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ECONOMICS OF SEDIMENTATION MANAGEMENT IN LARGE RESERVOIRS

[Draft Paper, Biswo Poudel, UC Berkeley]

05/21/2010

Abstract

This paper develops a model of sedimentation management in reservoirs. It contributes to the existing literature on the topic in different ways. The model is set in stochastic setting and is rich enough to accomodate the study of different contingencies such as weather fluctuations, regulation changes due to global warming or other events that causes change in public perception regarding large reservoirs. More over, this model also uses a new numerical method to solve nonlinear optimization equation that in some cases is faster than existing models and requires less computation. Such nonlinear differential equations appear naturally in sediment management problems. Data from Tarbela reservoir in Pakistan, one of the most sediment prone reservoirs, is used to calibrate the model and our result suggests that given the specific assumptions regarding cost functions of sediment removal, the dam could be sustainably run.

1 Introduction

"*Among the many sessions of the Third World Water Forum, held in Kyoto, Japan in March 2003, there was one titled "Sedimentation Management Challenges for Reservoir Sustainability". Two main messages emerged from that session:(i) Whereas the last century was concerned with reservoir development, the 21st century will need to focus on sediment management; the objective will be to convert today's inventory of non-sustainable reservoirs into sustainable infrastructures for future generations. (ii)The scientific community at large should work to create solutions for conserving existing water storage facilities in order to enable their functions to be delivered for as long as possible, possibly in perpetuity.*" (Johnson, Ian (2003))

Reservoirs are one of the most common forms of nonrenewable resources, yet their economic studies have been rare. Engineering literatures emphasize that even when reservoirs were structurally sustainable,they could nevertheless become unsustainable due to sedimentation accumulation. The loss of storage due to sediment accumulation is nontrivial and alarming: Mahmood, K. (1987) reports that the annual capacity loss of world's reservoirs due to sediment accumulation is about 1%, though White(2001) recently put this figure at 0.5%~1% . A world bank report translated the loss as the need to add some 45 km^3 of storage per year worldwide, costing US$13 billion per year exclusive of environmental cost. China, which alone accounted for more dams construction than the rest of the world during $1950-1980$,fairs worse, mainly due to the nature of sediment rich Yellow river. Zhou(1993) reported that China's $82,000$ reservoirs are losing their capacity at the average annual rate of 2.3%. Three other frequently cited example of storage capacity loss are Welbedacht dam (built in 1973, it has lost more than 80% of its capacity), Mangaho River project in New Zealand (59% of capacity lost in 45 years of operation, bottom outlet buried under 13 m of silt after 25 years of operation)and Tarbela reservoir in Pakistan(built in 1974, has lost about 20% of its storage capacity). If sedimentation issue is not taken care of properly, reservoirs needs to be abandoned after the sedimentation reaches a critical level.But sedimentated sites can't be easily recycled for reuse. Such recycling efforts could be extremely costly. For example, according to Morris and Fan (1998),it would cost $83 billion to restore Lake Powell in Colorado river (at the rate of \$2.5/m^3) assuming one could find the disposal site to dump $33km^3$ of sand.Furthermore, there are not many proper sites for constructing reservoirs. Such sites certainly are not growing. Also, since the best sites (from the perspective of construction as well as operation and management (O&M) costs) were taken up earliest, alternative sites will be progressively costlier. These facts attest to the reservoir being nonrenewable resource. Ruud et al (1993) claim that Green House gases (GHG) emitted from the reservoirs are positively correlated with the area flooded. In particular, areas which flood either upland foress or peatlands in Canada are likely to produce more GHG. Studies like these further reduce the number of suitable sites for reservoir and provide further evidence of them being nonrenewable resources.

There are at least 50000 dams in the world that are more than15m tall, as reported by International Commission On Large Dams (ICOLD). However, the total number of dams in the world is much more. In particular, given that only 7% of dams in the United States are more than 15m tall(National Inventory of Dams, US Army Corp of Engineers), the total number of dams in the world could be more than 1 million. Lots of these dams are reaching their age. Furthermore, public's perception of large dams as a clean source of energy is also undergoing transformation, and their decomissioning is more frequently discussed topics now than ever. At the same time, one needs a rigorous framework to calculate the economic value of dam at the time it is decomissioned so such decomissioning could be justified by judging it from some economically rational framework. Such value of the reservoir at the time of its decomissioning is the salvage value of the dam. From an operator's point of view,the salvage value of dam is stochastic for several reasons: the impact of sedimentation on ecology and human health are not clearly understood. (Danielevsky (1993), Tolouie (1993)) reported that desiccated deposits of fine sediments could be eroded and transported by wind, causing health hazard to nearby population. Furthermore, Chen et al(1993) reported that the presence of sediment against dam could constitute earthquake hazard. The impact of sediment accumulation on ecology alteration (such as on habitat of salmon) and the impact of delta deposition on the probability of flooding are also actively researched field. In legal front, Thimmes et al(2005) reviewed recent court decisions on cases against dam operators and found that courts have issued reward against dam operators for the ecological damage caused during the dam operations, and overall conclude that judicial determinations of reasonable reservoir management and reasonable precautionary measures by landowners are generally highly speculative, controversial, and based on limited information. Pansic et al (1995) report that currently three major costs associated with dam decomissioning include sediment management (48%), environmental engineering(22%) and infrastructure removal (30%). Furthermore, regulatory agencies may continue to impose new conditions on the operators as the new information on the impact of dams arrive, including their impact on Green House Gas stock in the atmosphere.The cost of decomissioning could very well be astronomical if stringent conditions are applied to the operators in the future, and this is consistent with the overall uncertain time dam operators are living in rightnow.

It is clear that the periodical removal of sedimentation is an integral part of the operation of a sustainable dam.There are several techniques to remove sedimentation from reservoirs. We can roughly divide them into four types: erosion prevention, sediment routing, flushing and dredging. Erosion prevention can always be used with the latter. Erosion prevention schemes include watershed management issues such as encouraging people upstream to get involved in the practices that are not going to contribute to soil erosion (i.e. best management practices). The other alternative is trapping sand before it reaches reservoirs; for example, by constructing check dams, though they are not very effective. Sediment routing methods involve both sediment bypass (circumventing the dam) and sediment passthrough(sending sediment through the dams) methods. These methods involve emptying reservoirs periodically or just before the flood.

Flushing involves opening a low level outlet to temporarily establish riverine flow through which eroded sediment is flushed. Flushing is distinct from routing as the former deals with settled sediment and involves release of sediment at the season which is different from the season used by sediment routing which releases sediment when they arrive. The timing aspect of sediment release also makes flushing not very popular among environmentalists. Dredging involves mechanically digging up the coarse deposit and removing them from the reservoir. A detailed description of these methods can be found in Morris and Fan (1998) and is also presented in the next chapter.The challenge in finding the optimal sedimentation technology is that any such prescription necessarily relies on the topography of the region and on such minute details as the size of sediment (fine, coarse, big boulder etc) and hence an economic model has to make tradeoff between the accuracy of representation and simplicity of modelling so that one achieves desired tractability to come up with reasonable insights.

Our goal in this paper is to formally represent the reservoir management problem, taking into account the stochastic nature of salvage value of the dam at the time of its decomissioning. The formalization also provides us the following three major insights: (1) ranking of different sedimentation removal techniques from the perspective of their impact on the age of dam (we differentiate between economic life of dam, usable life of dam and life of dam in general, using the terminology of Murthy(1977)) is facilitated. (2) optimal sedimentation management is retrieved as a result of a control problem of the operator and (3) the value of the dam at any point. At the end, we are also able to discuss sustainability issue of the reservoir.

We contribute to the literature in the following way: this paper is the first one to look at the sedimentation issue as a discrete continuous model in a stochastic framework. We provide detailed study of techniques and discuss qualitative properties of key thresholds that trigger different decision makings (such as in sedimentation management). We also provide a new method that modifies Judd(1992)'s projection method in solving the nonlinear equations that are results of optimizing decision of the operator. We use data from Tarbela dam in Pakistan to calibrate our model. We conclude that for some given cost functions, the dam could be sustainably run.

2 Literature Review

Economic studies of sediment removal techniques so far have been very rare. In 2003, the world bank's resource economics group developed a policy maker's manual-type report, called RESCON. Their work provides a brief survey of sedimentation technique and a "look-up table" type Excel based software to facilitate the economic and engineering evaluation of different sedimentation strategies. Another work by Palmieri et al (2001) used the RESCON software to show impact of different sediment strategies on sediment removal policy and life of the dam. Huffaker et al (2006) provided a detailed economic study of hydrosuction dredging sediment removal system. In particular, Huffaker et al construct a multi-state model of endogenous reservoir operations and apply singular perturbation solution methods that reduce dimensionality of the optimality system and facilitates the solution of the optimal system. They uncover a phenomenon called "sediment perching" due to which increased sedimentation in the reservoir makes the sediment control mechanims more effective in the long term. Though they take into account the positive effect of sediment perching on dredging cost, they fail to note that sediment perching alters the natural pattern of sediment flow downstream and may cause undesirable environmental cost and their estimate of the benefit of sediment perching may therefore be upward biased. The sediment perching phenomenon, in our view, is similar to the (S, s) decision making proposed by Arrow et al(1951): one waits until the stock is down to a certain value before replenishing the stock.

Literature has focussed on the debate of whether a dam should have "design life" or whether it should be run sustainably by using life cycle management strategy. Intergenerational equity requires that a dam either be run sustainably or the generation (or generations) that benefit from the dam pay for its decomissioning cost (for example by raising a fund to be used by future generation). Palmieri et al (2003) discuss about a method to generate such fund in a reservoir in China. In addition to these studies, Keohane et al (2006) proposed a SFQ model which they suggested could be used in the context of reservoir management. In their model, stock and flow both must be controlled to promote the quality, which in the context of reservoir management problems requires the control of both sediment flow and sediment stock to maintain the quality of the reservoir and reservoir products.Their result implies that if the dam operator has the choice of both sediment removal and restoration, then the threshold that triggers restoration in the absence of choice regarding sediment removal will be lower than the case in which planner has the option to remove sediment. On the other hand, the feasibility of restoration will reduce the optimal sedimentation removal each period. The author seem to treat restoration as if the asset being restored is renewable.

Exact timing of a decomissioning of a dam is not an issue studied in literature. However, the issue is similar to much studied machine replacement problem in finance and economics. The major study in the literature was due to Rust(1987), who studied the decision of an administrator making decision on repair or replacement of GMC bus engines. A dam administrator is in a way similar to Harold Zurcher, the bus administrator: making a decision on repair (i.e. sediment removal) or decomissioning, but most likely, without the option of replacement. Furthermore, with dam, the concept of sustainably running it is more important, where as with the bus, it is not even considered.

3 Sedimentation removal techniques:An overview

3.1 Engineering Classification

Sedimentation removal is an integral part of the operation of a reservoir. It is also a major instrument to make a reservoir sustainable. There are three major groups of sedimentation removal techniques:

(1) Erosion Control and Watershed Management: This class of technique include the investment in erosion control upstream so that the river doesn't carry a lot of sediment into the reservoir. This method is mainly focussed in rehabilitation of degraded soil and watershed upstream. Literature in sedimentation management emphasize that such management strategies be carried out with the help of landowners upstream as their noncooperation result in the failure of erosion control programs. Sediment management and erosion control techniques may use methods ranging from basic land use changes to the complicated high fixed cost structural methods such as construction of terraces, diversion channels, grassed waterways, check dams. Nonstructural methods include agronomic measures which rely on the regenerative properties of vegetables. Other methods in use include operational measures such as scheduling construction to minimize the area of exposed soil. Land use changes doesn't involve fixed cost, and may not result in reduced sedimentation yield immediately downstream. Faulkner and McIntyre (1996) reported that there were no change in sediment yield even 20 years after the transition to less erosive land use. There are several basic agricultural engineering techniques in erosion control (see, Schwab (1993) for a detailed study on it). In the United States, Best Management Practices(BMP) are recommended for erosion control.

From the economic point of view, these methods can be divided into two classes: (1) structural methods are fixed cost method with low annual maintance cost and (2) nonstructural methods have no fixed cost, but have relatively higher annual maintanance cost. They also differ in their efficacy: it is recognized that the nonstructural methods can never lead to zero sedimentation yield downstream.

Erosion control is also topography dependent. In countries like Nepal, which is situated in the tectonically active Himalayas, erosion control in the watershed is not considered technically feasible in several possible reservoir sites. This is the same case in Tarbela, the reservoir about which we study in detail later.

(2)Sediment Routing: Sediment routing techniques often "route" sediment either through the dam itself or from a diversion constructed to bypass the dam. The implementation of such techniques is considered to be site specific and could be cheap at some sites. At some sites, costly modification of dam is required to implement it. Routing techniques try to identify the part of the incoming water that has sediment and prevent it from depositing in the reservoir, often either by letting the sediment pass through the reservoir or around storage or intake areas. Sediment routing , in general, is considered useful in small reservoir.

Morris and Fan (1998) provides two major subgroups for sediment routing techniques:

(i) Sediment Pass-through: This group includes techniques such as seasonal drawdown of reservoir, flood drawdown by hydrograph prediction or rule curve and venting turbid density currents. Of these , seasonal drawdown often involves a season of emptying of a reservoir so that the channel is eroded along the thalweg and sediment is removed. Since lower level outlets are open through out the flood season, sediment deposition during flood season, which constitutes the main fraction of sediment during a year, is routed out of the reservoir. Sanmenxia Reservoir in China is a major example of the reservoir that employs this method. Flood drawdown method involves lowering the pool level and increasing flow velocities during individual flood events. Large reservoirs that employ hydrograph control release water from the rising limb of the hydrograph and refill the reservoir with water from the hydrograph recession. This method requires real time prediction of the inflowing hydrograph to guide gate operation. Similarly, small reservoirs may use rule curve to guide gate operation. The method of turbid density currents exploits the differences between two different types of fluid.

(ii) Sediment Bypass : These are mainly of three types: on-channel, off- channel and subsurface bypass. On-channel type involve the construction of large capacity channel or tunnel to bypass sediment-laden flow around an instream storage reservoir. Such a channel obviates the requirement of constructing large capacity spillway at the main dam. Example of such method is Nagle reservoir in South Africa. Off channel bypass technique requires the construction of impoundment off the main river . These off channel reservoir avoid sediment laden water by either partially or fully excluding the floodwater, by constructing intake structure to exclude coarse sediment and by using diversion dam to trap sediments. An example, as given by Wu(1991) , is Sun Moon reservoir of Taiwan in which 49.5 percent of total streamflow was diverted to the reservoir but only 3.5 percent of the stream sediment was diverted. Subsurface sediment bypass includes those storage that takes benefit of the fact that coarse sediment contain voids which can store water and in some situation, a trap dam may be constructed to accumulate coarse sediment, extracting water from the subsurface storage by a pipe through the base of the dam and which extends through the permeable deposits. Baurne(1984) claims that evidence from Libya suggess these types of structures have been used there for at least 1000 years.

(3) Sediment Flushing: Flushing is a technique that has drawn attention of some economists. It is limited to small reservoirs, with typical capacity inflow ratio (which is the ratio of total reservoir volume to mean annual inflow) of less than 0.3. Although there are a variety of flushing techniques, they all involve lowering of reservoir level and letting flood sweep away the sediment deposited in the reservoir. The three steps of flushing are therefore often called drawdown, erosion and refill (lowering reservoir level, letting flood water erode the deposition and then refilling the reservoir again). In a lot of reservoir such flushing is done seasonally. Seasonal emptying of reservoir is considered good if the demand of water is also seasonal (for example, Jensepei reservoir in Taiwan uses seasonal emptying of reservoir that is mainly used to supply water for a sugar mill that operates only six month every year). The problem with flushing is that it releases sediment at a higher rate than normal alluvial pattern and it causes a lot of problem downstream. The thick water with sediment is not suitable for any hydropower dam downstream, and may temporarily disable solid handling capacity of water purifying plants downstream. High sediment concentration in water also smothers benthic organism and clog gills. It may cause anoxia which can kill lots of organism in stream. Persistence of sediment in streambed may increase floodrisk and may cause desertification downstream.

A related important concept in literature is flushing efficiency which is basically the ratio of sediment to the the volume of water used during the flushing (i.e. $Flushing\ efficiency = \frac{Deposit\ volu\mathrm{me}\ eroded}{Water\ Volume\ Used}$. High flushing efficiency doesn't necessarily mean the method is desirable as flushing efficiency is different for coarse and fine sediment. Furthermore, the desirability of flushing efficiency also depends on the types of users downstream. In a related study, Basson(1997) provided a rule of thumb for flushing.Using Basson's index $K_w = \frac{S}{MAR}$, where S=Storage Capacity of a reservoir and MAR is Mean Annual Runoff , Basson's rule of thumb was to use flushing techniques (which is a sediment routing technique) for K_w<0.2 .

Flushing often involves channel formation and management. Once channel of flushing is formed, some reservoirs use mechanical assistance to increase the efficacy of the operation. An example is the use of bulldozer to push the sediment into flushing channel of San Gabrield debris basin in Los Angeles. Morris dam also did something similar, but later discontinued it because of the ecological impact downstream. It is conventionally agreed upon (Morris and Fan, 1998) that while fine sediments are taken care of relatively well by flushing, coarse sediments and boulder tend to accumulate over time and reservoirs employing flushing needs to look at the accumulation of coarse sediments too. Furthermore, behavior of coarse sediment and impact of flushing downstream is an area of research that has received scant attention.

(4) Sediment Excavation and Dredging: Excavation are costly options and most of the time, they are the only options once sediments are firmly deposited in the reservoir. Excavation option often depend on sediment volume, grain size, geometry of deposit, available disposal and reuse options and water level and environmental criterion. Dredging is an operation in which sediment is lifted from the bottom of the surface of a waterbody and is deposited elsewhere. In the United States, $500Mm^3$ sediment is dredged every year. Dry excavation involves completely emptying the reservoir, desiccating the surface and deposits and using earth moving equipment to remove the silt from the surface. Hydraulic excavation will require dewatering dredge slurry after it has been removed from the water surface, so that it can be removed in conventional hauling equipments to dump elsewhere. In small ponds in the united states, there have been some use of explosives to excavate sediments, but such use is rare among the large ponds.

Dredging as a long term strategy for reservoir management is possible only if a good dumping site can be found. Although in many mountainous regions, the river downstream is considered the natural target for dumping dredged materials, such dumping is considered environmentally undesirable.

There is a related method called Hydrosuction removal system(HSRS) that uses the hydrostatic head at the dam to provide energy for sediment removal. HSRS is of interest because there has been one major economic study of this method in detail (Huffaker et al(2006)). This method is similar to dredging, but it applies the hydraulic head available at the dam as the energy for dredging and is considered cheaper than dredging. HSRS consists of a barge that controls the flow in the suction and discharge pipe and can be used to move the suction end of the pipe around. The pipe's upstream end is located at the sediment level in the reservoir and the downstream end is draped over the dam to discharge sediment to downstream. Because of this, its applicability is limited to shorter reservoir. This method is normally considered energy conserving,and environmentally friendly.

3.2 Economic Classification

To properly use these methods in our formal model, we provide an alternative classification for sedimentation removal techniques based on their cost function. It is done for modelling expediency, but it also serves to emphasize the obvious that economics is not an engineering in its approach. We classify the techniques by the "economics" of them, and show how at times the approaches that belong to the same engineering class end up belonging to the different economic groups. For the discussion below, we assume that the sediment yield per unit time into the reservoir is M.

(i) Prevention strategies:These strategies involve the method of rewarding farmers upstream for their effort. Suppose reservoir owner rewards upstream farmers for their effort in erosion control.Let p_e be the price of water saved by erosion control effort e. Effort costs l per unit and produces reduction efficiency $R(e)$, where e is the amount of effort applied, R is a continuously differentiable function and $0 \leq R(e) \leq 1$.Throughout this paper, we will be assuming this function to be strictly concave so that an unique optima exists. For this specification to make sense, we define $\lim_{e \to 0} R'(e) = \infty$, $\lim_{e \to \infty} R'(e) \leq 0$. This assumption, along with intermediate value theorem, guarantees that there exists an optimal e such that $p_e R'(e) = l$. Farmer's wage, l, could be determined in the labor market exogenous to a dam operator or ,alternatively, farmers may have the bargaining power and may exact the wage which is equal in margin to the cost the dam operator could incur by using alternative strategy. At the end of the strategy, the sediment yield per unit time to the reservoir will be $(1 - R(e^*))M$ where e^* is an optimal e employed by the operator.

(ii) Fix and proportional cost strategies: These cost strategies involve constructing a structure that helps in reducing sedimentation yield to the reservoir, and then spending on annual maintanance of reservoir which is linear in sedimentation removed. For example, one may construct terraces, diversion channels, grassed waterways, and check dams upstreams and then spend some money

on erosion control each year. The hydrosuction dredging strategy studied by Huffaker et al, in which cost is represented in terms of water lost in removing sediment, is also proportional cost strategy.

Both (i) and (ii) are independent of the sedimentation level in the reservoir. We define sedimentation level dependent strategies as follows:

(iii) Quadratic cost strategies: These are the strategies which depend on both sediment level and total amount of sediment removed. For low level of sedimentation, the average cost of sediment removed is very high, where as it is lower for higher level of sedimentation. These strategies need to trade off the reduced cost with the reduced storage , and therefore one may generally find optimal level of both sedimentation level and optimal siltation removal in these strategies. These methods represent excavation and dredging strategies. The higher level of sedimentation is also needed if the dumping site needs to be found each time this strategy is used.

(iv) Uncertain cost strategies: These strategies include strategies where a part of the cost is uncertain. For example, when flushing is used to remove sedimentation from the reservoir, the downstream impact on the environment could be uncertain as high concentration of sediment tend to affect biodiversity downstream. It also disturbs the natural pattern of sediment flow in the river. Since environmental impacts are hard to quantify, we consider such strategy as an uncertain cost strategy.

(v) Fix and Quadratic Cost strategies: These are the strategies which involve a fixed cost at the start of the operation and a quadratic cost function for removal of siltation. Note that routing may involve construction of off-channel reservoir or flood gates at the start of the operation which is fixed in its nature and then construction of channel to remove the siltation through low level outlets which is quadratic in its nature.

The table below presents a summary of our classifications:

Economic Strategies	**Corresponding Engineering Strategies**
Prevention Strategies	Erosion Control, Watershed Management
Fix and Proportional Cost Strategies	Erosion Control (using terraces, check dams), flushing
Quadratic Cost Strategies	Dredging (Excavation) Strategies
Uncertain Cost Strategies	Flushing when environmental & other costs are uncertain
Fix and Quadratic Cost Strategies	Routing

4 Cost of the decomissioning of a Dam

Public's perception of dam as a clean source of energy has undergone some changes recently. In particular, the role of a dam as an emitter of green house gas has been asserted by researchers such as Ruud et al (1993) and Duchemin et al (1995). Duchemin et al studied methane and carbon dioxide emission in two hydroelectric reservoirs in northern Quebec for two years and found "above average emission fluxes". Their result showed the emission flux (measured in $mg\ m^{-2}d^{-1}$)to be five to eight times less than what Ruud et al found out. Though Duchemin et al found the emission was on a much smaller scale than conventional thermal power plants equivalent amounts of energy, studies done in Brazil's Balbina reservoir (Fearnside(1995), Irion et al (1987)) show that the reservoir produces more greenhouse gas than coal fired equivalent due to the vegetation inundated by the reservoir. Such results have made it difficult for large reservoirs to qualify for carbon credit in carbon markets (Whittington (2007)), eventhough the small hydropower with no forest inundation often qualify for it.

If large dams are sources of substantial emission, then their actual cost to the society is likely to be uncertain for long, since there is significant uncertainty related to the 'damage function' : damage to the society due to GHG induced increase in temperature. Hence the dam operator may know the cost of decomission at any moment, but the cost in the future is uncertain. This calls for the modification in assumption of Palmieri et al (2001) that the salvage value of the dam is fixed and constant. This also provides motivation to learn how sediment removal rate will be changed under such scenario.

There are two main reasons why a reservoir is decomissioned: the owners may find it economically infeasible or the regulatory agencies may demand that the reservoir is decomissioned. In the United States, Federal Energy Regulatory Commission(FERC) stated in its statement (made on 12/04/1994) that it has the right to decomission a project when considering its relicensing request. When a dam is decommissioned, there are three major issues: (1) what should be done regarding the dam? (2) what should be done regarding the sediment deposited in reservoir?(3) How should environmental restoration be carried out? The dam could be left as it is, partially breached or completely removed.The sediments could be left as it is if dam is left as it is. The other choices regarding sediment management are to allow natural erosion, construction of a channel through the deposits while leaving off chanel sediment as it is, and removal by mechanical excavation or hydraulic dredging.Some agencies may demand that the dam operator restore early fluvial condition. In such case, the dam operator may incur extra costs, apart from sediment management and infrastructure removal.It is reasonable to assume that the change in the cost related to (1) and (2) are relatively known and deterministic, but the change in the salvage cost related to (3) will be uncertain. Such uncertainties point to the need to study dams in stochastic settings.

5 Problem Formulation

Let K be capacity of existing dam, and $s(t)$ be the level of sedimentation at time t. The reservoir receives sedimentation at the rate of M per unit time. M is similar to "sediment yield" by a basin to the reservoir in engineering literature.

In stead of using sedimentation level $s(t)$ as a state variable, we use water level, $w(t)$, as the state variable. The two variables are equivalent in the sense that $w(t) = K - s(t)$. We recognize the uncertainty in water level by writing the equation of motion of water storage capacity as $dw = -(M - c)dt + \sigma dz$, where the uncertainty is additive and driven by Wiener process z and the standard deviation parameter, σ. This formulation is consistent with the observation that the arrival of sediment, and therefore the water storage available, is partly deterministic, as we know they arrive at some rate per year, and partly random. Let the implicit price of water, net of any dam related damage, be p. Assumption of nonstochastic p implies that there is no demand uncertainty at our abstraction level.We abstract from geological models regarding sedimentation removal technologies and assume the sedimentation removal function is $f(c(t), w(t))$, where c is the amount of sedimentation removed. We assume that $\frac{df}{dc} > 0, \frac{d^2f}{dc^2} \geq 0$.

The "profit function" $\pi(w(t), c(t))$ gives profit at time t, when water level is at $w(t)$ and the planner decides to remove $c(t)$ units of sediment. Assumption of risk neutral planner implies that $\pi(w(t), c(t)) = pw(t) - f(c(t), w(t))$. The planner's maximization problem is given by

$\max\limits_{c} \int_0^\infty e^{-\beta t}\pi(w, c)dt$

$dw = -(M - c)dt + \sigma dz$ ....(1)

with $w(0) = K$.

Where the additive uncertainty represents the uncertainty related to the amount of M. In particular, it captures temporal variability in sediment yield. One may also consider M as fixed and interpret the variance as capturing the variation in the rainfall which changes the level of water surface in a large reservoir.Notice that the control decision and value functions are always written as a function of w, i.e. are $\mathcal{F}_t^w-$adapted.

The functional equation of the social planner's problem can be described as follows:

$V(w) = \max\limits_{c}\ \pi(w, c)dt + e^{-\beta dt}EV(w')$....................$(1')$

After expanding the right hand side using Ito's calculus and following standard procedure, one gets

$\beta V(w) = \max\limits_{c}[\pi(w, c) - (M - c)V_w + \frac{\sigma^2}{2}V_{ww}]$..................(2)

(2) implies that the optimal amount of sedimentation removal must satisfy $\frac{\partial \pi}{\partial c} = -V_w$. Hence, a general rule of thumb should be that the dam operator should remove the sediment until the marginal loss of profit due to sedimentation removal is equal to the marginal increase in the value of the dam because of increased water storage capacity. Notice that this result is independent of any functional assumption on π or V.

Note that $w(t) - c(t)$ is often termed "sedimentation effect on yield" in engineering literature. Technically, in using storage capacity directly, we are abstracting from the engineering notion of reservoir yield, which one calculates by using Gould's gamma function.Here, we also assume $\int_0^\infty (pw - f(c, w))d\tau < \infty$ $\forall T$ to ensure that the problem is well posed.

To solve (2) more precisely, however, one must determine the precise form of cost functions. Since our strategy is to differentiate the sedimentation removal strategies based on their underlying cost functions, we use the classification of section (3) to calculate the value under different strategies.

5.1 Prevention Strategies

This strategy relies on watershed management. It rewards the farmers for their effort to reduce the erosion yield. The owner of the reservoir pays to the farmers. This is the simplest strategy, and in this case the problem is that of (pure) optimal stopping problem.

We use notation introduced above, and denote Reduction Effort function as $R(e)$. At optimal effort level, the firm incurs the cost of e^*l and the siltation yield at the reservoir is now changed from M to $(1 - R(e^*))M$.

The modified problem now is

$\max_T \int_0^T e^{-\beta t}\ (pw(t) - e^*l)dt$

$dw = -(1 - R(e^*))M\ dt + \sigma dz$ ....$(1')$

with $w(0) = K$.

Even without calculating explicit solutions, one notices that as long as $1 - R(e) > 0$, the dam will have a finite life, as sedimentation level increases by $(1 - R(e^*))M$ each period, and this value is independent of the level of s. Since $\frac{K}{(1-R(e^*))M} < \infty$, the flow becomes negative in finite time as both e^* and l are constant. Hence the dam will have a finite life.

Using optimal stopping method, it is straightforward to calculate the optimal T that will mandate the decomissioning of the dam. We use the method given in page 225, Oskendahl (2007), to prove that the optimal decomissioning of dam takes place at the time period when $w(t)=\frac{el}{p}$.The derivation of this is straightforward. The characteristic function of $\Theta(w,t) = e^{-\beta t}\ (pw(t) - e^*l)$ is zero when $(pw(t) - e^*l) = 0$. Thus the decomissioning takes place when $w(t)=\frac{e^*l}{p}$. The following observations are immediate:

1. An increase in the implicit price of water implies that decomissioning of dam takes place at the higher level of the stock of sedimentation level.

2. The increase in the labor cost to protect erosion, ceteris paribus,increases the required water level to run the dam effectively. More efficient labor force may decrease e , even though the labor cost for them may be higher than l. The cumulative impact of such case is ambiguous.

5.2 Fix And Proportional Cost Strategies

In sediment routing methods, the major cost is in setting up the initial routing structure. For example, creating a diversion from the river to the reservoir costs some fixed amount of money. Once such diversions are created ,the amount of sediments routed to the reservoir could be as low as 3% of what would be without such diversion (as in the case of Taiwan's Sun Moon reservoir mentioned in Wu(1991)). Such a small amount of sediment could be removed either by using "flushing" during the flood season, or by using human labor itself. If, for example, removed by flushing water, the cost of removing would be linear as it would be the product of the price of an amount of water needed to flush the sediments out.

A study that provides a clear indication of linearity of cost function is due to Basson's (1997). To remove X units of sediment, the volume of water required in sluicing operations is found to be $(3+17K_w)X$, where K_w is defined to be the ratio of storage capacity to mean annual runoff. For a constant K_w, it should be clear that the total cost to remove X unit of water is $p(3+17K_w)X$, a linear equation. Inspired by this example, we assume $\pi(w(t), c(t)) = pw(t) - \alpha c(t)$; where $0 \leq c \leq K - w$.

Equation (2) now becomes

$\beta V(w) = \max_c [pw - MV_w + c(V_w - \alpha) + \frac{\sigma^2}{2} V_{ww}]$.......(3)

Linearity of equation (3) implies that the sediment removal function is a bang bang in its nature. In particular, the maximizing c is given by

$c = 0$ if $(V_w - \alpha) < 0$

$K - w$ if $(V_w - \alpha) > 0$

Any amount between 0 and $K - w$ if $(V_w - \alpha) = 0$

if $(V_w - \alpha) \leq 0$, we need to solve for the following value function

$\beta V(w) = pw - MV_w + \frac{\sigma^2}{2} V_{ww}$.........................(4)

The solution for (4) can be given explicitly as follows:

$v(w) = -\frac{Mp}{\beta^2} + \frac{pw}{\beta} + Ae^{\alpha_1 w}$

where $\alpha_1 = \frac{M+\sqrt{M^2+2\sigma^2\beta}}{\sigma^2}$ and A is a constant yet to be determined. Notice that the solution has two parts,$Ae^{\alpha_1 w}$ is the solution of homogenous part of (4), where we ignored the part involving negative root of function $\frac{\sigma^2}{2}\psi(\psi - 1) - M\psi - \beta = 0$ to avoid indeterminacy of value function when $w = 0$. From (4), It is also immediate that the value of the reservoir when $w = 0$ (i.e. when the dam is filled up with sedimentation) is given by $A - \frac{Mp}{\beta^2}$.

We postpone the discussion on the precise identification of the value of A until after the proposition 1.

Precise determination of A involves using a value matching condition: i.e. the value of the function that solves

$\beta V(w) = pw - MV_w + (K - w)(V_w - \alpha) + \frac{\sigma^2}{2} V_{ww}$...........(5)

And equating (4) and (5) at the point where switching function, $V_w - \alpha$, equals zero.

Proposition 1: *For the dam with the initial storage size K, sediment arrival per unit time M, and proportional cost reward function given by $\pi(w(t), c(t)) = pw(t) - \alpha c(t)$ where p is price per unit of water and w is effective storage capacity, the following statements are true:*

1. If sedimentation accumulation is high enough that water storage capacity is less than $\frac{\ln(\frac{p+\alpha\beta}{A\beta\alpha_1})}{\alpha_1}$, where α_1 is the positive root of the equation $\frac{\sigma^2}{2}\alpha^2 - M\alpha - \beta = 0$, it is not worthwhile to remove the sedimentation from the reservoir. In such case, once the threshold value for sediment accumulation is crossed, the reservoir is quickly filled up with the sedimentation and is abandoned.

2. The value of the dam at the time is $v(w) = \frac{-Mp}{\beta^2} + \frac{pw}{\beta} + Ae^{\alpha_1 w}$.

Proof:

The proof of (2) has been derived above in the discussion. The proof of (1) is straightforward and is a consequence of the calculation of value function.

Note that $V_w = A\alpha_1 e^{\alpha_1 w} + \frac{p}{\beta}$. For this value to be less than α, $w < \frac{\ln(\frac{\alpha\beta - p}{A\beta\alpha_1})}{\alpha_1}$, assuming $\frac{\alpha\beta-p}{A\beta\alpha_1} > 0$.

The following discussion involves an assumption that that the planner knows precisely the two values of the dam: its initial value $V(K) = V_K$, its salvage value, i.e. the value when the dam is completely filled up by the sediments, $V(0) = V_0$. Notice that assuming the precise knowledge of these two values also means that we can pin down A. It turns out that, $A = V_0 + \frac{Mp}{\beta^2}$.

When $(V_w - \alpha) > 0$,

$\beta V(w) = pw - MV_w + (K-w)(V_w - \alpha) + \frac{\sigma^2}{2} V_{ww}$............(5′)

This equation can be solved using power series expansion.

Let $V(w) = \sum_{i=0}^{\infty} a_i w^i$.

Then expressed in terms of a_0, a_1, the recursive representation of coefficients of the value function is given as follows:

$a_2 = \frac{[\alpha k + \beta a_0 - (K-M)a_1]}{\sigma^2}$

$a_3 = \frac{-(\alpha+p)+(1+\beta)a_1 - 2(K-M)a^2}{3\sigma^2}$

...

..

$a_n = \frac{(n+\beta-2)a_{n-2} - (n-1)(K-M)a_{n-1}}{(\sigma^2 n(n-1)/2)}$.

Proposition 2: *For the dam discussed above, the following must be true:*

Define $w^* = \max\{0, \frac{1}{\alpha_1} \ln\left(\frac{\alpha - \frac{p}{\beta}}{(V_0 + \frac{Mp}{\beta^2})\alpha_1}\right)\}, A = V_0 + \frac{Mp}{\beta^2}$.

1. If the water storage capacity is less than w^*, sedimentation removal is not economically feasible.

2. The value of the dam, $V(w)$, is given by the following expression:

$V(w) = -\frac{Mp}{\beta^2} + \frac{pw}{\beta} + Ae^{\alpha_1 w}$ *if* $w \le w^*$

$V(w) = \sum_{i=0}^{\infty} a_i w^i$ *else*

where the following conditions determine a_0 and a_1

$V_K = a_0 + a_1 K + a_2 K^2 + \ldots$

$V_{w^*} = a_0 + a_1 w^* + a_3 w^{*2} + \ldots$

and determination of a_0 and a_1 determines $a_i, i > 2$, as follows:

$a_2 = \frac{[\alpha k + \beta a_0 - (K-M)a_1]}{\sigma^2}$

$a_3 = \frac{-(\alpha+p)+(1+\beta)a_1 - 2(K-M)a^2}{3\sigma^2}$

and for $n > 3$,

$a_n = \frac{(n+\beta-2)a_{n-2} - (n-1)(K-M)a_{n-1}}{(\sigma^2 n(n-1)/2)}$.

Proof: Obvious from the preceding discussions.

There are some obvious and intuitive insights confirmed by the preceding results. For example, $\frac{dw^*}{d\alpha} > 0$, i.e. higher unit cost of sedimentation removal increases the threshold water storage level that triggers sediment removal. Higher salvage value, V_o, increases overall value of the dam even when sedimentation is infeasible. The following proposition discusses the impact of uncertainty on the value of the dams:

Proposition 3. (Impact of uncertainty) *In reservoirs with low sediment arrival rate(M) and sufficiently low salvage value , the increase in uncertainty regarding the sediment arrival increases the threshold , w^*, which triggers inaction on sediment removal. In particular, for the dam above the following relationship holds:*

$\frac{dW^*}{d\sigma^2} > 0$ *if* $M \to 0$ *and* $e^{-1} < \frac{\alpha\beta - p}{A\beta\alpha_1} < \infty$; *and* $\frac{dW^*}{d\sigma^2} < 0$ *if* $M \to 0$ *and* $0 < \frac{\alpha\beta - p}{A\beta\alpha_1} < e^{-1}$; *where* $e = 2.71$*, the exponential constant.*

Proof: Since $w^* = \frac{1}{\alpha_1}\ln(\frac{-p+\alpha\beta}{A\beta\alpha_1})$ where $\alpha_1 = \frac{M+\sqrt{M^2+2\sigma^2\beta}}{\sigma^2}$, we have $\frac{dw^*}{d\alpha_1} = \frac{-(1+\ln\frac{-p+\alpha\beta}{A\beta\alpha_1})}{\alpha_1^2}$. Note that this expression is negative for $e^{-1} < \frac{\alpha\beta-p}{A\beta\alpha_1} < \infty$ and positive for $0 < \frac{\alpha\beta-p}{A\beta\alpha_1} < e^{-1}$.

It is deceptively difficult to sign $\frac{d\alpha_1}{d\sigma^2}$. However, when M is close to 0, we can sign this expression. Note that $\frac{d\alpha_1}{d\sigma^2} = \frac{\sigma^2\beta-(M+\sqrt{M^2+2\sigma^2\beta})\sqrt{M^2+2\sigma^2\beta}}{\sqrt{M^2+2\sigma^2\beta}\sigma^4}$. Therefore, $\lim_{M\to 0}\frac{d\alpha_1}{d\sigma^2} < 0$. The proposition follows after combining these two results.

On Decomissioning:

In general, the dam would be worthless if $V(w) = 0$, the value is increasing in w and sedimentation removal is no longer feasible at w at which $V(w) = 0$. For some dams around the world, in particular , Loess Plateau of China(Voegele(1997)), the salvage value has been positive, i.e. $V_0 > 0$, and premature decomissioning of dam is not considered (apparently, the salvage value is positive because in the highly erosion prone zone, the reservoir, once silted up, is expected to provide a fertile land valuable from the agricultural point of view). If it is not the case, then one looks at $V(w^*)$ where w^* is the level of water at which sedimentation removal is no longer considered feasible. If $V(w^*)$ > 0, but $V_0 < 0$, then the decomissioning of such dams should take place sometimes after the sedimentation removal is stopped. If $V(w^*) < 0$, then the dam is decomissioned even when sedimentation removal is still being carried out.

Another point of interest is the impact of global warming on decomissioning. Assuming that the global warming causes the fluctuation in temperature which affects the melting rate of snows in the Himalayas (or in similar way affects the source of water), it will be natural to assume that global warming may increases uncertainty parameter, and this implies, by proposition (3), that the threshold that triggers inaction on sediment removal will be higher. This in general implies reduced age of the dams.

Hooper(2007) observes that the impact of global warming is likely to be felt by increased erosion . This corresponds to the increased M in our model.Since $\frac{dw^*}{dM} = -sign|\alpha - \frac{p}{\beta}|$, this implies that increased M decreases w^* if initial value of $w^* > 0$. Hence sedimentation removal is stopped at lower water storage capacity level as M increases. It may be interpreted as the desire to extract as much water as possible before abandoning the dam.

5.3 Quadratic Cost Strategies

In this section, we are concerned with the sedimentation removal strategies that have convex shaped, quadratic cost functions. The convexity of cost function is with respect to the amount of sediment removed,c. In particular, we assume the cost function is given by $f(c,w) = \frac{1}{2}(c + \hat{c}G(w))^2$. Our assumption on cost function implies that marginal cost has a slope of unity and an intercept $\hat{c}G(w)$ at each w. Such assumptions are reasonable for the technology that requires different level of external effort for different level of water (or sediment). Notice that $\frac{\partial^2 f}{\partial c^2} = 1 > 0$. The cost function is convex in c, and at each level w, the function has minimum at $c = -\hat{c}G(w)$. This cost function recognizes the fact that if the sediment removal were a one-shot decision, then an optimizing sediment removal would depend on the level of sedimentation, i.e. w, and the dependence is given by a function $G(w)$. At $w = K$, the optimal sedimentation removal should be $c = 0$ as there is no sedimentation to remove, implying $G(K) = 0$. Similarly, if we admit the possibility of dam decomissioning, then at $w = 0$, the optimal sedimentation strategy should be $c = 0$, otherwise, it would always be optimal to remove some sediment from a "totally filled up" dam and the dam would never be recomissioned. These two arguments implies that $G(0) = G(K) = 0$ in static setting. However, such implications are no longer necessarily valid in dynamic setting.

We can rewrite the equation (2) as follows:

$\beta V(w) = \max_c [pw - \frac{1}{2}(c + \hat{c}G(w))^2 - (M - c)V_w + \frac{\sigma^2}{2}V_{ww}]$..........(6)

Equation (6) implies that the optimal amount of sedimentation removal is achieved by setting $c = \min\{\max(0, V_w - \hat{c}G(w)), K - w\}$. It is clear that this decision is not same as the decision that would be if the decision maker ignored the dynamic aspect of the decision making, in which case, $c = -\hat{c}G(w)$. In particular, whenever $V_w \geq 0$, one always removes (weakly) more sediment if he explicitly takes into account the dynamic aspect of sedimentation accumulation process,and this increment is given by V_w. In particular, the decision maker equates marginal cost of removing sediment with the gain in value due to increased water storage capacity. Replacing c in (6) by $c = V_w - \hat{c}G(w)$, one gets the following Dirichlet problem as an expression for the value function:

$\beta V(w) = pw + \frac{1}{2}(V_w)^2 - (M + \hat{c}G(w))V_w + \frac{\sigma^2}{2}V_{ww}$................(7)

$V(0) = V_0; V(K) = V_K$.

Equation (7) is a nonlinear equation, and not only it is hard to solve analytically, if not impossible, it is also not amenable to usual shooting methods that is used in solving nonlinear ordinary differential equations.

Before we solve for the value function, we discuss some qualitative properties of sediment removal function, c. The following observations are immediate:

Proposition 4: (i) $G_w > (<)\frac{V_{ww}}{\hat{c}}$ *implies* $\frac{\partial c}{\partial w} < (>)0$. *These conditions suggests if the graph of G is convex and* $\frac{V_w}{\hat{c}}$ *is downward sloping in such a way that G intersects the curve of* $\frac{V_w}{\hat{c}}$ *twice, once at* w_1 *and once at* w_2 *with* $0 < w_1 < w_2 < K$*,then ,the planner starts removing the sediment when the sediment level is* $K - w_1$*, and stops removing when it is* $K - w_2$.

Proposition 4 provides us a necessary condition on cost function which leads to the condition that the removal of sediment is desirable only after a certain level of sediment has been accumulated. It also clarifies the role of $G(w)$ in our formulation. A characterization of this is given in figure (2).

Notice that it is intuitive to think of G as a function which decreases as $w \to K$, as marginal cost of sedimentation removal should be decreasing as the level of sedimentation accumulates up initially. For example, if there are scarcely any sediments, removing them would require high cost, but it is reasonable to assume such cost to go down at least initially when sediment starts building up.Precise determination of critical value of w that triggers sediment removal can be expressed in terms of curvature of the unknown function, V. Notice that $c(K) = 0$, and hence for the water level near K, one gets $c(K - \varepsilon) = c(K) - c'(K)\varepsilon+$higher terms. Since $c(K) = 0$, and $c'(K) = V_{ww} - \hat{c}G_w|_{w=K}$, it is clear that $c(K - \varepsilon) > 0$ iff $V_{ww} - \hat{c}G_w|_{w=K} < 0, i.e.$ if $G_w > \frac{V_{ww}}{\hat{c}}$.

Two special cases are worth mentioning: if the graph of $G(w)$ doesn't touch the graph of $\frac{V_w}{\hat{c}}$, then either $G(w)$ is entirely above $\frac{V_w}{\hat{c}}$ or is entirely below $\frac{V_w}{\hat{c}}$. In the first case, sedimentation removal is never feasible and in the second case, one always removes sediments, no matter how much it has accumulated. The first case relates to the situation in which marginal cost of removing sedimentation is very high at each level of sedimentation. In the latter case, it is very low, and it makes economic sense to remove sedimentation at each level.

Before analyzing the numerical solution of equation(7), we note why this equation not amenable to three usual methods used in numerical analysis (i.e. Runge Kutta type shooting method, Finite Difference Methods and Rayleigh Ritz methods, for detailed discussion of these methods in solving higher order differential equations, see Stanoyevitch(2005)). Usually, Runge Kutta methods are considered better for linear differential equations, and our system is non-linear, because of the presence of V_w^2 term. The nonlinear shooting methods , which are used to derive numerical solutions of nonlinear boundary value problem like equation(7) above, don't work well in our case. In these methods, one starts from one boundary value , and chooses slope optimally to shoot for another boundary value. To see why such method doesn't work well, define an autonomous system of equations in the following way; Let $U = V_w, U_w = V_{ww}$. Near the origin, it is clear that

$$\begin{bmatrix} V_w \\ U_w \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{2\beta}{\sigma^2} & \frac{1}{\sigma^2}[2M + 2\hat{c}G - U] \end{bmatrix} \begin{bmatrix} V \\ U \end{bmatrix},$$

and the determinant of the Jacobian , $\frac{-2\beta}{\sigma^2}$,is negative. This implies that the corresponding equilibrium, i.e. the origin, is saddlepoint in this system. In such a system, using nonlinear shooting method is very slow in recovering the only solution curve that converges to the origin in this example, irrespective of how close we start our search. Similarly, finite difference methods are extremely slow to converge as they have large sparse matrices that eat up a lot of memory with storage and tends to be slow even when using special methods such as Thomas method to "handle" tridiagonal matrices that arises in such setting. The slow nature of convergence in finite difference method or nonlinear shooting methods is not unique to our problem, however. In the context of free boundary problems, see(Dangl et al (2004)) for a discussion on how solving such Bellman equations require locating the saddlepoint from the entire family of solution curve and why such tricky problem causes nonlinear shooting methods to fail. Rayleigh-Ritz methods are similar to finite element methods which are popular method to solve partial differential equations. We use a slight modification of such Rayleigh-Ritz (or Rayleigh-Ritz-Galerkin) methods which is more fitting in our context.The approach, given by Judd(1992), leads us to use Projection method which takes global approach in solving nonlinear Hamilton Jacobi Bellman equations such as the one in (7) and is a huge improvement over existing methods (see, Dangl et al. (2004)).

6 Numerical Results:

6.1 Method Description

We modify Judd (1992), and Caporale et al. (2010) methods to get the value function given in (7). Judd's method relies on the following major steps. First, one decides the "node" points where the functions are evaluated. The value function is considered to be linear combination of basis functions (their degree is often chosen arbitrarily). There are different options for basis functions, and often the major criterion for choosing them is their orthogonality. Mostly, the preferred basis function is Chebyshev functions. At each node, one evaluates the given equation, and derives error. Projection method depends on the observation that the projection of error function on any arbitrary basis function must be identically zero. This observation helps in coming up with the solution for coefficient of value function which is expressed as the linear combination of basis functions in which the weight of such combination is given by the coefficients.

The following method provides the sketch of our solution method. We refer to it as a Modified Projection Method(MPM). The details of projection methods are available in Judd(1992).

We assume that value function, V, that solves (7), takes the following form:

$\hat{V}(x) = \sum_{i=0}^{n} c_i \phi_i(x)$.........(8)

Here c_i are coefficients, $\phi_i(x)$ are basis functions. Our choice for $\{\phi_i(x)\}_{i=1}^{n}$ are Chebyshev basis functions of first kind. The nodes where we evaluate these functions are given by Chebyshev nodes as numerical analysis theory and empirical experience favor the use of Chebyshev nodes (Miranda et al (2002), page

119). These basis functions for the interval $[a, b]$ are defined as follows: first we normalize the interval $[a, b]$ into the interval $[-1, 1]$ and define $z = \frac{2(x-a)}{(b-a)} - 1$, where $x\epsilon[a, b]$ is a chebyshev node. The Chebyshev polynomials are defined recursively as follows: $\phi_j(x) = T_{j-1}(z)$; where $T_0(z) = 1, T_1(z) = z, .., T_n(z) = 2zT_{n-1}(z) - T_{n-2}(z)$. N Chebyshev nodes in the interval $[a, b]$ are given by $x_i = \frac{a+b}{2} + \frac{b-a}{2}\cos(\frac{N-i+0.5}{N}\pi), i = 1, 2, ..., N$. The derivative of $V(x)$ with respect to x is given by $V(x) = \sum_{i=0}^{N} c_i \frac{d\phi_i(x)}{dx}$. The derivative of nth Chebyshev basis function of the first kind is often expressed in terms of Chebyshev basis function of second kind(U) in the following way:$\frac{d\phi_n}{dx} = nU_{n-1}$. U is defined as follows: $U_0 = 1, U_1 = 2x$ and $U_n = 2xU_{n-1} - U_{n-2}$.

Judd's method defines an operator $\mathcal{F}$ over a function space B such that V, the solution of equation (7), is defined in B and that $\mathcal{F}(V) = 0$, where $\mathcal{F}(V) = pw + \frac{1}{2}(V_w)^2 - (M + \hat{c}G(w))V_w + \frac{\sigma^2}{2}V_{ww} - \beta V$. It should be clear that the function V that satisfies $\mathcal{F}(V) = 0$ and two boundary conditions, $V(0) = V_0, V(K) = V_K$, is the solution of our problem. In these methods, the number of basis functions to be used,n, is chosen progressively; i.e. we increase number of basis functions until nothing more is gained. Now, define $\mathcal{F}(V; \vec{c})$ as residual function given $\vec{c}$ is chosen as the appropriate coefficient of our approximation method. This involves first guessing c and iterating until the system of n equations $\left\langle \mathcal{F}(V; \vec{c}), \phi_i(x) \right\rangle_2$ are identically zero, where $\langle , \rangle_2$ denotes inner product, which generally takes the form,$\langle g_1(x), g_2(x)\rangle_2 = \int w(x)g_1(x)g_2(x)dx$, for some suitably chosen weight function $w(x)$. In this numerical setting, one is unlikely to get exactly zero, so one stops once the norm of the error is within a reasonable tolerance. The inner product is calculated by using Gaussian quadrature methods.

Our implementation of Judd's projection method uses the properties of coefficients of derivative of the value function expressed as the linear combinations of Chebyshev basis functions. Note that due to nonlinearity of (7), the projection method is extremely slow to run. To speed up the implementation, our method uses the following property of the coefficients of a value function. In particular, if $V(x) = C\Phi(x) = \sum_{i=0}^{\infty} c_i\phi_i(x)$, then $V^{(n)}(x) = \sum_{i=0}^{\infty} c_i^{(n)}\phi_i(x)$, where (n) refers to the degree of differentiation. For example,$n = 1$ refers to the first degree of differentiation, $n = 2$ refers to the second degree of differentiation. Define $\mathbf{C} = [c_0 \; c_1 \; .. \; .. \; c_N]^T$. We note that $c_r^{(n+1)} = 2\sum_{i=0}^{\infty}(r+2i+1)c_{r+2i+1}^{(n)}, r = 1, 2, .., N; c_i^n = 0$ for $i > N$. This leads to the formula for coefficient matrix for n^{th} derivative of $V(x)$ which is given by $V^{(n)}(x) = 2^n\aleph^n C\Phi$, where $\aleph$ is defined as follows:

$$\text{for odd } N,\ \aleph = \begin{bmatrix} 0 \; \frac{1}{2} \; 0 \; \frac{3}{2} \; 0 \; \frac{5}{2} \; \ldots \; \ldots \; \ldots \frac{N}{2} \\ 0 \; 0 \;\; 2 \;\; 0 \;\; 4 \; 0 \; \ldots \; \ldots \; \ldots 0 \\ 0 \; 0 \; 0 \; 3 \; 0 \; 5 \; \ldots \; \ldots \; \ldots \;\; N \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; \ldots \; \ldots \; \ldots \; N \\ 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; \ldots \; \ldots \; \ldots \; 0 \end{bmatrix}_{(N+1)\times(N+1)}$$

for even N,$\aleph = \begin{bmatrix} 0\ \frac{1}{2}\ 0\ \frac{3}{2}\ 0\ \frac{5}{2}\ \dots\ \dots\ \dots 0 \\ 0\ 0\ \ 2\ \ 0\ \ 4\ 0\ \dots\ \dots\ \dots N \\ 0\ 0\ 0\ 3\ 0\ 5\ \dots\ \dots\ \dots\ \ 0 \\ \dots\dots\dots\dots\dots\dots \\ 0\ 0\ 0\ 0\ 0\ 0\ \dots\ \dots\ \dots\ N \\ 0\ 0\ 0\ 0\ 0\ 0\ \dots\ \dots\ \dots\ 0 \end{bmatrix}_{(N+1)\times(N+1)}$

Then our goal is to find the degree of approximation, N, and corresponding coefficients for the solution function, V. There are therefore two loops, the first determines N and the second determines the coefficients, in MPM.

The algorithm , step by step, is detailed as follows:

(1) First, calculate $c_{1,}c_2$ which solves

$c_1\phi_1(0) + c_2\phi_2(0) = V_0$

$c_1\phi_1(K) + c_2\phi_2(K) = V_K$

This gives a tentative solution, $\hat{c}_1 = V_0$; $\hat{c}_2 = \frac{V_K - V_0}{K}$. Define $C_{old} = [c_1\ c_2]$;

{OUTER LOOP BEGINS}

(2) $N = 3$,Flag=True.

(3) Define tolerance

(4) Repeat until *Flag is True*

(i)Get N Chebyshev basis functions.

(ii)Get $N-2$ Chebyshev Nodes in the range $[0, K]$. Retain the endpoints as two other nodes.

{INNER LOOP BEGINS}

(iii) At each nodes $w_{i,}$ calculate $\hat{V}_w(w_i)$ using C_{old} and $N-1$ basis functions evaluated at nodes w_i.

(iv) Define matrix $\mathbf{P}'= [\frac{-\sigma^2}{2}(4\aleph^2) + (M + \hat{c}G(w_i) - \frac{\hat{V}_w(w_i)}{2})(2\aleph) + \beta\mathbf{I}_{N\times N}]$.

Define matrix $\mathbf{P}$ as the matrix that has two more rows appended to $\mathbf{P}'$. The two rows are basis functions evaluated at points 0 and K.

(v) solve for $\mathbf{C}$ in $\mathbf{P}\Phi\mathbf{C} = [p\mathbf{w}; V_{0;}V_K]$

(vi) use this C to calculate $V_w(w_i)$. Repeat (iv) and (v) until the value of V_w doesn't change.

{INNER LOOP ENDS}

(vii) set Flag = False if norm of difference between two values in some fixed predetermined nodes is smaller than the tolerance. Else set $N = N + 1$, and go to (2).

{OUTER LOOP ENDS}

The weakness of this algorithm is that it requires both processes that guide the loops to be convergent. We are still working on the precise determination of stability region for this algorithm, but when the algorithm works it seems to work very quickly. There are three tricky steps that require elaborate understanding: (1) In step vi, the difference in the value of V_w may not decrease over time as one gets new coefficient values. (2) In step v, the matrix that is to be inverted may be ill conditioned, in particular if the elements are either too small or too large and (3) In step vii, the convergence of V may not be achieved as N increases. We ignore such technical details for now, but users unfamiliar with the programming aspects involving matrix in solving differential equations and iterative convergent processes are advised to heed these aspects.

Our implementation of (7) will be a contribution to literature in itself for several reasons. We can use MPM to approximate the second order nonlinear differential function, and our problem is useful in both BVP and a free boundary problem (for example, when we search for the value of water storage that will lead the abandonment of the reservoir). The algorithm could be the part of a control theorist's toolbox to handle the situation where other more familiar methods are very slow. Most of the economists have so far used finite difference methods and limited their models to solving linear differential equations and a new tool like this should be an useful addition whenever convergence can be guaranteed.

6.2 Calibration

We calibrate the model using the data from Tarbela dam in the Indus River in Pakistan. Tarbela was constructed as a consequence of the World Bank facilitated mediation between India and Pakistan following a decade long water sharing crisis between the two south Asian countries. Tarbela dam is an important dam because the designers of the dam were aware of possible sedimentation impact during the construction phase. It is a large project, providing 11.48 billion cubic metres(bcm) storage and 3478 MW electricity and is often cited as a dam that is affected by sedimentation accumulation problem. In addition, it is one of the seven Focal Dams studied by World Commission on Dams (WCD) and hence is among the most studied dams in the world. Furthermore, this dam is also cited by major studies related to sedimentation management. We use data related to the Tarbela dam in calibrating our model, whenver it is possible.

The primitives of our models are the following, the total capacity of the dam, K, the implicit price per unit of stored water(p), annual sedimentation rate (M), its variance (σ^2), upstream wage rate that determines cost of sedimentation removal,l, Reduction Effort function , $R(e)$, intercept of marginal cost of sediment removal in quadratic cost strategy $(\hat{c}, G(w))$, and linear cost function parameter, α. Here, we discuss their determination in detail.

The Usable Storage (K) of Tarbela dam is 11.48 bcm. The mean arrival rate of sediment for $1974-1999$ is 0.105. The standard deviation of sediment arrival is 0.025. However, a note of the caution is that the sedimentation survey of Tarbela dam was started five years after the dam was operational, in 1979. Initially, it was estimated that the reservoir would silt up at the rate of 2% per year and the life of dam was predicted to be 50 years (Lieftinck Report(1968)). One observes that there are no obvious correlation between sediment arrival rate in one year and the previous or next year. In fact, the reduced sedimentation in $1979-1982$ is contributed to the reduced river flows, as those were dry years.

The cumulative economic benefit of water of Tarbella dam for $1975-1998$ in terms of 1998 dollars is estimated to be 2987.8 million dollars and in terms of 1965 dollars is estimated to be 577.9 million dollars (see Table 3.6, WCD(2000)).

We regard 1998 as base year, and regard the implicit price per bcm to be 260 million dollars. We use ten million dollars as a unit and set $p = 1$.

Tarbela dam lies close to the Himalayas. Indus River is joined by four other tributaries upstream, Shyok, Hunza, Gilgit and Siran rivers. Only Siran river drains monsoon influenced area of $10,200\ km^2$, which is only 6% of total catchment area of the Indus River.There is little prospect of afforestation and sedimentation management by utilizing best management practice upstream is limited to less than 6% of its catchment area. 94% of its catchment area is said to be either hyper arid or semi arid. Afforestation – a prevention strategy– was carried out with the help of forest department of North Western Frontier Province (NWFP) in collaboration with the WAPDA but it is considered that Tarbela watershed management program has a very limited impact (WCD(2000)).

Technological impossibility of making a conduit that could withstand the velocity of sediment in the reservoir (estimated to be $9 - 13$ meter per second in June-July) was apparently the major reason why the makers of Tarbela dam didn't opt to use the sluicing technology as a mean to remove sediments from the reservoir. At the time,these still conduits in the tunnels could withstand 6 m/s velocity only. 50% of annual sedimentation is carried during June-July, but sluicing them away also meant almost 60 days of no power production, and it was also politically considered undesirable at the time. (Lieftinck report (1968)). Other geological factors, such as broadness of the valley and depth of alluvium underlying the dam, were also cited in opting out of sluicing as an option. The only strategy being used by the dam operator rightnow is to progressively raise the minimum reservoir level , so that the sediment delta wouldn't reach the dam, but it is predicted that the live storage of the dam would be reduced at the faster rate due to this strategy.

The planners in Tarbela also ruled out dredging as an option. They estimated that removal of sediment would cost almost Rs 27 billion (approximately US$650 million) per year(WCD (2000)). The report doesn't provide the cost function used in calculating the estimate. The WCD report also mentions a proposed strategy to evacuate sediments by constructing high capacity outlets from the left or right bank for sluicing or flushing. However, none of the effective sedimentation strategies are in place. Our calibration therefore makes other assumptions about the cost functions.

Basson's (1990) conclusion was that the water volume required to remove one unit of sediment is typically $7 - 50$ units. The variance is primarily due to different surface conditions in different dams. Palmieri et al (2003) infer from this information a linear cost function given by $f(c, w) = (3 + 17K_w)c$, where K_w is the ratio of storage to mean annual runoff. The mean annual runoff of Indus River is estimated to be 80920 million m^3(TAMS(1998)) or 80 bcm approximately. Thus, K_w for Tarbela is 0.14, our linear function is given by $f(c, w) = 5.3c$. Hence we set linear cost function parameter $\alpha = 5.3$.

It is hard to exactly pinpoint what the quadratic cost function looks like in Tarbela. Evacuation of sediment could be roughly classified as a quadratic cost function which is convex in the amount of sediment removed, as limited amount of labor and machines are available to remove the sediment. Furthermore, we assume the cost function to be dependent on the level of sediment. In particular, if sediments are perched on the deeper level, it is reasonable to assume the marginal cost will be higher. Our assumption of cost function, $f(c,w) = \frac{1}{2}(c + \hat{c}G(w))^2$ implies the marginal cost function $c+\hat{c}G(w)$. This cost function implies an additive effect of water storage level. In particular, it assumes that the marginal cost function for a fixed w has a fixed intercept $\hat{c}G(w)$ and a slope of 1. We calculate the cost function for evacuation of sediment from the surface as given in WCD (2000) and use the following parameters for our calibration. Assuming the sediments are located at lower 20% part, we use the fact that it costs approximately $700 million to remove 1.1 bcm sediment to calculate $\hat{c} = 8$ for $G(w) = w$.

We set $V_K = 925.8$, as it cost $9258 million dollars in terms in 1998 dollars to construct the dam. We set $V_0 = 0$.

A complete list of parameters used for our calibration is given in Table(2).

6.3 Results

First we note that for the parameters given in Table (2), $\frac{1}{\alpha_1} \ln \left(\frac{\alpha - \frac{p}{\beta}}{(V_0 + \frac{Mp}{\beta^2})\alpha_1} \right) = -.0087 < 0$. This implies that $w^* = 0$. This suggests that for the case of the linear cost function, it is never advisable to give up removing the sediments from the reservoir. But since the control rule is bang-bang, it follows that, at any level of w, one will remove all the sediments accumulated at the reservoir. This implies that for this particular cost function, Tarbela is an economically sustainable reservoir. A tentative value function for $N = 4$ is given in Figure (2). Though the value is obviously way off, as we use $N = 4$ in stead of $N = \infty$, it is given in the figure as an illustration. It is also clear from the figure that increase in M reduces the value function, as claimed.

The crux of our computational work involved the quadratic cost function. Figure (3) provides the basic result for Tarbela. The control rule stipulates that for the current projection of cost, which is considered too expensive by WAPDA (WCD(2000)), the dam is still sustainable but it is profitable to leave the dam silted up until its storage is about $2.5bcm$. Part of the reason is our assumption of $G(w) = w$, which implies that the deeper we reach for sediment removal, the higher the fixed cost of removing sediment would be. Value function is observed not to be monotonic in water storage level, in particular the value is maximized near mid reservoir level. Given that Tarbela is now about 20% silted up, according to our assumption, it may reach its maximum value soon, and beginning to remove sedimentation at that level will be economically optimal.

Figure (4) and (5) implies that we observe a monotonic relationship of value and control rule with respect to increase or decrease in variance. The value function decreases at all level with the increase in variance, which is to be expected. The control rule decreases with the increase in variance. Uncertainty leads to less and less of sedimentation removal. One explanation of this result is that since uncertainty implies less value for each storage level, the planner will have less incentive to remove sediment as uncertainty increases.

As we noted earlier, global warming implies higher erosion. Under such drastic change, the value of reservoir changes. Our model shows that increase in the sedimentation rate decreases the value of the reservoir, in particular at the lower storage level. (Figure (6)). The cost of removing the storage is high at the lower level and in such case, increase sediment is likely to decrease the value of the reservoir. Moreover, as Figure (7) shows, the increased sediment arrival implies increased sediment removal at all level where sediment removal is optimal. The numerical experiments also confirm the intuition that increased p increases the value of dam and increases the sedimentation removal rate at all levels.

Discount rate features in our model in two important ways. The first is that discount rate has its traditional meaning regarding the patience of the society. It is expected that higher discount rate encourages individuals or society to consume more today. It also enters our paper in a different way (see next chapter). If the society faces uncertainty about the future of the reservoir, its decision making , under some assumption about the nature of such risk, is akin to increased discount rate. Figure (8) implies that increased discount rate increases sedimentation removal at the lower level water storage. Impatience in this case doesn't mean the policymaker will lessen the sedimentation removal. At all levels of water storage, increased impatience also increases the value of reservoir by a small amount. It is possible that uncertainty about the future makes people value the reservoir more.

7 STOCHASTIC SALVAGE VALUE

In the paper above, we assume $V(0)$ is known. $V(0)$ is the salvage value; the value of the dam when the dam is filled up by sediment. The assumption can be weakened in two major ways: one is the fact that a dam may be destroyed at a random time due to catastrophy or big flood, the arrival of which is unknown. The salvage value at the time may then be known, but since the arrival of such event is probabilistic, the overall setting for the problem has to be revised, since the decision maker now has to take into account an extra factor: the probabilistic event that the dam may be destroyed at any time. The other modification of the assumption is due to the fact that the planner, looking at the future from the current time t and current sedimentation level $w(t)$, may not know exact value of $V(0)$. This may happen because of the change in perception of public about the value of large dam or due to the arrival of new information regarding the role of the large reservoirs in events such as greenhouse gas accumulation whose exact impact are uncertain but being learned over time.

In our model, we tackle the issue of stochastic salvage value in the following way: at any time t, due to changing perception about the dams, there is a probability λ (a constant hazard rate) that government will introduce a new regulation which will reduce the profit from $\pi(t)$ to $(1-\tau)\pi(t)$. We simplify the problem by assuming that such tax is permanent, and irrevocable. Though simple, this covers both issues mentioned above. The risk of big catastrophe means $\tau = 1$ or a value close to 1. It covers the second case in that a planner may not know the value of $V(0)$, but assumes that the value of $V(0)$ is likely to change in a specific way(due to the introduction of a new tax).

Our next proposition provides an illustration of an extreme case in which a policymaker faces the closure of dam (either due to catastrophe or due to government regulation).

Proposition 5. *Assume a dam operator faces constant hazard rate, λ, of closure . The impact of such hazard rate is similar to the increase in the discount rate by hazard rate.*

Proof: This is entirely expected and the proof closely follows the one given by Yaari(1965) in life time saving analysis. First, since constant hazard rate implies exponential distribution of arrival rate , the survival rate ($S(t)$) is given by $e^{-\lambda t}$. We denote the probability density function of hazard as $f(t)(= \lambda e^{-\lambda t})$. The maximization problem is now given as

$\max_c \int_0^\infty [\int_0^t e^{-rs}\pi(c(s), w(s))ds]f(t)dt$

s.t. $\dot{w} = -(M - c)dt + \sigma dz$

with usual initial conditions.

We note that $\frac{d}{dt}(\int_0^t e^{-rs}\pi(c(s), w(s))ds) = e^{-rt}\pi(c(t), w(t))$. Let $u = \int_0^t e^{-rs}\pi(c(s), w(s))ds$; $du = e^{-rt}\pi(c(t), w(t))dt$; $v = F(t)$; $dv = f(t)dt$. This implies that $\int_0^\infty [\int_0^t e^{-rs}\pi(c(s), w(s))ds]f(t)dt = \int_0^\infty e^{-rs}\pi(c(s), w(s))ds - \int_0^\infty e^{-rs}F(s)\pi(c(s), w(s))ds = \int_0^\infty e^{-(r+\lambda)s}\pi(c(s), w(s))ds$ (using the fact that $S(t) = 1 - F(t)$). Hence the policymaker facing uncertain future will act just like before, except he would nice discount more.

8 Sedimentation Removal and Sustainability of Reservoirs

Sustainability of dam is a topic of interest when talking about the consumption of natural resources. If our use of natural resources precludes future generations from using these resources, then it may not be a just policy when viewed from the eyes of future generations. In particular, using Rawlsian notion of justice, when generations make decision using veil of ignorance about which generation they belong to, if they are unlikely to decide to make reservoirs, then such reservoirs are made unjustly (Rawls (1972)). Sidgwick put this statement differently by demanding anonymity in such utility ranking:i.e. the outcome of a preference ordering among different welfare paths between generations shouldn't differ based on which generation is making the decision . Unfortunately, the concept of intergenerational equity has yet to be incorporated satisfactorily in economics: the major instrument in NPV calculation is a positive discount rate that leads to the finite NPV of infinite stream of incomes. The formal models are inadequate in producing desirable results (Chichilinksy(1996)). Some sort of contradictions plague almost all proposed models such as paternalistic consumption models, paternalistic utility model, Chichilinsky's model (guaranteeing nondictatorship of present and nondictatorship of future) and Alvarez-cuadrado et al (2009)'s model that improves on Chichilinsky's model by guaranteeing optimal path of renewable resource extraction. The major problem lies in the inability of comparing these utility streams: undiscounted utility often has NPV equal to infinity and one can't Pareto rank them (see Diamond(1965), Svensson(1980)). Discounting on the other hand has always been very controversial, in particular in environmentally sensitive projects.

Our model uses discounting as a tool not only to ensure the finiteness of present value of the dam, but also in incorporating the stochasticity of future developments. Other studies of reservoirs have skipped the discussion of discounting altogether. Often,sustainability frontier of a dam is defined in terms of two ratios: K_w and K_t, where K_w is defined as above and K_t is the ratio of storage to annual sediment arrival. Basson(1997) and Palmieri et al (2003) both provide a lengthy discussion on sustainability frontier in terms of K_w, and K_t. If a nonsustainable outcome (in which a reservoir is allowed to silt up) is economically desirable to a sustainable outcome, then the reservoir can't be sustained. Such economic desirability is expressed in terms of net present value of the reservoir under the two conditions. If NPV of the reservoir under sustainable outcome is higher than the NPV under unsustainable outcome, then the reservoir is sustainable. Often, higher K_t is more likely to yield sustainable outcome than lower K_t for a given K_{w}.

Our problem formulation above allows for sustainability in different ways. For example, if in a linear cost model, $w^* = 0$, which was the case for Tarbela, one never stops removing sediment. Since sediment removal is a bang-bang decision in linear case, the reservoir is , at least from the economic point of view, sustainable. Similarly, in the case of quadratic cost function, one notes that $c = \min\{\max(0, V_w - \hat{c}G(w)), K - w\}$. If c is nonincreasing in w, and if there exists some $\bar{w}$ such that for $\forall w < \bar{w}, \max(0, V_w - \hat{c}G(w)) \geq K - w$, then a reservoir will never be silted up. The planner will always find it economically feasible to remove sediments before the reservoir is silted up. One can weaken the condition considerably, but we skip the issue for now.

9 Conclusion

We provide a new approach to model sedimentation management problem in large reservoir. This paper contributes to existing scant literature in what is being realized as an important topic in natural resources economics. Our model allows uncertainty in sedimentation accumulation, which is useful in understanding the impact of global warming or fluctuating weather if they contribute to the change in variance of sediment arrival rate (or sedimentation yield) in the reservoir. This paper also contributes by providing a new algorithm to solve a particular type of boundary value problem arising due to the quadratic nature of the cost function. Quadratic cost function leads to nonlinear second order value function. Though there are several existing algorithms to tentatively solve these equations, all of them have some deficiencies. Some , such as nonlinear shooting methods, could be very slow, while others such as finite difference method are computationally cumbersome. We provide a new nested method that is a slight modification of the projection method provided by Judd(1992).

We calibrate our model by using the data from Tarbela dam in Pakistan. Tarbela is one of the most vulnerable dams in the world rightnow, because of its apparently high rate of erosion. We find that the dam could be sustainably operated for a particular linear cost function, and also for the quadratic cost function. However, we note that there are other issues that could make these assertions weaker. Removing sediments, for example, would also require finding a proper place to dump those sediments.

Our model is simple and yet useful in understanding the issues sorrounding the reservoir management. We provided many comparative statics results such as impact of increased sediments, impact of change in discount rate and impact of increased uncertainties on both value function and control functions. In both our basic model and our definition of sustainability, we focus on the major role played by water storage level on the value of the reservoir or on the sustainability criterion.

Getting useful data on large dams is still very difficult. Dams also differ by their location, their political significance and their strategic and even psychological meaning in the host country. Each dam is also likely to have its own specific cost function of removing sediments from the reservoirs. Reservoir operators are just recently beginning to think about sustainability of the reservoirs. Tarbela's planners had originally planned the dam to operate for fifty years, a target they don't like to stick to anymore. While the planners are now beginning to weigh different options for sediment removal, our results show that they are not too late in implementing those strategies.

Future research in sediment management should look at the risk averseness of the planner. We use a risk neutral planner in our paper. Furthermore, since privately operated reservoirs are often licensed to run for a limited period (30 years in several rivers in Nepal), one would want to introduce a time dependent model to study the situation of private ownership. In this situation, optimization decision will yield a partial differential equation with time as one of the arguments. The welfare impact of allowing privately held reservoirs (in stead of government owned) is also important next step in this field. However, the most important of all is better understanding of cost function. Rightnow, the understanding of cost function in sediment removal problems is very limited and it hinders effective management of reservoirs. Also, a major weakness of the model is its assumption that V_0 and V_K are known. The calibration assumes that V_K is the cost of construction. It is an ad hoc assumption and probably is an underestimation of the value. A better understanding of such values could be derived only by, or at least in conjuction with, other economic techniques such as nonmarket valuation methods. Finally, in a lot of cases, a mixed model, in which different sedimentation strategies are used together are used. Modeling such a situation is more complex, but could be one of the topics of future research.

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11 Appendix:

Table 1.

Observation Periods	**Average Annual Sedimentation of Reservoir**	
	(mt)	*(bcm)*
1974-79	220	0.149
1974-81	204	0.138
1974-82	192	0.129
1974-83	188	0.127
1979-81	158	0.106
1979-82	110	0.074
1981-82	91	0.062
1982-83	153	0.103
1984-99	137	0.092
Average 1974-83		0.127
Average 1984-99		0.092
Average 1974-99		0.106

mt: million tonnes

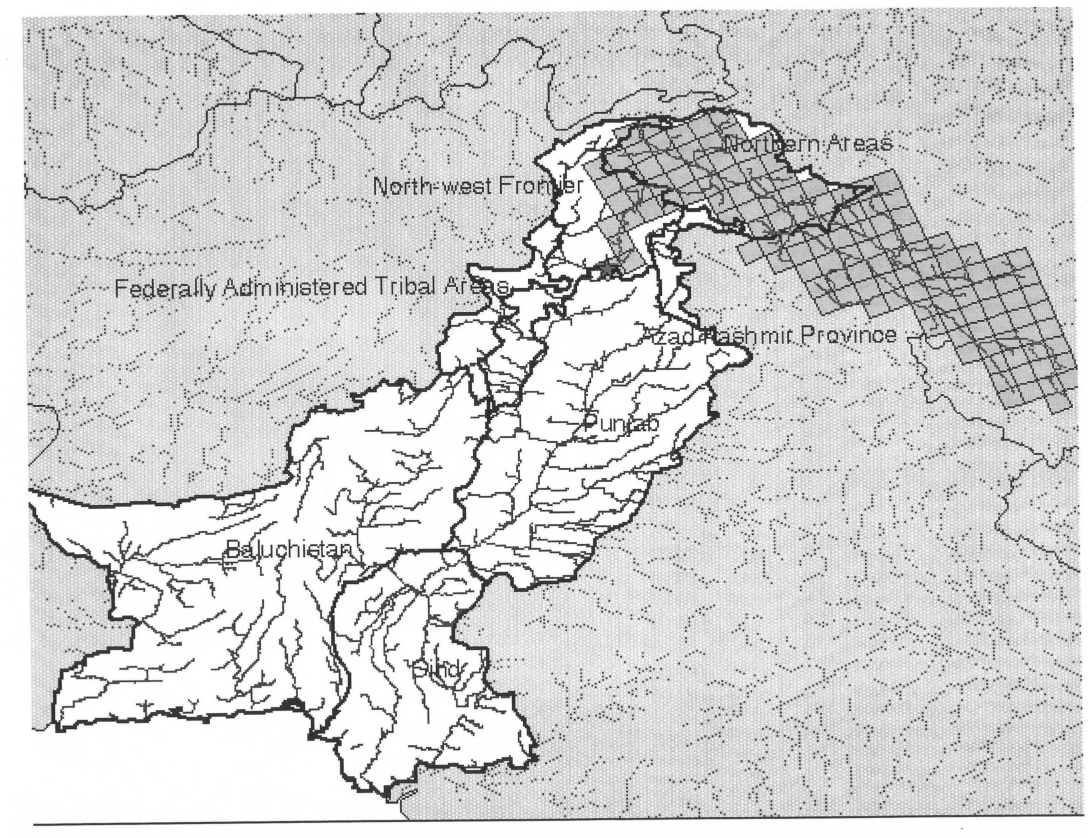
bcm: billion cubic meters

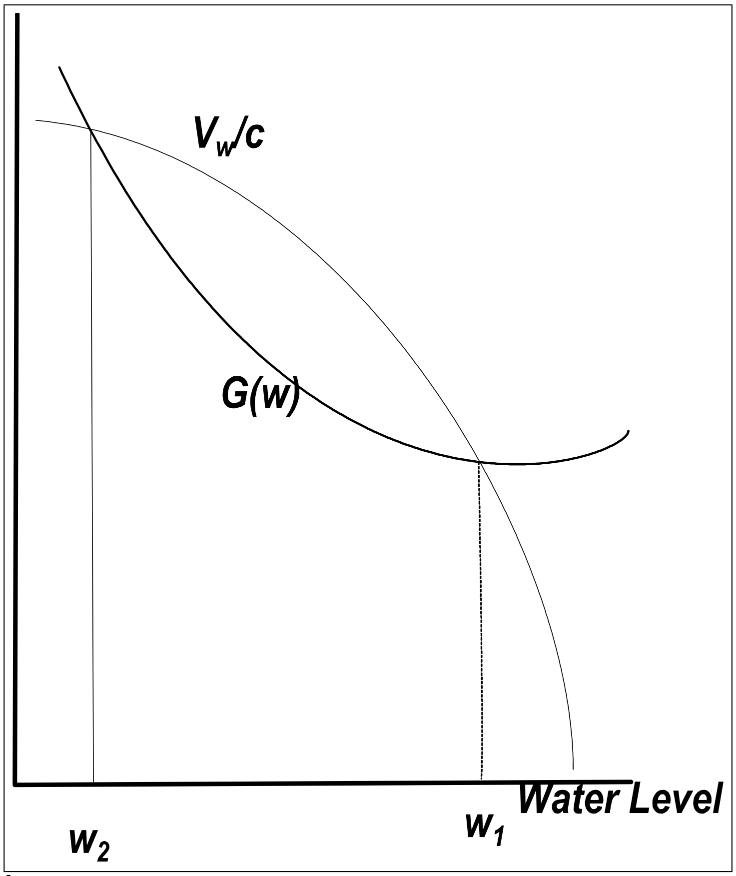
Source: Tarbela Dam Project Completion Report on Design and Construction, TAMS, 1984, WAPDA and Tarbela Dam Project, WAPDA, as reproduced in WCD(2000)

Table 2: Calibration Parameters

Parameters	**Value (units)**
K	11.48 (bcm)
M	0.105 (bcm)
σ	0.025 (bcm)
β	0.05
p	1 (\$ *ten million*)
l	—
$R(e)$	—
$\hat{c}$	8
$G(w)$	w
α	5.3
V_0	0
V_K	925.8 (\$ *ten million*)

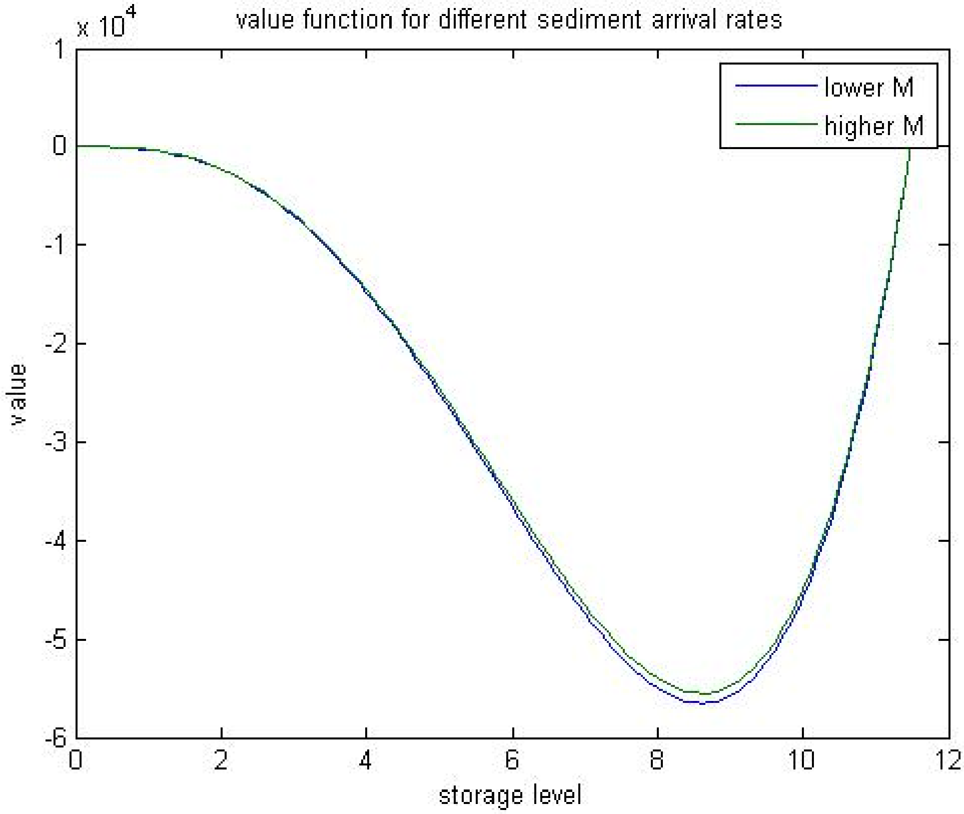
Geographical Location and Catchment Region of Tarbela Reservoir in Pakistan: $\star$ denotes the location of the Tarbela dam, where as shadowed area denote the catchment area. [**Source: fas.usda.gov**]





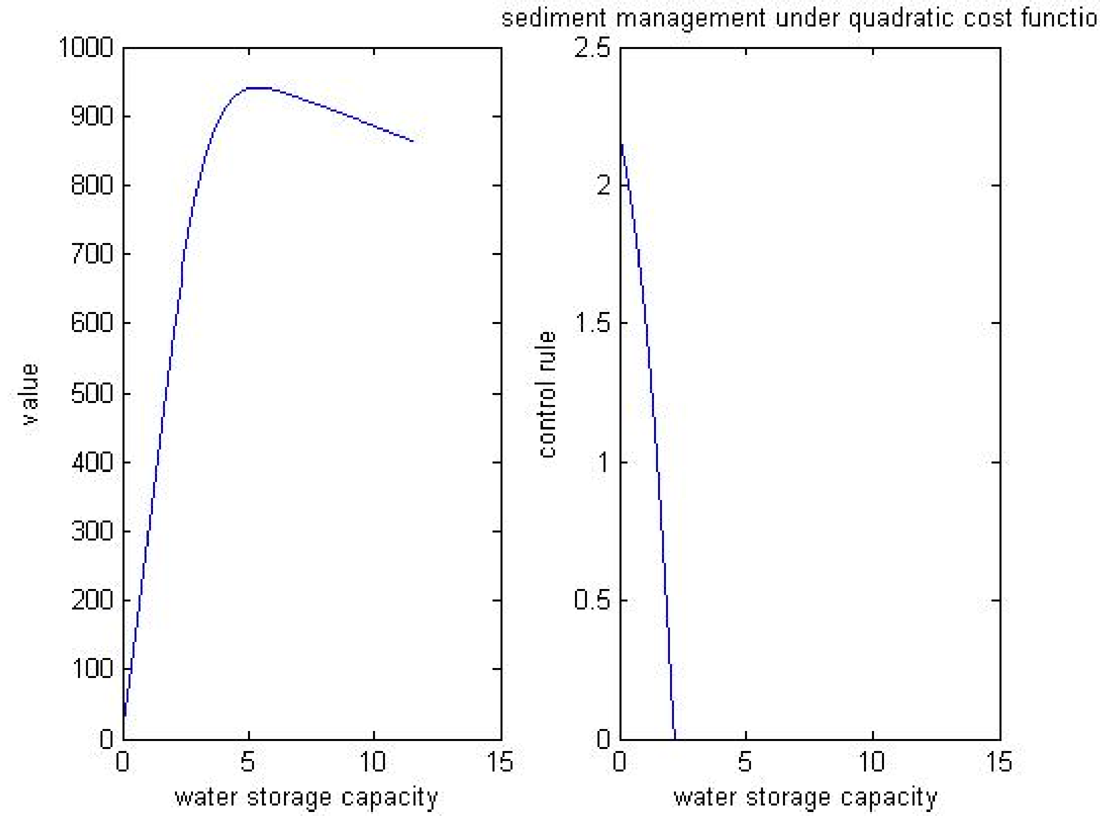
[**Figure 1**: Characterization of the necessary relationship in proposition 4].

Linear Cost Strategies: The value of the Tarbela dam is estimated for linear cost strategy, with $n = 4$ being the degree of approximation.



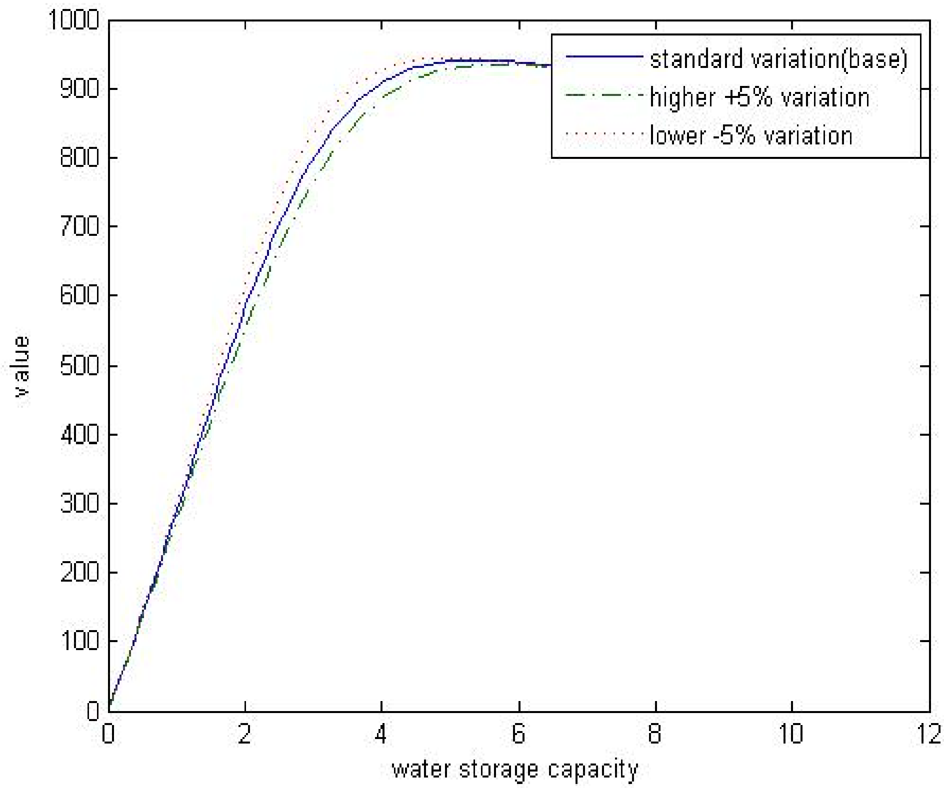
(**Figure 2**)

Quadratic Cost Strategy: For $G(w) = w$, and for given parameters, the value function and control rule for Tarbela are given as follows.



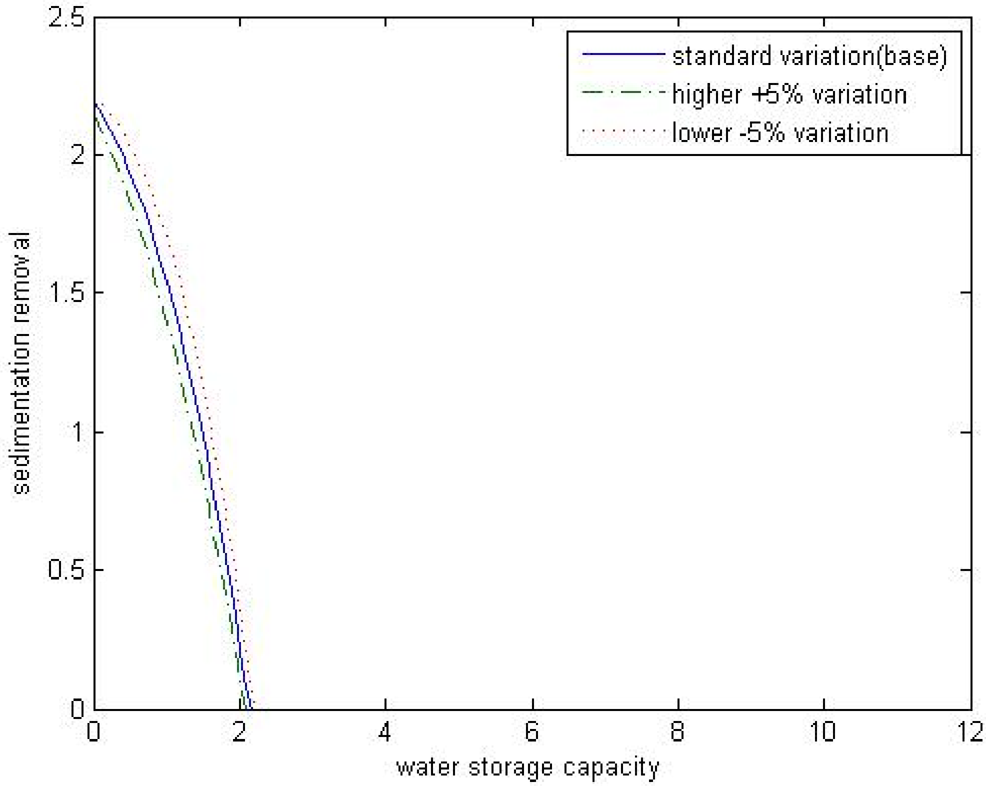
(**Figure 3**)

Sensitivity to Variation: Base case variation given in Table (1) ±5%



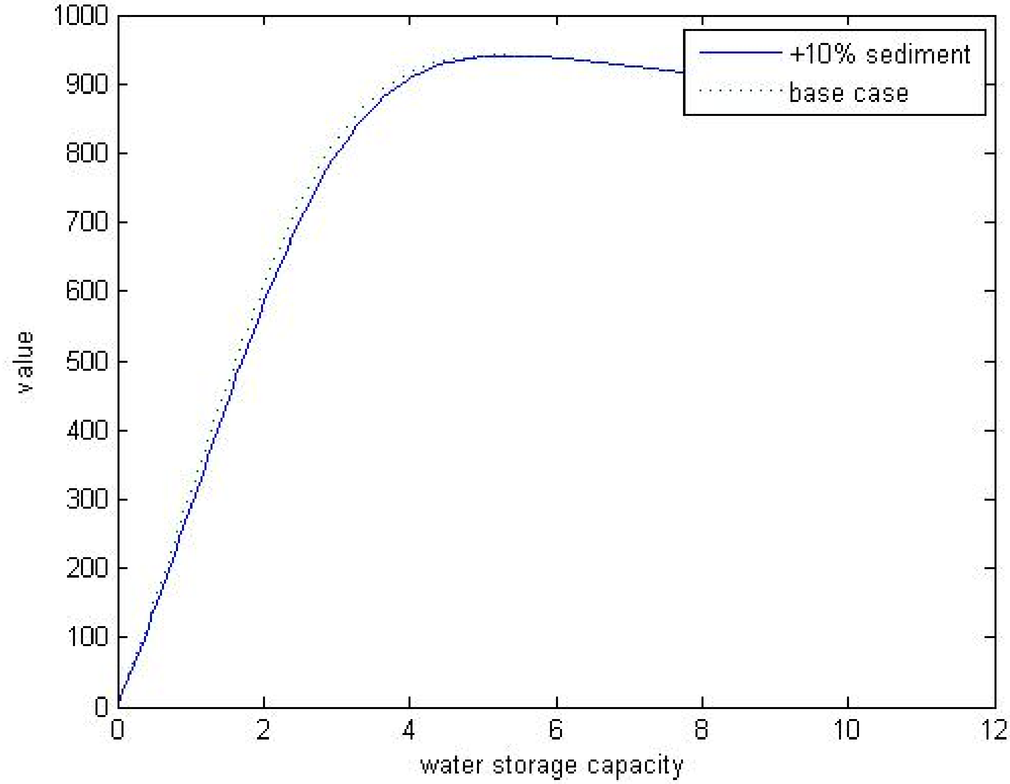
(Figure 4)

Sensitivity of control rule to Variation in sedimentation arrival: Base case variation given in Table (1) $\pm 5\%$



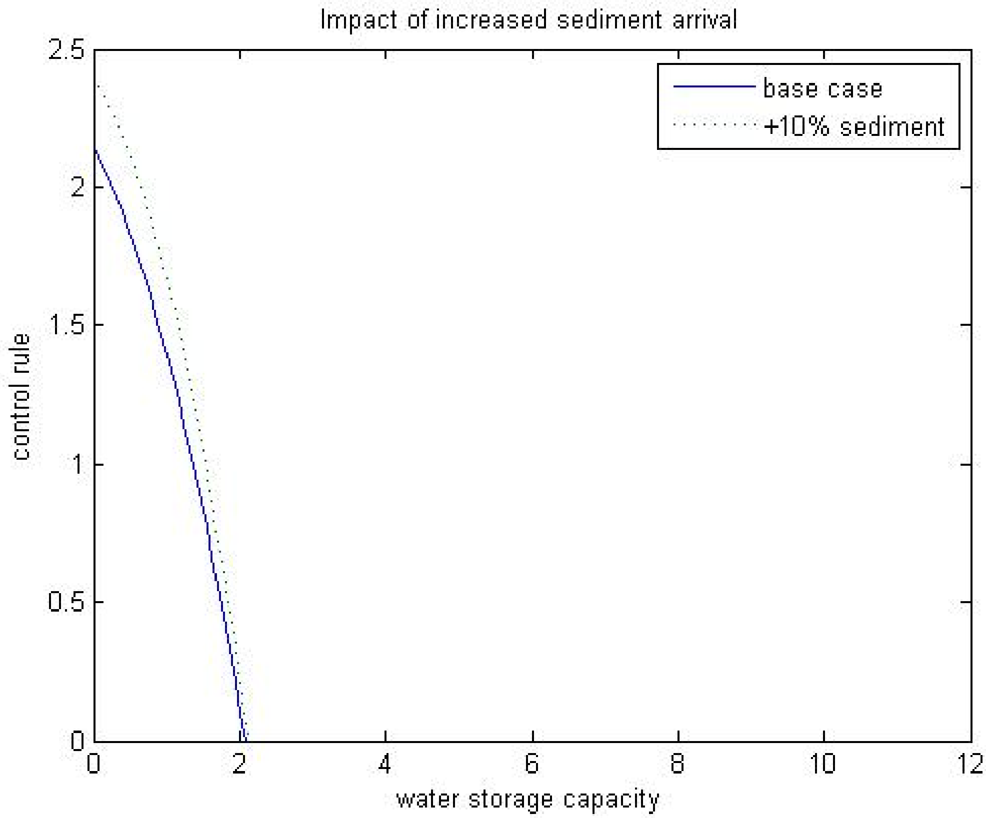
(**Figure 5**)

Relationship between the value function and Sedimentation Arrival rate



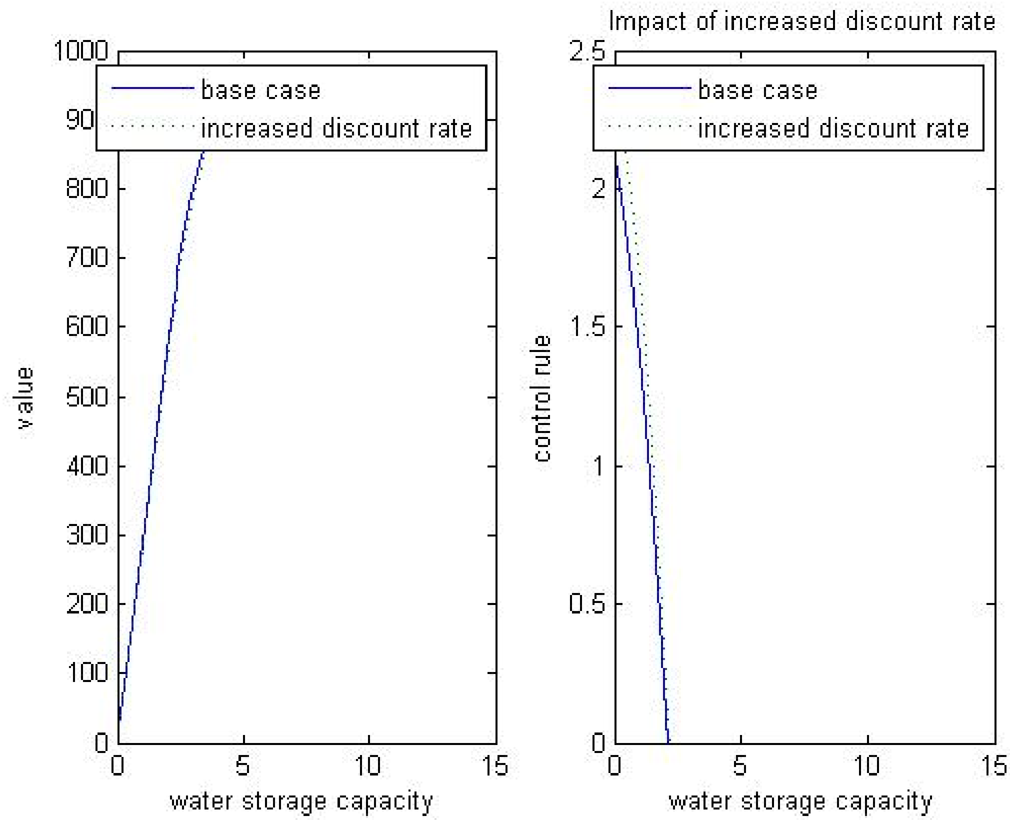
(**Figure 6**)

Relationship between control rule and sedimentation arrival rate:



(**Figure 7**)

Impact of Discount Rate



(**Figure 8**)