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Abstract— In this paper, we use game theory to model a general participation game for a distributed and dynamic network. Such networks may include sensing and control agents. The main problem we are interested in is how to achieve broad participation while aligning the incentives of all the participating agents. A consumer node is willing to invest an amount of rewards to get a set of networked sensor nodes, alternatively agents, to participate in some desirable activity; for example rewarding nodes by allocating more bandwidth to them. For the scope of this paper, the agents are heterogeneous sensors, and the consumer is a supervising sensor whether that be a human or a super-node. The consumer desires an accurate reading of a signal of interest; however, it may only communicate with its direct neighbors, or the sensors closest to it. Therefore, it must incentivize its neighbors to participate in further advertising and participating in the (sensing) activity. The neighbors then incentivize their neighbors to participate, and so on. We assume the commodity being traded to be the agent's participation in the activity; sensing a signal, and forwarding a query are some examples of participation. In the resulting game, agents choose their offers strategically. We prove the existence of equilibria for specific utility functions and simple network structures.

I. INTRODUCTION

In this paper we study the problem of incentivizing cooperation amongst a network of sensors. In a (wireless) sensor network, a set of heterogeneous sensors pool their resources in order to measure a quantity of interest. In general, a source node (a consumer) needs to receive information about a quantity that may not be directly accessible to it. The source node may then request the information from other nodes that will then propagate the request down to those that are best equipped to measure the quantity of interest. In order to measure and propagate signals however, sensor nodes need to expand some resource (power), and so they might not have an incentive to participate.

More generally, the problem we are interested in is that of incentivizing a set of nodes (alternatively agents or players) to participate in some activity that is of interest to a consumer. For example, the consumer might want to measure a signal, to solve a problem, or to distribute a good. The consumer receives a positive utility from every agent that participates, whereas an agent incurs a cost to participate. The consumer and the agents are located on a network and communication, in the form of trade, may only occur between directly connected agents. In essence, the consumer

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wants its direct neighbors to participate, and wishes that they further incentivize their direct neighbors to participate, and so on. We now present several instances of the problem:

- Distributed sensing and control: In a wireless sensor/control network setting, a number of spatially distributed sensors are assigned the task of monitoring and measuring a signal of interest. The nodes are usually limited in their resources and must cooperate in order to communicate an accurate measurement to the supervising node. The supervising node is thus a consumer of information and must incentivize as many nodes as possible to assist in obtaining an accurate measurement. In this setting, sensor nodes may be rewarded for example by allocating more bandwidth to them.

- Advertisement and/or search in social networks: Consider a consumer that wishes to broadly advertise a piece of information within a social network and has access to a small set of source agents. The consumer might be a company wanting to buy Ad space on users' pages for example and is willing to invest in doing so [1]. A similar problem is social information retrieval (or search) [2], [3] in which a user wishes to get a query answered and the user receives a utility from the answer(s). In both of these settings, the agents participating in the advertisement or the search incur a participation cost and trade is constrained by an agent's position in the network.

- Propagating reachability information: A similar setting to the previous one is advertisement of reachability information as is the case with the majority of routing protocols in communication networks. The Border Gateway Protocol (BGP) [4], for example, is a path-vector protocol and is the de-facto protocol for Internet interdomain routing. BGP is intrinsically about distributing route information about destinations, which are Internet Protocol (IP) prefixes, to establish paths in the network. A destination wishes that its route information be globally advertised and the participating agents incur a cost for doing so. The agents, Autonomous Systems (AS), are economic entities that act selfishly when implementing their internal policies and particularly the

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[1]Notice that in today's social networks, information spreads quickly starting from a set of sources. The agents in this case have a local incentive to propagate the information to their circle. Other models exist as well for information propagation based on utility [1] and they assume that agents will exchange information only when they benefit from the exchange (in this case gain information). In contrast, the problem setting we are interested in assumes that distribution is subsidized by the consumer.

decisions that relate to route selection and propagation [5].

- Group problem solving: In an effort to explore the power of social networking in team problem solving, DARPA has recently announced a network challenge [6] to solve a geographically distributed problem that requires social collaboration. The problem was to find the locations of 10 balloons dispersed across the continental United States. A timely solution to the problem requires broad participation and while DARPA, the consumer, may and is willing to subsidize the solution, it has access to a limited set of agents. DARPA's utility increases with participation while the agents incur a cost to participate.

In this work, we use game theory to model a general participation game. The main problem we are interested in is how to achieve broad participation while aligning the incentives of all the participating agents. The consumer, denoted by d, is willing to invest some initial amount of resource r_d to get the agents to participate in some desirable activity. However, d may only communicate with its direct neighbors. Therefore, d must incentivize its neighbors to participate in further advertising the activity, who then incentivize their neighbors to participate, and so on. We assume in this paper the commodity that may be traded to be the agent's participation in the activity (whichever form participation may take). In the resulting game, agents choose their prices (offers) strategically and they are rewarded proportionally to the size of their participation subtree (defined shortly). We assume *full information* since our main goal is to study the existence of equilibria rather than how to reach the equilibrium.

We have initially studied the BGP game in [7], [8]. This work extends the initial results of [7] by considering a fundamentally different assumption on tie breaking under competition. First, we define the general game model in section II. Section III then presents the equilibria results for two simple graph structures: 1) the line (and the tree) graphs which involve no competition, and 2) the ring graph which involves competition.

II. THE MODEL

We consider a graph $G = (V, E)$ where V is a set of n nodes (alternatively termed players, or agents) each identified by a unique index $i = \{1, \ldots, n\}$, and the consumer d, and E is the set of edges or links. Denote by $B(j)$ the set of direct neighbors of node j. The game proceeds as follows: d first advertises the activity to its neighbors promising them a reward $r_d \in \mathbb{Z}^+$ which directly depends on d's utility from participation. A player j in turn receives offers from its neighbors where each neighbor i's offer takes the form of a reward r_{ij}. A reward r_{ij} that a player i offers to some direct neighbor $j \in B(i)$ is a contract stating that i will pay j an amount that is a function of r_{ij} for each sale that player j makes. Let $R(j)$ be the set of all offers that are known to player j through advertisements, $|R(j)| \leq |B(j)|$. After receiving the offers, player j strategizes by selecting

an offer $r_{ij} \in R(j)$, and deciding on a reward $r_{jl} < r_{ij}$ to send to each *candidate* neighbor $l \in B(j)$ that it has not received a competing offer from. Note then that $r_{lj} < r_{jl}$ where $r_{lj} = 0$ means that j did not receive an offer from neighbor l. Player j then pockets the difference $r_{ij} - r_{jl}$. The process repeats up to some depth that is directly dependent on the initial investment r_d as well as on the strategies of the players. In the sensor network setting, this translates into an agent obtaining a bandwidth allocation from some upstream agent and further allocating part of it in incentives to the downstream nodes. We intentionally keep the reward model abstract at this point, but will revisit it later in the discussion when we define more specific utility functions. Let c_j denote player j's local cost of participation. Finally, denote by P_j the *upstream* reward path from j to d, $P_j \in \mathcal{P}(j)$ the latter being the set of all simple paths from j to d. The reward path P_j is defined recursively as $P_j = (j, i)P_i$ when j selects i's offer r_{ij}, where the notation $(j, i)P_i$ is the path formed by concatenating link (j, i) with path P_i (rewards flow on path P_j from d towards j). Define node i to be an *upstream* node relative to node j when $i \in P_j$. The opposite holds for a *downstream* node.

Assumptions: To keep our model tractable, we take the following simplifying assumptions:

1) the graph is at steady state for the duration of the game i.e. we do not consider topology dynamics;
2) the players are indistinguishable to the consumer d i.e. d receives the same marginal utility from every participation;
3) the advertised rewards are integers and are strictly decreasing with depth i.e. $\forall\ j, l, r_{jl} \in \mathbb{Z}^+$, and $r_{jl} < r_{ij}$ when player j accepts offer r_{ij}. We let 1 unit be the cost of advertisement [2] (a similar assumption was taken in [2] to avoid the degenerate case of never running out of rewards, referred to as "Zeno's Paradox");
4) a node that does not participate has a utility of zero;
5) the local participation cost is constant with $c_i = c = 1$.

Strategy Space: After receiving a set of offers $R(i)$ from neighboring nodes, a *pure strategy* $s_i = (r_{ji}, r_i)$, $s_i \in S_i$ of an autonomous player i comprises two decisions as follows:

- Choose a single "best" offer $r_{ji} \in R(i)$ (where "best" is defined shortly in Theorem 1);
- Choose a reward vector $r_i = [r_{ij}]_j$ offering a reward r_{ij} to each candidate neighbor j.

A strategy profile $\mathbf{s} = (s_1, \ldots, s_n)$ and a reward r_d define an outcome of the game [3]. Every outcome determines a set of paths to d given by $T_d = (P_1, \ldots, P_n)$. A utility function $u_i(\mathbf{s})$ for player i associates every outcome with a real value in $\mathbb{R}$. We use the notation s_{-i} to refer to the strategy profile of all players excluding i, where $\mathbf{s} = (s_i, s_{-i})$. The Nash equilibrium is defined as follows:

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[2] In our setting, this cost might be the bandwidth lost due to attenuation.

[3] We abuse notation hereafter and we refer to the outcome with simply the strategy profile $\mathbf{s}$ where it should be clear from context that an outcome is defined by the tuple $< \mathbf{s}, r_d >$. Notice that a strategy profile may be associated with an outcome if we model r_d as an action. We refrain from doing so to make it explicit that r_d is not strategic.

Definition 1: A Nash Equilibrium (NE) is a strategy profile $\mathbf{s}^* = (s_1^*, \ldots, s_n^*)$ such that no player can move profitably by changing her strategy, i.e. for each player i, $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \forall s_i \in S_i$.

Utility: We experiment with a simple class of utility functions which rewards a player linearly based on volume of sales. This model incentivizes participation and potentially requires a large initial investment from d. More clearly, let $N_i(\mathbf{s}) = \{j \in V\setminus\{i\}|i \in P_j\}$ be the set of nodes nodes downstream of i, and define $\delta_i = |N_i(\mathbf{s})|$. When $s_i = (r_{ki}, r_i)$, the utility of player i from an outcome or strategy profile $\mathbf{s} = (s_i, s_{-i})$ is:

$$u_i(\mathbf{s}) = (r_{ki} - c_i) + \sum_{\{j|P_j=(j,i)P_i\}} (r_{ki} - r_{ij})(\delta_j + 1) \quad (1)$$

The first term $(r_{ki} - c_i)$ of (1) is incurred by every participating player and is the one unit of reward from the upstream parent on the chosen reward path minus the local cost. Based on the fixed cost assumption, we often drop this first term when comparing player payoffs from different strategies since the term is always positive when $c = 1$. The second term of (1) (the summation) is the total profit made by i where $(r_{ki} - r_{ij})(\delta_j + 1)$ is i's profit from the sale to neighbor j (which depends on δ_j). A rational selfish node will always try to maximize its utility when picking $s_i = (r_{ki}, [r_{ij}]_j)$. There is an inherent tradeoff between $(r_{ki} - r_{ij})$ and (δ_j) when trying to maximize the utility in Equation (1) in the face of competition as shall become clear later. A higher offered reward r_{ij} allows the player to compete (and possibly increase δ_j) but will cut the profit margin. Finally, we implicitly assume that the consumer d receives a constant marginal utility of r_d from each player that accepts to participate - the marginal utility of participation - and declares r_d truthfully to its direct neighbors (i.e. r_d is not strategic).

Before proceeding with the model, we present the following two results (see [8] for the proof):

Theorem 1: In order to maximize her utility, player i always chooses the highest offer r_{ji} where $r_{ji} \geq r_{ki}, \forall r_{ki} \in R(i)$.

Thus, we shall represent player i's strategy hereafter simply with the rewards vector, i.e. $s_i = (r_i)$, and it should be clear that player i always chooses the highest offer. When the rewards are equal however, we assume that a player breaks ties uniformly (i.e. with equal probability of choosing any of the equal offers). The uniform tie-breaking assumption differentiates this work from the work in [7]. From Theorem 1 and the strictly decreasing rewards assumption, it may be shown that an outcome T_d of the game is always a tree. This second result allows us to focus on the existence of equilibria.

A. The Static Multi-Stage Game with fixed schedule

We restrict the analysis of equilibria to the simple line and ring graphs. In order to apply the correct solution concept, we fix the *schedule* of play (i.e. who plays when?) based on the inherent order of play in the model. We resort to the

multi-stage game with observed actions [9] where stages in our game have no temporal semantics. Rather, stages identify the network positions which have strategic significance due to the strictly decreasing rewards assumption. Formally, and using notation from [9], each player i plays only once at stage $k > 0$ where k is the distance from i to d in number of hops. At every other stage, the player plays the "do nothing" action. The game starts at stage 1 after d declares r_d. Players at the same stage play simultaneously, and we denote by $a^k = (a_1^k, \ldots, a_n^k)$ the set of player actions at stage k, the stage-k action profile. Further, denote by $h^{k+1} = (r_d, a^1, \ldots, a^k)$, the *history* at the end of stage k which is simply the initial reward r_d concatenated with the sequence of actions at all previous stages. We let $h^1 = (r_d)$. Finally, $h^{k+1} \subset H^{k+1}$ the latter being the set of all possible stage-k histories. When the game has a finite number of stages, say $K+1$, then a terminal history h^{K+1} is equivalent to an outcome of the game (which is a tree T_d) and the set of all outcomes is H^{K+1}. The pure-strategy of player i who plays at stage $k > 0$ is a function of the history and is given by $s_i : H^k \to \mathbb{R}^{m_i}$ where m_i is the number of direct neighbors of player i that are at stage $k + 1$ (implicitly, a player at stage k observes the full history h^k before playing). We resort to the multi-stage model (the fixed schedule) on our simple graphs to eliminate the synchronization problems and to focus instead on the existence of equilibria. The key concept here is that it is the *information sets* [9] that matter rather than the time of play i.e. since all the nodes at distance 1 from d observe r_d before playing, all these nodes belong to the same information set whether they play at the same time or at different time instants.

Starting with r_d (which is h^1), it is clear how the game produces actions at every later stage based on the player strategies, resulting in a terminal action profile or outcome. Hence, given r_d, an outcome in H^{K+1} may be associated with every strategy profile $\mathbf{s}$ and so the definition of Nash equilibrium remains unchanged (see [9] for definitions of *Nash equilibrium*, *proper subgame*, and *subgame perfection*). In our game, each stage begins a new subgame which restricts the full game to a particular history. For example, a history h^k begins a subgame $G(h^k)$ such that the histories in the subgame are restricted to $h^{k+1} = (h^k, a^k)$, $h^{k+2} = (h^k, a^k, a^{k+1})$, and so on. Hereafter, the general notion of equilibrium we use is the Nash equilibrium and we shall make it clear when we generalize to subgame perfect equilibria. We are only interested in pure-strategy equilibria [9] and in studying the existence question as the incentive r_d varies.

III. Equilibria on the Line Graph, the Tree, and the Ring Graph

In the general game model defined thus far, every outcome (including the equilibrium) depends on the initial reward/utility r_d of the advertiser. In the same spirit as [2] we inductively construct the equilibrium for the line graph of Figure 1(a) given the utility function of Equation (1). We present the result for the line which may be directly extended
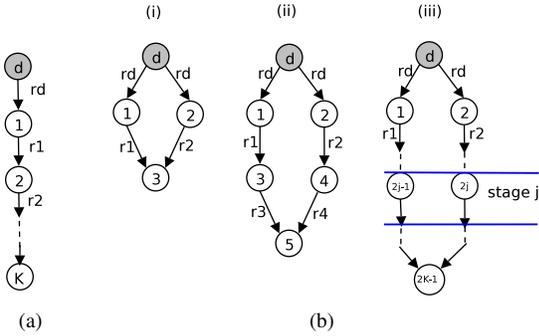
Fig. 1. (a) Line graph: a player's index is the stage at which the player plays; d advertises at stage 0; $K = n$; (b) Ring graph with even number of players: (i) 2-stage game, (ii) 3-stage game, and general (iii) K-stage game.

to trees. Before proceeding with the construction, notice that for the line, $m_i = 1$ for all players except the leaf player since each of those players has a single downstream neighbor. In addition, $\delta_i = \delta_j + 1, \forall i, j$ where j is i's child ($\delta_i = 0$ when i is a leaf). We shall refer to both the player and the stage using the same index since our intention should be clear from the context. For example, the child of player i is $i + 1$ and its parent is $i - 1$ where player i is the player at stage i. Additionally, we simply represent the history $h^{k+1} = (r_k)$ for $k > 0$ where r_k is the reward offered by player k (player k's action). The strategy of player k is therefore $s_k(h^k) = s_k(r_{k-1})$ which is a singleton since $m_i = 1$. For completeness, let $r_0 = r_d$. This is a *perfect information* game [9] since a single player moves at each stage and has complete information about the actions of all players at previous stages. Backward induction may be used to construct the subgame-perfect equilibrium. We construct the equilibrium strategy s^* inductively as follows: first, for all players i, let $s_i^*(x) = 0$ when $x \leq c$ (where c is assumed to be 1). Then assume that $s_i^*(x)$ is defined for all $x < r$ and for all i. Obviously, with this information, every player i may compute $\delta_i(x, s_{-i}^*)$ for all $x < r$. This is simply due to the fact that δ_i depends on the downstream players from i who must play an action or reward strictly less than r. Finally, for all players i we let $s_i^*(r) = \arg\max_x (r - x)\delta_i(x, s_{-i}^*)$ where $x < r$.

Theorem 2: The strategy profile s^* is a subgame-perfect equilibrium.
See [8] for the proof. The proof may be directly extended to the tree since each player in the tree has a single upstream parent as well and backward induction follows in the same way. On the tree, the strategies of the players that play simultaneously at each stage are also independent.

A. Competition: the ring

We present next a negative result for the ring graph. In a ring, each player has a degree $= 2$ and $m_i = 1$ for all players except the leaf player. We consider rings with an even number of nodes due to the direct competition dynamics. Figure 1(b) shows the 2-, the 3-, and general K-stage versions of the game. In the multi-stage game, after

observing r_d, players 1 and 2 play simultaneously at stage 1 offering rewards r_1 and r_2 respectively to their downstream children, and so on. We refer to the players at stage j using ids $2j - 1$ and $2j$ where the stage of a player i, denoted as $l(i)$, may be computed from the id as $l(i) = \lceil \frac{i}{2} \rceil$. Note that the player at stage K (with id $2K - 1$) breaks ties uniformly with probability $\frac{1}{2}$, and this is public information. For the 2-stage game in Figure 1(b)(i), it is easy to show that an equilibrium always exists in which $s_1^*(r_d) = s_2^*(r_d) = (r_d - 1)$ when $r_d > 1$ and 0 otherwise. This means that player 3 enjoys the benefits of *perfect competition* due to the Bertrand-style competition [9] between players 1 and 2. We now present the following negative result,

Claim 1: The 3-stage game induced on the ring (of Figure 1(b)(ii)) does not admit a subgame-perfect equilibrium. Particularly, there exists a class of subgames for $h^1 = r_d > 7$ for which there is no Nash equilibrium.

Proof: The proof makes use of a counterexample. Using the backward induction argument, notice first that the best strategy of players 3 and 4 is to play a *Bertrand-style competition* as follows: after observing $a^1 = (r_1, r_2)$, player 3 plays $r_3 = \min(r_1 - 1, r_2)$ and symmetrically player 4 plays $r_4 = \min(r_2 - 1, r_1)$. Knowing the strategies of players 3 and 4 and the uniform tie breaking strategy of player 5, players 1 and 2 choose their strategies simultaneously and no equilibria exist for $r_d > 7$ due to oscillation of the best-response dynamics. This may be shown by examining the strategic form game, in normal/matrix form, between players 1 and 2 (in which the expected utilities are expressed in terms of r_d). We briefly show the subgame for $r_d = 8$ and we leave the elaborate proof as an exercise for the interested reader. Figure 2 shows the payoff matrix of players 1 and 2 for playing actions $r_1 \in \{1, 2, 3, 4\}$ (rows) and $r_2 \in \{1, 2, 3, 4\}$ (columns), respectively, eliminating strictly dominated actions. The player's utility is taken to be $u_i = (r_d - r_i)\delta_i$ ignoring the first term of Equation (1). When $r_1 = r_2 > 1$, the expected utility is $\mathbb{E}\{u_1\} = \mathbb{E}\{u_2\} = \frac{1}{2}(r_d - r_1)(2) + \frac{1}{2}(r_d - r_1)(1)$. Clearly, no pure strategy

r1 \ r2	1	2	3	4
1	7 , 7	7 , 12	7 , 10	7 , 8
2	12 , 7	9 , 9	6 , 10	6 , 8
3	10 , 7	10 , 6	7.5 , 7.5	5 , 8
4	8 , 7	8 , 6	8 , 5	6 , 6

Fig. 2. The payoff matrix of players 1 and 2 for the 3-stage game on the ring of Figure 1(b)(ii) when $r_d = 8$.

Nash equilibria exist. ∎

The value $r_d > 7$ signifies the breaking point of equilibrium or the reward at which a player at stage 1, when maximizing its expected utility, will always oscillate between competing for player 5 (by playing large r_2) or not competing (by playing small r_2). This negative result for the game induced on the 3-stage ring may be directly extended to the general game for the K-stage ring by observing that a class of subgames $G(h^{K-2})$ of the general K-stage game are identical to the

3-stage game. While the full game does not always admit an equilibrium when $K > 2$ stages, we shall show next that there always exists an equilibrium for a special class of subgames.

B. Growth of Incentives and Equilibria

We next answer the following question: Find the minimum incentive r_d^*, as a function of the depth of the network K (equivalently the number of stages in the multi-stage game), such that there exists an equilibrium outcome for the subgame $G(r_d^*)$ that is a spanning tree i.e. maximum participation. We seek to compute the function f such that $r_d^* = f(K)$.

First, we present a result for the line, before extending it to the ring. On the line, K is the number of players i.e. $K = n$, and $f_l(K)$ grows exponentially with depth K as follows:

Lemma 1: On the line graph, we have $f_l(0) = 0$, $f_l(1) = 1$, $f_l(2) = 2$, and $\forall \ k > 2$,

$$f_l(k) = (k-1)f_l(k-1) - (k-2)f_l(k-2) \quad (2)$$

The strategy profile $\mathbf{s}^*$ such that a player at stage j plays $s_j^*(h^j) = f_l(K - j)$ is a Nash equilibrium for the subgame $G(r_d^*)$.
See [8] for the proof of Lemma 1.

Theorem 3: On the ring graph, we have $f_r(0) = 0$, $f_r(1) = 1$, $f_r(2) = 2$, and $\forall \ k > 2$,

$$f_r(k) = (2k-3)f_r(k-1) - 2(k-2)f_l(k-2) \quad (3)$$

The strategy profile $\mathbf{s}^*$ such that players at stage j perfectly compete by playing $s_{2j-1}^*(h^j) = s_{2j}^*(h^j) = f_r(K - j)$ is a Nash equilibrium for the subgame $G(r_d^*)$.

Proof: Using induction on k, f_r is trivially true for $k = 1, 2$. For the base case $k = 3$, it is straightforward to see that players at stage 1 may not deviate profitably from playing $f_r(2) = 2$ given $r_d^* = f_r(3) = 4$ (expected utility $= 3$). Assume that $f_r(x)$ holds for all x-stage games such that $x < k$, construct the k-stage game from the $(k-1)$-stage game by adding a stage between d and stage 1 in the $(k-1)$-stage game. In the new k-stage game, by definition of f_r, no player at stage j, $2 \leq j \leq k$ may deviate profitably from playing $f_r(k - j)$. What remains is to compute $f_r(k)$ in the k-stage game and to show that the players at stage 1 may not deviate profitably from playing $f_r(k - 1)$ given $h^1 = f_r(k)$. To compute the minimum incentive $r_d^* = f_r(k)$ such that players at stage k have an incentive to compete (hence resulting in spanning tree outcome), we solve the following inequality which directly results in Equation 3,

$$[f_r(k) - f_r(k-1)]\frac{2k-3}{2} \geq [f_r(k) - f_l(k-2)](k-2) \quad (4)$$

$$\implies f_r(k) \geq (2k-3)f_r(k-1) - 2(k-2)f_l(k-2) \quad (5)$$

The LHS in (4) is the expected utility of a player at stage 1 when both players at stage 1 are competing by playing $f_r(k - 1)$, whereas the RHS is utility from not competing by playing $f_l(k - 2)$. From inequality 5, the minimum such incentive results in equation 3. Next, given $r_d^* = f_r(k)$, we

show that the players at stage 1 may not unilaterally deviate from playing $f_r(k - 1)$. We prove this claim for player 1 knowing that the same argument applies for player 2. By definition of f_r, player 1 may not play a $r_1 < f_r(k - 1)$ since $f_r(k - 1)$ is the minimum incentive to get a spanning tree outcome in subgame $G(h^2)$. On the other hand, player 1 benefits from playing $r_1 > f_r(k - 1)$ only if by doing so she increases δ_1 and hence eliminates the competition. We show that this may not happen by first computing the minimum reward $r_1 = \phi(k - 1)$ required for player 1 to eliminate competition and increase δ_1, and then showing that player 1 may not benefit by playing r_1 given $r_d^* = f_r(k)$. We recursively compute the minimum reward $\phi(k-j)$ that a player at stage j must play to eliminate competition knowing that the competing player at stage j is playing $f_r(k - j)$ as follow:

$$[\phi(k) - \phi(k-1)](k-1) > [\phi(k) - f_r(k-1)]\frac{2k-3}{2} \quad (6)$$

where the LHS is the utility from eliminating competition and increasing δ to $(k-1)$. Equation 6 implies the minimum reward ϕ is given by,

$$\phi(k) = 2(k-1)\phi(k-1) - (2k-3)f_r(k-1) + 1 \quad (7)$$

where $\phi(1) = 2, \phi(2) = 3$. From Equations 3, and 7, it may be shown that in the k-stage game player 1 may not benefit by playing $\phi(k - 1)$ given $r_d^* = f_r(k)$ by showing that the first inequality below holds,

$$[f_r(k) - f_r(k-1)]\frac{2k-3}{2} \geq [f_r(k) - \phi(k-1)](k-1)$$
$$2(k-1)\phi(k-1) \geq f_r(k) + (2k-3)f_r(k-1)$$
$$\phi(k) + (2k-3)f_r(k-1) - 1 \geq f_r(k) + (2k-3)f_r(k-1)$$
$$\phi(k) > f(k)$$

The second, third, and last inequalities above follow directly from the first. The last inequality holds by definition of ϕ which concludes the proof. ∎

The result of Theorem 3 may be interpreted as follows: if the consumer were to play strategically assuming she has a marginal utility of at least r_d^* and is aiming for maximum participation, then $r_d^* = f_r(K)$ would be her Nash strategy in the game induced on the K-stage ring, $\forall \ K > 2$ (given $\mathbf{s}^*$).

IV. CONCLUSION

Broad participation of networked nodes to perform an activity is often of value to some consumer node. Selfish networked nodes however do not necessarily have the right incentives to participate, especially when participation comes at a cost. We have presented a general network participation game and studied the existence of equilibria for simple network structures. While we have presented a general game, the results may be directly applied to sensor network settings. A consumer wishes to achieve broad participation from a set of sensor nodes. The consumer incentivizes the nodes by allocating bandwidth to them. The nodes themselves further incentivize their neighbors to participate by allocating parts of their bandwidth downstream and so on. We presented equilibrium results for the simplest possible class of graphs:

the tree and the ring. We showed that a subgame perfect equilibrium always exists for the game induced on the tree, while no such equilibrium exists for the game induced on the ring graph due to oscillation of best-response dynamics under competition. While the full game does not have a subgame perfect equilibrium, we show that there always exists a Nash equilibrium for a special class of subgames. This required us to first quantify the growth of rewards, or in other words the minimum consumer incentive r_d such that there exists an equilibrium outcome which is a spanning tree (i.e. such that the consumer d achieves maximum participation).

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