Modified inverse first-order reliability method (I-FORM) for predicting extreme sea states

Aubrey Celia Eckert-Gallup

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Modified inverse first-order reliability method (I-FORM) for predicting extreme sea states

by

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B.S., Mathematics, University of New Mexico, 2012

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Abstract

Environmental contours describing extreme sea states are generated as the input for numerical or physical model simulations as a part of the standard current practice for designing marine structures to survive extreme sea states. Such environmental contours are characterized by combinations of significant wave height \( H_s \) and energy period \( T_e \) or peak period \( T_p \) values calculated for a given recurrence interval using a set of data based on hindcast simulations or buoy observations over a sufficient period of record. The use of the inverse first-order reliability method (I-FORM) is standard design practice for generating environmental contours. This thesis develops enhanced methodologies for data analysis prior to the application of the I-FORM, including the use of principal component analysis (PCA) to create an uncorrelated representation of the variables under consideration as well as new distribution and parameter fitting techniques. These modifications are shown to contribute to the development of more accurate and reasonable representations of extreme sea states for use in survivability analysis for marine structures.
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1 Introduction

The purpose of this work is to develop a refined methodology for determining values of parameters describing extreme sea states that can be subsequently used in survivability models for wave energy converter technology. The development of devices that can be used to convert wave energy to power is a complex problem that necessarily couples an understanding of hydrodynamics, mechanics, and electrical engineering. Evans (1981), published during the self-described ‘renaissance for wave power’ that began in the late 1970’s, presents a classic description of this problem, including expressions of important governing equations. The first step to understanding this problem is to characterize ocean waves in order to determine how the power that they contain might best be converted into electricity for human use. The characterization of ocean waves and sea states is described in Section 1.1. A wide variety of wave energy converters have been designed in an attempt to conquer this problem. Broad categories of these devices are detailed in Section 1.2. In order for such devices to be commercially viable and reliable sources of electric power, their survivability in extreme operating conditions must be studied. The current state of practice of wave energy converter survivability analysis and a description of traditional methods for determining extreme events at a specific site are given in Section 1.3.

1.1 Characterization of ocean waves and sea states

Ocean waves are random processes characterized by variations in height, period, and direction (National Data Buoy Center, 1996; Thorpe, 1999). A key difficulty in understanding how power can be absorbed from ocean waves, described by Falcão (2010), is that this power is governed by variability over several time scales including wave-to-wave interactions that occur over seconds, sea state dynamics, which are defined over hours or days, and seasonal variations. Although a wave field used in the simulation of a particular ocean environment for the purposes of determining a possible power output would theoretically be best derived from data
capturing the entire time series of the wave train over a specified area, collecting this data over a significant period of time is neither instrumentally nor computationally feasible. For this reason, time series data collected and analyzed at a single point is used as a representative measure of the conditions, or sea state, of the ocean area in the vicinity of the measurement point over a given period of time (National Data Buoy Center, 1996; Thorpe, 1999; Mollison, 1994). Sea states are defined as short-term descriptions of the wave field over a specific area and a discrete period of time in which characteristic descriptive parameters, such as significant wave height, energy period, and wave spectra, can be reasonably assumed to be constant (Faltinsen, 1990). These parameters will be described in more detail below.

Real-time data is collected by buoys deployed at varying water depths in offshore and coastal areas around the globe. The sea state data used in this work was collected and processed by the National Data Buoy Center (NDBC), part of the National Oceanic and Atmospheric Administration’s (NOAA) National Weather Service (NWS). The calculation of parameters provided by the NDBC, including significant wave height and energy period, requires the application of time-series and spectral analysis techniques to measurements of motion that are collected by each buoy over a specific period of time (National Data Buoy Center, 1996). These parameters can then be used to describe the sea state over the period of time in which the data was collected, usually one to three hours. The data used in this work was collected for sea states with a one hour duration.

Significant wave height is theoretically defined as the average height of the highest one-third of the waves occurring over a specific period of time (National Data Buoy Center, 1996; Britton and Lilly, 1981). This measure is termed ‘significant’ because it is the most stable measurement of wave motion and can be used to estimate the total energy of the wave train and/or the height of the highest waves that might be expected (Britton and Lilly, 1981). This theoretical measure is estimated using data collected by the NDBC as the wave elevation variance of a nondirectional wave spectrum, an estimation method that has been validated by large amounts of data derived from varying wave conditions (National Data Buoy Center, 1996).
Energy period, or average period, represents the average wave period over a specified length of time, where period is classically defined as the time in seconds required for the crests of two successive waves to pass a fixed point (Mollison, 1994). This can be estimated from the characteristic wave spectrum of a particular sea state (National Data Buoy Center, 1996). Peak period is defined as the reciprocal of the frequency at which the wave energy density of a particular spectrum is at a maximum, or more simply as the wave period associated with the highest energy (National Data Buoy Center, 1996).

1.2 Wave energy converter technology

Wave energy converters (WECs) are devices designed to capture the energy of ocean waves and convert this energy into electric power. Thousands of designs for wave energy converters have been proposed and are characterized by their particular methods for capturing energy, adaptations for various ocean environments, and utilization of differing types of power take-off mechanisms (Falcão, 2010).

Several predominant classifications of the many different types of WECs can be defined; these include attenuators, point absorbers, oscillating water columns, oscillating wave surge converters, and overtopping devices (Drew et al., 2009; Falcão, 2010; Babarit et al., 2012). Attenuators float on the ocean surface and generate energy through the relative motion of connected components (Drew et al., 2009). Point absorbers can be either submerged or floating, using pressure changes or surface motion, respectively, to generate power (Drew et al., 2009). Oscillating water column devices are partially submerged structures in which air, trapped over the water surface, is pushed through a turbine attached to an electric generator by the oscillating motion of ocean waves (Falcão, 2010). Oscillating wave surge converters create energy through the relative motion of a hinged device, propelled by wave surge, with a fixed axis on the sea floor (Drew et al., 2009). Overtopping devices utilize classic hydropower principles by capturing water at the wave crest, storing this in a reservoir that is higher than the ocean surface, and using the potential energy of this reservoir to drive hydraulic turbines (Falcão,
Within each of these WEC categories, individual devices may employ varying power take-off mechanisms, including air turbines, hydraulic turbines, high-pressure oil-hydraulics, etc., along with accompanying electrical equipment such as rotating electric generators (Falcão, 2010). Babarit et al. (2012) provides a case study of eight specific devices that fall within these varying WEC categories and characterizes each of these devices by their design components and their ability to produce power in differing sea states.

Each type of WEC is paired with a suite of concerns that may determine its survivability in a real ocean environment. The dynamics of the environment that the WEC is deployed in, including ocean surface conditions related to wind, waves, and current, play a strong role in the ability of a WEC to survive its deployment (Coe and Neary, 2014). The effect of mooring forces during both normal and extreme conditions must also be considered (Fitzgerald and Bergdahl, 2008). In addition, in order for a WEC to most efficiently absorb wave energy, the device should operate at near resonance conditions, meaning that the frequency of the device’s oscillatory motion should match that of the ocean waves that propel it (Falcão, 2010). This poses a danger to WECs as the resonance effects in sea states characterized by specific combinations of significant wave height and energy period might lead to WEC failure (Coe and Neary, 2014; Babarit et al., 2012). In order for WECs to be commercially viable, they must be able to both perform reliably under regular operating conditions and avoid catastrophic damage during extreme operating conditions (Coe and Neary, 2014).

### 1.3 Wave energy converter survivability analysis

The current practice for designing marine structures to survive extreme sea states is to apply nonlinear time domain numerical simulations to predict structural response to a short-term extreme wave or wave group. Extreme wave design generally includes the following steps as outlined in Coe et al. (2014): (1) consideration of hindcast simulations or buoy observations of sufficient duration and appropriate location; (2) application of extreme value theory and models used for extrapolation to events more extreme than those observed in a shorter period of record;
(3) generation of environmental contours consisting of pairs of significant wave height \( (H_s) \) and either peak period \( (T_p) \) or energy period \( (T_e) \) that elicit extreme structural responses for a given return period; (4) identification of one or more extreme sea states, which can be used with a wave spectrum to reconstruct a single extreme wave or wave group as input for numerical or physical model simulation. Note that in this study, the energy period, \( T_e \), is considered because it is widely used in wave energy applications (Lenee-Bluhm et al., 2011) and has been found to be more robust than the peak period \( (T_p) \).

Vanem and Bitner-Gregersen (2014) summarize methods for generating environmental contours. These methods include the traditional inverse first-order reliability method (I-FORM) utilizing the Rosenblatt transformation (Rosenblatt, 1952), described in detail by Winterstein et al. (1993), and newer methods that avoid the Rosenblatt transformation by employing Monte Carlo simulations of a joint probability model (Huseby et al., 2013). Silva-González et al. (2013) use the Nataf transformation to capture the correlation between inputs and create environmental contours using the marginal distributions and correlation coefficients of the environmental variables in a particular problem of interest. The I-FORM continues to be standard design practice for generating environmental contours used for estimating extreme sea states of a given recurrence interval or return period (DNV, 2014). Environmental loads associated with these extreme sea states are used to design various marine structures, including ships (DNV, 2002), dynamic risers (DNV, 2001), position moorings (DNV, 2010a), offshore floating platforms (DNV, 2010b), and wave energy converters (WECs) (DNV, 2008).

The use of the traditional I-FORM is clearly developed in Haver and Winterstein (2008) and has been applied by Berg (2011) in accordance with the recommendations in the DNV standard on position mooring (DNV, 2010a). The first step of the basic application of the I-FORM described in these works includes fitting distributions and parameter models to the significant wave height \( (H_s) \) and energy period \( (T_e) \) observations such that their interdependency is captured. The treatment of this interdependency in Haver and Winterstein (2008) includes a joint probability distribution using parameters developed by Nygaard and Johannessen (2000). The
The second step of the traditional I-FORM application uses these fitted distributions to estimate extreme sea states by expressing the probability level of the return period of interest (e.g., 100 years) as an isoline, or circle with a fixed radial distance from the most likely center point, in the standard normal space. This isoline is then transformed into the input space of interest using the given probability level as an input into the inverse of the probability distributions defined for the input variables. This traditional application, implemented by Berg (2011), is described and analyzed in more detail in Appendix A.

Examples of this application can be found in Dallman and Neary (2014) and Vanem and Bitner-Gregersen (2014). In some cases, environmental contours for a 100-year return period calculated using this traditional application fail to cover the observations taken over a relatively short (8- to 10-year) period of record. This problem was addressed in Dallman and Neary (2014) and Berg (2011) by inflating the contour resulting from the I-FORM using an arbitrary percentage value. The shortcomings of the environmental contours generated using the traditional methodology along with attempts to correct these problems as a post-processing step clearly show that this methodology requires further exploration and modification in order to generate more realistic contours.

The need for modifications to the treatment of input variables prior to the use of the I-FORM approach is addressed using empirical orthogonal functions (EOFs) by Forristall and Cooper (1997) in order to create more appropriate design water current profiles. In this paper, EOFs are used to simplify the time series data for ocean currents into a series of simpler modes, reducing the original vertical profile. These simplified input variables are then evaluated using the I-FORM to generate design current profiles. The use of EOFs by Forristall and Cooper (1997) shows that it is important to enhance methods for addressing the complexity of input data before applying the probability distribution fittings required by the I-FORM. The importance of capturing the correlations between input variables is also addressed by Silva-González et al. (2013). In the current work, input data complexity will be reduced using principal component analysis.
This thesis describes techniques for improving the traditional I-FORM through principal component analysis (PCA) and enhanced extreme value models for the subsequent uncorrelated random variables. Following the use of PCA to reduce the correlation between the input variables, the I-FORM is applied with modifications to the traditional distribution and parameter fitting models for the subsequent uncorrelated variables. These modifications are shown to contribute to the development of more accurate and reasonable representations of environmental contours for extreme sea states. This analysis uses data collected at four buoys: National Data Buoy Center (NDBC) 46212 offshore of northern California in 40 m depth, NDBC 46022, also offshore of northern California in 391.4 m depth, NDBC 51202 offshore of Oahu in 82 m depth, and NDBC 46050 offshore of Oregon in 128 m depth. More detailed information about each of these study sites, including maps of their locations, can be found in Appendix B.

The remainder of this thesis is organized as follows. Section 2 describes the use of PCA to create a set of uncorrelated variables that will be used to generate better-fitting environmental contours. Section 3 discusses an improved methodology for creating distribution and parameter models for the variables of interest so that the I-FORM can be applied. Section 4 details the application of the I-FORM to the problem under consideration and presents the results of the implementation of this methodology. Section 5 provides concluding remarks and describes recommendations for future work.
2 Principal component analysis

The concept of principal components was initially introduced by Pearson (1901) and more formally developed by Hotelling (1933). PCA provides a powerful transformation that works to reduce the dimensionality of a problem by considering that higher order components have a low impact on the variance of the data. The underlying goal of PCA is to develop a new orthogonal basis in which the variables will be (1) uncorrelated and (2) sorted such that the first variable represents the direction in which the data has the largest variance and each subsequent variable leads to the next largest variance. Traditionally, the variables in the new basis are called principal components while the values associated to each variable in this new basis for each point are called z-scores. The mathematical tools used to generate this new basis are based on classical linear (matrix) algebra applied to the covariance matrix, taking advantage of its structure as a square, symmetrical, and non-singular matrix (Jackson, 1991). The application of PCA to the current problem is mathematically analogous to the use of the empirical orthogonal functions utilized in Forristall and Cooper (1997) for the purpose of generating water current profiles.

2.1 Motivation

The wave data across four sites of interest was studied and compared as a first step in the process of improving upon the traditional application of the I-FORM to the problem at hand. This study included the creation of a representation of data density in order to understand the underlying patterns and trends masked by a traditional scatterplot representation of the data. The density around each point was calculated based on the number of observations included within a fixed neighborhood. Examples of the data density calculated for each of the four sites of interest are shown in Figure 1. These density representations help to characterize the developmental patterns present in the data by showing the differences in frequencies across the entire data set. The results of this study motivated the use of PCA to capture the relation
between the sea state variables of interest.

Figure 1: Representation of data density for four study sites (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

2.2 Application of PCA to the current problem

Principal component analysis was used to remove the correlation between energy period and significant wave height for each dataset. The application of PCA generates two new variables that will be called component one \( C_1 \) and component two \( C_2 \), where \( C_1 \) represents the component with the largest variance. The application of PCA to the original \( H_s \) and \( T_e \) data yields a
coefficient matrix defining a linear combination that allows for rotation into the principal component space. The rotational axes defined by this linear combination are shown in Figure 2. The general form of this coefficient matrix is shown below:

\[
V = \begin{bmatrix}
v_{1,1} & v_{1,2} \\
v_{2,1} & v_{2,2}
\end{bmatrix}
\]  

(1)

where \(v_{2,2} = -v_{1,1}\) and \(v_{1,2} = v_{2,1}\). For \(i = 1,\ldots,n\) where \(n\) is the number of observations in the data set under consideration, the equations for each component based on the application of the coefficient matrix above for each point \(p_i = (H_{s_i}, T_{e_i})\) are shown in Equations 2 and 3:

\[
U = \begin{bmatrix}
H_{s_1} & T_{e_1} \\
\vdots & \vdots \\
H_{s_n} & T_{e_n}
\end{bmatrix}
\]  

(2)

\[
C = UV = \begin{bmatrix}
C_{1,1} = H_{s_1}v_{1,1} + T_{e_1}v_{2,1} & C_{2,1} = H_{s_1}v_{1,2} + T_{e_1}v_{2,2} \\
\vdots & \vdots \\
C_{1,n} = H_{s_n}v_{1,1} + T_{e_n}v_{2,1} & C_{2,n} = H_{s_n}v_{1,2} + T_{e_n}v_{2,2}
\end{bmatrix}
\]  

(3)
Figure 2: Representation of the axes of the new basis developed using principal component analysis for NDBC 46212.

In order to fulfill the requirements for subsequent elements of the extreme event analysis (i.e., fitting probability distributions to the data), the rotated components must also be shifted upwards along the y axis to ensure that they are positive. This is achieved by applying a shift $s$ where $s = |\min(C_2)| + 0.1$. The final equations for the components defined by a single original data point $p_i = (H_s, T_e)$ are then given by:

$$C_{1i} = H_s v_{1,1} + T_e v_{2,1}$$  \hspace{1cm} (4)

$$C_{2i} = H_s v_{1,2} + T_e v_{2,2} + s$$  \hspace{1cm} (5)

The components defined by these equations are used throughout the remainder of the extreme event analysis until they are transformed back into the original space defined by the sea state variables $H_s$ and $T_e$. PCA is a bijective transformation, meaning that there is one and only one way to transform back into the original input space and that the inverse transformation is also a simple linear combination. Given a point $p_i = (C_{1i}, C_{2i})$ in the principal component space, the transformation back to the corresponding point in the original input space defined by $H_s$ and $T_e$ is:

$$p_i = (H_s, T_e) = (C_{1i} - s, C_{2i})$$
$T_e$ is defined by:

$$H_s = \frac{C_{11} v_{1,1} + (C_{21} - s) v_{1,2}}{v_{1,1}^2 + v_{1,2}^2}$$  \hspace{1cm} (6)$$

$$T_e = \frac{C_{12} v_{1,2} - (C_{22} - s) v_{1,1}}{v_{1,1}^2 + v_{1,2}^2}$$  \hspace{1cm} (7)$$

These equations are used to transform the calculated values for the extreme event contour into the input space following the application of the I-FORM described in the remainder of this paper.

### 2.3 Discussion

There is a definite correlation between energy period and significant wave height, as shown in Figures 1 and 2. Dissociating these two variables and treating them independently, as is done in the traditional approach to extreme sea state characterization, underestimates the inherent dependency that these variables share. The principal components in the new basis will be uncorrelated. Therefore, the application of PCA will capture some of the dependency between the initial variables $H_s$ and $T_e$. Correlation is only one aspect of the dependency between these two variables. Thus, some dependency that is not captured by PCA remains. This dependency is expressed in the nonlinear development of data density over time, as shown in Figure 1. This remaining dependency will be taken into account in the distribution fitting portion of the I-FORM application, and will be discussed in Section 3.2.
3 Distribution and parameter model fitting

Following the rotation of the input variables into the principal component space using PCA, distribution and parameter models must be developed for the subsequent component variables so that the I-FORM can be applied to develop environmental contours describing extreme sea states. These distribution and parameter models are developed such that they capture some of the remaining dependency between $C_1$ and $C_2$ that was not taken into account through the application of PCA.

3.1 Distribution fitting for Component 1

Following the rotation of the dataset into the principal component space, the cumulative distribution function (CDF) of $C_1$ is fitted with an inverse Gaussian distribution using a maximum likelihood estimation technique applied in Matlab. This component was chosen for the initial fitting because it has the largest variance, as can be seen in Figure 2. The inverse Gaussian distribution was used both because it provides a good fit of the CDF shape of $C_1$ observed in the datasets under study and because of the simplicity of its defining parameters in terms of interpretation. The equation for the CDF of the inverse Gaussian distribution is given by:

$$F(x) = \Phi\left(\sqrt{\frac{\lambda}{x}} \left(\frac{x}{\mu} - 1\right)\right) + \exp\left(\frac{2\lambda}{\mu}\right) \Phi\left(-\sqrt{\frac{\lambda}{x}} \left(\frac{x}{\mu} + 1\right)\right) \tag{8}$$

where $\Phi$ represents the CDF of the standard normal distribution and $\mu$ and $\lambda$ can be roughly interpreted as representing the location and scale of the inverse Gaussian distribution (Folks and Chhikara, 1978). The results of the inverse Gaussian fitting of the CDF of $C_1$ for all four study sites are shown in Figure 3.
This figure also shows the distribution fitting at each site at the highest quantiles. This area is the most important in terms of goodness of fit for the application of the I-FORM to follow because this area will be evaluated during the extrapolation of these distributions as the final environmental contour is constructed. While the inverse Gaussian distribution fits the CDF of $C_1$ for three of the study sites well, the fitting is not as good as expected for NDBC 51202, especially at the highest quantiles. This fitting issue predicts a smaller value for $C_1$ than is
observed at a given quantile for this site, resulting in an underestimation of extreme events by the final environmental contour, as seen in Figure 11(c). The selection of a more generic distribution or a mixed distribution to fit the variation in behavior for $C_1$ over the selection of study sites might contribute to the creation of more accurate extreme sea state contours across a wider variety of study sites and remains as an area of possible future improvement.

### 3.2 Binning Component 2 according to Component 1

The values of $C_2$ are sorted according to their corresponding $C_1$ values and split into bins containing 250 observations up to the last group, which contains all remaining points. This number was chosen based on the original sample sizes of the study sites under consideration because it is large enough to generate a good fit and small enough so that the influence of $C_1$ on $C_2$ can be captured with a fine enough discretization. The sensitivity of the root mean square error of the distribution parameter fitting models, described in the following section, and the final environmental contour to the size of the bins of $C_2$ is discussed in detail in Section 3.4. This binning scheme and the distribution and parameter fittings that follow help to capture some of the remaining dependency between $C_1$ and $C_2$.

The CDFs for all bins of $C_2$ are shown for all study sites in Figure 4.
The CDF of $C_2$ for each bin must be fitted with a probability distribution in order for the I-FORM to be applied. The parameters of the $C_2$ distributions are fit as functions of the representative value of $C_1$ for each bin in order to represent the dependency between $C_1$ and $C_2$ that has not yet been captured. Based on the shape of the CDFs for each bin of $C_2$, a normal distribution was chosen to fit each CDF of $C_2$. This fit seems appropriate considering the symmetry of the data, shown in Figure 4. The parameters that define the normal distribution are the mean, $\mu$, and standard deviation, $\sigma$. The equation for the CDF of the normal distribution is given by:

$$\Phi\left(\frac{x - \mu}{\sigma}\right) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{x - \mu}{\sigma \sqrt{2}} \right) \right]$$  \hspace{1cm} (9)
where erf represents the error function (Johnson et al., 1995). The CDFs for selected bins of $C_2$ along with their corresponding normal distribution fits for all study sites are shown in Figure 5. These normal distribution fits were generated using a maximum likelihood estimation technique for $\mu$ and the square root of the unbiased estimate of the variance for $\sigma$.

![Figure 5](image)

**Figure 5:** CDFs for selected bins of Component 2 with normal distribution fits for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

### 3.3 Fitting functions for Component 2 distribution parameters $\mu$ and $\sigma$

The sets of $\mu$ and $\sigma$ values that are created following the fitting of a normal distribution to the CDFs of $C_2$ are shown as functions of the mean value for $C_1$ for each bin at each study site in Figure 6.
Figure 6: Estimates of the Component 2 normal distribution parameters $\mu$ (top) and $\sigma$ (bottom) as a function of Component 1 for each bin for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

Based on the trends observed in the data, it was determined that a simple linear approximation could be used to fit $\mu$ as a function of $C_1$ and a constrained quadratic approximation could be used to fit $\sigma$. The fitting functions for $\mu$ and $\sigma$ are as follows:

$$f_\mu(C_1) = m_1 C_1 + m_2$$  \hspace{1cm} (10)$$

$$f_\sigma(C_1) = s_1 C_1^2 + s_2 C_1 + s_3$$  \hspace{1cm} (11)$$

In order to ensure that the quadratic fit for $\sigma$ remains positive for small values of the mean of
$C_1$, the least squares fit for this parameter is subjected to the following constraints:

\[
\begin{align*}
    s_3 & \geq 0 & (12) \\
    s_3 - \frac{s_2^2}{4s_1} & \geq 0 & (13)
\end{align*}
\]

These constraints ensure that both the y-intercept and the minimum of this quadratic model remain greater than or equal to zero. The fitting function for $\mu$ was fit to the data using a basic linear least squares technique. The $\sigma$ function was fit using a penalty function that applies the constraints given above to a least squares objective function evaluated using a direct search, derivative-free method applied in Matlab. The final models for $\mu$ and $\sigma$ at each study site for the binning scheme described above are shown in Figure 6 along with the RMSE for each fit.

Although these models do not perfectly represent the variations present in the data, they allow for smooth extrapolations that create more practically applicable environmental contours. This is especially true when the data appears to be unstructured and may contain multimodal distributions, as is seen in Figure 1(c).

### 3.4 Bin size sensitivity study

A sensitivity study was conducted to determine the impact of the size of the $C_2$ bins on both the root mean square error (RMSE) of the fitting models for $\mu$ and $\sigma$ and the final environmental contour at one site (NDBC 46212). Bin sizes were tested at intervals of 25 data points from a bin size of 50 points up to 10000 points. For each of these bins, the RMSEs for the fitting models of $\mu$ and $\sigma$ are shown in Figures 7 and 8, respectively.

In order to stabilize the response of the parameter fitting functions with respect to the size of each bin, the values for $\mu$ and $\sigma$ for the last bin were not included in the calculation of each fitting model. This ensures that the trends found in the majority of each parameter dataset, rather than possible extreme values created by a small number of data points in the last bin, are used to generate the coefficients for each parameter model. The responses of the RMSE
for the models of $\mu$ and $\sigma$ when the parameters of the last bin of $C_2$ are included in the fitting calculation are shown in blue in Figures 7 and 8 and are compared to the same responses when the last bin is not included, which are shown in green. The number of points in the final bin for each fixed bin size is also shown in these figures on a secondary $y$-axis. This shows that peaks

![Figure 7](image_url)

**Figure 7:** Root mean square error (RMSE) for the $\mu$ fitting function as a function of bin size shown when the last bin is included in the fitting (blue) and when it is not (green) along with the number of points in the final bin as a function of bin size (red) for NDBC 46212.

in the RMSE for both $\mu$ and $\sigma$ when the final bin is included in the fitting calculation often correspond to points at which there are very few points in the final bin. When these points are no longer included, the RMSE stabilizes for the $\mu$ fitting function and reaches a value between 0.025 and 0.05. The RMSE also stabilizes for the $\sigma$ fitting function up to a bin size of approximately 5500 data points when the final bin is not included in the fitting calculation. After this point, some peaks in the RMSE still remain. These peaks may correspond to numerical instabilities in the fitting algorithm caused by the large number of points included in each bin, which create a small number of points with little continuity for the $\sigma$ function to fit. This provides a justification for choosing a bin size (e.g., 250) that is large enough to contain a representative number of points for the normal distribution fit but small enough that the parameter values for
Figure 8: Root mean square error (RMSE) for the $\sigma$ fitting function as a function of bin size shown when the last bin is included in the fitting (blue) and when it is not (green) along with the number of points in the final bin as a function of bin size (red) for NDBC 46212.

Each parameter modeling function are continuous as a function of the mean value of $C_1$ for each bin, allowing for the generation of a close-fitting parameter model.

The final contours for each modeled bin size up to a bin size of 5000 points are shown in Figure 9. These contours show very little variation with a maximum significant wave height difference at a single contour point of 0.26 meters and maximum energy period difference of 0.08 seconds. This shows that when the fitting functions for $\mu$ and $\sigma$ have stable, relatively low RMSEs for a given bin size, the resulting contour will not be significantly different than the same contour for a slightly different bin size.
Figure 9: Final environmental contours for bins modeled at increments of 25 data points from a bin size of 50 points to a bin size of 5000 points for NDBC 46212.

4 I-FORM Application

Following the creation of an inverse Gaussian distribution fit for component one and the development of models fitting the parameters of the normal distributions of the bins of $C_2$ as a function of the mean value of $C_1$ for each bin, the inverse first-order reliability method (I-FORM) can be applied to construct an environmental contour for a given return period.

4.1 Contour creation

In the standard FORM approach, a threshold value is considered and its likelihood is estimated in the standard normal space. The inverse FORM approach, described by Haver and Winterstein (2008), starts with an exceedance probability (i.e., a return period of 100 years) that defines the radius $\beta$ of an isoline in the standard normal space. Equation 14 shows the calculation of the probability for a given return period ($t_r$), given in years, based on the measurement
interval ($t_s$), given in hours. The subsequent calculation of $\beta$ is shown in Equation 15:

\[
p = \frac{1}{365 \times \frac{24}{t_s} \times t_r}
\]

\[
\beta = \Phi^{-1}(p) = \sqrt{2} \text{erf}^{-1}(2p - 1)
\]

where $\Phi^{-1}$ is the inverse CDF of the standard normal distribution. This isoline is then transposed into the original uncertain input space in order to evaluate the range of extreme values. Numerically, a discretization is used on the angle $\theta$ over the isoline to generate a set of points $U_j = (u_{1j}, u_{2j})$, represented as the following parametric function:

\[
\begin{align*}
    u_{1j} &= \beta \cos(\theta_j) \\
    u_{2j} &= \beta \sin(\theta_j)
\end{align*}
\]

where $\theta_j = \frac{2\pi j}{k}$, $j = 1, ..., k$ (16)

For each value of $i$, the quantile position $Q_j = (q_{1j}, q_{2j})$ of the chosen probability of likelihood is calculated in both directions using the inverse of the standard normal cdf:

\[
\begin{align*}
    q_{1j} &= \Phi^{-1}(u_{1j}) \\
    q_{2j} &= \Phi^{-1}(u_{2j})
\end{align*}
\]

where $\Phi^{-1}$ is the inverse of the equation for the standard normal CDF. The resulting quantile $q_{1j}$ is then used to evaluate the inverse of the inverse Gaussian CDF to obtain a value for $C_1$:

\[
C_{1j} = F^{-1}(q_{1j})
\]

where $F$ is the equation for the inverse Gaussian distribution with parameters that were fit to the distribution of $C_2$ obtained from the original input data, described in Section 3.1. This value of $C_1$ is used to evaluate the normal parameter models in order to obtain a particular distribution
of $C_2$:

$$\mu_j = m_1C_{1j} + m_2$$

$$\sigma_j = s_1C_{1j}^2 + s_2C_{1j} + s_3$$

(19)

The quantile position $q_{2j}$ is used to evaluate the inverse of the particular normal CDF defined using the parameters calculated above in order to obtain a value for $C_2$ at this point:

$$C_{2j} = \sigma_j \sqrt{2} \left[ \text{erf}^{-1}(2q_{2j} - 1) \right] + \mu_j$$

(20)

This process is performed for each value along the discretization of the isoline in the standard normal space to create a complete environmental contour in the principal component space. The values of each point on this contour must then be transformed from the principal component space into the original sample space defined by the variables $H_s$ and $T_e$ using Equations 6 and 7, respectively. The extreme sea state contours resulting from this methodology at the four study sites under consideration are shown in Figure 10.
4.2 Results

The environmental contours generated using the modified I-FORM at each of the four study sites are shown with the corresponding contours created using the traditional methodology (e.g., Berg (2011)) in Figure 11. The contours that are developed under the new methodology are displayed along with a representation of the density of each data set under consideration, calculated as described in Section 2.1.
Figure 11: Extreme sea state contours created by the traditional methodology (left) and the new methodology presented in this paper shown with data density (right) for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

This representation helps to emphasize the importance of considering the conjoint influence
of energy period and significant wave height in order to develop extreme sea state contours that reflect the density patterns in each dataset. The contours created using the new methodology appear to follow the shape of trends present within the data, a great improvement on the contours created using the traditional methodology. This allows for coverage in the area of the input space in which both energy period and significant wave height are high, a possible area of importance for survivability analysis of marine structures. The new shape of these contours is similar to the shape of environmental contours obtained by Silva-González et al. (2013), which also provide a better coverage of the input space of interest than the contours obtained using the traditional methodology.

At the most basic level, the extreme sea state contours created using the new methodology show a much better coverage of the data for the given period of record at each of the study sites under consideration. It is expected that, given a period of record on the order of tens of years, the extreme contour for a return period on the order of hundreds of years should include all of the observed data. This is true for all of the study sites with the exception of NDBC 51202. At this site, the complex relations between $H_s$ and $T_e$ are not entirely captured by the application of both principal component analysis and the distribution and parameter fitting. Investigation into the patterns that are visible in this dataset, shown in Figure 1(c), found that the long fingers of observations stretching beyond the majority of the points belong to individual storm events. These storm events are consistent with the large annual winter storms in the North Pacific that have been found to have a strong impact on significant wave height in general (Gulev and Grigorieva, 2006) and on significant wave height in Hawaii in particular (Stopa et al., 2011). The extreme event contour at this site is perturbed by these storms because they are relatively too infrequent to be captured by the methodologies proposed in this work.

In addition, the distribution fitting functions described in Section 3 do not seem as appropriate for the CDFs for Component 1 and Component 2 at this site when compared to the other three sites, as shown in Figures 3 and 5. This suggests that the distribution of these variables, and the corresponding distributions of significant wave height and energy period, do
not follow the same patterns as the distributions of the same variables at the other three sites. This difference may stem from the variations in ocean hydrodynamics amongst continental and island landforms. NDBC 51202 is located off of the coast of the island of Oahu while the other three stations are located along the western coast of the continental United States, as shown in Figure 17. The wave dynamics at this island location are subject to the interacting phenomena of wave diffraction and refraction by which wave energy is propagated around an island when waves reach it and wave direction is changed due to depth variations, causing a wave train approaching from a single direction to wrap around an island’s perimeter (Bascom, 1980). Furthermore, the topography of the Hawaiian Islands has been found to modify trade-wind flow, causing local wind acceleration that may also increase the wave energy found at this location (Stopa et al., 2011). The combination of all of these effects, which are unique to the Hawaiian Islands, helps to explain the different patterns found in the data from this location that contribute to inaccuracies in the final environmental contour generated for this site.

Possible improvements to the environmental contour at this site could be made by (a) collecting more data, (b) using surrogate models, or (c) including data from a similar site. The improvement of the calculated contour at sites with these types of sea state patterns remains as an area of future work beyond the scope of the current analysis.

The maximum significant wave heights for a return period of 100 years predicted using the new methodologies proposed in this work are significantly higher than the maximum values obtained using the traditional methodology implemented as described by Berg (2011) and shown in Figure 11. This particular implementation of the traditional approach uses probability distribution models that do not fit the data as well as they should for extreme value extrapolation. In an effort to find support for the much higher maximum $H_s$ values obtained in the present study, the traditional methodology was implemented with updated distribution fitting techniques that simply found better fits for the input data. The results of this exploration are shown in Figure 12.
Figure 12: 100-year extreme sea state contour obtained using the traditional methodology (blue), using an updated version of the traditional methodology with better distribution fits (green) and using the new methodologies proposed in this work (red) for NDBC 46212.

These show that the improvement of distribution fitting in the implementation of the traditional methodologies can result in much higher values for the maximum significant wave height, supporting the maximum values found in this work.
5 Conclusion

Overall, this paper presents a significant improvement to the original method of calculating an extreme contour of sea states. The proposed modifications, utilizing principal components, better represent the measured data and provide a more reasonable estimation of environmental contours and extreme sea states. This can better prepare developers of marine structures for survivability analysis and can be applied in many areas of interest. The increase in the maximum significant wave height expected to occur for a given return period derived from the new contours developed in this work presents a significant development in the field of marine survivability analysis.

There are several areas that represent possibilities for future enhancements to the methodology developed in this paper. First, the use of principal component analysis to create an orthogonal decomposition of the data such that the values are uncorrelated in each direction only addresses one aspect of the complex relation between energy period and significant wave height, as can be seen in Figure 11(c) and (d). A more complex decomposition taking into account the varying relations among variables across the study sites shown in these representations of density (e.g., curvature) could lead to a better representation of the data and, therefore, a more accurate approximation of the extreme sea state contour for a given return period. However, it is unclear how orthogonality could be preserved under this new paradigm. Second, the selection of a more generic distribution or a mixed distribution to fit the variation in behavior for $C_1$ over the selection of study sites along with refinements in the models developed for the normal distribution parameters $\mu$ and $\sigma$ for $C_2$ might also contribute to the creation of more accurate extreme sea state contours for all sites. Finally, the use of principal component analysis can easily be extended to include multiple dimensions, with each new set of components ultimately presented as a simple linear combination of the original variables. In future work, this could be used to consider additional variables (e.g., wind and water current speed) related to the problem of interest.
Appendices

Appendix A: Review of implementation of traditional methodology

As the first step in the development of the new methodologies presented in this work, a code developed in Matlab by Berg (2011) utilizing the traditional methodology described in the standards of practice was examined and tested. The method developed in this existing code consists of a two-step process used to characterize extreme sea states. The first step includes fitting distributions to the significant wave height ($H_s$) and energy period ($T_e$) observations. The second step uses these fitted distributions to estimate extreme sea states.

The approach presented in the original code uses the traditional monodimensional fitting technique that is presented in the literature (Haver and Winterstein, 2008), in which $H_s$ and $T_e$ are fitted with probability distributions so that the I-FORM can be applied. A least squares technique is applied to a three-parameter Weibull distribution in order to fit the cumulative distribution function (CDF) of the $H_s$ data. An optimization is performed on the two classical scale ($\lambda$) and shape ($k$) parameters of a Weibull distribution (Johnson et al., 1995) along with a third parameter ($\alpha$) that serves as an offset on $x$. This third parameter allows the Weibull distribution to be shifted along the $x$ axis, as is shown in Figure 13 below.
Figure 13: Parameters of the three-parameter Weibull distribution used to fit significant wave height.

The CDF and inverse CDF of the 3-parameter Weibull distribution, described in Teimouri and Gupta (2013) are given by:

\[
F(x) = \begin{cases} 
1 - e^{-(\frac{x-\alpha}{\lambda})^k} & x \geq 0 \\
0 & x < 0 
\end{cases} 
\]  
(21)

and

\[
F^{-1}(q) = \alpha + \lambda (- \ln(1 - q))^{1/k} & q \in (0, 1) 
\]  
(22)

In order to increase the speed of the fitting calculation, the observations are first grouped into a set of 49 bins using a constant significant wave height increment. The subsequent three-parameter Weibull fitting resulting from this binning approach for NDBC 46212 is shown in Figure 14.
Figure 14: Traditional three-parameter Weibull fitting for the entire CDF of significant wave height (top) and zoom in on the highest quantiles (bottom) for NDBC 46212.

The binning approach used in this methodology slightly underestimates the distribution; for a given wave height, the quantile value is overestimated, meaning that the likelihood of occurrence for a certain wave height being equal to or less than the quantile value is lower for the binned data than the likelihood that is actually observed. While the resulting three-
parameter Weibull fitting provides a good representation of the binned distribution, its accuracy drops for the highest quantiles, as is shown in Figure 14 above. The consequence of this loss of accuracy is that, for a high value of significant wave height, the fitted distribution will associate a higher quantile value, and therefore a lower likelihood of occurrence, than was actually observed. Thus, this underestimation may lead to a prediction of maximum $H_s$ for a 100-year return period that is smaller than the $H_s$ values observed in the period of record, creating an inaccurate extreme event contour.

The energy period data is split into bins based on a discrete decomposition of the domain of $H_s$. For each bin, the approach for fitting the $T_e$ data is similar to that used for $H_s$. The $T_e$ values in each bin are fitted with a lognormal distribution using a least squares optimization on the traditional lognormal parameters $\mu$ and $\sigma$, which represent the mean and the standard deviation of the log (Johnson et al., 1995). This generates sets of $\mu$ and $\sigma$ that can be represented as a function of the $H_s$ values corresponding to each bin. The sets of lognormal parameters $\mu$ and $\sigma$ are fit with models that describe their behavior as a function of significant wave height. The data fitting models used for the $\mu$ and $\sigma$ parameters, proposed in Haver and Winterstein (2008) and based on work presented in Nygaard and Johannessen (2000), are given by:

\[
\mu(H_s) = m_1 + m_2(H_s)^{m_3} \tag{23}
\]

\[
\sigma^2(H_s) = s_1 + s_2 \exp(-s_3 H_s) \tag{24}
\]

These models are fit to the respective sets of $\mu$ and $\sigma$ again using a least squares technique and are shown in Figure 15.
Figure 15: Fitting functions for $\mu$ and $\sigma$ created using code implementing traditional methodology for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

The binning scheme that is used as a basis for the distribution fitting of $T_e$ creates uneven samples with very few observations of $T_e$ for the highest discrete intervals of $H_s$ at all sites. Thus, while the fitting functions shown in Figure 15 above follow the general trend of both datasets, they are not appropriate for either parameter at the highest intervals of $H_s$, contributing to inaccuracies in the final extreme sea state contour.

Following the creation of fitting models for $\mu$ and $\sigma$, the I-FORM is used to calculate the extreme sea state contour. The result of this application using the fitting models described above for the four sites under consideration are shown in Figure 16 below.
Figure 16: 100-year extreme sea state contour created using code implementing traditional methodology for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

The 100-year contour at each of these sites does not include all of the eight to ten years of data observed at each site and also does not include the area of the input set in which both energy period and significant wave height are high, an area of importance for the analysis of survivability for marine structures as is described in Section 4.2. These contours have been expanded by both ten and twenty percent in order to create a better coverage of the data from the period of record under consideration. Although these expansions do allow for the inclusion of some points falling outside of each contour, they are simply arbitrary multiplications of the initial contour and do not reflect a robust mathematical description of a reliability contour.
Appendix B: Description of study sites

This appendix describes each of the study sites considered in this thesis in detail. A map showing the locations of all four study sites is shown in Figure 17 below.

![Map showing the location of all four study sites](image)

**Figure 17:** Map showing the location of all four study sites.
The data collected at this station that is used in this thesis contains 76608 hourly observations of significant wave height and energy period. This data was collected from January 22, 2004 to December 31, 2012 in a water depth of 40 m using a Datawell directional buoy (SCRIPPS Institute of Oceanography, 2015b). This buoy was removed from service in April, 2013 (National Data Buoy Center, 2015c).
Figure 19: Map showing the location of NDBC 46022 near Eureka, CA.

The data collected at this station that is used in this thesis contains 122467 hourly observations of significant wave height and energy period. This data was collected from January 1, 1996 to December 31, 2012 in a water depth of 391.4 meters using a three-meter discus buoy fitted with an ARES payload (on-board computer system). The watch circle radius of this buoy is 641 yards. This buoy is owned and maintained by the National Data Buoy Center (National Data Buoy Center, 2015a).
The data collected at this station that is used in this thesis contains 105192 hourly observations of significant wave height and energy period. This data was collected from January 1, 2001 to December 31, 2012 in a water depth of 82 meters using a Datawell directional buoy (SCRIPPS Institute of Oceanography, 2015a). This buoy is owned and maintained by the Pacific Islands Ocean Observing System (National Data Buoy Center, 2015d).
The data collected at this station that is used in this thesis contains 126614 hourly observations of significant wave height and energy period. This data was collected from January 1, 1996 to December 31, 2012 in a water depth of 128 meters using a three-meter discus buoy fitted with an AMPS payload. The watch circle radius of this buoy is 281 yards. This buoy is owned and maintained by National Data Buoy Center (National Data Buoy Center, 2015b).
Appendix C: Wave energy converters

This appendix provides brief descriptions and diagrams for each of the main categories of wave energy converters detailed in Section 1.2.

Attenuators

Attenuators float on the ocean surface and generate energy through the relative motion of connected components (Drew et al., 2009).

Figure 22: Attenuator wave energy converter diagram (Office of Energy Efficiency and Renewable Energy, 2015).

Figure 23: Attenuator wave energy converter deployment (Drew et al., 2009).
Point absorbers

Point absorbers can be either submerged or floating, using pressure changes or surface motion, respectively, to generate power (Drew et al., 2009).

**Figure 24:** Floating point absorber wave energy converter diagram (Office of Energy Efficiency and Renewable Energy, 2015).

**Figure 25:** Submerged point absorber wave energy converter deployment (Drew et al., 2009).
Oscillating water columns

Oscillating water column devices are partially submerged structures in which air, trapped over the water surface, is pushed through a turbine attached to an electric generator by the oscillating motion of ocean waves (Falcão, 2010).

Figure 26: Oscillating water column wave energy converter diagram (Office of Energy Efficiency and Renewable Energy, 2015).

Oscillating wave surge converters

Oscillating wave surge converters create energy through the relative motion of a hinged device, propelled by wave surge, with a fixed axis on the sea floor (Drew et al., 2009).

Figure 27: Oscillating water surge converter diagram (Drew et al., 2009).
**Overtopping devices**

Overtopping devices utilize classic hydropower principles by capturing water at the wave crest, storing this in a reservoir that is higher than the ocean surface, and using the potential energy of this reservoir to drive hydraulic turbines (Falcão, 2010).

![Wave overtopping reservoir diagram](image1.png)

**Figure 28:** Wave overtopping reservoir diagram (Office of Energy Efficiency and Renewable Energy, 2015).

![Wave overtopping converter deployment](image2.png)

**Figure 29:** Wave overtopping converter deployment (Drew et al., 2009).
Appendix D: Thesis presentation

The purpose of this work is to develop a new methodology for determining values of parameters describing extreme sea states that can be used in survivability models for wave energy converters.

Presentation Overview

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Wave Energy Converters

A brief review of technologies

Device Overview

Thousands of designs have been proposed and are uniquely characterized by particular energy capture methods, environmental adaptations, and power take-off variations.

- Attenuators – float on ocean surface and generate energy through relative motion of connected components [2]

- Point Absorbers – floating or submerged, use pressure changes or surface motion to generate power [2]

- Oscillating Water Columns – Air trapped over the ocean surface pushed through a turbine by wave oscillations [3]
Device Overview

- Oscillating Wave Surge Converters – create energy through relative motion of hinge device attached to sea floor and driven by wave surge [3]
- Overtopping Devices – utilize classic hydropower principals to capture potential energy at the wave crest [3]

Important Considerations

- WECs must be able to both perform reliably (low O&M costs) and avoid catastrophic damage
- Environmental dynamics play a strong role in ability of a WEC to survive deployment
- Structural response varies under different environmental loading conditions
- Forces caused by relative motion of WEC components, mooring, etc.
- WECs may operate near resonance conditions in order to efficiently absorb energy
- Resonance in extreme sea states may lead to failure
- Survivability analysis requires simulation of response under events with magnitude related to a given return period (i.e., 10 years)
- How do we use short term data to find the sea state variables that characterize these extreme events?

Characterizing Ocean Waves

Data collection and important measures

- Buoy data from the National Data Buoy Center (NDBC), part of National Oceanic and Atmospheric Administration’s National Weather Service
- Weather and sea state data collected and post-processed by the NDBC to determine important parameters
- Sea state – short term (1 hour) description of the wave field in a specific area in which descriptive characteristics are assumed to be constant

Parameter Definitions

- Significant Wave Height – average height of the highest third of the waves, can also be found through spectral analysis of wave train [meters]
- Energy Period – average wave period over a specified length of time [seconds]
- Peak Period – period of waves with the highest energy [seconds]

Study Sites

- Buoy data from the National Data Buoy Center (NDBC), part of National Oceanic and Atmospheric Administration’s National Weather Service
- Weather and sea state data collected and post-processed by the NDBC to determine important parameters

- Sea state – short term (1 hour) description of the wave field in a specific area in which descriptive characteristics are assumed to be constant
Study Sites

NDBC 46212 – Northern California
- 40 meter depth
- 76608 hourly observations from January 22, 2004 to December 31, 2012

NDBC 46022 – Northern California
- 591.4 meter depth
- 122467 hourly observations from January 1, 1996 to December 31, 2012

NDBC 51202 – Hawaii
- 82 meter depth
- 105192 hourly observations from January 1, 2001 to December 31, 2012

NDBC 46050 – Oregon
- 40 meter depth
- 126614 hourly observations from January 1, 1996 to December 31, 2012

Risk Analysis and the I-FORM

Description of statistical methods

Risk Analysis

Three questions are posed in a classical risk analysis:

Q1: What can happen?
Q2: How likely is it to happen?
Q3: What are the consequences if it does happen?

This work seeks to answer Q1 and Q2 by finding a contour of variables that describes extreme events related to a given likelihood using the inverse first-order reliability method (I-FORM).

Contour defines pairs of variables whose combination is related to an extreme event.
### Statistics Terminology

- **CDF** – Cumulative distribution function
- **Probability** – the likelihood of an event occurring in time. Probability of 0 means that it will never happen. Probability of 1 means that it is guaranteed to happen.
- **Reliability** – the likelihood that a certain event will not occur in a given time period, what we are trying to design for.

### The I-FORM Method

- Define Probability & Reliability
- Transform into Standard Normal Space
- Discretize Reliability Isoline
- Calculate Quantiles
- Evaluate Variables
- Define Contour

#### Example

- A randomly sampled from inverse Gamma distribution and \( B \) from lognormal distribution
- Assume this data is collected every hour (\( t_s = 1 \) hr) and we want to calculate the 100-year contour (\( t_r = 100 \) yr)

\[
\begin{align*}
\alpha &= 1 - 365 \cdot \beta \cdot \sigma^2 \\
\beta &= 1.64 \times 10^{-8} \\
\sigma &= 1.0 \times 10^{-8} \\
\beta &= 0.19599988
\end{align*}
\]

#### Example

- Each standard normal variable corresponds to a quantile of the standard normal CDF
- Each quantile value corresponds to a value of each variable on their individual CDFs

#### Example

- \( \beta \) is the distance from the most likely point
- Combinations of variables – each combination has an equal probability while each individual variable has a different probability.
The I-FORM Method

- Define Probability & Reliability
- Transform into Standard Normal Space
- Discretize Reliability Isolines
- Calculate Quantiles
- Evaluate Variables
- Define Contour

- All pairs of points create a contour in the input space

876,000 random samples (100 years of hourly data) should fall mostly within the contour. Each point has a 0.000114% chance to be outside the contour.

Application to the Problem of Interest

What do we need in order to use this method to create an extreme sea state contour?

- Data
  - Representative collection period
  - How long does this need to be?
- Probability distributions for the parameters of interest
  - Significant wave height
  - Energy period
- Seems simple but:
  - These variables are not independent
  - Depending on one another and on other variables (wind, coastal hydrodynamics, etc.)
  - Complex dependencies impact the effectiveness of this method

Original Environmental Contour

- Environmental contours derived from methodology presented in key papers that are widely cited (Flouz and Winterstein, 2008) and applied in design standards for offshore structures
- Parameters derived from work in the North Sea (Nigard and Johannessen, 2008)

Standard Practice

Review of traditional approach

Description of Methods

- Code written at Hanka by Berg (2011) based on standard literature
- Significant wave height binned and fit with a 3-parameter Weibull distribution
- Energy period binned using discrete intervals of significant wave height and fit with lognormal distribution
- Lognormal parameters \( \mu \) (mean) and \( \sigma \) (standard deviation) fit as functions of significant wave height
- This step attempts to address the dependency between these variables
Analysis of Approach

- Significant wave height binning
- Distribution fitting inappropriate at highest quantiles
- Energy period binning
- Bins at highest intervals of significant wave height may only have a few data points, meaning distribution fitting is meaningless
- Distribution parameters for these intervals are fairly arbitrary, creating problems in the fitting functions for $\mu$ and $\sigma$

Need for Improvement

First Attempts

- Fit entire significant wave height distribution instead of using bins
- Develop double 3-parameter Weibull fit so that upper quantiles will fit better
- Change binning scheme for energy period so that each bin has a large number of observations
- Update parameter models to functions based on inverse tangent and sine

First Attempts

- More points inside the contour
- Higher extreme significant wave height
- And then we received more data

Iterative Process

- Complex fitting functions including Fourier series model applied

Iterative Process

- Fitting the data too closely creates problems with extrapolation required for the I-FORM
New Methodologies
Presentation of current developments

Representing Data Evolution

 Data density shows overall trends in data evolution through time

Representing Data Evolution

NDBC 46212 - California
NDBC 46022 - California
NDBC 51202 - Hawaii
NDBC 46050 - Oregon

Principal Component Analysis

 Removes the correlation between the two variables
 Linear combination of coefficients used for rotation
 Can be easily reversed or applied to related data (contour)

Component 1 Distribution Fits

 Inverse Gaussian distribution chosen to fit Component 1 CDF

Component 2 Bins

 Component 2 binned in groups of 250 with average Component 1 as representative value
Component 2 Distribution Fits

- Normal distributions used to fit Component 2 CDF for each bin

Component 2 Normal Parameters

- Linear fit for $\mu$ and constrained quadratic fit for $\sigma$

Results

New extreme sea state contours

Results

NDBC 46212 – Northern California

Contour generated using new methodologies.

NDBC 46022 – Northern California

NDBC 51202 – Hawaii

NDBC 46050 – Oregon

NDBC 46050 – Oregon

Results

Contour generated using new methodologies.

NDBC 46022 – Northern California

NDBC 46050 – Oregon
Results

Original contour created using traditional methods.
Contour generated using new methodologies.

NDBC 51202 – Hawaii

Monthly sea state evolution shows long fingers of storms.
These infrequent and complex dependencies are not captured by the new methodologies that are applied, perturbing the final contour.

Comparison

NDBC 51202 – Hawaii, monthly sea state evolution.
NDBC 46212 – California, monthly sea state evolution.

Proximity of Hawaii to winter storms in the North Pacific generate different weather and storm patterns.
Hydrodynamics allow wave energy to propagate around the island through processes of refraction/diffraction.
Hawaiian islands modify trade wind flow, causing local wind acceleration.

Differences in hydrodynamics, location generate variations in component distributions.
Inverse Gaussian distribution fitting for Component 1 is not as good, especially at higher quantiles, leading to an underestimation of extreme events.

Comparison

NDBC 51202 – Hawaii, Component 1 distribution.
NDBC 46212 – California, Component 1 distribution.

Differences in hydrodynamics generate variations in component distributions.
Shape of distributions of binned Component 2 values show that variable dependencies differ significantly at this site.

Future Work

Data decomposition with increased complexity to take into account nonlinear dependencies.
Dynamic/generic distribution fitting to accommodate variations in data trends.
Inclusion of additional dimensions (i.e., wind speed) to better describe sea state conditions.
Application of sampling techniques/uncertainty analysis for numerical simulations of WECs under extreme conditions.
Questions?

References


References


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