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**Abstract:** The objective of the paper ARE to introduce single-valued trapezoidal neutrosophic numbers(SVTrNNs), which is a special case of single-valued neutrosophic numbers and to develop a ranking method for ranking SVTrNNs. Some operational rules as well as cut sets of SVTrNNs have been introduced. The value and ambiguity indices of truth, indeterminacy, and falsity membership functions of SVTrNNs have been defined. A new ranking method has been proposed by using these two indices and applied the ranking method to multi attribute decision making problem in which the ratings of the alternatives over the attributes are expressed in terms of TrNFNs. Finally, an illustrative example has been provided to demonstrate the validity and applicability of the proposed approach.

**Keywords:** Single-valued neutrosophic number(SVNN), Single-valued trapezoidal neutrosophic number, Value index, Ambiguity index, Ranking of SVTrNNs, Multi attribute decision making.

## 1 Introduction

Fuzzy set [1] is capable of dealing with imprecise or vague information in decision making process, whose basic component is a membership function lying in the unit interval [0, 1]. Fuzzy number [2, 3] is a fuzzy subset of real numbers representing the expansion of assurance. Fuzzy numbers can be used to represent vagueness in multi-attribute decision making (MADM) [4, 5, 6, 7], data mining, pattern recognition, medical diagnosis, etc. However, in fuzzy numbers independence of non-membership function is not considered although it is equally important to represent imprecise numerical values in a flexible way. Intuitionistic fuzzy number [8], a generalization of fuzzy numbers, can present ill-known information with membership and non-membership function in the case where the available information is not sufficient to be expressed with fuzzy numbers. Shu et al.[9] defined a triangular intuitionistic fuzzy number(TIFN) and applied to fault tree analysis on printed board circuit assembly. Wang [10] extended TIFN to the trapezoidal intuitionistic fuzzy number(TrIFN) in a similar way as that of the fuzzy number. The concept of ranking of intuitionistic fuzzy numbers [11, 12, 13, 14, 15] has been employed in MADM under intuitionistic fuzzy environment. Li [16] proposed a ranking method for TIFNs by defining a ratio of value index to ambiguity index of TIFNSs and applied it to MADM problem. Zeng et al.[17] extended this ranking method by incorporating TrIFN and utilized it in MADM problems. For intuitionistic fuzzy number, indeterminate information is partially lost although hesitant information is taken into account by default. Therefore, indeterminate information should be considered in decision making process.

Smarandache [18, 19] defined neutrosophic set that can handle indeterminate and inconsistent information. Wang et al.[20] defined single valued neutrosophic set (SVNS), an instance of neutrosophic set, which simply represents uncertainty, imprecise, incomplete, indeterminate and inconsistent information. However, the domain of SVNSs is a discrete set where the truth membership degrees, indeterminacy membership degrees, and the falsity membership membership degrees are only expressed with fuzzy concept like “very good”,“good”, “bad”, etc. Taking the universe as a real line, we can develop the concept of single valued neutrosophic number (SVNN) whose domain is to be considered as a consecutive set. Hence, we can consider SVNNs as a special case of single-valued neutrosophic sets. These numbers can express ill-known quantity with uncertain numerical values in decision making problems. The nature of truth membership, indeterminacy membership, and falsity membership functions of SVNN may have different shape such as triangular shaped, trapezoidal shaped, bell shaped, etc. In the present study, we present only the case of trapezoidal shaped and leave others for future work. We define single-valued trapezoidal neutrosophic numbers (SVTrNN) in which its truth membership, indeterminacy membership, and falsity membership functions can be expressed as trapezoidal fuzzy numbers. Recently, the research on SVNNs has received a little attention and several definitions of SVNNs and its operational rules have been proposed. Ye [21] studied multiple attribute decision making problem by introducing trapezoidal fuzzy neutrosophic set. In his study Ye [21] also defined score function, accuracy function, and some operational rules of trapezoidal fuzzy neutrosophic sets. Biswas et al. [22] defined trapezoidal fuzzy neutrosophic number and their membership functions. Biswas et al. [22] also proposed relative expected value and cosine similarity measure for solving multiple attribute decision making problem.

Ranking method of SVTrNNs can play an important role in decision making problems involving indeterminate information which is beyond the scope of fuzzy numbers, intuitionistic fuzzy numbers. Literature review reflects that little attention has been received to the researchers regarding ranking method of SVTrNNs. Recently, Deli and Şubaş [23] proposed a ranking method for generalized SVTrNNs and presented a numerical example to solve multi-attribute decision making problem in neutrosophic environment. In the present study, We define normalized SVTrNNs and develop a ranking method of SVTrNNs to solve multi attribute decision making problem in neutrosophic environment.

Rest of the paper has been organized as follows: Section 2 provides some basic definitions of fuzzy sets, fuzzy numbers, single-valued neutrosophic sets. In Section 3, we propose SVNNs, SVTrNNs and study some of their properties. In Section 4, we present some arithmetic operations of SVTrNNs. Section 5 is devoted to present the concept of value index and ambiguity index of SVTrNNs and a ranking method of SVTrNNs. In Section 6, we formulate MADM model with the proposed ranking method of TrNNs. Section 7 presents an illustrative example. In Section 8, we present concluding remarks and future scope of research.

## 2 Preliminaries

In this Section, we recall some basic concepts of fuzzy sets, fuzzy number, single valued neutrosophic set.

**Definition 1.** *[1, 3] A fuzzy set $\tilde{A}$ in a universe of discourse $X$ is defined by $\tilde{A}$=$\{\langle x, \mu_{\tilde{A}}(x)\rangle \,|x \in X\}$, where, $\mu_{\tilde{A}}(x)$: $X \to [0,1]$ is called the membership function of $\tilde{A}$ and the value of $\mu_{\tilde{A}}(x)$ is called the degree of membership for $x \in X$.*

*The $\alpha-$cut of the fuzzy set $A$ is the crisp set $A_\alpha$ given by $A_\alpha = \{x \in X : \mu_A(x) \geq \alpha\}$, $\alpha \in [0,1]$.*

**Definition 2.** *[3] A fuzzy set $\tilde{A}$ of the real line $\mathbb{R}$ with membership function $\mu_{\tilde{A}}(x) : \mathbb{R} \to [0,1]$ is called a fuzzy number if*

1. *$\tilde{A}$ is normal, i.e. there exists an element $x_0$ such that $\mu_{\tilde{A}}(x_0) = 1$,*
2. *$\tilde{A}$ is convex, i.e. $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\big(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\big)$ for all $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0,1]$,*
3. *$\mu_{\tilde{A}}$ is upper semi continuous, and*
4. *the support of $\tilde{A}$, i.e. $S(\tilde{A})$=$\{x \in X : \mu_{\tilde{A}}(x) > 0\}$ is bounded.*

**Definition 3.** *[2] A fuzzy number $\tilde{A}$ is called a trapezoidal fuzzy number(TrFN), if its membership function is defined by*

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 0, & otherwise. \end{cases}$$

Figure 1: Trapezoidal fuzzy number $\tilde{A}$

*The TrFN $\tilde{A}$ is denoted by the quadruplet $\tilde{A}$=$(a_1, a_2, a_3, a_4)$ where $a_1$,$a_2$,$a_3$,$a_4$ are the real numbers and $a_1 \leq a_2 \leq a_3 \leq a_4$. The value of $x$ at $[a_2, a_3]$ gives the maximum of $\mu_{\tilde{A}}(x)$, i.e., $\mu_{\tilde{A}}(x)$ =1; it is the most probable value of the evaluation data. The value of $x$ outside the interval $[a_1, a_4]$ gives the minimum of $\mu_{\tilde{A}}(x)$, i.e., $\mu_{\tilde{A}}(x) = 0$; it is the least probable value of the evaluation data. Constants $a_1$ and $a_4$ are the lower and upper bounds of the available area for the evaluation data. The $\alpha-$cut of TrFN $\tilde{A}$=$(a_1, a_2, a_3, a_4)$ is the closed interval*

$$\begin{aligned} A_\alpha &= [L^\alpha(\tilde{A}), R^\alpha(\tilde{A})] \\ &= [(a_2-a_1)\alpha + a_1, -(a_4-a_3)\alpha + a_4], \alpha \in [0,1]. \end{aligned}$$

**Definition 4.** *[20] A single valued neutrosophic set $\tilde{A}$ in a universe of discourse $X$ is given by*

$$\tilde{A} = \Big\{\langle x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\rangle | x \in X\Big\},$$

*where, $T_{\tilde{A}} : X \to [0,1]$, $I_{\tilde{A}} : X \to [0,1]$ and $F_{\tilde{A}} : X \to [0,1]$, with the condition*

$$0 \leq T_{\tilde{A}}(x) + I_{\tilde{A}}(x) + F_{\tilde{A}}(x) \leq 3, \text{ for all } x \in X.$$

*The numbers $T_{\tilde{A}}(x)$, $I_{\tilde{A}}(x)$ and $F_{\tilde{A}}(x)$ respectively represent the truth membership, indeterminacy membership and falsity membership degree of the element $x$ to the set $\tilde{A}$.*

**Definition 5.** *An $(\alpha, \beta, \gamma)$-cut set of SVNS $\tilde{A}$, a crisp subset of $\mathbb{R}$ is defined by*

$$\tilde{A}_{\alpha,\beta,\gamma} = \big\{x | T_{\tilde{A}}(x) \geq \alpha, I_{\tilde{A}}(x) \leq \beta, F_{\tilde{A}}(x) \leq \gamma\big\} \quad (1)$$

*where, $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$, $0 \leq \gamma \leq 1$, and $0 \leq \alpha+\beta+\gamma \leq 3$.*

**Definition 6.** *A single-valued neutrosophic set $\tilde{A}$=$\Big\{\langle x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\rangle | x \in X\Big\}$ is called neut-normal, if there exist at least three points $x_0, x_1, x_2 \in X$ such that $T_{\tilde{A}}(x_0) = 1$, $I_{\tilde{A}}(x_1) = 1$, $F_{\tilde{A}}(x_2) = 1$.*

**Definition 7.** *A single-valued neutrosophic set $\tilde{A}$=$\Big\{\langle x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\rangle | x \in X\Big\}$ is a subset of the* *real line, called neut-convex if for all $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0,1]$ the following conditions are satisfied*

1. $T_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\big(T_{\tilde{A}}(x_1), T_{\tilde{A}}(x_2)\big)$;
2. $I_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq max\big(I_{\tilde{A}}(x_1), I_{\tilde{A}}(x_2)\big)$;
3. $F_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq max\big(F_{\tilde{A}}(x_1), F_{\tilde{A}}(x_2)\big)$.

*That is $\tilde{A}$ is neut-convex if its truth membership function is fuzzy convex, indeterminacy membership function is fuzzy concave and falsity membership function is fuzzy concave.*

# 3 Single-valued neutrosophic number and some arithmetic operations

Single valued neutrosophic set is a flexible and practical tool to handle incomplete, indeterminate or uncertain type information. However, it is often hard to express this information with the truth membership degree, the indeterminacy degree, and the falsity degree represented by the exact real values. Thus extension of SVNSs is required to deal the issues.

**Definition 8.** *A single-valued neutrosophic set $\tilde{A}$=$\Big\{\big\langle x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\big\rangle | x \in X\Big\}$, subset of the real line, is called single-valued neutrosophic number if*

1. *$\tilde{A}$ is neut-normal,*
2. *$\tilde{A}$ is neut-convex,*
3. *$T_A(x)$ is upper semi continuous, $I_A(x)$ is lower semi continuous, and $F_A(x)$ is lower semi continuous, and*
4. *the support of $\tilde{A}$, i.e. $S(\tilde{A})$=$\{x \in X : T_{\tilde{A}}(x) > 0, I_{\tilde{A}}(x) < 1, F_{\tilde{A}}(x) < 1\}$ is bounded.*

Thus for any SVNNs $\tilde{A}$, there exist twelve numbers $a_{11}$, $a_{21}$, $a_{31}$, $a_{41}$, $b_{11}$, $b_{21}$, $b_{31}$, $b_{41}$, $c_{11}$, $c_{21}$, $c_{31}$, $c_{41} \in \mathbb{R}$ such that $c_{11} \leq b_{11} \leq a_{11} \leq c_{21} \leq b_{21} \leq a_{21} \leq a_{31} \leq b_{31} \leq c_{31} \leq a_{41} \leq b_{41} \leq c_{41}$ and the six functions $T^L_{\tilde{A}}(x)$, $T^R_{\tilde{A}}(x)$,$I^L_{\tilde{A}}(x)$, $I^R_{\tilde{A}}(x)$,$F^L_{\tilde{A}}(x)$, $F^R_{\tilde{A}}(x) : \mathbb{R} \to [0,1]$ to represent the truth membership, indeterminacy membership,and falsity membership degree of $\tilde{A}$. The three non decreasing functions $T^L_{\tilde{A}}(x)$, $I^L_{\tilde{A}}(x)$, and $F^L_{\tilde{A}}(x)$ represent the left side of truth, indeterminacy, and falsity membership functions of a SVNN $\tilde{A}$ respectively. Similarly, the three non increasing functions $T^R_{\tilde{A}}(x)$, $I^R_{\tilde{A}}(x)$, $F^R_{\tilde{A}}(x)$ represent the right side of truth membership, indeterminacy, and falsity membership functions of a SVNN $\tilde{A}$, respectively.

Then the truth membership, indeterminacy membership and falsity membership functions of $\tilde{A}$ can be defined in the following form:

$$T_{\tilde{A}}(x) = \begin{cases} T^L_{\tilde{A}}(x), & a_{11} \leq x \leq a_{21}, \\ 1, & a_{21} \leq x \leq a_{31}, \\ T^U_{\tilde{A}}(x), & a_{31} \leq x \leq a_{41}, \\ 0, & otherwise. \end{cases} \tag{2}$$

$$I_{\tilde{A}}(x) = \begin{cases} I^L_{\tilde{A}}(x), & b_{11} \leq x \leq b_{21}, \\ 1, & b_{21} \leq x \leq b_{31}, \\ I^U_{\tilde{A}}(x), & b_{31} \leq x \leq b_{41}, \\ 0, & otherwise, \end{cases} \tag{3}$$

$$F_{\tilde{A}}(x) = \begin{cases} F^L_{\tilde{A}}(x), & c_{11} \leq x \leq c_{21}, \\ 1, & c_{21} \leq x \leq c_{31}, \\ F^U_{\tilde{A}}(x), & c_{31} \leq x \leq c_{41}, \\ 0, & otherwise. \end{cases} \tag{4}$$

The sum of three independent membership degrees of a SVNN $\tilde{A}$ lie between the interval $[0,3]$ i.e,

$$0 \leq T^U_{\tilde{A}}(x) + I^U_{\tilde{A}}(x) + F^U_{\tilde{A}}(x) \leq 3, x \in \tilde{A}.$$

**Definition 9.** *A single-valued trapezoidal neutrosophic number (SVTrNN) $\tilde{A}$ with the set of parameters $c_{11} \leq b_{11} \leq a_{11} \leq c_{21} \leq b_{21} \leq a_{21} \leq a_{31} \leq b_{31} \leq c_{31} \leq a_{41} \leq b_{41} \leq c_{41}$ is denoted as $\tilde{A}$= $\langle (a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41}) \rangle$ in the set of real numbers $\mathbb{R}$. The truth membership, indeterminacy membership and falsity membership degree of $\tilde{A}$ can be defined as follows:*

$$T_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_{11}}{a_{21} - a_{11}}, & a_{11} \leq x \leq a_{21}, \\ 1, & a_{21} \leq x \leq a_{31}, \\ \dfrac{a_{41} - x}{a_{41} - a_{31}}, & a_{31} \leq x \leq a_{41}, \\ 0, & otherwise. \end{cases} \tag{5}$$

$$I_{\tilde{A}}(x) = \begin{cases} \dfrac{x - b_{11}}{b_{21} - b_{11}}, & b_{11} \leq x \leq b_{21}, \\ 1, & b_{21} \leq x \leq b_{31}, \\ \dfrac{x - b_{31}}{b_{41} - b_{31}}, & b_{31} \leq x \leq b_{41}, \\ 0, & otherwise. \end{cases} \tag{6}$$

$$F_{\tilde{A}}(x) = \begin{cases} \dfrac{x - c_{11}}{c_{21} - c_{11}}, & c_{11} \leq x \leq c_{21}, \\ 1, & c_{21} \leq x \leq c_{31}, \\ \dfrac{x - c_{31}}{c_{41} - c_{31}}, & c_{31} \leq x \leq c_{41}, \\ 0, & otherwise. \end{cases} \tag{7}$$

For a SVTrNN $\tilde{A}$, $a_{21}$=$a_{31}$ for truth membership, $b_{21}$=$b_{31}$ for indeterminacy membership, and $c_{21}$=$c_{31}$ for falsity membership degree yield a single-valued triangular neutrosophic numbers which is a special case of SVTrNNs.

### 3.1 Cuts of single-valued trapezoidal neutrosophic numbers

Let $\tilde{A} = \big\langle(a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41})\big\rangle$ be the SVTrNN in the set of real numbers $\mathbb{R}$, where $T_{\tilde{A}}(x)$, $I_{\tilde{A}}(x)$, and $F_{\tilde{A}}(x)$ be the truth, indeterminacy and falsity membership functions.

**Definition 10.** *A $\alpha$-cut set of SVTrNN $\tilde{A}$=$\langle(a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41})\rangle$ is a crisp subset of $\mathbb{R}$ defined by $\tilde{A}_\alpha$= $\{x|T_{\tilde{A}}(x) \geq \alpha\}$, where $0 \leq \alpha \leq 1$. According to the definition of SVTrNN of $\tilde{A}$ and Definition 1, it can be shown that $\tilde{A}_\alpha$ is a closed interval. This interval is denoted by $\tilde{A}_\alpha$=$\big[L^\alpha(\tilde{A}), R^\alpha(\tilde{A})\big]$ and defined by*

$$\big[L^\alpha(\tilde{A}), R^\alpha(\tilde{A})\big] = \big[a_{11} + \alpha(a_{21} - a_{11}), a_{41} - \alpha(a_{41} - a_{31})\big]. \quad (8)$$

**Definition 11.** *A $\beta$-cut set of SVTrNN $\tilde{A}$=$\langle(a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41})\rangle$ is a crisp subset of $\mathbb{R}$ defined by $\tilde{A}_\beta$= $\{x|T_{\tilde{A}}(x) \leq \beta\}$, where $0 \leq \beta \leq 1$.*

*Similarly, the close interval is denoted by $\tilde{A}_\beta$=$\big[L^\beta(\tilde{A}), R^\beta(\tilde{A})\big]$ and defined by*

$$\big[L^\beta(\tilde{A}), R^\beta(\tilde{A})\big] = \big[b_{21} + \beta(b_{21} - b_{11}), b_{31} + \beta(b_{41} - b_{31})\big]. \quad (9)$$

**Definition 12.** *A $\gamma$-cut set of SVTrNN $\tilde{A}$=$\langle(a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41})\rangle$ is a crisp subset of $\mathbb{R}$ defined by $\tilde{A}_\alpha$= $\{x|T_{\tilde{A}}(x) \leq \gamma\}$, where $0 \leq \gamma \leq 1$. The close interval obtained from $\tilde{A}$ is denoted by $\tilde{A}_\gamma$=$\big[L^\gamma(\tilde{A}), R^\gamma(\tilde{A})\big]$ and defined by*

$$\big[L^\gamma(\tilde{A}), R^\gamma(\tilde{A})\big] = \big[c_{21} + \gamma(c_{21} - c_{11}), c_{31} + \gamma(c_{41} - c_{31})\big]. \quad (10)$$

The $(\alpha, \beta, \gamma)$-cut set of SVTrNN $\tilde{A}$=$\langle(a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41})\rangle$ can be defined by using Eqs.(8),(9), and (10) simultaneously.

**Definition 13.** *An $(\alpha, \beta, \gamma)$-cut set of SVTrNN $\tilde{A}$=$\langle(a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41})\rangle$ is a crisp subset of $\mathbb{R}$, which is defined by*

$$\begin{aligned}\tilde{A}_{\alpha,\beta,\gamma} &= \{x|T_{\tilde{A}}(x) \geq \alpha, I_{\tilde{A}}(x) \leq \beta, F_{\tilde{A}}(x) \leq \gamma\} \\ &= \left\{\begin{matrix}\big[L^\alpha(\tilde{A}), R^\alpha(\tilde{A})\big], \big[L^\gamma(\tilde{A}), R^\gamma(\tilde{A})\big], \\ \big[L^\gamma(\tilde{A}), R^\gamma(\tilde{A})\big]\end{matrix}\right\} \quad (11) \\ &= \left\{\begin{matrix}\big[a_{11} + \alpha(a_{21} - a_{11}), a_{41} - \alpha(a_{41} - a_{31})\big], \\ \big[b_{21} + \beta(b_{21} - b_{11}), b_{31} + \beta(b_{41} - b_{31})\big], \\ \big[c_{21} + \gamma(c_{21} - c_{11}), c_{31} + \gamma(c_{41} - c_{31})\big]\end{matrix}\right\} \quad (12)\end{aligned}$$

*where, $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$, $0 \leq \gamma \leq 1$, and $0 \leq \alpha + \beta + \gamma \leq 3$.*

*We observe for the $(\alpha, \beta, \gamma)$-cut set of SVTrNN $\tilde{A}$ that*

1. *$\frac{dL^\alpha(\tilde{A})}{d\alpha} > 0$, $\frac{dR^\alpha(\tilde{A})}{d\alpha} < 0$ for all $\alpha \in [0, 1]$, thus $L^1(\tilde{A}) \geq R^1(\tilde{A})$,*
2. *$\frac{dL^\beta(\tilde{A})}{d\beta} < 0$, $\frac{dR^\beta(\tilde{A})}{d\beta} > 0$ for all $\beta \in [0, 1]$, thus $L^0(\tilde{A}) \leq R^0(\tilde{A})$,*
3. *$\frac{dL^\gamma(\tilde{A})}{d\gamma} < 0$, $\frac{dR^\gamma(\tilde{A})}{d\gamma} > 0$ for all $\gamma \in [0, 1]$, thus $L^0(\tilde{A}) \leq R^0(\tilde{A})$.*

# 4 Some arithmetic operations of single-valued trapezoidal neutrosophic numbers

In this section, some arithmetic operations of SVTrNNs have been presented by using neutrosophic extension principle and $(\alpha, \beta, \gamma)$-cuts method.

### 4.1 Arithmetic Operations of single-valued neutrosophic numbers based on extension principle

The arithmetic operation $(*)$ of two SVTrNNs is a mapping of an input vector $X = [x_1, x_2]^T$ defined in the Cartesian product space $\mathbb{R} \times \mathbb{R}$ on to an output $y$ defined in the real space $\mathbb{R}$. Let $\tilde{A}$ and $\tilde{B}$ be two SVTrNNs, then their outcome of arithmetic operation is also an SVTrNN defined by the form

$$\big(\tilde{A} * \tilde{B}\big)(y) = \left\{\begin{matrix}\Big(y, \sup\limits_{y=x_1*x_2}\Big[\min\big(T_{\tilde{A}}(x_1), T_{\tilde{B}}(x_1)\big)\Big], \\ \inf\limits_{y=x_1*x_2}\Big[\max\big(I_{\tilde{A}}(x_1), I_{\tilde{B}}(x_1)\big)\Big], \\ \inf\limits_{y=x_1*x_2}\Big[\max\big(F_{\tilde{A}}(x_1), F_{\tilde{B}}(x_1)\big)\Big]\Big),\end{matrix}\right\}. \quad (13)$$

for all $x_1$, $x_2$ in $\mathbb{R}$.

To calculate the arithmetic operation of NTrFNs, it is sufficient to determine truth, indeterminacy and falsity membership function of resultant NTrFN as

$$T_{\tilde{A}*\tilde{B}}(y) = \sup_{y=x_1*x_2}\Big[\min\big(T_{\tilde{A}}(x_1), T_{\tilde{B}}(x_1)\big)\Big];$$
$$I_{\tilde{A}*\tilde{B}}(y) = \inf_{y=x_1*x_2}\Big[\max\big(I_{\tilde{A}}(x_1), I_{\tilde{B}}(x_1)\big)\Big]$$
$$\text{and } F_{\tilde{A}*\tilde{B}}(y) = \inf_{y=x_1*x_2}\Big[\max\big(F_{\tilde{A}}(x_1), F_{\tilde{B}}(x_1)\big)\Big].$$

### 4.2 Arithmetic operations of single-valued trapezoidal neutrosophic numbers based on $(\alpha, \beta, \gamma)$-cuts method

Some properties of SVTrNNs in the set of real numbers are presented here.

**Property 1.** *If $\tilde{A}$=$\big\langle(a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41})\big\rangle$ and $\tilde{B}$=$\big\langle(a_{12}, a_{22}, a_{32}, a_{42}), (b_{12}, b_{22}, b_{32}, b_{42}), (c_{12}, c_{22}, c_{32}, c_{42})\big\rangle$ be two SVTrNNs in the set of real numbers $\mathbb{R}$ then, $\tilde{C} = \tilde{A} \oplus \tilde{B}$ is also a SVTrNN and*

$$\tilde{A} \oplus \tilde{B} = \left\langle\begin{matrix}(a_{11} + a_{12}, a_{21} + a_{22}, a_{31} + a_{32}, a_{41} + a_{42}), \\ (b_{11} + b_{12}, b_{21} + b_{22}, b_{31} + b_{32}, b_{41} + b_{42}), \\ (c_{11} + c_{12}, c_{21} + c_{22}, c_{31} + c_{32}, c_{41} + c_{42})\end{matrix}\right\rangle. \quad (14)$$

*Proof.* Based on the extensible principle of single valued neutrosophic set and $(\alpha, \beta, \gamma)$-cut sets of $\tilde{A}$ and $\tilde{B}$ for $\alpha, \beta, \gamma \in [0, 1]$, it sufficient to prove that $A_{\alpha,\beta,\gamma} + B_{\alpha,\beta,\gamma} = (A + B)_{\alpha,\beta,\gamma}$. Using Eq.(12), the summation of $(\alpha,\beta,\gamma)$-cut sets of $\tilde{A}$ and $\tilde{B}$ is

$$\begin{aligned} &A_{\alpha,\beta,\gamma}+B_{\alpha,\beta,\gamma} \\ &=\begin{bmatrix} [a_{11}+\alpha(a_{21}-a_{11}), a_{41}-\alpha(a_{41}-a_{31})], \\ [b_{21}+\beta(b_{21}-b_{11}), b_{31}+\beta(b_{41}-b_{31})], \\ [c_{21}+\gamma(c_{21}-c_{11}), c_{31}+\gamma(c_{41}-c_{31})] \end{bmatrix} \quad (15) \\ &\quad+\begin{bmatrix} [a_{12}+\alpha(a_{22}-a_{12}), a_{42}-\alpha(a_{42}-a_{32})], \\ [b_{22}+\beta(b_{22}-b_{12}), b_{32}+\beta(b_{42}-b_{32})], \\ [c_{22}+\gamma(c_{22}-c_{12}), c_{32}+\gamma(c_{42}-c_{32})] \end{bmatrix} \\ &=\begin{bmatrix} [a_{11}+a_{12}+\alpha(a_{21}+a_{22}-a_{11}-a_{12}), \\ \quad a_{41}+a_{42}-\alpha(a_{41}+a_{42}-a_{31}-a_{32})], \\ [b_{21}+b_{22}+\beta(b_{21}+b_{22}-b_{11}-b_{12}), \\ \quad b_{31}+b_{32}+\beta(b_{41}+b_{42}-b_{31}-b_{32})], \\ [c_{21}+c_{22}+\gamma(c_{21}+c_{22}-c_{11}-c_{12}), \\ \quad c_{31}+c_{32}+\gamma(c_{41}+c_{42}-c_{31}-c_{32})] \end{bmatrix} \\ &=(A+B)_{\alpha,\beta,\gamma}. \end{aligned} \tag{16}$$

This establishes the property. □

**Property 2.** *If* $\tilde{A}=\langle(a_{11},a_{21},a_{31},a_{41}),(b_{11},b_{21},b_{31},b_{41}),(c_{11},c_{21},c_{31},c_{41})\rangle$ *be a SVTrNN in the set of real numbers* $\mathbb{R}$ *and* $k$ *be a real number then,* $k\tilde{A}$ *is also a SVTrNN and*

$$k\tilde{A}=\begin{cases} \left\langle \begin{array}{r}(ka_{11},ka_{21},ka_{31},ka_{41}),(kb_{11},kb_{21},kb_{31},b_{41}),\\ (kc_{11},kc_{21},kc_{31},kc_{41})\end{array}\right\rangle \\ \left\langle \begin{array}{r}(ka_{41},ka_{31},ka_{21},ka_{11}),(kb_{41},kb_{31},kb_{21},b_{11}),\\ (kc_{41},kc_{31},kc_{21},kc_{11})\end{array}\right\rangle, \end{cases} \tag{17}$$

*for* $k>0$ *and* $k<0$ *respectively.*

*Proof.* To establish this property, it has to be proved that $k\tilde{A}_{\alpha,\beta,\gamma}$=$(k\tilde{A})_{\alpha,\beta,\gamma}$.
From Eq.(12), the $(\alpha,\beta,\gamma)$-cut sets of $\tilde{A}$ multiplied with the real number $k>0$ can be taken as

$$\begin{aligned} k\tilde{A}_{\alpha,\beta,\gamma}&=k\begin{bmatrix} [a_{11}+\alpha(a_{21}-a_{11}), a_{41}-\alpha(a_{41}-a_{31})], \\ [b_{21}+\beta(b_{21}-b_{11}), b_{31}+\beta(b_{41}-b_{31})], \\ [c_{21}+\gamma(c_{21}-c_{11}), c_{31}+\gamma(c_{41}-c_{31})] \end{bmatrix} \\ &=\begin{bmatrix} [ka_{11}+\alpha(ka_{21}-ka_{11}), ka_{41}-\alpha(ka_{41}-ka_{31})], \\ [kb_{21}+\beta(kb_{21}-kb_{11}), kb_{31}+\beta(kb_{41}-kb_{31})], \\ [kc_{21}+\gamma(kc_{21}-kc_{11}), kc_{31}+\gamma(kc_{41}-kc_{31})] \end{bmatrix} \\ &=(k\tilde{A})_{\alpha,\beta,\gamma}. \end{aligned}$$

Similarly, it can be shown that $k\tilde{A}_{\alpha,\beta,\gamma}$=$(k\tilde{A})_{\alpha,\beta,\gamma}$ for real number $k<0$. The two results for $k>0$ and $k<0$ prove this property. □

Now we define some arithmetical operation of SVTrNN.

**Definition 14.** *If* $\tilde{A}=\langle(a_{11},a_{21},a_{31},a_{41}),(b_{11},b_{21},b_{31},b_{41}),(c_{11},c_{21},c_{31},c_{41})\rangle$ *be a SVTrNN in the set of real numbers* $\mathbb{R}$ *and* $k$ *be a real number, then the following operations are valid:*

1. $\tilde{A}\oplus\tilde{B}=\left\langle \begin{array}{c}(a_{11}+a_{12},a_{21}+a_{22},a_{31}+a_{32},a_{41}+a_{42}),\\ (b_{11}+b_{12},b_{21}+b_{22},b_{31}+b_{32},b_{41}+b_{42}),\\ (c_{11}+c_{12},c_{21}+c_{22},c_{31}+c_{32},c_{41}+c_{42})\end{array}\right\rangle$

2. $\tilde{A}\otimes\tilde{B}=\left\langle \begin{array}{c}(a_{11}a_{12},a_{21}a_{22},a_{31}a_{32},a_{41}a_{42}),\\ (b_{11}b_{12},b_{21}b_{22},b_{31}b_{32},b_{41}b_{42}),\\ (c_{11}c_{12},c_{21}c_{22},c_{31}c_{32},c_{41}c_{42})\end{array}\right\rangle$

3. $\lambda\tilde{A}=\left\langle \begin{array}{c}(\lambda a_{11},\lambda a_{21},\lambda a_{31},\lambda a_{41}),\\ (\lambda b_{11},\lambda b_{21},\lambda b_{31},\lambda b_{41}),\\ (\lambda c_{11},\lambda c_{21},\lambda c_{31},\lambda c_{41})\end{array}\right\rangle$

4. $\tilde{A}^{\lambda}=\left\langle \begin{array}{c}(a_{11}^{\lambda},a_{21}^{\lambda},a_{31}^{\lambda},a_{41}^{\lambda}),(b_{11}^{\lambda},b_{21}^{\lambda},b_{31}^{\lambda},b_{41}^{\lambda}),\\ (c_{11}^{\lambda},c_{21}^{\lambda},c_{31}^{\lambda},c_{41}^{\lambda})\end{array}\right\rangle$

# 5 Value and ambiguity index based ranking method for SVTrNNs

**Definition 15.** *Let* $\tilde{A}_\alpha$*,* $\tilde{A}_\beta$*, and* $\tilde{A}_\gamma$ *be the* $\alpha$*-cut,* $\beta$*-cut, and* $\gamma$*-cut sets of a SVTrNN* $\tilde{A}$ *. Then the value of truth(*$T_{\tilde{A}}(x)$*), indeterminacy(*$I_{\tilde{A}}(x)$*), and falsity(*$T_{\tilde{A}}(x)$*) membership degree of* $\tilde{A}$ *are respectively defined by*

$$V_T(\tilde{A})=\int_0^1\big(L^{\alpha}(\tilde{A})+R^{\alpha}(\tilde{A})\big)f(\alpha)d\alpha; \tag{18}$$

$$V_I(\tilde{A})=\int_0^1\big(L^{\beta}(\tilde{A})+R^{\beta}(\tilde{A})\big)g(\beta)d\beta; \tag{19}$$

$$V_F(\tilde{A})=\int_0^1\big(L^{\gamma}(\tilde{A})+R^{\gamma}(\tilde{A})\big)h(\gamma)d\gamma. \tag{20}$$

Weighting functions $f(\alpha)$, $g(\beta)$ and $h(\gamma)$ can be set according to nature of decision making in real situations. The function $f(\alpha)=\alpha(\alpha\in[0,1])$ gives different weights to elements in different $\alpha$-cut sets which make less the contribution of the lower $\alpha$-cut sets as these cut sets arising from values of $T_{\tilde{A}}(x)$ have a considerable amount of uncertainty. Thus, $V_T(\tilde{A})$ synthetically reflects the information on every membership degree and may be regarded as a central value that represents from the membership function point of view. Similarly, the function $g(\beta)=1-\beta$ has the effect of weighting on the different $\beta$-cut sets. $g(\beta)$ diminishes the contribution of the higher $\beta$-cut sets, which is reasonable since these cut sets arising from values of $I_{\tilde{A}}(x)$ have a considerable amount of uncertainty. $V_T(\tilde{A})$ synthetically reflects the information on every indeterminacy degree and may be regarded as a central value that represents from the indeterminacy function point of view. Similarly, the function $h(\gamma)=1-\gamma$ has the effect of weighting on the different $\gamma$-cut sets. $g(\gamma)$ diminishes the contribution of the higher $\gamma$-cut sets, which is reasonable since these cut sets arising from values of $F_{\tilde{A}}(x)$ have a considerable amount of uncertainty. $V_F(\tilde{A})$ synthetically reflects the information on every falsity degree and may be regarded as a central value that represents from the falsity membership function point of view.

Taking $f(\alpha)=\alpha$ in Eq.(18), the value of truth membership function can be obtained as:

$$\begin{aligned} V_T&=\int_0^1\Big(L_{\tilde{A}}(\alpha)+R_{\tilde{A}}(\alpha)\Big)f(\alpha)\,\mathrm{d}\alpha \\ &=\int_0^1\Big[a_{11}+\alpha(a_{21}-a_{11})+a_{41}-\alpha(a_{21}-a_{11})\Big]\alpha\mathrm{d}\alpha \\ &=\frac{1}{6}\big(a_{11}+2a_{21}+2a_{31}+a_{41}\big). \end{aligned} \tag{21}$$

Similarly, considering $g(\beta)=1-\beta$ in Eq.(19), the value of indeterminacy membership function can be defined as:

$$\begin{aligned} V_I &= \int_0^1 \Big(L_{\tilde{A}}(\alpha) + R_{\tilde{A}}(\alpha)\Big) g(\beta)\,\mathrm{d}\beta \\ &= \int_0^1 \Big[b_{21} - \gamma(b_{21} - b_{11}) + b_{31} + \gamma(b_{41} - b_{31})\Big](1-\beta)\mathrm{d}\beta \\ &= \frac{1}{6}\big(b_{11} + 2b_{21} + 2b_{31} + b_{41}\big). \end{aligned} \tag{22}$$

and by considering $h(\gamma) = 1 - \gamma$ in Eq.(20), the value of falsity membership function is defined by

$$\begin{aligned} V_F &= \int_0^1 \Big(L_{\tilde{A}}(\gamma) + R_{\tilde{A}}(\gamma)\Big) g(\gamma)\,\mathrm{d}\gamma \\ &= \int_0^1 \Big[c_{21} - \gamma(c_{21} - c_{11}) + c_{31} + \gamma(c_{41} - c_{31})\Big](1-\gamma)\mathrm{d}\gamma \\ &= \frac{1}{6}\big(c_{11} + 2c_{21} + 2c_{31} + c_{41}\big). \end{aligned} \tag{23}$$

**Definition 16.** *Let $\tilde{A}_\alpha$, $\tilde{A}_\beta$, and $\tilde{A}_\gamma$ be the $\alpha$-cut, $\beta$-cut, and $\gamma$-cut sets of a SVTrNN $\tilde{A}$ . Then the ambiguity of truth($T_{\tilde{A}}(x)$), indeterminacy($I_{\tilde{A}}(x)$), and falsity($T_{\tilde{A}}(x)$) membership function of a SVTrNN $\tilde{A}$ are respectively defined by*

$$A_T(\tilde{A}) = \int_0^1 \big(R^\alpha(\tilde{A}) - L^\alpha(\tilde{A})\big) f(\alpha) d\alpha; \tag{24}$$

$$A_I(\tilde{A}) = \int_0^1 \big(R^\beta(\tilde{A}) - L^\beta(\tilde{A})\big) g(\beta) d\beta; \tag{25}$$

$$A_F(\tilde{A}) = \int_0^1 \big(R^\gamma(\tilde{A}) - L^\gamma(\tilde{A})\big) h(\gamma) d\gamma. \tag{26}$$

It is observed that $R^\alpha(\tilde{A}) - L^\alpha(\tilde{A})$, $R^\beta(\tilde{A}) - L^\beta(\tilde{A})$, and $R^\gamma(\tilde{A}) - L^\gamma(\tilde{A})$ represent the length of the intervals of $\tilde{A}_\alpha$, $\tilde{A}_\beta$, and $\tilde{A}_\gamma$ respectively. Thus, $A_T(\tilde{A})$, $A_I(\tilde{A})$, and $A_F(\tilde{A})$ can be regarded as the global spreads of the truth, indeterminacy, and falsity membership function respectively. The ambiguity of three membership functions determine the measure of vagueness of $\tilde{A}$.

Now, putting the values of $\alpha$-cut of $\tilde{A}$ and $f(\alpha) = \alpha$ in Eq.(24), the ambiguity of membership function $T_{\tilde{A}}(x)$ can be determined as:

$$\begin{aligned} A_T(\tilde{A}) &= \int_0^1 \big(R^\alpha(\tilde{A}) - L^\alpha(\tilde{A})\big) f(\alpha)\mathrm{d}\alpha \\ &= \int_0^1 \Big[a_{41} - \alpha(a_{41} - a_{31}) - a_{11} - \alpha(a_{21} - a_{11})\Big]\alpha\mathrm{d}\alpha \\ &= \frac{1}{6}\big(-a_{11} - 2a_{21} + 2a_{31} + a_{41}\big). \end{aligned} \tag{27}$$

Similarly, putting the values of $\beta$-cut of $\tilde{A}$ and $f(\beta) = 1 - \beta$ in Eq.(25), the ambiguity of membership function $I_{\tilde{A}}(x)$ can be determined as:

$$\begin{aligned} A_T(\tilde{A}) &= \int_0^1 \big(R^\beta(\tilde{A}) - L^\alpha(\tilde{A})\big) f(\beta)\mathrm{d}\beta \\ &= \int_0^1 \begin{bmatrix} b_{31} + \beta(b_{41} - b_{31}) - b_{21} \\ + \beta(b_{21} - b_{11}) \end{bmatrix} (1-\beta)\mathrm{d}\beta \\ &= \frac{1}{6}\big(-b_{11} - 2b_{21} + 2b_{31} + b_{41}\big); \end{aligned} \tag{28}$$

and setting the values of $\gamma$-cut of $\tilde{A}$ and $f(\gamma) = 1 - \gamma$ in Eq.(26), the ambiguity of membership function $I_{\tilde{A}}(x)$ can be determined as:

$$\begin{aligned} A_T(\tilde{A}) &= \int_0^1 \big(R^\beta(\tilde{A}) - L^\alpha(\tilde{A})\big) f(\gamma)\mathrm{d}\gamma \\ &= \int_0^1 \begin{bmatrix} b_{31} + \gamma(b_{41} - b_{31}) - b_{21} \\ + \gamma(b_{21} - b_{11}) \end{bmatrix} (1-\gamma)\mathrm{d}\gamma \\ &= \frac{1}{6}\big(-c_{11} - 2c_{21} + 2c_{31} + c_{41}\big). \end{aligned} \tag{29}$$

**Definition 17.** *Let $\tilde{A} = \big\langle (a_{11}, a_{21}, a_{31}, a_{41}),\ (b_{11}, b_{21}, b_{31}, b_{41}),\ (c_{11}, c_{21}, c_{31}, c_{41})\big\rangle$ be a SVTrNN. A value index and ambiguity index for $\tilde{A}$ can be defined by*

$$V_{\lambda,\mu,\nu} = \lambda V_T + \mu V_I + \nu V_F \tag{30}$$

$$\begin{aligned} =&\frac{\lambda}{6}\Big(a_{11} + 2a_{21} + 2a_{31} + a_{41}\Big) \\ &+ \frac{\mu}{6}\Big(b_{11} + 2b_{21} + 2b_{31} + b_{41}\Big) \\ &+ \frac{\nu}{6}\Big(c_{11} + 2c_{21} + 2c_{31} + c_{41}\Big) \end{aligned} \tag{31}$$

$$A_{\lambda,\mu,\nu} = \lambda A_T + \mu A_I + \nu A_F \tag{32}$$

$$\begin{aligned} =&\frac{\lambda}{6}\Big(-a_{11} - 2a_{21} + 2a_{31} + a_{41}\Big) \\ &+ \frac{\mu}{6}\Big(-b_{11} - 2b_{21} + 2b_{31} + b_{41}\Big) \\ &+ \frac{\nu}{6}\Big(-c_{11} - 2c_{21} + 2c_{31} + c_{41}\Big) \end{aligned} \tag{33}$$

*where, the co-efficients $\lambda$, $\mu$, $\nu$ of $V_{\lambda,\mu,\nu}$ and $A_{\lambda,\mu,\nu}$ represent the decision makers' preference value with the condition $\lambda + \mu + \nu = 1$. The decision maker may intend to take decision pessimistically in uncertain environment for $\lambda \in [0, \frac{1}{3}]$ and $\mu + \nu \in [\frac{1}{3}, 1]$. On the contrary, the decision maker may intend to take decision optimistically in uncertain environment for $\lambda \in [\frac{2}{3}, 1]$ and $\mu + \nu \in [0, \frac{1}{3}]$. The impact of truth, indeterminacy, and falsity degree are same to the decision maker for $\lambda = \mu = \nu = \frac{1}{3}$. Therefore, the value index and the ambiguity index may reflect the decision makers attitude for SVTrNN.*

In the following, some properties regarding value and ambiguity index have been presented.

**Theorem 3.** *Let $\tilde{A}_1 = \big\langle (a_{11}, a_{21}, a_{31}, a_{41}),\ (b_{11}, b_{21}, b_{31}, b_{41}),\ (c_{11}, c_{21}, c_{31}, c_{41})\big\rangle$ and $\tilde{A}_2 = \big\langle (a_{12}, a_{22}, a_{32}, a_{42}),\ (b_{12}, b_{22}, b_{32}, b_{42}), (c_{12}, c_{22}, c_{32}, c_{42})\big\rangle$ be two SVTrNN in the set of real numbers $\mathbb{R}$. Then for $\lambda, \mu, \nu \in [0, 1]$ and $\psi \in \mathbb{R}$, the following results hold good.*

$$V_{\lambda,\mu,\nu}\big(\tilde{A}_1 + \tilde{A}_2\big) = V_{\lambda,\mu,\nu}\big(\tilde{A}_1\big) + V_{\lambda,\mu,\nu}\big(\tilde{A}_2\big) \tag{34}$$

$$V_{\lambda,\mu,\nu}\big(\phi\tilde{A}_1\big) = \phi V_{\lambda,\mu,\nu}\big(\tilde{A}_1\big) \tag{35}$$

*Proof.* From definition-14, the sum of two NTrFNs $\tilde{A}_1$ and $\tilde{A}_2$ can be written as follows:

$$\begin{aligned} \tilde{A} \oplus \tilde{B} = \Big\langle &(a_{11} + a_{12} - a_{11}a_{12}, a_{21} + a_{22} - a_{21}a_{22}, \\ &a_{31} + a_{32} - a_{31}a_{32}, a_{41} + a_{42} - a_{41}a_{42}), \\ &(b_{11}b_{12}, b_{21}b_{22}, b_{31}b_{32}, b_{41}b_{42}), \\ &(c_{11}c_{12}, c_{21}c_{22}, c_{31}c_{32}, c_{41}c_{42})\Big\rangle \end{aligned}$$

Now, by Eq.(31) the value index of the sum of two SVTrNNs $\tilde{A}_1$ and

$\tilde{A}_2$ can be written as follows:

$$V_{\lambda,\mu,\nu}(\tilde{A}_1+\tilde{A}_2) \quad (36)$$
$$=\lambda V_T(\tilde{A}_1+\tilde{A}_2)+\mu V_I(\tilde{A}_1+\tilde{A}_2)+\nu V_F(\tilde{A}_1+\tilde{A}_2) \quad (37)$$
$$=\begin{bmatrix} \frac{\lambda}{6}\Big[(a_{11}+a_{21})+2(a_{12}+a_{22})+2(a_{13}+a_{23})+(a_{14}+a_{24})\Big] \\ +\frac{\mu}{6}\Big[(b_{11}+b_{21})+2(b_{12}+b_{22})+2(b_{13}+b_{23})+(b_{14}+b_{24})\Big] \\ +\frac{\nu}{6}\Big[(c_{11}+c_{21})+2(c_{12}+c_{22})+2(c_{13}+c_{23})+(c_{14}+c_{24})\Big] \end{bmatrix} \quad (38)$$
$$=\frac{\lambda}{6}\Big(a_{11}+2a_{12}+2a_{13}+a_{14}\Big)+\frac{\lambda}{6}\Big(a_{21}+2a_{22}+2a_{23}+a_{24}\Big)$$
$$+\frac{\mu}{6}\Big(b_{11}+2b_{12}+2b_{13}+b_{14}\Big)+\frac{\mu}{6}\Big(b_{21}+2b_{22}+2b_{23}+b_{24}\Big)$$
$$+\frac{\nu}{6}\Big(c_{11}+2c_{12}+2c_{13}+c_{14}\Big)+\frac{\nu}{6}\Big(c_{21}+2c_{22}+2c_{23}+c_{24}\Big)$$
$$=V_{\lambda,\mu,\nu}(\tilde{A}_1)+V_{\lambda,\mu,\nu}(\tilde{A}_2) \quad (39)$$

For the second part of the theorem,

$$V_{\lambda,\mu,\nu}(\phi\tilde{A}_1) \quad (40)$$
$$=\lambda V_T(\phi\tilde{A}_1)+\mu V_I(\phi\tilde{A}_1)+\nu V_I(\phi\tilde{A}_1)$$
$$=\begin{bmatrix} \frac{\lambda}{6}(\phi a_{11}+2\phi a_{12}+2\phi a_{13}+\phi a_{14}) \\ +\frac{\mu}{6}(\phi b_{11}+2\phi b_{12}+2\phi b_{13}+\phi b_{14}) \\ +\frac{\nu}{6}(\phi c_{11}+2\phi c_{12}+2\phi c_{13}+\phi c_{14}) \end{bmatrix}$$
$$=\phi\begin{bmatrix} \frac{\lambda}{6}(a_{11}+2a_{12}+2a_{13}+a_{14}) \\ +\frac{\mu}{6}(b_{11}+2b_{12}+2b_{13}+b_{14}) \\ +\frac{\nu}{6}(c_{11}+2c_{12}+2c_{13}+c_{14}) \end{bmatrix}$$
$$=\phi V_{\lambda,\mu,\nu}(\tilde{A}_1) \quad (41)$$

Therefore, $V_{\lambda,\mu,\nu}(\tilde{A}_1+\tilde{A}_2)$=$V_{\lambda,\mu,\nu}(\tilde{A}_1)+V_{\lambda,\mu,\nu}(\tilde{A}_2)$ and $V_{\lambda,\mu,\nu}(\phi\tilde{A}_1)$= $\phi V_{\lambda,\mu,\nu}(\tilde{A}_1)$. □

**Theorem 4.** *Let* $\tilde{A}_1$=$\langle(a_{11},a_{21},a_{31},a_{41}),\ (b_{11},b_{21},b_{31},b_{41}),\ (c_{11},c_{21},c_{31},c_{41})\rangle$ *and* $\tilde{A}_2$ =$\langle(a_{12},a_{22},a_{32},a_{42}),\ (b_{12},b_{22},b_{32},b_{42}),(c_{12},c_{22},c_{32},c_{42})\rangle$ *be two SVTrNNs in the set of real numbers* $\mathbb{R}$. *Then for* $\lambda,\mu,\nu\in[0,1]$ *and* $\psi\in\mathbb{R}$, *the following equations hold good.*

$$A_{\lambda,\mu,\nu}(\tilde{A}_1+\tilde{A}_2)=A_{\lambda,\mu,\nu}(\tilde{A}_1)+A_{\lambda,\mu,\nu}(\tilde{A}_2) \quad (42)$$
$$A_{\lambda,\mu,\nu}(\phi\tilde{A}_1)=\phi A_{\lambda,\mu,\nu}(\tilde{A}_1) \quad (43)$$

*Proof.* From definition-14, the sum of two SVTrNNs $\tilde{A}_1$ and $\tilde{A}_2$ , the ambiguity index of the sum of two SVTrNNs $\tilde{A}_1$ and $\tilde{A}_2$ can be written as:

$$A_{\lambda,\mu,\nu}(\tilde{A}_1+\tilde{A}_2) \quad (44)$$
$$=\lambda A_T(\tilde{A}_1+\tilde{A}_2)+\mu A_I(\tilde{A}_1+\tilde{A}_2)+\nu A_F(\tilde{A}_1+\tilde{A}_2) \quad (45)$$
$$=\begin{bmatrix} \frac{\lambda}{6}\Big[-(a_{11}+a_{21})-2(a_{12}+a_{22})+2(a_{13}+a_{23})+(a_{14}+a_{24})\Big] \\ +\frac{\mu}{6}\Big[-(b_{11}+b_{21})-2(b_{12}+b_{22})+2(b_{13}+b_{23})+(b_{14}+b_{24})\Big] \\ +\frac{\nu}{6}\Big[-(c_{11}+c_{21})-2(c_{12}+c_{22})+2(c_{13}+c_{23})+(c_{14}+c_{24})\Big] \end{bmatrix} \quad (46)$$
$$=\frac{\lambda}{6}\Big(-a_{11}-2a_{12}+2a_{13}+a_{14}\Big)+\frac{\lambda}{6}\Big(-a_{21}-2a_{22}+2a_{23}+a_{24}\Big)$$
$$+\frac{\mu}{6}\Big(-b_{11}-2b_{12}+2b_{13}+b_{14}\Big)+\frac{\mu}{6}\Big(-b_{21}-2b_{22}+2b_{23}+b_{24}\Big)$$
$$+\frac{\nu}{6}\Big(-c_{11}-2c_{12}+2c_{13}+c_{14}\Big)+\frac{\nu}{6}\Big(-c_{21}-2c_{22}+2c_{23}+c_{24}\Big)$$
$$=A_{\lambda,\mu,\nu}(\tilde{A}_1)+A_{\lambda,\mu,\nu}(\tilde{A}_2)$$

For the second part of the theorem,

$$A_{\lambda,\mu,\nu}(\phi\tilde{A}_1) \quad (47)$$
$$=\lambda A_T(\phi\tilde{A}_1)+\mu A_I(\phi\tilde{A}_1)+\nu A_I(\phi\tilde{A}_1)$$
$$=\begin{bmatrix} \frac{\lambda}{6}(-\phi a_{11}-2\phi a_{12}+2\phi a_{13}+\phi a_{14}) \\ +\frac{\mu}{6}(-\phi b_{11}-2\phi b_{12}+2\phi b_{13}+\phi b_{14}) \\ +\frac{\nu}{6}(-\phi c_{11}-2\phi c_{12}+2\phi c_{13}+\phi c_{14}) \end{bmatrix}$$
$$=\phi\begin{bmatrix} \frac{\lambda}{6}(-a_{11}-2a_{12}+2a_{13}+a_{14}) \\ +\frac{\mu}{6}(-b_{11}-2b_{12}+2b_{13}+b_{14}) \\ +\frac{\nu}{6}(-c_{11}-2c_{12}+2c_{13}+c_{14}) \end{bmatrix}$$
$$=\phi A_{\lambda,\mu,\nu}(\tilde{A}_1) \quad (48)$$

Therefore, $A_{\lambda,\mu,\nu}(\tilde{A}_1+\tilde{A}_2)$=$A_{\lambda,\mu,\nu}(\tilde{A}_1)+V_{\lambda,\mu,\nu}(\tilde{A}_2)$ and $A_{\lambda,\mu,\nu}(\phi\tilde{A}_1)$= $\phi A_{\lambda,\mu,\nu}(\tilde{A}_1)$. □

**Proposition 1.** *Let* $\tilde{A}_1$=$\langle(a_{11},a_{21},a_{31},a_{41}),\ (b_{11},b_{21},b_{31},b_{41}),\ (c_{11},c_{21},c_{31},c_{41})\rangle$ *and* $\tilde{A}_2$ =$\langle(a_{12},a_{22},a_{32},a_{42}),\ (b_{12},b_{22},b_{32},b_{42}),(c_{12},c_{22},c_{32},c_{42})\rangle$ *be two SVTrNNs in the set of real numbers* $\mathbb{R}$. *Then ranking of two SVTrNNs* $\tilde{A}_1$ *and* $\tilde{A}_2$ *can be done by using the value and ambiguity of SVTrNN. The procedures have been defined as follows:*

P1. *If* $V_{\lambda,\mu,\nu}(A_1)\leq V_{\lambda,\mu,\nu}(A_1)$, *then* $\tilde{A}_1$ *is smaller than* $\tilde{A}_2$, *i.e,* $\tilde{A}_1\prec\tilde{A}_2$.

P2. *If* $V_{\lambda,\mu,\nu}(A_1)\geq V_{\lambda,\mu,\nu}(A_1)$, *then* $\tilde{A}_1$ *is greater than* $\tilde{A}_2$, *i.e,* $\tilde{A}_1\succ\tilde{A}_2$.

P3. *If* $V_{\lambda,\mu,\nu}(A_1)=V_{\lambda,\mu,\nu}(A_1)$ *and* $A_{\lambda,\mu,\nu}(A_1)\geq V_{\lambda,\mu,\nu}(A_1)$, *then* $\tilde{A}_1$ *is smaller than* $\tilde{A}_2$, *i.e,* $\tilde{A}_1\prec\tilde{A}_2$.

P4. *If* $V_{\lambda,\mu,\nu}(A_1)=V_{\lambda,\mu,\nu}(A_1)$ *and* $A_{\lambda,\mu,\nu}(A_1)\leq V_{\lambda,\mu,\nu}(A_1)$, *then* $\tilde{A}_1$ *is grater than* $\tilde{A}_2$, *i.e,* $\tilde{A}_1\succ\tilde{A}_2$.

P5. *If* $V_{\lambda,\mu,\nu}(A_1)=V_{\lambda,\mu,\nu}(A_1)$ *and* $A_{\lambda,\mu,\nu}(A_1)=V_{\lambda,\mu,\nu}(A_1)$, *then* $\tilde{A}_1$ *is equal* $\tilde{A}_2$, *i.e,* $\tilde{A}_1\approx\tilde{A}_2$.

**Theorem 5.** *Let* $\tilde{A}_1$=$\langle(a_{11},a_{21},a_{31},a_{41}),\ (b_{11},b_{21},b_{31},b_{41}),\ (c_{11},c_{21},c_{31},c_{41})\rangle$ *and* $\tilde{A}_2$ =$\langle(a_{12},a_{22},a_{32},a_{42}),$

$(b_{12}, b_{22}, b_{32}, b_{42}), (c_{12}, c_{22}, c_{32}, c_{42})\rangle$ *be two NTrFNs in the set of real numbers* $\mathbb{R}$.*If* $a_{11} > a_{42}$, $b_{11} > b_{42}$ *and* $c_{11} > c_{42}$ *then* $\tilde{A}_1 > \tilde{A}_2$.

*Proof.* We can obtain the following results from Eq.(21), (22) and (23):

$$V_T(\tilde{A}_1) = \frac{\lambda}{6}\big(a_{11} + 2a_{21} + 2a_{31} + a_{41}\big) > a_{11},$$
$$V_T(\tilde{A}_2) = \frac{\lambda}{6}\big(a_{12} + 2a_{22} + 2a_{32} + a_{42}\big) < a_{42}$$
$$V_I(\tilde{A}_1) = \frac{\lambda}{6}\big(b_{11} + 2b_{21} + 2b_{31} + b_{41}\big) > b_{11},$$
$$V_I(\tilde{A}_2) = \frac{\lambda}{6}\big(b_{12} + 2b_{22} + 2b_{32} + b_{42}\big) < b_{42}$$
and
$$V_F(\tilde{A}_1) = \frac{\lambda}{6}\big(c_{11} + 2c_{21} + 2c_{31} + c_{41}\big) > c_{11},$$
$$V_F(\tilde{A}_2) = \frac{\lambda}{6}\big(c_{12} + 2c_{22} + 2c_{32} + c_{42}\big) < c_{42}$$

With the relations $a_{11} > a_{42}$, $b_{11} > b_{42}$ and $c_{11} > c_{42}$, it follows that $V_T(\tilde{A}_1) > V_T(\tilde{A}_2)$, $V_I(\tilde{A}_1) > V_I(\tilde{A}_2)$, and $V_F(\tilde{A}_1) > V_F(\tilde{A}_2)$. Therefore from Eq.(30), we can obtain

$$\begin{aligned} V_{\lambda,\mu,\nu}(\tilde{A}_1) &= \lambda V_T(\tilde{A}_1) + \mu V_I(\tilde{A}_1) + \nu V_F(\tilde{A}_1) \\ &> \lambda V_T(\tilde{A}_2) + \mu V_I(\tilde{A}_2) + \nu V_F(\tilde{A}_2) = V_{\lambda,\mu,\nu}(\tilde{A}_2) \end{aligned}$$

This completes the proof. □

**Theorem 6.** *Let* $A_1$, $A_2$ *and* $A_3$ *be three SVTrNNs, where* $\tilde{A}_i = \langle(a_{1i}, a_{2i}, a_{3i}, a_{4i}), (b_{1i}, b_{2i}, b_{3i}, b_{4i}), (c_{1i}, c_{2i}, c_{3i}, c_{4i})\rangle$ *for* $i = 1, 2, 3$. *If* $\tilde{A}_1 > \tilde{A}_2$, *then* $\tilde{A}_1 + \tilde{A}_3 > \tilde{A}_2 + \tilde{A}_3$.

*Proof.* For $A_1$, $A_2$ and $A_3$, we can write the following results from Eq.(30):

$$V_{\lambda,\mu,\nu}(\tilde{A}_1 + \tilde{A}_2) = V_{\lambda,\mu,\nu}(\tilde{A}_1) + V_{\lambda,\mu,\nu}(\tilde{A}_2)$$
and
$$V_{\lambda,\mu,\nu}(\tilde{A}_2 + \tilde{A}_3) = V_{\lambda,\mu,\nu}(\tilde{A}_2) + V_{\lambda,\mu,\nu}(\tilde{A}_3).$$

Since $\tilde{A}_1 > \tilde{A}_2$, then we have

$$\begin{aligned} V_{\lambda,\mu,\nu}(\tilde{A}_1 + \tilde{A}_2) &= V_{\lambda,\mu,\nu}(\tilde{A}_1) + V_{\lambda,\mu,\nu}(\tilde{A}_2) \\ &> V_{\lambda,\mu,\nu}(\tilde{A}_2) + V_{\lambda,\mu,\nu}(\tilde{A}_3) \\ &= V_{\lambda,\mu,\nu}(\tilde{A}_2 + \tilde{A}_3). \end{aligned}$$

This completes the proof. □

# 6 Formulation of MADM model under SVTrNNs information

In this section, we present value and ambiguity based ranking method to MADM in which the ratings of alternatives over the attributes have been expressed with NTrFNs. Assume that for a MADM problem, $A = \{A_1, A_2, \ldots, A_m\}$ be a set of $m$ alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of $n$ attributes. The weight vector of the attributes provided the decision makers is $W = (w_1, w_2, \ldots, w_n)^T$, where $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$ and $w_j$ is the degree of importance for the attribute $C_j$. The rating of alternative $A_i$ with respect to attribute $C_j$ has been expressed with NTrFN $d_{ij} = \langle(a^1_{ij}, a^2_{ij}, a^3_{ij}, a^4_{ij}), (b^1_{ij}, b^2_{ij}, b^3_{ij}, b^4_{ij}), (c^1_{ij}, c^2_{ij}, c^3_{ij}, c^4_{ij})\rangle$, where $a^1_{ij}$, $a^2_{ij}$, $a^3_{ij}$, $a^4_{ij}$, $b^1_{ij}$, $b^2_{ij}$, $b^3_{ij}$, $b^4_{ij}$, $c^1_{ij}$, $c^2_{ij}$, $c^3_{ij}$, $c^4_{ij} \in \mathbb{R}$ and $c^1_{ij} \le b^1_{ij} \le a^1_{ij} \le c^2_{ij} \le b^2_{ij} \le a^2_{ij} \le a^3_{ij} \le b^3_{ij} \le c^3_{ij} \le a^4_{ij} \le b^4_{ij} \le c^4_{ij}$ for $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$. The component $(a^1_{ij}, a^2_{ij}, a^3_{ij}, a^4_{ij})$, $(b^1_{ij}, b^2_{ij}, b^3_{ij}, b^4_{ij})$, and $(c^1_{ij}, c^2_{ij}, c^3_{ij}, c^4_{ij})$ represent the truth membership degree, the indeterminacy membership degree and the falsity membership degree, respectively, of the alternative $A_i$ with respect to the attribute $C_j$.

In a MADM problem, the rating values $\tilde{d}_{ij} = \langle(a^1_{ij}, a^2_{ij}, a^3_{ij}, a^4_{ij}), (b^1_{ij}, b^2_{ij}, b^3_{ij}, b^4_{ij}), (c^1_{ij}, c^2_{ij}, c^3_{ij}, c^4_{ij})\rangle$ can be arranged in a matrix format, we call it neutrosophic decision matrix $D = \big(\tilde{d}_{ij}\big)_{m\times n}$ where,

$$\big(\tilde{d}_{ij}\big)_{m\times n} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \tilde{d}_{11} & \tilde{d}_{12} & \cdots & \tilde{d}_{13} \\ A_2 & \tilde{d}_{21} & \tilde{d}_{22} & \cdots & \tilde{d}_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_m & \tilde{d}_{m1} & \tilde{d}_{m2} & \cdots & \tilde{d}_{mn} \end{array} \tag{49}$$

for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. Here, value index and ambiguity index of SVTrNN have been applied to solve a MADM problem with SVTrNN by the following steps:

**Step 1.** Normalization of SVTrNNs based decision matrix

The decision matrix $\big(\tilde{d}_{ij}\big)_{m\times n}$ needs to be normalized into $\big(\tilde{r}_{ij}\big)_{m\times n}$ to eliminate the effect of different physical dimensions during final decision making process, where $\tilde{r}_{ij} = \langle(x^1_{ij}, x^2_{ij}, x^3_{ij}, x^4_{ij}), (y^1_{ij}, y^2_{ij}, y^3_{ij}, y^4_{ij}), (z^1_{ij}, z^2_{ij}, z^3_{ij}, z^4_{ij})\rangle$. Linear normalization technique has been used to normalize the decision matrix for the benefit type attribute (B) and cost type attribute (C) by the following formulas:

$$\begin{aligned} \tilde{r}_{ij} = \Bigg\langle &\Bigg(\frac{x^1_{ij}}{x^{4+}_j}, \frac{x^2_{ij}}{x^{4+}_j}, \frac{x^3_{ij}}{x^{4+}_j}, \frac{x^4_{ij}}{x^{4+}_j}\Bigg), \Bigg(\frac{y^1_{ij}}{y^{4+}_j}, \frac{y^2_{ij}}{y^{4+}_j}, \frac{y^3_{ij}}{y^{4+}_j}, \frac{y^4_{ij}}{y^{4+}_j}\Bigg), \\ &\Bigg(\frac{z^1_{ij}}{z^{4+}_j}, \frac{z^2_{ij}}{z^{4+}_j}, \frac{z^3_{ij}}{z^{4+}_j}, \frac{z^4_{ij}}{z^{4+}_j}\Bigg)\Bigg\rangle \text{ for } j \in B; \end{aligned} \tag{50}$$

$$\begin{aligned} \tilde{r}_{ij} = \Bigg\langle &\Bigg(\frac{x^{1-}_j}{x^1_{ij}}, \frac{x^{1-}_j}{x^2_{ij}}, \frac{x^{1-}_j}{x^3_{ij}}, \frac{x^{1-}_j}{x^4_{ij}}\Bigg), \Bigg(\frac{y^{1-}_j}{y^1_{ij}}, \frac{y^{1-}_j}{y^2_{ij}}, \frac{y^{1-}_j}{y^3_{ij}}, \frac{y^{1-}_j}{y^4_{ij}}\Bigg), \\ &\Bigg(\frac{z^{1-}_j}{z^1_{ij}}, \frac{z^{1-}_j}{z^2_{ij}}, \frac{z^{1-}_j}{z^3_{ij}}, \frac{z^{1-}_j}{z^4_{ij}}\Bigg)\Bigg\rangle \text{ for } j \in C \end{aligned} \tag{51}$$

where, $x^{4+}_j = \max_i(x^4_{ij})$, $y^{4+}_j = \max_i(y^4_{ij})$, $z^{4+}_j = \max_i(z^4_{ij})$, $x^{1-}_j = \max_i(x^1_{ij})$, $y^{1-}_j = \max_i(y^1_{ij})$, and $z^{1-}_j = \max_i(z^1_{ij})$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

**Step 2.** Aggregation of the weighted rating values of alternatives

According to definition-14, the aggregated weighted rating values of the alternatives $A_i (i = 1, 2, \ldots, m)$ can be determined as

$$\tilde{S}_i = \sum_{j=1}^{n} w_j \tilde{r}_{ij}, \tag{52}$$

respectively. Here, the aggregated weighted rating values $\tilde{S}_i (i = 1, 2, \ldots, m)$ are considered as SVTrNNs.

**Step 3.** Ranking of all alternatives

According to Eq.(52) and Proposition-1, ranking of all alternatives can be determined to the non-increasing order of SVTrNNs $\tilde{A}_i (i = 1, 2, \ldots, m)$ by using the value and ambiguity index of SVTrNN.

# 7 An illustrative Example

Consider a decision making problem in which a customer intends to buy a tablet from the set of primarily chosen five tablets $A_i(i = 1, 2, 3, 4, 5)$.The customer takes into account of the four attributes namely:

1. features ($C_1$);
2. hardware specification ($C_2$);
3. affordable price ($C_3$);
4. customer care ($C_4$).

Assume that the weight vector of the four attribute is $W$=$\{0.25, 0.25, 0.30, 0.20\}$ and the evaluations of the five alternatives with respect to the four attributes have been considered as SVTrNNs. Then we have a SVTrNNs based decision matrix $\left(\tilde{d}_{ij}\right)_{5\times4}$ presented in Table-1.

Table 1: NTrFNs based decision matrix

| | $C_1$ |
|---|---|
| $A_1$ | $\langle(0.5, 0.6, 0.7, 0.8), (0.1, 0.1, 0.2, 0.3), (0.1, 0.2, 0.2, 0.3)\rangle$ |
| $A_2$ | $\langle(0.3, 0.4, 0.5, 0.5), (0.1, 0.2, 0.2, 0.4), (0.1, 0.1, 0.2, 0.3)\rangle$ |
| $A_3$ | $\langle(0.3, 0.3, 0.3, 0.3), (0.2, 0.3, 0.4, 0.4), (0.6, 0.7, 0.8, 0.9)\rangle$ |
| $A_4$ | $\langle(0.7, 0.8, 0.8, 0.9), (0.1, 0.2, 0.3, 0.3), (0.2, 0.2, 0.2, 0.2)\rangle$ |
| $A_5$ | $\langle(0.1, 0.2, 0.2, 0.3), (0.2, 0.2, 0.3, 0.4), (0.6, 0.6, 0.7, 0.8)\rangle$ |
| | $C_2$ |
| $A_1$ | $\langle(0.1, 0.1, 0.2, 0.3), (0.2, 0.2, 0.3, 0.4), (0.4, 0.5, 0.6, 0.7)\rangle$ |
| $A_2$ | $\langle(0.2, 0.3, 0.4, 0.5), (0.1, 0.1, 0.2, 0.3), (0.2, 0.2, 0.3, 0.3)\rangle$ |
| $A_3$ | $\langle(0.1, 0.2, 0.2, 0.3), (0.2, 0.3, 0.3, 0.4), (0.4, 0.5, 0.6, 0.6)\rangle$ |
| $A_4$ | $\langle(0.5, 0.6, 0.7, 0.7), (0.2, 0.2, 0.2, 0.2), (0.1, 0.1, 0.2, 0.2)\rangle$ |
| $A_5$ | $\langle(0.5, 0.6, 0.6, 0.7), (0.1, 0.2, 0.3, 0.4), (0.2, 0.2, 0.3, 0.4)\rangle$ |
| | $C_3$ |
| $A_1$ | $\langle(0.3, 0.4, 0.4, 0.5), (0.1, 0.2, 0.2, 0.3), (0.2, 0.2, 0.3, 0.4)\rangle$ |
| $A_2$ | $\langle(0.2, 0.2, 0.2, 0.2), (0.1, 0.1, 0.1, 0.1), (0.6, 0.7, 0.8, 0.8)\rangle$ |
| $A_3$ | $\langle(0.2, 0.3, 0.4, 0.5), (0.2, 0.3, 0.3, 0.4), (0.3, 0.4, 0.4, 0.5)\rangle$ |
| $A_4$ | $\langle(0.3, 0.4, 0.4, 0.5), (0.1, 0.2, 0.2, 0.3), (0.1, 0.2, 0.3, 0.4)\rangle$ |
| $A_5$ | $\langle(0.6, 0.7, 0.8, 0.8), (0.2, 0.2, 0.3, 0.3), (0.1, 0.1, 0.2, 0.3)\rangle$ |
| | $C_4$ |
| $A_1$ | $\langle(0.4, 0.5, 0.6, 0.7), (0.2, 0.2, 0.3, 0.4), (0.1, 0.2, 0.3, 0.4)\rangle$ |
| $A_2$ | $\langle(0.4, 0.5, 0.6, 0.6), (0.2, 0.2, 0.3, 0.3), (0.2, 0.3, 0.4, 0.4)\rangle$ |
| $A_3$ | $\langle(0.2, 0.2, 0.3, 0.4), (0.3, 0.3, 0.3, 0.3), (0.3, 0.4, 0.5, 0.6)\rangle$ |
| $A_4$ | $\langle(0.1, 0.2, 0.3, 0.4), (0.2, 0.2, 0.3, 0.3), (0.5, 0.6, 0.7, 0.8)\rangle$ |
| $A_5$ | $\langle(0.2, 0.3, 0.4, 0.4), (0.1, 0.2, 0.3, 0.4), (0.3, 0.4, 0.4, 0.5)\rangle$ |

**Step 1.** Normalization of SVTrNNs based decision matrix

Using Eq.(50), the decision matrix $\left(\tilde{d_{ij}}\right)_{5\times4}$ has been normalized to the decision matrix $\left(\tilde{d_{ij}^N}\right)_{5\times4}$ by considering the selected four attributes as benefit type attributes. Then the normalized decision matrix $\left(\tilde{d_{ij}^N}\right)_{5\times4}$ can be obtained in Table-2.

**Step 2.** Aggregation of the weighted normalized rating values of alternatives

The weighted normalized rating values of the alternative $A_i(i = 1, 2, 3, 4, 5)$ can be determined by using Eq.(52). Table-3 shows the aggregated weighted normalized rating values of alternatives.

**Step 3.** Ranking of all alternatives

The value index and ambiguity index of NTrFNs $\tilde{A}_i(i = 1, 2, \ldots, m)$ are determined by using Definition-17 and Proposition-1 as

$$\begin{aligned}
V_{\lambda,\mu,\nu}(A_1) &= 0.5428\lambda + 0.5542\mu + 0.4536\nu;\\
V_{\lambda,\mu,\nu}(A_2) &= 0.6041\lambda + 0.4396\mu + 0.5365\nu;\\
V_{\lambda,\mu,\nu}(A_3) &= 0.5667\lambda + 0.7708\mu + 0.5898\nu;\\
V_{\lambda,\mu,\nu}(A_4) &= 0.5871\lambda + 0.7278\mu + 0.3656\nu;\\
V_{\lambda,\mu,\nu}(A_5) &= 0.6083\lambda + 0.6354\mu + 0.4542\nu;
\end{aligned}$$

and

$$\begin{aligned}
A_{\lambda,\mu,\nu}(A_1) &= 0.0802\lambda + 0.1417\mu + 0.0941\nu\\
A_{\lambda,\mu,\nu}(A_2) &= 0.0847\lambda + 0.0979\mu + 0.1260\nu\\
A_{\lambda,\mu,\nu}(A_3) &= 0.0933\lambda + 0.0875\mu + 0.0713\nu\\
A_{\lambda,\mu,\nu}(A_4) &= 0.0574\lambda + 0.1222\mu + 0.0677\nu\\
A_{\lambda,\mu,\nu}(A_5) &= 0.0625\lambda + 0.1729\mu + 0.0750\nu.
\end{aligned}$$

To rank the alternatives $A_i(i = 1, 2, 3, 4, 5)$, the value index and ambiguity index of each alternative have been examined for different values for $\lambda, \mu, \nu \in [0, 1]$. The results have been shown in the Table-4. For different values of $\lambda, \mu, \nu \in [0, 1]$, the ranking order of alternatives has been obtained as follows:

$$A_3 \succ A_5 \succ A_4 \succ A_2 \succ A_1.$$

Thus $A_5$ is the best alternative.

# 8 Conclusions

In the present study, we have introduced the concept of SVTrNN and defined some operational rules. We have also defined value index and ambiguity index of SVTrNN and established some of their properties. Then we have proposed a ranking method SVTrNN by using these two indices of SVTrNN. The proposed method has been applied to MADM problem with SVTrNN information. The method is simple, attractive and effective to determine the ranking order of alternatives used in neutrosophic MADM problems. The proposed concept can be easily extended to rank single-valued triangular neutrosophic numbers. The proposed MADM approach can be extended to solve the problem of medical diagnosis, pattern recognition, personal selection, etc.

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Table 2: SVTrNNs based normalized decision matrix

| | |
|---|---|
| | $C_1$ |
| $A_1$ | $\langle(0.6250, 0.7500, 0.8750, 1.000), (0.2500, 0.2500, 0.5000, 0.7500), (0.1429, 0.2857, 0.2857, 0.4286)\rangle$ |
| $A_2$ | $\langle(0.5000, 0.6667, 0.8333, 0.8333), (0.2500, 0.5000, 0.5000, 1.0000), (0.1250, 0.1250, 0.2500, 0.3750)\rangle$ |
| $A_3$ | $\langle(0.6000, 0.6000, 0.6000, 0.6000), (0.5000, 0.7500, 1.0000, 1.0000), (0.6667, 0.7778, 0.8889, 1.0000)\rangle$ |
| $A_4$ | $\langle(0.7778, 0.8889, 0.8889, 1.0000), (0.3333, 0.6667, 1.0000, 1.0000), (0.2500, 0.2500, 0.2500, 0.2500)\rangle$ |
| $A_5$ | $\langle(0.1250, 0.2500, 0.2500, 0.3750), (0.5000, 0.5000, 0.7500, 1.0000), (0.7500, 0.7500, 0.8750, 1.0000)\rangle$ |
| | $C_2$ |
| $A_1$ | $\langle(0.1250, 0.1250, 0.2500, 0.3750), (0.5000, 0.5000, 0.7500, 1.0000), (0.5714, 0.7143, 0.8571, 1.0000)\rangle$ |
| $A_2$ | $\langle(0.3333, 0.5000, 0.6667, 0.8333), (0.2500, 0.2500, 0.5000, 0.7500), (0.2500, 0.2500, 0.3750, 0.3750)\rangle$ |
| $A_3$ | $\langle(0.2000, 0.4000, 0.4000, 0.6000), (0.5000, 0.7500, 0.7500, 1.0000), (0.4444, 0.5556, 0.6667, 0.6667)\rangle$ |
| $A_4$ | $\langle(0.5556, 0.6667, 0.7778, 0.7778), (0.6667, 0.6667, 0.6667, 0.6667), (0.1250, 0.1250, 0.2500, 0.2500)\rangle$ |
| $A_5$ | $\langle(0.6250, 0.7500, 0.7500, 0.8750), (0.2500, 0.5000, 0.7500, 1.0000), (0.2500, 0.2500, 0.3750, 0.5000)\rangle$ |
| | $C_3$ |
| $A_1$ | $\langle(0.3750, 0.5000, 0.5000, 0.6250), (0.2500, 0.5000, 0.5000, 0.7500), (0.2857, 0.2857, 0.4286, 0.5714)\rangle$ |
| $A_2$ | $\langle(0.3333, 0.3333, 0.3333, 0.3333), (0.2500, 0.2500, 0.2500, 0.2500), (0.7500, 0.8750, 1.0000, 1.0000)\rangle$ |
| $A_3$ | $\langle(0.4000, 0.6000, 0.8000, 1.0000), (0.5000, 0.7500, 0.7500, 1.0000), (0.3333, 0.4444, 0.4444, 0.5556)\rangle$ |
| $A_4$ | $\langle(0.3333, 0.4444, 0.4444, 0.5556), (0.3333, 0.6667, 0.6667, 1.0000), (0.1250, 0.2500, 0.3750, 0.5000)\rangle$ |
| $A_5$ | $\langle(0.7500, 0.8750, 1.0000, 1.0000), (0.5000, 0.5000, 0.7500, 0.7500), (0.1250, 0.1250, 0.2500, 0.3750)\rangle$ |
| | $C_4$ |
| $A_1$ | $\langle(0.5000, 0.6250, 0.7500, 0.8750), (0.5000, 0.5000, 0.7500, 1.0000), (0.1429, 0.2857, 0.4286, 0.5714)\rangle$ |
| $A_2$ | $\langle(0.6667, 0.8333, 1.0000, 1.0000), (0.5000, 0.5000, 0.7500, 0.7500), (0.2500, 0.3750, 0.5000, 0.5000)\rangle$ |
| $A_3$ | $\langle(0.4000, 0.4000, 0.6000, 0.8000), (0.7500, 0.7500, 0.7500, 0.7500), (0.3333, 0.4444, 0.5556, 0.6667)\rangle$ |
| $A_4$ | $\langle(0.1111, 0.2222, 0.3333, 0.4444), (0.6667, 0.6667, 1.0000, 1.0000), (0.6250, 0.7500, 0.8750, 1.0000)\rangle$ |
| $A_5$ | $\langle(0.2500, 0.3750, 0.5000, 0.5000), (0.2500, 0.5000, 0.7500, 1.0000), (0.3750, 0.5000, 0.5000, 0.6250)\rangle$ |

Table 3: Aggregated rating values of attributes

| Alternative | Aggregated rating values of Attributes |
|---|---|
| $A_1$ | $\langle(0.4000, 0.4938, 0.5813, 0.7063), (0.3625, 0.4375, 0.6125, 0.8625), (0.2929, 0.3928, 0.5000, 0.6429)\rangle$ |
| $A_2$ | $\langle(0.4417, 0.5583, 0.6750, 0.7166), (0.3000, 0.3625, 0.4750, 0.6625), (0.3688, 0.4313, 0.5563, 0.8750)\rangle$ |
| $A_3$ | $\langle(0.4000, 0.5100, 0.6100, 0.7600), (0.5500, 0.7500, 0.8125, 0.9500), (0.4444, 0.5556, 0.6333, 0.7167)\rangle$ |
| $A_4$ | $\langle(0.4556, 0.5667, 0.6167, 0.7000), (0.4833, 0.6667, 0.8167, 0.9167), (0.2563, 0.3188, 0.4125, 0.4750)\rangle$ |
| $A_5$ | $\langle(0.4625, 0.5875, 0.6500, 0.7125), (0.3875, 0.5000, 0.7500, 0.9250), (0.3625, 0.3875, 0.4875, 0.6125)\rangle$ |

Table 4: Ranking results for alternatives

| Alternative | Value of $\lambda, \mu, \nu$ | Value index | Ambiguity index | Ranking order |
|---|---|---|---|---|
| $A_1$ | | 0.5027 | 0.1117 | |
| $A_2$ | | 0.5045 | 0.1107 | |
| $A_3$ | $\lambda = .10; \mu = .40;$ | 0.6599 | 0.0800 | $A_3 \succ A_5 \succ A_4 \succ A_2 \succ A_1$ |
| $A_4$ | $\nu = .50$ | 0.5327 | 0.0885 | |
| $A_5$ | | 0.5421 | 0.1129 | |
| $A_1$ | | 0.5125 | 0.1051 | |
| $A_2$ | | 0.5258 | 0.1046 | |
| $A_3$ | $\lambda = .30; \mu = .32;$ | 0.6408 | 0.0831 | $A_3 \succ A_5 \succ A_4 \succ A_2 \succ A_1$ |
| $A_4$ | $\nu = .38$ | 0.5480 | 0.0821 | |
| $A_5$ | | 0.5584 | 0.1026 | |
| $A_1$ | | 0.5168 | 0.1053 | |
| $A_2$ | | 0.5267 | 0.1029 | |
| $A_3$ | $\lambda = \frac{1}{3}; \mu = \frac{1}{3};$ | 0.6424 | 0.0840 | $A_3 \succ A_5 \succ A_4 \succ A_2 \succ A_1$ |
| $A_4$ | $\nu = \frac{1}{3}$ | 0.5602 | 0.0824 | |
| $A_5$ | | 0.5660 | 0.1035 | |
| $A_1$ | | 0.5283 | 0.1014 | |
| $A_2$ | | 0.5412 | 0.0969 | |
| $A_3$ | $\lambda = .50; \mu = .30;$ | 0.6325 | 0.0872 | $A_3 \succ A_5 \succ A_4 \succ A_2 \succ A_1$ |
| $A_4$ | $\nu = .20$ | 0.5850 | 0.0789 | |
| $A_5$ | | 0.5856 | 0.0981 | |
| $A_1$ | | 0.5361 | 0.0939 | |
| $A_2$ | | 0.5645 | 0.0915 | |
| $A_3$ | $\lambda = .70; \mu = .20;$ | 0.6098 | 0.0900 | $A_3 \succ A_5 \succ A_4 \succ A_2 \succ A_1$ |
| $A_4$ | $\nu = .10$ | 0.5931 | 0.0714 | |
| $A_5$ | | 0.5983 | 0.0858 | |

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