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FEASIBILITY ANALYSIS OF CHANNEL EQUALIZERS USING KHARITONOV-TYPE RESULTS

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1 ABSTRACT

Necessary and sufficient conditions for determining the robust location of the roots of a family of polynomials are presented. This family appears when multipath fading channels are considered due to the uncertainty in the channel impulse response. The uncertainty is reflected by means of confidence intervals for the channel parameters. A feasibility analysis of digital equalisers is performed in the frequency domain to answer the question of whether the poles are to be found in a pre-specified region of the complex plane. The resulting tests, which can be carried out in a graphical manner, are both analysis and design tools, that can help to decide the suitable length of a constrained equaliser.

2 INTRODUCTION

In many digital signal processing applications, one has to tackle the problem of having an uncertain response for which some common property needs to be determined. Usually, the margin of the uncertainty can be estimated, defining a bounded region in the parameter-space where the parameters are known to lie. In many cases, these bounds are based on some confidence intervals obtained for the parameters. Thus, the parameters will lie within these bounds with a certain probability. Of particular interest is the study of the so-called $\mathcal{D}$-stability, i.e., the determination of whether the zeros of the uncertain system lie inside a specific region of the complex plane.

In recent years, there has been a great deal of work concerning the robust-stability of uncertain systems. This activity was motivated by Kharitonov's work on continuous time systems [1]. Kharitonov's celebrated result states that the stability (all the zeros in the left half plane) of a family of uncertain polynomials contained in a hyperrectangle in the parameter space can be deduced from the stability of only four corner polynomials. This number does not depend on the order of the system; therefore, an infinite member family of polynomials is reduced to the test of four of them. Kharitonov's results remained unknown in the west until about 1983. Since then, his results have been extended in different ways. Although a discrete-time version of the main result does not hold, some special cases have been found for which a Kharitonov-like result holds. These results find applications in the analysis and design of robust controllers.

Of specific relevance to this paper is the frequency domain approach to the robust stability problem. This idea allows easy proofs of most of the known results that rely on simple geometry on the complex plane. In fact, the uncertainty set is evaluated along the boundary of the stability region obtaining the so-called evaluation set. This is then combined with the "zero-exclusion principle" to produce a stability test. However, most of the work on the stability analysis for uncertain polynomials has been developed only for real uncertain polynomials. While this approach is valid for many practical systems, it is sometimes appropriate to consider complex uncertainties. With this motivation, Chapellat et al. [2] have considered the problem of analysing the stability of (complex) disk polynomials. This case attempts to model the situation in which the coefficient perturbations are in a complex neighborhood of their nominal values (center polynomial).

Little work has been done to translate these analysis methods to the Signal Processing area. Amongst the steps in this direction we mention [3] where the robust stationarity of sparse predictors is studied and [4] where stability monitoring of recursive adaptive filters is accomplished by means of Kharitonov's polynomials.

In the present paper, we address the problem of studying the feasibility of digital communications equalizers for fading channels. This is done by analysing the zero locations of the possible discrete-time equivalent channels. Of course, it is necessary to obtain confidence bounds for the channel taps so as to characterise the uncertainty. In the more general case, the resulting uncertainty bounds happen to be complex rings, possibly not centered at the origin. The conditions obtained for robust stability determination are extremely easy and can be transformed into

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a frequency-domain graphical test.

3 UNCERTAINTY STRUCTURATION

For our analysis, we will consider a channel model that takes into account the effects of multipath propagation [5]. If the received signal consists of a continuum of multipath components (as it is the case for the tropospheric scatter channel), the channel impulse response can be defined by the attenuation that the signal experiments for every associated delay. If, instead, the channel consists of a finite number of paths, the results that follow are merely conservative. Since the distortion that the signal suffers is non-symmetrical about the carrier frequency, the low-pass equivalent channel is generally complex-valued.

Since the transmitted signal can be considered band-limited, the channel response can be sampled at twice the Nyquist frequency to yield a discrete equivalent. Furthermore, the channel response can be truncated such that M taps are taken as significant. We will assume that M is an even integer.

Central limit theorem arguments lead to the conclusion that the received demodulated signal consists of Gaussian quadrature components. Conversely, the channel response $c(t)$ can be viewed as having a Rayleigh-distributed envelope and a uniformly distributed phase over the interval $[0, 2\pi)$. Nonetheless, when strong scatterers are present, it is better to characterise the envelope p.d.f. as having a Rice distribution.

We will also assume that the channel discrete taps c_n parameters, considered uncorrelated, can be estimated accurately, so it is possible to obtain the corresponding confidence intervals. Different possibilities can be suggested for the determination of these confidence bounds. For instance, when the two quadrature components have zero mean and equal variance, the interval on the modulus of c_n can leave either a two-sided region or one tail to the right; the former gives a ring on the complex plane of each coefficient and the latter produces a complex ring. In both cases, the center is in general not at origin. When the mean of the coefficients in non-zero, the phase is no longer uniformly distributed. In this case, the consideration of constant p.d.f. contours leads to a disk-shaped confidence interval centered at the mean point.

Once the channel model has been outlined, we will set a common frame for the channel-zeros analysis. It can be readily seen that in the more general case, the complex taps c_n have annular confidence intervals in the complex plane, with centers d_n being not zero. If the inner radius of the ring is zero, then we have a disk for that coefficient. Since the channels are assumed to have M taps, every particular channel can be represented by a complex M-vector, $\mathbf{c} = [c_0, c_1, \cdots, c_{M-1}]$ where

$$c_i = d_i + a_i e^{j\phi_i} \qquad (1)$$

We need also to define the confidence set of channels $\mathcal{C}$ as

$$\mathcal{C} = \{\mathbf{c} : a_i^- \leq a_i \leq a_i^+, \phi_i \in [0, 2\pi), i = 0, \cdots, M-1\} \qquad (2)$$

where a_i^- and a_i^+ are the extremes of the obtained confidence intervals.

4 STABILITY ANALYSIS

We are interested on finding complex regions in which all the roots for every possible channel in $\mathcal{C}$ are contained. For this purpose, we define the open region $\mathcal{D}_r$ as

$$\mathcal{D}_r = \{z \in \mathbf{C} : |z| < r \leq 1 \text{ or } |z| > 1/r\} \qquad (3)$$

Let $q(t)$ denote the combined transmit-receive filter response which we assume has less than 100% excess bandwidth so it can be sampled without loss of information at $T/2$, where T is the signaling interval. Moreover, we make the practical assumption that the samples of $q(t)$ can be truncated after a certain time, so $q(nT+T/2) = 0, |n| > L$. We will then write $Q(z) = \sum_{n=-L}^{L} q(nT + T/2)z^{-n}$. It can be shown that for a T-spaced equalizer the discrete-equivalent channel response can be written as

$$H(z) = Q(z)C_1(z) + C_2(z) \qquad (4)$$

where $C_1(z) = \sum_{m=0}^{M/2-1} c_{2m+1} z^{-n}$ and $C_2(z) = \sum_{m=0}^{M/2-1} c_{2m} z^{-n}$ with $c_n = c(nT/2)$. We also need to define $D_1(z)$ and $D_2(z)$ as the Z-transform of respectively the sequences $d_{2m+1}, d_{2m}, m = 0, \cdots, M/2-1$ and the following functions

$$R_{1o}(z) = \sum_{m=0}^{M/2-1} a_{2m+1}^+ |z^{-m}| \qquad (5)$$

$$R_{2o}(z) = \sum_{m=0}^{M/2-1} a_{2m}^+ |z^{-m}| \qquad (6)$$

$$R_{1i}(z) = \max\{a_{2l+1}^- |z^{-l}| - \sum_{\substack{m=0 \\ i \neq l}}^{M/2-1} a_{2m+1}^+ |z^{-m}|, 0\} \qquad (7)$$

$$R_{2i}(z) = \max\{a_{2l}^- |z^{-l}| - \sum_{\substack{m=0 \\ i \neq l}}^{M/2-1} a_{2m}^+ |z^{-m}|, 0\} \qquad (8)$$

where

$$a_{2l+1}|z^{-l}| \doteq \max_{0 \leq m \leq M/2-1} \{a_{2m+1}^- |z^{-m}|\} \qquad (9)$$

$$a_{2l}|z^{-l}| \doteq \max_{0 \leq m \leq M/2-1} \{a_{2m}^- |z^{-m}|\} \qquad (10)$$

Our objective is to answer the following question. For each of the channels $H(z)$ determine whether all its zeros are contained in $\mathcal{D}_r$. This is solved by the next theorem

Theorem: $H(z)$ has all its zeros in $\mathcal{D}_r$ for every $\mathbf{c} \in \mathcal{C}$ if and only if the following three conditions hold

i) $H(z)$ has its zeros in $\mathcal{D}_r$ for one member of $\mathcal{C}$

ii) $\left|\frac{Q(re^{j\omega})D_1(re^{j\omega})+D_2(e^{j\omega})}{R_{in}(re^{j\omega})}\right| < 1$ *or*

$\left|\frac{Q(re^{j\omega})D_1(re^{j\omega})+D_2(re^{j\omega})}{R_{out}(re^{j\omega})}\right| > 1$ for all $\omega \in [0, 2\pi)$

iii) $\left|\frac{Q(e^{j\omega}/r)D_1(e^{j\omega}/r)+D_2(e^{j\omega}/r)}{R_{in}(e^{j\omega}/r)}\right| < 1$ *or*

$\left|\frac{Q(e^{j\omega}/r)D_1(e^{j\omega}/r)+D_2(e^{j\omega}/r)}{R_{out}(e^{j\omega}/r)}\right| > 1$ for all $\omega \in [0, 2\pi)$

where

$$R_{out}(z) = |Q(z)|R_{1o}(z) + R_{2o}(z) \qquad (11)$$

$$R_{in}(z) = \begin{cases} \max\{|Q(z)|R_{1i}(z) - R_{2o}(z), 0\} \\ \quad \text{if } |Q(z)|R_{1i}(z) > R_{2i}(z) \\ \\ \max\{R_{2i}(z) - |Q(z)|R_{1o}(z), 0\} \\ \quad \text{otherwise} \end{cases} \qquad (12)$$

The proof of this Theorem [6] follows from the fact that the set $H(re^{j\omega})$ is, for a fixed ω, an annulus centered at $Q(z)D_1(z) + D_2(z)$ with inner radius $R_{in}(z)$ and outer radius $R_{out}(z)$.

Note that when the inner radius of every ring is zero (i.e., we are dealing with disks), the test can be simplified since $R_{in}(z) = 0$. Also note that the conditions in the theorem can be readily transformed into a frequency-domain graphical test similar to those proposed in [7].

The given method may be transformed to search for the maximum r, if any, for which all the zeros of $H(z)$ or its reciprocals lie inside the region $\mathcal{D}_r$. Of course, when $r = 1$ our result allows a spectral nulls search on $H(e^{j\omega})$ for every possible channel $\mathbf{c}$. Indeed, conditions *ii*) and *iii*) in the theorem become identical and condition *i*) is not needed for $r = 1$. Condition *i*) may be checked in a graphical way by means of the following result:

Lemma: The system $H(z)$ has all its zeros inside $\mathcal{D}_r$ if and only if the net change of argument of $H(z)H^*(1/z^*)$ when z traverses $re^{j\omega}$, $\omega \in [0, 2\pi)$ is 0.

With the given method, many applications can be devised. We are currently using the previous Kharitonov-type results to study the feasibility of equalizers for the channel models mentioned above. When Block Least Squares Estimation is used, the length of the channel becomes a crucial factor since the complexity of the Viterbi decoder depends on it [8]. For channels with a number of significant taps, the Viterbi decoder becomes impractical. Then, it is necessary to reduce the number of states seen by the Viterbi algorithm. To overcome this problem, an equalizer may be cascaded so that the combined response channel-equalizer yields a desired impulse response which is shorter than that of the channel [9]. A major drawback in this strategy resides in the fact that the noise power at the input of the detector might be unlimited if one tries to equalise spectral nulls of the channel. Then, with the method proposed in the present paper, it is possible to determine whether any of the possible channels (those within the confidence intervals) will have spectral nulls (or near nulls). ¿From the definition of the desired region $\mathcal{D}_r$, the checking for spectral nulls can be generalized to a test for the zeros of $H(z)$ being sufficiently close to the unit circle. For instance, the desired response at the input of the Viterbi decoder must be carefully chosen if the channel is to have near spectral nulls, but this computational cost can be avoided if the proposed analysis determines that the spectrum does not vanish for any frequency.

When a symbol by symbol detection is to be used along with a Zero-Forcing equalizer, the proposed method can help to decide the proper length of the filter. As it is well known, if the equalizer has its poles close to the unit circle it will have a slowly decreasing impulse response. The reasonable length of the equalizer depends on how fast the different modes vanish for large n. Therefore, it is interesting from the systems designer point of view to determine if the poles of the equalizer are outside the $\mathcal{D}_r$ region.

5 NUMERICAL RESULTS

To illustrate the root clustering method proposed in the paper we will consider the following example, adapted from true channel measurements presented in [10]. The channel response was truncated at $M = 8$ samples and a strong scatterer affecting samples 1 to 3 was included. The confidence intervals for the modulus are considered as the median value $\pm$ the measured standard deviation. All the coefficient taps are normalized so that $c_0 = 1$. The transmit and receive filters are each a square root of a raised cosine filter with roll-off factor of 0.5. The channel parameters are, for $n = 0, \cdots, M-1$

$\{d_n\} = \{1, 0.5 - j0.18, 0.46 - j0.56, 0.13 + j0.17, 0, 0, 0, 0\}$
$\{a_n^+\} = \{0, 0.2, 0.1, 0.07, 0.0978, 0.1228, 0.059, 0.059\}$
$\{a_n^-\} = \{0, 0, 0, 0, 0.056, 0.058, 0.022, 0.022\}$

In order to follow the proposed method, we first pick a specific member of the family of channels and verify the zero locations. The zeros of $H(z)$ for that particular channel are calculated by a root-finding algorithm and tested to be in $\mathcal{D}_r$ or instead, the graphical way suggested in this paper can be used. Once condition *i*) is verified, we recognize that for this example $R_{in}(z) = 0$. Actually, this happens for most of the practical cases. Therefore, we need to check

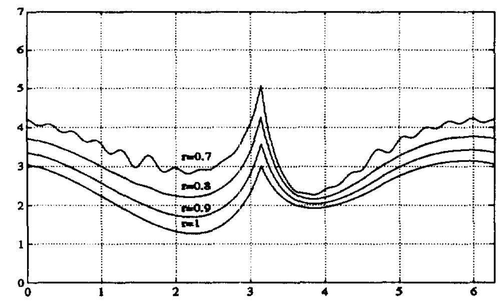

Figure 1: Condition *ii*) of theorem for different values of r

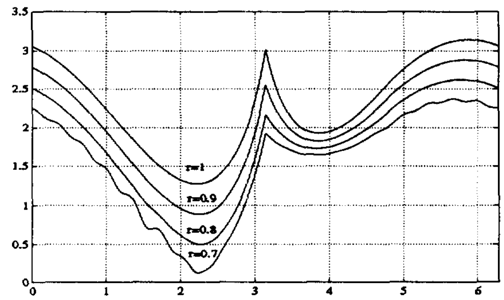

Figure 2: Condition *iii*) of theorem for different values of r

the second two parts of conditions *ii*) and *iii*). In Figure 1, the ratio $\left|(Q(re^{j\omega})D_1(re^{j\omega}) + D_2(re^{j\omega}))/R_{out}(re^{j\omega})\right|$ is plotted for different values of r. It can be seen that *ii*) holds for these values. Figure 2, shows the ratio $\left|(Q(e^{j\omega}/r)D_1(e^{j\omega}/r) + D_2(e^{j\omega}/r))/R_{out}(e^{j\omega}/r)\right|$. As it is easily seen *iii*) does not hold for $r = 0.9$ and smaller. As a consequence, $H(z)$ does not have spectral nulls for any possible channel, but for $r = 0.9$ there is some channel with zeros outside the region $\mathcal{D}_r$.

6 CONCLUSION

In this paper we have presented a powerful analysis method that relies on robust stability ideas to resolve the location of the poles of a Zero-Forcing Equaliser structure. The tests so obtained are extremely simple and based upon a gridding in the frequency domain. As a matter of fact, the results presented here have been modified to solve the problem for the MMSE FSE Equalisers [11], even though for that case it is necessary to discretize the coefficient set along with the frequency variable.

In work in progress, we are studying the availability of similar tests to different equaliser structures, including the DFE. Actually, a more general frame of reference is being studied to include problems of noise cancelling and deconvolution. Different uncertainty regions, such as ellipses, are also being taken into account to reflect the case of correlated gaussian quadrature channel taps.

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