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Towards A More Formal Understanding of *Anyāpoha*

BY

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DEDICATION

This thesis is dedicated to my parents Linda and Bruce Kasza and all teachers who believed in me when I did not believe in myself.

Towards A More Formal Understanding of *Anyāpoha*

by

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ABSTRACT

The aim of this thesis is to investigate if Dignāga's commitment that non-observation (*adarśanam*) of the reason (*adarśanam*) and property to be proven (*sādyā*) in the dissimilar example (*vyatireka dṛṣṭānta*) is alone sufficient to ground the exclusion of other referents (*anyāpoha*), as a valid inference for oneself (*svārthānumāna*) and proof for others (*parārthānumāna*). To answer this question, four formal accounts of Dignāga's view of the three characteristics (*Trairūpya*) of inference by Hayes, Katsura, Tillemans, and Oetke were consulted. I argue a formal logical account of *anyāpoha* shows that *vyatireka* as the lone sufficient example (*dṛṣṭānta*) in inference (*anumāna*) is false and does not lead to ascertainment (*niścaya*). Making a valid inference for oneself or a proof for others (*svārthānumāna* or *parārthānumāna*), requires that one must consider both examples (*anvaya* and *vyatireka dṛṣṭānta*) to gain ascertainment (*niścaya*) from the inference (*anumāna*).

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Introduction

Dignāga's *pramāṇavāda* theory is perhaps one of the most seminal influences on Buddhist epistemology and logic.¹ The doctrines of the three characteristics (*Trairūpya*) of inference (*anumāna*) and exclusion of other referents (*anyāpoha*) hold great importance to the understanding of the second *pramāṇa*, inference (*anumāna*). Formal explorations of inference (*anumāna*) that ground *anyāpoha* as the referent (*artha*) of a general term (*jātiśabda*), are similar but differ in their construction. Furthermore, the consequences of accepting the negative evidence of the dissimilar example (*vyatireka dr̥ṣṭānta*), based upon non-observation (*adarśanam*)² leads to questions about how exclusion of the other (*anyāpoha*) functions logically. I argue a logical formalization of *anyāpoha* shows that, if the *vyatireka* example (*dr̥ṣṭānta*) grounds *anyāpoha* by itself, then is not sufficient to produce ascertainment (*niścaya*) from inference (*anumāna*). The inclusion of the similar example (*anvaya dr̥ṣṭānta*) is necessary to gain ascertainment (*niścaya*) from that inference (*anumāna*) and thus be a valid inference for oneself and a proof for others (*svārthānumāna* and *parārthānumāna*) about the rule that has been given by someone who knows.³

An explication of the two types of cognition (*pramāṇas*), perception (*pratyakṣa*) and inference (*anumāna*), can be found in chapter one of Dignāga's *Pramāṇasamuccayavṛtti* (PSV), which provides us with the first glimpse into how Dignāga will argue for exclusion of other referents (*anyāpoha*):

¹ Mark Siderits et al., "Dignāga's Apoha Theory: It's Presuppositions and Main Theoretical Implications," in *Apoha: Buddhist Nominalism and Human Cognition* (New York, NY: Columbia University Press, 2011), pp. 64-83, 65. Hereafter, Dignāga's Apoha Theory.

² Ole Holten Pind, "Part 2," in *Dignāga's Philosophy of Language: Pramāṇasamuccayavṛtti V on anyāpoha*, ed. Ernst Steinkellner (Wien: Verlag der Österreichischen Akademie der Wissenschaften, 2015), 123. Hereafter Pind: Dignāga's Philosophy of Language.

³ Pind: Dignāga's Philosophy of Language, 183, 50b.

“PSV I 2a-b: the means of cognition are [immediate and mediate, namely,] perception (*pratyakṣa*) and inference (*anumāna*). They are only two because, “k2b-c, the object to be cognized has [only] two aspects.”⁴

In as much as there are only two means of cognition, perception (*pratyakṣa*) and inference (*anumāna*), they have divergent objects of cognition, perception (*pratyakṣa*) takes particulars as its object (*svalakṣaṇa/ bheda*) and inference (*anumāna*) takes universals as its object (*sāmānyalakṣaṇa/jāti*).⁵

A particular is the ineffable boundary that produces the horizon of knowing. A particular is the “absolutely distinct,” “point instant,” which is unintelligible and inexpressible.⁶ A particular, in being a “point instant,” is also seen as the event, which is the basis for the construction of the universal (*sāmānyalakṣaṇa, jāti*).⁷ In as much as they constitute the basis for the universal (*sāmānyalakṣaṇa, jāti*), the particulars accessible to perception (*pratyakṣa*) are free from the application of concepts, meaning they remain cognitively neutral or unmeaningful.⁸

However, it is suggested that the particular (*svalakṣaṇa*) is alone real and has the power of producing an effect (*arthakriyāśakti*).⁹ Further, particulars are rigorously real, in that a perception (*pratyakṣa*) that arises from a particular (*bheda*) is the same even when it is

⁴ Masaaki Hattori and Dignāga “Section 1. Exposition of the Theory of Perception,” in *Dignāga on Perception: Being the Pratyakṣapariccheda of Dignāga’s Pramāṇasamuccaya from the Sanskrit Fragment and the Tibetan Versions* (Cambridge, MA: Harvard University Press, 1968), 24. Hereafter, Hattori.

⁵ Hattori, 24.

⁶ Richard P. Hayes, *Dignāga on the Interpretation of Signs* (Kluwer Academic Publishers, 1987), 13. Hereafter, Hayes.

⁷ “M10_The Phenomenology of Meaning: Dinnāga to Ratnakīrti,” YouTube (YouTube, October 27, 2019), <https://www.youtube.com/watch?v=GHOEJWJW4sE&t=1386s>.

⁸ Hayes 24.

⁹ Hattori, 14.

broken and when there is mental abstraction.¹⁰ This concept plays an important role in explaining what Dignāga has in mind for the idea of the referent (*artha*), and what can be said about it with a general term (*jātiśabda*). As the referent (*artha*) is infinite (*arthasyānantye*)¹¹ and the cause of the general term (*jātiśabda*),¹² the referent (*artha*) could be represented by an unlimited number of terms; yet a general term (*jātiśabda*) doesn't produce doubt (*saṃśaya*) about its referent (*artha*) because the general term (*jātiśabda*) only denotes (*bhāṣate*) its own referent (*svārtham*) through the exclusion of other referents (*anyāpoha*).¹³

Traditionally, Buddhists have denied the metaphysical position that universals (*sāmānyalakṣaṇa*) have reality, and thus they are concepts that are mentally constructed, they have no bearing on the world beyond the mental states that are accessible to us through their pragmatic use.¹⁴ To understand why, Siderits and Chakrabarti suggest the theory that descriptive terms, like “cow” are considered to be real entities, but after all “no one can bind cowness with a rope, cut the tree's essence, or have lunch with humanity,” thus, the universals that allow for the production of these statements must have some imagined or constructed nature and cannot contain any substantial reality.¹⁵ In keeping true with this traditional understanding of the universal, Dignāga claims the exclusion of other referents (*anyāpoha*) is without division and without substance.¹⁶

¹⁰ Hayes, 96.

¹¹ Pind: Dignāga's Philosophy of Language, 119.

¹² Pind: Dignāga's Philosophy of Language, 121.

¹³ Pind: Dignāga's Philosophy of Language, 2, 131.

¹⁴ Hayes, 20.

¹⁵ Mark Siderits et al., “Apoha: Buddhist Nominalism and Human Cognition,” in *Apoha: Buddhist Nominalism and Human Cognition* (New York, NY: Columbia University Press, 2011), pp. 1-49, 3. Hereafter, Chakra and Siderits.

¹⁶ Pind: Dignāga's Philosophy of Language, 135.

Inference (*anumāna*) is the second kind of cognition in the *pramāṇavāda* tradition, and it takes universals as its objects. Dignāga’s aim is to explain how the structure of inference (*anumāna*) that we make for ourselves and a proof for others (*svārthānumāna* and *parārthānumāna*), without giving rise to doubt (*saṃśaya*) about a word’s application to its referent by excluding of other referents (*anyāpoha*). Inferences (*anumāna*) are conventionally real, which means there is no perception of them when they are broken into parts.¹⁷ Nevertheless, the parts of inference (*anumāna*) are known to us through the doctrine of *Trairūpya* (to be explained in depth shortly).

With an understanding of inference (*anumāna*), Dignāga claims in the first twelve verses of the PSV V that he can show how other theories of a universal term fail to accomplish denotation without some type of doubt (*saṃśaya*) arising from them. Dignāga claims his theory of *anyāpoha* avoids these problems in verses PSV V 34a-36d. In those verses, he claims, the general term (*jātiśabda*) “existent” (*sat*) does not denote (*bhāṣate*): 1. all its own particulars (*bhedānām*) due to ambiguity, nor 2. the general property (*jātimātre*), nor 3. the inherence relation (*tadyoga*), nor 4. the mere fact of possessing the general property (*jātimanmātra*).¹⁸ Exclusion of other referents (*anyāpoha*) is not a particular (*bheda*), so it does not apply to any particulars, it is not dependent, nor does it transfer its meaning, nor is it ambiguous, and it does not pervade any particulars because it has none. It can do this, because unlike the four other views of the general term (*jātiśabda*), *anyāpoha* is without division and without substance.¹⁹ This insight will give us an idea of what he has in mind when we start to generate our formal account of this conception of *Trairūpya*.

¹⁷ Hayes, 95.

¹⁸ Pind: Dignāga’s Philosophy of Language, 116, 128, 130, 132.

¹⁹ Pind: Dignāga’s Philosophy of Language, 133-135

Anyāpoha

Dignāga’s answer to the problem of the general term’s (*jātiśabda*) relationship between “language and world” is explained by the exclusion of other referents (*anyāpoha*) through means of the third characteristic, the dissimilar example (*vyatireka dr̥ṣṭānta*) of the *Trairūpya*.²⁰ It should be mentioned that the terms, ‘restriction’ (*niyama*), ‘pervasion’ (*vyāpti*), and ‘concomitance’ (*anubandha*) are all synonymous. Dignāga’s account of language claims that negative evidence based upon non-observation (*adarśanam*) is enough to account for ascertainment (*niścaya*) from an inference (*anumāna*).²¹ For this reason, and for the reasons that the general term (*jātiśabda*) cannot denote its referent as has been argued, Dignāga develops his account of exclusion of the other (*anyāpoha*).

First, let us consider how Dignāga interpreted inference’s (*anumāna*) role in *anyāpoha* as it is stated in the PSV V, with a hope to provide a more abstract account of *anyāpoha* as we move through what Dignāga says about the nature of reasoning and words. In the beginning of the *Pramāṇasamuccayavṛitti Apohapāriccheda* (PSV V), Dignāga claims that some have thought that verbal cognition is a cognition unto itself, separate from the two *pramāṇa*, perception (*pratyakṣa*) and inference (*anumāna*), but Dignāga thinks this is not the case since his view is translated by Hayes and Pind respectively as:

Hayes: PSV V 1.0.0: “Verbal communication is no different from inference as a means of acquiring knowledge. For it names its object in a way similar to [an inferential sign such as] the property of having been produced, [which indicates its object, namely, the property

²⁰ Ole Pind: Dignāga’s Apoha Theory, 65-66, 125. Dignāga-Hayes, 252.

²¹ Ole Pind: Dignāga’s Philosophy of Language, 123.

of being impermanent] by precluding what is incompatible [with the indicated property]."²²

Pind: PSV V 1.0.0: “Verbal cognition (*śābdam*) is not a means of cognition separate from inference (*anumānāt*). For it denotes (*bhāṣate*) its own referent (*svārtham*) by exclusion of other referents (*anyāpoha*) like [the general property] ‘being produced’ and the like.”²³

Said more simply, *anyāpoha* theory states that a word indicates its own object or referent (*svārtham*) through the exclusion of other incompatible referents (*anyāpoha*) by means of *Trairūpya* alone, just as the reason (*hetu*), smoke, indicates the property to be proven (*sādya*), fire, belonging to the object of inference (*anumeya*), a mountain. Further, the word, “cow” simply means that the object is not a non-cow.²⁴ The passage is provided in order to explain that verbal cognition, should be considered as subjected to the constraints of the *Trairūpya*, since the word behaves in the same way as those things which indicate their own form by means of *Trairūpya*.²⁵ Given the importance of *Trairūpya* to Dignāga’s account of the exclusion of other referents (*anyāpoha*) in his account of inference as verbal cognition, an account of *Trairūpya*’s tripartite characteristics is necessary to understand whether *vyatireka* can stand alone and do the job of *anyāpoha*.

The exclusion of other referents (*anyāpoha*), which Dignāga explains in PSV V, is his thinking about the role of non-observation (*adarśanam*) in producing valid inferences for oneself (*svārthānumāna*) and a proof for others (*parārthānumāna*) specifically in regard to the infinitely representable referent (*artha*) by means of a general term (*jātiśabda*) which

²² Hayes, 25

²³ Ole Pind Dignāga’s Philosophy of Language, 3.

²⁴ Dignaga-Hayes, 252.

²⁵ Ole Pind Dignāga’s Philosophy of Language, 219.

denotes it through exclusion of other referents (*anyāpoha*). To the degree that *Trairūpya* and *anyāpoha* represent Dignāga's thinking on the structure of *anumāna*, it is important to investigate the nature of his thought on the topic of *anumāna* through the lens of the *Trairūpya*, to establish a clearer understanding of the nature of *anumāna*'s ability to produce ascertainment (*niścaya*) by means of valid inferences for ourselves (*svārthānumāna*) and a proof for others (*parārthānumāna*) of the referent (*artha*) by means of a word (*śabda*).

Even though Dignāga was the first main proponent of the *pramānavāda* school and *anyāpoha* has been around in Buddhist schools of thought for at least a millennium and a half, there remains very little regarding a logical formalization of *Trairūpya* as it concerns *anyāpoha* in contemporary logical language. Yet, recently, there has been significant work by people like Katsura and Oetke in the formal development of *Trairūpya* in Dignāga's earlier chapters of the PSV II and IV that help explicate his commitments to the nature of inference (*anumāna*).

The literature on formal aspects of Dignāga's version of *Trairūpya* has produced different formulations of his account of the three characteristics of *Trairūpya*. All the formulations of the *Trairūpya* include accounts of both positive and negative pervasions (*anvaya* and *vyatireka vyāpti*). The inclusion of *anvaya* as a part of a valid inference (*anumāna*) becomes problematic for Dignāga in his development of *anyāpoha*, as he claims it shows that when words (*śabda*) are both observed (*darśanam*) when their co-referring terms are absent and not-observed (*adarśanam*) when their co-referring terms are present.²⁶ However, in other parts of his work Dignāga debates the primacy of each respective example (*dṛṣṭānta*), since

²⁶ Ole Pind Dignāga's Philosophy of Language, 125. Verse 35.

in earlier chapters he confronts the problem of the redundancy of the *vyatireka vyāpti* in PSV II 5cd.²⁷

²⁷ Claus Oetke, *Studies on the Doctrine of Trairūpya* (Wien: Arbeitskreis für Tibetische und Buddhistische Studien, Universität Wien, 1994), 61. Hereafter, Oetke.

Formal Accounts of *Trairūpya*

The roots of Dignāga's position come from the *Trairūpya* account of inference (*anumāna*) developed by the *Nyāya* school. Dignāga's account of inference (*anumāna*) has three characteristics. The first characteristic of the *trairūpya* is the membership of reason (*hetu*) in the object to be inferred (*anumeya*), which expresses the reason being a property of the topic (*pakṣa*) of a proposition (*pakṣadharmatva*). The second and third characteristics are the statements of the examples which prove an inseparable relation (*avinābhāva*),²⁸ positive concomitance (*anvaya anubandha*) and negative concomitance (*vyatireka annubandha*), all of which will be discussed in depth now from various perspectives of Hayes, Katsura, Tillemans, and Oetke. The goal of looking at these different perspectives of the three characteristics of inference (*Trairūpya*), is to produce a formal account of what Dignāga had in mind about how we can gain ascertainment (*niścaya*) about what we have been told by someone who knows by forming valid inferences (*svārthānumāna*) using the rule they provide.²⁹

Hayes' System

The most basic account of Dignāga's system was provided by Hayes, in his work on Dignāga's *Interpretation of Sign*, where he attempts to formalize the foundations of Dignāga's theory of language in what Hayes develops and names the "Hayes-Dignāga system of logic," hereafter (Hayes' system). This system provides the first semi-formal western account of Hayes' interpretation of Dignāga's theory of knowledge from a

²⁸ Shoryu Katsura and Ernst Steinkellner, "The Role of Drstanta in Dignāga's Logic," in *The Role of the Example (dṛṣṭānta) in Classical Indian Logic* (Arbeitskreis für Tibetische und Buddhistische Studien, Universität Wien, 2004), 137-141. Hereafter, Katsura, The Role of dṛṣṭānta in Dignāga's Logic.

²⁹ Dignāga's *Philosophy of Language*, 183. Verse 50b.

combination of the *Pramāṇasamuccaya*, and the *Hetucakranirṇaya* (HCN), which espouses Dignāga's theory of knowledge.³⁰

It is stated that the purpose of the HCN was to understand how relations between two classes of individuals are mediated by a third class of individuals.³¹ As was mentioned earlier, the first aspect of the work that we will see employed in understanding *anyāpoha* is the concept of pervasion, “whether one class contains another,” (*vyāpti*), whether the two classes are disjoint (*virodha*), or whether the two classes overlap (*vyabhicāra*),” which ultimately determines what kind of claims we can make about the identity of the two classes in question.³² The first pervasion (*vyāpti*) will act as a useful guide to the two types of examples (*dr̥ṣṭānta*) that Dignāga relies on to formulate his account of the *Trairūpya*, to produce ascertainment (*niścaya*) through valid inference for oneself and in turn a proof for others (*svārthānumāna* and *parārthānumāna*).

The two classes that Hayes used to explicate the logic of the similar (*anvaya*) and dissimilar (*vyatireka*) example (*dr̥ṣṭānta*) pulled from the HCN, are the possessors of the property to be used as evidence/reason (*hetu*), and the class of properties that is to be confirmed though evidence (*sādyadharmā*).³³ The domain of properties that consists of the subject of inference/inferable object (*pakṣa/anumeya*), is seen as, “a limited sample of the universe of individuals,” which contains the class of properties whose relationship to *sādyadharmā* is possible, i.e the reason (*hetu*) alone.³⁴ Further, two other “classes” of individuals are expressed in what Hayes describes as the induction domain. These are the

³⁰ Hayes, 61.

³¹ Hayes, 112-13.

³² Hayes, 113.

³³ Hayes, 113.

³⁴ Hayes, 113.

class of similar instances that are like the subject (*sapakṣa*) and a class of dissimilar instances (*asapakṣa/vipakṣa*) that are not like the subject.³⁵ He describes the relationship between the reason (*hetu*) and these two other classes as follows:

1. *Anvaya* = (Association): $\neg\text{PHS} > 0$
 - a. There exists at least one individual in the induction domain that is a member of the evidence (*hetu*) class and also a member of the subject-like class (*sapakṣa*).³⁶ So, we might see smoke on a mountain.
2. *Vyatireka* = (Dissociation): $\neg\text{PH}-\text{S} = 0$
 - a. There exists no individual that is a member of both the evidence (*hetu*) and the “un-subject-like” class (*asapakṣa*).³⁷ We will never find smoke on a lake.

Hayes uses association (*anvaya*) and dissociation (*vyatireka*) formulations from the *Trairūpya* to help explain the relations between the *hetu* and the *sapakṣa* or the *asapakṣa*.³⁸ In this effort, however, Hayes creates a great amount of ambiguity in his formulations of these two relations (*vyāpti*), in that he equivocates two definitions or mistakenly believes that the *sapakṣa* and *vipakṣa* act as the *sādyā*. This being that he defines “S” as the property to be confirmed (*sādyā*) but contradicts the claim in his definition of the *anvaya* and *vyatireka* when he claims the reason (*hetu*) is related to the subject or un-subject like classes, which he defines as the *sapakṣa* or the *asapakṣa*.

Hayes’ account of the *anvaya* and *vyatireka* leaves much to be desired since his use of informal language also leaves significant ambiguity as to what kind of relationship is being

³⁵ Hayes, 116

³⁶ Hayes, 118.

³⁷ Hayes, 118.

³⁸ Dignāga’s Apoha Theory, 70. We also see these terms referred to as, “Concordance (association, joint presence, *anvaya*) and difference (joint absence, *vyatireka*).”

drawn between the reason (*hetu*) and its effect/consequent/property to be proven (*sādyā*). They are ambiguous because they draw the relation between the *hetu* and the *sapakṣa* and the *hetu* and the *asapakṣa/vipakṣa*, meaning we can only infer the membership of smoke on a mountain or on a non-mountain but not the reason/evidence's (*hetu*) relation with the property to be proven (*sādyā*). In the natural language example of the two accounts, we can see that smoke is an indicator of fire on mountains but not fire on lakes, so thinking about *anvaya* and *vyatireka* in this way doesn't tell us anything about the nature of the invariable concomitance (*avinābhāva*) between the *hetu* and *sādyā*.

Importantly, the *anvaya* and *vyatireka* relations have been explored by several authors, since the *Trairūpya* has been used by Sanskrit Grammarians as a method of inquiry as well:

Anvaya and *vyatireka* are used to establish the meaningfulness of components and to ascribe individual meanings to components ... consists in observing the concurrent occurrence (*anvaya*) of a certain meaning and a certain linguistic unit and the absence of a meaning and a unit.³⁹

Fortunately, these semi-formal and muddled accounts of the subject (*pakṣa*), object to be inferred (*anumeya*), and examples (*drṣṭānta*), are unique to Hayes' understanding of Dignāga's *Trairūpya*. This means that differing accounts provided by other contemporary authors like Katsura, Oetke, and Tillemans, create a clearer although incongruent formal account of the nature of the parts of proof for oneself and a proof for others (*svārthānumāna* and *parārthānumāna*). Katsura and Oetke did most of the groundwork, with Katsura's commentary and translation of PSV IV and Oetke's commentary on PS II 5cd. These

³⁹ Hayes, 119.

descriptions vary, but include accounts of the *pakṣa* and the *anumeya*, as well as the two *drṣṭānta*'s, *anvaya* and *vyatireka*, and their descriptions of the inseparable relation (*avinābhāva*) between the reason (*hetu*) and the property to be proven (*sādyā*) in the *sapakṣa* and *vipakṣa* classes.

Katsura

Katsura provides a much clearer explication of the three characteristics that make up the *Trairūpya*. The first characteristic of the *Trairūpya* accounts for the ascertainment (*niścaya*) of the inferential mark in the state of affairs to be inferred (*anumeya* e.g. 'The top of a mountain').⁴⁰ The statement of the reason (*hetu*) is made in order to indicate that the reason (*hetu*) is a property of the state of affairs to be inferred (*anumeya*). Smoke is on top of the mountain. These examples (*drṣṭānta*) are defined in PSV IV 2:

An example is that [object] in which a reason (*hetu*) is shown to be followed by a property to be proved (*sādyā*) or to be absent in the absence of a property to be proved; it is of two kinds: 'similar' (*sādharmya*) and another (i.e., 'dissimilar' *vaidharmya*).⁴¹

The role of the statement of the example (*drṣṭānta-vacana*) in a proof, is to present the inseparable relation (*avinābhāva*) between a reason/proving property (*hetu*, *sādhanadharmā*), and the property to be proven (*sādyadharmā*).⁴² The same is true of the logical mark (*liṅga*, cause) and the logically marked (*liṅgin*, that which is to be inferred, effect) and the referent (*artha*) is considered to be the cause of the word (*śabda*) for Dignāga.⁴³ This relation explained by the example is also called restriction (*niyama*), pervasion (*vyāpti*) and

⁴⁰ Katsura, The Role of *drṣṭānta* in Dignāga's Logic, 136.

⁴¹ Katsura, The Role of *drṣṭānta* in Dignāga's Logic, 141 PSV IV 2.

⁴² Katsura, The Role of *drṣṭānta* in Dignāga's Logic, 139.

⁴³ Ole Pind: Dignāga's Philosophy of Language, 121.

concomitance (*anubandha*).⁴⁴ Like Hayes, Katsura argues that Dignāga's example statements (*dr̥ṣṭānta*) express the pervasion (*vyāpti*) of a proving property (*hetu*) by a property to be proven (*sādyā*), or the absence of the proving property (*hetu*) in the absence of the property to be proven (*sādyā*).

Further, we can say *anvaya* is an inseparable relation (*avinābhāvin*) when/if the reason, proving property, inferential mark (*hetu*, *sāadhanadharmā*, *liṅga*) (P) is present, then the property to be proven, inferentially marked (*sādyadharmā*, *liṅgin*)(Q) is present.⁴⁵ Katsura suggests that the similar and dissimilar example (*anvaya* and *vyatireka dr̥ṣṭānta*) can be presented in the logical form of the conditional, because the structure laid out in inference (*anumāna*) of reason (*hetu*) (P) and the property to be proven (*sādyā*) (Q) is that of a conditional statement:⁴⁶

Anvaya: When the smoke (*hetu*) (P) is present, then the property to be proved fire (*sādyā*) (Q) is present.⁴⁷

Yad P tad Q.

If x is P, then x is Q. $Px \rightarrow Qx$

Vyatireka: When the property to be proved (*sādyā*), fire, is absent ($\neg Q$), then the reason (*hetu*), smoke, is absent ($\neg P$).⁴⁸

Yad not-Q tad not-P.

⁴⁴ Katsura, The Role of *dr̥ṣṭānta* in Dignāga's Logic, 139.

⁴⁵ Katsura, The Role of *dr̥ṣṭānta* in Dignāga's Logic, 147.

⁴⁶ Katsura, The Role of *dr̥ṣṭānta* in Dignāga's Logic, 146.

⁴⁷ Katsura, The Role of *dr̥ṣṭānta* in Dignāga's Logic, 147.

⁴⁸ Katsura, The Role of *dr̥ṣṭānta* in Dignāga's Logic, 146.

If x is not-Q, then x is not-P. $\neg Qx \rightarrow \neg Px$ ⁴⁹

Katsura goes further to explain the structure of a valid proof for others (*parārthānumāna*) which is a mirror of the process that we have already completed for ourselves. The formulation is a formal account of proof 2 given in PSV IV, which we have adapted to include the smoke and fire example:⁵⁰

1. (Reason): Because the mountain possesses smoke, (Proposition): the mountain possesses fire.
2. Whatever possesses smoke is observed (*darśanam*) to possess fire. Like a plateau.
3. Whatever is not-observed (*adarśanam*) to possess fire, does not possess smoke. Like a lake.⁵¹

The formulation of the logical proof (*parārthānumāna*) in Dignāga's proof 2 indicates that we infer from a specific instance, namely the mountain's possession of smoke, that if the inferential mark (*hetu, liṅga*), smoke, is observed (*darśanam*), then the mountain's inferentially marked (*sādyā, liṅgin*), fire, will also be observed (*darśanam*). Further, in the absence of the observation (*adarśanam*) of fire, there is non-observation (*adarśanam*) of smoke.

Why are the accounts of *anvaya* and *vyatireka* as conditionals formulated as " $Px \rightarrow Qx$ " and " $\neg Qx \rightarrow \neg Px$ " and not in another way such as " $Px \rightarrow Qx$ and $\neg Px \rightarrow \neg Qx$," the latter expressing the stronger necessary and sufficient conditions that are normally

⁴⁹ Katsura, The Role of *dr̥ṣṭānta* in Dignāga's Logic, 146.

⁵⁰ Katsura, The Role of *dr̥ṣṭānta* in Dignāga's Logic, 143.

⁵¹ Katsura, The Role of *dr̥ṣṭānta* in Dignāga's Logic, 150.

used to establish identity or mutual codependence?⁵² Oetke claims, “there is evidence to the effect that the *Trairūpya*-cannon in the theories of Śāṅkarasvāmin and Dignāga was conceived as furnishing a necessary, but not a both necessary and sufficient criterion of the acceptability of logical reasons and indirectly of proofs and inferences.”⁵³ This seems evident by the fact that Dignāga explicitly claims that we should not accept the formulation of the necessary and sufficient condition in PSV IV 4. In that we have an account of Dignāga’s rejection of the use of the necessary and sufficient conditions together to form a valid inference for oneself and a proof for others (*svārthānumāna* and *parārthānumāna*), it seems that Dignāga knew about necessary and sufficient conditions but chose to reject them in favor of the contraposition of the similar pervasion (*anvaya vyāpti*) when formulating the dissimilar pervasion (*vyatireka vyāpti*).⁵⁴

The verse PSV IV 4 commits Dignāga to the position that the statements $Qx \rightarrow Px$, $\text{not-}Qx \rightarrow \text{not-}Px$, as similar and dissimilar examples respectively, lead to contradiction when reasoning about eternity and production. Due to the problems created by sufficient and necessary conditions, it appears that Dignāga wrote of this account of reasoning, because it leads to doubt (*saṃśaya*) in the case of eternity and production, (i.e. the relation of necessity and sufficiency does not hold for being a result and sensation, nor for eternity and not being a product), since in each case the property to be proved, proves its opposite.⁵⁵ As a response to this realization in PSV IV 4, Dignāga developed his theory by defining the *vyatireka* as the contraposition of *anvaya*. We see this contraposition reflected in his formal account of a proof for others (*parārthānumāna*) presented in proof 2 of PSV IV which Katsura has taken

⁵² Katsura, *The Role of dr̥ṣṭānta in Dignāga’s Logic*, 154.

⁵³ Oetke, 72

⁵⁴ Katsura, *The Role of dr̥ṣṭānta in Dignāga’s Logic*, 155.

⁵⁵ Katsura, *The Role of dr̥ṣṭānta in Dignāga’s Logic*, 155.

the time to formalize in quantificational logic, a theme that will continue in our contemporary accounts of *Trairūpya*:

a = *anumeya* = mountain Q = Fire P = Smoke

[reason] Pa: Because the mountain possesses smoke, [proposition] Qa: a mountain possesses fire.

Similar example (*anvaya dr̥ṣṭānta*): $(\forall x)(Px \rightarrow Qx) \ \& \ (\exists x)((Px \ \& \ Qx) \ \& \ (x \neq a))$

We can observe the following two natural language examples for Katsura's the *anvaya* relation.

- a. Whatever is observed (*darśanam*) to possess smoke possesses fire, e.g., like a plateau.
- b. For all "x" if "x" is observed (*darśanam*) to have smoke then "x" is observed (*darśanam*) to have fire, and for some "x," "x" is observed (*darśanam*) to have smoke and "x" is observed (*darśanam*) to have fire and "x" is not a mountain.

Dissimilar example (*vyatireka dr̥ṣṭānta*): $(\forall x)(\neg Qx \rightarrow \neg Px) \ \& \ (\exists x)((\neg Px \ \& \ \neg Qx) \ \& \ (x \neq a))$

We can observe the following two natural language examples for the *vyatireka* relation.

- a. Whatever is not-observed (*adarśanam*) to possess fire, does not possess smoke, e.g., like a lake.
- b. For all "x" if "x" is not-observed (*adarśanam*) to have fire, then "x" is not-observed (*adarśanam*) to have smoke, and for some "x," "x" is not-observed (*adarśanam*) to have smoke and "x" is not-observed (*adarśanam*) to have fire and "x" is not a mountain.⁵⁶

⁵⁶ Katsura, The Role of *dr̥ṣṭānta* in Dignāga's Logic 150.

Unfortunately, the variable “x” is never defined in any of the texts but given its context the variable “x” appears to refer to the state of affairs present to perception (*pratyakṣa*) that is like the object to be inferred (*anumeya*) which one wishes to make an inference about.

If we look carefully, the two examples are not in contraposition, since the statements of the *sapakṣa* and *vipakṣa* classes are added to their respective *anvaya* and *vyatireka* relations by being given their own formal statements. Katsura interprets Dignāga as expressing each class by adding a statement to convey the *sapakṣa*, “like a plateau” and *vipakṣa*, “like a lake,” with the existentially quantified statements that make up the second half of his formal accounts. Given the explicit inclusion of a concrete example of the *sapakṣa* or *vipakṣa* in his formal accounts of the *anvaya* and *vyatireka dṛṣṭānta* it seems that our formal account should include a reference to them in order to draw out a conclusion from the conditional statements that precede them and to gain ascertainment (*niścaya*) of the observation (*darśanam*) following the rule defined by the reason (*hetu*) and the proposition (*pakṣadharmatva*).

Tillemans

Tillemans’ work with the *Trairūpya*, provides insights into the reason why the *pakṣa* should be excluded from the domain of inference of the *anvaya* and *vyatireka* relations and which version of *anvaya* and *vyatireka* we should accept. In his consideration of the formal account of *anvaya* and *vyatireka*, he addresses the claim made by Gillion and Love which assumes that the two are equivalent if the *pakṣa* is not a member of the consequent P of the *anvaya* and not mentioned in the *vyatireka*:

1. $(\forall x)((Hx \rightarrow (Sx \ \& \ \neg Px) \leftrightarrow (\forall x)(\neg Sx \rightarrow \neg Hx))^{57}$
- a. For anything “x,” if “x” is observed (*darśanam*) to have the reason (*hetu*), smoke, then it will be observed (*darśanam*) to have the property to be proven (*sādyā*), fire, and not the *pakṣa*,” is the equivalent of, “for anything “x,” If “x” is not-observed (*adarśanam*) to have the property to be proven (*sādyā*), fire, then it will not be observed (*adarśanam*) to have the reason (*hetu*), smoke.”

There seem to be several issues with including the *pakṣa* as a general term, in one and not the other, and in the consequent. The *pakṣa* “Px” means that, “x” is observed (*darśanam*) to have the *pakṣa* condition,’ and in the *anvaya* relation the reason (*hetu*), infers the *sādyā* and is not the *pakṣa*, but we know the reason (*hetu*) is a member of the *pakṣa*. Further, we know that the *pakṣa* is never a member of the *vipakṣa* by means of *vyatireka* by the nature of a *vipakṣa* being dissimilar. However, the *pakṣa* is not expressed in the *vyatireka* relation as a negation in the antecedent with the *sādyā*. If we are to explicitly state that the reason is occurring in the *pakṣa*, but “x” is not, it seems unreasonable to come to the conclusion that the *pakṣa* should not be symbolized as being excluded from the *sapakṣa* and *vipakṣa* in both *anvaya* and *vyatireka*. Thus, it seems important to make the move that others make in the formal account of the theory by switching the *pakṣa* from a general term to a singular term.

We can either express the relation between the variable when it “x” is not a member of the *pakṣa* with $\neg Px$, or express the relationship as $x \neq p$, where “p” stands for the singular instance of an individual constant, the mountain (*anumeya*), having the possibility to express the reason (*hetu*), smoke. If we express the *pakṣa* in the stronger way as a general term

⁵⁷ Tillemans Tom J F., “Chapter 5: On Sapakṣa,” in *Scripture, Logic, Language: Essays on Dharmakīrti and His Tibetan Successors* (Boston: Wisdom Publications, 1999), 114. Hereafter, *Scripture, Logic, Language*.

(*jātiśabda*), it creates issues of the reason (*hetu*) inferring the lack of a *pakṣa*. Also, if the *pakṣa* is included as a general term, then we see that it should be included in the third characteristics of the *Trairūpya* the *vyatireka*, which creates the issue of the reason (*hetu*) inferring the lack of a *pakṣa*. If we are to avoid the problems created by the *pakṣa* being included as a general term, then we can represent the *pakṣa* as an individual constant “p” for the object to be inferred (*anumeya*), mountain, accompanied by the reason (*hetu*), smoke, since it is a particular instance, and we are trying to use the reason to draw similar *sapakṣa* and dissimilar *vipakṣa* examples by means of *anvaya* and *vyatireka*. We can express the absence of the *pakṣa* from the *anvaya* and *vyatireka* relations as $x \neq p$. When we use $x \neq p$ it shows us more clearly that the *anvaya* and *vyatireka* relations are equivalent by contraposition:

$$2. (\forall x)((x \neq p \ \& \ Hx) \rightarrow Sx) \leftrightarrow (\forall x)((x \neq p \ \& \ \neg Sx) \rightarrow \neg Hx)^{58}$$

- a. For anything “x,” if “x” is not identical to the *pakṣa* and it is observed (*darśanam*) to have the reason (*hetu*), smoke, then it will also be observed (*darśanam*) to have the property to be proved (*sādyā*), fire, is equivalent to, if anything “x” is not-identical to the *pakṣa* and it is not-observed (*adarśanam*) to have the property to be proven (*sādyā*), then it will also not be observed (*adarśanam*) to have the reason (*hetu*).

This reading of equivalence by contraposition has created many difficulties in understanding Dignāga’s commitments to *Trairūpya*, since although he was aware of the concept of the necessary and sufficient conditions in PSV IV 4, he favors a standard account

⁵⁸ Scripture, Logic, Language, 114.

of the conditional “If P, then Q,” and its equivalent contrapositive for the forms of *anvaya* and *vyatireka* to produce valid inferences for oneself and a proof for others (*svārthānumāna* and *parārthānumāna*). Thus, we can see that Tillemans suggests we read the *anvaya* and *vyatireka* conditions as stated above in 2 and 2a of this section.

Oetke

Oetke begins his formal account of Dignāga with a grounding in the *Vādaividhi* of Vashubahdu,⁵⁹ and the *Nyāyapraveśa*, a *Nyāya* text which was composed around the same time or soon after Dignāga.⁶⁰ He and Tillemans have used this framework, although accepting different statements from it, to try and formally explain what Dignāga’s position is on *anvaya* and *vyatireka*, given his commitments to various formal accounts of the *Trairūpya* that he lays out in the PSV II 5cd. Oetke presents two definitions for us to consider when we think about what kind of inseparable relation (*avinābhāva*) is being drawn between the “probans” (reason, *hetu*) and the “probandum” (property to be proven, *sādyā*):

“Def 1: The logical reason is the pronouncement of a property which does not occur without that which is inseparably connected with, a probandum.”

“Def 2: Inference is the observation (*darśanam*) of an object not occurring without the probandum for someone who knows that.”⁶¹

⁵⁹ Oetke, 11.

⁶⁰ Oetke, 16.

⁶¹ Oetke, 11.

“Def.1: *tādṛgavinābhāvidharmopadarśanam hetuḥ |*”

“Def 2: *nāntarīyakārthadarśanam tadvido 'numānam |*”

In the first definition, the proponent is committed to the restrictive stance that, if the property *sādyā* is absent, then the property *hetu* will also be absent. In other words, if we see smoke on a mountain, then we have the indication that there is fire on the mountain since the two always occur in inseparable relation (*avinābhāva*) to one another. The first definition also expresses this relationship between properties, i.e. smoke is a property of fire and thus the definition indicates that the smoke and fire are properties of the mountain as well. The use of properties to describe the entities that make up the inference is less desirable for Dignāga's purposes, since he explicitly rejects that a general term (*jāti*) denotes (*bhāṣate* particulars (*bhedānām*)).⁶²

Definition two provides a more inductive meaning to the nature of inferences. Inference is the act of observing the *hetu* to not occur without the *sādyā*. Said more clearly, when there is a person who knows smoke does not occur without fire, if the reason (*hetu*) smoke is observed (*darśanam*) on the object to be inferred (*anumeya*), the mountain, then the property to be proven (*sādyā*), fire, is said to be observed (*darśanam*) as well. We are speaking about a particular instance of experience that depends on the observer having been shown the referent's (*artha*) connection to the word (*śabda*) by someone to whom the connection is already known.⁶³ Thus, it appears that the second, inductive definition is more favorable to our understanding of the position that Dignāga accepts, since the meaning of a word is primarily the ability of *vyatireka* to produce the non-observation (*adarśanam*) of other referents.⁶⁴

⁶² Ole Pind, 8. Verse 2c

⁶³ Ole, Pind, 183 50b.

⁶⁴ Ole Pind, 125, vṛitti.

Oetke develops a similar formal account to the one developed by Katsura in his description of the inseparable connection (*avinābhāva*) that we see described in the definitions provided by the *Vādavidhi* of Vasubandhu. Here, we can easily see the reason for the uses of “H” to indicate the probans (reason, proving property, *hetu*, *liṅga*, *sāadhanadharmā*) and “S” to indicate the probandum (effect, property to be proven, *sādyā*, *liṅgin*, *sādyadharmā*) and the immediate similarities to the account provided by Katsura of the *anumeya* and *vyatireka vyāpti*:

1. $\neg(\exists x)(Hx \ \& \ \neg Sx)$

- a. It is not the case, that there is an “x” such that “x” is observed (*darśanam*) to have the reason (*hetu*), smoke, and “x” is not-observed (*adarśanam*) to have the property to be proven (*sādyā*), fire.

2. $(\forall x)(Hx \rightarrow Sx)$

- a. If anything “x” is observed (*darśanam*) to have the reason (*hetu*), smoke, then “x” is observed (*darśanam*) to have the property to be proven (*sādyā*), fire.

3. $(\forall x)(\neg Sx \rightarrow \neg Hx)$.⁶⁵

- a. If anything “x” is not-observed (*adarśanam*) to have the property to be proven (*sādyā*), fire, then “x” is not-observed (*adarśanam*) to have the reason (*hetu*), smoke.

In these formal logical accounts, as was alluded to above, we can see that each of these statements are equivalent. Making the claim, ‘There is a non-observation (*adarśanam*) of something which possess the *hetu* and which does not possess the *sādyā*,’ is the same as

⁶⁵ Oetke, 14

saying, ‘for all observations (*darśanam*), if something possesses the *hetu*, then it possesses the *sādyā*. However, we can’t gather from these statements anything regarding the *anumeya*, the *sapakṣa* or *vipakṣa*, because the *anumeya* establishes the object of inference that one knows. We are trying to draw inference for the *hetu* and *sādyā* in examples (*dr̥ṣṭānta*) of instances that are similar and dissimilar to that first experience of smoke at that specific object of inference (*anumeya*) and apply it more generally.

We can gather from PSV IV 2 and the formal accounts above that the reason (*hetu*) should be in anything that is similar by virtue of the fact that it is observed (*darśanam*) to exhibit the property to be proven (*sādyā*) in those things similar to the *pakṣa* but not identical with it.⁶⁶ The *vipakṣa* is not-observed (*adarśanam*) to exhibit the proving property (*hetu*) or to be proved (*sādyā*). So, in this case, it seems clear that, if “x” is a *vipakṣa*, then “x” will not be a member of the *pakṣa* or *sapakṣa* and it will not contain the *sādyā* either.⁶⁷

Although these formal logical descriptions are an attempt only to explicate the definition of inseparable relation (*avinābhāva*), they bear the identical formulation to the first parts of Katsura’s formal accounts of *anvaya* and *vyatireka*. This identity leads me to think that the vacillation between the primacy of the *anvaya* and *vyatireka* in different parts of Dignāga’s writing in the PSV II 5cd and PSV V 34’s *vṛitti* as well as elsewhere in PSV V, explains what Dignāga was thinking about the nature of his commitments to the structure of the *Trairūpya* that had him confused about what a statement of each *dr̥ṣṭānta* really contained. Further, the equivalence indicates why so much work has been done to show that the two are not equivalent, since such a robust system seems to be misunderstood if it comes

⁶⁶ Oetke, 22

⁶⁷ Oetke, 22

out to be saying the same thing in both the *anvaya* and *vyatireka* relations. However, in the case of Dignāga at least, it appears that even if equivalence of the two *dṛṣṭānta* is avoidable, the use of *anvaya* in valid inferences for oneself and a proof for others (*svārthānumāna* and *parārthānumāna*) seems not to be.

Oetke claims that his interpretation of Dignāga's PSV II 5cd, avoids the complications that are present in the apparent equivalence of the *anvaya* and *vyatireka vyāpti*.⁶⁸ PSV II 5cd presents an account of Dignāga rejecting that the *vyatireka* is redundant in light of the statement of the *pakṣa* and *anvaya* in his version of the *Trairūpya*. The first sentence characterizes the doctrine of *Trairūpya* concisely, in that it is meant to describe what the three characteristics accomplish:

“A property indicator must be present in the object of inference and in what is similar to it, and absent in what is not similar to it.”⁶⁹

The reason (*hetu*) is present in the object of inference (*anumeya*) and what is similar to it, the *sapakṣa*, and absent in what is not similar to it, the *vipakṣa*. It is said further, that the second characteristic (*anvaya*) is restrictive in such a way that the proving property (*hetu*) is present only in what is similar to the object of interference (*anumeya*), the *sapakṣa*, by means of its positive pervasion (*anvaya vyāpti*) with the property to be proven (*sādyā*).⁷⁰ The restriction of the reason (*hetu*) to what is similar to the object of inference (*anumeya*), the *sapakṣa*-domain by means of positive concomitance (*anvaya*) only denotes (*bhāṣate*) the presence of the reason (*hetu*) and the property to be proven (*sādyā*) at that specific referent.

⁶⁸ Oetke, 56-57.

⁶⁹ Oetke, 56

⁷⁰ Oetke, 56.

It may be argued by an opponent that, if the *anvaya* relation **only** (*eva*) denotes (*bhāṣate*) the restriction of a reason (*hetu*) to the *pakṣa* and *sapakṣa* alone, then by implication the reason is never included in the *vipakṣa*. If this is true, it seems that the statement of (*anvaya*) alone is sufficient to show the object only occurs in the *pakṣa* and *sapakṣa* to do the work of refuting other referents that are not that singular referent meant to be denoted (*bhāṣate*) by the word.⁷¹ Thus, the ability of the restriction of the reason, the *anumeya* and *sapakṣa* seems to create a problem, namely the superfluous generation of the *vipakṣa* condition, in that the *vipakṣa* is just a restatement of the *sapakṣa*.⁷² However, this restriction is tempered by the statement (*tattulye sadbhava eva*) that the logical reason is just similar and not necessarily present in all similar instances of the object of inference (*anumeya*).⁷³

The weak existential import implied by using the term *sadbhāvaḥ*, means that the logical reason occurs in at least one instance of the *sapakṣa* instead of the logical reason occurring at all instances of the *sapakṣa*. This means that the *anvaya* should not be considered to have universal import. If the *anvaya* does not have universal import and the reason (*hetu*) is excluded from all instances of *vipakṣa*, the *vyatireka vyāpti* needs to be stated to determine the reason (*hetu*) and (*sādyā*) are not observed (*adarśanam*) in any instances of the *vipakṣa*. So, while *pakṣa* and *anvaya* are sufficient to show that the logical reason occurs in the *pakṣa* and elsewhere only in *sapakṣa*, which in turn excludes most instances of the reason from the *sapakṣa*, it is argued that the dissimilar example (*vyatireka dṛṣṭānta*) of the

⁷¹ Oetke, 58.

⁷² Oetke, 58.

⁷³ Oetke, 56.

hetu and the *sādyā* outside *vipakṣa* must be included in order to determine that the *hetu* does not occur in the *vipakṣa* at all.⁷⁴

Oetke describes what he believes is the correct interpretation of the formulation of the *dr̥ṣṭānta* of *Trairūpya* that produce a valid inference for oneself or a proof for others (*svārthānumāna* and *parārthānumāna*). The first characteristic, the *pakṣa*, is always considered to be an instance where only the logical reason occurs.⁷⁵ Regarding the second characteristic, the *anvaya dr̥ṣṭānta*, Oetke claims, based on his reading of PS II 5cd, that we should take the *anvaya* of the *sapakṣa* as formalized by:

$$4. (\exists x)(x \neq p \ \& \ (Hx \ \& \ Sx))^{76}$$

- a. Something “x” is not identical with the *pakṣa*, and “x” is observed (*darśanam*) to exhibit the reason (*hetu*) and the property to be proven (*sādyā*).

In this case, we see that the *hetu* and the *sādyā* are not in a conditional relationship, but rather a conjunction, restricting their observation (*darśanam*) to one another in a specific instance. If one can establish that the *hetu* is only a part of the *pakṣa*, and *hetu* and *sādyā* though *anvaya vyāpti* are only a part of the *sapakṣa*, then we cannot conclude that the logical reason is contained in only those two from the evidence we have alone in the *pakṣa* and *anvaya*. Thus, the superfluous *vyatireka* relation is avoided because it is needed to show that there are no instances where the reason occurs in the dissimilar relation. So, in that the *vyatireka* attempts to express the non-observation (*adarśanam*) of the property to be proven (*sādyā*) with the non-observation (*adarśanam*) of the proving property (*hetu*), in the case of

⁷⁴ Oetke, 63.

⁷⁵ Oetke, 67.

⁷⁶ Oetke, 24.

the *vipakṣa*, the *vyatireka* should be expressed as a conditional similar to what is presented by Katsura in his formal account of PSV IV proof 2. Thus, Oetke gives us two different accounts of how we could understand the *vyatireka* as given his reading of PS II 5cd:

5. $(\forall x)((x \neq p \ \& \ \neg Sx) \rightarrow \neg Hx)$ ⁷⁷

- a. If anything, which is not identical with the *pakṣa* and is not-observed (*adarśanam*) to have the property to be proven (*sādyā*), fire, is not-observed (*adarśanam*) to have the reason (*hetu*), smoke.

6. $(\forall x)((x \neq p \ \& \ \neg Sx) \rightarrow \neg Hx) \ \& \ (\exists x)(x \neq p \ \& \ \neg Sx)$ ⁷⁸

- a. If anything, which is not identical with the *pakṣa* and is not-observed (*adarśanam*) to have the property to be proven (*sādyā*), fire, it is not-observed (*adarśanam*) to have the reason (*hetu*), smoke, and there is at least one thing that is not identical with the *pakṣa* that is not-observed (*adarśanam*) to have the property to be proven (*sādyā*), fire.

Both 5 and 6 in this section, represent the restrictive interpretation of *vipakṣa* that we have mentioned above, since they exclude the *pakṣa* from both the observation (*darśanam*) at hand and from anywhere the *sādyā* is not-observed (*adarśanam*), in the case of 6. With these formulations in hand, Oetke seems to be in agreement with Dignāga when Dignāga argues against those who claim the problem of the *vyatireka* of the *hetu* and *sādyā* to the *vipakṣa* is redundant, because it is implied by the statement of the reasons restricted relation with the *pakṣa* and the *sapakṣa*.⁷⁹ If the *anvaya* is a conjunction with an existential quantifier, then the

⁷⁷ Oetke 27.

⁷⁸ Oetke 27.

⁷⁹ Oetke 67.

vyatireka vyāpti must be expressed in order to determine that the reason (*hetu*) is absent in the *vipakṣa*. To infer that the occurrence of the non-observation (*adarśanam*) of the reason (*hetu*), the occurrence of the property to be proven (*sādyā*), fire, must also not be observed. In other words, that both the *hetu* is observed (*darśanam*) to occur when the *sādyā* is and the *hetu* is not observed to occur in the lack of it's *sādyā*. Unfortunately, as discussed above, this type of reasoning requires that we support the idea of the necessary and sufficient conditions, which has been explicitly rejected by Dignāga in PSV IV 4.

A Combined View

Now, given the account of *vyatireka* just provided by Oetke, we can see that there are some notable differences between his and Katsura's accounts:

Katsura: $(\forall x)(\neg Qx \rightarrow \neg Px) \ \& \ (\exists x)((\neg Px \ \& \ \neg Qx) \ \& \ (x \neq a))$.⁸⁰

- a. If anything is not-observed (*adarśanam*) to possess fire, it is not-observed (*adarśanam*) to possess smoke, e.g., like a lake.
- b. If anything is not observed (*adarśanam*) to have the property to be proven (*sādyā*), fire, it is not observed (*adarśanam*) to have the reason (*hetu*), smoke, and there is at least one thing that is not observed (*adarśanam*) to have the reason (*hetu*), smoke, and not observed (*adarśanam*) to have the property to be proven (*sādyā*), fire, and that thing is not identical with the *anumeya*.

Oetke: $(\forall x)((x \neq p \ \& \ \neg Sx) \rightarrow \neg Hx) \ \& \ (\exists x)(x \neq p \ \& \ \neg Sx)$ ⁸¹

- a. If anything which is not identical with the *pakṣa* and is not observed (*adarśanam*) to have the property to be proven (*sādyā*), fire, it is not observed (*adarśanam*) to have the reason (*hetu*), smoke, and there is at least one thing that is not identical with the *pakṣa* that is not observed (*adarśanam*) to have the property to be proven (*sādyā*), fire.

First, the *anumeya* is restricted only by the second existential statement in the case of Katsura, where Oetke instead restricts the *pakṣa* (the reason (*hetu*) occurrence in the (*anumeya*)). It seems that although Katsura's inclusion of the lack of identity of the object of

⁸⁰ Katsura, 150.

⁸¹ Oetke, 27.

inference “x,” with the object of inference already known “a,” is more correct than the *pakṣa* interpretation, its distribution across both statements is needed to be clear that we are not referring to the object of inference that we already know.

Katsura and Tillemans both commit to formal accounts of *anvaya* and *vyatireka* that are very close to the second *vyatireka* condition “6” of Oetke. The universal qualifier means that the *anvaya* and *vyatireka* relations implicate all instances of the reason (*hetu*), smoke, are invariably concomitant (*avinābhāva*) with the property to be proven (*sādyā*), fire, and separate from the *pakṣa*, but also there is observed (*darśanam*) at least one instance in which reason (*hetu*) and the property to be proven (*sādyā*) are found to be in conjunction. Oetke’s logical commitments in his formulation of *anvaya* confines him to the conception that what Dignāga means by expressing the *anvaya dṛṣṭānta* is actually a conjunction. So an *anvaya dṛṣṭānta* only expresses the observation (*darśanam*) of smoke and fire together, apart from the first instance of the reason’s (*hetu*) occurrence with the object of inference (*anumeya*). Given the evidence from Katsura’s and Tillemans’ and Oetke’s formulation of *vyatireka*, it seems more reasonable to consider the *anvaya* and *vyatireka dṛṣṭānta* as conditional statements that function as members of a proof.

Thus, we can revise both the *anvaya* and *vyatireka dṛṣṭānta* to exclude the object to be inferred (*anumeya*) and not the reasons (*hetu*), smoke, occurrence in the (*anumeya*), mountain, the (*pakṣa*). We can formulate a proof that remains true to the commitments that Dignāga makes in PSV IV proof 2 as well as Katsura, Oetke, and Tillemans formulations to guide a fresh formal account of a valid inference for oneself or a proof for others (*svārthānumāna* and *parārthānumāna*):

1. $(\forall a)(Pa \rightarrow Qa)$
 - a. Every instance of the mountain/If the (*anumeya*) mountain possesses smoke, the (*anumeya*) mountain possess fire.

2. $(\forall x)((x \neq a \ \& \ Px) \rightarrow Qx) \ \& \ (\exists x)(x \neq a \ \& \ Px)$
 - a. If anything, which is not identical with the *anumeya* and is observed (*darśanam*) to have the reason (*hetu*), smoke, then it will have the property to be proven (*sādyā*), fire, and there is at least one thing that is not identical with the *anumeya* that is observed (*darśanam*) to have the reason (*hetu*) smoke.

3. $(\forall x)((x \neq a \ \& \ \neg Qx) \rightarrow \neg Px) \ \& \ (\exists x)(x \neq a \ \& \ \neg Qx)$
 - a. If anything, which is not identical with the *anumeya* and is not observed (*adarśanam*) to have the property to be proven (*sādyā*), fire, then it is not observed (*adarśanam*) to have the reason (*hetu*), smoke, and there is at least one thing that is not identical with the *anumeya* that is not observed (*adarśanam*) to have the property to be proven (*sādyā*), fire.

The question as to whether the first and second conditions of the *Trairūpya* indicate that the logical reason occurs only in *pakṣa*, and the *sapakṣa*-domain is all that is required to restrict the reason to the *sapakṣa*-domain and exclude it from the *vipakṣa* seems to be dismissible given the three statements above. However, the pushback to the argument that the *pakṣa* and *anvaya* are sufficient to denote (*bhāṣate*) the absence of the reason (*hetu*) from the *vipakṣa* challenges the idea that *vyatireka* is alone sufficient to accomplish the exclusion of other referents (*anyāpoha*). This seems clearer if we consider *anyāpoha* to be logically

formalized by 1 and 3 above. Given our formal account of *vyatireka*, we can only conclude via modus ponens that when fire is absent, smoke is absent, or if we perform contraposition first, modus tollens about half of the reasoning that we need in order to gain ascertainment from our inferences (*anumāna*).

Conclusion

The best way to test a method of reasoning that is meant to produce ascertainment (*niścaya*) and avoid doubt (*saṃśaya*), is to put that reasoning to a test. If we take the *avinābhāva* of the reason (*hetu*), smoke, and the property to be proven (*sādyā*), fire, to be a general law, then we can use *anvaya* and *vyatireka* to conclude that we have in fact determined that the object of inference that we are not familiar with “x,” contains membership in those things that fit the rule and does not contain membership in those things that do not fit the rule. Our general rule has been provided by someone who knows that invariable connection (*avinābhāva*) between smoke and fire. So, we know that ‘If there is smoke then there is always fire,’ is a valid inference for oneself and others (*svārthānumāna* and *parārthānumāna*).

If we are given a set of instances that we would need to confirm our rule, we need to look at the instances where the reason (*hetu*) is observed (*darśanam*) with the property to be proven (*sādyā*) and the case where if we do not observe (*adarśanam*) the property to be proven (*sādyā*), then we do not observe (*adarśanam*) the reason (*hetu*). In other words, we confirm that we have both the modus ponens and tollens relations. A clear example of this type of reasoning is the Wason 4-card selection task, in which a simple rule is given by someone who knows that there is a relation between vowels and even numbers. In this case, one is asked to confirm the rule ‘if the card is observed to have a vowel on one side, then it is observed to have an even number on the other side,’ by choosing which cards to flip over from a set of four cards observed to have A, K, 2, 7 showing. In this case, to confirm the rule, we must select two different cards, the A and the 7. We select the A because, if we observe an A (vowel), then we should observe an even number on the other side. We select 7

because, if we don't observe an even number, then we should not observe a vowel on the other side.

For the same reasons, we must use both *anvaya* and *vyatireka* to confirm our previous observation of the reason (*hetu*) and the proposition (*pakṣadharmatva*) in the (*anumeya*) which defines the rule. Given the example above about the reasoning imbedded in the *Trairūpya* to confirm a rule provided by someone who knows, it seems impossible within these constraints to form a valid inference for oneself and a proof for others (*svārthānumāna* and *parārthānumāna*) without the use of both *anvaya* and *vyatireka dr̥ṣṭānta*. We can see in a formal account of the properties of the *anvaya* and *vyatireka dr̥ṣṭānta* that they are not equivalent, and both are needed to form a valid inference for oneself or a proof for others (*svārthānumāna* or *parārthānumāna*). Thus, in the case of the exclusion of other referents (*anyāpoha*) as denoting the general term (*jātiśabda*), it is false that we can use *vyatireka* alone or primarily to create inferences (*anumāna*) as Dignāga suggests. Therefore, the use of *anvaya* is necessary in an inference for oneself or proof for others (*svārthānumāna* or *parārthānumāna*) to prove the invariable relation (*avinābhāva*) of the general rule between the *hetu*, *sādyā*, and *anumeya* provided by someone who already knows.

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