficient similarity measure can be defined as follows:

$$V(X, Y) = \lambda \frac{2XY}{\|X\|^2 + \|Y\|^2} + (1-\lambda) \frac{X.Y}{\|X\| \|Y\|}$$

$$= \lambda \frac{2\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2 + \sum_{i=1}^n y_i^2} + (1-\lambda) \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2} \sqrt{\sum_{i=1}^n y_i^2}} \quad (4)$$

Proposition 8 [86]

Variational co-efficient similarity measure satisfies the following properties:

- 1. $0 \leq V(X, Y) \leq 1$
- 2. $V(X, Y) = V(Y, X)$
- 3. $V(X, Y) = 1$, for $X = Y$ i.e, $x_i = y_i (i = 1, 2, \dots, n)$ for every $x_i \in X$ and $y_i \in Y$.

4. Various similarity measures for single valued neutrosophic sets.

Assume $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ be two SVNNS in a universe of discourse $X = (x_1, x_2, \dots, x_n)$. $T_A, I_A, F_A \in [0,1]$ for any $x_i \in X$ in N_A or $T_B, I_B, F_B \in [0,1]$ for any $x_i \in X$ in N_B can be considered as a vector representation with three elements. Let $w_i \in [0,1]$ be the weight of each element x_i for $i = 1, 2, \dots, n$ such that $\sum_{i=1}^n w_i = 1$, then Jaccard, Dice and cosine similarity measures can be presented as follows:

Definition 4.1[10] Jaccard similarity measure between $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ can be defined as follows:

$$Jac(N_A, N_B) = \frac{(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))}{\frac{1}{n} \sum_{i=1}^n \left(\frac{[(T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2] + [(T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2] - \{T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)\}}{2} \right)} \quad (5)$$

Proposition 9 [10]

Jaccard similarity measure satisfies the following properties:

- 1. $0 \leq Jac(N_A, N_B) \leq 1$;
- 2. $Jac(N_A, N_B) = Jac(N_B, N_A)$;
- 3. $Jac(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every $x_i (i = 1, 2, \dots, n)$ in X .

Definition 4.1.1 [10] Weighted Jaccard similarity measure between $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ can be defined as follows:

$$Jac_w(N_A, N_B) = \frac{(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))}{\sum_{i=1}^n w_i \left(\frac{[(T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2] + [(T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2] - \{T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)\}}{2} \right)} \quad (6)$$

Proposition 10 [10]

Weighted Jaccard similarity measure satisfies the following properties:

- 1. $0 \leq Jac_w(N_A, N_B) \leq 1$;
- 2. $Jac_w(N_A, N_B) = Jac_w(N_B, N_A)$;
- 3. $Jac_w(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every $x_i (i = 1, 2, \dots, n)$ in X .

Definition 4.2 [11]

Dice similarity measure between $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ is defined as:

$$Dic(N_A, N_B) = \frac{1}{n} \sum_{i=1}^n \frac{2 \left[\begin{matrix} T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) \\ + F_A(x_i)F_B(x_i) \end{matrix} \right]}{\sqrt{\left[\begin{matrix} (T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2 \\ + (T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2 \end{matrix} \right]}} \quad (7)$$

Proposition 11 [11]

Dice similarity measure satisfies the following properties:

1. $0 \leq Dic(N_A, N_B) \leq 1$;
2. $Dic(N_A, N_B) = Dic(N_B, N_A)$;
3. $Dic(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every $x_i (i = 1, 2, \dots, n)$ in X .

Definition 4.2.1 [11]

Weighted Dice similarity measure between $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ can be defined as follows:

$$Dic_w(N_A, N_B) = \sum_{i=1}^n w_i \frac{2 \left[\begin{matrix} T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) \\ + F_A(x_i)F_B(x_i) \end{matrix} \right]}{\sqrt{\left[\begin{matrix} (T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2 \\ + (T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2 \end{matrix} \right]}} \quad (8)$$

Proposition 12 [11]

Weighted Dice similarity measure

1. $0 \leq Dic_w(N_A, N_B) \leq 1$;
2. $Dic_w(N_A, N_B) = Dic_w(N_B, N_A)$;
3. $Dic_w(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every $x_i (i = 1, 2, \dots, n)$ in X .

Definition 4.3 [12]

Cosine similarity measure between $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ can be defined as follows:

$$Cos(N_A, N_B) = \frac{1}{n} \sum_{i=1}^n \frac{(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))}{\sqrt{\left[\begin{matrix} (T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2 \\ + (T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2 \end{matrix} \right]}} \quad (9)$$

Proposition 13 [12]

Cosine similarity measure satisfies the following properties:

1. $0 \leq Cos_w(N_A, N_B) \leq 1$;
2. $Cos_w(N_A, N_B) = Cos_w(N_B, N_A)$

3. $Cos_w(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every $x_i (i = 1, 2, \dots, n)$ in X .

Definition 4.3.1 [12]

Weighted cosine similarity measure between $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ can be defined as follows:

$$Cos_w(N_A, N_B) = \sum_{i=1}^n w_i \frac{(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))}{\sqrt{\left[\begin{matrix} (T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2 \\ + (T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2 \end{matrix} \right]}} \quad (10)$$

Proposition 14 [12]

Weighted cosine similarity measure satisfies the following properties:

1. $0 \leq Cos_w(N_A, N_B) \leq 1$;
2. $Cos_w(N_A, N_B) = Cos_w(N_B, N_A)$
3. $Cos_w(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every $x_i (i = 1, 2, \dots, n)$ in X .

Jaccard and Dice similarity measures between two neutrosophic sets $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ are undefined when $T_A(x_i) = I_A(x_i) = F_A(x_i) = 0$ and $T_B(x_i) = I_B(x_i) = F_B(x_i) = 0$ for all $i = 1, 2, \dots, n$. Similarly the cosine formula for two neutrosophic sets $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ is undefined when $T_A(x_i) = I_A(x_i) = F_A(x_i) = 0$ or $T_B(x_i) = I_B(x_i) = F_B(x_i) = 0$ for all $i = 1, 2, \dots, n$.

5 Variational similarity measures for rough neutrosophic sets

The notion of rough neutrosophic set (RNS) is used as vector representations in 3D-vector space. Assume that $X = (x_1, x_2, \dots, x_n)$ and $Y = (y_1, y_2, \dots, y_n)$ be two n-dimensional vectors with positive co-ordinates. Jaccard, Dice, cosine and cotangent similarity measures between two vectors are stated as follows.

Definition 5.1 [21] Jaccard similarity measure under rough neutrosophic environment

Assume that $A = \left(\langle \underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i) \rangle, \langle \bar{T}_A(x_i), \bar{I}_A(x_i), \bar{F}_A(x_i) \rangle \right)$ and $B = \left(\langle \underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i) \rangle, \langle \bar{T}_B(x_i), \bar{I}_B(x_i), \bar{F}_B(x_i) \rangle \right)$ in $X = (x_1, x_2, \dots, x_n)$ be any two rough neutrosophic sets. Jaccard similarity measure [21] between rough neutrosophic sets A and B can be defined as follows:

$$Jac_{RNS}(A, B) =$$

$$\frac{1}{n} \sum_{i=1}^n \frac{(\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i))}{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \\ & - [\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)] \end{aligned} \right]} \quad (11)$$

Proposition 15 [21]

Jaccard similarity measure [21] between A and B satisfies the following properties:

1. $0 \leq Jac_{RNS}(A, B) \leq 1$;
2. $Jac_{RNS}(A, B) = Jac_{RNS}(B, A)$;
3. $Jac_{RNS}(A, B) = 1$; iff $A = B$
4. If C is a RNS in Y and $A \subset B \subset C$ then, $Jac_{RNS}(A, C) \leq Jac_{RNS}(A, B)$, and $Jac_{RNS}(A, C) \leq Jac_{RNS}(B, C)$

Definition 5.1.1 [21]

If we consider the weights of each element x_i , weighted rough Jaccard similarity measure [21] between rough neutrosophic sets A and B can be defined as follows:

$$Jac_{WRNS}(A, B) = \sum_{i=1}^n w_i \frac{(\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i))}{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \\ & - [\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)] \end{aligned} \right]} \quad (12)$$

$w_i \in [0,1], i = 1, 2, \dots, n$ and $\sum_{i=1}^n w_i = 1$. If we take $w_i = \frac{1}{n}$,

$i = 1, 2, \dots, n$, then $Jac_{WRNS}(A, B) = Jac_{RNS}(A, B)$

Proposition 16 [21]

The weighted rough Jaccard similarity [21] measure between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0 \leq Jac_{WRNS}(A, B) \leq 1$;
2. $Jac_{WRNS}(A, B) = Jac_{WRNS}(B, A)$;
3. $Jac_{WRNS}(A, B) = 1$; iff $A = B$
4. If C is a WRNS in Y and $A \subset B \subset C$ then, $Jac_{WRNS}(A, C) \leq Jac_{WRNS}(A, B)$, and $Jac_{WRNS}(A, C) \leq Jac_{WRNS}(B, C)$

Definition 5.2 [21] Dice similarity measure under rough neutrosophic environment

In this section, Dice similarity measure and the weighted Dice similarity measure for rough neutrosophic sets have been stated due to Pramanik and Mondal [21].

Suppose that

$$A = \left(\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i), \overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i) \right) \text{ and}$$

$$B = \left(\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i), \overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i) \right) \text{ be any}$$

two rough neutrosophic sets in $X = (x_1, x_2, \dots, x_n)$. Dice similarity measure between rough neutrosophic sets A and B can be defined as follows:

$$DIC_{RNS}(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{2 \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right]}{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \end{aligned} \right]} \quad (13)$$

Proposition 17 [21]

Dice similarity measure [21] satisfies the following properties.

1. $0 \leq DIC_{RNS}(A, B) \leq 1$;
2. $DIC_{RNS}(A, B) = DIC_{RNS}(B, A)$;
3. $DIC_{RNS}(A, B) = 1$; iff $A = B$
4. If C is a RNS in Y and $A \subset B \subset C$ then, $DIC_{RNS}(A, C) \leq DIC_{RNS}(A, B)$, and $DIC_{RNS}(A, C) \leq DIC_{RNS}(B, C)$,

For proofs of the above mentioned four properties, see [21].

Definition 5.2.1

If we consider the weights of each element x_i , a weighted rough Dice similarity measure between rough neutrosophic sets A and B can be defined as follows:

$$DIC_{WRNS}(A, B) = \sum_{i=1}^n w_i \frac{2 \left[\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i) \right]}{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \end{aligned} \right]} \quad (14)$$

$w_i \in [0,1], i = 1, 2, \dots, n$ and $\sum_{i=1}^n w_i = 1$. If we take $w_i = \frac{1}{n}$,

$i = 1, 2, \dots, n$, then $DIC_{WRNS}(A, B) = DIC_{RNS}(A, B)$

Proposition 18 [21]

The weighted rough Dice similarity [21] measure between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0 \leq DIC_{WRNS}(A, B) \leq 1$;
2. $DIC_{WRNS}(A, B) = DIC_{WRNS}(B, A)$;
3. $DIC_{WRNS}(A, B) = 1$; iff $A = B$
4. If C is a RNS in Y and $A \subset B \subset C$ then, $DIC_{WRNS}(A, C) \leq DIC_{WRNS}(A, B)$, and $DIC_{WRNS}(A, C) \leq DIC_{WRNS}(B, C)$.

For proofs of the above mentioned four properties, see [21].

Definition 5.3 [20]

Cosine similarity measure can be defined as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two rough neutrosophic sets. The cosine similarity measure is a fundamental measure used in information technology. Pramanik and Mondal [20]

defined cosine similarity measure between rough neutrosophic sets in 3-D vector space.

Assume that

$A = \langle (T_A(x_i), I_A(x_i), F_A(x_i)), (\bar{T}_A(x_i), \bar{I}_A(x_i), \bar{F}_A(x_i)) \rangle$ and $B = \langle (T_B(x_i), I_B(x_i), F_B(x_i)), (\bar{T}_B(x_i), \bar{I}_B(x_i), \bar{F}_B(x_i)) \rangle$ in $X = (x_1, x_2, \dots, x_n)$ be any rough neutrosophic sets. Pramanik and Mondal [20] defined cosine similarity measure between rough neutrosophic sets A and B as follows:

$$C_{RNS}(A, B) = \frac{\delta T_A(x_i)\delta T_B(x_i) + \delta I_A(x_i)\delta I_B(x_i) + \delta F_A(x_i)\delta F_B(x_i)}{\frac{1}{n} \sum_{i=1}^n \sqrt{\frac{(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2}{(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2}}} \quad (15)$$

Here, $\delta T_A(x_i) = \frac{T_A(x_i) + \bar{T}_A(x_i)}{2}$, $\delta T_B(x_i) = \frac{T_B(x_i) + \bar{T}_B(x_i)}{2}$,
 $\delta I_A(x_i) = \frac{I_A(x_i) + \bar{I}_A(x_i)}{2}$, $\delta I_B(x_i) = \frac{I_B(x_i) + \bar{I}_B(x_i)}{2}$,
 $\delta F_A(x_i) = \frac{F_A(x_i) + \bar{F}_A(x_i)}{2}$, $\delta F_B(x_i) = \frac{F_B(x_i) + \bar{F}_B(x_i)}{2}$

Proposition 19 [20]

Let A and B be rough neutrosophic sets. Cosine similarity measure [20] between A and B satisfies the following properties.

1. $0 \leq C_{RNS}(A, B) \leq 1$;
2. $C_{RNS}(A, B) = C_{RNS}(B, A)$;
3. $C_{RNS}(A, B) = 1$; iff $A = B$
4. If C is a RNS in Y and $A \subset B \subset C$ then, $C_{RNS}(A, C) \leq C_{RNS}(A, B)$, and $C_{RNS}(A, C) \leq C_{RNS}(B, C)$.

Definition 5.3.1 [20]

If we consider the weights of each element x_i , a weighted rough cosine similarity measure between rough neutrosophic sets A and B can be defined as follows:

$$C_{WRNS}(A, B) = \frac{\sum_{i=1}^n w_i (\delta T_A(x_i)\delta T_B(x_i) + \delta I_A(x_i)\delta I_B(x_i) + \delta F_A(x_i)\delta F_B(x_i))}{\sum_{i=1}^n w_i \sqrt{\frac{(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2}{(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2}}} \quad (16)$$

$w_i \in [0, 1]$, $i = 1, 2, \dots, n$ and $\sum_{i=1}^n w_i = 1$. If we take $w_i = \frac{1}{n}$, $i = 1, 2, \dots, n$, then $C_{WRNS}(A, B) = C_{RNS}(A, B)$

Proposition 20 [20]

The weighted rough cosine similarity measure [20] between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0 \leq C_{WRNS}(A, B) \leq 1$;
2. $C_{WRNS}(A, B) = C_{WRNS}(B, A)$;
3. $C_{WRNS}(A, B) = 1$; iff $A = B$
4. If C is a WRNS in Y and $A \subset B \subset C$ then, $C_{WRNS}(A, C) \leq C_{WRNS}(A, B)$, and $C_{WRNS}(A, C) \leq C_{WRNS}(B, C)$.

For proofs of the above mentioned four properties, see [20].

Definition 5.4 [19] **Cotangent similarity measures of rough neutrosophic sets**

Assume that

$A = \langle (T_A(x_i), I_A(x_i), F_A(x_i)), (\bar{T}_A(x_i), \bar{I}_A(x_i), \bar{F}_A(x_i)) \rangle$ and $B = \langle (T_B(x_i), I_B(x_i), F_B(x_i)), (\bar{T}_B(x_i), \bar{I}_B(x_i), \bar{F}_B(x_i)) \rangle$ in $X = (x_1, x_2, \dots, x_n)$ be any two rough neutrosophic sets. Pramanik and Mondal [19] defined cotangent similarity measure between rough neutrosophic sets A and B as follows:

$$COT_{RNS}(A, B) = \frac{1}{n} \sum_{i=1}^n \left\langle \cot \left[\frac{\pi}{12} \left(\frac{3 + |\delta T_A(x_i) - \delta T_B(x_i)|}{|\delta I_A(x_i) - \delta I_B(x_i)| + |\delta I_A(x_i) - \delta I_B(x_i)|} \right) \right] \right\rangle \quad (17)$$

Here, $\delta T_A(x_i) = \frac{T_A(x_i) + \bar{T}_A(x_i)}{2}$, $\delta T_B(x_i) = \frac{T_B(x_i) + \bar{T}_B(x_i)}{2}$,
 $\delta I_A(x_i) = \frac{I_A(x_i) + \bar{I}_A(x_i)}{2}$, $\delta I_B(x_i) = \frac{I_B(x_i) + \bar{I}_B(x_i)}{2}$,
 $\delta F_A(x_i) = \frac{F_A(x_i) + \bar{F}_A(x_i)}{2}$, $\delta F_B(x_i) = \frac{F_B(x_i) + \bar{F}_B(x_i)}{2}$

Proposition 21 [19]

Cotangent similarity measure satisfies the following properties:

1. $0 \leq COT_{RNS}(A, B) \leq 1$;
2. $COT_{RNS}(A, B) = COT_{RNS}(B, A)$;
3. $COT_{RNS}(A, B) = 1$; iff $A = B$
4. If C is a RNS in Y and $A \subset B \subset C$ then, $COT_{RNS}(A, C) \leq COT_{RNS}(A, B)$, and $COT_{RNS}(A, C) \leq COT_{RNS}(B, C)$.

Definition 5.4.1

If we consider the weights of each element x_i , a weighted rough cotangent similarity measure [19] between rough neutrosophic sets A and B can be defined as follows:

$$COT_{WRNS}(A, B) = \sum_{i=1}^n w_i \left\langle \cot \left[\frac{\pi}{12} \left(\frac{3 + |\delta T_A(x_i) - \delta T_B(x_i)|}{|\delta I_A(x_i) - \delta I_B(x_i)| + |\delta I_A(x_i) - \delta I_B(x_i)|} \right) \right] \right\rangle \quad (18)$$

$w_i \in [0, 1]$, $i = 1, 2, \dots, n$ and $\sum_{i=1}^n w_i = 1$. If we take $w_i = \frac{1}{n}$, $i = 1, 2, \dots, n$, then $COT_{WRNS}(A, B) = COT_{RNS}(A, B)$

Proposition 22 [19]

The weighted rough cosine similarity measure between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0 \leq COT_{WRNS}(A, B) \leq 1$;
2. $COT_{WRNS}(A, B) = COT_{WRNS}(B, A)$;
3. $COT_{WRNS}(A, B) = 1$; iff $A = B$
4. If C is a WRNS in Y and $A \subset B \subset C$ then, $COT_{WRNS}(A, C) \leq COT_{WRNS}(A, B)$, and $COT_{WRNS}(A, C) \leq COT_{WRNS}(B, C)$

Definition 5.5 (Variational co-efficient similarity measure between rough neutrosophic sets)

Let $A = \langle (\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)), (\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)) \rangle$ and $B = \langle (\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)), (\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)) \rangle$ be two rough neutrosophic sets. Variational co-efficient similarity measure between rough neutrosophic sets can be presented as follows:

$$Var_{RNS}(A, B) = \frac{1}{n} \left[\frac{\lambda \sum_{i=1}^n \left\{ \frac{2 \left\{ \delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) \right\} + \delta F_A(x_i) \delta F_B(x_i)}{\left[\left(\delta T_A(x_i) \right)^2 + \left(\delta I_A(x_i) \right)^2 + \left(\delta F_A(x_i) \right)^2 \right] + \left[\left(\delta T_B(x_i) \right)^2 + \left(\delta I_B(x_i) \right)^2 + \left(\delta F_B(x_i) \right)^2 \right]} \right\}}{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)} + (1-\lambda) \sum_{i=1}^n \left\{ \frac{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)}{\left[\left(\delta T_A(x_i) \right)^2 + \left(\delta I_A(x_i) \right)^2 + \left(\delta F_A(x_i) \right)^2 \right] + \left[\left(\delta T_B(x_i) \right)^2 + \left(\delta I_B(x_i) \right)^2 + \left(\delta F_B(x_i) \right)^2 \right]} \right\} \right] \quad (19)$$

Here, $\delta T_A(x_i) = \frac{T_A(x_i) + \overline{T}_A(x_i)}{2}$, $\delta T_B(x_i) = \frac{T_B(x_i) + \overline{T}_B(x_i)}{2}$,
 $\delta I_A(x_i) = \frac{I_A(x_i) + \overline{I}_A(x_i)}{2}$, $\delta I_B(x_i) = \frac{I_B(x_i) + \overline{I}_B(x_i)}{2}$,
 $\delta F_A(x_i) = \frac{F_A(x_i) + \overline{F}_A(x_i)}{2}$, $\delta F_B(x_i) = \frac{F_B(x_i) + \overline{F}_B(x_i)}{2}$

Proposition 23

The variational co-efficient similarity measure $Var_{RNS}(A, B)$ between two rough neutrosophic sets A and B, satisfies the following properties:

1. $0 \leq Var_{RNS}(A, B) \leq 1$;
2. $Var_{RNS}(A, B) = Var_{RNS}(B, A)$;
3. $Var_{RNS}(A, B) = 1$; if $A = B$ i.e.,

$\delta T_A(x_i) = \delta T_B(x_i)$, $\delta I_A(x_i) = \delta I_B(x_i)$, and
 $\delta F_A(x_i) = \delta F_B(x_i)$, for every $x_i (i = 1, 2, \dots, n)$ in X .

Proof.

(1.) It is obvious that $Var_{RNS}(A, B) \geq 0$. Thus it is required to prove that $Var_{RNS}(A, B) \leq 1$.

From rough neutrosophic dice similarity measure it can be written that

$$0 \leq \frac{1}{n} \sum_{i=1}^n \left\{ \frac{2 \left[\frac{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i)}{\left[\left(\delta T_A(x_i) \right)^2 + \left(\delta I_A(x_i) \right)^2 + \left(\delta F_A(x_i) \right)^2 \right] + \left[\left(\delta T_B(x_i) \right)^2 + \left(\delta I_B(x_i) \right)^2 + \left(\delta F_B(x_i) \right)^2 \right]} \right]}{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)} \right\} \leq 1 \quad (20)$$

and from rough neutrosophic cosine similarity measure it can be written that

$$0 \leq \frac{1}{n} \sum_{i=1}^n \frac{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)}{\sqrt{\left[\left(\delta T_A(x_i) \right)^2 + \left(\delta I_A(x_i) \right)^2 + \left(\delta F_A(x_i) \right)^2 \right] + \left[\left(\delta T_B(x_i) \right)^2 + \left(\delta I_B(x_i) \right)^2 + \left(\delta F_B(x_i) \right)^2 \right]}} \leq 1 \quad (21)$$

Combining Eq.(20) and Eq.(21), we obtain $Var_{RNS}(A, B) =$

$$\left[\frac{\lambda \sum_{i=1}^n \left\{ \frac{2 \left\{ \delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) \right\} + \delta F_A(x_i) \delta F_B(x_i)}{\left[\left(\delta T_A(x_i) \right)^2 + \left(\delta I_A(x_i) \right)^2 + \left(\delta F_A(x_i) \right)^2 \right] + \left[\left(\delta T_B(x_i) \right)^2 + \left(\delta I_B(x_i) \right)^2 + \left(\delta F_B(x_i) \right)^2 \right]} \right\}}{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)} + (1-\lambda) \sum_{i=1}^n \left\{ \frac{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)}{\left[\left(\delta T_A(x_i) \right)^2 + \left(\delta I_A(x_i) \right)^2 + \left(\delta F_A(x_i) \right)^2 \right] + \left[\left(\delta T_B(x_i) \right)^2 + \left(\delta I_B(x_i) \right)^2 + \left(\delta F_B(x_i) \right)^2 \right]} \right\} \right] \quad (22)$$

$= \lambda + (1-\lambda) = 1$

Thus, $0 \leq Var_{RNS}(A, B) \leq 1$;

(2.) $Var_{RNS}(A, B) =$

$$\left[\frac{\lambda \sum_{i=1}^n \left\{ \frac{2 \left\{ \delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) \right\} + \delta F_A(x_i) \delta F_B(x_i)}{\left[\left(\delta T_A(x_i) \right)^2 + \left(\delta I_A(x_i) \right)^2 + \left(\delta F_A(x_i) \right)^2 \right] + \left[\left(\delta T_B(x_i) \right)^2 + \left(\delta I_B(x_i) \right)^2 + \left(\delta F_B(x_i) \right)^2 \right]} \right\}}{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)} + (1-\lambda) \sum_{i=1}^n \left\{ \frac{\delta T_B(x_i) \delta T_A(x_i) + \delta I_B(x_i) \delta I_A(x_i) + \delta F_B(x_i) \delta F_A(x_i)}{\left[\left(\delta T_B(x_i) \right)^2 + \left(\delta I_B(x_i) \right)^2 + \left(\delta F_B(x_i) \right)^2 \right] + \left[\left(\delta T_A(x_i) \right)^2 + \left(\delta I_A(x_i) \right)^2 + \left(\delta F_A(x_i) \right)^2 \right]} \right\} \right]$$

$= Var_{RNS}(B, A)$

(3.) If $A = B$ i.e.,

$\delta T_A(x_i) = \delta T_B(x_i)$, $\delta I_A(x_i) = \delta I_B(x_i)$, and
 $\delta F_A(x_i) = \delta F_B(x_i)$, for every $x_i (i = 1, 2, \dots, n)$ in X ,

$Var_{RNS}(A, A) =$

$$\left[\begin{aligned} & \lambda \sum_{i=1}^n \frac{2 \left\{ \begin{aligned} & \delta T_A(x_i) \delta T_A(x_i) + \delta I_A(x_i) \delta I_A(x_i) \\ & + \delta F_A(x_i) \delta F_A(x_i) \end{aligned} \right\}}{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \end{aligned} \right]} \\ & + (1-\lambda) \sum_{i=1}^n \frac{\delta T_A(x_i) \delta T_A(x_i) + \delta I_A(x_i) \delta I_A(x_i) + \delta F_A(x_i) \delta F_A(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right]}} \end{aligned} \right]$$

$$= \frac{1}{n} [n\lambda + n(1-\lambda)] = 1$$

These results show the completion of the proofs of the three properties.

Definition 5.6 (Weighted variational co-efficient similarity measure between rough neutrosophic sets)

Let $A = \langle (T_A(x_i), I_A(x_i), F_A(x_i)), (\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)) \rangle$ and $B = \langle (T_B(x_i), I_B(x_i), F_B(x_i)), (\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)) \rangle$ be any two rough neutrosophic sets. Rough variational co-efficient similarity measure between rough neutrosophic sets A and B in 3-D vector space can be presented as follows:

$$Var_{WRNS}(A, B) =$$

$$\left[\begin{aligned} & \lambda \sum_{i=1}^n w_i \frac{2 \left\{ \begin{aligned} & \delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) \\ & + \delta F_A(x_i) \delta F_B(x_i) \end{aligned} \right\}}{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \end{aligned} \right]} \\ & + (1-\lambda) \sum_{i=1}^n w_i \frac{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right]}} \end{aligned} \right] \tag{23}$$

If $w = \left[\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n} \right]^T$, then Eq.(23) is reduced to Eq.(19).

Proposition 24

The weighted variational co-efficient similarity measure also satisfies the following properties:

1. $0 \leq Var_{WRNS}(A, B) \leq 1$;
2. $Var_{WRNS}(A, B) = Var_{WRNS}(B, A)$;
3. $Var_{WRNS}(A, B) = 1$; if $A = B$ i.e., $\delta T_A(x_i) = \delta T_B(x_i)$, $\delta I_A(x_i) = \delta I_B(x_i)$, and $\delta F_A(x_i) = \delta F_B(x_i)$, for every $x_i (i = 1, 2, \dots, n)$ in X .

Proof:

(1.) It is obvious that $Var_{WRNS}(A, B) \geq 0$. Thus it is required to prove that $Var_{WRNS}(A, B) \leq 1$.

From rough neutrosophic weighted dice similarity measure, it can be written that

$$0 \leq \frac{1}{n} \sum_{i=1}^n w_i \frac{2 \left\{ \begin{aligned} & \delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) \\ & + \delta F_A(x_i) \delta F_B(x_i) \end{aligned} \right\}}{\sqrt{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \end{aligned} \right]}} \leq 1 \tag{24}$$

and from rough neutrosophic weighted cosine similarity measure it can be written that

$$0 \leq \frac{1}{n} \sum_{i=1}^n w_i \frac{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)}{\sqrt{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \end{aligned} \right]}} \leq 1 \tag{25}$$

Combining Eq.(24) and Eq.(25), we obtain $Var_{WRNS}(A, B) =$

$$\left[\begin{aligned} & \lambda \sum_{i=1}^n w_i \frac{2 \left\{ \begin{aligned} & \delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) \\ & + \delta F_A(x_i) \delta F_B(x_i) \end{aligned} \right\}}{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \end{aligned} \right]} \\ & + (1-\lambda) \sum_{i=1}^n w_i \frac{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)}{\sqrt{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \end{aligned} \right]}} \end{aligned} \right] \tag{26}$$

$$\leq \lambda + (1-\lambda) = 1$$

Thus, $0 \leq Var_{WRNS}(A, B) \leq 1$;

(2.) $Var_{WRNS}(A, B) =$

$$\left[\begin{aligned} & \lambda \sum_{i=1}^n w_i \frac{2 \left\{ \begin{aligned} & \delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) \\ & + \delta F_A(x_i) \delta F_B(x_i) \end{aligned} \right\}}{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \end{aligned} \right]} \\ & + (1-\lambda) \sum_{i=1}^n w_i \frac{\delta T_A(x_i) \delta T_B(x_i) + \delta I_A(x_i) \delta I_B(x_i) + \delta F_A(x_i) \delta F_B(x_i)}{\sqrt{\left[\begin{aligned} & \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \\ & + \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \end{aligned} \right]}} \end{aligned} \right]$$

$$= \left[\begin{aligned} & \lambda \sum_{i=1}^n w_i \frac{2 \left\{ \begin{aligned} & \delta T_B(x_i) \delta T_A(x_i) + \delta I_B(x_i) \delta I_A(x_i) \\ & + \delta F_B(x_i) \delta F_A(x_i) \end{aligned} \right\}}{\left[\begin{aligned} & \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \\ & + \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \end{aligned} \right]} \\ & + (1-\lambda) \sum_{i=1}^n w_i \frac{\delta T_B(x_i) \delta T_A(x_i) + \delta I_B(x_i) \delta I_A(x_i) + \delta F_B(x_i) \delta F_A(x_i)}{\sqrt{\left[\begin{aligned} & \left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2 \right] \\ & + \left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right] \end{aligned} \right]}} \end{aligned} \right]$$

$$= Var_{WRNS}(B, A)$$

(3.) If $A = B$ i.e.,

$\delta T_A(x_i) = \delta T_B(x_i), \quad \delta I_A(x_i) = \delta I_B(x_i), \quad \text{and}$
 $\delta F_A(x_i) = \delta F_B(x_i), \text{ for every } x_i (i = 1, 2, \dots, n) \text{ in } X,$

$$Var_{WRNS}(A, A) = \left[\begin{array}{l} 2 \left\{ \frac{\delta T_A(x_i) \delta T_A(x_i) + \delta I_A(x_i) \delta I_A(x_i)}{\left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2 \right]} \right\} \\ + (1-\lambda) \sum_{i=1}^n w_i \frac{\delta T_A(x_i) \delta T_A(x_i) + \delta I_A(x_i) \delta I_A(x_i) + \delta F_A(x_i) \delta F_A(x_i)}{\sqrt{[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2]}} \end{array} \right]$$

$$= \left[\lambda \sum_{i=1}^n w_i + (1-\lambda) \sum_{i=1}^n w_i \right] = 1$$

These results show the completion of the proofs of the three properties.

6. Multi attribute decision making based on rough neutrosophic variational coefficient similarity measure

In this section, a rough variational co-efficient similarity measure is employed to multi-attribute decision making in rough neutrosophic environment. Assume that $A = \{A_1, A_2, \dots, A_m\}$ be the set of alternatives and $C = \{C_1, C_2, \dots, C_n\}$ be the set of attributes in a multi-attribute decision making problem. Assume that w_j be the weight of the attribute C_j provided by the decision maker such that each $w_i \in [0,1]$ and $\sum_{j=1}^n w_j = 1$ However, in real situation decision maker may often face difficulty to evaluate alternatives over the attributes due to vague or incomplete information about alternatives in a decision making situation. Rough neutrosophic set can be used in MADM to deal with incomplete information of the alternatives. In this paper, the assessment values of all the alternatives with respect to attributes are considered as the rough neutrosophic values (see Table 1).

Table1: Rough neutrosophic decision matrix

$$D_{RNS} = \langle \underline{d}_{ij}, \bar{d}_{ij} \rangle_{m \times n} =$$

	C_1	C_2	...	C_n
A_1	$\langle \underline{d}_{11}, \bar{d}_{11} \rangle$	$\langle \underline{d}_{12}, \bar{d}_{12} \rangle$...	$\langle \underline{d}_{1n}, \bar{d}_{1n} \rangle$
A_2	$\langle \underline{d}_{21}, \bar{d}_{21} \rangle$	$\langle \underline{d}_{22}, \bar{d}_{22} \rangle$...	$\langle \underline{d}_{2n}, \bar{d}_{2n} \rangle$
...
A_m	$\langle \underline{d}_{m1}, \bar{d}_{m1} \rangle$	$\langle \underline{d}_{m2}, \bar{d}_{m2} \rangle$...	$\langle \underline{d}_{mn}, \bar{d}_{mn} \rangle$

Here $\langle \underline{d}_{ij}, \bar{d}_{ij} \rangle$ is the rough neutrosophic number for the i -th alternative and the j -th attribute.

Definition 6.1: Transforming operator for SVNNS [80]

The rough neutrosophic decision matrix (27) can be transformed to single valued neutrosophic decision matrix whose ij -th element α_{ij} can be presented as follows:

$$\alpha_{ij} = \left\langle \frac{\underline{d}_{ij} + \bar{d}_{ij}}{2} \right\rangle_{m \times n}, \text{ for } i = 1, 2, 3, \dots, m;$$

$$j = 1, 2, 3, \dots, n. \tag{28}$$

Step1. Determine the neutrosophic relative positive ideal solution

In multi-criteria decision-making environment, the concept of ideal point has been used to help identify the best alternative in the decision set.

Definition 6.2 [51].

Let H be the collection of two types of attributes, namely, benefit type attribute (P) and cost type attribute (L) in the MADM problems. The relative positive ideal neutrosophic solution (RPINS) $Q_S^+ = [\delta q_S^+, \delta q_S^+, \dots, \delta q_S^+]$ is the solution of the decision matrix $D_S = \langle \delta T_{ij}, \delta I_{ij}, \delta F_{ij} \rangle_{m \times n}$ where, every component of Q_S^+ has the following form:

for benefit type attribute, every component of Q_S^+ has the following form:

$$q_S^+ = \langle \delta T_j^+, \delta I_j^+, \delta F_j^+ \rangle$$

$$= \left\langle \max_i \{ \delta T_{ij} \}, \min_i \{ \delta I_{ij} \}, \min_i \{ \delta F_{ij} \} \right\rangle \text{ for } j \in P \tag{29}$$

and for cost type attribute, every component of Q_S^+ has the following form

$$q_S^+ = \langle \delta T_j^+, \delta I_j^+, \delta F_j^+ \rangle$$

$$= \left\langle \min_i \{ \delta T_{ij} \}, \max_i \{ \delta I_{ij} \}, \max_i \{ \delta F_{ij} \} \right\rangle \text{ for } j \in L \tag{30}$$

Step 2. Determine the weighted variational co-efficient similarity measure between ideal alternative and each alternative.

The variational co-efficient similarity measure between ideal alternative Q_S^+ and each alternative A_i for $i = 1, 2, \dots, m$ can be determined by the following equation as follows:

$$Var_{WRNS}(Q_S^+, D_S) =$$

$$\left[\begin{array}{l} 2 \left\{ \frac{\delta T_j^+ \delta T_{ij} + \delta I_j^+ \delta I_{ij} + \delta F_j^+ \delta F_{ij}}{\left[(\delta T_j^+)^2 + (\delta I_j^+)^2 + (\delta F_j^+)^2 \right]} \right\} \\ + (1-\lambda) \sum_{i=1}^n w_i \frac{\delta T_j^+ \delta T_{ij} + \delta I_j^+ \delta I_{ij} + \delta F_j^+ \delta F_{ij}}{\sqrt{[(\delta T_j^+)^2 + (\delta I_j^+)^2 + (\delta F_j^+)^2]}} \end{array} \right] \tag{31}$$

Step3. Rank the alternatives.

According to the values obtained from Eq.(31), the ranking order of all the alternatives can be easily determined. Highest value indicates the best alternative.

Step 4. End.

7 Numerical example

In this section, rough neutrosophic MADM regarding investment problem is considered to demonstrate the applicability and the effectiveness of the proposed approach. However, investment problem is not easy to solve. It not only requires oodles of patience and discipline, but also a great deal of research and a sound understanding of the market, mathematical tools, among others. Suppose an investment company wants to invest a sum of money in the best option. Assume that there are four possible alternatives to invest the money: (1) A_1 is a computer company; (2) A_2 is a garment company; (3) A_3 is a telecommunication company; and (4) A_4 is a food company. The investment company must take a decision based on the following three criteria: (1) C_1 is the growth factor; (2) C_2 is the environmental impact; and (3) C_3 is the risk factor. The four possible alternatives are to be evaluated under the attribute by the rough neutrosophic assessments provided by the decision maker. These assessment values are given in the rough neutrosophic decision matrix (see the table 2).

Table2. Rough neutrosophic decision matrix

$$D = \langle \underline{N}_{ij}(P), \bar{N}_{ij}(P) \rangle_{4 \times 3} =$$

	C_1	C_2	C_3
A_1	$\langle (0.1, 0.2, 0.2), (0.3, 0.2, 0.2) \rangle$	$\langle (0.6, 0.4, 0.3), (0.8, 0.2, 0.3) \rangle$	$\langle (0.3, 0.2, 0.3), (0.5, 0.2, 0.1) \rangle$
A_2	$\langle (0.2, 0.4, 0.3), (0.4, 0.2, 0.3) \rangle$	$\langle (0.6, 0.3, 0.3), (0.8, 0.1, 0.1) \rangle$	$\langle (0.1, 0.4, 0.3), (0.3, 0.2, 0.3) \rangle$
A_3	$\langle (0.3, 0.2, 0.3), (0.5, 0.2, 0.1) \rangle$	$\langle (0.5, 0.2, 0.3), (0.7, 0.2, 0.1) \rangle$	$\langle (0.0, 0.2, 0.4), (0.2, 0.2, 0.2) \rangle$
A_4	$\langle (0.0, 0.4, 0.4), (0.2, 0.2, 0.2) \rangle$	$\langle (0.5, 0.4, 0.4), (0.7, 0.2, 0.2) \rangle$	$\langle (0.2, 0.3, 0.3), (0.4, 0.1, 0.1) \rangle$

The known weight information is given as follows:
 $W = [w_1, w_2, w_3]^T = [0.3, 0.3, 0.4]$ and $\sum_{i=1}^3 w_i = 1$.

Step1. Determine the types of criteria.

First two types i.e. C_1 and C_2 of the given criteria are benefit type criteria and the last one criterion i.e. C_3 is the cost type criteria.

Step2. Determine the relative neutrosophic positive ideal solution

Using Eq. (29), Eq.(30), the relative positive ideal neutrosophic solution for the given matrix defined in Eq.(32) can be obtained as:

$$Q_S^+ = [(0.4, 0.2, 0.2), (0.7, 0.2, 0.2), (0.1, 0.3, 0.3)]$$

Step3. Determine the weighted variational similarity measure

The weighted variational co-efficient similarity measure is determined by using Eq.(28), Eq.(31) and Eq.(32). The results obtained for different values of have been shown in the Table-3.

Table-3. Results of rough variational similarity measure for different values of λ , $0 \leq \lambda \leq 1$

Similarity measure method	Values of λ	Measure values	Ranking order
$Var_{WRNS}(Q_S^+, D_S)$	0.10	0.8769; 0.9741; 0.9917; 0.8107	$A_3 > A_2 > A_1 > A_4$
	0.25	0.8740, 0.9739 0.9905 0.8078	$A_3 > A_2 > A_1 > A_4$
	0.50	0.8692; 0.9735; 0.9887; 0.8028	$A_3 > A_2 > A_1 > A_4$
	0.75	0.8643; 0.9730; 0.9868; 0.7979	$A_3 > A_2 > A_1 > A_4$
	0.90	0.8614; 0.9728; 0.9857; 0.7949	$A_3 > A_2 > A_1 > A_4$

Step 4. Rank the alternatives.

According to the different values of λ , the results obtained in Table-3 reflects that A_3 is the best alternative.

8. Comparisons of different rough similarity measure with rough variation similarity measure

In this section, four existing rough similarity measures - namely: rough cosine similarity measure, rough dice similarity measure, rough cotangent similarity measure and rough Jaccard similarity measure - have been compared with proposed rough variational co-efficient similarity measure for different values of λ . The comparison results are listed in the Table 3 and Table 4.

Table-4. Results of existing rough neutrosophic similarity measure methods.

Rough similarity measure methods	Values of s	Measure values	Ranking order
$JAC_{WRNS}(Q_S^+, D_S)$ [21]	...	0.7870, 0.9471; 0.9739; 0.6832	$A_3 > A_2 > A_1 > A_4$
$DIC_{WRNS}(Q_S^+, D_S)$ [21]	...	0.8595; 0.9726; 0.9873; 0.7929	$A_3 > A_2 > A_1 > A_4$
$C_{WRNS}(Q_S^+, D_S)$ [20]	...	0.8788; 0.9738; 0.9920; 0.9132	$A_3 > A_2 > A_4 > A_1$
$COT_{WRNS}(Q_S^+, D_S)$ [19]	...	0.8472; 0.9358; 0.9643; 0.8103	$A_3 > A_2 > A_1 > A_4$

Conclusion

In this paper, we have proposed rough variational coefficient similarity measures. We also proved some of their basic properties. We have presented an application of rough neutrosophic variational coefficient similarity measure for a decision making problem on investment. The concept presented in the paper can be applied to deal with other multi attribute decision making problems in rough neutrosophic environment.

References

[1] L. A. Zadeh. Fuzzy sets. *Information and Control*, 8(3) (1965), 338-353.

[2] K. Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1986), 87-96.

[3] F. Smarandache. *A unifying field in logics, neutrosophy: neutrosophic probability, set and logic*. American Research Press, Rehoboth, 1998.

[4] F. Smarandache. Neutrosophic set- a generalization of intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics*, 24(3) (2005), 287-297.

[5] F. Smarandache. Neutrosophic set-a generalization of intuitionistic fuzzy set. *Journal of Defense Resources Management*, 1(1) (2010), 107-116.

[6] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, Single valued neutrosophic sets. *Multispace and Multistructure*, 4 (2010), 410-413.

[7] Z. Pawlak. Rough sets. *International Journal of Information and Computer Sciences*, 11(5) (1982), 341-356.

[8] S. Broumi, F. Smarandache, and M. Dhar. Rough neutrosophic sets. *Italian Journal of Pure and Applied Mathematics*, 32 (2014), 493-502.

[9] S. Broumi, F. Smarandache, and M. Dhar. Rough neutrosophic sets. *Neutrosophic Sets and Systems*, 3 (2014), 60-66.

[10] J. Ye. Vector similarity measures of simplified neutrosophic sets and their application in multi-criteria decision making. *International Journal of Fuzzy Systems*, 16(2) (2014), 204 - 211.

[11] S. Ye and J. Ye. Dice similarity measure between single valued neutrosophic multi-sets and its application in medical diagnosis. *Neutrosophic Sets and Systems*, 6 (2014), 50-55.

[12] J. Ye. Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. *Artificial Intelli-*

gence and Medicine, (2014), doi: 10.1016/j.artmed.2014.12.007.

[13] J. Ye. Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment. *Journal of Intelligence and Fuzzy Systems*, (2014), doi: 10.3233/IFS-141252.

[14] J. Ye and Q. S. Zhang. Single valued neutrosophic similarity measures for multiple attribute decision making. *Neutrosophic Sets and Systems*, 2(2014), 48-54.

[15] J. Ye. Clustering methods using distance-based similarity measures of single-valued neutrosophic sets. *Journal of Intelligence Systems*, (2014), doi: 10.1515/jisys-2013-0091.

[16] P. Biswas, S. Pramanik, and B. C. Giri. Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. *Neutrosophic Sets and Systems*, 8 (2015), 48-58.

[17] K. Mondal and S. Pramanik. Neutrosophic refined similarity measure based on tangent function and its application to multi attribute decision making. *Journal of New Theory*, 8 (2015), 41-50.

[18] K. Mondal and S. Pramanik. Neutrosophic refined similarity measure based on cotangent function and its application to multi attribute decision making. *Global Journal of Advanced Research*, 2(2) (2015), 486-496.

[19] S. Pramanik and K. Mondal. Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. *Journal of New Theory*, 4(2015), 90-102.

[20] S. Pramanik and K. Mondal. Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. *Global Journal of Advanced Research*, 2(1) (2015), 212-220.

[21] S. Pramanik and K. Mondal. Some rough neutrosophic similarity measure and their application to multi attribute decision making. *Global Journal of Engineering Science and Research Management*, 2(7) (2015), 61-74.

[22] K. Mondal and S. Pramanik. Tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. *Critical Review*, 11(2015), 26-40.

[23] K. Mondal, S. Pramanik, and F. Smarandache. Hypercomplex rough neutrosophic similarity measure and its application in multi-attribute decision making. *Critical Review*, 13 (2017). In Press.

[24] Mondal, S. Pramanik, and F. Smarandache. Several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in decision making. In F. Smarandache, & S. Pramanik (Eds), *New trends in neuro-*

- sophic theory and applications, Brussels, Pons Editions, 2016, 93-103.
- [25] L. Hwang and K. Yoon. Multiple attribute decision making: methods and applications, Springer, New York, 1981.
- [26] C. L. Hwang and K. Yoon. Multiple attribute decision making: methods and applications. A State of the Art Survey. Springer-Verlag, Berlin, 1981.
- [27] M. Ehrgott and X. Gandibleux. Multiple criteria optimization: state of the art annotated bibliography survey. Kluwer Academic Publishers, Boston, 2002.
- [28] L. Hwang and M. J. Li. Group decision making under multiple criteria: methods and applications. Springer-Verlag, Heidelberg, 1987.
- [29] R. R. Yager. On ordered weighted averaging aggregation operators in multicriteria decision making. IEEE Transactions on Systems, Man and Cybernetics B, 18(1) (1988), 183–190.
- [30] T. Kaya and C. Kahraman. Multi criteria decision making in energy planning using a modified fuzzy TOPSIS methodology. Expert System with Applications, 38(2011), 6577–6585.
- [31] M. Merigó and A. M. Gil-Lafuente. Fuzzy induced generalized aggregation operators and its application in multi-person decision making. Expert Systems with Applications, 38 (2011), 9761–9772.
- [32] L. Lin, X. H. Yuan, and Z. Q. Xia. Multicriteria fuzzy decision-making based on intuitionistic fuzzy sets. Journal of Computers and Systems Sciences, 73(1) (2007), 84-88.
- [33] H. W. Liu and G. J. Wang. Multi-criteria decision making methods based on intuitionistic fuzzy sets. European Journal of Operational Research, 179(2007), 220-233.
- [34] Z. S. Xu and R. R. Yager. Dynamic intuitionistic fuzzy multi-attribute decision making. International Journal of Approximate Reasoning, 48(1) (2008), 246–262.
- [35] F. E. Borana, S. Genç, M. Kurtb, and D. Akay. A multicriteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Expert Systems with Applications, 36 (2009), 11363–11368.
- [36] S. Pramanik and D. Mukhopadhyaya. Grey relational analysis based intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. International Journal of Computer Applications, 34(10) (2011), 21-29.
- [37] K. Mondal and S. Pramanik. Intuitionistic fuzzy multicriteria group decision making approach to quality-brick selection problem. Journal of Applied Quantitative Methods, 9(2) (2014), 35-50.
- [38] S. P. Wan and J. Y. Dong. A possibility degree method for interval-valued intuitionistic fuzzy multi-attribute group decision making. Journal of Computer and System Sciences, 80(2014), 237–256.
- [39] K. Mondal and S. Pramanik. Intuitionistic fuzzy similarity measure based on tangent function and its application to multi-attribute decision making. Global Journal of Advanced Research, 2(2) (2015), 464-471.
- [40] P. P. Dey, S. Pramanik, and B. C. Giri. Multi-criteria group decision making in intuitionistic fuzzy environment based on grey relational analysis for weaver selection in Khadi institution. Journal of Applied and Quantitative Methods, 10(4) (2015), 1-14.
- [41] J. Ye. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. International Journal of General Systems, 42 (2013), 386–394.
- [42] J. Ye. Single valued neutrosophic cross-entropy for multicriteria decision making problems. Applied Mathematical Modelling, 38 (3) (2013), 1170–1175.
- [43] J. J. Peng, J. Q. Wang, H. Y. Zhang, and X. H. Chen. An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. Applied Soft Computing, 25 (2014), 336–346.
- [44] A. Kharal. A neutrosophic multi-criteria decision making method. New Mathematics and Natural Computation, 2014, 10 (2) (2014), 143-162.
- [45] P. Biswas, S. Pramanik, and B. C. Giri. Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. Neutrosophic Sets and Systems, 2(2014), 102–110.
- [46] P. Biswas, S. Pramanik, and B. C. Giri. A new methodology for neutrosophic multi-attribute decision making with unknown weight information. Neutrosophic Sets and Systems, 3 (2014), 42–52.
- [47] J. Ye. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. Journal of Intelligent and Fuzzy Systems, 26 (2014), 2459–2466.
- [48] K. Mondal and S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education under simplified Neutrosophic environment. Neutrosophic Sets and Systems, 6 (2014), 28-34.
- [49] J. J. Peng, J. Q. Wang, J. Wang, H. Y. Zhang, and X. H. Chen. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. International Journal of Systems Science, 47 (10) (2016), 2342-2358.
- [50] R. Sahin and P. Liu. Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information. Neural Computing and Applications, 27(7) (2016), 2017–2029.
- [51] P. Biswas, S. Pramanik, and B. C. Giri. TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. Neural Computing and Applications, 27 (3) (2016), 727-737.
- [52] J. Ye. Trapezoidal neutrosophic set and its application to multiple attribute decision-making. Neural Computing and Applications, 26 (2015), 1157–1166.
- [53] J. Ye. Bidirectional projection method for multiple attribute group decision making with neutrosophic numbers. Neural Computing and Applications, (2015), doi: 10.1007/s00521-015-2123-5.
- [54] S. Pramanik, S. Dalapati, and T. K. Roy. Logistics center location selection approach based on neutrosophic multicriteria decision making. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications, Brussels, Pons Editions, 2016, 161-174.
- [55] S. Pramanik, D. Banerjee, and B.C. Giri. Multi-criteria group decision making model in neutrosophic refined set and its application. Global Journal of Engineering Science and Research Management, 3(6) (2016), 12-18.

- [56] P. Biswas, S. Pramanik, and B. C. Giri. Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. *Neutrosophic Sets and Systems*, 12 (2016), 20-40.
- [57] P. Biswas, S. Pramanik, and B. C. Giri. Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. *Neutrosophic Sets and Systems*, 12 (2016), 127-138.
- [58] Z. Tian, J. Wang, J. Wang, and H. Zhang. Simplified neutrosophic linguistic multi-criteria group decision-making approach to green product development. *Group Decision and Negotiation*, (2016) doi: 10.1007/s10726-016-9479-5.
- [59] Z. Tian, J. Wang, J. Wang, and H. Zhang. An improved MULTIMOORA approach for multi-criteria decision making based on interdependent inputs of simplified neutrosophic linguistic information. *Neural Computing and Applications*, 27 (3) (2016), 727-737.
- [60] N. P. Nirmal and M. G. Bhatt. Selection of material handling automated guided vehicle using fuzzy single valued neutrosophic set-entropy based novel multi attribute decision making technique implementation and validation. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*, Brussels, Pons Editions, 2016, 105-114
- [61] J. Q. Wang, Y. Yang, and L. Li. Multi-criteria decision-making method based on single-valued neutrosophic linguistic Maclaurin symmetric mean operators. *Neural Computing and Applications*. Doi:10.1007/s00521-016-2747-0.
- [62] K. Mandal and K. Basu. Multi criteria decision making method in neutrosophic environment using a new aggregation operator, score and certainty function. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*, Brussels, Pons Editions, 2016,141-160.
- [63] P.P. Dey, S. Pramanik, and B.C. Giri. An extended grey relational analysis based multiple attribute decision making in interval neutrosophic uncertain linguistic setting. *Neutrosophic Sets and Systems*, 11 (2016), 21-30.
- [64] P. Chi, and P. Liu. An extended TOPSIS method for the multi-attribute decision making problems on interval neutrosophic set. *Neutrosophic Sets and Systems*, 1 (2013), 63–70.
- [65] H. Zhang, P. Ji, J. Wang, and X. Chen. An improved weighted correlation coefficient based on integrated weight for interval neutrosophic sets and its application in multi-criteria decision making problems. *International Journal of Computational Intelligence Systems*, 8(6) (2015), 1027–1043.
- [66] S. Broumi, J. Ye, And F. Smarandache. An extended TOPSIS method for multiple attribute decision making based on interval neutrosophic uncertain linguistic variables. *Neutrosophic Sets and Systems*, 8 (2015), 22-31.
- [67] P. P. Dey, S. Pramanik, and B. C. Giri. Extended projection-based models for solving multiple attribute decision making problems with interval-valued neutrosophic information. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*, Brussels, Pons Editions, 2016, 127-140.
- [68] S. Pramanik and K. Mondal. Interval neutrosophic multi-attribute decision-making based on grey relational analysis. *Neutrosophic Sets and Systems*, 9 (2015), 13-22.
- [69] S. Pramanik, P. P. Dey, and B. C. Giri. TOPSIS for single valued neutrosophic soft expert set based multi-attribute decision making problems. *Neutrosophic Sets and Systems*, 10 (2015), 88-95.
- [70] I. Deli, M. Ali, and F. Smarandache. Bipolar neutrosophic sets and their application based on multi-criteria decision making, *Proceedings of the 2015 International Conference on Advanced Mechatronic Systems*, Beijing, China, 249-254.
- [71] P. P. Dey, S. Pramanik, and B. C. Giri. TOPSIS for solving multi-attribute decision making problems under bi-polar neutrosophic environment. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*, Brussels, Pons Editions, 2016, 65-77.
- [72] P. P. Dey, S. Pramanik, and B. C. Giri. Generalized neutrosophic soft multi-attribute group decision making based on TOPSIS. *Critical Review*, 11 (2015), 41-55.
- [73] P. P. Dey, S. Pramanik, and B. C. Giri. Neutrosophic soft multi-attribute decision making based on grey relational projection method. *Neutrosophic Sets and Systems*, 11, 98-106.
- [74] P. P. Dey, S. Pramanik, and B. C. Giri. Neutrosophic soft multi-attribute group decision making based on grey relational analysis method. *Journal of New Results in Science*, 10 (2016), 25-37.
- [75] P. K. Maji. Neutrosophic soft set. *Annals of Fuzzy Mathematics and Informatic*, 5(1) (2013), 157-168.
- [76] P. K. Maji. Weighted neutrosophic soft sets approach in a multi-criteria decision making problem. *Journal of New Theory*, 5 (2015), 1-12.
- [77] P. Biswas, S. Pramanik, and B. C. Giri. Some distance measures of single valued neutrosophic hesitant fuzzy sets and their applications to multiple attribute decision making. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*, Brussels, Pons Editions, 2016, 27-34.
- [78] P. Biswas, S. Pramanik, and B. C. Giri. GRA method of multiple attribute decision making with single valued neutrosophic hesitant fuzzy set information. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*, Brussels, Pons Editions, 2016, 55-63.
- [79] R. Sahin and P. Liu. Distance and similarity measures for multiple attribute decision making with single valued neutrosophic hesitant fuzzy information. In F. Smarandache, & S. Pramanik (Eds), *New trends in neutrosophic theory and applications*, Brussels, Pons Editions, 2016, 35-54.
- [80] K. Mondal and S. Pramanik. Rough neutrosophic multi-attribute decision-making based on rough accuracy score function. *Neutrosophic Sets and Systems*, 8 (2015), 16-22.
- [81] K. Mondal and S. Pramanik. Rough neutrosophic multi-attribute decision-making based on grey relational analysis. *Neutrosophic Sets and Systems*, 7(2015), 8-17.

- [82] S. Pramanik, P. Biswas, and B. C. Giri. Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. *Neural Computing and Applications*, (2015), doi:10.1007/s00521-015-2125-3.
- [83] P. Jaccard. Distribution de la flore alpine dans le Bassin des quelques regions voisines. *Bull de la Societe Vaudoise des Sciences Naturelles*, 37(140) (1901), 241-272.
- [84] L. R. Dice. Measures of amount of ecologic association between species. *Ecology*, 26 (1945), 297-302.
- [85] G. Salton and M. J. McGill. *Introduction to modern information retrieval*. Auckland, McGraw-Hill, 1983.
- [86] X. Xu, L. Zhang, and Q. Wan. A variational coefficient similarity measure and its application in emergency group decision making. *System Engineering Procedia*, 5(2012), 119-124.

Received: November 20, 2016. Accepted: December 15, 2016