Neutrosophic Sets and Systems

Volume 11 Article 4

1-1-2016

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J. Martina Jency

I. Arockiarani

NSS

University of New Mexico

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J. Martina Jency[1] and I. Arockiarani[2]

[1, 2] Nirmala College for Women, Coimbatore- 641018, Tamilnadu, India. E-mail: martinajency@gmail.com

Abstract. In this paper, we design a model based on *adjustable and mean potentiality approach* to single valued neutrosophic level soft sets. Further, we introduce the notion of *weighted single valued neutrosophic soft set* and investigate its application in decision making.

Keywords: Soft sets, single valued neutrosophic soft sets, weighted single valued neutrosophic soft sets.

1 Introduction

The classical methods are not always successful, because the uncertainties appearing in these domains may be of various types. While a wide range of theories, such as probability theory, fuzzy set theory, intuitionistic fuzzy set theory, rough set theory, vague set theory, and interval mathematics, are well-known mathematical approaches to modelling uncertainty, each of this theories has its inherent difficulties, as pointed out by Molodtsov [21]. The possible reason for their inconveniences is the inadequacy of the parameterization tool. Consequently, Molodtsov initiated the soft set theory as a completely new approach for modelling vagueness and uncertainty, free from the ponderosity affecting existing methods [20]. This theory has been useful in many different fields, such as decision making [7, 8, 10, 13, 15, 23] or data analysis [32].

Up to date, the research on soft sets has been very active and many important results have been achieved in theory. The concept and basic properties of soft set theory were presented in [14, 21]. Practically, Maji et al. introduced several algebraic operations in soft set theory and published a detailed theoretical study. Firstly, Maji et al. [15] applied soft sets to solve the decision making problem with the help of rough approach. Arockiarani et al. [4] extended the (classical) soft sets to single valued neutrosophic (fuzzy neutrosophic) soft sets. Zadeh introduced the degree of membership/truth (*t*), in 1965, and defined the fuzzy set. Atanassov introduced the degree of nonmembership/falsehood (*f*), in 1986, and defined the intuitionistic fuzzy set. Smarandache introduced the degree of indeterminacy / neutrality (*i*) as an independent component, in 1995 (published in 1998), and he defined the neutrosophic set on three independent components (*t*,*i*,*f*) = (truth, indeterminacy, falsehood). He coined/invented the words “neutrosophy”, and its derivative - “neutrosophic”, whose etymology is: *Neutrosophy* (from Latin "neuter" - neutral, Greek "sophia" – skill / wisdom), as a branch of philosophy, studying the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophy considers a proposition, theory, event, concept, or entity "A" in relation to its opposite, "Anti-A", and that which is not "A", "Non-A", and that which is neither "A", nor "Anti-A", denoted by "Neut-A". Neutrosophy is thus a generalization of dialectics. Neutrosophy is the basis of neutrosophic logic, neutrosophic set, neutrosophic set, neutrosophic probability and neutrosophic statistics. In 2013, Smarandache refined the single valued neutrosophic set to *n* components: $t_1, t_2, \ldots; i_1, i_2, \ldots; f_1, f_2, \ldots$.

In this paper, we present an adjustable approach and mean potentiality approach to single valued neutrosophic soft sets by using single valued neutrosophic level soft sets, and give some illustrative examples. The properties of level soft sets are as well discussed. Also, we introduce the weighted single valued neutrosophic soft sets and investigate its application in decision making.

2 Preliminaries

Definition 2.1 [11]

Let X be a space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A.

For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0,1]$. When X is continuous, a SVNS A can be written as A,

$$\int_X \langle T_A(x), I_A(x), F_A(x)\rangle / x, x \in X .$$

When X is discrete, a SVNS A can be written as

$$A = \sum_{i=1}^{n} \langle T(x_i), I(x_i), F(x_i) \rangle / x_i, x_i \in X$$

Definition 2.2 [20]

Let U be the initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Consider a non-empty set A, $A \subset E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \rightarrow P(U)$.

Definition 2.3 [4]

Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, $A \subset E$. Let $P(U)$ denote the set of all single valued neutrosophic (fuzzy neutrosophic) sets of U. The collection (F, A) is termed to be the (fuzzy neutrosophic) single valued neutrosophic soft set over U, where F is a mapping given by $F: A \rightarrow P(U)$.

3 An adjustable approach to single valued neutrosophic soft sets based decision making

Definition 3.1

Let $\varpi = \langle F, A \rangle$ be a single valued neutrosophic soft set over U, where $A \subseteq E$ and E is a set of parameters. For $r, s, t \in [0,1]$, the (r, s, t) - level soft set of ϖ is a crisp soft set $L(\varpi; r, s, t)$ = $\langle F_{(r,s,t)}, A \rangle$ defined by $F_{(r,s,t)}(e)$ = $L(F(e);r,s,t)$ $)=\{x \in U \,/\, T_{F(e)}(x) \geq r, I_{F(e)}(x) \geq s, F_{F(e)}(x) \leq t\}$, for all $e \in A$.

Here $r \in [0,1]$ can be viewed as a given least threshold on membership values, $s \in [0,1]$ can be viewed as a given least threshold on indeterministic values, and $t \in [0,1]$ can be viewed as a given greatest threshold on non-membership values.

For real-life applications of single valued neutrosophic soft sets based decision making, usually the thresholds r, s, t are chosen in advance by decision makers and represent their requirements on "membership levels", "indeterministic levels" and "non-membership levels" respectively.

To illustrate this idea, let us consider the following example.

Example 3.2

Let us consider a single valued neutrosophic soft set $\varpi = \langle F, A \rangle$ which describes the "features of the air conditioners" that Mr. X is considering for purchase. Suppose that there are five air conditioners produced by different companies in the domain $U = \{x_1, x_2, x_3, x_4, x_5\}$ under consideration, and that $A = \{e_1, e_2, e_3, e_4\}$ is a set of decision parameters. The e_i $(i = 1,2,3,4)$ stands for the parameters "branded", "expensive", "cooling speed" and "after sale product service", respectively.

Suppose that $F(e_1)$ = {<X_1,0.7,0.3,0.1>, <X_2,0.8,0.3, 0.1>, <X_3,0.9,0.4,0.05>, <X_4,0.6,0.3,0.2>, <X_5,0.5,0.4, 0.2>}, $F(e_2)$ = {<X_1,0.6,0.25,0.1>, <X_2,0.9,0.3,0.05>, <X_3, 0.8,0.3,0.05>, <X_4,0.6,0.2,0.4>, < X_5, 0.7,0.2,0.3>}, $F(e_3)$ = {<X_1,0.75,0.35,0.1>, <X_2,0.7,0.4,0.15>, <X_3,0.85,0.5, 0.1>, <X_4,0.5,0.4,0.3>, <X_5,0.6,0.45,0.2>}, $F(e_4)$ = {<X_1,0.65,0.3, 0.2>, <X_2,0.85,0.5,0.15>, <X_3,0.9,0.6,0.1>, <X_4,0.7,0.4, 0.2>, <X_5,0.6,0.3,0.1>}.

The single valued neutrosophic soft set $\varpi = \langle F, A \rangle$ is a parameterized family {$F(e_i)$, i=1,2,3,4} of single valued neutrosophic sets on U and $\langle F, A \rangle$ = {branded air conditioners = $F(e_1)$, expensive air conditioners = $F(e_2)$, High cooling speed air conditioners = $F(e_3)$, Good after sale product service = $F(e_4)$}. Table 1 gives the tabular representation of the single valued neutrosophic soft set $\varpi = \langle F, A \rangle$.

U	e_1	e_2	e_3	e_4
X_1	(0.7,0.3,0.1)	(0.6,0.25,0.1)	(0.75,0.35,0.1)	(0.65,0.3,0.2)
X_2	(0.8,0.3,0.1)	(0.9,0.3,0.05)	(0.7,0.4,0.15)	(0.85,0.5,0.15)
X_3	(0.9,0.4,0.05)	(0.8,0.3,0.05)	(0.85,0.5,0.1)	(0.9,0.6,0.1)
X_4	(0.6,0.3,0.2)	(0.6,0.2,0.4)	(0.5,0.4,0.3)	(0.7,0.4,0.2)
X_5	(0.5,0.4,0.2)	(0.7,0.2,0.3)	(0.6,0.45,0.2)	(0.6,0.3,0.1)

Table 1: Tabular representation of the single valued neutrosophic soft set $\varpi = \langle F, A \rangle$.

Now we take r = 0.7, s = 0.3, t = 0.2, then we have the following:

$L(F(e_1)$;0.7,0.3,0.2) = {X_1, X_2, X_3},
$L(F(e_2)$;0.7,0.3,0.2) = {X_2, X_3},
$L(F(e_3)$;0.7,0.3,0.2) = {X_1, X_2, X_3},
$L(F(e_4)$;0.7,0.3,0.2) = {X_2, X_3, X_4}.

Hence, the (0.7,0.3,0.2)-level soft set of $\varpi = \langle F, A \rangle$ is $L(\varpi$;0.7,0.3,0.2) = $\langle F_{(0.7,0.3,0.2)}, A \rangle$, where the set-valued mapping $F_{(0.7,0.3,0.2)}$: $A \rightarrow P(U)$ is defined by $F_{(0.7,0.3,0.2)}(e_i)$ = $L(F(e_i)$;0.7,0.3,0.2), for i=1,2,3,4. Table 2 gives the tabular representation of the (0.7,0.3, 0.2)-level soft set of $L(\varpi$;0.7,0.3,0.2).

U	e_1	e_2	e_3	e_4
X_1	1	0	1	0
X_2	1	1	1	1
X_3	1	1	1	1
X_4	0	0	0	1
X_5	0	0	0	0

Table 2: Tabular representation of the (0.7,0.3,0.2)-level soft set of $L(\varpi;0.7,0.3,0.2)$

Now, we show some properties of the (r,s,t) - level soft sets.

Theorem 3.3

Let $\varpi = \langle F, A\rangle$ be a single valued neutrosophic soft set over U, where $A \subseteq E$ and E is a set of parameters. Let $L(\varpi; r_1, s_1, t_1)$ and $L(\varpi; r_2, s_2, t_2)$ be (r_1, s_1, t_1)- level soft set, and (r_2, s_2, t_2)- level soft set of ϖ respectively, where r_1,s_1,t_1 , $r_2,s_2,t_2 \in [0,1]$. If $r_2 \le r_1$, $s_2 \le s_1$ and $t_2 \ge t_1$, then we have $L(\varpi; r_1, s_1, t_1) \tilde{\subset} L(\varpi; r_2, s_2, t_2)$.

Proof

Let $L(\varpi; r_1, s_1, t_1)$ =<$F_{(r_1,s_1,t_1)}$, A>, where $F_{(r_1,s_1,t_1)}(e)$ = $L(F(e); r_1, s_1, t_1)$ = $\{x \in U / T_{F(e)}(x) \ge r_1, I_{F(e)}(x) \ge s_1, F_{F(e)}(x) \le t_1\}$ for all $e \in A$.

Let $L(\varpi; r_2, s_2, t_2)$ =<$F_{(r_2,s_2,t_2)}$, A> where $F(r_2,s_2,t_2)(e)$ = $L(F(e); r_2, s_2, t_2)$ =$\{x \in U / T_{F(e)}(x) \ge r_2, I_{F(e)}(x) \ge s_2, F_{F(e)}(x) \le t_2\}$ for all $e \in A$. Obviously, $A \subseteq A$.

In the following, we will prove that for all $e \in A$, $F_{(r_1,s_1,t_1)}(e) \subseteq F(r_2,s_2,t_2)(e)$. Since $r_2 \le r_1$,$s_2 \le s_1$ and $t_2 \ge t_1$, then, for all $e \in A$, we have the following $\{x \in U / T_{F(e)}(x) \ge r_1, I_{F(e)}(x) \ge s_1, F_{F(e)}(x) \le t_1\} \subseteq \{x \in U / T_{F(e)}(x) \ge r_2, I_{F(e)}(x) \ge s_2, F_{F(e)}(x) \le t_2\}$. Since $F_{(r1,s_1,t_1)}(e)=\{x \in U / T_{F(e)}(x) \ge r_1, I_{F(e)}(x) \ge s_1, F_{F(e)}(x) \le t_1\}$ and $F(r_2,s_2,t_2)(e)=\{x \in U / T_{F(e)}(x) \ge r_2, I_{F(e)}(x) \ge s_2, F_{F(e)}(x) \le t_2\}$, thus we have $F_{(r_1,s_1,t_1)}(e) \subseteq F(r_2,s_2,t_2)(e)$. Therefore, $L(\varpi; r_1, s_1, t_1) \tilde{\subset} L(\varpi; r_2, s_2, t_2)$.

Theorem 3.4

Let $\varpi = \langle F, A\rangle$ and $\zeta = \langle G, A\rangle$ be a single valued neutrosophic soft sets over U, where $A \subseteq E$ and E is a set of parameters. $L(\varpi; r, s, t)$ and $L(\zeta; r, s, t)$ are (r,s,t) - level soft sets of ϖ and ζ , respectively, where r, s, $t \in [0,1]$. If $\varpi \tilde{\subset} \zeta$ then we have $L(\varpi; r, s, t) \tilde{\subset} L(\zeta; r, s, t)$.

Proof

$L(\varpi; r, s, t)$ = <$F_{(r,s,t)}$,A>, where $F_{(r,s,t)}(e)$ = $L(F(e);r,s,t)$ =$\{x \in U / T_{F(e)}(x) \ge r, I_{F(e)}(x) \ge s, F_{F(e)}(x) \le t\}$, for all $e \in A$.
Let $L(\zeta; r, s, t)$ = <$G_{(r,s,t)}$,A> where $G_{(r,s,t)}(e)$ = $L(G(e);r,s,t)$ =$\{x \in U / T_{G(e)}(x) \ge r, I_{G(e)}(x) \ge s, F_{G(e)}(x) \le t\}$, for all $e \in A$. Obviously, $A \subseteq A$.

In the following, we will prove that, for all $e \in A$, $F_{(r,s,t)}(e) \subseteq G_{(r,s,t)}(e)$. Since $\varpi \tilde{\subset} \zeta$, then we have the following $T_{F(e)}(x) \le T_{G(e)}(x)$, $I_{F(e)}(x) \le I_{G(e)}(x)$, $F_{F(e)}(x) \ge F_{G(e)}(x)$ for all $x \in U$ and $e \in A$. Assume that $x \in F_{(r,s,t)}(e)$. Since $F_{(r,s,t)}(e) = \{x \in U / T_{F(e)}(x) \ge r, I_{F(e)}(x) \ge s, F_{F(e)}(x) \le t\}$, then we have that $T_{F(e)}(x) \ge r$, $I_{F(e)}(x) \ge s$, $F_{F(e)}(x) \le t$. Since $T_{F(e)}(x) \le T_{G(e)}(x)$, $I_{F(e)}(x) \le I_{G(e)}(x)$, $F_{F(e)}(x) \ge F_{G(e)}(x)$, thus $T_{G(e)}(x) \ge r$, $I_{G(e)}(x) \ge s$, $F_{G(e)}(x) \le t$. Hence, $x \in \{x \in U / T_{G(e)}(x) \ge r, I_{G(e)}(x) \ge s, F_{G(e)}(x) \le t\}$. Since $G_{(r,s,t)}(e) = \{x \in U / T_{G(e)}(x) \ge r, I_{G(e)}(x) \ge s, F_{G(e)}(x) \le t\}$, then we have $x \in G_{(r,s,t)}(e)$. Thus, we have that $F_{(r,s,t)}(e) \subseteq G_{(r,s,t)}(e)$. Consequently, $L(\varpi; r, s, t) \tilde{\subset} L(\zeta; r, s, t)$.

Note 3.5

In the definition of (r,s,t) - level soft sets of single valued neutrosophic soft sets, the level triplet (or threshold triplet) assigned to each parameter has always constant values $r,s,t \in [0,1]$. However, in some decision making problems, it may happen that decision makers would like to improve different threshold triplets on different parameters. To cope with such problems, we need to use a function instead of a constant value triplet as the thresholds on membership values, indeterministic values and non-membership values respectively.

Definition 3.6

Let $\varpi = \langle F, A\rangle$ be a single valued neutrosophic soft set over U, where $A \subseteq E$ and E is a set of parameters. Let λ: $A \to I^3$ (I= [0,1]) be a single valued neutrosophic set in A which is called a threshold single valued neutrosophic set. The level soft set of ϖ with respect to λ is a crisp soft set $L(\varpi;\lambda) = \langle F_\lambda, A\rangle$ defined by $F_\lambda(e) = L(F(e);\lambda(e)) = \{x \in U / T_{F(e)}(x) \ge T_\lambda(e), I_{F(e)}(x) \ge I_\lambda(e), F_{F(e)}(x) \le F_\lambda(e)\}$, for all $e \in A$. To illustrate this idea, let us consider the following examples.

Example 3.7

Based on the single valued neutrosophic soft set $\varpi = \langle F, A\rangle$, we can define a single valued neutrosophic set mid_ϖ:$A \to [0,1]^3$, by

$$T_{mid_{\varpi}}(e) = \frac{1}{|U|} \sum_{x \in U} T_{F(e)}(x),$$

$$I_{mid_{\varpi}}(e) = \frac{1}{|U|} \sum_{x \in U} I_{F(e)}(x),$$

$$F_{mid_{\varpi}}(e) = \frac{1}{|U|} \sum_{x \in U} F_{F(e)}(x)$$

for all $e \in A$.

The single valued neutrosophic set mid_{ϖ} is called the mid-threshold single valued neutrosophic soft set ϖ. Further, the level soft set of ϖ with respect to the mid-threshold single valued neutrosophic set mid_{ϖ}, namely $L(\varpi; mid_{\varpi})$ is called the mid-level soft set of ϖ and simply denoted by $L(\varpi; mid)$.

Consider the problem in Example 3.2 with its tabular representation given by Table 1. It is clear that the mid-threshold of <F,A> is a single valued neutrosophic set

$mid_{<F,A>}$={<e_1,0.7,0.34,0.13>,<e_2,0.72,0.25,0.18>, < e_3,0.68,0.42,0.17> < e_4,0.74,0.42,0.15>}.

The mid-level soft set of <F,A> is a soft set L(<F,A>;mid) and its tabular representation is given by Table 3.

U	e_1	e_2	e_3	e_4
X_1	0	0	0	0
X_2	0	1	0	1
X_3	1	1	1	1
X_4	0	0	0	0
X_5	0	0	0	0

Table 3: Tabular representation of mid-level soft set $L(\langle F,A\rangle, mid)$

Example 3.8

Let $\varpi = \langle F, A\rangle$ be a single valued neutrosophic soft set over U, where $A \subseteq E$ and E is a set of parameters. Then, we can define:

(i) a single valued neutrosophic set $topbottom_{\varpi} : A \to I^3$

$T_{topbottom_{\varpi}}(e) = \max_{x \in U} T_{F(e)}(x)$, $I_{topbottom_{\varpi}}(e) = \max_{x \in U} I_{F(e)}(x)$,

$F_{topbottom_{\varpi}}(e) = \min_{x \in U} F_{F(e)}(x)$ for all $e \in A$.

(ii) a single valued neutrosophic set $toptop_{\varpi} : A \to I^3$

$T_{toptop_{\varpi}}(e) = \max_{x \in U} T_{F(e)}(x)$, $I_{toptop_{\varpi}}(e) = \max_{x \in U} I_{F(e)}(x)$,

$F_{toptop_{\varpi}}(e) = \max_{x \in U} F_{F(e)}(x)$ for all $e \in A$.

(iii) a single valued neutrosophic set $bottombottom_{\varpi} : A \to I^3$

$T_{bottombottom_{\varpi}}(e) = \min_{x \in U} T_{F(e)}(x)$, $I_{bottombottom_{\varpi}}(e) = \min_{x \in U} I_{F(e)}(x)$,

$F_{bottombottom_{\varpi}}(e) = \min_{x \in U} F_{F(e)}(x)$ for all $e \in A$, where I=[0,1]

The single valued neutrosophic set $topbottom_{\varpi}$ is called the top-bottom-threshold of the single valued neutrosophic soft set ϖ, the single valued neutrosophic set $toptop_{\varpi}$ is called the top-top-threshold of the single valued neutrosophic soft set ϖ, the single valued neutrosophic set $bottombottom_{\varpi}$ is called the bottom-bottom-threshold of the single valued neutrosophic soft set ϖ.

In addition, the level soft set of ϖ with respect to the top-bottom-threshold of the single valued neutrosophic soft set ϖ, namely $L(\varpi; topbottom_{\varpi})$ is called the top-bottom-level soft set of ϖ and simply denoted by $L(\varpi; topbottom)$.

Similarly, the top-top-level soft set of ϖ is denoted by $L(\varpi; toptop)$ and the bottom-bottom-level soft set of ϖ is denoted by $L(\varpi; bottombottom)$.

Let us consider the problem in Example 3.2 with its tabular representation given by Table 1. Here,

$topbottom_{<F,A>}$={<e_1,0.9,0.4,0.05>, <e_2,0.9,0.3,0.05>, < e_3,0.85,0.5,0.1> < e_4,0.9,0.6,0.1>

is a single valued neutrosophic set and the top-bottom-level soft set of $\langle F,A\rangle$ is $L(< F, A >; topbottom)$, see below.

U	e_1	e_2	e_3	e_4
X_1	0	0	0	0
X_2	0	1	0	0
X_3	1	0	1	1
X_4	0	0	0	0
X_5	0	0	0	0

Table 4: Tabular representation of top-bottom-level soft set $L(\langle F,A\rangle;$topbottom)

Also, the top-top-threshold of $\langle F,A\rangle$ is a single valued neutrosophic set $toptop_{\langle F,A\rangle}$={<$e_1$,0.9,0.4,0.2>, <$e_2$,0.9,0.3, 0.4>, <$e_3$,0.85,0.5,0.3>, <$e_4$,0.9,0.6,0.2>} and the top-top-level soft set of $\langle F,A\rangle$ *is* $L(< F, A >; toptop)$.

Its tabular representation is given by Table 5.

U	e_1	e_2	e_3	e_4
X_1	0	0	0	0
X_2	0	1	0	0
X_3	1	0	1	1
X_4	0	0	0	0
X_5	0	0	0	0

Table 5: Tabular representation of top-top-level soft set $L(\langle F,A\rangle$toptop)

It is clear that the bottom-bottom-threshold of $\langle F,A\rangle$ is a single valued neutrosophic set $bottombotttom_{<F,A>}$={<e_1,0.5,0.3,0.05>, <e_2,0.6,0.2,0.05>, <e_3,0.5,0.35, 0.1>, < e_4,0.6,0.3,0.1>} and the bottom-bottom level soft set of $\langle F,A\rangle$ is $L(<F,A>;bottombottom)$.

Its tabular representation is given by Table 6.

U	e_1	e_2	e_3	e_4
X_1	0	0	1	0
X_2	0	1	0	0
X_3	1	1	1	1
X_4	0	0	0	0
X_5	0	0	0	1

Table 6: Tabular representation of bottom-bottom-level soft set $L(\langle F,A\rangle$;bottombottom)

Remark 3.9

In Example 3.8, we do not define the bottom-top-level soft set of a single valued neutrosophic soft set, that is, we do not define the following single valued neutrosophic set $bottomtop_{\varpi}: A\rightarrow I^3$,

$T_{bottomtop\varpi}(e)=\min_{x\in U} T_{F(e)}(x)$, $I_{bottomtop\varpi}(e)=\min_{x\in U} I_{F(e)}(x)$,

$F_{bottomtop\varpi}(e)=\max_{x\in U} F_{F(e)}(x)$ for all $e\in A$.

The reason is the following: The bottom-top threshold is dispensable since it indeed consists of a lower bound of the degree of membership and indeterministic values and together with an upperbound of the degree of non-membership values. Thus, the bottom–top–threshold can always be satisfied.

Let us consider the Example 3.2, where the bottom–top–threshold of $\langle F,A\rangle$ is a single valued neutrosophic set $bottomtop_{<F,A>}$={<e_1,0.5,0.3,0.2>, <e_2,0.6,0.2,0.4>, <e_3,0.5, 0.35,0.3>, < e_4,0.6,0.3,0.2>} and the bottom-top-level soft set of $\langle F,A\rangle$ is a soft set $L(<F,A>;bottomtop)$ with its tabular representation given by Table 7.

U	e_1	e_2	e_3	e_4
X_1	1	1	1	1
X_2	1	1	1	1
X_3	1	1	1	1
X_4	1	1	1	1
X_5	1	1	1	1

Table 7: Tabular representation of bottom-top-level soft set $L(\langle F,A\rangle$;bottomtop)

From Table 7, we can see that all the tabular entries are equal to 1. In other words, the bottom-top-threshold can always be satisfied.

Now, we show some properties of level soft sets with respect to a single valued neutrosophic soft set.

Theorem 3.10

Let $\varpi=\langle F,A\rangle$ be a single valued neutrosophic soft set over U, where $A\subseteq E$ and E is a set of parameters. Let λ_1: $A\rightarrow I^3$ (I=[0,1]) and λ_2: $A\rightarrow I^3$ (I=[0,1]) be two threshold single valued neutrosophic sets. $L(\varpi;\lambda_1)=\langle F_{\lambda 1},A\rangle$ and $L(\varpi;\lambda_2)=\langle F_{\lambda 2},A\rangle$ are the level soft sets of ϖ with respect to λ_1 and λ_2, respectively. If $T_{\lambda 2}(e)\leq T_{\lambda 1}(e)$, $I_{\lambda 2}(e)\leq I_{\lambda 1}(e)$ and $F_{\lambda 2}(e)\geq F_{\lambda 1}(e)$, for all $e\in A$, then we have $L(\varpi;\lambda_1)\ \tilde{\subset}\ L(\varpi;\lambda_2)$.

Proof

The proof is similar to Theorem 3.3.

Theorem 3.11

Let $\varpi=\langle F,A\rangle$ and $\zeta=\langle G,A\rangle$ be two single valued neutrosophic soft sets over U, where $A\subseteq E$ and E is a set of parameters.

Let λ: $A\rightarrow I^3$ (I=[0,1]) be a threshold single valued neutrosophic set. $L(\varpi;\lambda)=\langle F_{\lambda},A\rangle$ and $L(\zeta;\lambda)=\langle G_{\lambda},A\rangle$ are the level soft sets of ϖ and ζ with respect to λ respectively. If $\varpi\ \tilde{\subset}\ \zeta$, then we have $L(\varpi;\lambda)\ \tilde{\subset}\ L(\zeta;\lambda)$.

Proof

The proof is similar to Theorem 3.4.

Theorem 3.12

Let $\varpi=\langle G,A\rangle$ be a single valued neutrosophic soft set over U, where $A\subseteq E$ and E be a set of parameters.

$L(\varpi;mid)$, $L(\varpi;topbottom)$, $L(\varpi;toptop)$, $L(\varpi;bottombottom)$ are the mid–level soft set, the top-bottom-level soft set, the top-top-level soft set and the bottom-bottom-level soft set of ϖ, respectively. Then, we have the following properties:

(i) $L(\varpi;topbottom)\ \tilde{\subset}\ L(\varpi;mid)$.

(ii) $L(\varpi;topbottom)\ \tilde{\subset}\ L(\varpi;toptop)$.

(iii) $L(\varpi;topbottom)\ \tilde{\subset}\ L(\varpi;bottombottom)$.

Proof

(i) Let $L(\varpi;topbottom)=\langle G_{topbottom},A\rangle$,where

$T_{topbottom\varpi}(e)=\max_{x\in U} T_{G(e)}(x)$, $I_{topbottom\varpi}(e)=\max_{x\in U} I_{G(e)}(x)$,

$F_{topbottom\varpi}(e)=\min_{x\in U} F_{G(e)}(x)$ for all $e\in A$.

Let $L(\varpi;mid)=\langle G_{mid},A\rangle$, where

$$T_{mid_{\varpi}}(e) = \frac{1}{|U|}\sum_{x\in U} T_{G(e)}(x),$$

$$I_{mid_{\varpi}}(e) = \frac{1}{|U|}\sum_{x\in U} I_{G(e)}(x),$$

$$F_{mid_{\varpi}}(e) = \frac{1}{|U|}\sum_{x\in U} F_{G(e)}(x)$$

for all $e\in A$. Obviously, $A\subseteq A$.

In the following we will prove that for all $e\in A$.

$G_{topbottom}(e) \subseteq G_{mid}(e)$.

Since $\max_{x\in U} T_{G(e)}(x) \geq \frac{1}{|U|}\sum_{x\in U} T_{G(e)}(x)$, $\max_{x\in U} I_{G(e)}(x) \geq \frac{1}{|U|}\sum_{x\in U} I_{G(e)}(x)$, $F_{G(e)}(x) \leq \frac{1}{|U|}\sum_{x\in U} F_{G(e)}(x)$,

then for al l $e\in A$ we have $T_{topbottom}(e) \geq T_{mid}(e)$, $I_{topbottom}(e) \geq I_{mid}(e)$, $F_{topbottom}(e) \leq F_{mid}(e)$. Thus, we have the following $\{x\in U \ / \ T_{G(e)}(x) \geq T_{topbottom}(e), I_{G(e)}(x) \geq I_{topbottom}(e), F_{G(e)}(x) \leq F_{topbottom}(e)\} \subseteq \{x\in U \ / \ T_{G(e)}(x) \geq T_{mid}(e), I_{G(e)}(x) \geq I_{mid}(e), F_{G(e)}(x) \leq F_{mid}(e)\}$. Since $G_{topbottom}(e)= \{x\in U \ / \ T_{G(e)}(x) \geq T_{topbottom}(e), I_{G(e)}(x) \geq I_{topbottom}(e), F_{G(e)}(x) \leq F_{topbottom}(e)\}$ and $G_{mid}(e)= \{x\in U \ / \ T_{G(e)}(x) \geq T_{mid}(e), I_{G(e)}(x) \geq I_{mid}(e), F_{G(e)}(x) \leq F_{mid}(e)\}$, then we have the following $G_{topbottom}(e) \subseteq G_{mid}(e)$. Therefore $L(\varpi; topbottom) \tilde{\subseteq} L(\varpi; mid)$.

Proof of (ii) and (iii) are analogous to proof (i).

Now, we show the adjustable approach to single valued neutrosophic soft sets based decision making by using level soft sets.

Algorithm 3.13

Step 1: Input the (resultant) single valued neutrosophic soft set $\varpi=\langle F,A\rangle$.

Step 2: Input the threshold single valued neutrosophic set $\lambda: A\rightarrow I^3$ (I=[0,1]) (or give a threshold value triplet $(r, s, t) \in I^3$ (I=[0,1]); or choose the mid-level decision rule; or choose the top-bottom-level decision rule; or choose the top-top-level decision rule; or choose the bottom-bottom-level decision rule) for decision making.

Step 3: Compute the level soft set $L(\varpi;\lambda)$ with the threshold single valued neutrosophic set λ (or the (r, s, t)–level soft set $L(\varpi;r,s,t)$; or the mid-level soft set $L(\varpi;mid)$; or choose the top-bottom-level soft set $L(\varpi;topbottom)$; or choose the top-top-level soft set $L(\varpi;toptop)$; or choose the bottom-bottom-level soft set $L(\varpi;bottombottom)$)

Step 4: Present the level soft $L(\varpi;\lambda)$(or $L(\varpi;r,s,t)$; $L(\varpi;mid)$; $L(\varpi;topbottom)$, $L(\varpi;bottombottom)$) in tabular form and compute the choice value c_i of o_i, for all i.

Step 5: The optimal decision is to select o_k if $c_k= \max_i c_i$.

Step 6: If k has more than one value, then any of o_k may be chosen.

Note 3.14

In the last step of Algorithm 3.13, one may go back to the second step and change the previously used threshold (or decision rule), as to adjust the final optimal decision, especially when there are too many "optimal choices" to be chosen.

To illustrate the basic idea of Algorithm 3.13, let us consider the following example.

Example 3.15

Let us consider the decision making problem (Example 3.2) involving the single valued neutrosophic soft set $\varpi=\langle F,A\rangle$ with its tabular representation given by Table 1.

If we deal with this problem by mid-level decision rule, we shall use the mid-threshold $mid_{\langle F,A\rangle}$ and thus obtain the mid-level soft set $L(\langle F,A\rangle,mid)$ with choice values having their tabular representation in Table 8.

U	e_1	e_2	e_3	e_4	*Choice values*
X_1	0	0	0	0	$c_1=0$
X_2	0	1	0	1	$c_2=2$
X_3	1	1	1	1	$c_3=4$
X_4	0	0	0	0	$c_4=0$
X_5	0	0	0	0	$c_5=0$

Table 8: Tabular representation of mid-level soft set $L(\langle F,A\rangle;mid)$ with choice values

From Table 8, it follows that the maximum choice value is $c_3=4$, so the optimal decision is to select X_3.

At the same time, if we deal with this problem by top-bottom-level soft set $L(\langle F,A\rangle,topbottom)$ we obtain the choice values given by Table 9.

U	e_1	e_2	e_3	e_4	*Choice values*
X_1	0	0	0	0	$c_1=0$
X_2	0	1	0	0	$c_2=1$
X_3	1	0	1	1	$c_3=3$
X_4	0	0	0	0	$c_4=0$
X_5	0	0	0	0	$c_5=0$

Table 9: Tabular representation of top-bottom-level soft set $L(\langle F,A\rangle;topbottom)$ with choice values

From Table 9, it is clear that the maximum choice value is $c_3=3$, so the optimal decision is to select X_3.

4 Weighted single valued neutrosophic soft sets based decision making

In this section, we will present an adjustable approach to weighted single valued neutrosophic soft sets based decision making problems.

Definition 4.1

Let *FN(U)* be the set of all single valued neutrosophic sets in the universe *U*. Let $A \subseteq E$ and *E* be a set of parameters. A weighted single valued neutrosophic soft set is a triple $\xi=\langle F, A, \omega\rangle$, where $\langle F, A\rangle$ is a single valued neutrosophic soft set over U , $\omega:A\to[0,1]$ is a weight function specifying the weight $w_j=\omega(e_j)$ for each attribute $e_j\in A$.

By definition, every single valued neutrosophic soft set can be considered as a weighted fuzzy soft set. The notion of weighted single valued neutrosophic soft sets provides a mathematical framework for modelling and analyzing the decision making problems in which all the choice parameters may not be of equal importance. These differences between the importance of parameters are characterized by the weight function in a weighted single valued neutrosophic soft set.

Algorithm 4.2 (an adjustable approach to weighted single valued neutrosophic soft sets based decision making problems)

Step 1: Input the weighted single valued neutrosophic soft set $\xi=\langle F, A, \omega\rangle$.

Step 2: Input the threshold single valued neutrosophic set $\lambda: A\to I^3$ (or give a threshold value triplet $(r, s, t) \in I^3$; or choose the mid-level decision rule; or choose the top-bottom-level decision rule; or choose the top-top-level decision rule; or choose the bottom -bottom-level decision rule) for decision making.

Step 3: Compute the level soft set $L(\langle F,A\rangle;\lambda)$ of ξ with respect to the threshold single valued neutrosophic set λ (or the (r, s, t)–level soft set $L(\langle F,A\rangle;r,s,t)$; or the mid-level soft set $L(\langle F,A\rangle;mid)$; or choose the top-bottom-level soft set $L(\langle F,A\rangle;topbottom)$; or choose the top-top-level soft set $L(\langle F,A\rangle;toptop)$; or choose the bottom-bottom-level soft set $L(\langle F,A\rangle;bottombottom)$).

Step 4: Present the level soft $L(\langle F,A\rangle;\lambda)$(or $L(\langle F,A\rangle;r,s,t)$; $L(\langle F,A\rangle;mid)$;$L(\langle F,A\rangle;topbottom)$, $L(\langle F,A\rangle;bottombottom)$) in tabular form and compute the choice value c'_i of o_i , for all *i*.

Step 5: The optimal decision is to select o_k if $c'_k=\max_i c'_i$.

Step 6: If *k* has more than one value then any of o_k may be chosen.

Note 4.3

In the last step of Algorithm 4.2, one may go back to the second step and change the previously used threshold (or decision rule), as to adjust the final optimal decision, especially when there are too many “optimal choices” to be chosen.

To illustrate the basic idea of Algorithm 4.2, let us consider the following example.

Example 4.3

Let us consider the decision making problem (Example 3.2). Suppose that Mr. X has imposed the following weights for the parameters in *A*: for the parameter “branded”, $w_1=0.8$, for the parameter “expensive”, $w_2=0.6$, for the parameter “cooling speed”, $w_3=0.9$, and for the parameter “after sale product service”, $w_4=0.7$. Thus, we have a weight function $\omega:A\to[0,1]$, and the single valued neutrosophic soft set $\varpi=\langle F, A\rangle$ in Example 3.2 is changed into a weighted single valued neutrosophic soft set $\xi=\langle F, A, \omega\rangle$. Its tabular representation is given by Table 10.

U	$e_1,w_1=0.8$	$e_2,w_2=0.6$	$e_3,w_3=0.9$	$e_4,w_{4=0.7}$
X_1	(0.7,0.3,0.1)	(0.6,0.25,0.1)	(0.75,0.35,0.1)	(0.65,0.3,0.2)
X_2	(0.8,0.3,0.1)	(0.9,0.3,0.05)	(0.7,0.4,0.15)	(0.85,0.5,0.15)
X_3	(0.9,0.4,0.05)	(0.8,0.3,0.05)	(0.85,0.5,0.1)	(0.9,0.6,0.1)
X_4	(0.6,0.3,0.2)	(0.6,0.2,0.4)	(0.5,0.4,0.3)	(0.7,0.4,0.2)
X_5	(0.5,0.4,0.2)	(0.7,0.2,0.3)	(0.6,0.45,0.2)	(0.6,0.3,0.1)

Table 10: Tabular representation of weighted single valued neutrosophic soft set $\xi=\langle F, A, \omega\rangle$.

As an adjustable approach, one can use different rules in decision making problem. For example, if we deal with this problem by mid-level decision rule, we shall use the mid-threshold $mid_{\langle F,A\rangle}$ and thus obtain the mid-level soft set $L(\langle F,A\rangle,mid)$ with weighted choice values having tabular representation in Table 11.

U	$e_1,w_1=0.8$	$e_2,w_2=0.6$	$e_3,w_3=0.9$	$e_4,w_4=0.7$	weighted choice value
X_1	0	0	0	0	$c'_1=0$
X_2	0	1	0	1	$c'_2=1.3$
X_3	1	1	1	1	$c'_3=3.2$
X_4	0	0	0	0	$c'_4=0$
X_5	0	0	0	0	$c'_5=0$

Table 11: Tabular representation of mid-level soft set $L(<F,A>;mid)$ with weighted choice values

It follows that the maximum weighted choice value is $c'_3=3.2$, so the optimal decision is to select X_3.

5 Mean potentiality approach

Definition 5.1

The potentiality of a single valued neutrosophic soft set (p_{fns}) is defined as the sum of all membership, indeterministic and non-membership values of all objects with respect to all parameters. Mathematically, it is defined as

$$p_{fns} = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} T_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} I_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} F_{ij} \right)$$

where T_{ij}, I_{ij}, F_{ij} are the membership, indeterministic and non-membership values of the i^{th} object with respect to the j^{th} parameter respectively, m is the number of objects and n is the number of parameters.

Definition 5.2

The mean potentiality (m_p) of the single valued neutrosophic soft set is defined as its average weight among the total potentiality.Mathematically, it is defined as

$$m_p = \frac{p_{fns}}{m \times n}.$$

Algorithm 5.3

Step 1: Input the (resultant) single valued neutrosophic soft set $\varpi = \langle F, A \rangle$**.**

Step 2: Compute the potentiality (p_{fns}) of the single valued neutrosophic soft set.

Step 3: Find out the mean potentiality (m_p) of the single valued neutrosophic soft set**.**

Step 4: Form m_p-level soft soft set of the single valued neutrosophic soft set in tabular form, then compute the choice value c_i of o_i, for all i.

Step 5: The optimal decision is to select o_k if $c_k = \max_i c_i$.

Step 6: If k has more than one value, then any of o_k may be chosen.

Example 5.4

Let us consider the problem in Example 3.2 with its tabular representation in Table 1.

U	e_1	e_2	e_3	e_4	Choice value
X_1	(0.7,0.3,0.1)	(0.6,0.25,0.1)	(0.75,0.35,0.1)	(0.65,0.3,0.2)	(2.7,1.2,0.5)
X_2	(0.8,0.3,0.1)	(0.9,0.3,0.05)	(0.7,0.4,0.15)	(0.85,0.5,0.15)	(3.25,1.5,0.45)
X_3	(0.9,0.4,0.05)	(0.8,0.3,0.05)	(0.85,0.5,0.1)	(0.9,0.6,0.1)	(3.45,1.8,0.3)
X_4	(0.6,0.3,0.2)	(0.6,0.2,0.4)	(0.5,0.4,0.3)	(0.7,0.4,0.2)	(2.4,1.3,1.1)
X_5	(0.5,0.4,0.2)	(0.7,0.2,0.3)	(0.6,0.45,0.2)	(0.6,0.3,0.1)	(2.4,1.35,0.8)

Table 12: Tabular representation of single valued neutrosophic soft set with choice values.

So, the potentiality is p_{fns} = (14.2,7.15,3.15).

The Mean potentiality $m_p = \frac{p_{fns}}{m \times n}$ is:

$$m_p = \left(\frac{14.2}{5 \times 4}, \frac{7.15}{5 \times 4}, \frac{3.15}{5 \times 4} \right) = (0.71, 0.36, 0.16).$$

Using this triplet, we can form the m_p-level soft set, which is shown by Table 13.

U	e_1	e_2	e_3	e_4	Choice values
X_1	0	0	0	0	$c_1=0$
X_2	0	0	0	1	$c_2=1$
X_3	1	0	1	1	$c_3=3$
X_4	0	0	0	0	$c_4=0$
X_5	0	0	0	0	$c_5=0$

Table 13: Tabular representation of m_p-level soft set with choice values.

From Table 13, it is clear that the maximum choice value is $c_3=3$, so the optimal decision is to select X_3.

Conclusion

In this paper, we introduced an adjustable and mean potentiality approach by means of neutrosophic level soft sets. Different level soft sets were derived by considering different types of thresholds, namely, *mid*, *topbottom*, *toptop*, *bottombottom*. In general, the final optimal decisions based on different level soft sets could be different. Thus, the approach discussed in this paper captures an important feature for decision making in an imprecise environment. Some of these problems are essentially humanistic, and thus, subjective in nature; there actually isn't a unique or uniform criterion for evaluating the alternatives. Hence, the decision making models presented in this paper make the approaches to single valued neutrosophic level soft sets based decision making more appropriate for many real world applications.

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Received: Feb. 10, 2016. Accepted: Mar. 22, 2016