3-31-2008

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Recommended Citation

Abdallah, Chaouki T.; Mohammad Hayajneh; and Walid Ibrahim. "Impact of Fading Wireless Channel on The Performance of Game Theoretic Power Control Algorithms for CDMAWireless Data." *IEEE/ACS International Conference on Computer Systems and Applications* (2008): 317-324. doi:10.1109/AICCSA.2008.4493552.

Impact of Fading Wireless Channel on The Performance of Game Theoretic Power Control Algorithms for CDMA Wireless Data

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Abstract

Our goal in this paper is to study the performance of the game-theoretic power control algorithms for wireless data introduced by Saraydar et al *[1] in two realistic channels:* **(a1)** *Fast flat fading channel and* **(a2)** *Slow flat fading channel. The fading coefficients under both* **(a1)** *and* **(a2)** *are studied under an appropriate small scale channel model that is used in the CDMA cellular systems, namely Nakagami channel model. To do so, we derive a closed-form expression of the average utility function which represents the number of bits received correctly at the receiver per one Joule expended. Then, using this expression we study the existence, uniqueness of Nash equilibrium (NE), and the social desirability of NE in the Pareto sense.*

1. Introduction

The mathematical theory of games was introduced by Von Neumann and Morgenstern in 1944 [10]. A core idea of game theory is how strategic interactions between rational agents (players) generate outcomes according to the players' preferences [7],[11]. Game theory thus forms a suitable framework to obtain more insight into the interactions of self-interested agents with potentially conflicting interests. A player in a non-cooperative game responds to the actions of other players by choosing a strategy (from its strategy space) in an attempt to maximize a utility function that quantifies its level of satisfaction.

In a cellular system each user desires to have a high SIR at the base station (BS) coupled with the lowest possible transmit power. It is important in such systems to have high SIR, as this will reflect a low error rate, a more reliable system, and high channel capacity, so that more users can be served per cell. It is also important to decrease the transmit power to lengthen battery life and to alleviate the near-far problem. In power control algorithms exploiting game theory however, the tendency of each user to maximize its utility function in response to other users' actions, leads to a sequence of power vectors that converges to a point where no user has incentive to individually increase its power. This operating point is called a NE Point. Due to the lack of cooperation between the users this point may not be efficient, in the sense that it may not be the most desirable social point [1]. In Pareto sense, the most desirable point is actually the power vector that Pareto dominates all other power vectors.

The power control problem for wireless data CDMA systems was first addressed in the game theoretic framework in [2],[1], then in [5],[6]. In this paper the work in [1], which only dealt with deterministic (nonfading) channels, is studied in a realistic wireless CDMA channels by considering the following two fading models: Nakagami fast flat fading and Nakagami slow flat fading channel models. Here we derive a closed-form expression of the average of the utility function proposed in [1]. We then evaluate the performance of their proposed game-theoretic algorithms through the existence, uniqueness and social desirability of NE operating point under the assumed channel models.

The remaining of this paper is organized as follows: In Section 2 we present the utility function and the system model studied in this paper. In Section 3 we evaluate the performance of the system for the channel models mentioned above. Non-cooperative power control game

(NPG) and Non-cooperative power control game with pricing (NPGP) are discussed briefly in Sections 4 and 5, respectively. We then point out the constraints on the new modified strategy spaces to guarantee the existence and uniqueness of NE points for NPG and NPGP under the assumed channel models in Section 6. Simulation results are outlined in Section 7, and our conclusions are given in Section 8.

2. Utility function and system model

In general utility functions are used to quantify the satisfaction level a player achieves by choosing an action from its strategy profile, given the other players' actions. A utility function thus maps the player's preferences onto the real line. A formal definition of a utility function may be found in [7]

In a CDMA cellular system, a number of users sharing the spectrum and air interface. Henceforth, each user's transmission adds to the interference of all users at the BS. Each user desires to achieve a high quality of reception at the BS, i.e., a high SIR, while using the minimum possible amount of power in order to extend the battery's life. The conflicting goal of each user to have a high SIR at the BS makes the game theoretic framework suitable for studying and solving the problem.

In this paper we consider the same system model and the same utility function of [1]: Single-cell direct sequence code division multiple access (DS-CDMA) system with N users, where each user transmits frames (packets) of M bits with L information bits. The rate of transmission is R bits/sec for all users. Let P_c represent the *average* probability of correct reception of a frame at the BS, and let p represent the average transmit power level. The utility function for a CDMA system is given by:

$$u = \frac{L\,R}{M\,p}\,f(\gamma) \tag{1}$$

where $f(\gamma)$ is an efficiency function that approximates P_c. Thus, u represents the number of information bits successfully received at the BS per joule of expanded energy. With the assumption of no error correction, and correct packet reception rate $\tilde{P}_c$, i.e., $P_c = E\{\tilde{P}_c\}$, is then given by $\prod_{l=1}^{M}(1-\tilde{P}_e(l))$, where $\tilde{P}_e(l)$ is the bit error rate (BER) of the lth bit at a given SIR γ_i. P_e is the average BER, that is $P_e = E\{\tilde{P}_e\}$ (c. f. (15)). It should be noted that the efficiency function $f(\gamma)$ has the same expression of P_c in terms of $\tilde{P}_e$, except that $\tilde{P}_e$ is replaced by $2\tilde{P}_e$ (see [1] for more details).

3. Evaluation of the performance

In this Section we derive a closed-form formulas of the average utility functions under Nakagami fast/slow channel models. The derived formulas are then used to study the existence and uniqueness of NE point in Section 6.

The SIR of the ith user (γ_i) at the BS is assumed to be large ($\gamma_i \gg 1$) to combat the fading effect, it is given by [9]:

$$\gamma_i = \frac{W}{R}\frac{p_i\,h_i\,\alpha_i^2}{\sum_{k\neq i}^{N} p_k\,h_k\,\alpha_k^2 + \sigma^2} \tag{2}$$

Where α_i is the path fading coefficient between ith user and the BS and is constant for each bit in a fast flat fading channels **(a1)**, while it is constant for each packet in a slow flat fading channels **(a2)**. W is the spread spectrum bandwidth, p_k is the transmitted power of the kth user, h_k is the path gain between the BS and the kth user, and σ^2 is the variance of the AWGN (additive-white-gaussian-noise) representing the background thermal noise in the receiver. For simplicity we express the interference from all other users as x_i, i.e.

$$x_i = \sum_{k\neq i}^{N} p_k\,h_k\,\alpha_k^2 \tag{3}$$

therefore (2) can be written as:

$$\gamma_i = \gamma_i(\alpha_i, x_i) = \frac{W}{R}\frac{p_i\,h_i}{x_i+\sigma^2}\ \alpha_i^2 = \gamma_i^{'}\alpha_i^2 \tag{4}$$

For a given α_i and x_i, the BER, $\tilde{P}(e|\alpha_i, x_i)$, of the ith user using BFSK is given by [9]:

$$\tilde{P}(e|\alpha_i, x_i) = \frac{1}{2}\,e^{-\frac{\gamma_i(\alpha_i,x_i)}{2}} \tag{5}$$

The average BER and average utility functions for this modulation scheme is evaluated next under the previously mentioned channel models.

3.1 Nakagami flat fading channel

Here, the fading coefficient α_i is modelled as a Nakagami random variable with a probability distribution given by [9]: $f^{\alpha_i}(\omega) = \frac{2m^m}{\Gamma(m)\Omega^m}\,\omega^{2m-1}\,e^{(-\frac{m}{\Omega})\omega^2};\ \ i = 1, 2, \cdots, N$ where $\Omega = E\{\alpha_i^2\}$ controls the spread of the distribution. The fading figure $m = \frac{\Omega^2}{E\{(\alpha_i^2-\Omega)^2\}}$ is a measure of the severity of the fading channel, where $m = \infty$ corresponds to a nonfading channel. In the following it is assumed that $\Omega = 1$. Then the distribution of γ_i for fixed x_i is given as: $f^{\gamma_i|x_i}(\omega) = \frac{1}{\Gamma(m)}\ \left(\frac{m}{\gamma_i^{'}}\right)^m\ \omega^{m-1}\ e^{-(\frac{m}{\gamma_i^{'}})\,\omega}$

3.1.1 Nakagami fast flat fading channel

We find the conditioned error probability $\tilde{P}(e|x_i)$ as:

$$\begin{aligned}\tilde{P}(e|x_i) &= \int_0^\infty \tilde{P}(e|\omega, x_i)\, f^{\gamma_i|x_i}(\omega) d\omega \\ &= \frac{1}{2\Gamma(m)} \left(\frac{m}{\gamma_i^{'}}\right)^m \int_0^\infty \omega^{m-1}\, e^{-(\frac{\gamma_i^{'}+2m}{2\gamma_i^{'}})\,\omega} d\omega \\ &= \frac{1}{2}\left(\frac{2m}{2m+\gamma_i^{'}}\right)^m \end{aligned} \tag{6}$$

For fixed m and $\gamma_i^{'} \gg 1$, (6) can be rewritten as:

$$\tilde{P}(e|x_i) \approx \frac{1}{2}\left(\frac{2m}{\gamma_i^{'}}\right)^m \tag{7}$$

To find the *average* P_e, we need to find the mean of $(x_i + \sigma^2)^m$. Here, x_i is a summation of independent random variables each distributed according to a Gamma density function. This makes the evaluation of $(x_i + \sigma^2)^m$ tedious and it may be easier to find an approximate density function of x_i. To do this, let us recall Esseen's inequality which estimates the deviation of the exact distribution of a sum of independent variables from the normal distribution [12].

Theorem 1 *let $Y_1, \cdots, Y_N$ be independent random variables with $EY_j = 0$, $E|Y_j|^3 < \infty$ $(j = 1, \cdots, N)$. Let $\sigma_j^2 = EY_j^2$, $B_N = \sum_{j=1}^N \sigma_j^2$, $L_N = B_N^{-3/2} \sum_{j=1}^N E|Y_j|^3$. Let $\psi_N(z)$ be the c.f. (cumulative distribution) of the random variable $B_N^{-1/2} \sum_{j=1}^N Y_j$. Then*

$$|\psi_N(z) - e^{-z^2/2}| \leq 16\, L_N\, |z|^3\, e^{-z^2/3} \tag{8}$$

Define $\tilde{Y}_k = p_k h_k \alpha_k^2$ and $Y_k = \tilde{Y}_k - p_k h_k$. By simple calculations we can find that $\tilde{Y}_k, (k = 1, \cdots, N)$ are Gamma distributed random variables, such that $f^{\tilde{Y}_k}(\omega) = \frac{(m/p_k h_k)^m}{\Gamma(m)}\, \omega^{m-1}\, e^{-(m/p_k h_k)\omega}$ and $E\tilde{Y}_k = p_k h_k$, which means that $Y_k, (k = 1, \cdots, N)$ are zero mean random variables. Note that $\sigma_k^2 = EY_k^2 = (p_k h_k)^2/m, \forall k = 1, \cdots, N$, and therefore, $B_N = \frac{1}{m}\sum_{k=1}^N (p_k h_k)^2$. It is fairly simple to find out that the third moment $E|Y_k|^3 = EY_k^3 = \frac{2(p_k h_k)^3}{m^2}$ $(Y_k \geq 0)$, and $L_N = \frac{2\sum_{k=1}^N (p_k h_k)^3}{\sqrt{m}\,(\sum_{k=1}^N (p_k h_k)^2)^{3/2}}$. For large N, L_N has a very small value, i.e., $L_N << 1$. Examining (8) for small values of z, L_N takes care of the bound and making it very small, while for large values of z, the exponential will decrease the bound and make it approach zero. In conclusion, we can approximate x_i as a Gaussian random variable with mean ζ_{x_i} and variance $\overline{\sigma}_{x_i}^2$ given by:

$$\begin{aligned}\zeta_{x_i} &= E\{x_i\} = E\left\{\sum_{k\neq i}^N \alpha_k^2 p_k\, h_k\right\} \\ &= \sum_{k\neq i}^N p_k\, h_k\, E\{\alpha_k^2\} = \sum_{k\neq i}^N p_k\, h_k \end{aligned} \tag{9}$$

and

$$\begin{aligned}\overline{\sigma}_{x_i}^2 &= E\{x_i^2\} - \zeta_{x_i}^2 \\ &= E\left\{\sum_{l\neq i}^N \sum_{k\neq i}^N p_l h_l p_k\, h_k\, \alpha_l^2\, \alpha_k^2\right\} - \zeta_{x_i}^2 \\ &= \frac{1}{m} \sum_{k\neq i}^N (p_k\, h_k)^2 \end{aligned} \tag{10}$$

where (10) was obtained using the fact that α_k and α_l are statistically independent for all $k \neq l$. So, we can write f^{x_i}, the PDF of x_i, as follows: $f^{x_i}(w) = \frac{\delta_i}{\sqrt{2\pi}\overline{\sigma}_{x_i}}\, e^{-\frac{(w-\zeta_{x_i})^2}{2\overline{\sigma}_{x_i}^2}}$, where $w \geq 0$ and $\delta_i = 2/(1 + Erf[\zeta_{x_i}/\sqrt{2}\,\sigma_{x_i}])$ is a scaling factor such that $f^{x_i}(w)$ is a valid PDF. $Erf[.]$ is the error function. By examining equations (9) and (10), one can see that $\zeta_{x_i} \gg \sigma_{x_i}$, therefore $\delta_i \approx 1$. Averaging (7) over $f^{x_i}(\omega)$ we obtain the average error probability P_e for high SIR below:

$$\begin{aligned}P_e &\approx \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m \int_0^\infty \left(x_i + \sigma^2\right)^m \\ &\quad \times \frac{1}{\sqrt{2\pi}\overline{\sigma}_{x_i}}\, e^{-\frac{(x_i-\zeta_{x_i})^2}{2\overline{\sigma}_{x_i}^2}}\, dx_i \\ &= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m \int_{\sigma^2}^\infty y^m \\ &\quad \times \frac{1}{\sqrt{2\pi}\overline{\sigma}_{x_i}}\, e^{-\frac{(y-(\zeta_{x_i}+\sigma^2))^2}{2\overline{\sigma}_{x_i}^2}}\, dy \\ &\approx \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m \int_0^\infty y^m \\ &\quad \times \frac{1}{\sqrt{2\pi}\overline{\sigma}_{x_i}}\, e^{-\frac{(y-(\zeta_{x_i}+\sigma^2))^2}{2\overline{\sigma}_{x_i}^2}}\, dy \end{aligned} \tag{11}$$

where we used the change of variable $y = x_i + \sigma^2$ and the last approximation in (11) used the fact that $\sigma^2 \ll 1$. By examining (11) one can see that it is the mth moment of a random variable y normally distributed with mean $\zeta_y = \zeta_{x_i} + \sigma^2$ and variance $\sigma_y^2 = \overline{\sigma}_{x_i}^2$. Therefore, the average P_e

is given by:

$$
\begin{aligned}
P_e &= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\,p_i\,h_i}\right)^m E\{y^m\} \\
&= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\,p_i\,h_i}\right)^m E\{((y-\zeta_y)+\zeta_y)^m\} \\
&= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\,p_i\,h_i}\right)^m \sum_{k=0}^{m}\binom{m}{k}\zeta_y^{m-k}\,C_k \\
&= 2^{m-1}\left(\frac{m}{\overline{\gamma_i}}\right)^m \sum_{k=0}^{m}\binom{m}{k}\frac{C_k}{\zeta_y^k} \qquad (12)
\end{aligned}
$$

where $\overline{\gamma_i}$ is the ratio of the mean of the received power from user i to the mean of the interference at the receiver and given by:

$$\overline{\gamma_i} = \frac{W}{R}\frac{p_i\,h_i}{\sum_{k\neq i}^{N} p_k\,h_k + \sigma^2}, \qquad (13)$$

and C_k is the kth central moment and it is given by [9]:

$$C_k = \begin{cases} 1.3\cdots(k-1)\,\overline{\sigma}_{x_i}^k & k \text{ even} \\ 0 & k \text{ odd} \end{cases}$$

By splitting up the summation in (12), we obtain:

$$
\begin{aligned}
\sum_{l=0}^{m}\binom{m}{l}\frac{C_l}{\zeta_y^l} &= 1 + \binom{m}{2}\frac{\overline{\sigma}_{x_i}^2}{(\sigma^2+\sum_{k\neq i}^{N} p_k\,h_k)^2} \qquad (14) \\
&+ \cdots + \binom{m}{m'}\frac{1.3\cdots(m'-1)\overline{\sigma}_{x_i}^{m'-1}}{(\sigma^2+\sum_{k\neq i}^{N} p_k\,h_k)^{m'}}
\end{aligned}
$$

where $m' = m$ if m is even and $m' = m-1$ if m is odd. Since $\overline{\sigma}_x^2$ (see (10)) is very small compared to ζ_{x_i} (see (9)), we can approximate the summation by its leading term which is 1. Therefore the average P_e at high SIR behaves like:

$$P_e \approx 2^{m-1}\left(\frac{m}{\overline{\gamma_i}}\right)^m \qquad (15)$$

And the average utility function of the ith user is given by:

$$u_i = \frac{L\;R}{M\;p_i}\left(1-2^m\left(\frac{m}{\overline{\gamma_i}}\right)^m\right)^M \qquad (16)$$

3.1.2 Nakagami slow flat fading channel

$u_i(\mathrm{p}|x_i)$ can be determined as follows:

$$
\begin{aligned}
u_i(\mathrm{p}|x_i) &= \int_0^\infty u_i(\mathrm{p}|\omega,x_i) f^{\gamma_i|x_i}(\omega)d\omega \\
&= \int_0^\infty \frac{L\;R}{M\;p_i}\,(1-e^{-\omega/2})^M\,\frac{1}{\Gamma(m)} \\
&\times \left(\frac{m}{\gamma_i'}\right)^m \omega^{m-1}\,e^{-(\frac{m}{\gamma_i'})\,\omega}\;d\omega \qquad (17)
\end{aligned}
$$

By factorizing $(1-e^{-\gamma_i/2})^M$ and using the identity $\int_0^\infty y^n e^{-a\,y}\,dy = \frac{\Gamma(n+1)}{a^{n+1}}$ we obtain:

$$u_i(\mathrm{p}|x_i) = \frac{L\,R}{M\,p_i}\sum_{k=0}^{M}(-1)^k\binom{M}{k}\left(\frac{2\,m}{k\,\gamma_i'+2\,m}\right)^m \qquad (18)$$

For fixed m and high SIR, $\gamma_i' \gg 1$ (18) can be approximated as:

$$u_i(\mathrm{p}|x_i) \approx \frac{L\,R}{M\,p_i}\left[1+(\frac{1}{\gamma_i'})^m\sum_{k=1}^{M}(-1)^k\binom{M}{k}\left(\frac{2\,m}{k}\right)^m\right] \qquad (19)$$

Averaging (19) with respect to the distribution of x_i and using the same argument as in (11), (12) and (14) we end up with the final approximate averaged utility function given by:

$$
\begin{aligned}
u_i &\approx \frac{L\,R}{M\,p_i}\left[1+(\frac{1}{\overline{\gamma_i}})^m\sum_{k=1}^{M}(-1)^k\binom{M}{k}\left(\frac{2\,m}{k}\right)^m\right] \\
u_i &\approx \frac{L\,R}{M\,p_i}\left[1-\xi\;(\frac{1}{\overline{\gamma_i}})^m\right] \qquad (20)
\end{aligned}
$$

where $\xi = -\sum_{k=1}^{M}(-1)^k\binom{M}{k}\left(\frac{2\,m}{k}\right)^m > 0$.
In the following two sections, we introduce briefly both NPG and NPGP games.

4. Non-cooperative power control game (NPG)

Let $\mathcal{N} = \{1,2,\cdots,N\}$ be the index set of the users currently served in the cell and $\{P_j\}_{j\in\mathcal{N}}$ represent the set of strategy spaces of all users in the cell. Let $G = [\mathcal{N},\{P_j\},\{u_j(.)\}]$ denote a noncooperative game, where each user, based on local information, chooses a power level from a convex set $P_j = [p_{j-min}, p_{j-max}]$ and where p_{j-min} and p_{j-max} are the minimum and the maximum power levels in the jth user strategy space, respectively. Assuming that the power vector $\mathrm{p} = [p_1,p_2,\cdots,p_N]$ is the result of NPG, the utility of user j is given by [1]:

$$u_j(\mathrm{p}) = u_j(p_j,\mathrm{p}_{-j}) \qquad (21)$$

where p_j is the power of user j, and p_{-j} is the vector of powers transmitted by all other users. The right side of (21) emphasizes the fact that user j can only control his own power. We rewrite (1) for user j as:

$$u_j(p_j,\mathrm{p}_{-j}) = \frac{L\,R}{M\,p_j}\,f(\gamma_j) \qquad (22)$$

The formal expression for the NPG is given in [1] as:

$$G:\quad \max_{p_j\in \mathrm{P}_j} u_j(p_j,\mathrm{p}_{-j}),\;\; \text{for all } j\in\mathcal{N} \qquad (23)$$

This game will produce a sequence of power vectors until it converges to an NE operating point where all users are satisfied with their utility level. Unfortunately, NE point of NPG is not efficient in most cases. In order to reach Pareto dominant NE point, a pricing technique was introduced in [1]. The resulted game is called non-cooperative power control game with pricing (NPGP) and it is described briefly in the next Section.

5. NPGP

In NPGP each user maximizes the difference between its utility function and a pricing function. This aims to allow more efficient use of the system resources within the cell, as each user is made aware of the cost of aggressive usage of resources, and of the harm done to other users in the cell. We use here a linear pricing function, i.e., a pricing factor multiplied by the transmit power. The base station broadcasts the pricing factor to help the users currently in the cell reach a NE that improves the aggregate utilities of all users at power levels lower than those of the pure NPG. In other words, the resulting power vector of NPGP is Pareto dominant compared to the resulting power vector of NPG, but is still not Pareto optimal.

Let $G_c = [\mathcal{N}, \{\mathrm{P}_j\}, \{u_j^c(.)\}]$ represent an N-player noncooperative power control game with pricing (NPGP), where the utilities are [1]:

$$u_j^c(\mathrm{p}) = u_j(\mathrm{p}) - c\, p_j \text{ for all } j \in \mathcal{N} \tag{24}$$

where c is a positive number chosen to get the best possible improvement in the performance. Therefore, NPGP with a linear pricing function can be expressed as:

$$G_c: \quad \max_{p_j \in \mathrm{P}_j} \{u_j(\mathrm{p}) - c\, p_j\} \text{ for all } j \in \mathcal{N} \tag{25}$$

6. Existence and uniqueness of NE point

In this Section we show that NPG and NPGP introduced by [1] admit a unique NE points under the assumed channel models. However, to guarantee the existence and uniqueness of the NE point in both games, the terminals' strategy spaces defined in [1] should be constrained more. That is, some transmit power values which were allowed in a non-fading channel, may not be allowed under a fading channel. In the following, we refer to the unconstrained maximizing transmit power level of user i by p_i^{max}. P_i refers to the convex strategy space of user i.

Lemma 1 *In NPG under Nakagami fast flat fading channel with the average utility function u_i given in (16) with $m = 2$, the existence of a NE point is guaranteed if the strategy space is modified to the following convex set $P_i = \{p_i : \overline{\gamma_i} \in (\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max})\}$, where $\overline{\gamma}_{i-min} = \sqrt{8}\sqrt{2+5M-\sqrt{M\,(8+17M)}}$ and $\overline{\gamma}_{i-max} = \sqrt{8}\sqrt{2+5M+\sqrt{M\,(8+17M)}}$. The best response vector of all users $r^5(p) = (r_1^5(p),\, r_2^5(p),\, \cdots,\, r_N^5(p))$, where $r_i^5(p) = min(p_i^{max}, p_{i-max})$, and*

$$p_i^{max} = 4\sqrt{1+2M}\, I_i, \tag{26}$$

is a standard interference function, therefore by [8] NE point is unique.

Proof 1 *In all following proofs we make use of the classical results of game theory, where the existence of a NE point is guaranteed if the utility function is quasiconcave and optimized on a convex strategy space. Thus, to prove the existence of NE point, it is enough to prove that the utility function u_i is concave (a concave function on some set is also a quasiconcave function on the same set) in p_i given p_{-i} on the convex set $P_i = \{p_i : \overline{\gamma_i} \in (\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max})\}$. Let us find the first and second order derivatives of u_i in (16) after setting $m = 2$ with respect to p_i as follows:*

$$\frac{\partial u_i}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\,(\frac{16(2M+1)}{\overline{\gamma}_i^2} - 1)\,(1 - \frac{16}{\overline{\gamma}_i^2})^{M-1}, \tag{27}$$

then

$$\frac{\partial^2 u_i}{\partial p_i^2} = \frac{1}{M\,p_i^3(-16+\overline{\gamma}_i^2)^2}\,\left(2\,L\,R\,(1-\frac{16}{\overline{\gamma}_i^2})^M \times \left[256(1+M)(2M+1) - 16(2+5M)\,\overline{\gamma}_i^2 + \overline{\gamma}_i^4\right]\right)$$

and this implies that $\frac{\partial^2 u_i}{\partial p_i^2} < 0, \forall\, \overline{\gamma_i} \in (\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max})$, where $\overline{\gamma}_{i-min} = \sqrt{8(2+5M) - 8\sqrt{M\,(8+17M)}}$ and $\overline{\gamma}_{i-max} = \sqrt{8(2+5M) + 8\sqrt{M\,(8+17M)}}$. Henceforth, the strategy space should have the following convex set: $P_i = \{p_i : \overline{\gamma_i} \in (\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max})\}$ to guarantee that u_i is strict concave on P_i, then a NE exists. By setting (27) to zero we find the maximizing transmit power level that lies in the convex strategy space P_i is given as in (26).

To prove the uniqueness of NE point we need to prove that $r^5(p)$ is a standard function, see for example [8] for the definition of the standard function. To prove that $r^5(p)$ is a standard interference function we proceed as follows: The proof of positivity is trivial, since $P_i \subset \mathbb{R}^+$ and $r_i^1(p_{-i}) \in P_i, \forall\, i \in \mathcal{N}$, where $r_i^5(p_{-i}) = r_i^5(p)$. Also, it is obvious that $p_i^{max}(p) > p_i^{max}(\hat{p})$ for all i if $p > \hat{p}$ by looking at (26), henceforth the monotonicity of $r^5(p)$ is satisfied. To prove the scalability, it is enough to prove that $p_i^{max}(p_{-i})$ is a scalable function and then the scalability of $r^5(p)$ comes through. Let us rewrite equation (26) as follows: $p_i^{max}(p_{-i}) = \frac{4R\,\sqrt{2M+1}\,(\sum_{k\neq i}^N h_k p_k + \sigma^2)}{W\,h_i}$ then

$$p_i^{max}(\delta p_{-i}) = \frac{4R\,\sqrt{2M+1}\,(\,\delta\sum_{k\neq i}^N h_k p_k + \sigma^2)}{W\,h_i}, \tag{28}$$

while

$$\delta p_i^{max}(p_{-i}) = \frac{4\,\delta\,R\sqrt{2M+1}\,(\sum_{k\neq i}^{N} h_k p_k + \sigma^2)}{W\,h_i} \quad (29)$$

It is clear that $\delta p_i^{max}(p_{-i}) > p_i^{max}(\delta\,p_{-i})$, therefore $r^5(p)$ is a standard interference function, and the NE point is unique.

In the following lemmas we omit the proof of existence and/or uniqueness as they are similar to those of lemma 1.

Lemma 2 *In NPG under Nakagami slow flat fading channel with the average utility function u_i given in (20), a NE point is guaranteed if and only if the strategy space is the following convex set $P_i = \{p_i : \overline{\gamma_i} \in \left(1, \sqrt{6\,\xi}\right)\}$. The best response vector of all users $r^6(p) = (r_1^6(p),\, r_2^6(p),\, \cdots,\, r_N^6(p))$, where $r_i^6(p) = min(p_i^{max}, p_{i-max})$, and $p_i^{max} = \sqrt{3\xi}\; I_i$, is a standard interference function, therefore by [8] NE point is unique.*

Proof 2 *The first derivative and second order derivatives of u_i after setting $m = 2$ with respect to p_i are given by: $\frac{\partial u_i}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\left(\frac{3\,\xi}{\overline{\gamma}_i^2} - 1\right)$, and $\frac{\partial^2 u_i}{\partial p_i^2} = \frac{2\,L\,R}{M\,p_i^3}\left(1 - \frac{6\,\xi}{\overline{\gamma}_i^2}\right)$, therefore $\frac{\partial^2 u_i}{\partial p_i^2} < 0, \forall\, \overline{\gamma_i} \in \left(1, \sqrt{6\,\xi}\right)$. As a result, the convex strategy space should be $P_i = \{p_i : \overline{\gamma_i} \in \left(1, \sqrt{6\,\xi}\right)\}$ to guarantee the strict concavity of u_i and then the existence of a NE point is guaranteed.*

Now, we turn to the existence and uniqueness of NE point of NPGP under the assumed channel models discussed above.

Lemma 3 *In NPGP under Nakagami fast flat fading channel model with utility function $u_i^c = u_i - c\,p_i$, where u_i is given in (16), a NE point existence is guaranteed if and only if the strategy space is the convex set: $P_i = \{p_i : \overline{\gamma_i} \in \left(\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max}\right)\}$, where $\overline{\gamma}_{i-min} = \sqrt{8}\sqrt{2 + 5M - \sqrt{M\,(8+17M)}}$ and $\overline{\gamma}_{i-max} = 4\sqrt{1+2M}$. The best response vector of all users $r^{11}(p) = (r_1^{11}(p),\, r_2^{11}(p),\, \cdots,\, r_N^{11}(p))$, where $r_i^{11}(p) = min(p_i^{max}, p_{i-max})$, and*

$$p_i^{max} \approx \sqrt{\frac{L\,R}{2\,M\,c}}\sqrt{-1 + \sqrt{1 + \frac{64\,(1+2M)\,I_i^2\,M\,c}{L\,R}}} \quad (30)$$

is a standard interference function, therefore by [8] NE point is unique.

Proof 3 *The maximizer transmit power p_i^{max} is the feasible solution of $\frac{\partial\,u_i}{\partial\,p_i} - c = 0$, where $\frac{\partial\,u_i}{\partial\,p_i}$ is given in (27), and results in a polynomial of degree $2M+4$. It is a tedious and may be impossible to find a closed-form for the feasible solution of this polynomial. Recall that $\overline{\gamma_i} > 4$ to guarantee $u_i(p) > 0$, so the maximizer transmit power level p_i^{max} can be approximated by the feasible solution of the following equation.*

$$p_i^4 + \frac{L\,R}{M\,c}\,p_i^2 - \frac{16\,(1+2M)\,L\,R\,I_i^2}{M\,c} = 0 \quad (31)$$

The only feasible solution of the equation above is given by (30).

Lemma 4 *In NPGP under Nakagami slow flat fading channel model with utility function $u_i^c = u_i - c\,p_i$, where u_i is given in (20), a NE point existence is guaranteed if and only if the strategy space is the following convex set: $P_i = \{p_i : \overline{\gamma_i} \in \left(1, \sqrt{3\,\xi}\right)\}$. The best response vector of all users $r^{12}(p) = (r_1^{12}(p),\, r_2^{12}(p),\, \cdots,\, r_N^{12}(p))$, where $r_i^{12}(p) = min(p_i^{max}, p_{i-max})$, and*

$$p_i^{max} = \sqrt{\frac{L\,R}{2\,M\,c}}\sqrt{-1 + \sqrt{1 + \frac{12\,\xi\,I_i^2\,M\,c}{L\,R}}} \quad (32)$$

is a standard interference function, therefore by [8] NE point is unique.

Proof 4 *The maximizer transmit power level p_i^{max} is the feasible solution of the following equation.*

$$p_i^4 + \frac{L\,R}{M\,c}\,p_i^2 - \frac{3\xi\,L\,R\,I_i^2}{M\,c} = 0 \quad (33)$$

The only feasible solution of the equation above is as given by (32). It is simple to check that $r^{12}(p)$ with the maximizer power in (32) satisfies all the conditions of a standard interference function. Henceforth, the NE point is unique.

Observing lemmas 1-2, we see that the maximizing SIR γ_i^{max} for all users are the same: $\gamma_i^{max} = 4\sqrt{1+2M}\,\forall\, i \in \mathcal{N}$ under fast Nakagami fading channels, and $\gamma_i^{max} = \sqrt{3\,\xi}\,\forall\, i \in \mathcal{N}$ under slow Nakagami fading channels. For nonfading channels it was shown in [1] that $\gamma_i^{max} = 12.4, \forall\, i \in \mathcal{N}$. This implies, as expected, that in order to overcome the fading effect, users in fading channels have to target higher SIR values.

Next, we introduce the algorithms that converges to Nash equilibria points of NPG and NPGP. We need to keep in mind that the strategy space denoted by P_i in the algorithm differs according to the channel model. The algorithm is the same as in [1] except that the strategy spaces are modified to the forms given in lemmas 1-2 to guarantee the existence of NE point under the studied channel models.

Assume user j updates its power level at time instances that belong to a set T_j, where $T_j = \{t_{j_1}, t_{j_2}, \cdots\}$, with $t_{j_k} < t_{j_{k+1}}$ and $t_{j_0} = 0$ for all $j \in \mathcal{N}$. Let $T = \{t_1, t_2, \cdots\}$ where $T = T_1 \bigcup T_2 \bigcup \cdots \bigcup T_N$ with $t_k < t_{k+1}$ and define $\underline{p}$ to be the smallest power vector in the modified strategy space $P = P_1 \bigcup P_2 \bigcup \cdots \bigcup P_N$.

Algorithm 1 *Consider non-cooperative game G as given in (23) and generate a sequence of power vectors as follows:* **(a)** *Set the power vector at time* $t = 0$*:* $p(0) = \underline{p}$*, let* $k = 1$ **(b)** *For all* $j \in \mathcal{N}$*, such that* $t_k \in T_j$*: Given* $p(t_{k-1})$*, calculate* $p_j^{max}(t_k) = arg \max_{p_j \in \boldsymbol{P}_j} u_j(p_j, p_{-j}(t_{k-1}))$ **(c)** *If* $p(t_k) = p(t_{k-1})$ *stop and declare the Nash equilibrium power vector as* $p(t_k)$*, else let* $k := k + 1$ *and go to* **b**.

The next algorithm finds the best pricing factor c for NPGP, keeping in mind that the strategy space should be according to lemmas 3-4.

Algorithm 2 **(a)** *Set* $c = 0$ *and broadcast* c *to all users currently in the cell.* **(b)** *Use Algorithm 1 to obtain* u_j^c *for all* $j \in \mathcal{N}$ *at equilibrium.* **(c)** *Increment* $c := c + \Delta c$*,* Δc *is a positive constant, and announce c to all users, and then go to* **b**. **(d)** *If* $u_j^{c+\Delta c} \geq u_j^c$ *for all* $j \in \mathcal{N}$ *go to* **(c)***, else stop and declare the best c as* c_{Best}

7. Simulation results

We show the effects of time-varying, fast and slow Nakagami fading wireless channels on the equilibrium utilities and powers of NPG and NPGP proposed in [1].

The system under study is a single-cell DS-CDMA cellular mobile system with 9 stationary users, all are using the same data rate $R = 10^4$ bps and the same modulation scheme (non-coherent BFSK). The system parameters used in this study are given in Table 1. The distances between the 9 users and the BS are d $= [310, 460,\ 570, 660,\ 740, 810, 880, 940, 1000]$ in meters. The path attenuation between user j and the BS using the simple path loss model is $h_j = 0.097/d_j^4$, where 0.097 approximates the shadowing effect. The simulations results show that under Nakagami fast flat fading channels with spreading gain $W/R = 10^2$, users do not reach a NE point. Where all users except the nearest user to the BS are using the highest power level in the strategy space without achieving the maximizing SIRs.

Fig.1 demonstrates the equilibrium utilities and the equilibrium powers of NPG under a fast fading channels **(a1)** with spreading gain $W/R = 10^3$. All users were able to achieve their maximizing SIR under Nakagami channels. The equilibrium utilities and equilibrium powers of the NPGP under **(a1)** are shown in the left and right graphs of Fig.2, respectively. Results show that a Pareto improvement over NPG in Nakagami channels was obtained such that all users succeeded to attain SIRs more than their corresponding NPG SIRs.

Fig. 3 presents the effect of the Nakagami slow flat fading channels **(a2)** on the equilibrium utilities and powers. This figure shows that, unlike fast fading channels, all users succeeded to achieve the maximizing SIR $\gamma_i^{max} = \sqrt{3\xi} = 25.1182$.

Table 1. the values of parameters used in the simulations.

L, number of information bits	64
M length of the codeword	80
σ^2, AWGN power at the BS	5×10^{-15}
N, number of users in the cell	9
W/R, spreading gain	10^2, 10^3
m, fading figure	2
p_{i-max}, ith user's maximum power	1 Watts

As for the effect of slow fading channels on the outcomes of NPGP, equilibrium utilities and equilibrium powers, the simulations results showed that Pareto improvement (dominance) over NPG was not possible under Nakagami small scale models. At $c = c_{Best}$, simulation results showed that the best policy for all users is to target a fixed SIR, that is $\gamma_i^{max} = 25.1182$, which is exactly the same situation as in NPG. To demonstrate this result more clearly, we present Fig. 4 for Nakagami channel model. Fig. 4 shows that p_i^{max} given in (32) behaves with feasible values of I_i the same as $p_i^{max} = \sqrt{3\,\xi}\, I_i$ given in Lemma 2. Surprisingly, both figures suggest that NPGP with linear pricing does not admit a Pareto dominance over NPG in a Nakagami slow flat fading channels.

8. Conclusions

We studied a noncooperative power control game (NPG) and noncooperative power control game with pricing (NPGP) introduced in [1] under realistic channel models. We studied the impact of power statistical variation in Nakagami fast/slow flat fading channels on the powers and utilities vectors at equilibrium. The results showed that an equilibrium with an equal maximizing SIR is not attainable in both games with spreading gain ($W/R = 10^2$). In fast fading with spreading gain $W/R = 10^3$, fixed target SIR NPG admitted NE point only under Nakagami small scale models. Results demonstrated that in Nakagami slow flat fading channels, NPGP with linear pricing does not exhibit a Pareto dominance over NPG outcomes at equilibrium.

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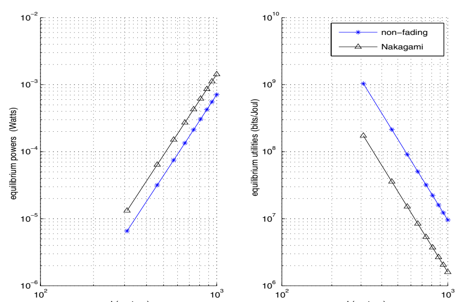

Figure 1. Equilibrium powers and equilibrium utilities of NPG for Nakagami fast flat fading (△) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 10^3$.

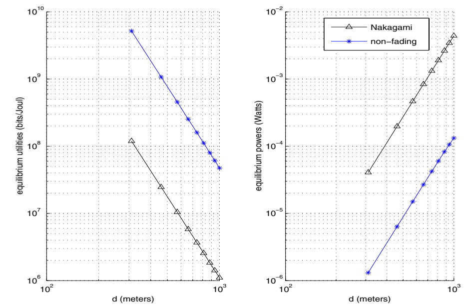

Figure 2. Equilibrium utilities and equilibrium powers of NPGP for Nakagami fast flat fading (△) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 10^3$.

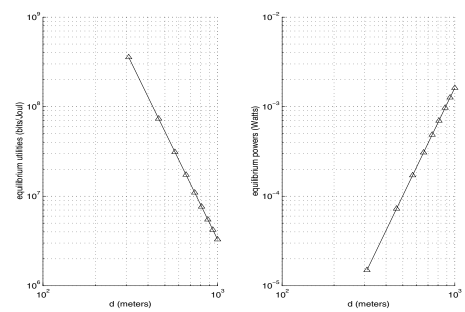

Figure 3. Equilibrium utilities and equilibrium powers of NPG in Nakagami slow flat fading (△) versus the distance of a user from the BS in meters with $W/R = 10^3$.

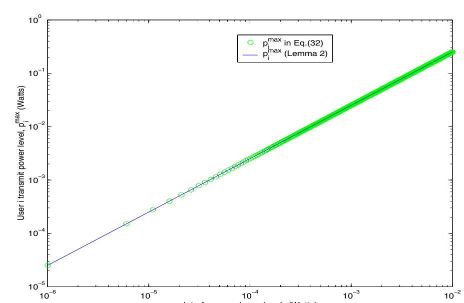

Figure 4. p_i^{max} as a function of I_i as in Eq. (32) (o), and the expression of $p_i^{max} = \sqrt{3\,\xi}\,I_i$ given in Lemma 2 (solid line) with $c = c_{Best}$.