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A journey into quantization in Astrophysics

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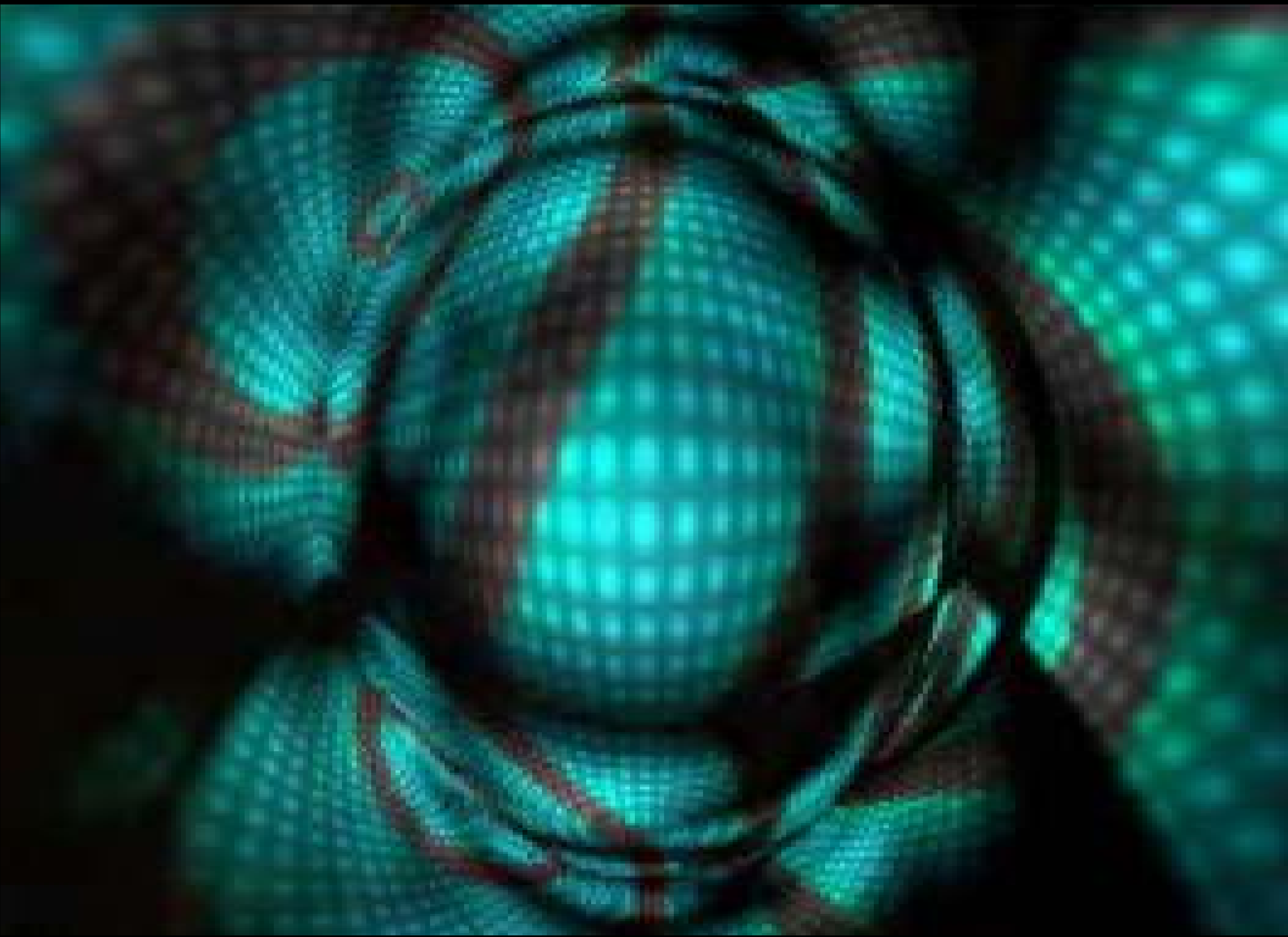
A Journey into Quantization in Astrophysics: A Collection of Scientific Papers



by Florentin Smarandache & Victor Christianto

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A collection of scientific papers

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Preface

The present book consists of 17 select scientific papers from ten years of work around 2003-2013. The topic covered here is quantization in Astrophysics. We also discuss other topics for instance Pioneer spacecraft anomaly.

We discuss a number of sub-topics, for instance the use of Schrödinger equation to describe celestial quantization. Our basic proposition here is that the quantization of planetary systems corresponds to quantization of circulation as observed in superfluidity. And then we extend it further to the use of (complex) Ginzburg-Landau equation to describe possible *nonlinearity* of planetary quantization.

Some of these papers have been published in journal form, but they were scattered around in a number of publications, so they are not easy to locate. So we decide to collect them all in one book for easy reading. Other papers included here have not been published before in journal or book form.

The present book is suitable for young astronomers and astrophysicists as well as for professional astronomers who wish to update their knowledge in the vast topic of quantization in astrophysics. This book is also suitable for college students who want to know more about this subject.

We would like to express our deep gratitude to many scientists who have inspired us along the way of this journey, including Profs. Robert M. Kiehn, Carlos Castro, Antun Rubcic, F. Winterberg, and Dr. Pavel Pintr, and also to a number of journal editors for their permissions to reprint our papers (*Apeiron, Progress in Physics, and Prespacetime Journal*). And special thanks to Multimedia Larga at Gallup, New Mexico, who supports this publication.

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What Gravity Is. Some Recent Considerations

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It is well-known, that when it comes to discussions among physicists concerning the meaning and nature of gravitation, the room temperature can be so hot. Therefore, for the sake of clarity, it seems worth that all choices were put on a table, and we consider each choice's features and problems. The present article describes a non-exhaustive list of such gravitation theories for the purpose of inviting further and more clear discussions.

1 Introduction

The present article summarizes a non-exhaustive list of gravitation theories for the purpose of inviting further and more clear discussions. It is well-known, that when it comes to discussions among physicists concerning the meaning and nature of gravitation, the room temperature can be so hot. Therefore, for the sake of clarity, it seems worth that all choices were put on a table, and we consider each choice's features and problems. Of course, our purpose here is not to say the last word on this interesting issue.

2 Newtonian and non-relativistic approaches

Since the days after Newton physicists argued what is the meaning of "action at a distance" (Newton term) or "spooky action" (Einstein term). Is it really possible to imagine how an apple can move down to Earth without a medium whatsoever?

Because of this difficulty, from the viewpoint of natural philosophy, some physicists maintained (for instance Euler with his impulsion gravity), that there should be "pervasive medium" which can make the attraction force possible. They call this medium "ether" though some would prefer this medium more like "fluid" instead of "solid". Euler himself seems to suggest that gravitation is some kind of "external force" acting on a body, instead of *intrinsic* force:

> "gravity of weight: It is a power by which all bodies are forced towards the centre of the Earth" [3].

But the Michelson-Morley experiment [37] opened the way for Einstein to postulate that ether hypothesis is not required at all in order to explain Lorentz's theorem, which was the beginning of Special Relativity. But of course, one can ask whether the Michelson-Morley experiment really excludes the so-called ether hypothesis. Some experiments after Michelson seem to indicate that "ether" is not excluded in the experiment setup, which means that there is Earth absolute motion [4, 5].

To accept that gravitation is external force instead of intrinsic force implies that there is distinction between gravitation and inertial forces, which also seem to indicate that inertial force can be modified externally via electromagnetic field [6].

The latter notion brings us to long-time discussions in various physics journals concerning the electromagnetic nature of gravitation, i.e. whether gravitation pulling force have the same properties just as electromagnetic field is described by Maxwell equations. Proponents of this view include Tajmar and de Matos [7, 8], Sweetser [9]. And recently Rabounski [10] also suggests similar approach.

Another version of Euler's hypothesis has emerged in modern way in the form of recognition that gravitation was carried by a boson field, and therefore gravitation is somehow related to low-temperature physics (superfluid as boson gas, superconductivity etc.). The obvious advantage of superfluidity is of course that it remains frictionless and invisible; these are main features required for true ether medium — i.e. no resistance will be felt by objects surrounded by the ether, just like the passenger will not feel anything inside the falling elevator. No wonder it is difficult to measure or detect the ether, as shown in Michelson-Morley experiment. The superfluid Bose gas view of gravitation has been discussed in a series of paper by Consoli et al. [11], and also Volovik [12].

Similarly, gravitation can also be associated to superconductivity, as shown by de Matos and Beck [29], and also in Podkletnov's rotating disc experiment. A few words on Podkletnov's experiment. Descartes conjectured that there is no gravitation without rotation motion [30]. And since rotation can be viewed as solution of Maxwell equations, one can say that there is no gravitation separated from electromagnetic field. But if we consider that equations describing superconductivity can be viewed as mere generalization of Maxwell equations (London field), then it seems we can find a modern version of Descartes' conjecture, i.e. *there is no gravitation without superconductivity rotation*. This seems to suggest the significance of Podkletnov's experiments [31, 32].

3 Relativistic gravitation theories

Now we will consider some alternative theories which agree with both Newton theory and Special Relativity, but differ either slightly or strongly to General Relativity. First of all, Einstein's own attempt to describe gravitation despite earlier gravitation theories (such as by Nordstrom [1]) has been inspired by his thought-experiment, called the "falling elevator" experiment. Subsequently he came up with conjecture that there is proper metric such that a passenger inside the elevator will not feel any pulling gravitation force. Therefore gravitation can be replaced by certain specific-chosen metric.

Now the questions are twofold: (a) whether the proper-metric to replace gravitation shall have non-zero curvature or it can be flat-Minkowskian; (b) whether the formulation of General relativity is consistent enough with Mach principle from where GTR was inspired. These questions inspired heated debates for several decades, and Einstein himself (with colleagues) worked on to generalize his own gravitation theories, which implies that he did find that his theory is not complete. His work with Strauss, Bergmann, Pauli, etc. (Princeton School) aimed toward such a unified theory of gravitation and electromagnetism.

There are of course other proposals for relativistic gravitation theories, such as by Weyl, Whitehead etc. [1]. Meanwhile, R. Feynman and some of his disciples seem to be more flexible on whether gravitation shall be presented in the General-Relativity "language" or not.

Recently, there is also discussion in online forum over the question: (a) above, i.e. whether curvature of the metric surface is identical to the gravitation. While most physicists seem to agree with this proposition, there is other argument suggesting that it is also possible to conceive General Relativity even with zero curvature [13, 14].

Of course, discussion concerning relativistic gravitation theories will not be complete without mentioning the PV-gravitation theory (Puthoff et al. [15]) and also Yilmaz theory [16], though Misner has discussed weaknesses of Yilmaz theory [17], and Yilmaz et al. have replied back [18]. Perhaps it would be worth to note here that General Relativity itself is also not without limitations, for instance it shall be modified to include galaxies' rotation curve, and also it is actually theory for one-body problem only [2], therefore it may be difficult to describe interaction between bodies in GTR.

Other possible approaches on relativistic gravitation theories are using the fact that the "falling-elevator" seems to suggest that it is possible to replace gravitation force with certain-chosen metric. And if we consider that one can find simplified representation of Maxwell equations with Special Relativity (Minkowski metric), then the next logical step of this "metrical" (some physicists prefer to call it "geometrodynamics") approach is to represent gravitation with yet another special relativistic but with extra-dimension(s). This was first conjectured in Kaluza-Klein theory [19]. Einstein himself considered this theory extensively with Strauss etc. [20]. There are also higher-dimensional gravitation theories with 6D, 8D and so forth.

In the same direction, recently these authors put forth a new proposition using Carmeli metric [21], which is essentially a "phase-space" relativity theory in 5-dimensions.

Another method to describe gravitation is using "torsion", which is essentially to introduce torsion into Einstein field equations. See also torsional theory developed by Hehl, Kiehn, Rapoport etc. cited in [21].

It seems worth to remark here, that relativistic gravitation does not necessarily exclude the possibility of "aether" hypothesis. B. Riemann extended this hypothesis by assuming (in 1853) that the gravitational aether is an incompressible fluid and normal matter represents "sinks" in this aether [34], while Einstein discussed this aether in his Leiden lecture *Ether and Relativity*.

A summary of contemporary developments in gravitation theories will not be complete without mentioning Quantum Gravity and Superstring theories. Both are still major topics of research in theoretical physics and consist of a wealth of exotic ideas, some or most of which are considered controversial or objectionable. The lack of experimental evidence in support of these proposals continues to stir a great deal of debate among physicists and makes it difficult to draw definite conclusions regarding their validity [38]. It is generally alleged that signals of quantum gravity and superstring theories may occur at energies ranging from the mid or far TeV scale all the way up to the Planck scale.

Loop Quantum Gravity (LQG) is the leading candidate for a quantum theory of gravitation. Its goal is to combine the principles of General Relativity and Quantum Field Theory in a consistent non-perturbative framework [39]. The features that distinguish LQG from other quantum gravity theories are: (a) background independence and (b) minimality of structures. Background independence means that the theory is free from having to choose an apriori background metric. In LQG one does not perturb around any given classical background geometry, rather arbitrary fluctuations are allowed, thus enabling the quantum "replica" of Einstein's viewpoint that gravity is geometry. Minimality means that the general covariance of General Relativity and the principles of canonical quantization are brought together without new concepts such as extra dimensions or extra symmetries. It is believed that LQG can unify all presently known interactions by implementing their common symmetry group, the four-dimensional diffeomorphism group, which is almost completely broken in perturbative approaches.

The fundamental building blocks of String Theory (ST) are one-dimensional extended objects called strings [40, 41]. Unlike the "point particles" of Quantum Field Theories, strings interact in a way that is almost uniquely specified by mathematical self-consistency, forming an allegedly valid quantum theory of gravity. Since its launch as a dual resonance model (describing strongly interacting hadrons), ST has changed over the years to include a group of related superstring theories (SST) and a unifying picture known as the M-theory. SST is an attempt to bring all the particles and their fundamental interactions under one umbrella by modeling them as vibrations of super-symmetric strings.

In the early 1990s, it was shown that the various superstring theories were related by dualities, allowing physicists to map the description of an object in one superstring theory to the description of a different object in another superstring theory. These relationships imply that each of SST represents a different aspect of a single underlying theory, proposed by E. Witten and named M-theory. In a nut-shell, M-theory combines the five consistent ten-dimensional superstring theories with eleven-dimensional supergravity. A shared property of all these theories is the holographic principle, that is, the idea that a quantum theory of gravity has to be able to describe physics occurring within a volume by degrees of freedom that exist on the surface of that volume. Like any other quantum theory of gravity, the prevalent belief is that true testing of SST may be prohibitively expensive, requiring unprecedented engineering efforts on a large-system scale. Although SST is falsifiable in principle, many critics argue that it is un-testable for the foreseeable future, and so it should not be called science [38].

One needs to draw a distinction in terminology between string theories (ST) and alternative models that use the word "string". For example, Volovik talks about "cosmic strings" from the standpoint of condensed matter physics (topological defects, superfluidity, superconductivity, quantum fluids). Beck refers to "random strings" from the standpoint of statistical field theory and associated analytic methods (spacetime fluctuations, stochastic quantization, coupled map lattices). These are not quite the same as ST, which are based on "brane" structures that live on higher dimensional spacetime.

There are other contemporary methods to treat gravity, i.e. by using some advanced concepts such as group(s), topology and symmetries. The basic idea is that Nature seems to prefer symmetry, which lead to higher-dimensional gravitation theories, Yang-Mills gravity etc.

Furthermore, for the sake of clarity we have omitted here more advanced issues (sometimes they are called "fringe research"), such as faster-than-light (FTL) travel possibility, warpdrive, wormhole, cloaking theory (Greenleaf et al. [35]), antigravity (see for instance Naudin's experiment) etc. [36].

4 Wave mechanical method and diffraction hypothesis

The idea of linking gravitation with wave mechanics of Quantum Mechanics reminds us to the formal connection between Helmholtz equation and Schrödinger equation [22].

The use of (modified) Schrödinger equation has become so extensive since 1970s, started by Wheeler-DeWitt (despite the fact that the WDW equation lacks observation support). And recently Nottale uses his scale relativistic approach based on stochastic mechanics theory in order to generalize Schrödinger equation to describe wave mechanics of celestial bodies [23]. His scale-relativity method finds support from observations both in Solar system and also in exo-planets.

Interestingly, one can also find vortex solution of Schrödinger equation, and therefore it is worth to argue that the use of wave mechanics to describe celestial systems implies that there are vortex structure in the Solar system and beyond. This conjecture has also been explored by these authors in the preceding paper. [24] Furthermore, considering formal connection between Helmholtz equation and Schrödinger equation, then it seems also possible to find out vortex solutions of Maxwell equations [25, 26, 27]. Interestingly, experiments on plasmoid by Bostick et al. seem to vindicate the existence of these vortex structures [28].

What's more interesting in this method, perhaps, is that one can expect to to consider gravitation and wave mechanics (i.e. Quantum Mechanics) in equal footing. In other words, the quantum concepts such as ground state, excitation, and zero-point energy now can also find their relevance in gravitation too. This "classical" implications of Wave Mechanics has been considered by Ehrenfest and also Schrödinger himself.

In this regards, there is a recent theory proposed by Gulko [33], suggesting that matter absorbs from the background small amounts of energy and thus creates a zone of reduced energy, and in such way it attracts objects from zones of higher energy.

Another one, by Glenn E. Perry, says that gravity is diffraction (due to the changing energy density gradient) of matter or light as it travels through the aether [33].

We can remark here that Perry's Diffraction hypothesis reminds us to possible production of energy from physical vacuum via a small fluctuation in it due to a quantum indeterminancy (such a small oscillation of the background can be suggested in any case because the indeterminancy principle). On the average the background vacuum does not radiate — its energy is constant. On the other hand, it experiences small oscillation. If an engine built on particles or field interacts with the small oscillation of the vacuum, or at least "senses the oscillation, there is a chance to get energy from them. Because the physical vacuum is eternal capacity of energy, it is easy to imagine some possible techniques to be discovered in the future to extract this energy.

Nonetheless, diffraction of gravity is not a "new hot topic" at all. Such ideas were already proposed in the 1920's by the founders of relativity. They however left those ideas, even unpublished but only mentioned in memoirs and letters. The main reason was that (perhaps) almost infinitely small energy which can be extracted from such background per second. (In the mean time, there are other vaious proposals suggesting that it is possible to 'extract' energy from gravitation field).

About Glenn Perry and his theory. There is a drawback that that matter he called "aether" was not properly determined by him. In such a way like that, everything can be "proven". To produce any calculation for practical purpose, we should have exact data on the subject of this calculation, and compare it with actual experiments.

On the other hand, such an idea could be put into another field — the field of Quantum Mechanics. That is, to study diffraction not gravitational radiation (gravitational waves which is so weak that not discovered yet), but waves of the field of the gravitational force — in particular those can be seismic-like waves travelling in the cork of the Earth (we mean not the earthquakes) but in the gravitational field of the planet. These seismic-like oscillations (waves) of the gravitational force are known to science, and they aren't weak: everyone who experienced an earthquake knows this fact.

Other hint from wave aspect of this planet is known in the form of Schumann resonance, that the Earth produces vibration at very-low frequency, which seems to support the idea that planetary mass vibrates too, just as hypothesized in Wave Mechanics (de Broglie's hypothesis). Nonetheless, there are plenty of things to study on the large-scale implications of the Wave Mechanics.

5 Concluding remarks

The present article summarizes a non-exhaustive list of gravitation theories for the purpose of inviting further and more clear discussions. Of course, our purpose here is not to say the last word on this interesting issue. For the sake of clarity, some advanced subjects have been omitted, such as faster-than-light (FTL) travel possibility, warpdrive, wormhole, cloaking theory (Greenleaf et al.), antigravity etc. As to the gravitation research in the near future, it seems that there are multiple directions which one can pursue, with which we're not so sure. The only thing that we can be sure is that everything changes (Heraclitus of Ephesus), including how we define "what the question is" (Wheeler's phrase), and also what we mean with "metric", "time", and "space". Einstein himself once remarked that 'distance' itself is merely an illusion.

Acknowledgment

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Schrödinger Equation and the Quantization of Celestial Systems

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In the present article, we argue that it is possible to generalize Schrödinger equation to describe quantization of celestial systems. While this hypothesis has been described by some authors, including Nottale, here we argue that such a macroquantization was formed by topological superfluid vortice. We also provide derivation of Schrödinger equation from Gross-Pitaevskii-Ginzburg equation, which supports this superfluid dynamics interpretation.

1 Introduction

In the present article, we argue that it is possible to generalize Schrödinger equation to describe quantization of celestial systems, based on logarithmic nature of Schrödinger equation, and also its exact mapping to Navier-Stokes equations [1].

While this notion of macro-quantization is not widely accepted yet, as we will see the logarithmic nature of Schrödinger equation could be viewed as a support of its applicability to larger systems. After all, the use of Schrödinger equation has proved itself to help in finding new objects known as extrasolar planets [2, 3]. And we could be sure that new extrasolar planets are to be found in the near future. As an alternative, we will also discuss an outline for how to derive Schrödinger equation from simplification of Ginzburg-Landau equation. It is known that Ginzburg-Landau equation exhibits fractal character, which implies that quantization could happen at *any scale*, supporting topological interpretation of quantized vortices [4].

First, let us rewrite Schrödinger equation in its common form [5]

$$\left[i \frac{\partial}{\partial t} + \frac{\bar{\nabla}^2}{2m} - U(x) \right] \psi = 0 \tag{1}$$

or

$$i \frac{\partial \psi}{\partial t} = H \psi \,. \tag{2}$$

Now, it is worth noting here that Englman and Yahalom [5] argues that this equation exhibits logarithmic character

$$\ln \psi(x,t) = \ln \big(|\psi(x,t)| \big) + i \arg\big(\psi(x,t)\big) \,. \tag{3}$$

Schrödinger already knew this expression in 1926, which then he used it to propose his equation called "eigentliche Wellengleichung" [5]. Therefore equation (1) can be re-written as follows

$$2m \frac{\partial\big(\ln|\psi|\big)}{\partial t} + 2\bar{\nabla} \ln|\psi| \, \bar{\nabla} \arg\big[\psi\big] + \bar{\nabla}\bar{\nabla} \arg\big[\psi\big] = 0 \,. \tag{4}$$

Interestingly, Nottale's scale-relativistic method [2, 3] was also based on generalization of Schrödinger equation to describe quantization of celestial systems. It is known that Nottale-Schumacher's method [6] could predict new exoplanets in good agreement with observed data. Nottale's scale-relativistic method is essentially based on the use of first-order scale-differentiation method defined as follows [2]

$$\frac{\partial V}{\partial(\ln \delta t)} = \beta(V) = a + b\,V + \dots \,. \tag{5}$$

Now it seems clear that the natural-logarithmic derivation, which is essential in Nottale's scale-relativity approach, also has been described properly in Schrödinger's original equation [5]. In other words, its logarithmic form ensures applicability of Schrödinger equation to describe macro-quantization of celestial systems. [7, 8]

2 Quantization of celestial systems and topological quantized vortices

In order to emphasize this assertion of the possibility to describe quantization of celestial systems, let us quote Fischer's description [4] of relativistic momentum from superfluid dynamics. Fischer [4] argues that the circulation is in the relativistic dense superfluid, defined as the integral of the momentum

$$\gamma_s = \oint p_\mu \, dx^\mu = 2\pi N_v \hbar \,, \tag{6}$$

and is quantized into multiples of Planck's quantum of action. This equation is the covariant Bohr-Sommerfeld quantization of γ_s. And then Fischer [4] concludes that the Maxwell equations of ordinary electromagnetism can be written in the form of conservation equations of relativistic perfect fluid hydrodynamics [9]. Furthermore, the topological character of equation (6) corresponds to the notion of topological electronic liquid, where compressible electronic liquid represents superfluidity [25]. For the plausible linkage between superfluid dynamics and cosmological phenomena, see [16–24].

It is worth noting here, because vortices could be defined as elementary objects in the form of stable topological excitations [4], then equation (6) could be interpreted as Bohr-Sommerfeld-type quantization from *topological quantized vortices*. Fischer [4] also remarks that equation (6) is quite interesting for the study of superfluid rotation in the context of gravitation. Interestingly, application of Bohr-Sommerfeld quantization for celestial systems is known in literature [7, 8], which here in the context of Fischer's arguments it has special meaning, i. e. it suggests that *quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale* [4]. In our opinion, this result supports known experiments suggesting neat correspondence between condensed matter physics and various cosmology phenomena [16–24].

To make the conclusion that quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale a bit conceivable, let us consider the problem of quantization of celestial orbits in solar system.

In order to obtain planetary orbit prediction from this hypothesis we could begin with the Bohr-Sommerfeld's conjecture of quantization of angular momentum. This conjecture may originate from the fact that according to BCS theory, superconductivity can exhibit macroquantum phenomena [26, 27]. In principle, this hypothesis starts with observation that in quantum fluid systems like superfluidity [28]; it is known that such vortexes are subject to quantization condition of integer multiples of 2π, or $\oint v_s\,dl = 2\pi n\hbar/m$. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition [28]

$$\oint_\Gamma p\,dx = 2\pi n\hbar \tag{6a}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is [28]

$$\int_0^T v^2 d\tau = \omega^2 T = 2\pi\omega\,, \tag{7}$$

where $T = 2\pi/\omega$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$. Then we can write the force balance relation of Newton's equation of motion [28]

$$\frac{GMm}{r^2} = \frac{mv^2}{r}\,. \tag{8}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum, a new constant g was introduced [28]

$$mvr = \frac{ng}{2\pi}\,. \tag{9}$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form [28]

$$r = \frac{n^2 g^2}{4\pi^2 GMm^2}\,, \tag{10}$$

which can be rewritten in the known form of gravitational Bohr-type radius [2, 7, 8]

$$r = \frac{n^2 GM}{v_0^2}\,, \tag{11}$$

where r, n, G, M, v_0 represents orbit radii, quantum number ($n = 1, 2, 3, \dots$), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (11), we denote [28]

$$v_0 = \frac{2\pi}{g}\,GMm\,. \tag{12}$$

The value of m is an adjustable parameter (similar to g) [7, 8]. In accordance with Nottale, we assert that the specific velocity v_0 is 144 km/sec for planetary systems. By noting that m is meant to be mass of celestial body in question, then we could find g parameter (see also [28] and references cited therein).

Using this equation (11), we could predict quantization of celestial orbits in the solar system, where for Jovian planets we use least-square method and use M in terms of reduced mass $\mu = \frac{(M_1 + M_2)}{M_1 M_2}$. From this viewpoint the result is shown in Table 1 below [28].

For comparison purpose, we also include some recent observation by Brown-Trujillo team from Caltech [29–32]. It is known that Brown et al. have reported not less than four new planetoids in the outer side of Pluto orbit, including 2003EL61 (at 52 AU), 2005FY9 (at 52 AU), 2003VB12 (at 76 AU, dubbed as Sedna). And recently Brown-Trujillo team reported a new planetoid finding, called 2003UB31 (97 AU). This is not to include their previous finding, Quaoar (42 AU), which has orbit distance more or less near Pluto (39.5 AU), therefore this object is excluded from our discussion. It is interesting to remark here that all of those new "planetoids" are within 8% bound from our prediction of celestial quantization based on the above Bohr-Sommerfeld quantization hypothesis (Table 1). While this prediction is not so precise compared to the observed data, one could argue that the 8% bound limit also corresponds to the remaining planets, including inner planets. Therefore this 8% uncertainty could be attributed to macroquantum uncertainty and other local factors.

While our previous prediction only limits new planet finding until $n = 9$ of Jovian planets (outer solar system), it seems that there are sufficient reasons to suppose that more planetoids in the Oort Cloud will be found in the near future. Therefore it is recommended to extend further the same quantization method to larger n values. For prediction purpose, we include in Table 1 new expected orbits based

Object	No.	Titius	Nottale	CSV	Observ.	Δ, %
	1		0.4	0.43		
	2		1.7	1.71		
Mercury	3	4	3.9	3.85	3.87	0.52
Venus	4	7	6.8	6.84	7.32	6.50
Earth	5	10	10.7	10.70	10.00	−6.95
Mars	6	16	15.4	15.4	15.24	−1.05
Hungarias	7		21.0	20.96	20.99	0.14
Asteroid	8		27.4	27.38	27.0	1.40
Camilla	9		34.7	34.6	31.5	−10.00
Jupiter	2	52		45.52	52.03	12.51
Saturn	3	100		102.4	95.39	−7.38
Uranus	4	196		182.1	191.9	5.11
Neptune	5			284.5	301	5.48
Pluto	6	388		409.7	395	−3.72
2003EL61	7			557.7	520	−7.24
Sedna	8	722		728.4	760	4.16
2003UB31	9			921.8	970	4.96
Unobserv.	10			1138.1		
Unobserv.	11			1377.1		

Table 1: Comparison of prediction and observed orbit distance of planets in Solar system (in 0.1AU unit) [28].

on the same quantization procedure we outlined before. For Jovian planets corresponding to quantum number $n = 10$ and $n = 11$, our method suggests that it is likely to find new orbits around 113.81 AU and 137.71 AU, respectively. It is recommended therefore, to find new planetoids around these predicted orbits.

As an interesting alternative method supporting this proposition of quantization from superfluid-quantized vortices (6), it is worth noting here that Kiehn has argued in favor of re-interpreting the square of the wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [1]. From this viewpoint, Kiehn suggests that there is *exact mapping* from Schrödinger equation to Navier-Stokes equation, using the notion of quantum vorticity [1]. Interestingly, de Andrade and Sivaram [33] also suggest that there exists formal analogy between Schrödinger equation and the Navier-Stokes viscous dissipation equation:

$$\frac{\partial V}{\partial t} = \nu \nabla^2 V \,, \tag{13}$$

where ν is the kinematic viscosity. Their argument was based on propagation torsion model for quantized vortices [23]. While Kiehn's argument was intended for ordinary fluid, nonetheless the neat linkage between Navier-Stokes equation and superfluid turbulence is known in literature [34, 24].

At this point, it seems worth noting that some criticism arises concerning the use of quantization method for describing the motion of celestial systems. These criticism proponents usually argue that quantization method (wave mechanics) is oversimplifying the problem, and therefore cannot explain other phenomena, for instance planetary migration etc. While we recognize that there are phenomena which do not correspond to quantum mechanical process, at least we can argue further as follows:

1. Using quantization method like Nottale-Schumacher did, one can expect to predict new exoplanets (extra-solar planets) with remarkable result [2, 3];
2. The "conventional" theories explaining planetary migration normally use fluid theory involving *diffusion* process;
3. Alternatively, it has been shown by Gibson et al. [35] that these migration phenomena could be described via Navier-Stokes approach;
4. As we have shown above, Kiehn's argument was based on *exact-mapping* between Schrödinger equation and Navier-Stokes equations [1];
5. Based on Kiehn's vorticity interpretation one these authors published prediction of some new planets in 2004 [28]; which seems to be in good agreement with Brown-Trujillo's finding (March 2004, July 2005) of planetoids in the Kuiper belt;
6. To conclude: while our method as described herein may be interpreted as an oversimplification of the real planetary migration process which took place sometime in the past, at least it could provide us with useful tool for prediction;
7. Now we also provide new prediction of other planetoids which are likely to be observed in the near future (around 113.8 AU and 137.7 AU). It is recommended to use this prediction as guide to finding new objects (in the inner Oort Cloud);
8. There are of course other theories which have been developed to explain planetoids and exoplanets [36]. Therefore quantization method could be seen as merely a "plausible" theory between others.

All in all, what we would like to emphasize here is that the quantization method does not have to be the *true* description of reality with regards to celestial phenomena. As always this method could explain some phenomena, while perhaps lacks explanation for other phenomena. But at least it can be used to predict something quantitatively, i. e. measurable (exoplanets, and new planetoids in the outer solar system etc.).

In the meantime, it seems also interesting here to consider a plausible generalization of Schrödinger equation in particular in the context of viscous dissipation method [1]. First, we could write Schrödinger equation for a charged particle interacting with an external electromagnetic field [1] in the form of Ulrych's unified wave equation [14]

$$\begin{aligned}\big[(-i\hbar\nabla - qA)_\mu(-i\hbar\nabla - qA)^\mu\,\psi\big] &= \\ &= \left[-i2m\,\frac{\partial}{\partial t} + 2mU(x)\right]\psi\,.\end{aligned} \tag{14}$$

In the presence of electromagnetic potential, one could include another term into the LHS of equation (14)

$$\begin{aligned}\big[(-i\hbar\nabla - qA)_\mu(-i\hbar\nabla - qA)^\mu + eA_0\big]\psi &= \\ &= 2m\left[-i\,\frac{\partial}{\partial t} + U(x)\right]\psi\,.\end{aligned} \tag{15}$$

This equation has the physical meaning of Schrödinger equation for a charged particle interacting with an external electromagnetic field, which takes into consideration Aharonov effect [37]. Topological phase shift becomes its immediate implication, as already considered by Kiehn [1].

As described above, one could also derived equation (11) from scale-relativistic Schrödinger equation [2, 3]. It should be noted here, however, that Nottale's method [2, 3] differs appreciably from the viscous dissipative Navier-Stokes approach of Kiehn [1], because Nottale only considers his equation in the Euler-Newton limit [3]. Nonetheless, it shall be noted here that in his recent papers (2004 and up), Nottale has managed to show that his scale relativistic approach has linkage with Navier-Stokes equations.

3 Schrödinger equation derived from Ginzburg-Landau equation

Alternatively, in the context of the aforementioned superfluid dynamics interpretation [4], one could also derive Schrödinger equation from simplification of Ginzburg-Landau equation. This method will be discussed subsequently. It is known that Ginzburg-Landau equation can be used to explain various aspects of superfluid dynamics [16, 17]. For alternative approach to describe superfluid dynamics from Schrödinger-type equation, see [38, 39].

According to Gross, Pitaevskii, Ginzburg, wavefunction of N bosons of a reduced mass m^* can be described as [40]

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + \kappa\,|\psi|^2\,\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,. \tag{16}$$

For some conditions, it is possible to replace the potential energy term in equation (16) with Hulthen potential. This substitution yields

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + V_{\mathrm{Hulthen}}\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,, \tag{17}$$

where

$$V_{\mathrm{Hulthen}} = -Ze^2\,\frac{\delta\,e^{-\delta r}}{1 - e^{-\delta r}}\,. \tag{18}$$

This equation (18) has a pair of exact solutions. It could be shown that for small values of δ, the Hulthen potential (18) approximates the effective Coulomb potential, in particular for large radius

$$V^{\mathrm{eff}}_{\mathrm{Coulomb}} = -\frac{e^2}{r} + \frac{\ell\,(\ell+1)\,\hbar^2}{2mr^2}\,. \tag{19}$$

By inserting (19), equation (17) could be rewritten as

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + \left[-\frac{e^2}{r} + \frac{\ell(\ell+1)\hbar^2}{2mr^2}\right]\psi = i\hbar\,\frac{\partial\psi}{\partial t}. \tag{20}$$

For large radii, second term in the square bracket of LHS of equation (20) reduces to zero [41],

$$\frac{\ell(\ell+1)\,\hbar^2}{2mr^2} \to 0\,, \tag{21}$$

so we can write equation (20) as

$$\left[-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2 + U(x)\right]\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,, \tag{22}$$

where Coulomb potential can be written as

$$U(x) = -\frac{e^2}{r}\,. \tag{22a}$$

This equation (22) is nothing but Schrödinger equation (1), except for the mass term now we get mass of Cooper pairs. In other words, we conclude that it is possible to re-derive Schrödinger equation from simplification of (Gross-Pitaevskii) Ginzburg-Landau equation for superfluid dynamics [40], in the limit of small screening parameter, δ. Calculation shows that introducing this Hulthen effect (18) into equation (17) will yield essentially similar result to (1), in particular for small screening parameter. Therefore, we conclude that for most celestial quantization problems the result of TDGL-Hulthen (20) is *essentially* the same with the result derived from equation (1). Now, to derive gravitational Bohr-type radius equation (11) from Schrödinger equation, one could use Nottale's scale-relativistic method [2, 3].

4 Concluding remarks

What we would emphasize here is that this derivation of Schrödinger equation from (Gross-Pitaevskii) Ginzburg-Landau equation is in good agreement with our previous conjecture that equation (6) implies macroquantization corresponding to superfluid-quantized vortices. This conclusion is the main result of this paper. Furthermore, because Ginzburg-Landau equation represents superfluid dynamics at low-temperature [40], the fact that we can derive quantization of celestial systems from this equation seems to support the idea of Bose-Einstein condensate cosmology [42, 43]. Nonetheless, this hypothesis of Bose-Einstein condensate cosmology deserves discussion in another paper.

Above results are part of our book *Multi-Valued Logic, Neutrosophy, and Schrödinger Equation* that is in print.

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Numerical Solution of Time-Dependent Gravitational Schrödinger Equation

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In recent years, there are attempts to describe quantization of planetary distance based on time-independent gravitational Schrödinger equation, including Rubcic & Rubcic's method and also Nottale's Scale Relativity method. Nonetheless, there is no solution yet for time-dependent gravitational Schrödinger equation (TDGSE). In the present paper, a numerical solution of time-dependent gravitational Schrödinger equation is presented, apparently for the first time. This numerical solution leads to gravitational Bohr-radius, as expected. In the subsequent section, we also discuss plausible extension of this gravitational Schrödinger equation to include the effect of phion condensate via Gross-Pitaevskii equation, as described recently by Moffat. Alternatively one can consider this condensate from the viewpoint of Bogoliubov-deGennes theory, which can be approximated with coupled time-independent gravitational Schrödinger equation. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In the past few years, there have been some hypotheses suggesting that quantization of planetary distance can be derived from a gravitational Schrödinger equation, such as Rubcic & Rubcic and also Nottale's scale relativity method [1, 3]. Interestingly, the gravitational Bohr radius derived from this gravitational Schrödinger equation yields prediction of new type of astronomical observation in recent years, i.e. extrasolar planets, with unprecedented precision [2].

Furthermore, as we discuss in preceding paper [4], using similar assumption based on gravitational Bohr radius, one could predict new planetoids in the outer orbits of Pluto which are apparently in good agreement with recent observational finding.. Therefore one could induce from this observation that the gravitational Schrödinger equation (and gravitational Bohr radius) deserves further consideration.

In the meantime, it is known that all present theories discussing gravitational Schrödinger equation only take its time-independent limit. Therefore it seems worth to find out the solution and implication of time-dependent gravitational Schrödinger equation (TDGSE). This is what we will discuss in the present paper.

First we will find out numerical solution of time-independent gravitational Schrödinger equation which shall yield gravitational Bohr radius as expected [1, 2, 3]. Then we extend our discussion to the problem of time-dependent gravitational Schrödinger equation.

In the subsequent section, we also discuss plausible extension of this gravitational Schrödinger equation to include the effect of phion condensate via Gross-Pitaevskii equation, as described recently by Moffat [5]. Alternatively one can consider this phion condensate model from the viewpoint of Bogoliubov-deGennes theory, which can be approximated with coupled time-independent gravitational Schrödinger equation. To our knowledge this proposition of coupled time-independent gravitational Schrödinger equation has never been considered before elsewhere.

Further observation is of course recommended in order to verify or refute the propositions outlined herein.

All numerical computation was performed using Maple. Please note that in all conditions considered here, we use only gravitational Schrödinger equation as described in Rubcic & Rubcic [3], therefore we neglect the scale relativistic effect for clarity.

2 Numerical solution of time-independent gravitational Schrödinger equation and time-dependent gravitational Schrödinger equation

First we write down the time-independent gravitational Schrödinger radial wave equation in accordance with Rubcic & Rubcic [3]:

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\frac{8\pi m^2E'}{H^2}R+ +\frac{2}{r}\frac{4\pi^2GMm^2}{H^2}R-\frac{\ell(\ell+1)}{r^2}R=0\,. \tag{1}$$

When H, V, E' represents gravitational Planck constant, Newtonian potential, and the energy per unit mass of the orbiting body, respectively, and [3]:

$$H = h\left(2\pi f\,\frac{Mm_n}{m_0^2}\right), \tag{2}$$

$$V(r) = -\frac{GMm}{r}\,, \tag{3}$$

$$E' = \frac{E}{m}\,. \tag{4}$$

By assuming that R takes the form:

$$R = e^{-\alpha r} \tag{5}$$

and substituting it into equation (1), and using simplified terms only of equation (1), one gets:

$$\Psi = \alpha^e\,e^{-\alpha r} - \frac{2\alpha\,e^{-\alpha r}}{r} + \frac{8\pi GMm^2\,e^{-\alpha r}}{r\,H^2}\,. \tag{6}$$

After factoring this equation (6) and solving it by equating the factor with zero, yields:

$$RR = -\frac{2\left(4\pi GMm^2 - H^2\alpha\right)}{\alpha^2H^2} = 0\,, \tag{7}$$

or

$$RR = 4\pi GMm^2 - H^2\alpha = 0\,, \tag{8}$$

and solving for α, one gets:

$$a = \frac{4\pi^2 GMm^2}{H^2}\,. \tag{9}$$

Gravitational Bohr radius is defined as inverse of this solution of α, then one finds (in accordance with Rubcic & Rubcic [3]):

$$r_1 = \frac{H^2}{4\pi^2 GMm^2}\,, \tag{10}$$

and by substituting back equation (2) into (11), one gets [3]:

$$r_1 = \left(\frac{2\pi f}{\alpha c}\right)^2 GM\,, \tag{11}$$

which is equivalent with Nottale's result [1, 2], especially when we introduce the quantization number: $r_n = r_1 n^2$ [3]. For complete Maple session of these all steps, see Appendix 1.

Solution of time-dependent gravitational Schrödinger equation is more or less similar with the above steps, except that we shall take into consideration the right hand side of Schrödinger equation and also assuming time dependent form of r:

$$R = e^{-\alpha\,r(t)}\,. \tag{12}$$

Therefore the gravitational Schrödinger equation now reads:

$$\begin{aligned}&\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \frac{8\pi m^2 E'}{H^2}R + \\ &+\frac{2}{r}\,\frac{4\pi^2 GMm^2}{H^2}R - \frac{\ell(\ell+1)}{r^2}R = H\frac{dR}{dt}\,,\end{aligned} \tag{13}$$

or by using Leibniz chain rule, we can rewrite equation (15) as:

$$\begin{aligned}&-H\frac{dR}{dr(t)}\frac{dr(t)}{dt} + \frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \frac{8\pi m^2E'}{H^2}R + \\ &\qquad + \frac{2}{r}\,\frac{4\pi^2 GMm^2}{H^2}R - \frac{\ell(\ell+1)}{r^2}R = 0\,.\end{aligned} \tag{14}$$

The remaining steps are similar with the aforementioned procedures for time-independent case, except that now one gets an additional term for RR:

$$\begin{aligned}RR' = H^3\alpha\left(\frac{d}{dt}r(t)\right)r(t) - \alpha^2 r(t)H^2 + \\ + 8\pi GMm^2 - 2H^2\alpha = 0\,.\end{aligned} \tag{15}$$

At this point one shall assign a value for $\frac{d}{dt}r(t)$ term, because otherwise the equation cannot be solved. We choose $\frac{d}{dt}r(t) = 1$ for simplicity, then one gets solution for (17):

$$\begin{aligned}&a2 := \left\langle \begin{matrix}\alpha=\alpha,\ \pi=\pi,\ m=m,\ H=H,\ G=G,\ M=M, \\ t = RootOf(r(\_Z)\alpha H^3 - r(\_Z)\alpha^2H^2 + 8\pi^2 GMm^2 - 2\alpha H^2)\end{matrix}\right\rangle, \\ &\{\alpha=0,\ t=t,\ m=m,\ H=H,\ G=G,\ M=M,\ \pi=0\}, \\ &\{\alpha=0,\ \pi=\pi,\ t=t,\ m=m,\ H=H,\ M=M,\ G=0\}, \\ &\left\{\pi=\pi,\ t=t,\ m=m,\ H=H,\ M=M,\ \alpha=H,\ G=\tfrac{H^3}{4\pi^2Mm^2}\right\}, \\ &\{\alpha=\alpha,\ H=0,\ \pi=\pi,\ t=t,\ m=m,\ M=M,\ G=G\}, \\ &\{\alpha=0,\ \pi=\pi,\ t=t,\ m=m,\ H=H,\ G=G,\ M=0\}, \\ &\{\alpha=0,\ \pi=\pi,\ t=t,\ H=H,\ G=G,\ M=M,\ m=0\}, \\ &\{\alpha=\alpha,\ H=0,\ \pi=\pi,\ t=t,\ m=m,\ G=G,\ M=0\}, \\ &\{\alpha=\alpha,\ H=0,\ \pi=\pi,\ t=t,\ G=G,\ M=M,\ m=0\}\end{aligned}$$

Therefore one can conclude that there is time-dependent modification factor to conventional gravitational Bohr radius solution. For complete Maple session of these steps, see Appendix 2.

3 Gross-Pitaevskii effect. Bogoliubov-deGennes approximation and coupled time-independent gravitational Schrödinger equation

At this point it seems worthwhile to take into consideration a proposition by Moffat, regarding modification of Newtonian acceleration law due to phion condensate medium, to include Yukawa type potential [5, 6]:

$$a(r) = -\frac{G_\infty M}{r^2} + K\frac{\exp(-\mu_\phi r)}{r^2}(1+\mu_\phi r)\,. \tag{16}$$

Therefore equation (1) can be rewritten to become:

$$\begin{aligned}&\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \frac{8\pi m^2E'}{H^2}R + \\ &+\frac{2}{r}\,\frac{4\pi^2\big(GM - K\exp(-\mu_\phi r)(1+\mu_\phi r)\big)m^2}{H^2}R - \\ &-\frac{\ell(\ell+1)}{r^2}R = 0\,,\end{aligned} \tag{17}$$

or by assuming $\mu = 2\mu_0 = \mu_0 r$ for the exponential term, equation (17) can be rewritten as:

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\frac{8\pi m^2 E'}{H^2}R+ \\ +\frac{2}{r}\,\frac{4\pi^2\left(GM-Ke^{-2\mu_0}(1+\mu_0 r)\right)m^2}{H^2}R- \\ -\frac{\ell(\ell+1)}{r^2}R=0\,. \tag{18}$$

Then instead of equation (8), one gets:

$$RR''=8\pi GMm^2-2H^2\alpha-8\pi^2m^2Ke^{-\mu_0}(1+\mu)=0. \tag{19}$$

Solving this equation will yield a modified gravitational Bohr radius which includes Yukawa effect:

$$r_1=\frac{H^2}{4\pi^2(GM-Ke^{-2\mu_0})m^2} \tag{20}$$

and the modification factor can be expressed as ratio between equation (20) and (11):

$$\chi=\frac{GM}{(GM-Ke^{-2\mu_0})}\,, \tag{21}$$

for complete Maple session of these steps, see Appendix 3.

A careful reader may note that this "Yukawa potential effect" as shown in equation (21) could be used to explain the small discrepancy (around $\pm 8\%$) between the "observed distance" and the computed distance based on gravitational Bohr radius [4, 6a]. Nonetheless, in our opinion such an interpretation remains an open question, therefore it may be worth to explore further.

There is, however, an alternative way to consider phion condensate medium, i.e. by introducing coupled Schrödinger equation, which is known as Bogoliubov-deGennes theory [7]. This method can be interpreted also as generalisation of assumption by Rubcic-Rubcic [3] of subquantum structure composed of positive-negative Planck mass. Therefore, taking this proposition seriously, then one comes to hypothesis that there shall be coupled Newtonian potential, instead of only equation (3).

To simplify Bogoliubov-deGennes equation, we neglect the time-dependent case, therefore the wave equation can be written in matrix form [7, p. 4]:

$$\left[A\right]\left[\Psi\right]=0\,, \tag{22}$$

where $\left[A\right]$ is 2×2 matrix and $\left[\Psi\right]$ is 2×1 matrix, respectively, which can be represented as follows:

$$\left[A\right]=\begin{pmatrix} \frac{8\pi GMm^2e^{-\alpha r}}{r\hbar^2} & \alpha e^{-\alpha r}-\frac{2\alpha e^{-\alpha r}}{r} \\ \alpha e^{-\alpha r}-\frac{2\alpha e^{-\alpha r}}{r} & -\frac{8\pi GMm^2\,e^{-\alpha r}}{r\hbar^2} \end{pmatrix} \tag{23}$$

and

$$\left[\Psi\right]=\begin{pmatrix} f(r) \\ g(r) \end{pmatrix}. \tag{24}$$

Numerical solution of this matrix differential equation can be found in the same way with the previous methods, however we leave this problem as an exercise for the readers.

It is clear here, however, that Bogoliubov-deGennes approximation of gravitational Schrödinger equation, taking into consideration phion condensate medium will yield nonlinear effect, because it requires solution of matrix differential equation* (22) rather than standard ODE in conventional Schrödinger equation. This perhaps may explain complicated structure beyond Jovian Planets, such as Kuiper Belt, inner and outer Oort Cloud etc. which of course these structure cannot be predicted by simple gravitational Schrödinger equation [1, 2, 3]. In turn, from the solution of (22) one could expect that there are multitude of celestial objects not found yet in the Oort Cloud.

Further observation is also recommended in order to verify and explore further this proposition.

4 Concluding remarks

In the present paper, a numerical solution of time-dependent gravitational Schrödinger equation is presented, apparently for the first time. This numerical solution leads to gravitational Bohr-radius, as expected.

In the subsequent section, we also discuss plausible extension of this gravitational Schrödinger equation to include the effect of phion condensate via Gross-Pitaevskii equation, as described recently by Moffat. Alternatively one can consider this condensate from the viewpoint of Bogoliubov-deGennes theory, which can be approximated with coupled time-independent gravitational Schrödinger equation.

It is recommended to conduct further observation in order to verify and also to explore various implications of our propositions as described herein.

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Appendix 1 Time-independent gravitational Schrödinger equation

```
> restart;
> with (linalg);
> R: = exp (−(alpha*r));
```

$$R := e^{-\alpha r}$$

```
> D1R:=diff (R,r); D2R:=diff (D1R,r);
```

$$D1R := -\alpha\, e^{-\alpha r}$$
$$D2R := -\alpha^2 e^{-\alpha r}$$

```
> SCHEQ1:=D2R+D1R*2/r+8*piˆ2*m*E*R/hˆ2+8*piˆ2*G*M*mˆ2*R/(r*hˆ2)−l*(l+1)*R/rˆ2=0;
> XX1:=factor (SCHEQ1);
> #Using simplified terms only from equation (A*8, of Rubcic & Rubcic, 1998)
> ODESCHEQ:=D2R+D1R*2/r+8*piˆ2*G*M*mˆ2*R/(r*hˆ2)=0;
```

$$ODESCHEQ := \alpha^2 e^{-\alpha r} - \frac{2\alpha\, e^{-\alpha r}}{r} + \frac{8\pi^2 GMm^2 e^{-\alpha r}}{rH^2} = 0$$

```
> XX2:=factor (SCHEQ2);
```

$$XX2 := \frac{e^{-\alpha r}\left(\alpha^2 rH^2 - 2H^2\alpha + 8\pi^2 GMm^2\right)}{rH^2} = 0$$

```
> RR:= solve (XX2, r);
```

$$RR := -\frac{2(4\pi^2 GMm^2 - H^2\alpha)}{\alpha^2 H^2}$$

```
> #Then solving for RR=0, yields:
> SCHEQ3:= 4*piˆ2*G*M*mˆ2−hˆ2*alpha=0;
```

$$SCHEQ3 := 4\pi^2 GMm^2 - H^2\alpha = 0$$

```
> a:= solve (SCHEQ3, alpha);
```

$$a := \frac{4\pi^2 GMm^2}{H^2}$$

```
> #Gravitational Bohr radius is defined as inverse of alpha:
> gravBohrradius:=1/a;
```

$$r_{gravBohr} := \frac{H^2}{4\pi^2 GMm^2}$$

Appendix 2 Time-dependent gravitational Schrödinger equation

```
> #Solution of gravitational Schrodinger equation (Rubcic, Fizika 1998);
> restart;
> #with time evolution (Hagendorn's paper);
> S:=r (t); R:=exp (−(alpha*S)); R1:=exp (−(alpha*r));
```

$$S := r(t)$$
$$R := e^{-\alpha r}$$

```
> D4R:= diff (S,t); D1R:=−alpha*exp (−(alpha*S)); D2R:=−alphaˆ2*exp (−(alpha*S)); D5R:=D1R*D4R;
```

$$D4R := \frac{d}{dt}\, r\,(t)$$

$$D1R := -\alpha\, e^{-\alpha\, r(t)}$$

$$D2R := -\alpha^2 e^{-\alpha\, r(t)}$$

$$D1R := -\alpha\, e^{-\alpha\, r(t)} \frac{d}{dt}\, r\,(t)$$

```
> #Using simplified terms only from equation (A*8)
> SCHEQ3:= −h*D5R+D2R+D1R*2/S+8*piˆ2*G*M*mˆ2*R/(S*hˆ2);
> XX2:=factor (SCHEQ3);
```

$$XX2 := \frac{e^{-\alpha r(t)}\left(H^3\alpha \frac{dr(t)}{dt} r(t) - \alpha^2 r(t) H^2 - 2H^2\alpha + 8\pi^2 GMm^2\right)}{r\,(t) H^2} = 0$$

```
> #From standard solution of gravitational Schrodinger equation, we know (Rubcic, Fizika 1998):
> SCHEQ4:= 4*piˆ2*G*M*mˆ2−hˆ2*alpha;
```

$$SCHEQ4 := 4\pi^2 GMm^2 - H^2\alpha$$

```
> #Therefore time-dependent solution of Schrodinger equation may introduce new term to this gravitational Bohr radius.
> SCHEQ5:=(XX2*(S*hˆ2)/(exp (−(alpha*S))))−2*SCHEQ4;
```

$$ODESCHEQ5 := H^3\alpha \frac{dr(t)}{dt} r(t) - \alpha^2 r(t) H^2$$

```
> #Then we shall assume for simplicity by assigning value to d[r (t)]/dt:
> D4R:=1;
> Then we can solve again SCHEQ5 similar to solution of SCHEQ4:
> a2:=solve((hˆ3*alpha*(D4R)*S−alphaˆ2*S*hˆ2)+2*SCHEQ4);
```

$$a2 := \left\langle \begin{matrix} \alpha=\alpha,\ \pi=\pi,\ m=m,\ H=H,\ G=G,\ M=M, \\ t = RootOf(r(\_Z)\alpha H^3 - r(\_Z)\alpha^2 H^2 + 8\pi^2 GMm^2 - 2\alpha H^2) \end{matrix} \right\rangle,$$
$$\{\alpha=0,\ t=t,\ m=m,\ H=H,\ G=G,\ M=M,\ \pi=0\},$$
$$\{\alpha=0,\ \pi=\pi,\ t=t,\ m=m,\ H=H,\ M=M,\ G=0\},$$
$$\left\{\pi=\pi,\ t=t,\ m=m,\ H=H,\ M=M,\ \alpha=H,\ G=\frac{H^3}{4\pi^2 Mm^2}\right\},$$
$$\{\alpha=\alpha,\ H=0,\ \pi=\pi,\ t=t,\ m=m,\ M=M,\ G=G\},$$
$$\{\alpha=0,\ \pi=\pi,\ t=t,\ m=m,\ H=H,\ G=G,\ M=0\},$$
$$\{\alpha=0,\ \pi=\pi,\ t=t,\ H=H,\ G=G,\ M=M,\ m=0\},$$
$$\{\alpha=\alpha,\ H=0,\ \pi=\pi,\ t=t,\ m=m,\ G=G,\ M=0\},$$
$$\{\alpha=\alpha,\ H=0,\ \pi=\pi,\ t=t,\ G=G,\ M=M,\ m=0\}$$

```
> #Therefore one could expect that there is time-dependent change of gravitational Bohr radius.
```

Appendix 3 Time-independent gravitational Schrödinger equation with Yukawa potential [5]

```
> #Extension of gravitational Schrodinger equation (Rubcic, Fizika 1998);
> restart;
> #departure from Newton potential;
> R:=exp (−(alpha*r));
```

$$R := e^{-\alpha r}$$

```
> D1R:= diff (R,r); D2R:= diff (D1R,r);
```

$$D1R := -\alpha\, e^{-\alpha r}$$
$$D2R := -\alpha^2 e^{-\alpha r}$$

> SCHEQ2:=D2R+D1R*2/r+8*piˆ2*(G*M−K*exp (−2*mu)*(1+mu*r))*mˆ2*R/(r*hˆ2)=0;

$$ODESCHEQ := \alpha^2 e^{-\alpha r} - \frac{2\alpha e^{-\alpha r}}{r} + \\ + \frac{8\pi^2(GM - Ke^{-2\mu}(1+\mu r))m^2 e^{-\alpha r}}{rH^2} = 0$$

> XX2:=factor(SCHEQ2);
> RR1:=solve(XX2,r);

$$RR1 := -\frac{2(-H^2\alpha + 4\pi^2 GMm^2 - 4\pi^2 m^2 Ke^{-2\mu})}{-\alpha^2 H^2 + 8\pi^2 m^2 Ke^{-2\mu}}$$

> #from standard gravitational Schrodinger equation we know:
> SCHEQ3:=4*piˆ2*G*M*mˆ2−hˆ2*alpha=0;
> a:=solve(SCHEQ3, alpha);
> #Gravitational Bohr radius is defined as inverse of alpha:
> gravBohrradius:=1/a;

$$r_{gravBohr} := \frac{H^2}{4\pi^2 GMm^2}$$

> #Therefore we conclude that the new terms of RR shall yield new terms (YY) into this gravitational Bohr radius:
> PI:=(RR*(alphaˆ2*hˆ2)−(−8*piˆ2*G*M*mˆ2+2*hˆ2*alpha));
> #This new term induced by pion condensation via Gross-Pitaevskii equation may be observed in the form of long-range potential effect. (see Moffat J., arXiv: astro-ph/0602607, 2006; also Smarandache F. and Christianto V. Progress in Physics, v. 2, 2006, & v. 1, 2007, www.ptep-online.com)
> #We can also solve directly:
> SCHEQ5:=RR*(alphaˆ2*hˆ2)/2;

$$SCHEQ5 := \frac{\alpha^2 H^2(-H^2\alpha + 4\pi^2 GMm^2 - 4\pi^2 m^2 Ke^{-2\mu})}{-\alpha^2 H^2 + 8\pi^2 m^2 Ke^{-2\mu}}$$

> a1:=solve(SCHEQ5, alpha);

$$a1 := 0, 0, \frac{4\pi^2 m^2(GM - Ke^{-2\mu})}{H^2}$$

> #Then one finds modified gravitational Bohr radius in the form:
> modifgravBohrradius:=1/(4*piˆ2*(G*M−K*exp (−2*mu))*mˆ2/hˆ2);

$$r_{modified.gravBohr} := \frac{H^2}{4\pi^2 m^2(GM - Ke^{-2\mu})}$$

> #This modification can be expressed in chi-factor:
> chi:=modifgravBohrradius/gravBohrradius;

$$\chi := \frac{GM}{GM - Ke^{-2\mu}}$$

A Cantorian Superfluid Vortex and the Quantization of Planetary Motion

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This article suggests a preliminary version of a Cantorian superfluid vortex hypothesis as a plausible model of non-linear cosmology. Though some parts of the proposed theory resemble several elements of what have been proposed by Consoli (2000, 2002), Gibson (1999), Nottale (1996, 1997, 2001, 2002a), and Winterberg (2002b), it seems such a Cantorian superfluid vortex model instead of superfluid or vortex theory alone has *never* been proposed before. Implications of the proposed theory will be discussed subsequently, including prediction of some new outer planets in solar system beyond Pluto orbit. Therefore further observational data is recommended to falsify or verify these predictions. If the proposed hypothesis corresponds to the observed facts, then it could be used to solve certain unsolved problems, such as gravitation instability, clustering, vorticity and void formation in galaxies, and the distribution of planet orbits both in solar system and also exoplanets.

Keywords: multiple vortices, superfluid aether, nonlinear cosmology, gravitation instability, Bose-Einstein condensate, Cantorian spacetime, fluid dynamics.

Introduction

In recent years, there has been a growing interest in the quantum-like approach to describe orbits of celestial bodies. While this approach has not been widely accepted, motivating idea of this approach was originated from Bohr-Sommerfeld's hypothesis of quantization of angular momentum, and therefore it has some resemblance with Schrödinger's wave equation (Chavanis 1999, Nottale 1996, Neto *et al*. 2002). This application of wave mechanics to large-scale structures (Coles 2002) has led to several impressive results in terms of the prediction of planetary semimajor axes, particularly to predict orbits of exoplanets (Armitage *et al*. 2002, Lineweaver *et al.* 2003, Nottale *et al*. 1997, 2000, Weldrake 2002). However, a question arises as how to describe the physical origin of wave mechanics of such large-scale structures. This leads us to hypothesis by Volovik-Winterberg of superfluid phonon-roton as quantum vacuum aether (Volovik 2001, Winterberg 2002a, 2002b).

In this context, gravitation could be considered as result of diffusion process of such Schrödinger-like wave equation in the context of Euler-Newton equations of motion (Kobelev 2001, Neto *et al*. 2002, Rosu 1994, Zakir 1999, Zurek 1995). And large-scale structures emerge as condensed objects within such a quantum vacuum aether.

In the mean time, despite rapid advancement in theoretical cosmology development, there are certain issues that remain unexplainable in the presently available theories; one of these issues concern the origin and nature of gravitation instability (Coles 2002, Gibson 1999). Recent studies that have incorporated condensation, and void formation occurring on the non-acoustic density nuclei produced by turbulent mixing, appear to indicate that the universe is inherently *nonlinear* nature. Thus a very different nonlinear cosmology is emerging to replace the presently accepted linear cosmology model.

For instance, recently Gibson (1999) suggested that the theory of gravitational structure formation in astrophysics and cosmology should be revised based on *real* fluid behavior and turbulent mixing[i] theory, which leads us to nonlinear fluid model. His reasoning of this suggestion is based on the following argument: "*The Jeans theory of gravitational instability fails to describe this highly nonlinear phenomenon because it is based on a linear perturbation stability analysis of an inadequate set of conservation equations excluding turbulence, turbulent mixing, viscous forces, and molecular and* gravitational *diffusivity."* This is because Jeans' theory neglects viscous and nonlinear terms in Navier-Stokes *momentum equations*, thus reducing the problem of gravitational instability in a nearly uniform gas to one of linear acoustics.[ii]

In related work, Nottale (1996, 1997) argued that equation of motion for celestial bodies could be expressed in terms of a scale-relativistic Euler-Newton equation.[iii] By separating the real and imaginary part of Schrödinger-like equation, he obtained a generalized Euler-Newton equation and the continuity-equation (which is therefore now part of the dynamics), so the system becomes (Nottale 1997, Nottale *et al*. 2000 p. 384):

$$m.(\partial / \partial t + V.\nabla)V = -V(\phi + Q) \tag{1a}$$

$$\partial\rho / \partial t + div(\rho V) = 0 \tag{1b}$$

$$\Delta\phi = -4\pi G\rho \tag{1c}$$

It is clear therefore Nottale's basic Euler-Newton equations above, while including the inertial vortex force, neglect viscous terms ($-\nu\,\Delta\mathbf{V}$) in Navier-Stokes momentum equations,[iv] so his equations will obviously lead us to certain reduction of gravitational instability phenomena similar to Jeans' theory. Though Nottale's expression could offer a plausible explanation on the origin of *dark energy* (Ginzburg 2002, Nottale 2002a p. 20-22, Nottale 2002b p. 13-14), his expression appears to be not complete enough to explain other phenomena in a nonlinear cosmology, such as clustering, gravitation condensation and void formation.

Therefore the subsequent arguments will be based on a more complete form of Navier-Stokes equations including inertial-vortex force (Gibson 1999). Furthermore in the present article, two basic conjectures are proposed, i.e.

(i) in accordance with Thouless *et al*. (2001), it is proposed here: Instead of using the Euler-Lagrange equation, '*the nonlinear Navier-Stokes equations are applicable to represent the superfluid equations of motion'.* By doing so we can expect to obtain an extended expression of Nottale's Euler-Schrödinger equations (Nottale 1996, 1997, 2000, 2001, 2002a).

(ii) by taking into consideration recent developments in Cantorian spacetime physics, particularly by Castro *et al.* (2000, 2001) and Celerier & Nottale (2002), we propose that *modeling the universe using superfluid aether is compatible (at least in principle) with Nottale's scale relativity framework*. This is the second basic conjecture in this article.[v]

Accordingly, this article suggests that the nonlinear dynamics of Cantorian vortices in superfluid aether can serve as the basis of a nonlinear cosmological model. The term 'Cantorian' here represents the notion of 'transfinite set' introduced by Georg Cantor.[vi] Recently this term has been reintroduced for instance by Castro *et al.* (2000) and Castro & Granik (2001) to describe the exact dimension of the universe. As we know, a transfinite set is associated with the mapping of a set onto itself, producing a '*self-similar*' pattern. This pattern is observed in various natural phenomena, including turbulence and tropical hurricane phenomena.

Turbulence usually occurs when conditions of low viscosity and high-speed gradients are present. A turbulent fluid can be visually identified by the presence of *vortices*. As we know, a flow pattern, whose streamlines are concentric circles, is known as circular vortex (vortice). If the fluid particle rotates around its own axis, the vortex is called rotational. Such vortices continually form and evolve over time, giving rise to highly complex motions. In this context, vortices are defined as the curl of the velocity ($\nabla \times V$) in Navier-Stokes equations.[vii] Landau describes turbulence as a superposition of an *infinite* number of vortices, with sizes varying over all scales (this '*all scales*' notion leads us to Cantorian term). From the large scale vortices, energy is transmitted down to smaller ones without loss. The energy of the fluid is finally dissipated to the environment when it reaches the smallest vortices in the range of scales. The solutions to the velocity field are unique when the helicity $= v \,.\, \mathrm{curl}\ v = 0$; otherwise the solutions are not unique.

As we know, real fluid flow is never irrotational, though the mean pattern of turbulent flow outside the boundary layer resembles the pattern of irrotational flow. In rotational flow of real fluids, vorticity can develop as an effect of viscosity. Provided other factors remain the same, vortices can neither be created nor destroyed in a non-viscous fluid. Since the vortex moves with the fluid, vortex tube retain the same fluid elements and these elements retain their vorticity. The term 'vorticity' here is defined as the number of circulations in a certain area, and it equals to the circulation around an elemental surface divided by the area of the surface (supposing such vortex lattice exists within equal distance).[viii]

In quantum fluid systems like superfluidity, such vortices are subject to quantization condition of integer multiples,[ix] i.e. they are present in certain N number of atoms, as experimentally established in the superfluid phase of ^{4}He,

$$\oint v_s.dl = 2\pi.n\hbar / m_4 = n.\kappa_o \qquad (2)$$

where m_4 is the helium particle mass, and κ_o is the quantum of circulation (Nozieres & Pines 1990, Thouless *et al*. 2001). Furthermore, quantized vortices is a topological excited state, which takes form of circulation with equidistance distribution known as vorticity (Carter 1999, Kiehn 2001). Usually the Landau two-fluid model is used, with a normal and superfluid component. The normal fluid component always possesses some nonvanishing amount of viscosity and mutual friction; therefore it could exhibit quantum vorticity as observed in Ketterle's experiments.

A 'Cantorian vortice' can be defined in simple terms as tendency of the dynamics of both fluids and superfluids to produce multiple regions of vortex and circulation structures at various *scales* (Barge & Sommeria 1995, Castro *et al*. 2002, Chavanis 1999, Kobelev 2001, Nozieres & Pines 1990, Volovik 2000b, 2000c). In principle, the notion of Cantorian Superfluid Vortex suggests that there is a tendency in nature, particularly at the astronomical level scale, to produce mini vortices within the bigger vortices *ad infinitum*. Though some parts of the proposed theory resemble several elements of what have been proposed by Consoli (2000, 2002), Gibson (1999), Nottale (1996, 1997, 2001, 2002a), Volovik (2000a, 2000b, 2001), and also Winterberg (2002a, 2002b), to the author's present knowledge the idea of using a Cantorian superfluid vortex model instead of (ordinary) superfluid model or vortex theory alone has *never* been proposed before. The Cantorian term here implies that such a superfluid vortice is—in accordance with Landau's definition of turbulence—supposed to exist both as quantum vacuum aether background (micro phenomena) and as representation of various condensed objects such as neutron stars (macro phenomena). The proposed hypothesis results in a non-homogenous isotropic Euclidean flat-spacetime expanding universe at *all scales*, but *without* a cosmological constant. This cosmology constant nullity is somewhat in accordance with some recent articles, for instance by Guendelman *et al*. (2002), Volovik (2001), and Winterberg (2002a, 2002b).

Implications of the proposed model will be discussed subsequently, where first results of the method yield improved prediction of three new planets in outer planet orbits of the solar system beyond Pluto. If the predictions of the proposed hypothesis correspond to the observed facts, it is intuitively conjectured that the proposed theory could offer an improved explanation for several unexplainable things (at least not yet in a quantifiable form) in regards to the origin of gravitation instability, void formation, and unifying gravity and quantum theory.

A review of recent developments

Throughout the last century of theoretical physics since Planck era, physicists have investigated almost every conceivable idea of how geometry can be used or modified to describe physical phenomena. For instance, Minkowski refined his 4D spacetime-geometry to explain Einstein's STR. Others have come up with 5D (Kaluza-Klein), 6D, and then ten D, eleven D, and recently 26D (bosonic string theory as a dual resonance model in 26D; see Winterberg 2002a). It seems like the number of geometrical dimensions simply grow with time. We could also note a considerable amount of study has been devoted to geometry with infinite-dimension or Hilbert space.

However, recently it seems there is also a reverse drift of simplifying these high dimensional (integer) numbers, for instance by use of the replacement of the dual resonance model in 26D with QCD in 4D to describe nuclear forces; and by using of the aforementioned analogies between Yang-Mills theories and *vortex dynamics*, there is a suggestion that string theory should perhaps be reinstated by some kind of vortex dynamics at the Planck scale (Winterberg 2002a). Furthermore, Castro *et al*. (2000, 2001) have proposed that the *exact* dimension of the universe is only a bit higher than Minkowskian 4D (less than 5D). They arrived at this conclusion after reconciling Cantorian spacetime geometry with the so-called Golden Section. Therefore instead of proposing a trivial argument over which geometry is superior, this article proposing accepting the hypothesis that the Cantorian fractal spacetime dimension as proposed by Castro *et al.* (2000) can be the *real* geometric dimension of the universe. This fractal dimension will be called the Cantorian-Minkowski dimension. This conjecture is somewhat in accordance with a recent suggestion made by Kobelev (2001) that Newton equation is a diffusion equation of multifractal universe.

In the mean time, despite the fact that most theoretical physics efforts are devoted toward the proper expressions of fields, fields are not the only objects which one can think as occupying spacetime, there are also fluids. When there is no equation of state specified they are *more general* than fields (Roberts 2001).[x] In this regards quantum fluids, which are usually understood as a limited class of objects used to describe low-temperature physics phenomena, have in recent years been used to model various cosmological phenomena, for instance neutron stars (Andersson & Comer 2001, Elgaroy & DeBlassio 2001, Sedrakian & Cordes 1997, Yakovlev 2000). It is not surprising therefore that there is increasing research in using superfluid model to represent cosmology dynamics (Liu 2002, Roberts 2001, Volovik 2000a, 2000b, 2000c, 2001, Zurek 1995).

In this context, it is worth noting here some recent development in superfluidity research. This direction of research includes application of NLSE (Nonlinear Schrödinger equation) as a model of the Bose-Einstein condensate under various conditions (Quist 2002). There are also NLSE proposals representing Cantorian fractal spacetime phenomena (Castro *et al*. 2002). Experiments on Bose-Einstein condensates have now *begun to address vortex systems*. Superfluid turbulence issues and its explanation in terms of quantum *vortex dynamics* have become one of the most interesting physics research these days (Volovik 2000a, 2002b, Zurek 1995). For instance, recent experiments in the past few years showed that some turbulent flows of the superfluid phase of ^4He (helium II) are similar to analogous turbulent flow in a classical fluid (Thouless *et al.* 2001). In theoretical realm, there is also new interest in the relationship between the topology (broken by reconnections, hence release of energy) and the geometry of structure—sometimes known as *topological defects* in cosmology (Yates 1996, Zurek 1995)—which cannot be changed arbitrarily as done traditionally by topologists but changes according to the dynamics (NLSE or Navier-Stokes equation[xi]).

Winterberg (2002a) has suggested that the universe can actually be considered an Euclidean flat-spacetime provided we include superfluid aether quantum vacuum into the model. Winterberg's aether is a densely filled substance with an equal number of positive and negative Planck masses $m_P = \sqrt{(hc/G)}$ which interact locally through contact-type delta-function potentials. In the framework of this approach Winterberg (2002a, 2002b) has shown that quantum mechanics can be derived as an approximate solution of the Boltzmann equation for the Planck aether masses. The particle in his model is a formation appeared as result of the interaction between the positive and negative Planck masses similar to the phonon in a solid. This suggestion is seemingly in a good agreement with other study of gravity phenomena as long wave-length excitation of Bose-Einstein condensate by Consoli (2000, 2002). Consoli (2000) noted that the basic idea that gravity can be a long-wavelength effect induced by the peculiar ground state of an underlying quantum field theory leads to considering the implications of spontaneous symmetry breaking through an elementary scalar field. He pointed out that Bose-Einstein condensation implies the existence of long-range order and of a gap-less mode of the Higgs-field. This gives rise to a 1/r potential and couplings with infinitesimal strength to the inertial mass of known particles. If this is interpreted as the origin of Newtonian gravity one finds a natural solution of the hierarchy problem. In the spirit of Landau, Consoli (2000, 2002) has also considered similarity between his condensate model and superfluid aether hypothesis. Furthermore, he also suggested: "*all classical experimental tests of general relativity would be fulfilled in any theory incorporating the Equivalence Principle*."

Furthermore, recently Celerier & Nottale (2002) have shown that the Dirac equation can be derived from the scale relativity theory. Since the Dirac equation implies the existence of aether, this derivation can be interpreted as: modeling superfluid aether in the universe is compatible (at least in principle) with Nottale's scale relativity framework.[xii] Nottale's conjecture on the applicability of the Schrödinger equation to describe macroscopic phenomena (up to astronomic scale) seems also to imply the presence of a certain form of fluid (aether) as the medium of vacuum quantum fluctuation or a zero point field (Roberts 2001). And because the only type of matter capable of resembling such quantum phenomena macroscopically is Bose-Einstein condensate or its special case superfluid (Consoli 2000, 2002), then this leads us to a conjecture that the *aether medium is very likely a quantum fluid*.

Combining the character of these selected recent developments, this article suggests that the nonlinear wave dynamics of Cantorian vortices of superfluid aether can serve as the basis of a nonlinear cosmological model, which will be capable of describing various phenomena including a plausible mechanism of continuous particle generation in the universe. The preceding work (albeit somewhat controversial from the present accepted view) suggests that this alternative and nonlinear cosmological model shall include: (a) an aether, (b) Euclidean flat spacetime[xiii], (c) vortex dynamics, (d) superfluid (Bose-Einstein condensate), and (e) fractal phenomena—as the basis of real physical model and also the theoretical analysis of nonlinear cosmology. It is the opinion of this author that a proper combination will lead us to a consistent real model.

Therefore, in theoretical terms this article argues in favor of combining *Cantorian-Minkowski geometry with Nottale-Gibson-Winterberg's vortex of superfluid aether.* The proposed model results in a Euclidean flat spacetime with some fluctuations induced by fractal phenomena (expressed as a non-integer dimension in Cantorian universe) arising from multiple vortices. A real physically-observed model is chosen here instead of geometrical construct, because it will directly lead us to a set of experimental tests which can be used to determine if the model is not valid. With regards to superfluidity research, perhaps the conjectures of this article can be considered as extending Volovik's (2000a, 2000b, 2001) superfluid theory to Cantorian spacetime case.

A derivation of the basic vortex model and quantization of semimajor axes

The Schrödinger equation of wave mechanics can be interpreted as a description for the tendency of micro aggregates of matter to make structures. In this regards, Nottale (1993, 1996, 1997) put forth a conjecture that spacetime is *non-differentiable,*[xiv] which led to a fractal version of the Schrödinger-like equation capable of predicting the semimajor axes of both planetary-like systems as well as micro orbits at molecular level. This reasoning could be considered as an alternative interpretation of Ehrenfest Theorem.

However, such a quantum-like approach in a large-scale structure has not been widely accepted (Coles 2002), for the quantization of macroscopic systems is something outside the scope of known physics (Neto *et al*. 2002). Nevertheless, some possible origins for such effects have been outlined. For instance Bohr-Sommerfeld's hypothesis of quantization of angular momentum, appears to be more direct than the Schrödinger-like equation, at least for (planar case of) planetary orbits in the solar system. For a spherical case (for some exoplanet systems) we should derive solution of the Schrödinger-like equation.

As we know, for the wave function to be well defined and single-valued, the momenta must satisfy Bohr-Sommerfeld's quantization conditions (Van Holten 2001):

$$\oint_{\Gamma} p.dx = 2\pi.n\hbar \tag{3}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is

$$\int_0^T v^2.d\tau = \omega^2.T = 2\pi.\omega \tag{3a}$$

where $T = 2\pi/\omega$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$.

Then the force balance relation of Newton's equation of motion:

$$GMm / r^2 = mv^2 / r \tag{3b}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (3a), a new constant g was introduced (which plays the role of a gravitational analog of the Planck constant):

$$mvr = ng / 2\pi$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form:

$$r = n^2.g^2 / (4\pi^2.GM.m^2) \tag{5}$$

or

$$r = n^2.GM / v_o^2 \tag{6}$$

where r, n, G, M, v_o represents semimajor axes, quantum number (n = 1,2,3,…), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (6), we denote

$$v_o = (2\pi / g).GMm \tag{6a}$$

This result (6) is the same as Nottale's basic equation for predicting semimajor axes of planetary-like systems (Nottale 1996, Nottale *et al.* 1997, 2000). It can be shown that equation (6) could be derived directly from the Schrödinger equation for planar case (Christianto

2001), therefore it represents the solution of the Schrödinger equation for *planar axisymmetric cylindrical case*. The value of m is an adjustable parameter (similar to g). For a planetary system including exoplanets Nottale *et al*. (1997, 2000) has found the specific velocity v_o is $\pm$ 144 km/s. Therefore this equation (6) implies the semimajor axes distribution can be predicted from a sequence of quantum numbers. This equation (5) is also comparable with Neto *et al*.'s (2002) approach, where they propose $m = 2.1 \times 10^{26}$ kg (the average mass of the planets in solar system).

It is worth noting here Nottale *et al*. (1997, 2000) reported this equation (6) agrees very well with observed data including those for exoplanets, and particularly for *inner planet orbits* in the solar system. Indeed the number of exoplanets found has increased fivefold since their first study (Nottale *et al*. 2000). However, a question arises when we compare this prediction with *outer planet orbits* in the solar system, since this results in very low predictions compared with observed data, i.e. 52.6% for Jupiter, 36.3% for Saturn, 22.3% for Uranus, 17.2% for Neptune, and 15.6% for Pluto. Therefore, Nottale (1996) proposed to use a different value for v_o to get the distribution of outer planets (the so-called Jovian planets).

Nottale (1996) proposed a plausible explanation for this discrepancy by suggesting outer planets from Jupiter to Pluto are part of different systems since they apparently consist of *different* physical and chemical planetary compositions, so we can expect two different diffusion coefficients for them. Therefore he proposed the following relation to predict orbits of inner planets and outer planets (Nottale 1996, p. 51) $a = n.(n + \frac{1}{2}).a_o$. Nottale then suggested the proper values are $a_{o.inner} = 0.038025$AU for inner orbits and $a_{o.outer} = 1.028196$AU for outer orbits, and based on these values the discrepancy in predicting outer planet distribution can be reconciled.

While Nottale's (1996, p. 53) description on these different chemical and physical compositions, distribution of mass, and distribution of angular momentum seem to be at least near to right, he did not offer any explanation of *why* there are different chemical and physical compositions if these outer planets were generated by the same Sun in the past. Nottale's proposed equation was based on the second quantum number *l*, derived from Schrödinger-type equation for *spherical case.* However, it should be noted that while the second quantum number could plausibly explain the different orbits for outer planets, it cannot provide any explanation for their different chemical and physical compositions. Therefore, this leads us to a conjecture, i.e. these *differences of planetary distribution and different chemical and physical compositions of the outer planets in the solar system are the consequences of the interaction of a negative mass (star) with the Sun*.[xv] From this author's opinion, it seems only through using this conjecture we could explain why the outer planets are physico-chemically different from the inner planets. From this conjecture, then we reinterpreted Nottale's conjecture that Jupiter should be the second planet ($n = 2$) in the outer orbit system, to obtain predicted values of semimajor axes of those Jovian planets, based on the notion of reduced mass μ. The result of this approach will be described subsequently.

Another plausible explanation of the outer planets distribution has been suggested by Chavanis (1999) based on *two-fluids model*. However, while this suggestion is in good agreement with observation of outer planet orbits, in the author opinion it also does not offer a convincing argument for the difference of chemical and physical composition *if* those inner and Jovian planets were generated by the same Sun.

Now let's turn our attention to the implications of equation (6) in regards to the basic vortex model. If T is the orbit period of the above planet around the Sun, then by Kepler's third law,

$$r^3 \approx T^2 \approx (2\pi r / v)^2 \tag{7}$$

Or

$$v^2 r \approx 4\pi^2 = k_{spring}$$

where r, T, v, k_{spring} represents semimajor axes, orbit period, orbit velocity, and 'spring constant' of the dynamics system, respectively.[xvi] For gravity case, one obtains $k_{spring} = G.M$. We remark here this constant k_{spring} could be comparable with Nottale's (Nottale *et al*. 2000) notion of parameter $D = G.M/2\omega$; thus $k_{spring} = D.2\omega = D.2\alpha_g c$. This alternative expression comes from the definition of gravitation coupling constant $\alpha_g = \omega/c$, where $\alpha_g^{-1} = 2072 \pm 7$ (Nottale *et al*. 2000).

By observing the above expressions, we conclude that equation (8) has the same basic form of Nottale's equation (6). We also note here Nozieres & Pines (1990) suggested that a vortex structure exists in a superfluid if its velocity is *radius-dependent* ($v = f(1/r)$). Since from equation (8) the quadratic of velocity is radius-dependent $v^2 = (k/r)$, we propose here that equation (8) also implies a *special case* of vortex motion. Therefore, we conclude equation (6) also implies a vortex motion. This seems to be in agreement with Nottale *et al*.'s (1997, 2000) assertion that specific velocity $v_o = 144$ km/s represents a new fundamental constant observed from the planetary up to extragalactic scale.

In order to generalize further equation (6), we proposed using Kobelev's (2001) idea that Newton's equations may be treated as a diffusion process in a multi-fractal universe. Provided such a relationship exists, we could conclude that equation (6) implies a

Cantorian fractality of vortex structure in the universe. But a question arises here as to whether a scaling factor is required to represent equation of motion of celestial bodies at various scales using equation (6). Therefore, by using a fractional derivative method as described by Kolwankar (1998, eq. 2.9), then

$$d^q f(\beta x)/[dx]^q = \beta^q.\{d^q f(\beta x)/[d(\beta x)]^q\} \qquad (9)$$

where it is assumed that for $dx \to 0, d(\beta x) \approx dx$. Hence this author obtained (Christianto 2002b) a linear scaling factor for equation (6):

$$a_0 = \phi.n^2.GM / v_o^2 \qquad (10)$$

This equation implies :

$$v_1^2 = (v_o^2 / \phi_o) \qquad (11)$$

In other words, for different scaling reference frames, specific velocity v_1 may vary and may be influenced by a scale effect ϕ. To this author's present knowledge, such a scaling factor has never appeared before elsewhere; neither in Nottale's work (1996, 1997, 2001, 2002) nor in Neto *et al*. (2002). A plausible reason for this is that Nottale's and Neto *et al*.'s theory were intended to describe planetary orbits only.

A note on this interpretation is perhaps worth making. While of course this Cantorian fractality of vortex structure in the universe is not the only possible interpretation, we believe this is the nearest interpretation considering the turbulence phenomena.[xvii] It is known that turbulent flows seem to display *self-similar* statistical properties at length scales smaller than the scales at which energy is delivered to the flow (this sometimes referred to as 'multi-fractality' of turbulence). For instance, Kolmogorov argued that at these scales, in three dimensions, the fluids display universal statistical features (Bernard 2000, Foias *et al*. 2001 p. 17, Gibson 1991, Weinan 2000).

Turbulent flow is conventionally visualized as a cascade of large vortices (large scale components of the flow) breaking up into ever smaller sized vortices (fine-scale components of the flow) – the principal cascading entity is the 'enstrophy'.[xviii]

Recent observational data of the similar size of semimajor axes between solar system and exoplanet systems (a/M = 0.043 AU/M_o for n = 1; and a/M = 0.17 AU/M_o for n = 2) seems to indicate that those are clusters of celestial objects at the same hierarchy (scale) of quantized vortices (Armitage *et al*. 2002, Lineweaver *et al.* 2003, Neto *et al*. 2002, Nottale *et al*. 1997, 2000, Weldrake 2002). This seems to imply that the proposed Cantorian vortices interpretation is in good agreement with observed data.

Superfluid vortices model

It is worth discussing here the *rationale* for suggesting a Cantorian superfluid aether as a real physical model for nonlinear cosmology. This brings us back in time to where GTR was first introduced (in passing we note in pre-GTR era aether hypothesis was almost entirely abandoned because of the growing acceptance of STR; see Munera 1998).

It is known that in GTR there is no explicit description of the medium of interaction in space (aether), though actually this was considered by Einstein himself in his lecture in Leiden 1921, "*Ether and Relativity*" (Einstein 1921):

> *"..According to the general theory of relativity space without an ether is* ***unthinkable****; for in such a space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not*

be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it."

A perfect fluid in GTR is therefore could be thought of as a liquid medium with no viscosity and no heat induction. Such a perfect fluid is basically a special case of quantum liquid or superfluid (Nozieres & Pines 1990). We note the term 'special case' because the superfluid here should be able to represent non-ponderable (weightless) characteristic of the aether medium, though perhaps it could have motion.

It is clear therefore aether is inherently implied in a GTR geometrical construct (see also Consoli 2002). Furthermore, it is possible to explain the frame dragging phenomena in a GTR geometrical construct as it is actually a fluid vortex—with a massive object in its vortex centre (Prix 2000)—capturing a volume of surrounding fluid and entraining its rotation.

In Maxwell's hypothesis, aether is a frictionless fluid. Based on this conjecture Winterberg (2002a, 2002b) has proposed an aether model, which consists of a quantum fluid made up of Bose particles. This analogy leads to the Planckian aether hypothesis which makes the assumption the vacuum of space is a kind of plasma (see also Roberts 2001). The ultimate building blocks of matter are Planck mass particles obeying the laws of classical Newtonian mechanics, but there are also *negative* Planck mass particles. Furthermore, with the Planck aether having an equal number of positive and negative Planck mass particles, the cosmological constant is zero and the universe is Euclidean flat-spacetime. In its groundstate the Planck aether is a two component positive-negative mass superfluid with a *phonon-roton* energy spectrum for each component.

The theory of superfluid vortices is based upon various versions of the Landau's two-component fluid model (Godfrey *et al*. 2001), and is adequately described by many researchers (Kivshar *et al*. 1998, Quist 2002, Thouless *et al*. 2001, Tornkvist & Schroder 1997, Volovik 2000c, 2001, Zurek 1995). For applications to Cosmology, it is presumed that the "vacuum" is a superfluid-like continuum in which the formation of topological defects as "vortices" generates the stars and galaxies as components of the normal fluid. The diffusive and dissipative Navier-Stokes fluid equations, with constraints that lead to the Complex Ginzburg-Landau equations to describe the superfluid, form the basis of the mathematical model. The topological defects can be homogeneously defined, hence they are self-similar, and scale covariant. Such topological defect domains can support not only fractals but also quantum like integer values for their closed integrals.

The conceptual map (Figure 1) depicts how the various parts of the most recent theories could plausibly be used to form a Cantorian superfluid vortex model for nonlinear cosmology.

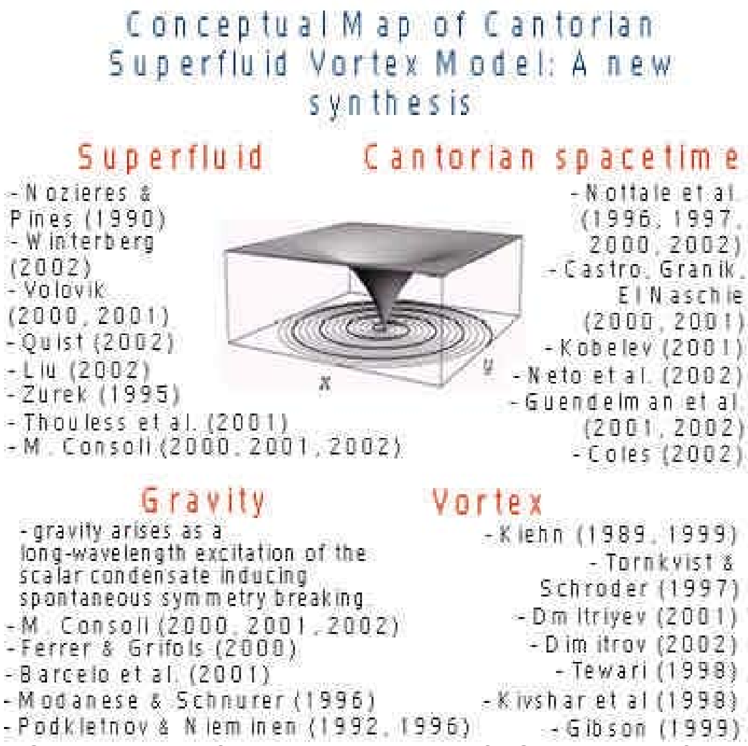

Figure 1. Conceptual map of the plausible synthesis of a Cantorian superfluid vortex model for nonlinear cosmology

Now we are going to illustrate how the equation of motion (6) is compatible with the proposed superfluid vortices model as described above. In other words, we will provide an argument to link the solution of the Schrödinger equation (6) with the solution of Navier-Stokes equations. Theoretically, R. Kiehn (1989, 1999) has shown that there is an exact mapping between the Schrödinger equation and Navier-Stokes equation, though without reference yet to its cosmological implications. Therefore now we extend his conjecture to a cosmological setting. In order to do this, we consider two approaches here:

- Gibson's (1999) Navier-Stokes model for cosmology;
- Godfrey *et al.*'s (2001) model of superfluid vortices.

First, we note here that Gibson (1999) has shown that his Navier-Stokes-Newton model yields the following solution:

$$v_r = m'.Gt / r^2 \tag{12}$$

where r, t, G, m', v_r represents semimajor axes, time elapsed, Newton gravitation constant, mass of the nucleus of orbit, and specific velocity, respectively. It is clear therefore that equation (12) admits mass growth rate as time elapsed, which is permitted by Gibson's Navier-Stokes model. Now we assert $v = 2\pi r / T$ or $r = vT / 2\pi = vt$, and substitute this value to one of r in equation (12). We get:

$$r = m'.G / v^2 \tag{13}$$

which is very similar to equation (6), except the expression for quadratic quantum number n^2. A plausible reason for this missing quantum number is that Gibson (1999) assumed a normal fluid in his model instead of quantum liquid. He also argued that equation (12) only governs the formation stage (such as spiral nebulae formation); while equation (13) is also applicable for present time provided we assert a quantum liquid for the system. Therefore we also conclude again that Nottale's equation (6) *actually implies a quantum liquid as medium of interaction.*

For the second method, we note here that according to Godfrey *et al*. (2001) the analytic form of an oscillating plane boundary layer flow of superfluid vortices can be derived from the Navier-Stokes equation, and the velocity u(*z,t*) is given by:

$$u = A.e^{-kz}.\cos(\omega t - kz) \tag{14}$$

where $k=\sqrt{(\omega/2\nu)}$, $\omega=2\pi/T$ is the angular frequency of oscillation, T is the period of oscillation, ν is the kinematic viscosity and A is an arbitrary constant. In the limit that the coupling of the superfluid and normal fluid components through mutual friction is negligible, we may take this oscillating velocity profile for the normal fluid, with the superfluid remaining at rest. Because we can assert velocity $u = dz/dt = d\Psi/dt$, therefore we can obtain Ψ and also its second differentiation $d^2\Psi/dt^2$. Hence we get:

$$d^2\Psi/dt^2 = -A.e^{-kz}.\sin(\omega t - kz).\omega \tag{15}$$

or

$$d^2\Psi/dt^2 + \omega^2.\Psi = 0 \tag{16}$$

which is the most basic form of the Schrödinger equation. In other words, we obtain the Schrödinger equation from a velocity expression derived from the Navier-Stokes equation for superfluid vortices (Godfrey *et al*. 2001). These two methods confirm Kiehn's (1989, 1999) conjecture that there is exact mapping between the Schrödinger equation and Navier-Stokes equation *regardless* of the scale of the system considered. This conclusion, which was based on a two-fluid model of superfluid vortices, is the *main result* of this article; and to this author's present knowledge this conclusion has never been made before for the astronomical domain (neither in Chavanis 1999, Neto *et al.* 2002, nor Nottale 1996, 1997, 2001, 2002). In this author opinion, Chavanis' article (1999) is the nearest to this approach, because he already considered two-fluid model for the Schrödinger equation (though without reference to superfluidity), though he did not mention the role of Navier-Stokes equations like Gibson (1999).

A distinctive feature of this proposed superfluid vortices approach is that we could directly compare our model with laboratory observation (Volovik 2001, Zurek 1995). For instance, using this model Godfrey *et al*. (2001) argued that the fluid at the edge of the disk moves a distance $4\phi_c R$ in a time T (with angular velocity $\omega = 2\pi/T$), thus having a critical dimensional linear velocity of

$$v_{disk} = 2\omega.\phi_c R/\pi \tag{17}$$

In this equation, ϕ_c represents critical amplitude where damping of the oscillations reduce to a value, which was interpreted as the damping due only to viscosity of the normal fluid component. In this regards, interpretation of the experiment is that superfluid boundary layer vortices are the cause of critical amplitude of oscillations observed. Therefore it seems we could expect to observe such critical amplitude for the motion of celestial objects. Of course for spherical orbit systems the equation of critical dimensional linear velocity is somewhat different from equation (17) above (Godfrey *et al*. 2001). To this author's present knowledge such theoretical linkage between critical amplitude of superfluid vortices and astronomical orbital motions has also never been made before; neither in Chavanis (1999), Nottale (1996, 1997, 2001, 2002), Volovik (2000a, 2000b, 2000c, 2001), nor Zurek (1995).

New planets prediction in solar system

Based on equation (6) and using Nottale's conjecture of Jupiter should be the second planet (n = 2) in the outer orbit system, we derive predicted and observed values of semimajor axes of those outer planets. Then by using Nottale's (1996, p. 53) conjecture for quantization of galaxy pairs, and minimizing the standard deviation (s) between these observed and predicted values, we can solve equation (6) for the reduced mass μ to get the most probable distribution for outer planet orbits:

$$\mu = (m_1.m_2)/(m_1 + m_2) \tag{18}$$

It is worth noting here, that a somewhat similar approach using reduced mass μ to derive planetary orbits has also been used by Neto *et al.* (2002), as follows:

$$-g^2/2\mu.\left(\partial^2\Psi/\partial r+\partial\Psi/r\partial r+r^{-2}.\partial^2\Psi/\partial\varphi^2\right)+V\Psi=E\Psi \quad (18a)$$

though Neto *et al*. (2002) did not come to the same conclusion as presented here. Result of this method (18) is presented in Table 1 below.

Msun = 0.198951 10^34 g k = 26.60 k.Msun = 5.29 10^34 g

Astroobject	Observed			Prediction based on Nottale-Ord				
	Orbit Size	r_{actual} (AU x 10)[1]	r_{actual} (10^6 km)	n	r_{pred} (10^6 km)	n	r_{pred}/r_{actual} (%)	$(r_{pred}-r_{act})^2$
				1	6.40			
				2	25.60	2		
Mercury	4	3.87	57.89	3	57.60	3	99.484	0.1
Venus	7	7.32	109.51	4	102.39	4	93.504	50.6
Earth	10	10.00	149.60	5	159.99	5	106.945	107.9
Mars	16	15.24	227.99	6	230.38	6	101.050	5.7
Hungarias[3]			314.00	7	313.58	7	99.865	0.2
Asteroid[4]		27.00	403.91	8	409.57	8	101.400	32.0
		31.50	471.23	9	518.36	9	110.001	2220.9
Jupiter	52	52.03	778.36	2	681.01	2	87.493	9476.8
Saturn	100	95.39	1427.01	3	1532.27	3	107.376	11078.9
Uranus	196	191.90	2870.78	4	2724.04	4	94.888	21534.7
Neptune	388	301.00	4502.90	5	4256.31	5	94.524	60806.4
Pluto	722	395.00	5909.12	6	6129.08	6	103.722	48384.8
Π_1				7	8342.36	7		
Π_2[5]				8	10896.14	8		
Π_3[6]				9	13790.43	9		

Table 1. Predicted orbit values of inner and outer planets in Solar system

From Table 1 above we obtain $\mu = 26.604.m_1$, for the minimum standard deviation $s = 0.76$AU.[xix] Inserting this μ value into equation (18) and solving it, we get the most likely companion mass of $m_2 = -(26.604/25.604).m_1$. Therefore we conclude it is *very likely* there is a negative-mass star (NMS) interacting with the Sun. This NMS has a mass value of very near to the Sun but with a negative sign, so this can be considered as the dim twin-companion star of the Sun. This is somewhat comparable to what some astronomers suggest of the hypothetical '*dark star'* (Damgov *et al*. 2002), though to this author's present knowledge none of the existing astronomic literatures has considered a negative-mass star as plausible candidate of the twin-companion of the Sun. Therefore thus far, this conclusion of the plausible presence of a large negative-mass object in the solar system could only be explained using *superfluid/superconducting model* (DeAquino 2002). [xx]

On the basis of this value of $\mu = 26.604.m_1$, we obtained a set of predicted orbit values for both inner planets and Jovian planets. For inner planets, our prediction values are very similar to Nottale's (1996) values, starting from n = 3 for Mercury; for n = 7 Nottale reported minor object called Hungarias; for Jovian planets from n = 2 for Jupiter up to n = 6 for Pluto our prediction values are also somewhat similar with Nottale's (1996) values. It is worth noting here, we don't have to invoke an *ad hoc* quantum number to predict orbits of Venus and Earth as Neto *et al*. (2002) did. We also note here that the proposed method results in prediction of orbit values, which are within a 7% error range compared to observed values, except for Jupiter which is within a 12.6% error range.

The departure of our predicted values compared to Nottale's predicted values (1996, 1997, 2001) appear in outer planet orbits starting from n = 7. We proposed some new predictions of the possible presence of three outer planets beyond Pluto (for n = 7, n = 8, n = 9) to be called here as Π_1, Π_2, Π_3 at orbits around 55.77 ± 1.24AU, 72.84 ± 1.24AU, and 92.18 ± 1.24AU, respectively. This prediction of most likely semimajor axes has taken into consideration standard deviation found above s = 0.76AU (Table 1). Two of these predicted orbits of outer planets are somewhat in agreement with previous predictions by some astronomers on the possible presence of outer planets beyond Pluto around ~50AU and around ~100AU (Horner *et al*. 2001). However, it is worth noting here, the predicted planet (for n = 8) at orbit 72.84 ± 1.24AU is purely based on equation of quantization of orbit (6) for Jovian planets. It is also worth noting here, that these proposed planets beyond Pluto are different from what is predicted by Matese *et al*. (1999), since Matese's planet is supposed to be somewhere around the outer Oort cloud.

Further remarks are worth considering here concerning predicted orbits at $n = 8$ and $n = 9$. We consider first for the case of inner orbits. It was suggested by Olber and also recently by Van Flandern in 1993 (Damgov *et al*. 2002) of a planet (or *planets*) existed until relatively recently between Mars and Jupiter, at the location where a missing planet is expected by the well-known Titius-Bode law (see Table 1 under column 'Orbit size'). As we know, Titius-Bode law was based on series of numbers 0,3,6,12,24,48,96… which then translated by factor 4. Thus we have series of 4,7,10,16,28,52,… which are supposed to be able to predict the orbit size of planets in solar system. This argument was subsequently supported by Nottale's equation except for orbits at $n = 7$ and $n = 9$, between Mars and Jupiter, which can be regarded as departure from the Titius-Bode law. However, while Nottale (1996, p. 51) has reported planets (or at least, recognizable objects) at $n = 8$ and $n = 9$ for *inner* orbit in solar system were observed, to our present knowledge no similar prediction has been made for $n = 8$ and $n = 9$ for outer orbits. Therefore new observational data is highly recommended to find the real semimajor axes of the proposed new outer planets beyond Pluto.

If these new outer planets correspond to the observational data, it is conjectured intuitively that the proposed Cantorian superfluid vortices model could offer an improved explanation for several things unexplainable (at least not yet in a observable and quantifiable form) thus far with regards to the origin of continuous particle generation, gravitation instability, and unifying gravity and quantum theory.

Notes on the superfluid experiments for cosmology: fractal superfluid

Zurek (1995) and Volovik (2000b) have proposed some aspects of superfluid analogies to describe various cosmological phenomena. However, extending this view towards Cantorian Superfluid Vortex hypothesis implies we should be able to observe fractal phenomena of superfluid and also Bose-Einstein condensate systems. While this has not become the accepted view, recent articles indicate such phenomena were *already* observed (Kivotides *et al.* 2001, 2001b, Ktitorov 2002).

In this regards, some recent observations have shown that the number of galaxies N(r) within a sphere of radius r, centered on any galaxy, is not proportional to r^3 as would be expected of a homogeneous distribution. Instead N(r) is proportional to r^D, where D is approximately equal to 2, which is symptomatic of distribution with fractal dimension D. It is interesting to note, that for $D = 2$, the cosmological gravitational redshift gives the linear distance-redshift relation and becomes an observable phenomenon (Mittal & Lohiya 2001). This non-integer dimension is known as Hausdorff dimension d_H, which can be computed to be within the range of $1.6 \sim 2.0$ up to the scale $1 \sim 200$ Mpc (Baryshev 1994, 1999). Furthermore, transition to homogeneity distribution has not been found yet. In this regards Anderson *et al.*[xxi] also admitted: "*These findings (of clustering and void formation) have become increasingly difficult to reconcile with standard cosmological theories, in which the approach to homogeneity at large-scales is central element.*" What more interests us here is that an extended version of Gross-Pitaevskii equation admits self-similar solutions and also it corresponds to Hausdorff dimension $d_H \sim 2$, which seems to substantiate our hypothesis that there is exact correspondence between cosmological phenomena and condensed matter physics.[xxii]

In principle, the proposed Cantorian Superfluid Vortex theory leads us to a fractal superfluid description of Euclidean flat-spacetime universe, which is scale-invariant and expanding at all scales, but without a cosmological constant (this was also suggested by Guendelman *et al.* 2002, Winterberg 2002a, 2002b). This Cantorian Superfluid Vortex model is inhomogeneous though it is perhaps isotropic (in accordance with Einstein-Mandelbrot Cosmological Principle; Mittal & Lohiya 2001). Gibson (1999) has also described how the nonlinear cosmology model based on Navier-Stokes equations could explain the hidden-universe problem. Furthermore, it seems that the superfluid vortice model could explain why the inner cylindrical core of earth rotates independently of the rest of the planet.[xxiii]

It seems therefore we could expect that further research will divulge more interesting fractal phenomena of Bose-Einstein condensate and superfluid systems (somewhat related to superfluid turbulence and its damping phenomena; Godfrey *et al*. 2001), which could lead us to further generalization of the proposed Cantorian Superfluid Vortex model.

A new method to predict quantization of planetary orbits has been proposed based on a Cantorian superfluid vortex hypothesis. It could be expected that in the near future there will be more precise nonlinear cosmology models based on real fluid theory.

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Endnotes

[i] Term 'turbulent mixing' here has been used in accord with Gibson's original terminology. Turbulence is defined as "an eddy-like state of fluid motion where the inertial-vortex forces of the eddies are larger than the viscous, buoyancy, electromagnetic or any other forces which tend to damp the eddies." Furthermore, natural flows at very high Reynolds, Froude, Rossby numbers in the ocean, atmosphere, stars and interstellar medium develop highly intermittent turbulent and mixing (Gibson 1991, also Foias et al. 2001).

[ii] For other publications of C. Gibson related to this issue, see arXiv.org: astro-ph/9904230, astro-ph/9904237, astro-ph/9904260, astro-ph/9904284, astro-ph/9904283, astro-ph/9904317, astro-ph/9911264, astro-ph/9904362, astro-ph/0003147, astro-ph/0002381, astro-ph/9810456, astro-ph/0003352, astro-ph/9904366, astro-ph/9908335.

[iii] See also Castro, Mahecha, Rodriguez (2002) for further discussion on this approach from the fractal diffusion viewpoint.

[iv] As we know $\rho(V.\nabla)V$ is the only nonlinear term in the Navier-Stokes equations; this term is also called the inertial (vortex) term. The Navier-Stokes equations are among the very few equations of mathematical physics for which the nonlinearity arises not from the physical attributes of the system but rather from the mathematical (kinematical) aspects of the system. In divergence free condition div **u** =0, the Navier-Stokes equations for a viscous, incompressible, homogenous flow are usually expressed as:

$$\partial u / \partial t - v.\Delta u + (u.\nabla)u + \nabla p = f,$$

$$\nabla .u = 0$$

where for notational simplicity, we represent the divergence of u by $\nabla.\mathbf{u}$, and for all practical purposes the density has been normalized to unity, ρ=1 (C. Foias et al., 2001). It shall be worthnoting, however, the origin of viscosity imposes a limit on the domain of validity of the Navier-Stokes equations. We should learn of some natural lengths characterizing the length scale region in which flow energy dissipation is dominated by viscous phenomena.

Therefore we find the significance of the Reynolds number emerges by comparing the inertial and dissipation terms of the Navier-Stokes equations. The inertial term dominates when:

$$\mathrm{Re} = L_* U_* / \nu >> 1$$

By setting the Re = +∞ (i.e. ν = 0), we obtain the case of inviscid flows. In this case, the divergence-free condition is retained but the momentum equation changes, resulting in the Euler equations for inviscid perfect fluids:

$$\partial u / \partial t + (u.\nabla)u + \nabla p = f,$$

$$\nabla . u = 0$$

Note here, some of the difficulties encountered in studying turbulent behavior, a largely inviscid regime, arise because of transition from Euler's equations to the Navier-Stokes equations necessitates a change from a first-order system to a second-order one in space (∇ to Δ) (C. Foias et al. 2001).

[v] We admit here the accepted viewpoint is superfluidity implies no dissipation (no turbulence is possible); the condensations –as long-lived states perhaps far from equilibrium – are indeed related to superfluidity, where the solutions are harmonic, so dissipative effects do not appear. Hence chaos can appear in the superfluid but not irreversible turbulence. However, recent research have begun to embrace this 'superfluid turbulence' issue (see Proceedings of the Isaac Newton Institute Workshop on Quantized Vortex Dynamics and Superfluid Turbulence, Cambridge, UK, Aug. 2000). They discussed for instance: hydrodynamic description of superfluid helium turbulence with quantum vortices; valuable comparison between the physics of Navier-Stokes and helium II turbulence; and a realistic possibility of experimental study of quantum turbulence in superfluid ^{3}He.

Other researchers have considered the possibility of superfluid turbulence phenomena, particularly for superfluid ^{3}He and He^4. Zurek (1995, 16) considered turbulent tangle of vortex lines. Volovik (2000b) considered ^{3}He-A effects to represent turbulent cosmic plasmas, though he admits these effects are less dramatic. Some experiments showing unusual properties damping and viscosity properties of helium II, indicating turbulence phenomenon, have also been reported by (Godfrey *et al*. 2001). Therefore we could expect under certain condition superfluid (helium) could exhibit such turbulence phenomena.

[vi] See also for instance *arXiv:math-ph/9909033*.

[vii] Inspired by Landau two-fluid theory, a number of researchers share a viewpoint that a **vortex** can be a singularity in a "background" fluid. The background fluid is the superconductor (or superfluid) which can admit circulation, but without vorticity and without dissipation. The defect "vortex" regions are then topological defects (Yates 1996), which, if not empty holes, are bounded regions of real vorticity, with a vorticity discontinuity on the boundary of the defect domain. The discontinuity implies a lack of differentiability. In the limit, these regions are taken to be "vortex" threads or strings, but this is only part of the story for there are other types of topologically bounded regions of "vorticity" which in many cases can have persistent lifetimes, and therefore represent "objects" in the background fluid (see Kiehn 2001). In this regards, an active community sponsored by ESF in Europe, COSLAB-VORTEX-BEC2000+ groups have combined to give a workshop in Bilbao this summer (2003), see http://tp.lc.ehu.es/ILE/bilbaocoslab.htm. It appears that the objective of COSLAB is to see how these objects in a laboratory superfluid may be considered as models of a cosmology (Zurek 1995, Volovik 2000b). In effect, the background is the "vacuum aether superfluid" and the stars and galaxies are the "condensed objects" within it.

[viii] Vorticity in cosmology has been considered in a recent article, C. Schmid, arXiv:gr-qc/0201095 (2002); while the idea of condensation may correspond to article by G. Chapline, arXiv:hep-th/9812129 (1998).

[ix] Such vortices sometimes are known as 'circulatory wave' or Wolter's vortex, see H. Rosu, arXiv:quant-ph/9506015 (1997).

[x] This argument can be considered as based on the simple observation, i.e. one can represent natural objects like gas or water as (kinematic) dynamics of

fluids, but not as fields. Therefore we could conclude the domains of application of fields are *less than* those of fluids.

[xi] It is known there exist exact solutions to the Navier-Stokes equations that – at constant vorticity- create bounded regions of fluid bubbles of isolated vorticity which are formed as the mean translational flow increases. It seems this could be an example of particle generation in dissipative media. It is perhaps also worth noting here, i.e. there does exist one-to-one correspondence between the Schroedinger equation and the Navier-Stokes equation for viscous compresible fluids, not just Madelung-Eulerian fluids (Kiehn 1989, 1999). The square of the wavefunction is the enstrophy of these fluids.

[xii] At this point, it is worthnoting here this previous works by Cartan have shown that Dirac equation can be generalized without any recourse to non-differentiability nor to an aether. Therefore, such aether interpretation could be considered merely as plausible alternative interpretation, somewhat in accordance with the previous works of Prokhovik, Rothwarf (1998), Consoli *arXiv:hep-ph/0109215* etc.

[xiii] Similar suggestion of *flat spacetime* universe has also been argued recently for instance by Moniz (*arXiv:gr-qc/0011098*) and K. Akama (*arXiv:hep-th/0007001, hep-th/0001113*).

[xiv] *Non-differentiable function* is defined here in simple term as function, which has a derivative nowhere. It is known there are such functions, which are continuous but nowhere differentiable. Some mathematicians propose Weierstrass function belongs to this group.

[xv] Alternatively, we could consider negative mass is inherent in the structure of the core of the Sun (*arXiv:physics/0205040*). This possibility has been discussed by DeAquino for the case of neutron stars. Otherwise, perhaps this negative mass could be considered as effects related to (ultra-cold superfluid neutron) boson stars as theorised by several authors.

[xvi] There is also known transformation (Kustaanheimo-Steifel) from the Kepler problem to the harmonic oscillator problem. An alternative expression was given by Tewari (1998).

[xvii] See also *Apeiron* Vol. 9 No. 2 (2002), though this article discusses atmospheric flows instead of the motion of celestial bodies.

[xviii] Mandelbrot also suggested turbulent velocity fields may have fractal structure with a non-integer Hausdorff dimension: a pattern of spiral with smaller spirals on them—and so on to increasingly smaller scales. This is in

accordance with Landau's (1963) turbulence definition as "superposition of an *infinite* number of vortices, or eddies, with sizes varying *over all scales.*" For discussion on possible limitations of such *scale symmetry* assumption, we refer to E.I. Guendelman, *arXiv:gr-qc/0004011, arXiv:gr-qc/9901067*.

[xix] This method uses *Ordinary Least Square* (OLS) theorem, or known as 'least square error' principle. However it shall be kept in mind, this OLS method has seven well-known premises known as "Gauss-Markov assumptions."

[xx] For discussion on the plausibility of the proposed Negative-Mass Star (NMS), see for instance F. De Aquino, *arXiv:physics/0205040* (2002a). In principle, he conjectures there is negative mass inside the vortex core of neutron stars. Therefore either we could observe a distant negative mass star as companion of the Sun, or perhaps the negative mass with mass approximately equivalent with the mass of the Sun is located inside the core of the Sun, as part of its inner structure. Alternatively, we could think such a negative mass as extension to Cantorian space of negative electron mass in Hall effect theory: $-eEm_h/m_e = +eE$ which can only hold if m_h=-m_e. See H. Myers, *Introductory solid state physics*, Taylor & Francis, 2nd ed. (1997), p. 266-267.

[xxi] Anderson, P.W., *et al., Europhys. Lett.* (), arXiv:astro-ph/0002054 (2000).

[xxii] Kolomeisky, E., *et al*., arXiv:cond-mat/0002282 (2000).

[xxiii] X. Song and P. Richards of Columbia University's Lamont-Doherty, http://www.ldeo.columbia.edu/song/pr/html

Comparison of Predictions of Planetary Quantization and Implications of the Sedna Finding

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In this article we compare some existing methods to predict quantization of planetary orbits, including a recent Cantorian Superfluid Vortex hypothesis by this author. It is concluded that there exists some plausible linkage between these methods within the framework of Quantum Cosmology hypothesis, which in turn may be due to gravitation-related phenomena from boson condensation.

Keywords: quantization of planetary orbits, Quantum Cosmology, vortices, boson condensation, gravitation

Introduction

As we know, in recent years there have been some methods proposed in order to predict the planetary orbits using quantum-like approach, instead of classical dynamics approach. These new approaches have similarity, that they extend the Bohr-Sommerfeld hypothesis of quantization of angular momentum to planetary systems. This application of wave mechanics to large-scale structures [1] has led to several impressive results in terms of prediction of planetary semimajor axes, particularly to predict orbits of exoplanets [2][3][4][5]. However, a question arises as to how to describe the physical origin of wave mechanics of such large-scale structures.

An interesting approach to explain this is by considering the known fact of scale-invariant spectrum [6], which is sometimes called as Harrison-Zel'dovich spectrum. For instance, Clayton & Moffat recently argued using variable light speed argument, that the Cosmic-Microwave Background Radiation (CMBR) anisotropy may be explained in terms of this kind of spectrum [7]. This notion of scale-invariant spectrum may also be related to noncommutative geometry representation of cosmology [8]. What is interesting here is that perhaps this scale-invariant spectrum may correspond to the fact mentioned before by G. Burbidge, i.e. if we supposed that if ρ is the density of visible matter in the universe and that He/H ratio by mass in it is 0.244, then the thermalized energy which has been released in producing He leads to blackbody temperature of T=2.76 ^{o}K. This value is astonishingly near to the value of 2.73 ^{o}K observed by COBE [9]. And because the CMBR's observed low temperature may be related to Bose-Einstein condensate, of course an interesting question is whether the universe resembles a large Bose-Einstein condensate in its entirety [10][11][12][13].

While at first glance this proposition appears quite fantastic, this can be regarded as no more than an observational implication of the notion of Quantum Cosmology hypothesis as proposed by some authors, including Vilenkin [14][15]. Provided this relationship corresponds to the facts, then it seems reasonable to hypothesize further that all predictions of planetary orbits using quantum-like approach shall somehow comprise the same theoretical implication, i.e. they correspond to the Quantum Cosmology hypothesis. Therefore it seems worth to compare these predictions here, which to this author's knowledge has not been made before, though a comparison of Titius-Bode law and a random stable solar system hypothesis is available elsewhere [16][19][20].

In this article we would compare the following approaches available in the literatures:

a. Nottale's Scale Relativity theory [4];
b. Chechelnitsky's Wave Universe theory [17];
c. Ilyanok's Macroquantum Condensate theory [12];
d. Neto *et al.*'s Schrödinger-type diffusion equation [18];
e. Cantorian Superfluid Vortices hypothesis.

We begin with a short description of each approach considered. It is worth noting here that this article does not attempt to examine validity of each of these theories, but instead we merely present what these authors intend to say as is. Therefore the original notations by these authors are kept intact.

Scale Relativity

Nottale [4] argued that equation of motion for celestial bodies could be expressed in terms of a scale-relativistic Euler-Newton equation, by separating the real and imaginary part of Schrödinger-like equation. Then he obtained a generalized Euler-Newton equation of (Ref .[4] p. 384):

$$m.(\partial / \partial t + V.\nabla)V = -V(\phi + Q) \tag{1}$$

$$\partial \rho / \partial t + div(\rho V) = 0 \tag{2}$$

$$\Delta \phi = -4\pi G \rho \tag{3}$$

Using these set of equations, Nottale came up with the generalised Schrödinger equation, by giving up the notion of *differentiability* of spacetime. For a Kepler potential and in the time-independent case, this equation reads (Ref [4] p. 380):

$$2D^2\Delta\Psi + (E/m + GM/r).\Psi = 0 \quad (4)$$

Solving this equation, he obtained that planetary orbits are quantized according to the law:

$$a_n = GMn^2 / v_o{}^2 \quad (5)$$

where a_n,G,M,n,v_o each represents orbit radius for given n, Newton gravitation constant, mass of the Sun, quantum number, and specific velocity (v_o=144 km/sec for Solar system and also exoplanet systems), respectively. Furthermore, according to Nottale, the ratio

$$\alpha_g = v_o / c \quad (6)$$

actually corresponds to gravitational coupling constant, similar to fine coupling constant in quantum electrodynamics. These equations form the basis of Nottale's Scale Relativity prediction of planetary orbits both in Solar system and also in exoplanet systems. The result of this equation (5) for the solar system is presented in Table 1.

Wave Universe

Chechelnitsky's Wave Universe hypothesis began with a fundamental wave equation, which reads as follows [17]:

$$\nabla\Psi + 2/d^2.[\varepsilon - U]\Psi = 0 \quad (7)$$

where for the solar system, U=-K/a; and K=1.327x10^{11} km^3/sec^{-2}, as the gravitational parameter of the Sun. The result of this equation is also presented in Table 1.

What is interesting here is that Chechelnitsky does not invoke argument of non-differentiability of spacetime, as Notale did. Furthermore, he also arrived at some Jovian planetary orbits beyond

Pluto, which obviously recommend an observation for verification or refutation.

Macroquantum condensate

Ilyanok & Timoshenko [12] took a bold step further by hypothesizing that the universe resembles a large Bose-Einstein condensate, therefore the distribution of all celestial bodies must also be quantized. This conjecture may be originated from the fact that according to BCS theory, superconductivity could exhibit macroquantum phenomena [21]. Therefore it seems also reasonable to argue that the universe resembles such macroquantum phenomena, at least in the context of Quantum Cosmology hypothesis [14][15].

According to Ilyanok and Timoshenko, the quantization of planetary orbits in solar system follows a formula of orbit radii and orbital velocity represented by [12]:

$$R_n = \left(n/3 + 2/3.(2m+1)\right)^2 .R_1 \tag{8}$$

$$v_n = 3v_1 / \left[n + 2(2m+1)\right] \tag{9}$$

where n,m are integers and v_1 and R_1 represents orbital velocity and orbit radius of Mercury, as follows:

$$n = 1,2,3,4,5,6,7,8,9$$

$$m = 0,0,0,0,1,2,3,4,5$$

$$v_1 = 3\alpha^2 c = 47.89307 km/\sec \tag{10}$$

$$R_1 = h/\left(\alpha^{12} m_p c\right) = 5.796x10^{10} m \tag{11}$$

where α, m_p, c each represents fine structure constant (1/137), proton mass, and the speed of light, respectively. The result of this method is presented in Table 1.

It seems worth noting here that at first glimpse this method appears similar to Nottale's quantization approach [4]. However, Ilyanok & Timoshenko attempt to build a direct linkage between fine structure constant and the quantization of planetary orbits, while Nottale puts forth a conjecture of gravitational coupling constant (6). It is perhaps also interesting to remark that Ilyanok & Timoshenko do not invoke argument of *nondifferentiability* of spacetime, which notion is essential in Notale's derivation. In a macroquantum condensate context, this approach seems reasonable, considering the fact that Bose-Einstein condensate with Hausdorff dimension $D_H\sim 2$ could exhibit fractality [22], implying a conjecture of nondifferentiability of spacetime perhaps is not required. The same fractality property has been observed in astrophysics [23][24][25], which in turn may bring us back to an explanation of the origin of multifractal spectrum as described by Gorski [6].

Neto *et al.*'s Schrodinger-type diffusion

In a recent article, Neto *et al.* considered an *axisymmetrical flat* analytical solution of Schrödinger-type equation involving an attractive central field, which is given by [18]:

$$-g^2/2\mu.\left(\partial^2\Psi/\partial r + r^{-1}.\partial\Psi/\partial r + r^{-2}.\partial^2\Psi/\partial\varphi^2\right) + V(r)\Psi = E\Psi \quad (12)$$

where g is a constant and μ is reduced mass. Then they derived a solution using separation of variables:

$$\Psi(r,\varphi) = f(r).\Phi(\varphi) \quad (13)$$

After a rescaling and defining $n = \mu GMm/g^2\beta$, and by using $V(r) = -GMm/r$, they obtained:

$$u'' + [-1/4 + n/\rho - (\ell^2 - 1/4)/\rho^2]u(\rho) = 0 \quad (14)$$

which is a confluent hypergeometric equation, referred as Whittaker's equation. This equation has a regular solution given by a hypergeometric series which converges if and only if,

$$n = |\ell| + 1/2 + k \quad \text{k=0,1,2,3} \tag{15}$$

from which condition they obtained the solution for f(r) in (13):

$$f(r) = c_1.(2\beta r - 1).\exp(-\beta r) \tag{16}$$

It is obvious therefore that in order to find the appropriate asymptotic expression of Schrödinger-type equation they invoke some arbitrary assumptions. Furthermore their result is based on *averaging* planetary masses, and also their equation (16) leads to prediction of planetary orbits which is equivalent the observed planetary data in Solar system except for Earth and Venus. Therefore, in order to reconcile with observed data, they have to invoke a second quantum number.

The result of their method is also presented in Table 1.

Cantorian superfluid vortex hypothesis

In principle the Cantorian superfluid vortex hypothesis as proposed by this author suggests that distribution of planetary systems can be modeled using superfluid vortices [26]. For a planar cylindrical case of solar system, this hypothesis leads to a known Bohr-Sommerfeld-type quantization of planetary orbits [27].

This hypothesis starts with observation that in quantum fluid systems like superfluidity, it is known that such vortices are subject to quantization condition of integer multiples of 2π, or $\oint v_s.dl = 2\pi.n\hbar / m_4$. Furthermore, such quantized vortices are distributed in equal distance, which phenomenon is known as vorticity. In large superfluid system, usually we use Landau two-fluid model, with normal and superfluid component. The normal fluid component always possesses some nonvanishing amount of viscosity and mutual friction. Similar approach with this proposed model has been considered in the context of neutron stars [28], and this proposed quantized vortice model may also be related to Wolter's vortex [29].

To obtain planetary orbit prediction from this hypothesis we could begin with the Bohr-Sommerfeld's conjecture of quantization of angular momentum. As we know, for the wave function to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition [30]:

$$\oint_{\Gamma} p.dx = 2\pi.n\hbar \tag{17}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is

$$\int_0^T v^2.d\tau = \omega^2.T = 2\pi.\omega \tag{18}$$

where $T=2\pi/\omega$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$.

Then we can write the force balance relation of Newton's equation of motion:

$$GMm / r^2 = mv^2 / r \tag{19}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (18), a new constant g was introduced:

$$mvr = ng / 2\pi \tag{20}$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form [26]:

$$r = n^2 . g^2 / (4\pi^2 . GM . m^2) \quad (21)$$

or

$$r = n^2 . GM / v_o^2 \quad (22)$$

where r, n, G, M, v_o represents orbit radii (semimajor axes), quantum number (n=1,2,3,…), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (22), we denote

$$v_o = (2\pi / g) . GMm \quad (23)$$

This result (23) is the same with Nottale's equation for predicting semimajor axes of planetary-like systems (5).The value of m is an adjustable parameter (similar to g). The result of this equation (22) is also presented in Table 1. While this method results in the same prediction with Nottale's equation (5) for inner orbits, this author uses a different approach for Jovian orbits. It is known that Nottale has to invoke a second quantum number for Jovian planets, while the Solar system is actually a *planar* cylindrical system [18], therefore a second quantum number seems to be superfluous. Therefore, instead of a second quantum number, in CSV hypothesis we describe outer Jovian planet orbits using a conjecture of reduced mass, μ [26].

Perhaps it would be more interesting if we note here that the same Bohr-Sommerfeld's quantization of orbits could also be treated using the viewpoint of quantum Hall liquid in the context of Chern-Simons theory [31][32]. According to L. Susskind [31] we could assume that the particles making up the fluid are electrically charged and move in a background magnetic field B. Furthermore he showed that the conservation law requires the "magnetic field" at each point y, to be time independent, and the analog of a vortex is a δ function magnetic field [31]:

$$\nabla x A = 2\pi . \rho_o q \delta^2 (y) \tag{24}$$

where q measures the strength of the vortex. The solution of this equation is unique up to a gauge transformation. In the Coulomb gauge,

$$\nabla . A = 0 \tag{25}$$

it is given by

$$A_i = q\rho_o \in_{ij} y_j / y^2 \tag{26}$$

To further understand the quasiparticle we must quantize the fluid. Assume the fluid is composed of particles of charge e. Then the momentum of each particle is [31]:

$$p_a = eB \in_a x_b / 2 \tag{27}$$

The standard Bohr-Sommerfeld quantization condition is

$$\oint_\Gamma p_a . dx_a = 2\pi . n \tag{28}$$

Inserting equation (27) into (28), then the quantization condition becomes [31]:

$$eB \oint_\Gamma (\in_a x_b / 2). dx_a = 2\pi . n \tag{29}$$

Using equation (26) then gives:

$$eBq = 2\pi n \tag{30}$$

Therefore an elementary quasiparticle (n=1) has electric charge:

$$e_{pq} = 2\pi\rho_o / B \tag{31}$$

which result agree with the quasiparticle charge from Laughlin's theory [31]. This expression could be extended to include a source. What interests us here from these relationships as described by Susskind is that it was understood recently that Bose-Einstein condensate in dilute atomic gases could be used to describe the physics of vortex matter when they undergo rotation [33]. Furthermore, there is a possibility that at larger angular velocity (ω) the vortex lattice melts and is replaced by a *quantum Hall liquid*. Exactly at this point, it seems we could find a plausible linkage between a quantum Hall liquid and quantization of planetary motion. And the electron fluid representation in quantum Hall liquid may correspond to the *'sea of electron'* terms of Dirac. In this regards, it is worth noting here that universality of quantum Hall liquid has been around in the literature for more than a decade [34], and and it has also been argued that Hall effect could also have some roles in star formation [35].

It may also be worth to remark here, that according to Obukhov [36] it is possible to explain the CMBR anisotropy from the viewpoint of rotating universe [37], which seems to support our conjecture that the universe in its entirety resembles a large rotating Bose-Einstein condensate. While of course this conjecture is not conclusive yet, it seems that CMBR anisotropy could become a test problem; i.e. to observe whether the proposed Bose-Einstein condensate vortices cosmology model could explain this phenomenon.

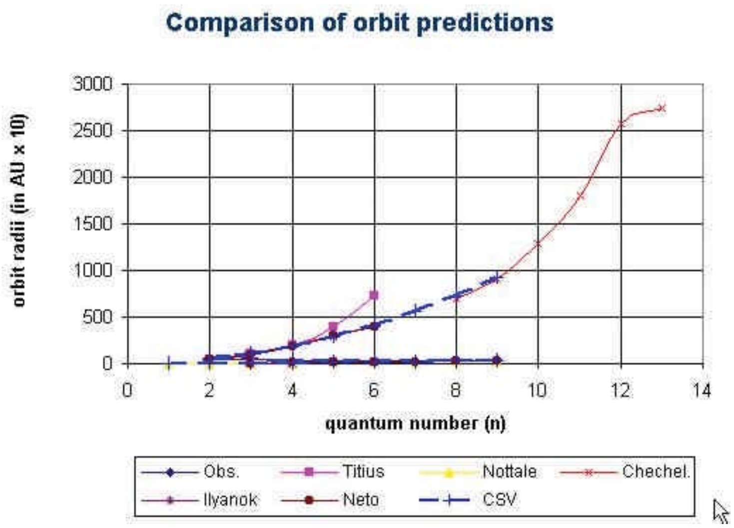
Comparison of predictions and implications of Sedna finding

Based on predicting methods as described above, a comparison table is presented in Table 1.

Astroobject	COMPARISON (in AU x 10' unit)								
	n	Obs.	Titius	Nottale	Chechel.	Ilyanok	Neto	CSV	
	1			0.4				0.43	
	2			1.7				1.71	
Mercury	3	3.87	4	3.9	3.9	3.9	3.9	3.85	
Venus	4	7.32	7	6.8	7.2	6.9	8.3	6.84	
Earth	5	10.00	10	10.7	10.0	10.8	10.0	10.69	
Mars	6	15.24	16	15.4	15.2	15.5	15.4	15.40	
Hungarias	7	20.99		21.0			19.4	20.96	
Asteroid	8	27.00		27.4	27.7		28.8	27.38	
Camilla	9	31.50		34.7			34.8	34.65	
Jupiter	2	52.03	52		52.0	52.1	50.3	45.52	
Saturn	3	95.39	100		95.2	110.2	93.4	102.43	
Uranus	4	191.90	196		191.6	189.8	183.0	182.09	
Neptune	5	301.00	388		300.7	291.0	302.0	284.52	
Pluto	6	395.00	722		393.7	413.7	399.0	409.70	
Π_1	7							557.65	
Π_2	8				700.0			728.36	
Π_3 (Sedna)	9	860			904.4			921.83	(March'04)
Π_4	10				1295				
Π_5	11				1810				
	14				4070				

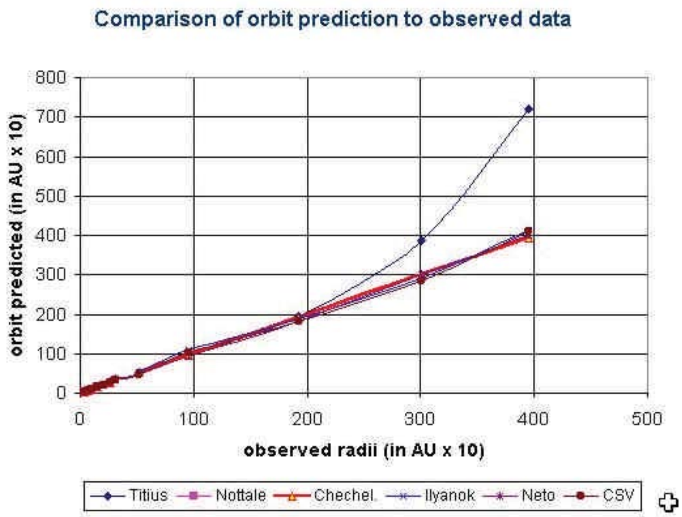
Table 1. Comparison of several methods of orbit prediction

It also seems interesting here to make graph plots for these data in Table 1. The two graphs presented below clearly show how prediction varies against quantum number (n), and against the observed data (Obs). Of course, for an exactly corresponding prediction values to observed data, we will get a gradient =1, corresponding to y=x+0.



Graph 1. Comparison of orbit predictions to quantum number

From Table 1 and its graphplots we observe that all methods compared are very near to the observed data, which seems to support our argument above of the similarity of wave mechanics approach for planetary quantization. We also note that Titius-Bode law overpredicts large orbits, at least for Pluto. Furthermore, there are only two methods which predict planetary orbits beyond Pluto, i.e. Chechelnitsky's Wave Universe hypothesis and the CSV hypothesis suggested by this author. Therefore it seems further observational data is required to verify or refute these predicted orbits beyond Pluto.



Graph 2. Comparison of orbit predictions to observed data

In this regard, it seems worth to put a recent observation of Sedna in this context of planetary quantization, corresponding to n=9 of Jovian planets in Table 1, though it does not mean that Sedna could not be explained in other ways than planetary quantization. As we know, Sedna has found by M. Brown *et al*. from Caltech [38] [39], having around 1770 km in diameter. This Sedna finding obviously leads to some interesting implications. First of all, in numerical terms this finding is very near to a quantum number n=9 as presented in Table 1, within error range of 6.7% as compared with CSV prediction of 92.2AU. Another recent article has also post-predicted this finding, though it was based on Jeans instability [40]. Other interesting aspect of this Sedna includes its very elliptical orbit.

In this article we compared and discussed some methods to predict planetary orbits based on wave-mechanics-type arguments. If the proposition described in this article corresponds to the facts, i.e. the wave mechanics description of celestial bodies correspond to a kind of Quantum Cosmology hypothesis, then it seems further theoretical development could be expected, for instance to extend noncommutative representation of Dirac equation to large scale structure of the universe [41]. Furthermore, a vortex interpretation of Schrödinger equation has also been suggested elsewhere [42][43]. While these are of course not the only plausible approaches, these seem quite interesting in order to find more precise cosmological theories, considering some recent remarkable observation of exoplanets as predicted by such a wave mechanics approach.

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On recent discovery of new planetoids in the solar system and quantization of celestial system

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The present note revised the preceding article discussing new discovery of a new planetoid in the solar system. Some recent discoveries have been included, and its implications in the context of quantization of celestial system are discussed, in particular from the viewpoint of superfluid dynamics. In effect, it seems that there are reasons to argue in favor of gravitation-related phenomena from boson condensation.

Keywords: quantization, planetary orbit, quantized superfluid, boson condensation, gravitation

Discovery of new planetoids

Discovery of new objects in the solar system is always interesting for astronomers and astrophysicists alike, not only because such discovery is very rare, but because it also presents new observation data which enables astronomers to verify what has been known concerning how our solar system is functioning.

In recent years a number of new planetoids have been reported, in particular by M. Brown and his team [1][2][3][4]. While new planet discoveries have been reported from time to time, known as *exoplanets* [9][10], nonetheless discovery of new planetoids in the solar system are very interesting, because they are found after a long period of silence after Pluto finding, around seventy years ago. Therefore, it seems interesting to find out implications of this discovery to our knowledge of solar system, in particular in the context of quantization of celestial system.

As we discussed in the preceding article [5], there are some known methods in the literature to predict planetary orbits using quantumwave-like approach, instead of classical dynamics approach. These new approaches have similarity, i.e. they extend the Bohr-Sommerfeld's quantization of angular momentum to large-scale celestial systems. This application of wave mechanics to large-scale structures [6] has led to several impressive results in particular to predict orbits of exoplanets [8][9][10]. However, in the present note we will not discuss again the physical meaning of wave mechanics of such large-scale structures, but instead to focus on discovery of new planetoids in solar system in the context of quantization of celestial system.

As contrary as it may seem to present belief that it is unlikely to find new planets beyond Pluto, Brown *et al*. have reported not less than four new planetoids in the outer side of Pluto orbit, including 2003EL61 (at 52AU), 2005FY9 (at 52AU), 2003VB12 (at 76AU, dubbed as *Sedna*. It is somewhat different to our preceding article suggesting orbit distance = 86AU in accordance with ref. [14]). And recently Brown and his team report new planetoid finding, dubbed as 2003UB31 (97AU). This is not to include *Quaoar* (42AU), which has orbit distance more or less near Pluto (39.5AU), therefore this object is excluded from our discussion. Before discovery of 2003UB31

(Brown himself prefers to call it '*Lila*'), Sedna has been reported as the most distant object found in the solar system, but its mass is less than Pluto, therefore one could argue whether it could be considered as a 'new planet'. But 2003UB31 is reported to have mass definitely greater than Pluto, therefore Brown argues that it is definitely worth to be considered as a 'new planet'. (Table 1)

Table 1. Comparison of prediction and observed orbit distance of planets in the Solar system (in 0.1AU unit)

Object	No.	Titius	Nottale	CSV	Observed	Δ (%)
	1		0.4	0.428		
	2		1.7	1.71		
Mercury	3	4	3.9	3.85	3.87	0.52
Venus	4	7	6.8	6.84	7.32	6.50
Earth	5	10	10.7	10.70	10.00	-6.95
Mars	6	16	15.4	15.4	15.24	-1.05
Hungarias	7		21.0	20.96	20.99	0.14
Asteroid	8		27.4	27.38	27.0	1.40
Camilla	9		34.7	34.6	31.5	-10.00
Jupiter	2	52		45.52	52.03	12.51
Saturn	3	100		102.4	95.39	-7.38
Uranus	4	196		182.1	191.9	5.11
Neptune	5			284.5	301	5.48
Pluto	6	388		409.7	395	-3.72
2003EL61	7			557.7	520	-7.24
Sedna	8	722		728.4	760	4.16
2003UB31	9			921.8	970	4.96
Unobserved	10			1138.1		
Unobserved	11			1377.1		

Moreover, from the viewpoint of quantization of celestial systems, these findings provide us with a set of unique data to be compared with our prediction based on CSV hypothesis [5]. It is therefore interesting to remark here that all of those new 'planetoids' are within 8% bound compared to our prediction (Table 1). While this result does not yield high-precision accuracy, one could argue that this 8% bound limit corresponds to the remaining planets, including inner planets. Therefore this 8% uncertainty could be attributed to macroquantum uncertainty and other local factors.

What's more interesting here is perhaps that some authors have argued using gravitational Schrödinger equation [12], that it is *unlikely to find new planets beyond Pluto* because density distribution becomes near zero according to the solution of Schrödinger equation [7][8][11]. From this viewpoint, one could argue concerning to how extent applicability of gravitational Schrödinger equation to predict quantization of celestial systems, despite its remarkable usefulness to predict exoplanets [9][10].

Therefore in the subsequent section, we argue that using Ginzburg-Landau equation, which is more consistent with superfluid dynamics, one could derive similar result with known gravitational Bohr-Sommerfeld quantization [13][15]:

$$a_n = GMn^2 / v_o^2 \tag{1}$$

where a_n,G,M,n,v_o each represents orbit radius for given n, Newton gravitation constant, mass of the Sun, quantum number, and specific velocity (v_o=144 km/sec for Solar system and also exoplanet systems), respectively [7][8].

Interpretation

In principle the Cantorian superfluid vortex (CSV) hypothesis [5] suggests that the quantization of celestial systems corresponds to superfluid quantized vortices, where it is known that such vortices are subject to quantization condition of integer multiples of 2π, or $\oint v_s.dl = 2\pi.n\hbar / m_4$ [5]. For a *planar cylindrical case* of solar system, this hypothesis leads to Bohr-Sommerfeld-type quantization of planetary orbits. It is also worthnoting here, while likelihood to find planetoid at around 90AU has been predicted by some astronomers, our prediction of new planets corresponding to n=7 (55.8AU) and n=8 (72.8AU) were purely derived from Bohr-Sommerfeld quantization [5].

The CSV hypothesis starts with observation that in quantum fluid systems like superfluidity, quantized vortices are distributed in equal distance, which phenomenon is known as vorticity. In a large superfluid system, we usually use Landau two-fluid model, with normal and superfluid component. Therefore, in the present note we will not discuss again celestial quantization using Bohr-Sommerfeld quantization, but instead will derive equation (1) from Ginzburg-Landau equation, which is known to be more consistent with superfluid dynamics. To our knowledge, deriving equation (1) from Ginzburg-Landau equation has *never* been made before elsewhere.

According to Gross, Pitaevskii, Ginzburg, wavefunction of N bosons of a reduced mass m* can be described as [17]:

$$-(\hbar^2 / 2m^*).\nabla^2\psi + \kappa|\psi|^2\psi = i\hbar.\partial\psi/\partial t \qquad (2)$$

For some conditions, it is possible to substitute the potential energy term ($\kappa|\psi|^2$) in (2) by Hulthen potential, which yields:

$$-(\hbar^2 / 2m^*).\nabla^2\psi + V_{Hulthen}.\psi = i\hbar.\partial\psi/\partial t \qquad (3)$$

where Hulthen potential could be written in the form:

$$V_{Hulthen} = -Ze^2.\delta.e^{-\delta r}/(1-e^{-\delta r}) \tag{4}$$

It could be shown that for small values of screening parameter δ, the Hulthen potential (4) approximates the effective Coulomb potential:

$$V_{Coulomb}^{eff} = -e^2/r + \ell(\ell+1).\hbar^2/(2mr^2) \tag{5}$$

Therefore equation (3) could be rewritten as:

$$-\hbar^2\nabla^2\psi/2m^* + \left[-e^2/r + \ell(\ell+1).\hbar^2/(2mr^2)\right]\psi = i\hbar.\partial\psi/\partial t \tag{6}$$

Interestingly, this equation takes the form of time-dependent Schrödinger equation. In the limit of time-independent case, equation (6) becomes similar with Nottale's time-independent gravitational Schrödinger equation from Scale relativistic hypothesis with Kepler potential [7][8][9]:

$$2D^2\Delta\Psi + (E/m + GM/r).\Psi = 0 \tag{7}$$

Solving this equation with Hulthen effect (4) will make difference, but for gravitational case it will yield different result only at the order of 10^{-39} m compared to prediction using equation (7), which is of course negligible. Therefore, we conclude that for most celestial quantization problems the result of TDGL-Hulthen (3) is *essentially* the same with the result derived from equation (7).

Furthermore, the extra potential to Keplerian potential in equation (5) is also negligible, in accordance with Pitkanen's remarks: "centrifugal potential $l(l+1)/r^2$ in the Schrödinger equation is negligible as compared to the potential term at large distances so that one expects that degeneracies of orbits with small values of l do not depend on the radius." [18]

It seems also worth noting here that planetoids 2003EL61 and 2005FY9 correspond to orbit distance of 52AU. This pair of planetoids could also be associated with Pluto-Charon pair. In the context of macroquantum phenomena of condensed matter physics, one could argue whether these pairs indeed correspond to macroobject counterpart of Cooper pairs [16]. While this conjecture remains open for discussion, we predict that more paired-objects similar to these planetoids will be found beyond Kuiper belt. This will be interesting for future observation.

Furthermore, while our previous prediction only limits new planetoids finding until n=9 of Jovian planets (outer solar system), it seems that there are more than sufficient reasons to expect that more planetoids are to be found in the near future. Therefore it is recommended to extend further the same quantization method to larger n values. For prediction purpose, we have included in Table 1 new expected orbits based on the same celestial quantization as described above. For Jovian planets corresponding to n=10 and n=11, our prediction yields likelihood to find orbits around 113.81 AU and 137.71 AU, respectively. It is recommended therefore, to find new objects around these predicted orbits.

In this note, we revised our preceding article suggesting that Sedna corresponds to orbit distance 86AU, and included recently found planetoids in the outer solar system as reported by Brown *et al*. While our previous prediction only limits new planet finding until n=9 corresponding to outer solar system, it seems that there are reasons to expect that more planetoids are to be found. While in the present note, we argue in favor of superfluid-quantized vortices, it does not mean to be the only plausible approach. Instead, we consider this discovery as a new milestone to lead us to find better cosmological theories, in particular taking into consideration some recent remarkable observation of exoplanets as predicted by wave mechanics approach.

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Possible CGLE signatures in solar system: Spiral gravity from spherical kinetic dynamics[1]

The present article discusses how some known phenomena in solar system, including the Lense-Thirring effect of anomalous precession, could be described using spherical kinetic dynamics approach. Other implications include a plausible revised version of the Bohr-Sommerfeld quantization equation described by Rubćić & Rubćić. Our proposition in this paper can be summarized as follows: by introducing time-incremental to the ordinary celestial quantization method (Nottale *et al.*), we can expect to observe signatures of CGLE (complex Ginzburg-Landau equation) in Solar system. Possible verification may include the use of Earth-based satellites, which go beyond traditional GTR tests such as precession of the first planet. Further observation to verify or refute this conjecture is recommended, plausibly using LAGEOS-type satellites.

Keywords: Lense-Thirring effect, Bohr-Sommerfeld quantization, quantized vortices, celestial quantization, LAGEOS satellite, boson condensation, signature of CGLE in solar system, spiral gravity

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Introduction

It is known that the use of Bohr radius formula to predict celestial quantization, based on Bohr-Sommerfeld quantization rules [2][3], has led to numerous verified observations [1]. While this kind of approach is not widely accepted yet, this could be related to wave mechanics equation to describe large-scale structure of the Universe [4], and also a recent suggestion to reconsider Sommerfeld's conjectures in Quantum Mechanics [5]. Some implications of this quantum-like approach include exoplanet prediction, which becomes a rapidly developing subject in recent years [6][7].

Rubćić & Rubćić's approach [2] is particularly interesting in this regard, because they begin with a conjecture that Planck mass ($m_p = \sqrt{\hbar c / 2\pi G}$) is the basic entity of Nature, which apparently corresponds to Winterberg's assertion that Planckian aether is comprised superfluid of phonon-roton pairs [8]. In each of these pairs, superfluid vortices can form with circulation quantized according to $\oint v_{+-}.dx = n\hbar / m_p$. This condition implies the Helmholtz vortex theorem, $d/dt\oint v_{+-}.dx = 0$. This relationship seems conceivable, at least from the viewpoint of likely neat linkage between cosmology phenomena and various low-temperature condensed matter physics [9][10][11]. In effect, celestial objects at various scales could also be regarded as spinning Bose-Einstein condensate; which method has been used for neutron stars [32].

Despite these aforementioned advantages of using quantum mechanical viewpoint to describe astrophysical phenomena, it is also known that all of the existing celestial quantization methods [1][2][3] thus far have similarity that they assume a circular motion, while the actual celestial orbits (and also molecular orbits) are elliptical.

Historically, this was the basis of Sommerfeld's argument in contrast to Bohr's model, which also first suggested that any excess gravitational-type force would induce a precessed orbit. Similar argument is used here as the starting premise of the present article, albeit for brevity we will not introduce elliptical effect yet [12].

Using a known *spherical kinetic dynamics* approach, some known interesting phenomena are explained, including the receding Moon, the receding Earth from the Sun, and also anomalous precession of the first planet (*Lense-Thirring effect*). Despite some recent attempts to rule out the gravitational quadrupole moment (J_2) contribution to this effect [13][14][15][16][17], it seems that the role of spherical kinetic dynamics [12] to describe the origin of Lense-Thirring effect has not been taken into consideration thus far, at least to this author's knowledge.

After deriving prediction for these known observed phenomena, this article will also present a revised version of quantization equation of L. Nottale [1] in order to take into consideration this spherical kinetic dynamics effect. Some implications are discussed, including possible time-incremental modification of ordinary Bohr-type quantization for solar system, which can take the form of spiral gravity. In turn, this 'spiralling gravity' phenomena can be considered as signatures of CGLE (complex Ginzburg Landau equation) in solar system.

Our paper starts from simple hypothesis that smaller celestial objects acquire its (spinning) energy from the larger systems. That is, Earth spinning motion gets its energy from the Sun. In turn, Solar system gets its spinning energy from its Galaxy center. One can say that this is just an astrophysics implications of turbulence dynamics (see Gibson et al. [22][23]), where *energy cascades from the larger scales down to the smaller scales.*

If this proposition described here corresponds to the facts, then one can say that it is possible to 're-derive' General Relativity phenomena from the viewpoint of Bohr-Sommerfeld quantization and spherical kinetic dynamics. Possible verification of this proposition may include the use of Earth-based satellites, which go beyond traditional GTR-tests such as precession of the first planet. Further observation to verify or refute this conjecture is recommended, plausibly using LAGEOS-type satellites (see Ciufolini and others [14]-[16]).

Spherical kinetic dynamics: Earth bulging from Earth geodynamics

Analysis of spinning dynamics of solid sphere with mass M (see Appendix I) yields:

$$\Delta M / \Delta t = -\dot{\omega}.MR^2\omega/(5.{c_s}^2) \qquad (1)$$

where c_s represents the sound velocity obeying [10b; p.4]:

$${c_s}^2(n) = (n/m)(d^2 \in / dn^2) \qquad (2)$$

For $\dot{\omega} = 0$ the equation (1) shall equal to zero, therefore this equation (1) essentially says that a linear change of angular velocity observed at the *surface* of the spinning mass corresponds to mass flux, albeit this effect is almost negligible in daily experience. But for celestial mechanics, this effect could be measurable.

If, for instance, we use the observed anomalous decceleration rate [30] of angular velocity of the Earth as noted by Kip Thorne [19]:

$$|\omega|/|\dot{\omega}| = 6x10^{11}\ years \qquad (3)$$

And using values as described in Table 1 for other parameters:

Table 1. Parameter values to compute kinetic expansion of the Earth

Parameter	Value	Unit
R_e	$6.38x10^6$	M
M_e	$5.98x10^{24}$	Kg
T_e	$2.07x10^6$	Sec
ω_e	$3.04x10^{-6}$	rad/s
c_s	0.14112	m/s

It is perhaps worthnoting that the only free parameter here is c_s =0.14112 m/sec. This value is approximately within the range of Barcelo *et al*.'s estimate of sound velocity (at the order of cm/sec) for gravitational Bose-Einstein condensate [11], provided the Earth could be regarded as a spinning Bose-Einstein condensate. Alternatively, the sound velocity could be calculated using equation (ii) in Appendix I, but this obviously introduces another kind of uncertainty in the form of determining temperature (T) inside the center of the Earth; therefore this method is not used here.

Then by inserting these values from equation (3) and Table 1 into equation (1) yields:

$$\Delta M / \Delta t \approx 3.76x10^{16} kg / year \tag{4}$$

Perhaps this effect could be related to a recent Earth bulging data, which phenomenon lacks a coherent explanation thus far [36].

Now we want to know how this mass accumulation affects the Earth surface and also its rotational period. Assuming a solid sphere, we start with a known equation [34]:

$$M = 4\pi.\rho_{sphere}.r^3 / 3 \tag{5}$$

where ρ_{sphere} is the average density of the 'equivalent' solid sphere. For Earth data (Table 1), we get ρ_{sphere}=5.50x10^6 gr/m^3. Using the same method with equation (8f), which will be discussed subsequently, equation (5) could be rewritten as:

$$M + \Delta M / \Delta t = 4\pi.\rho_{sphere}.(r + \Delta r / \Delta t)^3 / 3 \qquad (6)$$

or

$$\Delta r / \Delta t = \sqrt[3]{(M + \Delta M / \Delta t).3/(4\pi.\rho_{sphere})} - r \qquad (6a)$$

From equation (7) we get dr/dt=13.36 mm/year for Earth.

It would be worth here to compare this result with the known *Expanding Earth hypothesis* by Pannella, Carey, Vogel, Shields and others, who suggested that the Earth was only 60% of its present size in the Jurassic [49]. There is also a recent suggestion that Earth has experienced a slow down in spin rate during the past $9x10^8$ years.[2] To get a numerical estimate of Earth's radial increase each year, we quote here from Smoot [49]:

> "In order for this to happen, the lunar tides would have to slow down, which would affect the length of the lunar month. … an Earth year of 447 days at 1.9 Ga decreasing to an Earth year of 383 days at 290 Ma to 365 days at this time. However, the Devonian coral rings show that the *day is increasing by 24 seconds every million years,* which would allow for an expansion rate of about 0.5% for the past 4.5 Ga, all other factors being equal."

This observation seems to be in agreement with known 'facts' from *geochronometry* [50]:

> "It thus appears that the length of the day has been increasing throughout geological time and that the number of days in the year has been decreasing. At, the beginning of the Cambrian the length of the day would have been 21 h."

Now using this value of ΔT=24 sec/million years, T=23.9 hours, and rotational velocity $v = 2\pi R / T$, and assuming that rotational

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2 http://image.gsfc.nasa.gov/poetry/ask/a11765.html

velocity is the same throughout, then we could write in the same way with equation (6):

$$T.(1+\Delta T/T)=2\pi.R(1+\Delta R/R)/v \tag{7}$$

Inserting these values into equation (7) including Earth radius value from Table 1, we get ΔR=1.7766 mm/year for Earth, which is surprisingly of the same order of magnitude with the result from equation (6). Of course, some difference could be expected because this approximation was obtained from Devonian coral rings observation, which could contain some biases.[49]

In the subsequent sections we will discuss an alternative method to measure this effect more precisely. It is worth to note here that this result does not necessarily mean to support all arguments related to Expanding Earth hypothesis by Panella-Carey-Vogel-Shields, despite its calculated result can be quite similar, because nowhere they have considered quantization of motion [49].

Derivation of extended celestial quantization and prediction of the receding Moon

Now let suppose that this predicted value (4) is fully conserved to become inertial mass, and then we could rewrite Nottale's method of celestial quantization [1]. Alternatively, we could begin with the known Bohr-Sommerfeld quantization rule [3]:

$$\oint p_j.dq_j = n_j.2\pi.e^2/(\alpha_e.c) \tag{8a}$$

Then, supposing that the following substitution is plausible [3]:

$$e^2/\alpha_e \rightarrow GMm/\alpha_g \tag{8b}$$

where e,α_e,α_g represents electron charge, Sommerfeld's fine structure constant, and gravitational-analogue of fine structure constant, respectively. This corresponds to Nottale's basic equations $v_n = \alpha_g .c / n = v_o / n$ and v_o=144 km/sec [1]. And by introducing the gravitational potential energy [12]:

$$\Phi(r,\vartheta) = -GM / r.\left[1 - J_2.(a/r)^2.\left(3\cos^2\vartheta - 1\right)/2\right] \tag{8c}$$

where ϑ is the polar angle (collatude) in spherical coordinate, M the total mass, and a the equatorial radius of the solid.

Neglecting *higher order* effects of the gravitational quadrupole moment J_2 [13][14][15][16][17], then we get the known Newtonian gravitational potential:

$$\Phi = -GM / r \tag{8d}$$

Then it follows that the semi-major axes of the celestial orbits are given by [1][3]:

$$r_n = GMn^2 / v_o^{\ 2} \tag{8e}$$

where n=1,2,….is the principal quantum number.

It could be shown, that equation (8a) also corresponds to the conjecture of *quantization of circulation* [4b], which may correspond to the observation of quantized vortices dynamics, in particular in condensed matter physics (superfluidity etc.) [8][9][10][11] Therefore one can say that Bohr-Sommerfeld quantization has neat link with quantized vortice dynamics, just like Thompson's vortex hypothesis (before Rutherford). [51] In other words, our proposition for using Bohr-Sommerfeld quantization to describe celestial orbits may be just another implications of recent development in superfluid analogy in astrophysics, by Volovik *et al.*

By re-expressing equation (8e) for mass flux effect (5) by defining $M_{n+1} = M_n + \Delta M_n / \Delta t_n$, then the total equation of motion becomes:

$$(M + \Delta M / \Delta t) = (r + \Delta r / \Delta t).v_0^{\ 2} / (G.n^2) \tag{8f}$$

For $\Delta \to 0$, equation (8f) can be rewritten as:

$$dM / dt - \chi .dr / dt + M - r.\chi = 0 \tag{8g}$$

where

$$\chi = v_0^{\ 2} / (G.n^2) \tag{8h}$$

Now inserting (5a) into equation (8g), and dividing both sides by χ, yields:

$$dr / dt - M / \chi + r + \dot{\omega}.MR^2\omega / (\chi 5.c_s^{\ 2}) = 0 \tag{8i}$$

This equation (8i) can be rewritten in the form:

$$\dot{r} + r + \varphi = 0 \tag{8j}$$

by denoting $\dot{r} = dr / dt$ and

$$\varphi = -M / \chi .[1 - \dot{\omega}.R^2\omega / (5.c_s^{\ 2})] \tag{8k}$$

if we suppose a linear decceleration at the surface of the spinning mass. This proposition corresponds to the Expanding Earth hypothesis, because [49]:

> "In order for expansion to occur, the moment of inertia constraints must be overcome. An expanding Earth would necessarily rotate more slowly than a smaller diameter planet so that angular momentum would be conserved."

Equation (8j) and (8k) is obviously a first-order linear ODE equation [26], which admits exponential solution. In effect, this implies that the revised equation for celestial quantization [1][2] takes the form of *spiral motion.* This could also be interpreted as a plausible solution of diffusion equation in *dissipative* medium [33], which perhaps may also correspond to the origin of spiral galaxies formation [28]. And if this corresponds to the fact, then it could be expected that the spiral galaxies and other gravitational clustering phenomena [22b] could also be modeled using the same quantization method [39], as described by Nottale [1] and Rubćić & Rubćić [2].

To this author's knowledge these equations (8j) and (8k) have not been presented before elsewhere, at least in the context of celestial quantization. In the subsequent section we will discuss how this spiral path could be understood using Ginzburg-Landau equation.

Inserting result in equation (7) into (8e) by using n=3 and v_o=23.71 km/sec for the Moon [2] yields a receding orbit radius of the Moon as large as 0.0401 m/year, which is very near to the observed value ~ 0.04 m/year [20]. The quantum number and specific velocity here are also free parameters, but they have less effect because these could be replaced by the actual Moon orbital velocity using $v_n = v_0 / n$ [1].

While this kind of receding Moon observation could be described alternatively using oscillation of gravitational potential [30], it seems that the kinetic expansion explanation is more preferable particularly with regard to a known hypothesis of *continental drift* after A. Wegener [29][49]. Apparently, none of these effects could be explained using oscillation of gravitational field argument, because they are relentless effects.

Effect of varying M, instead of varying G

In this regard, it is interesting to note that Sidharth has argued in favor of varying G [21]. From this starting point, he was able to explain –among other things-- anomalous precession (Lense-Thirring effect) of the first planet and also anomalous Pioneer acceleration, which will be discussed in the subsequent section. In principle, Sidharth's basic assertion is [21]:

$$G = G_{\otimes}.(1 + t / t_{\otimes}) \tag{9}$$

It is worthnoting here that Barrow [40c] has also considered a somewhat similar argument in the context of varying constants:

$$G = G_{\otimes}.t_{\otimes} / (t - c) \quad (9a)$$

However, in this article we will use (9) instead of (9a), partly because it will lead to more consistent predictions with observation data. Alternatively, we could also hypothesize using Maclaurin formula:

$$G = G_{\otimes}.e^{t/t_{\otimes}} = G_{\otimes}.(1 + t/t_{\otimes} + (t/t_{\otimes})^2/2! + (t/t_{\otimes})^3/3! + ...) \quad (9b)$$

This expression is a bit more consistent with the exponential solution of equation (8j) and (8k). Therefore, from this viewpoint equation (9) could be viewed as first-order approximation of (9b), by neglecting second and higher orders in the series. It will be shown in subsequent sections, that equation (9) is more convenient for deriving predictions.

If we conjecture that instead of varying G, the spinning mass M varies, then it would result in the same effect as explained by Sidharth [21], because for Keplerian dynamics we could assert k=GM, where k represents the stiffness coefficient of the system. Accordingly, Gibson [22] has derived similar conjecture of *exponential* mass flux from Navier-Stokes gravitational equation, which can be rewritten in the form:

$$M = M_{\otimes}.e^{t/t_{\otimes}} = M_{\otimes}.(1 + t/t_{\otimes} + (t/t_{\otimes})^2/2! + (t/t_{\otimes})^3/3! + ...) \quad (10)$$

provided we denote for consistency [22]:

$$t_{\otimes} = \tau_g / 2\pi \quad (10a)$$

Using the above argument of Maclaurin series, equation (10) could be rewritten in the similar form with (9) by neglecting higher order effects:

$$M = M_{\otimes}.(1 + t/t_{\otimes}) \quad (11)$$

Now the essential question here is: which equation should be used, a varying G or varying M? A plausible reasoning could be given as follows: In a recent article Gibson & Schild [23] argue that their gravitational equation based on Navier-Stokes approach results in better explanation than what is offered by Jeans instability, which yields equation (10). Furthermore, R.M. Kiehn has also shown that the Navier-Stokes equation corresponds exactly to Schrödinger equation [27].

In the meantime, Bertschinger [22b] has discussed a plausible extension of Euler equation and *Jeans instability* to describe gravitational clustering, which supports Gibson's arguments of invoking viscosity term and also turbulence phenomena [22c, 22d]. Therefore, from kinematical gravitational instability viewpoint, apparently equation (11) is more plausible than equation (9), albeit the result will be similar for most (Newtonian) gravitation problems.

From equation (11) we could write for M at time difference $\Delta t = t_2 - t_1$:

$$M_2 = M_\otimes.(1 + t_2 / t_\otimes) \quad (12)$$

$$M_1 = M_\otimes.(1 + t_1 / t_\otimes) \quad (13)$$

from which we get:

$$\Delta M = (M_\otimes / t_\otimes).(t_2 - t_1) \quad (14)$$

Inserting our definition $\Delta t = t_2 - t_1$ yields:

$$\Delta M / \Delta t = (M_\otimes / t_\otimes) = k \quad (15)$$

For verification of this assertion, we could use equation (15) instead of (1) to predict mass flux of the Earth. Inserting the present mass of the Earth from Table 1 and a known estimate of Earth epoch of 2.2x10^9 years, we get k=0.272x10^{16} kg/year, which is approximately at the same order of magnitude (ratio=13.83) with equation (4).

Inserting equation (15) into equation (1), we get:

$$M_\otimes / t_\otimes \approx -\dot{\omega}.MR^2\omega / (5.c^2) \quad (16)$$

which is the basic conjecture of the present article.

Quantization of anomalous celestial precession

It is known that the Newtonian gravitation potential equation (8d) is only weak-field approximation, and that GTR makes a basic assertion that this equation is *exact*. And if gravitation could be related to boson condensation phenomena [9][10][11], then it seems worth to quote a remark by Consoli [9b; p.2]:

> "for weak gravitational fields, the classical tests of general relativity would be fulfilled in any theory that incorporates the Equivalence Principle."

And in the same paper he describes [9b; p.18]:

> "Einstein had to start from the peculiar properties of Newtonian gravity to get the basic idea of transforming the classical effects of this type of interaction into a metric structure. For this reason, classical general relativity cannot be considered a dynamical explanation of the origin of gravitational forces."

Furthermore, Consoli also argued that the classical GTR effects other than anomalous precession could be explained without introducing non-flat metric, as described by Schiff [9b; p.19], therefore it seems that the only remarkable observational vindication of GTR is anomalous precession of the first planet [37]. Therefore, it seems reasonable to expect that the anomalous precession effect could be predicted without invoking non-flat metric, which suggestion is particularly attributed to R. Feynman, who '*believed that the geometric interpretation of gravity beyond what is necessary for special relativity is not essential in physics*' [9d]. It will be shown that a consistent approach with equation (10) will yield not only the anomalous celestial precession, but also a conjecture that such an anomalous precession is quantized.

By using the same method as described by Sidharth [21], except that we assert varying mass M instead of varying G – in accordance with Gibson's solution [22]--, and denoting the average angular velocity of the planet by

$$\dot{\Omega} \equiv 2\pi / T \tag{17}$$

and period T, according to Kepler's Third Law:

$$T = 2\pi . a^{3/2} / \sqrt{GM} \tag{18}$$

Then from equation (10), (17), (18) we get:

$$\dot{\Omega} - \dot{\Omega}_o = -\dot{\Omega}_0 . t / t_{\otimes} \tag{19}$$

Integrating equation (19) yields:

$$\varpi(t) = \Omega - \Omega_o = -(\pi / T) . t^2 / t_{\otimes} \tag{20}$$

which is average precession at time 't'. Therefore the anomalous precession corresponds to the epoch of the corresponding system. For Mercury, with T=0.25 year, equation (20) yields the average precession per year at time 't':

$$\varpi(t)_{Mercury} = \Omega - \Omega_0 = -4\pi . t^2 / t_{\otimes} \tag{21}$$

Using again $t_{\otimes} = 2x10^{10}\ year$ as the epoch of the solar system and integrating for years n=1 … 100, equation (21) will result in total anomalous precession in a century:

$$\varpi(n) = \sum_{n=1}^{n=100} \varpi(n) = 43.86''\ percentury \tag{22}$$

It would be more interesting in this regard if we also get prediction of this effect for other planets using the same method (20), and then compare the results with GTR-prediction (using Lense-Thirring effect). Table 2 presents the result, in contrast with observation by Hall and also prediction by Newcomb, which are supposed to be the same [25].

Table 2. Comparison of prediction and observed anomalous precession

Celestial Object	Period, T	$\omega_{prediction}$	Hall/ Newcomb	Diff.	GTR/ Thirring	Diff.
	(year)	(arcsec/cy)	(arcsec/cy)	(%)	(arcsec/cy)	(%)
Mercury	0.25	43.86	43.00	2.03	42.99	-0.05
Venus	0.57	19.24	16.80	14.54	0.8	-95.2
Earth	1.00	10.96	10.40	5.46	3.84	-63.1
Mars	1.88	5.83	5.50	6.02	1.36	-76.0
Jupiter	4346.5	$2.52\text{x}10^{-3}$				
Saturn	10774.9	$1.02\text{x}10^{-3}$				
Uranus	30681.0	$3.57\text{x}10^{-4}$				
Neptune	60193.2	$1.82\text{x}10^{-4}$				
Pluto	90472.4	$1.21\text{x}10^{-4}$				

It is obvious from Table 2 above that the result of equation (20) appears near to GTR's prediction and observation by Hall for the first planet, but there is substantial difference between GTR and observation for other planets particularly Venus. In the mean time, average percentage of error from prediction using equation (20) and observation (Hall) is 7.01%. The numerical prediction for Jovian planets is negligible; though perhaps they could be observed provided there will be more sensitive observation methods in the near future.

It is perhaps also worthnoting here, that if we use the expression of quantization of period [3]:

$$T = 2\pi.GM.n^3 / v_0^3 \tag{23}$$

where $v_0 = \alpha_g.c = 144 km/s$ in accordance with Nottale [1]. Inserting this equation (23) into (20), yields:

$$\varpi(t)_{precess} = \Omega - \Omega_0 = -(v_0{}^3 / 2GMn^3).t^2 / t_\otimes \tag{24}$$

or

$$T_{precess} = 2\pi / \varpi(t)_{precess} = -4\pi.t_\otimes GMn^3 / v_o^3 t^2 \tag{24a}$$

These equations (24) and (24a) imply that the anomalous precession of Lense-Thirring type should also be *quantized*. Apparently no such an assertion has been made before in the literature.

It would be interesting therefore to verify this assertion for giant planets and exoplanets, but this is beyond the scope of the present article.

A plausible test using LAGEOS-type satellites

In this regard, one of the most obvious methods to observe those effects of varying spinning mass M as described above is using LAGEOS-type satellites, which have already been used to verify Lense-Thirring effect of Earth. What is presented here is merely an approximation, neglecting higher order effects [12][16][31].

Using equation (8c) we could find the rotational effect to satellite orbiting the Earth. Supposed we want to measure the precessional period of the inclined orbit period. Then the best way to measure quadrupole moment (J_2) effect would be to measure the ϑ component of the gravity force (8c):

$$g = 1/r.\partial V/\partial\vartheta = -3GM.a^2 J_2.\sin\vartheta.\cos\vartheta/r^4 \qquad (25)$$

This component of force will apply a *torque* to the orbital angular momentum and it should be averaged over the orbit. This yields a known equation, which is often used in satellite observation:

$$\omega_p/\omega_s = -3a^2 J_2.\cos i/2r^2 \qquad (26)$$

where i is the inclination of the satellite orbit with respect to the equatorial plane, a is Earth radius, r is orbit radius of the satellite,

ω_s is the orbit frequency of the satellite, and ω_p is the precession frequency of the orbit plane in inertial space. Now using LAGEOS satellite data [31] as presented in Table 3:

Table 3. LAGEOS satellite parameters

Parameter	Value	Unit
R_{LAGEOS}	$12.265x10^{6}$	M
i_{LAGEOS}	109.8	^{o}
T_{LAGEOS}	13673.4	sec
ω_s	$4.595x10^{-4}$	rad/s
J_2	$1.08x10^{-3}$	

Inserting this data into equation (26) yields a known value:

$$\omega_p = 0.337561° / day \tag{27}$$

which is near enough to the observed LAGEOS precession = 0.343^{o}/day.

Now let suppose we want to get an estimate of the effect of Earth kinetic expansion to LAGEOS precession. Inserting (r+dr/dt) from equation (6) to compute back equation (26) yields:

$$\Delta\omega_p = \omega_{p,n+1} - \omega_{p,n} = 1.41x10^{-9}° / day = 2.558 arc\sec / year \tag{28}$$

Therefore, provided the aforementioned propositions correspond to the facts, it could be expected to find a secondary precession of LAGEOS-satellite around 2.558 arcsecond/year. To this author's knowledge this secondary effect has not been presented before elsewhere. And also thus far there is no coherent explanation of those aforementioned phenomena altogether, except perhaps in [21] and [30].

As an alternative to this method, it could be expected to observe Earth gravitational acceleration change due to its radius increment. By using equation (8d) and (5):

$$\ddot{r}(t) = GM / r^2 = 4\pi.G.\rho_{sphere}.r / 3 \quad (29)$$

From this equation, supposing there is linear radius increment, then we get an expression of the rate of change of the gravitational acceleration:

$$\dddot{r}(t) = \Delta\ddot{r} / \Delta t = 4\pi.G.\rho_{sphere}.(r + \Delta r / \Delta t) / 3 - \ddot{r}(t) \quad (30)$$

Therefore, it would be interesting to find observation data from LAGEOS to verify or refute this equation.

Ginzburg-Landau equation and solar system: possible signatures of spiral gravity

The pattern formation is often described as result of diffusion reaction. And the most popular equation in these pattern-formation studies is CGLE (complex Ginzburg-Landau equation). These reaction-diffusion systems govern almost all phenomena in Nature from the smallest quantum entities to galaxies [40][41]. E. Goldfain has also considered CGLE with possible application in description of elementary particle masses [52].

In this regards, a considerable attempt has been made towards a better understanding of partial differential equations of parabolic type in *infinite space*. A typical equation is known as CGLE, which is commonly described as follows [42]:

$$\partial_t A = (1 - i\alpha)\Delta A + A - (1 + i\beta).A|A|^2 \quad (31)$$

The most interesting characteristics of CGLE is its superspiral solution [43], or ‘scroll waves’ pattern [44]. This equation could also lead to a kind of *'dark soliton’, which* is quite related to NLSE (nonlinear Schrödinger equation) [45].

A relative periodic orbit of the CGLE with drift (φ, S) and period T contains solutions that satisfy for all t [46]:

$$A(x,t) = e^{i\varphi}.A(x+S,t+T) \qquad (32)$$

The corresponding solution of the system of ODEs derived from CGLE thus satisfies [46]:

$$a_m(t) = e^{i\varphi}.e^{imS}.a_m(t+T) \qquad (33)$$

for all m and t. This equation could be reintroduced in the form [46]:

$$a_m(t) = e^{-t.Lg/T}.b(t/T) \qquad (34)$$

Where b is periodic with the period one, and

$$L_g = diag(i\varphi + imS) \qquad (34a)$$

Alternatively, solution of CGLE could be found in terms of MAW (modulated amplitude waves) with expression as follows [43]:

$$A(r,t) = a(z).e^{i\phi(z)}.e^{i(qr-\omega t)} \qquad (35)$$

Interestingly, this could be related to an extended solution of Bohr-radius-type equation of celestial quantization. In accordance with equation (8i)-(8j)-(8k), we could extend Bohr-radius type expression of quantized orbit of celestial bodies in solar system in the form of spiral motion. Therefore, it seems plausible to assert that the form of equation (34) and (35) appears very similar with equations (8i)-(8j)-(8k). This seems to suggest a possibility that CGLE could be related to quantization of celestial bodies, in lieu of describing this macroquantization using Schrödinger-Euler-Newton like Nottale's Scale Relativity Theory [1]. In this regards, El Naschie has also noted the significance of spiral geometry to describe gravitation (sometimes called 'spiral gravity').

For observational verification, we could rewrite equation (8j) and (8k):

$$dr/dt = M/\chi.[1-\dot{\omega}.R^2\omega/(5.c_s^{\ 2})]-r \tag{36}$$

and inserting equation (15), we get:

$$dr/dt = M/\chi.[1+M_{\otimes}/(t_{\otimes}.M)]-r \tag{37}$$

A plausible test of this conjecture could be made by inserting the result from equation (14) into equation (8e) and using $M_{\otimes} = 1.98951x10^{33}\, g$ and $t_{\otimes} = 2x10^{10}\, year$ as the epoch of the solar system [21], and specific velocity v_o=144 km/sec [1], then from equation (37) we get a receding Earth orbit radius from the Sun at the order of:

$$\Delta r_{Earth}/\Delta t = 6.03m/\,year \tag{38}$$

Interestingly, there is an article [24] hypothesizing that the Earth orbit is receding from the Sun at the order of 7.5 m/year, supposing Earth orbit radius has been expanding as large as 93x10^6 miles since the beginning of the solar epoch. (Of course, it shall be noted that there is large uncertainty of the estimate of solar epoch, see for instance Gibson [22]).

Therefore, it is suggested here to verify this asumption of solar epoch using similar effect for other planets. For observation purposes, some estimate values were presented in Table 4 using the same approach with equation (37).

Table 4. Prediction of planetary orbit radii (r) increment

Celestial object	Quantum number (n)	Orbit increment (m/yr)
Mercury	3	2.17
Venus	4	3.86

Earth	5	6.03
Mars	6	8.68

Concluding remarks

If physical theories could be regarded as continuing search to find systematic methods to reduce the entropy required to do calculation to minimum; then the fewer free parameters in a theory and the less computation cost required, the better is the method. Accordingly, in this article some twelve phenomena can be explained using only few free parameters, including:

- The Moon is receding from the Earth [20];
- Earth's angular velocity decrease (Kip Thorne, G. Smoot, J. Wells) [19];
- Planets are receding from the Sun [24];
- Lense-Thirring effect for inner planets, corresponding to Hall/Newcomb's observation;
- Celestial orbit prediction in solar system [1][2][3];
- Exoplanets orbit prediction [1][3];
- Pioneer-type anomalous acceleration [21];
- A plausible origin of increasing day length (24 second each million years);
- A plausible origin of continental drift effect [29];
- A plausible origin of spiral motion in spiral nebulae [22];
- Prediction of possible extra precession of LAGEOS satellite [31];
- Prediction of angular velocity decrease of other planets.

As a plausible observation test of the propositions described here, it is recommended to measure the following phenomena:

- Lense-Thirring effect of inner planets, compared to spherical kinetic dynamics prediction derived herein;
- Annual extra precession of Earth-orbiting LAGEOS-type satellites;
- Receding planets from the Sun;
- Receding satellites from their planets, similar to receding Moon from the Earth – all these celestial objects take the form of spiral motion;
- Angular velocity decrease of the planets;
- Angular velocity decrease of the Sun.

It appears that some existing spacecrafts are already available to do this kind of observation, for instance LAGEOS-type satellites [31]. Further refinement of the method as described here could be expected, including using *ellipsoidal kinetic dynamics* [12] or using analogy with neutron star dynamics [32]. Further extensions to cosmological scale could also be expected, for instance using some versions of Cartan-Newton theory [38]; or to find refinement in predictions related to varying constants.

All in all, the present article is not intended to rule out the existing methods in the literature to predict Lense-Thirring effect, but instead to argue that perhaps the notion of '*frame dragging*' in GTR [14][16] could be explained in terms of *dynamical interpretation*, through invoking the spherical kinetic dynamics. In this context, the dragging effect is induced by the spinning spherical mass to its nearby celestial objects.

Provided all of these correspond to the observed facts, it seems plausible to suggest that it is possible to derive celestial quantization in terms of (complex) Ginzburg-Landau equation, instead of the known Schrodinger-Euler-Newton like in Nottale's Scale Relativistic Theory [1]. Because CGLE is also commonly used in the context of

Bose gas [43][48], then it seems also plausible to hypothesize that the subtle medium of subparticle structure may be described using Winterberg's superfluid phonon-roton model [8]. It is known that an essential feature of Winterberg's superfluid Planckian aether model is that the basic entity is comprised of pairs of Planck mass. Interestingly, similar hypothesis of Planck mass as the basic entity of Nature has also been suggested by Spaans, using topological arguments [47]. Other implications of this CGLE's superspiral quantization either in nuclei realm or cosmological prediction remain to be explored [48].

If this proposition described here corresponds to the facts, then one can say that it is possible to 're-derive' General Relativity phenomena from the viewpoint of Bohr-Sommerfeld quantization and spherical kinetic dynamics. Possible verification of this proposition may include the use of Earth-based satellites, which go beyond traditional GTR-tests such as precession of the first planet. Further observation to verify or refute this conjecture is recommended, plausibly using LAGEOS-type satellites

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Appendix I: Derivation of equation (1)

We start with some basic equations that will be used throughout the present article. It is assumed that the solar nebula is disk-shaped and is in hydrostatic equilibrium in the vertical direction. Let suppose that the disk has approximately Keplerian rotation, ω; then the half-thickness of the disk is given by [4d; p.4-5]:

$$d = c_s / \omega \tag{i}$$

and

$$c_s \approx \sqrt{kT/m} \tag{ii}$$

where d and c_s represents half-thickness of the disk and sound velocity, respectively.

In order to find the spherical kinetic dynamics contribution to Lense-Thirring effect, we begin with the spinning dynamics of solid sphere with mass M. Using the known expression [12; p.6, p.8]:

$$E_{kinetic} = -I_{zz}\omega^2 / 2 \tag{iii}$$

$$I_{sphere} = 2MR^2 / 5 \tag{iv}$$

where I_{zz}, ω, M, R represents angular momentum, angular velocity, spinning mass of the spherical body, and radius of the spherical body, respectively. Inserting equation (iv) into (iii) yields:

$$E_{kinetic} = -MR^2\omega^2 / 5 \tag{v}$$

This known equation is normally interpreted as the amount of energy required by a spherical body to do its axial rotation. But if instead we conjecture that '*galaxies get their angular momentum from the global rotation of the Universe due to the conservation of the angular momentum'* [34], and likewise the solar system rotates because of the corresponding galaxy rotates, then this equation implies that the rotation itself exhibits extra kinetic energy. Furthermore, it has been argued that the global rotation gives a natural explanation of the empirical relation between the angular momentum and mass of galaxies: $J \approx \alpha M^{5/3}$ [34]. This conjecture seems to be quite relevant in the context of Cartan torsion description of the Universe [18][38]. For reference purpose, it is worthnoting in this regard that sometime ago R. Forward has used an argument of non-Newtonian gravitation force of this kind, though in the framework of GTR (*Amer.J.Phys*. **31** No. 3, 166, 1963).

Let suppose this kind of extra kinetic energy could be transformed into mass using a known expression in condensed-matter physics [10b; p.4], with exception that c_s is used here instead of v to represent the sound velocity:

$$E_{kinetic}(n, p) = c_s . p = m_s . c_s^2 \qquad \text{(vi)}$$

where the sound velocity obeying [10b; p.4]:

$$c_s^2(n) = (n/m)(d^2 \in /dn^2) \qquad \text{(vii)}$$

Physical mechanism of this kind of mass-energy transformation is beyond the scope of the present article, albeit there are some recent articles suggesting that such a condensed-matter radiation is permitted [35]. Now inserting this equation (vi) into (v), and by dividing both sides of equation (v) by Δt, then we get the incremental mass-energy equivalent relation of the spinning mass:

$$\Delta m_s / \Delta t = -\omega.(\Delta\omega / \Delta t).MR^2 /(5.c_s^2) \qquad \text{(viii)}$$

By denoting $\dot{\omega} = \Delta\omega / \Delta t$, then this equation (viii) can be rewritten as:

$$\Delta m_s / \Delta t = -\omega.(\Delta\omega / \Delta t).MR^2 /(5.c_s{}^2) \qquad \text{(ix)}$$

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Gravitational Schrödinger equation from Ginzburg-Landau equation, and its noncommutative spacetime coordinate representation

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Despite known analogy between condensed matter physics and various cosmological phenomena, a neat linkage between low-energy superfluid and celestial quantization is not yet widely accepted in literature. In the present article we argue that gravitational Schrödinger equation could be derived from time-dependent Ginzburg-Landau (or Gross-Pitaevskii) that is commonly used to describe superfluid dynamics. The solution for celestial quantization takes the same form with Nottale equation. Provided this proposed solution corresponds to the facts, and then it could be used as alternative solution to predict celestial orbits from quantized superfluid vortice dynamics. Furthermore, we also discuss a representation of the wavefunction solution using noncommutative spacetime coordinate. Some implications of this solution were discussed particularly in the context of offering a plausible explanation of the physical origin of quantization of motion of celestial objects.

Keywords: superfluidity, Bose-Einstein condensate, vortices, gravitation, celestial quantization

Introduction

There has been a growing interest in some recent literatures to consider gravity as scalar field from boson condensation [1]. This conjecture corresponds to recent proposals suggesting that there is neat linkage between condensed matter physics and various cosmological phenomena [2,3]. In this regard, it is worth noting here that some authors have described celestial quantization from the viewpoint of gravitational Schrödinger-type wave equation [4]. Considering that known analogy between condensed matter physics and various cosmological phenomena, then it seems also plausible to describe such a celestial quantization from the viewpoint of condensed-matter physics, for instance using Gross-Pitaevskii (GP) or Ginzburg-Landau wave equation.

In the present article, we derived gravitational Schrödinger-type wave equation from various equations known in condensed matter physics, including Gross-Pitaevskii (GP) equation and also time-dependent Ginzburg-Landau (TDGL) wave equation. This method could be regarded as 'inverse' way from method discussed in Berger's article [5], suggesting that it is possible to extend Schrödinger equation to TDGL using De Broglie potential. Provided this neat linkage from TDGL/GPE and Schrödinger equation is verified by observation, then it seems to support a previous conjecture of a *plausible linkage between celestial quantization and quantized vortices* [4]. And then we discuss some issues related to describing cosmological phenomena in terms of diffusion theory of gravitational Schrödinger-type equation, though this issue has been discussed in the preceding articles [3,8,9]. Furthermore, following our argument that it is possible to find noncommutative representation of the wavefunction [4], and then we will discuss a plausible interpretation of the gravitational

Schrödinger equation in terms of *noncommutative spacetime* coordinate. This extension to noncommutative coordinate perhaps will be found useful for further research. And if this proposition corresponds to the astrophysical facts, then it can be used to explain the origin of quantization in astrophysics [7][8].

An alternative method to find solution of gravitational Schrödinger-type equations

The present author acknowledged that the proposed method on relating cosmological phenomena with condensed-matter/low-energy physics has not been widely accepted yet, though some of these approaches have been used to predict phenomena corresponding to neutron stars [12,39]. Furthermore, there is also a deeper question concerning the appropriateness of using and solving gravitational Schrödinger-type equations for depicting cosmological phenomena, beyond what is called as Wheeler-DeWitt (WDW) equation. It should be noted here that our derivation method is somewhat different from Neto *et al*.'s approach [14], because we use Legendre polynomials approach.

Now we are going to find solution of the most basic form of Schrödinger-type equation using Legendre polynomials, from which we will obtain the same expression with known Nottale's quantization equation [11]. We start with noting that Schrödinger equation is derived from a wave of the form:

$$\Psi = \alpha . \sin 2\pi x / \lambda \tag{1}$$

By deriving twice equation (1), then we get the most basic form of Schrödinger equation:

$$d^2\Psi / dx^2 + A.\Psi = 0 \tag{2}$$

where for planetary orbits, it can be shown [13, 5] that we get:

$$A = 4\pi / \lambda^2 = \omega^2 / v^2 = m\omega^2 /(2.KE) \quad (3)$$

Solution of equation (2) is given by:

$$\chi = C_1 .\exp(\rho / 2) + C \ .\exp(-\rho / 2) \quad (4)$$

But we shall reject the first term because it will result in infinity for large distance (ρ>>0). This suggests solution of the form [14]:

$$\chi = F(\rho).\exp(-\rho / 2) \quad (5)$$

Substituting (5) into (2), we get:

$$d^2F / d\rho^2 - dF / d\rho + A.F = 0 \quad (6)$$

Now we shall find the series solution to (6) and put:

$$F = \sum_{p=1}^{\infty} a_p .\rho^p \quad (7)$$

The lower limit of this summation is p=1 rather than p=0, otherwise F and therefore χ would not be zero at ρ=0. Thus [14]:

$$dF / d\rho = \sum_{p=1}^{\infty} p.a_p .\rho^{p-1} \quad (8)$$

$$d^2F / d\rho^2 = \sum_{p=1}^{\infty} (p+1) p.a_{p+1} .\rho^{p-1} \quad (9)$$

$$F / \rho^2 = a_1 .\rho^{-1} + \sum_{p=1}^{\infty} .a_{p+1} .\rho^{p-1} \quad (10)$$

By inserting these equations (7), (8), (9), and (10) into equation (6), and observing that *each power of* ρ *mush vanish*, and by inserting our defiition of variable A from equation (3) and inserting the kinetic energy definition $KE = GMm / 2r$, and then we could find the expression for orbital radii which is similar to Nottale's equation [11]:

$$r_o = n^2 .GM / v_o^2 \quad (11)$$

Therefore we observed that a solution using Legendre polynomials yields the same expression with Nottale's quantization equation [11]. It is also obvious that some assumptions must be invoked in order to find the proper asymptotic solution.

On celestial quantization from GPE and TDGL

In a preceding article we provided simplified derivation of equation of quantization of planetary orbit distance based on Bohr-Sommerfeld hypothesis of quantization of angular momentum [4], which could be considered as 'retro' version of Bohr-Sommerfeld quantization method in microphysics. As shown above, similar quantization result can be derived from generalized Schrödinger-Newton equation suggested by L. Nottale [11].

But this Schrödinger-type wave equation does not exactly correspond to the superfluid theory or condensed matter, therefore in the present article we will derive Schrödinger-type wave equation based on GP/TDGL equation, which is commonly used to describe superfluid medium [3]. It will be shown that the previous solution (11) based on gravitational Schrödinger-type equation is only an approximation of a more general GP/TDGL equation, becauses it neglects nonlinear effects like temperature dependent or screening potential. This conjecture of quantum vortice dynamics also corresponds to hypothesis by Winterberg of superfluid phonon-roton as Planckian quantum vacuum aether [9].

First, we will discuss how to get Schrödinger-type equation from GP equation, and then from TDGL. At subsequent section we will discuss other nonlinear Schrödinger-type equation from Chern-Simons theory.

a. Gross-Pitaevskii equation (GPE)

As we know, superfluid medium is usually described using GP equation, or sometimes known as nonlinear Landau-Ginzburg equation or nonlinear Schrödinger equation (NLSE) [12,2]. In the GP theory the ground state and weakly excited states of a Bose gas are described by the condensate wave function ψ=a.exp(iϕ) which is a solution of the nonlinear Schrödinger equation [6]:

$$i\hbar.\partial\psi/\partial t = -\hbar^2/2m.\nabla^2\psi + V\,|\psi|^2\,\psi \tag{12}$$

where V is the amplitude of two-particle interaction.

It has been argued [6], that two-fluid hydrodynamics relations can be derived from the hydrodynamics of an ideal fluid in presence of thermally excited *sound waves*, i.e. phonon scattering by a vortex line. In order to obtain a complete system of equations of the two-fluid theory, one should take into consideration phonon-phonon interaction, which is essential for the phonon distribution function being close to the equilibrium Planck distribution. It was shown in [1], that this sound wave of boson condensate system consists of phonons with *sound velocity* of $c_s^{\,2} = \partial P/\partial(\mu\rho) = \pi''\rho/\mu$.

Furthermore, the phonon scattering by a vortex line is analogous to the so-called Aharonov effect for electrons scattered by a magnetic-flux tube, which analogy becomes more evident if one rewrites the sound equation [6] in presence of the vortex as:

$$k^2\phi - \left(-i\vec{\nabla} + k\vec{v}_v/c_s\right)^2\phi = 0 \tag{13}$$

But the stationery Schrödinger equation for an electron in presence of the magnetic flux confined to a thin tube is given by [6]:

$$E\psi(\vec{r}) = 1/2m.\left(-i\hbar\vec{\nabla} - e\vec{A}/c\right)^2\psi(\vec{r}) \tag{14}$$

Here ψ is the electron wave function with energy E and the electromagnetic vector potential is connected with the magnetic flux ϕ by the relation similar to that for the velocity $\vec{v}_v$ around the vortex line [6]:

$$\vec{A} = \Phi.\left[\hat{z}x\vec{r}\right]/2\pi r^2 \tag{15}$$

In other words, we have outlined a logical mapping [6]: (i) from GP (NLSE) equation to the two-fluid hydrodynamics; (ii) from hydrodynamics to the phonon scattering equation; (iii) from phonon scattering to electron scattered by magnetic-flux tube, and (iv) from electron scattering back to the stationery Schrödinger equation. Now it is worthnoting here, that there is *exact solution* of Aharonov effect for electrons obtained by the partial wave expansion. To find the solution of equation (14), partial-wave amplitudes ψ_l should satisfy equations in the cylindrical system of coordinates (r,φ) [6]:

$$d^2\psi_l / dr^2 + 1/r.d\psi_l / dr - (1-\gamma)^2.\psi_l / r^2 + k^2.\psi_l = 0 \qquad (16)$$

where

$$E = k^2\hbar^2 / 2m \qquad (17)$$

or

$$k^2 = 2m.KE / \hbar^2 = 1/\lambda^2 \qquad (18)$$

where KE, $\hbar, \lambda$ denotes the kinetic energy of the system, Planck constant and wavelength, respectively. From this equation (16), then we shall find a solution, which at large distances has an *asymptotic* character expressed in exponential form of $\psi=\alpha.\exp(\beta)$, which is typical solution of Schrödinger-type equation; where α and β are functions of some constants.

Because equation (16) is an ordinary differential equation in planar cylindrical system of coordinates, we consider that this equation corresponds to the celestial quantization if we insert proper values of Newtonian equation [4]. Therefore in the subsequent derivation we will not follow the standard partial wave analysis method as described in [6], but instead we will use a method to find solution of ordinary differential equation of Schrödinger equation: $a=n^2.GM/v_o^2$, which is in accordance with Nottale's solution [11]. Here a, n, G, M, v_o, represents semimajor axes, quantum number (n=1,2,3,…), Newton gravitation constant, mass of nucleus of gravitation field, and specific velocity, respectively.

Solution of equation (16) is given by $\psi_1(r,\varphi)= R(r).F(\phi)$. Inserting this relation into (16), and separating the $F(\phi)$ terms, then we get the *ground state* expression of the system ($m^2=0$ case):

$$d^2R/dr^2 + 1/r.(dR/dr) - [(1-\gamma)^2/r^2 + k^2].R = 0 \qquad (19)$$

The solution for R(r) is given by :

$$R(r) = [e^{-\alpha.r} + e^{\alpha.r}] \qquad (19a)$$

In order to get the sought-after *asymptotic* solution for equation (16), we only use the negative expression of R(r), otherwise the solution will diverge to infinity at large distance r:

$$R(r) = e^{-\alpha.r} \qquad (20)$$

Therefore

$$dR(r)/dr = -\alpha e^{-\alpha.r} \qquad (21)$$

$$d^2R(r)/dr^2 = \alpha^2.e^{-\alpha.r} \qquad (22)$$

Inserting (19a)-(22) into equation (19) and eliminating the exponential term $e^{-\alpha.r}$, yield:

$$\alpha^2 = 1/r^2.\{\alpha r + (1-\gamma)^2 - r^2k^2\} \qquad (23)$$

Because equation (23) must be right for any value of r, then the right hand side of equation (23) between the {} brackets must equal to zero:

$$\alpha r + (1-\gamma)^2 - r^2k^2 = 0 \qquad (24)$$

Maple solution for equation (24) is included in the Appendix section, which yields for γ:

$$\gamma = 1 \pm \sqrt{\alpha^2 r^2 - \alpha r + k^2 r^2} \qquad (25)$$

The remaining part is similar to equation (10)-(11), by inserting kinetic energy definition for gravitational potential.

Therefore we conclude that the right term between the {} brackets yields a secondary effect to the equation of celestial quantization, except for some condition where this extra term vanishes. To this author's knowledge, this secondary effect has never been derived before; neither in Nottale [11], nor Neto *et al*. [13]. In our method, the secondary effect comes directly from the partial wave analysis expression of GP equation.

Therefore we obtain a generalised form of the equation of celestial quantization [11], which has taken into consideration the secondary interaction effect of GPE. The expected value for γ can be estimated by equating the right term between the {} brackets to one.[1] However, it is not too clear in what kind of conditions this right term in the bracket will disappear, therefore we are going to discuss another approach for deriving gravitational Schrödinger-type equation, i.e. using TDGL (time-dependent Ginzburg-Landau equation).

b. Time-dependent Ginzburg-Landau equation (TDGL)

It is known that Ginzburg-Landau (TDGL) equation is more *consistent* with known analogy between superfluidity and cosmological phenomena [2][3], and TDGL could also describe vortex nucleation in rotating superfluid [19]. According to Gross, Pitaevskii, Ginzburg, wavefunction of N bosons of a reduced mass m* can be described as [20]:

$$-(\hbar^2/2m^*).\nabla^2\psi + \kappa|\psi|^2\psi = i\hbar.\partial\psi/\partial t \tag{26}$$

It is worthnoting here that this equation is quite similar to Jones' nonlinear Schrödinger equation to describe gravitational systems [21]. For some conditions, it is possible to replace the potential energy term in equation (26) by Hulthen potential. This substitution yields:

$$-(\hbar^2/2m^*).\nabla^2\psi + V_{Hulthen}.\psi = i\hbar.\partial\psi/\partial t \tag{27}$$

where

$$V_{Hulthen} = -Ze^2.\delta.e^{-\delta r}/(1-e^{-\delta r}) \qquad (28)$$

This equation (27) has a pair of exact solutions. It could be shown that for small values of δ, the Hulthen potential (28) approximates the effective Coulomb potential, in particular for large radius:

$$V_{Coulomb}^{eff} = -e^2/r + \ell(\ell+1).\hbar^2/(2mr^2) \qquad (29)$$

Inserting (29) into equation (27) yields:

$$-\hbar^2\nabla^2\psi/2m^* + \left[-e^2/r + \ell(\ell+1).\hbar^2/(2mr^2)\right]\psi = i\hbar.\partial\psi/\partial t \qquad (30)$$

While this equation is interesting to describe neutron model, calculation shows that introducing this Hulthen effect (28) into gravitational equation will yield different result only at the order of 10^{-39} m compared to prediction using equation (11), which is of course negligible. Therefore, we conclude that for most celestial quantization problems the result of TDGL with Hulthen potential (28) is *essentially* the same with the result derived from equation (11).

Some implications to cosmology model

The approach described in the previous section using arguments based on condensed matter physics also implies that the linear and point-like topological defects also induce an effective metric, which can be interesting for the theory of gravitation. In this regards, the vortex can be considered as cosmic spinning string.[2]

Another question can be asked here, i.e. to how extent GP equation could be regarded as *exact representation* of cosmological phenomena, because there are arguments suggesting that GP equation is only an approximation [23]. For instance, Castro *et al.* [22] argued that GP equation of NLSE has some weakness, i.e. it does not meet Weinberg homogeneity condition.

Therefore, it becomes obvious that there is also a typical question concerning whether such Schrödinger-type wave function expression corresponds to vortices description in hydrodynamics. In this regard, it seems worth here to consider a more rigorous approach based on Chern-Simons hydrodynamics. Pashaev & Lee [24] reformulated the case of Abelian Chern-Simons gauge field interacting with Nonlinear Schrodinger field as planar Madelung fluid. In this regard, the Chern-Simons Gauss law has simple physical meaning of creation of the local vorticity for the fluid flow; which appears very similar to Kiehn's derivation using Navier-Stokes argument [17,27]. Then Pashaev & Lee [24] obtained the following nonlinear wave equation:

$$iD_0\Psi + D^2\Psi / 2m - U\Psi = (1-\hbar^2)/2m.(\Delta|\Psi|.\Psi/|\Psi|) \qquad (31)$$

where

$$D_0 = \partial + \in / c.A_0 \qquad (32)$$

$$D = \nabla + \in / c.A \qquad (33)$$

Then in terms of a new wave function

$$\chi = \sqrt{\rho}.\exp(iS/\hbar) \qquad (34)$$

they recovered the standard linear Schrödinger equation:

$$i\hbar D_0\chi + D^2\chi.\hbar/2m - U\chi = 0 \qquad (35)$$

Thus they concluded that for $\hbar \neq 0$ equation (34) is gauge equivalent to the Schrödinger equation, while for $\hbar = 0$ it reduces to nonlinear wave equation of classical mechanics. The semiclassical limit has been applied to defocusing NLSE [24]:

$$i\hbar\partial_t\chi + \Delta\chi.\hbar^2/2m + 2g|\chi|^2\chi = 0 \qquad (36)$$

which provides an analytical tool to describe shockwave in nonlinear optics and vortices in superfluid. In the formal semiclassical limit $\hbar \to 0$ (before shocks), *one neglects the quantum potential and fluid becomes the Euler system*. Introducing the local velocity field:

$$V = 1/m.[\nabla S + e/c.A] \qquad (37)$$

And then they obtained a hydrodynamical model defined by two equations:

$$\partial V / \partial t + (V\nabla)V = -\nabla(-2g\rho - \hbar^2 / 2m.\Delta\sqrt{\rho}/\sqrt{\rho})/m \qquad (38)$$

$$\nabla x V = e^2 \rho/(m\kappa c^2) \qquad (39)$$

Therefore we concluded that a more rigorous representation of quantum fluid admits vortice configuration. It is perhaps interesting to remark here, that these equations differ appreciably from Nottale's basic Euler-Newton equations [11]:

$$m.(\partial / \partial t + V.\nabla)V = -V(\phi + Q) \qquad (40)$$

$$\partial\rho/ \partial t + div(\rho V) = 0 \qquad (41)$$

$$\Delta\phi = -4\pi G\rho \qquad (42)$$

which of course neglect vortice configuration.

Upon generalizing the solution derived above, we could expect to see some plausible consequences in cosmology. For instance, that (i) there should be a kind of Magnus-Iordanskii type force observed in astrophysical phenomena, and (ii) that there should be *hollow tubes* inside the center of spinning large celestial bodies, for instance in the Sun and also large planets, including this Earth;[3] (iii) the universe is also very likely to rotate, in accord with recent observation by Nodland & Ralston [25];[4] (iv) the notion of gravitational constant could be related to cosmological temperature [3]; and (v) there exists *ergoregions* in the rotating centers of celestial objects where phonon particles are continuously created [26]. This phenomenon of phonon creation in the ergoregions may offer a rational basis of the observed continuous expansion of the universe. However, it shall be noted here that all of these plausible consequences to cosmology require further research.

Furthermore, some recent observations have concluded that our universe has fractality property. For clarity, the number of galaxies N(r) within a sphere of radius r, centered on any galaxy, is not proportional to r^3 as would be expected of a homogeneous distribution. Instead N(r) is proportional to r^D, where D is approximately equal to 2, which is symptomatic to distribution with fractal dimension D. It is interesting to note, for D=2, the cosmological gravitational redshift gives the linear distance-redshift relation and becomes an observable phenomenon [28]. This property is indicated by its Hausdorff dimension, which can be computed to be within the range of 1.6 ~ 2.0 up to the scale of 200 Mpc. Furthermore, transition to homogeneity distribution has not been found yet. In this regard, P.W.Anderson *et al*. [29] also remarked: "*These findings (of clustering and void formation) have become increasingly difficult to reconcile with standard cosmological theories, in which the approach to homogeneity at large-scales is central element*." It is worth noting here that perhaps this fractality property can be explained using boson condensate model with non-integer dimension. It has been argued that such a boson condensate system exhibits Hausdorff dimension $d_H \sim 2$ [30]. There is also article arguing in favor of relating the fractal dimension with fluctuation graph [31]:

$$D = 2 - \alpha/2 \quad \text{for } \alpha \leq 2 \tag{43}$$

where α is the time decay exponent. Furthermore, it was shown recently that an extended version of *GP equation admits self-similar* solutions and also it corresponds to Hausdorff dimension $d_H \sim 2$ [23], which seems to confirm our hypothesis that there is exact correspondence between cosmological phenomena and condensed matter physics [1,2].

Therefore this Hausdorff dimension argument seems to be a plausible restriction for a good cosmology theoretical model: *Any cosmology theory which cannot exhibit fractality property from its intrinsic parameters perhaps is not adequate to explain inhomogeneity of large scale structures in universe.*

It is also worthnoting here, that an alternative argument in favor of cosmology with d_H~2 has been considered recently by Roscoe [30], which corresponds to Mach principle. While his argument seems very encouraging and perhaps it is also deeply interwoven with arguments presented herein, it shall be noted that his argument suggests the universe *must* have a fractal dimension d_H~2, while in the context of condensed matter physics it can fluctuates around 1.6~2.0 as observed [7]. Furthermore, by making an allusion to Newton's argument, Roscoe also did not consider any physical origin of such fractal distribution of masses in the universe, except that it corresponds to the nature of quantum vacuum aether. Nonetheless, Roscoe's conjecture on the presence of universal clock is very interesting.

Furthermore, if the equation of quantization of celestial motion derived herein from GPE/TDGL equation corresponds to the observed astrophysical facts, then it implies that it seems possible now to conduct a set of laboratory experiments as replica of some cosmological objects [2], provided we take into consideration proper scale modeling (similitude) theories.

Noncommutative spacetime representation

In this section we are going to discuss an alternative representation of the abovementioned Schrödinger equation using noncommutative spacetime coordinate, based on Vancea [33]. According to Vancea, the stationary Schrödinger equation is constructed by analogy with the commutation case an has the following form [33]:

$$H(x,p) * \Psi(x) = E.\Psi(x) \tag{44}$$

Here the wavefunction Ψ belongs to the noncommutative algebra, A_*. If explicit form of Schrödinger equation is given by [33]:

$$\left[-\hbar/2M.\sum_{m=1}^{2N}\partial_m^2+V\right]*\Psi=E\Psi \tag{45}$$

where V(x) is an arbitrary function from A_* and M is the mass of particle. The star product in the kinetic term is equal to the commutative product. Therefore, following the commutative case, the coordinates x^s for k=1,2,…,2N is a variable, and the coordinate x^k for is fixed. Equation (45) could be rewritten in the form [33]:

$$\left[-\hbar^2\partial_k{}^2/2M+V_k*\Psi\right](x)=E\Psi(x) \tag{46}$$

Supposed that there are two solutions of the equation (45) denoted by Ψ_k and $\widetilde{\Psi}_k$. Then they are linearly dependent, i.e. there are two nonzero complex numbers c_k and $\widetilde{c}_k$, such that the following relations hold simultaneously

$$\Psi_k=-\widetilde{c}_k/c_k.\widetilde{\Psi}_k \tag{47a}$$

$$\partial_k\Psi_k=-\widetilde{c}_k/c_k.\partial_k\widetilde{\Psi}_k \tag{47b}$$

Now, by introducing the quantum prepotential defined as in the commutative case by the following relation

$$\widetilde{\Psi}_k\equiv\partial F^k\left[\Psi_k\right]/\partial\Psi_k \tag{48}$$

Then the relation between noncommutative coordinate x^k and wavefunction has the following form;

$$x^k=F^k\left[\Psi_k\right]-\widetilde{\Psi}_k/2*\Psi_k-f^k\left(x^s\right) \tag{49}$$

This result appears interesting because now our gravitational wavefunction (11) could be given spacetime coordinate representation. This would be interesting subject for further study of the connection between condensed matter wavefunction (GPE/TDGL) and spacetime metric.

Concluding remarks

In the present article, we derived an alternative derivation of celestial quantization equation based on GPE/TDGL equation. It was shown that the obtained solution is also applicable to describe various phenomena in cosmology, including inhomogeneity and clustering formation. In this regard, fractality property emerges naturally from the theoretical model instead of invoked; and it corresponds to the observed value [7] of Hausdorff dimension ranging from 1.6~2.0 in universe up to the scale of 200 Mpc.

It could be expected therefore that in the near future there will be more rigorous approach to describe this fractality phenomena both in boson condensate and also in astrophysics, from which we can obtain a coherent picture of their interaction. Another interesting issue for future research in this regard, is extending the solution derived herein to include superfluid turbulence and also finding its implications in astrophysics.

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Appendix

Thanks to a note by anonymous referee, a Maple solution is included here to find solution of Schrodinger type radial equation from GPE (24). This solution indicates that for an exponential solution to present, this requires that extra term of GPE must vanish.

```
>#Partial Wave analysis
>restart;
> with (linalg):

>R:=exp(-(alpha*r));
D1R:=diff(R,r);D2R:=diff(D1R,r);
```

$$R := \mathbf{e}^{(-\alpha\, r)}$$

$$D1R := -\alpha\, \mathbf{e}^{(-\alpha\, r)}$$

$$D2R := \alpha^2\, \mathbf{e}^{(-\alpha\, r)}$$

```
Formulate the partial wave equation referenced from
Sonin[6]

>SCHEQ:=D2R+D1R/r-(1-g)^2*R/r^2+(k)^2*R;
```

$$SCHEQ := \alpha^2\, \mathbf{e}^{(-\alpha\, r)} - \frac{\alpha\, \mathbf{e}^{(-\alpha\, r)}}{r} - \frac{(1-g)^2\, \mathbf{e}^{(-\alpha\, r)}}{r^2} + k^2\, \mathbf{e}^{(-\alpha\, r)}$$

```
>XX1:=factor(SCHEQ);
```

$$XX1 := \frac{e^{(-\alpha r)} (\alpha^2 r^2 - \alpha r - 1 + 2 g - g^2 + k^2 r^2)}{r^2}$$

```
For the assumed exponential solution to be true,
the bracket must vanish.
HENCE: the roots of the quadratic equation are:
EITHER (solving for g)
```

> **GG:=solve(XX1,g);KK:=solve(XX1,k);AA:=solve(XX1,alpha);**
>

$$GG := 1 + \sqrt{\alpha^2 r^2 - \alpha r + k^2 r^2},\ 1 - \sqrt{\alpha^2 r^2 - \alpha r + k^2 r^2}$$

```
or (solving for k)
```

$$KK := \frac{\sqrt{-\alpha^2 r^2 + \alpha r + 1 - 2 g + g^2}}{r},\ -\frac{\sqrt{-\alpha^2 r^2 + \alpha r + 1 - 2 g + g^2}}{r}$$

```
or (solving for alpha)
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$$AA := \frac{\frac{1}{2} + \frac{1}{2}\sqrt{5 - 8 g + 4 g^2 - 4 k^2 r^2}}{r},\ \frac{\frac{1}{2} - \frac{1}{2}\sqrt{5 - 8 g + 4 g^2 - 4 k^2 r^2}}{r}$$

End note:

[1] Another expression for γ was described in Ref. [37]:

$$\gamma = 16\sqrt{2\pi}.Ana^3.(a_h / a).\sqrt{T_c / T}.\sqrt{\hbar\omega / k_B.T}$$

though it is not yet clear whether this expression could be directly used for cosmological phenomena.

[2] This author acknowledged Prof. C. Castro and Prof. C. Beck for suggesting that there is plausible correspondence between superfluid vortice model and (random) string theory.

[3] X. Song and P. Richards of Columbia University's Lamont-Doherty, http://www.ldeo.columbia.edu/song/pr/html.

[4] Also S. Carneiro, arXiv:gr-qc/0003096; Y.N. Obukhov, arXiv:astro-ph/0008106.

Plausible Explanation of Quantization of Intrinsic Redshift from Hall Effect and Weyl Quantization

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Using phion condensate model as described by Moffat [1], we consider a plausible explanation of (Tifft) intrinsic redshift quantization as described by Bell [6] as result of Hall effect in rotating frame. We also discuss another alternative to explain redshift quantization from the viewpoint of Weyl quantization, which could yield Bohr-Sommerfeld quantization.

1 Introduction

In a recent paper by Moffat [1] it is shown that quantum phion condensate model with Gross-Pitaevskii equation yields an approximate fit to data corresponding to CMB spectrum, and it also yields a modified Newtonian acceleration law which is in good agreement with galaxy rotation curve data. It seems therefore interesting to extend further this hypothesis to explain quantization of redshift, as shown by Tifft *et al.* [2, 6, 7]. We also argue in other paper that this redshift quantization could be explained as signature of topological quantized vortices, which also agrees with Gross-Pitaevskiian description [3, 5].

Nonetheless, there is remaining question in this quantized vortices interpretation, i. e. how to provide explanation of "intrinsic redshift" argument by Bell [6]. In the present paper, we argue that it sounds reasonable to interpret the intrinsic redshift data from the viewpoint of rotating Hall effect, i. e. rotational motion of clusters of galaxies exhibit quantum Hall effect which can be observed in the form of "intrinsic redshift". While this hypothesis is very new, it could be expected that we can draw some prediction, including possibility to observe small "blue-shift" effect generated by antivortex part of the Hall effect [5a].

Another possibility is to explain redshift quantization from the viewpoint of Weyl-Moyal quantization theory [25]. It is shown that Schrödinger equation can be derived from Weyl approach [8], therefore quantization in this sense comes from "graph"-type quantization. In large scale phenomena like galaxy redshift quantization one could then ask whether there is possibility of "super-graph" quantization.

Further observation is of course recommended in order to verify or refute the propositions outlined herein.

2 Interpreting quantized redshift from Hall effect. Cosmic String

In a recent paper, Moffat [1, p. 9] has used Gross-Pitaevskii in conjunction with his *phion condensate fluid* model to describe CMB spectrum data. Therefore we could expect that this equation will also yield interesting results in galaxies scale. See also [1b, 1c, 13] for other implications of low-energy phion fluid model.

Interestingly, it could be shown, that we could derive (approximately) Schrödinger wave equation from Gross-Pitaevskii equation. We consider the well-known Gross-Pitaevskii equation in the context of superfluidity or superconductivity [14]:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + \big(V(x) - \gamma\,|\Psi|^{p-1}\big)\,\Psi, \tag{1}$$

where $p < 2N/(N-2)$ if $N \geqslant 3$. In physical problems, the equation for $p = 3$ is known as Gross-Pitaevskii equation. This equation (1) has standing wave solution quite similar to solution of Schrödinger equation, in the form:

$$\Psi(x,t) = e^{-iEt/\hbar} \cdot u(x) \tag{2}$$

Substituting equation (2) into equation (1) yields:

$$-\frac{\hbar^2}{2m}\Delta u + \big(V(x) - E\big)\,u = |u|^{p-1}\,u\,, \tag{3}$$

which is nothing but a time-independent linear form of Schrödinger equation, except for term $|u|^{p-1}$ [14]. If the right-hand side of this equation is negligible, equation (3) reduces to standard Schrödinger equation.

Now it is worth noting here that from Nottale *et al.* we can derive a gravitational equivalent of Bohr radius from generalized Schrödinger equation [4]. Therefore we could also expect a slight deviation of this gravitational Bohr radius in we consider Gross-Pitaevskii equation instead of generalized Schrödinger equation.

According to Moffat, the phion condensate model implies a modification of Newtonian acceleration law to become [1, p. 11]:

$$a(r) = -\frac{G_\infty M}{r^2} + K\,\frac{\exp\left(-\mu_\phi r\right)}{r^2}\left(1 + \mu_\phi r\right), \tag{4}$$

where

$$G_\infty = G\left[1+\sqrt{\frac{M_0}{M}}\right]. \tag{5}$$

Therefore we can conclude that the use of phion condensate model implies a modification of Newton gravitational constant, G, to become (5). Plugging in this new equation (5) into a Nottale's gravitational Bohr radius equation [4] yields:

$$r_n \approx n^2\,\frac{GM}{v_0^2}\left[1+\sqrt{\frac{M_0}{M}}\right] \approx \chi\cdot n^2\,\frac{GM}{v_0^2}\,, \tag{6}$$

where n is integer (1,2,3 ...) and:

$$\chi = \left[1+\sqrt{\frac{M_0}{M}}\right]. \tag{7}$$

Therefore we conclude that — provided the higher order Yukawa term of equation (4) could be neglected — one has a *modified* gravitational Bohr-radius in the form of (6). It can be shown (elsewhere) that using similar argument one could expect to explain a puzzling phenomenon of *receding Moon* at a constant rate of $\pm 1.5''$ per year. And from this observed fact one could get an estimate of this χ factor. It is more interesting to note here, that a number of *coral reef* data also seems to support the same idea of modification factor in equation (5), but discussion of this subject deserves another paper.

A somewhat similar idea has been put forward by Masreliez [18] using the metric:

$$ds^2 = e^{\alpha\beta}\left[dx^2+dy^2+dz^2-(icdt)^2\right]. \tag{8}$$

Another alternative of this metric has been proposed by Socoloff and Starobinski [19] using multi-connected hypersurface metric:

$$ds^2 = dx^2 + e^{-2x}\left(dy^2+dz^2\right) \tag{9}$$

with boundaries: $e^{-x} = \Lambda$.

Therefore one can conclude that the use of phion condensate model has led us to a form of expanding metric, which has been discussed by a few authors.

Furthermore, it is well-known that Gross-Pitaevskii equation could exhibit *topologically* non-trivial vortex solutions [4, 5], which also corresponds to quantized vortices:

$$\oint p\cdot dr = N_v\, 2\pi\hbar\,. \tag{10}$$

Therefore an implication of Gross-Pitaevskii equation [1] is that topologically quantized vortex could exhibit in astrophysical scale. In this context we submit the viewpoint that this proposition indeed has been observed in the form of Tifft's redshift quantization [2, 6]:

$$\delta r = \frac{c}{H}\,\delta z\,. \tag{11}$$

In other words, we submit the viewpoint that Tifft's observation of quantized redshift implies a quantized distance between galaxies [2, 5], which could be expressed in the form:

$$r_n = r_0 + n(\delta r)\,, \tag{12}$$

where n is integer (1,2,3, ...) similar to quantum number. Because it can be shown using standard definition of Hubble law that redshift quantization implies quantized distance between galaxies in the same cluster, then one could say that this equation of quantized distance (11) is a result of *topological* quantized vortices (9) in astrophysical scale [5]; and it agrees with Gross-Pitaevskii (quantum phion condensate) description of CMB spectrum [1]. It is perhaps more interesting if we note here, that from (11) then we also get an equivalent expression of (12):

$$\frac{c}{H}\,z_n = \frac{c}{H}\,z_0 + n\left(\frac{c}{H}\,\delta z\right) \tag{13}$$

or

$$z_n = z_0 + n(\delta z) \tag{14}$$

or

$$z_n = z_0\left[1+n\left(\frac{\delta z}{z_0}\right)\right]. \tag{15}$$

Nonetheless, there is a problem here, i. e. how to explain intrinsic redshift related to Tifft quantization as observed in Fundamental Plane clusters and also from various quasars data [6, 6a]:

$$z_{iQ} = z_f\left[N - 0.1 M_N\right] \tag{16}$$

where $z_f = 0.62$ is assumed to be a fundamental redshift constant, and $N(=1,2,3\ \ldots)$, and M is function of N [6a]. Meanwhile, it is interesting to note here similarity between equation (15) and (16). Here, the number M seems to play a rôle similar to second quantum number in quantum physics [7].

Now we will put forward an argument that intrinsic redshift quantization (16) could come from rotating quantum Hall effect [5a].

It is argued by Fischer [5a] that "Hall quantization is of necessity derivable from a *topological quantum number* related to this (quantum) coherence". He used total particle momentum [5a]:

$$p = mv + m\Omega\times r + qA\,. \tag{17}$$

The uniqueness condition of the collective phase represented in (9) then leads, if we take a path in the bulk of electron liquid, for which the integral of mv can be neglected, to the quantization of the sum of a *Sagnac flux*, and the magnetic flux [5a]:

$$\begin{aligned}\Phi &= q\oint A\cdot dr + m\oint \Omega\times r\cdot dr = \\ &= \iint B\cdot dS = N_v\, 2\pi\hbar\,.\end{aligned} \tag{18}$$

This flux quantization rule corresponds to the fact that a *vortex* is fundamentally characterised by the winding number N alone [5a]. In this regard the vortex could take the form of cosmic string [22]. Now it is clear from (15) that quantized vortices could be formed by different source of flux.

After a few more reasonable assumptions one could obtain a generalised Faraday law, which in rotating frame will give in a non-dissipative Hall state the quantization of Hall conductivity [5a].

Therefore one could observe that it is quite natural to interpret the quantized distance between galaxies (11) as an implication of quantum Hall effect in rotating frame (15). While this proposition requires further observation, one could think of it in particular using known analogy between condensed matter physics and cosmology phenomena [10, 22]. If this proposition corresponds to the facts, then one could think that redshift quantization is an imprint of generalized quantization in various scales from microphysics to macrophysics, just as Tifft once put it [2]:

> "The redshift has imprinted on it a pattern that appears to have its origin in microscopic quantum physics, yet it carries this imprint across cosmological boundaries".

In the present paper, Tifft's remark represents natural implication of topological quantization, which could be formed at any scale [5]. We will explore further this proposition in the subsequent section, using Weyl quantization.

Furthermore, while this hypothesis is new, it could be expected that we can draw some new prediction, for instance, like possibility to observe small "blue-shift" effect generated by the Hall effect from antivortex-galaxies [23]. Of course, in order to observe such a "blue-shift" one shall first exclude other anomalous effects of redshift phenomena [6]. (For instance: one could argue that perhaps Pioneer spacecraft anomaly's blue-shifting of Doppler frequency may originate from the same effect as described herein.)

One could expect that further observation in particular in the area of low-energy neutrino will shed some light on this issue [20]. In this regard, one could view that the Sun is merely a remnant of a neutron star in the past, therefore it could be expected that it also emits neutrino similar to neutron star [21].

3 An alternative interpretation of astrophysical quantization from Weyl quantization. Graph and quantization

An alternative way to interpret the above proposition concerning topological quantum number and topological quantization [5a], is by using Weyl quantization.

In this regards, Castro [8, p. 5] has shown recently that one could derive Schrödinger equation from Weyl geometry using continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{1}{\sqrt{g}}\,\partial_i\big(\sqrt{g}\rho v^i\big) \tag{19}$$

and Weyl metric:

$$R_{\mathrm{Weyl}} = (d-1)(d-2)\left(A_k A^k\right) - 2(d-1)\,\partial_k A^k\,. \tag{20}$$

Therefore one could expect to explain astrophysical quantization using Weyl method in lieu of using generalised Schrödinger equation as Nottale did [4]. To our knowledge this possibility has never been explored before elsewhere.

For instance, it can be shown that one can obtain Bohr-Sommerfeld type quantization rule from Weyl approach [24, p. 12], which for kinetic plus potential energy will take the form:

$$2\pi N\hbar = \sum_{j=0}^{\infty} \hbar^j S_j(E)\,, \tag{21}$$

which can be solved by expressing $E = \sum \hbar^k E_k$ as power series in $\hbar$ [24]. Now equation (10) could be rewritten as follows:

$$\oint p\cdot dr = N_v\, 2\pi\hbar = \sum_{j=0}^{\infty} \hbar^j S_j(E)\,. \tag{22}$$

Or if we consider quantum Hall effect, then equation (18) can be used instead of equation (10), which yields:

$$\begin{aligned}\Phi = q\oint A\cdot dr + m\oint \Omega\times r\cdot dr = \\ = \iint B\cdot dS = \sum_{j=0}^{\infty}\hbar^j S_j(E)\,.\end{aligned} \tag{23}$$

The above method is known as "graph kinematic" [25] or Weyl-Moyal's quantization [26]. We could also expect to find Hall effect quantization from this deformation quantization method.

Consider a harmonic oscillator, which equation can be expressed in the form of deformation quantization instead of Schrödinger equation [26]:

$$\left(\left(x + \frac{i\hbar}{2}\,\partial_p\right)^2 + \left(p - \frac{i\hbar}{2}\,\partial_x\right)^2 - 2E\right) f(x,p) = 0\,. \tag{24}$$

This equation could be separated to become two simple PDEs. For imaginary part one gets [26]:

$$\left(x\,\partial_p - p\,\partial_x\right) f = 0\,. \tag{25}$$

Now, considering Hall effect, one can introduce our definition of total particle momentum (17), therefore equation (25) may be written:

$$\big(x\,\partial_p - (mv + m\Omega\times r + qA)\,\partial_x\big) f = 0\,. \tag{26}$$

Our proposition here is that in the context of deformation quantization it is possible to find quantization solution of harmonic oscillator without Schrödinger equation. And because it corresponds to graph kinematic [25], generalized Bohr-Sommerfeld quantization rule for quantized vortices (22) in astrophysical scale could be viewed as signature of "*super-graph*"*quantization*.

This proposition, however, deserves further theoretical considerations. Further experiments are also recommended in order to verify and explore further this proposition.

Concluding remarks

In a recent paper, Moffat [1] has used Gross-Pitaevskii in his "phion condensate fluid" to describe CMB spectrum data. We extend this proposition to explain Tifft redshift quantization from the viewpoint of topological quantized vortices. In effect we consider that the intrinsic redshift quantization could be interpreted as result of Hall effect in rotating frame.

Another alternative to explain redshift quantization is to consider quantized vortices from the viewpoint of Weyl quantization (which could yield Bohr-Sommerfeld quantization).

It is recommended to conduct further observation in order to verify and also to explore various implications of our propositions as described herein.

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Numerical Solution of Quantum Cosmological Model Simulating Boson and Fermion Creation

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A numerical solution of Wheeler-De Witt equation for a quantum cosmological model simulating boson and fermion creation in the early Universe evolution is presented. This solution is based on a Wheeler-De Witt equation obtained by Krechet, Fil'chenkov, and Shikin, in the framework of quantum geometrodynamics for a Bianchi-I metric.

1 Introduction

It is generally aserted that in the early stage of Universe evolution, the quantum phase predominated the era. Therefore there are numerous solutions have been found corresponding to the Wheeler-DeWitt equation which governs this phase [2]. In the present paper we present another numerical solution of Wheeler-De Witt equation for a quantum cosmological model simulating boson and fermion creation in the early Universe evolution for a Bianchi-type I metric [1].

The solution is based on Wheeler-De Witt equation for a Bianchi-I metric obtained by Krechet, Fil'chenkov, and Shikin [1], in the framework of quantum geometrodynamics. Albeit the essence of the solution is quite similar from the solution given in [1] using Bessel function, in the present paper we present numerical result using Maxima. For comparison with other solutions of 1-d hydrogen problem, see [3] and [4].

2 Solution of Wheeler-DeWitt equation for boson and fermion creation

In the evolution of the Universe after inflation, a scalar field describing de Sitter vacuum was supposed to decay and its energy is converted into the energy of fermions and heavy vector-particles (the so-called X and Y bosons) [2].

In the framework of quantum geometrodynamics, and for a Bianchi-I metric, the Wheeler-De Witt equation has been obtained by Krechet, Fil'chenkov, and Shikin, which reduces to become (Eq. 23 in [1]):

$$T'' - \frac{2iC}{3\tau}\,T' - (E - V)\,T = 0. \tag{1}$$

where T'' and T' represent second and first differentiation of T with respect to r. The resulting equation appears quite similar to radial 1-dimensional Schrödinger equation for a hydrogen-like atom [3], with the potential energy is given by [1]:

$$U(r) = \frac{\beta}{\tau} + \frac{\varepsilon_o}{\tau^{4/3}}\,, \tag{2}$$

$$E = \frac{8}{3}\,\kappa\left(\frac{\Lambda}{\kappa} - \frac{M^2}{2\lambda}\right) \tag{3}$$

has here a continuous spectrum.

The solution of equation (1) has been presented in [1] based on modified Bessel function. Its interpretation is that in this quantum cosmological model an initial singularity is absent.

As an alternative to the method presented in [1], the numerical solution can be found using Maxima software package, as follows. All solutions are given in terms of E as constant described by (3).

(a) *Condition where* $V = 0$

```
'diff(y,r,2)−E*y−(2*%i*C/3/t)*y=0;    ode2(%o1,y,r);
```
(4)

The result is given by:

$$\mathrm{y{=}K1\,\sin(a){+}K2\,\cos(a)}\,, \tag{5}$$

where:

$$\mathrm{a{=}(r/\sqrt{3})\,\sqrt{-3E-2iC/t}}\,. \tag{6}$$

(b) *Condition where* $V \leqslant 0$

```
'diff(y,r,2)−E*y−(2*%i*C/3/t)*y−(b/t+e/t^(4/3))*y=0;
ode2(%o2,y,r);
```
(7)

The result is given by:

$$\mathrm{y{=}K1\,\sin(d){+}K2\,\cos(d)}\,, \tag{8}$$

where:

$$\mathrm{d{=}(r/(\sqrt{3}\,t^{2/3})\,\sqrt{-3Et^{4/3}-2iCt^{1/3}-3e-3bt^{1/3}}}\,. \tag{9}$$

As a result, the solution given above looks a bit different compared to the solution obtained in [1] based on the modified Bessel function.

3 A few implications

For the purpose of stimulating further discussions, a few implications of the above solution of Wheeler-DeWitt equation (in the form of 1-d Schrödinger equation) are pointed as follows:

(a) Considering that the Schrödinger equation can be used to solve the Casimir effect (see for instance Silva [5], Alvarez & Mazzitelli [6]), therefore one may expect that there exists some effects of Casimir effect in cosmological scale, in a sense that perhaps quite similar to Unruh radiation which can be derived from the Casimir effective temperature. Interestingly, Anosov [7] has pointed out a plausible deep link between Casimir effect and the fine structure constant by virtue of the entropy of coin-tossing problem. However apparently he did not mention yet another plausible link between the Casimir effective temperature and other phenomena at cosmological scale;

(b) Other implication may be related to the Earth scale effects, considering the fact that Schrödinger equation corresponds to the infinite dimensional Hilbert space. In other words one may expect some effects with respect to Earth eigen oscillation spectrum, which is related to the Earth's inner core interior. This is part of gravitational geophysical effects, as discussed by Grishchuk et al. [8]. Furthermore, this effect may correspond to the so-called Love numbers. Other phenomena related to variation to gravitational field is caused by the Earth inner core oscillation, which yields oscillation period $T \sim 3$–7 hours. Interestingly, a recent report by Cahill [9] based on the Optical fibre gravitational wave detector gave result which suggests oscillation period of around 5hours. Cahill concluded that this observed variation can be attributed to Dynamical 3-space. Nonetheless, the Figure 6c in [9] may be attributed to Earth inner core oscillation instead. Of course, further experiment can be done to verify which interpretation is more consistent.

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An Exact Mapping from Navier-Stokes Equation to Schrödinger Equation via Riccati Equation

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In the present article we argue that it is possible to write down Schrödinger representation of Navier-Stokes equation via Riccati equation. The proposed approach, while differs appreciably from other method such as what is proposed by R. M. Kiehn, has an advantage, i.e. it enables us extend further to quaternionic and biquaternionic version of Navier-Stokes equation, for instance via Kravchenko's and Gibbon's route. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In recent years there were some attempts in literature to find out Schrödinger-like representation of Navier-Stokes equation using various approaches, for instance by R. M. Kiehn [1, 2]. Deriving exact mapping between Schrödinger equation and Navier-Stokes equation has clear advantage, because Schrodinger equation has known solutions, while exact solution of Navier-Stokes equation completely remains an open problem in mathematical-physics. Considering wide applications of Navier-Stokes equation, including for climatic modelling and prediction (albeit in simplified form called "geostrophic flow" [9]), one can expect that simpler expression of Navier-Stokes equation will be found useful.

In this article we presented an alternative route to derive Schrödinger representation of Navier-Stokes equation via Riccati equation. The proposed approach, while differs appreciably from other method such as what is proposed by R. M. Kiehn [1], has an advantage, i.e. it enables us to extend further to quaternionic and biquaternionic version of Navier-Stokes equation, in particular via Kravchenko's [3] and Gibbon's route [4, 5]. An alternative method to describe quaternionic representation in fluid dynamics has been presented by Sprössig [6]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 From Navier-Stokes equation to Schrödinger equation via Riccati

Recently, Argentini [8] argues that it is possible to write down ODE form of 2D steady Navier-Stokes equations, and it will lead to second order equation of Riccati type.

Let ρ the density, μ the dynamic viscosity, and f the body force per unit volume of fluid. Then the Navier-Stokes equation for the steady flow is [8]:

$$\rho\left(v \cdot \nabla v\right) = -\nabla p + \rho \cdot f + \mu \cdot \Delta v \,. \tag{1}$$

After some necessary steps, he arrives to an ODE version of 2D Navier-Stokes equations along a streamline [8, p. 5] as follows:

$$u_1 . \dot{u}_1 = f_1 - \frac{\dot{q}}{\rho} + v \cdot \dot{u}_1 \,, \tag{2}$$

where $v = \frac{\mu}{\rho}$ is the kinematic viscosity. He [8, p. 5] also finds a general exact solution of equation (2) in Riccati form, which can be rewritten as follows:

$$\dot{u}_1 - \alpha \cdot u_1^2 + \beta = 0 \,, \tag{3}$$

where:

$$\alpha = \frac{1}{2v} \,, \qquad \beta = -\frac{1}{v}\left(\frac{\dot{q}}{\rho} - f_1\right) s - \frac{c}{v} \,. \tag{4}$$

Interestingly, Kravchenko [3, p. 2] has argued that there is neat link between Schrödinger equation and Riccati equation via simple substitution. Consider a 1-dimensional static Schrödinger equation:

$$\ddot{u} + v \cdot u = 0 \tag{5}$$

and the associated Riccati equation:

$$\dot{y} + y^2 = -v \,. \tag{6}$$

Then it is clear that equation (5) is related to (6) by the inverted substitution [3]:

$$y = \frac{\dot{u}}{u} \,. \tag{7}$$

Therefore, one can expect to use the same method (7) to write down the Schrödinger representation of Navier-Stokes equation. First, we rewrite equation (3) in similar form of equation (6):

$$\dot{y}_1 - \alpha \cdot y_1^2 + \beta = 0 \,. \tag{8}$$

By using substitution (7), then we get the Schrödinger equation for this Riccati equation (8):

$$\ddot{u} - \alpha\beta \cdot u = 0 \,, \tag{9}$$

where variable α and β are the same with (4). This Schrödinger representation of Navier-Stokes equation is remarkably simple and it also has advantage that now it is possible to generalize it further to quaternionic (ODE) Navier-Stokes equation via quaternionic Schrödinger equation, for instance using the method described by Gibbon *et al.* [4, 5].

3 An extension to biquaternionic Navier-Stokes equation via biquaternion differential operator

In our preceding paper [10, 12], we use this definition for biquaternion differential operator:

$$\diamondsuit = \nabla^q + i\,\nabla^q = \left(-i\,\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\right) + \\ + i\left(-i\,\frac{\partial}{\partial T} + e_1\frac{\partial}{\partial X} + e_2\frac{\partial}{\partial Y} + e_3\frac{\partial}{\partial Z}\right), \qquad (10)$$

where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols: $e_1 = i$, $e_2 = j$, $e_3 = k$): $i^2 = j^2 = k^2 = -1$, $ij = -ji = k$, $jk = -kj = i$, $ki = -ik = j$ and quaternion *Nabla operator* is defined as [13]:

$$\nabla^q = -i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\,. \qquad (11)$$

(Note that (10) and (11) include partial time-differentiation.)

Now it is possible to use the same method described above [10, 12] to generalize the Schrödinger representation of Navier-Stokes (9) to the biquaternionic Schrödinger equation, as follows.

In order to generalize equation (9) to quaternion version of Navier-Stokes equations (QNSE), we use first quaternion Nabla operator (11), and by noticing that $\Delta \equiv \nabla\nabla$, we get:

$$\left(\nabla^q\bar{\nabla}^q + \frac{\partial^2}{\partial t^2}\right)u - \alpha\beta\cdot u = 0\,. \qquad (12)$$

We note that the multiplying factor $\alpha\beta$ in (12) plays similar role just like $V(x) - E$ factor in the standard Schrödinger equation [12]:

$$-\frac{\hbar^2}{2m}\left(\nabla^q\bar{\nabla}^q + \frac{\partial^2}{\partial t^2}\right)u + \big(V(x) - E\big)\,u = 0\,. \qquad (13)$$

Note: we shall introduce the second term in order to "neutralize" the partial time-differentiation of $\nabla^q\bar{\nabla}^q$ operator.

To get *biquaternion* form of equation (12) we can use our definition in equation (10) rather than (11), so we get [12]:

$$\left(\diamondsuit\bar{\diamondsuit} + \frac{\partial^2}{\partial t^2} - i\,\frac{\partial^2}{\partial T^2}\right)u - \alpha\beta\cdot u = 0\,. \qquad (14)$$

This is an alternative version of *biquaternionic* Schrödinger representation of Navier-Stokes equations. Numerical solution of the new Navier-Stokes-Schrödinger equation (14) can be performed in the same way with [12] using Maxima software package [7], therefore it will not be discussed here.

We also note here that the route to *quaternionize* Schrödinger equation here is rather different from what is described by Gibbon *et al.* [4, 5], where the Schrödinger-equivalent to Euler fluid equation is described as [5, p. 4]:

$$\frac{D^2w}{Dt^2} - (\nabla Q)w = 0 \qquad (15)$$

and its quaternion representation is [5, p. 9]:

$$\frac{D^2w}{Dt^2} - q_b \otimes w = 0 \qquad (16)$$

with Riccati relation is given by:

$$\frac{D^q_a}{Dt + q_a \otimes q_a} = q_b \qquad (17)$$

Nonetheless, further observation is of course recommended in order to refute or verify this proposition (14).

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Less Mundane Explanation of Pioneer Anomaly from Q-Relativity

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There have been various explanations of Pioneer blueshift anomaly in the past few years; nonetheless no explanation has been offered from the viewpoint of Q-relativity physics. In the present paper it is argued that Pioneer anomalous blueshift may be caused by Pioneer spacecraft experiencing angular shift induced by similar Q-relativity effect which may also affect Jupiter satellites. By taking into consideration "aether drift" effect, the proposed method as described herein could explain Pioneer blueshift anomaly within ∼0.26% error range, which speaks for itself. Another new proposition of redshift quantization is also proposed from gravitational Bohr-radius which is consistent with Bohr-Sommerfeld quantization. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In the past few years, it is becoming well-known that Pioneer spacecraft has exhibited an anomalous Doppler frequency blueshifting phenomenon which cannot be explained from conventional theories, including General Relativity [1, 4]. Despite the nature of such anomalous blueshift remains unknown, some people began to argue that a post-einsteinian gravitation theory may be in sight, which may be considered as further generalisation of pseudo-Riemannian metric of general relativity theory.

Nonetheless, at this point one may ask: Why do we require a generalization of pseudo-Riemannian tensor, instead of using "patch-work" as usual to modify general relativity theory? A possible answer is: sometimes too much path-work doesn't add up. For instance, let us begin with a thought-experiment which forms the theoretical motivation behind General Relativity, an elevator was put in free-falling motion [8a]. The passenger inside the elevator will not feel any gravitational pull, which then it is interpreted as formal analogue that "inertial acceleration equals to gravitational acceleration" (Equivalence Principle). More recent experiments (after Eötvös) suggest, however, that this principle is only applicable at certain conditions.

Further problem may arise if we ask: what if the elevator also experiences lateral rotation around its vertical axis? Does it mean that the inertial acceleration will be slightly higher or lower than gravitational pull? Similarly we observe that a disc rotating at high speed will exert out-of-plane field resemble an acceleration field. All of this seems to indicate that the thought-experiment which forms the basis of General Relativity is only applicable for some limited conditions, in particular the $F = m\frac{dv}{dt}$ part (because General Relativity is strictly related to Newtonian potential), but it may not be able to represent the rotational aspects of gravitational phenomena. Einstein himself apparently recognizes this limitation [8a, p.61]:

> "... all bodies of reference K' should be given preference in this sense, and they should be exactly equivalent to K for the formation of natural laws, provided that they are in a state of *uniform rectilinear and non-rotary motion* with respect to K." (Italic by Einstein).

Therefore, it shall be clear that the restriction of *non-rotary motion* remains a limitation for all considerations by relativity theory, albeit the *uniform rectilinear* part has been relaxed by general relativity theory.

After further thought, it becomes apparent that it is required to consider a new kind of metric which may be able to represent the rotational aspects of gravitation phenomena, and by doing so extends the domain of validity of general relativity theory.

In this regard, the present paper will discuss the aforementioned Pioneer blueshift anomaly from the viewpoint of Q-relativity physics, which has been proposed by Yefremov [2] in order to bring into application the quaternion number. Despite the use of quaternion number in physical theories is very scarce in recent years — apart of Pauli matrix — it has been argued elsewhere that using quaternion number one could expect to unify all known equations in Quantum Mechanics into the same framework, in particular via the known isomorphism between Dirac equation and Maxwell equations [5].

Another problem that was often neglected in most treatises on Pioneer spacecraft anomaly is the plausible role of aether drift effect [6]. Here it can be shown that taking this effect into consideration along with the aforementioned Q-relativity satellite's apparent shift could yield numerical prediction of Pioneer blueshift within ∼0.26% error range, which speaks for itself.

We also suggest a new kind of Doppler frequency shift which can be predicted using Nottale-type gravitational Bohr-radius, by taking into consideration varying G parameter as described by Moffat [7]. To our knowledge this proposition of new type of redshift corresponding to gravitational Bohr-radius has never been considered before elsewhere.

Further observation is of course recommended in order to verify or refute the propositions outlined herein.

2 Some novel aspects of Q-relativity physics. Pioneer blueshift anomaly

In this section, first we will review some basic concepts of quaternion number and then discuss its implications to quaternion relativity (Q-relativity) physics [2]. Then we discuss Yefremov's calculation of satellite time-shift which may be observed by precise measurement [3]. We however introduce a new interpretation here that such a satellite Q-timeshift is already observed in the form of Pioneer spacecraft blueshift anomaly.

Quaternion number belongs to the group of "very good" algebras: of real, complex, quaternion, and octonion [2]. While Cayley also proposed new terms such as quantic, it is less known than the above group. Quaternion number can be viewed as an extension of Cauchy imaginary plane to become [2]:

$$Q \equiv a + bi + cj + dk\,, \tag{1}$$

where a, b, c, d are real numbers, and i, j, k are imaginary quaternion units. These Q-units can be represented either via 2×2 matrices or 4×4 matrices [2].

It is interesting to note here that there is quaternionic multiplication rule which acquires compact form:

$$1q_k = q_k 1 = q_k\,, \quad q_j q_k = -\delta_{jk} + \varepsilon_{jkn} q_n\,, \tag{2}$$

where δ_{kn} and ε_{jkn} represent 3-dimensional symbols of Kronecker and Levi-Civita, respectively [2]. Therefore it could be expected that Q-algebra may have neat link with pseudo-Riemannian metric used by General Relativity. Interestingly, it has been argued in this regard that such Q-units can be generalised to become Finsler geometry, in particular with Berwald-Moor metric. It also can be shown that Finsler-Berwald-Moor metric is equivalent with pseudo-Riemannian metric, and an expression of Newtonian potential can be found for this metric [2a].

It may also be worth noting here that in 3D space Q-connectivity has clear geometrical and physical treatment as movable Q-basis with behaviour of Cartan 3-frame [2].

It is also possible to write the dynamics equations of Classical Mechanics for an inertial observer in constant Q-basis. $SO(3, R)$-invariance of two vectors allow to represent these dynamics equations in Q-vector form [2]:

$$m \frac{d^2}{dt^2}\left(x_k q_k\right) = F_k q_k\,. \tag{3}$$

Because of antisymmetry of the connection (generalised angular velocity) the dynamics equations can be written in vector components, by conventional vector notation [2]:

$$m\left(\vec{a} + 2\vec{\Omega} \times \vec{v} + \vec{\Omega} \times \vec{r} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right)\right) = \vec{F}\,. \tag{4}$$

Therefore, from equation (4) one recognizes known types of classical acceleration, i.e. linear, coriolis, angular, centripetal. Meanwhile it is known that General Relativity introduces Newton potential as *rigid* requirement [2a, 6b]. In other words, we can expect — using Q-relativity — to predict new effects that cannot be explained with General Relativity.

From this viewpoint one may consider a generalisation of Minkowski metric into biquaternion form [2]:

$$dz = \left(dx_k + i dt_k\right) q_k\,, \tag{5}$$

with some novel properties, i.e.:

- temporal interval is defined by imaginary vector;
- space-time of the model appears to have six dimensions (6D);
- vector of the displacement of the particle and vector of corresponding time change must always be normal to each other, or:

$$dx_k dt_k = 0\,. \tag{6}$$

It is perhaps quite interesting to note here that Einstein himself apparently once considered similar approach, by proposing tensors with Riemannian metric with Hermitian symmetry [8]. Nonetheless, there is difference with Q-relativity described above, because in Einstein's generalised Riemannian metric it has 8-dimensions, rather than 3d-space and 3d-imaginary time.

One particularly interesting feature of this new Q-relativity (or rotational relativity) is that there is universal character of motion of the bodies (including non-inertial motions), which can be described in unified manner (Hestenes also considers Classical Mechanics from similar spinor language). For instance advanced perihelion of planets can be described in term of such rotational precession [2].

Inspired by this new Q-relativity physics, it can be argued that there should be anomalous effect in planets' satellite motion. In this regard, Yefremov argues that there should be a deviation of the planetary satellite position, due to discrepancy between calculated and observed from the Earth motion magnitudes characterizing cyclic processes on this planet or near it. He proposes [2]:

$$\Delta\varphi \approx \frac{\omega V_e V_p}{c^2}\, t\,, \tag{7}$$

or

$$\Delta\varphi' \approx -\frac{\omega V_e V_p}{c^2}\, t'. \tag{8}$$

Therefore, given a satellite orbit radius r, its position shift is found in units of length $\Delta l = r\,\Delta\varphi$. His calculation

Satellites	Cycle frequency ω, 1/s	Angular shift $\Delta\varphi$, $''$/100 yrs	Linear shift Δl, km/100 yrs	Linear size a, km
Phobos (Mars)	0.00023	18.2	54	20
Deimos (Mars)	0.00006	4.6	34	12
Metis (Jupiter)	0.00025	**10.6**	431	40
Adrastea (Jupiter)	0.00024	**10.5**	429	20
Amalthea (Jupiter)	0.00015	6.3	361	189

Table 1: The following table gives values of the effect for five fast satellites of Mars and Jupiter. Orbital linear velocities are: of the Earth $V_E = 29.8$ km/s, of Mars $V_P = 24.1$ km/s, of Jupiter $V_P = 13.1$ km/s; the value of the light velocity is $c = 299\,793$ km/s; observation period is chosen 100 years. Courtesy of A. Yefremov, 2006 [3].

for satellites of Mars and Jupiter is given in Table 1. Nonetheless he gave no indication as to how to observe this anomalous effect.

In this regard, we introduce here an alternative interpretation of the aforementioned Q-satellite time-shift effect by Yefremov, i.e. this effect actually has similar effect with Pioneer spacecraft blueshift anomaly. It is known that Pioneer spacecraft exhibits this anomalous Doppler frequency while entering Jupiter orbit [1, 4], therefore one may argue that this effect is caused by Jupiter planetary gravitational effect, which also may cause similar effect to its satellites.

Despite the apparent contradiction with Yefremov's own intention, one could find that the aforementioned Q-satellite time-shift could yield a natural explanation of Pioneer spacecraft blueshift anomaly. In this regard, Taylor [9] argues that there is possibility of a mundane explanation of anomalous blueshift of Pioneer anomaly (5.99×10^{-9} Hz/sec). The all-angle formulae for relativistic Doppler shift is given by [9a, p.34]:

$$v' = v_0 \gamma \frac{(1-\beta\cos\phi)}{\sqrt{1-\beta^2}}\,, \tag{9}$$

where $\beta = v/c$. By neglecting the $\sqrt{1-\beta^2}$ term because of low velocity, one gets the standard expression:

$$v' = v_0 \gamma (1-\beta\cos\phi)\,. \tag{9a}$$

The derivative with respect to ϕ is:

$$\frac{dv'}{d\phi} = v_0 \gamma \beta \sin\phi\,, \tag{10}$$

where $\frac{dv'}{d\phi} = 5.99\times10^{-9}$ Hz/sec, i.e. the observed Pioneer anomaly. Introducing this value into equation (10), one gets requirement of an effect to explain Pioneer anomaly:

$$d\phi = \frac{\arcsin(5.99\times10^{-9}\,\text{Hz})}{v_0\gamma\beta} = 1.4\times10^{-12}\,\text{deg/sec}. \tag{11}$$

Therefore, we can conclude that to explain 5.99×10^{-9} Hz/sec blueshift anomaly, it is required to find a shift of emission angle at the order 1.4×10^{-12} degree/sec only (or around 15.894$''$ per 100 years).

Interestingly this angular shift can be explained with the same order of magnitude from the viewpoint of Q-satellite angular shift (see Table 1), in particular for Jupiter's Adrastea (10.5$''$ per 100 years). There is however, a large discrepancy at the order of 50% from the expected angular shift.

It is proposed here that such discrepancy between Q-satellite angular shift and expected angular shift required to explain Pioneer anomaly can be reduced if we take into consideration the "aether drift" effect [6]. Interestingly we can use experimental result of Thorndike [6, p.9], saying that the aether drift effect implies a residual apparent Earth velocity is $v_{obs} = 15 \pm 4$ km/sec. Therefore the effective V_e in equation (8) becomes:

$$V_{e.eff} = v_{obs} + V_e = 44.8\,\text{km/sec}. \tag{12}$$

Using this improved value for Earth velocity in equation (8), one will get larger values than Table 1, which for Adrastea satellite yields:

$$\Delta\varphi_{obs} = \frac{\omega V_{e.eff} V_p}{c^2}\,t = \frac{V_{e.eff}}{V_e}\Delta\varphi = 15.935''/100\,\text{yrs}. \tag{13}$$

Using this improved prediction, the discrepancy with required angular shift only (15.894$''$ per 100 years) becomes $\sim$ 0.26%, which speaks for itself. Therefore one may conclude that this less mundane explanation of Pioneer blueshift anomaly with Q-relativity may deserve further consideration.

3 A new type of redshift from gravitational Bohr radius. Possible observation in solar system.

In preceding paper [10, 11] we argued in favour of an alternative interpretation of Tifft redshift quantization from the viewpoint of quantized distance between galaxies. A method can be proposed as further test of this proposition both at solar system scale or galaxies scale, by using the known quantized Tifft redshift [14, 15, 16]:

$$\delta r \approx \frac{c}{H}\,\delta z\,. \tag{14}$$

In this regards, we use gravitational Bohr radius equation:

$$r_n = n^2 \frac{GM}{v_0^2}\,. \tag{15}$$

Inserting equation (15) into (14), then one gets quantized redshift expected from gravitational Bohr radius:

$$z_n = \frac{H}{c}\, n^2\, \frac{GM}{v_0^2} \tag{16}$$

which can be observed either in solar system scale or galaxies scale. To our present knowledge, this effect has never been described elsewhere before.

Therefore, it is recommended to observe such an accelerated Doppler-freequency shift, which for big jovian planets this effect may be detected. It is also worth noting here that according to equation (16), this new Doppler shift is quantized.

At this point one may also take into consideration a proposition by Moffat, regarding modification of Newtonian acceleration law to become [7]:

$$a(r) = -\frac{G_\infty M}{r^2} + K\,\frac{\exp(-\mu_\phi r)}{r^2}(1+\mu_\phi r) \tag{17}$$

where

$$G_\infty = G\left[1+\sqrt{\frac{M_0}{M}}\,\right]. \tag{17a}$$

Therefore equation (16) may be rewritten to become:

$$z_n \approx \frac{H}{c}\, n^2\, \frac{GM}{v_0^2}\left[1+\sqrt{\frac{M_0}{M}}\,\right] \approx \chi\, \frac{H}{c}\, n^2\, \frac{GM}{v_0^2} \tag{18}$$

where n is integer (1, 2, 3, ...) and:

$$\chi = \left[1+\sqrt{\frac{M_0}{M}}\,\right]. \tag{18a}$$

To use the above equations, one may start by using Bell's suggestion that there is fundamental redshift $z = 0.62$ which is typical for various galaxies and quasars [14]. Assuming we can use equation (16), then by setting $n = 1$, we can expect to predict the mass of quasar centre or galaxy centre. Then the result can be used to compute back how time-variation parameter affects redshift pattern in equation (18). In solar system scale, time-varying radius may be observed in the form of changing Astronomical Unit [4].

This proposition, however, deserves further theoretical considerations. Further observation is also recommended in order to verify and explore further this proposition.

4 Concluding remarks

In the present paper it is argued that Pioneer anomalous blueshift may be caused by Pioneer spacecraft experiencing angular shift induced by similar Q-relativity effect which may also affect Jupiter satellites. By taking into consideration aether drift effect, the proposed method as described herein could predict Pioneer blueshift within $\sim$0.26% error range, which speaks for itself. Further observation is of course recommended in order to refute or verify this proposition.

Another new proposition of redshift quantization is also proposed from gravitational Bohr-radius which is consistent with Bohr-Sommerfeld quantization. It is recommended to conduct further observation in order to verify and also to explore various implications of our propositions as described herein.

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Quaternion Relativity, Ehrenfest paradox, and Finsler-Berwald metric: Reply to "Notes on Pioneer Anomaly explanation by Satellite-Shift Formula of Quaternion Relativity"

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In the present article we would like to make a few comments on a recent paper by A. Yefremov in this journal [1]. It is interesting to note here that he concludes his analysis by pointing out that using full machinery of Quaternion Relativity it is possible to explain Pioneer XI anomaly with excellent agreement compared with observed data, and explain around 45% of Pioneer X anomalous acceleration. We argue that perhaps it will be necessary to consider extension of Lorentz transformation to H4 of Finsler-Berwald metric, as discussed by a number of authors in the past few years. In this regard, it would be interesting to see if the use of extended Lorentz transformation could also elucidate the long-lasting problem known as Ehrenfest paradox. Further observation is of course recommended in order to refute or verify this proposition.

Introduction

We are delighted to read A. Yefremov's comments on our preceding paper [3], based on his own analysis of Pioneer anomalous 'apparent acceleration' [1]. His analysis made use of a method called Quaternion Relativity, which essentially is based on SO(1,2) form invariant quaternion square root from space-time interval rather than the interval itself [1][2]. Nonetheless it is interesting to note here that he concludes his analysis by pointing out that using full machinery of Quaternion Relativity it is possible to explain Pioneer XI anomaly with excellent agreement compared with observed data, and explain around 45% of Pioneer X anomalous acceleration [1].

In this regard, we would like to emphasize that our preceding paper [3] was based on initial 'conjecture' that in order to explain Pioneer anomaly, it would be necessary to generalize pseudo-Riemann metric of General relativity theory into broader context, which may include Yefremov's Quaternion Relativity for instance. It is interesting to note here, however, that Yefremov's analytical method keeps use standard Lorentz transformation in the form Doppler shift effect (eq. 6):

$$f = \frac{f'}{\sqrt{1-\left(\frac{v_D}{c}\right)^2}}\left(1-\frac{v_D}{c}\cos\beta\right) \tag{1}$$

While his method using relativistic Doppler shift a la special relativity is all right for such a preliminary analysis, in our opinion this method has a drawback that it uses 'standard definition of Lorentz transformation' based on 2-dimensional problem of *rod-on-rail* as explained in numerous expositions of relativity theory [5]. While this method of rod-on-rail seems sufficient to elucidate why 'simultaneity' is ambiguous term in physical sense, it does not take into account 3-angle problem in more general problem. This is why we pointed out in our preceding paper that apparently General Relativity inherits the same drawback from special relativity [3].

Another problem of special relativistic definition of Lorentz transformation is known as 'reciprocity postulate', because in special relativity it is assumed that: $x \leftrightarrow x', t \leftrightarrow t', v \leftrightarrow -v'$. [6] This is why Doppler shift can be derived without assuming reciprocity postulate (which may be regarded as the 'third postulate' of Special Relativity) and without special relativistic argument, see [7]. Nonetheless, in our opinion, Yefremov's Quaternion Relativity is free from this 'reciprocity' drawback because in his method there is difference between moving-observer and static-observer.[2]

An example of implications of this drawback of 1-angle problem of Lorentz transformation is known as Ehrenfest paradox, which can be summarized as follows: "According to special relativity, a moving rod will exhibit apparent length-reduction. This is usually understood to be an observational effect, but if it is instead considered to be a real effect, then there is a paradox. According to Ehrenfest, the perimeter of a rotating disk is like a sequence of rods. So does the rotating disk shatter at the rim?" Similarly, after some thought Klauber concludes that *'The second relativity postulate does not appear to hold for rotating systems'*.[8]

While of course, it is not yet clear whether Quaternion-Relativity is free from this Ehrenfest paradox, we would like to point out that an alternative metric which is known to be nearest to Riemann metric does exist in literature, and known as Finsler-Berwald metric. This metric has been discussed adequately by Pavlov, Asanov, Vacaru and others. [9][10][11][12].

Extended Lorentz-transformation in Finsler-Berwald metric

It is known that Finsler-Berwald metric is subset of Finslerian metrics which is nearest to Riemannian metric [12], therefore it is possible to construct pseudo-Riemann metric based on Berwald-Moor geometry, as already shown by Pavlov [4]. The neat link between Berwald-Moor metric and Quaternion Relativity of Yefremov may also be expected because Berwald-Moor metric is also based on analytical functions of the H4 variable [4].

More interestingly, there was an attempt in recent years to extend 2d-Lorentz transformation in more general framework on H4 of Finsler-Berwald metric, which in limiting cases will yield standard Lorentz transformation.[9][10] In this letter we will use extension of Lorentz transformation derived by Pavlov [9]. For the case when all components but one of the velocity of the new frame in the old frame coordinates along the three special directions are equal to zero, then the transition to the frame moving with velocity V1 in the old coordinates can be expressed by the new frame as [9, p.13]:

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} [F] & [0] \\ [0] & [F] \end{bmatrix} \begin{pmatrix} x'_0 \\ x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} \tag{2}$$

Where the transformation matrix for Finsler-Berwald metric is written as follows [9, p.13]:

$$[F]=\begin{pmatrix} \frac{1}{\sqrt{1-V_1{}^2}} & \frac{V_1}{\sqrt{1-V_1{}^2}} \\ \frac{V_1}{\sqrt{1-V_1{}^2}} & \frac{1}{\sqrt{1-V_1{}^2}} \end{pmatrix} \tag{3}$$

and

$$[0]=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{4}$$

Or

$$x_0 = \frac{x'_0 + Vx'_1}{\sqrt{1-V_1{}^2}}\, x_1 = \frac{Vx'_0 + x'_1}{\sqrt{1-V_1{}^2}} \tag{5}$$

And

$$x_2 = \frac{x'_2 + Vx'_3}{\sqrt{1-V_1{}^2}}\, x_3 = \frac{Vx'_2 + x'_3}{\sqrt{1-V_1{}^2}} \tag{6}$$

It shall be clear that equation (5) $(x'_0, x'_1) \leftrightarrow (x_0, x_1)$ coincides with the corresponding transformation of Special Relativity, while the transformation in equation (6) differs from the corresponding transformation of Special Relativity where $x_2 = x'_2$, $x_3 = x'_3$.[9]

While we are not yet sure if the above extension of Lorentz transformation could explain Pioneer anomaly better than recent analysis by A. Yefremov [1], at least it can be expected to see whether Finsler-Berwald metric could shed some light on the problem of Ehrenfest paradox. This proposition, however, deserves further theoretical considerations.

In order to provide an illustration on how the transformation keeps the Finslerian metric invariant, we can use Maple algorithm presented by Asanov [10, p.29]:

```
> c1:=cos(tau);c2:=cos(psi);c3:=cos(phi);
> s1:=sin(tau);s2:=sin(psi);s3:=sin(phi);
> l1:=c2*c3-c1*s2*s3;l2:=-c2*s3-
c1*s2*c3;l3:=s1*s2;
> m1:=s2*c3+c1*c2*s3;m2:=-s2*s3+c1*c2*c3;m3:=-
s1*c2;
> n1:=s1*s3;n2:=s1*c3;n3:=c1;
> F1:=(e1)^((l1+m1+n1+l2+m2+n2+l3+m3+n3+1)/4)*
(e2)^((-l1-m1-n1+l2+m2+n2-l3-m3-n3+1)/4)*
(e3)^((l1+m1+n1-l2-m2-n2-l3-m3-n3+1)/4)*
(e4)^((-l1-m1-n1-l2-m2-n2+l3+m3+n3+1)/4):
> F2:=(e1)^((-l1+m1-n1-l2+m2-n2-l3+m3-n3+1)/4)*
```

```
(e2)^((l1-m1+n1-l2+m2-n2+l3-m3+n3+1)/4)*
(e3)^((-l1+m1-n1+l2-m2+n2+l3-m3+n3+1)/4)*
(e4)^((l1-m1+n1+l2-m2+n2-l3+m3-n3+1)/4):
> F3:=(e1)^((l1-m1-n1+l2-m2-n2+l3-m3-n3+1)/4)*
(e2)^((-l1+m1+n1+l2-m2-n2-l3+m3+n3+1)/4)*
(e3)^((l1-m1-n1-l2+m2+n2-l3+m3+n3+1)/4)*
(e4)^((-l1+m1+n1-l2+m2+n2+l3-m3-n3+1)/4):
> F4:=(e1)^((-l1-m1+n1-l2-m2+n2-l3-m3+n3+1)/4)*
(e2)^((l1+m1-n1-l2-m2+n2+l3+m3-n3+1)/4)*
(e3)^((-l1-m1+n1+l2+m2-n2+l3+m3-n3+1)/4)*
(e4)^((l1+m1-n1+l2+m2-n2-l3-m3+n3+1)/4):
> a:=array(1..4,1..4):
for i from 1 to 4
do
for j from 1 to 4
do
a[i,j]:=diff(F||i,e||j);
end do:
end do:
> b:=array(1..4,1..4):
for i from 1 to 4
do
for j from 1 to 4
do
b[i,j]:=simplify(add(1/F||k*diff(a[k,i],e||j),
k=1..4),symbolic);
end do:
end do:
> print(b);
```

The result is as follows :

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This result showing that all the entries of the matrix are zeroes support the argument that the metricity condition is true. [10]

Concluding remarks

In the present paper we noted that it is possible to generalise standard Lorentz transformation into H4 framework of Finsler-Berwald metric. It could be expected that this extended Lorentz transformation could shed some light not only to Pioneer anomaly, but perhaps also to the long-lasting problem of Ehrenfest paradox which is also problematic in General Relativity theory, or by quoting Einstein himself:

"…it will require a de tour of general relativity framework as described herein." [5]

Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

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Numerical solution of Schrödinger equation with PT-symmetric periodic potential, and its Gamow integral

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Abstract
In a number of preceding papers we introduced a new PT-symmetric periodic potential, derived from biquaternion radial Klein-Gordon equation. In the present paper we will review our preceding result, and continue with numerical solution of Gamow integral for that periodic potential. And then we also compare with other periodic potentials which are already known, such as Posch-Teller or Rosen-Morse potential. We also discuss a number of recent development in the context of condensed matter nuclear science, in particular those experiments which are carried out by Prof. A. Takahashi and his team from Kobe University. There is hint to describe his team's experiment as 'mesofusion' (or mesoscopic fusion). We then analyze possibility to enhance the performance of Takahashi's mesofusion experiment under external pulse field. Further experiments are of course recommended in order to verify or refute the propositions outlined herein.

a. Introduction

In a number of preceding papers we introduced a new PT-symmetric periodic potential, derived from biquaternion radial Klein-Gordon equation. [1][2] In the present paper we will review our preceding result, and continue with numerical solution of Gamow integral for that periodic potential. And then we also compare with other periodic potentials which are already known, such as Posch-Teller or Rosen-Morse potential [9][10][11].

We also discuss a number of recent development in the context of condensed matter nuclear science, in particular those experiments which are carried out by Prof. A. Takahashi and his team from Kobe University [6][7]. There is hint to describe his team's experiment as 'mesofusion' (from mesoscopic fusion). We then analyze possibility to enhance the performance of Takahashi's mesofusion experiment under external pulse field.

Further experiments are recommended in order to verify or refute the propositions outlined herein.

b. PT-symmetric periodic potential and its Gamow integral

In this section, first we will review our preceding result on the periodic potential based on radial Klein-Gordon equation, and then we discuss its numerical solution for Gamow integral.

There were some attempts in literature to introduce new type of symmetries in Quantum Mechanics, beyond the well-known CPT symmetry, chiral symmetry etc. In this regards, in recent years there are new interests on a special symmetry in physical systems, called PT-symmetry with various ramifications.

It has been argued elsewhere that it is plausible to derive a new PT-symmetric Quantum Mechanics (PT-QM) which is characterized by a PT-symmetric potential [3][4]:

$$V(x) = V(-x)\,. \tag{1}$$

One particular example of such PT-symmetric potential can be found in sinusoidal-form potential:

$$V = \sin\alpha\,. \tag{2}$$

PT-symmetric harmonic oscillator can be written accordingly [3]. Znojil has argued too [4] that condition (1) will yield Hulthen potential:

$$V(\xi) = \frac{A}{\left(1-e^{2i\xi}\right)^2} + \frac{B}{\left(1-e^{2i\xi}\right)}\,. \tag{3}$$

In our preceding paper [2][5], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows:

$$\left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) + i\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\right]\varphi(x,t) = -m^2\varphi(x,t)\,, \tag{4}$$

Or this equation can be rewritten as:

$$\left(\Diamond\bar{\Diamond} + m^2\right)\varphi(x,t) = 0\,, \tag{5}$$

Provided we use this definition:

$$\begin{aligned} \Diamond = \nabla^q + i\nabla^q &= \left(-i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\right) \\ &+ i\left(-i\frac{\partial}{\partial T} + e_1\frac{\partial}{\partial X} + e_2\frac{\partial}{\partial Y} + e_3\frac{\partial}{\partial Z}\right) \end{aligned} \tag{6}$$

Where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols: e_1=i, e_2=j , e_3 =k):

$$\begin{aligned} &i^2 = j^2 = k^2 = -1,\ ij = -ji = k\,, \\ &jk = -kj = i\,, ki = -ik = j\,. \end{aligned} \tag{7}$$

And quaternion *Nabla operator* is defined as [2][5]:

$$\nabla^q = -i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z} \tag{8}$$

Note that equation (8) already included partial time-differentiation.

Therefore one can expect to use the same method described above to find solution of radial biquaternion KGE [2][5].

First, the standard Klein-Gordon equation reads:

$$\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)\varphi(x,t)=-m^2\varphi(x,t)\,. \tag{9}$$

At this point we can introduce polar coordinate by using the following transformation:

$$\nabla=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)-\frac{\ell^2}{r^2}\,. \tag{10}$$

Therefore by introducing this transformation (10) into (9) one gets (by setting $\ell=0$):

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+m^2\right)\varphi(x,t)=0\,. \tag{11}$$

Using similar method (10)-(11) applied to equation (5), then one gets radial solution of BQKGE for 1-dimensional condition [2][5]:

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}\right)-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}\right)+m^2\right)\varphi(x,t)=0\,, \tag{12}$$

Using Maxima computer package we find solution of (12) as a new potential taking the form of sinusoidal potential:

$$y=k_1\sin\left(\frac{|m|r}{\sqrt{-i-1}}\right)+k_2\cos\left(\frac{|m|r}{\sqrt{-i-1}}\right), \tag{13}$$

Where k_1 and k_2 are parameters to be determined. Now if we set k_2 =0, then we obtain the potential function in the form of PT-symmetric periodic potential (2):

$$V=k_1\sin(\alpha)\,, \tag{14}$$

Where $\alpha=\left(\frac{|m|r}{\sqrt{-i-1}}\right)$.

In a recent paper [8], we interpret and compare this result from the viewpoint of EQPET/TSC model which has been suggested by Prof. Takahashi in order to explain some phenomena related to Condensed matter nuclear Science (CMNS).

c. Schrödinger equation and Gamow integral of PT-symmetric periodic potential

Now let us consider a PT-Symmetric potential of the form:

$$V=k_1.\sin(\beta.r)\,, \tag{15}$$

where

$$\beta=\frac{|m|}{\sqrt{-i-1}}\,. \tag{16}$$

Hence, the respective Schrödinger equation with this potential can be written as follows:

$$\Psi''(r) = -k^2(r).\Psi(r) \tag{17}$$

Where

$$k(r) = \frac{2m}{\hbar^2}[E - V(r)] = \frac{2m}{\hbar^2}[E - k_1.\sin(b.r)] \tag{18}$$

For the purpose of finding Gamow function, in area near x=a we can choose linear approximation for Coulomb potential, such that:

$$V(x) - E = -\alpha(x - a), \tag{19}$$

Substitution to Schrödinger equation yields:

$$\Psi'' + \frac{2m\alpha}{\hbar^2}(x - a)\Psi = 0 \tag{20}$$

which can be solved by virtue of Airy function.

In principle, the Gamow function can be derived as follows:

$$\frac{d^2y}{dx^2} + P(x)y = 0 \tag{21}$$

Separating the variables and integrating, yields:

$$\int \frac{d^2y}{y} = \int -P(x).dx \tag{22}$$

Or

$$y.dy = \exp(-\int P(x).dx) + \mathrm{C}) \tag{23}$$

To find solution of Gamow function, therefore the integral below must be evaluated:

$$\gamma = \sqrt{\frac{2m}{\hbar^2}[V(x) - E]} \tag{24}$$

The general expression of Gamow function then is defined by:

$$\Gamma \approx \frac{1}{\eta^2} = \exp(-2\int_a^b \gamma(x)dx) \tag{25}$$

Therefore it should be clear that we can find different solutions for any given form of potential. In the present paper we will only consider a few potential, namely Takahashi's block-type potential (he called it STTBA model), and our PT-symmetric periodic potential. Rosen-Morse potential will be compared for the results only.

c.1. Takahashi's STTBA-block-type potential

For the case of Takahashi experiment [3][4][5], we can use b=5.6fm, and r_0=5fm, where the Gamow function is given by:

$$\Gamma = 0.218\sqrt{\mu}.\int_{r0}^{b}(V_b - E_d)^{1/2}.dr \tag{26}$$

Where he obtained V_b=0.256 MeV.

c.2. PT-symmetric periodic potential (14)

Here we assume that E=V_b=0.257MeV. Therefore the integral becomes:

$$\Gamma = 0.218\sqrt{\mu}.\int_{r0}^{b}(k_1\sin(\beta r) - 0.257)^{1/2}.dr \tag{27}$$

By setting boundary conditions:

(a) at r=0 then Vo=-Vb—0.257 MeV
(b) at r=5.6fm then V1=$k_1\sin(br) - 0.257$=0.257Mev,therefore one can find estimate of m.
(c) Using this procedure solution of the equation (11) can be found.

The interpretation of this Gamow function is the tunneling rate of the fusion reaction of cluster of deuterium (with the given data) corresponding to Takahashi data, with the difference that here we consider a PT-symmetric periodic potential.

c.3. Rosen-Morse potential [8]

Another type of potential which may be considered here is known as Rosen-Morse potential [9][10]:

$$v = -2b.\cot|z| + a(a+a).\csc^2|z|, \tag{28}$$

Where z=r/d. Therefore the Gamow function can be written, respectively:

$$\Gamma = 0.218\sqrt{\mu}.\int_{r0}^{b}((-2b.\cot|z| + a(a+a).\csc^2|z|) - 0.257)^{1/2}.dr \tag{29}$$

(This section is not complete yet).

Some new findings indicating Condensed matter nuclear science and Mesofusion

In this section, we can mention that the most obvious objection against cold fusion is that the Coulomb wall between two nuclei makes the mentioned processes extremely unlikely to happen at low temperature. We can also mention here that there are three known reaction types in thermo fusion:

a. D+D → ^{4}He+γ (23.8 MeV)
b. D+D → ^{3}He+n
c. D+D → ^{3}He+p

In this regards we would like to mention here some clear reasons why cold fusion cannot be analyzed in the classical framework of fission or 'thermo' fusion:

a. No gamma rays are seen;
b. The flux of energetic neutron is much lower than expected on basis of the heat production rate;
c. Lack of signature of D-D reaction;
d. Isotopes of Helium and also tritium accumulate to the Pd samples;
e. Cold fusion appears to occur more effective in Pd nano-particles [6][7];
f. The ratio of x to D atoms to Pd atoms in Pd particle must be in the critical range [0.85,0.90] for the process to occur.

Other strict experimental conditions may also be considered before we can expect repeatability of this process. In this regards, a recent experiment in Arata Hall, Osaka University, on May 22 2008 by Arata has clearly demonstrated that this process did happen. Because the experiment took place at Arata-Zhang laboratory, it then was referred to as Arata-Zhang experiment [6]. Other teams also produced excellent results, for example Prof. Takahashi and his Kobe University team [7].

The basic element of Takahashi's series of experiments is that a periodic potential of the Bloch wave type, as shown in the Figure 1 below.

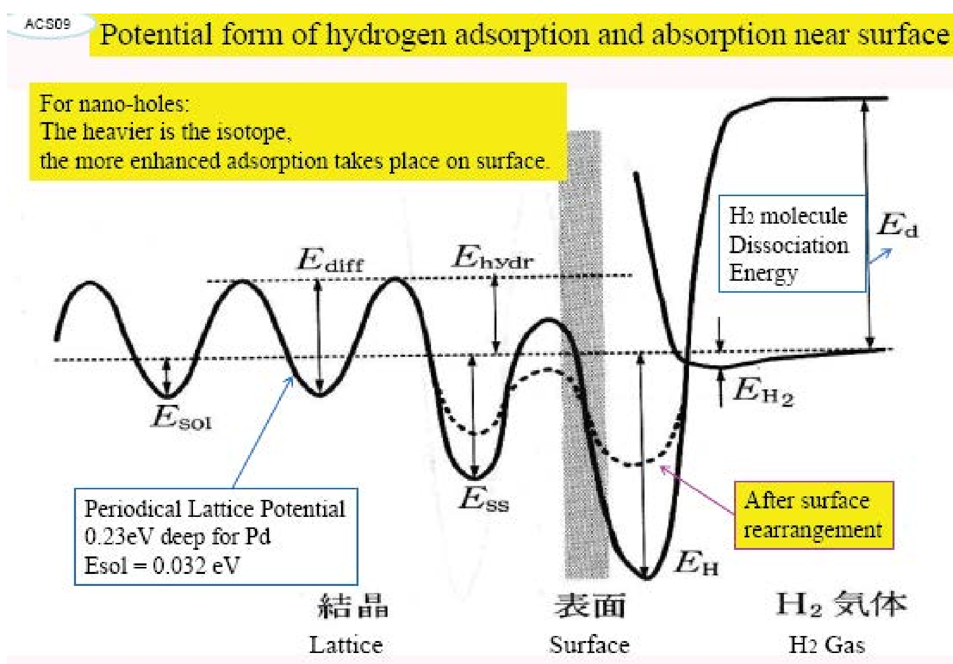

Figure 1. Lattice periodic potential used by Takahashi et al. [7]

From another line of reasoning, one can also consider this possibility of low-temperature fusion. Consider the heat production in our Earth, that some researchers consider it produced by nuclear fission or fusion. But considering that the Earth is lacking uranium (by statistical distribution), chance is that fission is unlikely, but the temperature inside the Earth is clearly much lower than the Sun, therefore the hotfusion is also unlikely to happen. Therefore apparently we can infer that inside the Earth, the heat is produced either as Condensate Nuclear transmutation (CMNS), or other types of low-energy nuclear reaction (LENR).

In other words, if we would like to keep ourselves a bit open-minded, then there other questions too which we don't find quick answer even in the natural processes surrounding us. This would mean as an indication that new types of transmutation processes should be taken into consideration as a possibility.

In this regards perhaps it would be useful to discuss a possible categorization of these new possibilities beyond standard (thermo) fusion process:

a. <u>CANR:</u> or chemically aided nuclear reaction, which essentially uses special types of chemical substance or enzymes [8]. For instance, see hydrino experiments (hydrino.org). Other chemists may prefer to use isoprenoids to create this new effect.
b. <u>LENR:</u> low-energy nuclear reaction [8], or some researchers may prefer to call it 'Lattice fusion Reaction', that is perhaps a more proper name for cold-fusion and other types of deuterium reaction which happens far below the Gamow energy. The name 'lattice fusion' also implies that the process includes neutron in some kind of solid-state physics. An indication that the fusion associated to LENR is outside the domain of standard fusion processes is lack of signature of D-D reaction, which would mean that perhaps the process is much more complicated (for instance Takahashi considered tetra-deuterium model). There is also indication of lacking of neutron emission during this process [7]. We will discuss more on these issues in subsequent section.
c. <u>Mesofusion (or mesoscopic fusion):</u> this belongs to experiments which can be associated to nano-Pd samples for instance by Takahashi and his team in Japan [6]. While this term is not well accepted yet, in our opinion this type of reactions will be much more common in particular for industrial applications, since nanometer devices are much more manageable rather than materials at the order of lepton or hadron scale.

Concluding remarks: Next steps

We would like to conclude this note with a number of some kinds of wish-list.

First of all, a rigorous theoretical framework is clearly on demand. This for instance, will include both to clarify the distinction between Mesofusion and Chromodynamics fusion, and also to consider new type of potentials.

And then, in terms of experiments it appears to be more interesting to introduce new types of tools in order to enhance the performance of these Mesofusion or Chromodynamics fusions. For instance, perhaps it would be interesting to see whether the performance can be improved by introducing either laser or external electromagnetic pulse, just like what has been done in the conventional thermo fusion.

All of these remarks are written here to emphasize that based on recent publication [5]-[8], we are clearly in the beginning of observing new types of fusion technologies, by harnessing our knowledge of hadron and chromodynamics theory.

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From fractality of quantum mechanics to Bohr-Sommerfeld's quantization of planetary orbit distance

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Abstract

In the present paper, we use periodic orbit quantization as suggested by Bohr-Sommerfeld in order to analyze quantization in astrophysical phenomena, i.e. planetary orbit distances. It is known that one can deduce Bohr-Sommerfeld quantization rules from Burger's turbulence [4], and recently such an approach leads to a subfield in physics known as quantum turbulence [5]. Further recommendation for generalizing Bohr-Sommerfeld quantization rules is also mentioned.

Introduction

It is known that quantum mechanics exhibits fractality at $d_F=2$, and an extensive report has been written on this subject and its related issues [1]. Moreover, a fractal solution of time-dependent Schrodinger equation has been suggested some time ago by Datta [2]. On the other side, if one takes a look at planetesimals in the case of planetary system formation, interstellar gas and dust in the case of star formation, the description of the trajectories of these bodies is in the shape of non-differentiable curves, and we obtain fractal curves with fractal dimension 2 [3]. This coincidence between fractality of quantum mechanics and fractal dimension of astrophysical phenomena seems to suggest that we can expect to use quantum mechanical methods such as wave mechanics and periodic orbit quantization to analyze astrophysical phenomena. Such an analysis has been carried out for example by Nottale and Celerier [3] in order to describe these phenomena from the viewpoint of macroscopic Schrodinger equation.

In the present paper, we use periodic orbit quantization as suggested by Bohr-Sommerfeld in order to analyze quantization in astrophysical phenomena, i.e. planetary orbit distances. It is known that one can deduce Bohr-Sommerfeld quantization rules from Burger's turbulence [4], and recently such an approach leads to a subfield in physics known as quantum turbulence [5]. Therefore, turbulence phenomena can also yield quantization, which also seems to suggest that turbulence and quantized vortice is a fractal phenomenon.

We will present Bohr-Sommerfeld quantization rules for planetary orbit distances, which will obtain the same result with a formula based on macroscopic Schrodinger equation.

Further recommendation for generalizing Bohr-Sommerfeld quantization rules is also mentioned.

Bohr-Sommerfeld quantization rules and planetary orbit distances

It was suggested in [6] and [7] that Bohr-Sommerfeld quantization rules can yield an explanation of planetary orbit distances of the solar system and exoplanets. Here, we begin with Bohr-Sommerfeld's conjecture of quantization of angular momentum. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition:

$$\oint_{\Gamma} p.dx = 2\pi.n\hbar, \tag{1}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is:

$$\int_0^T v^2.d\tau = \omega^2.T = 2\pi.\omega, \tag{2}$$

Where $T = \dfrac{2\pi}{\omega}$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$. Then we can write the force balance relation of Newton's equation of motion:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}. \tag{3}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (2), a new constant g was introduced:

$$mvr = \frac{ng}{2\pi}. \tag{4}$$

Just like in the elementary Bohr theory (just before Schrodinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form:

$$r = \frac{n^2.g^2}{4\pi^2.GMm^2}, \tag{5}$$

or

$$r = \frac{n^2.GM}{v_o^2}, \quad (6)$$

Where r, n, G, M, v_o represents orbit radii (semimajor axes), quantum number (n=1,2,3,…), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In equation (6), we denote:

$$v_0 = \frac{2\pi}{g} GMm. \quad (7)$$

The value of m and g in equation (7) are adjustable parameters.

Interestingly, we can remark here that equation (6) is exactly the same with what is obtained by Nottale using his Schrodinger-Newton formula [8]. Therefore here we can verify that the result is the same, either one uses Bohr-Sommerfeld quantization rules or Schrodinger-Newton equation. The applicability of equation (6) includes that one can predict new exoplanets (extrasolar planets) with remarkable result.
Furthermore, one can find a neat correspondence between Bohr-Sommerfeld quantization rules and motion of quantized vortice in condensed-matter systems, especially in superfluid helium [9]. In this regards, a fractional Schrodinger equation has been used to derive two-fluid hydrodynamical equations for describing the motion of superfluid helium in the fractal dimension space [10]. Therefore, it appears that fractional Schrodinger equation corresponds to superfluid helium in fractal dimension space.

Discussion and results

With the help of equation (6) one can describe planetary orbit distances of both the inner planets and Jovian planets in the solar system [7]. See Table 1. Moreover, we were able to predict three new planets in the outer-side of Pluto. This new prediction of three planets beyond the orbit distance of Pluto is made based on our method called CSV (Cantorian Superfluid Vortex) [7].

Table 1: Comparison of prediction and observed orbit distance of planets in Solar system (in 0.1AU unit)

Object	No.	Titius-Bode	Nottale [8]	CSV [7]	Observed	Δ (%)
	1		0.4	0.43		
	2		1.7	1.71		
Mercury	3	4	3.9	3.85	3.87	0.52
Venus	4	7	6.8	6.84	7.32	6.50
Earth	5	10	10.7	10.70	10.0	-6.95

Object	No.	Titius-Bode	Nottale [8]	CSV [7]	Observed	Δ (%)
Mars	6	16	15.4	15.4	15.24	-1.05
Hungarias	7		21.0	20.96	20.99	0.14
Asteroid	8		27.4	27.38	27.0	1.40
Camilla	9		34.7	34.6	31.5	-10.00
Jupiter	2	52		45.52	52.03	12.51
Saturn	3	100		102.4	95.39	-7.38
Uranus	4	196		182.1	191.9	5.11
Neptune	5			284.5	301	5.48
Pluto	6	388		409.7	395	-3.72
2003EL61	7			557.7	520	-7.24
(Sedna)	8	722		728.4	(760)	(4.16)
2003UB31	9			921.8	970	4.96
Unobserv.	10			1138.1		
Unobserv.	11			1377.1		

For inner planets, our prediction values are very similar to Nottale's (1996) values, starting from n = 3 for Mercury; for n = 7 Nottale reported minor object called Hungarias. It is worth noting here, we don't have to invoke several *ad hoc* quantum numbers to predict orbits of Venus and Earth as Neto *et al*. (2002) did [7]. We also note here that the proposed method results in prediction of orbit values, which are within a 7% error range compared to observed values, except for Jupiter which is within a 12.51% error range.

The departure of our predicted values compared to Nottale's predicted values (1996, 1997, 2001) appear in outer planet orbits starting from n = 7. We proposed some new predictions of the possible presence of three outer planets beyond Pluto (for n = 7, n = 8, n = 9) [7]. It is very interesting to remark here, that this prediction is in good agreement with Brown-Trujillo's finding (March 2004, July 2005) of planetoids in the Kuiper belt [13][14][15]. Although we are not sure yet of the orbit of Sedna, the discovery of 2003EL61 and 2003UB31 are apparently in quite good agreement with our prediction of planetary orbit distances based on CSV model.

Therefore, we can conclude that while our method as described herein may be interpreted as an oversimplification of the real planetary migration process which took place sometime in the past, at least it could provide us with useful tool for prediction [6b]. Now we also provide new prediction of other planetoids which are likely to be observed in the near future (around 113.8AU and 137.7 AU). It is recommended to use this prediction as guide to finding new objects (in the inner Oort Cloud).

What we would like to emphasize here is that the quantization method does not have to be the *true* description of reality with regards to celestial phenomena. As always this method could explain some phenomena, while perhaps lacks explanation for other phenomena. But

at least it can be used to predict something quantitatively, i.e. measurable (exoplanets, and new planetoids in the outer solar system etc.).
In the mean time, a correspondence between Bohr-Sommerfeld quantization rules and Gutzwiller trace formula has been shown in [11], indicating that the Bohr-Sommerfeld quantization rules may be used also for complex systems. Moreover, a recent theory extends Bohr-Sommerfeld rules to a full quantum theory [12].

Concluding remarks

In the present paper, we use periodic orbit quantization as suggested by Bohr-Sommerfeld in order to analyze quantization in astrophysical phenomena, i.e. planetary orbit distances. It is known that one can deduce Bohr-Sommerfeld quantization rules from Burger's turbulence [4], and recently such an approach leads to a subfield in physics known as quantum turbulence [5].

We presented Bohr-Sommerfeld quantization rules for planetary orbit distances, which will obtain the same result with a formula based on macroscopic Schrodinger equation.

Further recommendation for generalizing Bohr-Sommerfeld quantization rules is also mentioned.

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On Primordial rotation of the Universe, Hydrodynamics, Vortices and angular momenta of celestial objects

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Abstract

In the present paper, we make some comments on a recent paper by Sivaram & Arun in *The Open Astronomy Journal* 2012, 5, 7-11 with title: 'Primordial rotation of the Universe, Hydrodynamics, Vortices and angular momenta of celestial objects', where they put forth an interesting idea on the origin of rotation of stars and galaxies based on torsion gravity. We extend further their results by hypothesizing the presence of quantized vortices in relation with the torsion vector. If the hypothesis is proven and observed, then it can be used to explain numerous unexplainable phenomena in galaxies etc. The quantization of circulation can be generalized to be Bohr-Sommerfeld quantization rules, which are found useful to describe quantization in astrophysical phenomena, i.e. planetary orbit distances. Further recommendation for observation of the proposed quantized vortices of superfluid helium in astrophysical objects is also mentioned.

Introduction

Two recent papers by Sivaram & Arun, one in *The Open Astronomy Journal* 2012, 5, 7-11 [1], and one in arXiv [2] are found very interesting. They are able to arrive at the observed value of effective cosmological constant by considering background torsion in the teleparallel gravity. According to them: "*the background torsion due to a universal spin density not only gives rise to angular momenta of all structures but also provides a background centrifugal term acting as a repulsive gravity accelerating the universe, with spin density acting as effective cosmological constant.*"[1] The torsion is given by [1, p.10]:

$$Q = \frac{4\pi G\sigma}{c^3} \approx 10^{-28}\, cm^{-1}, \tag{1}$$

And the background curvature [1, p.10] is given by:

$$Q^2 \approx 10^{-56}\, cm^{-2}. \tag{2}$$

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In the meantime, a recent review of dark energy theories in the literature (including *teleparallel gravity*) has been given in [4], and present problems in the standard model general relativistic cosmology are discussed by Starkman [5]. These seem to suggest that a torsion model of effective cosmological constant based on teleparallel gravity as suggested by Sivaram and Arun (2012) seems very promising as a description of phenomena related to accelerated expansion of the Universe usually attributed to *'dark energy'* (as alternative to cosmological constant explanation).

However, Sivaram & Arun do not make further proposition concerning the connection between quantized vortices (Onsager-Feynman's rule) and the torsion vector. It will be shown here, that such a connection appears possible.

Here we present Bohr-Sommerfeld quantization rules for planetary orbit distances, which results in a good quantitative description of planetary orbit distance in the solar system [6][6b][7]. Then we find an expression which relates the torsion vector and quantized vortices from the viewpoint of Bohr-Sommerfeld quantization rules [3].

Further observation of the proposed quantized vortices of superfluid helium in astrophysical objects is recommended.

Bohr-Sommerfeld quantization rules and quantized vortices

The quantization of circulation for nonrelativistic superfluid is given by [1][3]:

$$\oint v dr = N \frac{\hbar}{m_s} \tag{3}$$

Where $N, \hbar, m_s$ represents winding number, reduced Planck constant, and superfluid particle's mass, respectively [3]. And the total number of vortices is given by [1]:

$$N = \frac{\omega . 2\pi r^2 m}{\hbar} \tag{4}$$

And based on the above equation (4), Sivaram & Arun [1] are able to give an estimate of the number of galaxies in the universe, along with an estimate of the number stars in a galaxy.

However, they do not give explanation between the quantization of circulation (3) and the quantization of angular momentum. According to Fischer [3], the quantization of angular momentum is a relativistic extension of quantization of circulation, and therefore it yields Bohr-Sommerfeld quantization rules.

Furthermore, it was suggested in [6] and [7] that Bohr-Sommerfeld quantization rules can yield an explanation of planetary orbit distances of the solar system and exoplanets. Here, we begin with Bohr-Sommerfeld's conjecture of quantization of angular momentum. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition:

$$\oint_{\Gamma} p.dx = 2\pi.n\hbar, \tag{5}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is:

$$\int_{0}^{T} v^2.d\tau = \omega^2.T = 2\pi.\omega, \tag{6}$$

Where $T = \frac{2\pi}{\omega}$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$. Then we can write the force balance relation of Newton's equation of motion:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}. \tag{7}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (6), a new constant g was introduced:

$$mvr = \frac{ng}{2\pi}. \tag{8}$$

Just like in the elementary Bohr theory (just before Schrodinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form:

$$r = \frac{n^2.g^2}{4\pi^2.GMm^2}, \tag{9}$$

or

$$r = \frac{n^2.GM}{v_o^2}, \tag{10}$$

Where r, n, G, M, v_o represents orbit radii (semimajor axes), quantum number (n=1,2,3,…), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In equation (10), we denote:

$$v_0 = \frac{2\pi}{g} GMm. \tag{11}$$

The value of m and g in equation (11) are adjustable parameters.

Interestingly, we can remark here that equation (10) is exactly the same with what is obtained by Nottale using his Schrodinger-Newton formula [8]. Therefore here we can verify that the result is the same, either one uses Bohr-Sommerfeld quantization rules or Schrodinger-Newton equation. The applicability of equation (10) includes that one can predict new exoplanets (extrasolar planets) with remarkable result.
Therefore, one can find a neat correspondence between Bohr-Sommerfeld quantization rules and motion of quantized vortices in condensed-matter systems, especially in superfluid helium [3]. Here we propose a conjecture that Bohr-Sommerfeld quantization rules also provide a good description for the motion of galaxies, therefore they should be included in the expression of torsion vector. We will discuss an expression of torsion vector of quantized vortices in the next section.

Torsion and quantized vortices

We cite here a rather old paper of Garcia de Andrade & Sivaram, 1998 [9], where they discuss propagation torsion model for quantized vortices. They consider the torsion to be propagating and it can be expressed as derivative of scalar field:

$$Q = \nabla \phi. \tag{12}$$

Therefore $\oint Q dS$ can be written as [9]:

$$\oint Q dS = \oint \nabla \phi dS = \int \nabla (\nabla \phi) dV \cong \int \nabla^2 \phi dV. \tag{13}$$

Also $\oint Q dS$ must have dimensions of length, and thus quantized as [9]:

$$\oint Q dS \cong \frac{n\hbar c}{M} \tag{14}$$

Now we invoke a result from the preceding section discussing Bohr-Sommerfeld quantization rules. Assuming that Bohr-Sommerfeld quantization rules also govern the galaxies motion as well as stars motion, then we can insert equation (11) into equation (14), to yield a new expression:

$$\oint Q dS \cong \frac{n\hbar c.2\pi Gm}{v_0 g} \tag{15}$$

Therefore, we submit a viewpoint that the torsion vector is also a quantized quantity, and it is a function of Planck constant, speed of light, Newton gravitation constant, vortex particle's mass, a specific velocity and an adjustable parameter, g. It is interesting to find out whether this proposition agrees with observation data or not.

The above proposition (15) connects torsion vector with gravitation constant, which seems to give a torsion description of gravitation. There are numerous other models to describe alternative or modified gravitation theories, for instance Wang is able to derive Newton's second law and Schrodinger equation from fluid mechanical dynamics. [10][11]

In the mean time, for discussion of galaxy disk formation, see [12]. And [13] gives alternative vortices argument for dark matter.

The proposed quantization of circulation as suggested by Sivaram and Arun [1] is based on a conjecture that the universe is formed by superfluid or condensed matter. For models describing further this proposition, see discussion in Brook [14].

Concluding remarks

In the present paper, we make some comments on a recent paper by Sivaram & Arun in *The Open Astronomy Journal* 2012, 5, 7-11 where they put forth an interesting idea on the origin of rotation of stars and galaxies based on torsion gravity. We extend further their results by hypothesizing the presence of quantized vortices in relation with the torsion vector. If the hypothesis is proven and observed, then it can be used to explain numerous unexplainable phenomena in galaxies etc.

Further recommendation for observation of the proposed quantized vortices of superfluid helium in astrophysical objects is also mentioned.

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A note on Quantization of Galactic Redshift and the Source-Sink model of Galaxies

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Abstract

This paper discusses shortly a Source-Sink model of galaxies and its implication to the observed quantization of galactic redshift. As shown elsewhere, the Newton law and Maxwell electromagnetic equations can be described by the Source-Sink model too. In this paper I shortly review the Source-Sink model of galaxies which can be interpreted further in the context of superfluid dynamics as described by Gross-Pitaevskii equation. And because it can be shown that radial Gross-Pitaevskii equation can yield ring soliton-like solutions, therefore I submit a hypothesis that the universe may likely have a centre in the form of ring soliton-like. This hypothesis requires further observation in order to verify or refute.

Introduction

In recent years there are some reports suggesting explanations for quantization of galactic redshift as observed by Tifft et al. One of those proposals is suggested by Firmin J. Oliveira, who submits a wave equation model based on Carmeli's Cosmological General Relativity in order to describe such a quantization of galactic redshift [1][2]. Despite its useful approach to describe this phenomenon of quantized redshift, Oliveira's approach apparently lacks a physical model to describe why there exists quantization of galactic redshift. Therefore we require a better approach which provides physical model of the phenomenon.

A Source-Sink model of Galaxies

Physical model of quantization of galactic redshift does exist, for example Hodge's Source-Sink model of Galaxies. Hodge argues that on the galactic scale the universe is inhomogeneous and redshift z is occasionally less than zero. He also argues that several differences among galaxy types suggest that spiral galaxies are Sources and that early type, lenticular, and irregular galaxies are Sinks of a scalar potential field.[3]

Then Hodge postulates the existence of a scalar potential ρ (erg) field with the characteristics to cause the observed differences in spiral and elliptical galaxies. The gradient of ρ is proportional to a force F_s (dyne) that acts on matter. [3] The SPM suggests F_s exerts a force to repel matter from spiral galaxies and to attract matter to early type galaxies.

The SPM also suggests the photon is a particle. The derivation of Planck's black body radiation equation includes the proposition that the energy of a photon is discrete and composed of a number N of basic energy/particle packets.[3] From this conjecture, some formulas for redshift of sink and source galaxies can be derived.

While of course the arguments of Hodge can be discussed further, it seems interesting that he can come up with a physical model in order to explain such a quantization of galactic redshift.

Furthermore, it may be interesting to note here that both Newton law and Schrodinger equations can be derived from similar assumption of source-sink model.[4][5] In this regard, Rahman has given a proof that classical electrodynamics can be derived from similar source-sink fluid model.[6]

While surely the aforementioned papers by Wang, Hodge and Rahman use different methods, all of them have the same assumption of the existence of source-sink particles. Therefore this approach seems quite promising to explore further.

Gross-Pitaevskiian interpretation

Now I would like to extend further Hodge's sink-source model of galaxies into the context of Gross-Pitaevskii model. We know that Gross-Pitaevskii equation is often used to describe superfluid dynamics. In one of his papers, Moffat has shown that quantum phion condensate model with Gross-Pitaevskii equation yields an approximate fit to data corresponding to CMB spectrum, and it also yields a modified Newtonian acceleration law which is in good agreement with galaxy rotation curve data.[7]

Furthermore, this author also has argued that Gross-Pitaevskii equations yields quantized vortice which can be used to explain the galactic intrinsic redshift phenomenon.[8] Therefore here I also argue that Hodge's Source-Sink model of galaxies can be related to Gross-Pitaevskiian description of superfluidity.

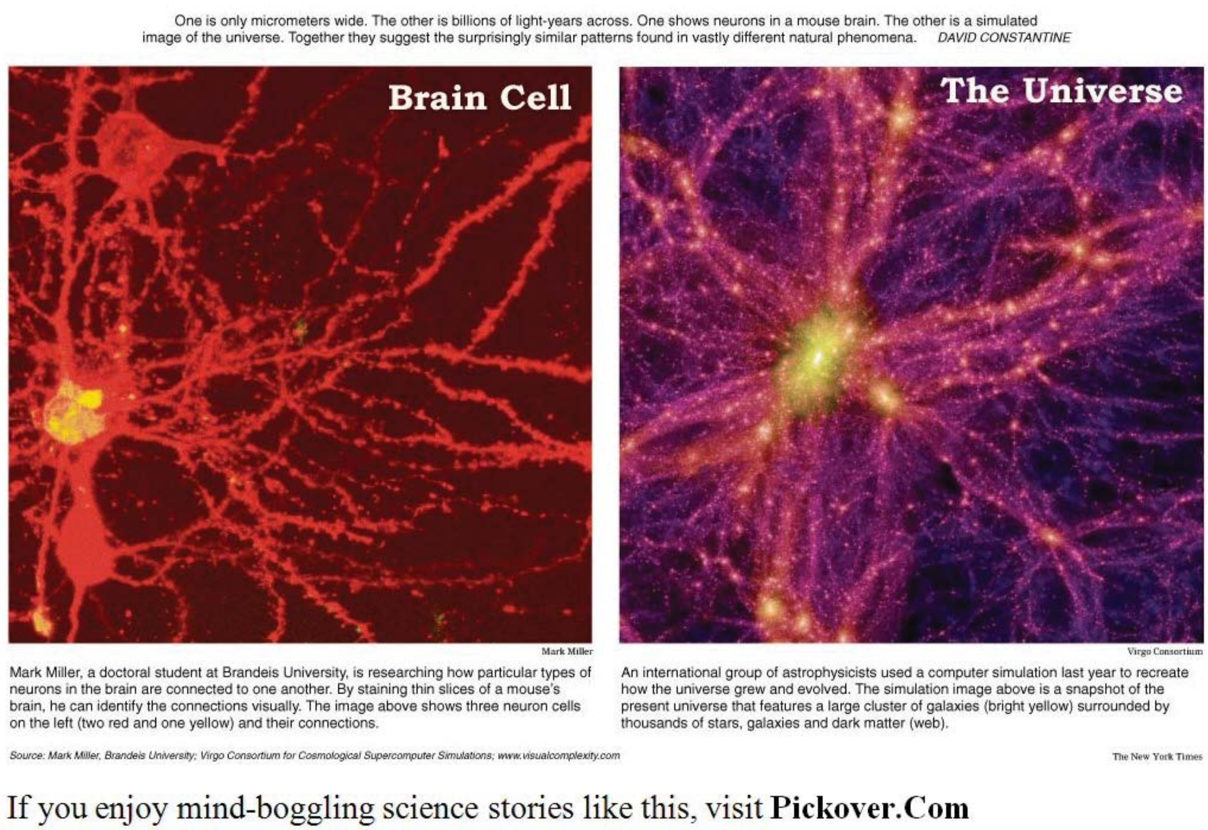
In this regard, I would like to mention a recent paper by Toikka, Hietarinta, and Suominen [9], which suggests that there can be ring soliton-like solutions of the cylindrically symmetric (i.e. radial) Gross-Pitaevskii equation with a potential. Extrapolating this result to the universe scale, I submit a hypothesis that the universe may likely have a centre in the form of ring soliton-like. This hypothesis requires further observation in order to verify or refute.

Interestingly, one can also note that Michael Peck has also suggested that the Universe may have a centre, using a revised model of General Relativity[10]. But of course it does not mean that I agree with all Peck's arguments.

Added note. Does the Universe have a centre?

In this section I would like to mention a picture suggesting similarity between brain's neuron and the structure of the universe. See Picture 1.[11][12][13]

(Note: One is only micrometers wide. The other is billions of light years across. One shows neurons in a mouse brain. The other is simulated image of the universe. Together they suggest the surprisingly similar patterns found in vastly different natural phenomena.- David Constantine)



Picture 1. Similarity between brain's neuron and the Universe

If the above picture holds true, then it seems to have a profound implication, suggesting that there can be a deep connection between brain's neuron and the structure of Universe.

While of course there is a question about the great differences between the structure of supercluster of galaxies and the structure of brain's neuron, one may recall that there are possible hints suggesting explanations about possible connection between brain's neuron and the structure of the Universe, for instance:

- Fractal theory suggests possible self-similarity between microscales and macroscales, see for instance the scale relativity theory of Nottale, 1997 [17]. See also Celerier & Nottale, 2005 [18].
- Spiral waves can be found in different scales from the microscales to the macroscales. These spiral waves may be a governing pattern in galaxy formation too, and these spiral waves are resulted from complex Ginzburg-Landau equations.
- A recent discovery of network cosmology by Dmitri Krioukov et al. suggests a deep similarity between brain, internet, and the Universe. [14] He finds theoretical link between hyperbolic metric and complex network. They write as follows: "Here we show that the causal network representing the large scale structure of spacetime in our accelerating universe is a power-law graph with strong clustering, similar to many complex networks such as the Internet, social, or biological networks. We prove that this structural similarity is a consequence of the asymptotic equivalence between the large-scale growth dynamics of complex networks and causal networks. This equivalence suggests that *unexpectedly similar laws govern the dynamics of complex networks and spacetime in the universe*, with implications to network science and cosmology."[14] (emphasis is added)
- Krioukov et al.'s finding may be related to the work of Serrano et al.[15], suggesting possible connection between self-similarity of complex networks and hidden metric space.

Regardless the differences of theoretical approaches as mentioned above, apparently we can agree about one thing from looking at Picture 1 above, that is both brain's neuron and the Universe have a centre. This can be generalized further as follows: that any complex network tends to have centre. Therefore apparently our hypothesis above that the universe can have a centre, which is based on Gross-Pitaevskiian description, now seems to be supported by recent finding based on complex network studies.

There is other study based on network analysis which also supports the idea that complex networks tend to have centre, that is a recent study about global corporate control which results in a conclusion that there are "core" corporate which hold control on majority of other corporate in the world.[16] This finding about core corporate seems also to suggest that there is a centre in the network of global corporate control.

We summarize therefore that based on recent findings based on complex network studies we can find clues to support our hypothesis that the Universe may likely have a centre. However, this hypothesis requires further observation in order to verify or refute.

Moreover, it seems that there remains a long way to prove that there exists deep connection between brain's neuron and the structure of Universe.

Concluding Remarks

This paper discusses shortly a Source-Sink model of galaxies and its implication to the observed quantization of galactic redshift. As shown elsewhere, the Newton law and Maxwell electromagnetic equations can be described by the Source-Sink model too. In this paper I shortly review the Source-Sink model of galaxies which can be interpreted further in the context of superfluid dynamics as described by Gross- Pitaevskii equation. And because it can be shown that radial Gross-Pitaevskii equation can yield ring soliton-like solutions, therefore I submit a hypothesis that the universe may likely have a centre in the form of ring soliton-like. This hypothesis requires further observation in order to verify or refute.

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A Journey into Quantization in Astrophysics:

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The present book consists of 17 select scientific papers from ten years of work around 2003-2013. The topic covered here is quantization in Astrophysics. We also discuss other topics for instance Pioneer spacecraft anomaly.

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