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Polynomial solutions for simultaneous stabilization

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ABSTRACT

In this paper, we present a new necessary and sufficient condition for simultaneous stabilization and new sufficient conditions for the existence of a simultaneously stabilizing controller, both derived from a polynomial approach. The additional requirements for the controller itself to be either stable or a Unit in H^∞ are also given. These new sufficient conditions are general in nature and are shown to reduce in special cases to several published papers. Examples illustrate the extensions.

Key Words. Simultaneous stabilization, strong stabilization, Unit stabilization, Hurwitz polynomials, exactly proper.

1 Introduction

In this paper, sufficient conditions for the existence of a controller, that stabilizes a set of n SISO plants: $P_1,\ P_2,\ \ldots,\ P_n$ (simultaneous stabilization), are derived using a polynomial approach. The new sufficient condition used here is that the differences formed from an artificial plant, P_0, with all of the n plants, $(P_0-P_1),\ (P_0-P_2),\ \ldots,(P_0-P_n)$ form minimum phase difference plants. It is shown that this condition is sufficient for (strong) simultaneous stabilization with a stable controller, when P_0 is minimum phase. It is also shown that this condition is sufficient for simultaneous stabilization with a unit in H^∞, when P_0 is stable and minimum phase.

From this new sufficient condition, a plethora of new easily testable conditions arise. These conditions are a generalization of Blondel's result [3] in that the difference plants are allowed to be strictly proper, when the high frequency sign of all the strictly proper difference plants are the same. These conditions are a generalization of Barmish and Wei's result [1] in that the high frequency sign condition of all the plants are relaxed, and the plants are allowed to be non-minimum phase.

The proofs of these new sufficient conditions are related to a new necessary and sufficient condition for simultaneous stabilization. Specifically, n plants are simultaneously stabilizable if and only if they can be simultaneously stabilized with an exactly proper controller.

This paper is organized in the following manner. New necessary and sufficient conditions for simultaneous stabilization are stated in Section 2. Section 3 contains new sufficient conditions for simultaneous stabilization. Section 4 converts these new sufficient conditions to testable conditions. Examples demonstrate their extension and application. The summary and conclusions are in Section 5.

2 Necessary and Sufficient Conditions for Simultaneous Stabilization

Tractable necessary and sufficient conditions for the simultaneous stabilization of more than two plants is an open problem. All existing forms of the necessary and sufficient conditions for this case are either an unsolved problem, or a translation from one unsolved problem into another. Vidyasagar and Viswanadham [13] showed that simultaneously stabilizing n MIMO plants is equivalent to *strongly stabilizing* $(n-1)$ plants, that is, stabilizing $(n-1)$ plants with a *stable* compensator, one with no right half plane (RHP) poles. When $(P_2 - P_1)$ is *minimum phase* (no RHP zeros) and *exactly proper* (zero relative degree), Blondel [2] showed that the n plants can be simultaneously stabilized if and only if $(n-2)$ plants formed from the difference plant numerators of $(P_2 - P_i)$ and $(P_1 - P_i)$ can be simultaneously stabilized with a unit.

In this section, a new necessary and sufficient condition for simultaneous stabilization of n continuous time plants is stated. Although the simultaneous stabilization problem still remains an open problem for more than two plants, this new necessary and sufficient condition restricts the search for existence of a simultaneously stabilizing controller to exactly proper controllers. This fact is used in the proofs of the sufficient conditions derived in Section 2.

It is assumed, without loss of generality, that the denominator polynomials, d_i, of all plants, $P_i = \frac{n_i}{d_i}$, are monic, and that the numerator polynomial, n_i, carries the high frequency coefficient for each plant. In most practical applications, the plants to be controlled are proper. It is assumed throughout this paper that the plants requiring simultaneous stabilization are proper. However, analogous derivations lead to similar results for improper plants. Some frequently used notation is first defined.

1. Given two plants $P_i = \frac{n_i}{d_i}$ and $P_j = \frac{n_j}{d_j}$, $n_{ij} = (n_i d_j - d_i n_j)$ represents the numerator of the difference of the two plants before denominator cancellations.
2. $n_{ij}^{+} = (n_i d_j + d_i n_j)$ represents the numerator of the sum of the two plants before denominator cancellations.
3. $o(n_i)$ = the order of $n_i(s)$ = the highest power of s in the polynomial $n_i(s)$.
4. $rd(P_i) = o(d_i) - o(n_i)$ the relative degree of P_i).
5. M_S = the set of strictly proper, minimum phase, rational functions.

The proofs in this paper are based upon the lemma proved by Barmish and Wei in [1], and variations of it.

Lemma 1 (Barmish and Wei [1]) *Given two polynomials, $g(s)$ and $h(s)$, of finite degree, $o(g)$ and $o(h)$ respectively, with fixed real coefficients, where*

1. *$h(s)$ is strictly Hurwitz with positive coefficients,*
2. *$g(s)$ is monic,*
3. *$o(g) \leq o(h) + 1$,*

then there exists $\epsilon_{max} > 0$ such that $\forall \epsilon : \; 0 < \epsilon < \epsilon_{max}$, the polynomial $f(s) = h(s) + \epsilon g(s)$ is strictly Hurwitz with positive coefficients.

A minor variation is the following lemma.

Lemma 2 *Given two polynomials, $g(s)$ and $h(s)$, of finite degree, $o(g)$ and $o(h)$ respectively, with fixed real coefficients, where*

1. *$h(s)$ is strictly Hurwitz with positive coefficients,*

2. *$g(s)$ is monic,*

3. *$o(g) \leq o(h)$,*

then there exists $\epsilon_{max} > 0$ such that $\forall\epsilon :\ 0 < \epsilon < \epsilon_{max}$, the polynomial $f(s) = h(s) - \epsilon g(s)$ is strictly Hurwitz with positive coefficients.

Proof of Lemma 2:

Hurwitz testing matrices H_ϵ^-, H, and $H^"$ are generated for $f(s)$, $h(s)$, and $g(s)$ respectively, as in Case 1 of the proof given by Barmish and Wei, but using $H_\epsilon^- = H - \epsilon H^"$ rather than $H_\epsilon^+ = H + \epsilon H^"$.

The norm of a matrix is understood to be the square root of the maximum eigenvalue of the product of the matrix multiplied by its conjugate transpose. Observing that $\|H_\epsilon^-\| = \|H - \epsilon H^"\| \geq \|H\| - \epsilon\|H^"\|$, and $\|H_\epsilon^+\| = \|H - \epsilon H^"\| \geq \|H\| - \epsilon\|H^"\|$ the remainder of the proof is identical. ■

Useful corollaries, which minimize the complexity of theorem proofs that follow, relax the monic requirements on $g(s)$ and the sign of the coefficients of $h(s)$.

Corollary 1 *Given two polynomials, $g(s)$ and $h(s)$, of finite degree, $o(g)$ and $o(h)$ respectively, with fixed real coefficients, where*

1. *$h(s)$ is strictly Hurwitz,*

2. *The sign of the coefficient of the highest order term of $g(s)$ is the same as the sign of the coefficients of all terms of $h(s)$,*

3. *$o(g) \leq o(h) + 1$,*

then there exists $\epsilon_{max} > 0$ such that $\forall\epsilon :\ 0 < \epsilon < \epsilon_{max}$, the polynomial $f(s) = h(s) + \epsilon g(s)$ is strictly Hurwitz and the sign of all of the coefficients of $f(s)$ are the same as the sign of all of the coefficients of $h(s)$.

Proof of Corollary 1:

Let g_0 represent the highest order coefficient of $g(s)$. Define $q(s)$ and $r(s)$ as

$$q(s) = \frac{1}{g_0} \cdot g(s), \quad r(s) = \frac{1}{g_0} \cdot h(s)$$

Then $q(s)$ is monic, $r(s)$ is strictly Hurwitz with positive coefficients, and $d_q \leq d_r + 1$, where d_q and d_r represent the degree of $q(s)$ and $r(s)$ respectively. From Lemma 1, there exists $\epsilon_{max} > 0$, such that $p(s) = q(s) + \epsilon r(s)$ is strictly Hurwitz with positive coefficients $\forall\epsilon :\ 0 < \epsilon < \epsilon_{max}$. Therefore, $f(s) = g_0 \cdot p(s)$ is also strictly Hurwitz and the sign of the coefficients of $f(s)$ are the same as the sign of the coefficients of $h(s)$. This completes the proof. ■

Corollary 2 *Given two polynomials, $g(s)$ and $h(s)$, of finite degree, $o(g)$ and $o(h)$ respectively, with fixed real coefficients, where*

1. *$h(s)$ is strictly Hurwitz,*

2. *$o(g) \leq o(h)$,*

then there exists $\epsilon_{max} > 0$ such that $\forall\epsilon :\ 0 < \epsilon < \epsilon_{max}$, the polynomial $f(s) = h(s) + \epsilon g(s)$ is strictly Hurwitz and the sign of all of the coefficients of $f(s)$ are the same as the sign of all of the coefficients of $h(s)$.

Proof of Corollary 2:

Let g_0 represent the highest order coefficient of $g(s)$. Define $q(s)$ and $r(s)$ as

$$q(s) = \frac{1}{g_0} g(s), \quad r(s) = \frac{1}{g_0} h(s)$$

Then $q(s)$ is monic, $r(s)$ is strictly Hurwitz, and $d_q \leq d_r$, where d_q and d_r represent the degree of $q(s)$ and $r(s)$ respectively. If the sign of g_0 is the same as the sign of the coefficients of $h(s)$, then from Lemma 1, there exists $\epsilon_{max} > 0$, such that $p(s) = q(s) + \epsilon r(s)$ is strictly Hurwitz with positive coefficients $\forall \epsilon : \ 0 < \epsilon < \epsilon_{max}$. If the sign of g_0 is the opposite of the sign of all the coefficients of $h(s)$, then from Lemma 2, there exists $\epsilon_{max} > 0$, such that $p(s) = q(s) - \epsilon[-r(s)] = q(s) + \epsilon r(s)$ is strictly Hurwitz with positive coefficients $\forall \epsilon : \ 0 < \epsilon < \epsilon_{max}$. Therefore, $f(s) = g_0 \cdot p(s)$ is also strictly Hurwitz and the sign of all of the coefficients of $f(s)$ are the same as the sign of all of the coefficients of $h(s)$. This completes the proof. ■

Theorem 1 *The n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, are simultaneously stabilizable if and only if the n plants are simultaneously stabilizable with an exactly proper controller.*

Proof:

See [4]. The sufficiency proof is obvious. The necessity proof is as follows.

A controller, $C(s)$, internally stabilizes a plant, $P(s)$, when the following four transfer functions are proper, have stable poles, and there are no RHP pole-zero cancellations in $P(s)C(s)$.

$$\frac{1}{1+P(s)C(s)}, \quad \frac{P(s)}{1+P(s)C(s)}, \quad \frac{C(s)}{1+P(s)C(s)}, \quad \frac{P(s)C(s)}{1+P(s)C(s)}$$

and when there are no RHP pole-zero cancellations in $P(s)C(s)$. This is discussed in [12], [2], and [7] among others. It should be noted, that the transfer function

$$\frac{1}{1+P(s)C(s)} \in H^\infty \Leftrightarrow \frac{P(s)C(s)}{1+P(s)C(s)} \in H^\infty$$

as can be seen from the simple subtraction.

$$1 - \frac{1}{1+P(s)C(s)} = \frac{P(s)C(s)}{1+P(s)C(s)}$$

Therefore, the system is internally stable when the three transfer functions

$$\frac{1}{1+P(s)C(s)}, \quad \frac{P(s)}{1+P(s)C(s)}, \quad \frac{C(s)}{1+P(s)C(s)} \tag{1}$$

are elements of H^∞, and when there are no RHP pole-zero cancellations in $P(s)C(s)$.

If there exists a controller that simultaneously stabilizes the n proper plants, then the controller is either improper, strictly proper, or exactly proper. If it is exactly proper, the proof is complete. The remainder of the proof consists of showing that if there exist a simultaneously stabilizing controller that is strictly proper or improper, then there also exist an exactly proper simultaneously stabilizing controller.

If $C^0_{sp} = \frac{n_c}{d_c}$ is a strictly proper controller of relative degree r_c, that simultaneously stabilizes the n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, then from Equation (1),

$$\frac{1}{(1+P_i C^0_{sp})}, \quad \frac{P_i}{(1+P_i C^0_{sp})}, \quad \frac{C^0_{sp}}{(1+P_i C^0_{sp})}$$

are all elements of H^∞, $\ \forall i = 1, 2, \ldots, n$ and P_i and C^0_{sp} have no pole-zero cancellations. Note that for a strictly proper controller to internally stabilize a plant, the plant must be proper.

Choosing

$$C_{sp}^{j} = C_{sp}^{j-1} \cdot (\epsilon_j s + 1), \ \forall j = 1, 2, \ldots, r_c$$

then from Corollaries 1 and 2, $\exists \ \epsilon_j > 0$ sufficiently small for each j, such that

$$\frac{1}{(1 + P_i C_{sp}^{j})}, \quad \frac{P_i}{(1 + P_i C_{sp}^{j})}, \quad \frac{C_{sp}^{j}}{(1 + P_i C_{sp}^{j})}$$

are also elements of H^∞. In fact, when $j = r_c$, $C_{sp}^{r_c}$ is exactly proper, and

$$(1 + C_{sp}^{r_c} P_i) \in H^\infty$$

and is exactly proper $\forall i = 1, 2, \ldots, n$. Therefore, $C_{sp}^{r_c}$ is an exactly proper controller that simultaneously stabilizes the plants.

Similarly, if $C_{ip}^{0} = \frac{n_c}{d_c}$ is an improper controller of relative degree $r_c = -rd(C_{ip}^{0})$, that simultaneously stabilizes the n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, then

$$\frac{1}{(1 + P_i C_{ip}^{0})}, \quad \frac{P_i}{(1 + P_i C_{ip}^{0})}, \quad \frac{C_{ip}^{0}}{(1 + P_i C_{ip}^{0})}$$

are all elements of H^∞, $\forall i = 1, 2, \ldots, n$ and P_i and C_{sp}^{0} have no pole-zero cancellations. Note that for an improper controller to internally stabilize a plant, the plant must be either improper or exactly proper.

Choosing

$$C_{ip}^{j} = C_{ip}^{j-1} \cdot \frac{1}{(\epsilon_j s + 1)}, \ \forall j = 1, 2, \ldots, r_c$$

then from Corollaries 1 and 2, $\exists \ \epsilon_j > 0$ sufficiently small for each j, such that

$$\frac{1}{(1 + P_i C_{ip}^{j})}, \quad \frac{P_i}{(1 + P_i C_{ip}^{j})}, \quad \frac{C_{ip}^{j}}{(1 + P_i C_{ip}^{j})}$$

are also elements of H^∞. In fact, when $j = r_c$, $C_{ip}^{r_c}$ is exactly proper, and

$$(1 + C_{ip}^{r_c} P_i) \in H^\infty$$

and is exactly proper $\forall i = 1, 2, \ldots, n$. Therefore, $C_{ip}^{r_c}$ is an exactly proper controller that simultaneously stabilizes the plants. ■

Theorem 1 is also true for improper plants. The following theorem and associated corollaries are restatements of Theorem 1 for proper plants and are used in the proofs of theorems for sufficient conditions.

Theorem 2 *The n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, are simultaneously stabilizable, if and only if there exists an exactly proper artificial plant, $P_0 = \frac{n_0}{d_0}$, such that*

1. *$(n_0 d_i - n_i d_0)$ is strictly Hurwitz $\forall i = 1, 2, \ldots, n$*
2. *$(P_0 - P_i)$ is exactly proper $\forall i = 1, 2, \ldots, n$*

Proof:

See [4]. This proof is simply derived by replacing the exactly proper simultaneously stabilizing controller with the substitution $C_{ep} = -\frac{1}{P_0}$. ■

Corollary 3 *The n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, are (strongly) simultaneously stabilizable with a stable controller, if and only if there exists an exactly proper, minimum phase artificial plant, $P_0 = \frac{n_0}{d_0}$, such that*

1. $(n_0 d_i - n_i d_0)$ *is strictly Hurwitz* $\forall i = 1, 2, \ldots, n$

2. $(P_0 - P_i)$ *is exactly proper* $\forall i = 1, 2, \ldots, n$

Proof:

See [4]. This proof is again complete by replacing the stable exactly proper simultaneously stabilizing controller with the substitution $C_{ep} = -\frac{1}{P_0}$. ■

Corollary 4 *The* n *proper plants:* $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, *are simultaneously stabilizable with a unit in* H^∞, *if and only if there exists a unit,* $P_0 = \frac{n_0}{d_0}$, *such that*

1. $(n_0 d_i - n_i d_0)$ *is strictly Hurwitz* $\forall i = 1, 2, \ldots, n$

2. $(P_0 - P_i)$ *is exactly proper* $\forall i = 1, 2, \ldots, n$

Proof:

This proof is obvious by replacing the simultaneously stabilizing Unit in H^∞ with $C_{ep} = -\frac{1}{P_0}$. ■

3 Sufficient Conditions for Simultaneous Stabilization

New sufficient conditions are presented in this section. These conditions are quite general in nature and offer a wide range of testable conditions, which are briefly discussed in Section 4. The main result of this section is stated in the following theorem. Since *strong* or *Unit* stabilization extends the number of plants which could be simultaneously stabilized by 1 or 2 respectively, the corollaries associated with the main result pertain to the plausibility of a strictly Hurwitz characteristic for the numerator or denominator polynomials of the simultaneously stabilizing controller.

Theorem 3 *The* n *proper plants:* $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, *are simultaneously stabilizable, if there exists an artificial plant,* $P_0 = \frac{n_0}{d_0}$, *not necessarily proper, such that*

1. $(n_0 d_i - n_i d_0)$ *is strictly Hurwitz* $\forall i = 1, 2, \ldots, n$

2. *The difference plants,* $(P_0 - P_i)$, *that are strictly proper, have the same high frequency sign*

Proof Outline:

This proof is split into 2 parts depending upon whether the maximum relative degree of all the difference plants, $r_d = \max\{r_i\} = \max\{rd(P_0 - P_i)\}$, is greater than the relative degree of P_0, $r_c = rd(P_0)$.

If $r_d > r_c$, then the plant $P_0^0 = P_0 = \frac{n_0^0}{d_0^0}$, is modified $(r_d - r_c)$ times as follows:

$$P_0^m = P_0^{m-1} \cdot \frac{(h_m + \epsilon_m a_m)}{h_m} = \frac{n_0^m}{d_0^m}, \ \forall m = 1, 2, \ldots, (r_d - r_c)$$

where

1. a_m is any arbitrary polynomial of finite order with the sign of the leading coefficient equal to the sign of the leading coefficient of $h_m n_{0i}^{m-1} n_0^{m-1}$, $\forall i \in K$.

2. $n_{0i}^{m-1} = n_0^{m-1} d_i - d_0^{m-1} n_i$

3. $K = \{i : (P_0 - P_i) \in M_S\}$

4. h_m is any strictly Hurwitz polynomial of degree equal to $o(h_m) = o(a_m) + r_{m-1} - r_c - 1$

5. $r_{m-1} = \max\{rd(P_0^{m-1} - P_i) \ \forall i = 1, 2, \ldots, n\}$

With this choice of modified plant, it is shown in [4] from Corollaries 1 and 2 that there exists an $\epsilon_m > 0$ sufficiently small such that the following conditions are true for each step, $m = 1, 2, \ldots, (r_d - r_c)$

1. $rd(P_0^m - P_i) = rd(P_0^{m-1} - P_i) - 1, \ \forall i \in J_{m-1}$
   $\Rightarrow \quad r_m = r_{m-1} - 1$
2. $rd(P_0^m - P_i) = rd(P_0^{m-1} - P_i), \ \forall i \notin J_{m-1}$
3. $n_{0i}^m := (n_0^m d_i - n_i d_0^m)$ is strictly Hurwitz $\forall i$
4. The high frequency signs of n_{0i}^m are the same $\forall i \in K$.
5. $rd(P_0^m) = rd(P_0^{m-1}) = r_c$

where $r_m = \max\{rd(P_0^m - P_i), \ \forall i = 1, 2, \ldots, n\}$ and $J_m = \{i : \ rd(P_0^m - P_i) = r_m\}$

After $(r_d - r_c)$ steps, there exists a proper plant, $P_0^{(r_d - r_c)}$, of relative degree r_c, such that $(n_0^{(r_d-r_c)} d_i - n_i d_0^{(r_d-r_c)})$ is strictly Hurwitz and $rd(P_0^{(r_d-r_c)} - P_i) \leq r_c, \forall i$, and that the high frequency sign of $(P_0^{(r_0-r_c)} - P_i)$ is the same $\forall i \in K$. If $r_d \leq r_c$, then these steps mentioned above can be skipped. If $r_c = 0$, then the proof is complete.

If $r_c > 0$ and $r_m = r_c$ or $r_d \leq r_c$, then we redefine the original plant as $P_c^0 = P_0^{(r_d-r_c)} = \frac{n_c^0}{d_c^0}$ when $r_m = r_c$ or $P_c^0 = P_0 = \frac{n_c^0}{d_c^0}$ when $r_d \leq r_c$. The plant P_c^0 is then modified r_c times as follows:

$$P_c^m = P_c^{m-1} \cdot \frac{(h_m + \epsilon_m a_m)}{h_m} = \frac{n_c^m}{d_c^m}$$

where

1. a_m is any arbitrary polynomial of finite order with the sign of the leading coefficient equal to the sign of the leading coefficient of $h_m n_{ci}^{m-1} n_c^{m-1}, \forall i \in K$.
2. $n_{ci}^{m-1} = n_c^{m-1} d_i - d_c^{m-1} n_i$
3. $K = \{i : (P_0 - P_i) \in M_S\}$
4. h_m is any strictly Hurwitz polynomial of degree equal to $o(h_m) = o(a_m) - 1$

With this choice of modified plant, it is shown in [4] from Corollaries 1 and 2 that there exists an $\epsilon_m > 0$ sufficiently small such that the following conditions are true for each step, $m = 1, 2, \ldots, r_c$

1. $rd(P_c^m - P_i) = rd(P_c^{m-1} - P_i) - 1, \ \forall i \in Q_m$
2. $rd(P_c^m - P_i) = rd(P_c^{m-1} - P_i), \ \forall i \notin Q_m$
3. $n_{ci}^m := (n_c^m d_i - n_i d_c^m)$ is strictly Hurwitz $\forall i$
4. The high frequency signs of n_{ci}^m are the same $\forall i \in K$.
5. $rd(P_c^m) = rd(P_c^{m-1}) - 1 = r_c - m$

where $Q_m = \{i : rd(P_c^m - P_i) = rd(P_c^m)\}$.

After r_c steps, we have an exactly proper plant, $P_c^{r_c}$, which forms exactly proper, minimum phase difference plants, $(P_c^{r_c} - P_i), \ \forall i = 1, 2, \ldots, n$. From Theorem 2, the controller $C = -\frac{1}{P_c^{r_c}}$ simultaneously stabilizes the n plants. ■

There is an equivalent interpretation of the sufficient conditions in Theorem 3 from a unit interpolation viewpoint. When a plant, P_0, exists, which forms minimum phase difference plants with all the other plants, $(P_0 - P_1),\ (P_0 - P_2), \ldots,\ (P_0 - P_n)$, then there exists a unit which interpolates to $\frac{d_1}{d_2}$, at all finite zeros of $(P_2 - P_1)$ in the RHP, and interpolates to $\frac{n_1}{n_2}$ at the zeros of $\frac{n_{12}}{n}$ in the RHP, while avoiding all other finite intersections with $\frac{n_{1i}}{n_{2i}}$, $\forall i = 3, 4, \ldots, n$ in the RHP. The high frequency sign conditions on the strictly proper difference plants are sufficient conditions for the unit to interpolate to the points at $s = \infty$ as well without causing the unit to intersect with any of these curves at any other point in the RHP. This is discussed in more detail in [4].

Corollary 5 *The n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, are (strongly) simultaneously stabilizable with a stable controller, if there exists a minimum phase artificial plant, $P_0 = \frac{n_0}{d_0}$, not necessarily proper, such that*

1. *$(n_0 d_i - n_i d_0)$ is strictly Hurwitz $\forall i = 1, 2, \ldots, n$*
2. *The difference plants, $(P_0 - P_i)$, that are strictly proper, have the same high frequency sign*
3. *If P_0 is strictly proper, then the high frequency sign of P_0 is the same as the high frequency sign of all of the difference plants, $(P_0 - P_i)$, that are strictly proper*

Proof:

The proof of this corollary follows that of Theorem 3 with the additional requirement of showing that for the very same plant modifications ϵ_m can also be chosen sufficiently small to ensure that n_0^m and n_c^m remain strictly Hurwitz polynomials. The proof then follows from Theorem 3. See [4] for more detail. ∎

Corollary 6 *The n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, are simultaneously stabilizable with a unit in H^∞, if there exists a minimum phase, stable artificial plant, $P_0 = \frac{n_0}{d_0}$, not necessarily proper, such that*

1. *$(n_0 d_i - n_i d_0)$ is strictly Hurwitz $\forall i = 1, 2, \ldots, n$*
2. *The difference plants, $(P_0 - P_i)$, that are strictly proper, have the same high frequency sign*
3. *If P_0 is strictly proper, then the high frequency sign of P_0 is the same as the high frequency sign of all of the difference plants, $(P_0 - P_i)$, that are strictly proper*

Proof:

The proof of this corollary follows that of Theorem 3 with the additional requirements of showing that for the very same plant modifications ϵ_m can also be chosen sufficiently small to ensure that n_0^m, n_c^m, d_0^m, and d_c^m remain strictly Hurwitz polynomials. The proof then follows from Theorem 4. See [4] for more detail. ∎

4 Conversions from Sufficient Conditions to Testable Conditions

In this section, the new sufficient conditions derived in Section 3 are used to create easily testable conditions. Examples illustrate some of the applications.

Theorem 4 *The n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, are simultaneously stabilizable, if one of the plants, $P_1 = \frac{n_1}{d_1}$, is such that*

1. *$(n_1 d_i - n_i d_1)$ is strictly Hurwitz $\forall i = 2, 3, \ldots, n$*
2. *The difference plants, $(P_1 - P_i)$, $i \neq 1$, that are strictly proper, have the same high frequency sign*

Proof: Assume there exists a proper plant, P_1, such that $n_{1i} := (n_1 d_i - n_i d_1)$ is strictly Hurwitz and the difference plants, $(P_1 - P_i)$, that are strictly proper, have the same high frequency sign $\forall i = 2, 3, \ldots, n$. Since each of the plants, P_i, are proper $\forall i$, $(P_1 - P_i)$ cannot be improper.

Consider the plant P_0,

$$P_0 = \frac{n_0}{d_0} = \frac{(n_1 h + \epsilon_0 a_0 n_{k1} n_j)}{(d_1 h + \epsilon_0 a_0 n_{k1} d_j)} \tag{2}$$

where

1. $j,\ k \in 2, 3, \ldots, n$. j and k may be equal, but are not required to be.

2. $$a_0 = \left\{ \begin{array}{ll} 1, & \text{when the high frequency sign of } n_{1i} > 0,\ \forall i \in K \\ -1, & \text{when the high frequency sign of } n_{1i} < 0,\ \forall i \in K \end{array} \right\}$$

3. h is any strictly Hurwitz polynomial of degree $o(h) \geq \max\{o_1, o_2\}$, where

$$\begin{aligned} o_1 &= o(n_{jk}) \\ o_2 &= o(n_{1k}) + o(n_{ji}) - o(n_{1i}),\ \forall i \neq 1, k, j \end{aligned}$$

The difference plants formed with this choice of P_0 become

$$(P_0 - P_i) = \frac{h n_{1i} + \epsilon_0 a_0 n_{k1} n_{ji}}{d_i (d_1 h + \epsilon_0 a_0 n_{k1} d_j)}$$

It is shown in [4] from Corollaries 1 and 2 that $\exists \epsilon_0 > 0$ sufficiently small such that each of these difference plants are minimum phase, and those that are strictly proper have the same high frequency sign. The proof then follows from Theorem 3. ■

Corollary 7 *The n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, are (strongly) simultaneously stabilizable with a stable controller, if one of the plants, $P_1 = \frac{n_1}{d_1}$, is minimum phase such that*

1. *$(n_1 d_i - n_i d_1)$ is strictly Hurwitz $\forall i = 2, 3, \ldots, n$*

2. *The difference plants, $(P_1 - P_i)$, that are strictly proper, have the same high frequency sign $\forall i = 2, 3, \ldots, n$*

3. *If the plant, P_1, is strictly proper, then the high frequency sign of P_1 is the same as the high frequency sign of the difference plants, $(P_1 - P_i)$, that are strictly proper.*

Proof:

The proof of this corollary follows that of Theorem 4 with the additional requirement of showing that for the very same plant formation ϵ_0 can also be chosen sufficiently small to ensure that n_0 remains a strictly Hurwitz polynomial. This requires that the the strictly Hurwitz polynomial, h, chosen in Equation (2), be of order $o(h) \geq \max\{o_1, o_2, o_3\}$, where

$$\begin{aligned} o_1 &= o(n_{jk}) \\ o_2 &= o(n_{1k}) + o(n_{ji}) - o(n_{1i}),\ \forall i \neq 1, k, j \\ o_3 &= o(n_{1k}) + o(n_j) - o(n_1) \end{aligned}$$

The proof then follows from Corollary 5. See [4] for more detail. ■

Corollary 8 *The n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, are simultaneously stabilizable with a unit in H^∞, if one of the plants, $P_1 = \frac{n_1}{d_1}$, is minimum phase, and stable such that*

1. *$(n_1 d_i - n_i d_1)$ is strictly Hurwitz $\forall i = 2, 3, \ldots, n$*
2. *The difference plants, $(P_1 - P_i)$, that are strictly proper, have the same high frequency sign $\forall i = 2, 3, \ldots, n$*
3. *If the plant, P_1, is strictly proper, then the high frequency sign of P_1 is the same as the high frequency sign of the difference plants, $(P_1 - P_i)$, that are strictly proper.*

Proof:

The proof of this corollary follows that of Theorem 4 with the additional requirement of showing that for the very same plant formation ϵ_0 can also be chosen sufficiently small to ensure that n_0 and d_0 remain strictly Hurwitz polynomials. This requires that the the strictly Hurwitz polynomial, h, chosen in equation 2 be of order $o(h) \geq \max\{o_1, o_2, o_3, o_4\}$, where

$$\begin{aligned}
o_1 &= o(n_{jk}) \\
o_2 &= o(n_{1k}) + o(n_{ji}) - o(n_{1i}), \;\; \forall i \neq 1, k, j \\
o_3 &= o(n_{1k}) + o(n_j) - o(n_1) \\
o_4 &= o(n_{1k}) + o(d_j) - o(d_1)
\end{aligned}$$

The proof then follows from Corollary 6. See [4] for more detail. ■

Example 1 *Find a simultaneously stabilizing controller for the following three plants.*

$$P_1 = \frac{n_1}{d_1} = \frac{1}{(s+1)}, \;\; P_2 = \frac{n_2}{d_2} = \frac{1}{(s^2 - s + 1)}, \;\; P_3 = \frac{n_3}{d_3} = \frac{-1}{3s}$$

It is easy to verify that $(P_3 - P_1)$ and $(P_3 - P_2)$ are minimum phase, strictly proper, and have the same high frequency sign.

$$\begin{aligned}
(P_3 - P_1) &= \frac{n_{31}}{d_3 d_1} = \frac{-(4s+1)}{3s(s+1)} \\
(P_3 - P_2) &= \frac{n_{32}}{d_3 d_2} = \frac{-(s^2+2s+1)}{3s(s^2 - s + 1)}
\end{aligned}$$

From Corollary 7, there exists a stable controller that simultaneously stabilizes all three plants. In this case, the plant forming minimum phase difference plants is of subscript 3. Therefore, by choosing $k = 1$, and $j = 2$, then the minimum order of h is $o(h) \geq 2$, and we can let

$$P_0 = \frac{(n_3 h_0 + \epsilon_0 n_{31} n_2)}{(d_3 h_0 + \epsilon_0 n_{31} d_2)}, \quad \text{where} \;\; h_0 = (2s^2 + 6s + 2), \;\; \epsilon_0 = 1$$

$$\Rightarrow \;\; P_0 = \frac{-(2s^2 + 10s + 3)}{(2s^3 + 21s^2 + 3s - 1)}$$

The corresponding difference plants become

$$\begin{aligned}
(P_0 - P_1) &= \frac{-(4s^3 + 33s^2 + 16s + 2)}{(2s^3 + 21s^2 + 3s - 1)(s+1)} = \frac{-(s+0.25)(s+0.258)(s+7.74)}{(2s^3 + 21s^2 + 3s - 1)(s+1)} \\
(P_0 - P_2) &= \frac{-(2s^4 + 10s^3 + 16s^2 + 10s + 2)}{(s^2 - s + 1)(2s^3 + 21s^2 + 3s - 1)} = \frac{-(s+0.38)(s+1)^2(s+2.62)}{(s^2 - s + 1)(2s^3 + 21s^2 + 3s - 1)} \\
(P_0 - P_3) &= \frac{-(4s^3 + 9s^2 + 6s + 1)}{(3s)(2s^3 + 21s^2 + 3s - 1)} = \frac{-(s+0.25)(s+1)^2}{(3s)(2s^3 + 21s^2 + 3s - 1)}
\end{aligned}$$

P_0 is strictly proper, minimum phase and has the same high frequency sign as all of the strictly proper minimum phase difference plants. Therefore, from Corollary 5, there exists a stable controller that simultaneously stabilizes all three plants. Since the relative degree of all three difference plants is less than or equal to the relative degree of P_0, the next modified plant can be of the form

$$P_0^1 = P_0 \cdot (\epsilon_1 s + 1)$$

where $a_1 = s$ and $h_1 = 1$. Choosing $\epsilon_1 = 0.2$,

$$\Rightarrow \quad P_0^1 = \frac{-(0.4s^3 + 4s^2 + 10.6s + 3)}{(2s^3 + 21s^2 + 3s - 1)}$$

Forming the difference plants,

$$(P_0^1 - P_1) = \frac{-0.4(s^2 + 15.5s + 81.2)(s + 0.277)(s + 0.222)}{(s + 1)(2s^3 + 21s^2 + 3s - 1)}$$
$$(P_0^1 - P_2) = \frac{-0.4(s + 6.46)(s^2 + 1.72s + 4,54)(s^2 + 0.8s + 0.17)}{(s^2 - s + 1)(2s^3 + 21s^2 + 3s - 1)}$$
$$(P_0^1 - P_3) = \frac{-1.2(s + 0.26)(s + 7.17)(s^2 + 0.9s + 0.45)}{(3s)(2s^3 + 21s^2 + 3s - 1)}$$

$$C = -\frac{1}{P_0^1}$$

$$\boxed{C(s) = \frac{(5s^3+52.5s^2+7.5s-2.5)}{(s+5)(s+4.68)(s+0.32)}}$$

Example 2 *Find a simultaneously stabilizing controller for the following three plants.*

$$P_1 = \frac{n_1}{d_1} = \frac{(s-1)}{4(s^2 - 2s + 4)}, \quad P_2 = \frac{n_2}{d_2} = \frac{1}{3(s-5)}, \quad P_3 = \frac{n_3}{d_3} = -\frac{(s^2 - s + 1)}{6(5s^2 - s + 3)}$$

The difference plants formed from P_2 have strictly Hurwitz numerator polynomials. There is only one difference plant, which is strictly proper. Since P_2 is minimum phase, strictly proper, and has the same high frequency sign as the difference plant, which is strictly proper, it follows from Corollary 7 that these plants are strongly simultaneously stabilizable.

$$(P_2 - P_1) = \frac{n_{21}}{d_1 d_2} = \frac{(s^2 + 10s + 1)}{12(s-5)(s^2 - 2s + 4)}$$
$$(P_2 - P_3) = \frac{n_{23}}{d_2 d_3} = \frac{(s^3 + 4s^2 + 4s + 1)}{6(s-5)(5s^2 - s + 3)} = \frac{(s + 0.38)(s + 1)(s + 2.62)}{6(s-5)(5s^2 - s + 3)}$$

By choosing $k = 3$ and $j = 1$, it is easy to verify that the minimum order required for h to ensure existence of a sufficiently small ϵ_0 is $o(h) \geq 4$. Choosing P_0 as

$$P_0 = \frac{h_0 n_2 + \epsilon_0 a_0 n_{32} n_1}{h_0 d_2 + \epsilon_0 a_0 n_{32} d_1}, \quad \text{where} \quad a_0 = 1, \quad h_0 = [4s^2 + 5s + 6]^2, \quad \epsilon_0 = 1$$

$$\Rightarrow \quad P_0 = \frac{n_0}{d_0} = \frac{(13s^4 + 31s^3 + 73s^2 + 69s + 39)}{(36s^5 - 144s^4 - 381s^3 - 1023s^2 - 960s - 588)}$$

Forming the difference plants with P_0,

$$(P_0 - P_1) = \frac{(16s^6 + 200s^5 + 489s^4 + 830s^3 + 709s^2 + 420s + 36)}{4(s^2 - 2s + 4)(36s^5 - 144s^4 - 381s^3 - 1023s^2 - 960s - 588)}$$
$$(P_0 - P_2) = \frac{(3s^5 + 42s^4 + 135s^3 + 135s^2 + 42s + 3)}{3(s - 5)(36s^5 - 144s^4 - 381s^3 - 1023s^2 - 960s - 588)}$$
$$(P_0 - P_3) = \frac{(36s^7 + 210s^6 + 651s^5 + 1452s^4 + 1872s^3 + 1419s^2 + 636s + 114)}{6(5s^2 - s + 3)(36s^5 - 144s^4 - 381s^3 - 1023s^2 - 960s - 588)}$$

All difference plants and P_0 are minimum phase, and P_0 has the same high frequency sign as the two strictly proper difference plants $(P_0 - P_1)$ and $(P_0 - P_2)$. Therefore, from Corollary 5, the plants are strongly simultaneously stabilizable. Since the relative degree of all of the difference plants are less than or equal to the relative degree of P_0, the next modified plant may be of the form,

$$P_0^1 = \frac{n_0(\epsilon_1 s + 1)}{d_0}$$

where $a_1 = s$ and $h_1 = 1$. Choosing $\epsilon_1 = 0.01$,

$$P_0^1 = \frac{0.01(s + 100)(13s^4 + 31s^3 + 73s^2 + 69s + 39)}{(36s^5 - 144s^4 - 381s^3 - 1023s^2 - 960s - 588)}$$

The corresponding difference plants formed from P_0^1 are

$$(P_0^1 - P_1) = \frac{0.0036(s + 0.098)(s^2 + 28.57s + 311)(s^2 + 1.8s + 1.6)(s^2 + 1.3s + 1.4)}{(s^2 - 2s + 4)(s - 6.44)(s^2 + 1.16s + 2.693)(s^2 + 1.279s + 0.942)}$$
$$(P_0^1 - P_2) = \frac{0.0036(s + 1.27)(s + 1.76)(s^2 + 1.65s + 92.4)(s^2 + 0.4s + 0.004)}{(s - 5)(s - 6.44)(s^2 + 1.16s + 2.693)(s^2 + 1.279s + 0.942)}$$
$$(P_0^1 - P_3) = \frac{0.0369(s + 0.36)(s + 1.1)(s + 2.24)(s^2 + 0.94s + 5.26)(s^2 + 0.82s + 0.6)}{(s^2 - 0.2s + 0.6)(s - 6.44)(s^2 + 1.16s + 2.693)(s^2 + 1.279s + 0.942)}$$

P_0^1 is exactly proper and minimum phase, and all of the difference plants are exactly proper and minimum phase,

$$\Rightarrow \qquad C = -\frac{1}{P_0^1}$$

$$\boxed{C(s) = \frac{-276.9(s-6.44)(s^2+1.16s+2.693)(s^2+1.279s+0.942)}{(s+100)(s^2+1.071s+3.299)(s^2+1.314s+0.909)}}$$

Theorem 5 *Let $n_{1i} := (n_1 d_i - n_i d_1)$ and $n_{2i} := (n_2 d_i - n_i d_2)$. The n proper plants: $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, are simultaneously stabilizable, if*

1. *$(n_{2i} - n_{1i})$ is strictly Hurwitz $\forall i = 1, 2, \ldots, n$*
2. *The plants, $\frac{(n_{2i} - n_{1i})}{d_i(d_2 - d_1)}$, that are strictly proper, have the same high frequency sign*

Proof of Theorem 5:

Let $P_0 = \frac{(n_2 - n_1)}{(d_2 - d_1)}$. Then all of the difference plants formed with P_0 become

$$(P_0 - P_i) = \frac{(n_{2i} - n_{1i})}{d_i(d_2 - d_1)}$$

The remainder of the proof follows from Theorem 3. ∎

The following example illustrates, that with the use of Theorem 5, it is sufficient for only one of the difference plants to be minimum phase.

Example 3 *Find a simultaneously stabilizing controller for the following three plants.*

$$P_1 = \frac{-4}{(s-1)},\ P_2 = \frac{(s-1)}{(s+2)},\ P_3 = \frac{-7}{3(s-2)}$$

When forming all difference plants, only one is minimum phase.

$$(P_2 - P_1) = \frac{n_{21}}{d_2 d_1} = \frac{(s^2+2s+9)}{(s+2)(s-1)}$$
$$(P_3 - P_1) = \frac{n_{31}}{d_3 d_1} = \frac{(5s-17)}{3(s-1)(s-2)}$$
$$(P_3 - P_2) = \frac{n_{32}}{d_3 d_2} = \frac{-(3s^2-2s+20)}{3(s+2)(s-2)}$$

Since n_{21} is strictly Hurwitz, we can check $n_{23} - n_{13}$.

$$n_{23} - n_{13} = (3s^2 - 2s + 20) + (5s - 17)$$
$$n_{23} - n_{13} = 3(s^2 + s + 1)$$

$$\Rightarrow P_0 = \frac{(n_2 - n_1)}{(d_2 - d_1)} = \frac{(s+3)}{3}$$

Checking the difference plants with P_0 results in

$$(P_0 - P_1) = \frac{(s^2+2s+9)}{3(s-1)},\ (P_0 - P_2) = \frac{(s^2+2s+9)}{3(s+2)},\ (P_0 - P_3) = \frac{(s^2+s+1)}{3(s-2)}$$

Since the difference plants are all minimum phase and improper, and all of the plants are proper, and P_0 is minimum phase, it follows from Corollary 5 that the controller, $C = -\frac{1}{P_0}$, strongly simultaneously stabilizes all plants.

$$\boxed{C(s) = \frac{-3}{(s+3)}}$$

In this case, the simultaneously stabilizing controller is strictly proper, which, in general, is more difficult to achieve than an exactly proper simultaneously stabilizing controller, per Theorem 1.

Comments:

There is an interesting special case of Theorem 5. If n_{21} is strictly Hurwitz, $(P_0 - P_i)$ has the same relative degree as $P_0 = \frac{(n_2-n_1)}{(d_2-d_1)}$, $(n_{2i} - n_{1i}) = n_{21}$, $\forall i = 3, 4, \ldots, n$, and $\frac{(d_2-d_1)}{(n_2-n_1)}$ is proper, then $C = \frac{(d_1-d_2)}{(n_2-n_1)}$ simultaneously stabilizes the n plants. It is easy to verify that the closed loop characteristic polynomials of all plants, h_i, in this case are the same. This is Emre's result [8]. However, with the application of the more general theorems described in this paper, it is much easier to see all of the special conditions required for common closed loop poles.

There are two corollaries associated with Theorem 5 as well as with the three theorems that follow. Each corollary corresponds to the cases where the controller is stable, minimum phase or both. Since they are similar to Corollaries 5 and 6, they are omitted.

Theorem 6 *Let* $n_{1i} := (n_1 d_i - n_i d_1)$ *and* $n_{2i} := (n_2 d_i - n_i d_2)$. *The* n *proper plants:* $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, *are simultaneously stabilizable, if*

1. $(n_{2i} + n_{1i})$ *is strictly Hurwitz* $\forall i = 1, 2, \ldots, n$
2. *The plants,* $\frac{(n_{2i}+n_{1i})}{d_i(d_2+d_1)}$, *that are strictly proper, have the same high frequency sign*

Proof:

Let $P_0 = \frac{(n_2+n_1)}{(d_2+d_1)}$. Then all of the difference plants formed with P_0 become

$$(P_0 - P_i) = \frac{(n_{2i} + n_{1i})}{d_i(d_2 + d_1)}$$

The remainder of the proof follows from Theorem 3. ■

Theorem 7 *Let* $n_{1i}^+ := (n_1 d_i + n_i d_1)$ *and* $n_{2i} := (n_2 d_i - n_i d_2)$. *The* n *proper plants:* $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, *are simultaneously stabilizable, if*

1. $(n_{2i} - n_{1i}^+)$ *is strictly Hurwitz* $\forall i = 1, 2, \ldots, n$
2. *The artificial plants,* $\frac{(n_{2i} - n_{1i}^+)}{d_i(d_2+d_1)}$, *that are strictly proper, have the same high frequency sign*

Proof:

Let $P_0 = \frac{(n_2-n_1)}{(d_2+d_1)}$. Then all of the difference plants formed with P_0 become

$$(P_0 - P_i) = \frac{(n_{2i} - n_{1i}^+)}{d_i(d_2 + d_1)}$$

The remainder of the proof follows from Theorem 3. ■

Theorem 8 *Let* $n_{1i}^+ := (n_1 d_i + n_i d_1)$ *and* $n_{2i} := (n_2 d_i - n_i d_2)$. *The* n *proper plants:* $P_i = \frac{n_i}{d_i}$, $\forall i = 1, 2, \ldots, n$, *are simultaneously stabilizable, if*

1. $(n_{2i} + n_{1i}^+)$ *is strictly Hurwitz* $\forall i = 1, 2, \ldots, n$
2. *The artificial plants,* $\frac{(n_{2i} + n_{1i}^+)}{d_i(d_2-d_1)}$, *that are strictly proper, have the same high frequency sign*

Proof:

Let $P_0 = \frac{(n_2+n_1)}{(d_2-d_1)}$. Then the difference plants formed with P_0 become

$$(P_0 - P_i) = \frac{(n_{2i} + n_{1i}^+)}{d_i(d_2 - d_1)}$$

The remainder of the proof follows from Theorem 3. ■

By now, it should be obvious that there are many combinations of the n plant numerator and denominator polynomials, which could be used to test for sufficient conditions for simultaneous stabilization. The new sufficient conditions of this paper reduce in special cases to published results as summarized below.

1. By choosing $P_0 = \frac{\pm\epsilon}{(s+1)^r}$, where r is the maximum relative degree of all minimum phase, strictly proper plants with the same high frequency sign, then these new sufficient conditions are a generalization of the results published in [1], [5], [9], and [11].
2. By choosing $P_0 = \epsilon$, where the plants are all minimum phase and exactly proper, then these new sufficient conditions are equivalent to the results published in [10].
3. By choosing $P_0 = P_1$, where the difference plants formed from P_1 are all minimum phase and exactly proper, then these new sufficient conditions are a generalization of the results published in [3] and [6].
4. By choosing $P_0 = \frac{(n_2-n_1)}{(d_2-d_1)}$, then these new sufficient conditons are a generalization of the results published in [8].

5 Summary and Conclusions

In this paper, new necessary and sufficient conditions for the simultaneous stabilization of n plants were derived. It was shown that if a controller exists, there must be an exactly proper controller.

New sufficient conditions for the simultaneous stabilization of n plants were also presented. These conditions offer many new tests for the existence of a controller. When these new conditions are satisfied, the controller can be designed in a step by step procedure following the construction steps in the proof. These new sufficient conditions are a generalization of and establish a link to the results published in [1], [3], [5], [6], [8], [9], [10],and [11]. These new sufficient conditions could be used in conjunction with unit interpolation techniques to satisfy interpolation points at infinity.

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