University of New Mexico [UNM Digital Repository](https://digitalrepository.unm.edu/)

[Mathematics & Statistics ETDs](https://digitalrepository.unm.edu/math_etds) [Electronic Theses and Dissertations](https://digitalrepository.unm.edu/etds)

Spring 4-12-2017

The Effects of Different Teaching Methods on Student Attitude and Achievement in Calculus Recitations

Jocelyn Noelle Rios University of New Mexico

Follow this and additional works at: [https://digitalrepository.unm.edu/math_etds](https://digitalrepository.unm.edu/math_etds?utm_source=digitalrepository.unm.edu%2Fmath_etds%2F103&utm_medium=PDF&utm_campaign=PDFCoverPages)

Part of the [Educational Methods Commons,](https://network.bepress.com/hgg/discipline/1227?utm_source=digitalrepository.unm.edu%2Fmath_etds%2F103&utm_medium=PDF&utm_campaign=PDFCoverPages) [Mathematics Commons](https://network.bepress.com/hgg/discipline/174?utm_source=digitalrepository.unm.edu%2Fmath_etds%2F103&utm_medium=PDF&utm_campaign=PDFCoverPages), and the [Scholarship of Teaching](https://network.bepress.com/hgg/discipline/1328?utm_source=digitalrepository.unm.edu%2Fmath_etds%2F103&utm_medium=PDF&utm_campaign=PDFCoverPages) [and Learning Commons](https://network.bepress.com/hgg/discipline/1328?utm_source=digitalrepository.unm.edu%2Fmath_etds%2F103&utm_medium=PDF&utm_campaign=PDFCoverPages)

Recommended Citation

Rios, Jocelyn Noelle. "The Effects of Different Teaching Methods on Student Attitude and Achievement in Calculus Recitations." (2017). [https://digitalrepository.unm.edu/math_etds/103](https://digitalrepository.unm.edu/math_etds/103?utm_source=digitalrepository.unm.edu%2Fmath_etds%2F103&utm_medium=PDF&utm_campaign=PDFCoverPages)

This Thesis is brought to you for free and open access by the Electronic Theses and Dissertations at UNM Digital Repository. It has been accepted for inclusion in Mathematics & Statistics ETDs by an authorized administrator of UNM Digital Repository. For more information, please contact disc@unm.edu.

Jocelyn Rios

Candidate

Mathematics and Statistics

Department

This thesis is approved, and it is acceptable in quality and form for publication: A pproved by

the Thesis Committee:

Michael Nakamaye

Dr. Michael Nakamaye, Chair

Li Li

Dr. Li Li, Co-Chair

Erik Erhardt

Dr. Erik Erhardt, Member

The Effects of Different Teaching Methods on Student Attitude and Achievement in Calculus Recitations

by

Jocelyn Rios

B.A., Mathematics, Colorado State University - Pueblo, 2013

THESIS

Submitted in Partial Fulfillment of the Requirements for the Degree of

> Master of Science Mathematics

The University of New Mexico

Albuquerque, New Mexico

May, 2017

Dedication

This thesis is dedicated to Ellie, to remember that you can achieve anything with hard work, perseverance, and faith. Also, to my husband. This would have not been possible without your unwaivering love and support. And finally to my parents, who have constantly supported me throughout my education. To my mom for teaching me work ethic and always believing in me and to my dad for sparking my curiosity and encouraging "his daughter" to study in a STEM field.

Acknowledgments

First, I would like to thank my advisor and thesis chair, Dr. Michael Nakamaye, for his support and encouragement. I would also like to thank my statistics advisor Dr. Li Li for her support invaluable advice, and incredible patience. I would like to thank all of the wonderful professors I have had at UNM such as Dr. Erik Erhardt, Dr. Cristina Pereyra, Dr. Janet Vassilev, and Dr. Pedro Embid. Also I would like to thank all of my incredible and supportive friends in the math department for helping me through my masters program such as Adam Frederickson, Steven Kao, and Abhinav Aggarwall. Finally I would like to thank all of my peers who agreed to participate in my study. None of this would have been possible if it weren't for your cooperation and support!

The Effects of Different Teaching Methods on Student Attitude and Achievement in Calculus Recitations

by

Jocelyn Rios

B.A., Mathematics, Colorado State University - Pueblo, 2013

M.S., Mathematics, University of New Mexico, 2017

Abstract

Many universities offer recitation sections in their calculus sequences; however, little research has been conducted on TA best practices in the recitations. This study first aimed to observe the different teaching methods that TAs employed in their recitations and then measure their efficacy in improving student course attitudes and achievement, focusing on student-centered and concept-based instruction. In addition, this study aimed to assess the quality of professional development offered to recitation leaders. Using a quantitative-dominant mixed methods design, 12 TAs were observed throughout the semester and TAs and calculus students were surveyed. This study found that student-centered instruction was positively correlated with student course attitude ($r = 0.68$, p-value= 0.008); however conceptually focused instruction was not correlated with course attitude ($r = -0.30, p$ -value= 0.34). Also, different teaching methods did not have an effect on final exam passing rates; however, both student-centered (*p*-value= 0.00001) and concept-based instruction (*p*value= 0.0005), improved students' likelihood of predicting a passing course grade.

Finally, our study found that TAs rated the quality of professional development they received as slightly effective in preparing them to lead a recitation. Our study has found evidence to suggest that different teaching methods impact the recitation and that more research is needed to investigate recitation best practices. The implications of this study can be used to help departments design their recitation sections and improve professional development offered to recitation leaders.

References 108

List of Figures

List of Figures

4.11 Ordinal regression average effects plot of student-centered $.\;.\;.\;.\;.\;.\;.\; 74$

List of Tables

List of Tables

Glossary

- Student-Centered Instruction: A teaching approach that includes substituting active learning for lectures, holding students responsible for their learning, using self-paced and cooperative learning, and assigning openended problems requiring critical or creative thinking¹
- Teacher-Centered Instruction: A teaching method where the teacher is actively involved in teaching while the learners are in a passive, receptive mode listening as the teacher teaches².
- Conceptual Knowledge: Conceptual knowledge is knowledge which is connected to the other pieces of knowledge, and the holder of the knowledge also recognizes the connections. The connections between the pieces of knowledge are as important as the pieces themselves³.
- Procedural Knowledge: Procedural knowledge consists of the formal language of mathematics, and of rules, algorithms and procedures used to solve mathematical tasks³.

¹Felder and Brent, 1996 ²Jared and Grace, 2016 ³Mahir, 2009

Chapter 1

Introduction

1.1 Overview

According to the President's Council on Jobs and Competitiveness, between 1990 and 2010, the number of college graduates increased by 50%; however, college graduates in STEM fields such as engineering stayed markedly constant [PCJobs, 2011]. In 1984, 113,000 students graduated with a bachelor's degree in a STEM subject, yet in 2010, only 112,000 students graduated with a bachelor's degree in a STEM subject [Bressoud et al., 2012a]. In the last decade, the United States has graduated approximately 120,000 students with engineering degrees. In contrast, China and India produce approximately 1,000,000 engineering graduates every year. [PCJobs, 2011].

Stunted STEM graduation rates may partly be attributed to the lack of equity in STEM fields [Willoughby, 2000]. According to the national center for educational statistics, the percentage of men entering STEM fields is more than twice the percentage of women entering STEM fields [Chen and Weko, 2009]. Baine et al. states that minority students are "disproportionately underrepresented" in nearly all STEM dis-

ciplines [Alexander et al., 1997]. Willoughby comments that "If we exclude the vast majority of our students from studying any substantial mathematics, we not only exclude them from many of the higher-paying occupations but also create a future in which we will not have enough scientists, engineers, accountants, computer scientists, and so on to maintain a viable economy and national defense" [Willoughby, 2000].

The Department of Commerce predicts that the job market in STEM fields will grow 1.7 times faster than non-STEM job markets [Michael, 2012]. The CEO of Intel stated, "Looking forward, this nation is at risk of a significant shortfall of qualified experts in science and math to meet the country's needs" [PCJobs, 2011]. The President's Council of Advisers on Science and Technology has called for 1 million more STEM graduates in the next decade in order to remain a global leader in STEM innovation and enable a strong, competitive American economy [Michael, 2012].

To achieve this increase in STEM graduation rates, we must turn our attention towards post-secondary STEM education [Chen and Weko, 2009]. The national center for educational statistics reported that from 1995-2001, 23% of undergraduates selected a STEM major; however 47% of these students did not complete their degree in a STEM field [Chen and Weko, 2009]. Many of these courses have strikingly low passing rates [Watt et al., 2014]. Introductory classes for STEM majors should serve as a pump, sending students through their program and into the work field; however, many educators are concerned that these courses act as a filter instead [Watt et al., 2014].

Since calculus is fundamental for most stem fields, introductory calculus courses can serve as a major filter for undergraduates interested in pursuing STEM careers [McGivney-Burelle and Xue, 2013]. Weischenberg found that nationally 40% of college students failed their first-year mathematics class. [Watt et al., 2014]. In 2010, the MAA National Study of College Calculus surveyed over 13,000 students enrolled in calculus I from varying institution types. Bressoud et al., analyzing preliminary

results of this study, found that 54% of students believed they would receive an A in calculus, however only 22% received an A and 27% withdrew or failed calculus I [Bressoud et al., 2012b].

In addition, many talented students are switching into non-stem fields despite passing their introductory STEM courses [Seymour, 2002]. Seymour and Hewitt conducted a study interviewing students who switched from STEM majors to nonstem majors. Surprisingly, 81% of the students who switched majors passed Calculus I with a C or higher [Seymour and Hewitt, 1997]. The study examined the reasons people reported for leaving a STEM field and found that 90% of all students that switched their major attributed their decision to poor teaching in STEM courses. Also, 60% of students reported that they lost interest in STEM subjects [Seymour, 2002]. Students expressed dissatisfaction with the lack of conceptual emphasis in STEM courses and lack of "peer study group support". Finally, 20% of students mentioned that a factor in switching their major was the lack of quality teaching in recitations led by graduate Teaching Assistants (TAs) [Seymour and Hewitt, 1997].

Since calculus is a gate-way course for STEM majors, the quality of calculus instruction has been a source of national concern. Reform efforts have worked toward improving teaching practices and student learning in calculus. Faculty have been particularly concerned that standard teaching methods are too instructor-focused and lean too heavily on developing students' procedural skills [Hughes Hallett, 2006].

Researchers have called for the use of more engaging and effective teaching that encourages peer interaction, higher-ordered thinking, and students to actively participate in the learning process [ALPSM, 2016]. Even though substantial research suggests that student-centered learning strategies decrease course failure rates, increase diversity in STEM programs, and improve student learning, the Higher Education Research Institute at UCLA, found that 63% of STEM professors still use "extensive lecturing" when they teach [ALPSM, 2016, Berrett, 2012].

Some math departments and faculty have been critical of calculus reform efforts [Chappell and Killpatrick, 2007]. McGivney-Burelle and Xue claim that faculty may choose not to implement more engaging pedagogy into their teaching because of the increased time and resources it require. The lecture format of instruction is often still viewed as the best way to ensure that all course content is covered in the semester [McGivney-Burelle and Xue, 2013]. According to Chappell and Killpatrick, faculty opposed to creating a more conceptually rich calculus curriculum have argued that this will "watered down" calculus education. [Chappell and Killpatrick, 2007]. Seymour suggests that in departments where the norm for teaching is still the traditional lecture model, the recitation is the perfect opportunity to include more research-aligned teaching practices. This may help improve student learning and attrition rates in courses that use large lectures [Seymour, 2002].

1.2 Statement of the Problem

High attrition rates in STEM fields have been attributed to poor teaching practices [Seymour, 2002]. Since calculus is a foundational gate-way course for STEM majors, calculus educators play an import role in national efforts to improve STEM graduation rates [McGivney-Burelle and Xue, 2013].

There is growing research available in calculus education [Rasmussen et al., 2014], but little research focuses on the recitation component. The MAA report found that approximately one-third of all students taking calculus attend a recitation taught by a graduate TA [MAA, 2015]. Melnikova avers that calculus recitations may be "an important factor in student success and attrition rates" [Melnikova, 2015]. Recitations can offer students a more personalized learning environment and studies have shown that students consider TAs to be more approachable, informal, and enthusiastic [MAA, 2015, Kendall and Schussler, 2012]. When implemented effectively,

research suggests that students who attend recitation can outperform students who only attend a lecture; however, poor teaching in recitations has been reported as a reason for students leaving STEM fields [Watt et al., 2014, Koenig et al., 2007, Seymour and Hewitt, 1997]. This is convincing evidence that the quality of recitation instruction matters and should be included in current research efforts.

Menges and Weimer state that in order to plan an effective course, the instructor must first understand the course's purpose and intended outcomes. The instructor should then develop teaching strategies that are in alignment with both of these elements. [Menges and Weimer, 1996]. Some math departments implement "structured" recitations. In this setting, the recitations have uniform course objectives, and TAs are given standardized course material and a model of teaching methods to follow. Other math departments may lack a clear purpose and objectives for their recitation, leading to difficulties in having explicitly defined best practices for TAs.

This may also cause disparities in the way calculus students, TAs, and faculty perceive the calculus recitation. In fact, Meliknova found that TAs, students, and coordinators held different views about the purpose of the recitation. She partly attributed this misalignment of views to her department's lack of a "specifically stated purpose" for the recitation and lack of "preparation in how the [recitation] component should be structured" [Melnikova, 2015].

Without department guidelines, TAs are consequently responsible for developing objectives and teaching practices for their recitation [Melnikova, 2015]. This can be problematic since TAs generally lack prior teaching experience and have "limited teaching skills" [Speer et al., 2005, Kendall and Schussler, 2012]. Their exposure to different teaching pedagogies may be significantly limited to their past experiences as students, which are often passive, instructor-focused, and procedurally driven learning environments [Speer et al., 2005, Spike and Finkelstein, 2012].

Hickok found that TAs have a propensity towards using traditional teaching strategies and recitations may not be aligned with the goals of calculus education research [Hickok, 2016]. In particular, recitation instructors may gravitate towards a teacher-centered paradigm. Watt et al. observed that calculus recitations often exemplify passive learning environments where the focus is on the TA solving homework problems [Watt et al., 2014]. Recitations may also not not be placing enough emphasis on building students' conceptual understanding of calculus. Several observational studies showed that TAs focused almost exclusively on algebraic representations and only showed one approach to solving a problem [Watt et al., 2014, Nilklad, 2004]. Using a student-centered paradigm while highlighting conceptual understanding in the recitation may be a nice enhancement to the calculus sequence, especially since lectures typically utilize teacher-centered instruction and may not be able to dedicate as much time to emphasizing concepts.

Seymour claims that creating a more pedagogically thoughtful recitation depends considerably on providing effective training for TA's [Seymour and Hewitt, 1997]. Unfortunately, research suggests that TAs receive very little and often inadequate teacher training [DeChenne and Enochs, 2010]. Some departments may require TAs to attend an orientation session before beginning their teaching assignment. Other departments offer semester or year-long teaching seminars [Speer et al., 2005]. Although there has been a push to increase TA professional development [Belnap, 2005], a national study found that 45% of TAs still reported feeling unprepared to teach [DeChenne and Enochs, 2010].

Even math departments that provide TAs with more effective professional development may not offer training tailored to the unique and specific role of a recitation leader. Yet, Ellis found analyzing MAA data, that 53% of math TAs are assigned to lead a recitation session [Ellis, 2014]. Training that addresses the role of a recitation instructor and effective teaching methods for recitations is especially needed at

institutions where the recitation is not "structured".

Some research in both math and science education have examined effective teaching methods for recitations; however, in the literature available, the recitation designs considered were always implemented in structured courses. Thus, TAs were provided with activities for each recitation session that reflected the department's defined purpose and objectives. To the best of our knowledge, no research has been conducted that examines the teaching methods TAs employ in their recitations when the sections are not coordinated. Likewise, no studies have measured the effectiveness of these methods and their implications for TA training.

1.3 Purpose of the Study

The focus of this study is on measuring the efficacy of different teaching methods employed by recitation leaders using student course attitude and achievement. The calculus sequence at the University of New Mexico offers lectures with a recitation component led by a graduate TA. The recitations are not "structured" and the department has not formally defined their course purpose or intended outcomes. Thus, the responsibility typically falls on TAs to develop their own teaching strategies for leading the recitation.

Using a quantitative-dominant mixed methods design, this study aims to first observe the different teaching practices used in recitations, focusing on whether TAs are utilizing student-centered and conceptually rich instruction. This study then aims to use quantitative data analysis methods to measure the efficacy of studentcentered and conceptually focused instruction on improving student course attitude and achievement in calculus. Lastly, this study will assess how TAs perceive their professional development and infer new strategies to improve TA training.

Data for this study was collected in several phases. In the first phase, 12 recitation sections of calculus I, II, and II were observed using a teaching observation protocol. In the second phase, calculus students completed a survey that measured their attitudes toward the recitation. Students were also asked to predict their final grade in the class. Scores from the students' surveys were used to measure student course attitude. Students predicted final grade and each section's passing rate on the final exam were used to measure student achievement. Finally, TAs completed a survey assessing the quality of professional development they received.

The study focuses on answering the following research questions:

- 1. Are TAs using research-based teaching methods, such as student-centered instruction and emphasizing conceptual understanding, in the recitation? Are these methods more effective in improving student achievement and course attitudes than more traditional teaching methods?
- 2. Do TAs feel that they understand the purpose of the recitation? Do TAs feel prepared to effectively lead a recitation after completing the graduate student professional development offered by the department?

1.4 Origin of Research Questions

This study's research questions emanated from my own experience as a recitation instructor. As a new graduate student, my first TA assignment was to lead a Calculus II recitation. I felt that the purpose and objectives of the recitation were ambiguous and I was unsure of which teaching methods would be more successful in helping students succeed. Adding to my confusion, when I talked to other graduate TAs, everyone had a different approach to the recitation. There seemed to be a lack of conformity among the different recitation sections. The training I attended was

helpful overall in preparing me to teach undergraduate math courses, but didn't answer some of my questions about leading a recitation. Though I ultimately ended up developing my own recitation style, having research available that examined the effectiveness of different teaching methods for recitations would have been helpful.

Chapter 2

Literature Review

2.1 Historical Background

The introduction of recitations into the calculus sequence has an interesting history that stemmed from World War II and the Cold War. Before the 1940's, math faculty viewed their primary role in higher education as teachers, specifically focused on undergraduate education. A common teaching load for faculty was 15 credits a semester and class sizes were relatively small [Tucker, 2015]. The chair at Princeton in the 1950s, who on top of his administrative duties, also taught a full teaching load, commented that "The most important thing that the Princeton math department did was teach freshman calculus and so it was obvious that as chair I should lead that effort" [Tucker, 2015].

World War II and the Cold War greatly impacted mathematics in the United States [Tucker, 2015]. Many eminent mathematicians immigrated to the United States to escape war-torn Europe. The United States began to emerge as a world leader in mathematics research and Americans began to realize the contributions that mathematicians made during World War II [Tucker, 2015]. These changes had several

effects on undergraduate math programs. For instance, faculty began to redefine their role in higher education and their primary focus shifted from undergraduate education to research. The math community pushed to lower faculty teaching loads [Tucker, 2015].

Also, new excitement and appreciation for mathematics, paired with an expanded focus on broadening student diversity, caused drastic increase in undergraduate enrollment [Tucker, 2015, Willoughby, 2000]. University enrollment quadrupled from the 1950s to the 1970s [Tucker, 2015]. This increase in enrollment lead to challengingly large classroom sizes for introductory courses, and educators became concerned with the quality of teaching in large lectures [Riner, 1972].

A study published in 1966 stated, "Experiments suggest that fewer students raise questions or interpose comments in larger classes than in small. There may not be interaction between instructor and student necessary for a successful lecture. In view of the fact that colleges are facing doubling enrollments in the next few years, it seems imperative that research be conducted with larger classes" [Turner et al., 1966]. The study compared achievement between the control, a standard large lecture, and a lecture paired with small discussion sections consisting of 12-20 students. These sessions were led by a teaching assistant (TA). TAs were advised to use a variety of teaching methods and students were encouraged to work together in small groups. The study found that the treatment group preformed just as well on the final exam and "using recitations allowed for active student participation" [Turner et al., 1966]. The large demand for decreased faculty teaching loads coupled with large enrollment rates prompted the implementation of TA run recitations for undergraduate mathematics courses [Tucker, 2015, Riner, 1972].

In the 1970s, the recitation was again used to address another problem, improving equity in mathematics. Uri Triesman studied differences in Black-Americans and Chinese-Americans taking calculus at UC Berkley. He found one major differ-

ence: Chinese-Americans created learning communities to study and do homework together. Triesman attributed the higher failure rates among some minorities in mathematics to their lack of community learning support. His study found that black and Latino students who attended his engineered calculus workshop known as the Emerging Scholars Program "substantially" outperformed their peers. Universities began adopting calculus workshops which were based on the standard recitation model, but utilized student-centered and collaborative pedagogical practices [Alexander et al., 1997].

Historically, math departments have chosen to adopted the calculus recitation to offer students a more personalized learning environment and to promote equity in calculus by encouraging collaborative learning.

2.2 Today's Recitation

Today, the inclusion of recitations in undergraduate programs is still common, especially among large research universities [MAA, 2015]. In 2010, the MAA National Study of College Calculus surveyed over 13,000 students enrolled in calculus I from varying degree-granting universities. The universities were categorized into 14 different institution types, ranging from two year colleges to large PhD granting universities. The study found that one third of all calculus students reported attending a recitation. When the data was restricted to just PhD granting universities, 49% of students reported using the recitation/lecture mode for calculus I [MAA, 2015]. Figure 2.1 shows the MAA reported proportions of institutions that offer calculus recitations based on institution type.

The MAA National Study of College Calculus also did a case study examining five mathematics departments of PhD granting Universities that had highly effective calculus programs. Four out of the five mathematics departments offered either a

Figure 2.1: Mosaic plot showing the proportion of math departments that offer calculus recitations by institution type. The x-axis labels indicate the highest degree granted by the institution and the student population. This data was provided by the MAA National Study of College Calculus (2015).

recitation or lab component with the calculus lecture. The report asserts that these universities "demonstrate that it is possible to have successful Calculus I programs that utilize large-lecture formats" [MAA, 2015]. Also, a study by Anderson and Loftgarten reported that students who attended lecture-recitation had a 3% better chance of passing calculus than students who only attended the lecture. This suggests that recitations are an important component of undergraduate mathematics education and contribute to student success in calculus [Melnikova, 2015].

2.3 Concerns with Calculus Instruction

In recent decades calculus instruction has been "under scrutiny" due to complaints of poor teaching practices and high attrition rates [Hughes Hallett, 2006, Seymour, 2002]. National concerns about the quality of calculus education has led to what is commonly referred to as the "calculus reform movement". Many of the concerns can be summarized by the lack of student-centered instruction and the lack of conceptual understanding in the classroom [Hughes Hallett, 2006].

2.3.1 Student-Centered Instruction

Felder and Brent define student-centered instruction as a "a broad teaching approach that includes substituting active learning for lectures, holding students responsible for their learning, using self-paced or cooperative learning, ... [and] assigning openended problems requiring critical or creative thinking" [Felder and Brent, 1996]. Keller et. al found that students in calculus were overall more successful when student-centered instruction was used [Keller et al., 1999]. Felder and Brent assert that when implemented accordingly, student-centered instruction improves student engagement and motivation, and encourages more meaningful, retentive learning [Felder and Brent, 1996]. This may be important in tackling concerns with calculus instruction.

Student Engagement and Motivation

One challenge in improving the quality of calculus education is providing students with more engaging teaching. Seymour finds that most courses in STEM are still largely taught using teacher-centered instruction, despite research that suggests a student-centered paradigm better engages students [Seymour, 2002]. Bressoud sur-

veyed students in calculus at Penn State and found that only 45% of students reported listening, paying attention, or trying to understand during calculus lectures. One student said "the prof gives notes and does a few examples... I usually end up behind and start doodling" [Bressoud, 1994]. This suggests that a significant amount of students in class are not actually benefiting from the lecture.

High attrition rates in calculus courses have also been associated with unengaging instructional practices [Seymour and Hewitt, 1997]. The traditional instructorfocused approach to teaching calculus can cause capable students to become uninterested in mathematics [Seymour, 2002]. Data suggests that "losing interest" in STEM fields disproportionately affects women and minorities [Rosenthal, 1995, Alexander et al., 1997]. Rosenthal expressed his concerns about students losing interest in mathematics, describing that

"Most mathematicians agree that the best way to learn mathematics is by actively doing mathematics; by discussing it with others; and by synthesizing major ideas. However, in typical university mathematics classes in the USA, students passively watch a professor lecture at a blackboard... The students may attend a 'recitation session' of some sort to discuss solutions to homework problems, but such sessions are typically of limited success and are often considered secondary. It is understandable, then, that students often do not see mathematics as the dynamic, exciting, creative discipline that it is" [Rosenthal, 1995].

Unexciting teaching not only affects student engagement, but may also affect students' motivation to learn. When students classify mathematics as "boring", their intrinsic motivation is stymied [Middleton and Spanias, 1999]. In contrast, Jones theorizes that when student-centered instruction is used, students have more perceived control over their learning environment, and therefore are more likely to be

intrinsically motivated [Jones, 2009]. Student-centered instruction involves students in the learning process, gives them a sense of control over their education, and holds them responsible for their learning [Panitz, 1999]. Designing classroom activities that allow students to be in control has also been shown to increase students' appreciation of the subject they are studying [Middleton and Spanias, 1999, Felder and Brent, 1996].

Also, student-centered teaching methods such as collaborative learning allow students the opportunity to interact with their classmates [Rejniak, 2004]. Peer interaction promotes positive relationships among students and encourages the development of student learning communities [Callahan, 2008, Zhao and Kuh, 2004]. Instead of students competing against their peers, which is common in more traditional learning environments, collaborative learning supports students working together to reach a common goal [Callahan, 2008, Zakaria and Iksan, 2007]. Studies have also shown that collaborative learning promotes equity in the classroom and has been linked with improving students' motivation to learn [Rejniak, 2004, Panitz, 1999].

In the current literature, several studies have found that student-centered instruction positively affected student-engagement. Lucas found that using peer-interaction helped increase student participation in calculus I [Keller et al., 1999]. Also Deslaurieirs et. al compared physics students who attended a lecture vs. students who attended a class designed to focus on active learning. They found that students in the student-centered learning environment attended class more often and had higher engagement during each class period [Deslauriers et al., 2011].

Meaningful and Retentive Learning

Lectures are not only failing to engage and motivate students, but may also be ineffective for student learning. Research from different fields suggest that student-

centered, active engagement modes, when implemented in the classroom effectively, foster more meaningful learning and better knowledge retention than traditional lecture modes [Michael, 2006]. It is concerning that despite current research, the norm in most calculus classes is still teacher-centered instruction [Callahan, 2008].

In the typical teacher-centered paradigm, knowledge is transfered from the instructor to the students [Zakaria and Iksan, 2007]. According to Zakaria and Iksan, students consequently rely on "rote learning skills" and depend on the instructor to learn. Instructors generally look for correct answers and incorrect answers are not used constructively to foster meaningful learning [Zakaria and Iksan, 2007].

Based on Bloom's taxonomy, traditional lectures require students to use low level cognition such as recalling knowledge or comprehending content [Zakaria and Iksan, 2007]. In contrast, when students do their homework, they are required to use a high level of cognition such as analyzing and evaluating [McGivney-Burelle and Xue, 2013]. Developing problem solving skills requires students to practice in an environment where they can receive feedback [Michael, 2006]. McGivney asserts that the problem with lecturing is that students aren't given an opportunity to receive support from either peers or the instructor while engaging in higher ordered thinking. Instead, they are required to use problem solving skills when completing their homework without any opportunities for meaningful support [McGivney-Burelle and Xue, 2013].

In Bressoud's survey of calculus students from Penn University, he asked students how they typically study for their calculus course. Surprisingly, 90% of students reported going over homework problems when they study, and only 22% of students reported looking at their lecture notes. This may indicate that students are not engaging in meaningful learning under the traditional lecture pedagogy [Bressoud, 1994].

Also, faculty have voiced concerns that teacher-centered instruction in lectures leads to poor student retention of basic course knowledge [Tucker, 2015]. Engelbrecht et. al assessed engineering students after completing calculus and found that students' retention of basic skills declined considerably after two years [Engelbrecht et al., 2007]. Tucker reports stories of students not being able to differentiate x^2 after passing first semester calculus [Tucker, 2015].

In contrast, when student-centered instruction is used, the pedagogical focus shifts from teaching to learning [Michael, 2006]. When students are actively participating in the learning process, knowledge is not just being transferred but meaningfully constructed by the learner [Michael, 2006]. Student-centered teaching strategies, such as Socratic dialogue, allow students to correct their own thinking. These strategies have been shown to be more conducive to rich learning [Hake, 1992]. Craik and Lockhard hypothesize that when the brain processes information at a deeper level and more meaning is attached to the information, it is more likely to be stored in long term memory [Kvam, 2000].

Student-centered teaching approaches like peer interaction also help create effective learning. Zakaria and Iksan comments that collaborative learning allows students to "exchange resources", "question each others conclusions", and "defend their own ideas", which requires students to use higher-ordered thinking [Zakaria and Iksan, 2007]. Collaborative learning also affords student an opportunity to practice communicating mathematical ideas. Michael claims that "a central part of learning any discipline is learning the language of that discipline" [Michael, 2006]. Finally, once students graduate and enter STEM fields, many of them will be expected to work on teams as scientists or engineers. Learning how to effectively work with others is a skill that prepares students for the work force [Springer et al., 1999]. Collaborative learning benefits students in multiple ways resulting in more meaningful learning connections [Michael, 2006].

In the current literature, several studies have also found that student-centered instruction is associated with more meaningful, long-term learning. Code et al. offered two different single-topic calculus interventions twice throughout the semester. One intervention utilized student-centered teaching methods and the other used lecturing. Using a switch-replication design, students in the more innovative intervention scored higher on an exam testing the topics learned in the interventions [Code et al., 2014]. Also, Hake compared introductory physics courses using either interactive engagement strategies or the traditional lecture strategies, and found that interactive engagement strategies improved students problem-solving skills [Hake, 1998].

2.3.2 Conceptual Understanding

Educators have also exhibited concerns for student's lack of conceptual understanding and the over-emphasis of procedural competency in calculus courses [Hughes Hallett, 2006].

Mahir defines the difference between conceptual and procedural knowledge as "conceptual knowledge is knowledge which is connected to the other pieces of knowledge, and the holder of the knowledge also recognizes the connection. The connections between the pieces of knowledge are as important as the pieces themselves. Procedural knowledge consists of formal language of mathematics, and of rules, algorithms and procedures used to solve mathematical tasks" [Mahir, 2009].

Hughes Hallett claims that students are accustomed to answering questions such as "use method X to do Y" and consequentially struggle applying what they have learned, especially into other disciplines [Hughes Hallett, 2006]. This is particularly concerning since the MAA 2015 report found that 72% of students taking calculus I are non-mathematics majors, studying either a science or engineering [MAA, 2015]. From the instructors surveyed in the 2010 MAA study, 150 final exams were analyzed

and each question was coded. Bressoud reported that 78.7% of all exam questions were coded as "recall and apply procedures" and only 10% were coded as "apply understanding" [Bressoud et al., 2012b].

Watt et al. claims that calculus concepts are predominantly taught to students using "algebraic representations". However, students construct conceptual understanding differently depending on their cognitive processes. When students are shown multiple representations of a concept, such as geometric or numeric, they are able to develop a concept image [Watt et al., 2014]. Tall and Vinner define a concept images as "all of the cognitive structure in the individual's mind that is associated with a given concept" [Tall and Vinner, 1981]. When students build concept images by learning through multiple representations, they are more likely to construct a "sound mathematical framework" [Watt et al., 2014].

Focusing on procedural knowledge and only exposing students to one representation of a concept encourages students to "superficially" learn calculus, meaning that students memorize procedures to pass exams, but lack conceptual understanding [Tucker, 2015]. Mahir found when comparing students' knowledge of integration after calculus I, that students average score on procedural questions was 83%, but on questions that tested students conceptual understanding, the average score was 16% [Mahir, 2009]. Smith and Moore state,

"Researcher reports that [students] procedural knowledge (e.g., substituting values into continuous functions, factoring and canceling, using conjugates, employing L'Hopital's rule) is largely separate from their conceptual knowledge. Much of what our students have actually learned ... is a set of "coping skills" for getting past the next assignment, the next quiz, the next exam . . . Because the teacher knows that conceptual questions are rarely answered correctly, the vicious circle of procedural questions is set in motion" [Tall, 1992].
The MAA 2015 report found that 66% of math faculty agreed that "understanding ideas in calculus typically comes after achieving procedural fluency" [MAA, 2015]. The disproportional emphasis of procedural mastery in calculus may be due to the common view among faculty that conceptual understanding emanates from procedural competency [Code et al., 2014]. In contrast, Anderson's model of learning claims that learning "begins with actions on existing conceptual knowledge". Heibert and Lefevre argue that "procedural knowledge is meaningful only if it is linked to a conceptual base" [Engelbrecht et al., 2005]. Other studies have found that conceptual understanding and procedural skills are independent of each other [Mahir, 2009, Engelbrecht et al., 2005].

Many math departments have redesigned their calculus curriculum to place greater emphasis on conceptual understanding. [Hughes Hallett, 2006, Tucker, 2015]. In the current literature, several studies report on the effectiveness of their concept-based curricula. Chappel and Killpatrick showed that a more conceptually focused calculus course can be designed without sacrificing students' procedural abilities. They designed two calculus courses, one that was concept-based and the other procedurebased. Students in the concept-based course out-performed their peers on an exam that tests procedural fluency [Chappell and Killpatrick, 2007]. Duke University created Project CALC, a calculus course which focused on conceptual learning, multiple representations, and real-world problem solving. Bookman and Friedman showed that Project CALC students performed significantly higher on a problem solving exam than students in traditional calculus sections [Bookman and Friedman, 1994].

Since the recitation section is often a part of the calculus sequence, using teaching methods that are aligned with current research goals may nicely contrast with the more traditional lecture and be important in improving the quality of undergraduate calculus education.

2.4 Developing the Recitation

2.4.1 Purpose of the Recitation

As Menges and Weimer suggest, having a clear purpose and objectives for a course is fundamental in establishing effective teaching strategies [Menges and Weimer, 1996]. Therefore, it is important to consider the purpose of the recitation section. The MAA report finds that the purpose of the calculus recitation varies depending on the math department's goals, resources, and the needs of their students [MAA, 2015]. However, based on the historical background of the recitation, we can conclude that one of its purposes is to create a "small class feeling" despite the large-enrollment courses often encountered in calculus [MAA, 2015].

A calculus student in the MAA report delineates, "I'm in one of the sections where we have the big lecture with over 100 people as well as the smaller class [recitation section, with 30-40 people. So the big lecture, I don't raise my hand at all \dots Then when I get to the smaller lecture, that's the time when I can ask questions if I want to, make comments, go up to the board, maybe do a problem, and it's a lot more interactive" [MAA, 2015]. The MAA report concludes that a smaller forum such as a recitation or lab section is essential to foster direct communication between the instructor and the students [MAA, 2015].

2.4.2 Common Objectives and Teaching Methods for Recitations

Math departments' objectives for the calculus recitation may vary considerably, or may be somewhat ambiguous. Accordingly, standard teaching methods utilized by TAs may also vary. After reviewing the current literature, two models for the recita-

tion are commonly used.

The Homework-Session Model

Mathematics Departments may focus their recitation to give students an opportunity to discuss homework question. The Department of Mathematics at the University of Rutgers has posted,

"A major purpose of a recitation is to go over homework problems, so that much of the discussion is usually student driven. However, if students do not come forward with enough questions, the recitation instructor should choose and present additional homework problems, and/or review material in other ways, to make up a full-length class" [Rutgers, 2009].

Some researchers have been critical of the "homework-session" approach to the recitation. Alexander et al. address their concern about the quality of discussion and peer interaction being generated. Spike and Finkelstein describe the TA's role in this traditional recitation as merely "modeling problem solving" with little opportunity for students to actively learn [Spike and Finkelstein, 2012].

A More Research-Based Model

Other mathematics departments have different objectives for the recitation. One of the departments highlighted in the MAA case study stated that their recitation sessions are "designed to go beyond a question and answer session, instead providing opportunities for students to work together on more conceptually oriented problems related to the lecture" [MAA, 2015]. Brown University describes their recitation to students as:

"TAs briefly taking questions about the homework problems, and then for most of the session, you'll work in groups to solve problems from a worksheet . . . the best way to learn mathematics is by actively solving problems. Recitation is an opportunity for you to do so with other students, so that you can benefit from each other's perspectives. In addition, we believe that being able to communicate mathematics is as important as understanding it yourself, so discussing problems in small groups builds useful skills. Why doesn't the TA spend the entire recitation answering questions? Having the TA speak the whole time would defeat the purpose of recitation being a problem-solving environment . . . in recitation, we want you to focus primarily on group collaboration" [Katz, n.d.]

Spike and Finkelstein state that "in transformed environments, the expected role of the TA is shifting; TAs are no longer expected to model problem-solving at the front of the classroom, but instead engage small groups in Socratic dialogue and are attentive to student reasoning and sense-making" [Spike and Finkelstein, 2012]. This student-centered approach to the recitation gives students the opportunity to interact with their peers and use higher-ordered thinking.

2.5 Alignment of the Student and Instructor Perspectives

While research shows that student-centered and concept-based instruction are more effective for student learning, they are often at odds with students' expectations of the purpose of the recitation. Consequently, students' attitudes toward the recitation may not correlate with research-based best practices. This can lead to disconnect between the students' and TAs' perspectives of the recitation.

First, students' perspectives on the purpose of the recitation may be influenced by how they view their role as the student. Borasi and Rose state that many math students see their responsibility as a learner as "acquiring facts" that they can apply to their homework and exam problems [Borasi and Rose, 1989]. Kortemeyer observed introductory physics students' online conversations about homework and found that 47% of conversations were considered procedural and solution-oriented, whereas only 10% were considered conceptually-oriented [Kortemeyer, 2006]. Kortmeyers argues that online discussion provides educators with intuition about students' learning beliefs, since they are collected from an "authentic non-research setting" [Kortemeyer, 2006, Kortemeyer, 2007].

Because of students' beliefs about learning, there may be disparities among the way TAs and students view the purpose of the recitation. In Meliknova's dissertation, she examines the alignment of TA and student views of the recitation. Using the How People Learn (HPL) Framework, Melnikova describes the four lens for learning environments: learner-centered, knowledge-centered, assessment-centered, and community-centered, and then examines which lenses TAs and students considered relevant to the recitation.

Melnikova first describes the four lens in the context of the calculus recitation. In a calculus recitation emphasizing a learner-centered lens, TAs would focus on providing "a bridge from the calculus topics to the student by considering what the student knows and is able to do. The classroom discourse would include mathematical terminology paired with everyday discourse to explain calculus concepts". In a knowledge-centered recitation, TAs encourage students to "think mathematically and make sense of the calculus topics without focusing entirely on computation". An assessment-centered calculus recitation would focus on assessing students learning and providing them with meaningful feedback. Finally, a calculus recitation which emphasizes the community-centered lens, encourages students to form a learning

community where they have the opportunity to "work together, to discuss ideas, and share strategies" [Melnikova, 2015].

Melnikova then surveyed students and TAs, and found a misalignment in how they viewed the purpose of the calculus recitation in terms of the HPL framework. Students identified most strongly with the knowledge-centered and learner-centered lenses. Students reported expecting the recitation to help them apply what they have learned in the lecture to their homework. Students viewed the purpose of the recitation as a place where they "had the material explained to them in a way they understood and to be able to comfortably ask questions" [Melnikova, 2015]. TAs, on the other hand, ranked the community-centered lens as the most important, followed by the knowledge-centered lens. TAs expressed wanting to create a "comfortable learning environment where students could actively participate" [Melnikova, 2015].

In Meliknova's study, a student-centered instructional mode may best reflect TAs beliefs about the recitation being a community environment that actively engages the students. However, students' comments about wanting the "material explained to them" highlights more of a teacher-centered paradigm. This is not surprising that students expectations of the recitation are rooted in traditional teaching methods. Felder and Brent claim that students are accustomed to traditional, instructorcentered learning environments [Felder and Brent, 1996]. Borasi and Rose argue that math students have become satisfied with merely learning procedural manipulations without seeking deeper conceptual comprehension [Borasi and Rose, 1989].

In fact, Felder and Brent suggest that students' impressions of reformed teaching may be quite negative at first. Engelbrecht et al. state that when students are used to being "given" information, they are reliant on an "external authority" to validate their learning [Engelbrecht et al., 2009]. Students may at first perceive a less traditional learning environment as losing this support, since they are expected to be responsible for their learning, work collectively with their peers, and are required

to exert more intellectual effort during class [Felder and Brent, 1996].

When Project CALC, described earlier in section 2.3.2, was first implemented at Duke University, Bookman and Friedman reported students being "very upset" about the new course design. They hypothesized that Project CALC conflicts with students' "deeply held beliefs" about learning mathematics [Bookman and Friedman, 1998]. Common student complaints were that Project CALC required too much time outside of class and students felt they were not learning as much as from a traditional lecture course. Other complaints included that the course was too challenging and covered "too much theory". One students stated in the end of the year evaluation, "Please, do not make everything Project CALC. Computing math with numbers is much easier to understand than writing about it is" [Bookman and Friedman, 1998].

Despite complaints, students in Project Calc out-performed their peers who attended the traditional calculus lecture. This suggests that student course attitudes which often reflect their beliefs about learning, may not correlate with student achievement. For example, in Kortemeyer's study mentioned earlier in this section, students predominately engaged in online discussion that was solution-oriented. This shows that students considered procedural knowledge to be important. However, his study found a negative correlation between the percentage of solution-oriented discussion students engaged in and their end of the semester scores on the Force Concept Inventory [Kortemeyer, 2007].

Surprisingly, after two years of implementation, Bookman and Friedman found that students' attitudes towards Project Calc improved drastically. Students even reported that their favorite aspect of Project CALC was that they "understood the material rather than memorizing it" and they liked the real world applications [Bookman and Friedman, 1998]. Felder and Bret state that using research-based teaching has a "steep learning curve" for students, but students eventually learn to adapt [Felder and Brent, 1996].

Since the recitation is intended to be a tool to help students succeed, it is meaningful to consider students' perspectives on the purpose of the recitation and corresponding student attitude towards it. Melnikova argues that examining students' views on the purpose of the recitation can help to create more alignment between students' and TAs' beliefs. [Melnikova, 2015]. Also, Bookman and Friedman comment that understanding students' attitudes towards learning is valuable when creating an effective learning environment [Bookman and Friedman, 1998] Studies have even found that student course attitude can have an impact on student achievement [Reins, 2015]. However, student attitudes and beliefs may not be an indication of instructional best practices.

2.6 Literature Review of Recitation Instruction

After reviewing current literature in both math and physics education, a few studies have investigated effective teaching methods in recitation sections.

Mathematics

In the current literature, one study by Watt et al. examines the effects of implementing different recitation activities on student achievement. Watt et al. developed in three phases the instructional design for their department's recitation component offered in conjunction with the large-enrollment calculus lecture. They compared each phase to the control group, a small enrollment calculus section with no recitation component. In the first phase, students had the opportunity to attend an optional recitation session lead by an undergraduate mentor. In the second phase, students attended a mandatory traditional recitation session led by a graduate student. In the final phase, the researchers developed concept activities for the recitation. These

activities focused on conceptual understanding by using multiple representations. Watt et al. state that "newly created recitation activities focused on developing mathematics concepts via an integrated verbal, geometric, numeric, and algebraic understandings approach" [Watt et al., 2014]. During each recitation, students in every section completed the same concept activity in small groups while a graduate student encouraged discussion.

The study compared final exam scores, course failure/withdrawal rates, and the one-year retention rates of students in STEM majors. The study found that in all phases, students attending a recitation component outperformed the control group in all three measurement assessments. Also, students in phase III had significantly higher scores on the final exam, higher retention rates, and lower course failure rates than the control group and the other two phases of the recitation design [Watt et al., 2014]. This study provides evidence that recitations with student-centered instruction and conceptually challenging content is an effective design for student learning.

Numerous research projects have validated Dr. Triesman's Emerging Scholars Program, piloted at UC-Berkley. Triesman transformed the original recitation into a workshop-style session, led by a graduate student, which focused on active and peer learning [Alexander et al., 1997]. During each session students worked collaboratively on a worksheet of calculus problems. These problems were deliberately designed to challenge students' conceptual understanding, require students to work together, and help students fill in gaps in their mathematical background [Asera, 2001].

The workshop's success, especially among minority students, can be attributed to its use of innovative teaching methods. These methods allow students to observe their peers' problem solving approaches and create a community of "social support" [Asera, 2001]. Tresiman and Fullilove reported that between 1978-1982, the average percentage of African American students receiving a B+ or higher in the tradi-

tional lecture/recitation calculus course at UC-Berkley was 22%. In contrast, 54% of African American students that participated in the Emerging Scholars Program received a B+ or higher [Tresiman and Fullilove, 1990].

Calculus workshops based on the model of the Emerging Scholars Program have been implemented by over 100 universities nationally and replicas of Treisman's study have found similar results [UCBerkley, 1993]. Duncan and Dick examined the calculus workshop adopted by Oregon State University. They collected data from students enrolled in both the workshop and traditional lecture/recitation sections. Using students' SAT scores and previous data, they created a linear model to predict the course grade students would have received if they did not attend the workshop. Researchers found workshop students' grade points were 0.671 higher than the predicted course grade [Duncan and Dick, 2000].

Physics

The physics education community has been more involved in researching recitation teaching methods for introductory calc-based physics courses. Substantial research has worked towards developing curricula for introductory physics recitations, one of the most common being Tutorials in Introductory Physics [Finkelstein and Pollock, 2005]. Finkelstein and Pollock describe Tutorials in Introductory Physics as

"a research-based curriculum, designed to supplement the conventional calculus-based introductory physics class by changing practices in the smaller recitation sections. The explicit goals are to develop student conceptual understanding and scientific reasoning skills... Students work in small groups, with the instructors playing the of "learning coach," asking guiding questions in a Socratic manner" [Finkelstein and Pollock, 2005].

Tutorials in Introductory Physics is widely used in calculus-based physics recitations. In a replication study at CU-Boulder, Finkelstein and Pollock found that after implementing the tutorials in their physics recitations, students average scores on a concept inventory exam increased from 53% to 81% [Finkelstein and Pollock, 2005]. In fact, in the first study discussed, Watt et al. referenced ideas from Tutorials in Introductory Physics while designing phase III of their calculus recitation [Watt et al., 2014].

Our final study from the literature examines the effectiveness of different physics recitation designs using Tutorials in Introductory Physics. Koenig et al. studied four different teaching methods and their effectiveness in generating conceptual understanding in calculus-based physics recitations. In the first teaching style considered, the instructor lectured during the recitation by solving physics problems on the board. In the second teaching style, students worked individually on problems from Tutorials in Introductory Physics and at the end of the recitation the TA gave students the solutions. In style three, students worked on the tutorials in collaborative learning groups. TAs also provided students the solutions at the end of the recitation. Finally, in style four, students worked on the tutorials in collaborative learning groups while TAs used Socratic dialogue to guide students' learning [Koenig et al., 2007].

Koenig et al. measured students' conceptual understanding by administering a pre and post test. In addition, students completed a course attitude survey. The results indicate that students in the style four recitation had significantly higher scores on the post test and were more likely to correct false reasoning used on the pretest. Also, Koenig et al. found interesting results from the course attitudes survey. Students surprisingly favored the first style recitation, despite finding that style one was not the most effective model for student learning [Koenig et al., 2007]. This further suggests that student course attitude may not correlate with teaching best

practices for effective learning.

In current research, the purpose and intended outcomes for the recitation were clearly articulated and the department utilized a "structured" course approach to the recitation. Researchers either created activities for each session or used a standard text. There is currently a gap in the literature on the effectiveness of teaching methods utilized by TAs in non-structured calculus recitations.

2.7 Graduate Student Teacher Training

In the current research on recitation instruction summarized in the previous section, TAs were provided with teacher training uniquely tailored to prepare recitation leaders. For example, in the Koenig et. al. study, recitation leaders met weekly to review the course material, cover important teaching pedagogy, and practice using teaching techniques such as Socratic dialogue. Faculty also observed the recitations throughout the semester to evaluate TAs' uses of different teaching strategies [Koenig et al., 2007]. Seymour claims that creating a successful recitation which uses student-centered instruction and enriches student learning is contingent upon the level of training TAs receive. She states that research into TA professional development is "of the utmost importance in meeting the challenges of improving student learning in more traditional large lecture classes" [Seymour, 2002].

Graduate programs focus primarily on training their students to become researchers, not educators; even though as faculty, one of their primary responsibility will be teaching [Golde and Dore, 2001]. Ellis states that TA's will have a considerable impact on calculus education throughout their careers; yet most graduate programs offer minimal and often ineffectual teacher education for TAs [Ellis, 2014].

In recent decades, professional development opportunities have increased.Belnap

reports that 87% of universities offer TAs some form of professional development [Belnap, 2005]. Ellis describes three categories of TA training training programs: orientation programs, transitional programs, and establishment programs. Orientation programs require TAs to attend a few days of training before the semester where they learn basic information for their teaching assignment. Transitional programs are designed for first-year TAs and are usually semester of year long courses. TAs learn introductory information about teaching and pedagogy. In establishment programs, the focus is on providing students with specialized training specific to the course they are teaching. TAs attend this training every semester [Ellis, 2014]. Many math departments offer training programs to TAs that fall into one of these categories.

Although training programs have become more prevalent in higher education, TAs may still not be receiving adequate professional development [Belnap, 2005]. Belnap reports that TAs were "highly critical" of the training they received. They expressed concerns about the insufficiency of information covered in the training and the lack of opportunities to practice learned pedagogy. Belnap also reported that TAs were disappointed that their training was not more relevant to their specific teaching assignment [Belnap, 2005]. In particular, TAs assigned to the unique role of a recitation instructor may not be receiving the specialized training needed to effectively lead a calculus recitation.

Spike and Fikelstein theorize that in order to be an effective instructor, TAs must have content knowledge, pedagogical knowledge, and pedagogical content knowledge (Figure 2.2). These knowledge domains often overlap, especially in "transformed" classrooms [Spike and Finkelstein, 2012]. Training programs may concentrate on different domains, but a successful training program should focus on building all three types of knowledge domains. Ellis states that "different professional development programs focus on different types of knowledge depending on philosophies of the department". However, effective training program should focus on developing all

three types of knowledge domains [Ellis, 2014].

Figure 2.2: Venn diagram of the three types of knowledge domains for teachers. This figure is taken from Spike, B.T. and Finkelstein N.D. (2012). Preparing tutorial and recitation instructors: A pedagogical approach to focusing attention on content and student reasoning.

2.7.1 Content Knowledge

A common belief in higher education is that if an instructor has content knowledge, they will be able to successfully teach students [Harris et al., 2009]. Most departments assume that TAs already have adequate content knowledge and therefore content knowledge is not a common focus in professional development [Ellis, 2014]. But according to Kendall and Schussler, students reported that TAs have a general lack of subject knowledge [Kendall and Schussler, 2012]. Some programs incorporate content knowledge into their training by reviewing course material before the semester [Pentecost et al., 2012]. Pavelich and Streveler describe a component of their graduate teacher training workshop which focuses on improving TAs' content knowledge. As part of the training, TAs "experience active learning classes as stu-

dents with the workshop presenters acting as teachers". One of the beneifts of this model is that it allows TAs to review the course material before they are expected to teach it [Pavelich and Streveler, 2004]. However, Luft et al. states that an important component of effective TA training is emphasizing that content knowledge alone is not sufficient to be an effective instructor [Luft et al., 2004].

2.7.2 Pedagogical Knowledge

Most training programs focus on building pedagogical knowledge since TAs typically enter graduate school with little to no teaching experience [Ellis, 2014]. The majority of their pedagogical knowledge comes from their past educational experiences, which are typically in lecture modes. [Speer et al., 2005]. Therefore, it is important to focus training on introducing innovative teaching techniques such as student-centered instruction and active learning [Hickok, 2016]. Effective professional development influences TAs to "buy-in to reformed instruction" and can be measured as "a positive change in teachers' knowledge, beliefs, instructional practice, and student's success" [Hickok, 2016, Ellis, 2014].

In Belnap's dissertation, he examined what factors affect the development of TAs' teaching styles. Belnap surprisingly found that TAs did not consider the training program as a major factor in developing their teaching style. Several other studies have found similar results [Belnap, 2005]. Most TAs reported that their past teaching experiences have most significantly shaped their teaching methods. TAs also mentioned mentors such as colleagues and advisors as being influential in shaping their teaching.

Belnap suggests that training programs should not just transmit pedagogical information to TAs, but also provide them with opportunities to gain experience using these new teaching strategies. Belnap suggests including activities in TA train-

ing programs such as microteaching, simulated teaching environments, team teaching with an experienced instructor, and observing experienced instructors [Belnap, 2005]. Because professional development may not be as influential in shaping students' teaching beliefs, training programs should focus on motivating TAs to work on developing their own teaching style and providing them with resource such as mentors and multimedia material. [Belnap, 2005].

2.7.3 Pedagogical Content Knowledge

Spike and Fikelstein define pedagogical content knowledge as "specialized knowledge possessed by teachers that combines knowledge of content with knowledge of pedagogy" [Spike and Finkelstein, 2012]. Ellis describes pedagogical content knowledge as "knowing how students may understand specific content, various solutions they may arrive at, and struggles they have with the material" [Ellis, 2014].

Speer, Strickland, and Johnson state that "even experienced graduate students often lack knowledge of student learning of key ideas and have not developed strategies to support student learning of these topics" [Ellis, 2014]. In Kendall and Schussler's study, students complained that TAs did not always know how to answer student questions satisfactorily [Kendall and Schussler, 2012]. This research suggests that even experienced TAs could benefit from training which addresses pedagogical content knowledge.

Also, using student-centered instruction effectively requires TAs to possess pedagogical content knowledge [Ellis, 2014]. In this setting, the instructor must recognize students' prior knowledge, student thinking processes, and where students typically struggle in order to provide quality instruction [Ellis, 2014, Spike and Finkelstein, 2012]. Using teaching techniques such as Socratic dialogue and representing a concept in multiple ways also requires strong pedagogical content knowledge. Ellis states

that professional development which highlights pedagogical content knowledge is more valuable to TAs than just covering pedagogy.

Some training programs afford TAs the opportunity to gain pedagogical content knowledge before they are assigned to teach [Ellis, 2014]. In these programs, TAs serve as a apprentice with an experienced faculty member. They attend the course every week, meet with the instructor to discuss student difficulties, and hold office hours to learn "content specific" teaching techniques. In other programs, TAs team teach with an experienced instructor to gain pedagogical content knowledge before being given their own teaching assignment [Ellis, 2014].

Spike and Fikelstein describe their approach to TA professional development for physics recitation leaders. In the recitation, TAs are expected to use research-aligned teaching pedagogy. As part of the training, TAs are required to meet every week and discuss "potential student difficulties" and student reasoning. Experienced faculty guide TAs' discussions of potential student pitfalls. Spike and Fikelstein showed that their training program helped TAs to successfully identify student conceptual difficulties before each recitation [Spike and Finkelstein, 2012].

Training programs which discuss pedagogical content knowledge are more effective for TAs because the training directly relates to their specific teaching assignment [Ellis, 2014].

2.7.4 A Successful Model

After reviewing the literature, to the best of our knowledge, no current research has examined effective TA training models tailored to calculus recitation instruction; however, again drawing upon other disciplines, we describe one effective and practical model for training chemistry recitation leaders.

Pentecost et al. describe their department's general chemistry sequence, which includes a recitation component. Recently, the department has transitioned from a traditional question and answer recitation, to a student-centered recitation with standardized materials that focus on building conceptual understanding. To aid in their transition, Pentecost et al. have also created a new TA professional development program to support the recitation leaders. In addition to the general training required for all graduate students, recitation leaders attend a three day training before the semester. They describe the goals of the training program as focusing on "reviewing core concepts of general chemistry and training students to lead student-centered recitation sections" [Pentecost et al., 2012].

First, TAs learn about research in education, chemistry education, and important pedagogical approaches to teaching through an interactive workshop model. "Our strategy was to engage TAs in the type of learning environment they would be expected to cultivate in their own sessions." Then TAs reviewed the course content by working collaboratively through the recitation material. "The content review sessions were modeled after our intended recitation structure. The training leader facilitated discussion among TAs as they worked through the recitaiton material in small groups". After every session, TAs work to identify where students may have difficulties and how students may approach the material. This helps TAs to build pedagogical content knowledge. This training model is uniquely tailored toward recitation instructors, thoroughly covers all three knowledge domains, and does not require a large time commitment [Pentecost et al., 2012].

Chapter 3

Methodology

To answer this study's research questions, a quantitative-dominant mixed methods design was used. Johnson et al. describe quantitative-dominant mixed methods research as a "type of mixed research in which one relies on a quantitative, postpositivist view of the research process, while concurrently recognizing that the addition of qualitative data and approaches are likely to benefit most research" [Johnson et al., 2007]. Because of the complexity of math education research, purely quantitative studies often aren't descriptive or explanatory, especially when characterizing teaching methods. Researchers often utilize mixed method designs concurrently to enhance and illustrate the finding between the two methods [Ross and Onwuegbuzie, 2012]. In this study, a mixed methods design was used with priority given to quantitative data analysis, while integrating qualitative data for an increased understanding of the recitation environment and teaching methods used.

3.1 Participants

The University of New Mexico offers sections of calculus I, calculus II, and calculus III using the lecture-recitation format. The lecture is instructed by a faculty member and the recitation is instructed by a graduate student (TA). The lecture meets three days a week on Monday, Wednesday, and Friday for 50 minutes. Recitations are 75 minutes and are scheduled on Tuesdays and Thursdays. Half of the students in the lecture attend the Tuesday recitation and the other half attend the Thursday recitation. During the Fall 2016 semester, the math department offered 6 lecturerecitation sections of calculus I, 5 lecture-recitation sections of calculus II, and 5 lecture-recitation sections of calculus III.

The participant population in this study is comprised of two groups: graduate teaching assistants instructing a calculus recitation and the students enrolled in their recitation sections.

Graduate Student Teaching Assistants

Of the 16 graduate TAs assigned to instruct a calculus recitation in the Fall 2016 semester, 12 of the TAs agreed to participate in this study. Table 3.1 provides a description of our sample. We include the number of international TAs in our sample and the TAs' years of experience teaching for the UNM Department of Mathematics and Statistics.

Lable 0.1. Descriptive table of IA participatities						
Course	Number	International	Years of Experience			
Level	of TAs	TAs				
Calculus I						
Calculus II						
Calculus III						
Total						

Table 3.1: Descriptive table of TA participants

The UNM Math Department's TA professional development is a transitional program. Graduate students that are offered a teaching assistantship are required to attend a one semester graduate student teacher training seminar. This seminar is discussion-based and is lead by more experienced graduate students. TAs are required to attend 8 seminars throughout the semester and each seminar addresses a different topic. The topics covered are preparing for class, grading, quiz/exam writing, motivating students, evaluations, using technology, active learning, and writing a teaching philosophy.

Both math and statistics graduate students are required to attend the same seminar and their TA assignments range from grading, recitation leader, or the instructor of record for a course. Therefore, the seminar is designed to provide general information that is applicable to all TA assignments in both math and statistics. The seminar does not focus on covering material specific to calculus recitation leaders, such as effective teaching methods in recitation settings or calculus pedagogical content knowledge.

It is suggested that TAs assigned to instruct a calculus recitation cover the current or previous weeks' material and answer homework questions. TAs are expected to also give a short quiz in every recitation. The department does not have any official guidelines on what content to cover in the recitation, how to structure the recitation, or what teaching methods TAs should use. The department also does not have a formally stated purpose for the recitation section.

Calculus Recitation Students

The students enrolled in these 12 calculus recitation sections were also asked to participate in this study. Our sample consisted of the 388 students who completed the survey (Table 3.2).

	Table 0.4. Descriptive table of staddille participation								
	Course Level Number of Sections Number of Students								
Calculus I		109							
Calculus II		129							
Calculus III	h	150							
Total	19	388							

 $Table 3.2: Descriptive table of student participants$

The recitation sections in each course level (Calculus I, Calculus II, Calculus III) reported having similar average GPAs. The calculus courses offered at UNM are a coordinated sequence with a common syllabus, homework, and final exam.

3.2 Instruments and Data Sources

Data was collected from 4 sources:

- 1. Observational data was collected from teaching assistants using the Teaching Dimensions Observation Protocol.
- 2. TAs were surveyed using the GTA Professional Development Survey.
- 3. Calculus recitation students were surveyed using the Student Evaluation of Recitation Teaching Practices Survey.
- 4. The calculus recitation students' final exam scores.

Data collected from the GTA Professional Development Survey, the Student Assessment of Teaching Methods Survey, and students' final exam scores were quantitative. Both quantitative and qualitative data were collected during classroom observations using the Teaching Dimensions Observation Protocol.

Teaching Dimensions Observation Protocol

The Teaching Dimensions Observation Protocol (TDOP) was used to gather quantitative and qualitative observational data from the 12 calculus recitation sections involved in this study. Developed by the Wisconsin Center for Educational Research, the TDOP "is a descriptive classroom observation protocol that provides robust and nuanced depictions of instructional behavior." The protocol is unique in that it captures teaching practices without making any inferences about the quality of instruction [Hora et al., 2013].

The TDOP uses 39 codes categorized into 6 different dimensions (Instructional Practices, Student-Teacher Dialogue, Instructional Technology, Potential Student Cognitive Engagement, Pedagogical Strategies, and Students' Time-on-Task) to describe teaching practices.

A list and description of each code can be found in Appendix A. During an observation using the TDOP, the investigator digitally records the codes observed in two-minute intervals. Quantitative data was obtained by recording the frequencies of the codes observed during each calculus recitation section.

A few adaptations were made to the TDOP to better characterize the distinct environment of a calculus recitation. First, conceptual and procedural codes were added to describe the focus of content being covered in the recitation.

CON Focusing on Conceptual Knowledge: The content being discussed or the task being completed by either the students or the instructor focuses on building conceptual knowledge. This can include emphasis on conceptual development, multiple methods of solving a problem to build connections among mathematical ideas, the instructor linking the entry knowledge and skills of students to more formal concepts and procedures, reinforcing new skills and

procedures with prior knowledge and intuition, presenting topics numerically or graphically, or the instructor only valued correct answers if students could explain them. A question should be coded as CON if it deals with the underlying concepts of the problem [Chappell and Killpatrick, 2007, Kortemeyer, 2006].

PRO Focusing on Procedural Knowledge: The content being discussed or the task being completed by either the students or the instructor focuses on building procedural knowledge. This can include emphasis on procedures, skills and algorithms, basic procedural examples that do not require advanced manipulations, procedural examples of a greater level of technical difficulty, emphasizing algebraic solution methods over non-algebraic solution methods, not encouraging students to work a problem in more than one way, or the instructor did not expect students to explain the variety of methods they employed as they solved a problem. A question should be coded as PRO if it is an inquiry about a mechanism of solving a problem without mentioning the underlying concepts or reasoning [Chappell and Killpatrick, 2007, Kortemeyer, 2006].

The definition of these two codes were taken from Chappel and Killpatricks' description of the difference between procedure-based and concept-based learning environments in undergraduate calculus. The definition used to define a procedural or conceptual question was taken from Kortmeyer's classification of procedural and conceptual student discussion.

Also, the TDOP includes four codes to describe student engagement. Since class sizes in a recitation are generally smaller than a typical classroom, we removed the

Very High code and rescaled the definition of student engagement using only High, Medium, and Low.

The TDOP also allows the observer to write notes during the observation. In the observation notes, the researcher records descriptions of the interactions, content, and environment observed in each recitation. Examples include interaction between students and either the TA or their peers, the types of problems being solved, examples of interesting student questions, etc. These observation notes served as the source of qualitative data in our mixed methods design.

Graduate Professional Development Survey

TAs that agreed to participate in this study were also asked to complete the GTA Professional Development Survey assessing the quality of TA training they received and its efficacy in preparing them to successfully lead a calculus recitation. The survey was designed using the GTA Professional Development Instrument [DeChenne et al., 2012].

The GTA Professional Development instrument focuses on measuring TAs' "perception of their learning about important topics in teaching during GTA professional development programs" [DeChenne et al., 2012]. The instrument has two initial questions which capture overall satisfaction with professional development. The responses to these two questions are on 6-point scale where 1 represents "not effective" and 6 represents "very effective". The remaining 14 questions gauge TAs perceived learning about different concepts relevant to effective undergraduate teaching. Response are also on a 6-point scale where 1 represents "never learned"' and 6 represents "learned well".

A few adaptations to the survey were made to better assess TA learning about teaching topics pertinent to calculus recitation instruction. These topics were iden-

tified from the literature as important to calculus recitation instruction. Of the 14 content-related questions included in the GTA Professional Development survey, we kept 8 of the questions that measure general teaching topics. We added 6 new questions that are relevant to recitation instruction such as student-centered instruction, collaborative learning, and checking for conceptual understanding.

Two other questions were added to the survey that measured how well TAs felt they understood the purpose of the recitation and how responsible they felt for designing their section. The full survey can be found in Appendix C. All data collected from the GTA professional development survey was quantitative.

Student Evaluation of Recitation Teaching Practices Survey

The calculus students that participated in this study were asked to complete a survey assessing their overall satisfaction with the recitation and the teaching methods used. We developed this survey by combining questions from the Arizona Course Instructor Evaluation Questionnaire (CIEQ) and the Student Assessment of Teaching and Learning Questionnaire (SATL).

The CIEQ aims to measure the overall "quality and effectiveness of course elements" by using evaluators such as students' general course attitude, course content, method of instruction, and student interest and attention [Aleanoni, 1978]. The questionnaire has 20 items, with each item corresponding to one of the four subscales described above. Item responses are on a 4-point scale. Half of the questions are negatively stated where 1 represents strongly agree and 4 represents strongly disagree, and the other half are positively stated where 1 represents strong disagree is 4 represents strongly agree. The CIEQ allows you to obtain a mean response for each sub-scale by averaging the numerically coded responses to each question that corresponds to that sub-scale. An overall course attitude score can be obtained by

averaging the responses to all questions (1 represents poor course attitude and 4 represents excellent course attitude). In our survey, we chose to remove 5 questions that did not pertain to the recitation.

The second half of the survey used in this study was based off of the SATL questionnaire [Evans et al., 1993]. In this section, students were asked to grade the teaching methods used in the recitation, the overall quality of the recitation, and its contribution to their learning. The grades that students gave were in the form of percentages that correspond to a letter grade.

We also include in our survey four questions about students' GPA, grade in previous math class, anticipated grade in current math class, and how clearly they understood the department's vision/purpose for the recitation. The complete survey can be found in appendix C. All data collected from the Student Evaluation of Recitation Teaching Practices survey was quantitative.

Calculus Recitation Students' Final Exam Scores

A common final exam is given to students in calculus I, calculus II, and calculus III. The final exams are graded by the math department using a standard rubric. Passing was defined as scoring a 70% or better on the final exam. Each section's passing rate on the final exam was recorded.

3.3 Procedure and Time Frame

In this study, data was collected in several phases. In the first phase, data was collected from observing calculus recitation instructors using the TDOP. Each recitation section was observed by the student investigator for one week. Both the Tuesday and Thursday recitation were observed from each section during that week. TAs

were asked to participate in the study in September and observations began in mid-October and ended in late November. In the last two weeks of the semester, TAs were asked to complete the GTA Professional Development Survey online anonymously.

In the second phase, calculus students completed the Student Evaluation of Recitation Teaching Practices Survey. During the 13th week of the semester, TAs asked students in both their Tuesday and Thursday recitations to participate in this study by filling out a survey. TAs read from an instruction script which provided students with information about the study. The TAs then left the classroom after distributing the surveys and instructing students to place them in a manila envelope and seal the envelope once all students were finished. TAs delivered the sealed envelope containing the surveys to the main office of the math department and were instructed not to view their students' completed surveys.

The final exam passing rates for each section were also recorded after all the finals exams had been graded by the department.

3.4 Validity and Reliability

Several measures were taken to increase the reliability and validity of the study.

Validity

To increase the validity of this study, we choose to use an observation protocol developed by field experts with an established validity. The two codes that were added to the protocol were developed using definitions from previously published studies by experienced researchers. Both surveys used in this study were also based on instruments developed by field experts with a tested validity.

Also to increases the validity of this study, measures were taken to ensure voluntary participation of both TAs and their students. TAs were informed that choosing to participate in this study was optional and choosing not to participate would not be disclosed to anyone other than the research team. When TAs recruited their students to participate in the study, TAs were instructed to read from an instruction script which emphasized to students that participation in this study is optional.

Measures were also taken to ensure participants' privacy and confidentiality. Once the observations were completed, each TA was given a randomly generated pseudonym and all identifying information was removed from the data. Only the researcher viewed the data before it was de-identified. The electronic surveys that the TAs completed did not contain any identifying information.

In the instruction script, TAs informed their students that they will not see any of the survey results. The teaching assistants were instructed to leave the classroom after distributing the surveys. Students placed the surveys in a manila envelope and sealed the envelope once all students completed the survey. No identifying information was collected from the students' surveys. These measures were taken to help increase the validity of the data collected by ensuring participants that their privacy and confidentiality was of critical importance.

Reliability

To increase the reliability of the quantitative observational data, intra-rater reliability testing was conducted. I completed an observation of a recorded calculus lecture using the TDOP. After 10 days, I re-observed the same lecture. The TDOP's builtin reliability testing capabilities calculated a Cohens Kappa statistic for the two different observations. A Cohens Kappa statistic measures agreement between two variables, where $\kappa > 0.75$ indicates excellent agreement. I obtained a Cohens Kappa of 0.81 after comparing the two observations.

Also, to increase reliability the surveys used in this study were developed using instruments with tested reliability.

3.5 Analysis of Quantitative Data

Linear Regression and Ordinal Regression Models were developed to analyze the quantitative data collected.

3.5.1 Variables

Explanatory Variables

Conceptual

The TDOP conceptual code was used to measure the frequency that conceptbased content was observed in two-minute time intervals during the recitations, where a value of 1 indicates that conceptual was coded in every two-minute interval observed (Table 3.6, Figure 3.1). Data was not collected while students were completing a quiz. Since each section was observed twice, the two conceptual scores were averaged to obtain a final score of conceptual frequency per section $(n=12)$.

Table 3.6: Summary of conceptual variable

				Min. 1st Qu. Median Mean 3rd Qu. Max.	
$0.00 -$	0.04	0.08	0.12	0.21 0.27	

Figure 3.1: Boxplot of conceptual variable, the blue points represent the recitation observations and the red diamond is the mean.

Student-Centered Variable

Student-centered instruction is a broad teaching approach that encompass different pedagogical strategies. The variables from the TDOP Code Bank that reflected student-centered instruction were student presentation, peer interaction, problem solving, group work, desk work, and individualized instruction. We combined peer interaction and student presentation to create a new peer interaction variable. We also chose the variable working out problems as a code that describes teacher-centered instruction. This variable refers to the instructor working out computations on the board in front of students. Two education researchers were consulted as content experts to evaluate the validity of our choice of variables. A more complete description of these variables can be found in Appendix A.

Factor analysis was used to group these variables based on shared variances to create a new factor variable measuring student-centered instruction. Because some of the variables above were highly correlated, we choose to only include individualized instruction, peer interaction, problem solving, and working out problems into our analysis. The variables were analyzed using maximum-likelihood factor analysis with Varimax rotation and Kaiser criterion (Table 3.4).

The variables that we associated with student-centered instruction loaded above 0.7, and working out problems, which is negatively associated with student-centered

Variables	Factor Loadings
Individualized Instruction	0.698
Peer Interaction	0.778
Problem Solving	0.776
Worked Problem	-0.998

Table 3.7: Factor loadings for student-centered instruction

instruction, loaded at -0.99. The proportional variance was 0.67 and the Kaiser Criterion was 2.69. Since the Kaiser criterion was larger than one, this indicates the factor's validity [Yong and Pierce, 2013]. Each section was given a factor score describing how student-centered the TA's teaching methods were (Table 3.5, Figure 3.2). A positive score indicates alignment with student-centered instruction $(n=12)$.

Table 3.8: Summary of student-centered variable

		Min. 1st Qu. Median Mean 3rd Qu. Max.	
		-1.27 -0.76 -0.25 0.00 1.06 1.39	

Figure 3.2: Boxplot of student-centered variable, the blue points represent the recitation observations and the red diamond is the mean.

Quality of Instruction in the Lecture

In the Student Evaluation of Recitation Teaching Practices Survey, one of the questions recitation students answered was "How would you grade the quality of teaching in the lecture?". Students were asked to respond with a number corresponding to the traditional grading scale.

 $A = 90 - 100$ $B = 80-89$ $C = 70 - 79$ $D = 60 - 69$ $F =$ below 60

This numeric variable was used in the analysis to adjust for the quality of the lecture (n=388). We also averaged the students' scores to obtain an average lecture quality score for each section $(n=12)$. Table 3.6 shows both summaries for lecture quality.

Table 3.9: Summary of lecture quality per student and average per section

	Min.				1st Qu. Median Mean 3rd Qu.	Max.
Per Student		85.00	92.50	88.56	95.00	100.00
Average Score Per Section	61.67	85.84	87.43	86.69	90.92	95.06

The Recitation's Contribution to Students' Learning

In the Student Evaluation of Teaching Practices Survey, students were asked to grade the contribution of the recitation and lecture to their learning in the course using the same grading scale above. A variable was created that adjusted for how much the recitation contributed to students' learning (Table 3.7). This variable, Recitation's Contribution to Learning (RCL), was defined as RCL = $\frac{R}{R}$ $\frac{1}{R+L}$, where R is the grade students gave the recitation contribution and L is the grade students gave the lecture contribution (n=388).

Students Grade in Past Math Class

In the student assessment of teaching methods survey, one of the questions recitation students answered was "What was the grade you received in your last math

class?". The answers to this question were categorized as A, B, C, and D or lower. This categorical variable was coded as 3, 2, 1, and 0 respectively (Table 3.8). This variable was used in the analysis to adjust for students' mathematical backgrounds and abilities (n=388).

Table 3.11: Summary of students' past math grades

122	128	
		6%
		Percent 34% 36% 24%

Calculus Course Level

Indicator variables were created to account for the differences between Calculus I, Calculus II, and Calculus III (Table 3.9).

Table 3.12: Summary of calculus course level Calculus I Calculus II Calculus III Count 109 129 150 **Percent** 28% 33% 39%

Response Variables

Students' Course Attitude Score

Students' course attitude score was obtained by averaging the responses to items from the CIEQ, which was included in the Student Evaluation of Recitation Teaching Practices Survey. Response greater than 2 indicate a positive attitude towards the recitation and responses lower than 2 indicate a negative attitude toward the recitation (Table 3.10). Scores were averaged per section $(n=12)$.

Reported Student Engagement

Table 3.13: Summary of student course attitude						
	Min. 1st Qu. Median Mean 3rd Qu. Max.					
2.51	2.86	- 3.06	- 3.01	3.1	3.43	

One of the subscales measured by the CIEQ included in the Student Evaluation of Recitation Teaching Practices Survey was Students Interest and Attention. Response greater than 2 indicate positive student-reported engagement in the recitation and responses lower than 2 indicate negative student-reported engagement (Table 3.11). Scores were averaged per section (n=12).

Observed Student Engagement

The TDOP Students' Time-on-Task dimension was used to measure student engagement observed in the classroom. "High" student engagement was recorded whenever more than 75% of students were actively taking notes or looking at the instructor/course material. The total was divided by the number of 2-minute time intervals observed (Table 3.12). Since each section was observed twice; the two "high" student engagement scores were averaged to obtain a final score per section $(n=12)$.

Table 3.15: Summary of observed student engagement Min. 1st Qu. Median Mean 3rd Qu. Max. 0.08 0.27 0.40 0.46 0.65 0.83

Passing Rate on Final Exam

The percentage of students who scored a C or better on the final exam in each section was calculated (Table 3.13). One section did not take the standardized final

exam, resulting in one missing value which was removed from analysis involving passing rates $(n=11)$.

Students' Predicted Course Grade

In the Student Evaluation of Recitation Teaching Practices Survey, recitation students answered the question "What grade do you anticipate you will get in this course?". The answers to this question were categorized as A, B, C, and D or lower, which we coded as 3,2,1,0 respectively (Table 3.14). Students completed the survey two weeks before the final exam. Burns finds through his study that students' "accuracy of grade expectations may improve as a course progresses" [Burns, 2012]. Because of the close proximity of the survey distribution to the end of the semester, it is reasonable to assume the validity of student responses is improved $(n=388)$.

Table 3.17: Summary of student's predicted course grade

Count	76	154	-112	
Percent 21%		44%	-32%	

3.5.2 Models

Linear Regression Models

Simple Linear Regression

Simple linear regression was used to describe the relationship between different teaching methods and student course attitude.
Chapter 3. Methodology

The simple linear regression model is:

$$
\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X
$$

where $\hat{\beta}_0$ and $\hat{\beta}_1$ are chosen using Least Squares Estimation:

$$
\arg\min_{\beta_0, \beta_1} \sum_{i=1}^n \{Y_i - (\beta_0 + \beta_1 X_i)\}^2
$$

We fit separate linear regression models using students' course attitude as the response variable and student-centered instruction, working out problems, and conceptual frequency as the predictor variables. The worked problems variable is a measure of teacher-centered instruction. We also performed a simple linear regression to determine if a correlation exists between student-centered instruction and students' reported student engagement and observed student engagement. Adjusted $R²$ values were used in the analysis. Each variable was an average score per section $(n=12)$.

Normality of residuals was tested using the Shaprio-Wilk test for normality. The null hypothesis for this test is H_0 : the distribution is normal, against the alternative hypothesis H_A : the distribution is not normal. Equal variance of errors was tested using the Breush-Pagan test for homoscedasticity using the null hypothesis for this test is H_0 : constant variance, against the alternative hypothesis H_A : heteroscedasticity. Model adequacy was also assessed using residual plots. ANOVA analysis was used to test the hypothesis of zero slope, where $H_0: \beta_1 = 0$ vs. $H_A: \beta_1 \neq 0$.

LASSO Regression

Because of the small sample size of our data $(n=11)$ and number of covariates, least absolute shrinkage and selection operator regression (LASSO) was used to model the relationship between different teaching methods used in the recitation and each section's final exam passing rate. The glmnet function in R was used for LASSO. The explanatory variables included to describe teaching methods of interest were

Chapter 3. Methodology

student-centered instruction and conceptual frequency. Other explanatory variables were used to adjust for the quality of the lecture, the recitation contribution to students' learning, and the calculus course level.

The general LASSO model fits the same linear model

$$
\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \ldots + \hat{\beta}_p X_p
$$

but $\hat{\boldsymbol{\beta}}$ is solved for by minimizing the general least squares equation

$$
\hat{\beta} = \arg \min_{\beta_i} \left(\sum_{i=1}^n \{ Y_i - \beta_0 - \sum_{j=1}^p \beta_j X_{ij} \}^2 \right)
$$

subject to a constraint

$$
\sum_{j=1}^p |\beta_j| \le t.
$$

This constraint, called the penalty term, uses the ℓ_1 norm. For sufficiently large λ, the coefficients will be driven to zero. If $\sum_{j=1}^{p} |B_j^L| < t$, where $\hat{\beta}^L$ is the least squares estimate, then LASSO will produce the same coefficients as the least squares regression, but if $0 < t < \sum_{j=1}^p |B_j^o|$, the optimization problem is equivalent to

$$
\hat{\beta} = \arg \min_{\beta_i} \left(\sum_{i=1}^n \{ Y_i - \beta_0 - \sum_{j=1}^p \beta_j X_{ij} \}^2 + \lambda \sum_{j=1}^p |\beta_j| \right)
$$

with $\lambda > 0$. A one-to-one correspondence exists between parameter t and λ .

Cross-validation was used to select the λ that minimizes the cross validation error. Interaction terms were considered, but were found to not be statistically significant, and thus were not included in the final model. LASSO regression has no assumptions of normality.

Significance testing was not conducted on our model since there isn't a widely used standard test for significance in LASSO regression analysis.

Ordinal Regression Model

Proportional Odds Cumulative Logistic Regression (ordinal regression) was utilized to determine the relationship between different teaching methods used in calculus recitations and students' predicted final grade in their calculus course. The polr function was used in R. Any observations with missing data were removed before applying the model $(n=362)$. The conceptual variable was standardized before computing the model.

The general model is

$$
\log \left(\frac{P(Y \leq j)}{1 - P(Y \leq j)} \right) = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \dots + \hat{\beta}_n X_n
$$

where $P(Y \leq j)$ is the cumulative probability. The response variable, students' predicted final grade, is a categorical variable with 4 factors. Ordinal regression requires an assumption of parallel lines to reduce the model's parameters. Our model becomes

$$
\log\left(\frac{P(Y \le 0)}{1 - P(Y \le 0)}\right) = \hat{\alpha_1} + \hat{\beta_1}X_1 + \hat{\beta_2}X_2 + \dots + \hat{\beta_n}X_n
$$

$$
\log\left(\frac{P(Y \le 1)}{1 - P(Y \le 1)}\right) = \hat{\alpha_2} + \hat{\beta_1}X_1 + \hat{\beta_2}X_2 + \dots + \hat{\beta_n}X_n
$$

$$
\log\left(\frac{P(Y \le 2)}{1 - P(Y \le 2)}\right) = \hat{\alpha_3} + \hat{\beta_1}X_1 + \hat{\beta_2}X_2 + \dots + \hat{\beta_n}X_n
$$

We first fit the full model. Non-significant variables were dropped using AIC backwards selection at the $\alpha = 0.05$ level. Interaction terms were not considered because the variables student-centered instruction and conceptual frequency can be collapsed down to only 12 distinct observations.

The AIC criterion and standard deviance were used to assess the model's strength. A likelihood ratio goodness-of-fit test was used to compare our model with the intercept only model. This served as a global F test. Also, model adequacy was determined by testing the model's assumption of parallel lines. These assumptions were assessed by performing a likelihood ratio goodness-of-fit test comparing this model to the full multinomial logit model that does not use a parallel slopes assumption. Ordinal regression has no assumptions of normality or equal variance.

Analysis of the GTA Professional Development Survey

Eleven out of the twelve TAs that participated in this study completed the anonymous online GTA Professional Development Survey. The average response for each question was calculated and compared to the mean average response.

3.6 Analysis of Qualitative Data

The qualitative data in this study consisted of the observation notes collected during each classroom observation. Our analysis focused on identifying/interpreting patterns and themes in our observation notes. This analysis aimed to better describe recitation teaching practices, student and TA interactions, and the content covered in the recitation.

Chapter 4

Results

4.1 Quantitative Results

4.1.1 Simple Linear Regression Models

Simple linear regression was used to determine if correlations existed between different teaching methods and student course attitude. Student course attitude did not vary significantly by calculus level. The average student course attitude for Calculus I was 2.99, the average course attitude for Calculus II was 3.16, and the average course attitude for Calculus III average was 2.91. A one-way analysis of variance (ANOVA) was calculated on student course attitude by calculus level. The ANOVA analysis was not significant $(F = 1.03, p$ -value= 0.40). Therefore, the variable for calculus course level was not added into the model.

Figure 4.1: Correlation of student-centered instruction with student course attitude, $r = 0.68$, p-value = 0.008, with an $R^2 = 0.52$. The points are the 12 recitation observations. The blue line is the linear regression fit to the data points and the bands are the 95% confidence bans.

Correlation between Student-Centered Instruction and Student Course Attitude

Simple linear regression found that student-centered instruction and student course attitude were positively correlated with $r = 0.68$ and a p-value of 0.008. Our sample size was $n = 12$. The R^2 coefficient was 0.52 (Table 4.1, Figure 4.1).

Table 4.1: Simple linear regression parameter estimates for student-centered vs. course attitude.

Coefficients		Estimate Std. Error t-value p-value		
Intercept	3.01	0.05	55.85	8.21×10^{-14}
Student-Centered Instruction	0.18	0.06	3.27	0.008

The Shapiro-Wilk test for normality found a *p*-value of 0.73, indicating normality

of residuals. The Breush-Pagan test for homoscedasticity found a p -value of 0.71, indicating equal variance of errors. The residual plots also provided evidence of linearity, normality, independence, and homogeneity of variance.

Correlation between Working out Problems and Student Course Attitude.

Simple linear regression found that the frequency of working out problems and student course attitude were negatively correlated with $r = -0.68$ and a p-value of 0.02. Our sample size was $n = 12$. The R^2 coefficient was 0.46 (Table 4.2, Figure 4.2).

Table 4.2: Simple linear regression parameter estimates for worked problems vs. course attitude

Coefficients		Estimate Std. Error t-value p-value		
Intercept	3.51	0.18	19.56	2.76×10^{-9}
Worked Problems	-0.73	0.25	-2.92	0.02

Figure 4.2: Correlation of frequency of worked problems with student course attitude, $r = -0.68$, p-value = 0.02, with an $R^2 = 0.46$. The points are the 12 recitation observations. The blue line is the linear regression fit to the data points and the bands are the 95% confidence bans.

The Shapiro-Wilk test for normality found a p -value of 0.58, indicating normality of residuals. The Breush-Pagan test for homoscedasticity found a p -value of 0.85, indicating equal variance of errors. The residual plots provided evidence of linearity, normality, independence, and homogeneity of variance.

Regression with confidence bands

Figure 4.3: Correlation of student-centered instruction with observed student engagement, $r = 0.60$, p-value = 0.04, with an $R^2 = 0.35$. The points are the 12 recitation observations. The blue line is the linear regression fit to the data points and the bands are the 95% confidence bans.

Correlation between Student-Centered Instruction and Student Engagement

Two simple linear regression models were used to measure the correlation between student-centered instruction and student engagement. The first model used "high" student engagement from TDOP observational data as the response variable. This is a measure of observed student engagement. Simple linear regression found that student-centered instruction and "high" student engagement observed were corre-

Table 4.3: Simple linear regression parameter estimates for student-centered vs. observed student engagement.

Coefficients		Estimate Std. Error t-value p-value		
Intercept	0.46	0.06	7.95	1.24×10^{-5}
High Student Engagement	0.14	0.06	2.34	0.04

lated with $r = 0.60$ and a p-value of 0.04. Our sample size was $n = 12$. The R^2 coefficient was 0.35 (Table 4.3, Figure 4.3).

The second simple linear regression model used the student interest and attention CIEQ subscale score as the response variable. This is a measure of student-reported engagement in the recitation. Simple linear regression found that student-centered instruction and student interest and attention were correlated with $r = 0.74$ and a p-value of 0.006. Our sample size was $n = 12$. The R^2 coefficient was 0.55 (Table 4.4, Figure 4.4).

Table 4.4: Simple linear regression parameter estimates for student-centered vs. student-reported engagement.

Coefficients		Estimate Std. Error	t-value	p-value
Intercept	2.71	0.06	42.30	1.24×1.31^{-12}
Student Interest & Attention	0.24	0.07	3.52	0.006

The Shapiro-Wilk test for normality found a p -value of 0.66 and 0.87 for these two linear regression models respectively, indicating normality of residuals. The Breush-Pagan test for homoscedasticity found a p-value of 0.86 and 0.12 respectively, indicating equal variance of errors. The residual plots for both regression models also provided evidence of linearity, normality, independence, and homogeneity of variance.

Correlation between Conceptual and Student Course Attitude

Simple linear regression found that conceptual frequency and student course attitude were not correlated with $r = -0.30$ and a p-value of 0.34. Our sample size was $n = 12$.

Figure 4.4: Correlation of student-centered instruction with student-reported engagement, $r = 0.74$, p-value = 0.006, with an $R^2 = 0.55$. The points are the 12 recitation observations. The blue line is the linear regression fit to the data points and the bands are the 95% confidence bans.

The R^2 coefficient was 0.09 (Table 4.5, Figure 4.5).

The Shapiro-Wilk test for normality found a p -value of 0.70, indicating normality of residuals. The Breush-Pagan test for homoscedasticity found a p-value of 0.77, indicating equal variance of errors. The residual plots provided evidence of linearity, normality, independence, and homogeneity of variance.

Table 4.5: Simple linear regression parameter estimates for conceptual vs. student course attitude.

Coefficients Estimate Std. Error t-value				p-value
Intercept	3.11	0.12	25.48	1.99×10^{-10}
Conceptual	-0.79	0.80	-1.00	0.34

Figure 4.5: Correlation of conceptual frequency with student course attitude, $r =$ -0.30, p-value = 0.34, with an $R^2 = 0.09$. The points are the 12 recitation observations. The blue line is the linear regression fit to the data points and the bands are the 95% confidence bans.

4.1.2 Summary of LASSO Regression Model

A LASSO regression was used to predict each section's final exam passing rate based on the use of student-centered instruction and conceptual frequency in calculus recitations. Other covariates were included in the model to adjust for variables that affect passing rates. The LASSO model predicted that the final exam passing rate for Calculus III is 8.27% higher than the passing rate of Calculus I or Calculus II. This model does not suggest that student-centered and concept-based instruction have an effect on the final exam passing rate.

Using cross-validation, the lambda value which minimizes the cross-validation error was $\lambda = 3.04$. With this choice of lambda, the cross-validated mean squared error is 100.41 and the percent of deviance was less than 0.18. Figure 4.6 shows the

Figure 4.6: As λ increases, the cross-validation curve shows the change in the crossvalidated mean squared error. The blue dotted line corresponds to the value of λ which minimizes the mean-squared error and the second black dotted line is one standard error from the minimum λ value.

cross-validation curve used to select our value of λ .

With our choice of λ , the model degrees of freedom is 1. Thus, we have one nonzero coefficient remaining in the model. Figure 4.7 shows the nonzero coefficients that are retained in the model as λ varies. Similarly, Figure 4.8 shows the coefficients retained in the model as the ℓ_1 norm varies and Figure 4.9 shows the coefficients as the fraction of deviance explained by the model varies.

The LASSO regression model suggests that only the indicator variable for calculus III has a non-zero coefficient (Table 4.6). The LASSO model procedure does not perform significance testing for coefficients.

The final model fitted from the our data is:

 $Y = \hat{\beta}_0 + \hat{\beta}_1 X$

Figure 4.7: As λ increases, the plot shows the path of each coefficient against log λ . The dotted line indicates our choice of $\lambda = 3.04$. The x-axis is decreasing log λ .

where $X = 1$ for calculus III and $X = 0$ otherwise. The $\hat{\beta}_1$ coefficient is the non-zero parameter estimate predicted by the LASSO model, and Y is the final exam passing rate for each section. A section of Calculus III is predicted to have 8.27% higher passing rate than Calculus I or Calculus II.

Parameter	DF	Estimate
Intercept		29.71
Student-centered Instruction		
Recitation Contribution to Learning		
Quality of Lecture		
Calculus II		
Calculus III		8.27
Conceptual frequency		

Table 4.6: LASSO regression model parameter estimates

Figure 4.8: As λ decreases, the plot shows the path of each coefficient against the ℓ_1 norm, $\sum_{i=1}^{p} |\beta_i|$. A small penalty will result in coefficients being driven to zero.

Figure 4.9: As λ varies, the plot shows the path of each coefficient against the fraction of deviance explained. The dotted line indicates our choice of $\lambda = 3.04$.

Parameter	Estimate	Std. Error	t-value	p-value
Student-centered Instruction	1.21	0.28	4.39	1.14×10^{-5}
Conceptual Frequency	0.68	0.20	3.47	5.11×10^{-4}
Past math grade of C	-1.18	0.49	-2.41	0.02
Past math grade of B	-0.47	0.47	-1.00	0.32
Past math grade of A	0.64	0.47	1.35	0.18
Quality of Lecture	0.05	0.01	5.27	1.36×10^{-7}
Calculus II	-2.18	0.39	-5.65	1.58×10^{-8}
Calculus III	0.66	0.30	2.23	0.03
Intercepts				
Below $C \mid C$	-0.29	0.94	-0.31	0.76
B	2.83	0.93	3.06	2.25×10^{-3}
Β А	5.25	0.96	5.47	4.47×10^{-8}

Table 4.7: Ordinal regression parameter estimates

4.1.3 Summary of Ordinal Regression Model

The ordinal regression model, using proportional odds cumulative logistic regression, was used to determine the relationship between different teaching methods used in college calculus recitations and the likelihood that students' predicted final grades were an A, B, C, or lower than a C.

In the full model, the explanatory variables student-centered instruction, conceptual frequency, lecture quality, recitation contribution to learning, students' grade in their past math class, and calculus course level were included in the model. Using AIC backward selection, recitation contribution was dropped from the model $(p\text{-value} = 0.45).$

The final model was fitted, with $AIC = 744.95$ and residual deviance $= 722.95$ on 11 df (Table 4.7).

The final model fitted from our data is:

$$
\log \left(\frac{P(Y \leq j)}{1 - P(Y \leq j)} \right) = \hat{\alpha_i} + \hat{\beta_1} X_1 + \hat{\beta_2} X_2 + \hat{\beta_3} X_3 + \hat{\beta_4} X_4 + \hat{\beta_5} X_5 + \hat{\beta_6} X_6 + \hat{\beta_7} X_7 + \hat{\beta_8} X_8
$$

Parameter	Variable	Definition	Indicator
DΟ		Intercept	
β_1	x_1	Student-centered Instruction	
β_2	x ₂	Conceptual Frequency	
β_3	x_3	Past math grade	€
β_4	x_4	Past math grade	В
β_5	x_5	Past math grade	А
β_6	x_6	Quality of Lecture	
	x_7	Course Level	Calculus II
β_8	x_8	Course Level	Calculus III

Table 4.8: Ordinal regression final model parameters and variables.

Table 4.9: Odds ratio estimates and confidence intervals.

Parameter	Odds Ratio		Confidence Interval
	Estimate	2.5%	97.5%
Student-centered Instruction	3.34	1.96	5.77
Conceptual Frequency	1.98	1.35	2.91
Past math grade of C	0.31	0.12	0.80
Past math grade of B	0.63	0.25	1.57
Past math grade of A	1.89	0.75	4.80
Quality of Lecture	1.03	0.05	1.07
Calculus II	0.11	0.05	0.24
Calculus III	1.94	1.09	3.48

with $0 \leq j < 3$, and where the variables and coefficients are defined in Table 4.8.

Since each β_i is the log-odds of falling into or above category j, by exponentiating the coefficients we obtain the odds ratio estimates of falling into or above category j (Table 4.9). For our interpretation, we will consider the odds ratio of students predicting their final grade as being a B or higher $(j=2)$. Because of the proportional odds assumption, the odds ratio does not change depending on the category chosen.

Our model infers that students are 3.34 times more likely to predict their final grade as being either an A or a B in their calculus course with one unit increase of student-centered instruction, holding all other variables constant $(p$ -value=

Figure 4.10: Average effects of conceptual frequency in the recitation on the expected probabilities of predicted grade with 95% confidence bans

0.00001, $95\% CI[1.96, 5.77]$. We also predict that an increase of conceptual frequency, students will be 1.07 times more likely to predict their final grade as being either an A or a B (p-value= 0.0005 , $95\% CI[1.35, 2.91]$), holding all other variables constant in the model.

Also, we find that with one percent increase in the grade given to the quality of lecture instruction, the odds of students predicting their grade as being either an A or a B increases by 1.05. Students in calculus III are 1.94 times more likely to predict a grade of A or B in their calculus course. Students in calculus II are less likely of reporting a grade of A or B.

Figure 4.10 and 4.11 show the average effects of student-centered instruction and conceptual frequency, respectively, on the expected probabilities of students' predicted course grade, holding all other variables in the model constant.

Figure 4.11: Average effects of student-centered instruction in recitation on the expected probabilities of predicted grade with 95% confidence bans

A likelihood ratio goodness-of-fit test was performed to test if this model fits better than the intercept only model. The null hypothesis tested was H_0 : β_i = $0, 1 \leq i \leq 8$ against the alternate hypothesis $H_A: \beta_i \neq 0$, for some *i*. We obtained a test statistic of p-value of 0.00 on 16 df, indicating that at least one of the regression coefficients is not equal to zero.

To test the model assumption of parallel lines, a likelihood ratio test was used to compare our model to the full multinomial logit model. The null hypothesis tested was H_0 : the current model explains as much variance as the more complex model, against the alternate hypothesis H_A : the current model does not explain as much variance as the more complex model. A p -value of 0.27 was obtained on a difference of 16 degrees of freedom. This indicates that the assumption of parallel lines is reasonable and provides evidence of model adequacy.

Question	Response
Overall questions on GTA training	
Overall, how effective has the TA training you have receive at	3.91
UNM been in preparing you to teach a recitation?	
Overall, how effective has the TA training you have received been	3.45
in preparing you to work with students?	
Teaching Topics and Skills ²	
Grading	4.27
Interacting professionally one-on-one with your students	3.60
Motivating students	3.27
Working with culturally diverse students	2.82
Using student-centered instruction	3.55
Teaching students with different skill/knowledge levels	3.36
Facilitating collaborative/cooperative learning	3.27
Facilitating active learning activities	3.36
Checking for conceptual understanding	3.09
Facilitating group discussion	3.55
Using socratic dialogue	3.09
Using multiple representations for problems	3.73
Teaching Styles	4.20
Developing quizzes/exams	4.36

Table 4.10: Results of the GTA Professional Development Survey.

¹ Responses indicate 1 = not effective, 6 = very effective.

² Responses indicate 1 = never learned, $6 =$ learned well. The mean response to these content specific questions was 3.54.

4.1.4 Summary of GTA Professional Development Survey Results

The results of the online GTA Professional Development Survey are summarized in Table 4.10. We also summarize the results from additional questions about the purpose and design of the recitation in Table 4.11.

Table 4.11: Additional results from the GTA Professional Development Survey and the Student Evaluation of Teaching Methods Survey.

Question		Average Response	
	TAs	Students	
I was responsible for designing the recitation I instructed.	5.27		
I clearly understood the math department's purpose/vision	4.64	4.42	
for the <i>recitation</i> .			

Responses indicate $1 =$ strongly disagree, $6 =$ strongly agree.

4.2 Qualitative Results

4.2.1 TDOP Observation Notes

The observation notes recorded during each classroom observation were analyzed and categorized by themes.The qualitative data was used to describe patterns observed in the recitations.

Student-Centered vs. Teacher-Centered Instruction

Some TAs gravitated towards teacher-centered instruction. During the recitation, these TAs spent the majority of class time solving problems on the board. The problems TAs solved were either taken from students' homework questions, unassigned problems from the textbook, or problems TAs prepared before the recitation. Some TAs focused more on reviewing content from the lecture. Other TAs followed more of a "homework-session" approach to their recitation by concentrating on students' questions.

Other TAs in the study utilized more student-centered instruction. TAs asked students to solve problems either individually or in groups. In one recitation section, students formed teams and raced each other to the board to solve math problems. Most TAs provided individualized instruction to students as they worked on prob-

lems. One TA had their students work one step at a time on a difficult problem and checked that every student had completed each step correctly.

In a few recitation sections, students worked well together and displayed a strong interest in learning the material collaboratively. However, in many of the studentcentered recitations, TAs asked students to work in groups but did not assign students to groups. The majority of students ended up working individually. In one recitation, students were asked if they would prefer to work on problems in groups or take notes as the instructor solved the problems. The majority of students voted for the instructor to solve the problems instead of working collaboratively.

Student Engagement

In more traditional teacher-centered recitations, a common observation note was that students who sat in the front of the classroom appeared to be highly engaged in the recitation, while students that sat in the back of the classroom appeared fairly unengaged. Students that sat in the back were observed texting or working on homework from other classes during the recitation. In several recitations, the majority of student-teacher dialogue revolved around questions from students in the front of the classroom. The recitation seemed to be tailored towards the needs of the more vocal, extroverted students in the recitation. Some TAs created a more interactive teacher-centered environment as they solved problems on the board by fostering group discussion and encouraging student questions. Less students appeared off task when the recitation was more interactive.

Student-centered recitations appeared to have more students on task, but still did not engage everyone in the classroom. In one recitation section, there were 18 men and 2 women attending the class. During a collaborative learning activity, most of the class was actively engaging in groups; however, the two women were not included

in the groups that students formed. They did not appear to be working on problems related to the course during the recitation. In another recitation observation, one student was the only member of the class not asked to work in groups with the other students. He was also the most ethnically diverse student in the class. The TA asked him how he was doing and he said he was struggling to understand how to begin the first problem. He spent the remainder of the recitation working on the first problem, while all the other students working in groups were able to complete several problems during class.

Focusing on Conceptual vs. Procedural Knowledge

Most recitations emphasized procedural knowledge. In one recitation, the TA reviewed the Simpson and Trapezoidal Rule by only focusing on making sure that students understood how to "plug numbers" into the formulas. The TA did not talk about the underlying concepts behind the formula nor did they draw a picture to illustrate its geometric interpretation. In another recitation section, students worked on finding the linear approximation to a function by "plugging and chugging" without any explanation of what students were actually computing. Based on students questions, it was clear that they did not understand the difference between the tangent and secant line, which is a fundamental concept of calculus.

Students also seemed focused on learning procedural knowledge. For example, one recitation instructor asked if someone would volunteer to solve the integral $\int_0^1 \ln x \, dx$ on the board. A student volunteered and confidently wrote the correct answer on the board, showing every step of his work. Afterwards, the instructor asked him to explain his process and the student was unable to explain any of his work. He was not even able to identify the method of integration he used. The student explained that the instructor had previously solved the integral during the lecture and he memorized every step. When students asked about homework questions, a common way they

stated their questions was "I don't understand number 5". The TA then addressed the procedural aspects of solving the problem while the students took notes.

Other recitation instructors focused more on checking for students' conceptual understanding. Several TAs used Socratic dialogue rather than directly answering students questions. When more emphasis was placed on understanding concepts, I observed that students also asked more specific homework questions such as "in question 5, I don't understand how this method applies to this problem".

During the observations, six TAs showed students multiple ways of solving a problem. In another recitation section that covered Trapezoidal and Simpson's Rule, students were asked to work in groups to come up with a geometric interpretation of both formulas. Students presented their picture and interpretation to the class. Also, another TA had students integrate $f(t) = |3t - 5|$ on the interval $[-3, 3]$ using both integration techniques and using geometry to find the area under the curve.

Different Recitation Purposes/Objectives

TAs seemed to have different beliefs about the purpose of the recitation. Some TAs focused on tailoring the recitation to cover topics that students were struggling with. One TA asked students whether they would rather go over the previous week's exam or work on Lagrange multiplier questions. The TA listened to students' feedback on what content would be the most helpful for them to cover in the recitation. Other TAs focused the recitation on writing out a list of definitions and topics that students will need to know for the quiz at the end of the recitation period and the next exam.

Other TAs had a prepared plan for the class and their recitation was not as tailored to students' needs. One recitation leader spent a significant part of the recitation explaining to students why lower and upper Riemann sum approximations were under and over estimations of the area under a curve. However, it did not

appear that students were struggling to understand this content.

Other TAs demonstrated their pedagogical content knowledge for the course they were teaching. They focused on giving students tips and tricks for specifics areas where students might struggle. For example, one TA covered potential pitfalls students might have using the alternate series test, comparison test, and integral test.

Chapter 5

Discussion

5.1 Summary of Results

Our research aimed to answer the following questions:

- 1. Are TAs using research-based teaching methods, such as student-centered instruction and emphasizing conceptual understanding, in the recitation? Are these methods more effective in improving student achievement and course attitudes than more traditional teaching methods?
- 2. Do TAs feel that they understand the purpose of the recitation? Do TAs feel prepared to design a recitation and be an effective recitation leader after completing the graduate student professional development offered by the department?

This study used a quantitative-dominant mixed methods design. To analyze the quantitative data collected, linear regression and ordinal regression techniques were used to understand how different recitation teaching methods explained variability

in student course attitude and achievement in calculus. Qualitative data was also analyzed to better describe the learning environments observed in the recitations. To answer our research questions, we reference results from quantitative and qualitative data collected in this study.

Are TAs using research-based teaching methods? In this study, we focused on student-centered instruction and emphasizing conceptual content as examples of teaching methods that align with undergraduate calculus research trends. Specific methods include collaborative learning, active learning, multiple representations, and Socratic dialogue. Research suggests these methods improve student learning, interest and motivation, and promote equity in the classroom. Because the department of mathematics at UNM does not have specific guidelines on preferred methods of instruction in calculus recitations, TAs used a wide variety of teaching methods in this study.

Student-Centered vs. Teacher-Centered Instruction: The participants in this study were fairly evenly divided between exhibiting teacher-centered or student-centered instructional methods. Six of the twelve TAs in this study were observed working problems on the board more than 75% of the recitation time (time allocated for quizzes was not include in the observational data collected) and seven of the TAs received a negative score for student-centered instruction. Also, during six of the TA observations, problem solving was not coded, implying that students were not asked to actively solve a problem during the recitation. Therefore, approximately half of the TAs that participated in this study utilized more traditional teaching methods in their recitation.

In contrast, five of the TAs observed in this study had positive scores for studentcentered instruction. In three of the recitation sections, problem solving was coded more than 40% of the recitation time. Students were asked to actively solve problems either individually or in groups, while TAs provided individualized instruction. Also,

in four of the TA observations, peer-interaction was observed more than 40% of the time. It was evident that several TAs successfully created a community environment where students felt comfortable working together. Students displayed a noticeable level of excitement about problem solving collaboratively.

Desk work was implemented more frequently than group work. Several TAs asked students to work in groups; however, they did not assign groups or ensure that students actually worked together. Though several TAs considered classroom activities to be "group work", students were actually completing desk work during class. This may show that TAs have a fundamental lack of understanding of the definition and objectives of collaborative learning.

Because collaborative learning activities were not always implemented successfully, they also did not always serve as effective equitable teaching strategies. In one recitation section, the two women in the classroom were not being engaged in the collaborative learning activities. In another recitation observation, one of the few ethnically diverse students in the class was also not being included in group work and as a result he spent the recitation struggling to solve the first problem.

Focusing on Conceptual vs. Procedural Knowledge: The majority of the recitation content that TAs covered emphasized building students' procedural proficiency. Three-fourths of the TAs that were observed focused on procedures more than 80% of the recitation and in a few observations conceptual was not coded at all. Most TAs developed only algebraic understanding of concepts while ignoring verbal, geometric, or numeric approaches.

In a few TA observations, conceptual was coded more than 25% of class time and there were several great examples of conceptually rich instruction. Several TAs used Socratic dialogue and showed multiple ways to solve a problem, for example using both calculus and geometry. It is interesting to note that of the six TAs who

used multiple representations during their recitation, five of the TAs were international graduate students. This could indicate that other countries may place more importance on multiple representations in their mathematics education.

Summary: This study found that TAs used a wide variety of both researchbased and traditional teaching methods in their recitation. Some TAs used more student-centered instruction and focused on collaborative or active learning. Others implemented teacher-centered instructional strategies and followed more of a "homework-session" approach to the recitation. Although most recitations focused on the procedural aspects of calculus, some TAs made efforts to make connections and build students' conceptual knowledge.

Are these methods more effective in improving student achievement and course attitudes? Simple linear regression was used to describe the relationship between student-centered and concept-based instruction with student course attitude. A LASSO regression model was developed to determine the relationship between these teaching methods and different sections' final exam passing rates and ordinal regression was used to explain students' predicted grade.

Student Course Attitude: Linear regression was first used to examine the correlation between teaching methods such as student and teacher-centered instruction with student course attitude. The variable used to describe teacher-centered instruction was the observed frequency of worked problems in the recitation. This study showed that there is a relatively strong, positive correlation between student-centered instruction and student course attitude ($r = 0.68$, p -value= 0.008) and a relatively strong, negative correlation between teacher-centered instruction and student course attitude ($r = -0.68$, p-value= 0.02). The r coefficients have the same magnitude because of the way we constructed the student-centered instruction variable using factor analysis.

These correlation results were somewhat surprising since they did not support findings from previous research, which suggests that students may respond negatively to student-centered instruction. Especially in mathematics, where most courses are taught via lectures, research has shown that students are accustomed to more traditional teaching methods that do not require them to actively learn during class. In fact, in one of our observations, the TA asked students if they would rather work in groups on a list of problems, or take notes as the TA solved the problems on the board. Students overwhelmingly voted for the instructor-centered paradigm.

However, it is exciting to see that overall students did in fact prefer studentcentered instruction in the recitation. The recitation is meant to offer students a different, more personalized learning experience than the lecture. As Seymour argues, in departments that primarily use large lectures for introductory course, using student-centered pedagogy in the recitation is a nice "middle path" that can help better engage students in the course [Seymour, 2002]. Our findings also provide evidence that student-centered instruction is more engaging. In our data, studentcentered instruction positively correlated with student engagement in the recitation, both observed ($r = 0.60$, p -value= 0.04) and what students reported ($r = 0.74$, p -value= 0.006).

Similarly, linear regression was used to determine the correlation between conceptually focused instruction and student course attitude. The study found that concept focused instruction was not correlated with student course attitude ($r = -0.30$, p -value= 0.34). These results were not surprising, since studies have shown that students may not perceive conceptually focused math curricula favorably. Students left comments on the evaluation of teaching methods survey such as "Taught by concept and I struggle to learn like that" and "I believe that more of the recitation should spend time going over current homework problems".

In fact, many students seemed more interested in the "superficial learning" of procedures and rote skills. Instead of asking TAs more conceptually rich questions, students' questions commonly reflected their focus on completing the homework. Questions were often stated as "Can you do number 5 on the board?". In another example which highlights students' interests in memorizing procedures, one student was able to solved the integral $\int_0^1 \ln x \, dx$ on the board because he had each step memorized. When he was asked about his work, he could not explain any of his steps and was not aware that he was using integration by parts.

Student Achievement: Two models were developed with the data collected to help predict student achievement using student-centered instruction and conceptual frequency as predictors. Student achievement was measured by analyzing each section's passing rate on the standardized final exam and students' predicted final grade in the course at the end of the semester.

The LASSO regression model was used to analyze each section's final exam passing rate. Student-centered instruction and conceptual frequency both had coefficients that were shrunk to zero during the LASSO analysis. The only non-zero coefficient left in the model was the indicator variable for calculus III. The final exam passing rate for a section of calculus III is predicted to be 8.27% higher than calculus I or II. Since students in calculus III have already passed calculus I and II, and typically have more university experience, this difference in the passing rate is not surprising. Our results did not support current research findings that suggest student-centered and conceptually focused instruction improve final exam passing rates.

Several factors may rationalize why the model did not explain more of the variability among different sections' final exam passing rates. First, our sample size for the LASSO model was only 11 observations (one section did not take the standardized final exam and therefore was not included in this model). Also, the grades that students gave the quality of their lecture instruction were included in the model to

adjust for variability in the lecture. However, undoubtedly the lecture plays a large role in students' performance on the final exam, and controlling for this variable in an experimental design would have increased the validity of our model. Finally, the final exams for Calculus I, II, and III were not analyzed by the research team. Thus, we can not make any inference about whether the exams focused on testing students' procedural or conceptual knowledge. A recitation that focuses on concepts may not better prepare students for a final exam which tests procedural knowledge and vice versa.

An ordinal regression model was developed to analyze the probability that students predicted their final course grade to be a B or better based on the use of student-centered and conceptual focused instruction in the recitation. Unlike the LASSO model, our ordinal regression model found that using student-centered instruction increases the probability that students will predict their final grade as being a B or better by 3.6, holding all other variables constant (*p*-value $= 0.00001$). Even using the lower bound of the confidence interval, for one unit of increase in studentcentered instruction, students are still 1.96 times more likely to predict their final grade as being a B or better. Our model also found that an increase in conceptual material in the recitation increased the probability that students predicted their final course grade to be a B or better, with all other variables held constant $(p$ -value $= 0.0005$).

Since this model examined how students' perceive their performance in the course, the results may show that when student-centered learning or concept-based learning were used in the recitation, students felt more confident in their abilities.

Summary: Our study has found that while student course attitude was positively affected by the use of student-centered instruction in the recitation, focusing on conceptual understanding did not have a significant effect on student attitude. We also found that student-centered instruction and emphasizing concepts increased the

likelihood that students will predict their final grade in the course to be a B or higher. This is evidence that these teaching methods may positively impact student achievement. Our study did not find evidence, however, that supports the hypothesis that different teaching methods used in the recitation will influence final exam passing rates.

Do TAs feel that they understand the purpose of the recitation? From the GTA professional development survey, TAs' average response to the question "I clearly understood the department's purpose/vision for the recitation" was 4.64, where 1 represents strongly disagree and 6 represent strongly agree. Though their average response was slightly positive, it does does not reflect strong understanding of the purpose of the recitation section. Students average response to this question was 4.42, also indicating a lack of strong understanding of the department's intended purpose for the recitation.

It was also evident from analyzing the observation notes that different TAs viewed the purpose of the recitation differently. Some recitation instructors focused on meeting students' needs. For example, at the beginning of the recitation, one TA gave students options of possible material to review and let students decide which concepts would be the most beneficial to cover during the recitation.

In other recitations, TAs had prepared the topics and problems to be covered during the recitation beforehand. This seemed to have varying degrees of success with students. For example, one TA spent a significant portion of the recitation explaining why a lower/upper Riemann sum is an under/over approximation of the area under a curve. Though the TAs explanation was very thorough, it did not appear that students were struggling to understand this concept. TAs may be able to more effectively choose the material to cover in their recitation if they have an increased understanding of pedagogical content knowledge.

Other TAs focused on preparing students for quizzes and exams. In this scenario, TAs spent the majority of the recitation advising students on important material that would likely appear on the weekly quiz or upcoming exam. For example, one TA spent part of the recitation writing down definitions that students would need to know for their quiz at the end of the class.

Recitation students were also asked to answer the same questions about how clearly they understood the purpose/vision of the recitation on the survey they completed. Students' average score was 4.42, similar to TAs' average score. This again reflects that although students reported slightly positive agreement, they did not express strong understanding of the purpose of the recitation. One student wrote on their survey "Our TA is great but the recitation could be used better by the whole department".

Summary: TAs (and students) expressed that they did not strongly understand the department's purpose and vision for the recitation section. The various approaches to the recitation showed that TAs held different views on the purpose and objectives of the recitation.

Do TAs feel prepared to lead a recitation after completing GTA training? TAs' average response to the question "I was responsible for designing the recitation I instructed" was 5.27, where 1 represents strongly disagree and 6 represent strongly agree. Despite TAs not reporting a strong understanding of the purpose of the recitation, TAs did fairly strongly agree that they felt responsible for designing the recitation section they were assigned to instruct.

Seymour claims that a TAs' ability to design a successful recitation depends on the quality of TA professional development they receive. At UNM, TAs are required to attend a one semester teaching seminar their first year, which covers a wide variety of general teaching topics such as grading, exam writing, active learning. Since TAs

began the program in different years, they did not all attend the same seminar. TAs mean response to the question "Overall, how effective has the TA training you have received at the UNM been in preparing you to teach a recitation?'' was 3.91. Their mean response to the question "Overall, how effective has the TA training you have received been in preparing you to work with students?" was 3.45. Both responses reflect positive views about the effectiveness of the training in preparing them to lead a recitation and work with students; however, TAs' responses did not indicate that TAs felt their training was "very effective".

TAs' mean average response to the content specific questions was 3.54, where 1 represents never learned and 6 represents learned well. TAs reported that they best learned grading, developing quizzes/exams, and teaching styles (average responses were 4.27, 4.36, and 4.20 respectively). Other topics that received higher scores from TAs were interacting professionally one-on-one with your students, using student-centered instruction, and facilitating group discussion (3.60, 3.55, and 3.55 respectively). The only topic with an average score below 3 was working with culturally diverse students. During the observations, TAs struggled to make sure that all students were included in collaborative learning activities. This is further evidence that students may not be prepared to work with culturally diverse students. Other topics that scored lower than the average included using Socratic dialogue and checking for conceptual understanding (3.09, 3.09 respectively).

Summary: TAs rated the professional development they received slightly positively in effectively preparing them to lead a recitation, but did not consider their training to be highly effective. In several topics which are relevant to recitation instruction such as Socratic dialogue, checking for conceptual understanding, and working with culturally diverse students, TAs felt that they were not well instructed.However, other more general teacher tasks such as grading and writing quizzes TAs felt they learned better. This may indicate that the training they received was successful in

covering basic teaching knowledge, but wasn't as effective in preparing TAs for the specific role of a recitation leader.

Conclusions

The recitation leaders that participated in this study utilized a variety of both traditional and research-based teaching methods. Our study suggests that different teaching methods used in recitations have an impact on students' course attitudes and student achievement in calculus. In particular, our study showed that studentcentered instruction improved student course attitudes, student engagement, and their predicted final grade in the course. Recitation instruction focusing on concepts also improved students' predicted final grade, but did not impact student course attitudes. This study provides evidence that the teaching methods used in recitations are impactful and that a larger study should be conducted to investigate the efficacy of different TA employed teaching methods used in calculus recitations.

The math department in this study does not have a formally stated purpose or intended outcomes for the recitation component. As a result, TAs and students in this study both indicated that they did not strongly understand the purpose of the recitation section. TAs felt responsible for designing their own teaching strategies and although they rated the professional development they received as effective, it may not have adequately covered topics relevant to recitation instruction. This study also provides evidence that more research should be conducted on tailoring TA professional development to better meet the needs of recitation leaders.

5.2 Implications

National efforts have called for improvements in the quality of education in gate-way courses such as calculus. The MAA report found that many doctoral granting universities enhance their calculus sequence by offering recitation sections in conjunction with the lecture [MAA, 2015]. Though some departments have "structured" recitations, other departments leave the majority of the course organization to the TAs. The results of this study can be applied to help departments develop an explicit purpose and learning objectives for their recitation, which can then be translated to both students and TAs. Having a clear purpose for the recitation may help to align TAs' and students' expectations, and may improve the overall quality of the recitation component [Melnikova, 2015].

The implications of this study can also be applied to developing strategies for TA training that effectively address recitation instruction. This study provides some evidence that student-centered instruction improves student course attitude, student engagement, and achievement. Focusing on concepts in the recitation may also improve student achievement. This suggests that departments should incorporate student-centered instruction and the importance of building students' conceptual knowledge into their TA training programs.

At the University of New Mexico, almost all math graduate students are assigned to teach a calculus recitation at some point. Unlike some universities, UNM's math department offers their TAs professional development opportunities funded through an NSFA grant. First, new TAs are required to attend an orientation session before the semester begins. This sessions focuses on giving TAs a "crash course" on teaching and prepares them for their first day in the classroom. New TAs are also required to attend a semester long teaching seminar, which explores in more detail different teaching topics such as motivating students, active learning, and using technology
Chapter 5. Discussion

in the classroom. Overall, the training program has received positive feedback from graduate students; however, the seminar does not focus on content specific to recitation instruction. To help improve the quality of recitation instruction in the calculus courses at UNM, we provide recommendations based on the findings of this study.

Policy Recommendations

- 1. The math department should formally state the purpose of the recitation component and its intended learning outcomes (see to section 2.4). Subsequently, these course elements should be articulated to TAs during the training program. Also, students should be informed of the purpose and objectives of the recitation section by either discussing them the first day of the recitation and/or including them in the syllabus and course website. This will engender better alignment between students' and TAs' expectations of the recitation (see section 2.5). Also understanding the course purpose will help TAs to develop appropriate teaching strategies for their recitation.Finally, having a clear purpose for the recitation will increase consistency among the different recitation sections.
	- (a) The results from this study suggest that the purpose and objectives of the recitation should be built on a student-centered paradigm which focuses on conceptual understanding.
- 2. One simple model for recitation instructional development could be to offer an additional training session to recitation leaders before the semester begins. This training could concentrate on first explaining the purpose and intended student outcomes for the recitation. Then, the training could cover effective teaching methods for the recitation and address building TA pedagogical content knowledge. This supplementary training would not require much additional time or

resources to implement, but may significantly improve the quality and consistency in the recitation sections (see section 2.7.4).

- 3. Based on the results of this study, the TA teacher training offered at UNM could better serve recitation leaders by addressing methods which support students' conceptual understanding, help implement collaborative learning activities effectively, and work successfully with culturally diverse students.
	- (a) To check for conceptual understanding, TA training should discuss strategies such as Socratic dialgoue and multiple representations, along with emphasizing the benefits of building students' conceptual knowledge (see section 2.3.2).
	- (b) Training should also discuss ways to successfully implement collaborative learning activities. For example, training should include strategies for getting students to work together, assigning groups, and ensuring that all students are being engaged. UNM has a diverse student body and training TAs to work with diverse students is crucial. Thus, TA training should also include covering equitable teaching strategies, such as collaborative learning, and their importance in higher education.
	- (c) Lastly, TA training should emphasize developing TAs' pedagogical content knowledge of calculus. Discussing strategies with TAs for identifying areas where students struggle may help TAs to more efficiently use the time in their recitations (see section 2.7.2).

5.3 Limitations

Several limitations negatively affected this study, first and foremost being the number of TAs observed. With a sample size of only 12 observations, it is difficult to make

Chapter 5. Discussion

any inferences about the results found.

In addition, because an experimental design was not used, several confounding variables negatively affected the validity of this study. First, the quality of the lecture that students attend clearly influences their achievement in the course. Although we attempted to control for this variable by asking students to grade the quality of their lecture and then included their scores in our model, using an experimental design would have been more ideal. Students past math backgrounds affect their achievement in the course as well. We also tried to model for students' math backgrounds by including their past math grade as a covariate in our model; however, issuing pre and post tests would have been a better way to control for this variable and measure student achievement.

Also, the researcher was the instructor of the graduate student teacher training seminar for one semester. Two of the TAs that chose to participate in this study completed the seminar that year. Although the GTA Professional Development Survey was anonymous, this could have impacted the results from this study and introduced bias into the survey results.

Because of its added implications to the IRB process, we did not ask students for access to their final grades in the course. Instead we relied on students' predicted final grades. There is no guarantee that students' predicted grades accurately reflect the final grade they received in the course. This study essentially measured students' self-analysis of their achievement which may not necessarily align with their actual achievement in the course.

To assuage participants discomforts about being observed, we informed TAs of the dates they were going to be observed. The observational data collected may not reflect the TAs' typical teaching practices, which may have introduced bias into the data. Also, all surveys used in this study were based off of validated instruments; nonetheless, this does not guarantee that the survey accurately measured students' or TAs' views. Finally, the researcher exclusively gathered the observational data. Bias could have been introduced during the observations and in the analysis of the qualitative data collected.

5.4 Future Work

This study has provided evidence that different teaching methods may affect the quality of the recitation and that future research should investigate recitation instructional strategies. A larger experimental study should be conducted exploring the effectiveness of different teaching methods in college calculus recitations and their impact on students' attitude and achievement.

At the University of New Mexico, students in each section attend the same lecture, and half of the student attend either the Tuesday or Thursday recitation. This design naturally lends itself to an effective experimental study. An idea for a future study design would be to vary the methods of instruction in the Tuesday and Thursday recitation using an experimental block design. Tuesday's recitation could utilize a more traditional, teacher-centered paradigm and Thursday's recitation could employ student-centered instruction which focuses on conceptual richness. Since students in each sections will attend the same lecture, this design will allow for measuring differences in students' achievement while controlling for the quality of the lecture. Further studies should also examine the effectiveness of different TA training strategies on preparing recitation leaders.

Appendices

Appendix A TDOP Code Bank

The following list of codes includes only code definitions from the Teaching Dimensions Observational Protocol. A more extensive discussion of coding rules and different instructional scenarios can be found in the "TDOP Technical Manual" available on the TDOP website. A few codes were either adapted or added to the code bank for the purposes of this research. Please see section 3.2 for more details.

Appendix A. TDOP Code Bank

Code Bank (Basic Dimensions plus Optional Dimensions)

Teaching Methods

Teacher-focused instruction (teacher is the primary actor)

- L Lecturing: The instructor is talking to the students and not using visuals, demonstration equipment, actively writing, or asking more than 2 questions in a row in a Socratic manner.
- LW Lecturing while writing: The instructor is talking to the students while actively writing on a chalkboard, transparencies, digital tablet, or other material. The instructor must either be writing or referring to what they are writing (or have already written). This code also captures real-time drawing of graphics (e.g., molecular structure, physiological processes), and if the use of visual representations is of interest, this should be included in the notes section. (Note that this code also captures writing/drawing in front of students without speaking, as a separate code for silent writing was deemed superfluous).
- SOC-L Socratic lecture: The instructor is talking to the students while asking multiple, successive questions to which the students are responding. Student responses are either guiding or being integrated within the discussion. A minimum of 2 relevant student responses is required to use this code. (Note that SOC-L can be co-coded with other types of lecturing, such as LW, if the instructor is doing both writing AND interspersing his/her talk with questions).
- IND Individualized instruction: The instructor provides instruction to individuals or groups and not the entire class. This often occurs while the instructor is roaming the classroom, but students or small groups may also approach the instructor. This code is usually co-coded with SGW or DW (see below). It is important to recognize that this code should not be used to classify the types of student-teacher interactions that are occurring in a large class setting – instead, use this code only when students are engaged in SGW or DW and the instructor is directly interacting with one or more students.
- A Assessment: The instructor is explicitly gathering student learning data in class (e.g., tests, quizzes, or clickers).
- AT Administrative task: The instructor is discussing exams, homework, or other non-content related topics.
- WP Working out Problems: This code refers to the instructor working out computations or problems. These can include balancing a chemical equation, working out a mathematical proof, or designing equations or Punnett squares, etc. The intent of the code is to capture the working through of some sort of problems in front of students. (If this process is being written out, then this code will be co-coded with LW, and if students are being asked to participate in the problem-solving process via questions, code SOC-L)

Student-focused instruction (students are the primary actor)

- SGW Small group work/discussion: Students form into groups of 2+ for the purposes of discussion and/or to complete a task.
- DW Deskwork: Students complete work alone at their desk/chair.
SP Student presentation: Groups or individual students are given
- **Student presentation:** Groups or individual students are giving to the class or are otherwise acting as the primary speaker or instructor in the classroom. In this instance, only select this code and none others as long as the primary instructor is not actively taking the lead in teaching the class.

Student-Teacher Dialogue

Teacher-led dialogue

- IRQ Instructor rhetorical question: The instructor asks a question without seeking an answer and without giving students an opportunity to answer the question.
- IDQ Instructor display question: The instructor poses a question seeking information. These questions can: seek a specific fact, a solution to a closed-ended problem, or involve students generating their own ideas rather than finding a specific solution.
- ICQ Instructor comprehension question: The instructor checks for understanding (e.g., "Does that make sense?") and pauses for at least five seconds, thereby indicating an opportunity for students to respond.

Appendix A. TDOP Code Bank

Student-led dialogue

- SQ Student question: A student poses a question to the instructor that seeks new information (i.e. not asking to clarify a concept that was previously being discussed) and/or clarification of a concept that is part of the current or past class period.
- SR Student response to teacher question: A student responds to a question posed by the instructor, whether posed verbally by the instructor or through digital means (e.g., clicker, website).
- PI Peer interactions: Students speaking to one another (often during SGW, WCD, or SP).

Optional Dimensions

Potential Student Cognitive Engagement

- PRO Focusing on Procedural Knowledge: The content being discussed or the task being completed by either the students or the instructor focuses on building procedural knowledge. This can include emphasis on procedures, skills, and algorithms, basic procedural examples that do not require advanced manipulations, procedural examples of a greater level of technical difficulty, emphasizing algebraic solution methods over non-algebraic solution methods, not encouraging students to work a problem in more than one way, or the instructor did not expect students to explain the variety of methods they employed as they solved a problem. A question should be coded as PRO if it is an inquiry about a mechanism of solving a problem without mentioning the underlying concepts or reasoning.
- CON Focusing on Conceptual Knowledge: The content being discussed or the task being completed by either the students or the instructor focuses on building conceptual knowledge. This can include emphasis on conceptual development, multiple methods of solving a problem to build connections among mathematical ideas, the instructor linking the entry knowledge and skills of students to more formal concepts and procedures, reinforcing new skills and procedures with prior knowledge and intuition, presenting topics numerically or graphically, or the instructor only valued correct answers if students could explain them. A question should be coded as CON if it deals with the underlying concepts of the problem.
- CNL Making connections to own lives/specific cases: Students are given examples (either verbally through illustrative stories or graphically through movies or pictures) that clearly and explicitly link course material to popular culture, the news, and other common student experiences. Students may also be given specific cases or incidents in order to link an abstract principle or topic (e.g., flooding) with a more readily identifiable instance (e.g., 2013 floods in Boulder, Colorado). For this code to be used, the observer will need to make a judgment that the specific case is something meaningful to students, such as a local historic item or location, or a widely recognized incident. In general, a high bar is required here that is based on specificity and salience to students, such that showing a picture of a sedimentary rock will not be sufficient for this code, but if the picture was of the Grant Canyon and named as such, it would be coded as CNL. This code will be particularly important in biology (e.g., Dolly the sheep) and geoscience courses. PS Problem solving: Students are asked to actively solve a problem (e.g., balance a chemical equation, work out a mathematical equation/algorithm). This is evident through explicit verbal (e.g., "Please solve for X") or written requests (e.g., worksheets) to solve a problem. This is coded in relation to closed-ended exercises
- or problems where the instructor has a specific solution or end-point clearly in mind.
 CR Creating: Students are provided with tasks or dilemmas where the outcome is open-ended rather than fixed (e.g., students are asked to generate their own ideas and/or products rather than finding a specific solution). The task can be delivered verbally or in written form. This is coded in relation to open-ended exercises or problems where the instructor does not have a specific solution or end-point clearly in mind.

Appendix A. TDOP Code Bank

Pedagogical Strategies

- HUM Humor: The instructor tells jokes or humorous anecdotes; this code requires laughter from at least a couple of students.
- ORG Organization: The instructor writes or posts an outline of class (i.e., advance organizer) or clearly indicates a transition from one topic to the next verbally or through transitional slides. This transition from one topic to another can indicate a change in topics within a single class or from a previous class to the present class. These transitions must be verbally explicit statements to the class (e.g., ?Now we?re moving from meiosis to mitosis?) as opposed to ambiguous statements such as ?Now we?ll pick up where we left off on Monday.? This may also include statements concerning how concepts covered in different portions of the class (e.g., lecture, homework and lab) may overlap.
- EMP Emphasis: The instructor clearly states that something is important for students to learn or remember either for a test, for their future careers, or to just learn the material well.

Student Engagement

- HI HIGH STUDENT ENGAGEMENT: more than 75% of the students in the immediate area of the observer are either (a) actively taking notes, or (b) looking at the instructor/course materials.
- MED MEDIUM STUDENT ENGAGEMENT: 75% 25% of the students in the immediate area of the observer are either (a) actively taking notes, or (b) looking at the instructor/course materials.
- LO LOW STUDENT ENGAGEMENT: less than 25% of the students in the immediate area of the observer are either (a) actively taking notes, or (b) looking $\begin{tabular}{ll} Appendix~A. & TDOP Code Bank \\ \hline \textbf{HICH STUDENT ENGAGEN} & \textbf{HIGH STUDENT ENGAGEN} \\ & \textbf{HICH STUDENT ENGAGEN} \\ & \textbf{MED IUM STUDENT ENGA} \\ & \textbf{MED IUM STUDENT ENGA} \\ & \textbf{MED IUM STUDENT ENGA} \\ & \textbf{MED IUM STUDENT ENGAGEM} \\ & \textbf{LOL} & \textbf{LOW STUDENT ENGAGEM} \\ & \textbf{immediate area of the observer are} \\ & \text{at the instructor/course materials.} \end{tabular}$

Appendix B

Student Evaluation of Recitation Teaching Practices Survey

The following survey was used to assess students' course attitude. This was an adaptation of Evans, et al. (1993). Development of a Student Perceptions Instrument to Assess Contributions of the Learning Environment to the Enhancement of Student Learning in Higher Education Settings. Louisiana State University. and Aleanoni, Lawrence (1978). Arizona Course/Instructor Evaluation Questionnaire (CIEQ) - ResuIts Interpretation Manual. University of Arizona.

Appendix B. Student Evaluation of Recitation Teaching Practices Survey

Student Evaluation of Recitation Teaching Practices

PART I

PART II

PART III

Using the following grading scale, write in a number (not a letter grade) in response to each of the following questions:

 $A = 90 - 100$ $B = 80 - 89$ $C = 70 - 79$ $D = 60 - 69$ $F =$ below 60

Please circle the number below that indicates how much you agree or disagree with the following statement.

Appendix C

GTA Professional Development Survey

The following survey was used to measure TAs assessment of the teacher training they received. This was an adaptation of DeChenne, S. E., et al. (2012). Towards a measure of graduate student teaching professional development. Journal of Effective Teaching 12(1), 4-19.

Appendix C. GTA Professional Development Survey

GTA Professional Development Survey

PART I

1. Please circle the number that best reflects your answer.

PART II

2. Of the following teaching topics and skills, please rate how well you have learned these in TA training at the University of New Mexico.

PART III

3. Please circle the number below that indicates how much you agree or disagree with the following statements.

References

- [Aleanoni, 1978] Aleanoni, L. (1978). Arizona Course/Instructor Evaluation Questionnaire (CIEQ) - ResuIts Interpretation Manual.
- [Alexander et al., 1997] Alexander, B., Burda, A., and Susan, M. (1997). A Community Approach to Learning Calculus: Fostering Success for Underpresresented Ethnic Minorities in an Emerging Scholars Program. Journal of Women and Minorities in Science and Engineering, 3(3):145–199.
- [ALPSM, 2016] (2016). Active Learning in Post-Secondary Mathematics Education.
- [Asera, 2001] Asera, R. (2001). Calculus and Community: A History of the Emerging Scholars Program. A Report of the National Task Force on Minority High Acheivement, The College Board.
- [Belnap, 2005] Belnap, J. K. (2005). Putting TAs into Context: Understanding the graduate mathematics teaching assistant. dissertation, University of Arizona.
- [Berrett, 2012] Berrett, D. (2012). Lectures Still Dominate Science and Math Teaching, Sometimes Hampering Student Success. The Chronicle of Higher Education.
- [Bookman and Friedman, 1994] Bookman, J. and Friedman, C. P. (1994). A Comparison of the Problem Solving Performance of Students in Lab Based and Tradional Calculus. 4.
- [Bookman and Friedman, 1998] Bookman, J. and Friedman, C. P. (1998). Student Attitudes and Calculus Reform. School Science and Mathematics, 98(3):117–122.
- [Borasi and Rose, 1989] Borasi, R. and Rose, B. J. (1989). Journal Writing and Mathematics Instruction. Journal Writing and Mathematics Instruction, 20(4):347–365.
- [Bressoud, 1994] Bressoud, D. (1994). Student Attitudes in First-semester Calculus. MAA Focus, 14:6–7.
- [Bressoud et al., 2012a] Bressoud, D., Camp, D., and Teague, D. (2012a). Background to the MAA/NCTM Statement on Calculus.
- [Bressoud et al., 2012b] Bressoud, D. M., Carlson, M. P., Pearson, M. J., and Rasmussen, C. (2012b). Preliminary Results of the Study on Characteristics of Successful Programs in College Calculus. 12th International Congress on Mathematical Education.
- [Burns, 2012] Burns, D. (2012). Will I Do as Well on the Final Exam as I Expect? An Examination of Students' Expectations. Journal of the Scholarship of Teaching and Learning, $8(3):1-19$.
- [Callahan, 2008] Callahan, K. M. (2008). Academic-Centered Peer Interactions and Retention in Undergraduate Mathematics Programs. Journal of College Student Retention: Research, Theory & Practice, 10(3):361–389.
- [Chappell and Killpatrick, 2007] Chappell, K. and Killpatrick, K. (2007). Effects of Concept-Based Instruction on Students' Conceptual Understanding and Procedural knowledge of calculus. PRIMUS, 13(1):17–37.
- [Chen and Weko, 2009] Chen, X. and Weko, T. (2009). Students Who Study Science, Technology, Engineering, and Mathematics (STEM) in Postsecondary Education. Technical report, US Department of Education, National Center for Educational Statistics.
- [Code et al., 2014] Code, W., Piccolo, C., Kohler, D., and MacLean, M. (2014). Teaching methods comparison in a large calculus class. ZDM, 46(4):589–601.
- [DeChenne and Enochs, 2010] DeChenne, S. E. and Enochs, L. (2010). Measuring the Teaching Self-Efficacy of Science, Technology, Engineering, and Math Graduate Teaching Assistants. volume 12, page 4, Denver, CO. Journal of the Scholarship of Teaching and Learning.
- [DeChenne et al., 2012] DeChenne, S. E., Lesseig, K., Anderson, S., Li, S., Staus, N., and Barthel, C. (2012). Toward a Measure of Professional Development for Graduate Student Teaching Assistants. Journal of Effective Teaching, 12(1):4–19.
- [Deslauriers et al., 2011] Deslauriers, L., Schelew, E., and Wieman, C. (2011). Improved Learning in a Large-Enrollment Physics Class. Science, 332(6031):862–864.
- [Duncan and Dick, 2000] Duncan, H. and Dick, T. (2000). Collaborative Workshops and Student Academic Performance in Introductory College Mathematics Courses: A Study of a Treisman Model Math Excel Program. School Science and Mathematics, 100(7):365–373.
- [Ellis, 2014] Ellis, J. F. (2014). Preparing Future College Instructors: The Role of Graduate Student Teaching Assistants (GTAs) in Successful College Calculus Programs. dissertation, San Diego State University.
- [Engelbrecht et al., 2009] Engelbrecht, J., Bergsten, C., and Kågesten, O. (2009). Undergraduate students' preference for procedural to conceptual solutions to mathematical problems. International Journal of Mathematical Education in Science and Technology, 40(7):927–940.
- [Engelbrecht et al., 2005] Engelbrecht, J., Harding, A., and Potgieter, M. (2005). Undergraduate students' performance and confidence in procedural and conceptual mathematics. International Journal of Mathematical Education in Science and Technology, 36(7):701–712.
- [Engelbrecht et al., 2007] Engelbrecht, J., Harding, A., and Preez, J. D. (2007). Long-term Retention of Basic Mathematics Knowledge Skills with Engineering Students. European Jouranl of Engineering Education, 32(6):735–744.
- [Evans et al., 1993] Evans, L., Ellet, C., Culross, R., and Loup, K. (1993). Development of a Student Perceptions Instrument to Assess Contributions of the Learning Environment to the Enhancement of Student Learning in Higher Education Settings.
- [Felder and Brent, 1996] Felder, R. M. and Brent, R. (1996). Navigating the Bumpy Road to Student-Centered Instruction. College Teaching, 44(2):43–47.
- [Finkelstein and Pollock, 2005] Finkelstein, N. D. and Pollock, S. J. (2005). Replicating and understanding successful innovations: Implementing tutorials in introductory physics. Physical Review Special Topics - Physics Education Research, $1(1):010101.$
- [Golde and Dore, 2001] Golde, C. and Dore, T. (2001). At Cross Purposes: What the experiences of today's doctoral students reveal about doctoral education. Technical report, A survey initiated by the Pew Charitable Trusts.
- [Hake, 1998] Hake, R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. American Journal of Physics, 66(64).
- [Hake, 1992] Hake, R. R. (1992). Socratic Pedagogy in the Introductory Physics Laboratory. Physics Teacher, 30(9):546–52.
- [Harris et al., 2009] Harris, G., Froman, J., and Surles, J. (2009). The professional development of graduate mathematics teaching assistants. International Journal of Mathematical Education in Science and Technology, 40(1):157–172.
- [Hickok, 2016] Hickok, E. (2016). Teaching Practices of Graduate Teaching Assistants. thesis, San Jose State University. arXiv: 1602.07740.
- [Hora et al., 2013] Hora, M., Oleson, A., and Ferrare, J. (2013). Teaching Dimensions Obseration Protocol (TDOP) User's Manual. Technical report, University of Wisconsin-Madison, Wisconsin Center for Education Research.
- [Hughes Hallett, 2006] Hughes Hallett, D. (2006). What Have We Learned from Calculus Reform? The Road to Conceptual Understanding.
- [Johnson et al., 2007] Johnson, R. B., Onwuegbuzie, A. J., and Turner, L. A. (2007). Toward a Definition of Mixed Methods Research. Journal of Mixed Methods Research, 1(2):112–133.
- [Jones, 2009] Jones, B. D. (2009). Motivating Students to Engage in Learning: The MUSIC Model of Academic Motivation. International Journal of Teaching and Learning in Higher Education, 21(2):272–285.
- [Katz, n.d.] Katz, D. (n.d.). Calculus Recitation FAQ.
- [Keller et al., 1999] Keller, B., Russell, C., and Thompson, H. (1999). A Largescale Study Clarifying the Roles of TI-92 and Instructional Format on Student Success in Calculus. International Journal of Computer Algebra in Mathematics Education, 6(3):191–207.
- [Kendall and Schussler, 2012] Kendall, K. D. and Schussler, E. E. (2012). Does Instructor Type Matter? Undergraduate Student Perception of Graduate Teaching Assistants and Professors. CBE-Life Sciences Education, 11(2):187–199.
- [Koenig et al., 2007] Koenig, K. M., Endorf, R. J., and Braun, G. A. (2007). Effectiveness of different tutorial recitation teaching methods and its implications for TA training. Physical Review Special Topics - Physics Education Research, 3(1):010104.
- [Kortemeyer, 2006] Kortemeyer, G. (2006). An analysis of asynchronous online homework discussions in introductory physics courses. American Journal of Physics, 74(6):526–536.
- [Kortemeyer, 2007] Kortemeyer, G. (2007). Correlations between student discussion behavior, attitudes, and learning. Physical Review Special Topics - Physics Education Research, 3(1):010101.
- [Kvam, 2000] Kvam, P. H. (2000). The Effect of Active Learning Methods on Student Retention in Engineering Statistics. The American Statistician, 54(2):136–140.

References

- [Luft et al., 2004] Luft, J. A., Kurdziel, J. P., Roehrig, G. H., and Turner, J. (2004). Growing a garden without water: Graduate teaching assistants in introductory science laboratories at a doctoral/research university. Journal of Research in Science Teaching, 41(3):211–233.
- [MAA, 2015] MAA (2015). Insights and Recommendations from the MAA National Study of College Calculus. Technical report, Mathematical Association of America.
- [Mahir, 2009] Mahir, N. (2009). Conceptual and procedural performance of undergraduate students in integration. International Journal of Mathematical Education in Science and Technology, 40(2):201–211.
- [McGivney-Burelle and Xue, 2013] McGivney-Burelle, J. and Xue, F. (2013). Flipping Calculus. PRIMUS, 23(5):477–486.
- [Melnikova, 2015] Melnikova, Y. (2015). Alignment in Students, Teaching Assistants, and Instructors on the Purpose and Practice of Calculus I Labs. dissertation, Texas State University.
- [Menges and Weimer, 1996] Menges, R. and Weimer, M. (1996). Teaching on Solid Ground, Using Scholarship to Improve Practice. Jossey-Bass Publishers, San Francisco.
- [Michael, 2012] Michael, F. (2012). One Decade, One Million more STEM Graduates.
- [Michael, 2006] Michael, J. (2006). Where's the evidence that active learning works? Advances in Physiology Education, 30(4):159–167.
- [Middleton and Spanias, 1999] Middleton, J. and Spanias, P. (1999). Motivation for Ahcievement in Mathematics: Findings, Generalizations, and Criticisms of the Research. Journal for Research in Mathematics Education, 30(1):65–88.
- [Nilklad, 2004] Nilklad, L. (2004). College Algebra Students' Understanding and Algebraic Thinking and Reasoning with Functions. dissertation, Oregon State University.
- [Panitz, 1999] Panitz, T. (1999). The Case for Student Centered Instruction via Collaborative Learning Paradigms.
- [Pavelich and Streveler, 2004] Pavelich, M. J. and Streveler, R. A. (2004). An active learning, student-centered approach to training graduate teaching assistants. In Frontiers in Education, 2004. FIE 2004. 34th Annual, pages F1E–1–5 Vol. 2.
- [PCJobs, 2011] (2011). President's Council on Jobs and Competitiveness Announces Industry Leaders Commitment to Double Engineering Internships in 2012.
- [Pentecost et al., 2012] Pentecost, T., Langdonn, L., Asirvatham, M., Robus, H., and Parson, R. (2012). Graduate Teaching Assistant Training That Fosters Student-Centered Instruction and Professional Development. Journal of College Science Teaching, 41(6):68–41.
- [Rasmussen et al., 2014] Rasmussen, C., Marrongelle, K., and Borba, M. C. (2014). Research on calculus: what do we know and where do we need to go? ZDM, 46(4):507–515.
- [Reins, 2015] Reins, K. (2015). Mindsets, Attitudes, and Achievement in Undergraduate Statistics Courses Valorie L. Zonnefeld, Ed.D., Curriculum and Instruction. dissertation, University of South Dakota.
- [Rejniak, 2004] Rejniak, G. (2004). Improving Student Learning in Undergraduate Mathematics. thesis, University of Central Florida.
- [Riner, 1972] Riner, J. (1972). Individualizing Mathematics Instruction. The American Mathematical Monthly, 79(1):77–86.
- [Rosenthal, 1995] Rosenthal, J. S. (1995). Active learning strategies in advanced mathematics classes. Studies in Higher Education, 20(2):223–228.
- [Ross and Onwuegbuzie, 2012] Ross, A. and Onwuegbuzie, A. J. (2012). Prevalence of Mixed Methods Research in Mathematics Education. The Mathematics Educa*tor*, $22(1)$.
- [Rutgers, 2009] (2009). Mathematics Department Guide for Mathematics Recitation Instructors.
- [Seymour, 2002] Seymour, E. (2002). Tracking the processes of change in US undergraduate education in science, mathematics, engineering, and technology. Science Education, 86(1):79–105.
- [Seymour and Hewitt, 1997] Seymour, Elaine, S. and Hewitt, N. (1997). Talking about Leaving: Why Undergraduates Leave the Sciences.
- [Speer et al., 2005] Speer, N., Gutmann, T., and Murphy, T. J. (2005). Mathematics Teaching Assistant Preparation And Development. College Teaching, 53(2):75–80.
- [Spike and Finkelstein, 2012] Spike, B. T. and Finkelstein, N. D. (2012). Preparing tutorial and recitation instructors: A pedagogical approach to focusing attention on content and student reasoning. American Journal of Physics, 80(11):1020–1026.
- [Springer et al., 1999] Springer, L., Stanne, M. E., and Donovan, S. S. (1999). Effects of Small-Group Learning on Undergraduates in Science, Mathematics, Engineering, and Technology: A Meta-Analysis. Review of Educational Research, $69(1):21-51.$
- [Tall, 1992] Tall, D. (1992). Students' Difficulties in Calculus Plenary Presentation in Working Group 3. Quebec.
- [Tall and Vinner, 1981] Tall, D. and Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. Educational Studies in Mathematics, 12(2):151–169.
- [Tresiman and Fullilove, 1990] Tresiman, U. and Fullilove, R. (1990). Mathematics Achievement Among African American Undergraduates at the University of California, Berkeley: An Evaluation of the Mathematics Workshop Program. Jornal of Negro Edcuation, 59(3).
- [Tucker, 2015] Tucker, A. (2015). The History of the Undergraduate Program in Mathematics in the United States. American Mathematical Monthly, Centennial(October 2013).
- [Turner et al., 1966] Turner, V. D., Alders, C. D., Hatfield, F., Croy, H., and Sigrist, C. (1966). A Study of Ways of Handling Large Classes in Freshman Mathematics. The American Mathematical Monthly, 73(7):768–774.
- [UCBerkley, 1993] (1993). University of California-Berkeley The Professional Development Program.
- [Watt et al., 2014] Watt, J. X., Feldhaus, C. R., Sorge, B. H., Fore, G. A., Gavrin, A. D., and Marrs, K. A. (2014). The effects of implementing recitation activities on success rates in a college calculus course. Journal of the Scholarship of Teaching and Learning, $14(4):1-17$.
- [Willoughby, 2000] Willoughby, S. (2000). Perspectives on Mathematics Education. Yearbook - National Council of Teachers of Mathematics, 2000:1–15.
- [Yong and Pierce, 2013] Yong, A. G. and Pierce, S. (2013). A Beginner's Guide to Factor Analysis: Focusing on Exploratory Factor Analysis. Tutorials in Quantitative Methods of Psychology, 9(2):79–94.
- [Zakaria and Iksan, 2007] Zakaria, E. and Iksan, Z. (2007). Promoting Cooperative Learning in Science and Mathematics Education: A Malaysian Perspective, volume 3.

References

[Zhao and Kuh, 2004] Zhao, C.-M. and Kuh, G. D. (2004). Adding Value: Learning Communities and Student Engagement. Research in Higher Education, 45(2):115– 138.