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Abstract

This work presents an Internet-Like Protocol (ILP) to coordinate the formation of n second-order agents in a two dimensional (2D) space. The trajectories are specified trough via points and a desired formation at each point. Simulink is used to verify the response of the agents in the desired trajectories.

I. Introduction

The coordination and formation of multiple agents is a problem of particular interest to numerous research groups [1], [3], [4], [5]. Applications of such research abound in space (satellite formation), military (remotely-operated clusters of vehicles) and civilian applications The problem of distributed coordination and control has been theoretically studied using various approaches. In [1], a graph-theoretic approach was presented to explain the behavior of n particles in the plane in an attempt to justify the model presented in [2], which had proposed a discrete-time model illustrating the heading alignments of the n particles. Graph theory was also utilized in [3] to define cost functions that govern the movement of the n systems/agents. In [4], virtual potentials were discussed as an analysis tool, while in [5], local sensing and minimal communication was the main focus of the research.

In this paper we present a different approach to the distributed control and coordination problem, inspired by the Internet congestion control protocols [6]. We formulate the coordination and control of various agents as a problem of competing for a common resource. Despite such selfish behavior, it has recently been shown [7] that all users proportionally share in the resource and indirectly cooperate to maximize the global utility of all users. The supervisor of such behavior is a main controller which sets a price to be incurred by a user as a function of the resource usage and resource capacity, then transmits this price to the users. By doing so, all users receive the same feedback price, and the communication overhead is significantly reduced. The purpose of this paper is to show exactly how such algorithms may be adopted to the coordination and control of physical agents, and in particular to the case of two-dimensional mobile agents.

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This paper is organized as follows: In Section II we present the formation coordinator. In Section III, the trajectories generator is discussed, while section IV shows some of our simulation results. We conclude the paper in section V, with our conclusions and future research discussion.

II. THE ILP FORMATION COORDINATOR

In this section we discuss how Internet-Like Protocols (ILP) are adapted to our formation and coordination problem. In order to implement the formation coordinator we use the results of [8], [9] which were adopted in [6], to deal with n users sharing a resource of size C. The users update their resource usage according to a feedback signal called "price" of the resource, where a low price indicates resource availability while a high price reflects resource shortage. An equilibrium point is reached when the users share proportionally the resource. These results may be applied when a group of users or agents is required to converge to a formation and follow a given trajectory in the plane. In this application, each agent is modelled as a second-order dynamical system. Assuming that the x and y axes can be decoupled and managed separately, we use the system analyzed in [6] to create a Master controller which generates the reference positions for the various agents. This reference position is passed along to each agent, which in turns follows it without communicating with other agents. A position control is then implemented for each agent using a PD compensator, although other controllers may also be used. In order to generate a feedback error signal to the master controller, the actual outputs from all agents are sent to the master (main) controller where their sum is compared with the sum of the desired positions. The integral of their difference is used in the update equation for the feedback signal. In [6] this feedback signal is called price, while in our current application this signal can be seen as a position error. Figure 1 shows a block diagram of the position controller for each agent i.

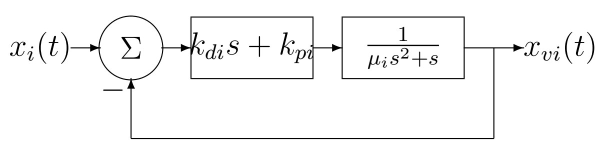

Fig. 1. *Block diagram of the second-order system.*

Let us focus our attention on the coordinator for the x axis, since the coordinator for the y axis is basically the same. The state variable x_i represents the reference position for the i^{th} agent, while the actual output from the agent is denoted by x_{vi}. The transfer function for each agent (with the position controller in place) is thus given by

$$\frac{X_{vi}(s)}{X_i(s)} = \frac{k_{di}s + k_{pi}}{\mu_i s^2 + (k_{di}+1)s + k_{pi}} \qquad 1 \leq i \leq n \tag{1}$$

where μ_i is the time constant of each agent, k_{pi} is the proportional gain, and k_{di} is the derivative gain in the corresponding controller. A state space representation of equation (1) is given in the following,

$$\begin{aligned}
\begin{bmatrix} \dot{x}_{vi_1}(t) \\ \dot{x}_{vi_2}(t) \end{bmatrix} &= \begin{bmatrix} -\frac{(k_{di}+1)}{\mu_i} & -\frac{k_{pi}}{\mu_i} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{vi_1}(t) \\ x_{vi_2}(t) \end{bmatrix} \\
&\quad + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_i(t) \\
x_{vi} &= \begin{bmatrix} \frac{k_{di}}{\mu_i} & \frac{k_{pi}}{\mu_i} \end{bmatrix} \begin{bmatrix} x_{vi_1}(t) \\ x_{vi_2}(t) \end{bmatrix}
\end{aligned} \tag{2}$$

The ILP coordinator then has the following structure (see [6]),

$$\begin{aligned}
\dot{x}_i(t) &= -x_i(t)p_x(t) + a_{xi} \qquad\qquad 1 \le i \le n \\
\dot{x}_{vi_1}(t) &= -\frac{k_{di}+1}{\mu_i}x_{vi_1}(t) - \frac{k_{pi}}{\mu_i}x_{vi_2}(t) + x_i(t) \\
\dot{x}_{vi_2}(t) &= x_{vi_1}(t) \\
\dot{p}_x(t) &= \gamma_x\left[\sum_{i=1}^{n}\left(\frac{k_{di}}{\mu_i}x_{vi_1}(t) + \frac{k_{pi}}{\mu_i}x_{vi_2}(t)\right) - C_x\right]
\end{aligned} \tag{3}$$

where a_{xi} is a user-defined parameter which represents the fraction of the resource C_x allocated for the i^{th} agent, and p_x is the position error feedback signal. The resource C_x is the sum of the desired positions in the formation at the via point for the agents . The parameter γ_x is a user-defined constant positive gain. Figure II shows a block diagram of the ILP coordinator for the x axis.

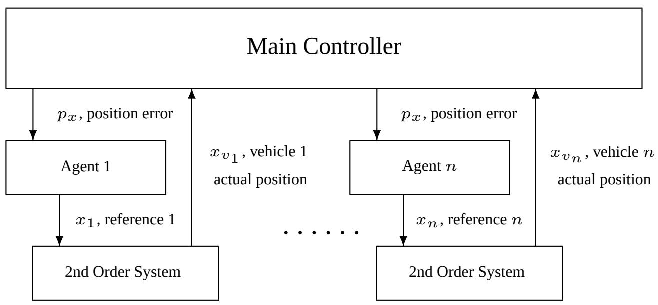

Fig. 2. *Block diagram of the x axis ILP coordinator.*

In a similar fashion, the y axis structure is given by

$$\begin{aligned}
\dot{y}_i(t) &= -y_i(t)p_y(t) + a_{yi} \qquad\qquad 1 \le i \le n \\
\dot{y}_{vi_1}(t) &= -\frac{k_{di}+1}{\mu_i}y_{vi_1}(t) - \frac{k_{pi}}{\mu_i}y_{vi_2}(t) + y_i(t) \\
\dot{y}_{vi_2}(t) &= y_{vi_1}(t) \\
\dot{p}_y(t) &= \gamma_y\left[\sum_{i=1}^{n}\left(\frac{k_{di}}{\mu_i}y_{vi_1}(t) + \frac{k_{pi}}{\mu_i}y_{vi_2}(t)\right) - C_y\right]
\end{aligned} \tag{4}$$

Proceeding similar as in [6], the equilibrium point of (3) is given by

$$\begin{aligned}
x_i^* &= \frac{a_{xi}}{p_x^*} = \frac{a_{xi}C_x}{\sum_{i=1}^{n} a_{xi}} \\
x_{vi_1}^* &= 0 \\
x_{vi_2}^* &= \frac{\mu_i}{k_{pi}}x_i^* \\
x_{vi}^* &= \frac{k_{di}}{\mu_i}x_{vi_1}^* + \frac{k_{pi}}{\mu_i}x_{vi_2}^* = x_i^* \\
p_x^* &= \frac{\sum_{i=1}^{n} a_{xi}}{C_x}
\end{aligned} \tag{5}$$

Likewise the equilibrium point of (4) is given by

$$\begin{aligned} y_i^* &= \frac{a_{yi}}{p_y^*} = \frac{a_{yi}C_y}{\sum_{i=1}^n a_{yi}} \\ y_{vi_1}^* &= 0 \\ y_{vi_2}^* &= \frac{\mu_i}{k_{pi}} y_i^* \\ y_{vi}^* &= \frac{k_{di}}{\mu_i} y_{vi_1}^* + \frac{k_{pi}}{\mu_i} y_{vi_2}^* = y_i^* \\ p_y^* &= \frac{\sum_{i=1}^n a_{yi}}{C_y} \end{aligned} \tag{6}$$

As we can see from (5) and (6), the equilibrium points of the agents actual coordinates x_{vi} and y_{vi} are given by

$$\begin{aligned} x_{vi}^* &= \frac{a_{xi}C_x}{\sum_{i=1}^n a_{xi}} \\ y_{vi}^* &= \frac{a_{yi}C_y}{\sum_{i=1}^n a_{yi}} \end{aligned} \tag{7}$$

Given the equilibrium points from (5) and (6), we can translate the system to the origin in order to analyze stability. The structures for x and y axes are similar, thus let us drop the x and y subindices and complete the analysis for one axis. We define the following translated variables

$$\begin{aligned} w_i(t) &= x_i(t) - \frac{a_i C}{S} \qquad\qquad 1 \le i \le n \\ w_{vi_1}(t) &= x_{vi_1}(t) \\ w_{vi_2}(t) &= x_{vi_2}(t) - \frac{\mu_i}{k_{pi}} \cdot \frac{a_i C}{S} \\ w_m(t) &= p_x(t) - \frac{S}{C} \end{aligned} \tag{8}$$

where $S := \sum_{i=1}^n a_i$, and $m = 3n+1$. Taking the time derivatives of the variables in (8) we obtain

$$\begin{aligned} \dot{w}_i(t) &= \dot{x}_i(t) \qquad\qquad 1 \le i \le n \\ \dot{w}_{vi_1}(t) &= \dot{x}_{vi_1}(t) \\ \dot{w}_{vi_2}(t) &= \dot{x}_{vi_2}(t) \\ \dot{w}_m(t) &= \dot{p}_x(t) \end{aligned} \tag{9}$$

We then re-write the translated system

$$\begin{aligned} \dot{w}_i(t) &= -\left(w_i(t) + \frac{a_i C}{S}\right) \cdot \left(w_m(t) + \frac{S}{C}\right) \\ &\quad + a_i \qquad 1 \le i \le n \\ \dot{w}_{vi_1}(t) &= -\frac{k_{di}+1}{\mu_i} w_{vi_1}(t) - \frac{k_{pi}}{\mu_i}\left(w_{vi_2}(t) + \frac{\mu_i}{k_{pi}} \cdot \frac{a_i C}{S}\right) \\ &\quad + w_i(t) + \frac{a_i C}{S} \\ \dot{w}_{vi_2}(t) &= w_{vi_1}(t) \\ \dot{w}_m(t) &= \gamma \left[\sum_{i=1}^n \left(\frac{k_{di}}{\mu_i} w_{vi_1}(t) + \frac{k_{pi}}{\mu_i}\left(w_{vi_2}(t) + \frac{\mu_i}{k_{pi}} \cdot \frac{a_i C}{S}\right)\right) - C\right] \end{aligned} \tag{10}$$

Finally simplifying to,

$$
\begin{aligned}
\dot{w}_i(t) &= -\frac{S}{C}w_i(t) - w_i(t)w_m(t) \\
&\quad -\frac{a_iC}{S}w_m(t) \qquad 1 \le i \le n \\
\dot{w}_{vi_1}(t) &= -\frac{k_{di}+1}{\mu_i}w_{vi_1}(t) - \frac{k_{pi}}{\mu_i}w_{vi_2}(t) + w_i(t) \\
\dot{w}_{vi_2}(t) &= w_{vi_1}(t) \\
\dot{w}_m(t) &= \gamma \sum_{i=1}^{n}\left(\frac{k_{di}}{\mu_i}w_{vi_1}(t) + \frac{k_{pi}}{\mu_i}w_{vi_2}(t)\right)
\end{aligned} \tag{11}
$$

Theorem 1: $\exists\, \gamma^*$ for which the system (11) is stable for $\gamma < \gamma^*$.
Proof: See [11].

III. Trajectory Generation

The trajectory for the agents in the two dimensional space (2D) is implemented in a similar way to point to point motion control in robotics [10]; it is composed of via points with a corresponding formation for each point. Given the initial positions, the agents converge to the desired formation at the via points. The formation at each via point is determined by the x and y position coordinates for each of the n agents. The positions are in turn defined by the a and C parameters. Thus it is necessary to update these parameters before each via point.
¿From (7), the desired equilibrium point can be obtained with different values of the parameter a, while keeping the same relation $\frac{a_i}{\sum_i a_i}$ for a given C. In section 2.1 of [6], we linearized the ILP system about its equilibrium point to get some insight into the effect of the parameters γ, C, and a in the performance of the system. We obtain that the eigenvalues locations are

$$
\begin{aligned}
\lambda_k &= -\frac{1}{C}\sum_{i=1}^{n} a_i \qquad 1 \le k \le n-1 && (12) \\
\lambda_n &= -\frac{1}{2C}\sum_{i=1}^{n} a_i + \frac{1}{2C}\sqrt{\sum_{i=1}^{n} a_i^2 + 2\sum_{i=1}^{n-1} a_i \sum_{j=i+1}^{n} a_j - 4C^3\gamma} \\
\lambda_{n+1} &= -\frac{1}{2C}\sum_{i=1}^{n} a_i - \frac{1}{2C}\sqrt{\sum_{i=1}^{n} a_i^2 + 2\sum_{i=1}^{n-1} a_i \sum_{j=i+1}^{n} a_j - 4C^3\gamma}
\end{aligned}
$$

We can see from (12) that the real part of the dominant eigenvalues is given by

$$
Real\ part = \frac{1}{2C}\sum_{i=1}^{n} a_i \tag{13}
$$

Using these linearization results, and assuming that γ is selected such that the imaginary part in the complex conjugate eigenvalues is close to zero, then using the theory of linear circuits, the time for convergence of the system is approximately

$$
T_c = 5 \cdot \frac{2C}{\sum_{i=1}^{n} a_i} \tag{14}
$$

Given the via point and the desired formation, we make the via point coincide with the location of one of the agents. As an example, in a triangle formation we assign the via point coordinates to the agent residing at the pointing vertex of the triangle. The other agents positions are calculated according to the desired formation. The C_x and C_y parameters are obtained from

$$\begin{aligned} C_x &= \sum_{i=1}^{n} x_i \\ C_y &= \sum_{i=1}^{n} y_i \end{aligned} \tag{15}$$

With the desired time of convergence T_c, the $\sum_{i=1}^{n} a_i$ can be calculated from (14) as

$$\begin{aligned} \sum_{i=1}^{n} a_{xi} &= \frac{10C_x}{T_{cx}} \\ \sum_{i=1}^{n} a_{yi} &= \frac{10C_y}{T_{cy}} \end{aligned} \tag{16}$$

The a_i parameters are then given by

$$\begin{aligned} a_{xi} &= \frac{x_i}{C_x} \sum_{i=1}^{n} a_{xi} \\ a_{yi} &= \frac{y_i}{C_y} \sum_{i=1}^{n} a_{yi} \end{aligned} \tag{17}$$

The following example illustrates the calculations of the ILP parameters given a desired via point.

Example 1: Let us consider the case of 3 agents and a triangle formation with the following (x, y) positions: agent1(1100,1100), agent2(1000,1000), agent3(1200,1000). Consider for the moment any length unit. Assuming that the desired time for convergence T_c is equal to 5 seconds in both axes and for the three via points, the parameters are then given by

$$
\begin{aligned}
C_x &= \sum_{i=1}^{3} x_i = 1100 + 1000 + 1200 = 3300 \\
C_y &= \sum_{i=1}^{3} y_i = 1100 + 1000 + 1000 = 3100 \\
\sum_{i=1}^{3} a_{xi} &= \frac{10C_x}{Tc_x} = \frac{10 \cdot 3300}{5} = 6600 \\
\sum_{i=1}^{3} a_{yi} &= \frac{10C_y}{Tc_y} = \frac{10 \cdot 3100}{5} = 6200 \\
a_{x1} &= \frac{x_1}{C_x} \sum_{i=1}^{3} a_{xi} = \frac{1100}{3300} \cdot 6600 = 2200 \\
a_{x2} &= \frac{x_2}{C_x} \sum_{i=1}^{3} a_{xi} = \frac{1000}{3300} \cdot 6600 = 2000 \\
a_{x3} &= \frac{x_3}{C_x} \sum_{i=1}^{3} a_{xi} = \frac{1200}{3300} \cdot 6600 = 2400 \\
a_{y1} &= \frac{y_1}{C_y} \sum_{i=1}^{3} a_{yi} = \frac{1100}{3100} \cdot 6200 = 2200 \\
a_{y2} &= \frac{y_2}{C_y} \sum_{i=1}^{3} a_{yi} = \frac{1000}{3100} \cdot 6200 = 2000 \\
a_{y3} &= \frac{y_3}{C_y} \sum_{i=1}^{3} a_{yi} = \frac{1000}{3100} \cdot 6200 = 2000
\end{aligned}
$$

IV. SIMULATION RESULTS

In this section we show the simulation results for different trajectories. Again, let us consider any length unit for both simulations. In the first simulation we used 6 agents with a triangle formation covering three via points. The via points and desired formations are described as follows:

Simulation 1:

Via point 1: agent1 (1200, 1200), agent2 (1100, 1100), agent3 (1300, 1100), agent4 (1000, 1000), agent5 (1200, 1000), agent6 (1400, 1000), $T_{c_x} = 5$ sec, $T_{c_y} = 5$ sec.

Via point 2: agent1 (2200, 2200), agent2 (2100, 2300), agent3(2100, 2100), agent4 (2000, 2400), agent5 (2000, 2200), agent6 (2000, 2000), $T_{c_x} = 5$ sec, $T_{c_y} = 5$ sec.

Via point 3: agent1 (3200, 1000), agent2(3300, 1100), agent3 (3100, 1100), agent4 (3400, 1200), agent5 (3200, 1200), agent6 (3000, 1200), $T_{c_x} = 5$ sec, $T_{c_y} = 5$ sec.

Using (15), (16) and (17), the parameters for the ILP at the via points are given by

Via point 1: $C_x = 7200$, $a_{x1} = 2400$, $a_{x2} = 2200$, $a_{x3} = 2600$, $a_{x4} = 2000$, $a_{x5} = 2400$, $a_{x6} = 2800$, $C_y = 6400$, $a_{y1} = 2400$, $a_{y2} = 2200$, $a_{y3} = 2200$, $a_{y4} = 2000$, $a_{y5} = 2000$, $a_{y6} = 2000$.

Via point 2: $C_x = 12400$, $a_{x1} = 4400$, $a_{x2} = 4200$, $a_{x3} = 4200$, $a_{x4} = 4000$, $a_{x5} = 4000$, $a_{x6} = 4000$, $C_y = 13200$, $a_{y1} = 4400$, $a_{y2} = 4600$, $a_{y3} = 4200$, $a_{y4} = 4800$, $a_{y5} = 4400$, $a_{y6} = 4000$.

Via point 3: $C_x = 19200$, $a_{x1} = 6400$, $a_{x2} = 6600$, $a_{x3} = 6200$, $a_{x4} = 6800$, $a_{x5} = 6400$, $a_{x6} = 6000$, $C_y = 6800$, $a_{y1} = 2000$, $a_{y2} = 2200$, $a_{y3} = 2200$, $a_{y4} = 2400$, $a_{y5} = 2400$, $a_{y6} = 2400$.

We substituted these parameters in a Simulink model of our ILP coordinator, Figure 3 shows the trajectories in the (x, y) plane followed by the agents, we traced lines to join the agents in the vertices of the triangle. As we can see from the figure, in a relatively short time, the agents are in their desired formation. The formation shape is maintained, even though some scaling may occur.

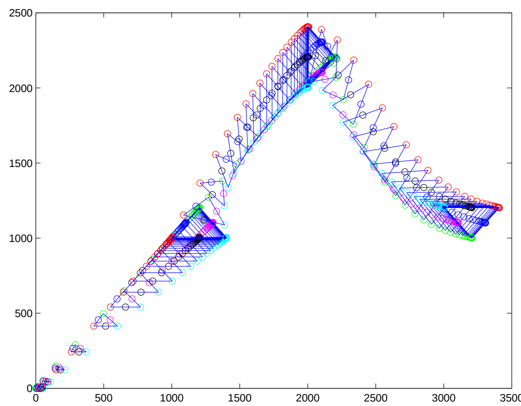

Fig. 3. *Trajectory followed by the six agents visiting 3 via points.*

For the second simulation we use 4 agents with a diamond formation covering 4 via points, The via points and desired formations are described as follows:

Simulation 2:

Via point 1: agent1 (2100, 3000), agent2 (2000, 3100), agent3 (2000, 2900), agent4 (1950, 3000), $T_{c_x} = 10$ sec, $T_{c_y} = 5$ sec.

Via point 2: agent1 (3000, 1900), agent2 (3100, 2000), agent3 (2900, 2000), agent4 (3000, 2050), $T_{c_x} = 5$ sec, $T_{c_y} = 10$ sec.

Via point 3: agent1 (1900, 1000), agent2 (2000, 900), agent3 (2000, 1100), agent4 (2050, 1000)), $T_{c_x} = 10$ sec, $T_{c_y} = 5$ sec.

Via point 4: agent1 (1000, 2100), agent2 (900, 2000), agent3 (1100, 2000), agent4 (1000, 1950)), $T_{c_x} = 5$ sec, $T_{c_y} = 10$ sec.

Using again (15), (16) and (17), the parameters for the ILP at the via points are given by

Via point 1: $C_x = 8050$, $a_{x1} = 2100$, $a_{x2} = 2000$, $a_{x3} = 2000$, $a_{x4} = 1950$, $C_y = 12000$, $a_{y1} = 6000$,

$a_{y2} = 6200$, $a_{y3} = 5800$, $a_{y4} = 6000$.

Via point 2: $C_x = 12000$, $a_{x1} = 6000$, $a_{x2} = 6200$, $a_{x3} = 5800$, $a_{x4} = 6000$, $C_y = 7950$, $a_{y1} = 1900$, $a_{y2} = 2000$, $a_{y3} = 2000$, $a_{y4} = 2050$.

Via point 3: $C_x = 7950$, $a_{x1} = 1900$, $a_{x2} = 2000$, $a_{x3} = 2000$, $a_{x4} = 2050$, $C_y = 4000$, $a_{y1} = 2000$, $a_{y2} = 1800$, $a_{y3} = 2200$, $a_{y4} = 2000$.

Via point 4: $C_x = 4000$, $a_{x1} = 2000$, $a_{x2} = 1800$, $a_{x3} = 2200$, $a_{x4} = 2000$, $C_y = 8050$, $a_{y1} = 2100$, $a_{y2} = 2000$, $a_{y3} = 2000$, $a_{y4} = 1950$.

We substituted these parameters in the simulink model of our ILP coordinator, Figure IV shows the trajectories in the (x, y) plane followed by the agents.

Note that the curve traced during the motion by the agents in formation can be controlled with the time of

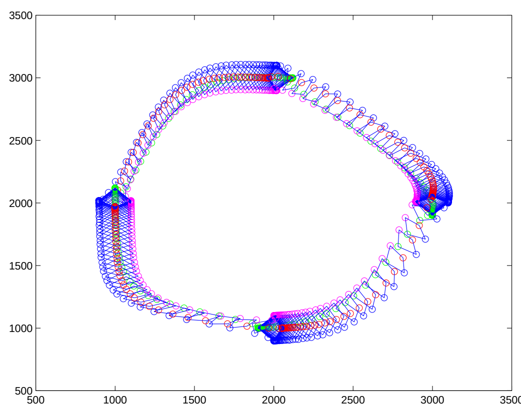

Fig. 4. *Trajectory followed by the four agents visiting 4 via points.*

convergence parameter. From figure 3, in the motion from via point 2 (above) to via point 3 (right) the time of convergence was selected the same for both axes, resulting in a trajectory similar to a decaying exponential. On the other hand, from figure IV, the time of convergence was selected such that the trajectories look like arches, trying to complete a circle with the four via points.

In order to show stability in the convergence of (11) for this particular example, we propose the following Lyapunov function

$$V(w) = \frac{1}{2}\vec{w}^T Q\vec{w} \tag{18}$$

where $Q > 0$ is a $m \times m$ identity matrix. $V(w) > 0$, and $V(\mathbf{0}) = \mathbf{0}$. Taking the time derivative of $V(w)$ results

$$\dot{V}(w) = \frac{1}{2}\left(\vec{w}^T I\vec{\dot{w}} + \vec{\dot{w}}^T I\vec{w}\right) = \sum_{i=1}^{3n} w_i(t)\dot{w}_i(t) + w_m(t)\dot{w}_m(t) \tag{19}$$

Substituting (11) in (19), yields

$$\begin{aligned}\dot{V}(w) &= \sum_{i=1}^{n}\Big[-\frac{S}{C}w_i^2(t) - w_i^2(t)w_m(t) \\ &\quad -\frac{a_iC}{S}w_i(t)w_m(t) - \frac{k_{di}+1}{\mu_i}w_{vi_1}^2(t) \\ &\quad -\frac{k_{pi}}{\mu_i}w_{vi_1}(t)w_{vi_2}(t) + w_{vi_1}(t)w_i(t) \\ &\quad +w_{vi_1}(t)w_{vi_2}(t) + \gamma\frac{k_{di}}{\mu_i}w_m(t)w_{vi_1}(t) \\ &\quad +\gamma\frac{k_{pi}}{\mu_i}w_m(t)w_{vi_2}(t)\Big] \end{aligned} \tag{20}$$

Substituting in (20) the parameters a_i, C, S, k_{pi}, k_{di}, μ_i, γ, and the vector w, from the first via point and the x axis, then $\dot{V}(w)$ results negative semidefinite and converges smoothly to zero, as shown in Figure 5.

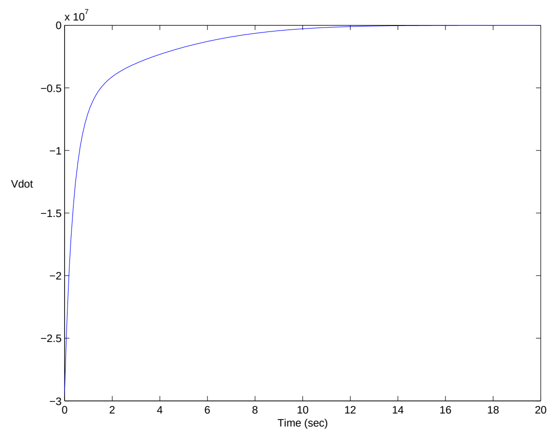

Fig. 5. *Lyapunov time-derivative for the x axis as a function of time.*

V. CONCLUSIONS

This paper presented an ILP to coordinate the movement and formation of n agents in the plane. The agents converged to the via points with the desired formation. The agents landed into the desired (approximate) formation a short time after the start of the motion, and the formation was completed before the arrival to the via points. This technique is powerful since it allows the distributed control and coordination of any number of agents, and since the actual implementation is robust [12]: If a number of agents drop out, the resources made available by the master controller are re-allocated amongst the remaining agents. On the other hand, if new agents were to join the group, resources will be allocated to the newcomers in an efficient manner.

We are currently investigating a discrete-time version of the algorithms discussed here, and attempting to include the effects of time delays in the network of agents. At the theoretical level, we are investigating Lyapunov-stability proofs of the complete system. At the experimental level, we are investigating a hardware implementation of a network of 3 agents.

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