Fall 11-9-2016

Probing Dark Matter-Neutrino Connection via Indirect Detection Experiments

Bradley Knockel
University of New Mexico

Follow this and additional works at: https://digitalrepository.unm.edu/phyc_etds

Part of the Cosmology, Relativity, and Gravity Commons, and the Elementary Particles and Fields and String Theory Commons

Recommended Citation
https://digitalrepository.unm.edu/phyc_etds/107

This Dissertation is brought to you for free and open access by the Electronic Theses and Dissertations at UNM Digital Repository. It has been accepted for inclusion in Physics & Astronomy ETDs by an authorized administrator of UNM Digital Repository. For more information, please contact disc@unm.edu.
Bradley Knockel
Candidate

Physics & Astronomy
Department

This dissertation is approved, and it is acceptable in quality and form for publication:

Approved by the Dissertation Committee:

Rouzbeh Allahverdi, Chairperson
Huaiyu Duan
Ylva Pihlström
Ivan Deutsch
Bhaskar Dutta
Probing Dark Matter-Neutrino Connection via Indirect Detection Experiments

by

Bradley Knockel

B.S., Physics and Math, University of New Mexico, 2009
M.S., Physics, University of New Mexico, 2012

DISSERTATION
Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy Physics
The University of New Mexico
Albuquerque, New Mexico

December, 2016
Acknowledgements

I suppose this PhD began with my first physics professor, Rob Duncan. He made physics come to life. Ever since that first physics class, I have been fascinated by understanding how things work using the way of thinking and problem solving called physics. Many great professors since, I am lucky to have received a great education.

After my master’s, I found myself in the world called research. I may not have continued in this new world if it were not for Mickey Odom, Alisa Gibson, Irene Pallardy, and especially my parents, Barbara Knockel and Perry Knockel, giving me the good advice, support, and encouragement to finish the marathon known as a PhD. I will always greatly value . . .

- being a part of something as huge and important as science pushing the boundaries of knowledge, for which I give credit to society and the drive of scientists. Solving original problems in clever ways that could be useful or interesting for others is a joy.

- being able to talk to knowledgeable (or at least curious) people about research or physics in general. To list everyone would take quite some time, and I hope you know who you are, and I thank you. What is the point of doing science and solving problems if you cannot discuss it? Talking with peers and experts about physics mysteries, the current state of a research field, or about related fields is great fun. Collaboration and communication are integral parts of research.

iii
my trusty laptop named pinkslime on whom countless lines of code and text have
been written and who has spent many days and nights running calculations.

To my brother, Jeff, and cousin, Augusta, it seems that I will be the first in my
family to get a PhD, so thanks for not beating me to it.

To Katie Richardson and Shashank Shalgar, each being a combination of a peer
and mentor, your unique guidance was invaluable towards the specifics of my research.
To Robert Lauer, your office door was always open when I had detailed questions
about how various detectors analyzed their data.

I owe much to Rouzbeh Allahverdi, my helpful and knowledgeable advisor. I
could not have done most of this work without him. Having foresight, he guided me
to do fundamental calculations that, in combination with me keeping them and other
documentation organized over the years, were essential in writing this dissertation.
Having patience, he provided research goals that seemed catered to my abilities and
preferences. He helped me present at conferences, apply for grants, meet with scien-
tists, join his collaborations, and would have happily aided me more if the world of
research was my top priority. Most importantly, he paid me! Guidance and guiding
are crucial for research, and I lucked out in this. I will try to remember to repay him
for all the coffee (and for an interesting tea called a Bob Marley at the Espresso Café)
one of these years...

To the rest of my dissertation committee—Huaiyu Duan, Ylva Pihlström, Ivan
Deutsch, and Bhaskar Dutta—who have been a part of this dissertation over the past
several years and who will have to read this dissertation and give thoughtful feedback,
you are an essential part of this process, and I thank you all. Your good-humored
willingness is appreciated.
Probing Dark Matter-Neutrino Connection via Indirect Detection Experiments

by

Bradley Knockel

B.S., Physics and Math, University of New Mexico, 2009
M.S., Physics, University of New Mexico, 2012
Ph.D., Physics, University of New Mexico, 2016

ABSTRACT

Various evidence reveals that dark matter is a primary component of this universe. The amount of dark matter is known, but its identity is a mystery. To determine its properties, efforts to detect and produce dark matter are underway. Dark matter annihilations throughout the galaxy may produce photons, neutrinos, and cosmic rays. Neutrino and photon detectors may then indirectly detect dark matter by detecting these annihilation products. The annihilation rate, dark matter mass, and dark matter scattering rate off of matter affect the signals received at Earth. These signals can therefore probe the identity of dark matter, especially if dark matter is a well-motivated Weakly Interacting Massive Particle. Another mystery in particle physics is how neutrinos acquire mass. To probe the identity of dark matter, considering only the simplest models may not be wise because there are many mysteries to solve. To consider the mysteries of neutrino mass and the identity of dark matter at the same time is a promising strategy. There are well motivated reasons to tie the two sectors together, and unique signals arise.

Three specific results have been obtained. First, the IceCube Neutrino Observatory’s ability to probe the unique annihilation channel to prompt neutrinos that may result when dark matter couples to neutrinos is competitive with direct detection, and is not highly dependent on annihilation rate if detecting neutrinos from dark matter that have been captured by the Sun. Second, detecting the neutrino mass
hierarchy at IceCube by detecting the annihilation of a scalar dark matter particle that exists in the context of a type-II seesaw-neutrino may also be possible in the context of measurements by other experiments. Third, in a type-I-seesaw scenario where dark matter annihilates to a few-GeV-mass right-handed neutrino inside the Sun, the right-handed neutrino can later decay outside the Sun giving unique strong signals that could set stringent constraints on allowed parameter space.
Contents

Acknowledgements iii

Abstract v

Preface xx

1 Dark Matter 1

1.1 The Standard Model ............................................. 1
1.2 What is Dark Matter? ............................................ 6
1.3 Gravitational Evidence .......................................... 9
  1.3.1 Rotation Curves ............................................ 10
  1.3.2 Gravitational Lensing ..................................... 11
  1.3.3 Modern Cosmological Measurements .................... 12
1.4 Particle Physics Candidates .................................. 18
  1.4.1 WIMP Miracle .............................................. 19
  1.4.2 One Explicit Example of WIMPs ......................... 21
1.5 Possible Types of Detection .................................. 23
1.6 Microscopic Parameters ...................................... 24
  1.6.1 DM-Nucleon Scattering Cross Sections .................. 25
  1.6.2 Annihilation Cross Section and Channels .............. 28
1.7 Indirect Detection ............................................. 31
CONTENTS

1.7.1 Detectable Annihilation Products ........................................ 31
1.7.2 Role of Indirect Detection ................................................. 32

2 Neutrinos ................................................................. 34

2.1 Introduction ................................................................. 34

2.2 Oscillations and Masses .................................................. 35
  2.2.1 Two-Neutrino-Flavor Oscillation ...................................... 35
  2.2.2 Three-Neutrino-Flavor Oscillation and Masses ..................... 37
  2.2.3 Matter Effects ........................................................... 41

2.3 Nature of Neutrino Mass .................................................. 43

2.4 Indirect Detection of WIMPs via Neutrinos .......................... 46
  2.4.1 Cross Sections for Neutrino-Matter Interaction .................... 46
  2.4.2 Neutrino Interactions Inside a Neutrino Detector ............... 48
  2.4.3 Neutrino Backgrounds ................................................ 49
  2.4.4 Averaging the Signal over Many Neutrinos ....................... 51

2.5 Possible DM Connection to Neutrinos ................................ 53

3 Prospects for Discovery of DM Annihilation to Prompt Neutrinos
  with IceCube ........................................................................ 55

3.1 Introduction ...................................................................... 55

3.2 Dark Matter Annihilation to Prompt Neutrinos: Motivations ...... 58
  3.2.1 Dark Matter with a Light Mediator .................................... 59
  3.2.2 $U(1)_{B-L}$ Extension of MSSM ....................................... 60

3.3 Neutrinos from Dark Matter Annihilation in the Sun .............. 61
  3.3.1 The Sun as a Source .................................................. 61
  3.3.2 IceCube as a Dark Matter Detector ................................ 64
  3.3.3 Neutrino Energy Distributions and Their Features ........... 65

3.4 Benchmarking Our Methodology ......................................... 69
## List of Figures

1.1 The SM describes the fundamental fermions (quarks and leptons) and fundamental bosons (the gauge bosons that mediate the fundamental forces and the Higgs boson) that comprise the universe (image credit: DESY at Hamburg). ................................................................. 4

1.2 The Milky Way is depicted with its DM halo. The red are clusters of ancient stars called globular clusters that are orbiting the Milky Way outside the disk. ................................................................. 8

1.3 In this galaxy, the stars beyond the disk unexpectedly *increase* in orbital speed at larger distances (image credit: NOAO, AURA, NSF, T.A.Rector). ................................................................. 11

1.4 The Bullet Cluster shows the result of two galaxies clusters colliding. The pink is X-rays from colliding gas, and the blue represents gravitational lensing of background stars rather than being emission from the cluster (image credit: NASA). ................................................................. 12

1.5 The energy budget of our universe is primarily dark energy, which is followed by dark matter. Einstein’s $E_0 = m$ is used for matter particles (image credit: Planck). ................................................................. 13
1.6 The surface of last scattering is the farthest we can presently see using electromagnetic radiation. The infrared and microwave light from this surface corresponds to a blackbody spectrum of 2.73 K, and the small deviations from this value in the figure are from photon-baryon acoustic oscillations 13.8 billion years ago. To obtain this image, the Doppler effect due to Earth’s motion has been removed, and, via a spectrum analysis, the Milky Way, which would be located at the vertically centered horizontal line, has been removed (image credit: Planck). 14

1.7 From the CMB data collected by the Planck satellite in Fig. 1.6, an analysis of which angular sizes have the strongest temperature fluctuations can be done. The sizes and strengths of the peaks can be predicted by various cosmological models and parameters, which can then be compared to the above data. The large-size uncertainty on the left side of the figure is due the low sample size of large regions in the sky (image credit: Planck). 15

1.8 Supersymmetry doubles the number of particles in the SM (plus a few more in the Higgs sector and more depending on the SUSY-breaking mechanism). The symmetry cannot be perfect, so the superpartners have a larger mass. The lightest stable superpartner may be the identity of DM (image credit: DESY at Hamburg). 22

1.9 Direct detection, indirect detection, and production all probe the same process. 23
1.10 $\sigma_{SI}$ exclusion curves from various direct-detection experiments. To make this plot the neutron and proton are assumed to have the same $\sigma_{SI}$. LUX’s 2016 results [16] improve upon their previous results by a factor of 4 for most $m_{DM}$. The dashed line (and surrounding shaded error bars) represents the expected sensitivity, which can be less sensitive than the final limits if less background is detected than expected (image credit: LUX).

1.11 Neutron’s $\sigma_{SD}$ exclusion curves from various direct-detection experiments. DAMA sees a potential DM signal in the region excluded by many other experiments. The curves labeled MSDM are collider limits assuming a specific model of DM (image credit: LUX’s paper [17]).

1.12 Proton’s $\sigma_{SD}$ exclusion curves from various direct-detection experiments. Dashed lines are for projected LZ (a future DM detector) sensitivity or, as will be discussed later, for annihilation-channel-dependent results of indirect detection from the Sun’s ability to capture DM via $\sigma_{SD}$ (image credit: LUX’s paper [17]).

1.13 Assuming branching fractions of 100% to various annihilation channels, Fermi presents these 95%-confidence exclusion curves using Milky Way dwarf spheroidal galaxies. The parameter space above the curves is ruled out (image credit: Fermi’s paper [18]).

1.14 Top: assuming branching fractions of 100% to various annihilation channels, IceCube presents these 90%-confidence exclusion curves using the Galactic Center (the limits based on actual data did better than the predicted detector sensitivities). Bottom: for the $\tau\tau$ channel, IceCube presents a comparison between its Galactic Center analysis, its halo analysis, and the results of other experiments. The parameter space above the curves is ruled out (image credit: IceCube’s paper [19]).
1.15 DM particles, $\chi$, annihilating via the $\tau\bar{\tau}$ channel. In this figure, one of the virtual $W$’s decays hadronically while the other decays leptonically. Time progresses to the right. 31

2.1 Two neutrino mass hierarchies are consistent with current data. The colors depict the PMNS mixing matrix (image credit: Snowmass 2013). 38

2.2 On the left (right) is the probability for a $\nu_e$ ($\nu_\mu$) to oscillate to any of the 3 flavors traveling in vacuum as a function of distance traveled. 39

2.3 This figure is the same as Fig. 2.2 except $\theta_{13} = 0$. The atmospheric (shorter) oscillations and solar (longer) oscillations are now easily seen. $|\nu_e(t)\rangle$ undergoes solar oscillations to $|\nu_\mu\rangle$ and $|\nu_\tau\rangle$, and $|\nu_\mu(t)\rangle$ and $|\nu_\tau(t)\rangle$ undergo atmospheric oscillations between $|\nu_\mu\rangle$ and $|\nu_\tau\rangle$ as well as solar oscillations to $|\nu_e\rangle$. 39

2.4 To complete the discussion of measurements of neutrino mass, upper bounds on neutrino mass also result if assuming that neutrinos are Majorana. $NS$ and $IS$ mean $NH$ and $IH$ respectively. If $NH$ is correct, IH may be ruled out using neutrinoless double-beta decay if the upper bound becomes lower in years to come (image credit: arXiv review paper by Bilenky and Giunti [26]). 41

2.5 Neutrino detectors must consider the above sources when attempting to indirectly detect DM (image credit: IceCube). 49

2.6 Left: air showers from cosmic rays produce $\mu$, $\nu_\mu$, and $\nu_e$ among other particles (image credit: CERN). Right: this up-going muon is either from a $\nu_\mu$ produced in the atmosphere on the other side of the Earth or from a $\nu_\mu$ produced in DM annihilation, but it cannot be an atmospheric $\mu$ (image credit: IceCube). 50
3.1 Neutrino energy distributions at the detector from annihilation of 100 GeV DM particles to $\nu_\mu$ (bottom, green), $W$ (middle, orange), and $\tau$ (top, brown) final states inside the Sun. ........................................ 67

3.2 Neutrino energy distributions at the detector from annihilation of 500 GeV DM particles to $\nu_e$ (bottom, red), $\nu_\mu$ (middle, green), and $\nu_\tau$ (top, blue) final states inside the Sun. ........................................ 69

3.3 Sensitivity plots for discovery of DM particles with 100% annihilation to $W$’s with one year of data from IC/DC: our analysis (middle, red), and the IceCube Collaboration published results in 2012 [39] (top, dotted) and 2011 [38] (bottom, dashed). .............................. 71

3.4 Sensitivity plots for $\sigma_{SD}$ for discovery of DM particles with various annihilation channels (top $W$, middle $\tau$, bottom flavor democratic prompt $\nu$’s) with one year of data from IC/DC. The grey band shows the direct detection bounds on $\sigma_{SD}$ from the COUPP experiment [63]. .......... 73

3.5 Sensitivity plots for $\sigma_{SI}$ for discovery of DM particles with various annihilation channels (top $W$, middle $\tau$, bottom flavor democratic prompt $\nu$’s) with one year of data from IC/DC. The grey band shows the direct detection bounds on $\sigma_{SI}$ from the LUX experiment [61]. .............. 74

3.6 Sensitivity plots for $\sigma_{SI}$ for discovery of DM particles preferentially annihilating to one neutrino flavor (top $\nu_e$, middle $\nu_\mu$, bottom $\nu_\tau$) with one year of data from IC/DC. The grey band shows the direct detection bounds on $\sigma_{SI}$ from the LUX experiment [61]. ............... 75

3.7 The same as Fig. 3.6 but with $1\sigma$ variation about the central value of $\Delta m^2_{\text{sol}}$ from PDG 2014 [24] for normal hierarchy scenario. ................. 76

3.8 The same as Fig. 3.7 but for inverted hierarchy scenario. ............... 76
3.9 The minimum value of annihilation rate $\langle \sigma_{\text{ann}} v \rangle$ for different channels that lead to equilibrium between capture and annihilation for sensitivity curves of $\sigma_{\text{SD}}$ in Fig. 3.4. ............................ 77

3.10 The number of standard deviations at which the $\nu_\mu$ channel can be distinguished from other channels with 10 years of data from IC/DC. 90% confidence is the 1.64 dashed line. ................................. 79

3.11 The same as Fig. 3.10 but with the $\tau$ channel as the target. ............... 79

3.12 The same as Fig. 3.10, but with 10 GeV smearing of the energy of contained muons. ................................................................. 80

3.13 Energy and angular cuts for our low-energy and high-energy analyses when attempting to discover DM. All annihilation channels use the same cuts. DM masses are from 50 GeV to 1 TeV. ......................... 86

3.14 The same as Fig. 3.13 but for discriminating between annihilation channels rather than discovery. ........................................... 87

4.1 Spectra of $\nu_\mu$ from DM annihilation at the Galactic Center for BP1 (left) with $m_{\text{DM}} = 500$ GeV and BP2 (right) with $m_{\text{DM}} = 700$ GeV. .. 101

4.2 Photon spectra from DM annihilation at the Galactic Center for BP1 (left) with $m_{\text{DM}} = 500$ GeV and BP2 (right) with $m_{\text{DM}} = 700$ GeV. .. 101

4.3 Spectra of $\nu_\mu$ (left) and $\gamma$ (right) from DM annihilation at the Galactic Center for BP3 and BP4. ................................................. 102

4.4 Spectra of $\nu_\mu$ from DM annihilation inside the Sun at the detector for $m_{\text{DM}} = 500$ (700) GeV in the left (right) panel. ......................... 105

4.5 Spectra of muons from DM annihilation at the Galactic Center for $m_{\text{DM}} = 500$ (700) GeV in the left (right) panels. Upper and lower panels show contained and through-going muon spectra with an angular cut of $5^\circ$. ......................................................... 106
4.6 Spectra of muons from DM annihilation in the Sun for \( m_{\text{DM}} = 500 \) (700) GeV in the left (right) panels. Upper and lower panels show contained and through-going muon spectra with an angular cut of \( 2^\circ \) respectively. .................................................. 107

4.7 Spectra of contained muons from DM annihilation at the Galactic Center (left) and inside the Sun (right) for \( m_{\text{DM}} = 500 \) GeV. The value of \( \sigma_{\text{ann}} \) used to obtain the muon flux is just below the current bounds from Fermi-LAT. The top line in both figures is due to the background arising from the atmospheric neutrinos. ................................. 110

5.1 The characteristic decay length is inside the photosphere (above the solid lines) and is within 200,000 km (above the dashed lines). The colors red, green, and blue correspond to the cases when \( N \) mixes dominantly with \( \nu_e, \nu_\mu \) and \( \nu_\tau \) respectively. The red and green curves almost coincide. .......................... 118

5.2 \( \nu_\mu \) (top) and photon (bottom) spectra from delayed decay of RH neutrinos produced in DM annihilation. In these plots, \( M_{\text{DM}} = 1 \) TeV and \( M_N = 2.5 \) GeV. The spectra are for \( N \) mixing with each of the \( \nu_e, \nu_\mu, \) and \( \nu_\tau \) flavors. .................................................. 120

5.3 \( \nu_\mu \) (top) and photon (bottom) spectra from delayed decay of RH neutrinos produced in DM annihilation. In these plots, \( M_{\text{DM}} = 1 \) TeV and \( M_N = 2.5 \) GeV. For normalization, we assume a total annihilation inside the Sun to be \( 1.5 \times 10^{19} \) per second. The “equal” curves is for the case when \( N \) mixing with all of the \( n_e, \nu_\mu \) and \( \nu_\tau \) flavors equally. The neutrino background is atmospheric neutrinos [175]. The shaded region around the Fermi data shows the uncertainty from Ref. [176]. 122
5.4 The 90% confidence level (C.L.) contours for $\sigma_{SD}$ using the Fermi-LAT limits on the photon signal from solar DM annihilation to light RH neutrinos. The shaded region is ruled out by PICO-60 limits on $\sigma_{SD}$ [177]. The characteristic length of the decays is inside the photosphere above the dashed line. BF is the branching fraction to RH neutrinos. . . . . 128

5.5 Same as Fig. 5.4 but for $\sigma_{SI}$. The shaded region is ruled out by LUX limits [16]. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 129

5.6 The 90% C.L. contours for $\sigma_{SD}$ using IceCube limits on the neutrino signal from solar DM annihilation to light RH neutrinos. The dashed line now marks when the characteristic length of the decays is inside 200,000 km. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 130

5.7 Same as Fig. 5.6 but for $\sigma_{SI}$. . . . . . . . . . . . . . . . . . . . . . . 131
List of Tables

4.1 Benchmark Points (BPs) and the corresponding DM relic density and
direct detection cross-sections. .............................. 98

5.1 Decay channels of a 2.5 GeV right-handed singlet neutrino via its mixing
into SM neutrinos. The partial widths are calculated at tree-level.
Depending on the mass of $N$, not all channels are kinematically al-
lowed. The final column differs because $N$ does not produce various
final states that contain a $\tau$. The $(q\bar{q})$ system in the hadronic modes
would hadronize into different combination of pions with respect to the
total electric charge. ................................................ 119
Preface

Getting a PhD is a unique journey. It is a pursuit of and creation of knowledge rather than of immediate money. At the end of the journey, one’s dissertation is basically a book written for one’s dissertation committee to show a knowledge of the field and ability to contribute towards research. Often, it is an important step towards becoming a professor, a caretaker of knowledge.

For me, this PhD means something slightly different. While I hope this dissertation to reflect ability to contribute towards research and reflect knowledge, teaching is my primary passion. I have sacrificed opportunities to work on my PhD because I respect knowledge and enjoy trying to solve problems that may have global and timeless importance, and PhD work has taught me how to carefully find necessary facts before acting. On a more practical level, it will hopefully open up new teaching and other opportunities, and it has allowed me to better understand what I enjoy doing. I hope my life to always include new challenges such as this PhD.

I am an advocate of diverse people working for a PhD if they are capable, especially in technical fields where there is money for student research, for I view a PhD as a more general goal than a step down a single path toward research. It is a unique job and experience that can benefit teachers, politicians, analysts, administrators, MDs, JDs, etc. Curiosity and a drive to solve problems are not always welcome by employers, but a PhD program encourages them. I would ideally like to find various positions in
engineering, industry, etc. throughout my life to gain more unique experiences where my drive to solve problems and love of computers can be useful.

I want to include my own vision for this dissertation in addition to satisfying the usual requirements. Unlike writing for a scientific journal, which has the value of being widely read and contributing to the world of research, a dissertation has no such value. I need this dissertation to be meaningful for me and my future as a teacher (I believe personalizing an education towards one’s career to be a goal of education), so my vision is that any new graduate student in physics—and therefore hopefully even a non-physics committee member—should, after reading it, be able to understand all the physics and procedures required to reproduce the research I have done on a very practical level (therefore anyone should be quickly able to expand on this work, which would imply the highest level of understanding of my work). For this reason, I have provided appendices, have made the first several sections of Chapter 1 pedagogical, and have tried to err on the side of explanations over jargon. I am writing as if to my past self in a way that documents my work for my future self. I hope to present a bit of my excitement for physics in an originally told “story” of the research I have done. Only this can make this long process bearable and possibly even fun, though it will make this dissertation a bit longer.
Chapter 1

Dark Matter

1.1 The Standard Model

The Standard Model of particle physics (SM) describes the fundamental particles such as electrons and photons, and describes how they interact via the fundamental forces: strong, electromagnetic, and weak. These particles and forces are very well understood through studies at particle accelerators. New physics—particles and interactions—must exist beyond the SM to explain various mysteries in particle physics, but experiments attempting to understand this new physics cannot yet identify its properties. Any understanding of what exists beyond the SM first requires an understanding of the SM. See [1] for a basic introduction.

A fundamental force is created when particles known as gauge bosons act as messengers between charged particles. If two particles such as electrons have electric charge, a gauge boson called the photon can be exchanged between the particles resulting in the electromagnetic force. If two particles such as the quarks in a proton
or neutron have color charge (strong charge), a gauge boson called the gluon can be exchanged between them resulting in the strong force (as can be inferred from its name, the strong force is the strongest known force and is what binds quarks together in protons and neutrons).

Most matter in daily life is made entirely of protons, neutrons, and electrons. Protons and neutrons are made of fundamental particles called quarks, which have color charge and electric charge. Due to their color charge, gluons hold them together. Quarks come in six flavors—up \((u)\), down \((d)\), charmed \((c)\), strange \((s)\), top \((t)\), and bottom \((b)\)—and the \(u\) and \(d\) flavors are stable, whereas the other flavors are massive enough to quickly decay into lighter flavors via the weak force shortly after being created. Electrons \((e)\) belong to a class of fundamental particle called leptons. Leptons do not have color charge. Leptons are either charged leptons (have electric charge) such as the \(e\) or are one of the corresponding neutrinos. The \(e\) is the stable charged lepton, and the more massive muon \((\mu)\) and tau \((\tau)\) also exist. The three types of neutrinos are electron neutrino \((\nu_e)\), muon neutrino \((\nu_\mu)\), and the tau neutrino \((\nu_\tau)\). All of these many particles are spin-1/2, so they are called the fundamental fermions.

Besides the fundamental fermions, fundamental bosons exist. These include the scalar (spin-0) Higgs boson and the vector (spin-1) gauge bosons. The gauge bosons are the photon \((\gamma)\), the gluon \((g)\), and the gauge bosons of the weak force: \(W^-\), \(W^+\), and \(Z^0\). The gauge bosons—and therefore the basic forces—arise from internal symmetries in the laws describing particles called local gauge symmetries. An example of a symmetry is the laws of physics being the same regardless of the position of the observer, which is called translational invariance. Symmetries are very important in physics as they generate laws of physics (for example, translation invariance generates the law of conservation of momentum). Translational invariance is a symmetry of spacetime, so it is not an internal symmetry like the local gauge symmetries. The gluons of the strong force arise due to insisting on a symmetry that keeps the laws
of physics the same after the group SU(3) is applied uniquely to each spacetime coordinate (locally) rather than globally to every spacetime coordinate the same way. This SU(3) group essentially “rotates” the three color charges—red, green, and blue—to new values. The weak and electromagnetic forces arise from U(1) and SU(2) symmetries of an electroweak force, where the U(1) “rotates” the quantum phase of each particle, and the SU(2) “rotates” pairs of left-handed-chirality particles (such as $e_L$ and $\nu_e$) to new mixtures of them (chirality states are states of the spacetime symmetry describing boosts called the Lorentz symmetry).

To maintain these local gauge symmetries, particles cannot have mass, which can be illustrated most simply by considering the chirality of massive particles. Assuming that a spin-1/2 particle has a unique antiparticle, relativistic quantum mechanics requires that it be a combination of the left- and right-handed chirality states, as Eq. (1.1) also demonstrates. However, the SU(2) symmetry of the electroweak force only applies to the left-handed chirality, so the right-handed component of the massive particles cannot transform under SU(2) in a way that cancels the transformation of the left-handed component to keep the physics the same (in the case of Eq. (1.1), the expression must be kept the same for there to be a symmetry). That is, these massive particles violate the symmetry.

The existence of a spin-0 Higgs field allows the symmetry to be maintained when mass exists. A spin-1/2 particle’s mass depends on how strongly the Higgs field interacts with it, and the properties of the Higgs field are such that the left-handed transformation is canceled. When the universe was less than a microsecond old, our region of the universe fell into a particular nonzero configuration of the Higgs field in a process called electroweak symmetry breaking. Since then, the nonzero Higgs field is the source of the mass of the spin-1/2 particles in the SM. The final particle of the SM, the Higgs boson ($H^0$), is the component of the Higgs field that remains after other components of the field give the $W$ and $Z$ bosons their mass, which, along
with splitting the electroweak force into the weak and electric forces, occurred during
electroweak symmetry breaking. All the particles of the SM are shown in Fig. 1.1.

![Standard particles](image)

**Figure 1.1:** The SM describes the fundamental fermions (quarks and leptons) and
fundamental bosons (the gauge bosons that mediate the fundamental forces and the
Higgs boson) that comprise the universe (image credit: DESY at Hamburg).

Quantum field theory is the theoretical framework of particle physics (unlike the
simpler relativistic quantum mechanics, it is complete and self-consistent). All par-
ticle types are quantum fields where a single field provides all particles of that type.
For example, all left-handed quarks of color red are quantum waves in the same
left-handed-red-quark field. The SM is given by specifying a particular Lagrangian
density, $\mathcal{L}$. The fields, interactions, and symmetries of a particular quantum field
theory can be understood by writing $\mathcal{L}$, which is a mathematical expression that can
be thought of as the input to quantum field theory before quantum field theory can
make calculations.

Perhaps the simplest example of $\mathcal{L}$ is that of a universe with only an electron (and
positron, the antielectron) field and a photon field. Using the standard notation of
relativistic quantum mechanics, the Lagrangian density (in Heaviside-Lorentz units)
The first two terms are the kinetic terms of the electron, which provide the kinetic energy (as the presence of derivatives suggests), with the left-handed \((L)\) and right-handed \((R)\) parts written separately. The next term defines the electron’s mass via its coupling \(\lambda_e\) to the Higgs field’s vacuum expectation value \(v\). Finally, the photon and its interaction with the electron field are defined in the remaining terms, where \(A_\mu\) is the electromagnetic potential. \(\mathcal{L}\) is a real number describing the energy per volume due to fields and having those units. Each term obeys Lorentz and CPT symmetries, and the entire expression for this particular \(\mathcal{L}\) obeys a \(U(1)\) symmetry (simultaneously considering \(e \to \exp[-i q \Lambda(x)] e\) and \(A_\mu \to A_\mu + \partial_\mu \Lambda(x)\) for arbitrary phase \(\Lambda(x)\), where \(x\) is shorthand for all four spacetime coordinates, leaves the form of \(\mathcal{L}\) unchanged).

The units commonly used in particle physics are natural units, which is when \(c = \hbar = 1\), and the convenient unit of energy at the scale of particles is the electronvolt (eV). Setting fundamental constant to the unitless 1 makes life much easier for many reasons. Equations such as \(E_0 = mc^2\) can now be simply written \(E_0 = m\). That is, the energy of a particle at rest, \(E_0\), is equal to the mass of the particle, which is the conceptual idea of the equation, and the speed of light squared only acted as a bothersome conversion between units. With our new equation, \(E_0 = m\), we say that the mass of the proton is 0.938 GeV (a proton has mass approximately equal to 1 GeV, which acts as a good reference when interpreting masses). Even though this is a unit of energy, this mass is interpreted as the energy of the mass of the proton, which is the relevant quantity when discussing mass. Even if the mass were given in kg, any particle physics calculation would immediately convert this mass to an energy, so we
might as well just think of mass as energy. With \( c = \hbar = 1 \), times and lengths can be measured in eV\(^{-1}\), so frequencies can be measured in eV, which makes perfect sense because frequencies and energy are highly related ideas in quantum mechanics. Also, \( k_B = 1 \) is used to write temperature in units of energy (\( k_B T \) has units of energy).

### 1.2 What is Dark Matter?

Neutrinos have neither electric charge nor color charge. For this reason, they are destined to be “ghost” particles in that they pass through matter and planets as light passes through glass. However, the weak force exists. All matter particles in the SM interact via the weak force, including neutrinos, so a neutrino will eventually interact with one of the neutrons, protons, or electrons in the matter it is passing through, though it may take many light years of matter for this to happen due to the weakness of the force. Only because there are so many neutrinos—65 billion per cm\(^2\) per second on Earth—can we detect enough to be able to study the nature of neutrinos.

What if a particle with mass (unlike the photon or gluon) was like the neutrino with no electric or color charge but either did not interact with the weak force or was not passing through Earth with a high enough flux to be detected? If it never or very rarely interacts, perhaps it is as good as not existing at all if we also cannot create this particle in labs. This matter could correctly be called dark in that it does not create, absorb, or interact with the photons that comprise light, and it has not interacted with any instruments on Earth enough to be detected. This matter could be called dark also due to its unknown nature that is explained only by unknown physics beyond the SM. It is both experimentally and theoretically dark, and it is appropriately called **dark matter** (DM).

In spite of our present inability to detect it, we know that DM exists. In fact, from cosmological measurements that will be discussed later, we know that the mass
density of DM in the universe is 5 times more than the mass density of “normal” matter [2]. In this context, we prefer to call the normal matter found in our planet and bodies baryonic matter. A baryon is any bound state of three valance quarks, which includes the familiar proton \((uud)\) and neutron \((udd)\). Since atoms and molecules are entirely composed of protons, neutrons, and electrons, and because the proton (or neutron) is \(\approx 2000\) times more massive than the electron, we can refer to all non-dark matter as baryonic. Baryons belong to a type of particle known as hadrons, which are bound states of quarks called baryons or mesons, where mesons are bound states of valence quark and antiquark pairs. The only stable hadrons are the proton and neutron (though a free neutron outside a nucleus decays to protons, and protons may eventually decay depending on the physics beyond the SM).

Not only do we know the mass density of DM, which is unsettlingly large, we know—to a limited extent—where it is! (See the next section for how we know.) DM is concentrated in large spherical blobs called halos that exist in and around galaxies as shown in Fig. 1.2. For example, the Milky Way has 20 times more DM than baryonic matter [3], though the exact distribution of the DM in and around the Milky Way is still undecided. Smaller galaxies often have an even larger ratio of DM to baryonic matter.

The only reason we can learn anything about DM has to do with there being so much of it that it can interact with the weakest known force: gravity. The electric repulsion between two protons is \(10^{36}\) times stronger than the gravitational attraction. Gravity is another fundamental force, but it uniquely has evaded our efforts to fully understand it in the modern quantum framework, so the SM does not describe gravity. To consistently describe both gravity and quantum mechanics is a lofty goal of physics, and modern string theories may be the correct solution. Einstein’s description of gravity, general relativity, is sufficient at all times and places except near very dense
objects such as neutron stars, black holes, or the universe itself before $10^{-43}$ s after the Big Bang\textsuperscript{1}.

Gravity is also unique in its accumulative effect. The weak force and the strong force are nuclear forces because they do not extend to distances beyond the scale of the atomic nucleus. The gauge bosons of the weak force are 80 or 90 times more massive than a proton, corresponding to energies of 80.4 GeV or 91.2 GeV. These energies are usually too high to allow these gauge bosons to exist for very long due to the time-energy uncertainty relation of quantum mechanics, which limits their range. For example, the time scale that an exchanged $W$ exists is $\frac{1}{80.4 \text{ GeV}} \sim 10^{-26}$ s, which is similar but different to an non-virtual $W$’s lifetime (that is, a $W$ with mass of 80.4 GeV) which depends on its decay channels. The strong force’s gluon is, in some sense, too strong in that it interacts with other gluons, so the force gets “knotted up” within baryons (or mesons) preventing the gluons from leaving. The energy scale is called the QCD scale and is $\approx 200$ MeV, so the range of the strong force is

\textsuperscript{1}The very weak field (or very large distance) limit of general relativity has yet to tested.
\[ \frac{1}{200 \text{ MeV}} \sim 10^{-15} \text{ m}. \] While not confined to nuclear scales, electromagnetism typically has limited range. For example, only a weak magnetic field can be detected from the entire planet Earth. This is because the universe is electrically neutral, so, over the size of a planet, positive charges cancel the negative charges. Gravity, on the other hand, is thought to extend universally because the “charge” of gravity is mass (technically the stress-energy tensor of general relativity) and negative mass is not believed to exist, so no cancellation occurs. While a person is on Earth, gravity is therefore a dominant force due to the large mass inside the Earth.

The nature of gravity being only relevant when enough mass has accumulated takes the study of DM to the field of astrophysics where large galactic structure can be affected by DM, and it takes the study of DM to the field of cosmology where the universe itself is influenced gravitationally by the existence of DM.

DM interacts gravitationally, but the story of DM will be uninteresting if no other interactions exist. What we call dark matter may always be called dark matter unless there is another interaction, possibly similar to the weak force, that allows it to be created in a lab or to be detected. Only then can we measure specific properties of DM particles such as spin, mass, etc. Someday, these measurements may allow us to solve the mystery of the identity of DM.

### 1.3 Gravitational Evidence

Standard cosmology and astrophysics are founded on the assumption that general relativity is the correct description of gravity—even at intergalactic distances.

All evidence for DM is related to gravity in some way, so it is natural to question the validity of our understanding of gravity. Einstein himself knew that general relativity was not the final theory. There is historical precedence to question the laws of gravity instead of proclaiming the existence of new matter. Neptune was discovered
by using Newtonian gravity to analyze the orbit of Uranus. That is, Neptune was new
matter that explained disturbances to the orbit of Uranus. However, sometimes the
laws of gravity must be changed. Mercury’s orbit was also found to be slightly different
than what Newtonian gravity predicted. Many people found evidence for new planets,
but, in the end, we discovered that Newtonian gravity is incorrect and Mercury’s orbit
is well described by Einstein’s general relativity (which affects Mercury more than
the other planets due to the especially strong gravitational fields nearest to the Sun).

Modifications to general relativity are not mutually exclusive with DM. New de-
grees of freedom arise in these models that are essentially DM, and additional non-
gravitational DM particles are often needed to make theory consistent with observa-
tions. Modifications to general relativity may explain the present acceleration of the
universe’s expansion (dark energy), while DM may explain the rest of the gravita-
tional mysteries. Good scientists must entertain all self-consistent possibilities that
are consistent with all the data. Future evidence may solve all these mysteries if
we presently work to understand all possibilities as best we can. This dissertation
will not consider modifications of general relativity because standard DM is presently
more compelling.

1.3.1 Rotation Curves

Historically, the first accurate clues for the existence of DM were found within ast-
trophysics in the 1930’s. As stars orbit their galaxies or as galaxies move inside a
galaxy cluster, their orbital velocities are affected by the amount of mass inside their
orbit. Fritz Zwicky measured that the galaxies near the farthest edge of a cluster
were moving far too quickly given the matter observed in the cluster. Similarly, oth-
ers have measured that orbital velocities of outermost stars in a galaxy are not only
unexpectedly fast but increase as distance from a galaxy increases. For example, M33
seems to contain DM [4], as shown in Fig. 1.3.
A simple application of Newton’s universal law of gravitation shows that, assuming the majority of the galaxy’s mass is in the main visible bulge and disk, the stars even farther than the disk should orbit more slowly as distance from the galaxy’s center increases. This is analogous to our solar system where the outermost planets orbit the Sun’s mass at a slower speed, which makes sense because gravity is weaker farther from the Sun.

A solution to this mystery is simple: the majority of the galaxy’s mass is not in the main visible bulge and disk but instead invisible and existing around (and inside) the disk in a DM halo. An outermost star orbits faster than stars that are slightly closer because its larger orbit contains more of the invisible mass than a closer orbit. An analogy would be if our Asteroid Belt were to be somehow made to be many times the mass of the Sun, which would cause the outer planets to orbit much faster than before (perhaps now faster than the inner planets) due to the increase of gravity as they now orbit both the Sun and the very massive Asteroid Belt.

1.3.2 Gravitational Lensing

A more direct way to observe the gravitational effects of DM is via gravitational lensing, which is when light bends around a massive galaxy or cluster due to the
mass’s gravity. We can map the location of mass by having computers measure the distortion of the light from background stars as it passes through and around the mass.

![The Bullet Cluster](image)

**Figure 1.4:** The Bullet Cluster shows the result of two galaxies clusters colliding. The pink is X-rays from colliding gas, and the blue represents gravitational lensing of background stars rather than being emission from the cluster (image credit: NASA).

The analysis of the Bullet Cluster in Fig. 1.4 is startling [5]. As is known from the X-ray emission in the center, two clusters have collided to form the Bullet Cluster. Most of the baryonic matter in the clusters was in the form of gas, which readily collides unlike the diffuse stars, causing the gas to heat to hundreds of millions of kelvins and emit X-rays. However, when using gravitational lensing to ask the question of where the mass is, the mass is in two blobs in opposite directions from the center and not primarily in the center! The standard explanation is that the DM in the clusters did not significantly collide with itself or with baryonic matter and just kept going.

### 1.3.3 Modern Cosmological Measurements

General relativity’s description of spacetime shows that the space in our universe dynamically changes. In fact, due to the nature of observations of redshift vs. luminosity distance of distant galaxies, we confirm that space itself is growing between any two galaxies. That is, we live in an expanding universe. A basic introduction to cosmology can be found in [6].
13.8 billion years ago, the beginning of this expansion occurred. This event is
called the Big Bang, though there was no explosion of matter as the name might
suggest, rather space itself grows between objects. If the objects are sufficiently far,
there is much space to grow, and this effect can be presently observed. The light from
the farthest visible galaxies has been traveling for over 13 billion years to reach us,
and this light we receive has redshifted into infrared during the course of its lengthy
travel.

The beginning of our universe was very different from today. Measurements of
the primordial abundances of the isotopes of hydrogen, helium, and lithium, which
were created when the universe was just a few minutes old, reveal that the expansion
began from a universe that was far hotter than the cores of stars. We also observe
the cosmological principle to be true (perhaps due to inflation), which says that the
distribution of galaxies on the largest scales is isotropic and homogeneous (the same
everywhere and in all directions). That is, the entire universe was like the core of a
star. As it rapidly expanded, it cooled due to the wavelengths of the particles being
redshifted (stretched by expansion) to larger wavelengths of lower energy.

Figure 1.5: The energy budget of our universe is primarily dark energy, which is
followed by dark matter. Einstein’s $E_0 = m$ is used for matter particles (image credit:
Planck).
The standard model of cosmology is the ΛCDM model, which is a universe that contains dark energy ($\Lambda$) and a type of DM called cold dark matter (CDM). CDM is DM that has moved nonrelativistically for most of the history of the universe. Hot dark matter would affect the relativistic degrees of freedom in the early universe, the time of matter-radiation equality, and formation of structures such as galaxies, so it can be ruled out. Fitting the ΛCDM model parameters to various cosmological data sets gives Fig. 1.5 [2].

Cosmic Microwave Background

![PLANCK](image)

**Figure 1.6:** The surface of last scattering is the farthest we can presently see using electromagnetic radiation. The infrared and microwave light from this surface corresponds to a blackbody spectrum of 2.73 K, and the small deviations from this value in the figure are from photon-baryon acoustic oscillations 13.8 billion years ago. To obtain this image, the Doppler effect due to Earth’s motion has been removed, and, via a spectrum analysis, the Milky Way, which would be located at the vertically centered horizontal line, has been removed (image credit: Planck).

When the universe was younger than 370,000 years, the entire universe was a plasma composed of mostly hydrogen and helium—and vastly more photons since the universe’s baryon-to-photon ratio is $\approx 5 \times 10^{-11}$. As the universe cooled, the plasma became a gas at an event called recombination at 370,000 years (though

---

2Dark energy causes the expansion of the universe to accelerate and is very different from DM, which acts to slow the expansion of the universe.
combination would be a more accurate name since the universe had never become a
gas before). The free electrons and nuclei of a plasma are more opaque to photons
than neutral gas atoms, so, for the first time, photons could freely travel without
colliding with charged particles in a plasma. In all directions around the Earth is a
sphere of last scattering, which is the surface from where the photons are just now
reaching Earth after traveling 13.8 billion years. Due to the finite speed of light,
telescopes see the past when the light was emitted. When we view this light, called
the cosmic microwave background (CMB), we view the universe at the oldest time
possible using light (in this case, microwave light) shown in Fig. 1.6. The photons
began as visible light at 3000 K, which has redshifted by a factor of 1100 down to the
present 2.73 K.

![Figure 1.7](imagecredit: Planck)

**Figure 1.7:** From the CMB data collected by the Planck satellite in Fig. 1.6, an
analysis of which angular sizes have the strongest temperature fluctuations can be
done. The sizes and strengths of the peaks can be predicted by various cosmological
models and parameters, which can then be compared to the above data. The large-
size uncertainty on the left side of the figure is due the low sample size of large regions
in the sky (image credit: Planck).

The importance of the CMB to modern cosmology is vast and shows the presence
of DM. We literally see (using microwaves) the universe as it was 13.8 billion years
ago (and we can learn about the matter that affects the light on its travel towards
Earth. The surface of last scattering is a snapshot in time and a spherical slice in space of density oscillations of the plasma. These acoustic oscillations are very similar to oscillations of the surface of a swimming pool, and the CMB is then similar to a photograph of these oscillations. On average, parts of the plasma are \( \sim 10^{-5} \) times hotter or colder than the mean due to being more contracted (more dense) or expanded. By studying the strengths and sizes of these oscillations at different angular sizes in the sky (shown in Fig. 1.7), we infer the presence of a large amount of DM gravitationally affecting the baryonic matter. Unlike baryonic matter which resists the contractive force of gravity with outward forces from photon pressure from heat, DM does not interact with the photons, so it only contracts rather than having an oscillation of alternating contraction from gravity and expansions from pressure. From the analysis of the oscillations, we not only measure the amount of DM, but we learn that DM was a significant component of the universe even 13.8 billion years ago!

One of the parameters that can be measured in the CMB analysis is the universe’s baryon-to-photon ratio. The numbers of baryons and CMB photons (per comoving volume of our expanding universe that is large enough to make net transport across boundaries negligible) each individually stay essentially constant since the first seconds of the universe because anti-baryons are no longer being thermally created. Because the CMB photons are measured, the baryon density of the universe can be inferred, from which we know that the amount of matter needed to provide the gravity of DM is much larger than baryonic matter. The CMB result of this ratio is more precise than and is consistent with both present-day measurements and measurements from the universe when a few minutes old (the latter of these are done by studying the amounts of deuterium and isotopes of helium in the universe that were created at that time).
Most baryonic matter in the universe is interstellar and intergalactic dust and gas rather than being inside luminous stars. The most obvious solution to the rotation curves is to propose that dust and gas are the cause. However, the Bullet Cluster is not consistent with this picture, and modern cosmological measurements of the CMB are the nail in the coffin.

Baryonic matter, even dust and gas, is not completely invisible like DM is. It can thermally emit electromagnetic radiation and can absorb light (especially higher frequencies) from background stars. For these reasons, we can measure the amount of gas and dust and include them in our measurements of baryonic matter. Massive astrophysical compact halo objects (MACHOs) are an opposing explanation to DM (the clever acronym contrasts WIMPs, which are a standard DM candidate). These massive objects are black holes, neutron stars, brown dwarfs, planets, etc. Due to their compact nature, they are very hard to detect and would not readily collide in the Bullet Cluster. However, a fraction of them can be detected either by the small amount of light they emit or by them passing between another object and Earth, so we can infer their contribution to the total mass density of our galaxy (or at least an upper bound). MACHOs do exist, but they do not supply enough mass, especially considering modern cosmological measurements which imply the existence of non-baryonic matter.

**Structure Formation**

The regions where DM was more dense eventually seeded the structures—galaxies, galaxy clusters, superclusters, filaments, etc.—that are present today. The gravity of the DM pulls baryonic matter along with it as it contracts under gravity. Without DM, baryonic matter from its initial state known from the CMB cannot have gravitationally contracted to form these structures in the past 13.8 billion years. Instead, the universe would be more like the uniform gas from which it formed. Remarkably,
with the addition of CDM, cosmological computer simulations show that the present universe results from the initial conditions [7].

Neutrinos only interact with the weak force and gravity. They are nearly massless (their mass is known to be much less than 1 eV as discussed in the next chapter), so they typically move at nearly the speed of light (massless). For this reason, the SM neutrinos (with a neutrino-mass mechanism specified in addition to the SM) cannot be responsible for the effects of DM, so physics beyond the SM is necessary. The production of neutrinos in the early universe is understood, so neutrinos would be called hot dark matter, which means that they have enough energy to move too quickly to allow the smaller structures to form by “blurring” the distribution of matter. Hot dark matter has been ruled out in favor of CDM.3

1.4 Particle Physics Candidates

Many theoretical models have been constructed to explain the nature of DM. Some are top-down approaches that start with a big idea, such as supersymmetry (SUSY), and DM is a result. Some models are bottom-up approaches that simply add a few fields and interactions to the Lagrangian density of the SM in order to explain experimental observations. These approaches are complementary because only a few fields or interactions of the entire top-down model may be responsible for observed phenomena. The most interesting models are those that have consequences that experiments may one day observe (this is science after all, so a theory—even a correct theory—is useless unless it can be observed to be correct).

The simplest way of having the required CDM is for the dark-matter particles to be in a category known as weakly interacting massive particles (WIMPs), which are, as the name suggests, particles that interact with the weak force or a force similar

3Also, due to their small mass, the total mass density of neutrinos—in spite of their large number density—is negligible and cannot explain DM.
to it. Their mass ($m_{DM}$) is weak scale (~100 GeV) but can be a couple orders of magnitude larger or smaller and still be undergoing annihilations that can be detected if the interaction strength ($\alpha^2$) balances the change in mass (assuming $m_{DM}$ is the dominant mass scale, the cross section for annihilation is proportional to $\alpha^2/m_{DM}^2$). WIMPs arise in many extensions of the SM [8, 9]. However, other possibilities than WIMPs exist. \footnote{Axions could be CDM even though they would have even less mass than neutrinos [10]. Sterile neutrinos may also be CDM [11].} This dissertation will discuss the potential for detecting WIMPs.

### 1.4.1 WIMP Miracle

DM and anti DM (possibly DM is its own antiparticle) may be annihilating throughout the universe. For a DM particle moving through other DM particles, the interaction rate is $\Gamma = n\sigma v$, where $n$ is the DM number density and $v$ is the relative velocity. For the total interaction rate of all particles per volume, we get

$$\frac{d\Gamma_{\text{total}}}{dV} = n^2\sigma v. \quad (1.2)$$

DM annihilations may have a velocity-dependent cross section, $\sigma_{\text{ann}}$, so we usually consider the average

$$\frac{d\Gamma_{\text{ann}}}{dV} = n^2\langle\sigma_{\text{ann}}v\rangle. \quad (1.3)$$

If DM particles are not their own antiparticle, the average, $\langle\sigma_{\text{ann}}v\rangle$, would be reduced.

In the early universe, the temperature was hot enough to create particle-antiparticle pairs of all particle types, including WIMPs. When the universe expanded and cooled, baryonic antimatter was completely annihilated by baryonic matter. A tiny fraction of baryonic matter was left due to an asymmetry (from baryogenesis) causing there to be slightly more matter than antimatter in the early universe. WIMPs may be
asymmetric [12]. However, WIMPs may freeze out before the annihilation can finish. That is, the universe’s expansion and cooling eventually separates the WIMPs until annihilation rate is negligible, so WIMPs freeze out before they can completely annihilate. The current observed energy density would be a leftover relic from the thermal history of the universe.

Assuming negligible asymmetry, the DM before freeze out would have a thermal number density that depends on the temperature of the universe ($T$) and the DM mass:

$$n \sim (m_{DM}T)^{\frac{3}{2}} e^{-m_{DM}/T}. \quad (1.4)$$

Even though the universe was primarily composed of relativistic particles, the above equation approximates the full statistical mechanical treatment assuming a nonrelativistic DM population (DM becomes nonrelativistic before the rest of the universe due to its large mass and the universe containing many massless photons). We must eventually check the freeze-out temperature we get with $m_{DM}$ to be sure that DM is nonrelativistic.

Freeze out occurred because the average particle interaction rate $n\langle \sigma_{\text{ann}} v \rangle$ lowered until it became on the order of the Hubble parameter (rate at which a unit volume of the universe expands), so

$$n_t \langle \sigma v \rangle_t \sim \frac{T_f^2}{M_P} \quad (1.5)$$

is when freeze out (f) occurs ($M_P$ is the reduced Planck mass $\sim 10^{18}$ GeV). For a range of WIMP masses and assuming that $\langle \sigma_{\text{ann}} v \rangle$ is large enough to give a detectable signal in upcoming decades, $T_f \sim \frac{m_{DM}}{25}$ solves the equations due to its logarithmic dependence on variables.
There is a constraint on $\langle \sigma_{\text{ann}}v \rangle$: the current observed energy density ($\rho_0$) must result from the freeze out (assuming single-component non-asymmetric DM). To a rough approximation, we can assume that the entire history of the universe has been dominated by relativistic particles (even though it became matter dominated and is presently dominated by dark energy) so that $a = \frac{T_0}{T}$ is true, where $a$ is the scale factor of the universe ($a$ gives the size of the universe, where the present scale factor is $a_0 = 1$). After freeze out, the number density of DM decreases due to the universe’s expansion (rather than being thermally produced):

$$n = \frac{n_0}{a^3} = \frac{\rho_0}{m_{\text{DM}}} \frac{T^3}{T_0^3},$$

(1.6)

where a subscript of 0 means the present-day value and no subscript means at freeze out. With this constraint at the time of freeze out,

$$\langle \sigma_{\text{ann}}v \rangle_t \sim 3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1},$$

(1.7)

which is shown as a function of $m_{\text{DM}}$ in Fig. 1.13 by the dashed curves. This value is consistent with the weak-scale, which provides cosmological motivation for WIMPs, and this consistency is called by some to be a “WIMP miracle” that gives good reason to consider WIMPs as the DM candidate.

Depending on the particle physics model, $\langle \sigma_{\text{ann}}v \rangle$ in the present day Milky Way may be many orders of magnitude smaller than the above value due to it possibly having a velocity dependence.

### 1.4.2 One Explicit Example of WIMPs

SUSY is a good example of a framework to make particle physics models that contain WIMPs that are stable (do not decay) over cosmological timescales. Since the laws of physics are symmetric under various spacetime transformations (rotations, boosts,
Figure 1.8: Supersymmetry doubles the number of particles in the SM (plus a few more in the Higgs sector and more depending on the SUSY-breaking mechanism). The symmetry cannot be perfect, so the superpartners have a larger mass. The lightest stable superpartner may be the identity of DM (image credit: DESY at Hamburg).

and translations of the Poincaré group), a final spacetime symmetry, supersymmetry, may also exist. Under a complicated spacetime transformation that converts fundamental fermions into fundamental bosons (and vice versa), the laws of physics in a supersymmetric universe would be the same. This can help solve the hierarchy problem of the SM, can explain the apparent high-energy unification of the SM couplings, could provide a path towards quantum gravity (by insisting that the universe be symmetric under local SUSY transformations), and provides candidate particles for WIMPs. The symmetry of SUSY cannot be perfect because we do not see the superpartners of known particles (which would have the same mass, charge, etc. but have different spin), which are shown in Fig. 1.8. Instead, there would need to be a SUSY-breaking mechanism that would make these superpartners massive enough so that they have not been sufficiently created in particle accelerators but still be present from the early universe to account for DM. See [13] for a review of SUSY.
1.5 Possible Types of Detection

WIMPs seeded the Milky Way and should have since stabilized (virialized) into elliptical orbits of various orientations and eccentricities forming a spherical DM halo. The satellite galaxies around the Milky Way have similar halos. The focus of DM experiments across the world is to attempt to discover these WIMPs.

With negligible self-interactions of CDM particles (CDM particles scattering off of each other), computer simulations of DM in galaxies show that the NFW profile well describes the density profile, which seems to agree with observations especially for high-mass galaxies [14, 15, 3]. Fitting the two profile parameters for the Milky Way gives mass per volume

$$\rho = \left( 0.4 \frac{\text{GeV}}{\text{cm}^3} \right) \left( \frac{20 \text{ kpc}}{r} \right) \left( 1 + \frac{r}{20 \text{ kpc}} \right)^{-2} \text{ cm}^{-3}, \quad (1.8)$$

where $r$ is the spherical distance from the Galactic Center that has a finite cutoff. The NFW profile describes a Galactic Center with a high-density cusp making this location promising for indirect detection. Our solar system is at $r \approx 8 \text{ kpc}$ giving $\rho \approx 0.5 \text{ GeV/cm}^3$ locally, which, depending on $m_{\text{DM}}$, means that there are many DM particles in any human body at a snapshot in time.

Figure 1.9: Direct detection, indirect detection, and production all probe the same process.
To constrain and hopefully eventually discover the identity of WIMPs in these halos, there are three approaches (see Fig. 1.9):

- Direct detection. Earth is moving through the DM halo as the Solar System orbits the Galactic Center (and as the Earth orbits the Sun), and direct detection attempts to detect the DM particles (hopefully sometimes interacting with atomic nuclei in the DM detector), which would depend on the local DM density and velocity distributions. Using standard assumptions about these distributions, the upper bounds from direct detection are described in subsection 1.6.1.

- Indirect detection. If DM is annihilating in the universe, photons, cosmic rays, and neutrinos may result and find their way to Earth detectors, which would be called an indirect detection because the Earth only detects the byproducts. Eq. (1.8) gives $n$ (after dividing $m_{DM}$) for Eq. (1.3), where $n^2$ is the relevant quantity. Subsection 1.6.2 describe this approach in more detail.

- Collider production. The most precise and controlled approach would be to create DM—or any other (unstable) particles in the dark sector—in a particle accelerator such as the Large Hadron Collider. A DM particle would appear as missing energy, and any particles from the dark sector can affect lifetimes, decays products, magnetic moments, etc. of SM particles.

As Fig. 1.9 suggests, these three approaches all probe the same interaction.

### 1.6 Microscopic Parameters

If the Lagrangian density that describes DM can be fully specified, the mystery of DM will have been completely solved. However, there are many diverse Lagrangian densities that give similar observable behavior, and parameterizing this behavior is a helpful bridge between theory and the three approaches of experiment. Certain
microscopic parameters—mass and cross sections—can be directly probed by experiments and can be predicted by theory, so they act as this bridge. As is commonly done, this section will assume single-component DM (that is, a single type of DM particle) for simplicity.

The most obvious parameter is $m_{\text{DM}}$, the mass of each DM particle. Since the total average mass density of DM in the universe is known, a smaller $m_{\text{DM}}$ means a larger average number density. Collider production has the best prospects for a good measurement of $m_{\text{DM}}$.

Assuming that DM decay does not occur or is negligible, the rest of the parameters quantify the probability that DM will interact with itself or other particles. Cross sections ($\sigma$) can be thought to be the cross-sectional area that a particle sees another particle to be as it approaches (a large cross section implies a large probability of interacting in a certain way). Given two particles, there may be many types of cross sections that all add to a total cross section.

### 1.6.1 DM-Nucleon Scattering Cross Sections

The spin-independent cross section ($\sigma_{\text{SI}}$) and the spin-dependent cross section ($\sigma_{\text{SD}}$) measure DM’s interaction with baryonic matter, namely with nucleons. In the present-day universe, DM is nonrelativistic, so spin is a useful quantum number (rather than helicity). The spin-dependent cross section arises when the sign (plus or minus) of the interaction depends on the relative spins, and the spin-independent are all other interactions. Direct detection requires either one of these to be nonzero. Spin-0 DM would obviously only have $\sigma_{\text{SI}}$.

The DM interaction with an entire nucleus is coherent (occurs via a quantum superposition with all nucleons). This is due to the interaction length being larger than that of a typical $\sim 10^{-14}$-meter nucleus. For $m_{\text{DM}} \gg m_{\text{target}}$, the momentum transferred is approximately $2 m_{\text{target}} v_{\text{DM}}$. A nucleon is 1 GeV, and $v_{\text{DM}} \sim 0.001$, so
Figure 1.10: $\sigma_{SI}$ exclusion curves from various direct-detection experiments. To make this plot the neutron and proton are assumed to have the same $\sigma_{SI}$. LUX’s 2016 results [16] improve upon their previous results by a factor of 4 for most $m_{DM}$. The dashed line (and surrounding shaded error bars) represents the expected sensitivity, which can be less sensitive than the final limits if less background is detected than expected (image credit: LUX).

this momentum transfer corresponds to $\sim 10^{-13}$ m. For smaller $m_{DM}$, the momentum transfer is even less, so coherence is guaranteed. Due to the sign of the spin-dependent interaction being dependent on relative spin, the result depends on the total spin of the nucleus ($J$), and the spin-independent interaction gets the usual mass-squared enhancement

$$\sigma_{SD,\text{nucleus}} \propto J (J + 1)\sigma_{SD}$$

$$\sigma_{SI,\text{nucleus}} \propto m_{\text{nucleus}}^2 \sigma_{SI}.$$  \hspace{1cm} (1.9)

The proton and neutron can theoretically have different $\sigma_{SI}$ and $\sigma_{SD}$ if DM interacts differently with different flavors of quark. Figs. 1.10, 1.11, and 1.12 show the $m_{DM}$-dependent exclusion curves for $\sigma_{SI}$, neutron’s $\sigma_{SD}$, and proton’s $\sigma_{SD}$, respectively, from
Figure 1.11: Neutron’s $\sigma_{SD}$ exclusion curves from various direct-detection experiments. DAMA sees a potential DM signal in the region excluded by many other experiments. The curves labeled MSDM are collider limits assuming a specific model of DM (image credit: LUX’s paper [17]).

Figure 1.12: Proton’s $\sigma_{SD}$ exclusion curves from various direct-detection experiments. Dashed lines are for projected LZ (a future DM detector) sensitivity or, as will be discussed later, for annihilation-channel-dependent results of indirect detection from the Sun’s ability to capture DM via $\sigma_{SD}$ (image credit: LUX’s paper [17]).
a number of experiments. The parameter space above any of the curves is ruled out since a detection would have been made there (a more sensitive experiment produces a lower curve).

1.6.2 Annihilation Cross Section and Channels

![Graph showing annihilation cross sections for different DM masses and DM annihilation channels.]

**Figure 1.13:** Assuming branching fractions of 100% to various annihilation channels, Fermi presents these 95%-confidence exclusion curves using Milky Way dwarf spheroidal galaxies. The parameter space above the curves is ruled out (image credit: Fermi’s paper [18]).

Besides the overall annihilation rate given by $\langle \sigma_{ann} v \rangle$, annihilations would be governed by annihilation channels (perhaps directly to SM particles) and the branching fractions to those channels. These branching fractions add to 1 and give the probabilities that an annihilation channel will occur. Using the $\gamma$ signal, Fig. 1.13 shows the $m_{DM}$-dependent exclusion curves for various annihilation channels assuming branching fractions of 100% (gamma-ray data from the direction of nearby satellite galaxies are examined, and no DM signal has yet been found). Using the $\nu$ signal, Fig. 1.14
Figure 1.14: Top: assuming branching fractions of 100% to various annihilation channels, IceCube presents these 90%-confidence exclusion curves using the Galactic Center (the limits based on actual data did better than the predicted detector sensitivities). Bottom: for the $\tau\tau$ channel, IceCube presents a comparison between its Galactic Center analysis, its halo analysis, and the results of other experiments. The parameter space above the curves is ruled out (image credit: IceCube’s paper [19]).
shows exclusion curves for various annihilation channels (neutrino data from the direction of Galactic Center is examined, and no DM signal has yet been found). Larger \( m_{\text{DM}} \) have weaker limits because they annihilate less frequently due to them having a lower number density.

Different annihilation channels give very different signals. Prompt neutrinos (those from the \( \nu\bar{\nu} \) channel) have \( E = m_{\text{DM}} \), the hardest energy distribution. Neutrinos from other channels produce softer energy distribution (though sometimes more neutrinos at these lower energies than prompt neutrinos).

The non-virtual \( WW \) channel provides a neutrino distribution that is intermediate in hardness because, in the \( W \)’s rest frame, the neutrinos have \( E = \frac{m_W}{2} \) (which can be greatly boosted up to \( E = m_{\text{DM}} \) if \( m_{\text{DM}} \gg m_W \), but the majority are not). The \( W \) decays leptonically \( \frac{1}{3} \) of the time (giving neutrinos) and hadronically \( \frac{2}{3} \) of the time (because the top quark cannot be produced from the \( W \) mass and because each remaining quark channel is 3 times more probable than a leptonic channel due to there being 3 colors). Photons are produced when the \( \pi^0 \) mesons resulting from hadronization decay to two photons. Also, low-energy photons are produced from bremsstrahlung (radiation of accelerating electrically charged particles) during the annihilation itself or as stable charged particles travel through surrounding medium.

An example of a soft neutrino distribution that produces many neutrinos is the \( \tau\tau \) channel. In the \( \tau \)’s rest frame, the neutrinos have \( E \leq \frac{m_W}{2} \) due to three-body production of a \( \nu_{\tau} \) and the 2 products of the virtual \( W \), which can include more neutrinos. Fig. 1.15 shows an example of an annihilation, and Fig. 3.1 (in Chapter 3) compares the results of different annihilation channels at an Earth detector from inside the Sun.

Even softer energy distributions exist. For example, the \( b\bar{b} \) channel only produces neutrinos from certain decays of its hadronization products (which may be stopped
Figure 1.15: DM particles, $\chi$, annihilating via the $\tau\bar{\tau}$ channel. In this figure, one of the virtual $W$'s decays hadronically while the other decays leptonically. Time progresses to the right.

before decaying if inside the Sun). Other channels such as $e\bar{e}$ produce no neutrinos but can produce photons through bremsstrahlung.

The energy due to relative motion of DM particles is not relevant. In the Sun’s core, the temperature is 1 keV, which is negligible for DM with mass of even a few GeV. DM in the galaxy has $v \sim 0.001$, corresponding to an increase of energy of only $\frac{1}{\sqrt{1-v^2}} = 100.00005\%$.

1.7 Indirect Detection

The focus of this dissertation is indirect detection. Possible sources for indirect detection are the Galactic Center, entire Milky Way halo, Sun, Earth, nearby satellite galaxies, nearby galaxy clusters, the universe itself (\(\gamma\)-ray background studies), or nearby interstellar space (cosmic ray studies).

1.7.1 Detectable Annihilation Products

Cosmic rays, photons, and finally the neutrinos that will be the focus of this dissertation may result from WIMP annihilation and be detected, each with its own challenges:
• Cosmic rays have electric charge, so the Milky Way’s magnetic field prevents them from freely moving in straight lines, but information can be gained by the amounts and types of cosmic rays reaching Earth from nearby interstellar space.

• High-energy photons are readily detected by orbiting $\gamma$-ray telescopes such as Fermi, and even higher energy photons (above a few 100 GeV) are readily detectable via ground-based detectors such as HAWC, but the astrophysical sources of interest for DM studies are often congested giving a complicated and poorly-understood $\gamma$-ray background.

• High-energy neutrinos are therefore a potentially better signal as they can leave the production region and can penetrate our atmosphere for ground detection, but neutrinos are very hard to detect for the same reason that they are not blocked by the Earth or the atmosphere. The situation is made worse by cosmic rays hitting the atmosphere producing the primary background for the necessarily large (due to the difficulty in detecting neutrinos) ground-based neutrino detectors.

The motion of Earth is not relevant for indirect detection (unlike direct detection). If a detector’s velocity around the Milky Way, velocity around the Sun, and velocity due to Earth’s rotation were to be added in the direction away from a source that is at rest relative to the galaxy, $v \lesssim 300 \text{ km/s} = 0.001$, which can only redshift the energy of a photon or neutrino to $\sqrt{\frac{1-v}{1+v}} = 99.9\%$.

1.7.2 Role of Indirect Detection

Even with a future indirect detection, measuring DM parameters may be quite difficult. For a hypothetical example, a neutrino signal from the Sun is found (after many years of gathering data) to be 3 standard deviations above the background.
This detection does not provide $m_{\text{DM}}$ because the branching fractions are not known and because of poor energy resolution of neutrinos. If, however, the Large Hadron Collider could measure $m_{\text{DM}}$, information regarding the product of branching fraction and capture cross section may be learned from the Sun’s signal, but there would be no way to know if $\sigma_{\text{SD}}$ or $\sigma_{\text{SI}}$ was at work. Even with equilibrium in the Sun, local DM density, and branching fractions assumed, the measurement of the capture cross section would have error bars of 33% given a 3-standard-deviation detection. Assuming that no improvements on the detector occur, Eq. (B.6) reveals it would have to be run for 4 times longer to double the result to a 6-standard-deviation discovery with a 17% error on the capture cross section.

The necessary goal then must be for all methods of detection to work in tandem to piece together the DM puzzle, guided by the theoretical models that relate the results from different types of detection. The puzzle may never be complete, but science never is.

Even if a detection is never made, we can continue to learn about what DM is not. Allowed regions of the parameter space can be further constrained, which can rule out theoretical models. As Thomas A. Edison said regarding his many failed attempts to build a light bulb, knowledge is valuable regardless of whether it is in the form one wished it to be (such as him learning that a large number of designs do not create a working light bulb).

In the case of WIMPs, there are good reasons to expect their existence, but, unlike attempting to invent a light bulb, if humanity does not find them, we will eventually stop. However, even this—perhaps unlikely—outcome is no cause for sadness because the mystery of the unknown has value and because persistent efforts will have caused much about technology and the universe to be learned. We can always proudly say that we looked.
Chapter 2

Neutrinos

A billion neutrinos go swimming: one gets wet.

Michael Kamakana

2.1 Introduction

From 1930—when Wolfgang Pauli predicted the existence of the neutrino to explain otherwise-missing energy in beta decays—to the 1960’s, there was no reason to think that neutrinos had mass. In the 1960’s, the Homestake Experiment discovered a deficit in the predicted $\nu_e$ flux from the Sun giving birth to what would be called the solar neutrino problem [20].

In 1998, the Super-Kamiokande neutrino observatory observed that neutrinos produced in the atmosphere oscillate to different flavors [21]. Specifically, they observed $\nu_\mu \rightarrow \nu_\tau$. If the flavor states are a mixture of the mass states$^2$, the quantum mechanical wave function of the mass states are the eigenstates of the Hamiltonian, so

---

$^1$There was a lack of data, so the neutrino was still commonly believed to be massless, which is partly why the SM, which was completed in the 1970’s, contained strictly massless neutrinos.

$^2$In the quark sector of the SM, the CKM matrix mixes the weak-force states and the mass states, and very similar mixing occurs here in the lepton sector—except via the PMNS matrix, which is not in the SM—where the weak-force states are called the flavor states (in the quark sector, the mass states are typically called the flavor states).
oscillation results (in quantum mechanics, oscillation results when the state is a mixture of the eigenstates of the Hamiltonian). Neutrinos not only have mass, but the flavors are a mixture mass states causing them to oscillate.

With subsequent experiments (and with an understanding of the matter effect of the Sun’s matter on neutrino oscillations), the solar neutrino problem no longer exists. Some $\nu_e$ from the Sun are oscillating to $\nu_\mu$ and $\nu_\tau$ preventing them from being detected as $\nu_e$. However, the nature of neutrino mass is now a leading unsolved problem in particle physics.

2.2 Oscillations and Masses

2.2.1 Two-Neutrino-Flavor Oscillation

Understanding three-neutrino-flavor oscillations is made much easier by first understanding a simpler two-neutrino case, so discussing this simpler case is standard (as in [22]).

If the $\nu_\tau$ flavor did not exist, a $2 \times 2$ unitary mixing matrix could relate the mass states to the flavor states

$$
\begin{pmatrix}
|\nu_e\rangle \\
|\nu_\mu\rangle
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
|\nu_1\rangle \\
|\nu_2\rangle
\end{pmatrix},
$$

(2.1)

where $|\nu_i\rangle$ have mass $m_i$ for $i = 1 \text{ or } 2$. The flavor states are what are produced and detected but are not the mass states. A general $2 \times 2$ unitary matrix has 4 parameters, but 3 phases can be removed because they do not affect observable probabilities.
Plane waves of constant $E$ produced as a $|\nu_e\rangle$ then propagate to a detector according to

$$|\nu_e(t)\rangle = e^{-i(Et - \vec{p}_1 \cdot \vec{x})} \cos \theta |\nu_1\rangle + e^{-i(Et - \vec{p}_2 \cdot \vec{x})} \sin \theta |\nu_2\rangle$$

$$\approx e^{-iE_t + iE L} \left( e^{-i \frac{m_2^2 L}{2E}} \cos \theta |\nu_1\rangle + e^{-i \frac{m_2^2 L}{2E}} \sin \theta |\nu_2\rangle \right),$$

where the overall phase $e^{-iE_t + iE L}$ can be ignored and where $L$ is the propagation length to the detector. To arrive at the approximation, the ultrarelativistic approximation for the dispersion relation, $p \approx E - \frac{m^2}{2E}$, was used. A subtlety is that the wave is produced and detected as a wave packet ($t$ and $L$ are not constants), which requires a more rigorous treatment that is beyond our purposes here [23].

The probability to measure $|\nu_e(t)\rangle$ as the other flavor is

$$P(\nu_e \rightarrow \nu_\mu) = |\langle \nu_\mu | \nu_e(t) \rangle|^2 = \sin^2(2\theta) \sin^2 \left( \frac{\Delta m^2_{21} L}{4E} \right)$$

where $\Delta m^2_{ji} \equiv m_j^2 - m_i^2$. As expected, this probability goes to zero as the mixing ($\theta$) or the mass difference ($\Delta m^2_{21}$) goes to zero. The oscillation length ($L_{osc}$) for $\sin^2$ is defined via $\sin^2(\pi L / L_{osc})$ giving

$$L_{osc} = \frac{4\pi E}{\Delta m^2_{21}} = 2.48 \text{ km} \cdot \frac{E}{\text{GeV}} \cdot \frac{eV^2}{\Delta m^2_{21}}$$

A few important conclusions can be made from Eq. (2.3). Firstly, the neutrino masses themselves cannot be known directly from oscillations since only the difference of squares appears in $\Delta m^2_{21}$. Secondly, the hierarchy of the masses (that is, which one is larger) cannot be known due to the symmetry of Eq. (2.3). However, neutrinos traveling in matter experience matter effects that are dependent on the hierarchy (matter effects will be discussed in subsection 2.2.3).
2.2.2 Three-Neutrino-Flavor Oscillation and Masses

The flavor states of neutrinos can be related to the mass states by the $3 \times 3$ Pontecorvo-Maki-Nakagawa-Sakata (PMNS) unitary mixing matrix

\[
\begin{pmatrix}
|\nu_e\rangle \\
|\nu_\mu\rangle \\
|\nu_\tau\rangle
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
|\nu_1\rangle \\
|\nu_2\rangle \\
|\nu_3\rangle
\end{pmatrix},
\]

where $|\nu_i\rangle$ have mass $m_i$ for $i = 1, 2, 3$.

The PMNS matrix is usually parametrized in terms of the three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) and a CP phase ($\delta$), and two Majorana CP phases ($\alpha_1, \alpha_2$) can also be included \[24]\textsuperscript{3}:

\[
U =
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \times \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1),
\]

where $c_{12} \equiv \cos \theta_{12}$, $s_{12} \equiv \sin \theta_{12}$, etc. A general $3 \times 3$ unitary matrix has 9 parameters, but, because overall phases do not affect observed probabilities, this number can be reduced to 4 or 6 for Dirac or Majorana neutrinos respectively (Dirac and Majorana neutrinos are discussed in Sec. 2.3).

\[3\text{Antineutrinos are mixed by the complex conjugate of the PMNS matrix, and any sterile neutrinos would increase the size of this matrix, but the mixings to these sterile neutrinos must be small enough to have avoided clear detection.}\]
The latest global analysis of the 3-neutrino oscillation data \cite{25} gives

\[ \Delta m_{21}^2 = 7.60 \times 10^{-5} \text{ eV}^2, \quad \theta_{12} = 34.6^\circ, \]
\[ \Delta m_{31}^2 = 2.48 \times 10^{-3} \text{ eV}^2 \text{ (NH)}, \quad \Delta m_{31}^2 = -2.38 \times 10^{-3} \text{ eV}^2 \text{ (IH)}, \]
\[ \delta = 1.41\pi \text{ (NH)}, \quad \delta = 1.48\pi \text{ (IH)}, \]
\[ \theta_{23} = 48.9^\circ \text{ (NH)}, \quad \theta_{23} = 49.2^\circ \text{ (IH)}, \]
\[ \theta_{13} = 8.6^\circ \text{ (NH)}, \quad \theta_{13} = 8.7^\circ \text{ (IH)}, \]

where NH and IH refer to the two possible mass hierarchies.

\[ \text{Figure 2.1: Two neutrino mass hierarchies are consistent with current data. The colors depict the PMNS mixing matrix (image credit: Snowmass 2013).} \]

Only the difference of masses squared—\( \Delta m_{21}^2 \) and \( |\Delta m_{31}^2| \approx |\Delta m_{32}^2| \)—are measured\(^4\). This leaves two possibilities, a “normal” hierarchy (NH), where \( m_1 < m_2 < m_3 \) or an inverted hierarchy (IH), where \( m_3 \) becomes the smallest mass, both shown in Fig. 2.1.

\(^4\)The sign of the solar mass difference is known from the matter effects of neutrinos leaving the Sun.
The two different mass differences each approximate the two-neutrino-flavor case due to their different scales. $\Delta m^2_{21}$ and $\theta_{12}$ are primarily responsible for the oscillations detected from the Sun, so they are often called $\Delta m^2_{\text{sol}}$ and $\theta_{\text{sol}}$ (solar). $|\Delta m^2_{31}| \approx |\Delta m^2_{32}|$ and $\theta_{23}$ are primarily responsible for oscillations detected from the atmosphere, so they are often called $\Delta m^2_{\text{atm}}$ and $\theta_{\text{atm}}$. Given a neutrino energy, the wavelength of the solar oscillations are longer than the atmospheric oscillations according to the ratio of the $\Delta m^2$ values. As for $\theta_{13}$ and $\delta$, they prevent the complete picture from being as simple as two separate oscillation types.

**Figure 2.2:** On the left (right) is the probability for a $\nu_e$ ($\nu_\mu$) to oscillate to any of the 3 flavors traveling in vacuum as a function of distance traveled.

**Figure 2.3:** This figure is the same as Fig. 2.2 except $\theta_{13} = 0$. The atmospheric (shorter) oscillations and solar (longer) oscillations are now easily seen. $|\nu_e(t)\rangle$ undergoes solar oscillations to $|\nu_\mu\rangle$ and $|\nu_\tau\rangle$, and $|\nu_\mu(t)\rangle$ and $|\nu_\tau(t)\rangle$ undergo atmospheric oscillations between $|\nu_\mu\rangle$ and $|\nu_\tau\rangle$ as well as solar oscillations to $|\nu_e\rangle$. 
The probability of a neutrino flavor oscillating to another flavor can be calculated using the same method as the two-neutrino-flavor case\(^5\). Fig. 2.2 shows the oscillations resulting from the data in Eq. (2.7). Fig. 2.3 shows the simpler oscillations resulting when \(\theta_{13} = 0\), and atmospheric oscillations between \(\nu_\mu\) and \(\nu_\tau\) and the solar oscillations between \(\nu_e\) and \(\nu_\mu, \nu_\tau\) are distinctly seen.

Because all experiments have failed to measure the neutrino masses, we instead only have upper bounds on the neutrino masses. The lightest mass state may be massless, in which case the heaviest neutrino would be 0.050 eV. Direct laboratory bounds on the individual masses come from studying tritium beta decays, giving an upper bound of 2.0 eV with 95% confidence [24]. When constructed, KATRIN—a future experiment which will make precision measurements of the electrons resulting from tritium beta decay—is expected to do much better, and KATRIN’s website says it will place an upper bound of 0.2 eV with 90% confidence assuming that the lightest neutrino is massless. The tightest upper bounds come from fits of cosmological data. Assuming standard cosmology, Planck reports an upper limit of \(\sum_i m_i = 0.23 \text{ eV}\) with 95% confidence after combining various cosmological data sets [2]. The tightest possible upper bound on the sum is 0.059 eV for NH and 0.098 eV for IH, so IH may one day be ruled out from a tight upper bound. If standard cosmology is correct, \(m_i \approx 0.08 \text{ eV}\) for all masses is the most mass allowed. This case is called degenerate because all the masses are very similar, especially for NH.

If neutrinos are Majorana (a possible type of neutrino discussed in Sec. 2.3), then neutrinoless double-beta decay searches can put a comparable limit that depends on unknown Majorana phases via an effective Majorana mass (\(|m_{\beta\beta}|\)) [27], as shown in Fig. 2.4. Double-beta decay rarely occurs, but has been detected. Neutrinoless double-beta decay has yet to be detected since it is suppressed due to the rate being proportional to the ratio \(|m_{\beta\beta}|^2/m_e^2\), where \(m_e\) is the mass of the electron.

---

\(^5\)One must be careful to not approximate \(\Delta m_{32}^2\) as \(\Delta m_{31}^2\), else odd things like negative probabilities occur.
CHAPTER 2. NEUTRINOS

Figure 2.4: To complete the discussion of measurements of neutrino mass, upper bounds on neutrino mass also result if assuming that neutrinos are Majorana. NS and IS mean NH and IH respectively. If NH is correct, IH may be ruled out using neutrinoless double-beta decay if the upper bound becomes lower in years to come (image credit: arXiv review paper by Bilenky and Giunti [26]).

2.2.3 Matter Effects

The matter effect, which is sometimes called the Mikheyev-Smirnov-Wolfenstein (MSW) effect, occurs when $\nu$ or $\bar{\nu}$ pass through ordinary matter where electrons outnumber positrons, muons, and taus causing $\nu_e$ to interact with matter more than the other flavors. This affects oscillations because the eigenstates of the Hamiltonian are no longer the mass states since the electrons act on the flavor states (specifically, on $\nu_e$).

Even though neutrinos interact very rarely, the lowest energy (forward) scatters are long-range enough to cause coherent interactions with the bulk matter, which greatly amplifies these otherwise rare interactions (analogous to visible photons being slowed by the bulk interaction of glass).
Neutral-current (NC) scattering is that which occurs via the $Z^0$. It occurs equally for all neutrino flavors (adding a multiple of the identity matrix to the Hamiltonian, which does not effect its new eigenstates), so oscillation is not affected. However, the charged-current (CC) scattering that occurs via the $W^\pm$ occurs for $\nu_e$ or $\bar{\nu}_e$ off electrons. Considering when this interaction is with the bulk of the material (that is, not hard scattering but a coherent forward scattering), ignoring constants, the Hamiltonian in the flavor basis is

$$H = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} \frac{1}{2E_\nu} & 0 \\ 0 & 0 & \Delta m^2_{31} \frac{1}{2E_\nu} \end{pmatrix} U^\dagger \pm \begin{pmatrix} \sqrt{2}G_F N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(2.8)

where the minus sign is for $\bar{\nu}$ oscillation (only in matter does $\bar{\nu}$ oscillate differently than $\nu$), and $N_e$ is the electron density. The first term originates due to a calculation similar to Eq. (2.2), and the second term can be found in Table 3.1 of [22]. Diagonalizing this Hamiltonian gives the new effective $\Delta m^2$ values (from the eigenvalues) and effective mixing angles (by comparing the parametrization in Eq. (2.6) with the resulting change of basis matrix).

As can be seen from Eq. (2.8), a large enough $N_e$ (or large $E_\nu$) causes the left term to be insignificant compared to $\sqrt{2}G_F N_e$, so $\nu_e$ becomes on eigenstate of the Hamiltonian. The remaining eigenstates still mix $\nu_\mu$ and $\nu_\tau$ flavors, which oscillate according to two-neutrino-flavor oscillation.

Propagation calculations can now be made. A neutrino that is produced in or suddenly enters matter should be expressed in terms of the eigenstates, which can then be propagated. For a changing matter density, this process can be quickly repeated by a computer by making a discretized volume (having many small regions of constant density). If the effective mixing angles change over characteristic distances that are much longer than the oscillation wavelength in matter, the calculation simplifies as the
eigenstates adiabatically become the new eigenstates. One should be careful because the effective mass states can rapidly change (called resonances; see section 3.3 of [22]), which can occur for antineutrinos if IH or neutrinos if NH.

2.3 Nature of Neutrino Mass

Since neutrinos only interact via the weak force, which involves only left-handed particles, the SM, which describes the elementary forces, does not have right-handed neutrinos ($\nu_R$). $\nu_R$ could not interact or be produced.

The SM Lagrangian density ($\mathcal{L}_{SM}$) contains massless neutrinos. All SM fundamental fermions except the neutrino have right-handed components, which are needed for them to acquire mass, so $\nu_R$ may exist, but neutrinos have other possibilities for acquiring mass.

When adding terms to $\mathcal{L}_{SM}$, one may begin by considering all possible terms that might describe the observed particles that obey Lorentz and CPT symmetries. Then, other symmetries (such as local gauge symmetries) are imposed to reduce the number of possible terms.

When adding terms to $\mathcal{L}_{SM}$ that can produce mass for neutrinos, the usual mass term for fermions (a Dirac mass term) can be added

$$\mathcal{L} \supset \lambda_{\nu} \Phi^0 \bar{\nu}_R \nu_L + \text{h.c.}, \quad (2.9)$$

which requires the existence of new fields, a $\nu_R$ for each of the $\nu_i$ (the above term would appear 3 times). $\Phi^0$ is the remaining degree of freedom of the Higgs field after electroweak symmetry breaking (its vacuum expectation value, $v$, would be responsible for the mass), and $\lambda_{\nu}$ is the (unitless) coupling between the Higgs field and the neutrino.
Because $\nu_R$ has no color or electroweak charges that must be conserved\(^6\), another mass term (a Majorana mass term) can be written

$$\mathcal{L} \supset \frac{M}{2} \nu_R^\dagger \sigma_2 \nu_R + \text{h.c.}, \quad (2.10)$$

which is obtained from Eq. (2.9) by doing $\nu_L \rightarrow \sigma_2 \nu_R^\ast$ since $\sigma_2 \nu_R^\ast$ transforms like a left-handed field (see Sec. 1.4 of [28]). $M$ is a mass matrix that would change the mass states from what they would have been if only Eq. (2.9) existed. This term cannot exist without Eq. (2.9) because then $\nu_R$ would have no interactions with any other particle and could not be produced.

Since $\sigma_2 \nu_L^\ast$ transforms like a right-handed field, we can also include the following term

$$\mathcal{L} \supset \Delta^0 \nu_L^\dagger \sigma_2 \nu_L + \text{h.c.} \quad (2.11)$$

at the price of creating a new object $\Delta^0$ to keep the term symmetric under global gauge transformations (that is, to make the term have 0 charge). This term is not possible for other SM particles because it would violate the electromagnetic U(1) symmetry (and a charged $\Delta^0$ would give mass to the photon via its vacuum expectation value).

$\nu_L$ has electroweak charges: the third component of weak isospin is $I_3 = 1/2$, and weak hypercharge is $Y = 2(Q - I_3) = -1$ (where $Q$ is electric charge, which is 0 here).

Since $\nu_L$ appears twice with no complex conjugate (which effectively changes the sign of the charge), $\Delta^0$ must cancel both of them, so it has $I_3 = -1$ and $Y = 2$.

Putting this all together, there are several possibilities, of which the most popular are:

- Pure Dirac neutrinos. If lepton number conservation is a symmetry enforced on $\mathcal{L}$ (the conservation can still be violated via a chiral anomaly in the early

---

\(^6\)However, $\nu_R$ does have lepton number, which would not be conserved by this term.
universe), Eq. (2.9) is the only mass term added, and the neutrino is just like every other SM fundamental fermion: Dirac. Like all Dirac fermions, these neutrinos are mixtures of $\nu_L$ and $\nu_R$, though the $\nu_R$ component of a relativistic neutrino is tiny. This $\nu_R$ component is “along for the ride” (often called sterile) because its only interaction with other fields is through Eq. (2.9).

- Pure Majorana neutrinos. If no new fields are added, Eq. (2.11) is the only mass term added, the neutrino is its own antiparticle, which is called a Majorana neutrino (perhaps motivated by not needing to create a $\nu_R$ field). More specifically, the right-handed helicity state of the neutrino acts as an antineutrino, which is also true of a relativistic Dirac antineutrino. Therefore, there are few potentially observable differences between Dirac and Majorana other than Majorana neutrinos being able to produce neutrinoless double-beta decay. If Eq. (2.11) is a non-renormalizable low-energy effective theory, $\Delta^0 = \Phi^0 \Phi^0 / \Lambda$ is possible, where $\Lambda$ is the energy scale of new high-energy physics, which is needed to give $\mathcal{L}$ the correct units (see section 2.2 of [22]). If $\Delta^0$ is instead part of a fundamental triplet as $I_3 = -1$ suggests, this is called type-II seesaw [29, 30, 31, 32, 33] and is renormalizable at the cost of creating a new triplet field.

- Effectively Majorana neutrinos. If Eqs. (2.9) and (2.10) both exist, this is called type-I seesaw [34, 35, 36]. If $M \ll \lambda_* v$, where $v$ is the vacuum expectation value of $\Phi^0$, this is uninteresting, and we effectively get only Eq. (2.9), so the neutrinos are Dirac with masses given by $\lambda_* v$. The whole point of this model is to have $M \gg \lambda_* v$. In this case, all 6 neutrino fields ($\nu_L$ and $\nu_R$, each in three flavors) are effectively Majorana. As expected, the three effectively-right-handed (sterile) neutrinos have large mass given by $M$ and are Majorana. The interesting result is that the three lightest neutrinos are also effectively left-handed Majorana.

---

7Instead of there being 2 neutrinos produced in the double-beta decay, a single neutrino is produced from one beta decay and absorbed as an antineutrino in the other, violating lepton number conservation as Majorana neutrinos may do.
neutrinos with masses given by \( \frac{(\lambda_\nu v)^2}{M} \), ignoring the matrix nature of \( \lambda_\nu \) and \( M \) (see subsection 2.3.1 of [22]). Type-I seesaw is attractive because \( \lambda_\nu \) no longer has to be very small to explain the very small left-handed neutrino masses (since a large \( M \) and squaring \( \lambda_\nu v \) help drive down the mass).

If these possibilities seem contrived or unnatural, consider that neutrinos seem contrived and unnatural due to their masses being many orders of magnitude smaller than other fundamental fermions and due to the PMNS matrix not being approximately diagonal like the CKM matrix.

\section*{2.4 Indirect Detection of WIMPs via Neutrinos}

WIMP annihilation in the Sun or galaxy may produce detectable neutrinos. To detect these neutrinos, neutrino scattering and absorption as well as the neutrino background must be understood. Also, to predict the neutrino signal, one must often average over many neutrinos.

\subsection*{2.4.1 Cross Sections for Neutrino-Matter Interaction}

Interaction of neutrinos with matter can affect their absorption and detection, so these interactions are important for understanding predicted indirect detection signals.

To begin with something more familiar, the electromagnetic force that governs the interaction of a photon of energy \( E \) with charged matter (or an electron’s interaction with charged matter) has a cross section of

\[ \sigma \sim \frac{\alpha_{\text{EM}}^2}{m_{\text{target}} E}, \]  

(2.12)
since the $E^2$ in the denominator of the center-of-mass-frame formula takes the form $E_1 E_2$ upon boosting to the lab frame. $\alpha_{\text{EM}} \approx \frac{1}{137}$ is the strength of the electromagnetic interaction. Given a cross section, the mean free path is $\frac{1}{n \sigma}$.

A neutrino traveling through dust or the Sun can interact before reaching the detector. Neutrinos, unlike photons, often escape. At a detector, the same cross sections that were responsible for absorption now provide detection. High-energy neutrinos above 1 GeV but below 1 TeV, which also happens to be the useful energy range of neutrinos that may be indirectly detected, have a cross section of

$$\sigma \sim G_F^2 m_{\text{target}} E_\nu$$

(2.13)

which differs from Eq. (2.12) due to the absence of a $E^{-4}$ contribution from a photon propagator since the $W$ and $Z$ propagators have mass and instead contribute the $M^{-4}$ (where $M$ is the mass of the $W$ or $Z$) to $G_F^2$, where $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$ gives the strength of the weak force. When interacting with the nucleus, the collisions with the quarks in the nucleons are inelastic causing the entire nucleon to respond, so $m_{\text{nucleon}}$ being the relevant mass is believable (Sec. 4.2.3 of [22] has a more detailed discussion). Clearly, high-energy neutrinos interact more with nucleons than electrons, and neutrinos with higher energy interact more than those with lower energy. See Sec. A.3 for how to calculate neutrino interactions in much more detail, in which Eq. (A.1) may be most interesting.

Inside the Sun or Earth before reaching the detector, neutrinos can interact with matter and still survive. A NC interaction does not absorb the neutrino but instead scatters it to a lower energy that depends on the angle of scattering. A CC interaction typically absorbs the neutrino by transforming it into a charged lepton, but, if the charged lepton is a $\tau$, the $\tau$ will subsequently decay creating one or more neutrinos essentially regenerating the initial neutrino at lower energies.
Neutrinos with energy above 1 TeV have a cross section less than Eq. (2.13) due to the $W$ or $Z$ propagator contributing more than a simple $M^{-4}$ to the cross section. This occurs when the center of mass energy squared is larger than $M_{Z,W}^2$.

### 2.4.2 Neutrino Interactions Inside a Neutrino Detector

A high-energy neutrino can strike the nucleons in a detector with cross section in Eq. (2.13). The result of this collision is independent of neutrino flavor for NC interactions, where the $Z$ boson gives energy to a nucleon producing hadronization that creates a shower in the material sending photons in all directions, which are detected by PMTs. For a CC interaction, a charged lepton is also produced whose type depends on the flavor of the neutrino.

- A $\nu_e$ can produce an electron that is stopped immediately due to bremsstrahlung.

- A $\nu_\mu$ can produce a $\mu$ (and a $\bar{\nu}_\mu$ can produce a $\bar{\mu}$) that uniquely leaves a long distinguishable track of Cherenkov radiation, which it can do because its mass is $\approx$200 times larger than the electron mass suppressing bremsstrahlung, allowing for angular resolution of the source of the neutrinos (all NC interactions and all other CC interactions produce spherical signals rather than tracks). See right panel of Fig. 2.6 for a muon track resulting from a CC $\nu_\mu$ interaction in ice. Ignoring bremsstrahlung at high muon energies, a useful rule of thumb is that muons and antimuons travel 5 m in ice per GeV of energy (see Sec. B.1 for more information on the passage of muons through a detector).

- A $\nu_\tau$ can produce a $\tau$ that decays very quickly. At extremely high energies, $\gtrsim$1 PeV, $\nu_\tau$ can produce a “track” made of two separated energy deposits from when the $\tau$ is produced and when the $\tau$ decays, but these neutrino energies are too high for WIMP annihilations to produce.
2.4.3 Neutrino Backgrounds

There are several sources of neutrino background (as shown in Fig. 2.5):

- ∼MeV-scale neutrinos. These are created in the Sun’s core from the nuclear processes that power the Sun, are produced from the nuclear reactions and from $e^-e^+$ annihilation inside supernovae and eventually escape the high-density and high-temperature plasma, or result from beta decay inside the Earth. At these relatively low energies, angular resolution is usually not possible, so a detector cannot simply look away. However, WIMPs belong to an entirely different energy scale ($\gtrsim$GeV), so these background neutrinos can easily be ignored by detectors.

- $\gtrsim$10-TeV astrophysical neutrinos. These are produced from active galactic nuclei, gamma-ray bursts, and interactions of ultra-high-energy cosmic rays with
Figure 2.6: Left: air showers from cosmic rays produce $\mu$, $\nu_\mu$, and $\nu_e$ among other particles (image credit: CERN). Right: this up-going muon is either from a $\nu_\mu$ produced in the atmosphere on the other side of the Earth or from a $\nu_\mu$ produced in DM annihilation, but it cannot be an atmospheric $\mu$ (image credit: IceCube).

the cosmic microwave background (active galactic nuclei produce neutrinos that are primarily $\sim$GeV, but the fluxes at these energies are negligible compared to atmospheric neutrinos). The fluxes of these neutrinos are very low (years of detection are needed by detectors such as the IceCube Neutrino Observatory to detect a significant number). Due to their very high energies, their rare detections are dramatic and easily distinguished from possible WIMP annihilations at lower energies.

- $\gtrsim$GeV atmospheric neutrinos. Having the same energies as the energies expected from WIMP annihilations, these neutrinos are the relevant background. The flux decreases at higher $E_\nu$, which would act to help the detection of larger
\( m_{\text{DM}} \) annihilation. Cosmic rays continuously strike nuclei in the upper atmosphere producing air showers of many particles (see left panel of Fig. 2.6). Many pions are produced. The charged pions predominantly decay via \( \pi^- \rightarrow \mu \nu_\mu \), and, except at very high energies when the \( \mu \) decay after being stopped by the Earth, the muons decay via \( \mu \rightarrow \nu_\mu e\nu_e \). In addition to the neutrinos, the \( \mu \) themselves can interact with the detector, so detectors are often a couple km underground to reduce this problem. If a detector is collecting muon track data (from the CC \( \nu_\mu \rightarrow \mu \) interaction in the detector), the \( \mu \) from the pion decay can be the dominant background for sources above the horizon (for example, the Galactic Center is always above IceCube’s horizon). Ideally, the source is below the horizon allowing the down-doing \( \mu \) to be ignored (as shown in right panel of Fig. 2.6).

### 2.4.4 Averaging the Signal over Many Neutrinos

When detecting many neutrinos from a distant or spatially extended source, oscillations are not relevant in that the flavors at the detector only depend on the PMNS matrix and the initial neutrino flavors (\( \Delta m^2 \) is no longer a relevant parameter). The average state of these neutrinos at a detector can easily be calculated via

\[
\langle P(\nu_\alpha \rightarrow \nu_\beta) \rangle = \sum_{i=1}^{3}|U_{\alpha i}|^2|U_{\beta i}|^2 = (XX^T)_{\alpha\beta}
\]

where

\[
X \equiv \begin{pmatrix}
|U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\
|U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2 \\
|U_{\tau1}|^2 & |U_{\tau2}|^2 & |U_{\tau3}|^2
\end{pmatrix}.
\]
to get the following probabilities:

\[
\text{Normal Hierarchy : } \langle P \rangle = \begin{pmatrix}
0.5380 & 0.1979 & 0.1759 \\
0.1979 & 0.4240 & 0.3779 \\
0.2640 & 0.3779 & 0.3579 \\
\end{pmatrix}
\]

\[
\text{Inverted Hierarchy : } \langle P \rangle = \begin{pmatrix}
0.5377 & 0.2009 & 0.2613 \\
0.2009 & 0.4223 & 0.3766 \\
0.2613 & 0.3766 & 0.3620 \\
\end{pmatrix}
\]

If \( L_{\text{osc}} \ll \Delta L \), where \( \Delta L \) is the size of the source, neutrino oscillations can be calculated according to Eq. (2.14). The Galactic Center certainly meets this condition. This condition can also be met if the detector is moving relative to the source. For example, if detecting over many months from the Sun, the Earth’s variable distance from the Sun causes \( \Delta L \) to include the difference between Earth’s minimum and maximum distance. In this case, neutrinos less than 200 GeV have wavelengths that may be short enough to use Eq. (2.14), though the safest procedure is to consider all distances each weighted by the time the Earth spends at that distance.

The mass states of neutrinos from WIMP annihilation in the Galactic Center arrive as spatially separated wave packets, so Eq. (2.14) can be used, but neutrinos from the Sun do not meet this criteria. WIMPs in the Sun have kinetic energy \( T \sim 10^{-6} \) GeV. WIMPs in the Galactic Center have kinetic energy \( T \sim 10^{-6} m_{\text{DM}} \). When these WIMPs annihilate, the resulting quantum-mechanical wave packet has length \( \sim \frac{1}{T} \). The wave packets of different mass states separate due to having different velocities. Eventually, the wave packets separate completely, which, assuming that the interaction in the detector has good enough spatial resolution, prevents the interference between the mass states that causes oscillation. Ignoring spreading of the wave packets of individual mass states, a quick calculation reveals that this occurs when
$\frac{1}{7} \ll \frac{\Delta m^2}{E^2_\nu} D$, where $D$ is the distance the neutrino has traveled. This condition is satisfied for the Galactic Center but not for the Sun.

A final remark is that the energy resolution when detecting neutrinos is not perfect, so, if $D \gg L_{\text{osc}}$, neutrinos with even a tiny difference in energy will practically arrive at random phases in their oscillation. The detector then observes averaged neutrino oscillations. This condition for having averaged oscillations is $\frac{\Delta E_\nu}{E_\nu} \gg \frac{L_{\text{osc}}}{D}$ while also having the fractional change in energies between neutrinos, $\frac{\Delta E_\nu}{E_\nu}$, be smaller than what the detector can differentiate. Most annihilation channels in the Sun are therefore effectively averaged, but prompt neutrinos are not necessarily averaged. Even WIMP annihilations from the Galactic Center to prompt neutrinos are effectively averaged due to thermal motions of WIMPs (and for the reasons in the above paragraphs).

### 2.5 Possible DM Connection to Neutrinos

To probe the identity of DM, considering only the simplest models of WIMPs may not be wise because there are many mysteries for particle physics to solve (DM, neutrino mass, etc.), so the nature of physics beyond the SM is likely more interesting than the simplest models. To consider multiple mysteries at the same time is a promising strategy.

The remaining chapters will explore the phenomenology of physics beyond the SM that links WIMPs and neutrinos. There are well motivated reasons to tie the two sectors together, and unique signals arise. Chapter 3 will explore the IceCube Neutrino Observatory’s ability to probe the unique annihilation channel to prompt neutrinos that may result when DM couples to neutrinos. Chapter 4 will consider the possibility of detecting the neutrino mass hierarchy at IceCube by detecting the annihilation of a scalar DM particle that exists in the context of a type-II seesaw-neutrino. Chapter 5 will consider a type-I-seesaw scenario where DM annihilates to
a few-GeV-mass right-handed neutrino inside the Sun, which later decays outside the Sun.
Chapter 3

Prospects for Discovery of DM Annihilation to Prompt Neutrinos with IceCube

They call me Hollywood

_The Big Show_

The work of this chapter was done in collaboration with Rouzbeh Allahverdi, Alexander Friedland, and Katherine Richardson. We wish to thank Yu Gao and Carsten Rott for many valuable discussions. This work is supported by LANL IGPP grant 162497-1. R.A. acknowledges partial support by the National Science Foundation (NSF) under Grant No. PHYS-1066293 and the hospitality of the Aspen Center for Physics.

3.1 Introduction

Indirect searches investigate annihilation of DM to various final states (neutrinos, photons, charged particles) through astrophysical observations. Neutrinos are especially interesting as they are least affected from the production source, which can
be the Sun, the Galactic Center, or extragalactic. Neutrino telescopes like IceCube (IC)\(^1\), which has an interior with better resolution called DeepCore (DC), can probe the properties of DM by detecting a neutrino signal produced from DM annihilation.

The (effective) interactions between the DM and neutrinos remain largely unexplored experimentally, yet it may have important consequences for cosmology (for example, see [37]). It is natural to use the neutrino signal from DM annihilation to shed light on interactions between the DM and neutrinos. Neutrino telescopes can then be considered as a powerful means to extract information about the unknown dark sector (DM) through the known dark sector (neutrinos).

In this chapter, we show that IC/DC sets tight constraints on models with direct DM-neutrino interactions. In particular, we present the sensitivity limits for discovery of DM at IC/DC from solar WIMP annihilation to prompt neutrinos. The IceCube Collaboration has published sensitivity limits for DM annihilation to gauge boson and charged fermion final states inside the Sun, which are the dominant annihilation channels for the popular and extensively studied neutralino DM in the minimal supersymmetric standard model (MSSM). However, there are well-motivated extensions of the SM relating DM to the neutrino sector in which prompt neutrinos arise as the dominant annihilation final state.

Our study of solar WIMP annihilation to prompt neutrinos is further motivated by its complementarity to that from galactic and extragalactic DM annihilation. The neutrino flux from galactic/extragalactic DM annihilation depends on the DM annihilation rate, for which the IceCube Collaboration has published sensitivity limits. On the other hand, the neutrino flux from DM annihilation inside the Sun depends on the DM-nucleon elastic scattering cross section (once equilibrium between DM capture and annihilation is achieved). Annihilation of solar WIMPs can therefore yield a detectable signal for annihilation rates much below the current experimental limits.

\(^1\)IceCube is different from Ice Cube, who is a rapper.
In fact, we show that it provides the only possibility for DM discovery in models with very small annihilation rates that escape other indirect searches.

We perform a “theorist fit” by using detector characteristics such as effective volume and effective area that the IceCube Collaboration has provided. We use both contained and through-going muons from conversion of neutrinos at IC/DC in our analysis. By taking the angular and energy resolution into account, we implement optimal energy and angular cuts to obtain sensitivity limits for the $\nu$, $\tau$, and $W$ final states. We use the $W$ final state as a benchmark for comparison with published limits by the IceCube Collaboration and find reasonable agreement with these results [38, 39]. Our main result is the sensitivity limits for prompt neutrinos, which are stronger by a factor of a few than those for $W$’s. This is expected because of the harder neutrino energy distribution and agrees with the rough estimate made in our previous work [40].

Finally, we explore the possibility that the prompt neutrino final states may be distinguished from the $\tau$ and $W$ final states in the case that a discovery is made soon. By performing $\chi^2$ analysis, we find that discrimination at a statistically significant level is possible for DM mass in the 50 GeV–1 TeV range with many years of data from IC/DC. Determining the flavor of prompt neutrinos might also be possible due to the $\nu_\tau$ regeneration effect but it is more challenging as the effect only becomes important for DM masses above 500 GeV.

This chapter is organized as follows. In Sec. 3.2 and Sec. 3.3, we briefly discuss motivations for considering DM annihilation into prompt neutrinos and explain the DM signal in neutrino telescopes. In Sec. 3.4, we describe our methodology and benchmark it by using the sensitivity limits published by the IceCube Collaboration for the $W$ final state. In Sec. 3.5, we present our results for the $\nu$ and $\tau$ final states. In Sec. 3.6, we investigate the prospect for distinguishing final states with prompt neutrinos of different flavors at a statistically significant level. Finally, we conclude
the chapter in Sec. 3.7. The details of methodology used in our analysis are presented in the Sec. 3.8.

3.2 Dark Matter Annihilation to Prompt Neutrinos: Motivations

DM particles can in principle annihilate into any of the SM particles. The annihilation rate to a final state can be parameterized by \( \langle \sigma_{\text{ann}} v \rangle = a + b \langle v^2 \rangle \), where \( v \) is the relative velocity of annihilating particles. The two terms in this expression represent the \( s \)-wave and \( p \)-wave contributions respectively. Considering temperature at the Sun’s core \( T_C \sim 1 \text{ keV} \), thermal velocity of DM particles that annihilate inside the Sun is very low \( (v \sim 10^{-4} \text{ for a 100 GeV DM particle}) \). Therefore channels that proceed through the \( s \)-wave dominate annihilation (unless \( a \) is extremely small).

For the popular and extensively studied scenario of neutralino DM in the MSSM, annihilation is mainly into gauge boson and charged fermion final states, chiefly \( W \) and \( \tau \), whose subsequent decay yields secondary neutrinos. The neutrino signal from neutralino annihilation inside the Sun has been studied in various cases [41, 42, 43, 44, 45].

Neutralino annihilation to prompt neutrinos happens via gauge interactions where neutrinos are produced through the \( Z\nu\bar{\nu} \) vertex. It therefore produces a left-handed (LH) neutrino and a right-handed (RH) antineutrino. Since the neutralino is a Majorana fermion, we have \( a \propto m_{\nu}^2/m_{\text{DM}}^2 \) for the prompt neutrino channel. Therefore \( s \)-wave contribution to this annihilation mode is highly suppressed due to the tiny neutrino mass \( m_{\nu} \sim (0.1 \text{ eV}) \). As mentioned earlier, \( p \)-wave contribution is in general very small because of velocity suppression. In consequence, production of prompt neutrinos from neutralino annihilation is extremely suppressed.
However, DM annihilation into prompt neutrinos can be dominant when we go beyond the neutralino DM. Dirac fermions have a non-suppressed s-wave. If Majorana-fermion DM annihilation produces a $\nu \bar{\nu}$ pair, instead of a $\nu \bar{\nu}$ pair, it can proceed in the s-wave without any mass suppression. A detailed analysis of settings in which DM annihilation into neutrinos is enhanced has been given in [46]. This enhancement can happen when DM is related to the neutrino sector, with some specific models discussed in [47, 48, 49, 50] (also see [51, 52]).

Besides the theoretical motivation, the neutrino signal from DM annihilation into prompt neutrinos inside the Sun is also interesting from experimental point of view. These models result in a highly suppressed gamma-ray signal, which easily escapes the bounds set by Fermi [53]. DM annihilation in the Galactic Center produces neutrinos whose flux depends on the annihilation rate. However, the neutrino signal from galactic annihilation can be detected only for very large annihilation rates $\langle \sigma_{\text{ann}}v \rangle \sim 10^{-23}$ cm$^3$ s$^{-1}$ [54]. On the other hand, as we will see, neutrinos from solar WIMP annihilation may be detectable for annihilation rates as small as $\langle \sigma_{\text{ann}}v \rangle \sim 10^{-28}$ cm$^3$ s$^{-1}$. This is because in this case the flux of neutrinos is controlled by the DM-nucleon elastic scattering cross section, so long as equilibrium is achieved inside the Sun. As a result, the neutrino signal from the Sun provides the strongest probe of models in which DM dominantly annihilates into prompt neutrinos.

Here we discuss two explicit examples where DM annihilation mainly produces neutrino final states in more detail.

### 3.2.1 Dark Matter with a Light Mediator

Models where DM resides in a hidden/secluded sector with interactions that are mediated by a light gauge boson were studied in detail (for example, see [55, 56]) after PAMELA results on the excess of positrons in the cosmic ray [57]. It was realized that the exchange of a light mediator can give rise to long range forces
between DM particles and an enhanced annihilation rate that could account for the PAMELA results. More recently, such models have been considered in the context of self-interacting DM, where the long range force can give rise to a sufficiently large DM self-interaction that may alleviate small scale problems of the cold dark matter (CDM) scenario [58, 59].

Here we consider a specific model of this type [37], which is relevant to our discussion of prompt neutrino final states. In this model, the DM candidate $\chi$ is a Dirac fermion that is coupled to a light $U(1)$ gauge boson

$$\mathcal{L} \supset g_\chi \bar{\chi} \gamma^\mu V^\mu \chi + g_\nu \bar{\nu} \gamma^\mu V^\mu \nu. \quad (3.1)$$

These interactions keep the DM in kinetic equilibrium with neutrinos until a much later time than in the standard scenario wherein kinetic decoupling occurs around 1 second. In addition, the exchange of $V$ between DM particles gives rise to a large DM self interaction. Combination of these effects can then alleviate the small scale problems of the CDM scenario in this model. The dominant annihilation mode of DM is $\chi \chi \rightarrowVV$ with each $V$ subsequently decaying into a $\nu \bar{\nu}$ pair, thereby resulting in a $4\nu$ final state.

### 3.2.2 $U(1)_{B-L}$ Extension of MSSM

A minimal extension of the SM gauge symmetry involves a gauged $U(1)_{B-L}$ symmetry [60], where $B$ and $L$ denote the baryon number and lepton number respectively. The $B-L$ model is well motivated since anomaly cancellation automatically implies the existence of three RH neutrinos that can be used to explain neutrino mass and mixing. The simplest $B-L$ extension of MSSM contains a new gauge boson $Z'$, two new Higgs fields with $B-L$ charges, three heavy Majorana RH neutrinos $N$, and
their supersymmetric partners. The superpotential of the model is given by

\[ W = W_{\text{MSSM}} + W_{B-L} + h N H_u L, \]

where \( H_u \) and \( L \) denote the Higgs field that gives mass to up-type quarks and the LH leptons respectively (family indices are omitted for simplicity). The \( W_{B-L} \) term contains the new Higgs fields and \( N \), and its detailed form depends on the charge assignments of the \( B-L \) Higgs fields. In the following \( g_{B-L}, Q_L, \) and \( Q_B \) denote the \( U(1)_{B-L} \) gauge coupling and the \( B-L \) charges of leptons and baryons respectively (note that \( Q_L = -3Q_B \)).

A natural DM candidate in this model is the RH sneutrino \( \tilde{N} \) [51, 52]. The dominant annihilation channel of sneutrino DM is \( \tilde{N} \tilde{N} \to NN \) via \( t \)-channel exchange of \( \tilde{Z}' \) gaugino. The RH neutrinos thus produced quickly decay to LH neutrinos and the MSSM Higgs. Therefore the annihilation final state contains two neutrinos whose average energy is approximately \( m_{\text{DM}}/2 \).

3.3 Neutrinos from Dark Matter Annihilation in the Sun

In this Section we discuss various processes involving production of neutrinos from DM annihilation in the Sun, their propagation to the detection point, and their detection by IceCube.

3.3.1 The Sun as a Source

The Sun would be a particularly good source for annihilations producing neutrinos due to its closeness to Earth and its 4.6 billion year age to gather and concentrate DM for enhanced annihilation. The Sun is essentially a massive DM detector that
never needed any funding to be constructed. Unlike the Galactic Center whose signal depends on \(\langle \sigma_{\text{ann}} v \rangle\) and on branching fractions, the Sun also requires \(\sigma_{\text{SI}}\) or \(\sigma_{\text{SD}}\) to gather DM. Unlike direct detection, a sufficient \(\langle \sigma_{\text{ann}} v \rangle\) is required to annihilate the gathered DM, but, beyond this, \(\langle \sigma_{\text{ann}} v \rangle\) is not important, as will be shown presently.

The annihilation rate in the Sun depends on the number of DM particles \(N\) that have been collected since \(t = 0\), which is chosen to be 4.6 billion years ago when \(N = 0\). The equation

\[
\dot{N} = \Gamma_{\text{cap}} - 2 \Gamma_{\text{ann}}
\]  

(3.3)

describes the time change in \(N\) (\(\dot{N}\)) due to the total capture rate (\(\Gamma_{\text{cap}}\)) and the total annihilation rate (\(\Gamma_{\text{ann}}\)). The 2 is present because every annihilation removes 2 DM particles. Capture rate is assumed to be constant, but the annihilation rate depends on \(N\). A single DM particle has an annihilation rate that can be defined to be \(\frac{A}{2} N\), where \(A\) is a constant for a given \(m_{\text{DM}}\) and \(\langle \sigma_{\text{ann}} v \rangle\), so the total annihilation rate of all \(N\) DM particles is \(\Gamma_{\text{ann}} = \frac{A}{2} N^2\). Solving the resulting differential equation for \(N\) allows us to write

\[
\Gamma_{\text{ann}} = \frac{\Gamma_{\text{cap}}}{2} \tanh^2\left(\frac{t}{\tau_{\text{eq}}}\right)
\]  

(3.4)

where \(t = 4.6\) billion years and \(\tau_{\text{eq}}\) is the characteristic time scale to reach equilibrium, which is when \(\dot{N} = 0\). Near equilibrium,

\[
\Gamma_{\text{ann}} \approx \frac{\Gamma_{\text{cap}}}{2},
\]  

(3.5)

which makes perfect sense because for every 2 captures there is enough \(N\) already present to simultaneously have 1 annihilation.
\( \Gamma_{\text{cap}} \) obviously depends linearly on \( \sigma_{\text{SI}} \) and \( \sigma_{\text{SD}} \), and it also depends greatly on \( m_{\text{DM}} \). To be captured, a DM particle should ideally be moving with a velocity similar to that of the Sun and ideally collide “head on” with nuclei in the Sun that are thermally moving directly towards the DM so that the DM particle can have a final velocity close enough to that of the Sun in order to be captured by the Sun’s gravitation. That is, larger \( m_{\text{DM}} \) particles are harder to capture due to the dynamics of changing their velocity (and also due to their smaller number density in the Milky Way). \( \sigma_{\text{SI}} \) can be hundreds of times better than \( \sigma_{\text{SD}} \) at capturing DM in the Sun (even thousands of times better if \( m_{\text{DM}} > 1 \text{ TeV} \)), due to Eq. (1.9), which shows that \( \sigma_{\text{SI}} \) allows the heavier nuclei that are more capable of stopping high-mass DM to contribute more.

The annihilation constant \( C \) clearly depends linearly on \( \langle \sigma_{\text{ann}} v \rangle \), and also depends on \( m_{\text{DM}} \) because the mass determines the distribution of DM within the Sun. Once captured, DM thermalizes with the Sun after many more interactions with the Sun over billions of years. DM that settles in the Sun’s core will therefore have the core’s temperature of \( T \approx 1 \text{ keV} \) (in natural units of \( k_B = 1 \)) as its kinetic energy and will have gravitational potential energy \( U = \frac{2}{3} \pi G m_{\text{DM}} \rho r^2 \), where \( \rho \) is the mass density of core, which is assumed to be constant (true deep inside the core). The virial theorem that relates the average values of energy is \( K = U \), which, for \( \rho \approx 100 \text{ g/cm}^3 \) gives a typical DM radius of

\[
r \sim (0.1 R_\odot) \sqrt{\frac{\text{GeV}}{m_{\text{DM}}}}, \tag{3.6}
\]

where \( R_\odot \) is the solar radius. Because a higher DM number density increases the annihilation rate, \( C \) increases as \( m_{\text{DM}} \) increases. Note that for \( m_{\text{DM}} \gtrsim 3 \text{ GeV} \), there is essentially zero chance of DM randomly evaporating out of the Sun, which Eq. (3.3) assumed.
Combining this information can put Eq. (3.4) in terms of more useful quantities

\[ \Gamma_{\text{ann}} = a_1 (\sigma_{\text{SD}} + a_3 \sigma_{\text{SI}}) \tanh^2 \left[ a_2 \sqrt{\langle \sigma_{\text{ann}} v \rangle (\sigma_{\text{SD}} + a_3 \sigma_{\text{SI}})} \right], \quad (3.7) \]

where \( a_1, a_2, \) and \( a_3 \) are complicated functions of \( m_{\text{DM}} \) that are based on the physics of capture and annihilation, so, with increasing \( m_{\text{DM}} \), \( a_1 \) decreases, \( a_2 \) decreases slightly when \( m_{\text{DM}} > 30 \text{ GeV} \), and \( c \) increases. Perhaps considering a constant \( m_{\text{DM}} \) is when this equation is most useful as it allows one to convert between required \( \sigma_{\text{SD}} \) and \( \sigma_{\text{SI}} \) (via \( a_3 \)) and allows for an analysis of the \( \langle \sigma_{\text{ann}} v \rangle \) required for equilibrium. It spite of its great usefulness, this parametrization is not in the literature. For a quick way of calculating the functions \( a_1, a_2, \) and \( a_3 \), see Sec. A.1 and independently vary \( \sigma_{\text{SD}}, \sigma_{\text{SI}}, \) and \( \langle \sigma_{\text{ann}} v \rangle \).

With a nominal freeze-out annihilation rate of \( 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \), DM particles with a mass \( m_{\text{DM}} = 100 \text{ GeV} - 1 \text{ TeV} \) have reached equilibrium for spin-independent elastic scattering cross sections \( \sigma_{\text{SI}} \gtrsim 10^{-10} \text{ pb} \). This is well below the tightest limits currently set by the direct detection experiments such as LUX [61] and XENON100 [62]. The spin-dependent cross section needed to reach equilibrium is \( \sigma_{\text{SD}} \gtrsim 10^{-7} \text{ pb} \), which is also much smaller than the tightest experimental bounds at present set by direct detection experiments [63], IceCube [39], and collider searches [64, 65, 66].

### 3.3.2 IceCube as a Dark Matter Detector

Neutrino telescopes like IceCube access the neutrino signal from DM annihilation by recording Cerenkov light from relativistic charged particles in their volume. Muon neutrinos produce muons via charged-current interactions in the detector. The long muon tracks thus produced allow us to reconstruct the energy and determine directionality of the incoming neutrinos. Below \( \sim 1 \text{ TeV} \) the energy loss is mainly governed
by ionization, which results in a constant loss rate, and hence the length of the track is a proxy for the energy loss rate ($\sim 1$ GeV per 5 meters in ice).

Since completion in December 2010, IC has 86 strings instrumented with high quantum-efficiency digital optical modules (DOMs). Whereas the IC design is primarily focused on event energies above a TeV, IC/DC can in principle achieve an energy threshold as low as 10 GeV. This is because it consists of eight more densely instrumented strings (a total of 15 strings in DC array). As a result, IC/DC increases the IC effective volume at energies below 65 GeV and accounts for the majority of events recorded below 100 GeV. Further infills of the IC/DC array, such as PINGU, could extend the energy threshold to a few GeV and further increase the effective volume by a factor of two at 10 GeV [67].

### 3.3.3 Neutrino Energy Distributions and Their Features

In this work, we adopt a model-independent approach and consider DM annihilation to prompt $\nu$’s, $W$’s, and $\tau$’s. For charged fermions, we investigate only annihilation to $\tau$’s for the following reasons. $e$’s are stable and cannot produce any neutrinos via decay. $\mu$’s are stopped almost immediately in the Sun due to electromagnetic scattering. At Sun’s core $T_C \sim 1$ keV and $n_e \sim 10^{26}$ cm$^{-3}$ ($n_e$ being the number density of electrons), which results in a stoppage time that is shorter than the muon (unboosted) lifetime $\tau_\mu \approx 2 \times 10^{-6}$ s. As a result, $\mu$ decay will occur at rest and produces low energy neutrinos with a very soft energy distribution. The situation is different for $\tau$’s because of the much shorter lifetime $\tau_\tau \approx 3 \times 10^{-13}$ s. This implies that $\tau$’s decay when their energy is $\sim m_{\text{DM}}$, hence producing much more energetic neutrinos.

Also, all quarks except the $t$-quark hadronize before their subsequent decay. The $b$-quark produces neutrinos via three-body decay, which is similar to the $\tau$ but has a softer energy distribution and can be neglected. The lighter hadronizing quarks
decay effectively at rest, and hence result in very low energy neutrinos that cannot be detected at IceCube.

The $t$-quark decays to a $W$ boson and a $b$-quark before hadronization, and the subsequent $W$ decay produces a neutrino energy distribution comparable to that from the $W$ final state. As for the gauge boson final states, $W$ and $Z$ energy distributions are relatively similar, and hence we consider only $W$ final states in our analysis. We also note that the Higgs mainly decays to $b$-quarks, thus yielding a neutrino energy distribution similar to that from the $b$-quark final state, which is negligible.\(^2\)

In the following, we therefore focus on prompt neutrinos versus secondary neutrinos from DM annihilation to $W$ and $\tau$ final states. Neutrino undergo various processes from the production point to detection point including absorption and rescattering inside the Sun, vacuum oscillations between the Sun and the Earth, and finally conversion to muons at the detector. Thus we are interested in the energy distributions of neutrinos at the detector after all these effects have been taken into account. As an example, Fig. 3.1 depicts the theoretical prediction for neutrino energy distributions that result from the annihilation of 100 GeV DM particles in the Sun to prompt $\nu_\mu$’s, $W$’s and $\tau$’s.

The prompt neutrinos result in a monochromatic peak at the DM mass in the neutrino energy distribution. For a DM mass of 100 GeV, the energy distribution is largely insensitive to the choice of neutrino oscillation parameters and annihilating neutrino flavor. Here we depict annihilation to $\nu_\mu$, normal mass hierarchy, and $\theta_{13} = 10^\circ$ (which is consistent with recent results from various experiments [70, 71, 72, 73] that have confirmed a non-zero value of $\theta_{13}$).

The $W$ bosons produce neutrinos via two-body decay, which are softer than the previous case. Two-body decay of boosted $W$’s (produced from DM annihilation) re-

\(^2\)In addition to high-energy neutrinos from DM annihilation, nearly all final states produce pions and muons that come to rest inside the Sun very quickly. Decay at rest of pions and muons produces low-energy neutrinos with known energy distributions, which are out of the reach of IceCube, but can be probed at Super-Kamiokande [68, 69].
CHAPTER 3. DM Annihilation to Prompt Neutrinos at IC

Figure 3.1: Neutrino energy distributions at the detector from annihilation of 100 GeV DM particles to $\nu_\mu$ (bottom, green), $W$ (middle, orange), and $\tau$ (top, brown) final states inside the Sun.

results in relatively sharp kinematic edges in the neutrino energy distribution in Fig. 3.1. We note that $W$’s also decay to $b$-quarks, which in turn produce tertiary neutrinos. This results in additional contributions to the neutrino energy distribution below the lower kinematic cutoff.\(^3\)

The $\tau$’s produce neutrinos via three-body decays, which results in a much softer neutrino energy distribution that rises toward lower energies as seen in Fig. 3.1. However, there is no kinematic cutoff in the neutrino energy distribution in this case. Moreover, the overall number of neutrinos with high energy is larger than the case for $W$’s, as is evident from the figure, because $\tau$ decay is 100% leptonic.

For larger values of DM masses the neutrino energy distributions from $W$ and $\tau$ final states are qualitatively similar to those at 100 GeV. However, the features in the energy distributions of prompt neutrinos will change depending on their flavor because neutrinos undergo charged-current and neutral-current interactions with matter as they propagate through the sun. At the energies of interest, the cross section for

\(^3\)For $m_{\text{DM}} \lesssim 100$ GeV, DM annihilation produces $W$’s with both transverse and longitudinal polarizations in comparable numbers, with transverse $W$’s resulting in harder neutrinos. In this case, it is important to carry out a calculation that accounts for spin correlations of the final state particles and the helicity-dependence of their decays [74]. For $m_{\text{DM}} \gg m_W$, this is not necessary because $W$’s are mostly transverse in this regime.
these interactions is proportional to the neutrino energy $E_\nu$. The charged-current interactions convert $\nu_e$, $\nu_\mu$, $\nu_\tau$ to $e$, $\mu$, $\tau$ respectively. $e$’s and $\mu$’s are stopped immediately due to electromagnetic interactions, while $\tau$’s decays quickly before losing too much energy because of its very short lifetime of $3 \times 10^{-13}$ s. This decay produces a $\nu_\tau$, which has a lower energy than the original one. These interactions are more important for prompt neutrinos due to their harder energy distribution that is peaked at $m_{DM}$, which results in the suppression of the peak for all flavors. However, in the case of the $\nu_\tau$, the regeneration of neutrinos via three-body decay populates the energy distribution at energies well below the DM mass. Neutral-current interactions with matter, which affect all the flavors equally, further suppress the peak of the neutrino energy distribution. However, due to their smaller cross-section (by a factor of 3), they have a subdominant effect.

Neutrino absorption becomes significant when the absorption length of neutrinos $L_{\text{abs}}$ is roughly equivalent to the core size of the sun $R_C \sim 70,000$ km. Using the charged-current neutrino-nucleon cross section [75] and a core density of 150 g/cm$^3$, we find that $L_{\text{abs}} \sim 70,000$ km at energies $E_\nu \sim 300$ GeV. However, oscillations among different flavors should be taken into account as neutrinos propagate through the Sun. For $\nu_e$ the flavor and mass eigenstates are the same deep inside the Sun. Thus absorption via charged-current interactions start to suppress the flux of $\nu_e$ for DM masses above 300 GeV as discussed above. On the other hand, $\nu_\mu$ and $\nu_\tau$ flavors oscillate (mainly to each other) since they are not mass eigenstates. Matter effects inside the Sun are the same for these two flavors, and hence the oscillation length is set by the atmospheric mass splitting $L_{\text{osc}} = E_\nu/4\pi\Delta m^2_{\text{atm}}$. As long as $L_{\text{abs}} \gtrsim L_{\text{osc}}/4$, oscillations win over absorption, which implies that $\nu_\mu$ final states also feel the regeneration effect. Since $L_{\text{abs}} \propto E_\nu^{-1}$ and $L_{\text{osc}} \propto E_\nu$, at sufficiently high energies $L_{\text{abs}}$ drops below $L_{\text{osc}}/4$. This happens for a DM mass of about 500 GeV. Then $\nu_\mu - \nu_\tau$ oscillations cease to be effective, and $\nu_\mu$ gets absorbed similar to $\nu_e$ as shown
in Fig. 3.2. In consequence, only the $\nu_\tau$ final state retains a significant regeneration signature for DM masses above 500 GeV.

Figure 3.2: Neutrino energy distributions at the detector from annihilation of 500 GeV DM particles to $\nu_e$ (bottom, red), $\nu_\mu$ (middle, green), and $\nu_\tau$ (top, blue) final states inside the Sun.

3.4 Benchmarking Our Methodology

We have simulated the muon signal and modeled the IceCube detector’s detection efficiency and resolution, which allows us to determine the sensitivity curves for discovery of DM at 90% confidence. We use DarkSUSY [76] to acquire the muon signal and the background from atmosphere. The signal from DarkSUSY results from a simulation of (1) the capture and annihilation of DM in the Sun, (2) neutrino propagation and oscillation through the Sun, space, and the Earth, and (3) conversion into muons within the ice. DarkSUSY gives contained muons and through-going muons, which we use for our low-energy (“DeepCore”) analysis and our high-energy (“IceCube”) analysis respectively. We then use [77] to acquire effective volume and effective area expressions to model detector efficiency and detector resolution. As mentioned before, IC/DC can in principle achieve an energy threshold as low as 10 GeV. However, in order not to be too optimistic, we use 40 GeV energy resolution for the detector in both the low-energy and high-energy analyses. For simplicity and
ease of scaling our results, we consider a year of detection that corresponds to half a
year of useful detector runtime. For each DM mass and annihilation channel, we ask
what the capture cross section (\(\sigma_{SD}\) or \(\sigma_{SI}\)) must be for 90\% confidence, and produce
the corresponding sensitivity plots. See the Chapter Appendix (Sec. 3.8) for more
details.

To check our methodology, we first find the sensitivity plot for the \(W\) annihilation
channel, shown in Fig. 3.3, and compare it with the sensitivity limits published by the
IceCube Collaboration [38, 39]. At certain DM masses, our results differ by a factor of
two from the 2011 sensitivity curves for the \(W\) annihilation channel that the IceCube
Collaboration published [38]. We primarily compare to the 2011 sensitivity curves
from IceCube because these curves use effective volumes and areas of the detector
that have been published [77].

The more recent 2012 sensitivity curve [39] is also provided for comparison. These
limits are weaker by a factor of a few than the previous ones [38]. The change is
due to a more detailed analysis showing that light propagation and light yield for
low-energy muons are not as good as was thought previously. Resolution of this
discrepancy requires further studies of low-energy muons, which is beyond the scope
of our analysis.

It is seen that our sensitivity curve is in reasonable agreement with the ones
published by the IceCube Collaboration. This gives us confidence to use our method
for other final states particularly the prompt neutrinos that are the main subject of
our study. However, considering the discrepancy between the limits published by the
IceCube Collaboration for the \(W\) final state [38, 39], the sensitivity limits that we
derive for other final states should be taken as accurate to within a factor of a few.

To make these results more precise, it is important for the IceCube Collaboration to

\[^4\] Cosmic ray showers create a muon background that can be controlled by selecting for upward
going events since muons are stopped in the Earth. This limits observation of a DM signal from the
Sun to half the year, when the Sun is below the horizon (i.e., the Antarctic winter).

\[^5\] We thank Carsten Rott for explaining this point to us.
 resolve the issue with light propagation and light yield for low-energy muons first. This will presumably result in new effective volume and effective area expressions for IC/DC that can be used to obtain improved limits for all final states.

### 3.5 Dark Matter Annihilation to Prompt Neutrinos: IceCube Sensitivity Limits

#### 3.5.1 Sensitivity Limits on $\sigma_{SD}$

After verifying our methodology through the $W$ final state, we did the analysis for various annihilation channels (assuming 100% annihilation into a single channel in each case). We show the results in Fig. 3.4 along with the current limits on $\sigma_{SD}$ from the COUPP experiment [63].

The differences between the sensitivity curves from different annihilation channels result due to a combination of (1) the total number of neutrinos produced by the DM annihilation, (2) the energy distribution of the produced neutrinos, and, at high DM mass, (3) the $\nu_\tau$ regeneration inside the Sun.
The $W$ channel, while producing harder neutrinos, is more difficult to detect than the $\tau$ channel. This is because $\tau$ decay produces a larger number of energetic neutrinos as $W$ primarily decays into hadronic modes. As a result, the limit on $\sigma_{SD}$ for the $\tau$ channel is stronger by a factor of $\sim 3$, which is in agreement with the results in [78, 79].

The $\nu$ channels are the easiest to detect due to the much harder neutrino energy distribution that they produce, see Fig. 3.1. This however gets worse as $m_{DM}$ increases, which is due to the absorption of energetic neutrinos inside the Sun. As explained before, this effect becomes important for $\nu_e$ and $\nu_\mu$ flavor above $m_{DM} \sim 300$ GeV and $m_{DM} \sim 500$ GeV respectively. It is seen in Fig. 3.4 that these are roughly the values where the sensitivity curves for the $\nu_e$ and $\nu_\mu$ channels start to separate from that for the $\nu_\tau$ channel. At DM masses above 1 TeV (not shown here) the $\nu_\tau$ channel joins the $W$ and $\tau$ channels as a result of the regeneration effect, while the sensitivity limits for the $\nu_e$ and $\nu_\mu$ channels become very weak due to strong absorption effects inside the Sun.

It is seen that the sensitivity limits for the $\nu$ channels are about an order of magnitude stronger than those for the $W$ channel for $m_{DM} \lesssim 300$ GeV, while the difference becomes less for larger masses. This is in agreement with the estimates from our previous work [40] where we used the total muon count as a rough estimator. For sufficiently small values of $m_{DM}$ the total number of muons detected at the IceCube is larger by a factor of $\sim 8$ for the $\nu$ channels, while it decreases due to the absorption/regeneration effect at larger DM masses.

### 3.5.2 Sensitivity Limits on $\sigma_{SI}$

In Fig. 3.5 we show the IceCube sensitivity limits on $\sigma_{SI}$ along with the tightest direct detection bounds set by the LUX experiment [61]. It is seen that the limits on $\sigma_{SI}$ exhibit a behavior as a function of mass and the annihilation channel that
CHAPTER 3. DM Annihilation to Prompt Neutrinos at IC

Figure 3.4: Sensitivity plots for $\sigma_{SD}$ for discovery of DM particles with various annihilation channels (top $W$, middle $\tau$, bottom flavor democratic prompt $\nu$’s) with one year of data from IC/DC. The grey band shows the direct detection bounds on $\sigma_{SD}$ from the COUPP experiment [63].

is qualitatively similar to those on $\sigma_{SI}$. At a quantitative level, however, the limits on $\sigma_{SI}$ are stronger by a factor of $\sim 300 - 1000$ within the depicted mass range $m_{DM} = 100 \text{ GeV} - 1 \text{ TeV}$.

The difference is discussed in 1.6.1 and 3.3.1 (in Chapter 1). The SI contribution is primarily from iron. The SD contribution is primarily from hydrogen. For larger values of $m_{DM}$ the ratio approaches $\sim 1000$ because the role of heavy elements in stopping DM particles becomes more prominent as DM mass increases even though heavy elements such as iron account for only a few thousandths of the Sun’s mass [8].

It is remarkable that for prompt neutrinos the limits from IC/DC are competitive with the tightest direct detection bounds, especially for $m_{DM} \sim 300 - 500 \text{ GeV}$, while for $W$ and $\tau$ channels the direct detection bounds are much stronger. This, as explained above, is due to the much harder neutrino energy distribution of $\nu$ channels. As an important consequence, neutrino telescopes can provide meaningful constraints on the direct detection cross section in models with DM annihilation to prompt neutrinos even if $\sigma_{SD}$ is negligible. For example, the model in [51, 52] has $\sigma_{SD} = 0$, which makes the limits in Fig. 3.4 irrelevant. However, neutrino telescopes
can probe this model at a competitive level with direct detection experiments as it is evident from Fig. 3.5.

One comment is in order. The limits shown here have been obtained assuming that the DM has the same coupling to the up and down quarks. However, this need not be the case when one goes beyond the neutralino DM in MSSM [80, 81]. Depending on the nature of the isospin-violating couplings of DM to quarks, the curves in Figs. 3.4,3.5 can be shifted up or down [82]. As a result, in some cases, the IceCube limits can be tighter than what depicted here. Therefore, in the case of prompt neutrino final states, IceCube can be even more competitive with direct searches in constraining $\sigma_{\text{SI}}$.

![Sensitivity plots for $\sigma_{\text{SI}}$ for discovery of DM particles with various annihilation channels (top W, middle $\tau$, bottom flavor democratic prompt $\nu$’s) with one year of data from IC/DC. The grey band shows the direct detection bounds on $\sigma_{\text{SI}}$ from the LUX experiment [61].](image)

**Figure 3.5:** Sensitivity plots for $\sigma_{\text{SI}}$ for discovery of DM particles with various annihilation channels (top W, middle $\tau$, bottom flavor democratic prompt $\nu$’s) with one year of data from IC/DC. The grey band shows the direct detection bounds on $\sigma_{\text{SI}}$ from the LUX experiment [61].

### 3.5.3 DM Annihilation to Individual Neutrino Flavors

Rather than taking an average over neutrino flavors, one may study DM annihilation to individual neutrino flavors. Software called WimpSim is directly run (as opposed to using DarkSUSY’s interpolation of pre-run WimpSim data) to simulate this data
because DarkSUSY only interpolates between a handful of $m_{\text{DM}}$ points, and this interpolation does not correctly capture the effects of neutrino oscillations.

An oscillatory behavior in the curve for $\nu$ channels is seen in Fig. 3.6 at DM masses above 200 GeV. This is due to oscillations between $\nu_e$ and $\nu_\mu$, $\nu_\tau$ flavors governed by $\Delta m^2_{\text{sol}}$. The oscillation length $L_{\text{osc}}$ for neutrinos at the peak of energy distribution increases with $m_{\text{DM}}$, which leads to visible effects once $L_{\text{osc}}$ exceeds the variation in the Sun-Earth distance over a year. The oscillatory behavior persists for the $\nu_e$ channel up to $m_{\text{DM}} = 1$ TeV due to this effect. However, the curves for the $\nu_\mu$ and $\nu_\tau$ channels are smoother at large values of $m_{\text{DM}}$ since the regeneration effect results in low-energy neutrinos with a smaller $L_{\text{osc}}$ whose oscillations are averaged away.

![Figure 3.6: Sensitivity plots for $\sigma_{\text{SI}}$ for discovery of DM particles preferentially annihilating to one neutrino flavor (top $\nu_e$, middle $\nu_\mu$, bottom $\nu_\tau$) with one year of data from IC/DC. The grey band shows the direct detection bounds on $\sigma_{\text{SI}}$ from the LUX experiment [61].](image)

In addition to the oscillations, another main feature of Fig. 3.6 is that IC most sensitive to $\nu_\tau$ due to regeneration effect. IC is also sensitive to $\nu_\mu$ due to its oscillations to $\nu_\tau$ inside the Sun’s core. The $\nu_e$ channel does not oscillate to $\nu_\tau$ inside the core due to matter effects, so it sees no regeneration.

Due to the uncertainty in the measured values of neutrino parameters [24], Fig. 3.6 has error bars. Depending on the neutrino mass hierarchy, these errors are shown in Fig. 3.7 and Fig. 3.8 via varying $\Delta m^2_{\text{sol}}$. 
Figure 3.7: The same as Fig. 3.6 but with 1σ variation about the central value of $\Delta m^2_{\text{sol}}$ from PDG 2014 [24] for normal hierarchy scenario.

Figure 3.8: The same as Fig. 3.7 but for inverted hierarchy scenario.

### 3.5.4 Probing the Annihilation Cross Section

Our results for $\sigma_{\text{SD}}$ and $\sigma_{\text{SI}}$ have been obtained assuming that DM capture and annihilation inside the Sun reach equilibrium, i.e., that $t > \tau_{\text{eq}}$ in Eq. (3.4). This, however, requires that the DM annihilation rate $\langle \sigma_{\text{ann}} v \rangle$ be sufficiently large.

Using the $\sigma_{\text{SD}}$ in Fig. 3.4 and the annihilation rates output by DarkSUSY, one can find a lower bound on $\langle \sigma_{\text{ann}} v \rangle$ that is set by the equilibrium condition when $t = \tau_{\text{eq}}$. The plots in Fig. 3.9 show the minimum $\langle \sigma_{\text{ann}} v \rangle$ that would provide equilibrium in the Sun for different annihilation channels provided that there is discovery after one
It is seen that equilibrium can be achieved for very small values of \( \langle \sigma_{\text{ann}} v \rangle \sim 2 \times 10^{-29} - 3 \times 10^{-28} \text{ cm}^3 \text{ s}^{-1} \), with the exact value depending on the annihilation channel. This is much smaller than the nominal freeze-out value of 3 \( \times \) \( 10^{-26} \text{ cm}^3 \text{ s}^{-1} \).

Figure 3.9: The minimum value of annihilation rate \( \langle \sigma_{\text{ann}} v \rangle \) for different channels that lead to equilibrium between capture and annihilation for sensitivity curves of \( \sigma_{\text{SD}} \) in Fig. 3.4.

Models with the \( W \) and \( \tau \) annihilation channels easily escape the gamma-ray bounds for such small annihilation rates \([53]\), while scenarios with DM annihilation into prompt neutrinos generically satisfy the gamma-ray bounds. In both cases, neutrinos provide the only probe for indirect detection. However, the neutrino signal from galactic DM annihilation may be detected only for annihilation rates that are larger than the nominal value of 3 \( \times \) \( 10^{-26} \text{ cm}^3 \text{ s}^{-1} \) by few orders of magnitude \([54]\). Therefore, DM annihilation inside the Sun provides the only possibility for indirect detection of models with very small annihilation rates.

### 3.6 Distinguishing Between Annihilation Channels

Assuming that DM discovery from IC/DC data will be made soon, it is natural ask about the possibility of discriminating between scenarios that have different annihilation rates.

---

\(^6\)If DM discovery occurs with \( n \) years of data from IceCube, the curves in Fig. 3.9 will go up by a factor of \( \sqrt{n} \).
tion channels. For simplicity, we assume 100% annihilation to a given channel. Then we ask how one might be able to distinguish between two scenarios with different final states. Realistic models have mixed annihilation channels. Hence, as a next step, one can try to find the branching ratios for different annihilation final states.

We employ mass-dependent optimized energy and angular cuts to obtain the statistical significance for distinguishing two channels with 10 years of data from IC/DC (for details, see Sec. 3.8). For simplicity, we assume that the DM mass has been measured.

In Fig. 3.10 we show the results for distinguishing the $\nu_\mu$ channel from other channels. It is seen that the $\nu_\mu$ channel can be distinguished rather reliably from the $W$ and $\tau$ channels, but not from the $\nu_e$ and $\nu_\tau$ channels, over the entire mass range. This is due to the fact that the energy of prompt neutrinos has a distinct peak around $m_{DM}$. This results in higher energy muons that tend to be produced at a smaller angle, relative to the incident neutrinos, and hence IceCube’s angular resolution of $1^\circ$ allows us to use the angular distribution of muons to distinguish between the prompt neutrinos and other channels. It is seen that the statistical significance for distinguishing the $\nu_\mu$ and $\nu_\tau$ channels starts to increase at $m_{DM} \sim 500$ GeV. As mentioned earlier, the reason is that the regeneration effect acts as a discriminator between the $\nu_\tau$ channel and the $\nu_\nu, \nu_e$ channels at DM masses $m_{DM} \lesssim 500$ GeV.

In Fig. 3.11 we show the results for distinguishing the $\tau$ channel from other channels, which is in agreement with the previous figure (Fig. 3.10). Note that the ability to distinguish the $\tau$ and $W$ channels (albeit at a low significance) is due to the fact that their muon energy distributions do not have the same shapes as $\tau$’s producing softer neutrinos and hence softer muons.

In both figures we notice a dip for DM masses around 300 GeV. This is due to transition from low-energy analysis to high-energy analysis. At smaller values of $m_{DM}$, the low-energy analysis using contained muons in DC array wins gives a better
Figure 3.10: The number of standard deviations at which the $\nu_\mu$ channel can be distinguished from other channels with 10 years of data from IC/DC. 90% confidence is the 1.64 dashed line.

result than the high-energy analysis. For larger values of $m_{\text{DM}}$, on the other hand, the high-energy analysis that uses through-going muons at IC provides a better result than the low-energy analysis. The dip is an indicator of the fact that we have simply chosen the better of the two results throughout the entire mass range shown here.

Figure 3.11: The same as Fig. 3.10 but with the $\tau$ channel as the target.

To obtain these figures, we have smeared the energy of muon events by 40 GeV for both the low-energy and high-energy analyses. However, as low as 10 GeV resolution can be achieved in DC array because of its more densely instrumented strings once the before-mentioned issue with light propagation and light yield for low-energy muons is resolved. In Fig. 3.12 we show the results for distinguishing the $\nu_\mu$ channel from
other channels for 10 GeV energy smearing in the low-energy analysis. We notice a significant improvement at lower DM masses compared to Fig. 3.10. This is because the existing features in the energy distributions are affected less when we smear by a smaller amount. The peak in the energy distribution of prompt neutrinos, which is pronounced at DM masses below $\sim 300$ GeV, will be largely preserved in this case leading to an increased significance for discrimination of the $\nu$ final states. We also note that for 10 GeV energy smearing the $W$ and $\tau$ channels look significantly different at $m_{\text{DM}} \lesssim 100$ GeV. This is due to kinematic edges in the energy distribution of neutrinos from the $W$ final state that are well below $m_{\text{DM}}$ in this mass range (see Fig. 3.1).

![Figure 3.12:](image)

**Figure 3.12:** The same as Fig. 3.10, but with 10 GeV smearing of the energy of contained muons.

### 3.7 Conclusion

In this chapter, we have investigated prospects for discovery of DM annihilation into prompt neutrinos and discriminating it from other final states with the IC/DC by using the neutrino signal from annihilation of gravitationally captured DM particles in the Sun. This study is motivated both theoretically and experimentally. The neutrino final states can arise as the dominant annihilation channels in important classes of
models that connect DM to the neutrino sector. Moreover, solar annihilation of DM can be used to probe models with prompt neutrino final states for annihilation rates as small as \( \langle \sigma_{\text{ann}}v \rangle \sim 3 \times 10^{-28} \text{ cm}^3 \text{ s}^{-1} \), which escape detection by other indirect experiments. Despite this, no sensitivity limits for the prompt neutrinos from solar annihilation of DM has been currently provided by the IceCube Collaboration. With two years of IC/DC data awaiting analysis, theoretical and experimental motivations for studying prompt neutrino final states provide the grounds for a dedicated analysis by the IceCube Collaboration to put stringent bounds on direct annihilation into neutrinos.

To underline the importance of such a study, we have performed a “theorist fit” by taking detector characteristics like the effective volume and effective area into account. By implementing optimal energy and angular cuts, we have combined the results from the low-energy and high-energy analyses (using contained muons in DC array and through-going muons in IC respectively) to obtain sensitivity limits for the \( \nu \), \( \tau \), and \( W \) final states. We have used the latter as a benchmark for comparison with published limits by the IceCube Collaboration and found reasonable agreement.

We have shown that the IC/DC sensitivity limits for the prompt neutrinos are stronger by a factor of a few than those for the other final states, which is due to their harder neutrino energy distribution. As an interesting consequence, for this channel, IceCube limits on \( \sigma_{\text{SI}} \) are competitive with the bounds from direct searches. The situation can be even more promising once the IceCube Collaboration resolves the issue with light propagation and light yield for low-energy muons. We have also explored the possibility of distinguishing the \( \nu \) annihilation channels from the \( W \) and \( \tau \) channels in the case that IceCube makes discovery soon. In this case, for DM masses in the 50 GeV–1 TeV range, the \( \nu \) final states may be discriminated with multi-year data from IC/DC.
3.8 Chapter Appendix: Simulation and Analysis

Methodology

3.8.1 Acquiring Simulated Data for Signal and Background at IceCube

We acquire the simulated signal from DarkSUSY [76] version 5.0.5 for each DM mass and annihilation channel. The signal is for neutrinos starting in the Sun and ending at IceCube. Each annihilation channel we acquire corresponds to the situation of 100% annihilations going into that channel. For the background, we use the tables [83, 84] provided by DarkSUSY. We set initial $\sigma_{SI}$ or $\sigma_{SD}$ high enough to insure that the DM distribution in the Sun has reached equilibrium ($\sigma_{SD}$ and $\sigma_{SI}$ are the DM-nucleon cross sections).

Within DarkSUSY, we use the default halo model, a Navarro-Frenk-White (NFW) profile [15], and we use the default method for calculating capture rates. For simulation from the annihilation point to the detector, DarkSUSY uses WimpSim [85]. We tried the default WimpSim oscillation scenario with $\theta_{13} = 0^\circ$, and we currently use $\theta_{13} = 10^\circ$ (see Sec. A.2 for more details). Using the default WimpSim value $\theta_{13} = 0^\circ$ results in a negligible difference, as does trying an inverted mass hierarchy. We scale the data to only include the six months of Antarctic winter per year.

We are only interested in the muon ($\mu$) tracks, which result from charged-current interactions between $\nu_{\mu}$ and the ice. Our data also includes $\bar{\mu}$. We use the contained muons (the $\nu_{\mu}$-to-$\mu$ vertices per volume) and through-going muons per area (muon flux through a plane perpendicular to the Sun). The former is a function of muon energy at creation vertex, while the latter is a function of the muon energy at the plane. All data is also a function of angle relative to the axis defined by the Sun.
When asking DarkSUSY for data, we sample the angles every 0.1° out to 30°, and we sample 50 evenly spaced energies from 1% to 99% of the DM mass.

In our analysis, we eventually use effective volume and effective area plots provided by IceCube [77], which take detector efficiency and many other detector properties into account. The effective volume and effective area data are functions of $\nu_\mu$ energy rather than $\mu$ energy. Because it is much easier to apply these to the $\nu_\mu$ data, we never actually get solar $\mu$ data from DarkSUSY other than for testing purposes. Instead, we get the $\nu_\mu$ data and convert it to $\mu$ data by using the nucleon density $n$ and the cross section for neutrino-nucleon charged current interaction $\sigma(E_\nu)$ (averaged for ice) taken from [86]. To get the energy and angular distributions, we do an analysis of delta-function neutrino signals.

### 3.8.2 Modeling IceCube Detection of the Muon Tracks

To model the detector, we first determine if a muon will be detected, and we then smear the resulting muon in angle and energy to determine the energy and angle at which it will be detected (taking imperfect detector resolution into account). The goal is to acquire detector counts as a function of energy and angle. The smearing could easily be implemented as energy dependent, which we did not do since our goal is only to give a simple model of the detector and the energy dependence is unknown to us. We perform two analyses: low-energy analysis that uses contained muons with the $\nu_\mu$ conversion vertex to $\mu$ inside the DC array, and high-energy analysis that uses through-going muon tracks in the IceCube86 (excluding DC array).

For the low-energy analysis, we need an expression to multiply with the contained muon data to get detector counts, which we call the effective volume. We use the effective volume curves given in [77], which are the best published results at the moment. We fit $a_1 \ln(a_2 E_\nu + 1)$ to the 15-string DC with trigger curve and obtain
the following expression for the effective volume

\[
\frac{V_{\text{eff}}}{\text{km}^3} = 0.0862 \ln(0.00603 \frac{E_{\nu}}{\text{GeV}} + 1),
\]  

(3.8)

when being careful to weight the low-energy more when fitting the curve.

For the high-energy analysis, we need an expression to multiply with the through-going data by to get detector counts, which we call the effective geometric area. Assuming the detector’s thickness is much less than the muon’s track length, the effective geometric area is the cross-sectional area of the detector times the detector efficiency, where the detector efficiency is the detected events divided by the generated events, a number that would be 1 for a perfect detector. We use the effective area curve given in [77]. We divide this curve by \( n\sigma(E_{\nu}) \) to find the effective volume. After dividing out the energy-dependent length of the detector (physical length plus muon track length) from the effective volume, we obtain the effective geometric area:

\[
\frac{A_{\text{eff}}}{\text{km}^2} = \frac{1.31 \ln(0.00233(\frac{E_{\nu}}{\text{GeV}} - 70) + 1)}{1 + 3 \ln(\frac{E_{\nu}}{3.650 \text{GeV}} + 1)},
\]  

(3.9)

where \( 3 \ln(\frac{E_{\nu}}{3.650 \text{GeV}} + 1) \) is the muon track length in km if we use the approximation \( E_{\mu} \approx \frac{E_{\nu}}{3} \).

Having these effective volume and effective geometric area expressions, we apply them to the neutrino data and then convert the \( \nu_{\mu} \) (\( \bar{\nu}_{\mu} \)) data to the contained and through-going \( \mu \) (\( \bar{\mu} \)) data we need using the results of our “delta-function” analysis. We use the plots from the IceCube Collaboration for both particles and antiparticles with \( \nu_{\mu} \) conversion to \( \mu \) uniformly scaled by \( \frac{3.77}{3.30} \) to get the \( \bar{\nu}_{\mu} \) conversion to \( \bar{\mu} \). However, this is not precise as the neutrino and antineutrino energy distributions have different shapes. We nonetheless use our expression for both particles and antiparticles due to lack of separate data.
We smear the angular distribution by $\sigma = 1^\circ$ for both the low-energy and high-energy analyses, where $\sigma$ takes into account the angular smearing done by the detector. Our smearing function per solid angle is

$$\frac{dP}{d\Omega} = \frac{1}{2\pi\sigma^2} e^{-\frac{\theta'^2}{2\sigma^2}},$$

(3.10)

where $\theta'$ is the angle relative to the muon track (which itself has an angle $\theta$ relative to the direction of the Sun). This smearing function is isotropic and, for small $\sigma$, normalized. For how to implement angular smearing, see Sec. B.3. To prevent data loss, our data is out to $30^\circ$ before smearing then chopped at $20^\circ$ after smearing. For the background, angular smearing has no effect because we average over all solid angles, which creates data that is effectively isotropic.

Due to then unknown energy-dependent smearing (resolution has larger error bars—though perhaps not larger relative error—at higher muon energies), we simply smear the entire energy distribution by $\sigma_E = 40$ GeV for both the low-energy and high-energy analyses. We use

$$\frac{dP}{dE} = \frac{1}{\sigma_E\sqrt{2\pi}} e^{-\frac{(E-\mu)^2}{2\sigma_E^2}} \frac{2}{1 + \text{erf}(\frac{\mu}{\sigma_E\sqrt{2}})}$$

(3.11)

as our smearing function. It is a Gaussian that abruptly stops at 0 GeV and normalized from zero to infinity by adding an additional factor. To prevent data loss at our highest energy bin, we add an extra energy bin that contains the data that would otherwise be lost. Compared to angular smearing, the amount by which we smear the energy has small effect on our final results for discovery of DM, but energy smearing affects discrimination between annihilation channels. Angular smearing effects the high DM mass data the most, and energy smearing mostly effects the intermediate-mass data.
3.8.3 Optimizing the Analysis for Discovery of DM and for Discriminating Between Annihilation Channels

The optimization selects the best energy and angular cuts for discovery of DM at 90% confidence level or for best discrimination. We keep in mind that at sub-TeV energies IceCube measures the energy of a muon based on the track length, hence we do not allow the energy cut to enter the region where the muon track is longer than the detector size. We apply a lower energy cut, above which all data is integrated. Also, we apply an upper angular cut, below which all data is integrated. The values of these cuts for discovery of DM are given in Fig. 3.13, and Fig. 3.14 contains the cuts for discriminating between annihilation channels. The cuts on actual data will probably need to be somewhat different due to some subtle issues involving the limitations of our $\nu_\mu$-to-$\mu$ conversion.

![Figure 3.13: Energy and angular cuts for our low-energy and high-energy analyses when attempting to discover DM. All annihilation channels use the same cuts. DM masses are from 50 GeV to 1 TeV.](image)

Above the energy cuts, we bin the energies for a $\chi^2$ analysis. Regardless of whether we are discovering DM or discriminating between channels, we use the same binning
Figure 3.14: The same as Fig. 3.13 but for discriminating between annihilation channels rather than discovery.

scheme. For the low-energy analysis, we use, in GeV, bins 10−50, 50−100, and 100−∞. The 40 GeV smearing advised our binning. However, for the high-energy analysis, the neutrinos scatter within the Sun and lose sharp features, so the best bin size is larger. We use 70−200 and 200−∞. When we try 10 GeV smearing for the low-energy analysis, our low-energy binning for discovery is unchanged, and our binning when discriminating annihilation channels is 10−20, 20−30, . . . , 100−∞.

DarkSUSY and WimpSim attempt to give a prediction of the average signals. However, we wish to simulate the full random statistical variations that would be seen by the IceCube experiment. We follow the first appendix of [87] to simply add a term to our $\chi^2$ expression to represent these statistical variations.

For discovery of DM, the background count ($B$) may be subtracted from the signal ($S$) by observing away from the Sun, off-source, as is done in Galactic-Center DM searches [54]. The total count will have uncertainty of $\sigma = \sqrt{S + B}$. If $\frac{S}{\sigma} = 1.64$, a discovery at 90% confidence will have been made (see Sec. B.4 for details). The energy distribution of the atmospheric neutrino background from cosmic rays is understood
theoretically to within 20% \cite{84} and is measured above 100 GeV to within 10% \cite{88}. Therefore, we make the conservative choice of using

\[ \frac{S}{\sqrt{S + 1.2B}} = 1.64 \quad (3.12) \]

as our discovery condition. We use a \( \chi^2 \) analysis, which reduces to this simpler approach. Conceptually, we find the energy and angular cuts that maximize the left-hand side of Eq. (3.12) for the low-energy and high-energy analyses for a given annihilation channel and DM mass.

Assuming that discovery at 90% confidence is made, we compare various annihilation channels to one chosen as the target for different values of DM mass. The representative channels chosen as target are the \( \tau \) channel (which is qualitatively similar to the \( W \) channel but softer) and the \( \nu_\mu \) channel (which is qualitatively similar to the \( \nu_e \) and \( \nu_\tau \) channels for DM mass below 1 TeV). We scale the data (both signal and background) by a factor of 10 from what is needed for discovery, which represents running the detector for 10 times as long (i.e., 10 years for discrimination vs 1 year for discovery). We then apply the cuts in Fig. 3.14, bin the energy as described previously, and do the \( \chi^2 \) analysis.
Chapter 4

Distinguishing Neutrino Mass Hierarchies Using DM Annihilation Signals at IceCube

I did it!

Brad Knockel (when his first paper was published)

The work of this chapter was done in collaboration with co-authors Rouzbeh Allahverdi, Bhaskar Dutta, Dilip Kumar Ghosh, and Ipsita Saha and was published in Journal of Cosmology and Astroparticle Physics [89].

This work was supported in part by NSF Grant No. PHY-1417510 (R.A. and B.K.) and DOE Grant DE-FG02-13ER42020 (B.D.). We thank the International Center for Theoretical Physics (ICTP), Trieste, Italy, where this work was initiated. D.K.G. and I.S. would like to thank Bhupal Dev for useful discussions. D.K.G also would like to thank Theory Unit of the Physics Department, CERN, Geneva, Switzerland where part of this work was done. R.A. and B.K. would like to thank Shashank Shalgar for valuable discussions. R.A. and B.D. thank the Center for Theoretical Underground
CHAPTER 4. Distinguishing Neutrino Mass Hierarchies Using DM and IC

Physics and Related Areas (CETUP* 2015) for its hospitality and for partial support during the completion of this work.

4.1 Introduction

Just like DM, the origin of neutrino mass and mixing defines a very interesting area of investigation beyond the SM (BSM) [22, 90]. These are the two encouraging avenues of new physics and a large number of BSM scenarios have already been proposed to address them. It will be even more interesting to investigate models where the two sectors are connected. In fact, such a connection arises naturally in a class of models where DM is tied to the neutrino sector. For example, in type-II seesaw plus a singlet scalar scenario [91], it has been shown that the neutrino mass hierarchy can influence the DM annihilation rate to charged leptons as well as neutrinos and subsequently provide a possible explanation of the positron fraction excess observed at the AMS-02 experiment [92]. Similar studies have been done in the $B-L$ extension of the minimal supersymmetric standard model (MSSM) where it is shown that annihilation of the right-handed sneutrino DM produces neutrino final states with a distinct feature [93].

There have been studies on the possibility of discovering the $B-L$ sneutrino DM at the Large Hadron Collider (LHC) [94, 95, 96]. The connection between the DM and neutrinos in such scenarios can also be probed through DM direct detection [97] and some indirect detection experiments [98]. Therefore, a combination of direct, indirect, and collider signatures could be useful to explore this connection (for a review, see [99]).

Our focus here is on the DM indirect detection signals as a means of testing the connection between DM and neutrinos. Neutrino telescopes like IceCube are able to examine these models through the signal arising from the DM annihilation into neutrinos at the Galactic Center [39] and inside the Sun [19]. Also, DM indirect
detection searches like the Fermi-LAT experiment [100] can probe these models via the photon signal coming from the DM annihilation at the Galactic Center. Unlike the photon flux, the neutrino flux arising from the DM annihilation at the Galactic Center has less astrophysical background uncertainty, which allows us to probe the exact nature of DM more accurately. It will be interesting to combine the results of IceCube and constraints from Fermi-LAT to probe models where the DM and the neutrino sectors are interconnected.

In this chapter, we study these issues within the above mentioned extension of the SM [91, 92]. In this model, the light neutrino masses and mixing angles are generated using the well-known type-II seesaw mechanism [31, 29, 30, 33, 32] that introduces $SU(2)_L$ triplet scalars. The neutral component of the triplet acquires a non-zero vacuum expectation value (vev) that generates tiny neutrino masses by breaking lepton number by two units and mixing among different neutrino flavors. Among the SM particles, these triplet scalars only couple to gauge bosons through gauge couplings and to leptons via Yukawa couplings. These Yukawa couplings are related to the triplet vev, which in turn determines different decay modes of these triplet scalars. It has been observed [101] that the triplets will dominantly decay to a pair of leptons if the vev is less than 0.1 MeV, otherwise the gauge boson final state will become dominant. In order to accommodate a stable DM candidate, the model is augmented by a SM singlet scalar with a $Z_2$ symmetry [91]. In the minimal case, the DM only couples to the Higgs and to the triplet scalars. For a vev smaller than 0.1 MeV, the annihilation of the DM produces a pair of triplets that will further decay to SM leptons (including neutrinos) with almost 100% branching fraction. Therefore, the flavor composition of the final states will depend upon the neutrino mass hierarchy that sets the Yukawa couplings. We exploit this feature and show how different neutrino mass hierarchies can be distinguished by using the muon signal arising from conversion of muon neutrinos from DM annihilation at the Galactic Center and inside
the Sun at IceCube. We note that the photon signal from the Galactic Center does not discriminate amongst different hierarchies because the differences are not significant due to the leptonic nature of final states.

This chapter is organized as follows. In Sec. 4.2, we discuss the model in brief. In Sec. 4.3, we present the analysis of the neutrino signal from DM annihilation at the Galactic Center and inside the Sun for normal and inverted hierarchies. We also calculate the corresponding muon spectra at the detector and compare them with the background. Finally, in Sec. 4.4 we present our discussion and conclusion.

### 4.2 Type-II Seesaw with a Scalar Singlet DM Candidate

The minimal version of the type-II seesaw model is an extension of the SM that includes a $SU(2)_L$ scalar triplet $\Delta$ with hypercharge $Y = 2$.

$$\Delta = \begin{pmatrix} \Delta^+ \\ \sqrt{2} \Delta^0 \\ \Delta^- \end{pmatrix}$$

In order to accommodate a CDM candidate, we introduce a SM singlet real scalar $D$. The stability of the DM candidate is ensured by imposing a $Z_2$ symmetry under which $D$ is odd while all other SM particles and the triplet $\Delta$ are even. The scalar potential of the model, including the relevant terms for the DM candidate $D$, is given
by \cite{92}

\begin{align}
V(\Phi, \Delta) &= -m_\Phi^2(\Phi^\dagger \Phi) + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2 + M_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) + \frac{\lambda_1}{2} \left[\text{Tr}(\Delta^\dagger \Delta)\right]^2 \\
&\quad + \frac{\lambda_2}{2} \left[\text{Tr}(\Delta^\dagger \Delta)\right]^2 - \text{Tr} \left[\left(\Delta^\dagger \Delta\right)^2\right] + \lambda_4 (\Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) \\
&\quad + \lambda_5 \Phi^\dagger [\Delta^\dagger, \Delta] \Phi + \left(\frac{\Lambda_6}{\sqrt{2}} \Phi^\dagger \sigma_2 \Delta^\dagger \Phi + \text{h.c.}\right), \tag{4.2}
\end{align}

\begin{align}
V_{\text{DM}}(\Phi, \Delta, D) &= \frac{1}{2} m_D^2 D^2 + \lambda_D D^4 + \lambda_\Phi D^2 (\Phi^\dagger \Phi) + \lambda_\Delta D^2 \text{Tr}(\Delta^\dagger \Delta). \tag{4.3}
\end{align}

where \( \Phi \) is the SM Higgs doublet. The couplings \( \lambda_i (i = 1 - 5) \) can be taken as real without any loss of generality. Due to spontaneous symmetry breaking, the non-zero vev of the Higgs doublet generates a tadpole term through interaction involving the \( \Lambda_6 \) coupling in Eq. (4.2). This in turn induces a non-zero vev \( (v_\Delta) \) for the neutral component of the triplet thereby breaking lepton number by two units. This triplet vev contributes to the SM gauge boson masses at the tree level which leads to a deviation in the electroweak \( \rho \) parameter. Hence, \( v_\Delta \) is constrained by the electroweak precision data that requires the \( \rho \) parameter to be close to the SM value of unity \cite{24} which eventually demands \( v_\Delta < 2 \) GeV \cite{102}. In addition to this, current experimental bounds on lepton flavor violating processes put a lower bound on \( v_\Delta \) \cite{103, 104} as

\[ v_\Delta M_\Delta \geq 150 \text{ eV GeV}. \tag{4.4} \]

It is to be mentioned here that we assume negligible mixing between the doublet and triplet scalars in our analysis \footnote{Detailed expressions for physical scalar mass eigenstates can be found in \cite{105}.} and with this assumption, the DM potential of Eq. (4.3) can be expressed in terms of physical scalars \( (h, H^\pm, H^{\pm\pm}, H^0, A^0) \) as

\begin{align}
V_{\text{DM}}(\Phi, \Delta, D) &= \frac{1}{2} m_{DM}^2 D^2 + \lambda_D D^4 + \lambda_\Phi v^2 D^2 h + \frac{1}{2} \lambda_\Phi D^2 h^2 + \\
&\quad \lambda_\Delta D^2 \left[H^{++} H^{--} + H^+ H^- + \frac{1}{2} \left(H_0^2 + A_0^2\right) + \frac{1}{2} v_\Delta H_0\right]. \tag{4.5}
\end{align}
Here, $m_{DM}^2 = m_D^2 + \lambda \phi v^2 + \lambda \Delta v_\Delta^2$ denotes the physical mass of the DM candidate. It is evident from Eq. (4.5) that the DM candidate can annihilate to a pair of Higgs and to a pair of exotic triplet scalar particles through the coupling parameters $\lambda \phi$ and $\lambda \Delta$.

In the limit $m_{DM} > m_\Delta$, where, $m_\Delta$ is the degenerate mass of all triplet scalars, the annihilation cross section of the DM candidate (non-relativistic) is expressed as

$$\langle \sigma v \rangle = \frac{1}{16\pi m_{DM}^2} \left[ \lambda \phi^2 \left( 1 - \frac{m_h^2}{m_{DM}^2} \right) + 6\lambda \Delta^2 \left( 1 - \frac{m_\Delta^2}{m_{DM}^2} \right) \right].$$

(4.6)

This should yield the correct relic abundance of the DM particle that lies within the $3\sigma$ limit of current Planck bound $\Omega_{DM} h^2 = 0.1199 \pm 0.0027$ [2]. Now, the final state of DM annihilation will depend upon the branching fraction of the Higgs and the triplet scalar decay. The triplet scalars can couple to the SM gauge bosons through gauge coupling and to SM lepton doublets through Yukawa couplings.

$$\mathcal{L}_Y = \mathcal{L}_Y^{SM} - \frac{1}{\sqrt{2}} (Y_\Delta)_{ij} L_i^T C i \sigma_2 \Delta L_j + h.c.$$ 

(4.7)

Here, $C$ is the charge conjugation operator and $L_i = (\nu_i, \ell_i)^T_L$ is the SU(2)$_L$ lepton doublet with $i$ being the three generation indices. Further, these Yukawa couplings can be obtained from the Majorana mass matrix of neutrinos that arises due to the non-zero triplet vev $v_\Delta$.

$$(M_\nu)_{ij} = v_\Delta (Y_\Delta)_{ij}, \quad \text{(4.8)}$$

$$Y_\Delta = \frac{M_\nu}{v_\Delta} = \frac{1}{v_\Delta} U^T M_\nu^{\text{diag}} U, \quad \text{(4.9)}$$

where $M_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$ and $U$ is the PMNS mixing matrix.
CHAPTER 4. Distinguishing Neutrino Mass Hierarchies Using DM and IC

With the recent global analysis data of Eq. (2.7) (in Chapter 2), we obtain the following structure of the Yukawa coupling matrix

\[
Y_{\Delta} = \frac{10^{-2} \text{eV}}{v_{\Delta}} \times \left( \begin{array}{ccc}
1.08 - 0.29i & -1.55 + 0.09i & 1.23 - 0.31i \\
-1.55 + 0.09i & 2.07 + 0.26i & -1.59 - 0.21i \\
1.23 - 0.31i & -1.59 - 0.21i & 2.59 + 0.0i \\
3.84 + 0.34i & 1.21 - 0.13i & -1.39 - 0.94i \\
1.21 - 0.13i & 2.97 - 0.35i & 1.98 - 0.65i \\
-1.39 - 0.94i & 1.98 - 0.65i & 2.66 + 0.01i \\
6.80 - 0.06i & -0.13 - 0.04i & 0. - 1.65i \\
-0.13 - 0.04i & 6.91 - 0.03i & 0. - 1.10i \\
0. - 1.65i & 0. - 1.10i & 6.71 + 0.10i
\end{array} \right) 
\]

(normal hierarchy)

\[
Y_{\Delta} = \frac{10^{-2} \text{eV}}{v_{\Delta}} \times \left( \begin{array}{ccc}
3.84 + 0.34i & 1.21 - 0.13i & -1.39 - 0.94i \\
1.21 - 0.13i & 2.97 - 0.35i & 1.98 - 0.65i \\
-1.39 - 0.94i & 1.98 - 0.65i & 2.66 + 0.01i \\
6.80 - 0.06i & -0.13 - 0.04i & 0. - 1.65i \\
-0.13 - 0.04i & 6.91 - 0.03i & 0. - 1.10i \\
0. - 1.65i & 0. - 1.10i & 6.71 + 0.10i
\end{array} \right) 
\]

(inverted hierarchy) (4.10)

\[
Y_{\Delta} = \frac{10^{-2} \text{eV}}{v_{\Delta}} \times \left( \begin{array}{ccc}
6.80 - 0.06i & -0.13 - 0.04i & 0. - 1.65i \\
-0.13 - 0.04i & 6.91 - 0.03i & 0. - 1.10i \\
0. - 1.65i & 0. - 1.10i & 6.71 + 0.10i
\end{array} \right) 
\]

(degenerate case).

For normal hierarchy (NH) and inverted hierarchy (IH), we choose the lightest neutrino mass eigenvalue to be zero. For the degenerate case, we impose the 95%-confidence limit from Planck on the sum of all light neutrino masses \(\sum_i m_i < 0.23 \text{ eV} \) [2] and take \(m_1 \simeq m_2 \simeq m_3 = 0.07 \text{ eV} \). However, the degenerate scenario does not offer any significant information in our study as we will see in later section.

The triplet scalars decay to leptonic final states, \(H^{++} \rightarrow \ell^+\ell^+\), \(H^+ \rightarrow \ell^+\nu_\ell\), \(H_0/A_0 \rightarrow \nu\nu\) with almost 100% branching ratio when \(v_{\Delta} \leq 0.1 \text{ MeV}\). On the other hand, for larger \(v_{\Delta}(> 0.1 \text{ MeV})\), gauge boson final states become dominant with the decay modes \(H^{++} \rightarrow W^+W^+, H^+ \rightarrow W^+Z, H_0/A_0 \rightarrow ZZ/W^+W^- \) \(^2\). For almost same order of \(\lambda_\phi\) and \(\lambda_{\Delta}\) within the range for producing correct relic density, the DM can annihilate to three different final states (depending upon the branching ratios i.e on \(v_{\Delta}\) for triplets):

(i) neutrino and charged lepton final states.

(ii) \(W^\pm W^\mp, ZZ\) finals states mainly from triplet scalar decays; \(WW^*, ZZ^*\) finals states arise from the decay of the SM Higgs boson.

(iii) \(b, \tau\) final states from the decay of the SM Higgs boson.

\(^2\)Throughout out our analysis we assume degenerate triplet scalars.
We choose two extreme values of $v_\Delta$, namely, 1 eV and 1 GeV, respectively to incorporate the effect of triplet scalars decay to only leptonic final state or only to non-leptonic final states. Our study is mainly focused on the study of neutrino flux coming from DM annihilation which can be obtained from all the above three final states. However, for triplet scalars decaying into leptonic final states includes the contribution of Yukawa couplings $Y_\Delta$ which is a function of neutrino mass hierarchy and we expect to see the difference in the neutrino flux between different hierarchies, say Normal (NH) or Inverted (IH). In the following sections, we will explicitly show how our analysis depends upon the neutrino mass hierarchy.

In this regard, it is to be noted that the triplet scalars can produce interesting signals at the colliders. The strongest limit on the scalar masses comes from the current searches at the LHC for the signature of doubly charged scalar where they can be pair produced via Drell-Yan and Vector Boson fusion and then decay to a pair of same-sign leptons. The experimental lower bound on $m_{H_{\pm\pm}}$ has been set by the ATLAS experiment [106] from a pair of isolated lepton for ($H_{\pm\pm} \rightarrow e^\pm e^\pm$) and ($H_{\pm\pm} \rightarrow \mu^\pm \mu^\pm$) decay modes at the center of mass energy 8 TeV. The 95%-confidence lower limit on the doubly charged Higgs mass for the same-sign isolated muons final state exclude mass range below 516 GeV. However, the experimental lower limit is based on the assumption that the double charged scalar decay to the dimuon channel with 100% branching ratio, which is not the case in our scenario. In this model, the double charged scalar decay to dimuon channel with at most 30% branching ratio and thus the lower limit on $m_{H_{\pm\pm}}$ can be relaxed. Following Fig. 5 of [106], we set the degenerate mass of the triplets ($m_\Delta$) as 400 GeV for our analysis.
4.3 Analysis

In this section, we demonstrate our findings on the DM annihilation at the Galactic Center and inside the Sun and the possibilities of using the respective neutrino signals to distinguish the NH and IH scenarios. For our analysis purpose, we choose to work with four benchmark points, shown in Table 4.1, by fixing $v_{\Delta}$ for two DM masses. We choose the DM mass to be greater than 400 GeV to get on-shell triplet scalars in the pair production processes of DM annihilation. Our choice of $v_{\Delta}$ has already been justified in the previous section. We choose $\lambda_\Delta$ and $\lambda_\Phi$ couplings in the model (see Eq. (4.3)) such that the correct DM abundance is obtained via thermal freeze-out with nominal value for thermally averaged annihilation cross section $\langle \sigma_{\text{ann}}v \rangle \approx 3 \times 10^{-26} \text{ cm}^3/\text{s}$. $\lambda_\Phi$ also enters into the direct detection cross section ($\sigma_{\text{SI}}$) through the $t$-channel exchange of the SM Higgs. Values of $\lambda_\Phi$ that satisfy thermal relic abundance yield a direct detection cross section well below the current limits from LUX experiment [61]. The choice of $\lambda_\Delta$ and $\lambda_\Phi$ and the corresponding relic density and direct detection cross section are also listed in Table 4.1. The correct DM relic abundance can also be produced via non-thermal mechanisms [107, 108, 109], in which case a larger DM annihilation cross section is allow. We will discuss the implications of a larger annihilation cross section at the end of this section.

As has been mentioned earlier, the DM annihilation will give rise to neutrino fluxes of different flavors that will help in distinguishing the structure of Yukawa coupling and, hence, the hierarchy of neutrino mass. IceCube detects neutrinos by recording the Cherenkov light from relativistic charged particles in its volume. Muon neutrinos ($\nu_{\mu}$) produce muon ($\mu$) tracks via the charged current interactions in the detector. On the other hand, electron neutrinos ($\nu_e$) and tau neutrinos ($\nu_\tau$) result in hadronic and electromagnetic cascade events in the ice. Since the cascades are localized, they carry no directional information, and hence are not as good for performing a meaningful DM search over the background at energies of interest. For this reason, we focus on
\(\nu_\mu\) fluxes that arrive on the Earth. We also estimate the photon flux for all scenarios in our model. Cosmic ray showers create a muon background that can be controlled by selecting only the upward going events since muons are stopped in the Earth. This limits the observation of DM signal to the time when the source is below the horizon. With atmospheric muons thus eliminated, the most significant contribution to the remaining background comes from atmospheric neutrinos \([84]\). In addition to this, a portion of the detector may be used as a veto to observe the contained muon events with a conversion vertex inside the instrumented volume, as in the case of DeepCore array in IceCube. The veto procedure virtually eliminates the contribution to the background from atmospheric muons by selecting only contained vertices. This increases the potential observation time to the full year when the source is both above and below the horizon.

We use both contained and through-going muons in our analysis. For contained muon tracks, the vertex at which \(\nu_\mu\) is converted to a muon, is within the detector volume. Through-going muons represent those events that go through a surface inside the detector but may have been produced outside it. To acquire the muon spectra, we use three simple methods:

1. WimpSim \([85]\) automatically provides muons converted from solar neutrinos.
2. DarkSUSY \([76]\) gives muons from atmospheric neutrinos (which we average to make an isotropic approximation).

<table>
<thead>
<tr>
<th>Benchmarks</th>
<th>(\nu_\Delta)</th>
<th>(m_{DM}) (GeV)</th>
<th>(m_\Delta) (GeV)</th>
<th>(\lambda_\Delta)</th>
<th>(\lambda_\phi)</th>
<th>relic density</th>
<th>(\sigma_{SI}) (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP1</td>
<td>1 eV</td>
<td>500</td>
<td>400</td>
<td>0.055</td>
<td>0.04</td>
<td>0.117</td>
<td>(2.25 \times 10^{-10})</td>
</tr>
<tr>
<td>BP2</td>
<td>1 eV</td>
<td>700</td>
<td>400</td>
<td>0.075</td>
<td>0.05</td>
<td>0.122</td>
<td>(1.79 \times 10^{-10})</td>
</tr>
<tr>
<td>BP3</td>
<td>1 GeV</td>
<td>500</td>
<td>400</td>
<td>0.055</td>
<td>0.04</td>
<td>0.117</td>
<td>(2.25 \times 10^{-10})</td>
</tr>
<tr>
<td>BP4</td>
<td>1 GeV</td>
<td>700</td>
<td>400</td>
<td>0.075</td>
<td>0.05</td>
<td>0.122</td>
<td>(1.79 \times 10^{-10})</td>
</tr>
</tbody>
</table>

Table 4.1: Benchmark Points (BPs) and the corresponding DM relic density and direct detection cross-sections.
3. Finally, GENIE [110] provides muons converted from neutrinos coming from the Galactic Center. 

GENIE provides the spectrum of contained muons produced from $\nu_\mu$s arriving at the detector. To find the total number of muon events, we use the total deep-inelastic charged-current cross sections from reference [86] to find the average $\nu_\mu$-to-$\mu$ and $\bar{\nu}_\mu$-to-$\bar{\mu}$ cross sections. Modeling the Galactic Center as an isotropic circle with radius of 5° and ignoring detector angular resolution, we obtain a 5° optimal cut, whereas the solar muons have a 2° optimal cut. To find the through-going muons, we propagate the muons using the parameters of Table 4 of reference [111].

In order to generate the energy spectra of the SM particles that are produced at the annihilation point, we first create the model file for CalcHEP using FeynRules 2.0 [112] and then use CalcHEP 3.6.23 [113] and PYTHIA-6 [114] to get the spectra (see Sections A.5, A.6, and A.7 for more information on this software). These spectra are valid both at the Galactic Center and inside the Sun. In the following we elaborately present our result for these two distinct DM annihilation points.

4.3.1 Signals from DM Annihilation in the Galactic Center

To calculate the fluxes near the Galactic Center, we run the indirect detection module of micrOMEGAsv4.1.8 [115] and measure the fluxes for angle of sight $0^\circ \leq \Psi \leq 5^\circ$ (see Sec. A.4 for more information on micrOMEGAs). We use the Navarro-Frenk-White (NFW) [14] profile $\rho(r) = \rho_0 (r/r_s)^{-1}/(1+r/r_s)^2$ with $r_s = 20$ kpc, $\rho_0 = 0.4$GeV/cm$^3$. If the annihilation of our proposed DM candidate happens near the Galactic Center, then the neutrinos will encounter averaged oscillations on their way to the Earth, so the initial-flavor dependent probability of a flavor reaching the detector will be given by Eqs. (2.14) and (2.15) (in Chapter 2). We have checked that varying all neutrino oscillation parameters within the 3$\sigma$ allowed range about their best fit central value
CHAPTER 4. Distinguishing Neutrino Mass Hierarchies Using DM and IC

results in \( \mathcal{O}(3-4\%) \) change in the neutrino flux from DM annihilation at the Galactic Center. This is much smaller than the difference due to different hierarchies.

Following are the intriguing features that emerged from Figs. 4.1, 4.2 and 4.3.

- Fig. 4.1 displays the \( \nu_\mu \) fluxes for NH and IH for BP1 and BP2 which is for \( v_\Delta = 1 \) eV. As previously argued, at such low vev, the triplet scalars generate leptonic final state with almost 100% branching ratios and mainly the DM annihilating to two on-shell triplet scalars contribute to the annihilation cross section, hence to the \( \nu_\mu \) flux. Now, it is to be observed that for \( m_{DM} = 500 \) GeV, the flux rises at around 100 GeV and then falls near 400 GeV and similarly, the rise and fall occur at 60 GeV and 640 GeV for \( m_{DM} = 700 \) GeV. This is not surprising and can be understood by the kinematics of the DM annihilation. The triplets are produced on-shell with some boost that comes from the mass difference between the DM and the triplets. Thus the two-body final states render a box like feature with end points \( m_{DM}(1 \pm \beta)/2 \) where \( \beta = \sqrt{1 - m_\Delta^2/m_{DM}^2} \).

- We also see from Fig. 4.1 that there exists a significant difference between the NH and the IH scenarios. This is due to the fact that in the NH case, we get more taus (\( \tau \)) from the triplet decays which produces \( \nu_\tau \) and the \( \nu_\tau \) further gets converted into \( \nu_\mu \). For both DM mass, the neutrino flux can be easily a factor of two higher for the NH scenario than the IH. The neutrino flux due to degenerate case will be in between the flux generated for NH and IH cases. This is simply because in the degenerate case, we get less \( \tau \)s from the triplet decays compared to the NH case but more than the IH case. This is true for all our discussions to follow. Therefore, the degenerate case will not provide any new insight and so we restrain ourselves from doing any further remark on this case.

- In Fig. 4.2, we present the diffused photon fluxes for NH and IH cases. In our model, the photon flux arises from external charged lepton legs, final state
radiation, and secondary decay of charged leptons that are directly produced from triplet decay. For both the benchmark points, we observe that the photon flux does not distinguish between NH and IH cases (due to the leptonic nature of final states).

• With the increase in $v_\Delta$, the branching ratio of the triplet scalars to leptons reduces and the decay modes with gauge boson final states open up and at sufficiently large $v_\Delta$, $\text{BR}(H^{\pm\pm} \to W^\pm W^\pm) \simeq 100\%$, dominating over the leptonic final states. Hence, for $v_\Delta = 1$ GeV, the dependence on neutrino mass hierarchy gets washed away. Moreover, the contribution to neutrino fluxes coming
from DM annihilating to Higgs will become more pronounced in this case. In Fig. 4.3, we show neutrino fluxes for BP3 and BP4 and evidently these do not distinguish between NH and IH cases. However, the shapes of neutrino spectra are different compared to the previous $\nu_{\Delta} = 1$ eV cases as shown in the Fig. 4.1. As expected, the flux is higher in the previous case because the leptonic final states originate from direct triplet decay. The photon flux, however, does not draw any discrimination be it between $\nu_{\Delta}=1$ eV and 1 GeV cases or between NH and IH scenarios.

![Figure 4.3: Spectra of $\nu_\mu$ (left) and $\gamma$ (right) from DM annihilation at the Galactic Center for BP3 and BP4.](image)

### 4.3.2 Signals from DM Annihilations Inside the Sun

DM particles that pass through the Sun lose energy due to the DM-nucleon scattering and get gravitationally trapped [116, 117, 118, 119, 120, 121, 74]. Subsection 3.3.1 (in Chapter 3) describes the capture and annihilation in the Sun. We recall that $\Gamma_{\text{ann}} \approx \frac{\Gamma_{\text{cap}}}{2}$ as long as $t > \tau_{\text{eq}}$. In this case, DM capture by and annihilation in the Sun reach equilibrium, and the total annihilation rate $\Gamma_{\text{ann}}$ is set by the capture rate $\Gamma_{\text{cap}}$. In our model, the capture rate of DM particles by the Sun is related to $\sigma_{\text{SI}}$. For values of $\sigma_{\text{SI}}$ that we have chosen, the nominal thermal freeze-out value
\( \langle \sigma_{\text{ann}} v \rangle = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \) and as a result, the equilibrium condition is satisfied for the age of the Sun.

DM annihilation final states that are most relevant for producing neutrinos inside the Sun are prompt neutrinos from \( H^0/A^0 \) and \( H^+/H^- \) decays, \( W, Z \) and \( \tau \) particles from \( H^+/H^- \) and \( H^{++}/H^{--} \) decays. Other final states (like \( e, \mu \), and lighter quarks) lose their energy and stop almost immediately, which results in production of neutrinos at energies below detection threshold. The charged current interactions inside the Sun convert \( \nu_e, \nu_\mu, \nu_\tau \) to \( e, \mu, \tau \) respectively. Using the charged current neutrino-nucleon cross section [86], we find that neutrino absorption becomes important (i.e., \( L_{\text{abs}} < R_C \), with \( R_C \) being the core radius of the Sun) at energies \( E_\nu > 300 \) GeV. For \( \nu_e \), the flavor and mass eigenstates are the same deep inside the Sun, and hence absorption suppresses the flux of \( \nu_e \) at energies above 300 GeV. On the other hand, \( \nu_\tau \) absorption produces \( \tau \) particles that decay quickly before losing too much energy because of very short lifetime of \( \tau \). This decay produces \( \nu_\tau \) at a lower energy, and hence this ‘regeneration’ effect populates the spectrum at energies well below the DM mass. Since \( \nu_\mu \) and \( \nu_\tau \) undergo oscillations inside the Sun that is dominantly set by the atmospheric mass splitting \( \Delta m^2_{\text{atm}} \), we have \( L_{\text{osc}} \propto E_\nu/\Delta m^2_{\text{atm}} \). As long as \( L_{\text{abs}} \gtrsim L_{\text{osc}}/4 \), oscillations mix \( \nu_\mu \) and \( \nu_\tau \) efficiently and \( \nu_\mu \) final states also feel the regeneration effect. When \( L_{\text{abs}} \) drops below \( L_{\text{osc}}/4 \), which happens at \( E_\nu \sim 500 \) GeV, oscillations cease to be effective and \( \nu_\mu \) gets absorbed similar to \( \nu_e \). As a consequence, only the \( \nu_\tau \) final state retains a significant regeneration signature at energies above 500 GeV.\(^3\)

For neutrinos of sufficiently low energies, vacuum flavor oscillations between the Sun and the Earth are averaged over half a year when the Sun is below the horizon at the south pole. In particular, below 100 GeV, the oscillation length set by the

\(^3\)Neutrinos also have neutral current interactions with matter inside the Sun that results in energy loss of the neutrinos from all flavors further shifting their spectra toward lower energies. However, the cross section for neutral current interactions is a factor of 3 smaller than that for charged current interactions, which makes them subdominant.
solar mass splitting $m_{\text{sol}}$ is less than the approximately 3 million kilometer change in the Earth-Sun distance over half a year. The situation changes at energies above 100 GeV where solar neutrino oscillations are not averaged out anymore. This significantly affects oscillations between $\nu_e$ and $\nu_\mu/\nu_\tau$, and hence the $\nu_\mu$ spectrum at the detector, at high energies.

We use DarkSUSY 5.1.1 [76] to simulate production of neutrinos in the Sun, their propagation to the South Pole, and the interaction of $\nu_\mu$ with ice at many different energies for all the SM channels. DarkSUSY does this by interpolating an older WimpSim [85] simulation. Here we run WimpSim 3.05 directly for the prompt neutrino channels, sample energies spaced at 10 GeV, and use oscillation parameters found in [25].

We then combine the energy spectra of all the channels at the production point with how they propagate, giving us the final neutrino spectra at the detector.\footnote{Since $\bar{\nu}$-nucleon cross section is about two times smaller than the $\nu$-nucleon cross section [122, 75], the number of absorbed $\bar{\nu}$ inside the Sun is smaller. This results in a larger $\bar{\nu}$ flux at the detector from DM annihilation inside the Sun.} It should be noted that variation of all neutrino oscillation parameters within the $3\sigma$ allowed range results in $\mathcal{O}(3-4\%)$ change in the final neutrino spectra from the Sun, similar to the Galactic Center case, which is insignificant.

In Fig. 4.4 we show the spectra of $\nu_\mu$ at the detector for $\nu_\Delta = 1$ eV (BP1, BP2) and for $\nu_\Delta = 1$ GeV (BP3) cases. It is seen that at energies below 400 GeV the NH scenario results in a larger number of $\nu_\mu$ and eventually muons than the IH scenario. The reason is similar to the Galactic Center case and can be explained by the triplet scalar decay patterns. For $\nu_\Delta = 1$ eV, as mentioned above, triplet decays mainly produce $\nu_\tau$, $\nu_\mu$ and $\tau$, $\mu$ final states in the NH scenario. In the IH scenario, on the other hand, triplet decays mainly produce $\nu_e$ and $e$ final states. As far as charged lepton final states are concerned, only $\tau$ decays before losing a significant fraction of energy and produces neutrinos. Regarding triplet decays to neutrinos, $\nu_\tau$ (and to a
lesser extent $\nu_\mu$) is the most relevant flavor that survive at energies above 300 GeV due to regeneration effect. In consequence, for $v_\Delta = 1$ eV, a larger number of $\nu_\mu$ arrive at the detector in the NH scenario. However, the neutrino spectra in the two scenarios do not differ when $v_\Delta = 1$ GeV as in earlier case. Comparing the neutrino signal from DM annihilation in the Sun (Fig. 4.4) and at the Galactic Center (Fig. 4.1), we notice some differences. First, the kinematic cuts in Fig. 4.1 are not prominent in Fig. 4.4. This is due to the fact that absorption/regeneration and down scattering inside the Sun erases sharp features in the spectra. More precisely, only a little bump develops at low energies for $m_{DM} = 500$ GeV due to the boosted triplet scalars, shown in right panel of Fig. 4.4, unlike the box-like structure that can clearly be observed in the Galactic Center neutrino flux. At higher energies, instead of a bump only the wiggles can be noticed which are the effect of neutrino oscillation. Furthermore, the difference between the spectra in NH and IH scenarios in Fig. 4.4 is smaller than that in Fig. 4.1. This is because stopping of muons and partial absorption of $\nu_\mu$ inside the Sun decreases the overall flux of $\nu_\mu$ arriving at the detector. Nevertheless, the NH scenario yields a larger flux than the IH scenario when $v_\Delta = 1$ eV.

We emphasize that the neutrino signal from DM annihilation inside the Sun is complementary to that from galactic DM annihilation since it is set by $\sigma_{SI}$ instead.
of $\sigma_{\text{ann}}$. Comparing Figs. 4.1 and 4.4, we see that for the model parameters given in Table 4.1 the signal from the Sun is about an order of magnitude stronger than that from the Galactic Center even though $\sigma_{SI}$ is well below the current LUX bound. As we will discuss later, both of these signals can be enhanced further.

### 4.3.3 Muon Spectra at the Detector

We now present the muon spectra obtained from the conversion of $\nu_\mu$ at the detector, which is the observed signal at neutrino telescopes. For simulation purpose, we assume a detector that has the same capability as the IceCube DeepCore array. In Fig. 4.5,

![Graphs showing muon spectra](image)

**Figure 4.5:** Spectra of muons from DM annihilation at the Galactic Center for $m_{\text{DM}} = 500$ (700) GeV in the left (right) panels. Upper and lower panels show contained and through-going muon spectra with an angular cut of $5^\circ$.

we show the spectra of contained and through-going muons in the detector from DM annihilation at the Galactic Center. In this case, the signal comes from a region...
around the Galactic Center that has $5^\circ$ angular extension. Hence, in order to optimize the signal to background ratio, we have imposed a $5^\circ$ angular cut on the muons relative to the center of the galaxy.

![Figure 4.6](image)

**Figure 4.6:** Spectra of muons from DM annihilation in the Sun for $m_{\text{DM}} = 500$ (700) GeV in the left (right) panels. Upper and lower panels show contained and through-going muon spectra with an angular cut of $2^\circ$ respectively.

In Fig. 4.6, we show the spectra of contained and through-going muons from DM annihilation inside the Sun. Since the Sun is a point-like source, the optimal signal to background ratio is obtained for an angular cut of $2^\circ$, in this case. The peaks in both figures are due to the imposed angular cuts that eliminate muons produced from $\nu_\mu$ conversion below a certain energy. For through-going muons the peak occurs at a lower energy, which can be understood by noting that the measured energy of a through-going muon is in general less than the actual energy at the production point.
As expected, the muon spectra have less features than the neutrino spectra shown in Figs. 4.1-4.4. Nevertheless, the difference between $v_{\Delta} = 1 \text{ eV}$ and $v_{\Delta} = 1 \text{ GeV}$ cases, as well as the NH and IH scenarios in the former case, are clearly visible in the muon spectra arising from both the Galactic Center and the Sun. Similar to the neutrino spectra, substantial deviation among the cases is seen in the signal arising from DM annihilation at the Galactic Center than in the Sun. However, we also note that the absolute value of the flux from the Galactic Center is smaller by about one order of magnitude than that from the Sun.

The muon spectra shown in Figs. 4.5 and 4.6 are obtained assuming the nominal thermal freeze-out value of $\langle \sigma_{\text{ann}} v \rangle$. However, the Fermi-LAT experiment on the detection of gamma rays coming from Milky Way dwarf spheroidal galaxies [18] allows larger values on the DM annihilation cross section for the values of DM mass that we have considered here. One can see from Fig. 8 of [18] that for $b\bar{b}$ and $W^+W^-$ final states $\langle \sigma_{\text{ann}} v \rangle$ can be larger than the thermal freeze-out value by a factor of 4 (6) when $m_{\text{DM}} = 500 \ (700) \ \text{GeV}$. In BP1 and BP2, where the distinction between the NH and IH scenarios is significant, these final states arise from DM annihilation to the SM Higgs that is controlled by the coupling $\lambda_\Phi$ of Eq. (4.3). As explained before, $\lambda_\Phi$ also controls $\sigma_{\text{SI}}$ in our model. Hence raising $\langle \sigma_{\text{ann}} v \rangle$ by the above factors, due to a larger value of $\lambda_\Phi$, also results in an increase in $\sigma_{\text{SI}}$ compared with that given in Table 4.1, which is still consistent with the LUX bounds [61]. For the $\tau^+\tau^-$ final state, $\langle \sigma_{\text{ann}} v \rangle$ can be larger than the thermal freeze-out value by a factor of 10 (17) when $m_{\text{DM}} = 500 \ (700) \ \text{GeV}$\footnote{It should be noted that the Fermi-LAT bound cannot be directly applied to our model since we have four-body annihilation final states in this case. Moreover, we have a combination of different final states instead of 100% of just one final state. Nevertheless, the Fermi-LAT limit provides a reasonable upper bound on $\sigma_{\text{ann}}$ in this case too.}. This final state arises due to DM annihilation to the triplet Higgs, which is controlled by the coupling $\lambda_\Delta$ of Eq. (4.3). Increasing $\lambda_\Delta$ results in an increase in $\sigma_{\text{ann}}$, but does not affect $\sigma_{\text{SI}}$.\footnote{It should be noted that the Fermi-LAT bound cannot be directly applied to our model since we have four-body annihilation final states in this case. Moreover, we have a combination of different final states instead of 100% of just one final state. Nevertheless, the Fermi-LAT limit provides a reasonable upper bound on $\sigma_{\text{ann}}$ in this case too.}
Therefore, by invoking non-thermal mechanisms for DM production in the early universe, we can obtain considerably larger neutrino fluxes from DM annihilation at the Galactic Center and inside the Sun. The resulting enhancement in the muon events, compared to Figs. 4.5 and 4.6, leads to a better prospect for detection of the neutrino signal against the atmospheric background. In Fig. 4.7, we show the spectra of contained muons due to DM annihilation at the Galactic Center (left panel) and inside the Sun (right panel) for annihilation cross sections that are just below the Fermi-LAT limits, where $m_{\text{DM}} = 500$ GeV, and the background from atmospheric neutrinos is also shown for comparison. Contained muons from DM annihilation inside the Sun provide the best detection opportunity. Within energy interval 100–400 GeV, the total number of contained muons for the NH and IH scenarios is 11 and 9 respectively (compared with 95 for the background). The maximum signal to background ratio, 17% and 15% for the NH and IH scenarios respectively, occurs at an energy of 255 GeV. For a neutrino telescope with the same capability as the IceCube DeepCore array, $3\sigma$ discovery of NH and IH scenarios takes 8 and 12 years respectively. To distinguish the different neutrino mass hierarchies, one should go beyond the simple number count and perform a careful shape analysis, which is beyond the scope of this paper.

There are proposals for directly measuring the mass hierarchy by using atmospheric neutrinos in future extensions of neutrino telescopes such as PINGU (Precision IceCube Next Generation Upgrade) [123]. It is possible to exclude the wrong mass ordering by this method (as well as in neutrino beam experiments) at the $3\sigma$ level within the next 10–15 years [124]. Our approach, which exploits an interesting connection between DM and neutrinos, is complementary to this direct method and makes another case for the future neutrino telescopes with much larger volume like IceCube-Gen2 [125].
Figure 4.7: Spectra of contained muons from DM annihilation at the Galactic Center (left) and inside the Sun (right) for $m_{\text{DM}} = 500$ GeV. The value of $\sigma_{\text{ann}}$ used to obtain the muon flux is just below the current bounds from Fermi-LAT. The top line in both figures is due to the background arising from the atmospheric neutrinos.

4.4 Summary and Conclusions

In this chapter, we have considered an extension of the SM that explains neutrino masses and mixing angles via the type-II seesaw mechanism and includes a singlet scalar as a viable DM candidate. The DM interacts with the triplet scalar that generates light neutrino masses thereby linking the two sectors. We have studied the neutrino signal from DM annihilation at the Galactic Center and inside the Sun in this model that can be detected at IceCube. Our main results are summarized as follows:

- In the region of the parameter space where the triplet scalar dominantly decays to leptonic final states, the flux of $\nu_\mu$ from DM annihilation depends upon the neutrino mass hierarchy (normal, inverted, or degenerate). The photon flux, on the other hand, is practically the same for different hierarchies due to the leptonic nature of final states.

- The difference in the flux of $\nu_\mu$ at the detector for NH and IH is visible. The NH scenario produces a larger flux because of its mass pattern. This holds for both signals from DM annihilation at the Galactic Center and inside the Sun.
• The difference is more significant in the neutrino signal arising from DM annihilation at the Galactic Center. This is mainly due to interaction of neutrinos with matter inside the Sun that results in a moderate attenuation of the $\nu_\mu$ flux at the detector in the latter case.

• The same holds for the contained and through-going muons that are produced from the conversion of $\nu_\mu$s at the detector. The muon spectra have less features than the neutrino spectra. Nevertheless, these muons can be detected over the atmospheric background by a detector that has the same capability as the IceCube/DeepCore array with multiyear data.

• For annihilation cross sections just below the Fermi-LAT limits, future extensions of IceCube are able to discover the neutrino signal and distinguish the different neutrino mass hierarchies with multiyear data.

The LHC in tandem with direct detection experiments, IceCube, and Fermi-LAT will be able to probe this minimal extension of the SM that utilizes a new Higgs sector. Finally, we would like to conclude by pointing out the fact that this analysis can be applied to other models that link DM to the neutrino sector.
Chapter 5

Indirect Signals from Solar Dark Matter Annihilation to Long-Lived Right-Handed Neutrinos

Some leaves hang on longer, but eventually they all fall.

Marty Rubin

The work of this chapter was done in collaboration with co-authors Rouzbeh Allahverdi, Yu Gao, and Shashank Shalgar. Soon after the writing of this dissertation, a paper sharing the title of this chapter will appear on arXiv.

The work of R.A. and B.K. is supported in part by NSF Grant No. PHY-1417510. Y.G. thanks the Mitchell Institute for Fundamental Physics and Astronomy (MIFPA) for support.
5.1 Introduction

Signals from dark matter (DM) annihilations inside the Sun [116, 117, 118, 121, 119] have been extensively studied in the context of DM indirect detection searches. DM particles, upon scattering off solar medium, can become gravitationally trapped and start annihilating into standard model (SM) particles after their numbers build up at the center of the Sun. Neutrinos thus produced can escape from the Sun and those with energies above weak scale can be detected by active Cherenkov detectors like IceCube [126] and Antares [127]. Low-energy neutrinos from stopped pions may also be used as a probe of DM annihilation inside the Sun [68, 69]. Provided that equilibrium between the capture and annihilation of DM particles inside the Sun is established, the flux of neutrinos is determined by the capture rate [8]. It can therefore be used to constrain the DM-nucleon elastic scattering cross section. The bounds thus set are much tighter than those form direct detection experiments for spin-dependent (SD) interactions [126], while for spin-independent (SI) interactions direct detection experiments, like LUX [16] and PandaX [128], often set much stronger limits.

DM annihilation inside the Sun may also result in a photon signal in case that it produces relatively long-lived intermediates states that can escape from the Sun before decaying. Such a scenario can arise in various new physics models, with the dark photon [129], secluded [130], inelastic [131], boosted [132], portal [133] DM models as examples. Since the solar direction is dark in high energy cosmic gamma rays, any photonic signal in that direction offers a stringent constraint.

DM particles may annihilate to the right-handed (RH) neutrinos, which in turn decay to the SM particles, at a significant rate in simple extensions of the SM. A minimal and well-motivated example is the supersymmetric extension of the SM that includes a gauged $U(1)_{B-L}$ symmetry [60] (where $B$ and $L$ are baryon number and lepton number respectively). Anomaly cancellation then implies the existence of three RH neutrinos and allows us to write the Dirac and Majorana mass terms for
the neutrinos to explain the mass and mixing of the light neutrinos. This model provides two new DM candidates, the lightest neutralino in the $B - L$ sector and the lightest RH sneutrino, both of which may dominantly annihilate into the RH neutrinos. The decay of these RH neutrinos can then lead to interesting indirect detection signals [52, 134, 93].

RH neutrinos with a mass much below the weak scale undergo three-body decay via off-shell $W$ and $Z$ bosons through their small mixing with the left-handed (LH) neutrinos. For masses in the 1-few GeV range, RH neutrinos produced from solar DM annihilation can readily obtain a long ($\sim 1 - 10$ seconds) lifetime. This implies that a significant fraction of the RH neutrinos can decay outside the Sun, resulting in distinct neutrino and photon signals compared with the usual scenario where DM annihilation produces SM particles inside the Sun. These signals can be used to limit the DM-nucleon elastic scattering cross sections. As we will see, the Fermi-LAT and IceCube data together constrain the SI cross sections better than direct detection experiments for DM masses in the $\sim 200 - 5000$ GeV range.

In this chapter, we adopt a model-independent approach to study the neutrino and photon signals from solar DM annihilation into long-lived RH neutrinos. In Section 5.2, we briefly discuss the case for light RH neutrinos. As we discuss in Section 5.3, depending on its mass, long-lived RH neutrinos can yield characteristic spectra in the neutrino and gamma-ray signals. In Section 5.4, we perform an analysis of the signals at IceCube and Fermi-LAT and obtain constraints in the parameter space consisting of the RH neutrino mass and DM mass. Finally, we conclude the chapter and discuss future prospects in Section 5.5.
5.2 Long-lived right-handed neutrinos

In the simplest case of Type-I seesaw [34, 35, 135, 36], the RH neutrino \( N \) with an arbitrary Majorana mass \( M_N \) mix with the SM neutrinos through a Yukawa term as follows

\[
\Delta \mathcal{L} \supset y_D (L^\dagger \cdot i\tau_2 H) N + \text{h.c.},
\]

(5.1)

where we ignore flavor indices for simplicity. After the Higgs field acquires a vacuum expectation value (vev), the Yukawa term induces a mixing \( \theta \sim \frac{y_D}{M_N} \nu \) between \( N \) and the LH neutrino \( \nu \). We choose the nominal value for the mixing such that it gives rise to the light neutrino mass \( m_\nu \):

\[
\theta \approx \left( \frac{m_\nu}{M_N} \right)^{1/2}.
\]

(5.2)

The mass of each light mass eigenstate \( m_\nu \) receives contributions from mixing with the three RH neutrinos. Hence, for a given RH neutrino, \( \theta \) may be larger or smaller than the nominal value in above. In the former case, mixings from different RH neutrinos must cancel out to give the right value of \( m_\nu \). In the latter case, the other RH neutrinos should make the main contribution to \( m_\nu \).

Although the magnitude of \( M_N \) is often assumed to be much larger than the electroweak scale, some or all of \( N \)'s can have a mass around or below the electroweak scale. This, for example, can happen in the split seesaw scenario [136]. Values of \( M_N \) around the electroweak scale are phenomenologically very interesting as they provide an opportunity for experimental discovery of the RH neutrinos thereby potentially unveiling the mechanism of neutrino mass generation. Masses up to 500 GeV are accessible at the LHC [137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151], and the prospect would be even better at a high-energy lepton collider [152, 137, 153, 154, 155, 156, 157, 158].
RH neutrinos with a mass $M_N < 5$ GeV bring in new possibilities to test them experimentally. They can be searched for in meson decays at $B$ and $K$ factories [159, 160, 161], fixed target experiments [162, 163, 164], and the SHiP experiment [165] proposed at CERN. The implications of such light RH neutrinos for the neutrinoless double-beta decay have also been studied [166]. Here we focus on RH neutrinos with a mass in the $\sim 1 - 5$ GeV range.

Given the eV [167] or sub-eV [168, 169] scale of current neutrino mass limits, heavy neutrinos of a GeV scale mass would imply a nominal mixing $\theta \sim 10^{-6} - 10^{-5}$, see Eq. (5.2). Since this mixing is generally small, from now on we use $N$ to denote the heavy mass eigenstate after the mixing.

$N$ can decay into to SM neutrinos and the Higgs via its dominant singlet component, as well as SM gauge bosons and leptons via its small mixing with LH neutrinos. If $M_N > m_H$, the Higgs decay channel will dominate$^1$. For $m_{W,Z} < M_N < m_H$, the gauge boson channels are dominant. In the case that $M_N < m_{W,Z}$, as we consider here, three-body decays via off-shell $W$ and $Z$ bosons will be the main channels$^2$. The $N$ decay is then dominated by the weak interaction from its SM lepton component, resulting in the following boosted decay width:

$$\Gamma_N \propto \theta^2 G_F^2 M_N^5 \frac{M_N}{M_{DM}}.$$ (5.3)

Here $G_F M_N^5$ is the rest frame decay width (up to a phase space factor), $M_N/M_{DM}$ is due to the Lorentz boost, and $\theta$ is the $N - \nu$ mixing. The detailed expression for the partial widths of leptonic and semi-leptonic decay modes of $N$ are given in [170, 171].

After using the nominal value for mixing in Eq. (5.2), the decay lifetime is found to

---

$^1$Indirect detection signals of DM annihilation to RH neutrinos for this case have been studied in the context of the supersymmetric $U(1)_{B-L}$ extension of the SM [52, 134, 93].

$^2$Three-body decays via off-shell Higgs are subdominant due to small Yukawa couplings of the Higgs to fermions.
be:

$$\tau_N \propto \frac{M_{\text{DM}} m_\nu}{M_N^5}. \quad (5.4)$$

We discuss the details of calculating $\tau_N$ for light RH neutrinos in the next Section. In Fig. 5.1, we show the $N$ lifetime contours in the $M_N - M_{\text{DM}}$ plane that correspond to two characteristics decay lengths: the Sun’s photosphere $R_\odot \approx 700,000$ km, and an ‘escape’ $R \approx 200,000$ km for neutrinos with less than TeV energy. RH neutrino decays outside the photosphere $R_\odot$ give rise to a photon signal. On the other hand, decays happening outside the 200,000 km radius $R$ produce neutrinos that propagate largely unaffected by interactions with the solar medium, while the associated photons are absorbed. We have checked that neutrinos with energy $E_\nu \lesssim 1$ TeV that are produced at distances larger than this experience less than 10% attenuation before completely leaving the Sun (more details on this later on).

The lifetime contours in Fig. 5.1 are shown for three cases when $N$ mixes dominantly with one of the $\nu_e$, $\nu_\mu$, and $\nu_\tau$ flavors according to Eq. (5.2) where $m_\nu \sim m_{\text{atm}} \approx 0.05$ eV. The third case (mixing with $\nu_\tau$) results in a longer lifetime because of the phase space suppression of the $W$-mediated decay channel to $\tau$. The first two cases (mixing with $\nu_e$ and $\nu_\mu$) essentially result in the same lifetime as $M_N \gg m_\mu$.

As seen in Fig. 5.1, boosted lifetimes of 1 to 10 seconds can be readily obtained for $M_N$ of few GeV and $M_{\text{DM}} \sim 200 - 5000$ GeV. We, however, note that the rest frame lifetimes are in principle much shorter than 1 second. Since RH neutrinos can be produced with a significant abundance in the early universe (and may even reach thermal equilibrium), this ensures that their decay does not pose any threat to big bang nucleosynthesis (BBN).
CHAPTER 5. Solar DM Annihilation to Long-Lived RH Neutrinos

Figure 5.1: The characteristic decay length is inside the photosphere (above the solid lines) and is within 200,000 km (above the dashed lines). The colors red, green, and blue correspond to the cases when $N$ mixes dominantly with $\nu_e$, $\nu_\mu$ and $\nu_\tau$ respectively. The red and green curves almost coincide.

5.3 Signals from delayed decays

We now study the photon and neutrino signals from solar annihilation of DM into RH neutrinos within the DM mass range shown in Fig. 5.1\textsuperscript{3}. In order to calculate the photon and neutrino spectra, first we have to find the major decay modes of light $N$ and their corresponding branching fractions.

The decay is dominated by weak gauge interaction and the largest partial width is taken by virtual $W$ channel, $N \to lW^*$, where $W^*$ splits into either $\nu$ and a charged lepton, or a quark-antiquark pair. $N \to \nu Z^*$ has a smaller partial width as $Z^*$ due to larger $Z$ mass. We list all the significant modes in Table 5.1. Note that for a light RH neutrino mass, the partial decay widths of $N$ can be kinematically affected by the mass of final state particles. Also, since $N$ is a Majorana fermion, it decays into a given final state as well as its $CP$-conjugate final state at the same rate.

In the three-body decays of $N$, the SM neutrinos/leptons take a significant fraction of the total energy. The $\mu$, $\tau$ leptons in the final state can further decay into neutrinos.

\textsuperscript{3}The photon signal from galactic DM annihilation and from dwarf spheoridals for heavier $N$ are discussed in [172, 173].
### Table 5.1: Decay channels of a 2.5 GeV right-handed singlet neutrino via its mixing into SM neutrinos. The partial widths are calculated at tree-level. Depending on the mass of $N$, not all channels are kinematically allowed. The final column differs because $N$ does not produce various final states that contain a $\tau$. The $(q\bar{q})$ system in the hadronic modes would hadronize into different combination of pions with respect to the total electric charge.

<table>
<thead>
<tr>
<th>Decay Mode</th>
<th>$\nu_e$ mixing case</th>
<th>$\nu_\mu$ mixing case</th>
<th>$\nu_\tau$ mixing case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N \to l(q\bar{q})$</td>
<td>44.6%</td>
<td>44.3%</td>
<td>1.7%</td>
</tr>
<tr>
<td>$N \to \nu_l(q\bar{q})$</td>
<td>19.9%</td>
<td>20.1%</td>
<td>53.5%</td>
</tr>
<tr>
<td>$N \to l(\nu_l\bar{q})$</td>
<td>13.1%</td>
<td>13.2%</td>
<td>1.1%</td>
</tr>
<tr>
<td>$N \to l\nu_l\bar{l}$</td>
<td>7.8%</td>
<td>7.6%</td>
<td>0%</td>
</tr>
<tr>
<td>$N \to \nu_l(l'\bar{l'})$</td>
<td>1.6%</td>
<td>1.7%</td>
<td>8.6%</td>
</tr>
<tr>
<td>$N \to \nu_l(\nu_l\bar{\nu}_l)$</td>
<td>6.5%</td>
<td>6.6%</td>
<td>17.6%</td>
</tr>
<tr>
<td>$N \to \nu_l\nu_l\bar{\nu}_l$</td>
<td>6.5%</td>
<td>6.6%</td>
<td>17.6%</td>
</tr>
</tbody>
</table>

Since $N$ has an energy $M_{DM} \gg M_N$, the charged leptons and neutrinos from $N$ decay acquire large energeties due to the Lorentz boost. Neutrinos and photons produced from these energetic leptons (also from the hadronization and shower in semileptonic decays) yield the high energy neutrino and gamma ray signals for indirect searches.

The neutrino signal, due to IceCube detection thresholds and the fact that the neutrino scattering cross-section increases with energy, receives most of the contribution from high energy part of the neutrino spectrum. In our case, the ‘hard’ part of the spectrum is dominated by the neutrinos that directly emerge from the three-body decay of $N$. Secondary neutrino arise from the charged lepton and pion decays in the final state, but their contribution is subleading due to the much lower energy after several decay steps. We note that the hard part of the spectrum is not suppressed for delayed $N$ decays, unlike the standard scenario where neutrinos from DM annihilation are produced inside the Sun. For DM mass above few hundred GeV, the neutrino signal can be detected by IceCube.

The photon signal has two major components: (1) the charged lepton’s bremsstrahlung radiation that yields a soft power-law shaped spectrum, and, (2) the neutral pion decay $\pi^0 \to \gamma\gamma$ that arises abundantly in final states with a $\tau$ and also directly from the
$N \rightarrow \nu Z^*$ decay channel. The bremsstrahlung contribution is mostly determined by a logarithmic dependence on the mass of the leading lepton in energy. As the leading lepton energy spectrum is universal (at least in the kinematically unsuppressed case), the bremsstrahlung strength can be directly inferred from the lepton flavor composition in $N - \nu$ mixings. In general, the bremsstrahlung produced far fewer energetic photons than pion decay, but dominates the low energy part of the photon spectrum.

**Figure 5.2:** $\nu_\mu$ (top) and photon (bottom) spectra from delayed decay of RH neutrinos produced in DM annihilation. In these plots, $M_{DM} = 1$ TeV and $M_N = 2.5$ GeV. The spectra are for $N$ mixing with each of the $\nu_e$, $\nu_\mu$, and $\nu_\tau$ flavors.

The neutral pion contribution needs to be addressed more carefully. Apart from the usual energy fragmentation in $\tau$ decays that is well implemented in analysis tools, the GeV-scale mass of $N$ itself introduces decay channels like $N \rightarrow l/\nu +$ pions, where neutral pions arises directly in the hadronic decays of $W^*$ and $Z^*$. Depending on the phase space in the hadronization process, $\pi^0$ can either emerge from a multi-pion final
state of a charged $q\bar{q}'$ system, or from the decays of excited states like $\eta$, at non-trivial branching fractions. A thorough exploration of the hadronization modes in $N$ decay would be beyond the scope of the current chapter. In our numerical simulation of the prompt photon spectra, we choose our benchmark $M_N$ to be the same as $m_\tau$, where the string fragmentation in the PYTHIA package successfully reproduces the multi-pion branching fractions as those in the semileptonic $\tau$ decay. We therefore limit ourselves to $M_N \gtrsim m_\tau$, as shown in Fig. 5.1.

We implement the neutrino mixing with FeynRules [112] and calculate the DM annihilation process with CalcHEP [113]. Final state radiation and showering are processed with PYTHIA 8 [174]. Since the delayed decays of $N$ occur in vacuum, we require all unstable particles, in particular muons and mesons, to fully decay in the final state. We rely on PYTHIA’s fragmentation scheme for the pion multiplicity in semileptonic $N$ decays. Fig. 5.2 shows the neutrino and gamma ray spectra from delayed decays of $N$ produced in annihilation of 1 TeV DM particles. We picked $M_N = 2.5$ GeV

In the calculation of the signal flux, we make the assumption that the DM capture and annihilation inside the Sun have reached an equilibrium. For a given $\sigma_{SD}$ or $\sigma_{SI}$, we use DarkSUSY [76] to calculate the minimum annihilation rate $\langle \sigma_{\text{ann}} v \rangle$ for equilibrium. Fig. 5.3 illustrates sample spectra that correspond to $\sigma_{SI} = 8.5 \times 10^{-45}$ cm$^2$ for $M_{DM} = 1$ TeV, which saturates the latest LUX bound [16].

As the $N$ decay occurs between the solar center and the Earth, and the source intensity decreases exponentially over distance. The large Lorentz boost along the line-of-sight forces most of the $N$ decay products to fall along the forward direction, pointing away from the Sun. Very soft photons may still deviate away from the line-of-sight direction. We adopt an half-cone radius cut of 1.5° as given in Fermi-LAT observation on solar disk [176], and integrate over the decay’s source intensity within.
Figure 5.3: $\nu_\mu$ (top) and photon (bottom) spectra from delayed decay of RH neutrinos produced in DM annihilation. In these plots, $M_{DM} = 1$ TeV and $M_N = 2.5$ GeV. For normalization, we assume a total annihilation inside the Sun to be $1.5 \times 10^{19}$ per second. The “equal” curves is for the case when $N$ mixing with all of the $n_e$, $\nu_\mu$, and $\nu_\tau$ flavors equally. The neutrino background is atmospheric neutrinos [175]. The shaded region around the Fermi data shows the uncertainty from Ref. [176].

We have checked this 1.5° angular cut sufficiently covers the source intensity for all the photons in the relevant energy range (in Fermi-LAT’s energy reach).

Note that in case of a shorter boosted lifetime, a fraction of decays may still happen inside the Sun. For gamma rays we take a simple cut at the Sun’s photosphere radius $R_\odot \approx 700,000$ km, and consider the decays outside the photosphere contribute to the signal. However, the situation is more complicated for the neutrino signal. Since attenuation of the neutrino flux is energy dependent, the distortion in the energy spectral shape depends on the DM mass, the prompt spectral shape, and the distance inside the solar medium. Due to flavor oscillations, the attenuation effects also differ...
between different neutrino flavors. In the chapter, we mainly focus on sufficiently long-lived \( N \) that decays outside the Sun, so we make an approximation that cuts off \( N \) decays below \( R \approx 200,000 \) km and consider neutrino fluxes that emerge from outside this radius as unattenuated. This selection is based on the fact that at TeV energies and below, the flux of neutrinos produced above this distance is suppressed by less than 10%.

The decays occurring at smaller distances are conservatively considered to occur at the Sun’s core, so WimpSim [85] could be used. We have run annihilation directly to all flavors of neutrino and antineutrino at a larger number of masses between 10 GeV and 5000 GeV, which could be used to construct general annihilation channels. Overall, this approximation is a reasonable simplification even though it may underestimate the neutrino signal for the intermediate \( N \) decay lengths close to \( R \approx 200,000 \) km.

We implement neutrino oscillations with the latest global analysis of the 3-neutrino oscillation data [25]. Averaging of oscillations due to the Earth’s orbital eccentricity during the southern winter when the Sun is below the horizon is also included. Because IceCube has angular sensitivity for muon tracks that result from conversion of \( \nu_\mu \)’s only, the different flavor schemes in \( N - \nu \) mixing can yield quite different \( \nu_\mu \) fluxes at the Earth.

## 5.4 Experimental constraints

Assuming an equilibrium between DM capture and annihilation in the Sun and using the signal spectra calculations in the previous section, we derive constraints on \( \sigma_{SI} \) and/or \( \sigma_{SD} \) from Fermi-LAT and IceCube limits on the photon and neutrino signals respectively. For the DM mass range considered here, we assume the nominal annihilation cross-section \( \langle \sigma_{\text{ann}} v \rangle = 3 \times 10^{-26} \) cm\(^3\) s\(^{-1}\), which achieves the capture-


annihilation equilibrium for the scattering cross-sections $\sigma_{SI}$ and $\sigma_{SD}$ that are shown in Figs. 5.4, 5.5, 5.6, and 5.7. In these figures, the three panels illustrate exclusive $N - \nu_e$ (upper), $N - \nu_\mu$ (middle) and $N - \nu_\tau$ (lower) mixing cases separately, where in each case $N$ only mixes with one SM neutrino flavor.

The photons that decay in the detectable energy range were compared to the highest energy bin of Fermi-LAT data in [176]. This final bin captures the ability to distinguish between our signal and the current Fermi-LAT observation. Including more bins hurts the statistics since Fermi’s data increases much more than our simulated data (and a chi-squared analysis is not appropriate because we do not have a background to add to our simulated signal). The contours for $\sigma_{SD}$ (Fig. 5.4) and $\sigma_{SI}$ (Fig. 5.5) show the value of respective cross sections ruled out at 90% C.L. occurring when $S_{DM} - S_{Fermi} = 1.64 \sigma_{Fermi}$, where $S_{DM}$ is the photon signal from DM annihilation in Fermi-LAT highest-energy bin, $S_{Fermi}$ is what Fermi-LAT detects in their highest energy bin, and $\sigma_{Fermi}$ is Fermi’s published 1$\sigma$ uncertainty in that bin. We could do better if the expected photon background from the Sun were theoretically predicted. The shaded regions in Figs. 5.4, 5.5 are ruled out by the current direct detection experiment limits from PICO-60 [177] and LUX [16] respectively.

We see that the 90% C.L. values in Figs. 5.4, 5.5 get tighter as $M_N$ decreases for a given $M_{DM}$. This is because of an increase in $\tau_N$, which results in a larger fraction of RH neutrinos decaying outside the photosphere. Increasing $M_{DM}$ with $M_N$ kept constant also initially results in stronger bounds as larger Lorentz boosts lead to longer lifetimes for $N$. However, this reverses and the contours quickly drop beyond with further increase in $M_{DM}$. The reason being that for higher DM masses the number of photons whose energy is below that detected by Fermi [176] decreases, and also the total flux from the Sun is suppressed because a smaller number of DM particles are captured. We note that, as expected, the tightest bounds are obtained
in the case that $N$ mixes with $\nu_{\tau}$ as hadronic $\tau$ decays result in abundant production of photons.

To constrain the neutrino signal, we use the 90% C.L. IceCube bound in Ref. and compare the muon event rates per volume per year for detection. The bound on $\sigma_{SI}, \sigma_{SD}$ assuming DM $DM \rightarrow NN$ annihilation channel can be obtained by scaling these scattering cross-sections so that they yield the same muon event rates as the channels given in Ref. [126]. We choose the $\tau\bar{\tau}$ channel to compare the muon event rates, which we computed with the DarkSUSY package (we ran GENIE [110] to get the muon signal from $N$ decay, and DarkSUSY’s muon spectra are within 4% of what GENIE calculates). For various DM masses $M_{DM} = [200, 500, 1000, 2000, 5000]$ GeV, we have found that $[505, 125, 60, 50, 40]$ km$^{-3}$ yr$^{-1}$ contained muons above an energy cut of $[75, 100, 150, 175, 200]$ GeV are needed respectively. The energy cut was chosen to optimize the statistics by reducing background, but the cut (as well as the annihilation channel) had very small effect on our final results. We have scaled $\sigma_{SD}$ and $\sigma_{SI}$ to obtain the same number of muons (plus antimuons) in our case. The contours for $\sigma_{SD}$ (Fig. 5.6) and $\sigma_{SI}$ (Fig. 5.7) show the value of respective cross sections ruled out at 90% C.L. according to IceCube’s sensitivity. As we see in Fig. 5.6, IceCube does much better than direct detection experiments in the case of $\sigma_{SD}$ as PICO-60 limits are much weaker. The situation is different for $\sigma_{SI}$, see Fig. 5.7, where the shaded regions are already ruled out by the LUX results [16].

We see that the 90% C.L. values in Figs. 5.6, 5.7 get tighter as $M_N$ decreases for a given $M_{DM}$ because it leads to a longer $\tau_N$ resulting in a larger fraction of RH neutrinos decaying outside the 200,000 km radius. However, the contours above and below the dashed line (denoting the lifetime corresponding to a characteristic radius of 200,000 km) behave differently with increasing $M_{DM}$ when $M_N$ is kept constant. For those above the line, a significant fraction of $N$ decays happen inside this radius. Therefore, the bound initially gets weaker as a larger Lorentz boost in the energy
of neutrinos results in more interactions with the solar medium that suppresses the neutrino signal. This reverses for larger values of \( M_{\text{DM}} \) for which a larger fraction of RH neutrinos decay outside the 200,000 km radius. The situation is opposite for the contours below the dashed line, for which the majority \( N \) decays occur outside this radius. Increasing \( M_{\text{DM}} \) initially results in more energetic neutrinos, and hence a stronger neutrino signal at IceCube. Further increase in \( M_{\text{DM}} \), however, leads to a smaller neutrino flux as a smaller number of DM particles are captured. As in Figs. 5.4, 5.5, the tightest bounds arise in the case that \( N \) mainly mixes with \( \nu_\tau \) because of additional neutrinos from \( \tau \) decay. In fact, see Fig. 5.6, IceCube can do better than direct detection experiments in constraining \( \sigma_{\text{SI}} \) in this case.

We see in Figs. 5.4, 5.6 that the neutrino signal alone rules out the entire parameter space considered here in the case of \( \sigma_{\text{SD}} \). In the case of \( \sigma_{\text{SI}} \), the photon signal rules out a sizable region of the parameter space for the \( N - \nu_e \) and \( N - \nu_\mu \) mixing cases. The photon and neutrino signals combined together rule out about half of the parameter space for the entire DM mass range in the case of \( N - \nu_\tau \) mixing (the former for \( M_{\text{DM}} \) up to \( \sim 2.5 \) TeV and the latter for \( M_{\text{DM}} \sim 2.5 - 5 \) TeV). The region ruled out by the photon signal can be extended to higher DM masses by future data from gamma ray observatories, like HAWC [178] and DAMPE [179], that can detect photons with higher energies than those detectable by Fermi-LAT.

## 5.5 Conclusions

In this chapter, we have performed a study of indirect detection signals from solar annihilation of DM into RH neutrinos \( N \) with a mass \( M_N \sim 1 - 5 \) GeV. These RH neutrinos dominantly decay via off-shell \( W \) and \( Z \) due to their small mixing with the LH neutrinos. For DM masses \( M_{\text{DM}} \sim 200 - 5000 \) GeV, and nominal value of mixing expected in Type-I seesaw, the RH neutrinos can have a lifetime \( \tau_N \sim 1 - 10 \)
s and escape the Sun before decaying. The delayed decays then give rise to a photon signal in the direction of the Sun, as well a neutrino signal that is not attenuated by absorption and scattering in the solar medium.

The strongest signals are obtained in the case that RH neutrinos produced from DM annihilation mainly mix with $\nu_\tau$. Then, for $M_N > m_\tau$, delayed decays of $N$ produce taus whose decay produces more photons (due to their hadronic decays) and neutrinos than muons and electrons. We have used the Fermi-LAT and IceCube limits on the photon and neutrino signals, respectively, in the direction of the Sun to constrain the product of the branching fraction of DM annihilation to RH neutrinos and the DM-nucleon elastic scattering cross sections at 90\% C.L.

The Fermi-LAT sets stringent bounds on both $\sigma_{SI}$ and $\sigma_{SD}$. For DM masses up to $\sim 2.5$ TeV the limits on $\sigma_{SI}$ are significantly tighter than the most stringent ones from direct detection experiments [16, 128]. The IceCube also sets limits on $\sigma_{SI}$ that are stronger than those in [16, 128], for DM masses in the $2.5 - 5$ TeV range, in the case that $N$ mixes mainly with $\nu_\tau$. Both Fermi-LAT and IceCube set bounds on $\sigma_{SD}$ that are much tighter than the strongest limits from direct detection experiments [177].

The neutrino signal from delayed decays of light RH neutrinos can probe the DM-nucleon elastic scattering cross sections for DM masses up to several TeV. This is much better than the usual scenario where neutrinos produced from solar DM annihilation are highly suppressed because of absorption and scattering in the Sun. The photon signal can also lead to stronger constraints at larger values of $M_{DM}$ by using data from experiments like HAWC that are sensitive to gamma rays with higher energies than those detectable by Fermi-LAT.
Figure 5.4: The 90% confidence level (C.L.) contours for $\sigma_{SD}$ using the Fermi-LAT limits on the photon signal from solar DM annihilation to light RH neutrinos. The shaded region is ruled out by PICO-60 limits on $\sigma_{SD}$ [177]. The characteristic length of the decays is inside the photosphere above the dashed line. BF is the branching fraction to RH neutrinos.
Figure 5.5: Same as Fig. 5.4 but for $\sigma_{SI}$. The shaded region is ruled out by LUX limits [16].
Figure 5.6: The 90\% C.L. contours for $\sigma_{SD}$ using IceCube limits on the neutrino signal from solar DM annihilation to light RH neutrinos. The dashed line now marks when the characteristic length of the decays is inside 200,000 km.
Figure 5.7: Same as Fig. 5.6 but for $\sigma_{SI}$. 
Chapter 6

Conclusions

A conclusion is simply the place where you got tired of thinking.

Dan Chaon, Staying Awake

The identity of DM and the exact nature of the neutrino sector are prominent mysteries in particle physics and cosmology. Experimental searches for DM are trying to determine if or how it scatters off nucleons and to determine if or how it self annihilates. Developing the theoretical models that interrelate potential observations is an essential aspect of this search. There are various models to explain neutrino mass, and there are many experiments attempting to measure masses and to measure the mixing parameters more precisely.

Indirect detection of DM provides an excellent means of probing the possible connection between DM and neutrinos:

- If WIMPs annihilate primarily to neutrinos and the present-day DM annihilation rate may be suppressed, the Sun may have still captured enough DM to annihilate sufficiently for near-future detection at IceCube with $\sigma_{\text{SD}}$ sensitivity competitive with direct detection. In many years assuming DM mass is known and $\langle \sigma_{\text{ann}} v \rangle \gtrsim 10^{-28} \text{ cm}^3 \text{ s}^{-1}$, the neutrino annihilation channels may be distin-
guished from $W$-like annihilation channels at IceCube solely from the shapes of the energy distributions of solar data. See Chapter 3.

- In the type-II seesaw mechanism (an extension of the SM that allows neutrinos to have mass), a scalar triplet may primarily decay to the most massive neutrino flavor. A scalar DM particle naturally couples to this scalar triplet, so an annihilation of DM to the scalar triplet can essentially be an annihilation to the most massive neutrino flavor. While being consistent with LHC bounds, direct detection limits, and a presently-non-observed photon signal from DM annihilation in the Galactic Center, future determination of the neutrino mass hierarchy is conceivable at detectors like IceCube (with the complimentary help of the LHC and neutrinoless double-beta decay experiments to determine model parameters). A different amount of muons will be produced in IceCube’s ice depending on initial neutrino flavor at production in Sun or Galactic Center. Beta decay experiments and oscillation experiments may also determine the hierarchy, but using DM annihilation is arguably the most unique approach. See Chapter 4.

- In the type-I seesaw mechanism for giving neutrinos mass, DM may primarily annihilate to right-handed neutrinos. If produced inside the Sun, $\tau$-mass right-handed neutrinos can easily decay outside the Sun’s photosphere. At these masses, they decay to left-handed neutrinos and an off-shell $Z$ or to charged leptons and an off-shell $W$. The photons and (left-handed) neutrinos that result from this decay suffer no absorption from the Sun. The neutrino signal is—as is typical of DM annihilation inside the Sun producing neutrinos—competitive with direct detection experiments, especially if DM primarily scatters off nucleons via $\sigma_{SD}$. However, the interesting and most promising signal is that of the photons, which would be completely absorbed by the Sun if not for the long
lifetime of the right-handed neutrino. Using Fermi-LAT’s data from the Sun, \( \sigma_{\text{SD}} \) larger than \( 10^{-42} \) cm\(^2\) can be ruled out (for \( m_{\text{DM}} = 1000 \) GeV), and \( \sigma_{\text{SI}} \) larger than \( 10^{-45} \) cm\(^2\) can be ruled out (for \( m_{\text{DM}} = 1000 \) GeV). See Chapter 5 for detailed results for a range of \( m_{\text{DM}} \) and right-handed neutrino masses.

With the tools available (see appendices), the indirect-detection signals at Earth from a wide range of scenarios can be calculated. However, a challenge is for various detectors to understand and publish their detectors’ capabilities in a way that theorists—with the understanding of the relevant background/noise—can quickly convert their calculated signals to a probability of detection. This challenge can be overcome in various ways. Chapter 4 largely avoided this challenge by simply providing the muon rates in ice from signal and background (because general future detectors were of interest). Chapter 5 simulated the signals of published IceCube results to understand how many muons per volume per year are needed above an energy cut to make a detection at 90% confidence, which was used to find the cross sections needed for more exotic signals to be detected. In Chapter 3, an even more precise method—modeling effecting volume/area and the energy and angular resolution of the detector—was used to calculate IceCube’s capacity to explore interesting scenarios. However, this was still a crude “theorist fit.” I hope for a future where the approach of Chapter 3 is simple and accurate. Recently, the IceCube collaboration has published the effective areas and volumes for neutrinos and for muons for the old 79-string IceCube experiment\(^1\) that include energy-dependent energy resolution (see the first 3 figures of [126]), which are great improvements.

As detectors continue to improve, the possible DM parameters will be further constrained. HAWC will extend our ability to detect gamma rays from Fermi-LAT’s \( \sim 200\)-GeV upper range to \( \sim 100 \) TeV [180]. IceCube may get improvements (such as PINGU and IceCube-Gen2) to better study neutrinos. As for direct detection, the

\(^1\)http://icecube.wisc.edu/science/data/ic79-solar-wimp
LZ experiment will improve sensitivity to both $\sigma_{SD}$ and $\sigma_{SI}$. New particle accelerators will be built (and the LHC will keep taking data) to further constrain $\sigma_{SD}$ for models of DM.
Appendix A

Software Overview

Never trust a computer you can’t throw out a window.

Steve Wozniak

Besides physics classes, learning how to do research, and learning about a specific field of research, being able to quickly harness the power of computers has been invaluable. Below is a brief summary of how to get software to do our calculations.

If email, file sharing, image editing (GIMP!), and running precompiled software (such as the highly useful Mathematica or MATLAB) is all that is needed, there is no need to continue. However, a computer with a coding environment is required for the more specific needs of scientific computation. OS X (which is Unix) with MacPorts or Linux is assumed for the following, and learning these systems is highly recommended due to their powerful and easily-expandable command-line terminals/shells and due to the standard use of Linux/Unix for scientific computing. LaTeX, command-line compilation abilities for C++, and GFortran should be installed and learned as soon as possible and mastered as needed. Having a good text editor and being able to write clever Bash scripts for the terminal/shell can help immensely. Many tools will hopefully be discovered along the way (gnuplot, the GRABIT package for MATLAB, pdftk, etc.). Of course, practically nothing works the first time, so the most useful
skill is dissecting a procedure (with the help of Google and colleagues) until something works. If luck is present, the software being used is written by a physicist who had both the time and knowledge to write good code instead of an undocumented kludge.

Many—but certainly not all—of the main software packages used within particle physics and dark matter studies are presented below, specialized for our purposes. A goal is to quickly be able to do basic “hello world” tasks with them to become quickly familiar with core functionality (modifying something that works for another purpose can be much easier than getting anything to happen at all), and such pedagogical tasks can be very difficult to find. Another goal is to discuss various undocumented observations, hacks, and tricks on a very practical level, which can be the difference between failure and success when trying to achieve a goal.

As Steve Wozniak’s quote at the beginning of this appendix could suggest, the power (and sometimes fun) of computers occurs when tinkering with and breaking them. Tinkering is essential as it shows us how to enhance a computer’s abilities and helps us understand the physics behind the results a computer may provide, which is the true goal (after which, I suppose the computer can be thrown out a window).

### A.1 DarkSUSY

DarkSUSY [76] is a software package for doing SUSY DM calculations, but we instead want to generalize it away from SUSY to do a model-independent study of DM annihilation in the Sun. Much detail is provided here for this relatively-easy-to-use software so that it can serve as an example for later software.

To install, download the current version from darksusy.org. Only source code and data are downloaded, which must be compiled to be run (creates executable files specialized for the compiling computer). To extract, configure, and compile, run
something similar to the following commands (doing this in our user’s home directory is a good idea):

```
tar zxvf darksusy-5.1.3.tar.gz
cd darksusy-5.1.3
./conf.gfortran
make
make docs
```

The `test` folder contains the example Fortran (.f) programs that can be edited and then compiled according to the rules in `makefile`. For example, to compile and run `dstest-isasugra.f`, run the following commands inside the `test` folder:

```
make dstest-isasugra
./dstest-isasugra
```

After playing with the codes in the `test` folder and when now making the code for a research project, a great idea for convenience of use and backing up is to setup a folder for the project outside all the various software folders by creating `makefile(s)` in the research folder using `test/makefile` as a template.

To generalize DarkSUSY, the base DarkSUSY code must be extended. It is possible to add .f files to the folders in the `src` folder, modify the `makefile` in that folder to include the new .f files, then, in the main DarkSUSY folder, run `make`.

A basic and useful extension is to be able to manually set annihilation branching fractions (that is, to prevent DarkSUSY from using the annihilation branching fractions calculated from a specific SUSY model). To do this, we can create a file called `src/an/dssigmavReset.f` that contains:

```
subroutine dssigmavReset
  implicit none
  integer i
  include 'dsprep.h'
  do i=1,29
    sigv(i) = 0.0d0
  enddo
end
```
Then create a file called src/an/dssigmavSingle.f that contains something like:

```fortran
subroutine dssigmavSingle(ch, br, mysigmav)
implicit none
integer ch
real*8 br, mysigmav
include 'dsprep.h'

newmodelsigmav=.false.
sigmav=mysigmav
if (ch.eq.1) then
  sigv(1) = sigmav*br ! H1 H1 = H0
elseif (ch.eq.2) then
  sigv(2) = sigmav*br ! H1 H2 = H0 h0
  .
  .
elseif (ch.eq.29) then
  sigv(29) = sigmav*br ! Z gamma (1-loop)
else
  write(*,*) 'Channel not programmed into dssigmavSingle.f'
endif
end
```

where the annihilation channels (given by the variable `ch`) are described in src/an/dssigmav.f, and the variable `br` is the branching fraction to that channel.

We can use these new subroutines to calculate neutrino fluxes and muon rates at detector due to self-annihilation in the Sun, now with custom DM mass and custom annihilation channels. The following program, createSignal.f, can be saved anywhere, but putting it in the `test` folder then editing the `makefile` is easiest.

```fortran
program createSignal
implicit none
real*8 mx, sigsip, sigsdp, sigv
integer ch
include 'dsmssm.h'
mx=200d0   !DM mass in GeV
sigsip=0   !spin-independent cross section in cm^2
sigsdp=1.0d-41 !spin-dependent cross section in cm^2
```
The code represented by the above dots would call the dsntdiffrates subroutine, which would be inside loops over energies and angles. These loops would also write results to a file (more loops over other parameters can be created; then just relax while the computer does all the work). See the DarkSUSY manual in docs folder or online for the description of dsntdiffrates. However, dsntdiffrates cannot take advantage of the cross sections that have been set. For this reason, a new modified version of src/nt/dsntdiffrates.f should be created that allows for the variables sigsip and sigsdp to be passed and used. The program createSignal.f now is completely free from needing a SUSY model (unless we want to change other parameters, which can become more complicated).

Though not in the documentation, a careful analysis revealed that angular values of 0.1°, 0.2°, 0.3°, etc. should be sampled. Energy should be sampled at 0.01%, 0.03%, 0.05%, …, 0.99% of the DM mass.

Another very useful basic program to write is getAnnihilations.f, which returns the annihilation rate per year in Sun (or Earth) as a function of DM mass given capture cross sections and \( \langle \sigma_{\text{ann}} v \rangle \). The important line is the call to dsntannrate.
If solar equilibrium is desired, a trick is to set $\langle \sigma_{\text{ann}} v \rangle$ to a large value such as 1 cm$^3$/s. Another trick is to then set the desired capture cross section to 1 cm$^2$ so that the annihilation rates created in ann.csv can easily be multiplied by the desired capture cross section in subsequent analysis without having to recompile and re-run the Fortran code.

Dsntdiffrates only provides muon neutrinos at detector. It is possible to create a new version of dsntdiffrates (and dsntmuonyield which dsntdiffrates calls) that allows the flavor of neutrinos at detector to be chosen. However, the .bin files in share/DarkSUSY/ must contain the data, so .bin files of the correct types (see src/wa/dswayield.f
for types) are needed, where the type is given by the final number in the .bin file’s filename. Older versions of DarkSUSY have these .bin files, which we can download and copy over. Or, if the older version is not yet compiled, the corresponding .dat.gz files (software called WimpSim created files that were later converted to these .dat.gz files) can instead be copied over then share/DarkSUSY/tab-install.pl can create the .bin files. Finally, the variable wabase in src/wa/dswacom.f should be correctly set.

Atmospheric backgrounds can be obtained via the dshonda and dsatm_mu functions, but one should ideally instead use the newest (un-oscillated) neutrino data found at www.icrr.u-tokyo.ac.jp/~mhonda/ then use software called GENIE to interact these neutrinos in the ice.

Finally, data files have been created. It is highly recommended to get data files into sophisticated programs such as Mathematica, MATLAB, or open-source alternatives as soon as possible for any further analysis. Fortran (or C++) are not ideal for analysis due to being designed with computer hardware in mind rather than with analysis in mind. Some minimalistic people prefer SciPy Stack (Python libraries) for analysis.

A.2 WimpSim

Given any mass, neutrino parameters, and annihilation channel, WimpSim [85] simulates the propagation of neutrinos through the Sun, space, and Earth (or just through Earth if they originated there). Due to the code being a Monte Carlo, running WimpSim can take weeks for the best results.

DarkSUSY contains pre-run WimpSim data for a single set of neutrino parameters at sampled DM mass points. Sometimes, WimpSim should be directly run to study how the neutrino parameters affect neutrino spectra at detector. Another reason to directly run WimpSim is because DarkSUSY interpolates between mass points,
which is not exactly correct in the case of prompt-neutrino annihilation channels, which require more mass points than DarkSUSY provides because of the sensitivity of a spiked energy distribution to small changes in parameters (which average out for wider energy distributions of the other annihilation channels).

To install, download the code and PDF documentation from copsosx03.fysik.su.se/wimpsim/ then extract. Over the years, this URL has been unstable, but the website always has eventually appeared somewhere. Before compiling, a few things must be done for which I use GFortran (I have no idea why the proprietary ifort is the default Fortran compiler).

We need to install the other software on the website’s download page. To compile PYTHIA, run

```
gfortran -c pythia-6.4.26.f -o pythia.o
```

To compile Nusigma, first change `ifort` to `gfortran` in `mains/makefile` and `src/makefile`, and `mains/makefile` also needs the DarkSUSY’s main folder location and `pythia.o` locations to be correctly set. Nusigma can now be compiled by running

```
make
```

To configure WimpSim’s compilation, edit the `makefile` to have the correct locations of the main DarkSUSY folder, the main Nusigma folder, and `pythia.o`. Then, change `ifort` to `gfortran` in the following files: `wimpann/src/makefile`, `wimpann/mains/makefile`, `contrib/sla-2.5.5/makefile`, `wimpevent/src/makefile`, and `wimpevent/mains/makefile`. WimpSim can now be compiled by running

```
make
```

The next task is to edit the initialization scripts. By editing `scr/wasetup.pl` and `scr/wesetup.pl` (both must be edited), we can select the desired DM masses and annihilation channels (channel numbers are found in WimpSim’s PDF documentation), and running data from annihilations in the Earth can eventually be prevented via a good amount of edits. Also, I recommend fixing the lines that say
WimpSim was coded to run annihilation channels of particle-antiparticle pairs. However, it may be necessary or interesting to have a more general functionality of being able to run the particle and antiparticle separately. To do this in the case of prompt-neutrino channels (12, 13, and 14), hack the file wimpann/src/gennu.f by—assuming WimpSim3.05—commenting out lines 43 through 46 then changing line 47 to say $N=1$. Run `make` to compile WimpSim again, and the software has now been updated to run only neutrinos. To do just antineutrinos, change `kf1` in line 40 to `kf2` and recompile WimpSim. This hack can be generalized to all annihilation channels by running the prompt-neutrino channels many times for all $m_{\text{DM}}$ and, in later analysis, weighting them appropriately with the channel’s neutrino energy distribution at production in the Sun.

We can now setup the run folders by running the following commands

```
$ scr/wasetup.pl myfolder in/osc-pdg2012.in 1000000
$ scr/wesetup.pl myfolder -90 0 2 D0V 2013 D0A 2013 3 1 0
```

where all the options (including the contents of the .in file) can be customized (see the tops of the .pl files for documentation). The first script sets up WimpAnn, which is for simulating neutrinos up to the detector, and the second script sets up WimpEvent, which is for taking the resulting neutrinos and simulating their interactions with a detector that can have a variable distance from the Sun depending on the specified dates. Luckily, future compilations of WimpSim will not affect the fully-self-sufficient run folder that has been created.
To finally do the runs, I recommend creating and running a Bash script similar to

```bash
#!/bin/bash
cd ~/wimpsim-3.05/myfolder
scr/run-wa-rev.one &
scr/run-wa.one
scr/run-we-rev.one &
scr/run-we.one
```

where the `rev.one` scripts are used in parallel with the regular scripts in order to take advantage of my laptop’s two CPU cores.

Due to the possibly very long run times, the ability to stop or pause a run is useful, but it is tricky because new processes/runs are continuously created by the `.one` scripts, so simply stopping the current run only begins the next one (and leaves the interrupted run in an unfinished state). If not in a huge rush, just change the name of the `runs` folder so that the computer will finish the current run(s) in `runs-running` folder then stop. After it stops, change the folder name back. Running the `.one` scripts again will pick up where it left off! If a run is instead forcibly stopped, restarting it is more complicated than simply putting the files in the `runs-running` folder back into the `runs` folder because WimpAnn has already created a .dat file and WimpEvent has already performed gunzip.

Final data can be found in WimpEvent’s `dat-we` folder. WimpSim’s website has a great description of the format of the data if we navigate to the Results page then the Data files page. WimpAnn results are no longer needed and can take up much disk space since the data is for every neutrino rather than being like WimpEvent’s statistical results. Consider deleting the `dat-wa` folder if needed.

### A.3 GENIE

GENIE [110] is a Monte Carlo for simulating neutrino interactions with a detector.
Installing is tricky. Download GENIE’s manual at genie-mc.org and read through the website’s installation instructions. To get the installation to work on Ubuntu (which is a Linux distribution that uses Debian’s .deb packages), I first installed the following packages (the dev at the end is important and crucial for making things work): liblog4cpp5-dev, libxml2-dev, liblhapdf-dev, libgsl0-dev, and subversion. PYTHIA 6 is needed, but do not download it. Instead, download build_pythia6.sh from GENIE’s website which will download the appropriate version and compile it in the exact way that GENIE requires (an entire folder containing many folders is created instead of just pythio.o being created).

A recent version of ROOT’s source code must first be downloaded and compiled. ROOT is the very powerful software used at the Large Hadron Collider to analyze huge amounts of data. To get it to compile on Ubuntu, I had to install the following packages: libx11-dev, libxpm-dev, libxft-dev, and libxext-dev. ROOT will now compile, but, for GENIE to compile afterwards, my procedure for compiling ROOT is

```
sudo apt-get install libgsl0-dev # for MathMore to compile
cd ~/root/
./configure linuxx8664gcc --enable-pythia6 --enable-mathmore \
    --with-pythia6-libdir=/home/brad/pythia/v6.424/lib
make -j 3
```

where the option on the make command allows it to use more CPU threads to finish in less than an hour. It is now important to add the following to the shell’s config file (perhaps ~/.bashrc) to let ROOT configure the terminal/shell

```
cd root
  bin/thisroot.sh
cd
```

Restart the terminal/shell, and type root to try it out! Note that typing .q quits.

Before downloading and compiling GENIE, add something like the following to the shell’s config file
GENIE="/home/brad/GENIE"
export GENIE
LHAPATH="/usr/share/lhapdf/PDFsets"
export LHAPATH
PATH=PATH:/home/brad/GENIE/bin
LD_LIBRARY_PATH=LD_LIBRARY_PATH:$GENIE/lib:/home/brad/pythia/v6.424/lib
LD_LIBRARY_PATH=LD_LIBRARY_PATH:/usr/lib:/usr/lib/x86_64-linux-gnu

GENIE can be downloaded via

```
svn list http://genie.hepforge.org/svn/generator/branches
svn co http://genie.hepforge.org/svn/generator/branches/R-2.8.6/GENIE
```

and finally compiled for my Ubuntu setup by doing

```
cd $GENIE
./configure --with-lhapdf-lib=/usr/lib/x86_64-linux-gnu \
  --with-lhapdf-inc=/usr/include \
  --with-pythia6-lib=/home/brad/pythia/v6.424/lib \
  --with-libxml2-inc=/usr/include/libxml2 \
  --with-libxml2-lib=/usr/lib/x86_64-linux-gnu \
  --with-log4cpp-inc=/usr/include/log4cpp \
  --with-log4cpp-lib=/usr/lib
make 2>&1 | tee makelog
```

Searching makelog for instances of the word *error* can be very helpful to diagnose problems.

The neutrino cross sections are saved in .xml files called spline files in the *data* folder. At the energies of interest for detecting neutrinos from DM annihilation, cross sections for only protons and neutrons are needed (Compton wavelength of high-energy neutrinos are small enough to not see the nucleus as a combined entity, especially considering that these high-energy neutrino are interacting with single quarks inside the a nucleon). To create these cross sections up to a default max energy of 1000 GeV, do

```
cd $GENIE/data
gmkspl -p 12,-12,14,-14,16,-16 -t 1000010010,1000000010 -n 1000 \n  -e 1000 -o xsec0allFlavors.xml
```
APPENDIX A. SOFTWARE OVERVIEW

See GENIE’s manual for more information. To analyze them, the following Bash scripts are very useful, and I call them readSplines and readSplines2.

```bash
#!/bin/bash
#Will read the splines.xml files you input and will output folders
#of the same name in the current directory and fill it with
#Mathematica-readable .dat files (1 spline becomes 1 file)

for i in "$@"; do # loop over all input files
    foldd='basename "$i" .xml';
    mkdir "$foldd"
    cd "$foldd"
    filee=0
goo=false
    while read p; do # loop over all lines
        # case 1: close current output file
        if [[ "$p" = '<\spline>' ]]; then
            goo=false
        # case 2: write to output file if open
        elif $goo; then
            a1=$(echo $p | sed 's/\.*<E>/// | sed 's/<\E>.*// | tr -d \[[[:space:]]\])
            a2=$(echo $p | sed 's/\.*<xsec>/// | sed 's/<\xsec>.*// | tr -d \[[[:space:]]\])
            echo "$a1 $a2" >> $filee.dat
        # case 3: open new output file
        elif [[ "$p" = '<\spline>' ]]; then
            ((filee++))
            rm -f $filee.dat
            goo=true
            echo "creating file $foldd/$filee.dat"
        fi
    done <"$i"
    cd ..
done

if [ $# -eq 0 ]; then
    echo "$(tput bold)usage:$(tput sgr0) readSplines <file list>" 
fi
```

The other Bash script is
#!/bin/bash
#Will read off the spline LABELS in the .xml files you input and
#output folders of the same name in the current directory each
#containing labels.txt

filee="labels.txt"
for i in "$@";do # loop over all input files
  foldd=`basename "$i" .xml`
  mkdir "$foldd"
  cd "$foldd"
  rm -f $filee
  echo "creating file $foldd/$filee"
  while read p; do # loop over all lines
    if [[ "$p" =~ ^<spline ' ]]; then
      a1=$(echo $p | sed 's/./genie:::/' | sed 's/" nknots.*/' | \ 
      tr -d '[:space:]')
      echo $a1 >> $filee
    fi
  done <"$i"
  cd ..
done

if [ $# -eq 0 ];then
  echo "$(tput bold)usage:$(tput sgr0) readSplines2 <file list>"
fi

From an analysis of the resulting files, one can learn that many of the cross sections (with neutrons and protons at least) are uniformly zero. Others grow then quickly level off to a constant value at high energies. The most relevant cross sections are those that increase linearly with neutrino energy (ignoring energies below a few GeV) like Eq. (2.13). These are deep inelastic scattering (DIS) cross sections, though a small handful of linearly-increasing cross sections are not DIS. By summing up all the DIS cross sections for a flavor of neutrino or antineutrino and weighting the neutron cross sections by 8/18 and the proton cross sections by 10/18 (to represent average nucleon
cross section of $\text{H}_2\text{O}$), the CC and NC cross sections per nucleon are

\[
\begin{align*}
\sigma_{CC} &= (6.11 \times 10^{-39} \text{ cm}^2) \frac{E}{\text{GeV}} \\
\bar{\sigma}_{CC} &= (3.51 \times 10^{-39} \text{ cm}^2) \frac{E}{\text{GeV}} \\
\sigma_{NC} &= (1.96 \times 10^{-39} \text{ cm}^2) \frac{E}{\text{GeV}} \\
\bar{\sigma}_{NC} &= (1.18 \times 10^{-39} \text{ cm}^2) \frac{E}{\text{GeV}},
\end{align*}
\]

where the overline ($\bar{\sigma}$) denotes the cross section for an antineutrino. GENIE’s cross sections can only be approximate to $\sim 10\%$ due to theoretical uncertainties of the quark structure functions inside the nucleons and due to the inability of reactor experiments to reach GeV-scale [175].

For detection of DM neutrinos, high energy neutrinos are expected. By default, GENIE can only calculate cross sections for neutrino energies up to 1 TeV. Above this energy, neutrino cross sections do not exactly increase linearly with energy. This is because the four-momentum transfer (depends on the center-of-mass energy) is no longer non-negligible compared to the mass of the $W$ and $Z$ bosons, so the interaction is no longer a simple single vertex with the Fermi constant as its coupling. However, to a good approximation, a linear extension of the cross sections is acceptable if energies are not too much larger than 1 TeV. At 10 TeV, the linear extension is about 50\% larger than the actual cross section, which is not a problem if only very few neutrinos arrive near 10 TeV, as is the case for the atmospheric background (this error certainly better than ignoring all neutrinos above 1 TeV).

GENIE cannot handle these energy values without a bit of effort. In config/User-PhysicsOptions.xml, change the value of the line

\[
<\text{param} \text{ type=“double” name=“GVLD-Emax”} 1000.000 </\text{param}>
\]

to something like 10001.000. By using the above analysis codes on the cross sections generated beyond 1 TeV, we learn that many of the cross sections extrapolate linearly.
However, some extrapolate in bizarre ways. Because even a few odd cross sections can ruin everything, the following Bash script, which I call extendSplines, can take spline files that go up to 1 TeV and extend them.

```bash
#!/bin/bash
# Will read the splines.xml file you input and outputs extended .xml file by a simple linear extension from 1000 GeV to a bit beyond 10000 GeV.
# The splines, func, and this script should be in current directory.

if [ $# != 2 ]; then
    echo "$(tput bold)usage: $(tput sgr0) extendSplines <file> <knots in input>"
else
    i="$1"
    knots=$2
    # where to take the previous value for extrapolation
    num=$((knots-1))
    # additional knots needed for extrapolation (must agree with func)
    new=$((knots/5+5))
    filee='basename "$i" .xml'
    filee="$filee-extended.xml"
    rm -f $filee
    j=0
    while IFS=""; read p; do # loop over all lines
        ((j++))
        if [[ "$p" == '<spline' ]]; then
            j=0
            temp=$(echo $p | sed "s/$knots"/$(new+knots)/")
            p=$temp
            fi
        echo "$p" >> $filee
        if [ $j -eq $num ]; then
            a1=$(echo $p | sed 's/.*<E>/ / | sed 's/<\xE>/.*/' | \ tr -d '[:space:]
            a2=$(echo $p | sed 's/.*<xsec>/ / | sed 's/<\xE>/.*/' | \ tr -d '[:space:]
        elif [ $j -eq $knots ]; then
            b1=$(echo $p | sed 's/.*<E>/ / | sed 's/<\xE>/.*/' | \ tr -d '[:space:]
            b2=$(echo $p | sed 's/.*<xsec>/ / | sed 's/<\xE>/.*/' | \ tr -d '[:space:]
```
where the following function called \textit{func} is needed to do the extrapolation:

```python
#!/usr/bin/env python
# usage: func a1 a2 b1 b2 knots
# used in extendSplines

import sys

a1 = float(sys.argv[1])
a2 = float(sys.argv[2])
b1 = float(sys.argv[3])
b2 = float(sys.argv[4])
knots = float(sys.argv[5])
# print "%.3f %.3e %.3e" % (a1, a2, b1, b2, knots)

# energy is the change in log(energy) past log(1000)=3,
# and is between 0 and 1
# I use an if statement for the weird single case of numerical
# problems in input
# I do a linear extrapolation, which is a rough approximation
# at TeV energies
if (b2-a2) >= -1.0e-15:
    for k in range(int(knots/5.5+5.5)):
        energy = 5/knots * (k+1)
        print "<knot> <E> %.1f <xsec> %.9e <xsec> <knot>" \ 
            "%( 10**(3+energy) , b2 + (b2 - a2)/(b1 - a1) * (10**(3+energy)-1000) )"
else:
    for k in range(int(knots/5.5+5.5)):
        energy = 5/knots * (k+1)
        print "<knot> <E> %.1f <xsec> %.9e <xsec> <knot>" \ 
            "%( 10**(3+energy) , b2 + 4.4526e-14 * (10**(3+energy)-1000) )"
```

```bash
./func $a1 $a2 $b1 $b2 $knots >> $filee
fi
done <"$i"
fi
```
To now run the neutrino interactions given an input energy distribution, do something like the following, which interacts muon neutrinos (not muon antineutrinos) from 5000-GeV-mass-DM annihilations with H$_2$O:

```
gevgen --n 100000 --p 14 --t 1000010010[0.5556],1000000010[0.4444] \ 
  --e 0.5000 --f input.dat --seed 1721827 \ 
  --cross-sections $GENIE/data/xsec0allFlavors--extended.xml
```

where input.dat is a file of two columns (separated by a space), where the first is energy in GeV, and the second is neutrino flux (normalization and units are irrelevant as is method of sampling). See GENIE’s manual for more info on the use of `gevgen`. Various files will be created, but the one with all the data is gntp.0.ghep.root, a ROOT file.

To run the atmospheric background (from, for example, Honda’s data), the low-energy bound (in the `-e` option) should not be 0 to prevent the spike from causing fatal errors (and from messing up the statistics of the high-energy tail). Instead, something like 20 GeV should be a good starting energy. To help the high-energy tail, doing the highest-energy part of the energy distribution separately can be wise.

To read the distributions, code must be written to read and analyze the ROOT data. The following is an analysis file I wrote to find muons and antimuons and output their energy and angular distributions, which are normalized to 1, and to print out summary statistics to standard output:

```
// usage: getMuons -f <file> -n <max energy in GeV>
// Outputs the muon (and antimuon) counts as a function of energy (1 GeV bins) and
// angle (1 degree bins).
// If no -f specified, it defaults to gntp.0.ghep.root
// If no -n specified, it defaults to 1000

// I divide by the number of muons+antimuons (a bit less than the number of events,
// and a bit more than the number of CC events) to get a
// muon distribution normalized to 1

// based on $GENIE/src/test/gtestEventLoop.cxx
```
#include <string>

#include <TSystem.h>
#include <TFile.h>
#include <TTree.h>
#include <TIterator.h>

#include "EVGCore/EventRecord.h"
#include "GHEP/GHepParticle.h"
#include "Ntuple/NtpMCFORMAT.h"
#include "Ntuple/NtpMCTreeHeader.h"
#include "Ntuple/NtpMCEventRecord.h"
#include "Messenger/Messenger.h"
#include "PDG/PDGCodes.h"
#include "Utils/CmdLnArgParser.h"

using std::string;
using namespace genie;

#include <cmath>
#include <TH1.h>
#include <TH2.h>
#include <iostream>
#include <fstream>

using std::cin;
using std::cout;
using std::endl;
using namespace std;

string gOptInpFilename="gntp.0.ghep.root";
int EBinNumb = 1000;
int ABinNumb = 180;
int nev;
int n;
int n2;
int qel=0;
int deep=0;
int res=0;
int co=0;
int coe=0;
int es=0;
int nuee=0;
int imd=0;
int imda=0;
int ibd=0;
int gr=0;
int amng=0;
int mec=0;
int dif=0;
int em=0;
int weak=0;
int cc=0;
int nc=0;
int mix=0;
int nmuo=0;
int nantimu=0;
string typ="aa";
string typ2="aaa";
double ang1;
double energ; // muon energy at creation
double energ2; // initial neutrino energy
double valu;
double tote;

int main(int argc, char ** argv)
{
    CmdLnArgParser parser(argc, argv);
    if( parser.OptionExists('f') ) gOptInpFilename = parser.ArgAsString('f');
    if( parser.OptionExists('n') ) EbinNumb = parser.ArgAsInt('n');

    TTree * tree = 0;
    NtpMCTreeHeader * thdr = 0; // not sure if I need this variable
    TFile file(gOptInpFilename.c_str(), "READ");
    tree = dynamic_cast<TTree*>( file.Get("gtree") );
    thdr = dynamic_cast<NtpMCTreeHeader*>( file.Get("header") );
    if(!tree) return 1;
    NtpMCEventRecord * mcrec = 0;
    tree->SetBranchAddress("gmcrec", &mcrec);

    TH1D *h1 = new TH1D("h1","energy in GeV",EbinNumb,0,EbinNumb);
    TH1D *h2 = new TH1D("h2","angle in degrees",AbinNumb,0,AbinNumb);
TH2D *h3 = new TH2D("h3","both",EbinNumb,0,EbinNumb,AbinNumb,0,AbinNumb);
nev = tree->GetEntries();
cout << endl << endl << endl;

// Loop over all events
for(int i = 0; i < nev; i++) {
    tree->GetEntry(i);
    EventRecord & event = *(mrec->event);

    typ="";
typ2="";
    // these are found in $GENIE/src/Interaction/ProcessInfo.h
    if(event.Summary()->ProcInfo().IsQuasiElastic()) {qe++;
typ2="QES";};
    if(event.Summary()->ProcInfo().IsDeepInelastic()) {deep++;
typ2="DIS";};
    if(event.Summary()->ProcInfo().IsResonant()) {res++;
typ2="RES";};
    if(event.Summary()->ProcInfo().IsCoherent()) {co++;
    if(event.Summary()->ProcInfo().IsCoherentElas()) {coe++;
    if(event.Summary()->ProcInfo().IsElectronScattering()) {es++;
typ2="ES";};
    if(event.Summary()->ProcInfo().IsNuElectronElastic()) {nu++;
    if(event.Summary()->ProcInfo().IsInverseMuDecay()) {imd++;
    if(event.Summary()->ProcInfo().IsIMDAAnnihilation()) {imda++;
    if(event.Summary()->ProcInfo().IsInverseBetaDecay()) {ibd++;
    if(event.Summary()->ProcInfo().IsGlashowResonance()) {gr++;
    if(event.Summary()->ProcInfo().IsAMNuGamma()) {am++;
    if(event.Summary()->ProcInfo().IsMEC()) {mec++;
    if(event.Summary()->ProcInfo().IsDiffractive()) {dif++;
    if(event.Summary()->ProcInfo().IsEM()) {em++;
    if(event.Summary()->ProcInfo().IsWeak()) {weak++;
    if(event.Summary()->ProcInfo().IsWeakCC()) {cc++;
typ2="CC";}
    if(event.Summary()->ProcInfo().IsWeakNC()) {nc++;
typ2="NC";}
    if(event.Summary()->ProcInfo().IsWeakMix()) {mix++;

    // Loop over all particles in this event
    GHepParticle * p = 0;
    TIter event_iter(&event);
n=0;
n2=0;
    while(((p=dynamic_cast<GHepParticle*>(event_iter.Next()))){
        if(p->Pdg() == kPdgNuMu || p->Pdg() == kPdgAntiNuMu) {
            // if(p->FirstMother() == -1){
n2++;  
if(n2==1) energ2 = p->E();
}

if(p->Status()==kIStStableFinalState){
    // codes found in $GENIE/src/PDG/PDGCodes.h
    // particle p is in $GENIE/src/GHEP/GHepParticle.h
    if(p->Pdg() == kPdgMuon || p->Pdg() == kPdgAntiMuon){
        n++; 
        energ = p->E();
        angl = atan2(sqrt(pow(p->Px(),2)+pow(p->Py(),2)), p->Pz())*180/M_PI;
        // cout << p->Name() << " with E = " << energ << " GeV" << endl;
        // cout << energ << endl;
        // cout << angl << endl;
        h1->Fill(energ);
        h2->Fill(angl);
        h3->Fill(energ,angl);
        if(p->Pdg() == kPdgMuon) nmuo++;
        if(p->Pdg() == kPdgAntiMuon) nantimuon++;
    }
}

if(n>1) cout << typ << " event with " << n << " muons/antimuons: " << i << endl;

mcrec->Clear();
}

cout << endl;
cout << "total events: " << nev << endl;
cout << "quasi-elastic events: " << qel << endl;
cout << "deep-inelastic events: " << deep << endl;
cout << "Resonant events: " << res << endl;
cout << "Coherent events: " << co << endl;
cout << "CoherentElas events: " << coe << endl;
cout << "ElectronScattering events: " << es << endl;
cout << "Nu Electron Elastic events: " << nuee << endl;
cout << "Inverse Mu Decay events: " << imd << endl;
cout << "IMD Annihilation events: " << imda << endl;
cout << "Inverse Beta Decay events: " << ibd << endl;
cout << "GlashowResonance events: " << gr << endl;
cout << "AMNuGamma events: " << amng << endl;
cout << "MEC events: " << mec << endl;
cout << "diffractive events: " << dif << endl;
cout << "EM events: " << em << endl;
cout << "weak events: " << weak << endl;
cout << "CC events: " << cc << endl;
cout << "NC events: " << nc << endl;
cout << "Mixed events: " << mix << endl;
cout << "muons: " << nmuo << endl;
cout << "antimuons: " << nantimuo << endl;
cout << endl;

hl->Scale(1/ (double) (nmuo+nantimuo));

for(int i = 1; i <= EbinNumb; i++){
  valu=hl->GetBinContent(i);
  myfile1 << valu << endl;
  tote+=valu;
}

myfile1.close();
cout << "energy normalization check: " << tote << endl;
cout << "energy underfill: " << hl->GetBinContent(0) << endl;
cout << "energy overfill: " << hl->GetBinContent(EbinNumb+1) << endl;
cout << endl;

ofstream myfile2;
myfile2.open ("angle.csv");
tote=0.0;
for(int i = 1; i <= AbinNumb; i++){
  valu=hl->GetBinContent(i);
  myfile2 << valu << endl;
  tote+=valu;
}
myfile2.close();
cout << "angle normalization check: " << tote << endl;
cout << "angle underfill: " << h2->GetBinContent(0) << endl;
cout << "angle overfill: " << h2->GetBinContent(AbinNumb+1) << endl;
cout << endl;

ofstream myfile3;
myfile3.open("both.csv");
tote=0.0;
for(int i = 1; i <= EbinNumb; i++){
  valu=h3->GetBinContent(h3->GetBin(i,1));
  myfile3 << valu;
  tote+=valu;
  for(int j = 2; j <= AbinNumb; j++){
    valu=h3->GetBinContent(h3->GetBin(i,j));
    myfile3 << "," << valu;
    tote+=valu;
  }
  myfile3 << endl;
}
myfile3.close();
cout << "normalization check: " << tote << endl;
cout << "underfill: " << h3->GetBinContent(0) << endl;
cout << "overfill: " << h3->GetBinContent(h3->GetBin(EbinNumb,AbinNumb)+1) << endl;
cout << endl;

file.Close();
return 0;
}

Finally, sophisticated and powerful software such as Mathematica can load the resulting data (in this case, contained muon plus antimuon data) to analyze it. However, the data is that of normalized-to-1 distributions. To get the correct normalization, the input flux of neutrinos and their corresponding energy-dependent cross sections can be used to calculate the average cross section of the neutrinos (within the energy range specified when running gevgen). If units of km$^{-3}$ are desired, simply calculate the number of nucleons in a km$^3$ of the detector, and multiply the average cross section by this number to get the cross section of an entire km$^3$. 
Through-going muons (muon flux) through an imaginary surface in a detector can be calculated from these contained muons (muons created per volume). However, GENIE cannot do it. See Sec. B.1 for the procedure.

Comparing these results with DarkSUSY or WimpSim as a consistency (sanity) check is wise, as frequent consistency checks always are (even if the user has done everything correctly, bugs in the software exist).

### A.4 micrOMEGAs

The micrOMEGAs [181] software is for taking complete model files for a DM Lagrangian density (the inputs) and doing calculations for what could be detected at Earth. For extended (incomplete) models, I have not found much use for this software.

Go to lapth.cnrs.fr/micromegas/ to download the software and manual. To compile the software and setup a working folder, simply run

```
make
./newProject myproject
```

Fill myproject/work/models/ with input model files. Run myproject/work/calchep to check the model. To do calculations edit the codes called `main` and run them via something like

```
cd ~/.micromegas_4.2.5/myproject
make main=main.c    # or use main.F or whatever
./main data.par
```

### A.5 FeynRules

FeynRules [112] is a Mathematica package for creating the model files (that software such as micrOMEGAs, MadGraph, and CalcHEP need) from a particle-physics Lagrangian. Because it is a Mathematica package, it is very easy to use, but Mathemat-
ica can cost money. LanHEP is perhaps an alternative to FeynRules if Mathematica is not available.

Download the software and manual from feynrules.irmp.ucl.ac.be, then simply extract the software. The compressed .tar.gz file I downloaded had the faux pax of not having a root folder containing all the many other folders and files, so be careful to first create a root folder for FeynRules to be extracted into. No compilation is required because Mathematica is awesome (cross platform).

To create model files for the SM, run Models/SM/SM.nb, which can be done via something like

```
mathematica -singleLaunch ~/feynrules/Models/SM/SM.nb
```

Then, simply edit the path to be FeynRules’s root folder, and run the first few lines up to and possibly excluding the `LoadRestriction` line. To create model files, search for the CalcHEP lines and run them, which will create a folder inside Models/SM/ with the model files.

For custom Lagrangians, either read the manual or, if only minor extensions are desired, a very useful trick is to edit the SM’s model files by hand. To do the latter, add new particle definitions into prtcles1.mdl, add new parameters and functions into vars1.mdl and func1.mdl, and add the new vertices into lgrng1.mdl. Use known particles, parameters, vertices, etc. as examples, and Chapter 8 of the `CalcHEP` manual has a good description of the format of these files.

### A.6 CalcHEP

CalcHEP [113] calculates tree-level cross sections and annihilation rates given model files that are input.
To install, download software and manual at theory.sinp.msu.ru/~pukhov/calchep.html then extract the software. To compile on Ubuntu, do the following

```
sudo apt-get install libX11-dev
make
```

To run the software, do something like

```
./mkWORKdir myfolder
cd myfolder
./calchep
```

The software is a graphical user interface with interactive menus rather than requiring the user to write code. Read Chapter 3 of the manual to figure out some of its nuances. The most unexpected of which are for editing a table: press `esc` when finished, press `enter` for a new line, and press `ctrl` and `D` at the same time to delete a line (which will be done often after accidentally pressing `enter` instead of `esc`).

Accomplishing simple things can be very difficult at first. Doing the following procedures should remove much of this problem. The first task for any calculation is to select the model files. Some default model files are in myfolder/models/, and custom model files are copied here to protect the original (see section 3.9 of manual to load in custom files). Before entering a process, play with how to view all the highly useful information that can be quickly calculated in *Numerical Evaluation* and how to edit the model.

If more detail of a particular process is needed, go to *Enter Process*. The following example will calculate the electron’s energy distribution from muon decay assuming the SM was loaded. For the process, enter “mu- -> vm, e-, ve~” or “m -> nm, e, Ne” depending on if the CalcHEP or FeynRules version of the model files are used (particle names should match the key displayed in CalcHEP, which should match the model files). Square the diagram (viewing or selecting diagrams before and after square can be helpful). Select *Symbolic calculations*. Then select *C code → C-compiler*. 
Select *Monte Carlo simulation*. Increase *nSess* and *nCalls* for less error (or just run more times in succession). Select *Set Distributions*, and enter “E(e-)|0|0.1057” or “E(e)|0|0.1057” (depending on model) where the final energy was chosen to be the muon mass (see sections 5.6 and 5.11 of manual for setting distributions). After running *Start integration* one or more times, values of the total width (Γ) are displayed, and distribution results will be saved in results/distr_.#. Distributions can be viewed now via *Display Distributions* or later via a terminal/shell by

```
bin/show_distr results/distr_#
```
during which the plot can be exported as a simple .txt file (for later input to software such as Mathematica), results/plot_#.txt, where there are a max of 300 bins for a 1D histogram (equally spaced over the energy range of distribution), and the values represent \( \frac{d\Gamma}{dE} \).

The results of this muon example seem reasonable if using CalcHEP’s built-in SM (perhaps due to the W width being defined in models-vars1.mdl). However, using model files from FeynRules causes divergences. I have found divergences and numerical problems to actually be the norm, so calculations from the Monte Carlo should always be questioned. To improve the situation, regularization can be done.

Before entering the Monte Carlo menu, go to *Phase space mapping* then *Regularization*. Then enter something like “13|MW|WW|2”, where the 3 may need to be changed to refer to the muon neutrino keeping in mind that particle numbering is as appears above the regularization table and *not* how initially entered (see section 5.10 of manual for more details), and MW and WW should reflect the prtcls.mdl file for the model.

The initial state of a process with two initial particles can be set when calculating cross sections, but, for calculating decays, there is no ability to boost the decay within CalcHEP. However, the .txt files containing energy-distribution data can be boosted during the subsequent analysis (see Sec. B.2).
A.7 PYTHIA

PYTHIA [174] is powerful software for generating simulated events in particle accelerators such as the Large Hadron Collider. By controlling the beam energy of electron-positron collisions (for example, setting it to the mass of the particles in the process of interest then boosting as described in Sec. B.2), the software is generally useful beyond particle accelerators. For our purposes, the SM can be extended by inputting the masses and branching fractions of new particles (calculated from CalcHEP for example). PYTHIA then decays products until stable particles (photons, neutrinos, electrons, and whatever else is set to be stable) are all that remain. Throughout this entire process, higher-order processes such as hadronization and bremsstrahlung are calculated.

There are two versions: PYTHIA 6 is in Fortran, and the current since 2007 is PYTHIA 8 in C++. I recommend using the latter (even if the former has already been installed to satisfy another software’s dependencies). To install, download from home.thep.lu.se/ torbjorn/Pythia.html, extract, then run

```
make
```

Documentation comes with the software and can be accessed by opening share/Pythia8/htmldoc/Welcome.html in a web browser. The PDF manuals are also locally stored and are accessed by clicking the hyperlinks in Separate documents. Even though this is a local website, web browsers can bookmark the location (the local website is static, which can be preferred over the online website).

There are example codes in the examples folder. To run the first of them, do something like

```
cd examples
make main01
./main01 > main01.log
```
The first step in running anything nontrivial is to create an .lhe run file. The standard way is to use software called MadGraph, which would have to be learned and produces much more complicated output than PYTHIA uses, so modifying existing .lhe files is a good alternative, which is made even easier by the file below. Even though the format of these .lhe files is not well documented, trial and error and common sense have resulted in the following .lhe file. This file defines a new $\tau$-mass particle (called 700014) and its decay channels in the $\langle$slha$\rangle$ tag, sets up an electron-positron beam with energy equal to the $\tau$ mass in the $\langle$init$\rangle$ tag, and lists events to be run.

```xml
<LesHouchesEvents version="1.0">
<header>
<slha>
Block mass
  700014 1.777000e+00
DECAY  700014 1.66e-17 # from CalcHEP
  7.23E-02  3  12  3 -3  # ve2 -> ve, s, s^-
  4.13E-03  3  11  4 -3  # ve2 -> e-, c, s^-
  2.07E-02  3  11  2 -3  # ve2 -> e-, u, s^-
  7.66E-02  3  12  1 -1  # ve2 -> ve, d, d^-
  2.67E-04  3  11  4 -1  # ve2 -> e-, c, d^-
  3.95E-01  3  11  2 -1  # ve2 -> e-, u, d^-
  5.94E-02  3  12  2 -2  # ve2 -> ve, u, u^-
  1.61E-02  3  12  13 -13  # ve2 -> ve, mu-, mu^+
  1.35E-01  3  14  11 -13  # ve2 -> vm, e-, mu^+
  8.22E-02  3  12  11 -11  # ve2 -> ve, e-, e^+
  3.46E-02  3  12  16 -16  # ve2 -> ve, vt, vt^-
  3.46E-02  3  12  14 -14  # ve2 -> ve, vm, vm^-
  6.93E-02  3  12  12 -12  # ve2 -> ve, ve, ve^-
Block QNUMBERS 7000014
  1 0  # 3 times electric charge
  2 2  # number of spin states (2S+1)
  3 1  # colour rep (1: singlet, 3: triplet, 8: octet)
  4 1  # Particle/Antiparticle distinction (0=own anti)
</slha>
</header>
<init>
  -11 11 0.1777E+01 0.1777E+01 0 0 0 0 3 1
</init>
</LesHouchesEvents>
```
In a text editor, I prefer to multiply the events until there are thousands of identical events because PYTHIA is faster reading events in an .lhe file than it is reloading an .lhe file (a combination of both approaches can give the desired number of events). Having the events be identical is not only easier but can be preferred if the newly defined particle was created with consistent nonzero momentum allowing for an angular analysis. Several values throughout the .lhe file are irrelevant to the results but must nonetheless be present. The PYTHIA documentation (especially the PDF worksheet that has particle codes in Appendix A) is all very helpful.

If giving the final particles’ momentum in the .lhe files, there is a bug that appeared in version 8.215 (and will possibly be in future versions). When using $p = \sqrt{E^2 - m^2}$ to calculate the momentum, round down the final few decimal values to prevent the event from being rejected.
Below is a sample C++ program that loads in the above .lhe file and outputs a file containing the resulting photon energy distribution, $dN_\gamma/dE_\gamma$. This code also turns the decay of various particles on or off depending on the value of \textit{decay}, which is useful depending on the physical context or depending on if more advanced statistics are desired.

```cpp
#include "Pythia8/Pythia.h"
using namespace Pythia8;

int nEvent1 = 100; // number of loops
int nEvent2 = 1000; // number in the LHE file
int nEvent = nEvent1*nEvent2;

double masss = 1.777; // from the LHE files (in GeV)
int binnum = 100; // cannot be larger than 1000

// input file
string fileee = "e1777GeV.lhe";

// decay hadrons and muon?
string decay = "true";

// output file
string stringg = "photon.dat";

int main() {

    // initialize the histogram
    Hist photo("photon energy", binnum, 0, masss);

    for (int iLoop = 0; iLoop < nEvent1; ++iLoop) {

        Pythia pythia;
        pythia.readString("Random:setSeed = on");
        pythia.readString("Random:seed = 0");
        pythia.readString("Beams:frameType = 4");
        pythia.readString("PartonLevel:ISR = off");
        pythia.readString("Next:numberShowEvent = 1"); // 1 is default
        // the following decay by default
        pythia.readString("331:mayDecay = +decay");
```
pythia.readString("413:mayDecay = "+decay);
pythia.readString("−413:mayDecay = "+decay);
pythia.readString("113:mayDecay = "+decay);
pythia.readString("−113:mayDecay = "+decay);
pythia.readString("313:mayDecay = "+decay);
pythia.readString("−313:mayDecay = "+decay);
pythia.readString("323:mayDecay = "+decay);
pythia.readString("−323:mayDecay = "+decay);
pythia.readString("3122:mayDecay = "+decay);
pythia.readString("−3122:mayDecay = "+decay);
pythia.readString("3212:mayDecay = "+decay);
pythia.readString("−3212:mayDecay = "+decay);
pythia.readString("3222:mayDecay = "+decay);
pythia.readString("−3222:mayDecay = "+decay);
pythia.readString("−423:mayDecay = "+decay);
pythia.readString("423:mayDecay = "+decay);
pythia.readString("−421:mayDecay = "+decay);
pythia.readString("421:mayDecay = "+decay);
pythia.readString("−433:mayDecay = "+decay);
pythia.readString("433:mayDecay = "+decay);
pythia.readString("−431:mayDecay = "+decay);
pythia.readString("431:mayDecay = "+decay);
pythia.readString("411:mayDecay = "+decay);
pythia.readString("−411:mayDecay = "+decay);
pythia.readString("311:mayDecay = "+decay);
pythia.readString("−311:mayDecay = "+decay);
pythia.readString("211:mayDecay = "+decay);
pythia.readString("−211:mayDecay = "+decay);
pythia.readString("2112:mayDecay = "+decay);
pythia.readString("−2112:mayDecay = "+decay);
pythia.readString("13:mayDecay = "+decay);
pythia.readString("−13:mayDecay = "+decay);

// pi0
pythia.readString("110:mayDecay = "+decay); // pi0
pythia.readString("2100:mayDecay = "+decay); // ud_0
pythia.readString("−2100:mayDecay = "+decay); // ud_0
pythia.readString("310:mayDecay = "+decay); // K_S0

// the following do not decay by default
pythia.readString("130:mayDecay = "+decay); // K_L0
pythia.readString("211:mayDecay = "+decay); // charged pi
pythia.readString("−211:mayDecay = "+decay); // charged pi
pythia.readString("321:mayDecay = "+decay); // charged kaon
pythia.readString("−321:mayDecay = "+decay); // charged kaon
pythia.readString("2112:mayDecay = "+decay); // neutron
pythia.readString("−2112:mayDecay = "+decay); // neutron
pythia.readString("13:mayDecay = "+decay); // muon
pythia.readString("−13:mayDecay = "+decay); // muon
pythia.readString("Beams:LHEF = "+filee);
pythia.init();
// cout >> particleData.isParticle(7000014) >> endl;

for (int iEvent = 0; iEvent < nEvent2; ++iEvent) {
    // generate the next pythia.event
    if (!pythia.next()) {
        cout << "iEvent = " << iEvent << " is bad" << endl;
        continue;
    }

    // comb through each pythia.event[i] to fill histogram
    for (int i = 0; i < pythia.event.size(); ++i) {
        if (pythia.event[i].isFinal()) {
            int a = pythia.event[i].id();
            if (a == 22) { photo.fill(pythia.event[i].e()); }
        }
    }

    // display histogram because why not?
    cout << photo;
    // make the data be per event
    photo/=nEvent;
    // make the data be "per GeV" rather than "per bin"
    photo/=(masss/(binnum+1.));
    // write out to data files
    photo.table(stringg);
}
return 0;
Appendix B

Methods of Calculation

The formulation of the problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill.

\[ \text{Albert Einstein} \]

Various calculations are useful when studying signals from the indirect detection of DM. Being able to invent methods such as the following is often necessary and even fun.

B.1 Calculating Through-Going Muons From Contained Muons

Through-going muons (muon flux) through an imaginary surface in a detector can be calculated from contained muons (muons created per volume). Because muons interact electromagnetically when traveling through a material, antimuons behave the same as muons in materials after they are created. Through-going muons are significant at IceCube when \( E_\nu \gtrsim 250 \text{ GeV} \) due to the large track length of the
resulting muons, causing more muons to enter the detector than to be created inside DeepCore.

To convert, divide a column of volume in front of the imaginary surface of the detector into many discrete slices as if it were a long loaf of bread with one end of the loaf being the surface of the detector. At each slice, use the energy loss in ice from Table 3 of [111]

\[
\frac{dE}{dx} = - \left( \frac{246.6 \text{ GeV}}{\text{km}} \right) - (0.4324 \text{ km}^{-1}) \ E \tag{B.1}
\]

to calculate the energy that muons plus antimuons have at the imaginary surface (the first term is ionization, which dominates at low energies, and the second term is bremsstrahlung). Then, add up the contributions from the slices. Depending on the muon’s initial energy \( E_0 \), the length of the bread loaf \( L \) changes according to

\[
L = \ln \left( 1 + \frac{0.4324}{246.6 \text{ GeV} \ E_0} \right) \frac{\text{km}}{0.4324}.
\]

The angle of the incoming muon (or antimuon) is not relevant in this conversion. The bread-loaf analogy may cause one to expect that muons arriving at various angles of incidence to the end of the loaf require careful treatment. However, the detector is not a single plane that faces in a single direction. Instead, detectors look in all directions simultaneously, and a plane exists through the detector that is perpendicular to each direction.

### B.2 Boosting Energy Distributions

Data of particle distributions may be in the wrong reference frame, or the generating software may be asked to generate a distribution in a single reference frame so that
APPENDIX B. METHODS OF CALCULATION

future analysis can convert the data to any desired frame. In these situations, one must have a method for boosting a energy (and angular) distributions.

The formula for boosting an isotropic energy distribution of any type of particle (for example, for when a decaying particle producing neutrinos is itself boosted) is

\[
\frac{dN}{dE} = \frac{1}{2} \int \frac{dN}{dE'} \frac{p}{p'} d(\cos \theta) \tag{B.2}
\]

where prime (') denotes center-of-mass (unboosted) frame, \( p = \sqrt{E^2 - m^2} \) is the magnitude of the 3D momentum of the particles, and \( \theta \) is the angle between the boost direction and the particle’s velocity. To use this formula, the integrand must be a function of unprimed variables (\( E \) and \( \theta \) only) for which the Lorentz transformation

\[
E' = \gamma (E - v p \cos \theta)
\]

is needed. The integral can be numerically optimized by using this Lorentz transformation to calculate the possible \( \cos \theta \) range as a function of \( E \) for the cases when this range is smaller than -1 to 1 (for example, at very large boosts). This formula is most likely implemented correctly if total particle number \( N \) is conserved and if mono-energetic (delta-function) isotropic decay products boost to flat distributions.

To derive this boosting formula for when the particles are massless (photons) or have negligible mass (neutrinos), we begin with the Lorentz transformations

\[
E' = \gamma (E - v E \cos \theta)
\]

\[
E = \gamma (E' + v E' \cos \theta')
\]

from which a transformation of angles can be derived

\[
\cos \theta' = \frac{\cos \theta - v}{1 - v \cos \theta}.
\]
APPENDIX B. METHODS OF CALCULATION

The Jacobian to convert from \( E'(E, \Omega) \) and \( \Omega'(E, \Omega) \) to \( E \) and \( \Omega \), where \( \Omega \) is the solid angle, simplifies because angular transformations are independent of energy (that is, since \( \frac{\partial \Omega'}{\partial E} = 0 \)), giving

\[
\frac{\partial^2 N}{\partial E \partial \Omega} = \frac{\partial^2 N}{\partial E' \partial \Omega'} \left| \frac{\partial E'}{\partial E} \frac{\partial \Omega'}{\partial \Omega} \right|.
\]

Integrating angles and taking advantage of the initial distribution being isotropic gives

\[
\frac{dN}{dE} = \frac{1}{2} \int \frac{dN}{dE'} \left| \frac{\partial E'}{\partial E} \frac{\partial \Omega'}{\partial \Omega} \right| d(\cos \theta).
\]

From here, obtaining the final form of the formula, Eq. (B.2), now requires only taking derivatives and algebra.

The assumption that the particles are relativistic such that \( E \gg m \) and \( E' \gg m \) was heavily used in the previous derivation. If the particles have arbitrary mass, the same procedure as above can be used to derive the same formula, Eq. (B.2).

If the source is not isotropic or if angular information is desired, the most general equation is

\[
\frac{\partial^2 N}{\partial E \partial \Omega} = \frac{\partial^2 N}{\partial E' \partial \Omega'} \frac{p}{p'}.
\]  

(B.3)

B.3 Angular Smearing

Convolving a signal with a detector’s limited resolution can be called smearing. If a signal exists at a single precise value, a detector would instead detect some “smeared” distribution around this value. Including these statistical effects of the detector can be wise.
Smearing has other uses. If having simulated muon data from a point source in the sky (the muons themselves are produced in the detector at many angles) but instead wanting to model a distributed source (such as the Galactic Center), smearing can be used where the smearing function is the angular distribution in the sky rather than a detector’s resolution of the muon angles (which can be be captured through a second smearing).

Angular smearing is more difficult than smearing energy. To smear energy, simply convolve the data with a smearing function. That is, take the contents of each energy bin of initial data and spread them to the nearby energy bins of the smeared data according to the smearing function. Care should be taken at $E = 0$ to prevent losing data.

Angular smearing is difficult due to the nontrivial 2D geometry of a sphere. Angular bins are annuli (rings) centered around $\theta = 0$, which is the source in the sky, and the smearing function is centered at the particle’s angle, which is defined as $\theta' = 0$ (that is, primed angles are those that are relative to the particle’s trajectory rather than being relative to the source in the sky). If $\theta_1$ is the angle between the source and the particle’s track and if $\theta_2$ is the angle between the source and the detected (smeared) track, the smearing angle between the actual and detected tracks is

$$\theta' = \arccos \left[ \cos(\theta_1) \cos(\theta_2) + \sin(\theta_1) \sin(\theta_2) \cos(\phi_2 - \phi_1) \right], \quad (B.4)$$

where $\phi$ is the azimuthal angle in the unprimed coordinate system. Note that, when $\phi_1 = \phi_2$, $\theta'$ is simply $|\theta_2 - \theta_1|$ as expected.
To derive Eq. (B.4), begin with the transformation to the primed coordinates

\[
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_1 & 0 & -\sin \theta_1 \\
0 & 1 & 0 \\
\sin \theta_1 & 0 & \cos \theta_1
\end{pmatrix}
\begin{pmatrix}
\cos \phi_1 & \sin \phi_1 & 0 \\
-\sin \phi_1 & \cos \phi_1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}.
\]

The first rotation is about the z axis by angle $\phi_1$, and the second rotation is about the new y axis by angle $\theta_1$. Aesthetically, there should be a third rotation about the new z axis by angle $-\phi_1$, but this will not affect $z'$. Substitute the spherical-to-Cartesian coordinate transformation

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} =
\begin{pmatrix}
r \sin \theta_2 \cos \phi_2 \\
r \sin \theta_2 \sin \phi_2 \\
r \cos \theta_2
\end{pmatrix},
\]

and then solve for $z' = r \cos \theta'$.

Given a smearing function $dP/d\Omega$ that gives the probability $P$ that a particle will smear to solid angle $\Omega$ as a function of $\theta'$, we can now calculate the probability of any of the $\theta_2$ bins containing the smeared particle given an initial $\theta_1$. To do this, use Eq. (B.4) to make $dP/d\Omega$ a function of unprimed angles then integrate over all solid angles in the annulus that is represented by the $\theta_2$ bin

\[
P = \int_{\theta_2 - \frac{\Delta \theta_2}{2}}^{\theta_2 + \frac{\Delta \theta_2}{2}} \int_0^{2\pi} dP/d\Omega \ d\phi \ \sin \theta \ d\theta
\]

\[\approx \sin \theta_2 \ \Delta \theta_2 \ \int_0^{2\pi} dP/d\Omega \ d\phi \quad \text{(B.5)}\]

where $\Delta \theta_2$ is the width of the bin and $\phi \equiv \phi_2 - \phi_1$.

To efficiently implement this, calculate a 2D matrix of these $P$ values where there is a row for every $\theta_1$ bin (using the angular value at the center of the bin) and where
APPENDIX B. METHODS OF CALCULATION

every column is a $\theta_2$ bin. The smeared data is found by weighting the rows by the number of particles in the initial data’s bins then adding all the rows together.

Initial data is likely a function of both energy and angle, so simply do the entire procedure for each energy. If $dP/d\Omega$ is not a function of energy, the 2D matrix can be reused to greatly speed up the calculation.

To prevent errors, do the usual things. Treat the conversions between radians and degrees carefully, check number conservation, and make plots throughout this process.

B.4 Calculating Probabilities

B.4.1 Discovering a Signal over Background

To discover a signal ($S$) over a background ($B$), use

$$\text{confidence} = \frac{S}{\sqrt{S+B}},$$

(B.6)

where confidence is the number of standard deviations ($\sqrt{S+B}$ is the standard deviation) that the expected $B$ differs from $S+B$. $\frac{S}{\sqrt{B}}$ is not the desired formula partly because it is less conservative (a higher confidence is returned though it is approximately the same if $B$ is large though). Eq. (B.6) describes a future detection of $S+B$ by an experimentalist who then asks what the probability that this detection is $B$, whereas $\frac{S}{\sqrt{B}}$ is for evaluating the chance that a false signal $S$ will appear given an expected $B$. If a detector’s error is predominantly systematic, the denominator of Eq. (B.6) should instead be the systematic error bars (assuming that these errors are a normal distribution).

If a desired confidence is expressed as a probability ($P$) instead of as the number of standard deviations ($\text{confidence}$), there are two choices. The standard two-tailed
APPENDIX B. METHODS OF CALCULATION

approach gives the desired confidence via the inverse error function

\[ \text{confidence} = \sqrt{2} \operatorname{erf}^{-1}(P), \]  

(B.7)

which has the advantage of being conservative (gives a higher desired confidence than the one-tailed approach) and is more consistent with the \( \chi^2 \) method described later. However, since the relevant question here is, “Is the \( S + B \) larger than expected \( B \)” the case of being smaller is not interesting, so a one-tailed approach may be more appropriate giving a desired confidence of

\[ \text{confidence} = \sqrt{2} \operatorname{erf}^{-1}(2P - 1). \]

When using this one-tailed approach, no signal (that is, \( \text{confidence} = 0 \)) corresponds to 50% probability of being a signal (a two-tailed approach gives 0%). This is correct in that a detector seeing the expected background with no statistical fluctuations can say with no certainty whether there is a signal or not (50% of the time there should be an under-fluctuation in \( B \) in which case an \( S \) would be needed to achieve the observed expected value). However, whether or not a signal exists may not be the best question. Perhaps the relevant information is the number of standard deviations apart \( S + B \) is from the expected \( B \), so Eq. (B.7) is the best, in which case a detection of 90% confidence (\( P = 0.9 \)) requires that \( S + B \) be 1.645 standard deviations above the expected \( B \).

Optimizing the cuts on the data for all of the variables is important. Choose a region (or regions) where the total \( \frac{S}{\sqrt{S+B}} \) within the region is maximized, which may requiring a search over various cuts. If the shape but not the strength of the signal is known, then doing this search does not hurt one’s confidence because the optimal cuts can be figured out then used on real data (that is, on not-simulated data with statistical fluctuations). However, if the experimentalist does a blind search over many
cuts on the real data, certain cuts will, by chance, produce a large \( \frac{data - B_{expected}}{\sqrt{data}} \) even if no signal exists. This should be taken into account when interpreting confidence values. Quoted confidence values are only for a single cut and can be too optimistic if one wants to interpret them more generally.

A more complicated alternative that gives practically equivalent results is to use Eq. (B.8) in the following section, where \( S_1 \to S \) and \( S_2 \to 0 \). The benefit of this new approach is that the optimization of energy cuts can be replaced with a simpler binning (perhaps at the cost of getting lower statistics if too many many bins are chosen).

### B.4.2 \( \chi^2 \) Analysis Between Simulated Signals

A detector may detect \( S_1 + B \) and then may ask for the probability that this is not \( S_2 + B \), where \( S_2 + B \) is a theoretical fit that may or may not be the cause of the observed \( S_1 + B \). To do this, an analysis of the data’s distribution (shape) must be done. For example, if a detector’s energy resolution is \( \pm 40 \) GeV, consider using 40-GeV bins in the energy region of interest then compare the two signals bin by bin. \( \chi^2 \) quantifies the difference between the shape of the two signals

\[
\chi^2 = \sum_{\text{bins}} \frac{(S_1 - S_2)^2}{S_1 + B},
\]

where \( S_1 + B \) represents the square of the standard deviation and can be replaced with the square of the error bars from systematic error if the systematics dominate (assuming that these errors are a normal distribution). The \( S_2 + B \) goodness of fit of the data is improved with a smaller \( \chi^2 \).

An experimentalist, after subtracting \( B \) (measured independently) from their data, has \( S_1 \), which includes statistical fluctuations. However, simulated data for \( S_1 \) does not have these statistical fluctuations. To predict the \( \chi^2 \) that an experimen-
talist will measure, these statistical fluctuations must also be simulated. Appendix A of [87] shows that modifying the $\chi^2$ equation is all that is needed to simulate these fluctuations, giving the expected value of $\chi^2$ to be

$$\langle \chi^2 \rangle = N_{\text{d.o.f.}} + \sum_{\text{bins}} \frac{(S_1 - S_2)^2}{S_1 + B}$$

\[ (B.8) \]

The number of degrees of freedom ($N_{\text{d.o.f.}}$) is the number of bins minus the number of fitted parameters ($N_{\text{parameters}}$). $N_{\text{parameters}} = 0$ unless $\langle \chi^2 \rangle$ is optimized over parameters. For example, $a S_2 + b B$ would be 2 parameters, $a$ and $b$. We see that, only after modeling the statistical fluctuations, there is always a nonzero $\langle \chi^2 \rangle$ even if comparing to an exactly correct model.

The probability that $S_2 + B$ does not fit the data can be calculated from $\langle \chi^2 \rangle$ and $N_{\text{d.o.f.}}$ by evaluating the cumulative distribution function of the corresponding $\chi^2$ distribution at the $\langle \chi^2 \rangle$ value. The binning should be chosen to optimize this probability.
References


REFERENCES

particle annihilations in the Sun, Phys. Rev. 81 (Apr., 2010) 085004, [0912.3137].


[52] R. Allahverdi, S. Bornhauser, B. Dutta and K. Richardson-McDaniel, 
*Prospects for indirect detection of sneutrino dark matter with IceCube, Phys. Rev. 80* (Sept., 2009) 055026, [0907.1486].


REFERENCES


REFERENCES


[79] ANTARES Collaboration, First results on dark matter annihilation in the Sun using the ANTARES neutrino telescope, JCAP 11 (Nov., 2013) 032, [1302.6516].


REFERENCES


REFERENCES


REFERENCES


[169] M.-M. Zhao, Y.-H. Li and X. Zhang, Constraining neutrino mass and extra relativistic degrees of freedom in dynamical dark energy models using Planck 2015 data in combination with low-redshift cosmological probes: basic extensions to $\Lambda$CDM cosmology, 1608.01219.


[173] M. Escudero, N. Rius and V. Sanz, Sterile Neutrino portal to Dark Matter II: Exact Dark symmetry, 1607.02373.

[175] M. Honda, M. S. Athar, T. Kajita, K. Kasahara and S. Midorikawa, 
Atmospheric neutrino flux calculation using the NRLMSISE-00 atmospheric 

[176] K. C. Y. Ng, J. F. Beacom, A. H. G. Peter and C. Rott, First observation of 
time variation in the solar-disk gamma-ray flux with Fermi, Phys. Rev. 94 
(July, 2016) 023004, [1508.06276].

et al., Dark matter search results from the PICO-60 CF$_3$ I bubble chamber, 
Phys. Rev. 93 (Mar., 2016) 052014, [1510.07754].

[178] HAWC collaboration, M. L. Proper, J. P. Harding and B. Dingus, First 
Limits on the Dark Matter Cross Section with the HAWC Observatory, PoS 
ICRC2015 (2016) 1213, [1508.04470].

[179] DAMPE collaboration, C. Feng, D. Zhang, J. Zhang, S. Gao, D. Yang, 
Y. Zhang et al., Design of the Readout Electronics for the Qualification Model 
of DAMPE BGO Calorimeter, in Proceedings, 19th Real Time Conference 

Arteaga-Velázquez et al., Sensitivity of the high altitude water Cherenkov 
detector to sources of multi-TeV gamma rays, Astroparticle Physics 50 (Dec., 
2013) 26–32, [1306.5800].

[181] G. Bélanger, F. Boudjema, A. Pukhov and A. Semenov, micrOMEGAs3: A 
program for calculating dark matter observables, Computer Physics 
Communications 185 (Mar., 2014) 960–985, [1305.0237].