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Abstract. Neutrosophic set is very useful to express uncertainty, impreciseness, incompleteness and inconsistency in a more general way. It is prevalent in real life application problems to express both indeterminate and inconsistent information. This paper focuses on introducing a new similarity measure in the neutrosophic environment. Similarity measure approach can be used in ranking the alternatives and determining the best among them. It is useful to find the optimum alternative for multi criteria decision making (MCDM) problems from similar alternatives in neutrosophic form. We define a function based on hypercomplex number system in this paper to determine the degree of similarity between single valued neutrosophic sets and thus a new approach to rank the alternatives in MCDM problems has been introduced. The approach of using hypercomplex number system in formulating the similarity measure in neutrosophic set is new and is not available in literature so far. Finally, a numerical example demonstrates how this function determines the degree of similarity between single valued neutrosohic sets and thereby solves the MCDM problem.

Keywords: Hypercomplex similarity measure, Neutrosophic fuzzy set, Decision Making.

1 Introduction

Zadeh introduced the degree of membership/truth (t) in 1965 and defined the fuzzy set which is an extension of ordinary or crisp set as the elements in the fuzzy set are characterised by the grade of membership to the set. Atanassov introduced the degree of nonmembership/falsehood (f) in 1986 and defined the intuitionistic fuzzy set. An intutionistic fuzzy set is characterized by a membership and nonmembership function and thus can be thought of as the extension of fuzzy set. Smarandache introduced the degree of indeterminacy/neutrality (i) as independent component in 1995 (published in 1998) and defined the neutrosophic set [1]. He has coined the words "neutrosophy" and "neutrosophic". In 2013 he refined the neutrosophic set to n components: $t_1, t_2, \dots t_j; i_1, i_2, \dots, i_k; f_1, f_2, \dots, f_l$, with j+k+l = n > 3. A neutrosophic set generalizes the concepts of classical set, fuzzy set and intutionistic fuzzy set by considering truth-membership function, indeterminacy membership function and falsity-membership function. Real life problems generally deal with indeterminacy, inconsistency and incomplete information which can be best represented by a neutrosophic set.

Properties of neutrosophic sets, their operations, similarity measure between them and solution of MCDM problems in neutrosophic environment are available in the literature. In [2] Wang et al. presented single valued neutrosophic set (SVNS) and defined the notion of inclusion, complement, union, intersection and discussed various properties of set-theoretic operators. They also provided in [3] the set-theoretic operators and various properties of interval valued neutrosophic sets (IVNSs). Said Broumi and Florentin Smarandache introduced the concept of several similarity measures of neutrosophic sets [4]. In this paper they presented the extended Hausdorff distance for neutrosophic sets and defined a series of similarity measures to calculate the similarity between neutrosophic sets. In [5] Ye introduced the concept of a simplified neutrosophic set (SNS), which is a subclass of a neutrosophic set and includes the concepts of IVNS and SVNS; he defined some operational laws of SNSs and proposed simplified neutrosophic weighted averaging (SNWA) operator and simplified neutrosophic weighted geometric (SNWG) operator and applied them to multi criteria decision-making problems under the simplified neutrosophic environment. Ye [6] further generalized the Jaccard, Dice and cosine similarity measures between two vectors in SNSs. Then he applied the three similarity measures to a multi criteria decision-making problem in the simplified neutrosophic setting. Broumi and Smarandache [7] defined weighted interval valued neutrosophic sets and found a cosine similarity measure between two IVNSs. Then they applied it to problems related to pattern recognition.

Various comparison methods are used for ranking the alternatives. Till date no similarity measure using hypercomplex number system in neutrosophic environment is available in literature. We introduce hypercomplex number in similarity measure. In this paper SVNS is represented as a hypercomplex number. The distance measured between so transformed hypercomplex numbers can give the similarity value. We have used hypercomplex numbers as discussed by Silviu Olariu in [8]. Multiplication of such hypercomplex numbers is associative and commutative. Exponential and trigonometric form exist, also the concept of analytic function, contour integration and residue is defined. Many of the properties of two dimensional complex functions can be extended to hypercomplex numbers in n dimensions and can be used in similarity measure problems. Here in lies the robustness of this method being another application of complex analysis.

The rest of paper is structured as follows. Section 2 introduces some concepts of neutrosophic sets and SNSs. Section 3 describes Jaccard, Dice and cosine similarity measures. In section 4 three dimensional hypercomplex number system and its properties have been discussed. We define a new function based on three dimensional hypercomplex number system for similarity measure to compare neutrosophic sets in section 5. Section 6 demonstrates application of hypercomplex similarity measures in Decision-Making problem. In section 7, a numerical example demonstrates the application and effectiveness of the proposed similarity measure in decision-making problems in neutrosophic environment. We conclude the paper in section 8.

2 Neutrosophic sets

2.1 Definition

Let U be an universe of discourse, then the neutrosophic set A is defined as $\mathrm{A} = \{< x: \mathrm{T_A(x)}, \mathrm{I_A(x)}, \mathrm{F_A(x)} >\}$, where the functions $\mathrm{T}, \mathrm{I}, \mathrm{F}: \mathrm{U} \to]\,^{-}0, 1^{+}[$ define respectively the degree of membership (or Truth), the degree of indeterminacy and the degree of non-membership (or falsehood) of the element $\mathrm{x} \in \mathrm{U}$ to the set A with the condition $^{-}0 \le \mathrm{T_A(x)} + \mathrm{I_A(x)} + \mathrm{F_A(x)} \le 3^{+}$.

To apply neutrosophic set to science and technology, we consider the neutrosohic set which takes the value from the subset of $[0, 1]$ instead of $]\,^{-}0, 1^{+}[$ i.e., we consider SNS as defined by Ye in [5].

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2.2 Simplified Neutrosophic Set

Let X is a space of points (objects) with generic elements in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $\mathrm{T_A(x)}$, an indeterminacy membership function $\mathrm{I_A(x)}$, and a falsity membership function $\mathrm{F_A(x)}$ if the functions $\mathrm{T_A(x)}, \mathrm{I_A(x)}, \mathrm{F_A(x)}$ are singletons subintervals/subsets in the real standard $[0, 1]$, i.e. $\mathrm{T_A(x)}: \mathrm{X} \to [0, 1], \mathrm{I_A(x)}: \mathrm{X} \to [0, 1]$, $\mathrm{F_A(x)}: \mathrm{X} \to [0, 1]$. Then a simplification of the neutrosophic set A is denoted by $\mathrm{A} = \{< x: \mathrm{T_A(x)}, \mathrm{I_A(x)}, \mathrm{F_A(x)} >, x \in X\}$.

2.3 Single Valued Neutrosophic Sets (SVNS)

Let X is a space of points (objects) with generic elements in X denoted by x. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, for each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0, 1]$. Therefore, a SVNS A can be written as $A_{SVNS} = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$.

For two SVNS, $A_{SVNS} = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$ and $B_{SVNS} = \{< x: T_B(x), I_B(x), F_B(x) >, x \in X\}$, the following expressions are defined in [2] as follows: $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x), I_A(x) \ge I_B(x), F_A(x) \ge F_B(x)$. $A_{NS} = B_{Ns}$ if and only if $T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)$. $A^c = < x, F_A(x), 1 - I_A(x), T_A(x) >$

For convenience, a SVNS A is denoted by $A = < T_A(x), I_A(x), F_A(x) >$ for any x$\in X$; for two SVNSs A and B; the operational relations are defined by [2],

(1)$A \cup B =$
$< \max(T_A(x), T_B(x)), min(I_A(x), I_B(x)), min(T_A(x), T_B(x))$
$>$

(2)$A \cap B =<$
$min(T_A(x), T_B(x)), max(I_A(x), I_B(x)), max(T_A(x), T_B(x)) >$

3 Jaccard, Dice and cosine similarity

The vector similarity measure is one of the most important techniques to measure the similarity between objects. In the following, the Jaccard, Dice and cosine similarity measures between two vectors are introduced

Let $X = (x_1, x_2, \dots, x_n)$ and $Y = (y_1, y_2, \dots, y_n)$ be the two vectors of length n where all the coordinates are positive. The Jaccard index of these two vectors is defined as

$$J(X, Y) = \frac{X.Y}{\|X\|_2{}^2 + \|Y\|_2{}^2 + X.Y} = \frac{\sum_{i=1}^{n} x_i . y_i}{\sum_{i=1}^{n} x_i{}^2 + \sum_{i=1}^{n} y_i{}^2 - \sum_{i=1}^{n} x_i . y_i},$$

where $X.Y = \sum_{i=1}^{n} x_i . y_i$ is the inner product of the vectors X and Y.

The Dice similarity measure is defined as

$$J(X, Y) = \frac{2X.Y}{\|X\|_2{}^2 + \|Y\|_2{}^2} = \frac{2\sum_{i=1}^{n} x_i . y_i}{\sum_{i=1}^{n} x_i{}^2 + \sum_{i=1}^{n} y_i{}^2}$$

Cosine formula is defined as the inner product of these two vectors divided by the product of their lengths. This is the cosine of the angle between the vectors. The cosine similarity measure is defined as

$$C(X,Y) = \frac{X.Y}{\|X\|_2{}^2.\|Y\|_2{}^2} = \frac{\sum_{i=1}^{n} x_i.y_i}{\sum_{i=1}^{n} x_i{}^2.\sum_{i=1}^{n} y_i{}^2}$$

It is obvious that the Jaccard, Dice and cosine similarity measures satisfy the following properties

$$(\mathrm{P}_1)\ 0 \le \mathrm{J(X,Y), D(X,Y), C(X,Y)} \le 1$$

$$(\mathrm{P}_2)\ \mathrm{J(X,Y) = J(Y,X), D(X,Y) = D(Y,X) \text{ and } C(X,Y) = C(Y,X)}$$

$$(\mathrm{P}_3)\ \mathrm{J(X,Y) = 1, D(X,Y) = 1 \text{ and } C(X,Y) = 1 \text{ if } X = Y}$$

i.e., $x_i = y_i (i = 1, 2, \ldots, n)$for every $x_i \in X$ and $y_i \in Y$. Also Jaccard, Dice, cosine weighted similarity measures between two SNSs A and B as discussed in [6] are

$$WJ(A,B) = \sum_{i=1}^{n} w_i \frac{\begin{matrix} T_A(x_i)T_B(x_i) \\ +I_A(x_i)I_B(x_i) \\ +F_A(x_i)F_B(x_i) \end{matrix}}{\begin{matrix} (T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2 \\ +(T_B(x_i))^2 + (T_A(x_i))^2 + (T_A(x_i))^2 + (T_A(x_i))^2 \\ -T_A(x_i)T_B(x_i) - T_B(x_i)T_C(x_i) - T_C(x_i)T_A(x_i) \end{matrix}}$$

$$WD(A,B) = \sum_{i=1}^{n} w_i \frac{2\begin{pmatrix} T_A(x_i)T_B(x_i) \\ +I_A(x_i)I_B(x_i) \\ +F_A(x_i)F_B(x_i) \end{pmatrix}}{\begin{matrix} (T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2 \\ +(T_B(x_i))^2 + (T_A(x_i))^2 + (T_A(x_i))^2 + (T_A(x_i))^2 \end{matrix}}$$

$$WC(A,B) = \sum_{i=1}^{n} w_i \frac{\begin{pmatrix} T_A(x_i)T_B(x_i) \\ +I_A(x_i)I_B(x_i) \\ +F_A(x_i)F_B(x_i) \end{pmatrix}}{\begin{matrix} \sqrt{(T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2} \\ \sqrt{(T_B(x_i))^2 + (T_A(x_i))^2 + (T_A(x_i))^2 + (T_A(x_i))^2} \end{matrix}}$$

4 Geometric representation of hypercomplex number in three dimensions

A system of hypercomplex numbers in three dimensions is described here, for which the multiplication is associative and commutative, which have exponential and trigonometric forms and for which the concepts of analytic tricomplex function, contour integration and residue is defined. The tricomplex numbers introduced here have the form $u = x + hy + kz$, the variables x, y and z being real numbers. The multiplication rules for the complex units h, k are $h^2 = k, k^2 = h, 1.h = h, 1.k = k, hk = 1$ as discussed in [8]. In a geometric representation, the tricomplex number u is represented by the point P of nates (x, y, z). If O is the origin of the x, y, z axes, (t) the trisector line $x = y = z$ of the positive octant and Π the plane $x + y + z = 0$ passing through the origin (O) and perpendicular to (t), then the tricomplex number u can be described by the projection S of the segment OP along the line (t), by the distance D from P to the line (t), and by the azimuthal angle ϕ in the Π plane. It is the angle between the projection of P on the plane Π and the straight line which is the intersection of the plane Π and the plane determined by line t and x axis, $0 \le \phi \le 2\pi$. The amplitude ρ of a tricomplex number is defined as $\rho = (x^3 + y^3 + z^3 - 3xyz)^{1/3}$, the polar angle θ of OP with respect to the trisector line (t) is given by $tan\,\theta = \frac{D}{S}, 0 \le \theta \le \pi$ and the distance from P to the origin is $d^2 = x^2 + y^2 + z^2$. the tricomplex number $x + hy + kz$ can be represented by the point P of coordinates (x, y, z). The projection S = OQ of the line OP on the trisector line $x = y = z$, which has the unit tangent$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $is\ S = \frac{1}{\sqrt{3}}(x + y + z)$. The distance D = PQ from P to the trisector line $x = y = z$, calculated as the distance from the point P(x, y ,z) to the point Q of coordinates $\left[\frac{x+y+z}{3}, \frac{x+y+z}{3}, \frac{x+y+z}{3}\right]$, is $D^2 = \frac{2}{3}(x^2 + y^2 + z^2 - xy - yz - zx)$. The quantities S and D are shown in Fig. 1, where the plane through the point P and perpendicular to the trisector line (t) intersects the x axis at point A of coordinates $(x + y + z, 0, 0)$, the y axis at point B of coordinates $(0. x + y + z, 0)$, and the z axis at point C of coordinates $(0, 0, x + y + z)$. The expression of ϕ in terms of x, y, z can be obtained in a system of coordinates defined by the unit vectors $\xi_1 = \frac{1}{\sqrt{6}}(2, -1, -1)$, $\xi_2 = \frac{1}{\sqrt{2}}(0, -1, -1), \xi_3 = \frac{1}{\sqrt{3}}(1, 1, 1)$and having the point O as origin. The relation between the coordinates of P in the systems ξ_1, ξ_2, ξ_3 and x, y, z can be written in the form:

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$(\xi_1, \xi_2, \xi_3) = \left(\frac{1}{\sqrt{6}}(2x - y - z), \frac{1}{\sqrt{2}}(y - z), \frac{1}{\sqrt{3}}(x + y + z)\right)$. Also $\cos\phi = \frac{2x-y-z}{2\sqrt{(x^2+y^2+z^2-xy-yz-zx)}}$

$$\sin\phi = \frac{\sqrt{3}(y-z)}{2\sqrt{(x^2+y^2+z^2-xy-yz-zx)}}$$

The angle θ between the line OP and the trisector line (t) is given by $\tan\theta = \frac{D}{S}$

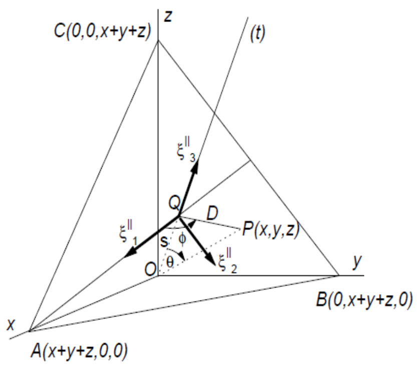

Figure 1: Tricomplex variables s, d, θ, ϕ for the tri-comlex number $x+hy+kz$, represented by the point P(x, y, z). The azimuthal angle ϕ is shown in the plane parallel to Π, passing through P, which intersects the trisector line (t) at Q and the axis of coordinates x, y, z at the points A, B, C. The orthogonal axis: $\xi_1^{||}$,$\xi_2^{||}$, $\xi_3^{||}$ have the origin at Q. The axis Q$\xi_1^{||}$ is parallel to the axis O$\xi_1^{||}$, the axis Q$\xi_2^{||}$ is parallel to the axis O$\xi_2^{||}$, and the axis Q$\xi_3^{||}$ is parallel to the axis O$\xi_3^{||}$, so that, in the plane ABC, the angle ϕ is measured from the line QA.

5 Hypercomplex similarity measure for SVNS

We here define a function for similarity measure between SVNSs. It requires satisfying some properties of complex number in three dimensions to satisfy the prerequisites of a similarity measure method. In this sense, we can call the function to be defined in three dimensional complex number system or hypercomplex similarity measurement function.

Definition I: Let $A = \{x, T_A(x), I_A(x), F_A(x)\}$ and $B = \{x, T_B(x), I_B(x), F_B(x)\}$ are two neutrosophic sets in $X = \{x\}$; then the similarity function between two neutrosophic sets A and B is defined as

$$S(A,B) = \frac{1}{2}\left[\frac{(1+D_{\theta_1}D_{\theta_2})^2}{1+D_{\theta_1}{}^2+D_{\theta_2}{}^2+D_{\theta_1}{}^2D_{\theta_2}{}^2} + \frac{(1+D_{\phi_1}D_{\phi_2})^2}{1+D_{\phi_1}{}^2+D_{\phi_2}{}^2+D_{\phi_1}{}^2D_{\phi_2}{}^2}\right],$$ where

$$D_{\theta_1} = \frac{\sqrt{(T_A(x)-I_A(x))^2+(I_A(x)-F_A(x))^2+(F_A(x)-T_A(x))^2}}{(T_A(X)+I_A(x)+F_A(x))}$$

$$D_{\theta_2} = \frac{\sqrt{(T_B(x)-I_B(x))^2+(I_B(x)-F_B(x))^2+(F_B(x)-T_B(x))^2}}{(T_B(X)+I_B(x)+F_B(x))}$$

$$D_{\phi_1} = \frac{\sqrt{3}(I_A(x)-F_A(x))}{2T_A(x)-I_A(x)-F_A(x)}$$

$$D_{\phi_2} = \frac{\sqrt{3}(I_B(x)-F_B(x))}{2T_B(x)-I_B(x)-F_B(x)}$$

Also ($T_A(x)$, $I_A(x)$, $F_A(x)$ ≠ (0, 0, 0) and ($T_B(x)$, $I_B(x)$, $F_B(x)$) ≠ (0, 0, 0) .

Lemma I: Function S (A, B) satisfies the properties of similarity measure.

Proof: Let us consider $S_1(A,B) = \frac{1}{2}\left[\frac{1}{1+\tan^2(\theta_1-\theta_2)} + \frac{1}{1+\tan^2(\varphi_1-\varphi_2)}\right] = \frac{1}{2}\left[\frac{(1+\tan\theta_1\tan\theta_2)^2}{1+\tan^2\theta_1+\tan^2\theta_2+\tan^2\theta_1\tan^2\theta_2} + \frac{(1+\tan\varphi_1\tan\varphi_2)^2}{1+\tan^2\varphi_1+\tan^2\varphi_2+\tan^2\varphi_1\tan^2\varphi_2}\right]$. From (1), (2) and (3),

we get the value of $\tan\theta_1, \tan\theta_2, \tan\varphi_1, \tan\varphi_2$. If we take $\tan\theta_1 = D_{\theta_1}$, $\tan\theta_2 = D_{\theta_2}$, $\tan\varphi_1 = D_{\varphi_1}$, $\tan\varphi_2 = D_{\varphi_2}$, then $S_1(A,B) = S(A,B)$

Clearly the function $S_1(A,B)$ satisfies the properties

$$(p_1)\ 0 \le S_1(A,B) \le 1$$

$$(P_2)\ S_1(A,B) = S_1(B,A)$$

(P_3) *When* $A = B$, $\theta_1 = \theta_2$ *and* $\phi_1 = \phi_2$, *i.e.*, $S_1(A,B) = 1$, *if* $A = B$.

6 Application of Hypercomplex similarity Measures in Decision-Making

In this section, we apply hypercomplex similarity measures between SVNSs to the multicriteria decision-making problem. Let $A = A_1, A_2, \dots, A_m$ be a set of alternatives and $C = C_1, C_2, \dots, C_n$ *be a set of criteria.* Assume that the weight of the criterion $C_j (j = 1,2,\dots,n)$ entered by the decision-maker is w_j, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. The m options according to the n criterion are given below:

$$\begin{array}{cccccc}
 & C_1 & C_2 & C_3 & \dots & C_n \\
A_1 & C_1{}^{(A_1)} & C_2{}^{(A_1)} & C_1{}^{(A_1)} & \dots & C_n{}^{(A_1)} \\
A_2 & C_1{}^{(A_2)} & C_2{}^{(A_2)} & C_3{}^{(A_2)} & \dots & C_n{}^{(A_3)} \\
A_3 & C_1{}^{(A_3)} & C_2{}^{(A_3)} & C_3{}^{(A_3)} & \dots & C_n{}^{(A_3)} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
A_m & C_1{}^{(A_n)} & C_2{}^{(A_3)} & C_3{}^{(A_3)} & \dots & C_n{}^{(A_m)}
\end{array}$$

Generally, the evaluation criteria can be categorized into two types: benefit criteria and cost criteria. Let K be a set of benefit criteria and M be a set of cost criteria. In the proposed decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit criteria and a minimum operator for the cost criteria to determine the best value of each criterion among all alternatives. Therefore, we define an ideal alternative

$$A^* = \{C_1{}^*, C_2{}^*, C_3{}^*, \ldots, C_n{}^*\}$$

Where for a benefit criterion

$C_j{}^* = \{max_i T_{C_j}{}^{(A_i)}, min_i I_{C_j}{}^{(A_i)}, min_i F_{C_j}{}^{(A_i)}\}$ while for a cost criterion,

$C_j{}^* = \{min_i T_{C_j}{}^{(A_i)}, max_i I_{C_j}{}^{(A_i)}, max_i F_{C_j}{}^{(A_i)}\}$

Definition II: We define hypercomplex weighted similarity measure as

$WS_K(A_i, A^*) = \sum_{j=1}^{n} W_j S\left(C_j{}^{(A_i)}, C_j{}^*\right), (i = 1, 2, 3, \ldots, m)$ Lemma II: $WS_K(A_i, A^*), (i = 1, 2, 3, \ldots, m)$ satisfies properties $P_1, P_2, P_3.$

Proof: Clearly $\sum_{j=1}^{n} w_j S\left(C_j{}^{(A_i)}, C_j{}^*\right) \geq 0$ and since from the property of hypercomplex similarity measure $S\left(C_j{}^{(A_i)}, C_j{}^*\right) \leq 1$, $\sum_{j=1}^{n} w_j S\left(C_j{}^{(A_i)}, C_j{}^*\right) \leq \sum_{j=1}^{n} w_j = 1$, so $0 \leq WS_K(A_i, A^*) \leq 1$. Thus P_1 is satisfied.

Since $S\left(C_j{}^{(A_i)}, C_j{}^*\right) = S\left(C_j{}^*, C_j{}^{(A_i)}\right), WS_K(A_i, A^*) = WS_K(A^*, A_i)$. Thus P_2 is satisfied.

When $C_j{}^{(A_i)} = C_J{}^*$, Using the property of hypercomplex similarity measure

$S\left(C_j{}^{(A_i)}, C_j{}^*\right) = 1$, So $\sum_{j=1}^{n} w_j S\left(C_j{}^{(A_i)}, C_j{}^*\right) = \sum_{j=1}^{n} w_j = 1$ if $C_j{}^{(A_i)} = C_j{}^*$.

So P_3 is also satisfied.

Through the similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected.

7 Numerical Example

In a certain network, there are four options to go from one node to the other. Which path to be followed will be impacted by two benefit criteria C_1, C_2 and one cost criteria C_3 and the weight vectors are 0.35, 0.25 and 0.40 respectively. A decision maker evaluates the four options according to the three criteria mentioned above. We use the newly introduced approach to obtain the most desirable alternative from the decision matrix given in table 1.

C_1, C_2 are benefit criteria, C_3 is cost criteria. From table 1 we can obtain the following ideal alternative:

$$A^* = \{(0.7, 0, 0.1), (0.6, 0.1, 0.2), (0.5, 0.3, 0.8)\}$$

	A_1	A_2	A_3	A_4
C_1	$(0.4, 0.2, 0.3)$	$(0.6, 0.1, 0.2)$	$(0.3, 0.2, 0.3)$	$(0.7, 0, 0.1)$
C_2	$(0.4, 0.2, 0.3)$	$(0.6, 0.1, 0.2)$	$(0.5, 0.2, 0.3)$	$(0.6, 0.1, 0.2)$
C_3	$(0.8, 0.2, 0.5)$	$(0.5, 0.2, 0.8)$	$(0.5, 0.3, 0.8)$	$(0.6, 0.3, 0.8)$

Table 1: Decision matrix (information given by DM)

Measure method	*measure value*	*Ranking order*
Weighted Jaccarad similarity measure	$WJ(A_1, A^*) = 0.7642$ $WJ(A_2, A^*) = 0.9735$ $WJ(A_3, A^*) = 0.8067$ $WJ(A_1, A^*) = 0.9962$	$A_4 > A_2 > A_3 > A_1$
Weighted Jaccarad similarity measure	$WD(A_1, A^*) = 0.8635$ $WD(A_2, A^*) = 0.9864$ $WD(A_3, A^*) = 0.8738$ $WD(A_4, A^*) = 0.9981$	$A_4 > A_2 > A_3 > A_1$
Weighted cosine similarity measure	$WD(A_1, A^*) = 0.8773$ $WD(A_2, A^*) = 0.9882$ $WD(A_3, A^*) = 0.8939$ $WD(A_4, A^*) = 0.9986$	$A_4 > A_2 > A_3 > A_1$
Weighted hypercomplex similarity measure	$W_kS(A_1, A^*) = 0.7211$ $W_kS(A_2, A^*) = 0.9857$ $W_kS(A_3, A^*) = 0.8090$ $W_kS(A_4, A^*) = 0.9895$	$A_4 > A_2 > A_3 > A_1$

7.1 Generalization of hypercomplex similarity measure

In this section 7, we formulate a general function for similarity measure using hypercomplex number system. This can give similarity measure for any dimension. Before formulating it, we should have a fare knowledge of hypercomplex number in n-dimensions [8] for which the multiplication is associative and commutative, and also the concepts of analytic n-complex function, contour integration and residue is defined. The n-complex number $x_0 + h_1x_1 + h_2x_2 + \cdots + h_{n-1}x_{n-1}$ can be represented by the point A of coordinates $(x_0, x_1, \ldots, x_{n-1})$ where$h_1, h_2, \ldots, h_{n-1}$ are the hypercomplex bases for which the multiplication rules are$h_jh_k = h_{j+k}$ if $0 \leq j + k \leq n - 1, and\ h_jh_k = h_{j+k-n}$ if $n \leq j + k \leq 2n - 2$, where $h_0 = 1$. If O is the origin of the n dimensional space, the distance from the origin O to the point A of coordinates $(x_0, x_1, \ldots, x_{n-1})$ has the expression $d^2 = x_0{}^2 + x_1{}^2 + x_2{}^2 + \cdots + x_{n-1}{}^2$. The quantity d will be called modulus of the n-complex number $u = x_0 + h_1x_1 + h_2x_2 + \cdots + h_{n-1}x_{n-1}$. The modulus of an n-complex number u will be designated by $d = |u|$. For even number of dimensions $(n \geq 4)$ hypercomplex number is characterized by two polar axis, one polar axis is the normal through the origin O to the hyperplane $v_+ = 0$ where $v_+ = x_0 + x_1 + \cdots + x_{n-1}$ and the second polar axis is the normal through the origin O to the hyperplane $v_- = 0$ where $v_- = x_0 - x_1 + \cdots + x_{n-2} - x_{n-1}$. Whereas for an odd number of dimensions, n-complex number is of one polar axis, normal through the origin O to the hyperplane $v_+ = 0$.

Thus, in addition to the distance d, the position of the point A can be specified, in an even number of dimensions, by two polar angles θ_+, θ_-, by n/2-2 planar angles ψ_k, and by $\frac{n}{2} - 1$ azimuthal angles ϕ_k. In an odd number of dimensions, the position of the point A is specified by d, by one polar angle θ_+, by planar angles ψ_k, and by $\frac{n-1}{2}$ azimuthal angles ϕ_k. The exponential and trigonometric forms of the n-complex number u can be obtained conveniently in a rotated system of axes defined by a transformation

Which, for even n,

$$\begin{bmatrix} \xi_+ \\ \xi_- \\ \vdots \\ \xi_k \\ \eta_k \\ \vdots \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & \cdots & \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} \\ \frac{1}{\sqrt{n}} & -\frac{1}{\sqrt{n}} & \cdots & \frac{1}{\sqrt{n}} & -\frac{1}{\sqrt{n}} \\ \vdots & \vdots & & \vdots & \vdots \\ \sqrt{\frac{2}{n}} & \sqrt{\frac{2}{n}}\cos\frac{2\pi\kappa}{n} & \cdots & \sqrt{\frac{2}{n}}\cos\frac{2\pi(n-2)k}{n} & \sqrt{\frac{2}{n}}\cos\frac{2\pi(n-1)k}{n} \\ 0 & \sqrt{\frac{2}{n}}\sin\frac{2\pi k}{n} & \cdots & \sqrt{\frac{2}{n}}\sin\frac{2\pi(n-2)k}{n} & \sqrt{\frac{2}{n}}\sin\frac{2\pi(n-1)k}{n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ \vdots \\ \vdots \\ x_{n-1} \end{bmatrix}$$

Here $k = 1,2, \dots, \frac{n}{2} - 1$.

And for odd n,

$$\begin{bmatrix} \xi_+ \\ \xi_- \\ \vdots \\ \xi_k \\ \eta_k \\ \vdots \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & \cdots & \frac{1}{\sqrt{n}} \\ \frac{2}{\sqrt{n}} & \sqrt{\frac{2}{n}}\cos\frac{2\pi}{n} & \cdots & \sqrt{\frac{2}{n}}\cos\frac{2\pi(n-1)}{n} \\ 0 & \sqrt{\frac{2}{n}}\sin\frac{2\pi}{n} & \cdots & \sqrt{\frac{2}{n}}\sin\frac{2\pi(n-1)}{n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{2}{\sqrt{n}} & \sqrt{\frac{2}{n}}\cos\frac{2\pi k}{n} & \cdots & \sqrt{\frac{2}{n}}\cos\frac{2\pi(n-1)k}{n} \\ 0 & \sqrt{\frac{2}{n}}\sin\frac{2\pi k}{n} & \cdots & \sqrt{\frac{2}{n}}\sin\frac{2\pi(n-1)k}{n} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ \vdots \\ \vdots \\ x_{n-1} \end{bmatrix}$$

Here $k = 1,2, \dots, \frac{n-1}{2}$

Definition III: Let $A = (x_0, x_1, x_2, \dots, x_{n-1})$ and $B = (y_0, y_1, y_2, \dots, y_{n-1})$ be two n dimensional complex numbers. Then similarity measure between A and B is defined as, when n is odd

$$S(A,B) = \frac{1}{n-1}\left[\frac{1}{1+\tan^2\left(\theta_+{}^{(A)} - \theta_+{}^{(B)}\right)} + \sum_{k=1}^{\frac{n-1}{2}} \frac{1}{1+\tan^2\left(\phi_k{}^{(A)} - \phi_k{}^{(B)}\right)} + \sum_{k=2}^{\frac{n-1}{2}} \frac{1}{1+\tan^2\left(\psi_{k-1}{}^{(A)} - \psi_{k-1}{}^{(B)}\right)}\right]$$

And when n is even,

$$S(A,B) = \frac{1}{n-1}\left[\frac{1}{1+\tan^2\left(\theta_+{}^{(A)} - \theta_+{}^{(B)}\right)} + \frac{1}{1+\tan^2\left(\theta_-{}^{(A)} - \theta_-{}^{(B)}\right)} + \sum_{k=1}^{\frac{n}{2}-1} \frac{1}{1+\tan^2\left(\phi_k{}^{(A)} - \phi_k{}^{(B)}\right)} + \sum_{k=2}^{\frac{n}{2}-1} \frac{1}{1+\tan^2\left(\psi_{k-1}{}^{(A)} - \psi_{k-1}{}^{(B)}\right)}\right]$$

Here $\tan\theta_+ = \frac{\sqrt{2}\rho_1}{v_+}, \tan\theta_- = \frac{\sqrt{2}\rho_1}{v_-}, \cos\phi_k = \frac{v_k}{\rho_k}, \sin\phi_k = \frac{\breve{v}_k}{\rho_k}$, ${\rho_k}^2 = {v_k}^2 + {\breve{v}_k}^2$, $v_+ = \sqrt{n}\xi_+$, $v_- = \sqrt{n}\xi_-$, $v_k = \sqrt{\frac{n}{2}}\,\xi_k$, $\breve{v}_k = \sqrt{\frac{n}{2}}\,\eta_k$, $\tan\psi_{k-1} = \frac{\rho_1}{\rho_k}$, and also $0 \le \theta_+ \le \pi$, $0 \le \theta_- \le \pi$, $0 \le \varphi_k \le 2\pi$ and $0 \le \xi_k \le \frac{\pi}{2}$.

It is very clear that $S(\mathrm{A}, \mathrm{B})$ satisfies the three properties of similarity measure.

8 Conclusion

In this paper we first introduced a new method of similarity measure between single valued neutrosohpic sets using hypercomplex number. We set up an example of decision making problem which requires finalizing an optimal path based on some certain criteria. We compared the result of our introduced similarity measure with those of other methods. We can conclude that we can efficiently apply the introduced similarity measure approach in decision making problems and any other similarity measure problems. Later, we proposed a general function for similarity measure.

The proposed similarity measure is based on the concept of hypercomplex number. We can relate the similarity measure with hypercomplex number system. Thus, it opens a new domain of research in finding the solutions of decision making problems related to the network problems by the use of similarity measures based on hypercomplex number system.

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