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Paradigms and Logical Morphology in Wittgenstein's Philosophy

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Paradigms and Logical Morphology in Wittgenstein's Philosophy

by

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DISSERTATION

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Dedication

The amount of chiaroscuro an idea harbors is the only index of its profundity.

– Emil Cioran

Yet insofar as everything I am saying has been said many times before and with great force, it seems impossible that the world has continued each time such words have rung out… If I now repeat it insofar as I am able, because I do so in a manner to amuse no one – without philosophical dignity or artistic correctness, but rather as a poor pedestrian measuring the terrain with his steps – if I do not pay entrance in any established categories or establish precedents for some new category, even in the best cases, I will have created… a doctoral thesis.

– Carlo Michelstaedter

Acknowledgments

Pouring the famed juice of the Danube
Into the depth of my head,
I shall drink and remember
The cry of the bright ones: “I am coming at you!”

– Velimir Khlebnikov

What I have written here – if it has actually been written well – stands as nothing other than a testament to the loving memory of my father. In fact, when, or better, if, I ever manage to put down something that is worth the very same ink and paper on which it happens to be scribbled, then all the credit must go to him, to his spirit. In the cases where this might occur, however rare they may be, whatever I write, I write for the both of us.

In this plane of existence, here and now, the immeasurable debt of my gratitude belongs to the two most important women in my life: my wife and my mother, who are listed in an order that I can only leave to the psychoanalysts to unravel. Likewise, since the two of us bear another kind of irrevocable bond, I also have to give an individual shout-out to my twin brother, who was my best man, and will always be my best friend.

In addition, Santos Rivera and Elliot Madison deserve the highest praise for always and invariably performing the ostensibly impossible task of helping me piece together whatever shards were left strewn about in the aftermath of my periodic bouts of catastrophic self-doubt.

Besides that, I can only give a broad and sweeping “thank you” to all of the countless and regrettably unnamed thinkers, writers, artists, and comrades who have allowed me to stand at their side in their undaunting struggle for truth and humanity.

Finally, for all those who have ever insulted my intelligence, more than likely due to a thinly disguised jealousy for the magnificent hue of my aristocratic skin tone, let it be known that this is only the initial phase in my settling of scores. As Shalamov had once wisely instructed, it is only by taking up the old Slavic tradition, which he attributes to the tempestuous legacy of Sviatoslav, that revenge can finally arrive at its rightful conclusion.

– Idris Robinson, 2023

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ABSTRACT

This dissertation investigates the notion of a paradigm and its related morphological method through the lens of the philosophy of Ludwig Wittgenstein (1889–1951). At once exegetical and philosophical, it is a project that aims to trace the progression of Wittgenstein's thought, from his early to his mature phase, in order to arrive at a twofold conclusion regarding the ontological status of paradigms and a closely related approach to logic grounded in morphology.

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Introduction

This dissertation investigates the notion of a paradigm and its related morphological method through the lens of the philosophy of Ludwig Wittgenstein (1889–1951). At once exegetical and philosophical, it is a project that aims to trace the progression of Wittgenstein's thought, from his early to his mature phase, in order to arrive at a twofold conclusion regarding the ontological status of paradigms and a closely related approach to logic grounded in morphology.

A good entrance into the discussion would be to simply start by going over some of the terminology. In this regard, it should be noted that Wittgenstein conceives of a paradigm, in accordance with the standard definition of the term as "A pattern or model, an exemplar; (also) a typical instance of something, an example."[1] Likewise, he synonymously refers to a paradigm as an example, model, sample, standard, schema, archetype (*Urtyp*), proto-picture (*Urbild*), and proto-phenomenon (*Urphänomen*).[2] However, going far beyond the ordinary conception of a paradigm, Wittgenstein will also employ the notion in the course of dispelling some of the most deep-seated philosophical confusions about the nature of language and existence as such. To be more specific, for Wittgenstein, a paradigm also serves as a "means of presentation [*Mittel der Darstellung*]" and an "instrument of language [*Instrument der Sprach*]," thereby conditioning what can be meaningfully said about the existence of an object and the

[1] *Oxford English Dictionary*, 2nd ed. (Oxford: Oxford University Press, 2022), s.v. "Paradigm, n. 1."

[2] See, for example, Ludwig Wittgenstein, *Philosophical investigations*, ed. P. M. S. Hacker and Joachim Schulte, trans. G. E. M. Anscombe, Rev. 4th ed (Malden, MA: Wiley-Blackwell, 2009), §§50–57, 73–74, 86, 134, 141, 163, 654. From here on, abbreviated as *PI*.

properties ascribed to it.[3] Wittgenstein will then attempt to settle a wide range of philosophical problems by elucidating how paradigms, which stand at the midpoint between conceptual presentation and mundane objects, confer form and structure upon the facts, propositions, and activities in a given context, system, or language-game.

In close connection with paradigms, I will also investigate Wittgenstein's mature approach to philosophy by way of his appropriation of a particular kind of morphological method, which involves the comparing and the contrasting of the *formal* attributes of schemas, propositions, objects, and various human activities. As it will be understood throughout this dissertation, morphology can be broadly defined as an investigation or examination of forms, patterns, or structures.[4] Thus, strictly speaking, morphology is a *lōgos* of *morphē*; or, in other words, a logical account of form. Moreover, the morphological method will encompass various kinds of analogical reasoning, as it will frequently demand a delineation of the similarities and differences that can be distinguished between various forms.

As the primary vehicle for the comparison and the exhibition of form, the paradigm will thereby play a central role in this methodological framework and can even be distinguished as the morphological tool *par excellence*. Indeed, in what is perhaps the earliest systematic account of a paradigm, Plato indicates how the basic practice of comparing similarities and differences through physical juxtaposition is etymologically

[3] Following the increasing consensus around Stanley Cavell's suggested translation, I will consistently render "*Darstellung*" as "presentation," rather than "representation," as is recommended in *This New yet Unapproachable America: Lectures after Emerson after Wittgenstein*, University of Chicago Press ed, Frederick Ives Carpenter Lectures 1987 (Chicago: University of Chicago Press, 2013), 22, 38–39.
[4] Kristijan Krkač, *A Custodian of Grammar: Essays on Wittgenstein's Philosophical Morphology* (Lanham: University Press of America, Inc, 2012), 77.

implicit within the compound of *parā* ("beside," "near," "contrary to") and *deīknumi* ("to show," "to point out," "display"):

> Place them beside [*parā*] what is not yet distinguished; and by comparing them alongside each other [*parabāllontas*], show [*edeiknūnai*] that the same nature and similarities are in both complexes. Until the things that are being conceived correctly have been shown [*deichthēi*] place them beside in comparison [*paratithēmena*] with all of things that are not being comprehended; and being shown [*deichthēnta*] in this way they become paradigms [*paradeīgmata*]… So then, we have indeed sufficiently grasped this now that there is, in fact, the creation of a paradigm [*paradeīmatōs*]. (*Statesman*, 278a–d)[5]

In the course of elaborating his notorious standard meter example, Wittgenstein further confirms the morphological aspects of the *Paradigma* by succinctly defining it as "something with which comparisons [*verglichen*] are made (*PI*: §50)."

Hence, the main thesis of this dissertation is twofold. *First*, I will argue that Wittgenstein's development of the notion of a paradigm throughout his *Nachlass* is an engagement with first philosophy and presents a novel solution to the age-old problems of being and existence. This implies that certain quietest, "end of philosophy" readings of Wittgenstein amount to a misinterpretation. Similarly, it is also a mistake to liken the Wittgensteinian paradigm to the relativist conceptions that have become prevalent after Thomas Kuhn's *The Structure of Scientific Revolutions*.[6] In fact, rather than standing at the apex of linguistic idealism, conventionalism, or any other associated anti-philosophical trend, I show how Wittgenstein can be interpreted as striving to surpass his earlier solution to the classic dichotomy between realism and idealism in the *Tractatus*

[5] My translation.
[6] Thomas S. Kuhn, *The Structure of Scientific Revolutions*, 3rd ed (Chicago, IL: University of Chicago Press, 1996).

Logico-Philosophicus (5.64)[7] by instead providing a thoroughly realist and even materialist account of a paradigm.[8] As is intimated by the base "*-phänomen*" in *Urphänomen* (*PI*: §654), it is something that can, perceptively, be either touched, seen, heard, smelled, or, just as well, tasted: turning the Platonic *eīdos* on its head, a paradigm is, therefore, an idea brought down from the formal realm, to our more familiar corporeal domain, so that it is now, finally and fully, materialized.

Second, I will argue that Wittgenstein's other fundamental breakthrough can be aptly described as "logicizing morphology."[9] In this regard, my thesis aims to substantiate and trace the implications of what might initially appear to be a somewhat equivocal formulation. For now, we can at least preliminarily comprehend its meaning by taking seriously Wittgenstein's own description of his methodology, and also by observing how he was historically poised at the intersection of two disparate traditions of thought.

The first tradition maintains that philosophical inquiry *must always* be conducted by means of logic. Accordingly, from the initial steps of his philosophical journey, Wittgenstein had indeed situated himself in this tradition, as a precocious student of Gottlob Frege's and Bertrand Russell's advancements in formal symbolic logic and as a

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[7] Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, trans. C. K. Ogden et al., Side-By-Side-By-Side Edition, Version 0.63 (July 14, 2022) (London, UK: Kegan Paul, 1922). From here on, abbreviated as *TLP*. Also, many of the translations below have been modified, since I have relied on the original German as well as the two English translations included in the Side-By-Side-By-Side Edition.

[8] Following what Wittgenstein set forth in his October 15th, 1916 pre-Tractarian notebook entry, I take solipsism to be a more extreme and individualized variant of the more philosophically general idealist position: "Idealism singles men out from the world as unique, solipsism singles me alone out…" in *Notebooks 1914–1916*, ed. Georg Henrik von Wright and G. E. M. Anscombe, trans. G. E. M. Anscombe, 2. ed., repr (Oxford: Blackwell, 1979), 85. From here on, abbreviated as *NB*.

[9] I borrow this phrase from Juliet Floyd, "Chains of Life: Turing, *Lebensform*, and the Emergence of Wittgenstein's Later Style," *Nordic Wittgenstein Review* 5, no. 2 (2016): 61; and "*Lebensformen*: Living Logic," in *Language, Form(s) of Life, and Logic*, ed. Christian Martin (De Gruyter, 2018), 78.

rabid critic of the Aristotelian shadow cast upon logic for millennia.[10] Furthermore, we must also acknowledge that the mature Wittgenstein was not at all speaking metaphorically when he claimed that "What I give is the morphology of the use of an expression."[11] That is to say, Wittgenstein's persistent fixation on symbolic expression bids us to carefully heed his constant reminders that, from the very beginning until his late period, his task was that of a logician.[12] Indeed, this way of understanding Wittgenstein will prove to be in line with the growing body of scholarship that attempts to evaluate his merits, even after his post-Tractarian return to "professional" philosophy, as a contribution to the philosophy of logic, instead of the now outdated ordinary language philosophy.[13]

As for the second tradition, Wittgenstein must be seen as an inheritor of the subterranean current of morphology by way of his well-acknowledged debt to Johann Wolfgang von Goethe and Oswald Spengler.[14] To this extent, he can be grouped within the ranks of thinkers on the Continent that simultaneously began to apply the once-obscure, comparative, and analogical methods of morphology to their respective domains

[10] See, for instance, Wittgenstein's first philosophical publication, "Review: P. Coffey, The Science of Logic," *The Cambridge Review* 34, no. 853 (1913): 351.

[11] Norman Malcolm, *Ludwig Wittgenstein: A Memoir*, 2nd ed (Oxford: Oxford University Press, 1984), 43.

[12] For instance, see *PI* (§§108, 242, & 345).

[13] Floyd, "Chains of Life"; Sebastian Sunday Grève, "Logic and Philosophy of Logic in Wittgenstein," *Australasian Journal of Philosophy* 96, no. 1 (2018); and Oskari Kuusela, *Wittgenstein on Logic as the Method of Philosophy: Re-Examining the Roots and Development of Analytic Philosophy*, First edition (Oxford, United Kingdom: Oxford University Press, 2019).

[14] In addition to the numerous scattered references, Wittgenstein's most sustained treatments of Goethe and Spengler are, respectively, found in the *Remarks on Colour*, ed. Gertrude Elizabeth Margaret Anscombe, trans. Linda L. McAlister and Margarete Schättle, 1. publ., [repr.] (Oxford: Blackwell, 1977); and Ludwig Wittgenstein, "Remarks on Frazer's Golden Bough," in *Philosophical Occcasions*, ed. James Klagge and Alfred Nordmann, trans. John Beversluis (Indianapolis: Hackett Publishing Company, 1993). From here on, abbreviated respectively as *RC* and *GB*.

in the interwar years.[15] During the same period, Wittgenstein was also to witness the "new" formalized mathematical logic, which he so vigorously championed in his youth, have its grand ambitions deflated. In addition to the limitations that he himself came to detect in his own exposition of logic in the *Tractatus*, he would also respond to Gödel's and Turing's negative resolution of the *Entscheidungsproblem*. To speak with some flourish, we could therefore say that Wittgenstein sought to raise morphology to the rigorous heights of logic at the very moment when the pretensions of mathematical logic were being demoted from Cantor's paradise. As a result, Wittgenstein fundamentally transformed each of the two traditions, logic and morphology, by allowing one to supplement the other.

Said another way, a concise and accurate summary of what I demonstrate in my dissertation can also be gleaned by attending to the intricate and often complementary relationship between logic and ontology in both Wittgenstein's *oeuvre* as well as in the Western philosophical tradition as a whole.[16] As other commentators have suggested, it is best to conceive of Wittgenstein's innovations in logic as the third revolution in the field of inquiry, following sequentially after the momentous developments instigated first by Aristotle and second by Frege.[17] Accordingly, I argue that it is precisely this form of logic as morphology, which can be distilled from Wittgenstein's later method, that

[15] Andrea Pinotti lists Aby Warburg, Walter Benjamin, Vladimir Propp, Leo Frobenius, Lucian Blaga, André Jolles, and Ernst Cassirer, alongside Spengler and Wittgenstein, as fellow partisans in the 20th century interbellum revival of morphology in his insightful lecture, entitled "Origin vs. Genesis: Warburg and Benjamin in the Footsteps of Goethe's Morphology" (Warburg, Benjamin and Kulturwissenschaft Conference, The Warburg Institute, School of Advanced Study University of London, UK, 2012), https://youtu.be/xZnz_ZXAav4.

[16] Hence, one of Wittgenstein's earliest programmatic declarations, from 1913, succinctly states: "Philosophy consists of logic and metaphysics: logic is its basis (*NB*, 106)."

[17] Kuusela, *Wittgenstein on Logic as the Method of Philosophy*, 1ff.

constitutes the third revolution. Furthermore, it should also be remembered that both the first and second revolutions in logic respectively brought with them two corresponding ontological frameworks: for Aristotle, this was a reality fundamentally comprised of hylomorphic substances; whereas, for Frege, it was a proposed model of reality instead made up of functions and objects. As for the third revolution inaugurated by Wittgenstein, the paradigm stands as the ontological correlate of the last milestone in logic. In this way, Wittgenstein's paradigm can be understood as a refinement of the various *Urphänomene* posited by Goethe in his morphological *Naturphilosophie*, insofar as the same crossing of idea and thing is given a more austere ontological characterization by divesting it of any of its lingering metaphysical residue.

But alas, as the poet once purportedly said: "The journey, not the arrival, matters."

Thus, the first chapter opens this dissertation with an examination of the logic set forth in the *Tractatus* cast through the medium of Paul Livingston's interpretation of the early Wittgenstein. There, I argue that the inherent tension that Livingston detects throughout the *Tractatus* can be more rigorously delineated in terms of the well-known exegetical debate between the standard and the resolute interpretations. More specifically, this tension can be elaborated by contrasting the standard and the resolute interpretations' respective emphases on universality and necessity versus particularity and contingency, especially with regard to how each proposes to account for the requirements imposed by the *Tractatus*'s method of logical analysis. Next, I demonstrate how two fundamental Tractarian logical requirements, the demand for simple names, and for a general form of a proposition, involve prohibitions against the kind of reflexive

language-use implicit in the formation of paradoxical constructions. Finally, I end the chapter by showing how the mature Wittgenstein's widely celebrated reconsiderations acknowledging the boundless diversity of human language also correspond with removing these prohibitions against reflexivity.

The second chapter moves from logic to ontology. It investigates the young Wittgenstein's well-known attempt to dissolve the distinction between solipsism and realism as it unfolds amidst a series of elucidations on the contours of Tractarian subjectivity (*TLP*: 5.5–5.64). Even measured in terms of the early Wittgenstein's habitual recourse to aphorism, the *Tractatus*'s discussion of the subject–object divide is extraordinarily terse. For this reason, chapter two remains largely exploratory, conjectural, and schematic. Accordingly, its focus is narrowed, almost exclusively, towards two principal lines of reasoning: the first is comprised of the various clarificatory thought-experiments, proposed at *TLP* (5.6–5.64), that aim to dispel any pretense of attributing a substantial existence to the philosophical subject; and second, the engagement with theories of judgment and the related analysis of expressions of propositional attitudes bearing the form of "[subject] A judges [proposition] *p*," "A believes that *p*," or "A thinks *p*," etc., at *TLP* (5.541–5.5422). As for my reading of the first strand of remarks, I offer a novel interpretation of Wittgenstein's efforts to mutually collapse idealism and realism into one another by situating them at the culmination of a legacy in 19th-century German philosophy, which includes Fichte, Schelling, and Schopenhauer, as they each sought to wrest realist conclusions from idealist presuppositions. As for my reading of the second strand of remarks, I argue that Wittgenstein's account, at *TLP* (5.542), does not succeed in its self-professed goal of

reducing expressions of propositional attitudes, like "A judges *p*," into propositions of the form "'p' says *p*." My suggestion is that Wittgenstein's decomposition fails because the term "says," in the resulting proposition, does not ostensibly appear to be a truth-functional operator, as is dictated by an analysis in accord with the general form of a proposition.[18] What is more, the previous chapter already casts suspicion on the general form of a proposition itself, as this requirement is shown to be founded on the basis of a dubious and largely ad-hoc prohibition that blocks reflexivity. I henceforth conclude the chapter by discussing how Wittgenstein's inability to abide by the constraints that he himself imposed on such an analysis leaves open for him the problem of coordinating the relationship between the subject and the world, idealism and realism.

The main crux of the dissertation arrives with the third chapter in its sketch of a morphologically enhanced approach to logical inquiry, i.e., a logical morphology. In short, the chapter demonstrates how the comparative techniques of morphology can emerge from the inherent incompleteness or inconsistency that was revealed within the logical symbolization of mathematics throughout the 1930s. In this way, I suggest that Wittgenstein's evaluation of the various negative responses to the *Entscheidungsproblem* is precisely what allows us to grasp the stakes of both his early and late outlooks on logic. That is, we find that the previously observed shortcoming in the early Tractarian method of logic bears significant formal similarities with what is shared between Gödel's incompleteness and Turing's uncomputability results; whereas, I then show how, later, the morphological method of logic arises from these same logico-mathematical considerations. The most simple and elegant way for me to carry this out is to focus on

[18] Paul M. Livingston also deserves credit here for bringing this shortcoming to my attention.

Wittgenstein's various elucidations of Cantor's diagonal argument since it is widely acknowledged and understood that both Gödel's and Turing's claims rely on diagonalization as a common basis undergirding their respective results. In Wittgenstein's reflections on the employment of diagonalization, the contradiction at the heart of the strategy is often equated with the previously observed tensions in Tractarian logic, yet also tends to involve the production of novel concepts that demand morphological examination. For instance, the diagonal proof that real numbers cannot be ordered into a countable series entails that the uncountable set of real numbers actually comprises a new theoretical system that can be analogically compared and contrasted with other number systems, like the integers and the irrational numbers. In my view, this is undoubtedly why Wittgenstein places the topic of numbers at center stage in the more general methodological sections of the *Philosophical Investigations*: different number systems constitute a "complicated network of similarities overlapping and criss-crossing" (*PI*: §66ff) and therefore offer a prime example of how language-games should be morphologically judged against one another in terms of Goethe, Spengler, and Wittgenstein's shared notion of a family resemblance.

The fourth chapter pivots back from thinking to being by investigating the ontological status of a paradigm. Here, the attempted Tractarian combination of realism and idealism is achieved, such that the paradigm is made out to fully encompass these two otherwise classically antithetical positions. In this intermingling of opposing sides, an account of a paradigm, as the culmination of Goethe's *Urphänomen*, is furnished insofar as it incorporates within itself the entirety of the Platonic division between idea and thing, concept and object, form and matter, *eīdos kaī ti*, *morphē kaī hÿle*.

Approached from another direction, a paradigm can also be comprehended as a kind of generalization of how the real numbers, in the previous chapter, were derived from applying the diagonal procedure to a countable system of numbers. That is to say, we come to see how a variety of paradigms, like the standard meter, the standard sepia, and even the human body as an archetypal picture (*PI*: §§50, iv.25) can emerge from tensions, pressures, and contradictions inherent within a given language-game. Moreover, it is argued that the realism of a paradigm extends itself into that of a robust materialism, as is likely already evident in the substantive material composition of metric rulers, color swatches, and the physiological complexities of human anatomy. However, beyond these three more readily acceptable examples of corporeal manifestation, what is far more significant is how this Wittgensteinian account of a paradigm allows us to snatch mathematics from its carefully guarded idealist shelter by providing a fully materialist account of mathematical proof.

Finally, the dissertation closes by bringing these results to bear on a more contemporary debate regarding the origins and the nature of subjectivity, language, and human consciousness involving Adrian Johnston, Paul Livingston, Lorenzo Chiesa, and Slavoj Žižek. More specifically, I show how the aforesaid morphological account of paradigms can accommodate the two conflicting poles of the debate by the way it aligns with the most central aspects of Johnston's psychoanalytically inflected German idealism and Livingston's competing meta-formal realism. On the one hand, I argue that logical morphology can provide a rigorously formal explanation of emergence and origination in a manner analogous to Livingston's valorization of Gödel's and Turing's results. However, in response to Johnston's objections against Livingston, the benefit of the

morphological method is that its manifold variety of comparative techniques cannot be simply reduced to taking Gödel's distinction between incompleteness and inconsistency to be, what the former has described as, "a universal philosophical master-matrix."[19] On the other hand, I argue that the emergence of a paradigm is in step with the materialist requirements that Johnston imposes on his transcendental materialist theory of subject formation. However, in response to Livingston's and Chiesa's objections against Johnston, I show how this materialist account of paradigms avoids the alleged monist and dualist pitfalls that the former attribute to the latter. The conclusion signals directions for a new research program that would putatively bring together Livingston's and Johnston's respective emphases on the logical and the biological by focusing on the algorithmic and informational characterizations of life, living systems, and abiogenesis.[20]

[19] Adrian Johnston, "Confessions of a Weak Reductionist: Responses to Some Recent Criticisms of My Materialism," (Unpublished Version, 2015), 36–37.

[20] I need to give a huge thank you to the up and coming writer, Zane Perdue, for his invaluable help in the editing process. Without him, this dissertation would have never shaped up in the way that it has.

Chapter 1: Paradox, Contradiction, and Reflexivity in the Tractatus: On Paul Livingston's Interpretation of the Early Wittgenstein

> One looks for some other word, but the ideas are always just as dark, just as simple and singularly painful.
>
> – Pierre Reverdy

In a series of articles and book-length contributions, Paul Livingston has developed a novel interpretation of Wittgenstein's philosophy that emphasizes the importance of paradoxes, contradictions, inconsistencies, tensions, and ambiguities.[21] Indeed, it is well-documented that some of Wittgenstein's earliest engagements in philosophical reflection were incited by the difficulties arising from Russell's paradox.[22] Ultimately, this interest would persist for Wittgenstein throughout his life, albeit under different auspices; even a cursory glance at his final remarks demonstrate that his concerns with logico-philosophical aporias never subsided.[23]

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[21] Paul M. Livingston, "The Breath of Sense: Language, Structure, and the Paradox of Origin," *Konturen* 2, no. 1 (2010): https://doi.org/10.5399/uo/konturen.2.1.1311; *The Logic of Being: Realism, Truth, and Time* (Evanston, Illinois: Northwestern University Press, 2017), 52, 114, 155–176; "Naturalism, Conventionalism, and Forms of Life: Wittgenstein and the 'Cratylus,'" *Nordic Wittgenstein Review* 4, no. 2 (2015); *Philosophy and the Vision of Language* (New York: Routledge, 2008), 1–28, 49–76, 123–226; *The Politics of Logic: Badiou, Wittgenstein, and the Consequences of Formalism*, 27 (New York: Routledge, 2012), 1–20, 131–186; "Wittgenstein and Parmenides," October 15, 2009, https://www.academia.edu/923400/Wittgenstein_and_Parmenides; and "Wittgenstein Reads Heidegger, Heidegger Reads Wittgenstein: Thinking Language Bounding World," in *Beyond the Analytics-Continental Divide: Pluralist Philosophy in the Twenty-First Century*, ed. Andrew Cutrofello, Paul M. Livingston, and Jeffrey A. Bell (New York: Routledge, 2015).

Following what R. M. Sainsbury stipulates in his comprehensive study on the matter, I assume a fairly broad definition of the term "paradox" as any "apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises" (2009, 1). Thus, this usage of the term carries a meaning that can cover various instances of contradictions, inconsistencies, ambiguities, and tensions, so long as they are inferred with apparently acceptable patterns of reasoning from apparently acceptable premises.

[22] Ray Monk, *Ludwig Wittgenstein: The Duty of Genius*, A Penguin Book Biography (New York, NY: Penguin Books, 1991), 32–33; Michael D. Potter, *Wittgenstein's Notes on Logic* (Oxford; New York: Oxford University Press, 2009), 16–17, 20ff, 186ff.

[23] Ludwig Wittgenstein, *Last writings on the Philosophy of Psychology. Vol. 1: Preliminary studies for part 2 of "Philosophical investigations" / ed. by G. H. von Wright and Heikki Nyman. Transl. by C. G.*

Beyond its exegetical significance, Livingston's focus on paradox and contradiction has allowed him to participate in a more contemporary discussion with the proponents of paraconsistent logic, who often cite Wittgenstein as a forerunner to their approach.[24] However, an admitted disregard for interpretative nuance is a recurrent shortcoming of these latter readings of Wittgenstein.[25] With greater sensitivity to detail, Livingston instead offers a more sophisticated and comprehensive sketch of Wittgenstein's philosophical progression by showing how the results of the early *Tractatus* phase harbored implicit contradictions and ambiguities that would only later become more pronounced and explicit:

> [O]nce we follow Wittgenstein in taking the possibility of rational criticism of metaphysical illusion to consist wholly in an immanent reflection on the possibilities of sense, we will not understand the possibility of its principles having force unless we can see this force, itself, as grounded in a certain kind of performative ambiguity or contradiction, a structure of overdetermination by means of which, in its application, the impossible is prohibited as the substantial content it cannot be, and the necessary is required as the empty generality that itself, being empty, cannot be required… It is this structure of overdetermination between necessity and requirement, impossibility and prohibition, at the root of the operation of any rational standard of sense that Wittgenstein had not perhaps completely, I suggested, recognized at the time of the *Tractatus*, insofar as the explicit remarks on the *Tractatus* themselves can be seen to rely on this

Luckhardt and, ed. Georg Henrik von Wright and Heikki Nyman, trans. C. G. Luckhardt and Maximilian A. E. Aue, vol. 1 (Oxford, UK Cambridge, USA: Blackwell, 1990), §174–175, 290, 525; *On Certainty*, ed. Georg Henrik von Wright and Gertrude Elizabeth Margaret Anscombe, trans. Gertrude Elizabeth Margaret Anscombe and Dennis Paul (Oxford: B. Blackwell, 1979), §392; *Remarks on the Philosophy of Psychology. Vol. 1*, ed. G. E. M. Anscombe and G. H. von Wright, trans. G. E. M. Anscombe, (Oxford: Blackwell, 1980), §37–44, 65, 246, 503, 885, 1132; *Remarks on the Philosophy of Psychology, Vol. 2*, ed. G. H. von Wright and Heikki Nyman, trans. C. G. Luckhardt and Maximilian A. E. Aue (Chicago : Oxford: University of Chicago Press ; Basil Blackwell, 1980), §290. From here on, abbreviated respectively as *LW I*, *OC*, *RPP I*, *RPP II*.

[24] Francesco Berto, *There's Something about Gödel: The Complete Guide to the Incompleteness Theorem* (Malden, MA: Wiley-Blackwell, 2009), 189-213; Graham Priest, *Beyond the Limits of Thought* (Cambridge, UK: Cambridge University Press, 1995), 197–213, 229–245; Graham Priest, *In Contradiction: A Study of the Transconsistent*, Expanded ed (Oxford : Oxford: Clarendon Press ; Oxford University Press, 2006), 27n4, 203–204; Graham Priest, "Wittgenstein's Remarks on Gödel's Theorem," in *Wittgenstein's Lasting Significance*, ed. Max Kölbel and Bernhard Weiss (New York, NY: Routledge, 2004), 207–27.

[25] Berto, *There's Something about Gödel*, 193–194; Priest, *Beyond the Limits of Thought*, 203, 230.

> crossing or confusion between the descriptive and the prescriptive if they are to have any ability to *lead* us to criticize our own "metaphysical" remarks as nonsense and so come to recognize and repudiate the temptations from which they arise.[26]

That is, according to Livingston, the logical laws, standards, and rules in the *Tractatus*, which serve to distinguish sense from nonsense, contain within themselves an implicit ambiguity that he describes as a form of overdetermination: on the one hand, they prescribe what already necessarily is; on the other hand, they proscribe what already has been dismissed as impossible.[27] Later, what was once merely latent will become overt in certain paradoxes and contradictions surfacing in some of the mature Wittgenstein's most well-known formulations: specifically, the nullification of the law of the excluded-middle in his account of the standard meter paradigm (*PI*: §50) and the notable paradox inherent in the so-called rule-following considerations (*PI*: §201).

Anyone familiar with the ongoing debates in Wittgenstein scholarship would surely see that the overdetermined ambiguity that Livingston locates in the status of Tractarian laws, standards, and rules closely parallels what has been endorsed by the two main interpretative currents in the secondary literature. Prescription and proscription, backed by a substantive, normative force, align with the standard interpretation, whereas a more descriptive, therapeutic method, which replaces prescription and proscription with tautological emptiness, aligns with the resolute interpretation. Without a doubt,

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[26] Livingston, "Wittgenstein and Parmenides, 12." See also, Livingston, *The Politics of Logic*, 136ff.

[27] In this regard, overdetermination can be defined as any case in which a given outcome is conditioned by two or more different determinants, yet each determinant is nevertheless a sufficient condition for that outcome on its own and by itself. Accordingly, what is upheld by such a law, standard, or rule would be deemed overdetermined because the outcome has already been sufficiently conditioned by necessity, but nevertheless a requirement is issued as an additional determinant. For a roughly similar definition, notwithstanding how it is construed in solely causal terms, see David Lewis, "Counterfactual Dependence and Time's Arrow," *Nous* 13, no. 4 (1979): 463ff. For the notion of overdetermination that guides Livingston's inquiry, see Reiner Schürmann, *Broken Hegemonies* (Bloomington, IN: Indiana University Press, 2003).

Livingston is aware of these discussions, as well as some of the implications that they might have for his reading of the early Wittgenstein.[28] Nevertheless, there remain moments when Livingston drifts, rather one-sidedly, in the direction of a highly problematic and widely disputed standard metaphysical approach, especially in the course of demonstrating the ambiguities and tensions so crucial to his interpretation.[29]

In what follows, I aim to preserve the valuable insights that can be gleaned from Livingston's engagement with Wittgenstein by providing them with a more rigorous and exegetically sound foundation. Therefore, in the first section, I will explain how the ambiguous overdetermination that besets Tractarian laws, standards, and rules actually coincides with how the respective strengths and weaknesses of the standard and resolute interpretations tend to reciprocally complement one another. The standard interpretation can adequately account for the universality and necessity that are typically attributed to logical laws, standards, and rules as Tractarian logical requirements. However, despite this strength, the standard interpretation nevertheless fails to abide by the fundamental dictates of the Tractarian method. It is now largely acknowledged that the resolute interpretation's fidelity to the method of the *Tractatus* is a more exegetically reliable and philosophically consistent reading of what the young Wittgenstein was actually pursuing in terms of his clarificatory ambitions. Yet, in correctly underlining the particularity and contingency by which the Tractarian method proceeds, the resolute interpretation

[28] Paul M. Livingston, "'Meaning Is Use' in the *Tractatus*," *Philosophical Investigations* 21, no. 1 (2004): 34–67; *The Politics of Logic*, 140–141; *Philosophy and the Vision of Language*, 19, 262; "Wittgenstein and Parmenides," 9–12.

[29] For example, Livingston relies on a rendition of the so-called argument for substance at *TLP* (2.02–2.021) in order to demonstrate what he regards as one of the main tensions at the heart of the *Tractatus* in both "Wittgenstein and Parmenides, 5–8"; and *The Politics of Logic*, 133–135. Admittedly, Livingston is following G. E. M. Anscombe's irresolute standard derivation of the metaphysical existence of simple objects in *An Introduction to Wittgenstein's Tractatus* (New York, NY: Harper & Row, 1959), 46–50.

inevitably deprives itself of any of the available resources that could be drawn on in order to account for the kind of universality and necessity that Wittgenstein did indeed impute to logic throughout his early period. What is revealed through this back-and-forth exchange is that the blatant paradoxes proposed by standard interpretations instead turn out to be the "deep tensions," as has been observed by certain resolute commentators.[30] What is more, these tensions support Livingston's reading, as they are veritably identical to the same overdetermined ambiguities implicit in Tractarian laws, standards, and rules.

Shortly after discovering the paradox that bears his name, Bertrand Russell went on to provide the canonical explanation of the connection between certain reflexive or self-referential uses of language and a series of closely related paradoxes and contradictions by demonstrating how the former unavoidably gives rise to the latter.[31] Accordingly, it should then come as no surprise that Livingston's interpretation would also closely examine how Wittgenstein himself handled issues pertaining to self-reference.[32] For my part, I will show, in the second section, how two of the most fundamental Tractarian logical requirements are also implicated in prohibitions against the kinds of reflexivity that trigger paradoxes and contradictions. In the course of outlining these arguments, I will place due emphasis on the general form of a proposition, which ought to be regarded as one of the most pivotal logical requirements in the unfolding of the *Tractatus*, but nevertheless remains markedly absent in Livingston's

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[30] Warren Goldfarb, "Metaphysics and Nonsense: On Cora Diamond's *The Realistic Spirit*," *Journal of Philosophical Research* 22 (1997): 64, 67; Oskari Kuusela, "The Dialectic of Interpretations: Reading Wittgenstein's Tractatus," in *Beyond the Tractatus Wars: The New Wittgenstein Debate*, ed. Rupert Read and Matthew A. Lavery (New York, NY: Routledge, 2011), 124, 130.

[31] Bertrand Russell, "Mathematical Logic as Based on the Theory of Types," *American Journal of Mathematics* 30, no. 3 (1908): 224–225.

[32] Paul M. Livingston, "Agamben, Badiou, and Russell," *Continental Philosophy Review* 42, no. 3 (2009): 310, 323; *The Politics of Logic*, 3–62.

exposition. Furthermore, I will demonstrate how the same prohibitions against reflexivity both saddle logical laws, standards, and rules with overdetermined ambiguities and restrict their expression exclusively to Tractarian showing.

Finally, in the third section, I will attempt to extend Livingston's interpretation by explaining how key features of Wittgenstein's mature thought developed by abdicating previously held prohibitions against reflexivity and, as a result, led him to embrace more overt paradoxes and contradictions. As for my own contribution to this account of Wittgenstein's philosophical evolution, I will argue that his newfound tolerance for certain reflexive, paradoxical, and contradictory constructions is in step with an overall relaxing of the constraints he had formerly imposed on significant instances of discourse, which, in turn, helped him to recognize that human language was far more complex and diverse than he had once assumed during his *Tractatus* stage.

1.1: Standard vs. Resolute Interpretations

Let us begin by following Kant's path from Aristotle's *De Interpretatione* (17b, 21–23) and thereby settle upon some preliminary definitions of universality and particularity, necessity and contingency.[33] In the first *Critique*, "strict universality" is defined as that in which "no exception at all is allowed to be possible… but is valid absolutely."[34] The logical form of universality can then be characterized to fit in a syllogism as "All *Fs* are

[33] Immanuel Kant, *Critique of Pure Reason*, trans. Paul Guyer and Allen William Wood (Cambridge: Cambridge University Press, 2000), A1/B3–A6/B6, A67/B92–A84/B117; *Lectures on Logic*, trans. J. Michael Young (Cambridge ; New York: Cambridge University Press, 1992), 101–114.
[34] Kant, *Critique of Pure Reason*, B4.

Gs." So, if universality is that which holds for *all* and *every*, then particularity would be that which admits exception. Accordingly, particularity then bears the logical form of "Some *Fs* are *Gs*."

Additionally, in the first *Critique*, Kant provides a good working definition of necessity as that which is "thus and so [and] could not be otherwise."[35] Its respective forms could then be presented as "Necessarily, *Fs* are *Gs*." In the *Jäsche Logik*, a judgment is said to have achieved "the dignity of necessity," and is thereby considered apodictic, upon the inclusion of the term "must."[36] So, if necessity is that which *must* be so, then contingency or possibility is that which admits some other alternative. The corresponding form could thus be presented as "Possibly, *Fs* are *Gs*." Similarly, a judgment that admits possibility can be indicated by the inclusion of the term "may."[37]

Finally, Kant observes that universality and necessity "belong together inseparably," such that the two are, indeed, logically equivalent.[38] In the analytic tradition, universality and necessity are often run together as the generality and the normative force of logical laws. Since Wittgenstein's critiques of this aspect of the tradition, especially that of Frege and Russell, are rather sophisticated,[39] it is instead better to simply recognize that the various logical requirements maintained within the

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[35] Ibid., B3.
[36] Kant, *Lectures on Logic*, 109.
[37] Ibid.
[38] Kant, *Critique of Pure Reason*, B4.
[39] For example, see *TLP* (3.325, 4.1272, 5.132, 5.4, 5.42). See also, Kuusela, *Wittgenstein on Logic as the Method of Philosophy*, 13–72; Marie McGinn, *Elucidating the* Tractatus*: Wittgenstein's Early Philosophy of Logic and Language* (Oxford: Clarendon Press; Oxford University Press, 2009), 53–74; Peter M. Sullivan, "The Totality of Facts," *Proceedings of the Aristotelian Society (Hardback)* 100, no. 1 (June 2000): 175–177.

Tractarian method meet the criteria for universality and necessity specified above. Cora Diamond explains this as "the laying down of logical requirements":

> This is not a view about what there is, external to language or thought, but about what they essentially are (despite appearances), and about what we can do, what it *must* be possible to do. The belief that there *must* be a certain kind of logical order in our language...
>
> We do make sense, our propositions stand in logical relations to each other. And such-and-such *is required* for that to be so… [T]he requirements… are internal to the character of language as language, in there being a general form of sentence, in *all* sentences having this form.[40]

So, on this conception, language's accordance with logical laws, standards, and rules is to be understood in terms of how the language we have already learned to speak will always turn out to be in conformity with certain logical requirements. As is suggested by the above-mentioned use of "all" and "must," these logical requirements are universal and necessary insofar as logico-linguistic clarification will always reveal that *all* the significant propositions that comprise a language *must* satisfy what is laid down by these requirements. Furthermore, an additional benefit of attending to logical requirements is that it initially allows for a kind of exegetical agnosticism since both Peter Hacker's standard interpretation and James Conant's and Cora Diamond's resolute interpretation acknowledges their universality and necessity.[41]

Given the extensive list of logical requirements that are operative within the Tractarian method, it will be helpful to narrow down our investigation by deciding upon three exemplary candidates that could respectively serve as a representative for a logical

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[40] Cora Diamond, *The Realistic Spirit: Wittgenstein, Philosophy, and the Mind* (Cambridge, Mass.: MIT Press, 2011), 18–19. Some emphasis is mine.

[41] James Conant and Cora Diamond, "On Reading the *Tractatus* Resolutely: Reply to Meredith Williams and Peter Sullivan," in *Wittgenstein's Lasting Significance*, ed. Max Kölbel and Bernhard Weiss (New York, NY: Routledge, 2004), 77–80. P. M. S. Hacker, *Insight and Illusion: Themes in the Philosophy of Wittgenstein*, Rev. ed (Oxford: Clarendon Press; Oxford University Press, 1986), 51, 58, 72, 113.

law, a standard, or a rule.[42] In the case of the laws of logic, the only viable choice is the general form of a proposition (*TLP*: 4.5ff). Upon considering how Wittgenstein reduces inferential laws solely to internal relations between propositions (5.13–5.133) and how these internal relations are presented by logical operators (5.2–5.242), the general form of a proposition can then be distinguished as the "sole logical constant" (5.46–5.47) capable of embodying all of the logical links between propositions. In the case of standards, the word itself does not occur as a technical term in the *Tractatus*. However, it is suitable to accept its immediate ancestor, the requirement for simple names and its related logical prototypes (*Urbilden*) of simplicity, since this demand clearly prefigures later formulations about paradigms like the standard meter (*TLP*: 3.2ff, 5.351; *PI*: §46–51).[43] Finally, logical rules, which are the rules (*Regeln*) of logical or grammatical syntax in the *Tractatus* (3.334ff), will not be given the same attention, because they themselves do not stand as logical requirements. Nevertheless, the rules of logical syntax will have a decisive role, as they participate in driving the motivations behind the other two logical requirements. Therefore, the universality and the necessity endowed in each of the two logical requirements that concern us can then be expressed as follows: *all* propositions *must* have the form of the general form of a proposition (*TLP*: 4.5–4.52, 5.47–5.472); and *all* propositions *must* be analyzable into elementary propositions consisting of simple names (4.22–4.2211).

[42] For a comprehensive list of logical requirements, see Conant and Diamond, "On Reading the *Tractatus* Resolutely," 78–80."

[43] See also, Cora Diamond, "How Long Is the Standard Meter in Paris," in *Wittgenstein in America*, ed. T McCarthy and S. C. Stidd (Oxford, UK: Clarendon Press, 2001), 108; Jean-Philippe Narboux, "Simplicity and Rigidity: Reading Section 50 of Wittgenstein's Philosophical Investigations After Kripke," in *Finding One's Way Through Wittgenstein's Philosophical Investigations: New Essays on §1–88*, ed. J.P Narboux and E. Bermon (Cham, Switzerland: Spring, 2017), 105–140.

Another reason for specifically selecting these two requirements is due to how they manifest what is likely the most fundamental dualistic distinction running throughout the *Tractatus*:

> 4.0312: The possibility of propositions is based upon the principle of the representation of objects by signs.
>
> My fundamental thought [*Grundgedanke*] is that the "logical constants" do not represent; that the *logic* of the facts cannot be represented.[44]

That is, on the one side, there is the form and content internal to a proposition, i.e., simple names, and, on the other, the emptiness of the sole propositional connective, i.e., the general form of a proposition. I propose that a given interpretative position can be evaluated on the basis of how well it can account for these two logical requirements.

1.1.1: The Standard Interpretation

The standard interpretation is well-suited to account for the universality and the necessity of the two logical requirements by appealing to a theory of language grounded in a logical, atomistic metaphysics. More specifically, according to Hacker, there is a fundamental isomorphic relationship between representation and what is represented that is secured by the requirement for simple names:

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[44] See also, Anscombe, *An Introduction to Wittgenstein's Tractatus*, 25–26; Eli Friedlander, *Signs of Sense: Reading Wittgenstein's Tractatus* (Cambridge, Mass: Harvard University Press, 2001), 92ff; Jean-Philippe Narboux, "Showing, the Medium Voice, and the Unity of the Tractatus," *Philosophical Topics* 42, no. 2 (2014): 230; Thomas Ricketts, "Wittgenstein Against Frege and Russell," in *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*, ed. Erich Reck (Oxford, UK: Oxford University Press, 2002), 227–228; Sullivan, "The Totality of Facts, 188–190."

> The "harmony between language and reality" was explained in the picture theory in the terms of the idea of simple unanalyzed names (logically proper names) which refer to sempiternal objects that concatenate to form facts.[45]

The necessity and universality of this requirement is an immediate consequence of how the denotation of names is guaranteed by these metaphysical objects:

> There *must* be unanalyzable objects if *language* is to be related to the world, and they *must* be indestructible. For only thus can the *need* for a firm anchor for *language* be met… They *must* exist in order that it be possible for us to say something false yet *meaningful*…[46]

As is indicated by the above use of "must," "need," and "language," the capacity for *every* true or false significant proposition (i.e., language in *TLP*: 4.011) to represent reality is ensured by simple names *necessarily* standing for simple objects.

Likewise, as the essence of *all* propositions, the general form of a proposition is also universal and necessary:

> This variable, the general propositional form, gives the essence of the proposition; also the essence of all description, and hence too the essence of the world. It is of the essence of reality that it consists of the existence and non-existence of states of affairs. It is the essence of a proposition to describe states of affairs; it is the nature of states of affairs that they may exist (obtain) or not exist (not obtain). The proposition, therefore, is a logical picture of a possibility – which may or may not be instantiated.[47]

Once again, the requirement is guaranteed by a logical, atomistic metaphysics. As is suggested by the necessity of a shared essence between propositions and the world, there

[45] Hacker, *Insight and Illusion*, 61.
[46] Ibid., 65–66, my emphasis.
[47] P. M. S. Hacker, "When the Whistling Had to Stop," in *Wittgensteinian Themes: Essays in Honour of David Pears*, ed. D. Charles and W. Childs (Oxford, UK: Clarendon Press, 2001), 142.

is an isomorphic correspondence between how names concatenate into propositions and how objects concatenate into states of affairs (*Sachverhalten*) (*TLP*: 4.1, 4.2–4.2211).

The problem, however, is that the standard interpretation's successes in accounting for the universality and necessity of the two logical requirements are directly proportional to its exegetical failures. Since these shortcomings have by now become very well-rehearsed objections, it is enough to mention the common criticism that, whether the standard interpretation's theory of language grounded in a logical, atomistic metaphysics is said or shown, it nevertheless disregards the young Wittgenstein's repeated rejections of philosophical theories as nonsense:

> 4.003: Most of the propositions and questions to be found in philosophical works are not false but nonsensical. Consequently, we cannot give any answers to questions of this kind but can only point out that they are nonsensical. Most of the propositions and questions of philosophers arise from our failure to understand the logic of our language.[48]

Since it is futile to construct theories that amount to nonsense, Wittgenstein's novel solution is to instead propound the rather idiosyncratic methodology set forth in the *Tractatus*:

> 4.112: … Philosophy is not a body of doctrine [*Lehre*, "theory"] but an activity…
>
> Philosophy does not result in "philosophical propositions," but rather the clarification of propositions…
>
> 6.53: The correct method in philosophy would really be… to say nothing except what can be said… i.e. something that has nothing to do with philosophy – and then, whenever someone else wanted to say something metaphysical, to demonstrate to him that he had failed to give a meaning to certain signs in his propositions…

[48] See also, Conant and Diamond, "On Reading the *Tractatus* Resolutely," 47–49.

Rather than producing theoretical explanations, philosophy conducted in the Tractarian mode involves a form of logical clarification that is both descriptive and therapeutic in the sense that it tailors itself to a series of *particularities*: the application of the method is *particular* to the individual interlocutors, their sensibilities, confusions, and the propositions and pseudo-propositions traded in a dialogic exchange. Given the manifold factors at play, the outcome of each interaction is an utterly *contingent* matter because it depends on how and even whether the interlocutors engage with one another.

1.1.2: The Resolute Interpretation

Conversely, the resolute interpretation takes seriously Wittgenstein's repudiation of philosophical theorizing by promoting an austere conception of nonsense as opposed to the substantial conception permitted by the standard interpretation:

> The substantial conception distinguishes between two different kinds of nonsense: mere nonsense and substantial nonsense. Mere nonsense is simply unintelligible – it expresses no thought. Substantial nonsense is composed of intelligible ingredients combined in an illegitimate way – it expresses a logically incoherent thought. According to the substantial conception, these two kinds of nonsense are logically distinct: the former is mere gibberish, whereas the latter involves (what [standard] commentators of the *Tractatus* are fond of calling) a "violation of logical syntax". The austere conception, on the other hand, holds that mere nonsense is, from a logical point of view, the only kind of nonsense there is…[49]

Since any theory of language or metaphysics, including whatever might be attributed to the *Tractatus*, is regarded in the austere fashion as mere nonsense, the resolute

[49] James Conant, "The Method of the Tractatus," in *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*, ed. Erich Reck (Oxford, UK: Oxford University Press, 2001), 381.

interpretation instead gives due weight to both the particularity and the contingency implicit in the Tractarian method by respectively emphasizing how it unfolds on a "case-by-case basis" and the "piecemeal character" of its outcome:

> There is no *once-for-all* demonstration in the *Tractatus* that propositions of *such-and-such sorts* are nonsensical: the task of clarifying propositions is a *one-by-one* task. Only the activity of philosophical clarification, or of attempting philosophical clarification, can reveal whether, in a *particular* case, there is or isn't something that we mean.[50]

That is, the method proceeds only in response to the particularity of propositions, the task, and the case of dialogic exchange. However, since its results are not guaranteed, the affair itself is contingent.

The resolute interpretation, despite its fidelity to the particularity and contingency of the Tractarian method, nevertheless maintains that logical requirements are both universal and necessary; but this necessity and universality is merely attributed to a mistaken and inadvertent lapse on Wittgenstein's part into a dubious metaphysics:

> The following [are] metaphysical commitments (underlying the conception advanced in the *Tractatus* of how the activity of clarification must proceed) …
>
> - *Every* proposition can be analyzed
> - There is a general form of proposition and *all* propositions have this form …
> - Logical analysis will reveal *every* proposition to be either an elementary proposition [i.e., a concatenation of simple names] or the result of truth-operations on elementary propositions …
>
> The italicized expressions in each of the above propositions indicate the occurrence of a moment of (what would count by the latter Wittgenstein's lights as) *metaphysical insistence* – a moment in which a requirement is laid down. The metaphysical commitments at issue here are, however, not

[50] Cora Diamond, "Saying and Showing: An Example from Anscombe," in *Post-Analytic Tractatus*, ed. Barry Stocker (London, UK: Routledge, 2004), 165–166. Emphasis mine.

> of a sort that early Wittgenstein, at the time of writing the *Tractatus*, would have taken to be metaphysical.[51]

As indicated by the above use of "all" and "every," the resolute commentators acknowledge that Wittgenstein had regarded logical requirements, including those for simple names and a general propositional form, as universal, and, on the basis of Kant's logical equivalence, necessary.

As several commentators otherwise sympathetic to the resolute interpretation have correctly noted, the priority that this exegetical line grants to the particularity and contingency inherent in the Tractarian method is ultimately irreconcilable with the universality and necessity of its logical requirement.[52] Indeed, as Kuusela has astutely observed, the aforementioned lapse into a disguised metaphysics does not go far enough in sufficiently explaining how Tractarian logical requirements could acquire such a wide scope and normative force:

> For it is unclear how offering piecemeal [i.e., particular and contingent] clarifications could ever commit Wittgenstein to such general [i.e., universal and necessary] views about language that Conant and Diamond attribute to him. This is a point where there appears to be a gap in their reading: Conant and Diamond attribute to Wittgenstein a general claim about the nature (logic) of language, but don't explain how clarification as he understands it could result in a commitment to such a claim… Indeed, even though there seems to be very good evidence that the later Wittgenstein thought his early philosophy *was* committed to a tacit doctrine [i.e., a latent metaphysics] about language, this can't alone settle the matter… Nevertheless, an account *in terms of the text of the Tractatus is still required* to explain how exactly the commitment found its

[51] Conant and Diamond, "On Reading the *Tractatus* Resolutely," 82–84, their emphasis.

[52] Goldfarb, "Metaphysics and Nonsense," 64ff; Kuusela, "The Dialectic of Interpretations," 127–133; P. M. Sullivan, "'The General Propositional Form Is a Variable' (Tractatus 4.53)," *Mind* 113, no. 449 (January 1, 2004): 47–51; "On Trying to Be Resolute: A Response to Kremer on the *Tractatus*," *European Journal of Philosophy* 10, no. 1 (2002): 49–52, 56ff.

> way into his philosophy… The lack of such an account is what I mean by the gap in Conant's and Diamond's interpretation.[53]

Since Conant and Diamond insist that "reliance on such [external] remarks [from the later period] cannot serve as a surrogate for having an independently coherent and textually plausible account of what he [Wittgenstein] was up to in the *Tractatus*,"[54] this canonical version of the resolute interpretation cannot meet its own philosophico-exegetical standards. At this point, it should also be clear how this "gap" in the resolute interpretation, which both Kuusela and Goldfarb identify as a kind of "deep tension," closely corresponds to the ambiguities that Livingston distinguishes in the *Tractatus*.[55] It does not all resemble the blatant paradoxes and inconsistencies that standard commentators have so often attributed to the text's final culmination in nonsense.[56] Instead, the gap is a far more subtle conflict between the substantial force of logical requirements that can be accounted for by the standard interpretation versus the therapeutic emptiness of the Tractarian method that is associated with the resolute interpretation.

1.1.3: Debating Tractarian Syntax

Finally, the rules of logical syntax can be connected with the two previous logical requirements insofar as they distinguish or characterize logical types, such as operations,

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[53] Kuusela, "The Dialectic of Interpretations," 131.
[54] Conant and Diamond, "On Reading the *Tractatus* Resolutely," 86.
[55] Goldfarb, "Metaphysics and Nonsense," 64; Kuusela, "The Dialectic of Interpretations," 124.
[56] Hacker, *Insight and Illusion*, 25.

prototypes, objects, names, propositions, etc.[57] In this regard, Livingston's overdetermined ambiguity presents itself in how the standard and resolute interpretations disagree about Wittgenstein's use of "*gehorcht*" in the following explanation of the role of logico-syntactic rules:[58] "In order to avoid such errors, we must employ a sign-language… that obeys [*gehorcht*: "is governed by," "responds to," "hearkens," "heeds"] *logical* grammar – logical syntax (*TLP*: 3.325)." The standard interpretation conceives of logical syntax in terms of the early analytic tradition's conception of necessity as rules endowed with the normative force of prescription and proscription. Conversely, for the resolute interpretation, logico-syntactic rules are utterly vacant of any such force and are instead methodological aids that can be used in order to avoid the philosophical confusions that might arise due to equivocations between different logical types.

Consequently, there are several interpretative distinctions that can be drawn from the two competing conceptions of logico-syntactic rules. With the standard interpretation, rules determine a sign's logical type and its mode of combination with other signs. Accordingly, this (standard) conception of rules lends itself to a more compositional understanding of language: an isolated sign that is outside any propositional context can have a meaning and determinate logical type. This compositional outlook explains why the standard interpretation can permit substantial

[57] In the nomenclature of the *Tractatus*, logical types are typically referred to in terms of formal properties, relations, and concepts (*TLP*: 4.122–4.128). However, on a number of occasions, Wittgenstein does, in fact, make mention of "types" (3.331–3.332, 5.252, 6.123).

[58] James Conant, "Two Conceptions of *Die Uberwindung der Metaphysik*: Carnap and Early Wittgenstein," in *Wittgenstein in America*, ed. T McCarthy and S. C. Stidd (Oxford, UK: Clarendon Press, 2001), 38ff; Cora Diamond, "Logical Syntax in Wittgenstein's Tractatus," *The Philosophical Quarterly* 55, no. 218 (2005); P.M.S. Hacker, "Wittgenstein, Carnap and the New American Wittgensteinians," *The Philosophical Quarterly* 53, no. 210 (2003).

nonsense: a sign, with a meaning and determinate logical type, is an "intelligible ingredient" that can be identified outside of a propositional context and within nonsense.

The resolute interpretation, since rules do not fix a sign's logical type or mode of combination, tends to advocate for a more contextual approach: the meaning and the logical type of a constituent sign can only be determined from within the context of a significant proposition. Accordingly, the standard interpretation must reject substantial nonsense and exclusively acknowledge austere nonsense: there is nothing intelligible about a sign that occurs in a nonsensical construction, because outside of a propositional context, it has no meaning nor any determinate logical characteristics.

The strengths and weaknesses of the two rival interpretative stances on Tractarian syntax can be evaluated by considering their exegetical disagreement over what Wittgenstein meant in outlining "the correct method of philosophy" by urging us, "whenever someone else wanted to say something metaphysical, to demonstrate to him that he had given no meaning [*Bedeutung*] to certain signs [*Zeichen*] in his propositions [*Sätzen*] (*TLP*: 6.53)." Earlier in the text, Wittgenstein explains the method in greater detail:

> 5.473: Logic must take care of itself.
>
> A *possible* sign must be able to signify [*bezeichnen*]. Everything that is possible is also permitted. ("Socrates is identical" means nothing [*heißt… nichts*] because there is no property [*Eigenschaft*] called "identical." The proposition is nonsensical [*unsinnig*] because we have not made some arbitrary determination [*willkürliche Bestimmung*], but not because the symbol [*Symbol*] is, in itself, impermissible).
>
> In a certain sense, we cannot make mistakes in logic.

Returning to the example of "Socrates is identical" as nonsense, Wittgenstein further elaborates on the method:

> 5.4733: Frege says that every legitimately constructed proposition must have a sense [*Sinn*]. And I say every possible proposition is legitimately constructed, and if it has no sense, this can only be because we have given no *meaning* [Bedeutung] to some of its constituents [*Bestandteile*].
>
> (Even if we believe that we have done so.)
>
> Thus "Socrates is identical" says nothing [*sagt… nichts*], because we have given *no* meaning [*Bedeutung*] to the word "identical" as an *adjective* [Eigenschaftswort: "property-word"]. For when it occurs as the equal sign [*Gleichheitszeichen*], it symbolizes in an entirely different way – the signifying relation is a different one – therefore the symbols also are entirely different in the two cases: two symbols have only the sign in common, and that is an accident.

One the one hand, the strength of the standard interpretation is that it can account for the fact that Wittgenstein explicitly indicates both the logical characteristics and meaning of a sign within nonsense: he speaks of "identical" as an adjective (*Eigenschaftswort*) and specifies that its meaning has only been stipulated to designate a relation of identity (*Gleichheit*) but not as property (*Eigenschaft*). However, its weakness is that it is exegetically inconsistent with the above passage's emphasis on contextuality and mention of an austere conception of nonsense. On the other hand, the resolute interpretation's strength, in abiding by the text's preference for contextuality, reciprocally entails a weakness in compositionality. That is, the resolute interpretation cannot explain Wittgenstein's claim about the meaning and logical characteristics of the "identical" in the nonsensical construction.[59] In fact, if we strictly adhere to an austere conception of

[59] Consider, for instance, Conant's attempt to overcome this difficulty in "Two Conceptions of *Die Uberwindung der Metaphysik*," 41n45: "The point of [*TLP* (5.473)] – about "identical" naming an unspecified property – is to offer a suggestion intended to enable us, based on the surface grammar of this peculiar string, to find a way to see the symbol in the sign. There is an invitation present in the pattern of ordinary language for us to try to read the sign in this way (on the model of "Socrates is happy"). But we can only go so far in this direction. We can assimilate "Socrates is identical" to an established pattern (and thereby recognize the symbol in the sign); but we still do not yet know *what* the sentence says, because there is no established use of identical as a concept expression. When Wittgenstein talks in [5.473] about a property, he is talking about a method of *symbolizing*."

Surely, the import of a notion, like "surface grammar," ostensibly borrowed from twentieth-century linguistics and utterly alien to the *Tractatus*, should indicate that Conant is at pains here to square his account with the example in *TLP* (5.473). Nevertheless, the struggle is futile: if one strictly abides by the

nonsense, then it could be just as feasible that we have not made an arbitrary determination about the meaning of "Socrates." More generally, it is hard to see how anyone could go about detecting the mistake that would lead to nonsense when it consists of nothing more than a formless and contentless sequence of signs without any discernible logical characteristics.[60]

What is likely the most debilitating obstacle in resolving the matter stems from Wittgenstein's apparent reluctance in providing the reader of the *Tractatus* with any examples of what might be included in a corpus of Tractarian-style rules. The presumed reason for withholding any such examples is offered at *TLP* (3.334): "The rules of logical syntax must follow of themselves [*sich von selbst verstehen*: "go without saying," "be

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context principle, then there is no viable way to discern anything about the logical type or the meaning of what might appear to be the individual signs in a nonsensical construction. Likewise, no benefit is obtained by Conant's recourse to the alien notion of an "established pattern," which appears to be an attempt at smuggling something akin to the logical form of the significant proposition, i.e., "Socrates is happy," into the formless and nonsensical string, "Socrates is identical." Nevertheless, once we admit that "we still do not yet know *what* the sentence says," we must almost confess our inability to recognize anything like a symbol in a sign, whether it be, what he calls, a "surface" adjective or a genuine "concept expression." Said in another way, there is nothing indicative about the logical types of the signs in the context of a significant proposition that can then be transferred over into a nonsensical context. In fact, Sullivan, in "On Trying to Be Resolute," 48, has noticed that the resolute interpretation faces certain difficulties in discriminating between logical types even when dealing with propositions with sense, let alone in nonsensical constructions. Thus, the resolute interpretation's exceedingly strict contextualism must deny any logical characterization of "identical" as either an adjective or an equals-sign, but this directly conflicts with what Wittgenstein explicitly says in *TLP* (5.473).

[60] For my part, I believe that this is the correct way to substantiate Livingston's objection, in "Wittgenstein and Parmenides," 11, directed against the resolute interpretation's failure to develop something like an "error theory" that could account for the "illusion of sense" produced by a nonsensical construction. By contrasting the resolute interpretation's account of an illusion of sense with Kant's account of transcendental illusion in *Critique of Pure Reason* (A327/B384), Livingston develops this objection as follows: "We can, of course, normally understand illusion as mistaking one thing for another, as the misconception or misperception, due to peculiar features of our perceptual or intellectual capacities, of what is in fact there as having properties or features than it actually does. But if the resolute interpretation is correct, then the illusion involved in the grip of metaphysics cannot be an illusion in this sense. For it does not involve taking something that exists as something other than what it is, but rather taking *nothing* – words or sentences that in fact have no sense – as in fact being something, as in fact articulating (some) substantial content or sense." Since, as I have argued, there is no available way for the resolute interpretation to discern anything about the logical type of the putative signs in nonsense, there is nothing in a nonsensical construction with enough logical characteristics to seduce someone into mistaking nonsense for sense.

self-evident," "be self-explanatory"]." At best, I think we can conclude that, for the Wittgenstein of the *Tractatus,* logico-syntactic rules were exclusively intended to *show* rather than say anything about the logical type of a given sign.

1.2: The Two Tractarian Prohibitions against Reflexivity

In this section, I show how the two logical requirements are implicated in prohibitions against reflexivity that, in turn, block the formation of paradoxical constructions. In the case of simple names, the motivations behind the requirement are, at once, what also prevent the specific kind of self-reference that can generate the property version of Russell's paradox.[61] Whereas, in the case of the general form of a proposition, it is the argument for this formal concept itself that prevents the specific kind of self-reference that can generate the propositional version of Russell's paradox.[62]

1.2.1: The Property Version of the Paradox

[61] For what is perhaps the earliest distinction between the property and propositional versions of the paradox, see Bertrand Russell, *Principles of Mathematics* (London: Routledge, 2010), "Appendix B," pp. 534–540.

In Peter M. Sullivan, "A Note on Incompleteness and Heterologicality," *Analysis* 63, no. 277 (January 2003): 33, the property version of the paradox is set up by considering a "property *R* defined to apply to just those properties that do not apply to themselves": thus, "*R*(*F*) iff ~ *R*(*F*), so that, in particular, *R*(*R*) iff ~*R*(*R*)."

[62] Sullivan sets up the propositional version of the paradox as follows, in "The general propositional form is a variable," 50n2: "[F]or any class of propositions *m* there is, or seems to be, a proposition $\hat{}m =_{def} p(p\ m \supset p)$… It is, or appears to be, an intelligible question whether $\hat{}m\ m$… Call any proposition $\hat{}m$ a class assertion. If $\hat{}m\ m$, then $\hat{}m$ is a self-class-assertion (SCA). If $\hat{}m \supset m$, then $\hat{}m$ is a non-self-class-assertion (NSCA). Now, let $r = \{\hat{}m: \hat{}m \supset m\}$, that is, the class of NSCA's. Then: $\hat{}r\ r$ iff $\hat{}r\ \{\hat{}m: \hat{}m \supset m\}$ iff $\hat{}r \supset r$, that is: $\hat{}r$ is a SCA iff $\hat{}r$ is $\hat{}m$ such that $\hat{}m \supset m$ iff $\hat{}r$ is a NSCA's."

Wittgenstein's treatment of the property version of the paradox rests on three presuppositions. The first is his account of propositional functions (*TLP*: 3.31–3.318). The second presupposition is the so-called sign/symbol distinction (*TLP*: 3.32–3.325). Finally, third, is his closely related conception of the rules of logical syntax (*TLP*: 3.325–3.332).

With the three presuppositions in place, Wittgenstein then briskly dispenses with Russell's paradox in only a few sentences:

> 3.333: A function cannot be its own argument, because the sign for a function already contains the prototype [*Urbild*] of its own argument and it cannot contain itself.
>
> For let us suppose that the function $F(fx)$ could be its own argument; then there would be a proposition "$F(F(fx))$," and in this the outer function F and the inner function F must have different meanings [*Bedeutungen*], for the inner one has the form $\varphi(fx)$ and the outer one has the form $\psi(\varphi(fx))$. Only the letter "F" is common to both functions, but the letter by itself signifies nothing.
>
> This immediately becomes clear if instead of "$F(Fu)$" we write "$(\exists\varphi):F(\varphi u).\varphi u = Fu$."
>
> And herewith Russell's paradox is disposed of.

As the first sentence in the above remark suggests, the paradox can be prevented from the outset because the first presupposition sets forth a specifically Tractarian rendition of a propositional function as a *Satzvariable* that does not allow for the formation of reflexive propositional functions that can apply to themselves. As is alluded to in *TLP* (3.318), the *Satzvariable* combines aspects from both Frege's and Russell's conception of a propositional function so as to result in a constitutively anti-reflexive notation.

That is, one the one hand, what Wittgenstein alternatively refers to as either a "symbol [*Symbol*]" or an "expression [*Ausdruck*]" is that which "marks a form and a content [*kennzeichnet eine Form und einen Inhalt*]," as the "common characteristic trait

[*gemeinsame charakteristische Merkmal*] of a class of propositions (*TLP*: 3.31– 3.311)." So, like Russell, the value output by a Tractarian propositional function is a class of propositions, of which a particular expression makes a common contribution to each member of that class (3.311– 3.314).[63] In its very design, a *Satzvariable* anticipates its range in a manner that already begins to work towards preempting self-referential constructions: "An expression presupposes [*voraussetzt*] the forms of all the propositions in which it can occur"; "What values a *Satzvariable* can assume [*annehmen*] is stipulated [*festgesetzt*] (*TLP*: 3.311, 3.316)." On the other hand, like Frege, Tractarian propositional functions permit stratification into a nested hierarchy of increased order, like, for example, "$\psi(\varphi(fx))$": third-order propositional functions, like "ψ ()," can only take second-order propositional functions as arguments, like "$\varphi()$"; likewise, second-order propositional functions can only take first-order propositional functions; and finally, first-order propositional functions, like "*f*," exclusively take names.[64]

In the next paragraph, Wittgenstein commences to the second presupposition by relying on the sign/symbol distinction in order to disambiguate the two occurrences of the same sign for a propositional function in "$F(F(fx))$". By definition, a sign is a written orthographic mark or an audible spoken sound that is perceived alongside an expression or symbol.[65] In short, "A sign [*Zeichen*] is what can be perceived of a symbol (*TLP*:

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[63] See also, Hide Ishiguro, "Wittgenstein and the Theory of Types," in *Perspectives on the Philosophy of Wittgenstein*, ed. Irving Block (New York, NY: Blackwell, 1981), 49; McGinn, *Elucidating the* Tractatus, 132.

[64] Ishiguro, "Wittgenstein and the Theory of Types," 47–48; Sullivan, "The Totality of Facts," 186–187; José L. Zalabardo, *Representation and Reality in Wittgenstein's* Tractatus (Oxford: Oxford University Press, 2015) 154, 156–158. I have tried to maintain a similar agnosticism with regards to the ongoing debate as to whether or not Wittgenstein, at least by the time he had completed the *Tractatus*, intended for predicate and relation signs to name Tractarian objects. For an overview of the debate and the relevant secondary literature, see Diamond, *The Realistic Spirit*, 191.

[65] James Conant, "Wittgenstein on Meaning and Use," *Philosophical Investigations* 21, no. 3 (1998): 235; Livingston, "'Meaning Is Use' in the Tractatus," 38

3.32)." So, whereas a symbol is strictly determined by its logico-formal characteristics, its corresponding sign is determined by its sensually discernible attributes, which can be either seen or heard. Wittgenstein will therefore seek to show that the two inscriptions of the same sign, "*F*()", actually amounts to an instance of the following case: "Two different symbols can therefore have the same sign (written or spoken, etc.) in common with each other – they then signify [*bezeichnen*] in different ways (3.321)." That is, in order to dispel the apparent reflexivity in the construction, "*F*(*F*(*fx*))," Wittgenstein must demonstrate how the two "*F*()" signs have "two different modes of signification [*Bezeichnungsweisen*]," and thus "signify two [different] objects [*Gegenständen*] (3.322)," and thereby have "different meaning [*Bedeutung*] (3.323)."

Finally, we can distinguish the two symbols that share the same sign by appealing to the conception of logico-syntactic rules postulated in the third presupposition:

> 3.326: In order to recognize a symbol by its sign, we must attend to significant use [*sinnvollen Gebrauch*].
> 3.327: A sign determines a logical form only together with its logico-syntactic employment [*Verwendung*].

In other words, we can identify the formal characteristics of a given sign by considering how it is *used* or *employed* in a significant proposition. As both Livingston and Adrian Moore rightly explain, the univocity of a Tractarian expression or symbol entails that it is restricted to only one single logico-syntactic use.[66] Consequently, whatever might ostensibly appear to result from the reflexive nesting of the same propositional function,

[66] Livingston, "'Meaning Is Use' in the *Tractatus*," 36, 43, 58;
Livingston, *The Politics of Logic*, 143–144; A. W. Moore, "Being, Univocity, and Logical Syntax," *Proceedings of the Aristotelian Society (Hardback)* 115, no. 1pt1 (April 2015).

such as "$F(F(fx))$," will instead be comprised of different symbols. On the one hand, the outer "$F()$" is being employed as a third-order propositional function with a corresponding logical form of "$\psi(\varphi(fx))$." On other hand, the inner "$F()$" is being used as a second-order function with the form "$\varphi(fx)$." It follows that the inner and the outer "$F()$" must be two different symbols and therefore must also have different meanings. Furthermore, if we recall that the Fregean hierarchy implies that third-order functions can only take second-order functions and second-order functions can only take first-order functions, then the very possibility of reflexively nesting the same propositional function is, straightaway, ruled out. Therefore, since it is not at all possible for the same propositional function to take itself as an argument, the specific form of self-reference needed to generate the property version of Russell's paradox cannot even get off the ground.

It is worthwhile to note that the logico-syntactic classification of a symbol with its one and only specific univocal use imposes an extremely tight restriction on its formal role within a propositional context. José Zalabardo explains this constraint on symbols, as propositional constituents, in terms of their being "maximally specific with respect to form":

> Whenever, say, a binary relation R figures as a relating relation in an atomic complex, this complex will have the same structure, which we can naturally represent as xRy. For singular terms, however, the situation seems to be different. A singular term a that figures as the first relatum in a dual complex (aRb) can also figure as the second relatum in a different dual complex (cSa), or as the subject of a subject-predicate proposition (Pa), etc. There doesn't seem to be a single structure exemplified by all the propositions in which a singular term figures. This seems to

> force Wittgenstein to accept that aXy, xYa, Xa… are different expressions – i.e. that, for example, aRb, cSa, and Pa don't have any expression in common.[67]

The benefit of this maximum specificity is that the location of a symbol in a proposition distinguishes it from any other symbol located elsewhere in the same proposition, so that it rules out any occurrence of a propositional function taking itself as its own argument. However, this exceedingly strict connection between a symbol's formal characteristics and its symbolic identity leads to some rather counterintuitive consequences. For example, since a symbol is "presented by means of the general form of the propositions that it characterizes" (*TLP*: 3.313), the formal differences between "$\varphi(a, y)$," "$\phi(x, a)$," and "$\psi(a)$" entail that a different symbol occurs for each of the three instances of sign, "*a*." Worse yet, it also follows that the instances of sign "*a*" cannot have the same meaning.[68]

1.2.2: The Property Version of the Paradox and the Requirement for Simple Names

Prima facie, it is hard to see what any of this would have to do with simple names. Yet, another innovative suggestion from Livingston can help reveal the important connection between the dissolution of the property version of Russell's paradox and the requirement for simple names.[69] More specifically, Livingston attempts to draw out such a

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[67] Zalabardo, *Representation and Reality in Wittgenstein's Tractatus*, 128; see also, 156, 178, 230.

[68] Roughly put, the sign "apples" would have to be a different symbol in each of the following propositional contexts: "People consume apples," "Apples consume plant-food," and "Apples are red." Furthermore, maximal specificity with respect to form runs counter to the following rather intuitive claims: "It is impossible to dispense with propositions in which the same argument occurs in different positions. It is obviously useless to replace φ (a, a) by φ(a, b).a = b (*NB*: 102)."

[69] Livingston, *The Politics of Logic*, 144.

connection by juxtaposing two otherwise disparate threads of remarks in the *Tractatus*. The first is the so-called argument for substance:

> 2.02: Objects are simple…
> 2.021: Objects make up the substance of the world. Therefore, they cannot be composite.
> 2.0211: If the world had no substance, then whether a proposition had sense would depend on whether another proposition was true.
> 2.012: It would then be impossible to devise a picture of the world (true or false).

The second is a set of stipulations about logical syntax that can be found in the lead-up to paradox dissolution:

> 3.33: In logical syntax the meaning [*Bedeutung*] of a sign ought to never play a role; it must admit of being established without mentioning the *meaning* [Bedeutung] of a sign; it ought to presuppose only the description of the expressions.
> 3.331: From this observation we get a further view into Russell's "Theory of Types." Russell's error is shown that in establishing the rules for signs he has to mention the meaning [*Bedeutung*] of signs.

According to Livingston, what both threads of remarks share is an injunction against a proposition mentioning what a sign denotes, i.e., its designated object, referent, or meaning.

There is, however, an immediate interpretative difficulty that must be resolved before the two sequences of remarks can be coherently read together. The most obvious problem is that the argument for substance does not explicitly put forward an injunction against mentioning a sign's referent. Instead, Livingston seems to be calling upon a rather debatable tradition in the secondary literature that regards the injunction as an unsaid and tacit corollary of the argument's appeal to the independence requirement. Thus, more textual evidence is needed in order to verify that the injunction is actually a

consequence of the requirement that the sense of *every* proposition *must* be independent of the truth or falsity of any proposition. Simultaneously, the injunction is overtly put forward in the stipulations about establishing rules of logical syntax. Yet, the problem here is that no readily available justification is given as to why it must be forbidden for a rule to mention a sign's referent. In short, the exegetical dilemma is that wherever the injunction is present, its justification is absent, and vice versa.

For my part, I want to suggest that a solution to one horn of the dilemma can be found by turning to 1914's "Notes Dictated to G.E. Moore in Norway." There we find, conveniently couched between two postulations about logical syntax, a slightly different formulation of the independence requirement:

> What symbolizes in a symbol, is that which is common to all the symbols which could, in accordance with the rules of logic = syntactic rules for manipulation of symbols, be substituted for it.
>
> The question whether a proposition has sense (*Sinn*) can never depend on the *truth* of another proposition about a constituent of the first. E.g., the question whether (x) x = x has meaning (*Sinn*) can't depend on the question whether (∃x) x = x is *true*. It doesn't describe reality at all, and deals therefore solely with symbols; and it says that they must *symbolize*, but not *what* they symbolize. (*NB*: 117)

In this somewhat more forthcoming version of the requirement, we learn that sense must remain independent of the truth or falsity of what might be said about a propositional constituent (*Bestandteil*).[70] However, since Wittgenstein does not explicitly specify *what*

[70] For the use of different variations of the term "*Bestandteil*," see *TLP* (2.0201, 3.24, 3.315, 4.025, 5.4733, 6.12). Also, note that subscribing to this formulation does *not* necessarily rule out the possibility that the requirement for independent sense could be contravened in other ways besides mentioning a propositional constituent. Furthermore, since the independent requirement is often associated with a standard metaphysical interpretation of the argument for substance, it is worth pointing out that this rendition of the requirement is veritably identical with its consciously resolute construal by Diamond in *The Realistic Spirit*, 191. However, more work is needed in order to determine whether this line of interpretation could be compatible with Zalabardo's more recent critique of various interpretations of the argument for

cannot be said about a given propositional constituent, we will have to look to the example he provides in order to safely conclude that he is alluding to a constituent's designated object or meaning.

The example threatens to obscure more than it clarifies, given that constructions containing an equal sign or sign for identity (*Gleichheitszeichen*), such as "(x) x = x" and "(∃x) x = x," are notoriously rendered nonsensical in the *Tractatus* (4.242–4.243, 5.5303–5.5352). Taking this into consideration, it is important to remember that the examples in question from "Notes Dictated to Moore" predate Wittgenstein's arrival at the definitive analysis of identity that would later be included in the *Tractatus*.[71] Thus, in a 1913 letter to Russell, we find that Wittgenstein did, indeed, parse a construction from the example in a way that ascribes a sense to it: "A proposition like '(∃x) x = x' is, for example, really a proposition of physics… it is for physics to say [*sagen*] *whether any thing exists* [Ding gibt] (*NB*: 128)." Therefore, we can reliably infer that the proposition, "(∃x) x = x," is an assertion about the existence of a thing (*Ding*), entity (*Sachen*), or object (*Gegenstand*): in other words, a propositional constituent's meaning. It follows that the independent requirement, *pro* Livingston, entails an injunction, at least, against the truth or falsity of a proposition about the existence of an object standing as a condition for sense. However, it is abundantly evident from postulates about logical syntax surrounding this rendition of the independent requirement that intention is to exclude any mention of things in reality that might threaten sense or symbolization.

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substance in *Representation and Reality in Wittgenstein's Tractatus*, "Appendix II: The Empty-Name Reading of the Substance Passage," 243–254.

[71] See Potter, *Wittgenstein's Notes on Logic*, 54–56.

As for the other horn of the dilemma, we have already been brought part of the way to a solution due to the insertion of the independence requirement within the context of postulates about logical syntax. Yet, in the pre-Tractarian *Notebooks*, the most dedicated effort to justify the injunction against logico-syntactic rules mentioning objects and meaning occurs in the various analyses of constructions like "M is a thing" as a purported rule (*NB*: 109–111). In a discussion that anticipates his later, more definitive treatment of logical syntax and types in the *Tractatus*, Wittgenstein dismisses the construction, "M is a thing," as "superfluous … because what this tries to say is something which is already *seen* when you see 'M.'"[72] However, the mere redundancy of saying what is already shown is not enough to irrefutably conclude that a theory of types relying on semantic meaning and objects must necessarily be erroneous. The deeper justification for the injunction is unfurled in Wittgenstein's struggle to perform a complete analysis of putative propositions that mention objects:

> *Now* we shall see how properly to analyze propositions in which "thing", "relation", etc., occur.
>
> (1) Take φx. We want to explain the meaning of "In 'φx' a *thing* symbolizes." The analysis is:
>
> —
>
> (∃y). y symbolizes. y = "x". "φx"
>
> ["x" is the name of y: "φx" = "'φ' is at [the] left of 'x'" and *says* φx.]
>
> N.B. [i.e. *nota bene*] "x" can't be the name of this actual scratch y, because this isn't a thing: but it can be the name of a thing; and we must understand that what we are doing is to explain what would be meant by saying of an ideal symbol, which did actually consist in one thing's being to the left of another, that in it a thing is symbolized. (*NB:* 110)

It is rather obvious that Wittgenstein is at pains to convey what he wants to get across in the symbolic notation. Roughly put, what the analysis attempts to show is that it cannot

[72] *NB* (110); C.f., *TLP* (3.325–3.3442, 4.122–4.1274).

reach a brute ontological thing in its capacity to symbolize or be signified, because it is impeded by the sign that denotes it. In other words, we cannot render perspicuous in a *Begriffsschrift* the fact that an object, y, is symbolized by the sign "x." However, given his complications in making this clear, it is no surprise that this stretch of symbolic analysis did not find its way into the *Tractatus*.

In this section of the *Notebooks,* Wittgenstein will nevertheless go on to summarize the results of these symbolic manipulations with the claim that "In our language names are not things: we don't know what they are: all we know is that they are of a different type from relations, etc. etc. The type of a symbol of a relation is partly fixed by [the] type of [a] symbol of [a] thing, since a symbol of [the] latter type must occur in it. (*NB*: 111)."[73] What Wittgenstein is struggling to suggest is that we cannot take up into language and articulate the internal relation between the name sign "x" and an object, y, because that internal relation is not an object that can be spoken about. Fortunately, he has better luck at communicating this line of reasoning in the "Notes on Logic":

> Just as little as we are concerned, in logic, with the relation of a name to its meaning, just so little are we concerned with the relation of a proposition to reality, but we want to know the meaning of names and the sense of propositions – as we introduce an indefinable concept "A" by saying: "'A' denotes something indefinable", so we introduce e.g. the form of propositions aRb by saying: "For all meanings of 'x' and 'y', 'xRy' expresses something indefinable about x and y." (*NB*: 102)[74]

[73] These corrections are not mine and were presumably made either by G. E. Moore himself, or by the editors of the collection, namely G. E. M. Anscombe and G. H. Wright.

[74] For my part, I believe that this passage provides primary textual evidence to substantiate the following line of reasoning that Jakub Mácha attributes to the early Wittgenstein in *The Philosophy of Exemplarity: Singularity, Particularity, and Self-Reference*, (New York, NY: Routledge, 2023), 45: "Introducing formal concepts thus avoids the 'horse' paradox, and this circumvention is incorporated into the *Tractatus*'s conceptual framework. Names refer to objects. If there were a name X of a name Y, then Y would be an object. But objects are simple; they cannot mean anything. And so, there is no name of Y."

In other words, Wittgenstein advises against focusing on the relation between a name and an object, a sign and a thing, because it turns out to be an utterly indefinable (internal) relation.[75] Furthermore, notice that, since "'A' denotes something indefinable" is basically equivalent in form and content to "'x' is the name of y," the argument aims at rejecting such a putative rule of logical syntax.

As already mentioned, these labors are not recorded in the *Tractatus*, but a version of the following conclusion will nevertheless later appear in the finished product:

> It is to be remembered that names are not things, but classes: "A" is the same letter as "A". This has the most important consequences for every symbolic language. (*NB*: 102).

Not only does Wittgenstein now get a clear idea about what a name is, but the above clarifies an otherwise hopelessly cryptic remark in the *Tractatus*, which seemingly comes out of nowhere:

> 3.203: A name means an object. The object is its meaning. ("*A*" is the same sign as "*A*.") [*Der Name bedeutet den Gegenstand*. *Der Gegenstand ist seine Bedeutung*. („A" *ist dasselbe Zeichen wie* „A".)]

Wittgenstein claims that a name sign that symbolizes is itself a class because it is, more precisely, "the common characteristic feature of a class of propositions (*TLP*: 3.311)." In other words, a name is nothing other than the shared role it has in a class of propositions with sense.[76] I want to argue that this can account for the rare instance above in which

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[75] Although the early Wittgenstein will later drop any position that will ascribe a meaning (*Bedeutung*) to a proposition, roughly the same point is put forward in this rather confused remark a few lines down: "Neither the sense nor the meaning of a proposition is a thing. These words are incomplete symbols. (*NB*: 102)."

[76] McGinn, *Elucidating the Tractatus*, 121.

Wittgenstein happens to actually employ the use/mention distinction with systematic care: he is indicating that there is nothing that can be gleaned about what an isolated sign, like "*A*," signifies outside of its participation in a class of propositions.[77] Furthermore, I also want to suggest that the empty tautological construction, "'*A*' is the same sign '*A*,'" is the closest the reader of the *Tractatus* can get to an example of a logico-syntactic rule. Put differently, this is the symbolic rendering of the injunction that a rule cannot mention an object, referent, or meaning designated by a sign. Finally, I want to maintain that this is also the reason why *TLP* (3.334) declares that rules must "follow of themselves" or "go without saying" (*sich von selbst verstehen*), because they cannot say what is instead shown by how a sign symbolizes in a proposition with sense.

1.2.3: The Propositional Version of the Paradox

As Peter Sullivan has convincingly argued in a series of article-length contributions, there is more to paradox dissolution in the *Tractatus* than what prevails over its property version.[78] Indeed, Sullivan astutely observes that *TLP* (3.333) does not address putative cases of self-reference that could arise in the form of "$F(F(p))$" but only that which results from the nesting of propositional functions, such as "$F(F(fx))$."[79] Furthermore, in a rather elliptical argument, Sullivan attempts to show how the possibility of a

[77] For Wittgenstein's application of the use/mention distinction, see Potter, *Wittgenstein's Notes on Logic*, 4–5.
[78] Sullivan, "The Totality of Facts"; Sullivan, "On Trying to Be Resolute"; Sullivan, "'The General Propositional Form Is a Variable.'" See also, Gordon P. Baker, *Wittgenstein, Frege, and the Vienna Circle* (Oxford: Blackwell, 1988), 73.
[79] Sullivan, "The Totality of Facts," 187.

proposition that refers directly to propositions is actually inherent in the very design of a propositional function as a Tractarian *Satzvariable*:

> To conceive a proposition in accordance with this model, the proposition "$F(a)$" for instance, is to think of "a" as replaceable by other arguments, yielding a grasp of a function $F(\xi)$, which may then be thought replaceable by other arguments, yielding a function $(\phi)a$, which may then be thought replaceable by other arguments… *and so on*. This "and so on" betokens a generalization that can find no place in the series of type-bound generalization that it ranks, but whose understanding yet appears to be every bit as much internal to our grasp of the structure of propositions as they are.[80]

For my part, I contend that the same conclusion can be reached by the shorter route of simply recognizing that "A proposition is itself [*selbst*] an expression" (*TLP*: 3.31). And since a proposition itself is "a function of the expressions contained in it" (*TLP*: 3.318), there arises the threat of a proposition that could contain other propositions or itself. Either way, if it is permitted for a proposition to say something about propositions, then everything is in place for the propositional version of Russell's paradox.

According to Sullivan, the key assumption that blocks the formation of the propositional paradox is elaborated in a skein of Tractarian remarks, which consist of various formulations of the assertion that "Propositions… are themselves facts" (*NB*: 97):

> 3.14: What constitutes a propositional sign is that in it its elements (the words) stand to one another in a determinate [*bestimmte*] way.
>
> A propositional sign is a fact.
>
> 3.141: A proposition is not a mixture of words – (Just as a musical theme is not a mixture of notes.)
>
> A proposition is articulate.

[80] Sullivan, "On Trying to Be Resolute," 57.

> 3.142: Only facts express a sense, a class of names cannot…
> 3.1432: Instead of, "The complex sign '*aRb*' says that *a* stands to *b* in the relation *R*," we ought to put, "*That* '*a*' stands to '*b*' in a certain relation says *that aRB*."
> 3.144: Situations can be described but not *given names*.[81]

That is, a determinate combination of constituents into a propositional sign is but a specific instance of the more general combination of objects into a state of affairs (2–2.01, 2.031). Since they are inherently composite, "Facts cannot be named," and likewise "Propositions are not names" (*NB*: 96, 98). Consequently, a proposition cannot make assertion about propositions because propositions cannot be *used* as a name:

> There are *internal* relations between one proposition and another, but a proposition cannot have to another *the* internal relation which a *name* has to the proposition of which it is a constituent, and which ought to be meant by saying it "occurs" in it. In this sense one proposition cannot occur in another. (*NB*: 116)

And once it is established that a proposition cannot contain propositions, the propositional version of Russell's paradox ends up as flightless as the property version.

As has already been mentioned, the same argument that prohibits the kind of reflexivity that could generate the propositional version of the paradox also establishes both the fundamental thought that "there are no logical objects" (*TLP*: 4.441) and the general form of a proposition. This is, of course, made evident through notation for logical operators or truth operations (*TLP*: 5.2–5.4).[82] However, what are likely the most cogent arguments that could serve to motivate the notation are also found in the *Notebooks*:

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[81] See also, *TLP* (4.032). Furthermore, it is worth mentioning that various formulations of these remarks are interspersed throughout the dictated pre-Tractarian notes (*NB*: 93–119).
[82] See, Sullivan, "The Totality of Facts," 188–190.

> Symbols are not what they seem to be…. Similarly in "φx," "φ" looks like a substantive but is not one; in "~p," "~" looks like "φ" but is not like it. This is the first thing that indicates that there *may* not be logical constants. A reason against them is the generality of logic: logic cannot treat a special set of things. (*NL*: 98).
>
> Logical indefinables cannot be predicates or relations, because propositions, owing to sense, cannot have predicates or relations. Nor are "not" and "or," like judgment, *analogous* to predicates or relations, because they do not introduce anything new. (*NL*: 99).
>
> The false assumption that propositions are names leads us to believe that there must be logical objects: for the meaning of logical propositions will have to be such things. (*NL*: 107).[83]

As it was with the previous argument, since a proposition is a fact, whatever connects with a proposition must be insubstantial and without content. The *Grundgedanke* is an immediate upshot because logical connectives, like negation, conjunction, disjunction, and implication, must then be insubstantial and empty. Additional generality can be achieved by realizing that all logical constants are veritably the same insofar as they are all empty: "One could say that the sole logical constant was what *all* logical constants, by their very nature, have in common with one another. But that is the general form of a proposition" (*TLP*: 5.47).

1.2.4: Medial Showing

Since it has already been implicitly operating in the background of the two cases of paradox dissolution, it is now necessary to provide Tractarian showing with a more

[83] Notice Wittgenstein's reservations about whether each particular elucidation can provide the generality necessary for the *Grundgedanke* and the general form of a proposition. This is another way we can understand how the resolute interpretation's emphasis on particularity leads to difficulties in motivating such requirements. However, the resolute interpretation can adequately account for the dissolution of the property version of the paradox because it does not necessarily require the same kind of general conclusion. For more on this, see Sullivan, "On Trying to Be Resolute, " 54–64.

systematic account. Indeed, the early Wittgenstein is compelled to resort to showing because his respective treatments of the property and the propositional versions of the paradox must eschew strategies of argumentation that speak about propositions and expressions so as to avoid employing the very reflexive propositions that he aims to invalidate. However, there is an extensive and compelling list of resolute objections against the mostly standard interpretations that rely on some kind of purported methodological faculty for showing, as they are often criticized as nothing other than ad hoc attempts to smuggle in the very metaphysical pseudo-propositions that are repeatedly rejected throughout the *Tractatus*.[84]

Due to its intentionally resolute design, I rely mainly on Jean-Phillipe Narboux's account of showing in terms of the grammatical middle voice or medial diathesis.[85] For Narboux, the key to understanding Tractarian showing demands recognizing Wittgenstein's deliberate employment of the middle voice in phrases such as "mirrors itself [*spiegelt sich*]," "expresses itself [*ausdrückt sich*]," and "shows itself [*zeigt sich*]."[86]

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[84] For a good overview of the voluminous secondary literature resolutely criticizing interpretative accounts of Tractarian showing, see the overviews in Kussella, "The Dialectics of Interpretations," 123–126; and Sullivan, "On Trying to Be Resolute," 49–52.

[85] Narboux, "Showing, the Medium Voice, and the Unity of the *Tractatus*."

[86] Narboux astutely traces the analytic tradition's willful disregard for grammatical distinctions back to its foundational moment with Frege and his initial methodological gestures ("Showing," 214). Most notably, Frege insisted, in his 1879's *Begriffsschrift* (§§3–4, 9), that there was no logical significance in distinctions drawn on the basis of grammatical diathesis. See, Gottlob Frege, *Translations from the Philosophical Writings of Gottlob Frege*, ed. Peter Geach and Max Black, 2nd ed. (Oxford: Basil Blackwell, 1960), 2–4, 12–15. On the contrary, the Continental tradition has historically been more favorably disposed in availing itself of the grammatical resources concerning the vicissitudes of the voice. A prime example of Continental philosophy's sensitivity to grammatical diathesis is Heidegger's well-known discussion of the middle-voice declension of the Greek verb *phaīno* as *phaīnesthai*. See, Martin Heidegger, *Being and Time: A Translation of Sein und Zeit*, trans. Joan Stambaugh, SUNY Series in Contemporary Continental Philosophy (Albany, NY: State University of New York Press, 1996), 25–29. Also, for a sweeping study in the Continental tradition that draws on the grammatical peculiarities of the middle voice with important connections to Wittgenstein, see Schürmann, *Broken Hegemonies*, 28–48.

In fact, grammatical constructions of this sort repeatedly occur in the principal thread of remarks where showing takes center stage in the *Tractatus*:

> 4.12: Propositions can present [*darstellen*] the whole of reality, but they cannot present what they must have in common with reality in order to be able to present – the logical form.
>
> In order to be able to present logical form, we [*wir*] should have to be able to position ourselves outside logic, that is, outside of the world.
>
> 4.121: A proposition cannot present logical form; it mirrors itself [*spiegelt sich*] in it.
>
> What mirrors itself [*sich… spiegelt*] in [*in*] language, language cannot present.
>
> What expresses *itself* [sich… *ausdrückt*] in language, *we* [wir] cannot express by means of [*durch*] language.
>
> A proposition *shows* [zeigt] the logical form of reality.
>
> It displays [*weist*] it…
>
> 4.1211: Thus, one proposition "*fa*" shows that the object *a* occurs in its sense, two propositions "*fa*" and "*ga*" show that the same object is talked about in both of them.
>
> If two propositions contradict one another, then their structure shows it; similarly, if one follows from another, etc.
>
> 4.1212: What *can* be shown [*gezeigt*] *cannot* be said [*gesagt*]….
>
> 4.122: In a certain sense, we can talk about formal properties of objects and states of affairs or of properties of the structure of facts and in the same sense of formal relations and relations of structure…
>
> 4.124: The existence [*Bestehen*] of such internal properties, however, cannot be asserted by means of [*durch*] propositions, but it shows itself [*zeigt sich*] in the propositions that present those states of affairs and treats of those objects.

The remarks above unfold a series of corresponding binary contrasts: states of affairs in reality, on the one hand, and their logical form, on the other; what is presented *by* propositions versus what is presented *in* propositions; what *we* express *by* language and what is expressed *in* language; and finally, the dichotomy can be rendered more succinctly in the opposition between saying and showing. Furthermore, Narboux explains that this distinction entails two completely heterogeneous modes of expression:

showing expresses itself in a proposition, whereas saying expresses itself by means of a proposition.

According to Narboux, a major confusion driving the controversy between the standard and resolute interpretations is a certain widespread failure to recognize that Wittgenstein took care to make room for showing as a wholly distinct means of expression:

> Conspicuous as it might seem, ,this central feature of the distinction at stake in 4.121 is equally obscured by readings of the *Tractatus* that assume Tractarian "showing" (*zeigen*) to be readily predicable to nonsense and by readings that take the three concepts of "the unsayable" (*das Unsagbare*), "the utterable" (*das Unaussprechliches*), and "the inexpressible," to be more or less interchangeable in the *Tractatus*.[87]

Upon heeding this more precise discrimination, it is then clear that saying and showing are both in the ambit of the expressible:

> In effect, resolute readers and standard readers alike tend to use interchangeably the terms "inexpressible," "unsayable," "ineffable," "unutterable," as if they were more or less synonymous by the lights of the *Tractatus*. This loose way of talking might be in some ways as damaging as the loose way of talking about saying and showing on which some versions of the standard readings trade. For there is every reason to think that Wittgenstein is at pains to mark distinct notions by means of the corresponding German terms. Thus, while "unsayable" straightforwardly renders "*unsagbar*," "inexpressible" is a most natural way of referring to what does not admit of expression in the Tractarian sense of the word (*Ausdruck*), and "unutterable" (or for that matter, "ineffable") well renders "*unaussprechlich*." But… while what can be shown (*gezeigt werden kann*) can in a way be deemed "unsayable," since it is "not sayable" (*kann nicht gesagt werden*) (4.1212), it can hardly be said to be "inexpressible," since it is precisely to the extent that it "expresses itself" (*sich ausdrückt*) that what shows forth can't be said. It is not at all

[87] Ibid., 211.

> inexpressible: it is inexpressible *by a proposition* (not sayable), because [it is] inexpressible *by us*.[88]

So, on one hand, *contra* certain versions of the standard interpretation, showing is neither a substitute nor an alternative to saying but a wholly different faculty for expression. Yet, on the other hand, *contra* certain versions of the resolution interpretation, showing cannot be dismissed as unintelligible or nonsensical, since it is capable of expressing.

Furthermore, the distinction between saying and showing parallels the distinction as to whether an agent is involved in the particular mode of expression:

> The key to 4.121, and more generally to the 4.121 sequence as a whole, is the distinction just alluded to between two non-overlapping modes of expression, one which has *nothing* to do with us, while the other has *everything* to do with us…
>
> The ubiquity, throughout the book, of "show itself" (*sich zeigen*) and other verbal phrases of the same grammar such as "*sich ausdrücken*" and "*sich spielgeln*," confirms what the 4.12s already suggest: namely, that the notion of what is shown by a proposition is to be understood in terms of the notion of shows itself (*zeigt sich*) *in* (*am*) a proposition, rather than conversely. The distinction between what a proposition shows and what it says is derivative. What is said is essentially said *by* a proposition. Moreover, for a proposition to say that *p* is essentially *for us* to say that *p* by means of a proposition. By contrast, what is shown is not essentially shown "by means of" a proposition, and not at all shown by us.[89]

Saying requires an agent since it is we (*wir*), the human speakers of a language, who express ourselves by using propositions. For example, *TLP* (4.026b, 4.062c) are characteristic instances of Wittgenstein's repeated employment of the active diathesis, in which the subject of the sentence, "we [*wir*]," indicates "us [*un*]," the human speakers of a language, as the very agents of expression: "With propositions, we [*wir*] explain

[88] Ibid., 207.
[89] Narboux, "Showing, the Medium Voice, and the Unity of the *Tractatus*," 212.

ourselves [*un*]"; "For, a proposition is true, if we by means of it say [*wir*… *durch*… *sagen*] such as it itself stands as so."[90] However, showing lacks an agent, since a proposition is what expresses itself, on its own, without us. As *TLP* (4.121c) indicates, the contrast between the two modes of expression is made explicit in terms of agentive presence or absence, its involvement or lack thereof: "What expresses *itself* [sich… *ausdrückt*] in language, *we* cannot express by means of language [wir *nicht durch sie ausdrücken*]." In other words, the logical form that shows itself is a vehicle of expression that does not require us (*un*), the human speakers of a language, and we (*wir*) cannot be distinguished as the primary actants in showing. Accordingly, observe that, while the second clause in *TLP* (4.121c) draws on the active diathesis, so as to distinguish the sentential subject as an agent, "*we* [wir]"; the first clause instead draws on the medial diathesis in the construction, "expresses *itself* [sich… *ausdrückt*]," so as to preclude any distinguishable sentential subject that might be construed as an agent of showing.

In order to get a better handle on the terminology, it should be noted that the strictly grammatical notion of the middle voice or diathesis can be specified with respect to how the subject of a sentence stands in relation to the process referenced by its verb. In particular, Narboux follows the grammatical account advanced by the French linguist Émile Benveniste in his renowned study of the middle voice in ancient languages:

> The principle of a properly linguistic distinction, turning on the relationship between subject and process, is brought out quite clearly by this comparison. In the active, the verbs denote a process that is accomplished outside the subject. In the middle, which is the diathesis defined by the

[90] In the lead-up to *TLP* (4.12–4.122), see also *TLP* (2.1, 3.001, 3.11–3.12) for various other formulations that explain how it is that we, the human speakers of a language, are to be distinguished as the specifically linguistic agents of expression.

> opposition, the verb indicates a process centering in the subject, the subject being inside the process.[91]

That is, in instances of active diathesis, the subject of the sentence is an agent that is somehow distinct from the process designated by the verb; yet, in instances of medial diathesis, the subject/agent is thoroughly involved in the process, so much so that the former is internal to the latter. As Narboux explains it, "In the [active] case, as we might also put it, the subject *transcends* the process; in the [middle] case, the subject is *immanent* to the process."[92] Furthermore, it is possible to radicalize this account by effecting a complete agent-demotion, such that the subject's involvement in the process denoted by the medial verb encompasses the former to the extent that it entirely disappears within the latter.

Likewise, Narboux observes this grammatical distinction throughout the *Tractatus*, as the occurrence of an external agent in saying and its inclusion within showing tends to coincide respectively with the active and the medial diathesis:

> What shows itself (*zeigt sich*) by Tractarian lights is indeed essentially *involved in* the process through which it shows itself… [W]hat reflects [i.e., mirrors] itself (*spiegelt sich*) is essentially *embedded in* its reflection, what expresses itself (*ausdrückt sich*), essentially *embodied* in its expression (4.121). The crucial point is thus that what shows itself *cannot be abstracted from its expression*, or rather that it does not make sense so much as to *want to* abstract it from its expression….
>
> It is the distinction between what is and what is not our business – between our business and (so to speak) logic's business… The diathesis of "we express" (*wir ausdrücken*) is external [i.e., active]: what can be said is that whose expression we can bring about. The diathesis of "expressing itself" is internal [i.e., medial].[93]

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[91] Émile Benveniste, "Active and Middle Voice in the Verb," in *Problems in General Linguistics* (University of Miami Press, 1971), 148.
[92] Narboux, "Showing, the Medium Voice, and the Unity of the *Tractatus*," 220.
[93] Ibid., 221–222.

As is intimated above, the agent as what shows itself and the process of showing should not be interpreted as separate or distinct. In other words, Tractarian showing is better understood along the lines of the radicalization of Benveniste's account, such that the agent vanishes within the process. Accordingly, since Narboux maintains that what shows itself cannot be abstracted from showing, he also insists upon a similar kind of radical agent demotion: "[W]hat is said to show the form of reality (the proposition) is no agent, what is shown (or rather shows itself) *in* it, no *patient*"; "what shows itself, while neither agent [n]or patient of the process, is immanent to it."[94] Put in another way, the process of Tractarian showing usurps what shows itself because the former totally absorbs the latter.

As is suggested by its lack of a patient, there is an often-observed connection between the medial diathesis and the lack of a sentential object in intransitive constructions. Consequently, there is an association drawn, on the one hand, between the active voice and transitive verbs that designate a process from subject to object; whereas, on the other hand, the middle voice can be associated with intransitive verbs that do not require an object. On the basis of this connection, Benveniste explains the shift from the middle to the active voice by way of a correlative shift from an intransitive to a transitive verb:

> Let us suppose now that a typically middle verb… is endowed secondarily with an active form. As a result of this, there will be a change in the relationship of the subject to the process so that the subject, in becoming exterior to the process, will become the agent of it, and the process, no longer taking place within the subject, will be transferred to another term that will become the object of it. The middle will be converted into the transitive… Transitivity is a necessary product of this

[94] Ibid., 212, 213.

> conversion from middle to active. Thus, starting from the middle, actives are formed that are called transitive, or causative, or factives, and which are always characterized by the fact that the subject, placed outside the process, governs it thenceforth as agent, and that the process, instead of having the subject for its seat, must take an object as its goal.[95]

Therefore, it should follow, on this account, that middle diathesis coincides with intransitivity because the subject is enveloped in a process that is itself neither an object nor has an object as its final end.

In Wittgenstein's early phase, little explicit attention is devoted to the transitive/intransitive distinction. For instance, in the *Tractatus*, the only mention of a related grammatical concern occurs in a brief and passing example involving intransitive verbs called on in order to demonstrate how a symbol could be distinguished from its correlative sign (*TLP*: 3.232). In his later phase, however, we do come to find a much more sustained engagement with transitivity and intransitivity.[96] In fact, for Wittgenstein, the distinction is broader than just grammar, as transitivity and intransitivity take on a formal/logical significance. For example, in the *Brown Book*, Wittgenstein takes it even further than the typical grammatical concerns regarding the presence or the absence of a sentential object but instead draws the following distinction between transitivity and intransitivity on the grounds of whether or not an expression can undergo linguistic clarification: "On the one hand, we may say, it is used preliminarily to a specification, description, comparison; on the other hand, as we might describe as

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[95] Benveniste, "Active and Middle Voice in the Verb," 149.

[96] Ludwig Wittgenstein, *Preliminary Studies for the "Philosophical Investigations" Generally Known as The Blue and Brown Books*, Nachdr. (Malden, Mass.: Blackwell, 2007), 22, 29, 158–161, 178. Ludwig Wittgenstein, *Philosophical Grammar*, ed. Rush Rhees, trans. Anthony Kenny, Repr (Oxford: Blackwell, 2004), 11, 79. From here on, abbreviated respectively as *BB* and *PG*.

emphasis (*BB*, 158)." In this regard, a transitive object entails admitting of further translation; whereas, the intransitive case resists any additional elaboration.

As Narboux points out, however, the philosophical implications of intransitive verbal form are clearly operative in Wittgenstein's early phase with the medial constructions "*sich zeigen*" and "*sich zeigt*." In fact, Narboux even detects grammatical intransitivity in a pair of remarks that might ostensibly look otherwise:

> 4.022: A proposition *shows* [zeigt] its sense.
>
> A proposition *shows* how things stand *if* it is true. And it *says* [sagt] *that* they do so stand…
>
> 4.461: A proposition shows what [*was*] it says; a tautology and a contradiction show that they say nothing.

Narboux tends to read these remarks in terms of how the late Wittgenstein construed the distinction between transitivity and intransitivity. Accordingly, what a proposition shows, whether construed as its sense, how things stand, or what it says, must be understood intransitively because it cannot be further translated into another object of description, specification, or comparison. As Narboux explains:

> To say that the proposition shows its sense (4.022) is to say that we do not need to *make out* what it means. Likewise, to say that the proposition shows what it says (4.461) is to say that we do not need to *make out* what it says…. Therefore, in 4.461, "what it says" (*was er sagt*) does *not* in fact mean "*that which* it says" (*Das, was…*). For the proposition to show what it says is not for it to "show that which it says." Not being a nominal phrase, "what it says" (*was er sagt*) does not admit a paraphrasing apposition of the form "names, (the fact)…" It was used intransitively, as it were.[97]

[97] Narboux, "Showing, the Medium Voice, and the Unity of the *Tractatus*," 228.

That is, like logical form, what shows itself in remarks 4.12–4.122, as well as what shows in 4.022 and 4.461, cannot be further taken up into words, and, for that reason, it must be intransitive.

Given how this account of the medial diathesis entails intransitivity, it must therefore be distinguished from reflexivity, reflexive constructions, and reflexive diathesis. Indeed, the very definition of a reflexive verb, as a verb whose subject and object designate the same referent, implies its distinctiveness. As for the classic example of linguistic reflexivity, consider the sentence, "He killed himself." In this case, the subject, "he," and the object, "himself," designate the same person. Consequently, such reflexive constructions must be distinguished from intransitive medial constructions because the former require a subject and an object, whereas the latter must exclude both.

Also in the middle to late period, Wittgenstein explicitly mobilizes this linguistic distinction for its philosophical significance:

> Now this is a characteristic situation to find ourselves in when thinking about philosophical problems. There are many troubles which arise in this way, that a word has a transitive and an intransitive use, and that we regard the latter as a particular case of the former, explaining the word when it is used intransitively by a reflexive construction.
>
> Thus we say, "By 'kilogram' I mean the weight of one liter of water," "By 'A' I mean. 'B,' where B is an explanation of A." But there is also the intransitive use: "I said that I was sick of it and meant it." Here again, meaning what you said could be called "retracing it," "laying an emphasis on it." But using the word "meaning" in this sentence makes it appear that it must have sense to ask "What did you mean?" and to answer "By what I said I meant what I said," treating the case of "I mean what I say" as a special case of "By saying 'A' I mean 'B.'". In fact one uses the expression "I mean what I mean" to say, "I have no explanation for it." The question, "What does this sentence mean?," if it doesn't ask for a translation of *p* into other symbols, has no more sense than "What sentence is formed by this sequence of words?"
>
> … We often use the reflexive form of speech as a means of emphasizing something. And in all such cases our reflexive expressions can be "straightened out." (*BB*: 160–161)

That is, despite their resemblance, Wittgenstein distinguishes between intransitive and reflexive constructions on the basis that only the latter is transitive. Once again, the intransitive case resists translation and instead puts emphasis on what has already been stated, whereas the reflexive case is distinct because its transitivity entails a further translation into another specification, description, or comparison, etc.

Once again, in Wittgenstein's early period, there is no explicit mention of the fine-grained distinction between intransitivity and reflexive transitivity. Nevertheless, the distinction is mobilized in how intransitive showing is used to subvert reflexive self-reference in order to uphold the following as a general claim: "No proposition can say anything about itself…(*TLP*: 3.332)." More generally, the intransitivity of medial showing participates in the prohibition against reflexivity because it implies that whatever variation of logical form happens to be shown, it cannot also be named or spoken about:

> It is not simply form (*Form*), i.e. the possibility of structure (*Struktur*) (2.033), but also structure itself that shows itself according to the *Tractatus*. It can neither be named nor said (stated). In particular, the proposition is not articulated by anything. Nothing ties its elements (the names it comprises) into the whole that it is. Rather, the proposition articulates into its elements: it articulates itself.
>
> In effect, the structure of a state of things (*Sachverhalt*), that is to say the definite way in which things stand with each other (*verhalten sich . . . zueinander*) in a state of things (2.031), their definite way of hanging together (*zusammenhängen*) (2.032), is neither a "thing" nor a "situation." It is neither "nameable" nor "sayable." As the structure of a fact (*Tatsache*) is simply the structures of the state of things in whose obtaining and not obtaining it consists (2.034), it is likewise neither a thing nor a situation, and so is likewise neither nameable nor sayable. But the same thing can be said of the structure of a picture [i.e., a logical picture]. The structure of a picture is the structure of the fact into which it consists, insofar as it represents (*vorstellt*) things to be so and so: it is the definite way in which elements stand with each other (*sich zu einander verhalten*) in the state of things (*Sachverhalt*) in whose obtaining it con-sists, insofar as it

> represents (*stellt vor*) that things stand in that way (*dass sich die Sachen so zu einander verhalten*) (2.15).[98]

In other words, showing blocks naming or speaking about logical features (*Züge*), which we will take to encompass the various ways that logical form can be elaborated as either a structure, a logical prototype, a logico-syntactic rule, or an internal/formal property, relation, or concept (*TLP*: 3.34, 4.1221). In fact, intransitive medial showing is the unsaid premise in the dissolution of both the property and the propositional versions of Russell's paradox.

Starting specifically with the property verison, we find that medial showing prevails as the basis of each of the three presuppositions underlying its dissolution: 1) propositional functions; 2) the sign/symbol distinction; and 3) logico-syntactic rules. However, since an expression or symbol is "thus presented [*dargestellt*] by a variable whose values are the propositions that contain the expression [*Ausdruck*], (*TLP*: 3.313)," Wittgenstein's distinctive construal of a propositional function as a *Satzvariable* means that, if intransitive medial showing is implicit in the first presupposition, then it must carry over to the second and the third as well. Moreover, the preeminent importance of the first presupposition is also due to how the very design of a Tractarian propositional function precludes reflexivity from the outset because "[i]t corresponds to a logical form [*logischen Form*] – a logical prototype [*logischen Urbild*], (*TLP*: 3.315)" and "the sign for a function contains the prototype [*Urbild*] of its own argument and [therefore] it cannot contain itself (*TLP*: 3.333)." However, here we encounter the section's main problem upon observing that we cannot *say* that "a propositional function cannot contain

[98] Narboux, "Showing, the Medium Voice, and the Unity of the *Tractatus*," 229–30.

itself," because it would imply the very kind of reflexive language that is being dismissed.

What is needed in order to explain how this prohibition against reflexivity can instead be shown is provided in a strand of remarks that clarifies precisely how a propositional variable shows a logical feature:

> 4.1221: An internal property [i.e., formal property] of a fact we can also call a feature [*Zug*] of that fact. (In the sense in which we speak about facial features [*Gesichtszügen*]).
> 4.123: A property is internal if it is unthinkable that its object does not possess it…
> 4.126: In the sense in which we speak of formal properties we can also speak of formal concepts…
>
> That something falls under a formal concept as one of its objects, this cannot be expressed by means of a proposition. Instead, it shows itself [*zeigt sich*] in the sign for the object itself. (A name shows that it signifies an object, a numerical sign that it signifies a number, etc.)
>
> Formal concepts cannot, like proper concepts, be presented [*dargestellt*] by a function.
>
> For their traits [*Merkmale*], the formal properties, are not expressed by means of [*nicht durch... ausgedrückt*] functions.
>
> The expression [*Ausdruck*] of a formal property is a feature [*Zug*] of certain symbols.
>
> The sign for the traits [*Merkmale*] of a formal concept is, therefore, a characteristic feature [*charakteristischer Zug*] of all symbols whose meanings fall under the concept.
>
> The expression [*Ausdruck*] of a formal concept is therefore a propositional variable [*Satzvariable*] in which only its characteristic feature [*charakteristischer Zug*] is constant.

On the basis of the comparisons drawn in *TLP* (4.126d–e), we can infer another way to explain why we cannot say that a function cannot be its own argument: it would require that the function contained in such a proposition would be capable of expressing the purely formal relation between functions and the forms of its arguments. More specifically, due to its integral presuppositions and stipulations (*TLP*: 3.311, 3.316), a function must stand in an internal relation to arguments that have the form of a logical type of a lesser order. This means that self-reference cannot get off the ground, because the function cannot take itself but only that of a lower order. Moreover, since the implicit

presuppositions are in accord with a range of propositions that bear a "common characteristic trait [*gemeinsame charakteristische Merkmal*] (*TLP*: 3.311)," this internal relation is shown as an characteristic feature (*charakteristischer Zug*) of the formal concept of a function (i.e., the logical type of a function).

In fact, Wittgenstein's likening of a logical feature with that of a facial feature in *TLP* (4.1221) is an early instance of, what commentators refer to as, the "master simile" that will eventually occur quite frequently through his *Nachlass*.[99] It is no coincidence that Wittgenstein elaborates the master simile with greater detail in the same discussion on intransitivity that was recorded in *Brown Book*:

> Let this [drawing of a] face produce an impression on you… "Words can't exactly describe it," one sometimes says. And yet one feels that what one calls the expression of the face is something that can be detached from the drawing of the face. It is as though we could say: "This face has a particular expression: namely this" (pointing to something). But if I had to point to anything in this place it would have to be the drawing I am looking at. (We are, as it were, under an optical delusion which by some sort of reflection makes us think that there are two objects where there is only one. The delusion is assisted by our using the verb "to have," saying "The face *has* a particular expression". Things look different when, instead of this, we say: "This *is* a peculiar face". What a thing *is*, we mean, is bound up with it; what it has can be separated from it.) (*BB*: 162).

In the same way that the facial feature or expression cannot be detached or separated from the face that it is bound up with, it is inconceivable to detach or separate a certain logical feature from the logical type it characterizes. Similarly, like how the facial feature is intransitive because it cannot be translated into a further specification, description, or comparison, and instead its elaboration can only put emphasis right back

[99] Diamond, *The Realistic Spirit,* 243; Juliet Floyd, "Aspects of Aspects," *Cambridge Companion to Wittgenstein*, ed. Hans Sluga and David G. Stern (Cambridge, UK: Cambridge University Press, 2018), 362–363.

on the face; the logical feature is intransitive because it is not an object that could be named or state of affairs that could be articulated in a proposition and instead it is shown by the logical type it is embodied in. Finally, an attempt at comparing the facial feature with the face is mistaken because it confuses what is actually an intransitive emphasis on the face with the delusion of a reflexive and transitive comparison of the face with itself; the logical feature that is shown cannot be reflexively taken to a proposition because the intransitive nature of what is shown cannot be named nor spoken about.

As for the propositional version of Russell's paradox, the block on reflexivity is tantamount to what is shown in its main presupposition that "a propositional sign is a fact (*TLP*: 3.14)." The manner in which the objects in a state of affairs and the elements in a logical picture hang together so as to constitute the structure of fact (*TLP*: 2.03–2.034, 2.131–2.15):

> 3.14: What constitutes a propositional sign is that in it its elements (the words) stand to one another in a determinate [*bestimmte*] way…
> 3.1432: Instead of, "The complex sign '*aRb*' says that *a* stands to *b* in the relation *R*," we ought to put, "*That* '*a*' stands to '*b*' in a certain relation says *that aRB*."

As Narboux explains, what intransitively shows is how the words stand to one another in a way that cannot be reflexively named or stated:

> These sections [i.e., *TLP* (2.14, 2.15, 3.14, 3.1432)] can be said to make an "intransitive use" of one or both of the adjectives "determinate" (*bestimmt*) and "certain" (*gewiss*). The use of "determinate" (*bestimmter*) in 2.15 does not leave room for a description because it is simply not meant to introduce one. It is akin to the intransitive use of "peculiar" in the emphatic "This soap has a peculiar smell!" rather than to its transitive use in the specificatory "This soap has a peculiar smell: it is the kind we used as children." However, when we talk of the determinate way in which the elements of the [logical] picture stand with each other, we might "appear to ourselves to be on the verge of describing the way, whereas we aren't really opposing it to any other way."

> There can be a temptation to mistake the intransitive use of "determinate" (*bestimmt*) for a reflexive one, hence for a special case of the transitive use. The illusion that the structure of the picturing fact lies beyond the reach of description stems from the illusion that talking of "the determinate way in which the elements of the picture stand with each other" involves a sort of "reflexive comparison."[100]

Given how the words determinately stand to one another so as to structure a fact, they cannot be reflexively named or said but only medially shown.

1.3: From Pressures to Aporias

The seeds of Wittgenstein's newfound acceptance of paradoxes, inconsistencies, and reflexivity can be detected as early as the first indications of his philosophical shift with 1929's "Some Remarks on Logical Form."[101] As is well-known, the essay addresses difficulties concerning Wittgenstein's previous solution to the so-called color-incompatibility problem outlined at *TLP* (6.3751).[102] Since a detailed discussion of the shortcomings he found in his former analysis of color-ascription is beyond our present scope, it is enough simply to mention that Wittgenstein came to realize that the general form of a proposition's associated truth-functional notation was far too crude and restrictive to handle the nuances implicit in discourse about color. Thus, in his more comprehensive treatment of color-incompatibility in the *Big Typescript*, Wittgenstein

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[100] Narboux, "Showing, the Medium Voice, and the Unity of the *Tractatus*," 231. I have altered the quotation slightly by inserting "determinate" for Narboux's translation of variations of "*bestimmt*" as "definite."

[101] Ludwig Wittgenstein, James Carl Klagge, and Alfred Nordmann, *Philosophical Occasions, 1912-1951* (Indianapolis: Hacket Pub. Co, 1993), 29–35. From here on, abbreviated as *PO*.

[102] José Medina, *The Unity of Wittgenstein's Philosophy: Necessity, Intelligibility, and Normativity*, SUNY Series in Philosophy (Albany: State University of New York Press, 2002) 5ff; Monk, *Ludwig Wittgenstein*, 272–274.

came to admit that the general form of a proposition must be abandoned as a universal logical requirement:

> Of course the proposition "There is only room for *one* color in one location at one time" is a disguised grammatical proposition. Its negation isn't a contradiction, but it *contradicts* a rule of our normal grammar.
>
> The rules for "and", "or", "not", etc. that I have represented via the T-F notation are *part* of, but not *the entire*, grammar of these words.[103]

That is, given that *TLP* (6) had maintained that the general form of a truth-function was equivalent to the general form of a proposition, color-ascription propositions' recalcitrance to a truth-functional notation implies that this logical requirement can no longer remain universally valid for *all* or *every* proposition.

As would be expected given the centrality of the requirement, there are two important consequences that follow from the relinquishing of the general propositional form. *First*, leaving behind the uniform regimentation of language in the *Tractatus*, Wittgenstein comes to realize that significant human language is far more extensive and diverse than he had formerly assumed. For this reason, in another section of the *Big Typescript*, entitled "Proposition: Sense of a Proposition," the middle-period Wittgenstein repeatedly contrasts the general form of a proposition with a variety of different linguistic systems or rule-governed calculi:

> If I ask "What is the general form of a proposition?", then the counter-question can be: "Do we really *have* a general concept of a proposition, which we just want to formulate exactly?"
>
> … Something is a proposition only in a language.

[103] Ludwig Wittgenstein, C. Grant Luckhardt, and Maximilian Aue, *The Big Typescript, TS. 213*, German-English scholar's ed (Malden, MA: Blackwell Pub, 2005), 341e. From here on, abbreviated as *BT*.

> … What holds for the word "language" must also hold for the expression "system of rules". And therefore also for the word "calculus".
>
> … There are no such things as general discourses about the world and language. (*BT*: 50e–54e)

Later in the same section, Wittgenstein rephrases this conclusion by circumscribing what had formerly been the most general propositional form into now what is only a delimited subset of language:

> The statement: "A proposition is everything that can be true or false" defines the concept of a proposition, within a *specific system* of language, as what is an argument of a truth-function.
>
> And when we speak of what is essential to the form of a proposition as such, then *sometimes* it is truth-functions. When I said that the general form of a proposition is "Such and such is the case" this was precisely what I meant. (*BTS*: 61e, my emphasis)

In other words, the so-called general form of a proposition does *not* remain at all general but now can only hold for those particular systems or calculi governed by its rules of truth-functional connection.

As is well-known, Wittgenstein's recognition of the inherent multiplicity in language would then go on to become a central component of his mature thought in the *Investigations*:

> But how many kinds of sentence are there? Say assertion, question and command? – There are *countless* kinds; countless different kinds of use of all things we call "signs", "words", "sentences". And this diversity is not something fixed, given once and for all; but new types of language, new language-games, as we may say, come into existence, and others become obsolete and get forgotten…
>
> – It is interesting to compare the diversity of the tools of language and of the ways they are used, the diversity of kinds of word and sentence, with what logicians have said about the structure of language. (This includes the author of the *Tractatus Logico-Philosophicus*.) (*PI*: §23)

Also, the late Wittgenstein's acknowledgment of manifold linguistic plurality is cast in opposition to his earlier endorsement of the general form of a proposition:

> "You make things easy for yourself! You talk about all sorts of language-games, but have nowhere said what is essential to a language-game, and so to language: What is common to all these activities, and makes them into language or parts of language. So you let yourself off the very part of the investigations that once gave you the most headache, the part about the *general form of the proposition* and of language".
>
> And this is true. – Instead of pointing out something common to all that we call language, I'm saying that these phenomena have no one thing in common in virtue of which we use the same word for all – but there are many different kinds of *affinity* between them. (*PI*: §65)

And so, the only minor difference between the middle Wittgenstein and the more refined presentation of his later philosophy above is that the general form of a proposition will subsequently tend to be set against a more sophisticated conception of assorted language-games in addition to systems and calculi.

The *second* important consequence of dropping the general form of a proposition is that reflexive propositions and expressions, in particular, come to be accepted within the myriad of new linguistic varieties that are now under consideration. Accordingly, in the same section of the *Big Typescript*, the middle-period Wittgenstein frequently allows self-referential assertions about language, propositions, and even the rules of logical syntax or grammar:

> For when I speak about language – word, sentence, etc. – I have to speak in everyday language. – But is there any other? …
>
> That in explaining language (in our sense) I already have to use full-blown language (and not, say, a preparatory, provisional one) shows that all I can do is to present external facts about language…
>
> Your questions refer to words, so I have to talk about words. (*BT*: 58e)

As Wittgenstein moves into his mature period, an even greater emphasis is put on self-referential examination of language itself:

> We see that what we call "proposition", "language", has not the formal unity that I imagined, but is a family of structures more or less akin to one another. – But what becomes of logic now?
>
> … The sense [*Sinn*] in which philosophy of logic speaks of sentences and words is no different from that in which we speak of them in ordinary life when we say, for example, "What is written here is a Chinese sentence", or "No, that only looks like writing, it's actually just ornamental", and so on.
>
> We're talking about the spatial and temporal phenomenon of language, not about some non-spatial, atemporal non-entity… But we talk about it as we do about pieces in chess when we are stating the rules [*Spielregeln*] for their moves, not describing their physical properties… (*PI*: §108)

Wittgenstein's abandonment of his former preconceived notion of formal unity is but another way of saying that the requirement for a general form of a proposition has now been relinquished. Furthermore, what is of the utmost importance are the methodological consequences drawn from surrendering the uniformity imposed on propositions by encasing them within a general form. *Contra TLP* (5.54), it is now permitted for a proposition to occur in another proposition in ways other than as merely the bases of truth-operations. In fact, more than merely admitting that there can be significant reflexive discourse about the propositions and expressions that comprise language, it is also distinguished as the very locus of how philosophical inquiry ought to be conducted by rigorously logical means.

Finally, *contra TLP* (3.34), if self-referential language is now acceptable, what is reserved for the rules of logical syntax must no longer "follow of themselves" or "go without saying." That is, for the late Wittgenstein, grammatical propositions can now *say* what rules of logical syntax before could only *show*:

> What *sort* of proposition is: "The class of lion is not a lion, but the class of classes is a class?" how could it be *used*? – So far as I can see, only as a grammatical proposition…
>
> One may say that the word "class" is used reflexively, even if for instance one accepts Russell's theory of types. For it is used reflexively there too…
>
> But suppose that one gives a particular lion (the king of the lions) the name "Lion"? Now you will say: But it is clear in that sentence, "Lion is a lion" the word "lion" is being used in two different ways. (*Tractatus Logico-Philosophicus.*). But can't I use them as *one* kind of use….
>
> Even though "the class of lions is not a lion" seems like nonsense, to which one can only ascribe a sense out of politeness; still I do not want to take it like that, but as a proper sentence, if only taken right. (And so not as in the *Tractatus*.). Thus my conception is a different one here. Now this means that I am saying: there is a language-game with this sentence too.[104]

Once again, Wittgenstein is explicit in condoning what the *Tractatus* had beforehand precluded. Furthermore, what Wittgenstein's deliberations about syntactic use and employment disclose is that he has given up the constraints of both univocal use and maximal specificity with respect to form. Instead, we see Wittgenstein contemplating a wide variety of uses of the word "lion," but there is no suggestion that this *must* imply different symbols for the same sign, "lion."[105] With these constraints disavowed, the possibility emerges of constructing self-referential expressions that could induce the property version of the paradox that was bound up with the restrictions on syntax and the requirement for simple objects. In fact, Wittgenstein concludes the lion example with an admission of paradox that is characteristic of his mature thought: "Then it would be possible to set up the paradox that there isn't a definite number of all lions (*RFM VII*, §36)."

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[104] Ludwig Wittgenstein, *Remarks on the Foundations of Mathematics*, ed. Rush Rhees and Georg Henrik von Wright, trans. Gertrude Elizabeth Margaret Anscombe, 3d ed., rev (Oxford [Eng.]: B. Blackwell, 2001), VII. From here on, abbreviated as *RFM*.

[105] I want to leave open the question of primary and secondary meaning here, but I suspect that it is relevant. See, *PI* (§§282, xi.275–279); Diamond, *The Realistic Spirit*, "Secondary Sense," 225–241.

Chapter 2: Wittgenstein's Subversion of the Subject

Let us begin at the end, taking as our starting point the conclusion of the early Wittgenstein's dialectical manipulation of the divide between the subject and its object in the *Tractatus* (5.64):

> Here we see that solipsism strictly carried out coincides with pure realism. The I in solipsism shrinks to a point without extension and there remains the reality coordinated with it.

Furthermore, here we find what is perhaps the most explicit engagement in the *Tractatus* with any customary theoretical stance that can be recognizably situated within the Western philosophical tradition. Indeed, a later passage from the *Philosophical Remarks* confirms that Wittgenstein viewed the problem in terms of a confrontation with classical philosophical positions:

> From the very outset "Realism," "Idealism," etc., are names which belong to metaphysics. That is, they indicate that their adherents believe they can say something specific about the essence of the world. (*PR*: p. 86, §55)

Nevertheless, Wittgenstein's heterodox inclinations prevail, as he is driven to problematize the two standardly antithetical positions, idealism and realism, by attempting to show how they can be harmonized with one another.[106]

Following Eli Friedlander, I also take Wittgenstein at his own word by insisting that the author of the *Tractatus* did, in fact, wholeheartedly maintain that solipsism and realism each culminate in their unified and total correspondence:

[106] Once again, I take it that Wittgenstein regards solipsism as a more personal variant of idealism.

> In other words, realism is the truth of solipsism, or solipsism thought through is realism. This famous moment of the *Tractatus* [i.e., *TLP* (5.64)] is sometimes taken to mean that since two opposite metaphysical positions are identified, they are both shown to be nonsense. But, such a reading is not dialectical enough. Existence, or world is that wherein idealism and realism come to the same.[107]

As textual support for this line of interpretation, Friedlander observes that Wittgenstein uses the feminine article, "*die*," and the possessive determiner, "*mein*," twice over, once before and once after *TLP* (5.64), in the claim that "the world is my world (5.62, 5.641)" in order to signify the fusion of an independent reality and the solipsistic self. Furthermore, as the final remark in the *Tractatus*'s examination of the subjective and the objective makes abundantly clear, there is a significance in how the subject vanishes into its alignment with the world: "Thus, there is really a sense [*wirklich einen Sinn*] in which philosophy can talk [*Rede*] about the non-psychological I [*Ich*] (*TLP*: 5.641)."

Additional primary confirmation for a reading of *TLP* (5.64) that affirms the ultimate equivalence of solipsism and realism together can be found in an earlier and more forthcoming rendition of the remark in the pre-Tractarian *Notebooks*:

> This is the way I have traveled: idealism singles men out from the world as unique, solipsism singles me alone out, and at last I see that I too belong with the rest of the world, and so on the one side *nothing* is left over, and on the other side, as unique, *the world*. In this way idealism leads to realism if it is strictly thought out. (*NB*: 85, Oct. 15, 1916)

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[107] Eli Friedlander, "Logic, Ethics and Existence in Wittgenstein's Tractatus," 2018, https://www.academia.edu/34491395/Logic_Ethics_and_Existence_in_Wittgensteins_Tractatus, 10. See also, Eli Friedlander, "Missing a Step Up the Ladder:," *Philosophical Topics* 42, no. 2 (2014): 45–73, https://doi.org/10.5840/philtopics201442219, 57, 61–62.; *Signs of Sense: Reading Wittgenstein's Tractatus* (Cambridge, Mass: Harvard University Press, 2001), 193.

The above passage also summarizes how bringing idealism and realism to their mutual apex involves the dialectical disappearance of the subject.

Accordingly, this chapter's overarching question asks: What were the steps that led the young Wittgenstein toward the dissolution of the subjective self in order to stage such a radical confrontation with orthodox philosophy? Furthermore, since it is clear that bold moves of this kind could not have been inherited from the restrained example handed to him by Frege and Russell, the chapter will also examine how we might find some philosophical precedents for Wittgenstein's rather audacious maneuvers.

2.1: German Idealism and German Realism

I want to suggest that a close conceptual precursor to Wittgenstein's solution to the problem of the subject–object divide can be found in how J. G. Fichte construes his "critical" or "limited" transcendental idealism as a kind of "real-idealism" or "ideal-realism."[108] In order to grasp Fichte's blend of idealism and realism, it will be helpful to consider his notion of the *Anstoß* or check. For Fichte, the *Anstoß* functions as a counter to the all-encompassing centrifugal activity of the subject by making it turn back on itself in a centripetal return. Accordingly, Fichte's use of the term, *Anstoß*, tends to carry two

[108] Johann Gottlieb Fichte, *Foundations of Transcendental Philosophy (Wissenschaftslehre) Nova Methodo (1796/99)*, trans. Daniel Breazeale (Ithaca: Cornell University Press, 1992), 103, 164, 170, 182–184, 359; Johann Gottlieb Fichte, *Science of Knowledge; with the First and Second Introductions*, trans. Peter Heath and John Lachs (Cambridge: Cambridge University Press, 1982), 21–22, 64, 147, 162–164, 171, 177, 188–206, 240–250 ; Günter Zöller, "German Realism: The Self-Limitation of Idealist Thinking in Fichte, Schelling, and Schopenhauer," in *The Cambridge Companion to German Idealism*, ed. Karl Ameriks (Cambridge: Cambridge University Press, 2000), 204.

connotations: “obstacle” or “hindrance,” on the one hand; and “impulse,” “impetus,” or “stimulus,” on the other.[109]

As is well-known, Fichte’s theoretical project aimed to develop a transcendental idealism that could circumvent the widely debated conceptual snares surrounding the Kantian noumena. However, Fichte initially takes a rather drastic detour by advocating an extreme form of idealism in order to sidestep the pitfalls associated with a noumenal reality. That is to say, Fichte’s philosophical starting point involves positing the absolute “I” that he specifies as the origin and source of all that is. Yet, as Johnston explains, this idealist and subjective absolute “I” turns in on itself:

> [T]he foundational distinction between the “I” of subjectivity and the “not-I” of objective/external reality is instituted exclusively by the "act" (*Tathandlung*) of the subject “positing” itself, thereby revealing the “not-I” to be dependent upon the “I.” Not only… does the subject shape all of reality by being the unavoidable condition of possibility for the very cognition of reality itself – but, as per Fichte, the distinction between the "I" as agent of knowledge and the alterity of reality as what this agent knows is instituted solely with the advent of this agent itself.[110]

As the exegesis above indicates, it is hard not to see Fichte’s massaging of the subjective into the objective as presaging Wittgenstein’s later maneuvers. Likewise, Johnston summarizes Fichte’s conclusion as a merger of antitheses, thus bringing together idealism and realism:

> Fichte is able simultaneously to advance two seemingly contradictory theses: on the one hand, the self-positing “I” of idealist subjectivity is asserted to be the first principle, the axiomatic ground zero, of any viable philosophical system (this being Fichte’s idealism); on the other hand, despite the fact that this “I” conditions everything “not-I” (and that this “not-I,” as “not-I,” is brought into

[109] Adrian Johnston, *Žižek’s Ontology: A Transcendental Materialist Theory of Subjectivity* (Evanston, Ill: Northwestern University Press, 2008), 16.
[110] Johnston, *Žižek’s Ontology*, 17.

> existence in and through the being of the "I"), the negative alterity of the *Anstoss* with respect to subjectivity is indicative of the mysterious X subsisting apart from the subject (this being what might be called Fichte's "materialism" [i.e., realism]).[111]

So, despite the differences in philosophical contexts, it is quite apparent that Wittgenstein's strategy and outcome resemble Fichte's overall approach.

It is also worthwhile to note how the *Anstoß* plays a crucial role in allowing Fichte to avoid both the absurdities of solipsism and its opposite in a crude pre-critical realism:

> And yet, while the *Anstoss* is only ever encountered and conceptually/discursively grasped through mediations of subjective ideality, it nonetheless marks, via its negations of and resistances to the ideal subject's multiple efforts to posit itself as "All," those points at which the solipsistic self-enclosure of transcendental idealist subjectivity collides with an utterly asubjective Real.[112]

Similarly, for the young Wittgenstein, the subject is the point of departure, and the "truth of solipsism" is manifested in a comparable drive to envelop all and everything into "*my* world" (*TLP*: 5.62). However, the failure of Wittgenstein's multiple efforts to locate a stable position for the subject in the world also eventually leads to a return of the Real(ism). The only major difference is that whereas the Fichtean subject's back-and-forth collision with the *Anstoß* is what gives rise to the domain of the Real, the young Wittgenstein's attainment of realism is instead achieved through the subject's very disappearance.

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[111] Ibid., 18.
[112] Ibid., 17.

2.2: Wittgenstein and Schopenhauer

Their likenesses aside, it would nevertheless be rather perverse to maintain that a philosopher who reportedly bragged that he had never read a page of Aristotle could have been directly influenced by Fichte's obscure program for a renewed transcendental idealism. Yet, as Friedlander suggests, Wittgenstein's thought often intersects with many of the same topics, themes, and questions that were emblematic of the German idealist tradition.[113] Perhaps, Wittgenstein's extensive and probing cultural sensitivities may be responsible for this, as it is not far off to assume that he inherited currents from German idealism through his wide appreciation of German literature, music, and art. However, the most immediately philosophical explanation for the startling parallels between Wittgenstein and the German idealists is likely the former's lifelong interest in a student of Fichte's, namely, Arthur Schopenhauer.

Indeed, commentators have come to acknowledge that both the teacher and the student pursued a grand and sweeping idealism that nonetheless bottomed out in a realist outcome:

> Fichte, Schelling, and Schopenhauer, each in their own way, continue the Kantian project of articulating the grounds and bounds of reason. Like Kant, they seek to strike a balance between the idealist recognition that the natural and social worlds reflect reason's demands and interests and the realist insight that the world is more than the work of reason… To be sure, this reevaluation of reality does not amount to an outright cancellation of the idealist insistence on the constitutive role of reason. Rather, Kantian and post-Kantian idealism undergoes an emendation: the apparent self-sufficiency of reason is complemented, in fact, completed, by being traced back

[113] Friedlander, "Logic, Ethics and Existence in Wittgenstein's Tractatus," 10.

> to a dimension of ultimate origin or being that is beyond reason but without which there would be no reason.[114]

It is understandable to be wary of the above use of the term, "reason," given that some of the most contentious quarrels in 19th-century German philosophy concerned the very meaning and implications of *Vernunft*. Nevertheless, if "reason" is replaced by the "subject" or "subjectivity," then it is quite easy to see their shared project as the coordination of the subject and the object, idealism and realism.

What is all the more interesting is that Schopenhauer's habit of fawning over Kant and castigating Fichte tends to accomplish the opposite of its intended purpose by instead providing the primary source material that supports an interpretation of shared strategy, bringing idealism and realism together:

> My works had scarcely excited the attention of a few, when the dispute as to priority arose with regard to my fundamental idea… With regard to the matter itself, it may be observed that the root of my philosophy is to be found already in the Kantian… but generally in the fact that, whenever Kant brings the thing-in-itself somewhat nearer to the light, it always appears through its veil as *will*. I have expressly drawn attention to this in my "Criticism of the Kantian Philosophy," and accordingly have said that my philosophy is only his thought out to the end. Therefore we need not wonder if the philosophemes of Fichte and Schelling, which also start from Kant, show traces of the same fundamental idea, although they appear without sequence, continuity, or development, and accordingly may be regarded as a mere foreshadowing of my doctrine. In general, however, it may be said on this point that, before every great truth has been discovered, a previous feeling, a presentiment, a faint outline thereof, as in a fog, is proclaimed, and there is a vain attempt to grasp it just because the progress of the times prepared the way for it.[115]

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[114] Zöller, "German Realism," 202.

[115] Arthur Schopenhauer, *Parerga and Paralipomena: Short Philosophical Essays*, trans. E. F. J. Payne (Oxford: Clarendon Press; Oxford University Press, 2000),132–133.

As is well-known, Schopenhauer followed in Fichte's tracks by tackling the problem of the Kantian noumena, but the former's realist twist involved positing a universal will instead of a check. So, despite the relentless invectives that Schopenhauer spewed at his former philosophy professor, he nevertheless saw him, to some extent, as an intellectual ally who engaged in roughly the same project.

In order to complete the link from indirect to direct inheritance, it is widely acknowledged that the sections on the distinction between the subject and the object, in both the *Notebooks* and the *Tractatus*, display what is undoubtedly the most pronounced Schopenhauerian influence in all of Wittgenstein's *oeuvre*.[116] As Wittgenstein's biographer Ray Monk explains, this is especially the case when he tends to metaphorically muse on the vanishing of the subject in poetic terms: "Wittgenstein's remarks on the will and the self are, in many ways, simply a restatement of Schopenhauer's 'Transcendental Idealism' with its dichotomy between the 'world as idea' the world of space and time, and the 'world as will,' the *noumenal*, timeless, the world of the self."[117] For example, when Wittgenstein responds negatively to the rhetorical question, "Where *in* the world is a metaphysical subject to be noticed?":

> 5.633: You say this case is altogether like that of the eye and the field of vision. But you do *not* see the eye.
>
> And from nothing *in the field of vision* can it be concluded that it is seen from an eye.

It is no coincidence that the same figurative language is deployed in a delineation of subjectivity in the second volume of *The World as Will and Representation*: "But the I is

[116] The literature on Wittgenstein and Schopenhauer is voluminous. It is perhaps enough to say that Hacker's *Insight and Illusion*, 81-100 is often credited as landmark in sparking interest in topic.
[117] Monk, *Wittgenstein*, 144.

the dark point of consciousness… and the eye sees everything except itself."[118] Likewise, the same can be said, if not more so, for the *Notebooks*: "It is true: Man *is the* microcosm: I am my world (*NB*: 84)." This remark, in which a shorter version would go on to become *TLP* (5.63), also has its roots in Schopenhauer: "For the will to life, the essence in itself, appears whole and undivided in every individual, and the microcosm is the same as the macrocosm."[119]

2.3: On Why "*P*" Can't Say *P*

Let us now try to endow these metaphors with some analytic rigor by turning to Wittgenstein's full-fledged attempt to dissolve anything like a substantial subject located in the world at *TLP* (5.54–5.542):

> 5.54: In the general propositional form, a proposition occurs [*vorkommt*] in other propositions only as bases of truth-operations.
>
> 5.541: At first glance, it appears as if there were also a different way in which one proposition could occur in another.
>
> Especially in certain propositional forms of psychology, such as "A believes that *p* is the case" or "A thinks *p*", etc.
>
> Here it appears superficially as if the proposition *p* stood in some kind of relation to an object A.
>
> (And in modern theory of knowledge (Russell, Moore, etc.) these propositions have actually been construed this way).
>
> 5.542: But it is clear [*klar*] that "A believes that *p*," "A thinks *p*," and "A says [*sagt*] *p*" are of the form "'*p*' says [*sagt*] *p*": and this does not involve a correlation of a fact and an object, but rather a correlation of facts by means of a correlation of their objects.

[118] Arthur Schopenhauer, *The World as Will and Representation, Vol 2*, trans. Judith Norman, Alistair Welchman, and Christopher Janaway (Cambridge: Cambridge University Press, 2018), 507.
[119] Schopenhauer, *The World as Will and Representation, Vol 2*, 606.

In short, the correct analysis of a judgment containing a propositional attitude, such as "A thinks *p*," or "A judges *p*," should result in "'*p*' says *p*." It will be quite complicated to demonstrate how to get from the former to the latter. Nevertheless, it is worth the investment given the philosophical significance returned by its overall conclusion at *TLP* (5.5421):

> This also shows [*zeigt*] that there is no such thing [*Unding*] as the soul – the subject etc. – as conceived in the superficial psychology of the present day.
>
> Indeed, a composite soul would no longer be a soul.

That is, since "'*p*' says *p*" stands as a determinate combination of objects, its complexity rules out the very existence of a substantial and unified subject. What's more, such a disqualification of the subject would entail that "[Wittgenstein] had… reached a point where Schopenhauerian solipsism and Fregean realism were combined in the same point of view."[120]

In *TLP* (5.54), the first presupposition of this demonstration recalls the earlier section's discussion (1.2) of how the general form of a proposition also serves as a prohibition against the reflexive construction of one proposition in another. Put succinctly, a propositional sign is a fact and therefore is not a name (*TLP*: 3.14–3.144). Said in another way, a proposition cannot contain another proposition in the same way that it contains a name. In the terminology of the *Tractatus*, it is only by way of operation that a proposition can stand in an internal relation to another. In other words, "Operations and functions must not be confused with one another (*TLP*: 5.25)."

[120] Monk, *Wittgenstein*, 144.

In the next remark, *TLP* (5.541), Wittgenstein launches into a critique of theories of judgment that fail to heed this distinction. That is to say, these theories mistakenly assume that propositional forms of psychology, such as "A judges *p*," express a relation between a substantial subject, designated by object A, in a connection with a proposition, *p*. Once again, we are still on familiar terrain because the alleged relation between object A and proposition *p* is rejected on the grounds of the general form of a proposition: "Logical indefinables cannot be predicates or relations, because propositions owing to sense, cannot have predicates or relations (*NB*: 99)." In other words, a proposition cannot have a predicate or stand-in relation to an object, because a proposition is a fact and facts cannot stand beside symbols (cf. *NB*: 116). What we are meant to take away from *TLP* (5.541) is that whatever "judges" might be in constructions like "A judges *p*," it cannot be an external relation. As Wittgenstein puts in *NB* (95): "This is obviously not a relation in the ordinary sense."

What is rarely observed, however, is that there is another way to criticize a propositional attitude like "A judges *p*" in *TLP* (5.541) that is, in fact, more in line with the previous remark in *TLP* (5.54). It can be made apparent by considering a more informative version of the later remark from the *Notebooks*: "There are *internal* relations between one proposition and another; but a proposition cannot have to another *the* internal relation which a *name* has to the proposition of which it is a constituent, and which ought to be meant by saying it 'occurs' in it (*NB*:116)." In other words, the general form of a proposition excludes cases in which one proposition might contain another as it would a name. More specifically, this means that another objection should be leveled against the alleged relation between the proposition, "A judges *p*," on the one

hand, and the proposition, "*p*," on the other. That is to say, what is presumably specious about these theories of judgment is that it appears to allow proposition *p* to occur in other propositions like "A judges *p*" but not as bases in a truth-operation.

Finally, at *TLP* (5.542), Wittgenstein performs an analysis, such that "A judges *p*" is rendered into "'*p*' says *p*" because the latter avoids the mistake of positing a relation between a name and a proposition, an object and a fact, or simply "A" and "*p*." The solution involves canceling out the pseudo-relation that is supposed to be indicated by "judges" and replacing the entire formulation with the proposition "'*p*' says *p*" because the latter would be either a relation between a proposition, '*p*' and a fact, *p*, or relation between two facts, '*p*' and *p*. Wittgenstein then tells us that the relation between two facts can be construed as a relation between objects: "[T]his does not involve a correlation of a fact and an object, but rather a correlation of facts by means of a correlation of their objects." However, despite his claims otherwise, after "A judges *p*" has been rejected, it is not at all immediately "clear [*klar*]" what motivated the first and principal step to "'*p*' says *p*."

Following Sullivan, I take it that the best explanation of the analysis was probably the first; namely, what Frank Ramsey had put forward in his 1923 review of the *Tractatus*.[121] In Sullivan's reading of Ramsey, the key to understanding the opaque transition in the analysis is *TLP* (2.1513): "the pictorial relation which makes it a picture belongs to the picture." Since the pictorial relation is already contained in the proposition '*p*,' Ramsey can conclude: "For if a thought or proposition token 'p' says p, then it is

[121] F. P. Ramsey, "Critical Notice of L. Wittgenstein's *Tractatus Logico-Philosophicus*," *Mind* 32, no. 128 (1923): 469-471; Peter M. Sullivan, "What Is Squiggle? Ramsey on Wittgenstein's Theory of Judgement," in *Ramsey's Legacy*, ed. Hallvard Lillehammer and D. H. Mellor, Mind Association Occasional Series (Oxford : New York: Clarendon Press ; Oxford University Press, 2005), 53–70.

called true if p and false if, ~p."[122] In other words, since the pictorial relation is already somewhere within "'*p*' says *p*," it can be reduced to *p,* as a proposition that pictures a fact.

I admit that this is probably the most philosophically convincing interpretation, but there is an exegetical issue with this solution, as it is not strictly in accord with what little we have regarding how Wittgenstein perceived the problem: "The relation of 'I believe p' to 'p' can be compared to the relation of '"p" says (*besagt*) p' to p: it is just as impossible that I should be a simple as that "p" should be (*NB*: 119)." Yet, despite the obscurity of this remark, the most that we can gather from it is that, once again, there can be no external relation between an object and a fact. Besides this, I am unaware of any other evidence indicating, even remotely, that Wittgenstein condoned the reduction advanced by the Ramsey-Sullivan reading.

For my part, I want to argue that, somewhat like the color-exclusion problem, *TLP* (5.542) actually amounts to a failed attempt at accomplishing the analysis. To some extent, I am in agreement with the rare few commentators who have noticed that, for several reasons, "'*p*' says *p*" remains ill-formed and therefore would demand further analysis.[123] To conclude this section, I will put forward four reasons why the analysis of "A judges *p*" to "'*p*' says *p*" remains incomplete.

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[122] Ramsey, "Critical Notice," 460.

[123] See, for example, Michael Cohen, "Tractatus 5.542," *Mind* 83, no. 331 (1974): 442–44; Anthony Kenny, *Wittgenstein*, 4. print (Cambridge, Mass: Harvard Univ. Press, 1981). However, I depart from these commentators by rejecting that the nonsense that results from the analysis shows what can't be said. I take *TLP* (5.1362), "'A knows that *p* is the case' is senseless if *p* is a tautology," as strong enough evidence to conclude that Wittgenstein assumed that propositional attitudes had sense, since if *p* was not a tautology then it would presumably be significant.

The *first* reason is a consequence of the alternative reading I proposed of *TLP* (5.541). That is, *contra TLP* (5.54), '*p*' is a proposition that does, in fact, occur in another proposition, namely "'*p*' says *p*," but not as a base of a truth-operation. Put bluntly, whatever relation is symbolized by the sign "says" in "'*p*' says *p*," it does not seem to be a logical operator or truth-operation. Therefore, we should conclude that the analysis fails to abide by its own initial requirement.

Second, another good reason to assume that "says" is not a truth-operation is that it already occurs in an example that had previously been dismissed as misleading. Notice that "A says *p*" is already presented in the list of rejected propositional attitudes in *TLP* (5.542): "But it is clear [*klar*] that 'A believes that *p*,' 'A thinks *p*,' and 'A says [*sagt*] *p*' are of the form '"*p*" says [*sagt*] *p*."' In other words, if the verb "says" is carried over from propositions "A says *p*" to "'*p*' says p," then it is just as likely for it to be a relation that stands to an object as it is to a proposition.

Third, I want to suggest that, even if it is assumed that "says" could somehow be a truth-operation, it is likely to have the form of a tautology, such as "'*p*' ⊃ *p*." Although I have yet to see this connection drawn out in the secondary literature, it is rather evident to me that the closest equivalent to "'*p*' says *p*" is found in a closely preceding remark at *TLP* (5.5351):

> There are certain cases in which one is tempted to use expressions of the form "*a* = *a*" or "*p* ⊃ *p*" and the like [*u. dgl. zu.*]. In fact, this happens when one wants to talk about a prototype [*Urbild*], e.g. about proposition, thing, etc. Thus in Russell's *Principles of Mathematics* "*p* is a proposition" – which is nonsense [*Unsinn*] – was given the symbolic rendering "*p* ⊃ *p*" and placed as a hypothesis in front of certain propositions in order to exclude from their argument-places everything but propositions…

Since "'*p*' says *p*" does, at least superficially, look like the nonsensical concatenation of signs "*a* = *a*" or "*p* ⊃ *p*," there are grounds to infer that, if "says" did turn out to be a truth-operator, then "'*p*' says *p*" would accordingly be nonsensical. Furthermore, the same problem that Russell is being criticized for trying to resolve with argument-places in *TLP* (5.5351) is implicitly also driving the analysis of propositional attitudes since the restriction in *TLP* (5.54) against what can "occur" in a proposition excludes what can be taken in an argument-place by way of the general form of a proposition.

Finally, the *fourth* reason for doubting the success of the analysis at *TLP* (5.542) builds on the previous criticisms of the general form of a proposition in section 1.3. We have already seen that Wittgenstein would later reject the prohibition against reflexivity at *TLP* (5.54) that relies on the general form of a proposition to block propositions from occurring in each other. Therefore, we are justified in having suspicions about the wider trajectory of the analysis aimed at dissolving the subject, because the general form of a proposition will eventually be abandoned as a far too restrictive and artificial logical requirement.

2.4: On Limits and Other Obstacles

If the above analysis had been successful, thereby disintegrating the subject and reciprocally collapsing idealism and realism into one another, then attaining this goal would have also involved casting the subject to the very limits (*Grenzen*) of the world:

> 5.6: *The limits* [Grenzen] *of my language* mean the limits of my world.
> 5.61: Logic pervades the world: the limits of the world are also its limits…

> 5.62: That the world is *my* world, shows itself in that the limits of language (*the* language which I alone understand) mean the limits of *my* world….
> 5.632: The subject does not belong to the world but it is the limit [*Grenze*] of the world…
> 5.641: The philosophical I is not the man, not the human body, or the human soul of which psychology treats, but the metaphysical subject, the limit – not part of the world.

In step with Sullivan's interpretation of the 5.6s as an undermining of Kantian transcendental idealism, Marie McGinn explains that the banishing of the subject to the outskirts of all that is would also have been tantamount to realizing that the relation between the subject and the world is the same as the internal relation between logic and the world:

> To say that the thinking subject is a limit of the world is… simply to recognize that whenever there is representation of the world there is a correlate notion of an active subject who projects propositional signs onto reality… Wittgenstein's aim is to make clear that the idea of logic as the limit of the world cannot be understood in the sense of a boundary, but is to be understood in the sense that there is no representation of the world without logic. It is now clear that the idea of the subject as a limit of the world has to be understood in the same way. The world is not conceivable independently of the propositions that stand in a projective relation to the world; whenever there is representation of the world in propositions there is a subject who is in a position to say "I think…". Thus, the notions of the subject, logic, and the world stand as correlates to another; they cannot be understood, or made intelligible, independently of one another. This is not – either in the case of logic or in the case of the subject – to draw a boundary round the world. It is, as Peter Sullivan says, not to impose a restriction on the world at all. Rather, it is to recognize that the notion of the world has no content independently of the notion of what is described in the true propositions of *my* language.[124]

Let us go through this explanation step by step: *First*, logic as a limit, for the young Wittgenstein, understood in terms of a *Grenze*, is not a boundary or restriction. There is nothing beyond the limits of logic in the same way that there is no such thing as an

[124] McGinn, *Elucidating the Tractatus*, 276; See also, Peter M. Sullivan, "The 'Truth' in Solipsism and Wittgenstein's Rejection of the A Priori," *European Journal of Philosophy* 4 (1996).

illogical thought (*TLP*: 3.03–3.031, 5.4731). Thus, the claim that "there is no representation of the world without logic" corresponds to "the limits of the world are also the limits of logic (5.61)." *Second*, the subject is akin to logic insofar as it is at the limits of the world because there is also no representation of the world without a subject. Accordingly, for McGinn, the subject is positioned at the limit, along with logic, due to its active role in speaking, thinking, or more precisely, representing the world by projecting propositional signs onto reality. *Third* and finally, the subject, the limits of the world, and logic are, in some way, correlative, or better, coterminous with one another.

Therefore, McGinn concludes that the three conceptually coincide because the subject, the limits of the world, and logic cannot be coherently grasped independently of one another:

> The upshot of the general reflections is that there is no intelligible notion of the world that is independent of the idea of what is thinkable in a language that represents states of affairs: the thinkable represents a limit on the other side of which there is simply nonsense…
>
> The world – my world – is simply what is described in the true propositions of my language. There is a question of what is the case in the world – i.e. of which propositions are true and which are false – but there is no other, or further question of whether the world I describe in the true propositions of my language corresponds to something wholly outside of language.[125]

Once again, in aligning the limits of thought and language with the limits of the world, we can comprehend how idealism comes to coincide with realism, as there is nothing intelligibly beyond what is subjectively said about the objective world.

[125] Marie McGinn, *Elucidating the Tractatus*, 271, 272.

In order to further orient how this notion of a *Grenze* opposes a kind of boundary or restriction, Peter Sullivan engages Wittgenstein's handling of a limit with that of Descartes and Kant:

> In both his early and later work Wittgenstein is concerned with understanding the limits of thought. By the notion of a *limit* here is meant something set by, so essentially equivalent to, the essential nature or form of what it limits. It is a notion used when one says that a space is limited by its geometry (to take Wittgenstein's favorite analogy from the *Tractatus*). This notion of a limit is not a contrastive one. There is nothing thought-like excluded by the limits of thought for lacking thought's essential nature, just as there are no points excluded from space for being contra-geometrical. But thinking in general is contrastive: in general, that is, thinking something to be the case is thinking it to be the case rather than not. That is the broadest reason, if only the initial reason, why thought about limits is apt to portray them instead as *limitations*, boundaries that separate what has a certain nature from what does not… [T]he crucial step in embracing or resisting idealism consists in succumbing to or resisting the construal of limits as limitations. This step is not enough on its own. Descartes construed the limits both of thought and of possibility as limitations, and he was no idealist. But having accepted that construal he did have to offer some account of how the limitations came to be what they are, and in particular of how the limits of thought and of possibility should so happily coincide. Kant mocked the explanation Descartes offered. And in this Wittgenstein was always with Kant: *if* there had to be any account of such things, then an idealist account was the only contender he could take seriously.[126]

We are given additional terminology, such that the limit of the world is to be properly conceived as a non-contrastive limit because there is nothing over its edge, whereas the opposing notion of a boundary, restriction, or what Sullivan refers to as a limitation, is instead contrastive. Presumably, Sullivan is alluding to Kant's objections against Descartes' problematic idealism in the *Critique of Pure Reason* (B274, A364) so as to elucidate how the distinction between non-contrastive limits and contrastive restrictions are inevitably implied in any encounter with the topic of idealism. However, it is also

[126] Peter M. Sullivan, "Synthesizing without Concepts," in *Beyond the Tractatus Wars: The New Wittgenstein Debate*, ed. Rupert J. Read and Matthew A. Lavery (New York: Routledge, 2011), 171–172.

rightly noted that attending to this distinction alone is not sufficient to tackle the problem of the ideal and the real, the subject and the object.

For my part, I want to suggest that we can comprehend just how sparse Wittgenstein's notion of a limit is by comparing it to Kant's more nuanced discriminations between a *Grenze* and a *Schranke*.[127] Kant's most sustained and focused discussion of the distinction is located in the pages of the *Prolegomena*:

> *Grenzen* (in extended things) always presupposes a space that is found outside of a certain fixed location, and that encloses that location; *Schranken* requires nothing of the kind, but are mere negations that affect a magnitude insofar as it does not possess an absolute completeness. Our reason [*Vernunft*], however, sees around itself as it were a space for the cognition of things in themselves, although it can never have determinate concepts of those things and is restricted [*eingeschänkt*] to appearance alone.[128]

Clearly, Kant also found geometrical analogies to be valuable in explaining the role of a limit, but he stretches this comparison in ways beyond what is set forth in the *Tractatus*. It is *Schranken* that most closely resembles Tractarian limits since it is a non-contrastive limit without any space designated outside of the delimited domain. However, the Kantian conception of *Grenzen* encompasses certain contrastive aspects of a restriction or a boundary that would most likely be rejected by the young Wittgenstein.

Additionally, it is enlightening to consider Kant's examples of what topics and fields of inquiry are respectively enclosed by *Schranken* and *Grenzen*. For instance, Kant

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[127] For a detailed comparison of Wittgenstein's and Kant's notion of a limit see, John J. Callanan, "The Boundary of Pure Reason," in *Kant's Prolegomena: A Critical Guide*, ed. Peter Thielke (Cambridge; New York, NY: Cambridge University Press, 2022), 133–53.

[128] Immanuel Kant, *Prolegomena to Any Future Metaphysics That Will Be Able to Come Forward as Science: With Selections from the Critique of Pure Reason*, ed. Gary C. Hatfield, Rev. ed, Cambridge Texts in the History of Philosophy (Cambridge, UK; New York: Cambridge University Press, 2004), 103–104; 4:352.

writes, “In mathematics and natural science human reason recognizes *Grenzen* but not *Schranken*,” because there are “concepts” beyond sensory appearances that neither math nor physics can address, i.e., metaphysics and morality (4:352–353). Not only would the young Wittgenstein repudiate any such speculation about concepts beyond such a limit, but he would also confine this internally demarcated realm to physics alone: “The totality of true propositions is the total natural science (or the totality of natural sciences) (*TLP*: 4.11).” As indicated above by how “reason” tends to “see around itself,” Kant also, to some extent, acknowledges a metaphysics that exceeds such limits:

> … [W]e noted *Grenzen* of reason with respect to all cognition of mere beings of thought; now, since the transcendental ideas nevertheless make the progression up to these *Grenzen* necessary for us, and have therefore led us, as it were, up to the contiguity of the filled space (of experience) with empty space (of which we can know nothing – the *noumena*), we can also determine the *Schranken* of pure reason; for in all *Schranken* there is something positive (e.g., a surface is the *Schranke* of corporeal space, yet is nonetheless itself a space; a line is a space, which is the *Schranke* of a surface; a point is the *Schranke* of a line, yet is nonetheless a locus in space), whereas *Grenzen* contain mere negations. (4:354)

Drawing a *Schranke* this way, by admitting a positive space on the other side of it, is specifically what Wittgenstein refuses at the outset of the *Tractatus*: “The book will… draw a limit to thinking, or rather – not to thinking… for, in order to draw a limit to thinking we should have to be able to think both sides of the limit (we should therefore have to be able to think what cannot be thought) (*TLP*: Preface).” So, as commentators have noted, it is the Tractarian non-contrastive limit’s radical austerity – in both the usual and the technical sense of the term – that has the benefit of resisting the pitfalls of transcendental idealism because there is nothing to be speculated about beyond the

propositions of natural science.[129] However, the sparseness of this conception of a limit also impedes what can actually be done with it or even said about it.[130]

Actually, however, the Kantian roots of the Tractarian non-contrastive limit are not to be found in the distinction between a *Grenze* and a *Schranke* in the *Prolegomena* but in the exposition of the principle of complete determination in the first *Critique*. Kant offers the reader two formulations of the principle: "Every thing… as to its possibility… stands under the principle… according to which, among all possible predicates of things, insofar as they are compared with their opposites, one must apply to it"; "The proposition… signifies not only that every given pair of opposed predicates, but also of every pair of possible predicates, one of them must always apply to it (A571/B579, A573/B601)." Said in a more contemporary and straightforward way, the principle of complete determination holds that "every object is completely determinate with respect to every pair of predicates *F* and ¬*F*, that is, every object is determinately either *F*, or ¬*F*."[131] However, Kant does not unconditionally advocate for this principle, but instead insists that it can only be upheld as a regulative ideal:

> It is self-evident that with this aim – namely, solely that of representing the necessary complete determination of things – reason does not presuppose the existence of a being conforming to the ideal, but only the idea of such a being, in order to derive from an unconditioned totality of complete determination the conditioned totality, i.e. that of the limited [*Eingeschränkten*]. For reason, the ideal is thus the original image (*prototypon*) of all things, which all together, as

[129] Sullivan, "Synthesizing without Concepts." 171–173.

[130] Contrasting the Tractarian subject with Lacan's concepts, Alain Badiou captures the former's bareness, in *Wittgenstein's Antiphilosophy*, trans. Bruno Bosteels (London; New York: Verso, 2011), 148–140: "Statement 5.64 is very close to Lacan… But unlike Lacan, Wittgenstein does not go on to produce a logical algebra of this vanishing [subject], nor a topology of the limit of being/existence… Nor does he engage with even that minimum of ontology, meant to support the algebraic and topological operations, to which Lacan ends up consenting despite his sarcasms."

[131] Nicholas F. Stang, "Kant on Complete Determination and Infinite Judgement," *British Journal for the History of Philosophy* 20, no. 6 (December 2012): 1119.

> defective copies (*ectypa*), take from it the matter for their possibility, and yet although they approach more or less nearly to it, they always fall infinitely short of reaching it. (A577/B605–A578/B606, translation slightly altered)

Although it may serve as a guide for reason, it would be mistaken to posit the existence of such a completely determined object. In other words, if the principle as a subjective ideal of pure reason is instead confused for a requirement imposed on the hypostasized existence of objective things, then this misstep would constitute a lapse into dogmatism. Accordingly, it should be no surprise that the principle has its roots in the pre-critical metaphysics of Wolff, Baumgarten, and Leibniz.[132]

Wittgenstein will eventually inherit this principle from Frege's appropriation of it as "the principle of completeness" in the *Grundgesetze der Arithmetik*:

> A definition of a concept (a possible predicate) must be complete; it has to determine unambiguously for every object whether it falls under the concept or not (whether the predicate can be applied to it truly). Thus, there must be no object for which, after the definition, it remains doubtful whether it falls under the concept, even though it may not always be possible, for us humans, with our deficient knowledge, to decide the question. Figuratively, we can also express it like this: a concept must be sharply limited [*scharf begrenzt*]. If one pictures a concept with respect to its extension as a region in a plane, then this is, of course, merely an analogy and must be treated with care, though it can be of service here. A concept without sharp limits would correspond to a region that would not have a sharp limit-line [*scharfe Grenzlinie*] everywhere but would, in places, be completely blurred, merging with its surroundings. This would not really be a region at all; and, correspondingly, a concept without sharp definition is wrongly called a concept. Logic cannot recognize such concept-like constructions as concepts; it is impossible to formulate exact laws concerning them. The law of excluded middle is in fact just the requirement, in another form, that concepts have sharp limits. Any object Δ either falls under the concept Φ or it does not fall under it: *tertium non datur*. Would, for example, the proposition, "Every square root of 9 is odd", have any graspable sense if *square root of 9* were a concept without sharp limits? Does the

[132] Béatrice Longuenesse, *Kant on the Human Standpoint*, Modern European Philosophy (Cambridge: Cambridge University Press, 2005), 211–214.

> question, "Are we still Christians?", indeed have a sense if it is not determined to whom the predicate *Christian* can be truly applied and from whom it must be withheld?[133]

In Frege's *Begriffsschrift*, the limits implicit in a completely determined concept are due to its requirement for sharp limits and definitions. However, we should notice that Frege's principle holds universally for *all* objects and *all* concepts. According to Kant's strictures, Frege's principle of completeness is therefore not merely a formal or a logical principle, like the principles of excluded middle and non-contradiction, but since it encompasses the content of all material possibilities, it is instead transcendental (A572/B600ff). Put frankly, Frege's endorsement of the principle comes dangerously close – if not, fully embracing – the very pre-critical metaphysics that Kant had already repudiated.

The path that we have been following terminates with the young Wittgenstein equating the requirement for simple names with a version of the principle of completeness at *TLP* (3.23): "The requirement for the possibility of simple signs is the requirement that sense be determinate." The requirement that a proposition *must* have determinate sense can be construed, in terms of Frege's formulation, as the demand that the truth or falsity of whether an object falls under a concept must always be unambiguously determined. The same demand for clarity is implicit in the demand for simple names: "A proposition has one and only one complete analysis"; "What a proposition expresses it expresses in a determinate and clearly [*bestimmte, klar*] specifiable way (3.25–3.251)." That is, if a proposition is to have a sense at all, then it

[133] Gottlob Frege et al., *Basic Laws of Arithmetic, Derived Using Concept-Script: Volumes I & II*, 1st ed (Oxford, U.K: Oxford University Press, 2013), 69–70. For consistency, I have altered the translation of *Grenze* as "limit" rather than "boundary."

must be a sharp, clear, definite, unambiguous, and determinate sense; otherwise, it is nothing other than nonsense.

Consequently, this brings us to the most fundamental aspect of Wittgenstein's notion of a limit (*Grenze*): it is the shift from a limit in thought to a limit in language, between a sharp and determinate sense on the one side and austere nonsense on the other. This is equivalent to the methodological step taken in the preface of the *TLP* that occurs after the possibility of delineating a limit in thought had been disavowed: "for, in order to draw a limit to thinking we should have to be able to think both sides of the limit… The limit can, therefore, only be drawn in language and what lies on the other side of the limit will be simply nonsense." Altogether, on this interpretation, Wittgenstein's notion of a *Grenze* ultimately reaches back to the principle of complete determination, insofar as it demands a sharp limit, with no gray area, between the completely determined sense of a proposition and utter nonsense.

As is indicated by the exceptionally long section of pages in his pre-Tractarian *Notebooks*, Wittgenstein had struggled extensively in order to find a plausible motivation for mutually upholding both the requirement for simple names and the notably severe requirement for the determinacy of sense (*NB*: 43–71). What is all the more revealing is that Wittgenstein's desperate search for justification takes place in the very metaphysical register that Kant had rejected and Frege had unknowingly fallen victim to. For example, in the middle of his exploration, Wittgenstein writes:

> And nothing seems to speak against infinite divisibility.
>
> *And it keeps on* forcing itself upon us that there is some simple indivisible, an element of being, in brief a thing [i.e., a simple Tractarian object].

> It does not go against our feelings, that *we* cannot analyze PROPOSITIONS so far as to mention the elements by name; no, we feel that the WORLD must consist of elements. And it appears as if that were identical with the proposition that the world must be what it is, it must be determinate. Or in other words, what vacillates is our determinations, not the world. It looks as if to deny things were as much as to say that the world can, as it were, be indeterminate in some such sense as that in which our knowledge is uncertain and indeterminate. (*NB*: 62)

In this passage, Wittgenstein is at pains to validate his intuition that the ontological determinacy of reality corresponds with its decomposition into simple objects. The pre-critical tenor of the passage is indicated at the very outset by noticing that Wittgenstein's musing about "infinite indivisibility" basically amounts to nothing other than an attempt to resolve Kant's second antinomy of pure reason. That is, on the one hand, "nothing seems to speak against" a world in which "[n]o composite thing… consists of simple parts, and nowhere in it does there exist anything simple (A435/B463)"; yet, on the other hand, "it keeps forcing itself upon us" that "[e]very composite substance in the world consists of simple parts, and nothing exists anywhere except the simple or what is composed of simples (A434/B462)." Furthermore, a backpedaling into a problematic metaphysics is perhaps even more obviously on display in the above use of rather brute ontological terms like "element," "thing," and "world."

Closely related problems will, unfortunately, persist even upon assuming a more top-down anti-metaphysical approach. Notice that the young Wittgenstein wants to follow Frege in blaming the possibility of indeterminacy on human fallibility and deficiency when he claims, "what vacillates is our determinations, not the world." Notice also how this is akin to Frege's assertion that "there must be no object for which… it remains doubtful whether it falls under the concept, *even though it may not always be*

possible, for us humans, with our deficient knowledge, to decide the question."[134] Also, like Frege's move from the sharp limits of a definition to that of objects and concepts themselves, Wittgenstein seems to reason that the requirement for determinacy placed upon language must be mirrored in reality as well: "And it appears as if that were identical with the proposition that the world must be what it is, it must be definite." In other words, it is the finite human that has indeterminate knowledge, while the world itself remains completely determinate.

Later, in his *Philosophical Investigations*, Wittgenstein will submit to critique precisely how this demand for sharp conceptual limits is then carried over to the world:

> And this is how we do use the word "game." For how is the concept of a game delimited? What still counts as a game, and what no longer does? Can you say where the limits are? No. You can draw some, for there aren't any *drawn* yet. (But this never bothered you before when you used the word "game".) …
>
> But this is not ignorance. We don't know the limits because none have been drawn. To repeat, we can draw a limit – for a special purpose. Does it take this to make the concept usable? Not at all! Except perhaps for that special purpose. (*PI*: §§68–69)

Against the backdrop of his later notion of a language-game, Wittgenstein takes up the example of the concept of a game so as to reconsider and ultimately reject the principle of complete determination. A principle or requirement that demands that a concept must have a sharp definition and limits for it to be a concept at all is deemed far too restrictive because we obviously use the concept of a game without any awareness of such limits. Furthermore, by denying that the concept of a game necessarily *must* have "fixed limits [*feste Grenzen*]," Wittgenstein denies the previously stipulated universality of the

[134] My emphasis.

requirement that *all* concepts must have limits. However, there *may* be *particular* instances where a limit might be erected around a concept, but this is a practice reserved "for a special purpose."

Eventually, Wittgenstein explicitly confronts Frege's formulation of the principle of completeness:

> One can say that the concept of a game is a concept with blurred edges. – "But is a blurred concept a *concept* at all?" – Is a photograph that is not sharp a picture of a person at all? Is it even always an advantage to replace a picture that is not sharp by one that is? Isn't one that isn't sharp often just what we need?
>
> Frege compares a concept to a region, and says that a region without clear boundaries can't be called a region at all. This presumably means that we can't do anything with it. – But is it senseless to say "Stay roughly here"? Imagine that I were standing with someone in a city square and said that. As I say it, I do not bother drawing any boundary, but just make a pointing gesture – as if I were indicating a particular spot. And this is just how one might explain what a game is… (*PI*: §§68–71)

Once again, we only need sharp limits in certain circumstances depending on the nature of the inquiry or the context of a discussion. Even though Wittgenstein's mention of a region is a response to Frege's untenably strict definition of a region, it should also be noted that the example is a direct reference to an object, i.e., a region, rather than a concept. That is to say, we do indeed encounter blurred things, like photographs and regions of space, and we speak about them accordingly. It can, therefore, be concluded that there are instances where we encounter indistinct and hazy distinctions both in language and throughout reality as a whole.

It follows that, along with dropping the requirement for simple objects and now complete determination, its accompanying Tractarian notion of a subject can no longer be

retained as the only feasible account. Of course, the Tractarian subject may be viable in a language-game that employs a symbolic logic that is similar to what had been unfurled within the dialectic of the *Tractatus*. However, there are perhaps other cases in which the subject could only be poised on a blurred limit or boundary that would be inconsistent with demands laid down in the *Tractatus*. That is to say, there is a possibility that the subject could be indeterminately situated both inside and outside of a world, or, better, between two language-games. Moreover, there is also the possibility that there may be no limit at all, and, accordingly, there would be no such Tractarian-style subject to even consider.

2.5: Closing Thoughts on Reflexive Subjectivity

In the reciprocal implosion of idealism and realism, it is worthwhile to observe how the two Tractarian logical requirements that were investigated in the first chapter serve as the driving presuppositions propelling the operation. On the one hand, the requirement for a general form of a proposition expels the substantial subject from the existent realm of objects and states of affairs to the very limits of the world. Whereas, on the other hand, the requirement for simple names and objects shrinks this limit into a sharp distinction between sense and nonsense.

Furthermore, what was also demonstrated in a previous section (1.2.4) is how Tractarian showing, as construed by way of the medial diathesis, draws the two logical requirements together. On the one hand, the general form of a proposition follows from how the words in a propositional sign are combined together in a way that can only be

shown through a medial and intransitive presentation (*TLP*: 3.14–3.144). On the other hand, the simplicity of a name that stands for a simple object also can only be medially shown and not said (*TLP*: 4.1211, 4.1272).

The focus on the account of showing based on the medial diathesis brings out another parallel in that it mutes the expressive capacity of the agentive human subject and expressivity is then totally passed over to logic and language. Narboux rather eloquently explains how the vanishing of the subject and the etching of the limit coincide in showing:

> [T]he section that motivates the distinction between what shows itself and what is said [i.e., *TLP*: 4.12ff]: in order to be able to present logical form, we would have to station ourselves on the far side of the limit…
>
> The diathesis that we have called, after Benvéniste, the "internal diathesis," captures alone the sense in which the limits of thought can be drawn *from within*. The limit (*Grenze*) may be said to "show itself " (*sich zeigen*) in what it limits (*begrenzt*) to this extent only, that it is internal to it. It bespeaks the essence of logic that it should be neither possible nor necessary for us so much as to want to take care of it: to undertake to "see to it" that logic is not handled in a way that sins against its nature.[135]

Yet, as was established in the previous chapter, the intransitivity of this account of medial showing directly entails a prohibition against reflexivity.

Therefore, we confront the well-known and persistent link in modern philosophy between the subject and its reflexive awareness of itself; but there is a lingering question about what is driving this lineage and what justifies the continued bond between the two.[136] Scholars have often traced this connection back to modernity's inaugural moment

[135] Narboux, "Showing, the Medium Voice, and the Unity of the *Tractatus*," 221–2.

[136] For an illuminating and comprehensive conceptual genealogy of subjectivity, including its reflexive aspects, see Frances Cooper, "The Subject and/of the Law: Yan Thomas and the Excess of History over

with Descartes positing a *cogito* that reflexively attributes the "I think…" to the very existence of the subjective, "I am." As for the German philosophical tradition, Kant is often considered responsible for securing the association between subjectivity and some kind of reflexive self-consciousness by maintaining that "The *I think* must be able to accompany all my representations (B132)."[137] However, there seems to be an ongoing debate within the scholarship as to how this Kantian thread was taken up in terms of the implicit reflexivity in the accounts of the subject provided by Fichte and Schopenhauer.[138]

What is important for our present concerns is that the question of subjective reflexivity is not at all up for debate to the young Wittgenstein. Indeed, since any proposition of the form "I think *p*" would be excluded due to its formal resemblance to the propositional attitude "A thinks *p*," there is simply no way to formulate any cogito-like propositions in the Tractarian symbolic notation. Moreover, the Tractarian logical requirements that impose prohibitions on the reflexive construction of propositions further entail that any recurring site of subjectivity as a reflexive turning back on oneself is barred from the outset. However, as the mature Wittgenstein will eventually relinquish any comprehensive prohibition on reflexivity, the question of the subject, its dissolution,

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Concept," in *Afterlives: Transcendentals, Universals, and Others*, ed. Peter Osborne, vol. 4 (CRMEP Book, 2022), 81–85.

[137] For a thoroughgoing study of Kant's canonical role in the reflexive underpinnings of subjectivity, see the entry on the subject in Barbara Cassin, ed., *Dictionary of Untranslatables: A Philosophical Lexicon*, trans. Steven Rendall et al. (Princeton: Princeton University Press, 2014), 1080–1083.

[138] On the one hand, there are commentators who have downplayed this subjective reflexivity in Fichte and Schopenhauer. For example, see Ronald Mather, "On the Mythology of the Reflexive Subject," *History of the Human Sciences* 10, no. 4 (November 1997): 65–82. On the other hand, there are other interpretations that emphasize the reflexivity of the subject in the German idealist tradition from Kant, through Fichte, up to Hegel. For example, see Slavoj Žižek, "Fichte's Laughter," in *Mythology, Madness, and Laughter: Subjectivity in German Idealism*, ed. Markus Gabriel and Slavoj Žižek (London: Continuum, 2009), 122–67.

and the resulting alignment of idealism and realism brought about by its disappearance must then be addressed anew. Hence, the issue will be taken up again in the fourth chapter to come.

Chapter 3: Morphology as Logic and Logic as Morphology

In this chapter, I demonstrate how Wittgenstein derives his morphological method from both the tensions that have already been observed in the *Tractatus* as well as certain coinciding results of the negative resolutions to the *Entscheidungsproblem* or decision problem. Specifically, the tension is brought out in terms of the distinction between compositionality and contextuality. I argue that, rather than ameliorating the tension, Wittgenstein valorizes and sharpens the conflict into a productive force for the institution and inauguration of new aspects of language. Similarly, I show how an analogously pitched opposition occurs in Wittgenstein's interpretation of Cantor's set-theoretic proof for the uncountability of the real numbers. In particular, we find that Wittgenstein's evaluation of Cantor's proof is in step with his newfound acceptance of paradox and contradiction. After showing how Wittgenstein's revised outlook on paradox and contradiction can help explain his insights with regard to Turing's results, the aporetic notion of the standard meter, and concept formation, the chapter closes by detailing how all this can be encompassed under the morphological method and its analogical comparison of form.

3.1: Compositionality vs. Contextuality

As has already been mentioned in section 1.1, the conflicting strengths and weaknesses between the standard and the resolute interpretations can be elaborated in terms of the tension within the *Tractatus* between compositionality and contextuality. In the parlance of the Anglo-American tradition, this distinction is customarily understood in terms of

Frege's competing proposals of the principle of compositionality and the context principle. The former principle of compositionality maintains that the sense of a proposition is a function of the meaning of its constituent words and their mode of combination. For example, in the *Begriffsschrift* (§9), Frege underlines compositionality by segmenting propositions into "one part a function, the other an argument"; "If an expression is thought of as variable in this way, it is split up into a constant part representing the totality of these relations and a symbol, imagined as replaceable by others, that stands for the object related by the relations."[139] On the other hand, the context principle maintains that the meaning of a word is determined by the significant proposition in which it occurs. Frege's definitive formulation of the context principle appears in the *Grundlagen*: "it is only in the context of a proposition that words have meaning."[140] Put simply, we could say that compositionality emphasizes the compositional parts over a whole proposition, whereas contextuality emphasizes a whole proposition over its segmentation into its compositional parts.

Since its importance will later become clearly evident, it is worthwhile to note that Enzo Melandri has exhaustively traced various manifestations of this distinction, which he describes in terms of the *chiasma* between nominal semantics and propositional semantics, throughout the entirety of the Western philosophical tradition.[141] As is

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[139] Gottlob Frege, *Translations from the Philosophical Writings of Gottlob Frege*, ed. Peter Geach and Max Black, 2nd ed. (Oxford: Basil Blackwell, 1960), 12.

[140] Gottlob Frege, *The Foundations of Arithmetic*, trans. J. L. Austin, 2nd ed. (Evanston, Illinois: Northwestern University Press, 1953), 73.

[141] Angelo Bonfanti, "Le Forme Dell'Analogia Studi Sulla Filosofia Di Enzo Melandri [The Forms of Analogy: A Study on the Philosophy of Enzo Melandri]."" (PhD Dissertation, Bergamo, Italy, Università Degli Studi Di Bergamo, 2014), 203–233.
I can't express enough of my gratitude to the brilliant philosopher and comrade, Angelo Bonfanti, for his utterly groundbreaking and masterful dissertation on Enzo Melandri's thought, which was later published in book-length format as *Le Forme Dell'analogia: Studi Sulla Filosofia Di Enzo Melandri*, I edizione (Ariccia (RM): Aracne editrice int.le S.r.l, 2016).

intimated by its connotations about names and naming, nominal semantics aligns with compositionality; yet, as the term "proposition" intimates, propositional semantics aligns closer with contextuality. Melandri quite eloquently articulates the distinction: "It is curious to note that the thesis of nominal semantics, which is as old as the world itself, has always been matched by the diametrically opposite thesis, which we will call propositional semantics."[142] In fact, Melandri argues that we can detect this chiasma as far back as the very moment in which the Presocratics, especially Heraclitus and Parmenides, began striving to free philosophical contemplation from the fog of *mūthos* by offering rival conceptions of the distinction between *ēpos* and *lōgos*, word and sentence.

In the *Tractatus*, there is a wealth of textual evidence to substantiate that Wittgenstein was committed to both principles. For example, consider the various formulations of the principle of compositionality:

> 3.14: What constitutes a propositional sign is that in it its elements [*Elemente*], the words [*Wörte*] stand to one another in a determinate [*bestimmte*] way.
>
> A propositional sign is a fact.
>
> 3.141: A proposition is articulate.
>
> 3.318: I conceive of a proposition – like Frege and Russell – as a function of the expressions [*Ausdrücke*] contained in it.
>
> 4.024: One understands it [i.e., a proposition] if one understands its constituents [*Bestandteile*].

I would also like to use this opportunity to, again, make a public call for an English translation of both Bonfanti's book as well as Enzo Melandri's magnum opus, *La linea e il circolo: studio logico-filosofico sull'analogia* (Macerata: Quodlibet, 2004).

[142] Melandri, *La linea e il circolo*, 51.

Notice that in each remark, the emphasis is put on the constituent words, elements, or expressions that together compose a proposition. Nevertheless, in the *Tractatus*, it is not difficult to find various formulations of the opposing context principle:

> 2.0122: It is impossible for words to occur in two different ways, alone and in a proposition.
> 3.3: Only [*Nur*] propositions have sense [*Sinn*]; only in the context of a proposition has a name meaning [*Bedeutung*].
> 3.34: An expression has meaning only in a proposition.
> 4.23: A name occurs in a proposition only in the context of an elementary proposition.

In these remarks, the emphasis is reversed, as it is the complete proposition with sense that asserts its dominance over the constituent words, names, and expressions. In fact, contextuality, for Wittgenstein, imposes the rather severe restriction that a name can *only* have substantial meaning within the wider context of a proposition. Given this tight constraint, it is widely acknowledged by commentators that the two principles, compositionality and contextuality, somehow stand in a kind of direct conflict with one another.

Let us clarify how this conflict reveals itself in more precise terms. We can begin by explaining the emphasis that compositionality puts on words and that contextuality puts on propositions in terms of priority. Following Silver Bronzo, we can define "priority" as a certain asymmetrical relation, such that "if A is prior to B, B cannot in turn be prior to A."[143] The inverse would then be a "symmetrical interdependence," such that "A depends on B and B depends A."[144] Accordingly, we can then use this definition of

[143] Silver Bronzo, "Context, Compositionality, and Nonsense in Wittgenstein's Tractatus," in *Beyond The Tractatus Wars: The New Wittgenstein Debate*, ed. Rupert Read and Matthew A. Lavery (New York: Routledge, 2011), 108n10.
[144] Bronzo, "Context, Compositionality, and Nonsense," 108n10.

priority to elucidate the contrast between the two principles: compositionality maintains that constituent words are prior to whole propositions; whereas contextuality maintains that the whole proposition is prior to its constituent words.

For Bronzo, the kind of priority that is of interest for the current topic is conceptual rather than temporal: "[We] are interested in the conditions of possibility – or, as I prefer to say, the conditions of intelligibility – of the phenomenon that they want to explain in terms of the conceptually prior item."[145] Relying on conceptual priority, the distinction between the two principles can now be drawn as follows:

> So contextualism would appear to be committed to holding that the meanings of complete sentences are intelligible without any reference to the meanings of the words composing them; those meanings are indeed explained in terms of the meanings of sentences, which must therefore be already given. Compositionalism, on the other hand, would appear to be committed to holding that the meanings of words are intelligible without any reference to the meanings of the sentences in which they occur; the latter sort of meaning is indeed explained in terms of the meanings of words, which, again, must be given in advance.[146]

Consequently, we can understand the conflictual tension between compositionality and contextuality as an opposition between the conceptual priority of either words or propositions.

As was discussed in an earlier section (1.1.3), the typical interpretative approach to this tension within the secondary literature is to bolster one principle, while minimizing the other. As for the standard interpretation, it is inclined to give prominence to compositionality, while relaxing contextuality. For example, Hans-Johann Glock veers towards compositionality and away from contextuality by opting for a much weaker

[145] Ibid., 90–91.
[146] Ibid., 91.

version of the context principle than what is actually proposed at *TLP* (3.3): a word just has to "be *capable* of occurring in a proposition… but it does *not* follow that the word has meaning only in the context of a proposition."[147] Recall also that the standard interpretation's proclivity for compositionality tends to imply a substantial conception of nonsense. In fact, this is exactly what Glock sets out to defend: "[B]oth early and late he [Wittgenstein] allowed that nonsense can result not just from a failure to assign a meaning [i.e., austerity], but also from combining meaningful expressions in a way that is prohibited by the rules for the use of these expressions [i.e., substantial nonsense]."[148] Since Glock's very aim is to advocate for a substantial conception of nonsense, its admission cannot be viewed as a philosophical or exegetical weakness. However, critics of his interpretation have otherwise argued that bending the stick this far in the direction of compositionality leads to an account of language that is far too bottom-up and atomistic to model any viable means of communication.[149]

Conversely, the resolute interpretation will strengthen contextuality, while correspondingly weakening compositionality and rejecting all kinds of nonsense, except of the austere variant. For example, Bronzo endorses a more resolute-style trade-off by "adopting a strong version of the context principle entailing the austere conception of nonsense: words have meaning *only* in the context of significant propositions."[150] However, arguably, compositionality is downplayed by stripping its priority and allowing for its symmetrical interdependence with compositionality: "No priority can be given to

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[147] Hans-Johann Glock, "All Kinds of Nonsense," in *Wittgenstein at Work: Method in the* Philosophical Investigations, ed. E. Ammereller and E. Fischer (London: Routledge, 2004), 229, some emphasis my own.
[148] Glock, "All Kinds of Nonsense," 222.
[149] Bronzo, "Context, Compositionality, and Nonsense," 102–104.
[150] Ibid., 101.

either the contents of propositions or to the meaning of words, if we don't want to lose sight of the phenomenon of language altogether."[151] Yet, as several standard interpreters have noted, if a resolute version of the context principle is accepted, which discounts individual words having any meaning in isolation, then it must counter-intuitively also reject some very ordinary and routine linguistic practices involving the use of words outside of any propositional context; such as, for example, the everyday occurrence of consulting a dictionary.[152] Furthermore, there are exegetical grounds that can be culled from within the primary literature that indicate that the Wittgenstein of the *Tractatus* had considered language to have enough compositionality to accommodate this fairly commonplace practice:

> 4.025: The translation of one language into another is not a process of translating each *proposition* of the one into a *proposition* of the other, but only the constituents of the propositions [*Satzbestandteile*] are translated.
>
> (And a dictionary translates not only substantives, but also verbs, adjectives, and conjunctions, etc.; and it treats them all the same way.)

Consequently, the contextualist resolute reading overcomes rigid atomism but does not manage to maintain enough compositionality to accommodate certain well-established, referential connections between an individual name in isolation and its meaning.

As was demonstrated in the first chapter, the tension between compositionality and contextuality was one of many observed between the standard and the resolute interpretations. Additionally, it was shown how the clarificatory exercise intended to reveal "Socrates is identical" as nonsense in *TLP* (5.473–5.4733) instead indicated how

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[151] Ibid., 90.

[152] Glock, "All Kinds of Nonsense," 226; P.M.S. Hacker, "Wittgenstein, Carnap and the New American Wittgensteinians," 17.

two competing interpretations traded off one's strengths for the other's weakness and vice versa. That is, on the one hand, the compositionality of the standard interpretation could account for Wittgenstein's thoughts about whether the individual word "identical" had a certain meaning, but it could not be made consistent with the endorsements of contextuality and austere nonsense in those same remarks. Yet, on the other hand, the resolute interpretation is, indeed, consistent with contextuality and austerity, but it could not account for what Wittgenstein clearly states about the individual word, "identical."

As Melandri has masterfully shown through his penetrating application of philosophical archeology, the very history of Western philosophy can be recapitulated through the dialectical back-and-forth of compositionality and contextuality, the nominal and the propositional.[153] According to Melandri, what is essentially the driving force propelling this oscillating dialectic is that, even though the two positions oppose one another, there is also an inherent implausibility when each position is taken by itself and to its extremes. Indeed, Wittgenstein scholarship has arrived at the same conclusion "by taking contextualism and compositionalism," when considered on their own, as something like a maximalist ideology, to be overwhelmingly unable to "provide an *intelligible* notion of *language*."[154] Namely, if a language was too compositional, then it would be too rigid and atomized for the construction of communicable sentences; but if a language was too contextual, it would lose its grip on reality and its sentences would fail to convey any information.

[153] Bonfanti, "Le Forme Dell'Analogia Studi Sulla Filosofia Di Enzo Melandri," 205ff.
[154] Bronzo, "Context, Compositionality, and Nonsense," 90.

For my part, I wish to follow Wittgenstein, at least in *ethos*, if not by the letter of his word, and opt for the most radical gesture presented by the dilemma. Specifically, I will argue in favor of both a strong compositionality and a strong contextuality, while acknowledging and accepting that the two principles remain in conflict with one another. In other words, if the compatibility of the two principles demands their interdependence, then my standpoint is the opposite: both compositionality and contextuality are meant to remain prior to one another and therefore radically incompatible. The textual evidence supporting this kind of pitched antagonism between the two principles can be found in Wittgenstein's most decisive reevaluation of his early approach to this and other related problems in the *Philosophical Investigations* (§§46–64). Moreover, this is in step with the interpretation that I advanced in the first chapter, which locates hidden tensions and ambiguities in the *Tractatus* that the later Wittgenstein will then reveal as overt contradictions, inconsistencies, and paradoxes in the later work of the *Investigations* period.

3.2: Compositionality vs. Contextuality in the Later Period

A more robust notion of priority is necessary in order to address how the distinction between compositionality and contextuality plays out in Wittgenstein's middle and later periods. That is, in addition to Bronzo's conceptual priority, we will have to also attend to the very same temporal priority that he explicitly rejects: thus, the principle of compositionality is then rendered as "*first* we grasp the meanings of words, *then* we proceed to put them together and obtain the meaning of a proposition"; whereas the

contextual principle is accordingly rendered: "*first* we understand the proposition, *then* we segment it into its constituent parts."[155] In other words, we will have to supplement the conditions of intelligibility by taking into consideration the sequential order of the occurrences of words and propositions.

There are exegetical grounds for taking into account temporal priority alongside conceptual priority, as it is implicit in several formulations of compositionality, even as early as the *Tractatus*:

> 4.027: It belongs to the essence of a proposition that it can communicate a *new* sense [neuen *Sinn*] to us.
> 4.03: A proposition must communicate a new sense with old expressions [*alten Ausdrücken*].

As the terms "old" and "new" indicate, compositionality is, in fact, presented in the *Tractatus* in terms of a sequential ordering that must begin *first* with constituent words, and *then* next comes the whole proposition.

As for the philosophical stakes, let us consider Bronzo's reason for selecting the "conceptual rather than the temporal": "It doesn't seem that either of the two rival positions [i.e., compositionality and contextuality] wants to make an empirical-psychological claim."[156] However, even though Wittgenstein remained consistent in his rejection of psychologism, he eventually came to realize, in a rather nuanced fashion, that empirical considerations did have a decisive role in logico-philosophical clarification. As Kuusela helpfully explains:

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[155] Ibid.
[156] Ibid.

> Wittgenstein also rejects his early view that empirical facts and generalities are simply irrelevant to logic and that logic is concerned with something ideal which is pure from anything empirical or that the object of investigation of logic is language as an abstract entity… A way to describe Wittgenstein's [late] position is to say that he wants to do justice to, and incorporate into his account of logic, the way in which natural facts about humans and their environment shape language and thought.[157]

For this reason, empirical and temporal factors will prove to be decisive in an examination of the two principles, as it becomes a concern, for logic, how whole propositions and their constituent words are learned and repeatedly used.

In fact, for the late Wittgenstein, these two factors, the empirical and the temporal, are intimately connected; he repeatedly directs his reader's attention to the crucial moments of the institution, development, and acquisition of language in its variety of uses. It is helpful here to consider how Livingston does well in underlining the significance of this shift in focus from the early to the late period:

> And it is beyond doubt that the Wittgenstein of the *Investigations* takes this problem – the problem of how signs get their application, how they get to be meant or used in the ways that they regularly are, of what this regularity means, and more generally of what is involved in talking or thinking of "the use of a sign" or the rules by which we characterize it, and what it means to learn these rules, to know them, to follow them or to dispute them – as one of the deepest and most significant problems that contemporary critical thought can indeed take up.[158]

However, deliberations of this sort are uncommon, if not absent, from the *Tractatus*. For this reason, the criticism that the mature Wittgenstein levels at the North African philosopher-saint could just as well have been directed at his younger self: "Augustine describes the learning of human language as if the child came into a foreign country and

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[157] Kuusela, *Wittgenstein on Logic as the Method of Philosophy*, 180–181.
[158] Livingston, *The Politics of Logic*, 140; "Wittgenstein and Parmenides," 19.

did not understand the language of the country; that is, as if he already had a language, only not this one (*PI*: §32)." In other words, for Wittgenstein, an insouciant disregard for how language is actually acquired tends to be connected with a certain "primitive idea of the way language functions (*PI*: §2)," in which both the *Tractatus* and the *Confessions* succumb but is subjected to sharp criticism in the later *Investigations* period. As a powerful corrective to his youthful neglect, the empirico-temporal vicissitudes involved in what the mature Wittgenstein tends to refer to as "instruction," or "preparation," are now deemed integral to the philosophical elucidation of the logic of our language (*PI*: §§6–7, 26, 31).

These considerations are foregrounded when Wittgenstein, in the *Philosophical Investigations*, is once again confronted by the tension between compositionality and contextuality in the course of reexamining his earlier commitment to the requirement for simple names (§46ff). Simple names tend to be the locus of both compositionality and contextuality in the *Tractatus*. For example, names are given both conceptual and temporal priority in the following declaration of the principle of compositionality: "One name [*Name*] stands for one thing, another for another thing, and they are combined with one another; so, the whole – like a living picture – represents a state of affairs (*TLP*: 4.0311)." Whereas, the most canonical formulation of the context principle is put forward by highlighting how a name is essentially derivative upon its insertion in a significant proposition: "only in the context of a proposition has a name [*Name*] meaning (*TLP*: 3.3)." Yet, in returning to the subject of simple names in the *Investigations*, Wittgenstein prompts his reader to reflect upon scenes of instruction with regard to how a person might learn how the one-letter names can stand for their correlated colored

squares in order to play the language-game of §48: "There are red, green, white and black squares. The words of the language are (correspondingly) "R," "G," "W," "B," and a sentence is a sequence of these words." Hence, we are introduced to a game that is designed to clarify a vision of language, where the complexity of language bottoms out in names that denote simple objects or primary elements.

Next, at *PI* (§49), Wittgenstein goes on to problematize the logical requirement for simple names by imagining a variety of scenarios that might involve the game:

> But what does it mean to say that we cannot explain [*erklären*] (that is, describe [*beschreiben*]) these elements [*Elemente*], but only name [*benennen*] them? Well, it could mean, for instance, that when in a limiting case a complex consists of only *one* square, its description is simply the name of the colored square.
>
> Here one might say – though this easily leads to all kinds of philosophical superstition – that a sign "R" or "B", etc., may sometimes be a word [*Wort*] and sometimes a proposition [*Satz*]. But whether it "is a word or a proposition" depends on the situation [*Situation*] in which it is uttered or written. For instance, if A has to describe complexes of colored squares to B, and he uses the word "R" *by itself*, we'll be able to say that the word is a description – a proposition. But if he is memorizing the words and their meanings, or if he is teaching someone else the use [*Gebrauch*] of the words and uttering them in the course of ostensive teaching, we'll not say that they are sentences. In this situation the word "R", for instance, is not a description; one *names* an element with it —— but that is why it would be strange to say here that an element can *only* be named! For naming and describing do not stand on the *same* level: naming is a preparation [*Vorbereitung*] for describing. Naming is not yet a move in a language-game – any more than putting a piece in its place on the board is a move in chess. One may say: with the mere naming of a thing, *nothing* has yet been done. Nor *has* it a name except in a game. This was what Frege meant too when he said that a word has a meaning only in the context of a proposition.[159]

The above passage is obviously a criticism of Wittgenstein's earlier claim that "Objects can only [*nur*] be *named*. Signs represent them. I can only speak *of* them. I cannot *assert*

[159] For consistency, I have altered the Anscombe, Hacker, and Schulte translation by rendering "*Satz*" as "proposition," instead of "sentence."

them. A proposition can only *say* how things are, not *what* they are (*TLP*: 3.221)." Instead of the previous stubborn use of "only," the situation (*Situation*) will now decide whether a sign is being used as a name or a proposition. Put in the specific terms of the language-game of §48, whether a sign, like "R," is classified as a simple name or a propositional description hinges on the circumstance and the occasion in which it is employed. Furthermore, the mature Wittgenstein's interest in language acquisition is brought to bear as teaching, learning, practicing, and memorizing are distinguished as viable situations in which "R" might be feasibly used as a name.

What is also remarkable about this specific remark in the dialectical unfolding of the *Investigation* is how it accentuates the tension between compositionality and contextuality. Notice, on the one hand, that the use of names in the scenes of instruction, especially with regard to ostensive teaching and learning, stresses the compositionality of language. In addition, Wittgenstein tells us that naming as "preparation [*Vorbereitung*]" is distinct, insofar as it stands on its own level. We could also say that teaching and learning with names and squares constitutes its own language-game. On the other hand, the use of propositions and descriptions instead stresses the contextuality of language. Likewise, the act of constructing propositions has its own level and ought to be considered a genuine move in the language-game of §48. The immediate problem for the reader of the *Investigations* is that we are not informed as to how the two levels, preparing and acting, naming and describing, compositionality and contextuality, are meant to interact.

Actually, if we follow *PI* (§49) to the letter, it seems that their connection is only that of outright conflict. More specifically, even though naming is designated as

"preparation for describing," it seems to fail to accomplish its specified goal. Notice that there are three instances in which Wittgenstein declares that naming proves itself to be unsuccessful: "Naming is not yet a move"; "*nothing* yet has been done"; and "*nor* has it a name except in a game." Strictly speaking, preparation alone did not even manage to establish the names, "R", "G", "W", and "B", for their respective colored squares. What actually occurred on the compositional level of naming turned out to be utterly futile.

For this reason, it is all the more striking that it is at least implied that the transition from compositional naming to contextual describing is still somehow effected. Indeed, this move to the next level is signaled by Wittgenstein's paraphrase of Frege's context principle: "a word [*Wort*] only [*nur*] has meaning [*Bedeutung*] in a propositional context (*Satzzusammenhang*)." However, it seems that we have come across a strange case in which a speaker has been initiated into an extremely contextual language that does not require learning a single word as a precondition. In other words, the construction of propositions and descriptions is somehow able to proceed despite the measures for the preparation of names having failed. Furthermore, if the level of naming was exceedingly compositional due to the ostensive teaching and learning, then the level of describing turns out to be just as contextual since the propositions are not relying on names that denote objects.

The impasse we have encountered is not a mistake on Wittgenstein's part but, instead, is the earlier tension detected between compositionality and contextuality, now facing off with one another in open and all-out war. Furthermore, for Wittgenstein, the conflict we are observing between the two principles will indicate an integral aspect of his fully developed outlook on language, its acquisition, and its institution. As he

colorfully describes elsewhere in the *Investigations*, it is a case in which a sign "hangs in the air" before its use has been fully determined (*PI*: §198). For example, the specific name, "R," for the red-colored square hangs in the air before it is employed in a proposition.

It is also no coincidence that the opposition between compositionality and contextuality at *PI* (§49) immediately precedes more paradoxical formulations, such as paradigmatic instruments of language like the standard meter (§§50–51, 55–60) and an early glimpse of the so-called rule-following considerations (§§53–54). For example, in a much earlier remark, we find Wittgenstein and Waismann drawing a similar connection between examples like the language-game of §48 and paradigms like the standard meter:

> For the red object to which I point in ostensively defining the word "red" is not a described object but a sample, a part of language. In the future it will serve me as an object of comparison, just like the standard meter, which is a paradigm and not a measured object.[160]

What is perhaps all the more revealing is a section from the *Remarks on the Foundations of Mathematics*, where a description of a somewhat similar deadlock in which the paradigm of the meter itself hangs in the air:

> If I were to see the standard meter in Paris, but was not acquainted with its institution of measuring and its connection with the standard meter – could I say, that I was acquainted with the concept of the standard meter?
>
> Is a proof also not a part of an institution in that way?
>
> A proof is an instrument – but why do I say an "instrument of language"?
>
> … What I always do seems to be – to emphasize the distinction between the determination of a sense and the employment of sense. (*RFM III*: §36–37)

[160] Ludwig Wittgenstein and Friedrich Waismann, *The Voices of Wittgenstein: The Vienna Circle*, ed. Gordon P. Baker (London: Routledge, 2003), 227; see also 269, 271; See also, Friedrich Waismann, *The Principles of Linguistic Philosophy*, ed. R Harre (London: MacMillan, 1965).

That is, in the same way that a name could not be considered such until it was used in the language-game of §49, the paradigm of the standard meter, which has now been identified as a sample and an instrument to language, cannot function until we become familiar with the institutionalized practice of measuring. The above remark also provides us with another way of understanding the distinction between two levels, naming and describing, preparing and acting, composition and context, as it is now also put in terms of the determination of sense and the employment of sense.

It also becomes clear that, for Wittgenstein, there is a strong commonality between simple names, the standard meter rod, and mathematical proofs: each precipitates a transition from one game or level to the next that leaves something in a kind of "suspended state" in between. In particular, what is important for our purposes is that Wittgenstein's philosophical inquiry into Cantor's diagonal proof for the uncountability of the real numbers evinces veritably the same explicit opposition between compositionality and contextuality in language. Moreover, the correspondence between composition/contextuality in language and set theory will have two other significant consequences: first, since the diagonal procedure forms the unified backdrop to the various negative resolutions of the *Entscheidungsproblem*, it allows us to see how Wittgenstein's later treatment of the specific tensions located in Tractarian logic can be grasped in terms of more general results regarding the formalization of mathematics; second, Wittgenstein's examination of the diagonal proof will also entail an acceptance of the paradoxes and contradictions that had been prohibited in the *Tractatus* phase. Hence, in the next two sections, I will show how, for Wittgenstein, mathematical proof, specifically the set-theoretic demonstration of the uncountability of the real numbers,

presents an elegant and perspicuous example of how in each case the gap is crossed between the two levels of preparation and description, naming and saying, compositionality and contextuality.

3.3. The Standard Rendition of Cantor's Diagonal Proof

Before turning to Wittgenstein's rather infamous comments on the application of the diagonal procedure in proving the uncountability of the real numbers, we should begin by considering Cantor's proof as it was first set forth in "On Elementary Questions in the Theory of a Manifold."[161] In addition, following Wittgenstein's practice, I will tend to rely mainly on the standard exposition of the proof as it is typically presented in analysis textbooks. For this reason, I have chosen Stephen Abbott's *Understanding Analysis* specifically, since its rendition of the diagonal proof is in close accord with what Cantor historically put forward in his 1897 essay.

And so, let us begin with a few very rough presuppositions by respectively defining the natural numbers, the integers, and rational numbers as follows: **N** = {1, 2, 3, 4, 5, …}; **Z** = {…, -3, -2, -1, 0, 1, 2, 3, …}; and **Q** = {all fractions p/q where p and q are integers with $q \neq 0$}. Furthermore, assume that the natural numbers and the integers are ordered; the integers are closed under addition and subtraction; and finally, the rational numbers meet both these conditions, plus multiplication and division, so that they form

[161] Georg Cantor, "On an Elementary Question in the Theory of a Manifold," in *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, ed. William Bragg Ewald (Oxford: Oxford University Books, 1996), 920–922.

an ordered field.[162] However, given Wittgenstein's criticisms of the various textbook treatment of the real numbers, it is best to leave open the question of how to define the real numbers, **R**.

Next, let us introduce some notation and terminology for functions. Given two sets *A* and *B*, a *function* from *A* to *B* is a rule or mapping that takes each element $a \in A$ and associates it with a single element of *B*. In this case, we write $f : A \rightarrow B$. The set *A* is called the *domain* of function *f*. The *range* of *f* is not necessarily equal to *B*, but refers to the subset of *B* given by $\{ b \in B : b = f(a)$ for some $a \in A$, such that $b = f(a)$.

A function $f : A \rightarrow B$ is called either injective or one-to-one (i.e., 1-1), if $a_1 \neq a_2$ in *A* implies that $f(a_1) \neq f(a_2)$. That is, $\forall a_1, a_2 \in A : a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$. A function $f : A \rightarrow B$ is called either surjective or onto, if given any $b \in B$, it is possible to find an element $a \in A$ for which $f(a) = b$. That is, $\forall b \in B : \exists a \in A$ such that $f(a) = b$. Said in informal terms, "The property of being 1-1 [i.e., injective] means that no two elements of *A* correspond to the same element *B*… and the property of being onto [i.e., surjective] ensures that every element of *B* corresponds to something in *A*…"[163] Finally, a function $f : A \rightarrow B$ is bijective or there is a one-to-one (i.e., 1-1) correspondence between sets *A* and *B*, if $f : A \rightarrow B$ is *both* injective (i.e., one-to-one) and surjective (i.e., onto). That is, $\forall b \in B : \exists! a \in A$ such that $b = f(a)$. In addition, if there exists a bijective function $f : A \rightarrow B$, then we can say that set *A* has the same cardinality *as B* or write $A \sim B$. Lastly, a set is *A* is called countable if $\mathbf{N} \sim A$. Conversely, an infinite set that is not countable is called an uncountable set.

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[162] See, Abbott, *Understanding Analysis*, 3, 299.
[163] Ibid., 25.

Everything is now in place for the standard proof of the uncountability of the real numbers. In order to do so, it is enough to show that an open interval of the real numbers $(0,1) = \{x \in \mathbf{R} : 0 < x < 1\}$ is itself uncountable. Assume for, *reductio ad absurdum*, that there exists a bijective function $f : \mathbf{N} \rightarrow (0, 1)$. In other words, we are assuming that (0, 1) is countable. Accordingly, for each $m \in \mathbf{N}$, $f(m)$ is a real number between 0 and 1, and we can represent using the decimal notation, such that, $f(m) = .a_{m1}a_{m2}a_{m3}a_{m4}a_{m5}\ldots$. This is intended to indicate that for each m, $n \in \mathbf{N}$, a_{mn} is the digit from the set $\{0,1,2, \ldots ,9\}$ that represents the nth digit in the decimal expansion of $f(m)$. The assumed bijection between **N** and (0, 1) can be summarized in the doubly indexed array:[164]

N		$(0,1)$								
1	$\longleftrightarrow$	$f(1)$	$=$	$.\mathbf{a_{11}}$	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	$\cdots$
2	$\longleftrightarrow$	$f(2)$	$=$	$.a_{21}$	$\mathbf{a_{22}}$	a_{23}	a_{24}	a_{25}	a_{26}	$\cdots$
3	$\longleftrightarrow$	$f(3)$	$=$	$.a_{31}$	a_{32}	$\mathbf{a_{33}}$	a_{34}	a_{35}	a_{36}	$\cdots$
4	$\longleftrightarrow$	$f(4)$	$=$	$.a_{41}$	a_{42}	a_{43}	$\mathbf{a_{44}}$	a_{45}	a_{46}	$\cdots$
5	$\longleftrightarrow$	$f(5)$	$=$	$.a_{51}$	a_{52}	a_{53}	a_{54}	$\mathbf{a_{55}}$	a_{56}	$\cdots$
6	$\longleftrightarrow$	$f(6)$	$=$	$.a_{61}$	a_{62}	a_{63}	a_{64}	a_{65}	$\mathbf{a_{66}}$	$\cdots$
$\vdots$		$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

Now define a real number $x \in (0, 1)$ with the decimal expansion, $x = .b_1b_2b_3b4 \ldots$ using the diagonal rule: $b_n = \{$ if $0 \leq a_{nn} < 9$, then $b_n = a_{nn} + 1$; if $a_{nn} = 9$, then $b_n = a_{nn} + 1\}$.[165] In other words, we go down the diagonal, $a_{11}, a_{22},\ldots a_{nn}$. And whenever $a_{nn} = 2$, then we

[164] Ibid., 33.

[165] In short, the rule adds 1 to a digit, unless it is 9 then it subtracts 1. For the sake of simplicity, I have selected this formulation of the diagonal rule, because it was frequently employed by Wittgenstein (see, *RFM II*: 9). In Abbott, *Understanding Analysis*, 33, the diagonal rule is formulated as follows: $b_n = \{$if $a_{nn} \neq 2$, then $b_n = 2$; if $a_{nn} = 2$, then $b_n = 3\}$.

make $b_n = 3$; and if $a_{nn} \neq 2$, then $b_n = 2$. Since the diagonal number $x = .b_1b_2b_3b4\ldots$ is different from every $f(m)$, we know that f cannot be a bijective function. Hence, $\boldsymbol{R}$ does not have the same cardinality as $\boldsymbol{N}$. Hence, $\boldsymbol{R}$ is uncountable.

3.4: Wittgenstein on Diagonalization

In turning to Wittgenstein's commentary on the proof throughout *RFM II*, it is crucial to recognize that he did not set out to find a mistake or somehow uncover that the proof was fallacious. As he puts it at *PI* (§125), "it is not the business of philosophy to resolve a contradiction by means of mathematical or logico-mathematical discovery, but to render perspicuous ["surveyable": *übersehbar*] the state of mathematics that troubles us." In other words, his principal aim is to demystify the imprecise discourse and the ideological obscurities that tend to linger around certain mathematical proofs, results, and, especially, the informal interpretations of the two.

For Wittgenstein, a routine practice of clarification involves setting other viable means for defining number systems and functions besides the now standard set-theoretic definitions. This does not amount to an allegation that standard set theory is somehow erroneous, but a perspicuous view of mathematics typically requires that a given subject or domain is viewed from a variety of perspectives. A frequent strategy employed with the intention of clarifying functions and systems of numbers is delineated in terms of the distinction between the extensionalist and non-extensionalist (i.e., intensionalist) points of view. In this regard, it will be useful to quote at length how Juliet Floyd and Felix Mühlhölzer distinguish between the two perspectives:

> With the term "extension" Wittgenstein has two things in mind. First, he will strictly distinguish between sequences of numbers that the extensionalist considers to be, in Cantor's sense, "finished" [fertig] entities or sets – these are the "extensions" – from the techniques or rules by means of which such entities may be produced, assessed, or accessed…
>
> For the non-extensionalist, on the other hand, it is the processes and structured conceptual motifs, the grammar or logic of the notions, we should be concerned with. The most important cases discussed in Wittgenstein's texts are given by the conception of a real number as a rule-governed calculational procedure for which we can see that for any given n it will generate n digits: for example, a recipe for generating more and more successive digits of √2….
>
> The second context in which Wittgenstein speaks of "extensions" is the context of sets, paradigmatically sets of numbers: **N**, **Q**, **R**, and subsets of them. Considered extensionally, the laws or rules or techniques through which we may approach them are taken as irrelevant, as their identity is only determined by the elements of which they consist. From Wittgenstein's non-extensionalist view, however, it is precisely these laws, rules or techniques we should take as primary.[166]

Clearly, the previous definitions of the number systems and functions assume an extensionalist standpoint. Typically, the extensionalist view takes for granted that the relevant mathematical entities are already there, waiting for us to discover them. On the other hand, Wittgenstein wants to persuade us to remember to also look from the non-extensionalist view by seeing how the relevant mathematics can be generated by the application of certain rules, methods, and procedures.

In the focused discussion of the diagonal procedure comprising *RFM II*, there is additional primary textual evidence that confirms an interpretation that juxtaposes the extensional and the non-extensional, rather than simply rejecting the former and accepting the latter:

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[166] Juliet Floyd and Felix Mühlhölzer, *Wittgenstein's Annotations to Hardy's Course of Pure Mathematics: An Investigation of Wittgenstein's Non-Extensionalist Understanding of the Real Numbers* (Cham, Switzerland: Springer, 2020), 31.

> 30. Cantor shows that if we have a system of extensions [*Extensionen*] it makes sense to speak of an extension that is different from them all. – But with that the grammar of the word "extension" is not yet determined.
> 31. Cantor gives a sense [*Sinn*] to the expression [*Ausdruck*] "extension which is different from all the extensions in a system," by proposing that an extension should be so called [*genannt*] when it can be proved that it is diagonally different from the expansions in a system.
> 32. Thus it can be *set* as a question: Find a number whose expansion [*Entwicklung*] is diagonally different from those in this system.[167]

In each of the above remarks, Wittgenstein clearly accepts the validity of what he refers to as extensions as "decimal expansions extensionally conceived," like that of the diagonal number that was presented in the previous section as, $x = .b_1b_2b_3b4\ldots$[168] Therefore, rather than intervening in the actual practices of mathematicians, Wittgenstein is instead concerned with the often informal use of terms, like "extension" and "expansion," that can cast ambiguities around the sheer manipulation of mathematical signs that comprise a calculation or a proof. More succinctly, the following is a good rule of thumb that could characterize Wittgenstein's philosophical approach to mathematics: "The result of a calculation expressed verbally ought to be regarded with suspicion (*RFM II*: §7)." In other words, with regard to mathematics, Wittgenstein asks us to examine what this is, or that is, "called [*gennant*]," how we use this or that "expression [*Ausdruck*]," as well as evaluate whether these expressions have a "sense [*Sinn*]" or are nonsense.

The indeterminate status of terms accompanying a calculation or proof tends to bring about the following series of questions about their use, sense, and meaning:

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[167] I follow the amended translation provided by Floyd and Mühlhölzer, *Wittgenstein's Annotations*, 187.
[168] Ibid., 187n105.

> Or: we can say this and give this as our reason.
>
> But if we do say it – what are we to do next? In what practice is this proposition anchored? It is for the time being a piece of mathematical architecture which *hangs in the air*, and looks as if it were, let us say, an architrave, but not supported by anything and supporting nothing. (*RFM II*, §35, my emphasis)

Accordingly, in the course of working through the diagonal proof, informal terms, such as "extension" and "expansion," can also "hang in the air" like an ostensively defined name for a square in *PI* (§49), or a ruler for those unacquainted with measuring. In the same way that the standard meter is simply a useless stick without any accompanying awareness of the practice of measuring, an extensional class of numbers may also lack an apparent use without the rules, methods, and procedures that are associated with a non-extensional perspective.

It should also be noted that Wittgenstein is careful to say both "not supported by anything" and "supporting nothing," which will indicate two ways of scrutinizing the logico-linguistic status of the words and expressions found in the context of mathematical proofs or calculations. On the one hand, a term might hang in the air, because it has not been adequately prepared or "anchored" with the proper grounding foundations. In this case, the term is "not supported by anything." Yet, on the other hand, a term might hang in the air because it is unclear what to do with it, as well as what else could be derived from it. So, in this case, the term is, instead, "supporting nothing." On the one hand, it will soon be shown that certain conclusions about the uncountability of the real numbers will "hang in the air," precisely because they are "supporting nothing"; whereas, on the other hand, certain *reductio* moves in the proof will instead "hang in the air," precisely because they are "not supported by anything."

Take, for example, how the standard, extensionally-oriented, version of the diagonal proof above puts forth the rule that bids us to progressively calculate the decimal expansion of b_n, along the diagonal: b_n = {if $0 \leq a_{nn} < 9$, then $b_n = a_{nn} + 1$; if $a_{nn} = 9$, then $b_n = a_{nn} + 1$}. Apropos such a rule, Wittgenstein asks us to consider an exchange in which an interlocutor is attempting to teach this stage of the diagonal proof to a pupil: "'I want to teach you a method as to how with an expansion you can sequentially dodge all these expansions ["*allen diesen Entwicklungen nach der Reihe ausweichen*"].' The diagonal procedure is such a method (*RFM II*, §8)."[169] As the consenting response to the interlocutor suggests – "The diagonal is such a method" – Wittgenstein accepts this as a rather unproblematic way of speaking about the rule. In fact, the interlocutor's mention of a step-by-step sequential method is uncontroversially non-extensional, as it is in accord with the once widely-acknowledged constructivist aspects of the diagonal proof.[170] In other words, teaching the diagonal procedure would involve explaining the application of the rule to the first decimal digit of the expansion, then second, then third, and so on: a progressive computation that would look like, $.b_1$, $.b_1b_2$, $.b_1b_2b_3$, $.b_1b_2b_3b_4\ldots$ However, in the three sentences that bring *RFM II* (§8) to a close, subtle suspicions become perceptible in the response to the interlocutor: "'Therefore it produces an expansion that is different from all these.' Is that right? – Yes; if, that is to say, you want to apply these words to the case described above." The problem here, for Wittgenstein, is that the interlocutor's "therefore" does not signify a

[169] Once again, for *RFM II* (§8), I am closely following the amended translation offered in Floyd and Mühlhölzer, *Wittgenstein's Annotations*, 143.

[170] Ibid, 131. Indeed, an authority, such as Abraham Fraenkel, whose name is venerated in the eponymous and thoroughly extensionalist, Zermelo–Fraenkel set theory, also happened to recognize the undeniably constructivist characteristics implicit within the diagonal proof in his "Zum Diagonalverfahren Cantors," *Fundamenta Mathematicae* 25 (1935): 45–50.

hard won inference: the gradual step-by-step computation of a decimal expansion in conformity with the diagonal rule does *not* actually entail the production of the complete diagonal number, $x = .b_1b_2b_3b4\ldots$, i.e., the entirety of "an expansion different from all these." In general, we can summarize the point here with the following diagnosis: "Let us not say – not: 'This method gives us a result,' but rather: 'it gives us an infinite series of results (*RFM II*, §5).'" In fact, the method cannot produce a single result that could encompass the complete and entire decimal expansion of the diagonal number. Therefore, the incredulous and thinly veiled sarcasm detected in the remark is a justified response to the interlocutor's claim that a piecemeal method could somehow engender an infinitely long result. Another appropriate reply would be to remind someone not to hold their breath.

It would be wrong, however, to assume that Wittgenstein's exacting attention to how we actually go about following something like the diagonal rule would further imply an outright finitist or constructivist position. Despite his mocking tone, Wittgenstein is actually assenting to the claim that there is "an expansion that is different from all these" with the acknowledgement, "Yes; if, that is to say, you want to apply these words to the case described above." That is, although he rejects the dubious inference from a finite method to an infinitely long result, Wittgenstein is suggesting that there is a more appropriate way to introduce this decimal expansion that would be consistent with the extensional presentation of the diagonal number as a whole. This was already alluded to in *RFM II* (§31): "Cantor gives a sense to the expression 'extension which is different from all the extensions in a system.'" So, instead of repudiating extensionalism *tout court*, we see that Wittgenstein is actually seeking the proper manner for introducing the

concepts that go along with "apply[ing] these words," such that "extension which is different from all these" has a less vague sense.

Moving two remarks ahead in *RFM II*, Wittgenstein advances a claim that his literary executors would have likely wished to suppress, as its publication has never ceased scandalizing the various commentators who would later review the passage:

> It means nothing to say [*heißt nichts zu sagen*]: "*Therefore* the X numbers are not countable [*nicht abzählbar*]." One might say something like this: I call the number-concept X uncountable [*unabzählbar*] if it has been stipulated [*festgesetzt*] that, whatever numbers falling under this concept you arrange in a series, the diagonal number of this series is also to fall under that concept. (*RFM* II, §10)[171]

However, upon having recovered from the initial shock, I'd like to suggest two perfectly reasonable interpretations of the passage. In both cases, the interpretation turns on a distinction that Mühlhölzer characterizes as "actually the basic motive behind Wittgenstein's philosophy in its entirety": "the difference between what is characteristic of mathematics, on the one hand, and empirical endeavors, on the other."[172] In other words, it will become clear that a convincing explanation of *RFM II* (§10) can be advanced upon by heeding certain distinctions between the empirical, factual, and physical versus the mathematical, conceptual, and logical.

The first interpretation concentrates on the first claim of the remark: "It means nothing to say: '*Therefore* the X numbers are not countable.'" Before we identify the issue at hand, let us first recall how we defined countability: a set or class *A* is called countable if $\mathbf{N} \sim A$. That is, set *A* is countable, just in the case that *A* has the same

[171] Floyd and Mühlhölzer, *Wittgenstein's Annotations*, 152–153. Note, the term "*abzählbar*" is rendered "countable," as opposed to "denumerable" as it is in the Anscombe translation.
[172] Ibid., 159.

cardinality as the natural numbers **N**. In other words, there is a bijective function from the natural numbers **N** to set *A*. Whereas, on the contrary, a class or set is called uncountable when it is simply not countable (*nicht abzählbar*): the set thus has a different cardinality, because there is no bijective function that can map the set to the natural numbers, **N**.

Even though countability and uncountability look like symmetrical opposites, there is an imbalanced difference between the two from a non-extensional perspective. That is, before Cantor's contributions, there had already been significant results regarding the countability of particular kinds of numbers. As Mühlhölzer puts it, "The notion of *countability* has mathematical substance far beyond the set theoretical language… because there are so many cases of countable sets in which the relevant function *f* from **N** to *M* can be given by easily usable algorithms, for example, if *M* is the set of the rationals or of the algebraic numbers."[173] Yet, the same cannot be said of the notion of uncountability: without Cantor's results, we did not have an accessible method to show that a specific set of numbers was uncountable. As a consequence, if we lack Cantor's diagonal proof of the uncountability of the real numbers, then we can only rely on a purely extensional and set-theoretic definition of uncountability as what is strictly not countable. Furthermore, in the absence of any viable methods, techniques, or procedures that could determinately specify a particular uncountable set, the situation suggests that there factually exists an extensionally pre-given set awaiting discovery. As Wittgenstein puts it, "The dangerous, deceptive thing about the formulation… 'The set… is not countable' is that it makes what is the determination of a concept, the formation of a

[173] Ibid., 156.

concept, look like a fact of nature (*RFM II*: §19)."[174] Since there is nothing significant or substantial that can be said about an uncountable set besides maintaining that it would be simply not countable, *RFM II* (§10)'s claim that "*Therefore* the X numbers are not countable" will remain senseless and empty until further specification is put forward.

For the second interpretation, consider how Wittgenstein alternatively proposes a conceptual definition of uncountability based on a stipulation: a particular concept of a kind of number is called "uncountable" if upon arranging these numbers into a countably ordered series the diagonal number of this series is also in accord with this concept. This definition is a somewhat non-extensional explanation of the steps taken in the above diagonal proof. For instance, the *reductio* step that presupposed a bijective function, for each $m \in \mathbf{N}$, $0 \leq f(m) < 1 \in \mathbf{R}$ can be translated into non-extensional terms by assuming a method that would arrange the real numbers into a countably ordered series. We then go about applying the rule, b_n = { if $0 \leq a_{nn} < 9$, then $b_n = a_{nn} + 1$; if $a_{nn} = 9$, then $b_n = a_{nn} + 1$} to a countably ordered series of real numbers and construct a diagonal that itself is a real number.

On the basis of the stipulation, we can conclude that the assertion in *RFM II* (§10) says nothing because the assertion itself amounts to tautology:

> At the same time, however, it makes the "therefore" in "Therefore the X numbers are not countable" totally empty because we now have to say that if it is the case that whatever numbers falling under the concept of X numbers you arrange in a series, the diagonal number of this series is also to fall under that concept, then it is the case that whatever numbers falling under the concept of X numbers you arrange in a series, the diagonal number of this series is also to fall

[174] Ibid., 164.

> under that concept. Wittgenstein has defined "uncountable" in precisely such a way that, trivially, the statement "Therefore the X numbers are not countable" in fact means nothing.[175]

In order to assuage the initial shock of deeming the definition of uncountability as senseless, it should be remembered that the young Wittgenstein of the *Tractatus* had similarly reduced the entirety of mathematics to tautologies: "Mathematical propositions express no thoughts (*TLP*: 6.21)"; "The logic of the world, which the propositions of logic show in tautologies, mathematics shows in equations (*TLP*: 6.22)." Additionally, we should try to erase 150 years of set theory from our memories and think of what it would be like to be initially introduced to this notion of uncountability. Without any prior familiarity with the concept itself, one is likely to wonder what has even been proven by showing that a diagonal can be constructively derived from an arrangement of real numbers listed in series. More specifically, we can conceivably imagine someone unacquainted with set theory being confused about what they are even supposed to do with such a number. Like many of the mathematicians of Cantor's day, for example, Leopold Kronecker himself, we might justifiably ask, "Is the question not really: What can this number be *used* for? True, that sounds strange. – But what it means is: in which mathematical surroundings is it placed (*RFM II*: §3)." Like the first set-theoretic and extensionalist definition of uncountability as not countable, the second tautological definition will also require further non-extensional methods, as well as a more enriched conceptual environment in order for it to acquire significance. And until a use and a context is provided, it is legitimate to ask in both cases: "What can the concept 'uncountable' be used for? (*RFM II*: §12)"

[175] Ibid., 154.

In that case, until we gain some awareness of how the concept of uncountability is to be applied, the situation is not much different from knowing an ostensively defined name without any corresponding ability to construct a proposition with it. That is to say, if we look back to our original problem, then we begin to see how simply making a stipulation about uncountability is basically equivalent to returning to the compositional moment at the level of naming or the language-game of preparation that consisted in the ostensive learning and teaching about the names of squares. What's more, there are strong similarities in some of the steps in the above proof for the uncountability of the real numbers and what occurred in the preparation with ostensive naming: it is not far off to say that we also ostensively named the decimal expansions by assigning each a natural number. Once we have derived a diagonal that cannot be assigned a natural number, we are at a loss as to what to do until we are ready to begin making moves in the language-game of the uncountable real numbers. So, on the one hand, there is a liminal moment where we are not sure what to mathematically do with the notion of uncountability; whereas, on the other hand, we are equally unsure of what to do with the name of a colored square. Likewise, we could also say that the concept of uncountability "hangs in the air" in the same way that the concept of a meter would if we were entirely ignorant of the metric system yet were handed the standard meter.

From this direction, we have encountered the kind of deadlock brought about by a concept, which in this case is uncountability, that is "supporting nothing (*RFM II*: §35)." In line with this metaphor, we might say that the concept of uncountability supports nothing because we don't know how to use it to derive additional theorems, or, for that matter, employ it in any other way that might further extend mathematics. However,

from the other, converse direction, it will now become apparent that the concept of uncountability is "not supported by anything" either (§35). Indeed, at *RFM II* (§16), Wittgenstein will proceed to pull the rug out from under us:

> The mistake begins when one says that the cardinal numbers can be ordered in a series. For what concept has one of this ordering? One has of course a concept of an infinite series, but here that gives us at most a vague idea, a lodestar for the formation of a concept. For the concept itself is *abstracted* from this and from some other series; or: the expression denotes a certain analogy between cases, and it can e.g. be used to delimit for the time being a domain that one wants to talk about.

Let us begin with what he calls the "cardinal numbers," which we would today call the natural numbers, **N**: 1, 2, 3, 4… Recall that the above extensional definition of countability meant that a class has the same cardinality as the natural numbers, such that we called a class or set *A* countable if $\mathbf{N} \sim A$. From a non-extensional standpoint, Wittgenstein seems to be in pursuit of what it is specifically about the natural numbers that is getting predicated of the other sets that we call countable. If read closely, we see him focusing on the fact that we already have a clear idea of what it means to order the natural numbers in a series, as that is perhaps the most basic notion of countability. In the first book of *RFM I* (§§1–4), Wittgenstein approaches this concept through the lens of instruction, learning, and institution by surveying what is involved in mastering the technique of counting with the natural numbers.[176] As will be discussed later, we could think of counting with the natural numbers as a paradigm from which we can try to understand what it means to order other number systems, like the integers, the rational numbers, the even numbers, or the algebraic numbers, into their own countable series. In

[176] Of course, the canonical discussion of this will eventually be that which begins at *PI* (§143ff).

other words, we could say that the non-extensional technique of counting with the natural numbers, 1, 2, 3, 4…, is what precedes our extensional notion of a bijection with the set of the naturals. In short, this basic activity with the natural (i.e., cardinal) numbers gives us the concept of what it is to produce a countably ordered series. That is, the natural numbers and other countable systems of numbers constitute what Wittgenstein describes as a delimited domain where we have some notion of how to order a particular kind of number into a series.

There are issues, however, when we try to extend the paradigm of counting with the natural numbers to analogically apply the concept of a countably ordered series beyond the previously mentioned systems to those of the real numbers:

> That, however, is not to say that the question "Can the set **R** be ordered in a series?" has a clear sense. For this question means e.g.: Can one do something with these formations that corresponds to the ordering of the cardinal numbers in a series? Asked: "Can the real numbers be ordered in a series?," the conscientious answer might be: "For the time being I can't form any precise idea of that". – "But you can order the roots and the algebraic numbers for example in a series; so you surely understand the expression!" – To put it better, I *have got* certain analogous formations, which I call by the common name "series." But so far I haven't any bridge from these cases to that of "all real numbers." Nor have I any general method of trying whether such-and-such a set "can be ordered in a series." (*RFM II*: §16)

In trying to apply the notion of a countably ordered series to the real numbers, we end up going beyond the delimited domain where the notion had once been confidently employed. Although we know what a countably ordered series for the naturals and the even numbers is, it is still unclear how this might apply to the reals: "The procedure exhibits something – which can in a very vague way be called the demonstration that *these* methods of calculation cannot be ordered in a series. And here the meaning of '*these*' is just kept vague (*RFM II*: §14)." For this reason, there is not even a yes or no

answer to the question of whether the real numbers can be feasibly ordered into a series, because we cannot comprehend what either option would be like. Said in morphological terms, we simply don't yet have a procedure for analogically comparing and contrasting a formal attribute of natural numbers with that of the real numbers.

RFM II (§16) is indeed an exceptionally long remark, but it is finally brought to a close with another exceptionally radical claim:

> Now I am shown the diagonal procedure and told: "Now here you have the proof that this ordering can't be done here." But I can reply: "I don't know – to repeat – what it is that can't be done here. Though I see you want to show a difference between the use of 'root', 'algebraic number,' etc., on the one hand, and 'real number' on the other. Let's say as follows: we call the roots 'real numbers' and so too the diagonal number constructed out of them. And likewise for all series of real numbers. For this reason it makes no sense to talk about a 'series of all real numbers', just because the diagonal number of the series is also called a 'real number'". – Would this not be somewhat similar to the case if one ordinarily called any row of books a book itself and now said: "It makes no sense to speak of 'the row of all books', since this row would itself be a book".

That is, in a similar way to how Wittgenstein previously asserted that it was nonsensical to claim that the real numbers were uncountable, he likewise claims, from the other direction, that speaking about the ordering of real numbers in a countable series is also nonsensical. The beginning of the above passage can be understood as reiterating that we still do not have a clear idea of both the question and the answer regarding whether or not the real numbers can be ordered into a countable series. In addition, we lack a precise conception of both the affirmative and the negative responses as well. This supports Wittgenstein's claim that it turns out to be nonsensical whether it is asserted that the real

numbers are countable or uncountable.[177] Consequently, even upon being shown the diagonal proof that purportedly demonstrates that the real numbers cannot be countably ordered into a series, the above reply is meant to convey that everything still remains obscure.

The real crux of the matter comes to the fore when reading the above passage through the lens of the step-by-step unfolding of the diagonal proof. Recall that in the extensional diagonal proof above, we tried to assign a natural number to coincide with each real number in the interval (0,1).[178] In other words, we assumed, for *reductio*, that each $m \in \mathbf{N}$, $f(m) = .a_{m1}a_{m2}a_{m3}a_{m4}a_{m5}....$ is a real number between 0 and 1; or, in short, there is a bijection $f : \mathbf{N} \rightarrow (0, 1)$. The non-extensional parallel of this would be to arrange the real numbers between 0 and 1 into a countably ordered series. Recall, further, that according to the non-extensional stipulation, if we can arrange the real numbers into a series and the diagonal of this series is also a real number, then the real numbers are, by definition, uncountable. This application of the diagonal procedure would therefore demonstrate that: "The diagonal number of a series of real numbers is a real number as well; therefore the real numbers cannot be ordered in a series."[179] In other words, the very success of the proof that the real numbers cannot be ordered into a countable series invalidates its own key premise that presupposes that the real numbers had been arranged into a countably ordered series: "according to this concept [i.e., the stipulated non-extensional definition of uncountability] it in fact *makes no sense*, as Wittgenstein now

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[177] This case is undoubtedly reminiscent of *TLP* (4.1241): "It is impossible to distinguish forms from one another by saying that one has this property and another that property: for this presupposes that it makes sense to assert either property to either form."

[178] The expansions could be derived with the rational numbers, instead of presupposing the real numbers, but this would take us too far off course from the present inquiry.

[179] Floyd and Mühlhölzer, *Wittgenstein's Annotations to Hardy's* Course on Pure Mathematics, 173.

says in §16, to talk about a 'series of all real numbers' because it is ruled out by this concept from the outset."[180] Whereas, from the extensionalist perspective, there is no challenge to the sense of the propositions and intelligibility of the concepts because the classes of natural and real numbers are always-already pre-established, so that the countability of the former and the uncountability of the latter follow from their different cardinalities. However, none of this is available to a non-extensionalist perspective because its step-by-step constructivist approach to the proof entails that the real numbers within the (0,1) will have to be arranged in a countably ordered series so as to further develop the diagonal number. We therefore encounter incoherence, not only because we have no rule, method, or procedure for arranging the real numbers into a countably ordered series, but the conclusion itself attempts to demonstrate that this cannot actually be done. In other words, the rule implicit in the premise, which bids us to arrange the real numbers into a countably ordered series, is utterly unintelligible from a non-extensionalist point of view. Consequently, it follows that any proposition about the "series of all [countably ordered] real numbers" would be nonsensical.

The *prima facie* implausibility of Wittgenstein's claim that the premise about countably ordering the real numbers is nonsensical is largely due to his rather idiosyncratic outlook on *reductio* arguments. As Floyd has noted, Wittgenstein's analysis in the *Tractatus* (*TLP*: 4.461ff, 6.2ff), which respectively reduces mathematical truth and falsity into logical tautologies and contradictions, can be regarded as an early instance of his persistent approach to the kind of mathematical absurdity construed in

[180] Ibid.

reductio ad absurdum arguments.[181] That is, both in Wittgenstein's early and late phases, falsity in mathematics encompasses an incoherence and unintelligibility that indicates a marked difference from false empirical statements, which nonetheless remain both conceivable and understandable. This is kind of intelligibility and lack thereof is at the forefront of Wittgenstein's explicit pronouncement on *reductio* arguments in *RFM V*:

> We can always imagine proof by *reductio ad absurdum* used in argument with someone who puts forward a non-mathematical assertion (e.g. that he has seen a checkmate with such-and-such pieces) which can be mathematically refuted.
>
> The difficulty which is felt in connection with *reductio ad absurdum* in mathematics is this: what goes on in this proof? Something mathematically absurd, and hence unmathematical? How – one would like to ask – can one so much as assume the mathematically absurd at all? That I can assume what is physically false and reduce it *ad absurdum* gives me no difficulty. But how to think the – so to speak—unthinkable?
>
> What an indirect proof says, however, is: "If you want *this* then you cannot assume *that*: for only the opposite of what you do not want to abandon would be combinable with *that*." (§28)

So, while there is no major difficulty in thinking physical absurdity in terms of false empirical propositions, mathematical absurdity is far more incoherent because it is not just false, but somehow defies thought as something un-mathematical or non-mathematical, and thereby illogical. This also recalls another recurring theme dating back to the *Tractatus*: "We cannot think anything illogical, for otherwise we should have to think illogically (*TLP*: 3.03)." Consequently, if we want *this* conclusion, which asserts that the real numbers cannot be ordered into a countable series, then you cannot any longer assume *that* illogical premise, which asserts that the real numbers can be ordered in such a way. In other words, upon accepting the conclusion, the premise is deemed

[181] Juliet Floyd, "On Saying What You Really Want to Say: Wittgenstein, Gödel and the Trisection of the Angle," in *From Dedekind to Gödel: The Foundations of Mathematics in the Early Twentieth Century*, ed. Jaakko Hintikka, vol. 251, Synthese Library (Kluwer Academic Publishers, 1995), 382–386.

unthinkable; or better, the signs that together constitute the supposed premise are nonsense.

The realization that the conclusion of the diagonal proof invalidates its own presuppositions mirrors the language-game of *PI* (§48)'s lack of success in naming the simple squares. We have already noted the striking resemblance between assigning each decimal expansion to a corresponding natural number and the ostensive naming in preparation for the language-game of §48. Recall further that what was meant to be achieved on the compositional level of naming had failed: "Naming is not yet a move"; "*nothing* yet has been done"; "*nor* has it a name except in a game (*PI*: §49)." For this reason, commentators that favor a compositional reading tend to reject this particular failure, and, accordingly, complain that its lack of success would entail a dismissal of *reductio* arguments.[182] However, roughly the same happenstance occurs in the diagonal proof's failure to *name* each real number with a natural number. That is, as already mentioned, the assigning of a natural number to each decimal expansion of a real number between (0,1) also failed; and "For this reason it makes no sense to talk about a 'series of all real numbers' (*RFM II*: §16)." Thus, we can conclude that, for Wittgenstein, the crux of a *reductio* argument is the impossibility of carrying out such a task that mirrors the failure to name each element.

Likewise, the conclusion that comes about by introducing the concept of the uncountable real numbers mirrors the advance onto the contextual level of active description with propositions in the language-game of §48. For example, it is possible to observe that the shift from the compositional to the contextual resembles the shift from

[182] For a version of this objection, see Glock, "All Kinds of Nonsense," 222.

the discreteness of the countable natural numbers to the kind of continuity associated with the real numbers. To some extent, this seems to be what Wittgenstein has in mind the following remark:

> One might say: Besides the rational points there are *diverse systems* of irrational points on the number line.
>
> There is no system of the real numbers – but also no super-system, no "set of the real numbers" of an infinite or higher order. (*RFM II*: §33)

For various mathematical and philosophical reasons, Wittgenstein is at pains here to convey how the irrational numbers can be different from one another as well as from the rational numbers. In this regard, Mühlhölzer is helpful in extracting the more important insights in *RFM II* (§33) from their somewhat misleading formulations:

> Seen in this light, perhaps Wittgenstein should have written that on the number line, besides the rational points given in this elementary way, there are diverse systems of irrational points *and of rational points*. The decisive issue concerns the diversity of systems of *real* numbers, i.e., of irrational and rational ones.
>
> It may [also] seem puzzling that Wittgenstein talks here about the number *line* and *points* on this line, whereas normally our use of the picture of this line is meant as representing the *homogeneity* of the continuum considered as a complete totality, which Wittgenstein is just disputing in §33… However, I regard Wittgenstien's claim that *there is no* super-system of the real numbers, no "set of the real numbers" of an infinity of a higher order, not only as somewhat strange but also as too coarse. It is not only too dogmatic but also smacks of "ontology," which is far removed from Wittgenstein's intent in his later philosophy.[183]

In other words, despite certain problematic issues with *RFM II* (§33), the emphasis should be on how the real numbers are comprised of diverse and occasionally heterogeneous systems of rational and irrational numbers. Furthermore, much like how

[183] Floyd and Mühlhölzer, *Wittgenstein's Annotations to Hardy's* Course on Pure Mathematics, 189.

the movement from one level to the next, from preparation to active description, entails a qualitative conceptual change from a delimited set of names to a vast diversity of propositions, Wittgenstein also seems to be indicating a qualitative conceptual change from countable numbers, like natural and rational numbers, to a vast diversity of systems of uncountable numbers, like the different kinds of irrationals. The likeness between the linguistic and the mathematical examples gains further corroboration by rendering the latter in geometric terms: the rational points are like discrete names; and the irrationals comprise a variety of continuous segments in a manner similar to how the contextual propositions were ultimately irreducible to distinct words.

In suggesting that the premise and the conclusion, which respectively held that the uncountability or the countability of the real numbers were both nonsensical, it may seem as if Wittgenstein does actually reject Cantor's diagonal proof. Yet, on the contrary, after *RFM II* (§16), Wittgenstein repeatedly endorses various verbal articulations of the conclusion of Cantor's demonstration. Wittgenstein's approval, however, is conditional upon the specific ways in which this conclusion ought to be expressed and interpreted. For example, at *RFM II* (§20), Wittgenstein reiterates the conclusion but emphasizes its thoughtful articulation:

> The following proposition sounds humble [*bescheiden*: "modest," "sober"]: "If one calls [*nennt*] something a series of real numbers, the expansion given by the diagonal procedure is also called [*heißt*] a 'real number,' and one in fact says [*sagt*] that it is different [*verschieden*] from all the members of the series."[184]

[184] Translation slightly modified.

Wittgenstein's use of "call [*nennt*]," "calling [*heißt*]," and "says [*sagt*]" bespeak his more cautious presentation of Cantor's results. This focus on what is said throughout the proof, such as when "one calls [*nennt*] something a series of [countably ordered] real numbers," avoids the problems associated with extensionally positing the existence of a series of reals, since such a formulation will be eventually excluded as unintelligible and nonsensical. The same measured shrewdness is exhibited in the conclusion by articulating it with regard to what we say (*sagen*) about a diagonal number that is also a real number.

Furthermore, what is verbally marked as "different [*verschieden*]" about this real diagonal number is meant to indicate how a new determination of sense coincides with the introduction of the new concept of the uncountable real numbers. Indeed, this conceptual difference is rehearsed, over and over again:

> If it were said: "The reflection about the diagonal method shows you that the *concept* 'real number' has much less analogy with the concept 'cardinal number' [i.e., natural number] than we, seduced by certain analogies, are inclined to believe," that would have a good and honest sense. But just the *opposite* happens [i.e., in extensional interpretation]: in that the "set" of all real numbers is allegedly compared in magnitude with the set of cardinal numbers [i.e., natural numbers]. The difference in kind between the two conceptions is represented, by a skewed form of expression, as difference of extension. I believe and hope that a future generation will laugh at this hocus pocus. (*RFM II*: §23)
>
> Cantor's diagonal procedure does not show us a real number different from all in the system, but it gives sense to the mathematical proposition that the number so-and-so is different from all those of the system. Cantor could say: you can prove that a number is different from all the numbers in the system by proving that it differs in the first place from the first number, in its second place from the second number, and so on.
>
> Cantor is saying something about the multiplicity [*Multiplizität*] of the concept "real number different from all the ones of a system". (*RFM II*: §29)

> For the point of a new technique of calculation [e.g., the diagonal procedure] is to supply us with a *new* picture, *a new form of expression*; and there is nothing so absurd as to try and describe this new schema, this new kind of scaffolding, by means of the old expression. (*RFM II*: §46)

As the above remarks indicate, *RFM II* can be convincingly read as nothing other than a sequence of reiterations that describe how Cantor's diagonal proof involves the production and the formation of the concept of the real numbers as a system that is qualitatively different in kind from the other countable number systems. As Wittgenstein more succinctly put it, in the initial stages of his philosophical return, "The set is of a different kind. It isn't 'no longer' countable, it's simply not countable!"[185] Therefore, 1937–1938's *RFM II* can be read as simply filling out the details of this conceptual distinction.

3.5: Paradox and Concept

It is worthwhile to reflect upon how *RFM II* (§16) concludes with a rather puzzling statement that compares a series of real numbers with a row of books:

> For this reason it makes no sense to talk about a "series of all real numbers", just because the diagonal number of the series is also called a "real number". – Would this not be somewhat similar to the case if one ordinarily called any row of books a book itself and now said: "It makes no sense to speak of 'the row of all books,' since this row would itself be a book."

Put simply, if the initial series of real numbers, taken in the *reductio* step, is likened to a row of books, then the diagonal number can be likened to defining the row of books itself

[185] *PR* (§174). Once again, I have altered the translation slightly by translating "*abzählbar*" as "countable" rather that "denumberable."

as a book. Thus, it presumably ought to follow that much like how the diagonal numbers challenged the very coherence of positing a countably ordered series of *all* the real numbers, defining the row itself as a book should demonstrate a similar incoherence in the attempt to contemplate an arrangement of *all* the books in the world as an ordered row. Unfortunately, in the context of *RFM II*, Wittgenstein does not, however, tell us anything else about what we should deduce from the association he has drawn between a series of real numbers and the row of all books.

For this reason, it will be helpful to consider another example of an analogy with the diagonal method recorded in MS-162a, in which books have been replaced by boxes:

> "You cannot put [*legen*] all boxes of the world in a box." Why? Because they are too many?
> – I will prove to you that there are an infinite number of boxes; because no box, no matter how big you make it, can contain [*enthalten*] / house [*beherbergen*] all boxes (MS 162a, p. 101).

It should follow that, since the box containing all the boxes in the world is itself a box, the box containing all the boxes ought to be ruled out much like the row of all books was. However, there is a slight difference here: if the row of books was likened to a series of real numbers, then, as is insinuated by the above use of "put [*legen*]," "contain [*enthalten*]," and "house [*beherbergen*]," the boxes can instead be likened to classes that include other classes as members or subsets. Indeed, as it was with the classes of countable and uncountable numbers, Wittgenstein's mention of the amount and the size of the boxes is meant to indicate the absurdity of a purely extensional interpretation of what he takes to also be a conceptual dilemma.

The example of the row of books and the collection of boxes parallels yet another remark that has already been encountered in a previous section (1.3):

> What *sort* of proposition is: "The class of lions is not a lion, but the class of classes is a class"? How is it verified? How could it be *used*? – So far as I can see, only as a grammatical proposition. To draw someone's attention to the fact that the word "lion" is used in a fundamentally different way from the name of a lion; whereas the class word "class" is used like the designation [*Bezeichnung*] of one of the classes, say the class *lion*. (*RFM VII*: §36).

In some sense, we have come full circle from the series of all real numbers, to the amusing examples of the row of all books and the box containing all boxes, to return back to the more serious case of the class of all classes. Yet, what is also striking about the similarity, specifically between the case of the boxes and that of the classes, is that they both tend to imply a more obvious kind of set-theoretic paradox: the box containing all boxes and classes containing all classes seem to both be inherently self-undermining in a far more comparable fashion.

Nevertheless, every case that has been examined throughout this section has involved some degree of reflexivity. For example, the diagonal number is itself a real number derived from the series of real numbers, and the class containing all classes should itself be one of those classes. Wittgenstein is, of course, also signaling the topic of reflexivity by addressing his mature notion of grammatical propositions: a proposition "that expresses a rule for the use of words."[186] In fact, the focus on self-reference is explicitly corroborated in the next paragraph of the remark:

> One may say that the word "class" is used reflexively, even if for instance one accepts Russell's theory of types. For it is used reflexively there too. (*RFM VII*: §36)

[186] Hacker, *Insight and Illusion*, 213.

As was mentioned in the first chapter, after relaxing the Tractarian prohibitions against reflexivity, Wittgenstein would replace the strictly showable rules of logical syntax with grammatical propositions that reflexively speak about propositions themselves. Whereas the logico-syntactic rules in the *Tractatus* were deemed senseless insofar as they are "self-evident," "follow of themselves," or "go without saying [*sich von selbst verstehen*] (*TLP*: 3.334)," his mature approval of sayable rules is indicated by his willingness to admit that a grammatical proposition could be ascribed a sense:

> Even though "the class of lions is not a lion" seems like nonsense [*Unsinn*], to which one can only ascribe a sense [*Sinn*] out of politeness; still I do not want to take it like that, but as a proper proposition [*Satz*], if only it is taken right. (And so not as in the *Tractatus*.) Thus my conception is a different one here. Now this means that I am saying: there is a language-game with this proposition too. (*RFM VII*: §36, translation slightly altered)

That is to say, there are contexts, like, for example, language-games of preparation and instruction, in which logico-grammatical propositions can significantly express syntactic rules of use for words like "lion" and "class."

In step with surrendering a comprehensive injunction against reflexivity, Wittgenstein's shift in his conception about syntactic rules of logic and grammar also involves loosening the severe restrictions that had once been imposed on signs and symbols in the *Tractatus*, such as their maximum specificity with respect to form and their univocity.[187] Indeed, there are passages in *RFM VII* (§36) that should be read as direct responses to the discussion in *TLP* (3.32–3.334) on rules:

[187] Livingston, *Politics of Logic*, 141–146; Zalabardo, *Representation and Reality in Wittgenstein's* Tractatus, 156.

> But suppose that one gives a particular lion (the king of lions) the name "Lion"? Now you will say: But it is clear that in the sentence "Lion is a lion" the word "lion" is being used in two different ways. (*Tractatus Logico-Philosophicus*.) But can't I count them as *one* kind of use?

That is, instead of the two instances of "lion" in the proposition, "Lion is a lion," being comprised of two different symbols, a proper name and a common noun, as it would have been in *TLP* (3.323), the late Wittgenstein can now accept that there might be language-games in which each instance of "lion" is somehow used as the same symbol.

Furthermore, as was mentioned in the first chapter, the implicit prohibitions against reflexivity and symbolic plurality in the *Tractatus* were implemented so as to avoid certain paradoxical and contradictory constructions. However, the mature Wittgenstein's reconsideration of his previous position is made evident in *RFM VII* (§36), as he comes to accept the paradoxes that follow from acknowledging significant cases of linguistic reflexivity. In fact, there are roughly two paradoxes that can be gleaned from the remark.

As for the first, let us consider Wittgenstein's explicit endorsement of a paradox at the very end of the remark, which is then immediately reaffirmed with another:

> Imagine a language in which the class of lions is called "the lion of all lions," the class of trees "the tree of all trees," etc. – Because people imagine all lions as forming *one* big lion. (We say: "God created man [*Gott hat den Menschen geschaffen*].")
>
> Then it would be possible to set up the paradox that there isn't a definite number of all lions. And so on… (*RFM VII*: §36)
>
> We might ask: What role can a proposition like "I always lie" have in human life? And here we can imagine a variety of things. (*RFM VII*: §37)

Once again, like the boxes and the classes, there seems to be something paradoxical in the formulations about the lions, the trees, and the relation between God and man. For

example, it appears to be self-undermining for the big lion to both contain all lions and be itself a lion. In a remarkably astute interpretation of this remark, Mácha provides some insight into what Wittgenstein has in mind with regard to this group of paradoxes:

> This passage, which for some reason has received little attention from commentators, discusses language-games involving self-membered classes. In "God created man" (in German with the definite article: "*den* Menschen," accusative singular), "man" refers to all men and at the same time to one particular man, Adam. The quotation from *Genesis* 1:27 continues: "In his own image, in the image of God (he) created him." Both the first man and man, the class of men, are created as an image of God. The first man is himself an image of man; he is, so to speak, an archetype of man – a paradigmatic sample. In the same way, the "lion of all lions" can refer to the paradigm of a lion or the "song of songs" to a paradigmatic song.[188]

Therefore, what Mácha helps us see is that examples, from the real numbers to Genesis, are instances of paradigmatic samples, like the standard meter. Also, in morphological terms, these paradigms can be referred to as "archetypes" or "*Urtypen*."

What's more, for Wittgenstein, these paradigms are obviously connected with the determination of a new sense and the formation of concepts, since he deems them structurally similar to how diagonalization brought about the concept of the uncountable real numbers. In addition, the biblical verse about how "God created man" confers a more literal emphasis on the theme of conceptual production. Yet, the less mystified version of this would be the institution of the practice of measuring with the standard meter.

This interpretation of *RFM VII* (§36) also addresses some lingering questions that have been carried over from the first chapter. For instance, it was demonstrated in the first chapter how the *Tractatus* encompasses a tension, between universality and

[188] Mácha, *The Philosophy of Exemplarity*, 49.

particularity, that manifests itself in a variety of ways and through which the standard and resolute interpretations oppose one another. In this chapter, we examined the opposition between compositionality and contextuality as a specific manifestation of this tension. However, what this account of a paradigm reveals is how the implicit tension between universality and particularity is taken up explicitly within the paradigm itself. With regards to an example put forward in *RFM VII* (§36), Mácha explains how a paradigm stands at the confluence of universality and particularity.

> [E]xamples [i.e., paradigms] are not mere instances… Every example modifies the particular, the concrete universality. By exemplifying a particular feature, an example involves every member of its species (i.e., every member of a set or collection)... Wittgenstein's point is… that it is not childish to say that the particular is similar to the general.[189]

So, for example, the lion of all lions is both one particular lion and also universally refers to all lions, Since Mácha's acknowledge his accordance with Livingston's account of a paradigm, it is not surprising that the latter characterizes the notion in a similar fashion:

> Owing to this role, for instance, the standard meter stick must be treated, ambiguously, as both *one* object among others (it is this that makes it usable as an object of comparison at all) and, at the same time, as occupying the elevated and exceptional position of the general, what in being comparable to *any* other sets the terms by which any other individual can be judged. It is this paradoxical position—as we might say, not the position of the particular (the meterstick itself) or the universal, but rather the position of their crossing…[190]

From this view, and given the easing of the restrictions on logical syntax, the conjuncture of universality and particularity also suggests that the paradigm traverses the Fregean distinction between concept and object or a first-order function and its arguments. In

[189] Ibid., 49.
[190] Livingston, *Politics of Logic*, 140.

other words, the standard meter embodies the concept of a meter in the very object of a ruler itself.

As the earlier Tractarian tension between universality and particularity eventually comes to the fore, there is a similar occurrence with the corresponding tension between necessity and contingency. That is to say, when the mature Wittgenstein reviews his previous outlook on the logical requirement for simple names and objects through the lens of the language-game of §48 and the standard meter, there is a combination of necessity and contingency embodied within each:

> We can put it like this: This sample is an instrument of the language [*Instrument der Sprache*], by means of which we make color statements. In this game, it is not something that is presented [*Dargestelltes*], but is a means of presentation [*Mittel der Darstellung*]. – And the same applies to an element in language-game (48) when we give it a name by uttering the word "R" – in so doing we have given that object a role in our language-game; it is now a *means* of presentation. And to say "If it did not exist, it could have no name" is to say as much and as little as: if this thing did not exist, we could not use it in our language-game. – What looks as if it *had* to exist is part of the language. It is a paradigm [*Paradigma*] in our game; something with which comparisons are made. And this may be an important observation; but it is none the less an observation about our language-game – our mode of presentation. (*PI*: §50)

In other words, the simple objects, as paradigms, are necessary insofar as they are required for the contingent employment of language. Once again, Livingston's account of the standard meter proves instructive in bringing out how a paradigm encompasses both sides of this classical philosophical opposition within itself:

> This structurally necessary place of paradox, it is important to note, can by no means be dissipated or resolved simply by drawing a distinction between perspectives "internal" and "external" to our language-games or practices. For in fact the singular place of the standard appears from neither of these two perspectives; to take it as either one is to submit it to the logic of the ordinary run of objects which it in fact underlies. From *outside* the practice, the standard is simply another

> particular, undistinguished and essentially undifferentiated from any other. From *inside*, the standard does not exist as an object at all; it is useful only as a *contingent* means of reference to the law of generality which, clothed with the mystica laura of *necessity*, must always already have been in place. Neither perspective captures… the rational force of any standard in its actual application to cases… by means of which the standard is, simultaneously, both *particular* and *universa*l, both irreducible fact and the normative basis of general law.[191]

For similar reasons, Wittgenstein will often describe these aspects of language as both arbitrary and non-arbitrary (*PI*: §372, 492–497).

As the primary text itself indicates, Wittgenstein considers these examples of paradigms, like the lion of all lions, and the archetypical human, to be closely related to paradox. Indeed, this is made incontrovertibly clear in the above quoted passage from *RFM VII* (§36): "Then it would be possible to set up the *paradox* that there isn't a definite number of all lions."[192] On top of all this, Wittgenstein alludes to another biblical verse in putting forward a version of the Epimenides paradox in the next remark: "What role can a proposition like 'I always lie' have in human life? And here we can imagine a variety of things (*RFM VII*: §37)."[193] Nevertheless, careful scrutiny is still demanded, as it will turn out to be of pivotal consequence what specific kind of paradox is being assessed and what is its precise relevance to the issues at hand. This brings us to the second paradox in *RFM VII* (§36): "the class of [all] classes is a class."

Clearly, each of the formulations that has been looked over throughout this section violates Russell's vicious-circle principle, the canonical version of which is set forth in the *Principia Mathematica*:

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[191] Livingston, "Wittgenstein and Parmenides," 21, some emphasis my own.
[192] My emphasis.
[193] See also, Russell, "Mathematical Logic as Based on the Theory of Types," 222.

> The principle which enables us to avoid illegitimate totalities may be stated as follows: "Whatever involves all of a collection must not be one of the collection"; or, conversely: "If, provided a certain collection had a total, it would have members only definable in terms of that total, then the said collection has no total." We shall call this the "vicious-circle principle," because it enables us to avoid the vicious circles involved in the assumption of illegitimate totalities.[194]

So, for example, a formulation like "the class of all classes is a class" is certainly an illegitimate totality, as this collection of classes has itself as a member. Yet, despite how the principle is often discussed, the authors of the *Principia* tend to speak about it as a means for avoiding paradoxes and are, accordingly, rather measured about how it specifically relates to contradictions: "Such arguments [i.e., "arguments which are condemned by the vicious-circle principle"], *in certain circumstances, may* lead to contradictions…"[195] There is perhaps, then, a need to abide by a more fine-grained distinction between a wide array of paradoxes and contradictions in particular.

Observe, for example, that the expression "the class of [all] classes is a class" is subtly different from the canonical class version of Russell's paradox:

> Let *w* be the class of all those classes which are *not* members of themselves. Then, whatever class *x* may be, "*x* is a *w*" is equivalent to "*x* is *not* an *x*." Hence, giving to *x* the value *w*, "w is a w" is equivalent to "w is not a w."[196]

As is indicated by the two above instances of "not," the class version of Russell's paradox contains a negation, whereas there is no negation present in the initial formulation of the class of all classes in *RFM VII* (§36). Likewise, the Epimenides paradox also lacks any explicit negation: at best, it could be compared with what

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[194] Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, 3 Vols, Second Edition. (Cambridge: Cambridge University Press, 1962), 37.
[195] Whitehead and Russell, *Principia Mathematica*, 38, my emphasis.
[196] Russell, "Mathematical Logic as Based on the Theory of Types," 222, some emphasis mine.

Aristotle regarded as the privative negation of a term, as opposed to the complete negation of a proposition, such as it would be in "It is *not* the case that I always tell the truth."[197]

Indeed, Russell himself always distinguished between the two formulations about classes, as they marked distinct phases in the progression of his investigation into symbolic mathematical logic:

> I was led to this contradiction by considering Cantor's proof that there is no greatest cardinal number. I thought, in my innocence, that the number of all the things there are in the world must be the greatest possible number, and I applied his proof to this number to see what would happen. This process led me to the consideration of a very peculiar class. Thinking along the lines which had hitherto seemed adequate, it seemed to me that a class sometimes is, and sometimes is not, a member of itself. The class of teaspoons, for example, is not another teaspoon, but the class of things that are not teaspoons, is one of the things that are not teaspoons. There seemed to be instances which are not negative; for example, the class of all classes is a class. The application of Cantor's argument led me to consider the classes that are not members of themselves; and these, it seemed, must form a class. I asked myself whether this class is a member of itself or not. If it is a member of itself, it must possess the defining property of the class, which is to be not a member of itself. If it is not a member of itself, it must not possess the defining property of the class, and therefore must be a member of itself. Thus each alternative leads to its opposite and there is a contradiction.[198]

That is, reflecting on his philosophical evolution, Russell identifies his research into Cantor's theorem, the theorem's relation to the class of all classes, and the formulation of his eponymous and contradictory paradox as distinct steps. Likewise, in Russell's *Principles of Mathematics*, published in 1903, which we know Wittgenstein read closely

[197] *De Interpretatione*, 21b.
[198] Bertrand Russell, *My Philosophical Development*, (Abingdon Oxon: Routledge Classics, 2023), 63.

and attentively, the former actually explains the same progression, yet with greater technical and philosophical detail.[199]

Hence, for example, throughout *RFM II*, Wittgenstein does not ever mention the canonical class version of Russell's paradox, but, instead, Cantor's use of his eponymous theorem to show that there is no largest number is dispersed throughout the series of remarks:

> For how do we make use of the proposition: "There is no greatest cardinal number"? When and on what occasion would it be said? This use is at any rate quite different from that of the mathematical proposition "25 x 25 = 625." (§24)

Consequently, if Wittgenstein had a paradox in mind throughout *RFM II*, it would most likely be what could be derived from positing the class of all classes. Accordingly, it is by no means a coincidence that the closest rendition of the paradox that Wittgenstein is concerned with in *RFM VII* (§36) is what Floyd has elaborated as the "positive version" of Russell's paradox. Floyd describes what makes the formulation distinctly positive as follows:

> [For] the "positive" Russell paradox, that is, the issue of the set of all sets that *are* members of themselves. This is the exact complement, so to speak, of the usual Russell set of all sets that are *not* members of themselves. Think of it as the *positive* Russell set. In a certain sense, S "comes before" Russell's set, is more primordial, for there is no use of negation within its definition. And it is not contradictory.[200]

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[199] Russell, *Principles of Mathematics*, 102, 107, 115, 361, 367. See also, Monk, *Wittgenstein*, 30–33.
[200] Juliet Floyd, "Wittgenstein and Turing," in *Philosophy of Logic and Mathematics: Proceedings of the 41st International Ludwig Wittgenstein Symposium*, ed. Gabriele M. Mras, Paul Weingartner and Bernhard Ritter (Berlin, Boston: De Gruyter, 2020): 285.

What is illuminating about the positive version of Russell's paradox is that it logically mirrors precisely how Russell actually progressed chronologically. Furthermore, in accordance with the definition above, if the class of all classes is, indeed, a class, then it *should* be a member of itself.[201]

Floyd has presented two versions of the argument for the paradox, one with and one without a diagonal, but for our purposes we will focus on the latter:

> Define:
>
> $S = \{x \mid x \in x\}$.
>
> Now ask
>
> Is $S \in S$?
>
> And the answer is:
>
> If Yes, then $S \in S$.
>
> If No, then $S \notin S$.
>
> So we have that:
>
> $S \in S \Longleftrightarrow S \in S$.[202]

In other words, if we try to determine whether the class of all classes contains itself, then we get an empty answer: it is something "if it does, then it does and if it doesn't, then it doesn't." As Floyd explains it:

> For all that we can deduce here is that:
>
> $S \in S \Longleftrightarrow S \in S$, and also $S \notin S \Longleftrightarrow S \notin S$.

[201] By taking up Floyd's positive version of Russell's paradox, I depart from both Livingston and Mácha who tend to focus on the canonical version.

[202] Floyd, "Wittgenstein and Turing," 285. For the other diagonal version of the positive version of Russell's paradox, see Juliet Floyd, "Wittgenstein's Diagonal Argument: A Variation on Cantor and Turing," in *Epistemology versus Ontology: Essays on the Philosophy and Foundations of Mathematics in Honour of Per Martin-Löf*, ed. Peter Dybjer, Sten Lindström, Erik Palmgren, and Göran Sundholm (Dordrecht: Springer, 2012), 39–40.

> We are caught in a kind of circular thought of the form, "it is whatever it is". This is surely not incoherent or inconsistent. The trouble is deeper: the thought cannot be implemented or applied.[203]

In other words, such an attempt to decide whether the class of all classes contains itself leads to a paradoxically circular scenario that does not offer any information towards one choice or the other. However, what is crucial about this positive rendition of the paradox is that it does not engender the outright contradiction that arises from the canonical version of the paradox, because the former lacks the negation that is included in the latter.

What is also of interest is how Floyd observes in the class of all classes or the positive Russell set what we have recognized as an important feature in paradigms:

> An apparently unproblematic way of thinking is applied, but two different ways of thinking about *S* are involved. They are at first blush buried, just as in Russell's usual form of the paradox, but they are there, and they are separable, viz., there is thinking of *S* as an object or element that is a member of other sets, and the thinking of *S* in terms of a concept, or defining condition.[204]

Indeed, there is a striking parallel with paradigms, like the standard meter, which encompasses both the concept of a meter and the object of a ruler.

Furthermore, if the line of reasoning's conclusion, $S \in S \Longleftrightarrow S \in S$, and also $S \notin S \Longleftrightarrow S \notin S$, is thought of terms of a rule, then another suggestive similarity is encountered:

> We have here what might be regarded… as a kind of performative or empty rule. You are told to do something depending upon what the rule tells you to do, but you cannot do anything, because you get into a loop or tautological circle.

[203] Floyd, "Wittgenstein and Turing," 285.
[204] Floyd, "Wittgenstein's Diagonal Argument," 40.

That is, the emptiness of the conclusion resembles how, in Wittgenstein's discussion of the uncountability of the real numbers, he was often brought back to his earlier Tractarian account of mathematics as tautologies. For example, recall that in the *Tractatus* that:

> 6.2322: The identity of the meaning [*Bedeutung*] of two expressions cannot be *asserted*. For in order to assert anything of their meaning I must know their meaning: and by knowing their meaning, I know whether they mean the same or different…
> 6.234: Mathematics is a method of logic.

However, the major difference, for the mature Wittgenstein, is that, like the conclusion about the uncountability of the real numbers or stipulation about the class of all classes, there are language-games in which these putatively tautological claims can be asserted with sense.[205]

To summarize, we have now seen how the tensions in the early *Tractatus* gives way to the overt paradoxes of the later - especially upon taking into account how the composition of *RFM II* is basically contemporaneous with the first drafts of the *Investigations*. Accordingly, we have arrived at the appropriate point to turn our attention to another challenge leveled at symbolic mathematical logic around the same. In particular, it is a good moment to consider Wittgenstein's thoughts on the *Entscheidungsproblem*, as we will also find that they bear a close association with his positive version of Russell's paradox and its relation to rules.

3.6: Wittgenstein on the *Entscheidungsproblem*

[205] *RFM III* (§41)

Wittgenstein's repeated questions into the presumed use of the diagonal proof for the uncountability of the real numbers should not obscure the fact that he was indeed aware of how both Gödel and Turing had employed the diagonal procedure in their respective negative answers to the decision problem.[206] In fact, roughly a decade after the composition of *RFM II*, Wittgenstein will return to these issues by formulating his own version of Turing's renowned argument in "On Computable Numbers, with an Application to the *Entscheidungsproblem*."[207] It begins by translating Turing's eponymous machine into that of a particular sort of language-game:

> Turing's "Machines." These machines are *humans* who calculate. And one might express what he says also in the form of *games*. And the interesting games would be such as brought one *via* certain rules to nonsensical instructions. I am thinking of games like the "racing game". One has received the order "Go on in the same way" when this makes no sense, say because one has got into a circle. For any order makes sense only in certain positions. (Watson) (*RPP I*: §1096)

That is to say, Wittgenstein proposes a language-game that will encapsulate Turing's renowned comparison between "a man in the process of computing a real number to a machine which is only capable of a finite number of conditions."[208] Accordingly, the rules of this game would then correspond to the machine instructions, i.e., its standard description or S.D.[209] As is well-known, it is integral to Turing's proof that each machine is assigned with an integer or description number, D.N: "The integer represented by this

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[206] For a comprehensive study on the application of diagonalization in both Gödel's and Turing's results, see Raymond M. Smullyan, *Diagonalization and Self-Reference*, Oxford Logic Guides 27 (Oxford ; New York: Clarendon Press, 1994).
[207] I rely on the heavily annotated version of the paper presented in Charles Petzold, *The Annotated Turing: A Guided Tour through Alan Turing's Historic Paper on Computability and the Turing Machine* (Indianapolis, IN: Wiley Pub, 2008).
[208] Petzold, *The Annotated Turing*, 68.
[209] Ibid., 138–142.

numeral may be called a description number (D.N) of the machine [and] D.N determine the S.D and the structure of the machine uniquely."[210] Each description number, therefore, represents a rule for the development of a decimal expansion.

What is peculiar about this particular game is that following its rules eventually leads the player to a rule comprised of nonsense: the instruction to "Go on the same way" in the course of a circle, much like what occurred in the positive version of Russell's paradox. Additionally, this happenstance also recalls a well-known passage from the *Investigations*:

> Here the fundamental fact is that we lay down rules, a technique, for playing a game, and that then, when we follow the rules, things don't turn out as we had assumed. So that we are, as it were, entangled in our own rules.
>
> This entanglement in our rules is what we want to understand: that is, to survey [*übersehen*]. (*PI*: §125)

Another way to put it, then, is that the game entangles itself in a circle much like a computer caught within an infinite loop.

In the next remark from *RPP I*, the specifics of what Juliet Floyd refers to as "Wittgenstein's Diagonal Argument"[211] is set forth in detail:

> A variant of Cantor's diagonal proof:
>
> Let $v = \varphi(k,n)$ be the form of the laws for the expansion of decimal fractions. v is the nth decimal place of the kth expansion. The law of the diagonal then is: $v = \varphi(n, n)$ =def. $\varphi'(n)$.
>
> It is to be proven that $\varphi'(n)$ cannot be one of the rules $\varphi(k,n)$. Assume it is the 100th. Then we have the formation rule
>
> of $\varphi'(1)$: [runs] $\varphi(1, 1)$
>
> of $\varphi'(2)$: [runs] $\varphi(2, 2)$

[210] Ibid.

[211] Floyd, "Wittgenstein's Diagonal Argument," 35–39; "Wittgenstein and Turing," 288–292.

etc.,

but the rule for the formation of the 100th place of $\varphi'(n)$ is/becomes [runs] $\varphi(100, 100)$, that is, it tells us only that the 100th place is supposed to be equal to itself, and so for $n = 100$ is *not* a rule. (I have namely always had the feeling that the Cantor proof did two things, while appearing to do only one.)

The rule of the game runs "Do the same as…" – and in the special case it becomes "Do the same as you are doing."[212]

As Floyd has convincingly shown throughout her meticulous studies of Turing's and Wittgenstein's mutual influence upon one another, the above diagonal argument is a variant of the second pass at a diagonal proof in "On Computable Numbers" that seeks to show "there is no machine that will determine whether a particular integer is a Description Number of a circle-free machine."[213]

What Wittgenstein's diagonal proof shows us is that his non-extensionalist interpretation of the proof of the uncountability of the real numbers can be deployed for Turing's negative results. The function, $\varphi(k,n)$, ought to be understood as a non-extensionalist rule, method, or procedure for developing the decimal expansion of a real number. As Floyd instructively explains:

Wittgenstein considers first a list or series of rules – or, as he also say, "laws" – for the expansion of forms of decimal representations of "computable" real numbers

$a_{k1}a_{k2}a_{k3}$ ....

He calls this list $\varphi(k, ...)$. According to his notation, $\varphi(k, n)$ is the nth decimal place determined by the kth rule in the list.[214]

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[212] *RPP I* (§1097). However, I follow the notation from *MS-135* (60r–60v). According to Floyd and Mühlhözer, *RPP I* (§1097) actually has its origins in *MS-135* (60r–60v) and *TS-229* (§1764) in *Wittgenstein's Annotations to Hardy's Course of Pure Mathematics*, 145, 145n34. The remark is repeated almost verbatim in *Z* (§694).

[213] Petzold, *The Annotated Turing*, 178; Floyd, "Wittgenstein's Diagonal Argument, " 36.

[214] Floyd, "Wittgenstein and Turing," 289.

Put in the notation from the above standard rendition of Cantor's diagonal proof (3.3) so as to construct a similar diagram, we would have to take the function $f(k)$ to be a purely non-extensional, i.e., intensional, function, such that for each $k \in \mathbf{N}$: $f(k) = .a_{k1}a_{k2}a_{k3}a_{k4}a_{k5}\ldots a_{kn}$.

Then, Wittgenstein gives the rule for computing the diagonal by moving from the binary function $\varphi(n, n)$ to a unary function $\varphi'(n)$:

> The rule for computing $\varphi'(n)$ is clear: go down the diagonal of this list, picking off the value of $[\varphi(n, n)]$. This rule appears to be perfectly comprehensible and is in *that* sense well defined.[215]
>
> For $n = 1$ it says: calculate the first decimal place provided by the law $\varphi(1, \ldots)$; for $n = 2$: calculate the second decimal place provided by the law $\varphi(2, \ldots)$;…[216]

Once again, each rule or law at least ostensibly appears to be well-defined and comprehensible enough to carry out.

A problem would arise, however, if the diagonal was somehow included amongst the list of the other rules for decimal expansions. That is to say, there would be an issue if we incrementally went down the list of rules or laws, $\varphi(1, \ldots)$, $\varphi(2, \ldots)$, $\varphi(3, \ldots)$…, and then eventually arrived at the diagonal rule itself. Wittgenstein asks us to suppose this occurrence by assuming that the rule for the diagonal was found at k=100, such that $\varphi(100, \ldots)$.

According to Wittgenstein, this would lead to a circular entanglement, "it tells us only that the 100th place is supposed to be equal to itself," that goes back and forth

[215] Floyd, "Wittgenstein's Diagonal Argument," 37. I have altered the quotation in accordance with the difference in notation.
[216] Floyd, "Wittgenstein and Turing," 290.

between $\varphi'(100)$ and $\varphi(100, 100)$. Once again, it will be helpful to learn from Floyd's explanation:

> There will be no trouble at all until we try to say *which* rule on our list, in particular, this instruction [i.e., the rule/law for developing the diagonal] is. Suppose (without loss of generality) that it is the 100th. Then at $n = 100$ we have the following command: calculate the 100th decimal place provided by the law $\varphi(100, ...)$. But we just presupposed that the law $\varphi(100, ...)$ is the *same* as $\varphi'(n)$! Therefore, this instruction, namely "Calculate $\varphi'(100)$ by calculating $\varphi(100, 100)$," is identical with the instruction: "Calculate $\varphi(100, 100)$ by calculating $\varphi(100, 100)$," which is empty. It is not a rule that we can follow as we can the others on the list, and in that sense it is "not a rule," as Wittgenstein says.[217]

Wittgenstein also describes this repetitive loop in terms that are clearly intended to highlight its tautological nature as: "Do the same as you are doing." As Floyd also indicates, this interpretation of the tautological rule as "do what you do" is obviously similar to the previous section's response to the positive version of Russell's paradox as "it is whatever it is."

Like many of the previous cases, there are cogent reasons for interpreting this tautological rule as also conceptually productive:

> Oddly, because it turns on a tautology, its conclusion is "positive": it "constructs" a formulable rule that cannot be literally identified with any of the rule-commands on the list of rules supposed to be given. The diagonal then gives one a positive way of creating something new, i.e., a directive that cannot be sensibly followed.[218]

Although it cannot be followed in that particular context with the given lists of rules for decimal expansions, we should still recall that Wittgenstein does, in fact, claim that "any

[217] Floyd, "Wittgenstein and Turing," 290.
[218] Floyd, "Wittgenstein's Diagonal Argument,"39.

order makes sense only in certain positions (*RPP I*: §1096)." Hence, as Wittgenstein tells us in *RFM VII* (§36), this empty and tautological rule can be "a proper [*rechten*] proposition, if only it is taken right [*richtig*], [as] there is a language-game with this proposition too."

Since the similarities between Wittgenstein's diagonal argument, the positive paradox, the class of all classes, paradigms like the standard meter, and the non-extensional interpretation of the uncountability proof have all been made apparent, it will be edifying to clarify the mechanics behind these analogical comparisons. As Floyd has convincingly argued, Wittgenstein viewed on the negative resolution to *Entscheidungsproblem* as an "object of comparison":

> And Wittgenstein did investigate the sense in which any model of computation such as Turing's could be said to give us a description of how humans (or human brains or all possible computing machines) actually work, when calculating. Turing offers, not a definition of "state of mind," but what Wittgenstein thought of as a "language game," a simplified model or snapshot of a portion of human activity in language, an *object of comparison* forwarded for a specific analytic purpose.[219]

Indeed, this thoroughly morphological term of art was to become central to the mature Wittgenstein's overall philosophical methodology:

> Our clear and simple language-games are not preliminary studies for a future regimentation of language – as it were, first approximations, ignoring friction and air resistance. Rather, the language-games stand there as *objects of comparison* [Vergleichsobjekte] which, through similarities [*Ähnlichkeit*] and dissimilarities [*Unähnlichkeit*], are meant to throw light on features of our language.
>
> For we can avoid unfairness or vacuity in our assertions only by presenting the model [*Vorbild*] as what it is, as an object of comparison [*Vergleichsobjekt*] – as a sort of yardstick [*Maßstab*]; not

[219] Floyd, "Wittgenstein's Diagonal Argument," 26.

> as a preconception to which reality *must* correspond. (The dogmatism into which we fall so easily in doing philosophy.) (*PI*: §§130–1)

As is characteristic of Wittgenstein's later philosophy, these "similarities [*Ähnlichkeit*]" and "dissimilarities [*Unähnlichkeit*] tend to "criss-cross [*kreuz und quer*] in every direction over a wide field of thought. (*PI*: Preface)." This is all the more so, Wittgenstein admits, when it comes to drawing philosophical comparisons: "we see a complicated network of similarities [*Ähnlichkeiten*] overlapping and crossing [*kreuzen*]: similarities in the large and in the small (*PI*: §66)."[220] The above mention of a "yardstick [*Maßstab*]," perhaps confusing, suggests what Plato spoke of in the *Statesman* (277d9), when he proposed that, "the paradigm itself is in need of a paradigm": the standard meter, as a yardstick, is itself an object of comparison that is being compared with other objects of comparison. Given this web of similarities, it will be useful to untangle a few threads in order to get an idea of how crucial such comparisons are for Wittgenstein's mature methodology.

According to Floyd, Wittgenstein used Turing's negative resolution to the *Entscheidungsproblem* in order to draw a comparison that could clarify what it is to take a step in a formal system of logic:

> For in Wittgenstein's mature philosophy there remains a unity and robustness in the logical, responding to the generality and mathematical robustness of Turing's analysis of what it is to take a "step" in a formal system of logic. For this is conceived by Wittgenstein in terms of step-by-step, partially-defined, rule-governed, symbolically articulated procedures and their backdrop in interlocutory exchanges and forms of life. This recovered, *realistic* unity, a kind of norm of elucidation for philosophy – the embedding of language-games in forms of life – is what prevents Wittgenstein's mature idea of logic from hardening into a dogmatically asserted totality of

[220] Translation slightly amended.

> propositions, a static, divided archipelago of conventional schemes, or an artificially ordered series of games.[221]

In other words, what Wittgenstein gleaned from how Turing compared "a man in the process of computing a real number to a machine" in "On Computable Numbers" is a clearer view of the key aspects involved in the procedure of calculation as such.[222] As Floyd argues, Turing's second pass at the diagonal proof in "On Computable Numbers" was decisive because this non-extensional variant meant that his results could be more comprehensively applied in a manner not beholden to the various competing approaches to logic in the first half of the 20th century, from Brouwer's intuitionism, Hilbert's formalism, or the logicism of Wittgenstein's prior teachers. Whatever our logical presuppositions, the elucidation acquaints us with what is to make a move or take a step in a system of logic.

Since Wittgenstein indeed employs variations of the term "*Regel*" throughout *RPP I* (§§1096–7), it should come as no surprise that the formulation of his own version of the diagonal argument could also be put to use as an object of comparison for, what is known in the secondary literature, as the rule-following considerations. Also, in her extended engagement with Wittgenstein's diagonal argument, Floyd convincingly points to the wider significance in the argument's derivation of such a novel and yet tautologically empty rule:

> [T]he idea of a routine everywhere defined from all perspectives is in a sense incomplete. The mechanism of the argument clearly depends upon our ability to *see* that a rule cannot be followed, rather than our getting one another to agree or disagree about the status or scope of the

[221] Floyd, "Wittgenstein and Turing," 269.
[222] Petzold, *The Annotated Turing*, 68.

> law of the excluded middle, or a general point of view on negation or contradictions. In this sense Wittgenstein's diagonal argument draws out something fundamental also to Turing's diagonal argument: that it is fundamental to our very idea of logic – more fundamental, in fact than the idea of any particular logical law holding or not holding – that we have a hold on everyday ways of applying rules, rule-following, and shareable commands. Logic does not need to depend upon community-wide agreement on philosophical theses or conventions about what is to count as a correct logical "law." It is not a question of consensus but of forms of life.[223]

Put simply, Wittgenstein's clarification of a step in a formal system can likewise be employed to elucidate what it is to follow a rule as such. Since Wittgenstein's rendition of Turing's results help clarify that following a rule can be seen as necessary and essential to the employment of any formal calculus or symbolic notation, it therefore also show that what it *is* to follow a rule can have aspects that exceed and escape symbolic rendering within such a system.

Therefore, Wittgnenstein's diagonal argument also develops a conception of a rule that remains incomplete, or not totally well-defined and circumscribed. As Livingston explains, it is precisely the negative or limitive results in response to the *Entscheidungsproblem* insofar as they show how there are certain problems that cannot be solved by a step-by-step, rule-guided algorithm – which draws us nearer, specifically, to the rule-following paradox:

> With respect to computability, the analogue is apparently to take Turing's result itself wholly negatively – that is, as showing that it is not possible, on pain of contradiction (or at least paradox), for our rational procedures to completely model themselves. This suggests that there will be, among these, some infinitary procedures that, although perfectly determinate, are not effectively computable. This by itself does not suffice to guarantee our access to these procedures. But such infinitary techniques, fixtures of human life that are not fixed, in their totality, by any

[223] Floyd, "Wittgenstein and Turing," 291.

> finite symbolism, may be just what Wittgenstein is alluding to when, resolving the rule-following paradox of the *Philosophical Investigations*.[224]

That is to say, the demonstration that there are problems that are unsolvable by a step-by-step and rule-guided algorithmic method does, in fact, share important and insightful resemblance with many of the major themes in the rule-following section of the *PI* (§§185–244). In particular, much like how it is shown that "it is not possible… for our rational procedures to completely model themselves," the aforesaid paradox at the heart of the rule-following considerations exposes the limitations in issuing rules to follow a rule.

Put in another way, a rule itself does not fully dictate exactly how it should be followed, and this can brought out by the way a rule can presumably be open to interpretation:

> But how can a rule teach me what I have to do at *this* point? After all, whatever I do can, on some interpretation, be made compatible with the rule." – No, that's not what one should say. Rather, this: every interpretation hangs in the air together with what it interprets, and cannot give it any support. Interpretations by themselves do not determine meaning. *PI* (§198)

I take this openness to a variety of interpretations to, at least, partially explain Wittgenstein's claim that: "My symbolical expression was really a mythological description of the use of a rule (*PI*: §221)." That is to say, there is only so much that an expression of a rule can instructively elaborate insofar as much of how it is followed is not or cannot be explained.

[224] Livingston, *Politics of Logic*, 164.

The permissive variability in how a rule could be interpreted leads precisely to what is referred to in the literature as the rule-following considerations:

> This was our paradox: no course of action could be determined by a rule, because every course of action can be brought into accord with the rule. The answer was: if every course of action can be brought into accord with the rule, then it can also be brought into conflict with it. And so there would be neither accord nor conflict here.
>
> That there is a misunderstanding here is shown by the mere fact that in this chain of reasoning we place one interpretation behind another, as if each one contented us at least for a moment, until we thought of yet another lying behind it. For what we thereby show is that there is a way of grasping a rule which is not an interpretation, but which, from case to case of application, is exhibited in what we call "following the rule" and "going against it".
>
> That's why there is an inclination to say: every action according to a rule is an interpretation. But one should speak of interpretation only when one expression of a rule is substituted for another. (*PI*: §201)

That is, if a rule was left open to interpretation, then it would be hard to see the point, since everyone would behave however they saw fit. Once again, like what we have inferred from the negative response to the *Entscheidungsproblem*, "it is not possible… for our rational procedures to completely model themselves." Wittgenstein diagnoses a similar problem above in how a rule set for another rule, such that "chain of reasoning [in which] we place one interpretation behind another." Wittgenstein eloquently reiterates the same point: "If that means 'Have I reasons?,' the answer is: my reasons will soon give out (*PI*: §211)"; "Once I have exhausted the justifications, I have reached bedrock, and my spade is turned (*PI*: §217)."

Therefore, rather than a futile regress of reasons, justifications, or interpretations, we determine whether a rule has been followed by observing how it is actually used in its "case to case of application (*PI*: §201)." In other words, accordance with a rule is shown in an "act, without reasons" and in "simply what I do (*PI*: §211, 217)." It is a lived

accordance, which can only be shown rather than said, that Wittgenstein notably referred to as a "form of life": "This is agreement not in opinions, but rather in form of life [*Lebensform*] (*PI*: §241)." Livingston characterizes this notoriously difficult concept: "Showable but not sayable, evident but not describable, forms of life are given, outside the assurance of any structure, in the immanence and heterogeneity of actual cases and the widely varied circumstances of an everyday life."[225] It is my contention that this is precisely where we find the morphological.

3.7: Logical Morphology, Family Resemblance, and Analogy

Frequently, in *RFM II*, Wittgenstein insists that an appropriate understanding of how different sets or systems of numbers relate with one another somehow involves "analogy [*Analogie*] (§§16, 22, 49)," "comparisons [*Vergleiche*] (§§4–5, 7, 22, 47, 49)," "similarities [*Ähnlichkeiten*] (§§16, 38, 50)," and "differences [*Unterschiede*] (§§16, 29–32, 34, 40, 49)." At *RFM II* (§49), Wittgenstein gives the chapter's most explicit endorsement of a certain analogical approach: "How do we compare [*vergleicht*] games? By describing them – by describing one as a variation of another – by describing them and *emphasizing* their differences [*Unterschiede*] and analogies [*Analogien*]." As we have seen in the previous sections, we can compare different number systems – for example, the natural numbers, integers, and the even numbers – by drawing similarities as to what is countably ordered in each of them as a series; but, in some cases, such as that of real numbers, the differences are too great to draw any such meaningful

[225] Livingston, *Politics of Logic*, 16.

connection.

Such practices tend to be unnecessary from the extensionalist perspective, because that viewpoint presupposes that all of the different systems of numbers are always-already there, collected together according to their varying complexity or cardinality. However, for the intensionalist or non-extensionalist perspective, which tends to lack preset collections at their disposal, analogical judgments are indispensable in order to perceive similarities, differences, and interactions between what are qualitatively different conceptual systems. Similarly, the implicit limitations revealed in formal systems of logic also suggests that a method of investigation based on comparison is necessary in order to clarify that which escapes the codified rules and definitions of such symbolic calculi.

As has already been suggested, the mature Wittgenstein's employment analogical reasoning comprises an essential component of his appropriation of the morphological method. In fact, the importance of analogy is indicated in this contemporary definition from a well-known proponent of morphology in the life sciences:

> Morphology is the doctrine of Gestalt..., above all in the biosciences, although disciplines of biochemistry and of the human sciences also use this concept. Morphology contains the methodology of scientific comparison, namely, the distinction between essential similarities (homologies) and accidental similarities (analogies).[226]

Likewise, in what is, to my knowledge, the only systematic book-length study on Wittgenstein's employment of morphology, Kristijan Krkač and Josip Lukin also emphasize analogy as a crucial aspect of this methodology:

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[226] Rupert Riedl, "The role of morphology in the theory of evolution," in *Dimensions of Darwinism*, ed. M. Grene, (Cambridge, UK: Cambridge University Press, 1983), 206.

> Therefore, Wittgenstein's philosophical morphology, as a method of making an overview of "use of our words" of a certain [form of life] and as a structure of a [form of life] (phenomena) by investigating analogies and disanalogies is the best method of representing the world.[227]

Furthermore, as the above uses of the term *Gestalt* and form of life suggest, morphology consists in the examination and analogical comparison of forms, patterns, or structures.

Since analogy and its associated forms of analogical reasoning have been severely neglected throughout the history of Western philosophy, it is necessary for us to get a better sense as to how these terms might be more precisely employed. According to Melandri, philosophy in the Classical era marks an important exception, as there was widespread consensus, from the Pythagoreans to Aristotle, that tended to understand *analogīa* in terms of proportion:

> In the context of ancient mathematics *ἀναλογία* means proportion, just as *λόγος* means ratio. In medieval Latin and modern English *ratio* translates precisely this special sense of *logos*, which we would rather say "ratio," "quotient," or "fraction." Thus, the *ἀναλογία* or *proportio* is defined as the equality of two *λόγοι* or *rationes*.[228]

Since analogy itself resists univocal definition, Melandri insists that its closest characterization would be to simply acknowledge that it ought to somehow bear the form or structure of a proportion. Indeed, this form is succinctly set forth in Aristotle's *Nicomachean Ethics*: "There is justice in, at least, four terms; and the ratio itself has split

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[227] Krkač, *A Custodian of Grammar*, 60.

[228] Enzo Melandri, *La linea e il circolo: studio logico-filosofico sull'analogia* (Macerata: Quodlibet, 2004), 231; Angelo Bonfanti, *Le Forme Dell'analogia: Studi Sulla Filosofia Di Enzo Melandri*, I edizione (Ariccia (RM): Aracne editrice int.le S.r.l, 2016), 104.

in two the same terms… in this way, the first is to the second term, so is the third to the fourth term (1131b1–10)." In short, $a : b = c : d$.

For Melandri, this formal framework can thereby serve as a reference point for more complex and varied analogies: "The key… is that through analogy we express a type of similarity between things or situations based on an equality of relationships; not on the simple sharing of attributes between two objects."[229] However, Melandri's perspective is actually a bit more nuanced and coincides with the distinction cited above between essential similarities and accidental similarities. In other words, he distinguishes between a brute similarity in attributes, on the one hand, and a systemic formal relationship, on the other, such that they establish the two limiting poles in which analogy must reside:

> The common matrix of these two forms of analogy [i.e., intensive and extensive] is "comparison," which is the foundation of both. The closer we get to the intentional-qualitative-attributive side, the more the analogy is imperfect and degrades into similarity or pure metaphoricity. The closer we get to the quantitative-extensional-proportional side, the closer the analogy comes to the mathematical model of numerical properties…
>
> In conclusion, it can be said that, for Melandri, the "form" of analogy is a concept that oscillates between the two extremes of qualitative comparison and mathematical relationships; and that its great or lesser precision derives from its location within the vast space that separates the two poles of attribution and proportionality.[230]

Thus, on the one hand, there can be simple instances of analogical comparison that involve nothing more than an intensive comparison of the common property shared by two things. Yet, on the other hand, there can be analogies construed between two

[229] Bonfanti, *Le Forme Dell'analogia*, 105.
[230] Ibid., 123, 128.

systems that are instead more formal, structural, or even numerical. Most importantly, we can gather a sense of analogy as what resides between the two extremes.

Given that Melandri was an astute reader of Wittgenstein as well as a proponent of the morphological method, it should come as no surprise that a similar outlook on the trade-off between these two opposing forms of analogical comparison is advanced in a section of the *Philosophical Investigations* with pronounced morphological overtones (*PI*: §§72–75). That is, while elaborating the morphological notion of a family resemblance, Wittgenstein gives an additional morphological nod to Goethe's *The Metamorphosis of Plants* by considering the example of a leaf as a paradigmatic sample:

> So if I'm shown various leaves and told "This is called a 'leaf,'" I get an idea of the shape of a leaf, a picture of it in my mind. – But what does the picture of a leaf look like when it does not show us any particular shape, but rather "what is common to all shapes of leaf"? What shade is the "sample [*Muster*] in my mind" of the color green – the sample of what is common to all shades of green?
>
> "But might there not be such 'general' samples? Say a schematic leaf, or a sample of *pure* green?" – Certainly! But for such a schema [*Schema*] to be understood as a *schema,* and not as the shape of a particular leaf, and for a snippet of pure green to be understood as a sample of all that is greenish, and not as a sample of pure green – this in turn resides in the way the samples are applied [*Anwendung*].
>
> Here also belongs the idea that someone who views this leaf as a sample of "leaf shape in general" will see it differently from someone who views it as, say, a sample of this particular shape. Well, this might be so – though it is not so – for it would only amount to saying that, as a matter of experience, someone who sees the leaf in a particular way will then use it in such-and-such a way or according to such-and-such rules. Of course, there is such a thing as seeing in this way or that; and there are also cases where whoever sees a sample like this will in general use it in this way, and whoever sees it otherwise in another way. (*PI*: §§ 73–74)

That is, what distinguishes a particular leaf as a paradigm, sample, or an object of comparison is its analogical use as a general schema in its *formal or structural relation* to

other particular leaves. However, it is admitted that the leaf's use in an analogy *may* be facilitated by its possessing a certain observable attribute, quality, or property.

Moving on to an exceptionally direct and comprehensive reflection on his approach philosophy as such, Wittgenstein tells us that the prominence that he places on analogy is both an heir to Goethe's morphological studies of the natural world and also constitutes the very basis of his wider and more general philosophical method:

> Philosophical investigations: conceptual investigations. The essential thing about metaphysics: that the difference between factual and conceptual investigations is not clear to it. A metaphysical question is always in appearance a factual one, although the problem is a conceptual one.
>
> What is it, however, that a conceptual investigation does? Does it belong in the natural history of human concepts? – Well, natural history, we say, describes plants and beasts. But might it not be that plants had been described in full detail, and then for the first time someone realized the analogies in their structure, analogies which had never been seen before? And so, that he establishes a new order among these descriptions. He says, e.g., "compare this part, not with this one, but rather with that" (Goethe wanted to do something of the sort) and in doing so he is not necessarily speaking of *derivation*; nonetheless the new arrangement *might* also give a new direction to scientific investigation. He is saying "Look at it like *this*" – and that may have advantages and consequences of various kinds. (*RPP I*: §§949–950; see also *Z*: 458)

Reading the first paragraph of the quote with Cantorian set theory in mind, it is not hard to see how Wittgenstein could line up the opposition between a conceptual and a factual investigation with that of a non-extensionalist and an extensionalist perspective. Certainly, the metaphysical backsliding that Wittgenstein diagnoses above is that "dangerous, deceptive thing" pointed in *RFM II* (§19) regarding how certain extensional formulations about classes tend to "make what is a determination of a concept, the formation of a concept, look like a fact of nature."

In contrast, the late Wittgenstein distinguishes his new, strictly correct, method of philosophy as a conceptual investigation that involves the same kind of analogical

comparisons, which we have already seen employed in the non-extensionalist approach to classes and numbers. What is crucial to recognize about the above passage is that the mention of what Goethe wanted to do in his botanical inquiries means that the analogical is identified with the morphological, and taken up as what it means to conduct philosophical research as such.

Furthermore, the aforementioned "new order of descriptions" and "new direction to scientific investigation" is clearly an allusion to what Wittgenstein elsewhere refers to as "perspicuous presentation."[231] As philosophical logic's ultimate goal of "complete clarity," perspicuous presentation is achieved when philosophical problems totally disappear.[232] Accordingly, at the heart of the methodological section of *Investigations,* Wittgenstein distinguishes perspicuous presentation as as his key objective in what surely his most definitive remark on both his own philosophical method and morphology as a whole:

> A main source of our failure to understand is that we don't have *an overview* ["*clear view*": übersehen] of the use of our words. – Our grammar is deficient in perspicuity [*Übersichtlichkeit*]. – A perspicuous presentation [*übersichtliche Darstellung*] precisely that kind of understanding which consists in "seeing connections [*Zusammenhänge sehen*]." Hence the importance of finding and inventing *intermediate links* [Zwischengliedern].
>
> The concept of perspicuous presentation [*übersichtlichen Darstellung*] is of fundamental significance for us. It characterizes the form of presentation [*Darstellungsform*] we give, how we look at matters. (Is this a "Weltanschauung"?) (*PI*: §122)[233]

[231] Eli Friedlander, "Wittgenstein and Goethe: The Life of Colors," 2011, https://www.academia.edu/49287803/Wittgenstein_and_Goethe_The_Life_of_Colors, 9.
[232] Cavell, *This New yet Unapproachable America,* 38–39.
[233] Translation slightly altered.

Elsewhere in Wittgenstein's *Nachlass*, other versions of this remark about perspicuous presentation are either implicitly, and often explicitly, connected to the morphology of Goethe and Spengler.[234] Moreover, Wittgenstein tells us that achieving the kind of understanding that is aligned with perspicuous presentation is involved in "seeing connections [*Zusammenhänge sehen*]." Kristijan Krkač and Josip Lukin explain this technical term of morphology by way of analogical comparison: "conceptual investigation which consists of finding similarities and analogies between many, and at first glance completely different and disconnected, cases (of usage of words), which is in fact morphology as a method."[235] In order to see connections between rather disparate phenomena, Wittgenstein recommends either discovering or developing "intermediate link [Zwischengliedern]" until a chain of similarities can unite what had previously appeared opposed to one another.

By assembling intermediate links, it is possible to eventually assemble a network of interconnected cases: this is what Wittgenstein in *RPP I* (§950) refers to as a "new order" and a "new arrangement." With a subtle allusion to his morphological critique of Frazer's *Golden Bough*, Wittgenstein describes the entwined result of interlocking these connections and links:

> There is a "why" to which permits not predictions. That's the way it is with animistic explanations, for instance. Many of Freud's explanations, or those of Goethe in his theory colors, are of this kind. The explanation gives us an analogy. And now the phenomenon no longer stands alone; it is connected with others, and we feel reassured. (*LWPP II*: 86)

[234] *PO* (133); *PR* (foreword); Krkač, *A Custodian of Grammar*, xix, xx, xxvi, xxviii, 4–5, 24, 40–47, 57–59, 60–152.
[235] Krkač, *A Custodian of Grammar*, 62.

Once again, this organization of what is already in order to facilitate its perspicuous presentation is a hallmark of Wittgenstein's mature philosophical approach: "Philosophy just puts everything before us, and neither explains nor deduces anything (*PI*: §126)."

Wittgenstein proposes a thoroughly morphological term for such a network of connections by referring them as a family resemblance:

> Instead of pointing out something common to all that we call language, I'm saying that these phenomena have no one thing in common in virtue of which we use the same word for all – but there are many different kinds of *affinity* [verwandt] between them. And on account of this affinity, or these affinities, we call them all "languages."...
>
> And the upshot of these considerations is: we see a complicated network of similarities overlapping and criss-crossing: similarities in the large and in the small [*Wir sehen ein kompliziertes Netz von Ähnlichkeiten, die einander übergreifen und kreuzen. Ähnlichkeiten im Großen und Kleinen*] ….
>
> I can think of no better expression to characterize these similarities than "family resemblances [*Familienähnlichkeiten*]"; for the various resemblances between members of a family – build, features, color of eyes, gait, temperament, and so on and so forth – overlap and crisscross in the same way. – And I shall say: "games" form a family. (*PI*: §§65–67)

Even though he may have encountered the notion by reading Schopenhauer, there is conclusive primary evident that demonstrates that Wittgenstein inherited his conception of a family resemblance from the following line found Spengler's morphological study of history, *The Decline of the West*: "In peasant art, Gothic and Baroque have been identical, and streets of old towns with their pure harmony of all sorts of gables and façades… show that the *family resemblance* between the members is far greater than they themselves realize."[236] Indeed, Wittgenstein relies on the same metaphor to describe

[236] Oswald Spengler, *The Decline of the West: Form and Actuality, Vol. 1*, trans. Charles Frances Atkinson (New York: Alfred A. Knopf, 1932), 202; see also, Ludwig Wittgenstein, *Culture and Value: a Selection from the Posthumous Remains*, ed. Georg Henrik von Wright, Heikki Nyman, and Alois Pichler, trans. Peter Winch (Oxford: Blackwell, 2004), 22. From here on, abbreviated as *CV*.

language in the *PI* (§18): "Our language can be regarded as an ancient city: a maze of little streets and squares…"

What is revealing is that, besides games of language and otherwise, Wittgenstein main example of a family resemblance are various systems of numbers that we encountered throughout *RFM II*:

> And likewise the kinds of numbers, for example, form a family. Why do we call something a "number"? Well, perhaps because it has – a direct – affinity with several things that have hitherto been called "number"; and this can be said to give it an indirect affinity with other things that we also call "numbers." And we extend our concept of number, as in spinning a thread we twist fiber on fiber. And the strength of the thread resides not in the fact that some one fiber runs through its whole length, but in the overlapping of many fibers.
>
> But if someone wanted to say, "So there is something common to all these constructions, namely, the disjunction of all their common properties" – I'd reply: Now you are only playing with a word. One might as well say, "There is a Something that runs through the whole thread – namely, the continuous overlapping of these fibers."
>
> "Right; so in your view the concept of number is explained as the logical sum of those individual interrelated concepts: cardinal numbers, rational numbers, real numbers, and so forth; and in the same way, the concept of a game as the logical sum of corresponding sub-concepts." ——This need not be so. For I can give the concept of number rigid boundaries in this way, that is, use the word "number" for a rigidly bounded concept; but I can also use it so that the extension of the concept is not closed by a boundary. (*PI*: §§67, 68)

The connections and links of similarities are now described as fibers running somewhat haphazardly through varying systems of numbers. The metaphor recalls how, in *RFM II* (§16), the "concept of an infinite series" was used with regard to the cardinal numbers, algebraic numbers, and roots as an "expression [that] stands for a certain analogy between cases." In other words, the connection between the three different kinds of number concepts has its basis in how each can similarly be ordered into a countable series. However, the same concept of a series could not forge the same kind of analogous

link to the real numbers: “so far I haven't any certain bridge from these cases to that of ‘all real numbers (*RFM II*: §16).’” Roughly, the same assertion about the lack of an analogical relation based on the concept of a series is reiterated at *RFM II* (§22): “If it were said: "Consideration of the diagonal procedure shews you that the *concept* 'real number' has much less analogy with the concept 'cardinal number' than we, being misled by certain analogies, are inclined to believe", that would have a good and honest sense.” Nevertheless, another kind of analogous connection could be established between the algebraic numbers, the roots, and the real numbers, but passes over the cardinal numbers, due to how only the former can have decimal expansions, but the latter does not. Thus, we can observe the greater or lesser gradient of analogy, as proportion, between the different cases.

In *RFM II* (§50), it is revealing how Wittgenstein compares family resemblances between numbers with his other preferred of a family resemblance between games like that of checkers and chess:

> “In draughts there isn't a King” – what does this mean? (It sounds childish.) Does it mean that none of the pieces in draughts is called “King”; and if we did call one of the pieces that, would there be a King in draughts? But what about *this* proposition: "In draughts all the pieces have the same rights, but not in chess"? Whom am I telling this? One who already knows both games, or else someone who does not yet know them. Here it looks as if the first one stands in no need of our information and the second can do nothing with it. But suppose I were to say: "See! In draughts all the pieces have the same rights,..." or better still: "See! In these games all the pieces have the same rights, in those not." But what does such a proposition do? It introduces a new *concept*, a new ground of classification. I teach you to answer the question: "Name games of the first sort" etc. But in a similar way it would be possible to set questions like: "Invent a game with a King".

It is not hard to see how checkers is meant to align with the natural numbers and chess with the real numbers. What is important for our present concerns a person coming to see the similarities and dissimilarities between games. Once that is achieved, the person can find links between checkers and other games based on pawns and invent links between chess and new games based on a king. Hence, after a suitable network of links is constructed, it is possible to gain a perspicuous view of what, for example, is called a number or a game.

As has already been introduced in the previous sections, besides seeing connections by way of the arrangement of phenomena, there is also the paradigmatic approach to morphology. In an extended passage that was a forerunner to both *PI* (§130)'s methodological proposal about objects of comparison and *PI* (§65–68)'s elaboration of the notion of a family resemblance, Wittgenstein explains how philosophy can employ paradigmatic comparisons within a morphological framework:

> Spengler could be better understood if he said: I am *comparing* different periods of culture with the lives of families; within the family there is a family resemblance, while you will also find a resemblance between members of different families; family resemblance differs from the other sort of resemblance in such & such ways etc. What I mean is: We have to be told the object of comparison, the object from which this approach is derived, so that prejudices do not constantly slip into the discussion. Because then we shall willy nilly ascribe what is true of the prototype the approach to the object to which we are applying the approach as well; & we claim "*it must always be*…"
>
> This comes about because we want to give the prototype's characteristics a foothold in the approach. But since we confuse prototype & object we find ourselves dogmatically conferring on the object properties which only the prototype necessarily possesses. On the other hand we think the approach will lack the generality we want to give it if it really holds only of the one case. But the prototype must just be presented for what it is; as characterizing the whole approach and determining its form. In this way it stands at the head & is generally valid by virtue of determining the form of approach, not by virtue of a claim that everything which is true only of it holds for all the objects to which the approach is applied. One should thus always ask when exaggerated

> dogmatic claims are made: What is actually true in this. Or again: In what case is that actually true.[237]

In applying the morphological approach in this direction, the paradigm or prototype, as object of comparison, is what presents the feature that will serve as the analogical similarity so as to establish the links within the family of phenomena under investigation. On again, we have already encountered this way of conducting a morphological inquiry, when the cardinal numbers were used as a paradigm for a countably ordered series in *RFM II* (§16):

> The mistake begins when one says that the cardinal numbers can be ordered in a series. For what concept has one of this ordering? One has of course a concept of an infinite series, but here that gives us at most a vague idea, a lodestar for the formation of a concept. For the concept itself is *abstracted* from this and from some other series; or: the expression denotes a certain analogy between cases, and it can e.g. be used to delimit for the time being a domain that one wants to talk about.

Put in the language of the above quote, the paradigm is the lodestar (e.g., the natural or the cardinal numbers), and the delimited domain is the family grouped by resemblance (e.g., the positive and negative integers, the even numbers, and other systems that can be countably ordered).

Interestingly, in a remark that occurs immediately after Wittgenstein set forth his own version of the diagonal argument in *RPP I* (§1098), he reaches back for an even more basic and fundamental paradigm of countable ordering:

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[237] Ludwig Wittgenstein, *Culture and Value: a Selection from the Posthumous Remains*, ed. Georg Henrik von Wright, Heikki Nyman, and Alois Pichler, trans. Peter Winch (Oxford: Blackwell, 2004), 22. From here on, abbreviated as *CV*.

> The concept of "ordering," e.g., the rational numbers, and of the "impossibility" of so ordering the irrational [i.e., the real] numbers. Compare this with what is called an ordering of digits. Likewise the difference between the "coordination [*Zuordnen*]" of a figure (or nut) with another and the "coordination" of all whole numbers with the even numbers. Everywhere a shifting of concepts. [Cf. Z 707.][238]

Presumably, this remark can be interpreted as starting from the most basic paradigm grasped by a child in the early stages of its mathematical education. For example, the child learns how to count digits in order and set figures like blocks or nuts in numerical order. Eventually, after mastering the technique of ordering numerical digits, as well as a few others, the child can then analogically extend the concept of ordering by comparing the paradigm of digits to the rational numbers. Likewise, the child that assigns a number to each figure can eventually apply this paradigm to a bijection between the whole and the even numbers.

Upon considering how, from a non-extensional standpoint, the conceptual differences between systems of numbers require comparison by way of analogical similarities and differences, the demand for analogy is further accentuated with regards to formal systems of logical calculi. For example, however identity might be expressed in one system, it would nevertheless seem careless to simply carry it over to another. Furthermore, since questions often arise as to which formal system of logical is better for a specific elucidatory purpose, it would be presumably implausible to rely on either of the two forms of identity in order compare the different systems of formal logic. In facing such a dilemma, this is where the real potential of the morphological method's application of analogical reasoning reveals itself.

[238] Once again, following Mühlhölzer, I assume that the remark actually should be referring to the reals instead of the irrationals.

What is undoubtedly innovative about Melandri's philosophical approach is how he marshalls analogy in order to mediate between formalized logical calculi. As Bonfanti explains, although markedly different, the two kinds reasoning, the formal and the analogical, nevertheless mutually reinforce one another:

> Analogy is… above all a region of rationality that is complementary to logic, it is a calculation with its own principles that are alternative to logical calculation. Alternative not in the sense that it can be considered as a self-sufficient calculus and a substitute for logic: analogical calculus exists only as a complement of logical calculus. Moreover, as we have already said about analogical argumentation (analogical inference), we should not understand analogical calculus starting from the paradigm of deductive calculus in classical logic. It is a reasoning procedure, and therefore a "calculation" in a broad sense, which is not subject to strict formal principles. These clarifications serve to avoid imagining the analogical calculus and the "logic" of analogy as a variant of the real logical calculus.[239]

Whereas I concur that these two forms of reasoning should both be deemed rational, I depart by further insisting, on grounds soon be explained, that both formal and analogical reasoning fall in the province of logic. Nevertheless, the key insight here is that a comparative analogical inference is, by no means, more primitive or inferior to that of a formal systematic deduction; instead, they are merely alternatives that have reciprocal strengths and weaknesses.

Furthermore, Melandri's distinction between the formal and the analogical can coincide with various dichotomies that have been encountered throughout this chapter. On the one hand, the following are characteristic features of formalized logical calculi: bivalence; the principle of non-contradiction; the principle of elementary identity; extensionality; extensiveness; discreteness; and finitude. Whereas, on the other hand, the

[239] Bonfanti, *Le Forme Dell'analogia*, 153.

following contrary characteristics are attributed to analogy: a continuous gradation between bivalence, a tolerance for contradiction, the principle of functional identity, intentionality, intensiveness, continuity, and infinitude. Thus, on the one hand, formal symbolic logic has distinguishing features that coincide with the following: the *Begriffsschriften* of Frege, Russell, and the early Wittgenstein; compositionality; the natural numbers; and the extensional approach to mathematics. Whereas, on the other hand, analogical comparison coincides with: morphology; contextuality, the real numbers; and the intensional or non-extensional approach to mathematics. However, Melandri contends that the formal and the analogical do not oppose one another as contradictories, like black and white, truth and falsity; instead, as complementary contraries, governed by analogy, there is gradation between the two sides.

The distinction between the principle of elementary identity in line with formalism and the principle of functional identity in line with analogy can resolves the problem of comparing of logical calculi. On the one hand, the former demands the kind of symbolic univocity that was observed in the *Tractatus*, which tends to impose an associated atomistic view on the reality it signifies. Nevertheless, whether logical, ontological, or both, the downside of elementary identity is that it either imposes far too stringent requirements on its domain of application or it must be confined to a severely limited domain itself. On the other hand, the latter is an identity based on the notion of analogy as a proportion. In this way, functional identity is an "identity… which compares two distinct domains starting from their identity of structure": an "identity between of relations… in which the objects that occupy the "nodes" of the relational

network are not determinative."[240] Since it is possible to compare the various logical features and relations in the structure of a formal system as "nodes," the functional identity of analogy offers a means for deciding between the two calculi.

Finally, since we have established that there are aspects of rule-following that escape symbolic codification, but nonetheless we are still able to recognize rule-guided accordance, it is, therefore, an empirical fact there are still obvious judgements of sameness and identity with regards to procedural regularity. As Wittgenstein, quite colorfully, puts it:

> Disputes do not break out (among mathematicians, say) over the question of whether or not a rule has been followed. People don't come to blows over it, for example. This belongs to the scaffolding from which our language operates (for example, yields descriptions). (*PI*: §240)

Due to the limitation of formal logic revealed throughout this chapter, the shared acknowledgement of evident cases of identically following a rule cannot be explained by the principle of elementary identity. Wittgenstein takes his interlocutor's repeated appeals to the principle to task by ridiculing the senseless of pseudo-propositions, such as "If someone sees a thing, he sees identity too (*PI*: §215)."

Wittgenstein nevertheless does admit that we are able to detect and respond to this kind of sameness and identity, despite its occurrence within the unsayable scaffolding that grounds language. As already mentioned, how we agree about identity in this regard is by way of a form of life:

> It is not only agreement in definitions, but also (odd as it may sound) agreement in judgements that is required for communication by means of language. This seems to abolish [*aufzuheben*]

[240] Ibid., 159.

> logic, but does not do so. – It is one thing to describe methods of measurement, and another to obtain and state results of measurement. But what we call "measuring" is in part determined by a certain constancy in results of measurement. (*PI*: §242)

However, even though it can't be said, this doesn't demand a retreat into the mysticism of the ineffable. By turning to morphology's analogical reasoning, we can show the formal, structural, and relational sameness by means of functional identity. Indeed, morphological analogy is able to pick where formal logic exposes its own shortcoming. However, as Wittgenstein insisted, this does not abolish logic, which is why I insisted that analogy was *both* rational *and* logical.

Chapter 4: Back to Rough Ground: Wittgenstein, Goethe, and the Materiality of the Paradigm

In an extended passage from an aborted collaborative project with Friedrich Waismann, Wittgenstein unambiguously acknowledges his philosophical debt to Goethe:

> Our thought here marches with certain views of Goethe's, which he expressed in the *Metamorphosis of Plants*… His conception of the original plant implies no hypothesis about the temporal development of the vegetable kingdom such as of Darwin. What then *is* the problem solved by this idea? It is the problem of synoptic [i.e., perspicuous] presentation. Goethe's aphorism "All the organs of plants are leaves transformed" offers us a plan in which we may group the organs of plants according to their similarities as if around some natural center. We see the original form of the leaf changing into similar and cognate forms, into the leaves of the calyx, the leaves of the petal, into organs that are half petals, half stamens, and so on. We follow this sensuous transformation of the type by linking up the leaf through intermediate forms with the other organs of the plant.
>
> That is precisely what we are doing here. We are collating one form of language with its environment, or transforming it in imagination so as to gain a view of the whole of space in which the structure of our language has its being.[241]

As has been well-established in the secondary literature, Wittgenstein often understood his own momentous methodological shift into his mature phase in terms of his sympathetic affinity with Goethe's morphological studies on the natural world.[242] In the passage above, Wittgenstein is specifically referring to Goethe's notion of an "original," "proto-," or "primal" plant (*Urpflanze*). In *The Metamorphosis of Plants*, Goethe defines

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[241] Ludwig Wittgenstein and Friedrich Waismann, *The Voices of Wittgenstein: The Vienna Circle*, ed. Gordon P. Baker (London: Routledge, 2003), 309–311.

[242] Fritz Breithaupt and Richard Raatzsch, eds., *Goethe and Wittgenstein: seeing the world's unity in its variety*, Wittgenstein-Studien 5 (Frankfurt am Main: P. Lang, Europäischer Verl. der Wissenschaften, 2003); Eli Friedlander, "Wittgenstein, Benjamin and Pure Realism," in *Wittgenstein and Modernism*, ed. Michael LeMahieu and Karen Zumhagen-Yekplé (Chicago ; London: The University of Chicago Press, 2017), 121ff. Ray Monk, *Ludwig Wittgenstein: The Duty of Genius*, A Penguin Book Biography (New York, NY: Penguin Books, 1991), 303–304, 509–512, 561–566.

this primal plant as: "a general term to describe this organ which metamorphized into such a variety of forms, a term descriptive of the standard against which to compare the various manifestations of its form."[243] As has already been suggested, we could therefore say that what the *Urpflanze* was for Goethe's botanical investigations is what the paradigm, schema, or object of comparison will be for Wittgenstein's investigations into the logic of our language.

During his lifetime and well after, Goethe was frequently met with incredulous responses to his claims about the ontological status of his various *Urphänomen*. In a well-known transcription of a debate with Schiller, Goethe was nonetheless adamant that an *Urphänome* ought to be properly conceived as a combination of both an intelligible idea and a perceptible thing:

> We reached his [i.e., Schiller's] house, and our conversation drew me in. There I gave an enthusiastic description of the metamorphosis of plants, and with a few characteristic strokes of the pen I caused a symbolic plant to spring up before his eyes. He heard and saw all this with great interest, with unmistakable power of comprehension. But when I stopped, he shook his head and said, "That is not an observation from experience. That is an idea." Taken aback and somewhat annoyed, I paused; with this comment he had touched on the very point that divided us… my old resentment began to rise in me. I collected my wits, however, and replied, "Then I may rejoice that I have ideas without knowing it, and can even see them with my own eyes."[244]

Goethe clearly comprehends his *Urpflanze* as a *real* thing, in terms of a certain realism that is capable of opposing a contrary form of idealism. In fact, Goethe puts his position with regard to Schiller in precisely this way: "He [i.e., Schiller] answered as a cultivated Kantian, and when my stubborn *realism* touched off a lively rejoinder we embarked on a

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[243] §120 of *The Metamorphosis of Plants* in Johann Wolfgang von Goethe, *Scientific Studies*, Edited by Douglas Miller, Goethe Edition, v. 12 (New York, N.Y: Suhrkamp, 1988), 96–97.
[244] From a "Fortunate Encounter" in Goethe, *Scientific Studies*, 20.

long struggle…"[245] What's more, for Goethe, the primal plant seems to be materialist insofar as he insists to Schiller that he is conducting an "empirical" study of "Nature." Indeed, this seems to be how Wittgenstein read Goethe as well, since the former recognized that the latter was actually interested in categorizing the various "sensuous transformation[s]" that a primal plant might undergo.

Thus, more generally, Goethe's *Urpflanze*, alongside his *Urgestein* (primordial mineral) and *Urtier* (primordial animal), are each instances of *Urphänomene* (original, archetypal, primordial, or proto phenomena) that can be characterized as: "[a] synthetic mediation of theoretical idea[s] and empirical knowledge"; "both real (experienced) and ideal (conceptualized)."[246] As we already seen with various paradigms, such as the lion of all lions, the archetypal human, and the standard meter, all of them exhibited a similar paradoxical crossing of concept and object. Furthermore, given that Wittgenstein can now mobilize the reflexivity that was barred in his early phase, the fusion of idealism and realism that was attempted in the dissolution of the subject can now be achieved due to the inherent reflexivity in a paradigmatic standard.

Striving beyond a "stubborn realism," even a summary glance at Goethe's voluminous corpus of scientific writings would indicate that his research was also infused with a materialist ethos.[247] Of course, the materialist streak in Goethe's thought is evident insofar as he is conducting investigations directed towards the natural world – in fact, different variations of the terms "*Materie*" and "*Material*" occur over four hundred

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[245] Ibid., my emphasis.

[246] Sebastian Meixner, "Urphänomen (Original/Primordial Phenomenon)," *Goethe-Lexicon of Philosophical Concepts* 2, no. 1 (2022): https://goethe-lexicon.pitt.edu/GL/article/view/46#.

[247] For Goethe's various pronouncements on matter and materialism, see *Scientific Studies*, 6, 35–36, 117, 122, 137, 155, 304.

times in his scientific *Nachlass*.[248] Nevertheless, Goethe goes further by explicitly advancing his own unique conception of matter and material that persistently serves as a guiding principle dictating the course of his scientific research.

For example, in his "Commentary on the Aphoristic Essay 'Nature,'" Goethe characterizes outlook on nature in the overly materialistic terms that follow:

> The missing capstone is the perception of the two great driving forces in all nature: the concepts of *polarity* [Polarität] and *intensification* [Steigerung], the former a property of matter [*Materie*] insofar as we think of it as material [*materiell*], the latter insofar as we think of it as spiritual. Polarity is a state of constant attraction and repulsion, while intensification is a state of ever-striving ascent. Since, however, matter can never exist and act without spirit [*Geist*], nor spirit without matter, matter is also capable of undergoing intensification, and spirit cannot be denied its attraction and repulsion. Similarly, the capacity to think is given only to someone who has made sufficient divisions to bring about a union, and who has united sufficiently to seek further divisions.[249]

What is important for us is that Goethe outlines a division that is set firmly *within* matter. Furthermore, he insists that "spirit" and "thought," the two watchwords of idealism, have no independent existence outside of matter. As Jennifer Caisley explains, "Even though they are separate concepts, *Materie* and *Geist* are also inherently and inextricably connected; hence, attempting to understand the terms by defining them as isolated notions does not do justice to Goethe's presentation of these concepts."[250] In fact, Caisley insists that the link between matter and spirit are so intertwined that they

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[248] Jennifer Caisley, "Matter (Materie)," *Goethe-Lexicon of Philosophical Concepts* 2, no. 1 (2022): https://goethe-lexicon.pitt.edu/GL/article/view/60

[249] Goethe, *Scientific Studies*, 6.

[250] Jennifer Caisley, "Matter (Materie)," https://goethe-lexicon.pitt.edu/GL/article/view/60

paradoxically comprise a kind of "monist duality, with its two distinct components nevertheless forming a single unit."[251]

Accordingly, one of the primary objectives of this chapter is persuasively argue that Wittgenstein conceives of the material aspect of a paradigm in a manner that echoes the inseparability of matter and formative *Geist* for Goethe. A fitting way to begin getting a handle on the materiality of the Wittgensteinian paradigm is by considering the following remark: "The aspects seem to belong to the structure of the inner materialization. (*LW II*: 13; c.f. *LW I,* 482)." As commentators have noted, the late Wittgenstein's exploration of aspects is an evolution of his earlier thoughts on a logical feature in *TLP* (4.1221): "An internal property of a fact we can also call a feature [*Zug*] of that fact. (In the sense in which we speak about facial features)."[252] Along lines of the master simile, in the same way that a logical feature was inseparable from an instantiation of its corresponding logical type, the material is inseparable from its instantiation in a paradigm. In other words, it is unthinkable to separate a paradigm from its material. As a continuation of the master simile comparing logical feature with facial features, it will become evident why the mature Wittgenstein repeatedly explains the material aspects of a paradigm in terms physiognomy.

4.1: Wittgensteinian Realism to Wittgensteinian Materialism

[251] Ibid.
[252] Floyd, "Aspects of Aspects," 362ff.

The realism typically attributed to the later Wittgenstein's thought is derived from an oft-repeated slogan from the *Remarks on the Foundations of Mathematics*: "Not empiricism and yet realism in philosophy, that is the hardest thing (*RFM VI*: 23)." This line is often interpreted as a methodological plea for more careful attention to detail in the midst of a philosophical investigation because the activity of philosophy itself evokes delusions and militates against reality. In the secondary literature, the classic account of this is Cora Diamond's endorsement of a certain *Realistic Spirit*:

> If, in philosophy, we seek to understand such things as how our thoughts can be true or false at all, what makes their logical character possible, what makes it possible for there to be adherence to a definition, we may take the details of what we say, what we do, to be irrelevant to the understanding we seek. What I call the realistic spirit aims not to provide that sort of understanding but to change our idea that it *is* what we seek.[253]

Rather than what is often construed as the standard philosophical realism about objective, free standing, and mind-independent entities, this idiosyncratic brand of Wittgensteinian realism amounts to a broad procedural directive to take into close consideration what other thinkers are liable to ignore.[254]

This focus on the relevant details will prove decisive for Diamond's study of the standard meter paradigm, as it allows her to correct certain misleading interpretations that neglect the actual practice of measuring with a ruler.[255] As an example, the realistic spirit is indeed operative in the following clarification of the metaphysical illusions obscure the

[253] Cora Diamond, *The Realistic Spirit: Wittgenstein, Philosophy, and the Mind* (Cambridge, Mass.: MIT Press, 2011), 13.
[254] See also, Friedlander, *Signs of Sense*, 192–193.
[255] Diamond, "How Long Is the Standard Meter in Paris."

interlocutor's understanding of what is actually involved in the practice of measuring with a ruler:

> "Put a ruler against this object; it does not say that the object is so-and-so long. Rather, it is in itself – I am tempted to say – dead, and achieves nothing of what a thought can achieve." It is as if we had imagined that the essential thing about a living human being was the outward form. Then we made a lump of wood into that form and were abashed to see the lifeless block, lacking any similarity to a living creature. (*PI*: §430)[256]

The interlocutor, in this case, conceives of a ruler as an inert object that is, at best, only indirectly connected with an ideal and abstract length read off of it. Diamond diagnoses the interlocutor's disregard for the use of the concrete physical object that is the ruler as a lapse into the unrealistic: "This picture can lead into philosophical confusion through its seeming to illustrate a kind of measurement which is totally independent of the whole business of actually putting objects alongside each other, reading properly off instruments, and so on."[257] Commenting specifically on *PI* (§430), Diamond also writes:

> What gives strength to the idea that the essential thing in measuring is the comparison with a definite length, conceived as "fitting" the stick used in measuring, is that the stick itself is just a piece of wood or metal; we might say of it, Wittgenstein notes, that in itself it is dead; *it* cannot say that the body measured is of such-and-such length. The stick appears as a mere means through which we reach something that is intrinsically a measure. And so, in our [problematic] philosophical view of the workings of the language of measurement, what appears central is establishing the referential connection to the "definite length"; the actual practice of using words and sticks of wood or metal rods in measuring things disappears from view.[258]

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[256] See also, *BT* (70e–71e).
[257] Diamond, "How Long Is the Standard Meter in Paris," 122.
[258] Ibid., 126.

The delusion of an abstract length that can be distinguished from a ruler as an inert object tends to arise when the concrete practice of measuring is ignored as irrelevant, thereby divorcing the length from any activity done with the ruler. For our purposes, what is crucial is that, in this case, the realism about practice implies materialism because this particular activity of measurement entails the use of an instrument composed of wood or metal.

Similar considerations have compelled Jakub Mácha to argue that the Wittgensteinian notion of a paradigm must be necessarily endowed with some kind of materiality:

> I derive the usage of the term "paradigm" from Wittgenstein. Let a paradigm of X be a *material object* together with the *practice* of applying this object in a given situation. I call the object of a paradigm a *paradigmatic sample*. The term "object" is understood in the broadest way possible. Paradigmatic samples are real material things, ranging from clearly defined objects such as the meter stick or a color plate to intricate structures such as formalizations of mathematical proofs or works of art.[259]

On this account, the realism of practice with what Mácha identifies as the "paradigmatic sample" or "the object of a paradigm" entails materialism.

It is all the more revealing that Mácha marshalls a passage for primary evidence in which Wittgenstein proposes an example of a ruler in order to explain how the realism of practice necessitates materialism:

> [T]his [spatio-temporal] nature must relate… as the corporeality [*Körperlichkeit*] of a measuring stick does to its extendedness, by means of which it measures. In the case of the measuring stick you can't say: "Yes, the measuring stick measures length in spite of its corporeality; but a measuring stick that had only length would be the ideal, would be the *pure* measuring stick." No,

[259] Mácha, *The Philosophy of Exemplarity*, 14.

> if a body [*Körper*] has length, then there can't be any length without a body – and even if I do understand that in a certain sense only the length of the measuring stick does the measuring, yet what I put into my pocket is still the measuring stick – the body and not the length. (*BT*: 352e)[260]

Underlining how measuring must include a rule and that ignoring the object's use gives way to philosophical confusion, Wittgenstein stresses that the materiality of the rod itself is essential by directing the interlocutor's attention to its "spatio-temporal nature," "corporeality," and the concrete existence of its "body." Put in another way, it is utterly unintelligible to conceive of a length somehow separated from a ruler's material corporeality.

In the secondary scholarship, the centrality of a paradigm's material constitution has unfortunately often gone overlooked.[261] However, for Wittgenstein, its materiality is essential to its preceding experimental phase, which is characterized by its emphasis on spatio-temporal, physico-empirical, and contingent phenomena. Taking up another example of paradigms for measuring, Wittgenstein provides a hypothetical scenario in which the standard length could be settled upon:

> Suppose that we found that all ordinary objects had lengths which were multiples of the length of this piece of chalk. Then nothing would be more natural than to choose this chalk as our unit of length. Our unit of length is in that case dependent upon experience, in the sense that it is experience which makes us choose it. But if we later came across objects whose lengths were not multiples of this piece of chalk, we should not give up that unit of length.
>
> One might say, "This piece of chalk is the unit of length," and mean that all objects have lengths which are multiples of this. In this case it is not independent of experience; it is an

[260] Translation slightly modified.

[261] For example, consider what is likely the earliest article-length study exclusively devoted to Wittgenstein's conception of a paradigm in C. G. Luckhardt, "Beyond knowledge: Paradigms in Wittgenstein's Later Philosophy," *Philosophy and Phenomenological Research* 39, no. 2 (1978): 245. There, Luckhardt lists three characteristic features of Wittgensteinian paradigms, but its material existence, which is likely its most obvious feature, is left out of the list and largely ignored throughout the wider article.

> experiential proposition. But one can use the sentence "This piece of chalk is the unit of length" in quite a different way, in order to say something about the way one is going to measure lengths.[262]

Even though the two propositions are used differently, the empirical proposition coincides with the experimental phase and the grammatical proposition with its codification as a standard, the materiality of the measuring paradigm was a necessity for both its prior selection and its later use. As Wittgenstein explains in *RFM VII* (§69): "But have we not determined the relative length of foot and meter experimentally? Yes; but the result was given the character of a rule." What is implicit here is that what the previous experimental stages yields and what is authorized as a paradigmatic standard is the same *thing*, but only its use differs.

For similar reasons, Wittgenstein's metaphor about depositing a paradigmatic standard in an archive turns on the same realistic insights that imply the use of a material object:

> I once said: A calculation could always be laid down in the archive of measurements. It can be regarded as a picture of an experiment. We deposit the picture in the archives, and say, "This is now regarded as a standard of comparison by means of which we describe future experiments." It is now the paradigm with which we compare...
>
> We have the meter rod in the archives. Do we also have an account of how the meter rod is to be compared with other rods? There might be a point sometimes in putting an account – say, a picture – of the way in which we compare them; or instruments used for this purpose. (*LFM*: 104–105)

[262] Ludwig Wittgenstein, *Wittgenstein's Lectures on the Foundations of Mathematics Cambridge, 1939*, ed Cora Diamond, (Ithaca, NY: Cornell University Press, 1976), 42. From here on, abbreviated as *LFM*.

Once again, it is the material object resulting from the physical experiment that is stored in the archive.

It is perhaps appropriate to mention that it is not solely for pedantic exegetical purposes that I have cited the two passages that imply the materiality of a paradigm, nor is it for the sake of consistency with previous accounts of a paradigm as a convergence of concept and object. Most importantly, the emphasis on the materiality of a paradigm is integral to its use as such. Consider, for example, Kuusela's accurate account of how the actual employment of a paradigm must involve certain non-idealist factors:

> Rather, as explained, Wittgensteinian logical models [i.e., paradigms] constitute modes of representing the function of relevant expressions. Instead of constituting a temporal factual statement, a clarificatory model is non-temporal, making no reference to any particular time and space, except when such a non-temporal model is put to use as an object of comparison with the purpose of comparing actual cases with it.[263]

As it was with the example of the chalk remaining a unit of measure despite observing empirical facts that presumably invalidated the initial reasons for selecting it as a paradigmatic length, Wittgenstein nonetheless asserts that these spatio-temporal circumstances are inconsequential once the experimental object has been distinguished and placed in the archive. However, Kuusela highlights a significant exception, as the concrete employment of the paradigm "with the purpose of comparing actual cases with it" has temporal and presumably spatial implications as well. Consequently, it is hard to imagine how the use of anything in time and space would not entail that it was some kind

[263] Kuusela, *Wittgenstein on Logic as the Method of Philosophy*, 190.

of corporeal object. So, for this reason, materiality must be regarded as an intrinsic and an inextricable aspect of a paradigm.

What is also telling is that the previous remark from the *Big Typescript* includes assertions about how language itself must be regarded as a "physical phenomenon (*BT*: 352e)," and these claims will later be incorporated into a decisive remark found in the methodological section of the *Philosophical Investigations*:

> The sense in which philosophy of logic speaks of sentences and words is no different from that in which we speak of them in ordinary life when we say, for example, "What is written here is a Chinese sentence", or "No, that only looks like writing; it's actually just ornamental", and so on.
>
> We're talking about the spatial and temporal phenomenon of language, not about some non-spatial, atemporal non-entity [*Unding*]. [Only it is possible to be interested in a phenomenon in a variety of ways] But we talk about it as we do about the pieces in chess when we are stating the rules for their moves, not describing their physical properties… (*PI*: §108b–d)

Indeed, it is almost inconceivable that Wittgenstein could be referring to a spatial and temporal thing (*Ding*) with physical properties, such as ornamental ink on paper without any logico-linguistic significance, but it could somehow still fall short of having the status of material body. Often, when contemplating a paradigmatic sample, like a color swatch, Wittgenstein decides that they can be taken up as a part that belongs to the language: "It is most natural, and causes least confusion, if we count the samples as tools of the language (*PI*: §16)." Yet, conversely, if human language consists of physical corporeal objects, then perhaps it would cause even less confusion to instead include language amongst a collection of material samples. Either way, a paradigm could not be what is, by helping us do what it has been designed to do, whether it be presenting length, color, or language itself, without its intrinsic material aspect availing to our use.

4.2: The Most Idealistic is the Material

If language and logic are to be located in the ambit of the material, then it would not be a far cry to include mathematics alongside them, despite the latter's persistent historical association with metaphysical idealism. In fact, throughout *RFM III*, Wittgenstein repeatedly traces the material aspects that are integral to mathematics by way of the demand that a proof must be reproducible. This reproducibility is often put in the most literal terms that could express a tangible process of construction or synthesis:

> "A proof must be capable of being taken in" means: we must be prepared to use it as our guide-line in judging.
>
> When I say "a proof is a picture" – it can be thought of as a cinematographic picture.
>
> We construct the proof once and for all. A proof must of course have the character of a model. The proof (the pattern of the proof) shows us the result of a procedure (the construction); and we are convinced that a procedure regulated in *this* way always leads to this configuration. (The proof exhibits a fact of synthesis to us.) (*RFM III*: §22)

The familiar language should suggest that Wittgenstein regards a proof as a kind of mathematical paradigm. Recall above that he has already placed a calculation in the archives alongside a paradigmatic standard of length. Yet, instead of an abstract set of tenets on paper, Wittgenstein tends to speak here of a proof much more like we speak of a manufactured object: a proof is used as a model that was the result of a synthetic procedure or construction.

In fact, after explicitly categorizing a proof as a paradigm, Wittgenstein somehow finds a way to further highlight the physicality in the construction of a proof:

> The proposition proved by means of the proof serves as a rule – and so as a paradigm…

> The proof constructs a proposition; but the point is *how* it constructs. Sometimes, for example, it first constructs a *number* and then comes the proposition that there is such a number. When we say that the construction must *convince* us of the proposition, that means that it must lead us to apply this proposition in such-and-such a way…
>
> What is in common between the purpose of a Euclidean construction, say the bisection of a line, and the purpose of deriving a rule from rules by means of logical inferences?
>
> The common thing seems to be that by the construction of a sign I compel the acceptance of a sign.
>
> Could we say: "mathematics creates new *expressions*, not new propositions"?
>
> Inasmuch, that is, as mathematical propositions are instruments taken up into the language once for all – and their proof shows the place where they stand.
>
> But in what sense are e.g. Russell's tautologies "instruments of language"?
>
> Russell at any rate would not have held them to be so. His mistake, if there was one, can however only have consisted in his not paying attention to their *application*.
>
> The proof makes one structure generate another.
>
> It exhibits the generation of one from others.
>
> That is all very well – but still it does quite different things in different cases! What is the interest of this transition?
>
> Even if I think of a proof as something deposited in the archives of language – who says *how* this instrument is to be employed, what it is for? (*RFM III*: §§28–29)

Not only is construction, generation, and production again underscored, but proof is designated as a paradigm, like the standard meter, and an instrument of language, like a color swatch, that are also to be placed in the archives.

Furthermore, a proof is said to be "taken up" into language like the sample color swatch, but once again the physicality of language is indicated by the mention of "the construction of a sign". Accordingly, there several moments in the *RFM III* when Wittgenstein's description of a proof mirrors what was said about language as a spatial and temporal phenomenon in *PI* (§108): "Thus we take the constructability (provability) of this symbol (that is, of the mathematical proposition) as a sign that we are to transform symbols in such and such a way (*RFM III*: §27)." In other words, what is constructed are

material signs and symbols that guide us in constructing other material signs and symbols.

What is likely the most exegetically convincing materialist pronouncement in *RFM III* is another instance of the identity of the object of a physical experiment and its final result:

> A proof shows us what OUGHT to come out. – And since every reproduction of the proof must demonstrate the same thing, while on the one hand it must reproduce the result automatically, on the other hand it must also reproduce the *compulsion* to get it.
>
> That is: we reproduce not merely the *conditions* which once yielded this result (as in an experiment), but the result itself. And yet the proof is not a stacked game, inasmuch as it must always be capable of guiding us.
>
> On the one hand we must be able to reproduce the proof *in toto* automatically, and on the other hand this reproduction must once more be *proof* of the result.
>
> "Proof must be surveyable": this aims at drawing our attention to the difference between the concepts of "repeating a proof," and "repeating an experiment." To repeat a proof means, not to reproduce the conditions under which a particular result was once obtained, but to repeat every step *and the result*. And although this shows that proof is something that must be capable of being reproduced *in toto* automatically, still every such reproduction must contain the force of proof, which compels acceptance of the result. (*RFM III*: §55)

The automatic reproduction of the result of the experiment, of course, suggests the reproduction of a material object in as a thing. This all the more so considering that Wittgenstein's go-to example of the transition from is experiment to proof involves adding apples:

> Proof, one might say, must originally be a kind of experiment – but is then taken simply as a picture.
>
> If I pour two lots of 200 apples together and count them, and the result is 400, that is not a proof that 200 + 200 = 400. That is to say, we should not want to take this fact as a paradigm for judging all similar situations.

> To say: "these 200 apples and these 200 apples come to 400" – means: when one puts them together, none are lost or added, they behave normally.
>
> "This is the model for the addition of 200 and 200" – not: "this is the model of the fact that 200 and 200 added together yield 400." The process of adding *did* indeed yield 400, but now we take this result as the criterion for the correct addition – or simply: for the addition – of these numbers.
>
> The proof must be our model, our picture, of how these operations have a *result*.
>
> The "proved proposition" expresses what is to be read off from the proof-picture.
>
> The proof is now our model of correctly counting 200 apples and 200 apples together: that is to say, it defines a new concept: "the counting of 200 and 200 objects together." Or, as we could also say: "a new criterion for nothing's having been lost or added."
>
> The proof *defines* "correctly counting together."
>
> The proof is our model for a particular *result's being yielded*, which serves as an object of comparison (yardstick) for real changes. (*RFM III*: §23–4)

We have a physical transformation that takes place, "The process of adding *did* indeed yield 400," and the "result" is a group of material objects that is now "the criterion for the correct addition." In fact, the outcome is referred to as an "*object* of comparison."

Once more, this gloss of *RFM III* on the requirement for the duplication of proofs may seem more like a pure exercise in interpretative exegesis, than actual philosophical reflection. Furthermore, much like how Wittgenstein's later comments on the so-called foundations of mathematics were only met with suspicions, if widely considered, his radical proposal for a materialized mathematics would probably elicit similar doubts. Nevertheless, as the course of time has lent due credence to Wittgenstein's thoughts on meta-mathematics and symbolic logic, it now seems as though a comparable trend is occurring that could corroborate his insistence upon the indispensability of material things in the use and application of mathematics.[264] In fact, Wittgenstein endorsement of

[264] See Paolo Mancosu, *The Philosophy of Mathematical Practice*, (Oxford ; New York: Oxford University Press, 2008).

realism and not empiricism occurs only a few sentence before he claims that "[A mathematical paradigm] is thus withdrawn from being checked by experience, but now serves as a paradigm for judging experience (*RFM VI*: 23)." Perhaps, then, we could say that realism in mathematical practice does not entail verification with experience (i.e. empiricism), but nevertheless the actual experiences in judgments comparing objects.

Finally, in the same way that Wittgenstein identified a proof as a material object of comparison designed for the repeated physical manipulations of other objects, he also maintains that only a material object that is physically comparable to a living human body can coherently be ascribed the otherwise consecrated idealist manifestation of a soul or consciousness. However, the paradigmatic object of comparison is bit more complicated in this case, because it involves much more complex objects than mere symbols and apples.

Relying on language-games of preparation and instruction instead of experiments for the development of the required objects of comparison, Kuusela interprets Wittgenstein, throughout the private-language section of *Philosophical Investigations*, as interested in:

> … the function of relevant expressions as determined and shaped by certain psychological and/or physical facts about human beings, such as their capacity to feel pain and to express it by means of sounds, facial, and other bodily expressions that belong to their pre-linguistic behavioral repertoire. On Wittgenstein's account, such facts relating to humans as bodily beings constitute the background for the subsequent use of linguistic expressions of pain as extensions of pre-linguistic expressions, without which the latter could not be made sense of or get off the ground… The reason why he evokes such facts in describing the function of relevant expressions is intimately connected with a key insight underlying the language-game method... Linguistic expressions of pain have a determinate function and role only as part of a life constituted by relevant natural historical facts from which the use of those expressions cannot be divorced.

> Because of this the examination of such facts can help to render perspicuous the function or role of relevant linguistic expressions (see section 5.3).[265]

Given the multitude of highly significant intricacies that shape all of our lives, it should be expected that such a fabric would introduce unavoidable complexities when it is employed as the backdrop of our philosophical inquiries. Thus, for our purposes, however, it will be necessary to hone in our focus on the sheer fact that the body of a human is a material object. Indeed, the emphasis on the living and breathing human body comprised of corporeal flesh is the main topic of the skein of remarks that begins with *PI* (§281):

> "But doesn't what you say amount to this: that there is no pain, for example, without pain-behavior?" – It amounts to this: that only of a living human being and what resembles (behaves like) a living human being can one say: it has sensations; it sees; is blind; hears; is deaf; is conscious or unconscious.

As I understand it, the gist of the remark is that consciousness can only be significantly ascribed to a human body or instead a material object that bears a strong enough resemblance to such a body. Otherwise, an attempt to attribute the sign typically used to denote human consciousness results in mere nonsense.

Moving further throughout this thread of remarks, we encounter another that seems to blatantly underline the conceptual link between human experience and the material human body:

> What gives us *so much as the idea* that beings, things, can feel?
>
> … And can one say of the stone that it has a mind, and *that* is what has the pain? What has a mind, what have pains, to do with a stone?

[265] Ibid., 184.

> Only of what behaves like a human being can one say that it *has* pains.
>
> For one has to say it of a body, or, if you like, of a mind which some body *has*. And how can a body *have* a mind? (*PI*: §283)

Despite the difficulty of this nuanced passage, it is nonetheless safe to say that it stands and falls on the distinction between two material objects: a rock and a human body. Furthermore, as odd as it might sound, it is a physical fact that a person's anatomy is both materially and structurally different from that of a stone and this difference entails that we only talk about the emotional life one of these two things.

Chapter 5: The Pathos of Real: Johnston, Livingston, Chiesa, and Žižek on Totality, Subjectivity, and Noumenal Reality

> Man's skull represents the same infinity for the movement of conceptions. It is equal to the universe, in it is contained all that it sees in it... Is not the whole universe that strange skull in which meteors, suns, comets and planets rush endlessly?
>
> – Kazimir Malevich

In this chapter, I aim to intervene in an ongoing debate regarding how to appropriately situate the fracture dividing the objective and the subjective involving Adrian Johnston, Paul Livingston, Lorenzo Chiesa, and Slavoj Žižek.[266] For each of these interlocutors, their primary concern is to provide a convincing and ontologically realist account of the origins and the nature of subjectivity, human language, and consciousness without shrinking from all the paradoxical and antinomic consequences that arise from its advent. Looked at from another direction, the discussion also encompasses issues pertaining to how the emergence and perpetuation of these symbolic, cognitive, and agentive features characteristic of subjectivity can be rendered concomitantly with the intrusion of gaps and contradictions within a wider ontological totality. For this reason, each thinker tends to put forward their respective account distinguishing objectivity and subjectivity in terms of both the Kantian distinction between the phenomenal and the noumenal realm,

[266] The best overview of the debate, which also puts forth a significant intervention in its own right, can be found in Lorenzo Chiesa, *The Not-Two: Logic and God in Lacan* (Cambridge, Massachusetts: MIT Press, 2016), especially chapter 2. A response to both Chiesa and Livingston is put forward in an unpublished version of Johnston, "Confessions of a Weak Reductionist: Responses to Some Recent Criticisms of My Materialism."

as well as within a Lacanian psychoanalytic framework that opposes the imaginary and symbolic to the register of the real.

Given the more sustained and pointed tenor of their engagement, I will tend to focus on where Johnston's transcendental materialism and Livingston's meta-formal realism are inclined to converge and diverge. Furthermore, I will attempt to show how the account of morphology and paradigms sketched in the previous chapters can accommodate, as well as shed light on, some of the most important aspects of both Johnston's and Livingston's respective theoretical standpoints. On the one hand, I argue that the development of logical morphology from the contradictions that arise within formal systems can align well with Livingston's uptake of Gödel's and Turing's logico-mathematical results. However, I contend that only a formal symbolic logic complemented by a morphology equipped with a materialist paradigm and a multitude of comparative techniques is then capable of allaying Johnston's objections that Livingston's reliance Gödel's distinction between incompleteness and inconsistency suffers from a kind of idealist Platonism that is far too reductive and simplistic to handle the variegated complexity of ontological questions. On the other hand, I argue that the initiation of a paradigm is in-step with the materialist requirements that Johnston imposes on any satisfactory theory of subject formation. However, in response to Livingston's and Chiesa's objections against Johnston, I show how this materialist account of paradigms avoids the alleged monist and dualist pitfalls that the former attribute to the latter.

Whether or not I am able to make a substantial contribution to the discussion, my primary aim will have been achieved insofar as I am able to simply provide a helpful

outline of its stakes and implications so that others can come along and do better. Nevertheless, I still take this opportunity to put forward a few suggestions about further avenues of research. Most notably, I will pose questions regarding the close similarities between philosophical accounts of the origins of subjectivity and those that concern life itself in terms of abiogenesis. In particular, I will try to fuse Livingston's and Johnston's respective emphases on the logical and the biological by focusing on the algorithmic and informational characterizations of life, living systems, and their creative origination.

5.1: Livingston's Meta-Formal Realism

Building from his previous logico-philosophical investigations, Livingston entered the debate by advancing a realist ontology based on the distinction he derives from Gödel's incompleteness theorems:

> [I]t is necessary to affirm that references to self-including or self-referring totalities are not somehow simply incoherent or nonsensical, but are themselves actually coherent and essential, even if (as the paradoxes demonstrate) they invoke the actually contradictory structure of the world as such. More broadly, the paradoxes can be formally understood as pointing toward the necessity of a metalogical choice or duality, on the level of references to wholes, between completeness and consistency. In the wake of the paradoxes, it is possible to affirm the completeness or totality of a whole of which one is a part, but only on pain of inconsistency at the limits. Alternatively, one can save consistency and deny completeness. This is the choice usually taken by mathematicians, following Russell, and also officially followed by Badiou. It amounts to holding that no situation is total, and that every definable or coherently considerable situation is thus capable of being supplemented or expanded by coming to include what it, as yet, does not. The solution maintains axiomatic consistency with respect to particular, constituted solutions, but it only makes sense if a coherent position can be presupposed from which it is possible to survey these situations, in general, from outside. Where, however, the stakes of political thought and action essentially involve the constitutive reference to a totality which essentially includes the

> position from which this thought and action themselves take place, and thus generates the essential paradoxes and antinomies of the totality of world as such, it is clear that the other term of the dual (that of completeness and inconsistency) better captures the underlying structural logic that is involved.[267]

As was put forward in detail in his book-length *The Politics of Logic*, the starting point of Livingston's theoretical project involves drawing a metalogical distinction between consistent incompleteness and inconsistent completeness. On one side of the duality, the more standard choice for consistency is today most often identified with Zermelo–Fraenkel set theory (ZFC), yet the same prohibitions against paradox and contradiction were also the driving motivation behind the development of Russell's theory of types. For politico-philosophical reasons, Alain Badiou also stands on the side of consistency by advocating a fundamental ontology that wholly coincides with ZFC. On the other hand, Livingston is quite willing to abandon consistency for completeness, as he does not consider contradictions to be harmful but rather an essential characteristic of a thoroughly realist ontology. Furthermore, as is indicated in the passage above, Livingston also opts for the completeness of a totality based on political grounds: "It is clear, moreover, that in response to the bogus universalism of global capitalism and its associated politics of narrowly electoral representational democracy, the only real possibility of an appropriate counter-claim rests in the appeal to a real universality of a single world to come."[268] In fact, for Livingston, this universalism is upheld as a necessary requirement for a rigorous ontological account of the so-called One-All of existence.

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[267] Paul M. Livingston, "Politics, Subjectivity and Cosmological Antinomy: Kant, Badiou and Žižek," Crisis & Critique 1, no. 2 (2014): 29.
[268] Livingston, "Politics, Subjectivity and Cosmological Antinomy," 27–28.

In addition, Livingston argues in favor of his own particular brand of realism, what he calls a "meta-formal realism," that is given its most sophisticated and sustained elaboration in a 2013 journal-length version of "Realism and the Infinite," and in greater depth in his 2017 book-length contribution, *The Logic of Being*.[269] Livingston is quick to explain that the realism he advocates is not meant to serve as the usual justification for the existence of a free-standing mind or language-independent entities. Instead, he seeks to capture and account for the Lacanian battle-cry that "the Real can be inscribed only on the basis of an impasse of formalization."[270] For Livingston, it is precisely this access to formalization that can be rigorously arrived at by way of the two metalogical choices derived from Gödel's incompleteness theorems: "[meta-formal realism's] primary source is not any empirical experience but rather the experience of formalization, both insofar as this experience points to the real-impossible point of the actual relation of thinkable forms to being and insofar as it schematizes, in results such as Gödel's, the intrinsic capacity of formalization problematically to capture and decompose its own limits." Further describing this realism in terms of formalization, Livingston writes:

> It is, rather, that both terms of Gödel's disjunction capture, in different ways, the structural point of contact between [human] capacities and what must, on either horn of the distinction, be understood as an infinite thinkable structure determined quite independently of anything that is, in itself, finite. Thus, each term of Gödel's disjunction reflects the necessity, given Gödel's theorems, that any specification of our relevant capacities involve their relation to a structural infinity about which we must be realist, i.e. which it is not possible to see as a mere production or creation of these capacities.[271]

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[269] Paul M. Livingston, *The Logic of Being: Realism, Truth, and Time* (Evanston, Illinois: Northwestern University Press, 2017); "Realism and the Infinite," Speculations IV (2013): 1–38.
[270] Jacques Lacan, Encore: The Seminar of Jacques Lacan Book XX, ed. Jacques-Alain Miller, trans. Bruce Fink (New York: Norton & Co., 1999), 93.
[271] Livingston, "Realism and the Infinite," 8.

In short, each of the two choices entails a connection between a finite human and an ontologically real and infinite form.

Let us briefly consider how Livingston seeks to substantiate this link between a person and the forms. On the one hand, the first choice of consistent incompleteness follows the more commonplace interpretation of Gödel's results as establishing the existence of mathematical truths beyond what could be derived within a formal system based upon a finite set of axioms and rules:

> If human mathematical thought can know the truth of statements about numbers which are beyond the capacity of any formal system to prove, then the epistemic objects of this knowledge are "realities" (i.e., truths) that also exceed any finitely determinable capacity of knowledge. It does not appear possible to take these truths as "creations" of the mind unless the mind is not only credited with infinite creative capacities, but understood as having actually already created all of a vastly infinite and in principle unlimitable domain.[272]

The second choice of inconsistent completeness underlines Gödel's derivation in a formal system of certain undecidable statements or unsolvable mathematical problems, thereby demonstrating that it is impossible for this system to prove its own consistency:

> If there are well-specified classes of mathematical problems that are not solvable, as classes, by any means whatsoever, neither by any specifiable formal system nor by human cognition itself, then these problems must be thought of as realities determined quite independently of our capacities to know them (or, indeed, to solve them).[273]

[272] Livingston, *The Logic of Being: Realism, Truth, and Time*, 135.
[273] Livingston, "Realism and the Infinite," 8.

In both cases, finite human thought and practice can reach, through a formal system, something that is both infinite and independently real.

5.2: The German Idealist Objections Against Meta-Formal Realism

In an earlier draft version of his response to his critics, entitled "Confessions of a Weak Reductionist," Johnston advances roughly two interconnected objections against Livingston's meta-formal realist account of ontology.[274] The *first* objection alleges that Livingston's account of the entirety of ontological existence as fundamentally an inconsistent and complete totality is not sufficiently materialist:

> [A]t least as far as I can tell, nothing clearly and convincingly separates this Gödelian panlogicism from a Platonic, idealist-*qua*-anti-materialist metaphysical realism of transcendent forms (such as the mathematical Platonism embraced by Kurt Gödel himself). Additionally, I would maintain that my critique of Badiou's claims to be a materialist…given his reliance upon ontological difference as per Martin Heidegger to justify identify pure mathematics (specifically, the Zermelo–Fraenkel plus the axiom of choice [ZFC] axiomatization of post-Cantorian trans-finite set theory) as fundamental ontology… directly entails the disqualification of Livingstonian formalism, itself also informed by Heideggerian thinking, as any kind of materialism.[275]

Johnston is right in terms of identifying Livingston's philosophical influence, since the former does endorse Badiou's version of Platonism and has now devoted an entire book-

[274] Johnston, "Confessions of a Weak Reductionist: Responses to Some Recent Criticisms of My Materialism."
[275] Johnston, "Confessions of a Weak Reductionist," 36–37. See also, Johnston, *Prolegomena.*

length study to Heidegger. However, I want to get around the stalemates that typically occur in disputes between opposing philosophical schools and emphasize the crux of the objection. That is, the real issue is that Livingston's specific employment of formal symbolic logic and metamathematics leaves his ontology susceptible to the charges of idealism as opposed to a thoroughgoing realism. In fact, according to Johnston, if Livingston's meta-formal realism is indeed realistic, it is a metaphysical realism with regard to abstract forms rather than material matter.

The *second* objection contends that characterizing all of that in terms of an inconsistent and complete totality is simply far too reductive and thereby oversimplifies the vast complexities that constitute the whole of reality.

> Similarly, just as Badiou presumes without demonstrating that all ontic regions (such as the organic kingdoms of the life sciences) somehow fundamentally boil down to set-theoretic forms… Livingston operates in fashions bearing witness to an assumption that Gödel's discoveries as regards incompleteness and inconsistency constitute a universal philosophical master-matrix upon which each and every branch of knowledge ultimately must rest (however, by contrast with Livingston's prioritization of mathematics, Badiou's metaphysics determines the mathematics he favors, not vice versa, with Badiouian ontology and phenomenology motivating the turns to set and category/topos theory respectively, and not the reverse).[276]

As it was previously, this objection was initially developed against Badiou's set-theoretic ontology and maintains that meta-formal realism is not refined enough to tackle the various contingencies and particularities that arise within the various local situations that comprise a larger totality.

[276] Johnston, "Confessions of a Weak Reductionist," 37.

5.3: Johnston's Transcendental Materialism

Johnston describes the aim of his transcendental materialist project as "seek[ing] to delineate how the negativity of the *Cogito*-like subjectivity (especially in its related Kantian, Schellingian, Hegelian, and Lacanian manifestations) is internally generated out of material being."[277] Likewise, in his book-length study of Žižek's synthesis of Lacanian psychoanalysis and German idealism, Johnston elaborates further on their shared theoretical endeavor:

> The transcendental materialist theory of the subject is materialist insofar as it asserts that the Ideal of subjective thought arises from the Real of objective being, although it is simultaneously transcendental insofar as it maintains that this thus-generated Ideal subjectivity thereafter achieves independence from the ground of its material sources and thereby starts to function as a set of possibility conditions for forms of reality irreducible to the explanatory discourses allied to traditional versions of materialism.[278]

Following the Hegelian dictum that "substance is essentially subject," the theory posits an objective and thoroughly material basis for the foundation of subjectivity. However, the subject's autonomous existence avoids the pitfalls of eliminative materialists since its associated features of consciousness cannot be wholly reduced to the material ground from which it emerged.

Against the backdrop of their shared commitments, Johnston has, however, increasingly begun to highlight a few crucial differences between his and Žižek's respective positions:

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[277] Adrian Johnston, "'Naturalism or Anti-Naturalism? No, Thanks — Both Are Worse!': Science, Materialism, and Slavoj Žižek," *Revue Internationale de Philosophie* 261, no. 3 (2012): 321.
[278] Johnston, *Žižek's Ontology*, 275.

> Žižek and I, starting from a set of shared convictions, offer different answers to these questions. Žižek's universe privileges the structures and dynamics of quantum physics… and depicts libidinal economies as revolving around the enigmatic "x" of a primal emptiness. My ontological perspective focuses on how biology and its branches provide indispensable bridges between natural substances and denaturalized subjects… and recasts the negativities of drives and desires as secondarily emergent vis-à-vis evolutionary natural history. These are distinct visions, although they are equally rooted in an ensemble of axioms, decisions, intuitions, and references that Žižek and I hold in common.[279]

As opposed to physics, Johnston demands that "the dirty, messy onticness of concrete entities" characteristic of biology and natural sciences is crucial for any thoroughgoing materialism.[280] Additionally, Johnston has criticized Žižek's intermittent recourse to primal emptiness or void for operating like a mysterious *trīton gēnos* or third kind:

> Of special relevance… are my hesitations with respect to his occasional talk of there being, in addition to the two dimensions of nature and culture, some sort of un-derived third vector (whether labeled the "night of the world," the "death drive," vanishing mediator," etc.) as the root-source of what comes to be subjectivity proper in and for itself ($). On my view… Žižek's periodic summoning of a mysterious neither-natural-nor-cultural, as an arguably under- or un-explained supplement to his ontology, are both incompatible with an authentically materialist materialism as well as superfluous considering his Lacanian renditions of nature and culture are equally "barred" Others (*qua* inconsistent, conflict-ridden, and so on).[281]

In removing the shadowy third realm in between nature and culture, Johnston develops his notion of a weak nature as that which is capable of giving rise to the symbolic register of culture that is aligned with subjectivity.

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[279] Adrian Johnston, *A New German Idealism: Hegel, Žižek, and Dialectical Materialism* (New York: Columbia University Press, 2018), xii–xiii.
[280] Adrian Johnston, *Prolegomena to Any Future Materialism. Volume One: The Outcome of Contemporary French Philosophy*, Diaeresis (Evanston, Ill: Northwestern University Press, 2013), 107.
[281] Johnston, "'Naturalism or Anti-Naturalism?," 326–327.

As is also indicated in the above passage, Johnston insists that the tensions, scissions, and inconsistencies that he detects running throughout nature and culture are enough to motivate the rise of subject from substance:

> [T]he transcendental materialism of a weak nature I advocate, itself profoundly marked by [an] interlinked ontology and theory of the subject, gestures at a vision of nature as itself monstrous, as self-distorting (insofar as it explains the emergence out of nature of humans *qua* deranged monsters rebelling against nature requires a much weirder picture of nature than standard, traditional species of naturalism usually offer). This vision has no need… for imagining the presence of a supernatural excess/surplus as a neither natural-nor-cultural third power miraculously sparking the *ex nihilo* irruption of peculiarly human subjectivities running amok down the path of denaturalization. Self-sundering natural-material substance is auto-disruptive enough to account for these explosions of unrest, of the restlessness of negativity.[282]

That is to say, the monstrosity of "self-distorting" and "auto-disrupting" inconsistencies that are implicit in nature is sufficient for the emergence of equally maladapted humans. According to Johnston, recent developments in biology, epigenetics, and cognitive science provide a wealth of evidence that justifies his conceptions of both the natural and human world as being riven with contradictions.

In fact, Johnston maintains that subjectivity's break with nature cannot be a complete severing of their connection because the natural foundation continues to exert influence upon the subjective excrescence. For Johnston, psychoanalysis itself cannot condone an extreme and total detachment between non-natural subjectivity and natural materiality, as many of the human problems and dysfunctions diagnosed by psychoanalysis are manifested in connection with the natural biological basis in the human body. Thus, if the non-natural subject became entirely disengaged from the

[282] Adrian Johnston, "'Naturalism or Anti-Naturalism? No, Thanks — Both Are Worse!': Science, Materialism, and Slavoj Žižek," *Revue Internationale de Philosophie* 261, no. 3 (2012): 344.

naturalized material of the body, then the result would be that our human condition would be so ideal that none of us would have to find our way on to the analyst's couch.

5.4: The Meta-Formal Critique of Transcendental Materialism

Since Livingston tends to evaluate a given theoretical framework based on how well it can accommodate a certain global universalist scope, he opts for ontological orientations that favor totalization and completeness. For this reason, Livingston's main critique of Žižek's and Johnston's Hegelian-Lacanianism is a wavering he detects between the two metalogical choices he has delineated – between consistent incompleteness and inconsistent completeness:

> This equivocation or alternation with respect to the actual structural basis of the "non-All" that Žižek sees as the actual basis for the structure of subjectivity as such is replicated in Johnston's discussion. Thus, for instance, Johnston portrays the "necessary lack of full closure" that characterizes "the domain of manifest, concrete reality" as, on the one hand, "nothing other than the subject itself" but also, on the other, as indicating a "Real of being as a groundless ground shot through with tensions and scissions," which further suggests (he says) that "Being '*is*' this very acosmos, this unstable absence of a cohesive, unifying One-All."[283]

Even though the charge here is about vacillating between two logico-ontological orientations, Livingston attempts to drive home the objection by underlining how Žižek's and Johnston's fidelity to the German idealist tradition, with its emphasis on a finite subject as the basis for freedom, entails that the latter two are closer to consistent incompleteness than inconsistent completeness.

[283] Livingston, "Politics, Subjectivity and Cosmological Antinomy," 39.

Outside of the battles amongst competing philosophical tradition, the real lynchpin of Livingston's critique seems to involve a twofold objection aimed at the specifics of Johnston's transcendental materialist ontology.[284] The overall tenor of this critique involves Livingston questioning how a link could be drawn between an external "more-than-material" subject in relation to the world of material being. As Livingston puts it, "The problem… involves a 'transcendental' exteriority of this subject to the (constituted) world itself that cannot be easily justified directly on jointly realist and materialist grounds."[285] In short, how can a material reality give rise to and connect with a non-material subject?

The *first* specific formulation of this critique problematizes the relation between the subject and material reality, but expressly in terms of the temporal moment of the former's emergence. In other words, this objection is directed at the "problematic empirical moment" or the very instant "in historical time" when "anthropogenesis" or "ontogenesis" occurs: "It is difficult to imagine that this 'moment' of genesis could ever actually be specifically located on the basis of empirical or historical evidence, or that any such location would or could actually settle the question of the ultimate *structural* basis of such apparent phenomena as those of free will and spontaneity."[286] As Chiesa echoes, the problem here is about the complications in isolating the "moment" when the exact necessary and sufficient conditions for consciousness had been fulfilled.[287]

[284] Ibid., 42–43.
[285] Ibid., 42.
[286] Ibid., 43.
[287] Chiesa, *Not-Two*, 73.

The *second* closely related objection is aimed at transcendental materialism's emergentist response to the subject–object divide:

> [T]he conception according to which psychological reality, along with freedom and subjectivity themselves, emerge *from* a world at first entirely innocent of them relies on an emergentist picture of the relationships of scientific disciplines that is itself difficult to motivate on the level of a fundamental ontology. If, in particular, the world is conceived as basically subject to materialist constraints that it does include, on a basic ontological level, then it is difficult to see how their subsequent "emergence" does not simply amount to the arbitrary addition of essentially non-material ontological element to a world at first lacking them.[288]

Specifically, Livingston is attempting to cast doubt on whether the scientific resources mobilized by the transcendental materialist perspective, from neuroscience to genetics as well as several other branches of biology, successfully assist in the development of a philosophical account that is capable of accounting for how the subject arises from a wholly materialist and naturalist backdrop. In addition, the critique seems to be comprehensively aimed at the plausibility of securing such a binary connection between transcendental subjectivity and an immanent objectivity.

An odd issue in the debate is that if Livingston critiques Johnston for his inability to secure a link between competing levels in a dualist model, Chiesa's critique of Johnston's transcendental materialism seems to be directed at a monist conflation. In what is likely the first contribution to touch on some of the finer points in the debate between meta-formal realism and transcendental materialism, Chiesa summarizes his agreements and disagreements with the latter perspective:

[288] Livingston, "Politics, Subjectivity and Cosmological Antinomy," 43.

> Johnston has the great merit of having stubbornly insisted on how a dialectical materialism inspired by Lacan should, despite some of his claims, investigate the genesis of the transcendental / symbolic differentiality, opposing any veto against enquiring into the origins of language… One must indeed interrogate this phenomeno-logical out-of-jointedness as itself real. Yet calling into question the apparent exceptionality of the human condition… should not necessarily lead us to hurriedly conclude that "nature itself is disordered and out-of-joint," "a disharmonious, self-sundering Real." This conclusion – whereby the human "dis-adapted" condition is universalized, and thus paradoxically exalted – only displaces our basic ontological problem, that of the immanent genesis of the transcendental: the barred symbolic is differential because the barred real always-already was so, but, if this is the case, (1) how does symbolic difference differ from real difference?; (2) where does the difference of the barred real qua "prior" to the difference of the barred symbolic come from?[289]

In other words, Chiesa argues that Johnston helps himself to an ad hoc assumption by supposing that the inherent inconsistency that is a necessary feature of the symbolic register of human consciousness and culture is reflected in the natural and material register of the real.

5.5: The Logical Materialism of Morphological Paradigms

It is my contention that logical morphology exemplified in the materialist paradigm can accommodate both meta-formal realism and transcendental materialism, while also responding to the above objections. I will begin my treatment of both theoretical platforms in order, beginning meta-formal realism and then moving to transcendental materialism. I will close by suggesting a further complementary suggestion that can mediate the differences between the two ontological systems.

[289] Chiesa, *Not-Two*, 72–3.

Clearly, as was indicated in the third chapter, this dissertation is in accord with Livingston's meta-formal realism by also recognizing the significance of the meta-logical contributions of Cantor, Turing, and Gödel. Even with Livingston presumably coming to accept both inconsistent completeness and incomplete consistency as compatible with his meta-formal realist project, logical morphology is equipped to employ both frameworks, since it would regard both sides of the dichotomy as objects of comparison. As Wittgenstein states in one of the remarks about objects of comparison which occurs at the center of the methodological section of the *Investigations* that, "Our clear and simple language-games are not preliminary studies for a future regimentation of language (*PI*: §130)." In other words, whether inconsistency or incompleteness are respectively exemplified in formalized paraconsistent logic or Zermelo–Fraenkel set theory, they will only be considered as objects of comparison that make no claim about how things in reality *must* be but are instead available to be used and discarded depending how they assist in resolving philosophical confusion.

In response to Johnston's objection, however, that alleges that meta-formal realism's distinction between inconsistency and incompleteness serves as an all-encompassing and comprehensive screen that blurs crucial philosophical complexities, my claim is that, by taking up the morphological solely as a paradigmatic object of comparison, it can avoid precisely these difficulties. Since there is no claim as to how reality must be or what it must conform to, the meta-logical duality in logical morphology is not upheld as a "universal philosophical master-matrix" that can explain everything. Instead, either side of the dichotomy is available for resolving philosophical questions, but may be discarded in cases where they obscure more than they clarify.

It is unclear to what extent the Wittgensteinian morphology that I have proposed is compatible with the Hegelian dialectical logic to which Johnston likely subscribes. Some hints can perhaps be gleaned from Johnston's interpretation of Hegel's panlogicism:

> Hegel indeed might be guilty of a certain variety of panlogism – and this insofar as one of the upshots of his Kant critique is that the organization and functioning of objective realities beyond subjects' concepts and logics, as knowable realities capable of being captured by these subjects, are organized and function in "conceptual"/"logical" ways (taking "conceptual"/"logical" in Hegel's broadened senses). To be knowable in and through subjects' thoughts, asubjective things must not be wholly alien and completely foreign to the forms and contents of thoughts. Therefore, if the forms and contents of the subjective thoughts of things known are logical and conceptual, then the dynamics and structures of these things themselves are similarly somehow logical and conceptual too. This might very well be panlogism of a particular (post-Kantian) type.[290]

As John McDowell has repeatedly insisted, whatever their differences, Wittgenstein and Hegel are consistent with one another insofar as both philosophers maintain that there is no grip on reality beyond its conceptual outfitting.[291] Furthermore, as Eckart Förster has sought to explain in his meticulous studies of the progression of German idealism, there are several aspects of Goethe's morphology that were carried over into Hegel's dialectical method.[292]

As for transcendental materialism, the account of the paradigm delineated throughout this dissertation, indeed, meets Johnston's materialist requirements. Furthermore, in the fourth chapter, we saw the material human body, as a paradigm,

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[290] Adrian Johnston, *Adventures in Transcendental Materialism: Dialogues with Contemporary Thinkers*. Speculative Realism, (Edinburgh: Edinburgh University Press, 2014) 42.
[291] John Henry McDowell, *Mind and World: With a New Introduction*, (Cambridge, Mass: Harvard University Press, 1996) 44.
[292] Eckart Förster, *The Twenty-Five Years of Philosophy: A Systematic Reconstruction*, (Cambridge, Mass: Harvard University Press, 2012) 288–292.

served to clarify the perplexities posed by human consciousness. Whereas it might appear that his focus on the physical lands him into an untenable behaviorism, Wittgenstein tells us that he does not seek to substantiate claims about the existence of consciousness any more than he did about extensional existence of classes of number:

> The confusion and barrenness of psychology is not to be explained by its being a "young science"; its state is not comparable with that of physics, for instance, in its beginnings. (Rather, with that of certain branches of mathematics. Set theory.) For in psychology, there are experimental methods and conceptual confusion. (As in the other case, conceptual confusion and methods of proof.)
>
> The existence of the experimental method makes us think that we have the means of getting rid of the problems which trouble us; but problem and method pass one another by.
>
> An investigation entirely analogous to our investigation of psychology is possible also for mathematics… (*PI*: xiv.371–372)

Even though this outlook seems to reject the cutting-edge empirical research that is central to the transcendental materialist project, it can be more interpreted sympathetically as a recommendation to arrange the preexisting data for its perspicuous presentation.

As for the two-fold critiques of the alleged dualism and monism inherent in transcendental materialism, it seems to me that the soundest means for tackling the issue is to call on a theoretical commonality between Livingston and Johnston. In particular, both sides of the debate have expressed approval regarding certain aspects of Giorgio Agamben's *Homo Sacer* project.[293] For example, Johnston employs Agamben's *zoē/bios*

[293] Giorgio Agamben, *Homo Sacer: Sovereign Power and Bare Life*, (Stanford, Calif: Stanford University Press, 1998). Adrian Johnston and Catherine Malabou, *Self and Emotional Life: Philosophy, Psychoanalysis, and Neuroscience*. Insurrections, (New York: Columbia University Press, 2013), 192–193. Livingston, *Politics of Logic*, 8, 11, 38–41, 57, 60.

distinction in order elaborate on current developments in neuroscience that the former marshalls to corroborate his transcendental materialist outlook:

> [A] combination of Agamben's handling of the *zoē-bios* distinction with my position which is neither naturalist nor antinaturalist, a position centered on a hypothesized failed dialectic of incomplete denaturalization that is constitutive of human forms of subjectivity, enables the following to be said apropos a Lacan-influenced neuro-psychoanalytic metapsychology of affect: In human beings, the *zoē* of bare emotional life - this life doesn't disappear with the advent of the *bios* of feelings and the array of their accompanying conditions of possibility, but is only partially eclipsed and absorbed by the mediating matrices giving shape to *bios* - is fractured… into unsublated brute, raw basic emotions (which manifest themselves solely in rare, extreme conditions) and sublated feelings as sociosymbolically translated emotions (or even, following Žižek, as affective states aroused by the gap between emotions and feelings). In Žižek's parlance, the life 1.0 of *zoē*, although inverted into the produced exception instead of the given rule in the never-finished denaturalization brought about by subjectification, resists being take up without remainder into the nonnatural or not-wholly-natural defiles of *bios* as life 2.0. The updates don't erase entirely the early versions, with bugs, glitches, and loopholes being generated by the unsynthesized layering of these material temporal-historical strata.[294]

Despite my sympathies for Agamben's inversion of the distinction between nature and culture, it is hard to see how this account could be squared with Johnston's transcendental materialist reading above. That is, if we follow Agamben by taking the pseudo-natural bare life of *zoē* to be produced by the human social collectivity of *bios*, then the status of positing a pre-social and actually natural *zoē* would be utterly indeterminate. That is to say, it appears that it would be impossible to distinguish between the *zoē* in the state of nature and *zoē* in the state of an emergency. Therefore, Livingston's worries about how to secure a connection between the two-sided dualism of the transcendental and the materialist levels in the transcendental materialist project appears to be apt.

[294] Johnston and Malabou, *Self and Emotional Life*, 194.

As for my suggestion as to how to mediate this disagreement, I can only make the faint proposal to follow the misplaced trail of morphology. For it is with morphology that the sharp lines of the formal and the blurred edges of the biological can thereby meet. Without space to the make the argument here, I will just mention that the morphological research in the life sciences initiated by D'Arcy Wentworth Thompson and was enthusiastically expanded by no other Alan Turing in his late masterpiece "The Chemical Basis of Morphogensis." So, even though remains *the* "secret law," it is often followed in contemporary research that spans the interests of both thinkers addressed in this chapter.

References

Works by Ludwig Wittgenstein:

Wittgenstein, Ludwig, C. Grant Luckhardt, and Maximilian Aue. *The Big Typescript, TS. 213*. German-English scholar's ed. Malden, MA: Blackwell Pub, 2005. (*BT*)

Wittgenstein, Ludwig. *Culture and Value: a Selection from the Posthumous Remains*. Edited by Georg Henrik von Wright, Heikki Nyman, and Alois Pichler. Translated by Peter Winch. Rev. 2. ed. with Engl. transl; Repr. Oxford: Blackwell, 2004. (*CV*)

———. *Last Writings on the Philosophy of Psychology. Vol. 1: Preliminary Studies for Part 2 of "*Philosophical Investigations.*"* Edited by Georg Henrik von Wright and Heikki Nyman. Translated by C. G. Luckhardt and Maximilian A. E. Aue. Vol. 1. Oxford, UK Cambridge, USA: Blackwell, 1990. (*LW I*)

———. *Wittgenstein's Nachlass. The Bergen Electronic Edition*. (2015). Wittgenstein Archive, University of Bergen, Bergen, Norway. Edited by the Wittgenstein-Archiv, University of Bergen under the direction of Claus Huitfeldt. Incorporates *Wittgenstein-Source Bergen Facsimile Edition*, ed. Alois Pichler, with the assistance of H. W. Kruger, D. C. P. South, T. M. Bruvik, A. Lindebjerg, V. Olstad, Wittgenstein-Archiv University of Bergen (www. wittgensteinsource.org). Originally published by Intelex Corporation and Oxford University Press as *The Published Works of Ludwig Wittgenstein* (1993), *The Collected Works of Ludwig Wittgenstein* (1998) *Wittgenstein Letters and Lectures* (2001), and *Wittgenstein: Tagebücher und Briefe* (2002). (*BEE*)

———. *Notebooks 1914–1916*. Edited by Georg Henrik von Wright and G. E. M. Anscombe. Translated by G. E. M. Anscombe. 2. ed., Repr. Oxford: Blackwell, 1979. (*NB*)

———. *On Certainty*. Edited by Georg Henrik von Wright and Gertrude Elizabeth Margaret Anscombe. Translated by Gertrude Elizabeth Margaret Anscombe and Dennis Paul. Oxford: B. Blackwell, 1979. (*OC*)

———. *Philosophical Investigations*. Edited by P. M. S. Hacker and Joachim Schulte. Translated by G. E. M. Anscombe. Rev. 4th ed. Malden, MA: Wiley-Blackwell, 2009. (*PI*)

Wittgenstein, Ludwig, James Carl Klagge, and Alfred Nordmann. *Philosophical Occasions, 1912-1951*. Indianapolis: Hacket Pub. Co, 1993. (*PO*)

Wittgenstein, Ludwig. *Philosophical Remarks*. Edited by Rush Rhees. Translated by Raymond Hargraves and Roger White. Reprint. Oxford: Blackwell, 1990. (*PR*)

———. *Remarks on Colour*. Edited by Gertrude Elizabeth Margaret Anscombe. Translated by Linda L. McAlister and Margarete Schättle. 1. publ., [Repr.]. Oxford: Blackwell, 1977. (*RC*)

———. *Remarks on the Foundations of Mathematics*. Edited by Rush Rhees and Georg Henrik von Wright. Translated by Gertrude Elizabeth Margaret Anscombe. 3d ed., Rev. Oxford: B. Blackwell, 2001. (*RFM*)

———. "Remarks on Frazer's *Golden Bough*." In *Philosophical Occasions*, edited by James Klagge and Alfred Nordmann, translated by John Beversluis. Indianapolis: Hackett Publishing Company, 1993. (*GB*)

———. *Remarks on the Philosophy of Psychology. Vol. 1*. Edited by G. E. M. Anscombe and G. H. von Wright. Translated by G. E. M. Anscombe. Repr. Vol. 1. Oxford: Blackwell, 1980. (*RPP I*)

———. *Remarks on the Philosophy of Psychology, Vol. 2*. Edited by G. H. von Wright and Heikki Nyman. Translated by C. G. Luckhardt and Maximilian A. E. Aue. Oxford: Basil Blackwell, 1980. (*RPP II*)

———. "Review: P. Coffey, *The Science of Logic*." *The Cambridge Review* 34, no. 853 (1913): 351. (*RC 1913*)

———. *Tractatus Logico-Philosophicus*. Translated by C. K. Ogden, Frank Ramsey, Brian McGuinness, and David Pears. Side-By-Side-By-Side Edition, Version 0.63 (July 14, 2022). London, UK: Kegan Paul, 1922. (*TLP*)

Wittgenstein, Ludwig, and Friedrich Waismann. *The Voices of Wittgenstein: The Vienna Circle*. Edited by Gordon P. Baker. London: Routledge, 2003. (*WWK 1979*)

Wittgenstein, Ludwig. *Wittgenstein's Lectures on the Foundations of Mathematics Cambridge, 1939*. Edited by Cora Diamond. Ithaca, NY: Cornell University Press, 1976. (*LFM*)

Wittgenstein, Ludwig. *Zettel*. Edited by Georg Henrik von Wright and G. E. M. Anscombe. Translated by G. E. M. Anscombe. Berkeley, Calif.: Univ. of California Press, 1970. (*Z*)

General References:

Abbott, Stephen. *Understanding Analysis*. 2nd ed. New York: Spring, 2015.

Agamben, Giorgio. *Homo Sacer: Sovereign Power and Bare Life*. Homo Sacer 1. Stanford, Calif: Stanford University Press, 1998.

———. *The Open: Man and Animal*. Meridian, Crossing Aesthetics. Stanford, Calif: Stanford University Press, 2004.

———. *The Signature of All Things: On Method*. New York: Cambridge, Mass: Zone Books; Distributed by the MIT Press, 2009.

Anscombe, G. E. M. *An Introduction to Wittgenstein's Tractatus*. New York, NY: Harper & Row, 1959.

Aristotle. *De Interpretatione*. Edited by L. Minio-Paluello. Oxford: Oxford University Press, n.d.

———. *Ethica Nicomachea*. Edited by J. Bywater. Oxford: Clarendon Press, 1894.

Badiou, Alain, Ray Brassier, and Alberto Toscano. *Theoretical Writings*. London; New York: Continuum, 2004.

———. *Wittgenstein's Antiphilosophy*. Translated by Bruno Bosteels. London; New York: Verso, 2011.

Baker, Gordon P. *Wittgenstein, Frege, and the Vienna Circle*. Oxford: Blackwell, 1988.

Benveniste, Émile. "Active and Middle Voice in the Verb." In *Problems in General Linguistics*, 145–53. University of Miami Press, 1971.

Berto, Francesco. *There's Something about Gödel: The Complete Guide to the Incompleteness Theorem*. Malden, MA: Wiley-Blackwell, 2009.

Bonfanti, Angelo. "Le Forme Dell'Analogia Studi Sulla Filosofia Di Enzo Melandri (The Forms of Analogy: A Study on the Philosophy of Enzo Melandri)." PhD Dissertation, Università Degli Studi Di Bergamo, 2014.

———. *Le Forme Dell'analogia: Studi Sulla Filosofia Di Enzo Melandri*. I edizione. Ariccia (RM): Aracne editrice int.le S.r.l, 2016.

Burns, Michael O'Neill, and Brian Smith. "Materialism, Subjectivity and the Outcome of French Philosophy: Interview with Adrian Johnston." *Cosmos and History: The Journal of Natural and Social Philosophy* 7, no. 1 (2011): 167–81.

Breithaupt, Fritz, and Richard Raatzsch, eds. *Goethe and Wittgenstein: Seeing the World's Unity in its Variety*. Wittgenstein-Studien 5. Frankfurt am Main: P. Lang, Europäischer Verl. der Wissenschaften, 2003.

Bronzo, Silver. "Context, Compositionality, and Nonsense in Wittgenstein's Tractatus." In *Beyond The Tractatus Wars: The New Wittgenstein Debate*, edited by Rupert Read and Matthew A. Lavery, 84–111. New York: Routledge, 2011.

Caisley, Jennifer. "Matter (Materie)." *Goethe-Lexicon of Philosophical Concepts* 2, no. 1 (2022): https://goethe-lexicon.pitt.edu/GL/article/view/60

Callanan, John J. “The Boundary of Pure Reason.” In *Kant’s Prolegomena: A Critical Guide*, edited by Peter Thielke, 133–53. Cambridge; New York, NY: Cambridge University Press, 2022.

Cantor, Georg. “On an Elementary Question in the Theory of Manifolds.” In *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, edited and translated by William Bragg Ewald, 920–922. Oxford: Oxford University Books, 1996.

Cassin, Barbara, Steven Rendall, and Emily S. Apter, eds. *Dictionary of Untranslatables: A Philosophical Lexicon*. Princeton: Princeton University Press, 2014.

Cavell, Stanley. *This New yet Unapproachable America: Lectures after Emerson after Wittgenstein*. University of Chicago Press ed. Frederick Ives Carpenter Lectures 1987. Chicago: University of Chicago Press, 2013.

Chiesa, Lorenzo. *The Not-Two: Logic and God in Lacan*. Cambridge, Massachusetts: MIT Press, 2016.

Cohen, Michael. “Tractatus 5.542.” *Mind* 83, no. 331 (1974): 442–44.

Conant, James. “The Method of the *Tractatus*.” In *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*, edited by Erich Reck, 374–462. Oxford, UK: Oxford University Press, 2001.

Conant, James, and Cora Diamond. “On Reading the *Tractatus* Resolutely: Reply to Meredith Williams and Peter Sullivan.” In *Wittgenstein’s Lasting Significance*, edited by Max Kölbel and Bernhard Weiss, 42–97. New York, NY: Routledge, 2004.

Conant, James. “Two Conceptions of *Die Uberwindung der Metaphysik*: Carnap and Early Wittgenstein.” In *Wittgenstein in America*, edited by T McCarthy and S. C. Stidd, 13–61. Oxford, UK: Clarendon Press, 2001.

———. “Wittgenstein on Meaning and Use.” *Philosophical Investigations* 21, no. 3 (1998): 222–50.

Diamond, Cora. “How Long Is the Standard Meter in Paris.” In *Wittgenstein in America*, edited by T. McCarthy and S. C. Stidd, 104–39. Oxford, UK: Clarendon Press, 2001.

———. “Logical Syntax in Wittgenstein’s *Tractatus*.” *The Philosophical Quarterly* 55, no. 218 (2005): 78–89.

———. *The Realistic Spirit: Wittgenstein, Philosophy, and the Mind*. Cambridge, Mass.: MIT Press, 2011.

———. “Saying and Showing: An Example from Anscombe.” In *Post-Analytic* Tractatus, edited by Barry Stocker, 151–66. London, UK: Routledge, 2004.

Fichte, Johann Gottlieb. *Foundations of Transcendental Philosophy (Wissenschaftslehre) Nova Methodo (1796/99)*. Translated by Daniel Breazeale. Ithaca: Cornell University Press, 1992.

———. *Science of Knowledge; with the First and Second Introductions*. Translated by Peter Heath and John Lachs. Cambridge: Cambridge University Press, 1982.

———. *The System of Ethics: According to the Principles of the Wissenschaftslehre*. Translated by Daniel Breazeale and Günter Zöller. Cambridge, UK: Cambridge University Press, 2005.

Floyd, Juliet. “Aspects of Aspects.” In *Cambridge Companion to Wittgenstein*, edited by Hans Sluga and David G. Stern, 361–88. Cambridge, UK: Cambridge University Press, 2018.

———. “Chains of Life: Turing, *Lebensform*, and the Emergence of Wittgenstein’s Later Style.” *Nordic Wittgenstein Review* 5, no. 2 (December 16, 2016): 7–89.

———. “*Lebensformen*: Living Logic.” In *Language, Form(s) of Life, and Logic*, edited by Christian Martin, 59–92. De Gruyter, 2018.

———. “On Saying What You Really Want to Say: Wittgenstein, Gödel and the Trisection of the Angle.” In *From Dedekind to Gödel: The Foundations of Mathematics in the Early Twentieth Century*, edited by Jaakko Hintikka, 251:373–426. Synthese Library. Kluwer Academic Publishers, 1995.

———. “Wittgenstein’s Diagonal Argument: A Variation on Cantor and Turing.” In *Epistemology versus Ontology: Essays on the Philosophy and Foundations of Mathematics in Honour of Per Martin-Löf*, edited by Peter Dybjer, Sten Lindström, Erik Palmgren, and Göran Sundholm, 25–44. Dordrecht: Springer, 2012.

———. “Wittgenstein and Turing.” In *Philosophy of Logic and Mathematics: Proceedings of the 41st International Ludwig Wittgenstein Symposium*, edited by Gabriele M. Mras, Paul Weingartner and Bernhard Ritter, 263–296. Berlin, Boston: De Gruyter, 2020.

Floyd, Juliet, and Felix Mühlhölzer. *Wittgenstein’s Annotations to Hardy’s* Course of Pure Mathematics*: An Investigation of Wittgenstein’s Non-Extensionalist Understanding of the Real Numbers*. Cham, Switzerland: Springer, 2020.

Förster, Eckart. *The Twenty-Five Years of Philosophy: A Systematic Reconstruction*. Cambridge, Mass: Harvard University Press, 2012.

Fraenkel, Abraham. “Zum Diagonalverfahren Cantors.” *Fundamenta Mathematicae* 25 (1935): 45–50.

Frances, Cooper. “The Subject and/of the Law: Yan Thomas and the Excess of History over Concept.” In *Afterlives: Transcendentals, Universals, and Others*, edited by Peter Osborne, 4:72–100. CRMEP Book, 2022.

Frege, Gottlob. *Basic Laws of Arithmetic, Derived Using Concept-Script: Volumes I & II*. Edited by Philip A. Ebert, Marcus Rossberg, and Crispin Wright. 1st ed. Oxford, U.K: Oxford University Press, 2013.

———. *The Foundations of Arithmetic*. Translated by J. L. Austin. 2nd ed. Evanston, Illinois: Northwestern University Press, 1953.

———. *Translations from the Philosophical Writings of Gottlob Frege*. Edited by Peter Geach and Max Black. 2nd ed. Oxford: Basil Blackwell, 1960.

Friedlander, Eli. “Logic, Ethics and Existence in Wittgenstein’s Tractatus,” 2018. https://www.academia.edu/34491395/Logic_Ethics_and_Existence_in_Wittgensteins_Tractatus.

———. “Missing a Step Up the Ladder:” *Philosophical Topics* 42, no. 2 (2014): 45–73.

———. *Signs of Sense: Reading Wittgenstein’s Tractatus*. Cambridge, Mass: Harvard University Press, 2001.

———. “Wittgenstein, Benjamin and Pure Realism.” In *Wittgenstein and Modernism*, edited by Michael LeMahieu and Karen Zumhagen-Yekplé, 114–29. Chicago; London: The University of Chicago Press, 2017.

———. “Wittgenstein and Goethe: The Life of Colors,” 2011. https://www.academia.edu/49287803/Wittgenstein_and_Goethe_The_Life_of_Colors.

Goethe, Johann Wolfgang von, Douglas Miller, and Johann Wolfgang von Goethe. *Scientific Studies*. Goethe Edition, v. 12. New York, N.Y: Suhrkamp, 1988.

Goldfarb, Warren. “Metaphysics and Nonsense: On Cora Diamond’s *The Realistic Spirit*.” *Journal of Philosophical Research* 22 (1997): 57–73.

Glock, Hans-Johann. “All Kinds of Nonsense.” In *Wittgenstein at Work: Method in the Philosophical Investigations*, edited by E. Ammereller and E. Fischer, 221–45. London: Routledge, 2004.

Grève, Sebastian Sunday. “Logic and Philosophy of Logic in Wittgenstein.” *Australasian Journal of Philosophy* 96, no. 1 (2018): 168–82.

Hacker, P. M. S. *Insight and Illusion: Themes in the Philosophy of Wittgenstein*. Rev. ed. Oxford: Clarendon Press; Oxford University Press, 1986.

———. “When the Whistling Had to Stop.” In *Wittgensteinian Themes: Essays in Honour of David Pears*, edited by D. Charles and W. Childs, 141–69. Oxford, UK: Clarendon Press, 2001.

———. “Wittgenstein, Carnap and the New American Wittgensteinians.” *The Philosophical Quarterly* 53, no. 210 (2003): 1–23.

Heidegger, Martin. *Being and Time: A Translation of Sein und Zeit*. Translated by Joan Stambaugh. SUNY Series in Contemporary Continental Philosophy. Albany, NY: State University of New York Press, 1996.

Ishiguro, Hide. “Wittgenstein and the Theory of Types.” In *Perspectives on the Philosophy of Wittgenstein*, edited by Irving Block, 43–60. New York, NY: Blackwell, 1981.

Johnston, Adrian. *Adventures in Transcendental Materialism: Dialogues with Contemporary Thinkers*. Speculative Realism. Edinburgh: Edinburgh University Press, 2014.

———. “Confessions of a Weak Reductionist: Responses to Some Recent Criticisms of My Materialism.” (Unpublished Version, 2015).

———. “’Naturalism or Anti-Naturalism? No, Thanks — Both Are Worse!’: Science, Materialism, and Slavoj Žižek.” *Revue Internationale de Philosophie* 261, no. 3 (2012): 321–46.

———. *A New German Idealism: Hegel, Žižek, and Dialectical Materialism*. New York: Columbia University Press, 2018.

———. *Prolegomena to Any Future Materialism. Volume One: The Outcome of Contemporary French Philosophy*. Diaeresis. Evanston, Ill: Northwestern University Press, 2013.

———. “The Weakness of Nature: Hegel, Freud, Lacan, and Negativity Materialized.” In *Hegel and the Infinite: Religion, Politics, and Dialectic*, edited by Slavoj Žižek, Clayton Crockett, and Creston Davis, 159–79. New York: Columbia University Press, n.d.

———. *Žižek’s Ontology: A Transcendental Materialist Theory of Subjectivity*. Evanston, Ill: Northwestern University Press, 2008.

Johnston, Adrian, and Catherine Malabou. *Self and Emotional Life: Philosophy, Psychoanalysis, and Neuroscience*. Insurrections. New York: Columbia University Press, 2013.

Kant, Immanuel. *Critique of Judgement*. Edited by Nicholas Walker. Translated by James Creed Meredith. Oxford: Oxford University Press, 2007.

———. *Critique of Pure Reason*. Translated by Paul Guyer and Allen William Wood. Cambridge: Cambridge University Press, 2000.

———. *Lectures on Logic*. Translated by J. Michael Young. Cambridge: Cambridge University Press, 1992.

———. *Prolegomena to Any Future Metaphysics That Will Be Able to Come Forward as Science: With Selections from the Critique of Pure Reason*. Edited by Gary C. Hatfield. Rev. ed. Cambridge Texts in the History of Philosophy. Cambridge, UK; New York: Cambridge University Press, 2004.

Kenny, Anthony. *Wittgenstein*. 4. print. Cambridge, Mass: Harvard Univ. Press, 1981.

Krkač, Kristijan. *A Custodian of Grammar: Essays on Wittgenstein's Philosophical Morphology*. Lanham: University Press of America, Inc, 2012.

Kuhn, Thomas S. *The Structure of Scientific Revolutions*. 3rd ed. Chicago, IL: University of Chicago Press, 1996.

Kuusela, Oskari. "The Dialectic of Interpretations: Reading Wittgenstein's *Tractatus*." In *Beyond the* Tractatus *Wars: The New Wittgenstein Debate*, edited by Rupert Read and Matthew A. Lavery, 121–48. New York, NY: Routledge, 2011.

———. *Wittgenstein on Logic as the Method of Philosophy: Re-Examining the Roots and Development of Analytic Philosophy*. First edition. Oxford, United Kingdom: Oxford University Press, 2019.

Lacan, Jacques. "Seminar on 'The Purloined Letter.'" Translated by Jeffrey Mehlman. *Yale French Studies*, no. 48 (1972): 39.

———. *Encore: The Seminar of Jacques Lacan Book XX*. Edited by Jacques-Alain Miller. Translated by Bruce Fink. New York: Norton & Co., 1999.

Lewis, David. "Counterfactual Dependence and Time's Arrow." *Nous* 13, no. 4 (1979): 455–76.

Livingston, Paul M. "Agamben, Badiou, and Russell." *Continental Philosophy Review* 42, no. 3 (2009): 297–325.

———. "The Breath of Sense: Language, Structure, and the Paradox of Origin." *Konturen* 2, no. 1 (October 11, 2010): 14. https://doi.org/10.5399/uo/konturen.2.1.1311.

———. *The Logic of Being: Realism, Truth, and Time*. Evanston, Illinois: Northwestern University Press, 2017.

———. "'Meaning Is Use' in the *Tractatus*." *Philosophical Investigations* 21, no. 1 (2004): 34–67.

———. “Naturalism, Conventionalism, and Forms of Life: Wittgenstein and the ‘Cratylus.’” *Nordic Wittgenstein Review* 4, no. 2 (2015): 7–38.

———. *Philosophy and the Vision of Language*. New York: Routledge, 2008.

———. *The Politics of Logic: Badiou, Wittgenstein, and the Consequences of Formalism*. New York: Routledge, 2012.

———. “Politics, Subjectivity and Cosmological Antinomy: Kant, Badiou and Žižek.” *Crisis & Critique* 1, no. 2 (2014): 22–51.

———. “Realism and the Infinite.” *Speculations* IV (2013): 1–38.

———. “Wittgenstein and Parmenides,” October 15, 2009. https://www.academia.edu/923400/Wittgenstein_and_Parmenides.

———. “Wittgenstein Reads Heidegger, Heidegger Reads Wittgenstein: Thinking Language Bounding World.” In *Beyond the Analytics-Continental Divide: Pluralist Philosophy in the Twenty-First Century*, edited by Andrew Cutrofello, Paul M. Livingston, and Jeffrey A. Bell, 222–48. New York: Routledge, 2015.

Longuenesse, Béatrice. *Kant on the Human Standpoint*. Modern European Philosophy. Cambridge: Cambridge University Press, 2005.

Luckhardt, C. G. “Beyond knowledge: Paradigms in Wittgenstein’s Later Philosophy.” *Philosophy and Phenomenological Research* 39, no. 2 (1978): 240–252.

Mácha, Jakub. *The Philosophy of Exemplarity: Singularity, Particularity, and Self-Reference*. New York, NY: Routledge, Taylor & Francis Group, 2023.

Malcolm, Norman. *Ludwig Wittgenstein: A Memoir*. 2nd ed. Oxford: Oxford University Press, 1984.

Mancosu, Paolo. *The Philosophy of Mathematical Practice*. Oxford ; New York: Oxford University Press, 2008.

Mather, Ronald. “On the Mythology of the Reflexive Subject.” *History of the Human Sciences* 10, no. 4 (November 1997): 65–82.

McDowell, John Henry. *Mind and World: With a New Introduction*. 1st Harvard University Press paperback ed. Cambridge, Mass: Harvard University Press, 1996.

McGinn, Marie. *Elucidating the* Tractatus*: Wittgenstein’s Early Philosophy of Logic and Language*. Oxford: Clarendon Press; Oxford University Press, 2009.

Medina, José. *The Unity of Wittgenstein's Philosophy: Necessity, Intelligibility, and Normativity*. Albany: State University of New York Press, 2002.

Meixner, Sebastian. "Urphänomen (Original/Primordial Phenomenon)." *Goethe-Lexicon of Philosophical Concepts* 2, no. 1 (2022): https://goethe-lexicon.pitt.edu/GL/article/view/46#

Melandri, Enzo. *La linea e il circolo: studio logico-filosofico sull'analogia*. Macerata: Quodlibet, 2004.

Monk, Ray. *Ludwig Wittgenstein: The Duty of Genius*. A Penguin Book Biography. New York, NY: Penguin Books, 1991.

Moore, A. W. "Being, Univocity, and Logical Syntax." *Proceedings of the Aristotelian Society (Hardback)* 115, no. 1pt1 (April 2015): 1–23.

Narboux, Jean-Philippe. "Showing, the Medium Voice, and the Unity of the Tractatus." *Philosophical Topics* 42, no. 2 (2014): 201–62.

———. "Simplicity and Rigidity: Reading Section 50 of Wittgenstein's *Philosophical Investigations* After Kripke." In *Finding One's Way Through Wittgenstein's* Philosophical Investigations*: New Essays on §1–88*, edited by J.P Narboux and E. Bermon, 95–159. Cham, Switzerland: Spring, 2017.

Oxford English Dictionary, 2nd ed. Oxford: Oxford University Press, 2022.

Petzold, Charles. *The Annotated Turing: A Guided Tour through Alan Turing's Historic Paper on Computability and the Turing Machine*. Indianapolis, IN: Wiley Pub, 2008.

Pinotti, Andrea. "Origin vs. Genesis: Warburg and Benjamin in the Footsteps of Goethe's Morphology." The Warburg Institute, School of Advanced Study University of London, UK, 2012. https://youtu.be/xZnz_ZXAav4.

Plato, *Platonis Opera*. Edited by John Burnet. Oxford, UK: Oxford University Press, 1903.

Potter, Michael D. *Wittgenstein's Notes on Logic*. Oxford; New York: Oxford University Press, 2009.

Priest, Graham. *Beyond the Limits of Thought*. Cambridge, UK: Cambridge University Press, 1995.

———. *In Contradiction: A Study of the Transconsistent*. Expanded ed. Oxford: Oxford: Clarendon Press; Oxford University Press, 2006.

———. "Wittgenstein's Remarks on Gödel's Theorem." In *Wittgenstein's Lasting Significance*, edited by Max Kölbel and Bernhard Weiss, 207–27. New York, NY: Routledge, 2004.

Ramsey, F. P. “Critical Notice of L. Wittgenstein’s *Tractatus Logico-Philosophicus*.” *Mind* 32, no. 128 (1923): 465–78.

Ricketts, Thomas. “Analysis, Independence, Simplicity, and the General Sentence-Form” *Philosophical Topics* 42, no. 2 (2014): 263–88.

———. “Wittgenstein Against Frege and Russell.” In *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*, edited by Erich Reck, 227–51. Oxford, UK: Oxford University Press, 2002.

Riedl, Rupert. “The role of morphology in the theory of evolution.” In *Dimensions of Darwinism*, edited by M. Grene, pp. 205–238. Cambridge, UK: Cambridge University Press, 1983.

Rogers, H. *Theory of Recursive Functions and Effective Computability*. Cambridge, Mass: MIT Press, 1987.

Rotten, Elisabeth. *Goethes Urphänomen und die platonische Idee*. Gieben, Weimar: Verlag von Alfred Töpelmann, 1913.

Russell, Bertrand. “Mathematical Logic as Based on the Theory of Types.” *American Journal of Mathematics* 30, no. 3 (1908): 222–62.

———. *My Philosophical Development*. Abingdon, Oxon: Routledge Classics, 2023.

———. *Principles of Mathematics*. London: Routledge, 2010.

Sainsbury, R. M. *Paradoxes*. 3rd ed. Cambridge: Cambridge University Press, 2009.

Schopenhauer, Arthur. *Parerga and Paralipomena: Short Philosophical Essays*. Translated by E. F. J. Payne. Oxford: Clarendon Press; Oxford University Press, 2000.

———. *The World as Will and Representation, Vol 2*. Translated by Judith Norman, Alistair Welchman, and Christopher Janaway. Cambridge: Cambridge University Press, 2018.

Schürmann, Reiner. *Broken Hegemonies*. Bloomington, IN: Indiana University Press, 2003.

Smullyan, Raymond M. *Diagonalization and Self-Reference*. Oxford Logic Guides 27. Oxford; New York: Clarendon Press, 1994.

Spengler, Oswald. *The Decline of the West: Form and Actuality, Vol. 1*. Translated by Charles Frances Atkinson. New York: Alfred A. Knopf, 1932.

Stang, Nicholas F. “Kant on Complete Determination and Infinite Judgement.” *British Journal for the History of Philosophy* 20, no. 6 (December 2012): 1117–39.

Sullivan, Peter M. "'The General Propositional Form Is a Variable' (*Tractatus* 4.53)." *Mind* 113, no. 449 (January 1, 2004): 43–56.

———. "A Note on Incompleteness and Heterologicality." *Analysis* 63, no. 277 (January 2003): 32–38.

———. "Synthesizing without Concepts." In *Beyond the Tractatus Wars: The New Wittgenstein Debate*, edited by Rupert J. Read and Matthew A. Lavery, 171–89. New York: Routledge, 2011.

———. "The Totality of Facts." *Proceedings of the Aristotelian Society (Hardback)* 100, no. 1 (June 2000): 175–92.

———. "The 'Truth' in Solipsism and Wittgenstein's Rejection of the A Priori." *European Journal of Philosophy* 4 (1996): 195–219.

———. "On Trying to Be Resolute: A Response to Kremer on the *Tractatus*." *European Journal of Philosophy* 10, no. 1 (2002): 43–78.

———. "What Is Squiggle? Ramsey on Wittgenstein's Theory of Judgement." In *Ramsey's Legacy*, edited by Hallvard Lillehammer and D. H. Mellor, 53–70. Mind Association Occasional Series. Oxford: New York: Clarendon Press; Oxford University Press, 2005.

Suppes, Patrick. *Axiomatic Set Theory*. New York: Dover Publications, 1972.

Waismann, Friedrich. *The Principles of Linguistic Philosophy*. Edited by R Harre. London: MacMillan, 1965.

Whitehead, Alfred North, and Bertrand Russell. *Principia Mathematica*, 3 Vols, Second Edition. Cambridge: Cambridge University Press, 1962.

Zalabardo, José L. *Representation and Reality in Wittgenstein's* Tractatus. Oxford: Oxford University Press, 2015.

Žižek, Slavoj. "Fichte's Laughter." In *Mythology, Madness, and Laughter: Subjectivity in German Idealism*, edited by Markus Gabriel and Slavoj Žižek, 122–67. London: Continuum, 2009.

———. *Hegel in a Wired Brain*. New York: Bloomsbury Academic, 2020.

Zöller, Günter. "German Realism: The Self-Limitation of Idealist Thinking in Fichte, Schelling, and Schopenhauer." In *The Cambridge Companion to German Idealism*, edited by Karl Ameriks. Cambridge: Cambridge University Press, 2000.