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# $N_\omega$ –CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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**Abstract**. Neutrosophic set and Neutrosophic Topological spaces has been introduced by Salama[5]. Neutrosophic Closed set and Neutrosophic Continuous Functions were introduced by Salama et. al.. In this paper, we introduce the concept of Nω- closed sets and their properties in Neutrosophic topological spaces.

**Keywords:** Intuitionistic Fuzzy set, Neutrosophic set, Neutrosophic Topology, $N_s$-open set, $N_s$-closed set, $N_\omega$- closed set, $N_\omega$-open set and Nω-closure.

## 1. Introduction

Many theories like, Theory of Fuzzy sets[10], Theory of Intuitionistic fuzzy sets[1], Theory of Neutrosophic sets[8] and The Theory of Interval Neutrosophic sets[4] can be considered as tools for dealing with uncertainities. However, all of these theories have their own difficulties which are pointed out in[8].

In 1965, Zadeh[10] introduced fuzzy set theory as a mathematical tool for dealing with uncertainities where each element had a degree of membership. The Intuitionistic fuzzy set was introduced by Atanassov[1] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of nonmembership of each element. The neutrosophic set was introduced by Smarandache[7] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set.

In 2012, Salama, Alblowi[5] introduced the concept of Neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of nonmembership of each element. In 2014 Salama, Smarandache and Valeri [6] were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions. In this paper, we introduce the concept of $N_\omega$- closed sets and their properties in neutrosophic topological spaces.

## 2. Preliminaries

In this paper, X denote a topological space $(X, \tau_N)$ on which no separation axioms are assumed unless otherwise explicitly mentioned. We recall the following definitions, which will be used throughout this paper. For a subset A of X, Ncl(A), Nint(A) and $A^c$ denote the neutrosophic closure, neutrosophic interior, and the complement of neutrosophic set A respectively.

**Definition 2.1.[3]** Let X be a non-empty fixed set. A neutrosophic set(NS for short) A is an object having the form $A = \{<x, \mu_A(x), \sigma_A(x), \upsilon_A(x)>: \text{for all } x \in X\}$. Where $\mu_A(x)$, $\sigma_A(x)$, $\upsilon_A(x)$ which represent the degree of membership, the degree of indeterminacy and the degree of nonmembership of each element $x \in X$ to the set A.

**Definition 2.2.[5]** Let A and B be NSs of the form $A = \{<x, \mu_A(x), \sigma_A(x), \upsilon_A(x)>: \text{for all } x \in X\}$ and $B = \{<x, \mu_B(x), \sigma_B(x), \upsilon_B(x)>: \text{for all } x \in X\}$ . Then

i. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \geq \sigma_B(x)$ and $\upsilon_A(x) \geq \upsilon_B(x)$ for all $x \in X$,

ii. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

iii. $A^c = \{<x, \upsilon_A(x), 1- \sigma_A(x), \mu_A(x)>: \text{for all } x \in X \}$,

iv. $A \cup B = \{<x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \upsilon_A(x) \wedge \upsilon_B(x): \text{for all } x \in X >\}$,

v. $A \cap B = \{<x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \upsilon_A(x) \vee \upsilon_B(x): \text{for all } x \in X >\}$.

**Definition 2.3.[5]** A neutrosophic topology(NT for short) on a non empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms:

i) $0_N, 1_N \in \tau$,

ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,

iii) $\cup G_i \in \tau$, for all $G_i: i \in J \subseteq \tau$

In this pair (X, τ) is called a neutrosophic topological space (NTS for short) for neutrosophic set (NOS for short) τ in X. The elements of τ are called open neutrosophic sets. A neutrosophic set F is called closed if and only if the complement of F($F^c$ for short) is neutrosophic open.

**Definition 2.4.[5]** Let (X, τ) be a neutrosophic topological space. A neutrosophic set A in (X, τ) is said to be neutrosophic closed(N-closed for short) if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is neutrosophic open.

**Definition 2.5.[5]** The complement of N-closed set is N-open set.

**Proposition 2.6.[6]** In a neutrosophic topological space (X, T), T = $\mathfrak{J}$ (the family of all neutrosophic closed sets) iff every neutrosophic subset of (X, T) is a neutrosophic closed set.

## 3. $N_\omega$ -closed sets

In this section, we introduce the concept of $N_\omega$-closed set and some of their properties. Throughout this paper (X, $\tau_N$) represent a neutrosophic topological spaces.

**Definition 3.1.** Let (X, $\tau_N$) be a neutrosophic topological space. Then A is called neutrosophic semi open set($N_s$-open set for short) if A $\subset$ Ncl(Nint(A)).

**Definition 3.2.** Let (X, $\tau_N$) be a neutrosophic topological space. Then A is called neutrosophic semi closed set($N_s$-closed set for short) if Nint(Ncl(A)) $\subset$ A.

**Definition 3.3.** Let A be a neutrosophic set of a neutrosophic topological space (X, $\tau_N$). Then,

i. The neutrosophic semi closure of A is defined as $N_s cl(A) = \cap$ {K: K is a $N_s$-closed in X and A $\subseteq$ K}

ii. The neutrosophic semi interior of A is defined as $N_s int(A) = \cup$ {G: G is a $N_s$-open in X and G $\subseteq$ A}

**Definition 3.4.** Let (X, $\tau_N$) be a neutrosophic topological space. Then A is called Neutrosophic ω closed set($N_\omega$-closed set for short) if Ncl(A) $\subseteq$ G whenever A $\subseteq$ G and G is $N_s$-open set.

**Theorem 3.5.** Every neutrosophic closed set is $N_\omega$-closed set, but the converse may not be true.

**Proof:** If A is any neutrosophic set in X and G is any $N_s$-open set containing A, then Ncl (A) $\subseteq$ G. Hence A is $N_\omega$-closed set.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.6.** Let X = {a,b,c} and $\tau_N = \{0_N, G_1, 1_N\}$ is a neutrosophic topology and (X, $\tau_N$) is a neutrosophic topological spaces. Take $G_1$ = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)>, A = <x, (0.2, 0.2, 0.1), (0, 1, 0.2), (0.8, 0.6,0.9)>. Then the set A is $N_\omega$-closed set but A is not a neutrosophic closed.

**Theorem 3.7.** Every $N_\omega$-closed set is N-closed set but not conversely.

**Proof:** Let A be any $N_\omega$-closed set in X and G be any neutrosophic open set such that A $\subseteq$ G. Then G is $N_s$-open, A $\subseteq$ G and Ncl (A) $\subseteq$ G. Thus A is N-closed.

The converse of the above theorem proved by the following example.

**Example 3.8.** Let X = {a,b,c} and $\tau_N = \{0_N, G_1, 1_N\}$ is a neutrosophic topology and (X, $\tau_N$) is a neutrosophic topological spaces. Let $G_1$ = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)> and A = <x, (0.55, 0.45, 0.6), (0.11, 0.3, 0.1), (0.11, 0.25, 0.2)>. Then the set A is N-closed but A is not a $N_\omega$-closed set.

**Remark 3.9.** The concepts of $N_\omega$-closed sets and $N_s$-closed sets are independent.

**Example 3.10.** Let X = {a,b,c} and $\tau_N = \{0_N, G_1, 1_N\}$ is a neutrosophic topology and (X, $\tau_N$) is a neutrosophic topological spaces. Take $G_1$ = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)>, A = <x, (0.2, 0.2, 0.1), (0, 1, 0.2), (0.8, 0.6,0.9)>. Then the set A is $N_\omega$-closed set but A is not a $N_s$-closed set.

**Example 3.11.** Let X = {a,b} and $\tau_N = \{0_N, G_1, G_2, 1_N\}$ is a neutrosophic topology and (X, $\tau_N$) is a neutrosophic topological spaces. Take $G_1$ = <x, (0.6, 0.7), (0.3, 0.2), (0.2, 0.1)> and A = <x, (0.3, 0.4), (0.6, 0.7), (0.9,0.9)>. Then the set A is $N_s$-closed set but A is not a $N_\omega$-closed.

**Theorem 3.12.** If A and B are $N_\omega$-closed sets, then A ∪ B is $N_\omega$-closed set.

**Proof:** If A ∪ B $\subseteq$ G and G is $N_s$-open set, then A $\subseteq$ G and B $\subseteq$ G. Since A and B are $N_\omega$-closed sets, Ncl(A) $\subseteq$ G and Ncl(B) $\subseteq$ G and hence Ncl(A) ∪ Ncl(B) $\subseteq$ G. This implies Ncl(A ∪ B) $\subseteq$ G. Thus A ∪ B is $N_\omega$-closed set in X.

**Theorem 3.13.** A neutrosophic set A is $N_\omega$-closed set then Ncl(A) – A does not contain any nonempty neutrosophic closed sets.

**Proof:** Suppose that A is $N_\omega$-closed set. Let F be a neutrosophic closed subset of Ncl(A) – A. Then A $\subseteq F^c$. But A is $N_\omega$-closed set. Therefore Ncl(A) $\subseteq F^c$. Consequently F $\subseteq (Ncl(A))^c$. We have F $\subseteq$ Ncl(A). Thus F $\subseteq Ncl(A) \cap (Ncl(A))^c = \phi$. Hence F is empty.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.14.** Let X = {a,b,c} and $\tau_N = \{0_N, G_1, 1_N\}$ is a neutrosophic topology and (X, $\tau_N$) is a neutrosophic topological spaces. Take $G_1$ = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)> and A = <x, (0.2, 0.2, 0.1), (0.6, 0.6, 0.6), (0.8, 0.9,0.9)>. Then the set A is not a $N_\omega$-closed set and Ncl(A) – A = <x, (0.2, 0.2, 0.1), (0.6, 0.6, 0.6), (0.8, 0.9,0.9)> does not contain non-empty neutrosophic closed sets.

**Theorem 3.15.** A neutrosophic set A is $N_\omega$-closed set if and only if Ncl(A) – A contains no non-empty $N_s$-closed set.

**Proof:** Suppose that A is $N_\omega$-closed set. Let S be a $N_s$-closed subset of Ncl(A) – A. Then $A \subseteq S^c$. Since A is $N_\omega$-closed set, we have Ncl(A) $\subseteq S^c$. Consequently $S \subseteq (Ncl(A))^c$. Hence $S \subseteq Ncl(A) \cap (Ncl(A))^c = \phi$. Therefore S is empty.
Conversely, suppose that Ncl(A) – A contains no non-empty $N_s$-closed set. Let $A \subseteq G$ and that G be $N_s$-open. If $Ncl(A) \nsubseteq G$, then $Ncl(A) \cap G^c$ is a non-empty $N_s$-closed subset of Ncl(A) – A. Hence A is $N_\omega$-closed set.

**Corollary 3.16.** A $N_\omega$-closed set A is $N_s$-closed if and only if $N_s cl(A)$ – A is $N_s$-closed.
**Proof:** Let A be any $N_\omega$-closed set. If A is $N_s$-closed set, then $N_s cl(A) - A = \phi$. Therefore $N_s cl(A)$ – A is $N_s$-closed set.
Conversely, suppose that Ncl(A) – A is $N_s$-closed set. But A is $N_\omega$-closed set and Ncl(A) – A contains $N_s$-closed. By theorem 3.15, $N_s cl(A) - A = \phi$. Therefore $N_s cl(A) = A$. Hence A is $N_s$-closed set.

**Theorem 3.17.** Suppose that $B \subseteq A \subseteq X$, B is a $N_\omega$-closed set relative to A and that A is $N_\omega$-closed set in X. Then B is $N_\omega$-closed set in X.
**Proof:** Let $B \subseteq G$, where G is $N_s$-open in X. We have $B \subseteq A \cap G$ and $A \cap G$ is $N_s$-open in A. But B is a $N_\omega$-closed set relative to A. Hence $Ncl_A(B) \subseteq A \cap G$. Since $Ncl_A(B) = A \cap Ncl(B)$. We have $A \cap Ncl(B) \subseteq A \cap G$. It implies $A \subseteq G \cup (Ncl(B))^c$ and $G \cup (Ncl(B))^c$ is a $N_s$-open set in X. Since A is $N_\omega$-closed in X, we have $Ncl(A) \subseteq G \cup (Ncl(B))^c$. Hence $Ncl(B) \subseteq G \cup (Ncl(B))^c$ and $Ncl(B) \subseteq G$. Therefore B is $N_\omega$-closed set relative to X.

**Theorem 3.18.** If A is $N_\omega$-closed and $A \subseteq B \subseteq Ncl(A)$, then B is $N_\omega$-closed.
**Proof:** Since $B \subseteq Ncl(A)$, we have $Ncl(B) \subseteq Ncl(A)$ and $Ncl(B) - B \subseteq Ncl(A) - A$. But A is $N_\omega$-closed. Hence Ncl(A) – A has no non-empty $N_s$-closed subsets, neither does Ncl(B) – B. By theorem 3.15, B is $N_\omega$-closed.

**Theorem 3.19.** Let $A \subseteq Y \subseteq X$ and suppose that A is $N_\omega$-closed in X. Then A is $N_\omega$-closed relative to Y.
**Proof:** Let $A \subseteq Y \cap G$ where G is $N_s$-open in X. Then $A \subseteq G$ and hence $Ncl(A) \subseteq G$. This implies, $Y \cap Ncl(A) \subseteq Y \cap G$. Thus A is $N_\omega$-closed relative to Y.

**Theorem 3.20.** If A is $N_s$-open and $N_\omega$-closed, then A is neutrosophic closed set.
**Proof:** Since A is $N_s$-open and $N_\omega$-closed, then $Ncl(A) \subseteq A$. Therefore Ncl(A) =A. Hence A is neutrosophic closed.

## 4. $N_\omega$-open sets

In this section, we introduce and study about $N_\omega$-open sets and some of their properties.

**Definition 4.1.** A Neutrosophic set A in X is called $N_\omega$-open in X if $A^c$ is $N_\omega$-closed in X.

**Theorem 4.2.** Let $(X, \tau_N)$ be a neutrosophic topological space. Then
(i) Every neutrosophic open set is $N_\omega$-open but not conversely.
(ii) Every $N_\omega$-open set is N-open but not conversely.

The converse part of the above statements are proved by the following example.

**Example 4.3.** Let X = {a,b,c} and $\tau_N = \{0_N, G_1, 1_N\}$ is a neutrosophic topology and $(X, \tau_N)$ is a neutrosophic topological space. Take $G_1$ = <x, (0.7, 0.6, 0.9), (0.6, 0.5, 0.8), (0.5, 0.6, 0.4)> and A = <x, (0.8, 0.6, 0.9), (1, 0, 0.8), (0.2, 0.2, 0.1)>. Then the set A is $N_\omega$-open set but not a neutrosophic open and B = <x, (0.11, 0.25, 0.2), (0.89, 0.7, 0.9), (0.55, 0.45, 0.6)> is N-open but not a $N_\omega$-open set.

**Theorem 4.4.** A neutrosophic set A is $N_\omega$-open if and only if $F \subseteq Nint(A)$ where F is $N_s$-closed and $F \subseteq A$.
**Proof:** Suppose that $F \subseteq Nint(A)$ where F is $N_s$-closed and $F \subseteq A$. Let $A^c \subseteq G$ where G is $N_s$-open. Then $G^c \subseteq A$ and $G^c$ is $N_s$-closed. Therefore $G^c \subseteq Nint(A)$. Since $G^c \subseteq Nint(A)$, we have $(Nint(A))^c \subseteq G$. This implies $Ncl((A)^c) \subseteq G$. Thus $A^c$ is $N_\omega$-closed. Hence A is $N_\omega$-open.
Conversely, suppose that A is $N_\omega$-open, $F \subseteq A$ and F is $N_s$-closed. Then $F^c$ is $N_s$-open and $A^c \subseteq F^c$. Therefore $Ncl((A)^c) \subseteq F^c$. But $Ncl((A)^c) = (Nint(A))^c$. Hence $F \subseteq Nint(A)$.

**Theorem 4.5.** A neutrosophic set A is $N_\omega$-open in X if and only if G = X whenever G is $N_s$-open and $(Nint(A) \cup A^c) \subseteq G$.
**Proof:** Let A be a $N_\omega$-open, G be $N_s$-open and $(Nint(A) \cup A^c) \subseteq G$. This implies $G^c \subseteq (Nint(A))^c \cap ((A)^c)^c = (Nint(A))^c - A^c = Ncl((A)^c) - A^c$. Since $A^c$ is $N_\omega$-closed and $G^c$ is $N_s$-closed, by Theorem 3.15, it follows that $G^c = \phi$. Therefore X = G.
Conversely, suppose that F is $N_s$-closed and $F \subseteq A$. Then $Nint(A) \cup A^c \subseteq Nint(A) \cup F^c$. This implies $Nint(A) \cup F^c = X$ and hence $F \subseteq Nint(A)$. Therefore A is $N_\omega$-open.

**Theorem 4.6.** If $Nint(A) \subseteq B \subseteq A$ and if A is $N_\omega$-open, then B is $N_\omega$-open.
**Proof:** Suppose that $Nint(A) \subseteq B \subseteq A$ and A is $N_\omega$-open. Then $A^c \subseteq B^c \subseteq Ncl(A^c)$ and since $A^c$ is $N_\omega$-closed. We have by Theorem 3.15, $B^c$ is $N_\omega$-closed. Hence B is $N_\omega$-open.

**Theorem 4.7.** A neutrosophic set A is $N_\omega$-closed, if and only if Ncl(A) – A is $N_\omega$-open.
**Proof:** Suppose that A is $N_\omega$-closed. Let $F \subseteq Ncl(A) - A$ Where F is $N_s$-closed. By Theorem 3.15, $F = \phi$. Therefore $F \subseteq Nint((Ncl(A) - A)$ and by Theorem 4.4, we have Ncl(A) – A is $N_\omega$-open.

Conversely, let A ⊆ G where G is a $N_s$-open set. Then Ncl(A) ∩ $G^c$ ⊆ Ncl(A) ∩ $A^c$ = Ncl(A) – A. Since Ncl(A) ⊆ $G^c$ is $N_s$-closed and Ncl(A) – A is $N_\omega$-open. By Theorem 4.4, we have Ncl(A) ∩ $G^c$ ⊆ Nint(Ncl(A) – A) = ϕ. Hence A is $N_\omega$-closed.

**Theorem 4.8.** For a subset A⊆X the following are equivalent:

(i) A is $N_\omega$-closed.

(ii) Ncl(A) – A contains no non-empty $N_s$-closed set.

(iii) Ncl(A) – A is $N_\omega$-open set.

**Proof:** Follows from Theorem 3.15 and Theorem 4.7.

## 5. $N_\omega$-closure and Properties of $N_\omega$-closure

In this section, we introduce the concept of $N_\omega$-closure and some of their properties.

**Definition 5.1.** The $N_\omega$-closure (briefly $N_\omega$cl(A)) of a subset A of a neutrosophic topological space (X, $\tau_N$) is defined as follows:

$N_\omega$cl(A) = ∩ { F ⊆ X / A ⊆ F and F is $N_\omega$-closed in (X, $\tau_N$)}.

**Theorem 5.2.** Let A be any subset of (X, $\tau_N$). If A is $N_\omega$-closed in (X, $\tau_N$) then A = $N_\omega$cl(A).

**Proof:** By definition, $N_\omega$cl(A) = ∩ { F ⊆ X / A ⊆ F and F is a $N_\omega$-closed in (X, $\tau_N$)} and we know that A ⊆ A. Hence A = $N_\omega$cl(A).

**Remark 5.3.** For a subset A of (X, $\tau_N$), A ⊆ $N_\omega$cl(A) ⊆ Ncl(A).

**Theorem 5.4.** Let A and B be subsets of (X, $\tau_N$). Then the following statements are true:

i. $N_\omega$cl(A) = ϕ and $N_\omega$cl(A) = X.

ii. If A ⊆ B, then $N_\omega$cl(A) ⊆ $N_\omega$cl(B)

iii. $N_\omega$cl(A) ∪ $N_\omega$cl (B) ⊂ $N_\omega$cl (A ∪ B)

iv. $N_\omega$cl (A ∩ B) ⊂ $N_\omega$cl(A) ∩ $N_\omega$cl (B)

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