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Power control algorithms in wireless communications

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1 Abstract
This paper presents a comprehensive review of the published algorithms on power control for cellular systems. The majority of the research is focused on Code Division Multiple Access (CDMA) systems, although a small fraction of the reviewed literature pertains to Frequency Division Multiple Access (FDMA) and Time Division Multiple Access (TDMA).

2 Introduction
Power control in cellular systems is applied to numerous communication architectures. For Frequency Division Multiple Access (FDMA) and Time Division Multiple Access (TDMA) power control reduces intercell interference and improves capacity.

For Code Division Multiple Access (CDMA) power control is a required part of the base station and mobile station designs. Power Control reduces interference due to multiple users in the same frequency band. A general term that characterizes this type of multiple access interference is the “near-far effect”. Due to propagation characteristics the signals from mobiles closer to the base station could overpower the signals from mobiles located farther away. With power control each mobile adjusts its own transmit power to ensure an adequate quality of service (QoS) or signal-to-interference ratio (SIR) at the base station. The mobile’s transmit power is either increased or reduced depending on instructions from the base station (uplink power control). By reducing a user’s transmit power, that particular component in the multiuser interference is also reduced. While power control reduces the average required transmit power of

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each mobile, it also improves system capacity. The former improves battery life and the latter allows for more system users.

3 Optimum/Global Power Control

Centralized power control is often referred to as optimum power control due to the upper performance bounds achieved by the algorithms. In such approaches, a central station controls all the links in a cellular system. The goal of centralized power control research is to develop upper bounds for power control algorithms. The results are used for comparison with the distributed power control algorithms implemented in cellular systems. The classic paper on the topic of centralized power control was written by Jens Zander in 1992 [1]. The paper investigates optimum transmitter power control for general cellular systems. The algorithms are for the downlink, base station to mobile station, but similar results were later developed for the uplink, mobile to base, in [2].

In [1], the goal of transmitter power control is the ability to control co-channel interference and the associated quality is measured based on the carrier to interference ratio (CIR). The author also proposes an “optimal” power control scheme that minimizes the interference and thus the outage probability, the probability that a CIR is below the set threshold. In addition to the optimum power control algorithm a Step Removal Algorithm (SRA) is proposed. SRA, when implemented, removes cells stepwise until all remaining mobiles can satisfy the required CIR. The derivations in the paper contain the link gain matrix for the mobile in cell $i$ to the base station in cell $j$, $G_{ij}$.

Specifically, let

$$CIR_i = \Gamma_i = \frac{P_i}{\sum_{j=1}^{Q} P_j \cdot Z_{ij}}, Z_{ij} = \frac{G_{ij}}{G_{ii}}$$

where $P_i$ denotes the power for user $i$, $Q$ is the number of co-channel sets, and $\Gamma_i$ and $Z_{ij}$ are stochastic variables.

The goal of transmitter power control is to choose all $P_i$’s such that all $\Gamma_i$’s are above the desired CIR threshold. In centralized power control the $P_i$’s required to achieve the $\Gamma_i$’s are determined via a central controller with the link gain matrix.

In [1] the term “global” power control algorithm (PCA) is used to describe the centralized PCA. Zander’s global PCA, $\Psi_G$, has access to the entire link gain matrix $Z = \{Z_{ij}\}$ and can control the entire power vector $P$, where $P = \Psi_G(Z)$. Optimum power control is achieved when a power vector is chosen such that $P \geq 0$ and $\Gamma_i \geq \gamma, \forall i$, for the achievable CIR. Zander then shows for given a stochastic link gain matrix $Z$ the maximum achievable CIR, defined as $\gamma^* = \max \{\gamma | \exists P \geq 0 : \Gamma_i \geq \gamma, \forall i\}$, is given by $\gamma^* = \frac{1}{\lambda^* - 1}$, $\lambda^*$ is the largest real eigenvalue of $Z$. The optimum power vector, $P^*$, is the eigenvector corresponding to $\gamma^*$. Since $Z$ consists of stochastic random variables $Z$ has full rank and must have one real positive eigenvalue $\lambda^*$ for which the corresponding
eigenvector is positive. With this result Zander proves that the global PCA is optimum by starting with the known link gain matrix \( Z \), and calculating the largest positive eigenvalue \( \lambda' \) and corresponding positive eigenvector \( P' \). If \( \lambda' \) and \( P' \) do not support the desired CIR, \( \gamma^* \), then a cell or subset of cells are removed, and the process is repeated until the desired CIR is achieved. The cell removal procedure helps achieve higher system capacity and the optimum power control.

A paper co-authored by Grandhi et.al.\[3\] presents a centralized power control approach that is based on signal strength measurements. This centralized power control algorithm converges to the same optimal solution presented in \[1\]. The centralized controller updates each mobile’s transmitter power so the received CIR is common to all links. This paper lays the foundation for a distributed algorithm that converges to the optimal, global power control solution.

Qiang Wu authored two papers on optimum power control \[4\],\[5\]. The first paper presents the optimum power control scheme (OPCS) designed for CDMA systems and develops the upper bounds for all transmitter power control schemes. Wu presents simulation results that show system capacity increases by 1.9 dB over an IS-95 system with perfect average power control. In \[5\] Wu builds upon the algorithms in \[4\] to develop an optimum power control algorithm for cellular systems with heterogeneous signal to interference ratio (SIR) thresholds; different SIR thresholds for different links. This is a plausible situation when dealing with both voice and data links that have varying data rates. In practical cellular systems the minimum required SIR thresholds are functions of the fading characteristics, data rates, and required Quality of Service (QoS). Systems with heterogeneous SIR thresholds allow the base stations (on the uplink) to assign each channel a unique SIR threshold. This allows each link to have the ideal minimum threshold which will improve QoS, reduce the power requirements on each mobile, and reduce the multiple access interference.

## 4 Distributed Power Control

The topic of distributed power control has been an active area of research since the early 1990’s. Many research topics relating to DPC have also become popular and very important to cellular communications. A few examples are stochastic power control, SIR estimation, time delay compensation, power control and capacity.

The architecture of distributed power control (DPC) is based on the idea that each base station tracks and updates the transmitted powers from the local mobile stations. Therefore power control is “distributed” to all base stations and a centralized controller is not required. The benefits of DPC are reduced computational complexity and the link gain matrix statistics are not required. The only information required for deterministic, distributed power control is knowledge of local SIRs and local link gains.

Zander authored one of the two pioneering papers on DPC. In \[6\] results are presented of imperfect CIR estimates due to multipath, fading, mobility,
and system noise. This distributed approach is based on a proportional control algorithm which has stability problems when the CIR target is high. Zander’s DPC incorporates distributed CIR balancing to overcome the stability issues and improve the system capacity. The system model presented in [6] is based upon the model and assumptions in [1]. The CIR at mobile \(i\), for the downlink from base \(j\), is given by (1) As in the previous papers [1],[2] the performance measure is the interference function \(F(\gamma) = \frac{1}{Q} \sum_{j=1}^{Q} \Pr \{ \Gamma_j \leq \gamma_o \}\) where \(\gamma_o\) is the minimum required CIR. The largest achievable CIR ratio is related to the link gain matrix \(Z\) as shown in Section 3. If \(P^*\) is computationally feasible then it is used to obtain the same CIR ratio in all mobiles. Thus the system is defined as “balanced”.

One main assumption in [6] is that the link gain matrix \(Z\) is a constant. Zander points out that \(Z\) is a stochastic process and the dimensions of \(Z\) change based on the time varying traffic conditions. But for initial analysis it is assumed that \(Z\) changes slowly compared to the algorithm’s update rate.

The distributed discrete-time power control algorithm (DPCA) is defined as \(P_i^{(v+1)} = \Psi_D \left( P_i^{(v)}, \Gamma_i^{(v)} \right)\). The formulation evolves into the distributed balancing (DB) algorithm. The algorithm assumes that the initial power vector \(P_o\) is positive, i.e. all components are \(>0\). From equation 1 the DB algorithm is defined as

\[
P_i^{(v+1)} = \beta P_i^{(v)} \left( 1 + \frac{1}{\Gamma_i^{(v)}} \right), \beta > 0.
\]

\[
\lim_{v \to \infty} P_i^{(v)} = P^*, \lim_{v \to \infty} \Gamma_i^{(v)} = \gamma^*.
\]

The second pioneering paper [7] on distributed power control was authored by Yates. The concept of “interference functions”, associated properties, and proofs of synchronous and asynchronous convergence are presented. The proposed power control scheme is based on iterative algorithms that can be applied to “dynamic” systems with time varying channel characteristics.

The iterative power control algorithm is based on the SIR requirements described by \(p \geq I(p)\) where \(p\) is a transmitter power vector, \(p = (p_1, ..., p_n)\), and \(p_i\) is the transmitter power of user \(i\). \(I(p)\) is the interference function, \(I_i(p) = (I_1(p), ..., I_n(p))\), \(I_i(p)\) is the interference that mobile \(i\) must overcome. This formulation leads to the iterative power control algorithm

\[
p(t+1) = I(p(t))
\]

Yates defines the interference function \(I(p)\) as standard if \(p \geq 0\) and the following three properties are satisfied.

1. Positivity \(\rightarrow I(p) > 0\)
2. Monotonicity \(\rightarrow \) If \(p \geq p'\), then \(I(p) \geq I(p')\)
3. Scalability $\rightarrow$ For all $\alpha > 1$, $\alpha I(p) > I(\alpha p)$

In [7] five systems are described with the iterative constraints placed on each. The first three systems, fixed assignment, minimum assignment, and macro diversity, have been previously studied and Yates applies the approach of interference functions to each. The last two system, Limited Diversity and Multiple Connection Reception, are presented by Yates and have pros and cons as compare to the first three systems. The goal of [7] is to investigate the systems in terms of the interference functions and provide a “framework” for understanding the convergence issues associated with each. With regards to the five systems the SIR of user $i$ at base station $j$ is defined as $p_{iuj}(p)$ where

$$p_{iuj}(p) = \frac{p_i h_{ij}}{\sum_{i \neq j} h_{ij} p_i + \sigma_j}$$

$\sigma_j$ is the receiver noise power.

For the Fixed Assignment (FA) system the SIR requirement for user $i$ is $p_{iuj}(p) \geq \gamma^*$. This is written as

$$p_i \geq I_{FA}^i(p) = \frac{\gamma^*}{u_{ij}(p)}$$

In this system the desired SIR $\gamma^*$ is common to all users, $\gamma^* = \gamma^*$. The goal of the FA system is to maximize $\gamma$ with respect to $p \geq I_{FA}^i(p)$.

The Minimum Power Assignment (MPA) system is a soft handoff system where user $i$ is assigned to the base station where it’s SIR is maximized. The SIR of user $i$ is maximized with respect to base station $j p_{iuj}(p) \geq \gamma^*$,

$$p_i \geq I_{MPA}^i(p) = \min_j \frac{\gamma^*}{u_{ij}(p)}$$

In this system it is assumed that the powers of all other users are fixed, user $i$ is then assigned to the base station where the minimum power is required to achieve the desired SIR, $\gamma^*_i$.

Macro Diversity (MD) is a diversity receiving method where the signal from user $i$ is received at each base station and then maximal ratio combining is applied. The MD algorithms is

$$p_i \geq I_{MD}^i(p) = \frac{\gamma^*}{\sum_k u_{ij}(p)}$$

Macro Diversity, Limited Diversity, and Multiple Connection Reception are variations where a number of base stations are used for diversity combining.

Following the developments of a standard interference function and the five power control systems are proofs of convergence for both synchronous and asynchronous power control algorithms. The proofs show that standard power control algorithms converge to a fixed point, with the assumption that $I(p)$ is feasible. The advantage of asynchronous power control is the each base/mobile
pair can update its own power independent of others. Therefore some pairs could update their power faster and perform more iterations.

Foschini and Miljanic co-authored a paper [8] in 1993 in which a distributed autonomous power control algorithm is presented and the convergence is proven. The authors develop a continuous time differential equation approach and a discrete time difference equation approach. Their work demonstrates that the power control algorithms are universal in the sense that the link gain matrix $Z$ does not affect the convergence to the optimum power vector $P^*$.  

4.1 Power Control for Cellular Systems

For effective power control there are many “requirements”. Distributed and asynchronous power updates are dictated by the general power control problem. The algorithms must be stable and converge to the “near” optimum solution. One key to power control is estimating and/or measuring the BER and SIR, which characterizes the channel quality. The BER/SIR measurement rate is determined by the rate at which the power control updates are made. Fast power control is used to suppress Rayleigh fading and slow power control is used for shadow fading. Errors can cause an increase in interference and an increase in outage probability. The estimation errors are due, in part, to channel and interference power information that is no longer current. The time delay between the measurements and power updates can cause instability, power oscillations, and estimation errors. Estimation errors and stochastic link gains are commonly disregarded in snapshot analysis, but when stochastic processes and estimation errors are the topics of research then the standard analysis techniques fail. The future of power control may lie in the area of estimation approaches and power update methods for stochastic environments.


The first half of [9] is devoted to cellular system architectures and general areas of power control. The second half of the paper is focused on distributed power control. Overall this article provides a good “framework” for the power control problem. The authors present six factors that cause interference in cellular systems. These factors are: 1) Signaling/modulation scheme associated with the chosen multiple access method, 2) Link orientation, 3) Environment morphology and topology, 4) Speed of the mobile terminals, 5) Cell hierarchy, and 6) Connection type. The authors in [9] view 1) as the most dominant cause of interference. Today the research on power control problem is usually centered on CDMA, therefore 1) is no longer the primary focus of power control research. The majority of research papers are topics associated with 3), 4), 5) and the dominant link analyzed is the uplink, (2).

The rate of convergence of distributed power control algorithms is an important topic of research. As new algorithms with faster convergence rates are
proposed, the ability to suppress fast Rayleigh fading increases. Since convergence rates and power update rates are related it is important to develop algorithms that are computationally simple which allows for fast convergence. In [10] Huang and Yates focus on the convergence rate of the Constrained Minimum Power Assignment (CMPA) algorithm. This algorithm implements iterative power control and assignments of base stations. The CMPA is based on CIR measurements and as the name implies has a constrained power level. Jantti and Kim propose a second-order power control algorithm in [11] and prove that the rate of convergence exceeds that shown in [10]. The second-order algorithm uses power levels from current and previous iterations for calculating the power update commands. The benefit from the second-order approach is a gain in the rate of convergence, specifically the authors show that the convergence is asymptotically faster than the convergence of the Distributed Constrained Power Control (DCPC) algorithm in [10].

Two second-order algorithms are developed, the unconstrained second-order power control (USOPC) algorithm and the constrained second-order power control (CSOPC) algorithm. Both algorithms are based on the successive over-relaxation iterative method (SOR) and evolved from the distributed power control (also known as distributed balancing algorithm [12]) and constrained distributed power control.

The USOPC algorithm is

\[
P_{i}^{(n+1)} = w \frac{\gamma_{i}^{(n)}}{\gamma_{i}^{(n)}} P_{i}^{(n)} + (1 - w) P_{i}^{(n-1)}, \quad n = 1, 2, ...
\]

If \( w = 1 \) then the USOPC becomes the DPC algorithm from [12]. For purposes of implementation \( w \) is a non-increasing sequence of control parameters. \( w(1) = w(2) < w(3) = w(4) < ... < w(2n+1) \) where \( P_{i}^{(0)} \) and \( P_{i}^{(1)} \) are arbitrary, \( \gamma_{i}^{(n)} \) is the target CIR value, and \( \gamma_{i}^{(n)} \) is the received CIR from mobile \( i \). The problems associated with the USOPC algorithm, which are: 1) power vectors from a feasible system may be out of the specified range of the transmitter, 2) the algorithm could compute \( P_{i}^{(n)} < 0 \). The CSOPC algorithm solves the problems with the USOPC by constraining the maximum value of \( P_{i}^{(n)} \). The CSOPC algorithm is

\[
P_{i}^{(n+1)} = \min \left\{ P_{i}, \max \left\{ 0, w^{(n)} \frac{\gamma_{i}^{(n)}}{\gamma_{i}^{(n)}} P_{i}^{(n)} + \left( 1 - w^{(n)} \right) P_{i}^{(n-1)} \right\} \right\}.
\]

For implementation the CSOPC algorithm is modified to the form

\[
P_{i}^{(n+1)} = \min \left\{ P_{i}, \max \left\{ 0, w^{(n)} \Delta_{i}^{(n)} P_{i}^{(n)} + \left( 1 - w^{(n)} \right) P_{i}^{(n-1)} \right\} \right\}
\]

where \( \Delta_{i}^{(n)} \) is the step size for each iteration and

\[
\Delta_{i}^{(n)} = \begin{cases} \Delta, & \gamma_{i}^{(n)} \leq \gamma_{i}^{t} \\ \frac{\Delta}{2}, & \gamma_{i}^{(n)} > \gamma_{i}^{t} \end{cases}
\]
If \( w = 1 \) then this algorithm is equivalent to the “bang-bang” power control (B-BPC) used in the IS-95 system.

Numerical simulations show that the USOPC algorithm converges asymptotically faster than the DPC algorithm if the optimal \( w \) is chosen asymptotically. The convergence speed increases as the algorithm approaches the optimal power vector \( P^* \). Simulations show that the CSOPC algorithm converges faster in terms of outage probability than the B-BPC algorithm. Also the modified CSOPC algorithm in (2) outperforms the B-BPC in terms of convergence rate.

Another paper focusing on distributed power control and system performance is authored by El-Osery and Abdallah [13]. In this paper a new state-space approach to the power control problem is presented. The Linear Quadratic Power Control (LQPC) algorithm is faster in computing transmitter power and has a higher capacity than the CSOPC algorithm in [11].

The state space representation of linear quadratic control can be developed by assigning each mobile to base station connection as a subsystem.

\[
s_i(n + 1) = \frac{p_i(n) + u_i(n)}{I_i(n)} = s_i(n) + v_i(n)
\]

\( I_i(n) \) is the received interference.

\[
I_i(n) = \sum_{i \neq j}^Q p_i \frac{G_{ij}}{G_{ii}} + \frac{\eta_i}{G_{ii}}
\]

The input to each subsystem is \( u_i(n) \) and

\[
v_i(n) = \frac{u_i(n)}{I_i(n)}
\]

\[ s_i(n) = \frac{p_i(n)}{I_i(n)} \]

The objective of the power control algorithm is for \( s_i(n) \) to track the desired SIR \( \gamma^* \). The “integrator of the error” is added to the system to eliminate steady state errors and include different \( \gamma^* \) at each mobile station. This state is

\[
\xi_i(n + 1) = \xi_i(n) + e_i(n), e_i(n) = s_i(n) - \gamma^*
\]

If \( x_i(n) \) is defined as

\[
x_i(n) = \begin{pmatrix} \xi_i(n) \\ e_i(n) \end{pmatrix}
\]

then the second-order linear state space system is

\[
x_i(n + 1) = \begin{pmatrix} \xi_i(n + 1) \\ s_i(n + 1) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x_i(n) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} v_i(n)
\]

and the feedback controller is

\[
v_i(n) = -(k_\xi \quad k_\gamma) x_i(n)
\]
The key to the LQPC algorithm is choosing the feedback gains, $k_\xi$ and $k_s$, so the system is asymptotically stable. After the feedback gains are found then the power update command is computed by

$$p_i(n + 1) = \min \{p_i, s_i(n + 1) I_i(n)\}$$

Simulations show that initially the LQPC algorithm has a higher outage probability than the CSOPC algorithm in [11]. After a few iterations the LQPC outage probability drops below the CSOPC’s. This shows that the LQPC requires a few iterations before it generates the optimal power assignments. In addition it is shown that LQPC has higher capacity over CSOPC. The simulation environment includes $P_{i,\text{max}} = 1$ watt, $\gamma^* = 7dB$, $BW = 1.2288MHz$, and $n_i = 10^{-12}$With this system LQPC supports 19 mobiles with zero outages while CSOPC supports 17 mobiles. If the maximum constrained power is increased, $P_{i,\text{max}} = 5$ watts, then LQPC supports 26 mobiles with zero outages while CSOPC supports 21 mobiles. In both cases LQPC converges to zero outage probability in fewer iterations than CSOPC.

5 Conclusion and Outstanding Issues

Power control in cellular systems is employed for a number of reasons: 1) reduce the near-far effect, 2) reduce the cumulative interference power, 3) improve battery lifetime via reducing transmitter power, and 4) improve system capacity. The first three topics have been covered in the first two sections of this report. Centralized power control produces the optimum results, but is unrealistic in a real world system. Distributed power control uses locally available information to update the local mobile’s transmit power. There are many versions of distributed power control, each has its pros and cons.

Additional topics in cellular system research are devoted to specific areas of power control; multiuser receivers [19], system capacity [24][25], time delay compensation [14][15], SIR/CIR estimation [17][18], stochastic power control [20]. These topics are open areas of research and offer performance improvements in terms of quality of service (QoS), outage probability, and system capacity. Receiver design and power control for IS-95 and 3G systems is an important area of cellular system research. As QoS and capacity demands increase new approaches must developed to enhance system performance. This includes using statistical analysis to understand and adapt to time varying channel conditions.

Multiuser receivers and power control are two topics that have been extensively researched independently over the past decade. Both power control algorithms and multiuser receivers are designed to combat the “near-far effect” in CDMA cellular systems. Researchers at the Wireless Information Network Laboratory, Rutgers University, have published research that combines the two topics. When interference from adjacent cells is included power control with multiuser receivers offers exceptional performance in terms of additional capacity and lower interference levels, refer to [21] and [22].
Yun and Messerschmitt [26] addressed the subject of variable quality of service (QoS) for different traffic types. In this paper a statistical power control approach is developed for cellular systems that employ various traffic types (different data rates and QoS requirements) through substreams. The transmitted power is modulated to support the varying QoS requirements for the different substreams. Auto-interference is a concept not widely discussed in power control literature. Godlewski and Nuaymi authored the only paper [27] devoted solely to the subject. Auto-interference is a term used to describe the effect when the total transmit power of user \( i \) is not entirely used for signal decoding. The fraction of the power rendered unusable is attributed to multipath, transmission nonlinearity, imperfect equalization, and intersymbol interference. If the concept of auto-interference is applied to standard power control algorithms then the maximum achievable CIR is lower than the optimal case.

Overall there is a wide ranging catalogue of power control algorithms for deterministic signals and systems. The future in this area of research lies in statistical analysis of stochastic systems. A good foundation is developing and many new areas are open for research.

References


