GENERATION AND USE OF FEMTOSECOND, GIGAWATT, NEAR INFRARED LASER PULSES FROM AN AMPLIFIED, MODE-LOCKED, Ti:SAPPHIRE LASER

David Anthony Valdes
David Valdés
Candidate

Physics and Astronomy
Department

This thesis is approved, and it is acceptable in quality and form for publication.

Approved by the Thesis Committee:

Dr. Jean-Claude Diels, Chairperson

Dr. Paul Schwoebel

Dr. Elohim Becerra

Dr. Francesca Cavallo
GENERATION AND USE OF FEMTOSECOND, GIGAWATT, NEAR INFRARED LASER PULSES FROM AN AMPLIFIED, MODE-LOCKED, Ti:SAPPHIRE LASER

by

DAVID VALDÉS

A.A., Liberal Arts, Central New Mexico Community College (formerly Albuquerque Technical Vocational Institute), 2002
B.A., Physics and Astrophysics, University of New Mexico, 2007

THESIS
Submitted in Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE
OPTICAL SCIENCE AND ENGINEERING

The University of New Mexico
Albuquerque, New Mexico

MAY 2019
DEDICATION

To my wonderful wife, Kim, who was patient and supportive in the attainment of my degree. And to my children, Zoë and Kilian, who I trust will benefit from this work.
ACKNOWLEDGEMENTS

I kindly and emphatically acknowledge the support, time, and dedication given to me by my advisor, Dr. Jean-Claude Diels, without whom I may not have achieved this goal.

I also thank my committee members Dr. Elohim Becerra, Dr. Paul Schwoebel, and Dr. Francesca Cavallo for graciously reviewing this work.

Lastly, I thank James Hendrie, Ning Hsu, and other students of Dr. Diels for their kind assistance with equipment and ideas.
GENERATION AND USE OF FEMTOSECOND, GIGAWATT, NEAR INFRARED LASER PULSES FROM AN AMPLIFIED, MODE-LOCKED, Ti:SAPPHIRE LASER

by

David Valdés

A.A., Liberal Arts, Central New Mexico Community College (formerly Albuquerque Technical Vocational Institute), 2002
B.A., Physics and Astrophysics, University of New Mexico, 2007
M.S., Optical Science and Engineering, University of New Mexico, 2019

ABSTRACT

This work modeled the early to middle successes achieved in the field of ultrafast, high peak power optics, beginning with the work of Nobel Prize winners Donna Strickland and Gérard Mourou in 1985 [1]. In our work, 100 fs light pulses of around 800 nm were generated by a Ti:Sapphire oscillator, then amplified to approximately 30 GW peak power using a chirped pulse amplification system that included regenerative and multi-pass amplifiers. As a verification of our pulses having high peak powers and ultrashort durations, they were then used to strike water, glass, and a Kerr Cell. Supercontinuum generation was observed as a result of striking the water and the glass. Moreover, using water produced a stable source of white light. Glass did not produce a stable source of white light due to material damage. In striking the Kerr Cell, we hoped to observe an induced voltage from the interaction of high power pulses with the CS$_2$ contained within. This was not observed. However, spectral components of green, yellow, and red were observed. In addition to the expected results, for several months during the experiment we
generated 55 fs pulses. This is an exciting result as pulses of this duration are thought by some to be impossible given the elements used for our system. The precise mechanisms that contributed to the sub-100 fs pulses are yet undetermined. This suggests interesting work for the future of this system given the extra stability it affords as compared to other modern systems.

A brief history of this type of laser system is given in Chapter 1 along with some key physical ideas. The components of our systems and principles of operation are discussed in Chapter 2. Data for each part of the experiment is given in Chapter 3. Experimental procedures and selected derivations for mathematical analysis are included in the appendices.
TABLE OF CONTENTS

LIST OF FIGURES ........................................................................................................... x

LIST OF TABLES .............................................................................................................. xii

PREFACE ............................................................................................................................ xiii

CHAPTER 1 INTRODUCTION ......................................................................................... 1

1.1 Brief history of this laser system .............................................................................. 1
1.2 The Creation of Ultrashort Pulses ........................................................................... 2
  1.2.1 Superposition and Pulses ................................................................................... 2
  1.2.2 Pulse Duration .................................................................................................... 4
  1.2.3 The Kerr Effect ................................................................................................ 6
  1.2.4 Dispersion .......................................................................................................... 9
1.3 Chirped Pulse Amplification (CPA) ........................................................................ 16
1.4 Applications of CPA ............................................................................................... 20

CHAPTER 2 SYSTEM, OPERATING PRINCIPLES, & RESULTS .............................. 21

2.1 System Overview Flowchart .................................................................................. 21
2.2 Mode-locked Ti:Sapphire Laser: The Seed .......................................................... 23
  2.2.1 Pump Source .................................................................................................... 24
  2.2.2 Ti:Sapphire Lasing Medium ............................................................................. 24
  2.2.3 Curved Mirrors ............................................................................................... 32
  2.2.4 Lens and Multiple Quantum Well .................................................................... 33
  2.2.5 Prism Pair ....................................................................................................... 34
  2.2.6 Cavity Ends .................................................................................................... 35
  2.2.7 Pulse Train Measurement ............................................................................... 35
  2.2.8 Autocorrelation ............................................................................................. 36
2.3 Stretcher .................................................................................................................. 41
2.4 Regenerative Amplifier .......................................................................................... 55
  2.4.1 Pulse Trapping ............................................................................................... 58
  2.4.2 Timing ............................................................................................................. 59
2.5 Multi-pass Amplifier .............................................................................................. 62
2.6 Compressor ............................................................................................................ 64
2.7 Targets......................................................................................................................66

CHAPTER 3 DATA...........................................................................................................70

3.1 Mode-locked Ti:Sapphire laser..................................................................................70
  3.1.1 Pump Laser – Millennia V Diode-Pumped, CW Visible Laser made by Spectra
        Physics .................................................................................................................70
  3.1.2 Ti:Sapphire Oscillator ......................................................................................70
3.2 Stretcher....................................................................................................................70
3.3 Regenerative Amplifier ..............................................................................................71
  3.3.1 Settings ............................................................................................................71
  3.3.2 Power Distribution ............................................................................................72
  3.3.3 Single Pulse Amplification Measurements .........................................................72
3.4 Multi-pass Amplifier .................................................................................................74
3.5 Compressor ..............................................................................................................74
3.6 Water Target ............................................................................................................74
3.7 Glass Target .............................................................................................................75
3.8 Interaction with Kerr Cell .........................................................................................75

APPENDICES.................................................................................................................77

APPENDIX A: Procedures ..............................................................................................77

A.1 Mode-locked Ti:Sapphire laser..................................................................................77
  A.1.1 Creating the Cavity ...........................................................................................77
  A.1.2 Measuring Pulses .............................................................................................81
A.2 Faraday Isolator .........................................................................................................85
A.3 Stretcher ...................................................................................................................86
  A.3.1 Stretcher Set-Up Procedure .............................................................................87
  A.3.2 Stretched Pulse Image Processing Procedure ..................................................90
A.4 Regenerative Amplifier .............................................................................................90
  A.4.1 Initial Set-Up Procedure ....................................................................................90
  A.4.2 Pumping Set-Up Procedure .............................................................................94
  A.4.3 Regenerative Amplifier Lasing .........................................................................95
  A.4.4 Timing Set-Up Procedure ................................................................................96
A.4.5 Single Pulse Isolation ................................................................. 99
A.5 Multi-pass Amplifier ................................................................. 101
A.6 Compressor and Water Target....................................................... 103
A.7 Glass Target.............................................................................. 106
A.8 Interaction with Kerr Cell............................................................. 107

APPENDIX B: Selected Derivations .................................................... 108

B.1 The Fourier Transform of a Gaussian............................................ 108
B.2 The Gaussian Integral.................................................................... 109
B.3 The Fourier Transform of $f(x) = \text{sech}^2(ax)$.............................. 110
B.4 The Inverse Fourier Transform: $g(t) = \mathcal{F}^{-1}\left\{\frac{\mathcal{F}\{\text{sech}^2(at)\}}{\mathcal{F}\{be^{-ct^2}\}}\right\}$ ........................................... 115

REFERENCES ...................................................................................... 126
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1:</td>
<td>Pulses by Superposition</td>
</tr>
<tr>
<td>Figure 2:</td>
<td>Intensity Profile (red), Electric Field Profile (black)</td>
</tr>
<tr>
<td>Figure 3:</td>
<td>Train of Pulses</td>
</tr>
<tr>
<td>Figure 4:</td>
<td>Seed Pulse (Autocorrelation)</td>
</tr>
<tr>
<td>Figure 5:</td>
<td>Kerr Lensing with Hard Aperture</td>
</tr>
<tr>
<td>Figure 6:</td>
<td>Soft Aperture Kerr Lensing</td>
</tr>
<tr>
<td>Figure 7:</td>
<td>Pulse Broadening due to GVD</td>
</tr>
<tr>
<td>Figure 8:</td>
<td>Phase and Group Velocities</td>
</tr>
<tr>
<td>Figure 9:</td>
<td>Hyperbolic Secant Pulses with Cosine Carrier Wave</td>
</tr>
<tr>
<td>Figure 10:</td>
<td>Hyperbolic Secant Squared Intensity Pulse Train</td>
</tr>
<tr>
<td>Figure 11:</td>
<td>PVD through a Prism</td>
</tr>
<tr>
<td>Figure 12:</td>
<td>Chirped Pulse Amplification Schematic</td>
</tr>
<tr>
<td>Figure 13:</td>
<td>Up-Chirped Pulse</td>
</tr>
<tr>
<td>Figure 14:</td>
<td>System Flow Chart</td>
</tr>
<tr>
<td>Figure 15:</td>
<td>Experimental Set-Up</td>
</tr>
<tr>
<td>Figure 16:</td>
<td>Mode-locked Ti:Sapphire Laser Cavity (Zoomed-in on the Crystal)</td>
</tr>
<tr>
<td>Figure 17:</td>
<td>Mode-locked Ti:Sapphire Laser Cavity (Closer to Scale)</td>
</tr>
<tr>
<td>Figure 18:</td>
<td>Self-Phase Modulation of a Gaussian Pulse</td>
</tr>
<tr>
<td>Figure 19:</td>
<td>SPM and Frequency Shift</td>
</tr>
<tr>
<td>Figure 20:</td>
<td>Phase, Frequency Shift, &amp; Chirp</td>
</tr>
<tr>
<td>Figure 21:</td>
<td>SPM - Hyperbolic Secant Electric Field</td>
</tr>
<tr>
<td>Figure 22:</td>
<td>Phase, Freq. Shift, &amp; Chirp for Hyperbolic Secant E-Field</td>
</tr>
<tr>
<td>Figure 23:</td>
<td>Astigmatism Compensation</td>
</tr>
<tr>
<td>Figure 24:</td>
<td>Illuminated Ti:Sapphire between Focusing Mirrors</td>
</tr>
<tr>
<td>Figure 25:</td>
<td>Pulse Train Measurement</td>
</tr>
<tr>
<td>Figure 26:</td>
<td>Intensity Autocorrelation Measurement (Misleading Time Scale)</td>
</tr>
<tr>
<td>Figure 27:</td>
<td>Autocorrelation Apparatus</td>
</tr>
<tr>
<td>Figure 28:</td>
<td>SHG for Autocorrelation</td>
</tr>
<tr>
<td>Figure 29:</td>
<td>Autocorrelation Apparatus Diagram</td>
</tr>
<tr>
<td>Figure 30:</td>
<td>Autocorrelation Time-Scale Determination</td>
</tr>
<tr>
<td>Figure 31:</td>
<td>Pulses Forming Autocorrelation Signal</td>
</tr>
<tr>
<td>Figure 32:</td>
<td>Shared Area of Passing Pulses</td>
</tr>
<tr>
<td>Figure 33:</td>
<td>Hand Sketch of Actual Stretcher</td>
</tr>
<tr>
<td>Figure 34:</td>
<td>Stretcher Diagram</td>
</tr>
<tr>
<td>Figure 35:</td>
<td>Stretched Pulse</td>
</tr>
<tr>
<td>Figure 36:</td>
<td>Seed Pulse</td>
</tr>
<tr>
<td>Figure 37:</td>
<td>Stretched Pulse Duration Approximation - Example 2</td>
</tr>
<tr>
<td>Figure 38:</td>
<td>Example 2 – Normalized: $\epsilon = 0.2\tau$</td>
</tr>
<tr>
<td>Figure 39:</td>
<td>Example 2 – Normalized: $\epsilon = 0.9\tau$</td>
</tr>
<tr>
<td>Figure 40:</td>
<td>Normalized Stretched Pulse Numeric Approximation, 1001 Terms</td>
</tr>
<tr>
<td>Figure 41:</td>
<td>Normalized Stretched Pulse Numeric Approximation, 10001 Terms</td>
</tr>
<tr>
<td>Figure 42:</td>
<td>Estimate of Normalized Physical Stretched Pulse</td>
</tr>
<tr>
<td>Figure 43:</td>
<td>Pulse Data Estimations by Smooth Functions</td>
</tr>
<tr>
<td>Figure 44:</td>
<td>Estimated Physical Pulse Compared to Data</td>
</tr>
<tr>
<td>Figure 45:</td>
<td>Faraday Isolator System Demonstrating Switching by Polarization</td>
</tr>
</tbody>
</table>
Figure 46: Regenerative Amplifier Diagram ............................................................ 56
Figure 47: Regenerative Amplifier Timing Control Diagram .................................. 59
Figure 48: Relative Timing of Timing Control Device Outputs ................................. 60
Figure 49: Amplified Pulses within Regenerative Amplifier ...................................... 60
Figure 50: Single Amplified Pulse after Regenerative Amplifier ............................... 61
Figure 51: Bow Tie Configuration for Multi-pass Amplifier ...................................... 62
Figure 52: Per Pass Voltage in Multi-Pass Amplifier ................................................ 64
Figure 53: Pulse Compression: Reversal of GVD due to Stretching ............................ 64
Figure 54: Sketch of Compressor Set-Up .................................................................. 65
Figure 55: High Energy Pulse Interaction with BK-7 Glass ........................................ 66
Figure 56: White Light ......................................................................................... 67
Figure 57: Supercontinuum Generation .................................................................... 67
Figure 58: Continuum Generation in Water .............................................................. 69
Figure 59: Single Pulse Amplification ....................................................................... 73
Figure 60: Amplification Inside the Regenerative Amplifier ...................................... 73
Figure 61: Spectrum Generated by Pulse Interaction with Water .............................. 75
Figure 62: Pulse Interaction with Bulk Glass ............................................................ 75
Figure 63: Pulse Interaction with a Kerr Cell ............................................................ 76
Figure 64: Prism at Minimum Deviation .................................................................... 80
Figure 65: Autocorrelator Apparatus Diagram ......................................................... 82
Figure 66: Switching with a Faraday Isolator System ............................................... 85
Figure 67: Hand Sketch of Actual Stretcher .............................................................. 87
Figure 68: Regenerative Amplifier Diagram ............................................................. 91
Figure 69: Numbered Bow Tie Configuration ......................................................... 101
Figure 70: Sketch of Compressor Set-Up ............................................................... 103
Figure 71: Numbered Compressor Set-Up ............................................................... 104
LIST OF TABLES

Table 1: Regenerative Amplifier Timing Sequence ............................................. 61
Table 2: Millennia Pump Laser Settings ............................................................ 70
Table 3: Seed Pulse Data .................................................................................. 70
Table 4: Stretched Pulse Data .......................................................................... 70
Table 5: Timing Control Device Settings ........................................................... 71
Table 6: Surelite Settings .................................................................................. 71
Table 7: Pockels Cell Settings .......................................................................... 71
Table 8: Surelite Power Distribution ................................................................. 72
Table 9: Amplification of Regenerative Amplifier .............................................. 72
Table 10: Energy per Pass in Multi-pass Amplifier .......................................... 74
PREFACE

In addition to fulfilling the requirements of my Master’s Thesis, this work is written to undergraduate students in optics and other physical sciences who wish to pursue graduate level coursework in optics. This work serves as an introduction to Ti:Sapphire based generation and amplification of ultrashort laser pulses. This can be used as a supplement to course materials in laser or optics courses. The interested student is encouraged to take the next step in pursuit of an advanced degree.
CHAPTER 1 INTRODUCTION

1.1 Brief history of this type of laser system

As with any component history of the laser, the history of this type of laser, or more appropriately, laser system, is detailed and interesting. Let me begin, however, by describing our laser system, in brief.

The output of our system is high energy, ultrashort, pulses. These pulses are then used to strike various targets of study. More specifically, our system begins with an optically pumped Ti:Sapphire mode-locked laser that outputs pulses of around 100 fs with roughly 100 mW average power. (Pulse durations of around 50 fs were recorded for several months during the project. This phenomenon is as of yet unaccounted for.) Pulses are separated by about 13.5 ns, which corresponds to 1.35 nJ/pulse. A Ti:Sapphire laser cavity such as ours is well described by G. Cerullo, et al. [2]. Please note in reading the referred material that laser cavities, including this type of laser cavity, are often called resonators or oscillators. The ultrashort pulses generated in our Ti:Sapphire cavity are then amplified using chirped pulse amplification (described later in this introduction). The output energy of the system is about 3.82 mJ/pulse. Assuming a return to the original pulse duration, the peak power per pulse is about 30 GW. After amplification the high energy pulses are used to strike and study various targets such as bulk glass, water, and a Kerr Cell. Glass and water targets were chosen to demonstrate supercontinuum generation, while the Kerr Cell was chosen to potentially observe orientation of the CS$_2$ molecules contained within. The result of striking various glass targets, and water, was indeed the generation of white light
(or supercontinuum). Striking the Kerr Cell showed non-linear effects in a dispersion of green, yellow, and red colors. It did not, however, register a voltage output as desired.

In general, Ti:Sapphire based mode-locked lasers have been employed since the 1990’s. During this time, and leading up to the early 2000’s, pulse durations of less than 5 fs had been achieved [3]. Precursors to this and other types of pulsed lasers began as early as the 1960’s with mode-locked He-Ne lasers (as mode-locking is a vital component to ultrashort pulses) [4]. Applications of this type of pulsed laser system are varied. Some examples are given at the end of this chapter.

1.2 The Creation of Ultrashort Pulses

1.2.1 Superposition and Pulses

Pulses, in general, are created by the superposition of multiple waves with differing frequencies that are in phase at some place and time. Figure 1 illustrates this concept. The orange wave at the bottom of the figure is the lowest frequency wave, and can be thought of as the fundamental mode of a laser cavity. Waves, above the orange wave, have successively increasing frequencies, and are harmonics of the fundamental mode. The multicolored sixth wave pattern from the bottom is the previous five individual waves shown on top of each other. The highest wave in the figure is the superposition of the five individual waves. The mathematics that describe this phenomenon can be found in several sources, e.g. [5].
Figure 1: Pulses by Superposition

Harmonic waves are shown from fundamental (bottom-orange) to 5th harmonic (purple). The superposition is shown at the top.

In general, the greater the number of harmonic waves that are in phase at some place and time, the narrower the resultant pulse. Pulse generation in the presence of dispersion is discussed in the Dispersion section of this chapter.

However, it is the intensity of a pulse that is measured by detectors, such as avalanche photodiodes, photo multiplier tubes, or even your eyes. Since intensity is proportional to the square of the electric field, what the detector output looks like is more like Figure 2.

Figure 2: Intensity Profile (red), Electric Field Profile (black)

Black solid curve: same as top of Figure 1. Red solid curve: black curve, squared

Created with DESMOS free online graphing calculator at https://www.desmos.com/calculator
In the Ti:Sapphire cavity there may be a million modes adding together to produce a resultant pulse at a certain time. The Ti:Sapphire cavity in our system produced the pulses shown in Figure 3. These pulses are shown, in sequence, in the time domain (also known as a train of pulses).

Figure 3: Train of Pulses

The time difference between pulses is \( \Delta t \approx 13.5 \) ns.

### 1.2.2 Pulse Duration (or Temporal Pulse Width)

Since the creation of ultrashort pulses requires the sum of many waves in phase, it is desirable to use a gain medium that yields the highest number of available modes. The number of available modes can be approximated by [6]

\[
N = \frac{\Delta \nu}{\nu_{fsr}}
\]  

where \( \Delta \nu = \text{gain bandwidth} \), and \( \nu_{fsr} = \text{frequency difference between modes} \)

Note that the subscript “fsr” means “free spectral range,” used by F. Pedrotti, L. S., Pedrotti, and L. M. Pedrotti [6]. The gain bandwidth is a property of the gain medium whereas the free spectral range is a property of the geometry of the laser cavity. Both terms are clearly important to achieve the highest number of modes. The relation between the length of the laser cavity and the free spectral range is [6]
\[ \nu_{fsr} = \frac{c}{2L} \]  
where \( c \) = speed of light in vacuum, and \( L \) = cavity length

If we assume that the small signal gain coefficient is twice the threshold gain coefficient for a particular gain medium, then we can also relate the pulse duration to the gain bandwidth by the following approximation [6].

\[ \Delta t_p \approx \frac{1}{\Delta \nu} \]  
where \( \Delta t_p \) = pulse duration measured at full width half maximum (FWHM)

Please note that the notation for Equations 1 through 3 vary by author.

Better estimates of the pulse width for particular pulse shapes and characteristics are given by Diels and Rudolph in [7]. For a hyperbolic secant pulse shape of the electric field, such as the approximate shape of our pulses, we have

\[ \Delta t_p \geq \frac{0.315}{\Delta \nu_p} \]  
where the equality holds in the absence of chirp (described later)

The intensity profile of one of our pulses, as shown by an intensity autocorrelation, is shown in Figure 4. This signal gives us the opportunity to determine the pulse width of each of our seed pulses.
The duration of each of our current seed pulses is around 100 fs as measured and analyzed using an autocorrelation technique. Further details are given in Chapter 2.

Voltage vs. time. Each small hash mark ≈ 40 fs.

1.2.3 The Kerr Effect

For a Ti:Sapphire laser cavity to exhibit mode-locking with narrow pulses, as shown in the pulse train of Figure 3 (mode-locking) and the autocorrelation of Figure 4 (narrow pulse), several factors must be considered. One factor that must be accounted for is the Optical Kerr Effect (or AC Kerr Effect). This effect causes an intensity-dependent refractive index (IDRI) where higher intensities of incident light cause greater indices of refraction [8]. In general, the intensity-dependent refractive index can be described by [9]

\[ n = n_0 + n_2 I \]  

where \( n_0 \) is the low intensity index, \( n_2 \) is the nonlinear index, and \( I \) is the intensity of the pump beam.

The nonlinear index of refraction, \( n_2 \), has values of \( 10^{-16} \) cm\(^2\)/W to \( 10^{-14} \) cm\(^2\)/W for transparent crystals and glasses [10].

The Optical Kerr Effect causes self-focusing in the spatial domain and self-phase modulation (SPM) in the temporal domain. In the spatial domain, consider a pump beam with Gaussian intensity profile, as shown in Figure 5. As the pump beam enters the Kerr medium (Ti:Sapphire in our case), the intensity-dependent refractive index causes more
bending of the light toward the optical axis where the intensity of the pump is the highest. This effect is therefore also called Kerr Lensing.

Figure 5: Kerr Lensing with Hard Aperture

Kerr Lensing, in concert with an aperture, causes mode-locking, and therefore pulses. The lensing will take place regardless of the hard aperture shown in Figure 5. For mode-locking to occur, however, either a “hard aperture” or “soft aperture” is required. For the hard aperture case shown in Figure 5, the reason for mode-locking may be self-evident as certain modes are blocked by a physical barrier, allowing in-phase modes to add, while blocking other modes that would ultimately prohibit the beat phenomenon described earlier. If the Ti:Sapphire crystal is of the appropriate size, the crystal itself can be used as the hard aperture.

Our system, however, looks more like that of Figure 6, dubbed “Soft Aperture” Kerr Lensing. Perhaps a better name would be “self-aperture” as the self-lensing due to the Kerr Effect causes a greater overlap of high intensity waves from the Ti:Sapphire with that of the pump beam. Since the higher intensity fields overlap the pump beam more than the
lower intensity fields, they experience greater gain and eventually predominate to form pulses [11].

Figure 6: Soft Aperture Kerr Lensing.

\[ \text{Green} = \text{pump beam, Blue} = \text{pulsed laser, Red} = \text{cw laser} \]

In either the hard or soft aperture case, Kerr Lensing is a primary feature in the generation of ultrashort pulses using Ti:Sapphire as the lasing medium. Both hard and soft aperture Kerr Lens Mode-locking (KLM) are discussed here [12].

A third means of obtaining mode-locking is by self-deflection of the beam at the face of a Brewster cut Ti:Sapphire crystal. Due to the Kerr effect, higher intensities deflect more. The laser cavity can be aligned for the path taken by the high intensity beam. Then, given a kick from a spring loaded mirror, or by means of saturable absorption, pulses will build and mode-locking will occur. Further details on the operating principals of our system are given in the following chapters.

So far we’ve been discussing the Kerr effect in the spatial domain. In the temporal domain, SPM causes an increase in the spectrum of frequencies that contribute to mode-locked pulses [13]. As discussed in the previous section, the greater the number of waves that contribute to each pulse, the narrower the pulses. However, this does not immediately
lead to the shortest pulses. Although SPM does not greatly change the temporal profile of a pulse while acting alone, it does cause modulation of the phase profile of the spectrum. In a complicated fashion, SPM works in concert with group velocity dispersion to assist in the production of the narrowest pulses [14].

1.2.4 Dispersion (a measure of spreading out)

Dispersion is the temporal analog of diffraction, and describes how light pulses spread in time during propagation through various media. There are two types of dispersion: phase velocity dispersion (PVD), which is also called chromatic dispersion, and group velocity dispersion (GVD). A description of both types, as they relate to pulses, is given by Geoffrey New [15]. In addition to New’s explanation, pulse broadening due to GVD is nicely represented in the next figure, used with permission by Jean-Claude Diels.

Figure 7: Pulse Broadening due to GVD

used with permission by Jean-Claude Diels
In this figure, the original pulse (green) is stretched and chirped, as shown by the multicolored pulse. Of the many frequencies that combine to form a pulse, the bluer light travels slower than the redder light through a particular medium, thereby producing temporal broadening of the pulse. (The idea of chirp will be addressed later.)

Taking a step back, a standard textbook approach in defining phase and group velocity is as follows. When harmonic waves are added together, such as in the laser cavity, the result can be a rapidly varying waveform with respect to a slowly varying waveform. In the following figure, two cosine waves are summed to illustrate a rapidly varying carrier wave, shown in blue, and a slowly varying envelope wave, shown in red. The carrier wave in this figure can be thought of as the superposition of two electric fields.

**Figure 8: Phase and Group Velocities**

![Envelop wave in red, carrier wave in blue.](https://www.desmos.com/calculator)

The blue carrier wave moves with phase velocity, $v_{\text{phase}}$, while the red envelope wave moves with group velocity, $v_{\text{group}}$.

Interestingly, phase and group velocities need not be the same. Moreover, since the transfer of energy from one object to another is governed by the group envelope, it is the group
velocity that is restricted to no more than the speed of light. The phase velocity is not subject to this restriction, and may even occur in opposite direction to that of the group velocity. The mathematical definitions for phase and group velocities for two closely spaced frequencies are [16]

\[
v_{\text{phase}} = \frac{\omega_p}{k_p} = \frac{\omega_1 + \omega_2}{k_1 + k_2} \cong \frac{\omega}{k} \tag{6}
\]

\[
v_{\text{group}} = \frac{\omega_g}{k_g} = \frac{\omega_1 - \omega_2}{k_1 - k_2} \cong \frac{d\omega}{dk} \tag{7}
\]

where \( \omega_1 \cong \omega_2 = \omega \)

\( v_{\text{phase}} = \frac{\omega}{k} \) and \( v_{\text{group}} \cong \frac{d\omega}{dk} \) also hold for the superposition of many waves over a range of frequencies [16]. The next figure shows pulses with an envelope wave of hyperbolic secant profiles and a corresponding cosine carrier wave to come to a closer approximation to the superposition of the electric fields within a mode-locked laser cavity in the production of ultrashort pulses.

Figure 9: Hyperbolic Secant Pulses with Cosine Carrier Wave

Created with DESMOS free online graphing calculator at https://www.desmos.com/calculator
Recall that it is the electric field intensities that are measured by detectors. The intensity profile of the pulses in Figure 9 are shown next in Figure 10.

Figure 10: Hyperbolic Secant Squared Intensity Pulse Train

Let us now go back to the dispersion that occurs in phase and group velocities. Beginning with phase velocity dispersion, and expanding equation (6) to include the index of refraction, we have

$$v_{phase} = \frac{\omega}{k} = \frac{c}{n(\omega)}$$

(8)

where $n(\omega) =$ the frequency dependent index of refraction

For normally dispersive media, the index of refraction increases with increasing frequency, thereby slowing the waves of higher frequencies with respect to waves of lower frequencies. In the laser this introduces a phase difference between varying frequencies that in turn alters the overall superposition of the electric fields. To give an example of phase velocity dispersion you can see, consider a simple prism. As white light propagates through the prism, the frequency dependent index of refraction describes varying paths through Snell’s Law. As light is separated into its constituent frequencies, its spectrum is
evident in the beautiful array of colors. Figure 11 shows phase velocity dispersion, and demonstrates why it’s also called chromatic dispersion.

Figure 11: PVD through a Prism

Photo Credit: D-Kuru/Wikimedia Commons

Laser light is, of course, nearly monochromatic; yet, we still use the language of redder or bluer when referring to wavelengths around the nominal wavelength of the laser. Thus, the dispersion of laser light through a prism still separates the “colors.” Though the dispersion of laser light through a prism is too little to detect with our eyes, it is important in the generation of ultrashort pulses, e.g. when a prism pair is used in the laser cavity. Phase velocity dispersion is commonly described by the instantaneous change in index with respect to frequency or with respect to wavelength, i.e. by \( \frac{dn}{d\omega} \) or \( \frac{dn}{d\lambda} \). \( \frac{dn}{d\lambda} < 0 \) is called ‘normal’ dispersion while \( \frac{dn}{d\lambda} > 0 \) is called ‘anomalous’ dispersion. In terms of frequency, \( \frac{dn}{d\omega} > 0 \) is called ‘normal,’ while \( \frac{dn}{d\omega} < 0 \) is called ‘anomalous.’

Group velocity dispersion describes the variation in speeds between groups of waves traveling through a particular media. This is important for pulses since several waves are summed to create each pulse. If some groups travel faster than others, then the resultant
pulse may spread out or even break apart all together. The definition for group velocity dispersion is the instantaneous rate of change of the inverse group velocity with respect to angular frequency [17]:

$$GVD = \frac{d}{d\omega} \left( \frac{1}{v_{\text{group}}} \right) = \frac{d}{d\omega} \left( \frac{dk}{d\omega} \right) = \frac{d^2k}{d\omega^2}$$

Using $k = \frac{\omega n}{c}$, we can also write the GVD as

$$GVD = \frac{d^2k}{d\omega^2}$$

GVD can also be represented in terms of the nominal wavelength by examining $\frac{dn}{d\omega}$ where $n = n(\lambda)$. Notice, 

$$\frac{dn}{d\omega} = \frac{dn}{d\lambda} \frac{d\lambda}{d\omega} = \frac{dn}{d\lambda} \frac{d\lambda}{d\omega} \left( \frac{2\pi c}{\omega n} \right) = \frac{dn}{d\lambda} \left[ \frac{2\pi c}{n} \frac{d}{d\omega} \left( \frac{1}{\omega} \right) \right] = \frac{dn}{d\lambda} \left[ -\frac{2\pi c}{n\omega^2} \right] = -\frac{\lambda}{\omega} \frac{dn}{d\lambda}$$

The inverse group velocity is then given by

$$\frac{dk}{d\omega} = \frac{1}{c} \left( \omega \frac{dn}{d\omega} + n \right) = \frac{1}{c} \left( \omega \left( -\frac{\lambda}{\omega} \frac{dn}{d\lambda} \right) + n \right) = \frac{1}{c} \left( n - \lambda \frac{dn}{d\lambda} \right) : n = n(\lambda)$$

The derivative of the inverse group velocity with respect to wavelength is

$$\frac{d}{d\lambda} \left[ \frac{1}{c} \left( n - \lambda \frac{dn}{d\lambda} \right) \right] = \frac{1}{c} \left[ \frac{dn}{d\lambda} - \lambda \frac{d^2n}{d\lambda^2} - \frac{dn}{d\lambda} \right] = \frac{-\lambda}{c} \frac{d^2n}{d\lambda^2}$$

Thus, we have a new definition of GVD in terms of wavelength:
\[ GVD_{\lambda} = \frac{-\lambda d^2n}{c \frac{d\lambda}{d^2n}} \]  \hspace{1cm} (11)

This is also called the Dispersion Parameter, typically used in the context of fiber optics:

\[ D_{\lambda} = \frac{-\lambda d^2n}{c \frac{d\lambda}{d^2n}} \]  \hspace{1cm} (12)

GVD is ‘positive’ when \( \frac{d^2n}{d\lambda^2} > 0 (D_{\lambda} < 0) \) and ‘negative’ when \( \frac{d^2n}{d\lambda^2} < 0 (D_{\lambda} > 0) \).

It may be of interest to the reader to specifically relate phase and group velocities.

Group velocity and phase velocity can be related using Equations (6) and (7). By substitution [16],

\[ v_{\text{group}} \approx \frac{d\omega}{dk} = \frac{d}{dk} (k v_{\text{phase}}) = k \frac{d v_{\text{phase}}}{dk} + v_{\text{phase}} \]

\[ v_{\text{group}} = k \frac{d v_{\text{phase}}}{dk} + v_{\text{phase}} \]  \hspace{1cm} (13)

Since \( v_{\text{phase}} = \frac{c}{n(\omega)} \),

\[ \frac{d v_{\text{phase}}}{dk} = \frac{d}{dk} \left( \frac{c}{n} \right) = c \left( -\frac{dn}{dk} \frac{1}{n^2} \right) = -\frac{c \ dn}{n^2 \ dk} \]

Substituting this result into Equation (13) gives

\[ v_{\text{group}} = k \left( -\frac{c \ dn}{n^2 \ dk} \right) + \frac{c}{n} = \frac{c}{n} \left( 1 - \frac{k \ dn}{n \ dk} \right) = v_{\text{phase}} \left( 1 - \frac{k \ dn}{n \ dk} \right) \]

\[ v_{\text{group}} = v_{\text{phase}} \left( 1 - \frac{k \ dn}{n \ dk} \right) \]  \hspace{1cm} (14)

Furthermore, since
\[
\frac{dn}{dk} = \frac{dn}{d\lambda} \frac{d\lambda}{dk}
\]

and

\[
\frac{d\lambda}{dk} = \frac{d}{dk} \left( \frac{2\pi}{k} \right) = -\frac{2\pi}{k^2}
\]

The group velocity is

\[
v_{\text{group}} = v_{\text{phase}} \left( 1 - \frac{k}{n} \left( -\frac{2\pi dn}{k^2 d\lambda} \right) \right) = v_{\text{phase}} \left( 1 + \frac{2\pi dn}{nk d\lambda} \right) = v_{\text{phase}} \left( 1 + \frac{\lambda dn}{n d\lambda} \right)
\]

\[
v_{\text{group}} = v_{\text{phase}} \left( 1 + \frac{\lambda dn}{n d\lambda} \right)
\]

1.3 Chirped Pulse Amplification (CPA)

CPA is an integral feature of the system described in this work, and thereby seems appropriate to introduce here. CPA has become the foundation of high energy ultrashort pulses. The idea, as it is used today, was introduced by Donna Strickland and Gérard Mourou in 1985, and would subsequently earn them the Nobel Prize in Physics in 2018 (also shared with Arthur Ashkin for an unrelated development) [18]. The paper that describes their original work can be found here [1]. T. Damm, et al. also contributed to our knowledge of the amplification of chirped pulses, publishing a work in the earlier part of the same year [19]. The difference between the work of Strickland and Mourou, and Damm, et al. was a matter of how and when to amplify pulses. The phenomenal development introduced by Strickland and Mourou was the addition of energy to a stretched pulse, allowing for much greater amplification while avoiding material damage to optical equipment. Strickland and Mourou attribute the idea to a similar technique successfully applied to radar. In their 1985 work, they report output pulses of 2 ps duration.
having 1 mJ of energy. This corresponds to a peak power per pulse of roughly 440 MW (440 x 10^{12} W). Since 1985 this has paved the way for ever increasing powers packed into ultrashort pulses. Peak powers have now reached the petawatt level (10^{15} W), with plans to reach the exawatt (10^{18} W) and even zettawatt regime (10^{21} W) [20]. This much power packed into a very small area will be used to explore the very nature of the physical universe as we know it, such as the production of electron-positron pairs from vacuum. Further applications of this frontier are listed in the next section.

As the name implies, the purpose of CPA is to amplify pulses. More specifically, CPA is used to prevent damage to equipment while getting the benefit of pulses with very high peak powers. Other benefits include the avoidance of self-phase modulation and gain saturation in downstream Ti:Sapphire crystals used for amplification. (Self-phase modulation is further discussed later in this work.) In CPA, the amplification sequence is (1) create pulses, (2) stretch pulses before amplification, (3) amplify pulses, then (4) recompress pulses. Michael Perry of Lawrence Livermore National Laboratory gave an overview of CPA that well illustrates the overall concept [21]. The schematic shows an original pulse, pulse stretching, amplification, recompression, and a high energy pulse as the result of the process.
Figure 12: Chirped Pulse Amplification Schematic

Pulses intended to be amplified are initially created by mode-locking the Ti:Sapphire oscillator. (Henceforth, I will call these “seed” pulses.) Since mode-locking is a central feature of the seed pulses, I refer you to a general description of mode-locking in this lab report [22]. Our system uses Kerr lens mode-locking (KLM) via the Ti:Sapphire crystal medium. This phenomenon was described by Spence, Kean, and Sibbett [23], and later by T. Brabec, et al. [12].

After the seed pulses are created, they are stretched. In the previous figure, stretching is accomplished by the first pair of gratings shown in the upper-right. The effect of the grating pair is to cause the bluer frequencies to travel a farther distance than the redder frequencies, which can also be described as group velocity dispersion (GVD). GVD causes a linear chirp which can be reversed by recompression of the pulse.
Before discussing the amplification phase, I’d like to briefly discuss the meaning of chirp. A pulse is said to be chirped if its instantaneous frequency is *time dependent*. If a pulse’s instantaneous frequency increases with time, the pulse is up-chirped (positive chirp). If the opposite is true, the pulse is down-chirped (negative chirp).

Figure 13: Up-Chirped Pulse

Electric field of a strongly up-chirped pulse, where the instantaneous frequency grows with time.

![Electric field of a strongly up-chirped pulse](https://www.rp-photonics.com/chirp.html)

We can now say that we are giving the pulse a linear up-chirp by changing the GVD instead of saying that we are stretching the pulse. Hence, the name given to the process, *Chirped Pulse Amplification*.

After the stretching phase, pulses are sent to one or more amplifiers. Our system uses two amplifiers, a regenerative amplifier and a multi-pass amplifier, in that order. Each of these amplifiers uses a Ti:Sapphire crystal. Details for these system components are given later in this work. From seed pulses to amplified pulses, our system produces an energy amplification factor (output/input) of about 7 million.

After amplification, pulses are recompressed using another grating pair, as shown at the bottom-left of Figure 12. Recompression of a pulse causes down-chirp (hopefully back to the unchirped state), by reversing the GVD introduced by stretching. Since the
compression occurs outside the system that created the pulse, damage to equipment is mitigated, and the desired outcome of a set of high energy pulses is achieved. Although the concept is simple, the optical elements and electronics involved can be somewhat complicated, and a bit touchy.

1.4 Applications of CPA

Chirped Pulse Amplification for optics was introduced in 1985, as previously mentioned [1]. Since this time, high energy pulses have been (or may be) used in a variety of applications, e.g. to precisely cut stainless steel, high density graphite, and a transparent polymer plastic [25], cutting the cornea of the eye for intrastromal surgery [26], creating “soft” X-rays and the potential for attosecond X-ray pulses, precise cutting (and perhaps dismantling) of high explosive materials, precise imaging [27], generating white light in glass, laser induced lightening (as shown by the engraved plate on the door to our lab) [28], remote sensing of the atmosphere [29], study of the nature of energy and matter [30], and more. Of particular interest is the Extreme Light Infrastructure in Europe, led by none other than Gérard Mourou [31]. Many of the technologies required for these applications were built on the platform and understanding of the mode-locked Ti:Sapphire laser.
CHAPTER 2 SYSTEM, OPERATING PRINCIPLES, & RESULTS

2.1 System Overview Flowchart

The figure below shows major systematic components with brief descriptions of some of the important elements within each system. Details for each system component are given in the following sections.

Figure 14: System Flow Chart

Seed Pulse Generation
- Ti:Sapphire Oscillator, including
- Continuous wave optical pump at 532 nm and approximately 2.5 W
- Multiple quantum well to encourage stable mode-locking of ultrashort pulses
- Prism pair to control group velocity dispersion

Stretcher
- Faraday Isolator, in coordination with polarizing beam splitters, to allow seed pulses into the stretcher, block stretched pulses from going back to their source, and to send stretched pulses to the regenerative amplifier
- Single diffraction grating operating as two using mirrors and a large lens

Regenerative Amplifier
- Thin film polarizer, quarter wave plate, and Pockels Cell with corresponding electronic controls, to select single pulses for amplification
- Ti:Sapp laser cavity as the amplifying apparatus
- Pulsed optical pump at 532 nm, approximately 1 W into the amplifier, to energize the Ti:Sapp crystal, which in turn gives up its energy to an injected pulse

Multi-pass Amplifier
- The same Faraday Isolator acting as a switch to allow stage one amplified pulses into this amplifier for second stage amplification
- Ti:Sapp crystal as the amplifying medium
- Uses the same pulsed pump as the regenerative amplifier, pumped with approximately 2.5 W power
- Total pump power for regenerative and multi-pass amplifier is approximately 3.5 W

Compressor
- Two diffraction gratings used with a mirror to recompress pulses, thereby achieving high energy ultrashort pulses

Targets
- Glass (BK7) to demonstrate supercontinuum generation
- Water to demonstrate supercontinuum generation
- Kerr Cell, for physical exploration
Figure 15: Experimental Set-Up
2.2 Mode-locked Ti:Sapphire Laser: The Seed

Our system begins with the generation of ultrashort pulses. This can be accomplished by exploiting the Kerr effect in conjunction with an appropriate tuning of the group velocity dispersion inherent in the excitation of the Ti:Sapphire non-linear medium and also caused by transit through other intracavity elements, including the air. This tuning is accomplished using an intracavity prism pair. Our system also employs a multiple quantum well for pulse stabilization and to cause self-starting KLM. (Many systems simply use a spring loaded mirror or piezo device for the “kick” needed for KLM to begin.)

The essential elements of our set-up is shown in the figure below. The flat mirror between the prisms is unnecessary to generate ultrashort pulses, but necessary to fit all the equipment on our optical bench, and coincidentally also fits nicely on this page. In the following sections I’ll explain the physical reason for each element, following the path of the input beam.

Figure 16: Mode-locked Ti:Sapphire Laser Cavity (Zoomed-in on the Crystal)

Not to scale
Standard z-cavity for Ti:Sapphire oscillators. Crystal fluorescence is shown along with prisms for GVD compensation.
2.2.1 Pump Source

We begin with a continuous wave optical input of 532 nm at approximately 2.5 Watts. Although Ti:Sapphire has an absorption peak at the pump wavelength of 490 nm, 532 nm is used because it is a wavelength that is efficiently obtainable with solid state diode lasers while only inducing a small absorption loss relative to the peak [32]. Specifically, our pump is the Millennia V Diode-Pumped, CW Visible Laser made by Spectra Physics. This pump uses a Nd:YVO₄ lasing medium, frequency doubled to provide the 532 nm output. The output of the pump is s-polarized. This polarization state is changed to p-polarized using a periscope before entering the Ti:Sapphire medium.

2.2.2 Ti:Sapphire Lasing Medium

The pump light (now p-polarized) is focused into the Ti:Sapphire crystal using a positive lens. The Ti:Sapphire crystal is Brewster cut to minimize reflection losses that would otherwise occur in a right-angle cut crystal. This cut also helps avoid the Fabry-Pérot effect. As the pump light propagates through the crystal, Kerr lensing occurs and red fluorescence can be seen in the absence of lasing, i.e. before the cavity is fully developed.
The crystal fluorescence can be conveniently used to help align cavity elements to achieve lasing. In specific, I overlapped the reflections from the curved mirrors surrounding the Ti:Sapphire crystal to bring the laser into near alignment.

The physical considerations for this medium are the (gain) bandwidth, self-phase modulation (SPM), and group velocity dispersion (GVD).

The spectral bandwidth of Ti:Sapphire can be as great as 63 THz (using 5 fs in Equation (4)). This is the current performance limit of the mode-locked Ti:Sapphire laser [3]. Additional information about the Ti:Sapphire crystal can be found here [32]. The spectral bandwidth is some fraction of the gain bandwidth. It is desirable, therefore, to use as much of the gain bandwidth as possible, by design.

Self-phase modulation in the Ti:Sapphire medium is an important effect and is due to an intensity-dependent index of refraction. To consider this effect, let’s follow the electric field through the crystal using $E = E_0 e^{i(\omega t - k z)}$. Due to the Kerr Effect, the index of refraction within the crystal is $n = n_0 + n_2 I$ (see Equation (5)). Then, as the pulse propagates, $k = \frac{\omega n}{c} = \frac{\omega}{c} (n_0 + n_2 I(t)) = \frac{\omega n_0}{c} + \frac{\omega n_2 I(t)}{c}$. Note the introduced time dependence on the intensity. Plugging this back into our electric field description gives

$$E = E_0 e^{i(\omega t - (\frac{\omega n_0}{c} + \frac{\omega n_2 I(t)}{c}) z)}$$

$$= E_0 e^{-\frac{(\omega n_0)}{c} z} e^{i(\omega t - (\frac{\omega n_2 I(t)}{c}) z)}$$

$$= E_0 e^{-\frac{(\omega n_0)}{c} z} e^{i\omega (t - (\frac{n_2 I(t)}{c}) z)}$$.

By including the time independent term in the electric field amplitude, we have

$$E = E_0' e^{i(\omega t - (\frac{\omega n_2}{c}) I(t))}$$

$$= E_0' e^{i(\omega t + \phi(t))}.$$
The time dependent phase, due to the Kerr nonlinearity, is then

$$\phi(z, t) = -\frac{\omega n_2 z}{c} I(t)$$

(16)

where \( I(t) = \text{time dependent intensity} \)

This description of the phase is also given by New [13].

Taking the first time derivative of the phase gives the frequency shift,

$$\frac{\partial \phi}{\partial t} = -\frac{\omega n_2 z}{c} \frac{\partial I}{\partial t}$$

(17)

The second time derivative of the phase gives the chirp (a description of how the frequency changes across the temporal pulse profile),

$$\frac{d^2 \phi}{dt^2} = \frac{d\omega}{dt} = -\frac{\omega n_2 z}{c} \frac{\partial^2 I}{\partial t^2}$$

(18)

To see this graphically, and somewhat generically, let’s choose a Gaussian intensity profile, e.g. let \( I = e^{-t^2} \). Then, \( E = E'_0 e^{i(\omega t - \frac{\omega n_2 z}{c} e^{-t^2})} \). The envelope of the electric field should be proportional to the square root of the intensity profile. Therefore, let \( E'_0 = \sqrt{e^{-t^2}} \).

Furthermore, by including the constraint of the envelope, and considering the real part of the complex field, we have

$$Re\{E\} = \sqrt{e^{-t^2}} \cos \left( \omega t - \left( \frac{\omega n_2 z}{c} \right) e^{-t^2} \right)$$

(19)

Real Electric Field Proportional to Gaussian Intensity Profile
The real electric field in Equation (19) can be modeled by \( E(t) = \sqrt{e^{-t^2}} \cos(at - be^{-t^2}) \).

Using simple to graph coefficients of \( a = 20 \) and \( b = 12.52 \), the idea of what’s happening to a pulse under self-phase modulation can be shown and is given in the next figure.

Figure 18: Self-Phase Modulation of a Gaussian Pulse

Red solid curve: \( E(t) = \sqrt{e^{-t^2}} \cos(20t - 12.52e^{-t^2}) \)

Blue dashed curve: \( E'_0(t) = \pm \sqrt{e^{-t^2}} \) (Gaussian pulse envelope)

Created with DESMOS free online graphing calculator at https://www.desmos.com/calculator
Using a reduced amplitude equation for the frequency shift, the graphs of the electric field and frequency shift are shown together in the next figure to highlight the relationship.

Figure 19: SPM and Frequency Shift

![Graph showing electric field and frequency shift](https://www.desmos.com/calculator)

Given our example, the frequency shift is \( \frac{\partial \phi}{\partial t} = 25.04te^{-t^2} \), and the chirp, or second phase derivative is, \( \frac{\partial^2 \phi}{\partial t^2} = 25.04e^{-t^2}(-2t^2 + 1) \). The next figure shows the phase, frequency shift, and chirp. Notice that the phase is proportional to the negative of the intensity profile.
Phase, frequency shift, and chirp of self-phase modulated Gaussian pulse

Green solid curve (phase): \( \phi = -12.52e^{-t^2} \propto -I(t) \)

Black dotted curve (frequency shift): \( \frac{\partial \phi}{\partial t} = 25.04te^{-t^2} \)

Blue solid curve (chirp): \( \frac{\partial^2 \phi}{\partial t^2} = 25.04e^{-t^2}(-2t^2 + 1) \)

The region of up-chirp roughly corresponds to the half-maximum intensity points of the intensity profile, and is greater than the combined down-chirps (for \( n_2 > 0 \)) [13]. One of the beneficial effects of SPM is the increase in frequencies that contribute to the overall electric field of the cavity, thereby reducing pulse duration. The next few graphs show an example of a hyperbolic secant electric field using the same reasoning, with \( I = \text{sech}^2(t) \).
Self-phase modulated hyperbolic secant pulse
Red solid curve: $E(t) = \text{sech}(t) \cos(20t - 12.52 \text{sech}^2(t))$
Blue dashed curve: $E'_0(t) = \pm \text{sech}(t)$

Created with DESMOS free online graphing calculator at https://www.desmos.com/calculator
Figure 22: Phase, Freq. Shift, & Chirp for Hyperbolic Secant E-Field

Phase, frequency shift, and chirp of self-phase modulated hyperbolic secant pulse

Green solid curve (phase): $\phi = -12.52 \text{sech}^2(t) \propto -I(t)$

Black dotted curve (frequency shift): $\frac{\partial \phi}{\partial t} = 25.04 \tanh(t) \text{sech}^2(t)$

Blue solid curve (chirp): $\frac{\partial^2 \phi}{\partial t^2} = 25.04 \text{sech}^2(t) (1 - 2 \tanh^2(t))$

Created with DESMOS free online graphing calculator at [https://www.desmos.com/calculator](https://www.desmos.com/calculator)

GVD is also present in the Ti:Sapphire crystal, is positive, and contributes to pulse broadening. However, this effect depends on the length of the crystal and can be significantly reduced. Nevertheless, it is part of the overall GVD of the laser cavity. If the GVD of the cavity can be held in precise balance with SPM, then the train of pulses shown in Figure 3 (Chapter 1) is formed. These pulses are solitons. The action of self-phase
modulation with dispersion to produce solitons is described by New [33]. GVD is carefully controlled for this experiments using a prism pair. This is further discussed in section 2.2.5.

2.2.3 Curved Mirrors (or Focusing Mirrors)

The next element is the first curved mirror which redirects the Ti:Sapphire output and the remainder of the pump beam towards the multiple quantum well, then upon back reflection, focuses the light into the Ti:Sapphire crystal. This light is intercepted by a second curved mirror, then redirected toward the prism pair and cavity end. The two mirrors form a confocal arrangement. These mirrors are typically spherical and therefore induce spherical aberration. In the paraxial approximation, however, this is not an issue for this experiment.

Astigmatism also occurs due to off axis striking of the mirrors that form the z-cavity configuration shown in Figures 15 and 16. Astigmatism compensation of the confocal arrangement formed by the two mirrors and Brewster cut Ti:Sapphire crystal is given by [34] (with minor correction). The analysis used for this formulation assumes Gaussian beam propagation. With this assumption, an equation that yields a good approximation for astigmatism compensation is given below Figure 23.

Figure 23: Astigmatism Compensation

\[ \theta = 2\alpha \]

Physical arrangement for the following astigmatism compensation equation. Each curved mirror has radius of curvature, R.
\[
\cos \alpha = \frac{-L_c(n^2 - 1)\sqrt{n^2 + 1} \pm \sqrt{4R^2n^8 + L_c^2(n^2 + 1)(n^2 - 1)^2}}{2Rn^4}
\] (20)

\[R = \text{radius of curvature (assuming both mirrors are the same)}\]
\[n = \text{index of refraction of the crystal}\]
\[L_c = \text{length of the Brewster cut crystal}\]

In addition to the purpose of focusing, these mirrors are used for the creation of the laser cavity in such a way as to provide the ability to add the prism pair shown in Figure 15. The path of light traversing both curved mirrors forms a Z or inverted z-shape, and is therefore called a “z-cavity” once all necessary elements are in place. The z-cavity clearly provides a way to add intracavity elements without disturbing the critical focusing of light into the Ti:Sapphire crystal. The position of the focusing mirrors relative to the Ti:Sapphire crystal is an important feature of KLM, and is described by G. Cerullo et al. for a cavity similar to ours [2].

Figure 24: Illuminated Ti:Sapphire between Focusing Mirrors

2.2.4 Lens and Multiple Quantum Well (MQW)

The positive lens after the first curved mirror serves the purpose of collecting all available Ti:Sapphire fluorescence to be handled by the MQW. The MQW is comprised of
two 9 nm thick layers of GaAs (absorbing material) separated by layers of non-absorbing GaAsAl grown on top of a mirrors structure. This particular MQW was created at the University of New Mexico’s Center for High Technology Materials (CHTM).

The function of the MQW is saturable absorption. Higher intensities than the saturation intensity of the absorber are transparent to the MQW. In this way, the MQW is used for passive mode-locking. However, the ultrashort durations of the final mode-locked pulses are due to the nonlinear Kerr effect, not due to the MQW. Thus, the mode-locking operation of the MQW serves to provide high intensity pulses to the Ti:Sapphire crystal, which then takes over in the production of ultrashort pulses via the Kerr effect with a soft aperture. Self-starting Kerr-lens Mode-locking (KLM) and cavity stabilization are also positive results of including the MQW in the system. The downside (if there is one) is that the pulse duration may be limited.

2.2.5 Prism Pair

Prism pairs can be used to compensate for the overall GVD occurring in the laser cavity, providing a method of employing either positive or negative GVD [35]. Since both positive and negative GVD are accessible, the prism pair can be used to sensitively tune the GVD of the cavity, thereby stabilizing pulse creation and propagation. In our cavity, negative dispersion from the prism pair compensates for positive dispersion from the Ti:Sapphire crystal and the air. The prism pair also provides chirp compensation in the creation of optical solitons. However, it should be noted that the exact chirp compensation is complicated since chirp caused by SPM is different than chirp caused by GVD. Chirp due to SPM is only positive in the central region of the intensity of the pulse that creates the modulation, whereas chirp due to GVD is linear across the pulse profile [13]. This
means that linear chirp due to propagation through the air, for example, is cleanly compensated by the prism pair, but chirp due to SPM is only partially compensated by the prism pair. However, there may be a way to more wholly compensate for the chirp due to SPM with the prism pair using an appropriate adjustment of vertical prism angles. This method is suggested in explanation of the accidental creation of approximately 55 fs pulses using this laser cavity, at which time the prisms where tilted approximately 30 degrees down with respect to the plane of the optical bench. A general theory for dispersion through multiple prisms is given by Duarte [36]. Please also see [35] for a description of group velocity dispersion in prism pairs.

2.2.6 Cavity Ends

One of the cavity ends is the multiple quantum well previously mentioned in this chapter. The other cavity end is the output coupler shown in Figure 15. This output coupler is a dielectric mirror that is 98% reflective.

2.2.7 Pulse Train Measurement

The pulse train shown in Figure 24 below is detected by a fast photodiode. In this experiment we’ve used two photodiodes at different times. For some time, we used a Hamamatsu S2381 Si APD (avalanche photodiode) driven at about +100 V. Currently we are using a Thor Labs FDS015 Si Photodiode driven at about +150 V. The output of the detector is viewed on a Tektronix 7854 Oscilloscope. Each pulse in the train is approximately 13 ns apart corresponding to a repetition rate of about 80 Mhz. These photodiodes are fast enough to clearly resolve the time between output pulses, but are not fast enough to resolve the duration of an individual pulse. Therefore, the pulse width of an individual pulse shown in Figure 25 is essentially the resolution limit of the detector.
2.2.8 Autocorrelation

Since photodiodes are not fast enough to measure pulse durations in the fs regime, the intensity autocorrelation is employed. The mathematics and principles of autocorrelations are described by Diels and Rudolph [37]. Figure 26 shows an intensity autocorrelation trace used for measurement. However, the time scale shown in the upper right of the figure is misleading, and cannot be used to determine the actual pulse width. If the scale shown was used, one would believe the pulse duration is 180 μs, measured at the FWHM! Instead, the apparatus used to create the autocorrelation (essentially a Michelson interferometer) is used to determine and apply an appropriate time scale, and will be discussed in the following paragraphs.
Figure 26: Intensity Autocorrelation Measurement (Misleading Time Scale)

Voltage vs. time. Each small hash mark ≈ 40 fs.

The autocorrelation trace, shown in Figure 26, is created with a Tektronix 7313 Oscilloscope and Hamamatsu Photomultiplier Tube, driven at about 300 V. The photomultiplier tube is shown on the left hand side of Figure 27 (with yellow tape).

Figure 27: Autocorrelation Apparatus

PMT, thin KDP, and positive lens shown on the left, BS in the center, corner cube with micrometer control to the right, and corner cube mounted on speaker at the bottom. The speaker moves at 24 Hz.
Figure 28: SHG for Autocorrelation

Second harmonic generation is shown as the blue dot on the business card.

Figure 28 shows a close up view of the photomultiplier tube blocked by a screen (business card) to show the second harmonic signal detected by the tube (blue dot on the business card in the shaded region). The second harmonic signal is generated in a thin slice of KDP. The thickness (or thinness) of the crystal is a critical feature in second harmonic generation [38]. The figure below shows a diagram of the autocorrelation apparatus.

Figure 29: Autocorrelation Apparatus Diagram
There are a couple of points to make about how the autocorrelation set-up creates the image shown in Figure 18, and how the image is used to measure the pulse duration of any of the pulses in the pulse train emanating from the Ti:Sapphire oscillator. Let’s begin with determining the pulse duration. It was previously mentioned that the time scale shown in the upper right of Figure 25 is misleading. To obtain a confident measure of the pulse duration, we used the micrometer translation stage to position the pulse peak on the left and right hand sides of the oscilloscope screen, as shown in Figure 29. By taking the difference between the lengths shown on the translation stage dial, we were able to appropriately scale the hash marks on the oscilloscope screen. If we call this difference in length $\Delta L$, then the mathematics to determine the pulse duration are:

$$FWHM_{autocorrelation} = \frac{h}{50} \Delta t$$  \hspace{1cm} (21)

where $h = \text{the number of hash marks denoting the FWHM, out of 50 total}$,

and $\Delta t = \frac{2 \Delta L}{c} = \text{the round trip time of light traversing length } \Delta L$

Assuming a hyperbolic secant squared pulse shape [39],

$$FWHM_{pulse} = \frac{FWHM_{autocorrelation}}{1.543}$$  \hspace{1cm} (22)

For the image shown in Figure 18, $\Delta L = 0.30 \text{ mm}$, and $h = 4$. This gives a pulse duration of

$$FWHM_{pulse} = \left(\frac{1}{1.543}\right) \left(\frac{4}{50}\right) \left(\frac{2 \times 0.30 \text{ mm}}{0.3 \text{ mm/ps}}\right) \approx 0.104 \text{ ps} = 104 \text{ fs}$$

Pulses as short as 55 fs were measured using this system and measurement technique, when the intracavity prisms had significant down angles with respect to the optical bench. Achieving this result again should be an object of future study. Figure 30 shows the
oscilloscope screen with the autocorrelation signal at the left and right hand side to illustrate the points at which micrometer measurements were taken. The reading on the micrometer dial was 8.79 mm for one side, and 8.49 mm for the other side, giving a length difference of 0.30 mm. (The micrometer dial is sufficiently marked to determine whether hundredths of a mm should be rounded up or down.)

Figure 30: Autocorrelation Time-Scale Determination

Voltage vs. time. Each small hash mark ≈ 40 fs.

To produce the autocorrelation image, a pulse arrives at the thin KDP from the stationary arm shown in Figure 29, while another pulse arrives periodically at the same place and time from the moving arm (corner cube with speaker).

Figure 31: Pulses Forming Autocorrelation Signal

Action of pulses from each arm of the autocorrelator coming together in the SHG signal.
The trace shown in Figure 26 is proportional to the amplitude of the shared area of the two pulses in Figure 31 as they pass each other, as illustrated in Figure 32.

Figure 32: Shared Area of Passing Pulses

![Interaction of pulses from each arm of the autocorrelator coming together in the SHG signal.](image)

2.3 Stretcher

Figure 33: Hand Sketch of Actual Stretcher

![Pulses are stretched ≈1000 times via four strikes on the grating shown on the right of the hand sketch, together with an appropriate combination of the other elements shown.](image)

Figure 34: Stretcher Diagram

![A pair of gratings disperses the spectrum and stretches the pulse by a factor of a thousand.](image)

The actual stretcher in use for this experiment is sketched in Figure 33, with an attempt to show the change in height and width of the light beam as it makes several passes. Distances between elements are shown in Figure 15 at the beginning of the chapter. At the output, the light beam is spatially recompressed, though significantly stretched in time, to send to the regenerative amplifier. Figure 34 shows a simple diagram of the concept. For our stretcher, we made use of a mirror to use one grating to accomplish the task of the two gratings shown in Figure 34. The labels in Figure 33 indicate the strike number of the beam, e.g. the first mirror the beam encounters is labeled M1, 9 because the light strikes this mirror first, and 9th in order of all mirror strikes. The grating is labeled G1, 2, 3, 4 to indicate that the light strikes it four times.

The train of pulses from the Ti:Sapphire oscillator feeds into the stretcher, through the Faraday Isolator switching system, as shown in Figure 15, and described at the end of this section. As mentioned in Chapter 1, the purpose for the stretcher is to add energy to the stretched pulses during the amplification phase without the worry of damage to optical equipment downstream, while also avoiding unwanted self-phase modulation and self-focusing. The unamplified stretched pulse intensity is approximately 1/10 of the seed pulse intensity.

Considering the temporal element, the duration of the stretched pulse in Figure 35 is instrumentally accurate, whereas the duration of the seed pulse in Figure 36 is completely inaccurate, and simply represents the resolution limit of the detector. This actually presents a nice way to be sure the detector is accurately measuring the stretched pulse. Since the seed pulses are short, ~100 fs, the photodiode simply can’t keep up, and the reading on the oscilloscope must therefore be resolution limited. If the stretched pulse is longer than the
resolution limit, we can say that the detector is indeed measuring the relative duration of the stretched pulse (relative to the actual pulse duration, determined by Fourier analysis).

If the stretched pulse and the seed pulse appear to have the same duration, then we cannot trust the equipment.

Figure 35: Stretched Pulse

Figure 36: Seed Pulse (Resolution Limit of Detector)

The detection system used to obtain the images in Figures 35 and 36 consist of a fast photodiode, Albis model PQW30A-S, driven at +10 V, with collimator attachment model CFC-8X-C, fiber connected to a Tektronix S-4 Sampling Head, viewed on a Tektronix 7854 Oscilloscope.

Final determination of the durations of the stretched and un-stretched pulses requires special equipment and analysis. The duration of an un-stretched seed pulse was described in the previous section (using autocorrelation). Now I present the determination of the duration of a stretched pulse. The image of the stretched pulse in Figure 35 is the convolution of the “delta” function response of the detector and the real stretched pulse, as shown on the next page.
\[ f(t) = d(t) \ast g(t) \]

where \( f(t) \) = the measured stretched pulse, 
\( d(t) \) = the delta response of the detector (or impulse response), 
and \( g(t) \) = the real stretched pulse

Determination of the real stretched pulse can be accomplished using Fourier transforms since

\[ \mathcal{F}\{f(t)\} = \mathcal{F}\{d(t) \ast g(t)\} = \mathcal{F}\{d(t)\} \cdot \mathcal{F}\{g(t)\} \]

Then,

\[ g(t) = \mathcal{F}^{-1}\left\{ \frac{1}{\mathcal{F}\{d(t)\} \cdot \mathcal{F}\{f(t)\}} \right\} \]

(The procedures used for obtaining the real stretched pulse are given in Appendix A. The real stretched pulse duration was determined to be approximately 80 ps. See next page.)

The most intuitive approach to determining the duration of the stretched pulse, \( g(t) \), is to simply apply the Discrete Fourier Transform (DFT) to a set of numeric data corresponding to each measured pulse, \( f(t) \), and \( d(t) \), then divide the transform of the measured stretched pulse by that of the measured delta response of the system, and take the inverse DFT as the physical stretched pulse, \( g(t) \). Indeed, this was my first approach. However, this produced only what looked like noise. This may be due to the small number of points describing each pulse (around 60 each), such that, only a small number of these points were in vertical alignment with each other (which I assume distorts the division of the spectrums). For data samples that are dense in points, I assume this problem would be mitigated.
Issues with a direct determination of the stretched pulse duration led me to explore ways to approximate using smooth functions and analytical techniques. The next few pages give examples of these techniques.

Definitions for the following examples are:

For \( h(t) \) = some time-domain function, and \( H(\omega) \) = the frequency-domain spectrum of \( h(t) \), the Fourier transform and its inverse, are defined as

\[
H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-i\omega t} dt = \mathcal{F}\{h(t)\}
\]

\[
h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{i\omega t} d\omega = \mathcal{F}^{-1}\{H(\omega)\}
\]

**Example 1:** Approximate the character of the real profile of the stretched pulse when the system response to an impulse is given as a shifted delta function. This represents the idealized case where the measurement system can take measurements on a time scale much shorter than the pulse being measured.

\[
let \ d(t) \approx \delta(t - a): a \neq 0
\]

\[
\mathcal{F}\{d(t)\} = \int_{-\infty}^{\infty} \delta(t - a) e^{-i\omega t} dt = e^{-i\omega a}
\]

So,

\[
g(t) = \mathcal{F}^{-1}\left\{\frac{\mathcal{F}\{f(t)\}}{e^{-i\omega a}}\right\} = e^{i\omega a} \mathcal{F}^{-1}\{\mathcal{F}\{f(t)\}\} = e^{i\omega a} f(t)
\]

\[
Re\{g(t)\} = \cos(\omega a) f(t)
\]

If \( f(t) = I_0 \, \text{sech}^2(1.76t/\tau_p) \), then

\[
Re\{g(t)\} = I_0 \cos(\omega a) \, \text{sech}^2(1.76t/\tau_p)
\] (23)
At $a = 0$ this demonstrates the perfect replication of the real pulse by the system. If $a \neq 0$, the real pulse would be a simple amplitude variation of the system output.

**Example 2:** Approximate the character of the real profile of the stretched pulse when the system response to an impulse is given as a Gaussian approximation of the delta function.

$$d(t) \approx \lim_{\epsilon \to 0} e^{-4\ln(2)t^2/\epsilon^2}$$

Since Gaussians are prevalent in the description of pulses, it may be instructive to demonstrate the Fourier transform of a generic Gaussian of the form $f(t) = ae^{-bt^2}$. This is shown in Appendix B, with the result that

$$F(\omega) = a \sqrt{\frac{\pi}{b}} e^{-\omega^2/4b}$$

(24)  

*Fourier Transform of the Gaussian, $f(t) = ae^{-bt^2}$*

This presents the interesting fact that the Fourier transform of a Gaussian is a Gaussian.

Using this result with our original choice of $d(t) \approx \lim_{\epsilon \to 0} e^{-4\ln(2)t^2/\epsilon^2}$ gives

$$F\{d(t)\} = \lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} e^{-4\ln(2)t^2/\epsilon^2} e^{-i\omega t} dt$$

$$= \lim_{\epsilon \to 0} \left[ \frac{\epsilon}{2} \sqrt{\frac{\pi}{\ln(2)}} e^{-\frac{(\epsilon \omega)^2}{16\ln(2)}} \right]$$

$$\approx \lim_{\epsilon \to 0} [0.738 \epsilon \cdot e^{-0.09(\epsilon \omega)^2}]$$

The stretched pulse profile is then

$$g(t) = F^{-1}\left\{ \frac{F\{f(t)\}}{\lim_{\epsilon \to 0} [0.738 \epsilon \cdot e^{-0.09(\epsilon \omega)^2}]} \right\}$$
For simplicity, let’s assume a Gaussian pulse shape for the measured pulse, i.e.

\[ f(t) = e^{-\frac{4\ln(2)t^2}{\tau^2}} \quad \tau = \text{pulse duration at the FWHM} \]

Then,

\[ F\{f(t)\} = \frac{\tau}{2} \sqrt{\frac{\pi}{\ln(2)}} e^{-\frac{(\omega\tau)^2}{16\ln(2)}} \]

Plugging this result back into our equation for the stretched pulse gives

\[ g(t) = F^{-1}\left\{ \frac{\tau}{2} \sqrt{\frac{\pi}{\ln(2)}} e^{-\frac{(\omega\tau)^2}{16\ln(2)}} \right\} \]

\[ = F^{-1}\left\{ \lim_{\epsilon \to 0} \left\{ \frac{\tau}{\epsilon} e^{-\frac{[(\tau\omega)^2-(\epsilon\omega)^2]}{16\ln(2)}} \right\} \right\} \]

\[ = \lim_{\epsilon \to 0} \left\{ F^{-1}\left\{ \frac{\tau}{\epsilon} e^{-\frac{\tau^2-\epsilon^2}{16\ln(2)}} \right\} \right\} \]

Using Equation (25), after some algebra and simplification, we have

\[ g(t) = \lim_{\epsilon \to 0} \left\{ \frac{2\tau}{\epsilon} \left( \frac{\tau^2 - \epsilon^2}{\ln(2)} \right)^{-\frac{1}{2}} e^{-\frac{(4\ln(2))}{\tau^2-\epsilon^2}t^2} \right\} \]

Unfortunately, this limit does not exist. However, if we impose the reasonable condition that \(0 < \epsilon < \tau\), then

\[ g(t) = \frac{2\tau}{\epsilon} \left( \frac{\tau^2 - \epsilon^2}{\ln(2)} \right)^{-\frac{1}{2}} e^{-\frac{(4\ln(2))}{\tau^2-\epsilon^2}t^2} \quad : \quad 0 < \epsilon < \tau \]
Comparative graphs of the measured pulse, $f(t)$, and the “real pulse,” $g(t)$, are shown in the next figure for $\epsilon = 0.2\tau$.

**Figure 37: Stretched Pulse Duration Approximation - Example 2**

Green curve (measured): $f(t) = e^{-4\ln(2)t^2}$

Red curve (real): $g(t) \approx 8.497e^{-2.88t^2}$

Created with DESMOS free online graphing calculator at https://www.desmos.com/calculator

In the following figure, the normalized real pulse is compared to the normalized measured pulse to demonstrate how close the duration of the real pulse is to that of the measured pulse when the duration of the system response to an impulse is 20% of the duration of the measured pulse.
Figure 38: Example 2 – Normalized: $\epsilon = 0.2 \tau$

Green curve (normalized measured pulse): $f(t) = e^{-4 \ln(2)t^2}$
Red curve (normalized real pulse): $g(t) \approx e^{-2.88t^2}$

Created with DESMOS free online graphing calculator at https://www.desmos.com/calculator

Next, comparative graphs are given for the case that the duration of the system response to an impulse is 90% of the duration of the measured pulse. In comparing the previous figure to the next figure, note that as the duration of the system response to an impulse is
increasingly small, the duration of the measured pulse is increasingly accurate in its representation of the real pulse, as would be expected from our analysis in the first example.

![Figure 39: Example 2 – Normalized: $\epsilon = 0.9\tau$](image)

Green curve (normalized measured pulse): $f(t) = e^{-4\ln(2)t^2}$

Red curve (normalized real pulse): $g(t) \approx e^{-14.6t^2}$

Created with DESMOS free online graphing calculator at [https://www.desmos.com/calculator](https://www.desmos.com/calculator)

**Example 3:** Approximate the character of the real profile of the stretched pulse when $f(t) = \text{sech}^2(at)$, and the system response is $d(t) = e^{-ct^2}$. This represents the case most like the measured data shown at the beginning of this discussion.

Then,

$$g(t) = \mathcal{F}^{-1}\left\{\frac{\mathcal{F}\{\text{sech}^2(at)\}}{\mathcal{F}\{e^{-ct^2}\}}\right\}$$
The Fourier transform of $\text{sech}^2(at)$ is shown below. The derivation is given in Appendix B. The Fourier transform of the Gaussian, $e^{-ct^2}$, was shown in the previous example. So, given

$$
\mathcal{F}\{\text{sech}^2(at)\} = \int_{-\infty}^{\infty} \text{sech}^2(at) e^{-i\omega t} dt = \frac{\omega \pi}{a^2} \text{csch} \left( \frac{\omega \pi}{2a} \right)
$$

we have,

$$
g(t) = \mathcal{F}^{-1} \left\{ \frac{\omega \pi}{a^2} \frac{\text{csch} \left( \frac{\omega \pi}{2a} \right)}{\sqrt{\frac{\pi}{c} e^{-\frac{\omega^2}{4c}}} \right\}
$$

The inverse Fourier Transform in the above equation is not trivial, and its derivation is shown in Appendix B.4. The inverse transform is

$$
g(t) = -4 \sqrt{\frac{c}{\pi}} \sum_{n=0}^{\infty} (-1)^n e^{-\left(\frac{a^2}{c}\right)n^2} ne^{-2atn}
$$

Relating this form to the full-width, half-maximum (FWHM) definition of the pulse width, we have

$$
a = \frac{1.76}{\tau_p} : \tau_p = \text{pulse width of } \text{sech}^2(at)
$$

and

$$
c = \frac{4 \ln(2)}{\epsilon^2} : \epsilon = \text{pulse width of the Gaussian, } e^{-ct^2}
$$

This gives

$$
g(t) = -\frac{8}{\epsilon} \sqrt{\frac{\ln(2)}{\pi}} \sum_{n=0}^{\infty} (-1)^n e^{-\left(\frac{1.76\epsilon n}{2\tau_p \sqrt{\ln(2)}}\right)^2} \cdot n \cdot e^{-\left(\frac{3.52tn}{\tau_p}\right)}
$$
The reader is encouraged to attempt to find a closed form solution of \( g(t) \). Using a 1001 terms of the sum in Equation 28, the numeric approximation gives the following graph. The left side of the graph clearly cuts off, seemingly before showing the full profile. As the number of terms are increased, only the right side remains, as shown in Figure 40.

Figure 40: Normalized Stretched Pulse Numeric Approximation, 1001 Terms

Figure 41: Normalized Stretched Pulse Numeric Approximation, 10001 Terms
It appears that the sum converges to that shown in the previous figure. Nevertheless, by symmetry, it also seems reasonable to construct the rest of the graph. Reading from the graph of Figure 42, the estimated pulse width of the physical pulse is 80 ps. This is about 84% of the pulse shown on the oscilloscope given approximately 95 ps for the pulse width of the stretched pulse signal on the oscilloscope, and approximately 43 ps for the pulse width of the impulse response of the system.

Figure 42: Estimate of Normalized Physical Stretched Pulse

Figure 43: Pulse Data Estimations by Smooth Functions
This analysis and estimation could be strengthened, perhaps, by using some type of asymmetric pulse to represent the impulse response of the system.

Before discussing the next major element in the overall system, I’d like to turn back to the Faraday Isolator switching system. The operating principles of the Faraday Isolator are given by Pedrotti, Pedrotti, and Pedrotti [40]. The isolator we employ has an attached half-wave plate to generate the polarization changes shown in the next figure, for the purpose of protecting the Ti:Sapphire oscillator from retro pulses, and to redirect light to and from various system components. In the next figure, notice that the isolator rotates the polarization angle by 45° regardless of propagation direction; whereas the half-wave plate rotates the polarization angle by 45°, dependent on propagation direction (as measured from a common origin).
2.4 Regenerative Amplifier

Now that the seed pulses have been stretched, it is safe to add energy. The regenerative amplifier uses a pulsed pump laser, separate from the pump used for the Ti:Sapphire Oscillator, to stimulate yet another Ti:Sapphire crystal. When a pulse from the stretcher arrives at this crystal, it will be significantly amplified, provided that the crystal is stimulated at the proper time. Moreover, the pulse is trapped to cause it to make several
passes, receiving even greater amplification before being released to move on to the next amplifier in the system. Several timing elements are used to create this ballet of timing and amplification. In the following paragraphs, the elements that comprise this amplifier are described.

In our case, the pump for the regenerative amplifier is the Surelite, manufactured by Continuum (in 1998). This laser uses a flash lamp to excite the Nd:YAG medium, and operates most efficiently around 8 Hz, about 1 millionth of our seed repetition rate. The pump is then frequency doubled using DKDP to provide the 532 nm output necessary to excite the Ti:Sapphire and cause amplification of the seed.

Figure 46: Regenerative Amplifier Diagram

Note the Brewster cut of the Ti:Sapphire crystal. Reasons for using this cut are described at the beginning of this chapter. Interaction of the pump and this crystal, together with the other components of the amplifier, form a laser cavity. This cavity can be
conveniently used to ensure good alignment of the components, i.e. if the cavity components are well aligned, lasing will occur when the quarter wave plate and Pockels cell are in proper configuration. Lasing, however, is not the goal of this amplifier. Rather, the goal is for a selected pulse to steal the energy given to the Ti:Sapphire by the pump, thereby becoming amplified.

Considering the path of an incoming pulse, the first element encountered is the thin film polarizer, followed by the quarter wave plate and Pockels cell. The thin film polarizer acts as a gate when set at the proper angle. Reflection occurs for s-polarized light, whereas, p-polarized light passes through and onto the Ti:Sapphire crystal.

Given the double pass of light that occurs by reflection from EM₁, the quarter wave plate can be set to provide a 90° shift in polarization, or a 0° shift in polarization. This is quite convenient when establishing precise alignment of the incoming and outgoing pulse, while also providing a mechanism of fine tuning for the entire amplifier.

The Pockels cell is used as a quarter wave plate given the proper amount of applied voltage. When the voltage is not applied, the cell does not affect the polarization of light. Moreover, the Pockels cell can respond quickly to changes in applied voltage, making it a nice mechanism for fast switching, which is precisely its role in this experiment. When the voltage is applied to the Pockels cell, a pulse is trapped in the regenerative amplifier for the duration of the applied voltage. Currently, the Pockels cell in our experiment is being used at +575 V Bias with +56 V applied voltage. When the voltage is stopped, the amplified pulse is released from the amplifier and allowed to move on to the next amplifier in the system. For a description of the operating principles of the Pockels cell, see [41] and [42].
2.4.1 Pulse Trapping

Trapping the pulse using the quarter wave plate and Pockels cell can be reasoned as follows: When incoming light passes the quarter wave plate, the light is given a 45° shift in polarization, and is now circularly polarized. This passes through the Pockels cell without an applied voltage, strikes the end mirror, passes back through the Pockels cell (still without an applied voltage), then through the quarter wave plate again, further rotating the light by 45° for a total rotation of 90° while simultaneously bringing the polarization state of the light back to linear. This has changed the incoming light from s-polarization to p-polarization, allowing it to pass through the thin film polarizer and into the regenerative amplifier. The pulse then passes through the Ti:Sapphire crystal, which has been energized by the pump, thereby stealing energy and being amplified. While this is happening, we apply voltage to the Pockels cell, causing it to act as a quarter wave plate. This makes the quarter wave plate and Pockels cell act as a half-wave plate when taken together. When the p-polarized pulse comes back through the thin film polarizer, it is rotated by 90° through the quarter wave plate/Pockels cell pair, strikes the end mirror, and is then rotated by 90° again when passing through the pair a second time. This keeps the pulse p-polarized for the duration of the applied voltage to the Pockels cell, thereby trapping the pulse in the amplifier, and allowing it to obtain maximum energy from the energized Ti:Sapphire crystal. When the applied voltage to the Pockels cell is stopped, the pulse then switches back to s-polarization after a double pass through the quarter wave plate/Pockels cell pair, and exits the amplifier by reflection off of the surface of the thin film polarizer.
2.4.2 Timing

Now that the overall mechanism of switching has been described, we can discuss the sensitive timing required to isolate and amplify a single pulse. The block diagram below shows the flow of timing and equipment used to accomplish this task.

Figure 47: Regenerative Amplifier Timing Control Diagram

Timing control is used to choose the optimal time to start and stop the Pockels Cell, thereby achieving maximum gain from the regenerative amplifier.
Figure 48: Relative Timing of Timing Control Device Outputs

![Image of Figure 48]

**Key**
- Blue = “Charge” (6 dB attenuation)
- Yellow = “Lamp” (3 dB attenuation)
- Red = “Trigger” (3 dB attenuation)
- Green = “Q-Switch” (20 dB attenuation). *See previous figure.*

Figure 49: Amplified Pulses within Regenerative Amplifier

![Image of Figure 49]

Figure 48 shows a snapshot of several pulses being amplified in the regenerative amplifier.

Using the pulse generators (as timing elements), a single pulse, within the set of pulses,
can be isolated for much greater amplification. Figure 50 shows an example of single pulse isolation. The scale of the oscilloscope must be changed as we get closer to the isolation of a single pulse as it’s amplitude is orders of magnitude greater than that of the pulses shown in Figure 49. In the amplification process, a trapped pulse makes about 66 passes through the pumped Ti:Sapphire crystal (given that the Pockels cell is activated for about 440 ns, and the cavity length is about 2 meters).

Figure 50: Single Amplified Pulse after Regenerative Amplifier

Table 1: Regenerative Amplifier Timing Sequence

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Approximate Relative Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surelite Flash Lamp Fires</td>
<td>0 s</td>
</tr>
<tr>
<td>Surelite Q-switch Fires</td>
<td>168.40 μs</td>
</tr>
<tr>
<td>Pockels cell START</td>
<td>170.29 μs</td>
</tr>
<tr>
<td>Pockels cell STOP</td>
<td>170.73 μs</td>
</tr>
</tbody>
</table>
2.5 *Multi-pass Amplifier*

After regenerative amplification, the flow of amplified pulses encounters yet another Ti:Sapphire crystal, pumped by the same source that pumps the regenerative amplifier.

Figure 51: Bow Tie Configuration for Multi-pass Amplifier

![Multi-pass Amplifier Diagram](image)

Figure 51 shows the 4 passes that occur in our multi-pass amplifier. The gain contribution of the multi-pass can be reasoned as follows. We can start by describing the instantaneous rate of change of energy density with respect to position in the crystal, similar to the intensity rate equation found in [43], as

\[
\frac{dW}{dz} = \frac{\alpha W}{1 + \frac{W}{W_s}}: W_s = \text{saturation energy density}, \alpha = \text{gain factor} \tag{29}
\]

Separating variables gives

\[
\frac{dW}{W} \left(1 + \frac{W}{W_s}\right) = \alpha dz
\]

By a quick change of variables, and integrating the left side from some initial energy density to some final energy density, over the length of the medium, we have
\[
\int_{W_0}^{W} \frac{dW'}{W'} \left( 1 + \frac{W'}{W_s} \right) = \int_0^L \alpha dz'
\]

Which gives

\[
\ln W' + \frac{W'}{W_s} \bigg|_{W_0}^{W} = \alpha z' \bigg|_0^L
\]

Therefore,

\[
\ln \left( \frac{W}{W_0} \right) + \frac{W - W_0}{W_s} = \alpha L \quad (30)
\]

Now, if \( W \gg W_s \), our original rate becomes

\[
\frac{dW}{dz} = \frac{\alpha W}{1 + \frac{W}{W_s}} \approx \frac{\alpha W}{W_s} = \alpha W
\]

yielding the result that \( W \approx W_s \cdot \alpha L + W_0 \), i.e. linear gain. However, if \( W \ll W_s \) then

\[
\frac{dW}{dz} = \frac{\alpha W}{1 + \frac{W}{W_s}} \approx \alpha W
\]

and we see that the gain is exponential, \( W \approx W_0 e^{\alpha L} \). The saturation energy density is given by \( W_s = \hbar \omega / 2 \sigma_0 \) where \( \sigma_0 \) = the emission cross-section [44]. We can estimate the saturation energy density with an emission cross-section of \( 5 \times 10^{-20} \text{ cm}^2 \). For a nominal frequency of 377 THz, we have

\[
W_s \approx \frac{(6.626 \times 10^{-34})(377 \times 10^{12})}{(2 \cdot 5 \times 10^{-20})} \approx 2.5 \text{ J/cm}^2
\]

Note that \( e^{\alpha L} \approx 1 + \alpha L \) for small \( L \). Thus, for small \( L \), \( W \approx W_0 e^{\alpha L} \approx W_0 (1 + \alpha L) \). In this case, we see linear gain in relation to the input energy density rather than the saturation
energy density when operating well below saturation. The voltage at each of the four passes in our multi-pass amplifier is shown in the next figure. The zeroth pass represents the input energy. (The energy conversion rate for this measurement is given below).

Figure 52: Per Pass Voltage in Multi-Pass Amplifier

Data was taken using a Gentec EO energy meter, model QE50SP-H-MB-QED-D0. The calibration conversion for this detector is 2.88 V/J.

2.6 Compressor

After all amplification processes are complete, the pulses are sent to the compressor to bring them back to as close to their original duration as possible. A section of the Chirped Pulse Amplification scheme of Figure 12 is shown in Figure 53, highlighting the grating pair used in compression.

Figure 53: Pulse Compression: Reversal of GVD due to Stretching

Our compressor configuration is almost identical to that shown in the previous figure. One small difference is that we use a half-wave plate to switch from s-polarization to p-polarization to maximize compression. The compressed pulse is then flown over the input mirror to strike the target, as shown in Figure 53 (clipped from Figure 15). “Gnormal” is the normal of the first grating. “B” and “R” indicate the “blue” and “red” portions of the pulse spectrum respectively. Since both portions are traveling at the same speed, and since red travels a longer distance than blue, the two portions of the spectrum get closer in time, thereby compressing the pulse. The double arrow below the second grating in Figure 54 indicates position control of the second grating. This allows a certain extent of control to the amount of compression.

Figure 54: Sketch of Compressor Set-Up

Pulse energy after recompression is approximately 3.82 mJ/pulse. As mentioned in chapter 1, assuming a return to the original pulse duration, this provides a peak power of roughly 30 GW. This was calculated using an equation for hyperbolic secant squared solitons [45]:

\[
P_{\text{peak}} \approx \frac{0.88E_p}{\tau_p}
\]

\[
P_{\text{peak}} \approx \frac{0.88(3.82 \text{ mJ})}{104 \text{ fs}} \approx 32.3 \text{ GW}
\]
2.7 **Targets**

Our first target of study is bulk BK7 glass. The result is supercontinuum generation, as shown by the rainbow of colors appearing on the paper screen in Figure 55. (Some properties of BK7 can be found here: [http://www.glassdynamicsllc.com/bk7.html](http://www.glassdynamicsllc.com/bk7.html).) The glass is positioned after high energy compressed pulses travel through a telescope used for beam collimation.

**Figure 55: High Energy Pulse Interaction with BK-7 Glass**

Using a thinner, 8 mm deep, glass target, Figure 56 shows the production of ultrashort white light laser pulses. This piece of glass is inserted within the telescope, at the focal length of the first lens. The output is then collected and collimated by the second lens. Figure 57 shows this light after dispersion through a prism. The beam of what remains of the source laser can be seen striking the left side of the figure, while the extra spectral elements of the supercontinuum can be seen in the dispersed colors.
The mechanism of supercontinuum generation is attributed to self-phase modulation primarily, in the case of ultrashort pulses, and to stimulated Raman scattering and four wave mixing for pulses in the picosecond regime [46]. According to Geoffrey New,

“In supercontinuum generation (SCG), an intense narrow-band incident pulse undergoes dramatic spectral broadening through the joint action of several third-order processes, resulting in an output of essentially white light. […] In bulk media, the threshold for SCG coincides with the threshold for self-focusing, and the increase in intensity that accompanies beam collapse promotes a family of third-
order processes including self-phase modulation, cross-phase modulation, self-steepening, and plasma formation.” [47]

According to Diels and Rudolph,

“The interplay of self-focusing […] and various nonlinear processes make the exact treatment of the continuum generation with short pulses extremely complex. […] Other nonlinear effects that are likely to contribute are parametric four wave mixing and Raman scattering. The strong anti-Stokes component […] is likely because of multiphoton excitation of the dielectric material followed by avalanche ionization. […] The resulting electron plasma in the conduction band produces a fast rise of a negative refractive index component that can explain the dominant broadening toward the shorter wavelengths. […] SPM is associated with self-focusing leading to extremely high intensities where the beam collapses. It is at this point where the continuum generating nonlinear processes are most effective.” [48]

Unfortunately, the glass target in Figure 56 also experienced damage. This was likely due to the beam collapse mentioned by Diels and Rudolph.

To mitigate material damage, while also efficiently generating a continuum, water was used in place of the glass. Figure 58 shows white light generated by striking a cuvette of water, placed at the focus of the telescope previously mentioned.
Figure 58: Continuum Generation in Water

Continuous white light generation appeared to be sustainable as long as the system was running. This could therefore be a viable source for reliable time-resolved spectroscopy, for example.

The figure shows white light containing the visible spectrum. This light is made of ultrashort pulses around 100 fs.
CHAPTER 3 DATA

3.1 Mode-locked Ti:Sapphire laser

3.1.1 Pump Laser – Millennia V Diode-Pumped, CW Visible Laser made by Spectra Physics

Table 2: Millennia Pump Laser Settings

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Operation</td>
<td>Current mode at 90% current</td>
</tr>
<tr>
<td>Power Output</td>
<td>2.15 – 2.33 W (as read on Control Module)</td>
</tr>
<tr>
<td>Diode 1 Temperature</td>
<td>14.7 °C</td>
</tr>
<tr>
<td>Diode 2 Temperature</td>
<td>14.7 °C</td>
</tr>
<tr>
<td>Wavelength</td>
<td>Frequency doubled to 532 nm</td>
</tr>
</tbody>
</table>

3.1.2 Ti:Sapphire Oscillator

Angles of the z-cavity folding mirrors and the distances between optical elements are shown in Figure 16. Other important quantities that characterize the oscillator are stated in the table below.

Table 3: Seed Pulse Data

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Pulse Width</td>
<td>52 fs (from 6/2/2016 to 3/20/2017)</td>
</tr>
<tr>
<td>Average Pulse Width</td>
<td>110 fs (from 12/9/2017 to 10/30/2018)</td>
</tr>
<tr>
<td>Average Power</td>
<td>98 mW (measured at 770 nm)</td>
</tr>
<tr>
<td>Pulse Separation</td>
<td>13.5 ns</td>
</tr>
<tr>
<td>Pulse Repetition Rate</td>
<td>74 MHz</td>
</tr>
</tbody>
</table>

3.2 Stretcher

Table 4: Stretched Pulse Data

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretched Pulse Width</td>
<td>100 ps (measured value on oscilloscope)</td>
</tr>
<tr>
<td>Stretched Pulse Width</td>
<td>80 ps (after image processing)</td>
</tr>
</tbody>
</table>
3.3  *Regenerative Amplifier*

3.3.1  Settings

Table 5: Timing Control Device Settings

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LAMP</td>
<td>124.835 ms</td>
</tr>
<tr>
<td>Q-SWITCH</td>
<td>0 000</td>
</tr>
<tr>
<td>CHARGE</td>
<td>124.826 ms</td>
</tr>
<tr>
<td>TRIGGER</td>
<td>0 000</td>
</tr>
<tr>
<td>LAMP firing frequency</td>
<td>8.011 Hz</td>
</tr>
</tbody>
</table>

Table 6: Surelite Settings

| Laser Type | Q-switched pulse laser, 1064 nm output. Our experiment incorporates a large DKDP crystal to frequency double the output to 532 nm. A half-wave plate is used before the crystal to optimize SHG in the crystal. The crystal is mounted on a mount with three dimensional control. |
| Stable Operation | Occurs after 1.5 – 2 hours of flashlamp firing |
| Stable Power | Approximately 3.8 W observed with energy setting of 3.4 kV |

Table 7: Pockels Cell Settings

| Applied Voltage | +530 V |
| Bias Voltage    | +56 V  |
| START time      | Surelite Q-switch + 1.933 μs |
| BNC Pulse Generator Settings |  |
| External Trigger Applied Voltage: | 1 V |
| Delay: | 1.933 μs |
| Width: | 10 μs |
| STOP time      | START time + 440 ns |
| HP Pulse Generator Settings |  |
| Top half, left to right: |  |
| Pulse Period: | Ext |
| Vernier: Approx. 6 marks clockwise (1st mark counted as 1) |  |
| Amplitude: | 4 – 16 (button depressed) |
| Vernier: Approx. 6 marks clockwise (1st mark counted as 1) |  |
| Bottom half, left to right: |  |
| Pulse Width: no selection made (no buttons depressed) |  |
In addition to the above settings, the quarter wave plate in our current set-up is set to 160°. This setting only applies to this particular quarter wave plate mounting. The general idea is that the quarter wave plate is set to allow maximum seed pulse signal into the regenerative amplifier when the Pockels cell is off. (Thus, when the Pockels cell is on, the pulse is trapped.)

3.3.2 Power Distribution

Table 8: Surelite Power Distribution

<table>
<thead>
<tr>
<th>Surelite output power</th>
<th>3.4 W</th>
</tr>
</thead>
<tbody>
<tr>
<td>To Regenerative Amplifier</td>
<td>0.9 W</td>
</tr>
<tr>
<td>To Multi-pass Amplifier (left)†</td>
<td>1.3 W (then through half-wave plate)</td>
</tr>
<tr>
<td>To Multi-pass Amplifier (right)†</td>
<td>1.2 W</td>
</tr>
</tbody>
</table>

† See Figure 15.

3.3.3 Single Pulse Amplification Measurements

Table 9: Amplification of Regenerative Amplifier

| Pre-amplified energy per pulse | 28.755 pJ |
| Amplified energy per pulse     | 694.4 μJ |
| Amplification factor           | 24 million |

The next two figures show the amplification of a single pulse from outside and inside the regenerative amplifier.
Figure 59: Single Pulse Amplification

The yellow trace is the amplified pulse. The blue trace is the signal sent to the Pockels Cell to control its start and stop times.

Figure 60: Amplification Inside the Regenerative Amplifier

The yellow trace shows pulses being amplified within the regenerative amplifier. The blue trace is the signal sent to the Pockels Cell to control its start and stop times.
3.4 Multi-pass Amplifier

Table 10: Energy per Pass in Multi-pass Amplifier

<table>
<thead>
<tr>
<th>Pass Number</th>
<th>Energy (mV)</th>
<th>Energy (mJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>0.694</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>2.083</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>3.819</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
<td>5.903</td>
</tr>
<tr>
<td>4</td>
<td>21</td>
<td>7.292</td>
</tr>
</tbody>
</table>

The power distribution from the Surelite pump laser to the Ti:Sapphire crystal in the multi-pass amplifier is shown from both sides in Figure 14, and in Table 8. The total gain factor from the multi-pass system is approximately 9.5.

3.5 Compressor

Compressed pulses have an energy of about 3.82 mJ/pulse as measured by a Gentec EO energy meter (model number given in Chapter 2). This translates to about 11 mW/pulse which gives about 30 GW peak power per pulse, assuming a return to the original pulse duration. This calculation is shown in Chapter 2.

3.6 Water Target

Water was placed at the focal point of the high power pulses, held in a cuvette. This produced a continuous source of white light, as shown in Figure 58 at the end of Chapter 2. This light, after being sent through a prism, is shown below.
Various levels of continuum generation were observed in bulk glass as differences in brightness and clarity of the spectrum. When the input beam was focused too tightly within the glass, long streaks of smoky looking damage occurred. When the beam was focused completely inside the glass, i.e. the focal point resided in the glass, damage appeared as localized “scratches” or cracks.

Supercontinuum generation was also observed in a much thinner piece of glass, as shown in Figure 56 near the end of Chapter 2.

**3.8 Interaction with Kerr Cell**

An electronic signal, in reaction to the high power pulses, was not observed at this time. Non-linear effects were observed as shown in the next figure.
Nonlinear interaction of high peak power, short duration pulses, as evidenced by the green, yellow and red colors shown on the screen.
APPENDIX A: PROCEDURES

A.1 Mode-locked Ti:Sapphire laser

A.1.1 Creating the Cavity

Lasing can be achieved with sufficient pump power focused into the Ti:Sapphire crystal, in concert with proper alignment of the mirrors. Our Ti:Sapphire oscillator starts lasing at about 1.5 Watts of pump power, and is sustained at about 2.5 Watts. In addition, using a multiple quantum well (MQW) as one of the end mirrors, as a passive gate saturable absorber, gives a high probability of mode-locking. Narrow pulses, however, require some fine tuning to be achieved. Major steps taken to achieve the results of this experiment are given below. For potential use in the lab, the steps are stated as instructions.

1. Use the pump beam to define a height standard which will then be used to initially set the height control of all mirrors in the cavity. We used a small Allen wrench bolted to a post to define a reasonable initial height. All beams were then subject to maintaining this height, thereby defining a plane of propagation. This plane is also used to describe p-polarization. Taking this step roughly eliminates one degree of freedom when trying to precisely align mirrors to obtain lasing. Hereafter, this practice will be assumed as a preliminary step for all other aspects of the experiment.

2. Ensure that the output of the completed cavity is p-polarized. If the pump output is p-polarized then nothing needs to be done. If the pump is s-polarized, the use a periscope to change the polarization. This will become important for later use of the Faraday isolator system.
3. Begin with the basic z-shape design of Figure 16 using the Ti:Sapphire crystal and two focusing mirrors. Use the appropriate astigmatism compensation angle, as prescribed by Equation 20. The Ti:Sapphire crystal and focusing mirrors must be set on a translation stage with fine controls for each element. In the beginning, the Ti:Sapphire crystal can be roughly placed at the focus of both mirrors. The Ti:Sapphire crystal should be cooled to optimize lasing and prevent damage. Place the multiple quantum well at the end of what will be the shorter arm of the cavity, at the focus of a positive lens.

4. Allow the pump beam, at low power, to penetrate the first focusing mirror, which will then focus into the Ti:Sapphire crystal, strike the second focusing mirror, then onto the positive lens and MQW. This light will be reflected back, ultimately striking the initial focusing mirror, then reflect out in the direction that will begin the longer arm of the cavity. Adjust focusing mirrors and the MQW until a z-shape is formed by beam reflections. If the pump beam is too bright, a glass filter can be used to reduce the intensity of the pump, leaving the fluorescence signal of the Ti:Sapphire readily visible using an infrared viewer.

5. Place a flat mirror in the output beam path of the z-shape to act as an end mirror. This mirror must allow laser light to penetrate once lasing is achieved. The MQW will act as the other end mirror. Lasing will not occur until there is sufficient pump power and precise alignment of the MQW, focusing mirrors, and flat mirror (described in the next step.)

6. Temporarily remove the positive lens that focuses on the MQW. Adjust the MQW and flat mirror to obtain straight paths to the focusing mirrors. Adjust the focusing mirrors
so that their shadows overlap and are centered on each other, as seen on the surface of a piece of paper (or some other screen) at the output of the z-cavity. Once this occurs, replace the lens, and ensure once again that the beams overlap by adjusting the lens position.

7. Increase the pump power and look for lasing. Lasing can be clearly seen on the surface of the flat mirror using an infrared viewer. (If you’re not sure if it’s lasing, it’s not lasing.) Minor adjustments will likely be necessary to achieve lasing. If the pump is decently focused into the Ti:Sapphire crystal by both focusing mirrors, then usually some minor horizontal adjustments of the flat mirror will give rise to the proper alignment conditions for lasing. If the horizontal alignment is very good, then very slight vertical adjustments will likely give rise to lasing. On the other hand, if the vertical adjustments are very good, then horizontal adjustments can be made fairly swiftly by moving the whole base of the mirror using your fingers. You may see lasing as a quick flash of red about the size of a pencil tip. After seeing this quick flash, clamp down the mirror and use the fine horizontal controls to achieve lasing. Unfortunately, the initial lasing conditions may be hard to achieve due to several mirrors all needing to be in good alignment at the same time. After the initial lasing is achieved, extending the long arm of the cavity is somewhat less difficult.

8. To extend the length of the laser cavity, place a flat parallel face mirror in the path of the laser beam (for alignment only). This mirror needs to allow the extended laser beam to penetrate, and be seen on the other side once lasing is re-obtained. 98% reflectivity is a good choice for this mirror, though not necessary. Adjust the mirror until the reflected beam exactly overlaps the incident beam. Be sure this mirror is clamped down
to avoid disturbances that lead to misalignment. Remove the current end mirror of the cavity. If the alignment of the mirror you just placed is good then lasing will occur, and will be evident on the surface of the new mirror using an infrared viewer, or may be viewable on a white screen (like the back of a business card) behind the mirror. Adjust the new mirror until the lasing signal is maximized. Repeat this step as necessary to continue to extend the length of the cavity.

9. As previously described in this work, group velocity dispersion (GVD) is a critical component for generating ultrashort pulses. To control GVD, a prism pair needs to be added to the long arm of the cavity. (Other prism arrangements or chirped mirrors can also be used to control GVD.) To add the first prism, place the prism in the path of the laser beam. The beam will be refracted through the prism, coming out at an angle with respect to the input beam (the deviation angle). To minimize chromatic dispersion, turn the prism until minimum deviation is achieved. This idea is exaggerated in the figure below for a given input beam path.

![Figure 64: Prism at Minimum Deviation](image)

In practice, the separation in “colors” is not visible. However, what is clearly visible is the position of the output beam on a “far away” screen. About half a meter is usually far enough. As you turn the prism you’ll noticed the spot on the screen moving until it hits a certain point, then it will proceed back from the direction in which it came. This
inflection point of the position of the beam spot is where the prism is at minimum deviation. At this point, clamp the prism down and use the fine controls on the prism mount to obtain the best estimate of minimum deviation.

10. To extend the cavity beyond the prism, place a flat mirror in the beam path, essentially repeating the procedure in step 7. In practice, I’ve found this to be a bit trickier than simply extending the cavity from one flat mirror to another, presumably due to the dispersion factor.

11. Continue to extend the cavity around other elements as necessary, using the procedure in step 7. For example, on our particular optical bench, a flat mirror was needed between the prims due to space constraints.

12. Place the second prism in the beam path, approximately 1 meter from the first, as measured along the beam path. Repeat step 9.

13. Add the output coupler as you would another flat mirror, according to step 7. The output coupler is the end mirror of the cavity.

14. Optimize the cavity by simultaneously observing the pulse train and autocorrelation as described in the next section.

A.1.2 Measuring Pulses

Two detection systems are used in this experiment to measure pulses. One system is used to show mode locking, determine the separation between pulses, and send a signal to the timing electronics for the regenerative amplifier, while the other is used to measure the width of each pulse.

To measure the pulse train (train of mode locked pulses), leakage from an intracavity mirror is redirected to an avalanche photodiode (APD). Section 2.2.7 describes
the specifications of the APD and oscilloscope used. A picture of the mode locked pulses is also given in section 2.2.7 in Figure 25. The signal from the APD is split between the oscilloscope and the timing control device for the regenerative amplifier.

To measure the pulse width, the output beam of the Ti:Sapphire oscillator is sent to the autocorrelator. The specifications of our system components along with the main ideas of the autocorrelation technique, and set up, are given in section 2.2.8. Referring to the diagram below, steps to achieve the autocorrelation signal are as follows:

Figure 65: Autocorrelator Apparatus Diagram

1. Send the seed (output of the Ti:Sapphire oscillator) to the location of the yet to be assembled autocorrelator. I preferred to use the holes in the optical bench as a guide. A flipper mirror was used so that when the duration of the seed pulses was not being measured, the mirror could be flipped down, and the seed allowed to move on to the stretcher assembly. This also gave me the benefit of using the full power of the seed pulses, as opposed to trying to use leakage from an intracavity mirror. The minor
downside to using the flipper is that the vertical and horizontal controls need to be slightly adjusted every time it is flipped up to use.

2. Center the speaker mounted mirror (corner cube) using the seed. Reflect the seed back to its source, at the same height, but at a slight horizontal angle to avoid injection into the oscillator. Our speaker mirror assembly is mounted in such a way to provide height control, which I found particularly useful in getting all the parts to work just right.

3. Place the 50/50 beam splitter in the path of the seed as shown in the diagram. Set the initial position so that the reflected beam is very close to 90° with respect to the incident beam. Again, I found the holes in the optical bench to be a useful guide. The beam splitter should be mounted on a stage with tilt control for fine tuning.

4. Set the adjustable mirror (corner cube) on a translation stage with micrometer control, as indicated in the diagram, roughly at a distance away from the beam splitter equal to that of the distance between the beam splitter and the speaker mounted mirror. Ensure that the reflected light is parallel to the incident light. This can be accomplished by checking for divergence of the two spots created by each arm of the system as viewed in the place where the photomultiplier tube will eventually end up, and at several points downstream of that location.

5. Place a positive lens of short focal length in the path of the reflected beams to the left of the beam splitter, as shown in the diagram. Ensure that the lens collects all available light from each arm of the system.

6. Place the thin KDP at the focal length of the lens. Once the KDP is in the proper position, phase matching conditions should be met to achieve second harmonic generation (SHG).
7. Place a red filter just after the KDP to view the brilliant blue signal, which can be seen as a blue dot on a business card. This will also reduce noise from the signal going to the photomultiplier tube. This filter is technically not necessary and is not shown in the diagram. We currently have a filter attached to the KDP crystal mount so that when we move it to achieve optimal SHG we still clearly see the blue signal.

8. Place a red filter in front of the photomultiplier tube, in place of or in addition to the one described in step 7, as shown in the diagram. This will reduce background noise from the unconverted seed signal, i.e. from the part of the beam that did not convert to the second harmonic. We have a filter directly attached to the input of the photomultiplier tube.

9. Place the photomultiplier tube in the best location to receive maximum signal, as shown in the diagram.

10. Make all necessary electrical connections, such as, the connection between the speaker and frequency generator, and the connections to the photomultiplier tube, power supply, and oscilloscope. Note that a resistor can be added to the path from the photomultiplier tube to the oscilloscope, in parallel, to change the resolution (see RC time constant). Our current added resistance is 100 kΩ (in parallel with 1 MΩ).

11. Set the speaker in motion using the frequency generator. We’ve found that the best frequency to view the autocorrelation signal is around 25 Hz.

12. Power up the photomultiplier tube and turn on the oscilloscope. Our photomultiplier tube power supply currently delivers about 300 V to generate a nice trace. This can be increased substantially when trying to view the initial autocorrelation, and is likely necessary while further optimizing the Ti:Sapphire oscillator. (Of course, follow
manufacturer guidelines for all equipment.) The oscilloscope time and voltage settings can be quickly probed to look for an autocorrelation trace. Our current settings are 200 mV per division and 200 μs.

A.2 Faraday Isolator

In Chapter 2, section 2.3, the Faraday isolator was mentioned at the end of the discussion on the stretcher. It’s role as a switching mechanism for the amplifier system can be seen in Figure 15. However, since the isolator physically comes before the stretcher, this is the appropriate time to describe the procedures for incorporating it into the experiment. To aid in following the steps, a portion of Figure 45 has been copied below.

Figure 66: Switching with a Faraday Isolator System

1. Place an iris around the beam coming from the Ti:Sapphire oscillator. This will come in handy when adjustment to the oscillator needs to be made after the isolator system is in place. (This is guaranteed to happen!)

2. Place the first polarizing beam splitter (PBS) in the orientation shown in the top part of the diagram. At first, set the PBS slightly off the desired angle to get a good reflection. The reflection can then be used to level the PBS by setting the height of the reflected beam the same as the incident beam (assuming you’ve already taken measures to ensure
a level beam from the oscillator.) Knowing that the PBS is level, rotate until you get maximum transmission and minimum reflection.

3. Place the second PBS shown in the top part of the diagram, leaving enough space for the Faraday isolator. (Our isolator came with an attached half-wave plate.) Repeat step 2 to ensure that the PBS is level and at the best angle.

4. Place an iris around the beam after the second PBS.

5. Place the Faraday isolator with attached half-wave plate, as shown in the diagram. Set the height and angle of the isolator so that the beam appears to go through and come out at the center of each side of the isolator. Rotate the half-wave plate to achieve maximum output after the second PBS. This ensures that the beam after the Faraday isolator system has remained p-polarized, and that light returning through the system will become s-polarized, preventing injection of the stretched pulse back into the oscillator, and sending it out to the regenerative amplifier.

A.3 Stretcher

The operating principles, equipment, and image processing to determine the stretched pulse duration are all described in section 2.3. This section describes the steps taken using our experimental set-up, as shown in the following hand sketch, which is a copy of Figure 33.
A.3.1 Stretcher Set-Up Procedure

Figure 67: Hand Sketch of Actual Stretcher

1. Note that the numbers in the diagram correspond to the order in which reflections occur. One set of numbers is given for all the mirrors, and one for the grating. In the case of the grating, it’s important that there are four strikes on the grating, hence the labeling.

2. Place an iris just before the stretching components. Not only does this aid in maintaining a level beam, it can be used to look for the output of the stretcher when nearly closed.

3. Place the small flat mirror, M1 in the diagram, in the beam path after the iris. Adjust its angle to roughly direct the reflected beam slightly upward, and to where the diffraction grating will be placed.

4. Place the diffraction grating according to the diagram. Set the pitch angle of the grating to reflect the diffracted beam over mirror M1, and roughly level. Set the azimuth angle of the grating to have the first order reflected beam (or highest energy beam) directed toward the other elements of the stretcher. Please note that the reflected beam must fly over mirror M1 enough to allow a later reflection to fly over mirror M1 and strike pulses are stretched ≈1000 times via four strikes on the grating shown on the right of the hand sketch, together with an appropriate combination of the other elements shown.
mirror M5, without the first reflection from the grating being interrupted. About an inch over M1 should suffice.

5. Place mirror M5 so that the top is roughly an inch above mirror M1, as shown in the diagram. This mirror will be used for later reflections, and should not intercept any reflected light from the grating at this time.

6. Place a large lens in the path of the beam reflected from the grating to roughly collimate the output beam, as shown in the diagram. The necessary degree of collimation depends on how far the beam will travel. The idea is to allow the beam to spread as much as possible while still being contained within the boundaries of all the mirrors. Also, the farther you can make the “blue” portion of the light travel with respect to the distance the “red” portion travels, the greater the stretching. The greater the stretching, the more energy can be packed into a single pulse without damaging equipment. The higher the energy in a stretched pulse, the higher the energy in its recompressed version at the end of the amplifier system. The other purpose for the lens is to refocus the stretched beam onto the grating and ultimately back out of the system spatially recompressed but temporally stretched.

7. Place large mirror M2 according to the diagram. Looking at the surface of this mirror with an infrared viewer, you will see a horizontal line. This can be the last mirror you use if your table is big enough, that is, if you reached the length that you desired. At this point you do not want the line to take up all the available space on the mirror. To facilitate the process of setting up the entire system, using only about half the available mirror space is recommended. Send the reflection of the line to the future location of mirror M3, if you plan to use the additional mirror.
8. If you’d like to extend the length of travel for the stretched beams, as we did, add mirror M3. The downside to adding mirrors is the inevitable loss of energy on reflection. This may be a critical issue and should be carefully considered.

9. For the sake of further description, I will assume use of the additional mirror M3.

10. Set mirror M3 to send the line back through the lens and onto the grating. Adjust the vertical control so that the reflection from the grating flies over mirror M1, but squarely strikes mirror M5.

11. Adjust the controls (and perhaps even position) to send the reflection from its surface back to the grating, to the place on the grating from which the beam came, as best as can be seen at this point.

12. Close the iris as much as possible without a significant loss of energy. Looking at the surface of the iris using an infrared viewer, on the side facing mirror M1, adjust the controls of mirror M5 until you see the output beam of the stretcher appear to go through the iris and back out to the Faraday isolator. If your estimate in step 11 was good, then only minor adjustments should be necessary.

13. Open the iris. The beam of stretched pulses should now be on its way back through the Faraday isolator system. If not, look for the beam on the surface of the first PBS it would encounter. Adjust mirror M1 to get the beam to go back through the isolator. When the beam has traveled back through the isolator, it can be seen at the output of the other PBS, as shown in Figure 44 on the bottom-right. Close the iris and repeat step 12 using mirror M5. Repeat this process until the beam is maximally transmitted back through the isolator system.
14. Use mirror M5 to slightly adjust the height of the beam that has gone back through the isolator and is on its way to the regenerative amplifier. The height of this beam should match the height of the beam that went through the isolator system in the first place.

A.3.2 Stretched Pulse Image Processing Procedure

Processing Steps

1. Create numeric files from images.
2. Normalize amplitudes of each numeric file.
3. Shift the time data of one or both pulses to obtain a common center.
4. Use an appropriate technique of Fourier analysis. The method I used is described in Chapter 2. Derivations and Matlab code are given in Appendix B.

After performing the image processing just described, the actual duration of our stretched pulse is approximately 80 ps.

A.4 Regenerative Amplifier

A.4.1 Initial Set-Up Procedure

The equipment and basic principles of operation are given in section 2.4. Steps for setting up our regenerative amplifier are as follows. A near copy of a previous figure is shown here for reference.
1. The input beam for the regenerative amplifier begins with the output of the right-most polarizing beam splitter of the Faraday isolator, i.e. the output of the stretching system.

2. Use the input beam to set the height of the TFP, just as would be done with any other mirror. The face of the TFP must be perpendicular to the plane of incidence to achieve the best results.

3. Direct the beam toward the thin film polarizer (TFP), as shown in the diagram, placing an iris along the way to aid in further alignment procedures. The beam path from the Faraday isolator to the TFP for our experiment is shown in Figure 15, at the beginning of Chapter 2. The direction of the input beam and the angle of the TFP must be such that the polarization of the input beam is preserved, namely $s$-polarization. An analyzer (polarized sheet) can be temporarily used to test the quality of the polarization when placed after reflection from the TFP surface. Set the analyzer to transmit $s$-polarization,
and adjust the angle of the TFP, with respect to the input beam, to get maximum transmission, or set the analyzer to p-polarization and adjust the TFP angle to get minimum transmission.

4. Adjust the input beam angle, with respect to the optical bench, and re-adjust the TFP, as described by step 3, until the reflected beam is traveling in a preferred direction as defined by your optical bench and where you want your equipment.

5. Place the quarter wave plate (QWP) and end mirror number 1 (EM₁) as shown in the diagram. Do not place the Pockels cell at this time, but leave plenty of room to do so. The beam from the TFP should travel through the center of the QWP. Set EM₁ so that the reflected beam goes back to its origin. This can be seen using the iris placed in step 3. Note that the QWP now acts as a half-wave plate (HWP) on the double pass of the beam, and therefore rotates the initial beam depending on the wave plate angle. This is precisely how it works as a continuous switch. At one angle the input and output beams (in from the Faraday isolator and out to the isolator respectively) experience zero rotation and the polarization states of the input and output beams remain the same. At another angle, a 45° turn from the zero state (theoretically), the input beam experiences a 90°, or half-wave rotation, thereby switching polarization states from s- to p-polarized on the double pass. Let’s call these QWP positions the zero wave and half-wave positions. At the zero wave position, transmission through the TFP should be minimum (zero transmission is, of course, the goal). At the half-wave position, transmission through the TFP should be maximum (about the same amplitude as the input beam). In the interest of the next step, leave the QWP in the zero wave position.
6. Place the Pockels cell in the beam path as shown in Figure 68 (without electrical connections in place at this time). Alignment of the Pockels cell is critical and requires adjustable motion in yaw, pitch, and roll. To get a good initial alignment, adjust the Pockels cell until the beam passes through free and clear in both directions. Continue to adjust the Pockels cell until the output beam returns to its point of origin at the Faraday isolator. If the alignment is excellent, the beam will propagate back through the Faraday isolator, then be redirected by the left-most polarizing beam splitter toward what will become the multi-pass amplifier. Again, for the big picture, please see Figure 15.

7. Precise alignment of the Pockels cell requires some additional procedures. Please see, for example, the referred publication by Gooch & Housego for further details [49]. After alignment procedures have been completed, turn off the Pockels cell power supply and adjust the QWP to the half-wave position. This will allow maximum transmission through the TFP, which will be beneficial for the next step.

8. Set the flat mirror as shown in the diagram.

9. Set the Brewster cut Ti:Sapphire crystal in the beam path. Ensure that the crystal is mounted in a housing that can be cooled. Our crystal is mounted in a copper housing that is in turn mounted in a larger water cooled aluminum housing. Indium foil can be used between the crystal and the copper for increased thermal conductivity. Set the angle of the crystal for maximum transmission. Brewster cut angles for Ti:Sapphire are described here [50].

10. Set end mirror number 2 (EM2), as shown in the diagram. Adjust the horizontal angle until the reflected beam travels back through the Ti:Sapphire crystal along the path
from which it came. The surface of the QWP is a nice location to view the reflected beam spot using an infrared viewer. If the horizontal angle of EM$_2$ is not correct, you will see two beam spots on the QWP. The position of EM$_2$ is excellent when the beam reflected from its surface travels all the way back through the Faraday isolator and is visible at the output of the left-most polarizing beam splitter.

This concludes the initial set-up. Pumping, fine tuning, timing, and single pulse isolation follow.

### A.4.2 Pumping Set-Up Procedure

1. Set up the pumping scheme for the amplifier. Figure 68 shows part of this scheme, but more detail is necessary as the pump is simultaneously used for the multi-pass amplifier. Figure 15 illustrates the dual use of this single pump.

2. Our pump outputs 1064 nm. This is then frequency doubled using a DKDP crystal. A half-wave plate is used between the 1064 nm output and crystal to achieve maximum SHG as polarization and orientation of the crystal are both critical.

3. Within the proceeding arrangement of optics, the combination of the half-wave plate and thin-film polarizer, shown at the top-left of Figure 15, is used to adjust the power distribution between the regenerative amplifier and the multi-pass amplifier. As the half-wave plate is turned, the original polarization of the pump laser is rotated. The thin-film polarizer will reflect s-polarization while transmitting p-polarization. Therefore, the relative ratio of s- to p-polarization of the light after the half-wave plate directly relates to the ratio of the power going to the regenerative and multi-pass amplifiers. Generally speaking, more power can be sent to the multi-pass amplifier due to its passive nature. Indeed, to achieve maximum amplification as a result of both
amplification processes, more power should be sent to the multi-pass amplifier. Currently, about 26% of the pump power is going to the regenerative amplifier.

A.4.3 Regenerative Amplifier Lasing

Precise alignment is required for trapped pulses to be amplified. This can be difficult without some kind of reference. A convenient reference can be lasing of the regenerative amplifier as it also works as a laser cavity, given an appropriate setting of the quarter wave plate, with the Pockels Cell off. When the amplifier is lasing, optimal alignment for amplification is close. Steps to achieve this lasing are as follows.

1. Set the pump power to its lowest setting, while still producing green light.
2. Use the pump control mirror shown in Figure 68 to align the pump beam with the seed signal just before and after the Ti:Sapphire crystal, then turn the pump off.
3. Set the quarter wave plate to minimize seed pulses into the regenerative amplifier (zero wave condition). This can be seen by viewing the seed signal just before the Ti:Sapphire crystal.
4. Alignment is reasonably close at this point. Turn the pump back on and slowly increase power while looking for lasing on the surface of one of the cavity optics. Do this with caution! Given that the pump light is focused into the crystal, damage can occur relatively easy if the pump power is too great. If lasing has not occurred at high pump power, adjust the horizontal control of the pump control mirror shown in the diagram. If lasing still does not occur, make a slight adjustment to the vertical control of the pump control mirror, then scan horizontal positions, iterating this procedure until a reasonable area of the crystal has been swept.
5. Once lasing occurs, make slight adjustments to the pump control mirror to maximize the signal.

6. Reduce pump power until lasing becomes weak, then re-adjust the pump control mirror to maximize the signal. Repeat this process until you reach the lowest pump power that still causes steady lasing.

7. Turn off the pump, and reset the quarter wave plate to allow maximum seed signal back into the regenerative amplifier. The system is now nearly, if not actually, optimally aligned to yield high amplification of a trapped pulse.

A.4.4 Timing Set-Up Procedure

Precision timing is required to amplify a single pulse. Changes in nanoseconds to the starting time of the Pockels cell, for example, affects the maximum achievable gain. The timing sequence also involves getting the maximum power from the pump source and therefore requires some thought about the optimal frequency of operation for the pump. The following steps optimize the pump power while isolating and amplifying single pulses. Please note, as a general precaution, to turn off power from one source before connecting to another.

1. To get the most power out of the Surelite pump laser, the frequency of the flashlamp firing needs to be around 8 Hz. In addition to this criteria, the timing between the firing of the flashlamp and the Q-switch must be optimized. Current settings on the timing control device are LAMP 124.835 ms and Q-SWITCH 0 000. The first 0 on the Q-SWITCH setting can be toggled between 0 and 1, but seems to make no difference in time whether it is set at 0 or 1. The next three Q-SWITCH settings roughly correspond to 0.1, 0.01, and 0.001 μs respectively. Given the current settings, LAMP fires 176.8
μs before Q-SWITCH (see step 3, however, for more timing details). Also, the firing repetition rate of the lamp is approximately 8.011 Hz (1/124.835 ms). {Experimental note: the timing control device experienced some malfunction, thereafter requiring an SLP-200+ Low Pass Filter to be added at the input to operate properly.}

2. Connect the LAMP output of the timing control device to the Surelite according to Surelite operation manual for external triggering. This requires soldering to a 9-pin connector. (Soldering to the 9-pin connector is not complete until step 4 is complete.)

3. Connect the Q-SWITCH output to the pulse inverter. Given that the Surelite takes the drop in voltage as a switch for both the flashlamp and the q-switch, this subtracts approximately 8 μs from the time of the firing of the LAMP to the firing of the Q-SWITCH, giving 168.4 μs between the two (measured).

4. Connect a BNC T-connector at the output of the pulse inverter. Then connect a cable from one side of the T-connector to the external q-switch input of the Surelite. This will require soldering to the same 9-pin connector as previously done for the LAMP output of the timing control device. Once this is complete the 9-pin connector can be connected to the Surelite. See the Surelite operators manual for using external triggering.

5. Connect a cable from the other side of the T-connector to the BNC Pulse Generator at the “TRIG IN” port. To use the generator, some voltage must be applied. Use the “TRIG” menu to apply voltage to the external trigger. We currently apply 1 V. In the “Timing” menu set the delay to 200 ns. (You’ll find a range of values between approximately 190 ns to 220 ns that may give optimal results.) In the same menu, you can select a pulse width. We currently use 1 μs.
6. Connect the output of the BNC Pulse Generator to a T-connector attached to the START port of the Pockels cell power supply. Our power supply is a Conoptics model 307 A., currently operating a +575 V output amplitude and +56 V bias amplitude. The Pockels cell itself is a Conoptics model 350-50.

7. Connect the other side of the T-connector on the START port of the Pockels cell power supply to the HP Pulse Generator. Make appropriate selections to run the generator at the period of the incoming pulse, and to invert the output of the generator. This inversion of the signal gives a way to control the precise time to stop the Pockels cell by controlling the width of the outgoing pulse using the vernier control.

8. Connect the output of the HP Pulse Generator to the Hamamatsu C1097 passive delay generator for fine timing control using up/down switches rather than a turn-knob. This element may not be necessary, but is helpful in our experiment as the slightest changes in the vernier knob of the HP Pulse Generator produces drastic changes in time (on the time scale in which we are working).

9. Connect the output of the passive delay generator to the multi-output TTL box shown in Chapter 2. Two outputs are sufficient.

10. Connect one of the outputs of the TTL box to the STOP port of the Pockels cell power supply.

11. Connect another TTL box output to an oscilloscope to trigger on the STOP time signal sent to the Pockels cell. This will be used in conjunction with a detector to view changes in the pulses running through the regenerative amplifier.
12. Steps 1-11 establish all the necessary connections and give a baseline for timing, but do not give enough information to isolate and amplify a single pulse. The next few steps in the next section indicate how to achieve this result.

A.4.5 Single Pulse Isolation

1. Set up a detector to sample the pulses coming out of the regenerative amplifier. Ours is set at the output of the amplifier, just before the entrance to the multi-pass amplifier. You may have already done this in aligning the Pockels cell previously, in which case there is not a need for an additional detector. A fast avalanche photodiode will do. Trigger the detector on the Pockels cell STOP signal coming from the TTL box (which you connected previously.)

2. Adjust the detector so that you can view seed pulses.

3. Ensure that the quarter wave plate in the regenerative amplification system is set to the right setting. Ours is set to 160°, but this is arbitrary and depends on how your quarter wave plate is mounted. In essence, you want the quarter wave plate to allow maximum seed signal into the regenerative amplifier cavity while the Pockels cell system is off. (Note that if your detector is set at the output of the regenerative amplifier then the seed pulse signal will be maximized when the quarter wave plate is in the zero wave condition and the Pockels cell is off, or when the quarter wave plate is set to the half-wave condition and the Pockels cell is on. This is because the signal, in both of these conditions, is bouncing off less mirrors and going through less media.)

4. Turn all timing equipment on.
5. Turn the Surelite laser on. (The system should lase if the quarter wave plate is rotated to the zero wave condition. If not, you may need to align the system again before proceeding.)

6. Allow the Pockels cell to remain on for as long as possible (while still being connected at the STOP port) by using the maximum pulse width available from the HP Pulse Generator (vernier knob turned all the way clockwise).

7. Adjust the START time of the Pockels cell by adjusting the delay of the BNC Pulse Generator until you see the system lasing as viewed by the detector. This will begin to look like the amplified set of pulses shown in Chapter 2 as the timing gets closer to optimum.

8. CAUTION. In the next step you will amplify a single pulse. You may achieve sufficient amplification to destroy your detector. Be sure to take appropriate measures, such as, adding neutral density filters to reduce the pulse amplitude at the detector.

9. Once you achieved maximum amplification of the set of pulses, slowly rotate the vernier knob on the HP Pulse Generator counter-clockwise to bring the STOP time of the Pockels cell closer to the START time. This will begin to cut off the set of amplified pulses. Continue to turn the knob until the set of amplified pulses completely disappears. Just after this time, as you continue to slowly turn the knob, you’ll see amplification of a single pulse present itself. As you get closer and closer to the optimal start time, this single amplified pulse will exhibit strong gain. At this time, toggle between the START and STOP times to maximize the isolated pulse. The single amplified pulse will look like that shown in Chapter 2, and have much greater amplitude than the set of pulses seen in the previous steps.
A.5 *Multi-pass Amplifier*

Figure 69: Numbered Bow Tie Configuration

1. Mark out a basic outline of mirror positions similar to the figure above. The specific geometry can be altered to fit the physical constraints of your system. Our precise setup is shown in Figure 15, including distances (to give an example of what an altered system may look like.) The important factors to achieve the greatest amplification are alignment of the beam to be amplified with the pump beam, and the relative beam waists of the beam to be amplified and the pump beam.

2. Turn the regenerative amplifier off.

3. Set the Ti:Sapphire crystal, and form the pump beam loop as shown in Figure 15. This is best done with the pump beam on its lowest possible setting while still producing light. Forming the loop will require lenses to focus the pump beam from each direction. The lenses should be of long focal length (on the order of 1 meter). Caution, do not set the focal point from either direction within the crystal. This could cause irreparable damage as the pump power is increased.
4. Ensure that the polarization of the pump beam is the same from both directions. We added a half-wave plate to one direction to accomplish this task. Our multi-pass amplifier is currently operating in s-polarization to match the polarization of the beam emerging from the regenerative amplifier.

5. Slowly bring up the power of the pump beam, adjusting the focal points of the lenses in the pump beam loop so that the pump beam is not too tightly focused in the crystal. The size of the beam waist of the amplified pulse from the regenerative amplifier is a good minimum approximation. Do not exceed the damage threshold of the crystal. Additionally, the pump beam from each direction should overlap so as to appear as a single beam through the crystal.

6. Turn the pump beam off.

7. Set mirror 1. Send the unamplified seed beam through the center of the crystal, as close to parallel with the pump beam as possible, and at the same inclination (hopefully zero) and elevation (hopefully constant).

8. Set mirror 2. Send the reflected light from mirror 2 to a detector in a convenient location. None of the pump beam should be part of this reflected light. We used an energy meter for detection for the following reasons: (1) an energy measurement can be directly related to peak power, (2) the detection area was large, and therefore did not require additional lenses to capture all the light, and (3) filters were not necessary as the detector was capable of handling the full output of the system.

9. Turn the pump beam on.

10. Adjust the position of the (now amplified) beam from the regenerative amplifier using mirror 1 to achieve maximum amplification.
11. Adjust the focal lengths of the pump beam from each direction to maximized amplification.

12. Repeat steps 10 and 11 until the maximum possible amplification is achieved. An amplification factor of 2 should be achievable even while operating at saturation.

13. Turn the pump off.

14. Set mirror 3. Adjust mirror 2 to cleanly strike mirror 3. Adjust mirror 3 to send the seed beam through the crystal and in the direction of the approximate future location of mirror 4. Set the detector in the place where mirror 4 will be placed.

15. Turn the pump on.

16. Make slight adjustments using mirror 3 until the signal at the detector is maximized. Do not adjust the focal lengths of the lenses for the pump beam as they have already been optimized.

17. Repeat steps 14-16, iterating the mirror number accordingly, to optimize amplification for each pass.

The output of the multi-pass amplifier should now be optimized. When operating at saturation, the total amplification should be roughly the amplification of the first pass times the number of passes.

A.6 Compressor and Water Target

Figure 70: Sketch of Compressor Set-Up

Pulses are recompressed by four strikes on the gratings with an appropriate geometry of the other elements, as shown in the next figure.
Refer to the above figures for the following instructions. The dashed line represents the second reflection from grating 1, back towards and flying over mirror 1 on its way to the target.

1. Set the half-wave plate to rotate the output beam of the multi-pass amplifier from s-polarization to p-polarization.

2. Set mirror 1 in place to redirect the light towards grating 1. Leave enough space between this mirror and the half-wave plate to allow reflection from grating 1 to pass mirror 1 and expand along its way toward grating 2. The reflection from mirror 1 to grating 1 should proceed at a slight incline. Mirror 1 is approximately 12.5 cm from the half-wave plate.

3. Set grating 1 so that the reflection from mirror 1 proceeds toward grating 2 in level flight. This will require a slight adjustment to the vertical angle of the grating. This reflection should be the 1st order reflection of the grating (which carries the most
energy). Adjust the horizontal angles of mirror 1 and grating 1 to maximize the energy of the beam reflected from grating 1 while sending the light on a straight and clear path toward grating 2. The gratings are separated by about 1.86 m.

4. Set grating 2 on a track, in the path of the light from grating 1. The track will allow smooth adjustments to the distance from grating 1 to 2. This distance will later be optimized for best performance. The current separation in gratings is approximately 1.86 m.

5. Set mirror 2, in the path of, but lower than, the light from grating 1, as shown in Figure 49. Although there is a slight horizontal angle from grating 2 to mirror 2 in the figure, the light going to mirror 2 in the actual experiment is virtually in line with the light from grating 1 to 2. Also, mirror 2 should be a large mirror so as to accept and reflect the expanded beam, which now looks like a horizontal line on the surface of a screen (piece of paper) in the vicinity of mirror 2 and grating 2. Mirror 2 is approximately 40 cm from grating 2.

6. Adjust the vertical angle of grating 2 to send the reflected beam down to mirror 2. Adjust the horizontal angle of grating 2 to reflect the 1st order, and to be approximately in line with the beam coming in from grating 1.

7. Adjust the horizontal and vertical angles of mirror 2 to reflect light back onto grating 2, but at a slightly higher position than the beam striking it from grating 1.

8. Adjust the vertical angle of mirror 2 so that the light reflected from grating 2 strikes grating 1 at a position slight higher than, but in line with, the original input beam from mirror 1.
9. Adjust the vertical angles of mirror 2 and grating 2 until the light reflected from grating 1 is in level flight above mirror 1. The beam reflected from grating 1 should now be recompressed, as seen as a spot on a screen (business card) as opposed to a horizontal line.

10. Place the long focal length lens in the path of the beam from grating 1 and traveling over mirror 1. Be cautious as the now recompressed and focused pulse has high peak energy.

11. Place the cuvette of water at or near the focal length of the long focal length lens. If grating 2 is in good position, non-linear behavior will be evident as white light generation. If white light is not observed, make slight adjustments to the position of grating 2 along its track. White light generation can therefore be used to make precise adjustments to the grating position.

12. Place the short focal length lens to capture and collimate the white light emanating from the water.

You now have a femtosecond pulsed source of white light at a repetition rate of about 8 Hz.

A.7 Glass Target

To produce the white light (or supercontinuum) described in the previous section, glass, such as BK7, may be inserted in the telescope described at the end of Chapter 2. However, plasma formation within the glass is likely if it is placed at the focal point of the long focal length lens. I recommend, therefore, to insert the glass much closer to the lens, then move it slowly toward the focal point until continuum generation begins. This is shown at the end of Chapter 2 using an 8 mm thick piece of glass.
Other non-linear behavior can also be observed in a large piece of bulk glass located within the telescope such as expanding multi-colored rings. Damage to a 15 cm length piece of glass has been observed as smoke trails from one end to the other.

A.8 Interaction with Kerr Cell

1. Remove optics after the first positive lens in the compressor set-up.

2. Replace the first positive lens in the compressor set-up with a cylindrical lens of an appropriate focal length (0.5 m or so).

3. Place Kerr Cell in the path of the focused beam. To attempt to measure an output voltage from the cell, place it so that the focused beam travels through the center of the metal plates.

4. NOISE. If significant noise is present, as was the case in our experiment, emanating from the Pockels cell power feed, turn off the laser and cover the Kerr Cell with copper mesh. Make sure the mesh is grounded to the table to remove any stray signals. We found that the cell had to be well covered. In addition to this, the coaxial line connecting the Kerr Cell to the oscilloscope had to be externally shielded by a tinned copper sleeve.

5. Create a small hole in the copper mesh surrounding the Kerr Cell to provide a path for the input beam.

6. Turn the laser system on, and attempt to measure a voltage.

7. Note: to view non-linear interactions as an output spectrum, remove the copper mesh surrounding the Kerr Cell, and observe on a screen.
APPENDIX B: SELECTED DERIVATIONS

B.1 The Fourier Transform of a Gaussian

Two important transform properties that will be used in this derivation are

\[ \mathcal{F}\left\{ \frac{df(t)}{dt}\right\} = i\omega F(\omega) \quad \text{and} \quad \mathcal{F}\{tf(t)\} = i \frac{dF(\omega)}{d\omega} \]

For a Gaussian centered at \( t = 0 \), let \( f(t) = ae^{-bt^2} \). Then,

\[ \mathcal{F}\left\{ \frac{df(t)}{dt}\right\} = \mathcal{F}\{-2b \cdot tf(t)\} \]

Using the transform properties stated above,

\[ i\omega F(\omega) = -2bi \frac{dF(\omega)}{d\omega} \]

Separating variables gives

\[ \frac{\omega}{-2b} d\omega = \frac{dF(\omega)}{F(\omega)} \]

Then,

\[ \int_{0}^{\omega} \frac{\omega'}{-2b} d\omega' = \int_{0}^{\omega'} \frac{dF(\omega')}{F(\omega')} \]

\[ -\frac{\omega^2}{4b} \left|_{0}^{\omega} \right. = \ln(F(\omega')) \left|_{0}^{\omega} \right. \]

\[ -\frac{\omega^2}{4b} = \ln(F(\omega)) - \ln(F(0)) = \ln\left(\frac{F(\omega)}{F(0)}\right) \]

Now,
\[
F(0) = \int_{-\infty}^{\infty} a e^{-bt^2} e^{-i(0)t} dt = \int_{-\infty}^{\infty} a e^{-bt^2} dt = a \sqrt{\frac{\pi}{b}} \quad \text{(see section B.2)}
\]

So,

\[
e^{-\frac{\omega^2}{4b}} = \frac{F(\omega)}{a \sqrt{\frac{\pi}{b}}}
\]

Therefore,

\[
F(\omega) = a \sqrt{\frac{\pi}{b}} e^{-\frac{\omega^2}{4b}}
\]

where \(F(\omega) = \text{the Fourier Transform of the Gaussian, } f(t) = ae^{-bt^2}\)

**B.2 The Gaussian Integral**

\[
\int_{-\infty}^{\infty} ae^{-bt^2} dt
\]

Following standard procedures, I will square the integral to demonstrate the expected result using the standard variables of the Cartesian and polar coordinate systems.

Let \(bt^2 = x^2\). Then,

\[
\left( \int_{-\infty}^{\infty} ae^{-bt^2} dt \right)^2 = \left( \frac{a}{\sqrt{b}} \int_{-\infty}^{\infty} e^{-x^2} dx \right)^2 = \frac{a^2}{b} \left( \int_{-\infty}^{\infty} e^{-x^2} dx \right)^2
\]

Now,

\[
\left( \int_{-\infty}^{\infty} e^{-x^2} dx \right)^2 = \int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-y^2} dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dxdy
\]

Since, in making the change to polar coordinates

\[r^2 = x^2 + y^2 \quad \text{and} \quad \iint_{-\infty}^{\infty} f(x, y) dA = \int_{0}^{2\pi} \int_{0}^{\infty} f(r) r dr d\theta\]

we have
\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \, dx \, dy = \int_{0}^{2\pi} \int_{0}^{\infty} e^{-r^2} r \, dr \, d\theta = \int_{0}^{2\pi} \int_{0}^{\infty} r e^{-r^2} \, dr
\]

\[
= 2\pi \int_{0}^{\infty} r e^{-r^2} \, dr
\]

Making the substitution \( u = -r^2 \),

\[
2\pi \int_{0}^{\infty} r e^{-r^2} \, dr = 2\pi \int_{0}^{\infty} \frac{e^u}{-2r} \, du = -\pi \int_{0}^{\infty} e^u \, du = -\pi e^u \Big|_{0}^{\infty} = -\pi [0 - 1] = \pi
\]

Bringing this back to the square integral gives

\[
\left( \int_{-\infty}^{\infty} e^{-x^2} \, dx \right)^2 = \pi
\]

Or,

\[
\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}
\]

Relating this to our original integral,

\[
\left( \int_{-\infty}^{\infty} a e^{-bt^2} \, dt \right)^2 = \frac{a^2}{b} \left( \int_{-\infty}^{\infty} e^{-x^2} \, dx \right)^2 = \frac{a^2 \pi}{b}
\]

Or,

\[
\int_{-\infty}^{\infty} a e^{-bt^2} \, dt = a \sqrt{\frac{\pi}{b}}
\]

B.3 The Fourier Transform of \( f(t) = \text{sech}^2(at) \)

Fourier transform definition:

\[
F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \, dt = \mathcal{F}\{f(t)\}
\]

Let \( f(t) = \text{sech}^2(at) \). Then,
\[ F(\omega) = \int_{-\infty}^{\infty} \text{sech}^2(at) e^{-i\omega t} \, dt = \int_{-\infty}^{\infty} \frac{e^{-i\omega t}}{\cosh^2(at)} \, dt = \int_{-\infty}^{\infty} \frac{e^{-i\omega t}}{\cos^2(iat)} \, dt \]

The complex plane and the Residue Theorem comes in handy in the evaluation of this integral. Therefore, let

\[ f(z) = \frac{e^{-i\omega z}}{\cos^2(iaz)} : z = \text{a complex variable} \]

This function contains poles where the denominator is zero.

\[ 0 = \cos^2(iaz) \]

\[ 0 = \cos(iaz) \]

\[ iaz = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots = \frac{(2n + 1)\pi}{2} : n = 0, 1, 2, \ldots \]

\[ z = \frac{1}{i} \frac{(2n + 1)\pi}{2a} = -\frac{i(2n + 1)\pi}{2a} = \text{poles of } f(z) \]

We can apply the Residue Theorem by first selecting an appropriate contour as \( R \to \infty \), i.e.

The blue squares represent the infinite poles along the imaginary axis. The contour integral is then
\[ \oint_C f(z) \, dz = \oint_{C_1} f(z) \, dz + \oint_{C_2} f(z) \, dz \]

To evaluate the integral of contour \( C_2 \), let \( z = Re^{-i\theta} : 0 \leq \theta \leq \pi \), as \( R \to \infty \). Then

\[ f(z) = \frac{e^{-i\omega z}}{\cos^2(iaz)} = \frac{e^{-i\omega Re^{-i\theta}}}{\cosh^2(aRe^{-i\theta})} \]

and

\[ dz = -iRe^{-i\theta} \, d\theta \]

So,

\[ \oint_{C_2} f(z) \, dz = \lim_{R \to \infty} \int_0^\pi e^{-i\omega Re^{-i\theta}} \frac{1}{\cosh^2(aRe^{-i\theta})} (-iRe^{-i\theta}) \, d\theta \]

\[ = \lim_{R \to \infty} \left[ \int_0^\pi e^{-i\omega R \cos \theta} e^{-\omega R \sin \theta} (-iRe^{-i\theta}) \frac{1}{\cosh^2(aRe^{-i\theta})} \, d\theta \right] \]

\[ = \lim_{R \to \infty} \left[ \int_0^\pi -ie^{-i(\theta + \omega R \cos \theta)} Re^{-\omega R \sin \theta} \frac{1}{\cosh^2(aRe^{-i\theta})} \, d\theta \right] \]

\[ = 0 \text{ (by Jordan's Lemma)} \]

Therefore, in the limit as \( R \to \infty \)

\[ \oint_C f(z) \, dz = \oint_{C_1} f(z) \, dz \]

Now, by the Residue Theorem (when the rotation is clockwise),

\[ \oint_C f(z) \, dz = -2\pi i \sum_{\text{poles}} \text{residues} \]

To determine the residues we take advantage of the Laurent expansion of \( f(z) \). This requires us to find the coefficient of the \( z - z_0 \) term of the expansion. In our case, since
there are an infinite number of poles, we will find the coefficient of the $z - z_n$ term of the expansion. This will define the residue of the nth pole. Therefore, let

$$\Delta z = z - z_n = \text{the neighborhood of the nth pole}$$

where $z_n = -\frac{i(2n + 1)\pi}{2a}$

Then,

$$f(z) = f(z_n + \Delta z) = \frac{e^{-i\omega(z_n + \Delta z)}}{\cos^2(ia(z_n + \Delta z))}$$

Focusing on the cosine term in the denominator,

$$\cos(ia(z_n + \Delta z)) = \cos\left(ia \left(-\frac{i(2n + 1)\pi}{2a} + \Delta z\right)\right) = \cos\left(\frac{(2n + 1)\pi}{2} + ia\Delta z\right)$$

$$= \cos\left(\frac{(2n + 1)\pi}{2}\right)\cos(ia\Delta z) - \sin\left(\frac{(2n + 1)\pi}{2}\right)\sin(ia\Delta z)$$

$$= -\sin\left(\frac{(2n + 1)\pi}{2}\right)\sin(ia\Delta z) \approx (-1)^{n+1}ia\Delta z \text{ as } \Delta z \to 0$$

So,

$$\cos^2(ia(z_n + \Delta z)) \approx \left[(-1)^{n+1}ia\Delta z\right]^2 = -a^2(\Delta z)^2$$

We now have

$$f(z_n + \Delta z) = \frac{e^{-i\omega(z_n + \Delta z)}}{-a^2(\Delta z)^2} = \frac{-a^{-2}e^{-i\omega z_n}e^{-i\omega\Delta z}}{(\Delta z)^2} \approx \frac{-a^{-2}e^{-i\omega z_n}(1 - i\omega\Delta z)}{(\Delta z)^2}$$

$$= \frac{-a^{-2}e^{-i\omega z_n} + a^{-2}e^{-i\omega z_n}i\omega\Delta z}{(\Delta z)^2} = \frac{-a^{-2}e^{-i\omega z_n} + a^{-2}e^{-i\omega z_n}i\omega}{\Delta z}$$

Or,

$$f(z) = f(z_n + \Delta z) = \frac{a^{-2}e^{-i\omega z_n}i\omega}{\Delta z} + \frac{-a^{-2}e^{-i\omega z_n}}{(\Delta z)^2}$$

The residue of the nth pole is then
The sum of all residues is

\[ \sum_{n=0}^{\infty} R_n = \sum_{n=0}^{\infty} \frac{a^{-2} e^{-i\omega z_n i\omega}}{a^2} = \frac{i\omega}{a} \sum_{n=0}^{\infty} e^{-i\omega z_n} = \frac{i\omega}{a^2} \sum_{n=0}^{\infty} e^{-i\omega \left( -\frac{(2n+1)\pi}{2a} \right)} \]

Now,

\[ e^{-i\omega \left( -\frac{(2n+1)\pi}{2a} \right)} = e^{-\frac{n\pi}{a}} e^{-\frac{\omega \pi}{2a}} \]

So,

\[ \sum_{n=0}^{\infty} R_n = \frac{i\omega}{a^2} \sum_{n=0}^{\infty} e^{-\frac{n\pi}{a}} e^{-\frac{\omega \pi}{2a}} = \frac{i\omega}{a^{2}} e^{-\frac{\omega \pi}{2a}} \sum_{n=0}^{\infty} \frac{e^{-\frac{\omega \pi}{2a}}}{a} \]

The sum can be determined as follows. Let \( x = -\frac{\omega \pi}{a} \)

\[ \sum_{n=0}^{\infty} e^{nx} = 1 + e^x + e^{2x} + \cdots = S \]

\[ e^{-x}S = e^{-x} + e^x + e^{2x} + \cdots = e^{-x} + S \]

Then, after basic manipulations,

\[ S = \frac{e^{-x}}{e^{-x} - 1} = \frac{e^{\frac{x}{2}}}{e^{\frac{x}{2}} - 1} = \frac{e^{-\frac{x}{2}}}{e^{-\frac{x}{2}} - e^{-\frac{x}{2}}} = -\frac{1}{2} \cdot \frac{2e^{-\frac{x}{2}}}{e^{\frac{x}{2}} - e^{-\frac{x}{2}}} = -\frac{e^{-\frac{x}{2}}}{2} \cdot \frac{2}{e^{\frac{x}{2}} - e^{-\frac{x}{2}}} \]

Or,

\[ S = -\frac{1}{2} e^{-\frac{x}{2}} \text{csch} \left( \frac{x}{2} \right) \]

Then

\[ \sum_{n=0}^{\infty} e^{-\frac{n\omega \pi}{a}} = -\frac{1}{2} e^{\frac{\omega \pi}{2a}} \text{csch} \left( -\frac{\omega \pi}{2a} \right) = \frac{1}{2} e^{\frac{\omega \pi}{2a}} \text{csch} \left( \frac{\omega \pi}{2a} \right) \]

This gives the sum of the residues as
\[
\sum_{n=0}^{\infty} R_n = \frac{i\omega}{a^2} e^{-\frac{\omega \pi}{2a}} \sum_{n=0}^{\infty} e^{-\frac{n\omega \pi}{a}} = \frac{i\omega}{a^2} e^{-\frac{\omega \pi}{2a}} \left[ \frac{1}{2} e^{\frac{\omega \pi}{2a}} \operatorname{csch} \left( \frac{\omega \pi}{2a} \right) \right] = \frac{i\omega}{2a^2} \operatorname{csch} \left( \frac{\omega \pi}{2a} \right)
\]

The contour integral in question is then

\[
\oint_C f(z) \, dz = -2\pi i \sum \text{residues} = -2\pi i \left[ \frac{i\omega}{2a^2} \operatorname{csch} \left( \frac{\omega \pi}{2a} \right) \right] = \frac{\omega \pi}{a^2} \operatorname{csch} \left( \frac{\omega \pi}{2a} \right)
\]

This brings us back to our original integral. As \( R \to \infty \)

\[
F(\omega) = \int_{-\infty}^{\infty} \operatorname{sech}^2(at) e^{-i\omega t} \, dt = \oint_{C_1} f(z) \, dz
\]

Therefore,

\[
F(\omega) = \int_{-\infty}^{\infty} \operatorname{sech}^2(at) e^{-i\omega t} \, dt = \frac{\omega \pi}{a^2} \operatorname{csch} \left( \frac{\omega \pi}{2a} \right)
\]

**B.4 The Inverse Fourier Transform:**

\[
g(t) = \mathcal{F}^{-1} \left\{ \mathcal{F}\{\operatorname{sech}^2(at)\} \right\}
\]

From section B.3,

\[
\mathcal{F}\{\operatorname{sech}^2(at)\} = \frac{\omega \pi}{a^2} \operatorname{csch} \left( \frac{\omega \pi}{2a} \right)
\]

From section B.1 (with a change of constants to avoid confusion),

\[
\mathcal{F}\{b e^{-ct^2}\} = b \sqrt{\frac{\pi}{c}} e^{-\frac{\omega^2}{4c}}
\]

Plugging these back into \( g(t) \) gives

\[
g(t) = \mathcal{F}^{-1} \left\{ \frac{\omega \pi}{a^2} \operatorname{csch} \left( \frac{\omega \pi}{2a} \right) \right\}
\]

\[
\left\{ b \sqrt{\frac{\pi}{c}} e^{-\frac{\omega^2}{4c}} \right\}
\]
\[ g(t) = \frac{\sqrt{\pi c}}{a^2 b} \mathcal{F}^{-1} \left\{ \frac{\omega e^{k\omega^2}}{\sinh \left( \frac{\omega \pi}{2a} \right)} \right\} : k = \frac{1}{4c} \]

To leave out the pre-factor in the continued derivation, let

\[ g(t) = \frac{\sqrt{\pi c}}{a^2 b} h(t) \]

Then,

\[ h(t) = \mathcal{F}^{-1} \left\{ \frac{\omega e^{k\omega^2}}{\sinh \left( \frac{\omega \pi}{2a} \right)} \right\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\omega e^{k\omega^2}}{\sinh \left( \frac{\omega \pi}{2a} \right)} e^{i\omega t} d\omega \]

To perform this integration, we will again take advantage of the complex plane and the Residue Theorem by letting

\[ f(z) = \frac{ze^{kz^2} e^{itz}}{\sinh \left( \frac{z \pi}{2a} \right)} \]

To determine where the denominator is equal to zero, notice

\[ \sinh \left( \frac{z \pi}{2a} \right) = -i \sin \left( i \frac{z \pi}{2a} \right) \]

Then,

\[ -i \sin \left( i \frac{z \pi}{2a} \right) = 0 \]

\[ i \frac{z \pi}{2a} = -n\pi : n = 0, 1, 2, ... \]
\[ z = \frac{-2an}{i} = i \cdot 2an = \text{poles of } f(z) \]

An appropriate contour for integration in the complex plane is shown below. The blue dots represent the infinite poles along the imaginary axis.

\[
\oint_C f(z)\,dz = \oint_{C_1} f(z)\,dz + \oint_{C_2} f(z)\,dz + \oint_{C_3} f(z)\,dz + \oint_{C_4} f(z)\,dz
\]

To evaluate the integral of contour \( C_2 \), let \( z = Re^{i\theta} \) as \( R \to \infty \). Then

\[
\oint_{C_2} f(z)\,dz = \lim_{R \to \infty} \int_0^{\pi} Re^{i\theta} e^{k(Re^{i\theta})^2} e^{ltRe^{i\theta}} iRe^{i\theta} d\theta
\]

Notice that

\[ e^{k(Re^{i\theta})^2} = e^{kR^2e^{i(2\theta)}} = e^{kR^2(\cos(2\theta)+i\sin(2\theta))} = e^{kR^2 \cos(2\theta)} e^{ikR^2 \sin(2\theta)} \]

and

\[ e^{ltRe^{i\theta}} = e^{ltR(\cos \theta+i \sin \theta)} = e^{ltR \cos \theta} e^{-tR \sin \theta} \]

So,

\[ e^{k(Re^{i\theta})^2} e^{ltRe^{i\theta}} = e^{(kR^2 \cos(2\theta)-tR \sin \theta)} e^{i(kR^2 \sin(2\theta)+tR \cos \theta)} \]

Since both \( \cos(2\theta) \) and \( \sin(2\theta) \) have an average of zero over \([0,\pi]\),
\[ e^{k(Re^{i\theta})^2}e^{itRe^{i\theta}} \rightarrow e^{(-tR\sin \theta)}e^{(tR \cos \theta)} \]

as the integration is performed from \([0, \pi]\). Then, as \(R \rightarrow \infty\)

\[ e^{(-tR \sin \theta)}e^{(tR \cos \theta)} \rightarrow 0 \]

and

\[ \frac{Re^{i\theta} \cdot iRe^{i\theta}}{\sinh \left( \frac{\pi}{2a}Re^{i\theta} \right)} \rightarrow 0 \]

Therefore,

\[ \oint_{C_2} f(z) \, dz = 0. \]

To evaluate the integral of contour \(C_4\), let \(z = re^{i\theta}\) as \(r \rightarrow 0\). Then

\[ \oint_{C_4} f(z) \, dz = \lim_{r \rightarrow 0} \int_{0}^{\pi} \frac{re^{i\theta}e^{k(r e^{i\theta})^2}e^{itre^{i\theta}}ire^{i\theta}}{\sinh \left( \frac{\pi}{2a}re^{i\theta} \right)} \, d\theta \]

\[ \rightarrow \int_{0}^{\pi} \lim_{r \rightarrow 0} \frac{re^{i\theta}e^{k(r e^{i\theta})^2}e^{itre^{i\theta}}ire^{i\theta}}{re^{i\theta}} \, d\theta \]

\[ \rightarrow \int_{0}^{\pi} \lim_{r \rightarrow 0} e^{k(r e^{i\theta})^2}e^{itre^{i\theta}}ire^{i\theta} \, d\theta = 0 \]

Therefore,

\[ \oint_{C_4} f(z) \, dz = 0 \]

This leaves us with

\[ \oint_{C} f(z) \, dz = \oint_{C_1} f(z) \, dz + \oint_{C_3} f(z) \, dz = 2\pi \cdot h(t) \]

By the Residue Theorem,

\[ \oint_{C} f(z) \, dz = 2\pi i \sum \text{residues} \]
Therefore,

\[ 2\pi \cdot h(t) = 2\pi i \sum_{\text{poles}} \text{residues} \]

Or,

\[ h(t) = i \sum_{\text{poles}} \text{residues} \]

To determine the residues we take advantage of the Laurent expansion of \( f(z) \). This requires us to find the coefficient of the \( z - z_0 \) term of the expansion. In our case, since there are an infinite number of poles, we will find the coefficient of the \( z - z_n \) term of the expansion. This will define the residue of the \( n \)th pole. Therefore, let

\[ \Delta z = z - z_n = \text{the neighborhood of the } n\text{th pole} \]

where \( z_n = i \cdot 2an \)

The expansion of \( f(z) \) is as follows:

\[ f(z) = f(z_n + \Delta z) = \frac{(z_n + \Delta z)e^{k(z_n+\Delta z)^2}e^{it(z_n+\Delta z)}}{\sinh\left(\frac{\pi}{2a}(z_n + \Delta z)\right)} \]

Focusing on the denominator,

\[ \sinh\left(\frac{\pi}{2a}(z_n + \Delta z)\right) = -i \sin\left(\frac{i \pi}{2a}(z_n + \Delta z)\right) = -i \sin\left(\frac{i \pi}{2a}z_n + i \frac{\pi}{2a} \Delta z\right) \]

\[ = -i \left[ \sin\left(\frac{i \pi}{2a}z_n\right) \cos\left(\frac{i \pi}{2a} \Delta z\right) + \cos\left(\frac{i \pi}{2a}z_n\right) \sin\left(\frac{i \pi}{2a} \Delta z\right) \right] \]

\[ = -i \left[ \sin\left(\frac{i \pi}{2a}(i \cdot 2an)\right) \cos\left(\frac{i \pi}{2a} \Delta z\right) + \cos\left(\frac{i \pi}{2a}(i \cdot 2an)\right) \sin\left(\frac{i \pi}{2a} \Delta z\right) \right] \]

\[ = -i \left[ \sin(-n\pi) \cos\left(\frac{i \pi}{2a} \Delta z\right) + \cos(-n\pi) \sin\left(\frac{i \pi}{2a} \Delta z\right) \right] \]

\[ = -i \left[ (-1)^n \sin\left(\frac{i \pi}{2a} \Delta z\right) \right] \]
\[
\approx -i \left[ (-1)^n \left( i \frac{\pi}{2a} \Delta z \right) \right] \text{ as } \Delta z \to 0
\]

Therefore,

\[
\sinh \left( \frac{\pi}{2a} (z_n + \Delta z) \right) \approx \frac{(-1)^n \pi \Delta z}{2a}
\]

The numerator of our expansion becomes

\[
(z_n + \Delta z) e^{k(z_n+\Delta z)^2} e^{it(z_n+\Delta z)} \to (z_n) e^{k(z_n)^2} e^{it(z_n)} \text{ as } \Delta z \to 0
\]

\[
(z_n) e^{k(z_n)^2} e^{it(z_n)} = i \cdot 2an \cdot e^{k(i2an)^2} e^{it(i2an)}
\]

\[
= i \cdot 2an \cdot e^{-k(2an)^2} e^{-t(2an)}
\]

Then,

\[
f(z) = f(z_n + \Delta z) = \frac{i \cdot 2an \cdot e^{-k(2an)^2} e^{-t(2an)}}{(-1)^n \pi \Delta z/2a}
\]

Which, after a little manipulation, gives

\[
f(z_n + \Delta z) = \frac{i \cdot (-1)^{n-1} \cdot 4a^2 n \cdot e^{-k(2an)^2} e^{-t(2an)}}{\Delta z}
\]

The nth residue is then

\[
R_n = i \cdot (-1)^n \pi^{-1} \cdot 4a^2 n \cdot e^{-k(2an)^2} e^{-t(2an)}
\]

The sum of the residues is

\[
\sum_{n=0}^{\infty} R_n = \sum_{n=0}^{\infty} i \cdot (-1)^n \pi^{-1} \cdot 4a^2 n \cdot e^{-k(2an)^2} e^{-t(2an)}
\]

\[
= \frac{i \cdot 4a^2}{\pi} \sum_{n=0}^{\infty} (-1)^n n e^{-k(2an)^2} e^{-t(2an)}
\]

\[
= \frac{i \cdot 4a^2}{\pi} \sum_{n=0}^{\infty} (-1)^n n e^{-\left(\frac{1}{4c}\right)(2an)^2} e^{-t(2an)}
\]
$$= \frac{i \cdot 4a^2}{\pi} \sum_{n=0}^{\infty} (-1)^n n e^{-(\frac{4a^2}{4c})n^2} e^{-2atn}$$

$$= \frac{i \cdot 4a^2}{\pi} \sum_{n=0}^{\infty} (-1)^n n e^{-(\frac{a^2}{c})n^2} e^{-2atn}$$

Given,

$$g(t) = \sqrt{\frac{\pi c}{a^2 b}} h(t) = \sqrt{\frac{\pi c}{a^2 b}} i \sum \text{residues}$$

We have

$$g(t) = \frac{\sqrt{\pi c}}{a^2 b} i \cdot \frac{4a^2}{\pi} \sum_{n=0}^{\infty} (-1)^n n e^{-(\frac{a^2}{c})n^2} e^{-2atn}$$

$$= -\frac{4}{b} \sqrt{\frac{c}{\pi}} \sum_{n=0}^{\infty} (-1)^n e^{-(\frac{a^2}{c})n^2} n e^{-2atn}$$

$$= -\frac{4}{b} \sqrt{\frac{c}{\pi}} \cdot \frac{d}{dt} \left\{ -\frac{1}{2a} \sum_{n=0}^{\infty} (-1)^n e^{-(\frac{a^2}{c})n^2} e^{-2atn} \right\}$$

$$= \frac{2}{ab} \sqrt{\frac{c}{\pi}} \cdot \frac{d}{dt} \left\{ \sum_{n=0}^{\infty} (-1)^n e^{-(\frac{a^2}{c})n^2} e^{-2atn} \right\}$$

Then, thus far

$$g(t) = \frac{2}{ab} \sqrt{\frac{c}{\pi}} \cdot \frac{d}{dt} \left\{ \sum_{n=0}^{\infty} (-1)^n e^{-(\frac{a^2}{c})n^2} e^{-2atn} \right\}$$

Producing a closed form solution of the remaining sum has been difficult. I leave it to the reader to attempt the feat. Nevertheless, a numeric approximation can be made from a previous form, namely

$$g(t) = -\frac{4}{b} \sqrt{\frac{c}{\pi}} \sum_{n=0}^{\infty} (-1)^n e^{-(\frac{a^2}{c})n^2} n e^{-2atn}$$
Relating this form to the full-width, half-maximum (FWHM) definition of the pulse width, we have

\[ a = \frac{1.76}{\tau_p} : \tau_p = \text{pulse width of sech}^2(at) \]

and

\[ c = \frac{4 \ln(2)}{\epsilon^2} : \epsilon = \text{pulse width of the Gaussian}, e^{-ct^2} \]

This gives the pre-factor of \( g(t) \) as

\[ -\frac{4}{b} \sqrt{\frac{c}{\pi}} = -\frac{4}{b} \sqrt{\frac{4 \ln(2)}{\pi \epsilon^2}} = -\frac{8}{b \epsilon} \sqrt{\frac{\ln(2)}{\pi}} \]

One of the exponents is

\[ -\left( \frac{a^2}{c} \right) n^2 = -\left( \frac{1.76}{\tau_p} \right)^2 \left( \frac{\epsilon^2}{4 \ln(2)} \right) n^2 = -\left( \frac{1.76 \epsilon n}{\tau_p^2} \cdot 4 \ln(2) \right) = -\left( \frac{1.76 \epsilon n}{2 \tau_p} \right)^2 \cdot \frac{1}{\ln(2)} \]

\[ = -\left( \frac{1.76 \epsilon n}{2 \tau_p \sqrt{\ln(2)}} \right)^2 \]

The other exponent is

\[ -2atn = -2 \frac{1.76}{\tau_p} \cdot tn = -\frac{3.52 tn}{\tau_p} \]

Therefore, in terms of the pulse widths defined above,

\[ g(t) = -\frac{8}{b \epsilon} \sqrt{\frac{\ln(2)}{\pi}} \sum_{n=0}^{\infty} (-1)^n e^{-\left( \frac{1.76 \epsilon n}{2 \tau_p \sqrt{\ln(2)}} \right)^2} \cdot \frac{3.52 tn}{\tau_p} \]

Finally, the “∙” symbol has been added below to highlight the individual factors of the sum.
\[g(t) = -\frac{8}{b\epsilon} \sqrt{\frac{\ln(2)}{\pi}} \sum_{n=0}^{\infty} (-1)^n \cdot e^{-\left(\frac{1.76n}{2\tau_p \sqrt{\ln(2)}}\right)^2} \cdot n \cdot e^{-\left(\frac{3.52tn}{\epsilon}\right)^2}\]

The Matlab code below was used to generate a numeric approximation of the above function given the normalized Gaussian amplitude of \(b = 1\). Original data was produced by taking pictures of the stretched pulse signal and system impulse response signals, which were then transferred to numeric files using free imaging software. These numeric files were normalized and centered before importing to Matlab.

```matlab
% Assign Matlab Variables from Data
s = ToMatlab.str_mV; % stretched pulse signal amplitude data
d = ToMatlab.d_mV; % delta response signal amplitude data (a.k.a. system impulse response)
ts = ToMatlab.str_ps; % stretched pulse time data
td = ToMatlab.d_ps; % delta response time data

% Curve Fitting - Stretched Pulse - Sech Squared Assumption
FWHM = 95; % beginning estimate of pulse width (in ps) - guess and check!
sfit = (sech(1.76*(ts-250)/FWHM)).^2; % possible fit, centered at 250 ps
H = 0.5*ones(1,length(ts));

% Curve Fitting - Delta Response - Gaussian Assumption
fwhm = 43; % beginning estimate of pulse width (in ps) - guess and check!
dfit = exp(-4*log(2)*((td-250)/fwhm).^2); % possible fit, centered at 250 ps
J = 0.5*ones(1,length(td)); % half-maximum

% Fit checks
figure;
subplot(2,1,1)
plot(ts,s,ts,sfit) % plot stretched pulse signal data against sech squared fit
xlim([min(ts),max(ts)])
title(['Stretched Pulse - Sech Squared Fit - FWHM Est. is ',num2str(FWHM),' ps'])
xlabel('ps')

subplot(2,1,2)
plot(td,d,td,dfit) % plot delta response signal data against Gaussian fit
xlim([min(td),max(td)])
title(['Delta Response - Gaussian Fit - FWHM Est. is ',num2str(fwhm),' ps'])
xlabel('ps')
```
% Common Time - Smooth Functions
% common time domain for smooth functions
% stretched pulse in common time
% delta response in common time
xlabel('ps')

% Original Data to Smooth Functions Comparison
figure;
subplot(2,1,1)
plot(ts,s,td,d)
title('Original Data')
xlabel('ps')
subplot(2,1,2)
plot(t,sc,t,dc)
title('Defined Functions')
xlabel('ps')

% Numeric Approximation - Analysis Shown in David's Thesis
dpw = fwhm; % pulse width of delta response in ps
spw = FWHM; % stretched pulse width in ps
T = (-5000:5000)'; % time domain in ps
k = (0:length(T)-1); % index for summation
Sn = (-1).^k.*exp(-(1.76*dpw/spw)^2*(k.^2)/(4*log(2))).*k.*exp(-2*T.*k*(1.76/spw)); % summand
P = (-8/spw)*sqrt(log(2)/pi); % pre-factor
S = sum(Sn,2); % sums each row of the summand, listed as a column vector
g = P*S; % brings back the pre-factor
gnorm = g/max(g); % normalized amplitude
figure;
plot(T,gnorm); % plot of estimated physical pulse by numeric approximation
xlim([-300,300])
title('Result of Numeric Approximation')
xlabel('ps')

% Define Physical Pulse by Symmetry
% right side of numeric approximation
T_right = T(T>=0); % time values on the right side of numeric approximation
T_left = T(T<=0); % time values on the left side of numeric approximation
% new definition of left side of numeric approximation based on symmetry
gnew = [g_left(g_left<1);g_right]; % new definition of the physical pulse based on symmetry
hm = 0.5*ones(1,length(T)); % half-max line to determine the FWHM of the physical pulse
T_sh = T+250; % shifted time variable to compare physical pulse with data that is centered at 250 ps
figure;
plot(T,hm,T,gnew) % plot half-max to determine FWHM of the estimated physical pulse
xlim([-300,300])
title('Estimated Physical Pulse')
xlabel('ps')

figure;
subplot(2,1,1)
plot(t,sc,t,dc,T,gnew) % plot all smooth function approximations
xlim([-250,250])
title('Comparison of Smooth Functions')
xlabel('ps')

subplot(2,1,2)
plot(ts,s,td,d,T_sh,gnew) % plot estimated physical pulse with all other data
xlim([0,500])
title('Comparison of Data to Estimated Physical Pulse')
xlabel('ps')
REFERENCES


