Interval Valued Fuzzy Neutrosophic Soft Structure Spaces

I. Arockiarani
I. R. Sumathi

Follow this and additional works at: https://digitalrepository.unm.edu/nss_journal

Recommended Citation

This Article is brought to you for free and open access by UNM Digital Repository. It has been accepted for inclusion in Neutrosophic Sets and Systems by an authorized editor of UNM Digital Repository. For more information, please contact amywinter@unm.edu.
Interval Valued Fuzzy Neutrosophic Soft Structure Spaces  

I.Arockiarani¹ & I.R.Sumathi²  

¹² Nirmala College for Women, Coimbatore- 641018 Tamilnadu, India. E-mail: sumathi_raman2005@yahoo.co.in  

Abstract. In this paper we introduce the topological structure of interval valued fuzzy neutrosophic soft sets and obtain some of its properties. We also investigate some operators of interval valued fuzzy neutrosophic soft topological space.  

Keywords: Fuzzy Neutrosophic soft set, Interval valued fuzzy neutrosophic soft set, Interval valued fuzzy neutrosophic soft topological space.  

1 Introduction  
Neutrosophic Logic has been proposed by Florentine Smarandache[14,15] which is based on non-standard analysis that was given by Abraham Robinson in 1960s. Neutrosophic Logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision undefined, incompleteness, inconsistency, redundancy, contradiction. The neutrosophic logic is a formal frame to measure truth, indeterminacy and falsehood. In Neutrosophic set, indeterminacy is quantified explicitly whereas the truth membership, indeterminacy membership and falsity membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors.  
Yang et al.[16] presented the concept of interval valued fuzzy neutrosophic soft sets by combining the interval valued fuzzy set and soft set models. Jiang,Y et al.[5] introduced interval valued intuitionistic fuzzy soft sets. In this paper we define interval valued fuzzy neutrosophic soft topological space and we discuss some of its properties.  

2 Preliminaries  
Definition 2.1[2]:  
A fuzzy neutrosophic set A on the universe of discourse X is defined as  
\[ A = \left\{ (x, T_A(x), I_A(x), F_A(x)) \mid x \in X \right\} \]  
where \(T, I, F: X \to [0, 1]\) and \(0 \le T_A(x) + I_A(x) + F_A(x) \le 3\).  

Definition 2.2[3]:  
An interval valued fuzzy neutrosophic set (IVFNS in short) on a universe X is an object of the form \(A = \left\{ (x, T_A(x), I_A(x), F_A(x)) \right\} \) where  
\[ T_A(x) = X \to \text{Int}([0,1]), I_A(x) = X \to \text{Int}([0,1]) \text{ and } F_A(x) = X \to \text{Int}([0,1]) \text{ (Int}([0,1]) \text{ stands for the set of all closed subinterval of } [0,1]) \text{ satisfies the condition} \]  
\[ \forall x \in X, \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3. \]  

Definition 2.3[3]:  
Let U be an initial universe and E be a set of parameters. IVFNS(U) denotes the set of all interval valued fuzzy neutrosophic sets of U. Let A\(\subseteq E\). A pair \((F, A)\) is an interval valued fuzzy neutrosophic soft set over U, where F is a mapping given by \(F : A \to \text{IVFNS}(U)\).  
Note : Interval valued fuzzy neutrosophic soft set is denoted by IVFNS set.  

Definition 2.4[3]:  
The complement of an INFNSS \((F, A)\) is denoted by \((F, A)^c\) and is defined as \((F, A)^c = (F^c, [A] \to \text{IVFNS}(U))\) is a mapping given by \(F^c(e) = \{x \in [0,1], F_{[F^c]}(x), F_{[F^c]}(x), F_{[F^c]}(x)\} \) for all \(x \in U\) and \(e \in \neg A\), \((I_{[F^c]}(x))^c = 1 - I_{[F^c]}(x) = [1 - I_F(x), 1 - I_F(x)]\).  

Definition 2.5[3]:  
The union of two IVFNSS \((F, A)\) and \((G, B)\) over a universe \(U\) is an IVFNSS \((H, C)\) where \(C = A \cup B, \forall e \in C\).  

Definition 2.6[3]:  
The intersection of two IVFNSS \((F, A)\) and \((G, B)\) over a universe \(U\) is an IVFNSS \((H, C)\) where \(C = A \cap B, \forall e \in C\).  

where  
\[ T_{B(\alpha)}(\beta) = \max\{T_{B(\alpha)}(\beta), T_{B(\alpha)}(\beta)\} \]  
\[ I_{B(\alpha)}(\beta) = \max\{I_{B(\alpha)}(\beta), I_{B(\alpha)}(\beta)\} \]  
\[ F_{B(\alpha)}(\beta) = \min\{F_{B(\alpha)}(\beta), F_{B(\alpha)}(\beta)\} \]
The intersection of two IVFNSS (F,A) and (G,B) over a universe U is an IVFNSS (H,C) where
\[ C = A \cap B, \forall e \in C. \]

\[
H_C(e) = \begin{cases} \min \{ F_{\mathcal{G}}(e), F_{\mathcal{H}}(e) \} & \text{if } e \in A - B \\ \max \{ F_{\mathcal{G}}(e), F_{\mathcal{H}}(e) \} & \text{if } e \in B - A \\ \min \{ F_{F}(e), F_{H}(e) \} & \text{if } e \in A \cap B \end{cases}
\]

where
\[
F_{\mathcal{G}}(e) = \min \{ F_{\mathcal{F}}(e), F_{\mathcal{G}}(e) \}
\]
\[
F_{\mathcal{H}}(e) = \max \{ F_{\mathcal{F}}(e), F_{\mathcal{H}}(e) \}
\]

3. INTERVAL VALUED FUZZY NEUTROSOPHIC SOFT TOPOLOGY

Definition 3.1:
Let \((F_A, E)\) be an element of IVFNS set over \((U, E)\), \(P(F_A, E)\) be the collection of all INFS subsets of \((F_A, E)\). A sub-family \(\tau\) of \(P(F_A, E)\) is called an interval valued fuzzy neutrosophic soft topology (short IVFNS-topology) on \((F_A, E)\) if the following axioms are satisfied:

(i) \((\varphi_A, E), (F_A, E) \in \tau\).

(ii) \([(f_1^A, E)/k \in K] \subseteq \tau \text{ implies } \bigcup_{k \in K} (f_1^A, E) \in \tau\).

(iii) If \((f_1, E), (g_A, E) \in \tau\) then \((f_1, E) \cap (g_A, E) \in \tau\).

Then the pair \((F_A, E), \tau\) is called interval valued fuzzy neutrosophic soft topological space (IVFNSTS). The members of \(\tau\) are called \(\tau\)-open IVFNS sets or open sets where \(\varphi_A: A \rightarrow \text{IVFNS}(U)\) is defined as \(\varphi_A(e) = \{ \langle x, [0,0],[0,0],[1,1]\rangle : x \in U, \forall e \in A \} \) and \(F_A: A \rightarrow \text{IVFNS}(U)\) is defined as \(F_A(e) = \{ \langle x, [1,1],[1,1],[0,0]\rangle : x \in U, \forall e \in A \} \).

Example 3.2:
Let \(U = [h_1, h_2, h_3], E = \{ e_1, e_2, e_3, e_4 \}, A = \{ e_1, e_2, e_3 \}. \)

\((F_A, E) = \{ e_1 = \langle [0,0],[0,0],[1,1]\rangle, \langle h_1, [1,1],[1,1],[0,0]\rangle, \langle h_2, [1,1],[1,1],[0,0]\rangle, \langle h_3, [1,1],[1,1],[0,0]\rangle \}
\)

\(e_2 = \langle [0,0],[0,0],[1,1]\rangle, \langle h_1, [1,1],[1,1],[0,0]\rangle, \langle h_2, [1,1],[1,1],[0,0]\rangle \}

\(e_3 = \langle [0,0],[0,0],[1,1]\rangle, \langle h_1, [1,1],[1,1],[0,0]\rangle, \langle h_2, [1,1],[1,1],[0,0]\rangle \}

\(\varphi_A(e) = \{ e_1 = \langle [h_1, [0,0],[0,0],[1,1]\rangle, \langle h_2, [0,0],[0,0],[1,1]\rangle \}
\)

\(F_A(e) = \{ \langle [0,0],[0,0],[1,1]\rangle, \langle h_1, [1,1],[1,1],[0,0]\rangle, \langle h_2, [0,0],[0,0],[1,1]\rangle, \langle h_3, [1,1],[1,1],[0,0]\rangle \}
\)

\((f_1^A, E) = \{ e_4 = \langle [h_1, [0,0],[0,0],[1,1]\rangle, \langle h_2, [0,0],[0,0],[1,1]\rangle \}
\)

\((f_2^A, E) = \{ e_4 = \langle [h_1, [0,0],[0,0],[1,1]\rangle, \langle h_2, [0,0],[0,0],[1,1]\rangle \}
\)

\((f_3^A, E) = \{ e_4 = \langle [h_1, [0,0],[0,0],[1,1]\rangle, \langle h_2, [0,0],[0,0],[1,1]\rangle \}
\)

\(\tau = \{ (\varphi_A, E), (F_A, E) \} \) is an interval valued fuzzy neutrosophic soft topological space.

Note: The subfamily \(\tau\) of \((\varphi_A, E), (F_A, E) \) is not an interval valued fuzzy neutrosophic soft topology on \((F_A, E)\) since the union \(f_1^A \cup f_2^A \neq f_3^A\) does not belong to \(\tau\).

Definition 3.3:
As every IVFNS topology on \((F_A, E)\) must contain the sets \((\varphi_A, E)\) and \((F_A, E)\), so the family \(\tau = \{ (\varphi_A, E), (F_A, E) \}\) forms an IVFNS topology on \((F_A, E)\). This topology is called indiscrinate IVFNS- topology and the pair \((F_A, E), \tau\) is called an indiscrinate interval valued fuzzy neutrosophic soft topological space.

Theorem 3.4:
Let \(\tau_i; i \in I\) be any collection of IVFNS-topology on \((F_A, E)\). Then their intersection \(\bigcap_{i \in I} \tau_i\) is also a topology on \((F_A, E)\).
**Proof:**

(i) Since \((\varphi, E), (F, A) \in \tau\) for each \(i \in I\), hence \((\varphi, E), (F, A) \in \bigcap_{i \in I} \tau_i\).

(ii) Let \(\{f^{k, i}_{F(A)} / k \in K\}\) be an arbitrary family of interval valued fuzzy neutrosophic soft sets where \(f^{k, i}_{F(A)} \in \bigcap_{i \in I} \tau_i\) for each \(k \in K\). Then for each \(i \in I\), \(f^{k, i}_{F(A)} \in \tau_i\) for \(k \in K\) and since for each \(i \in I\), \(\tau_i\) is a topology, therefore \(\bigcup_{k \in K} f^{k, i}_{F(A)} \in \tau_i\) for each \(i \in I\). Hence
\[
\bigcup_{k \in K} f^{k, i}_{F(A)} \in \bigcap_{i \in I} \tau_i.
\]

(iii) Let \((f_{A}, E), (g_{A}, E) \in \bigcap_{i \in I} \tau_i\), then \((f_{A}, E)\) and \((g_{A}, E)\) \(\in \tau_i\) for each \(i \in I\) and since \(\tau_i\) for each \(i \in I\) is a topology, therefore \((f_{A}, E) \cap (g_{A}, E) \in \tau_i\) for each \(i \in I\).
Hence \((f_{A}, E) \cap (g_{A}, E) \in \bigcap_{i \in I} \tau_i\). Thus \(\bigcap_{i \in I} \tau_i\) satisfies all the axioms of topology. Hence \(\bigcap_{i \in I} \tau_i\) forms a topology.

But the union of topologies need not be a topology, which is shown in the following example.

**Remark 3.5:**

The union of two IVFNS – topology may not be an IVFNS- topology. If we consider the example 3.2 then the subfamilies \(\tau_1 = ((\varphi, E), (F, A), (f^{1, i}_{F(A)}))\) and \(\tau_2 = ((\varphi, E), (F, A), (f^{2, i}_{F(A)}))\) are the topologies in \((F, A)\). But their union \(\tau_1, \tau_2 = ((\varphi, E), (F, A), (f^{1, i}_{F(A)}), (f^{2, i}_{F(A)}))\) is not a topology on \((F, A)\).

**Definition 3.6:**

Let \(((F, A), E), \tau)\) be an IVFNS-topological space over \((F, A)\). An IVFNS subset \((f_{A}, E)\) of \((F, A)\) is called interval valued fuzzy neutrosophic soft closed (IVFNS closed) if its complement \((f_{A}, E)^c\) is a member of \(\tau\).

**Example 3.7:**

Let us consider example 3.2, then the IVFNS closed sets in \(((F, A), E), \tau)\) are
\[
\begin{align*}
(\varphi, E)^c &= \{e_1 = (h_1, [1, 1], [0, 0]), h_2 = (h_2, [1, 1], [0, 0]), h_3 = (h_3, [1, 1], [0, 0]), h_4 = (h_4, [1, 1], [0, 0])\}, \\
(\varphi, E)^c &= \{e_2 = (h_2, [1, 1], [0, 0]), h_3 = (h_3, [1, 1], [0, 0]), h_4 = (h_4, [1, 1], [0, 0])\}, \\
(\varphi, E)^c &= \{e_3 = (h_3, [1, 1], [0, 0]), h_4 = (h_4, [1, 1], [0, 0]), h_5 = (h_5, [1, 1], [0, 0])\}, \\
(\varphi, E)^c &= \{e_4 = (h_4, [1, 1], [0, 0]), h_5 = (h_5, [1, 1], [0, 0]), h_6 = (h_6, [1, 1], [0, 0])\}, \\
(\varphi, E)^c &= \{e_5 = (h_5, [1, 1], [0, 0]), h_6 = (h_6, [1, 1], [0, 0]), h_7 = (h_7, [1, 1], [0, 0])\}.
\end{align*}
\]

**Theorem 3.8:**

Let \(((F, A), E), \tau)\) be an interval valued fuzzy neutrosophic soft topological space over \((F, A)\). Then

(i) \((\varphi, E)^c\) \((F, A), E)\) are interval valued fuzzy neutrosophic soft closed sets.

(ii) The arbitrary intersection of interval valued fuzzy neutrosophic soft closed sets is interval valued fuzzy neutrosophic soft closed set.

(iii) The union of two interval valued fuzzy neutrosophic soft closed sets is an interval valued fuzzy neutrosophic soft closed set.

**Proof:**

(i) Since \((\varphi, E), (F, A) \in \tau\) implies \((\varphi, E)^c\) and \((F, A), E)^c\) are closed.
Let \( \{ f^k_A, E \} / k \in K \) be an arbitrary family of IVFNS closed sets in \( (\mathcal{F}_A, E, \tau) \) and let 
\[
(f^R_A, E) = \bigcap_{k \in K} (f^k_A, E)
\]
\[
\bigcup_{k \in K} (f^k_A, E)^c
\]
\[
\bigcap_{k \in K} (f^k_A, E)^c \in \tau \text{ for each } k \in K, \text{ so}
\]
\[
\bigcup_{k \in K} (f^k_A, E)^c \in \tau.
\]
Hence \((f^A, E)^c \in \tau \). 
Therefore \((f^A, E) \in \epsilon \). 

Remark 3.9: The intersection of an arbitrary family of IVFNS – open set may not be an IVFNS- open and the union of an arbitrary family of IVFNS closed set may not be an IVFNS closed set.

Let us consider \( U = \{ h_1, h_2, h_3 \}; E = \{ e_1, e_2, e_3, e_4 \} \), 
\( A = \{ e_1, e_2, e_3 \} \) and let 
\( (F_A, E) = \{ e_1 = \{ h_1, \text{[1,1]}, \text{[1,1]}, \text{[0,0]} \}, \) 
\( h_2 = \{ h_{21}, \text{[1,1]}, h_{22}, \text{[1,1]}, h_{23}, \text{[0,0]} \} \) 
\( e_2 = \{ h_{e1}, \text{[1,1]}, h_{e2}, \text{[1,1]}, h_{e3}, \text{[0,0]} \} \) 
\( e_3 = \{ h_{e1}, \text{[1,1]}, h_{e2}, \text{[1,1]}, h_{e3}, \text{[0,0]} \} \) 
\( e_4 = \{ h_{e1}, \text{[0,0]}, h_{e2}, \text{[0,0]}, h_{e3}, \text{[1,1]} \} \) 
\( (\varphi_A, E) = \{ e_1 = \{ h_1, \text{[0,0]}, h_2, \text{[0,0]}, h_3, \text{[1,1]} \}, \) 
\( e_2 = \{ h_{e1}, \text{[0,0]}, h_{e2}, \text{[0,0]}, h_{e3}, \text{[1,1]} \} \) 
\( e_3 = \{ h_{e1}, \text{[0,0]}, h_{e2}, \text{[0,0]}, h_{e3}, \text{[1,1]} \} \) 
\( e_4 = \{ h_{e1}, \text{[0,0]}, h_{e2}, \text{[0,0]}, h_{e3}, \text{[1,1]} \} \) 

For each \( n \in \mathbb{N} \), we define

We observe that \( \tau = \{ (F_A, E), (\varphi_A, E), (f^R_A, E) \} \) is a IVFNS topology on \((F_A, E)\).

But \( \bigcap_{n=1}^{\infty} (f^A, E) \neq \tau \).

The IVFNS closed sets in the IVFNS topological space \((F_A, E, \tau)\) are \((F_A, E)^c\), \((\varphi_A, E)^c\) and \((f^R_A, E)^c\) for \( n = 1, 2, 3, \ldots \).

But \( \bigcup_{n=1}^{\infty} (f^A, E) \neq \tau \).

For each \( (f_A, E) \in \tau_1 \), \((f_A, E) \in \tau_2 \) is not an IVFNS-open set in IVFNS topological space \((F_A, E, \tau)\), since \( \bigcup_{n=1}^{\infty} (f^A, E) \neq \tau \).

**Definition 3.10:**

Let \((F_A, E), \tau_1 \) and \((F_A, E), \tau_2 \) be two IVFNS topological spaces. If each \((f_A, E) \in \tau_1 \) implies \((f_A, E) \in \tau_2 \), then \( \tau_2 \) is called interval valued fuzzy neutrosophic soft finer topology than \( \tau_1 \) and \( \tau_1 \) is called interval valued fuzzy neutrosophic soft coarser topology than \( \tau_2 \).
Example 3.11:
If we consider the topologies \( \tau_1 = \{(\varphi_A, E), (F_A, E), (f^1_A, E), (f^2_A, E), (f^3_A, E), (f^4_A, E)\} \) as in example 3.2 and \( \tau_2 = \{(\varphi_A, E), (F_A, E), (f^1_A, E), (f^3_A, E)\} \) on \( (F, E) \). Then \( \tau_1 \) is interval valued fuzzy neutrosophic soft finer than \( \tau_2 \) and \( \tau_2 \) is interval valued fuzzy neutrosophic soft coarser topology than \( \tau_1 \).

Definition 3.12:
Let \((F_A, E), (F, E)\) be an IVFNS topological space of \((F_A, E)\) and \(B\) be a subfamily of \(\tau\). If every element of \(\tau\) can be expressed as the arbitrary interval valued fuzzy neutrosophic soft union of some element of \(B\), then \(B\) is called an interval valued fuzzy neutrosophic soft basis for the interval valued fuzzy neutrosophic soft topology \(\tau\).

Example 3.13:
In example 3.2 for the topology \( \tau = \{(\varphi_A, E), (F_A, E), (f^1_A, E), (f^2_A, E), (f^3_A, E), (f^4_A, E)\} \) the subfamily 
\( B = \{(\varphi_A, E), (F_A, E), (f^1_A, E), (f^2_A, E), (f^3_A, E)\} \) of \( P(F_A, E) \) is a basis for the topology \(\tau\).

Definition 3.14:
Let \( \tau \) be the IVFNS topology on \((F_A, E) \in IVFNS(U,E)\) and \((f_A, E)\) be an IVFNS set in \( P(F_A, E) \) is a neighborhood of a IVFNS set \((g_A, E)\) if and only if there exist an \(\tau\)-open IVFNS set \((h_A, E)\) such that \((g_A, E) \subseteq (h_A, E) \subseteq (f_A, E)\).

Example 3.15:
Let \( U = \{h_1, h_2, h_3\}, E = \{e_1, e_2, e_3, e_4\}, A = \{e_1\} \) in an IVFNS topology \( \{\{\varphi_A, E\}, (F_A, E), (h_A, E)\} \) where \((F_A, E) = \{e_1 = \{h_1, [1, 1.1], [1, 1], [0, 0.1]\},\) 
\(h_2 = \{h_1, [1, 1.1], [0, 0.1]\},\) 
\(h_3 = \{h_1, [1, 1.1], [1, 1.1], [0, 0.1]\}\}
\((h_A, E) = \{e_1 = \{h_1, [0, 0.1], [0, 0.1], [1, 1]\},\) 
\(h_2 = \{h_1, [0, 0.1], [1, 1], [0, 0.1]\},\) 
\(h_3 = \{h_1, [1, 1.1], [0, 0.1], [1, 1]\}\}\}
\((f_A, E) = \{e_1 = \{h_1, [0, 0.1], [0, 0.1], [1, 1]\}\}\}
\((g_A, E) = \{e_1 = \{h_1, [0, 0.1], [0, 0.1], [1, 1]\}\}\}
\((f_A, E) = \{e_1 = \{h_1, [0, 0.1], [0, 0.1], [1, 1]\}\}\}
\((g_A, E) = \{e_1 = \{h_1, [0, 0.1], [0, 0.1], [1, 1]\}\}\}

The IVFNS set 
\((f_A, E) = \{e_1 = \{h_1, [0, 0.1], [0, 0.1], [1, 1]\}\}\}
\((h_A, E) = \{e_1 = \{h_1, [0, 0.1], [0, 0.1], [1, 1]\}\}\}
\((g_A, E) = \{e_1 = \{h_1, [0, 0.1], [0, 0.1], [1, 1]\}\}\)

by Theorem 3.16:
\[(g_A, E) \subseteq (h_A, E) \subseteq (f_A, E)\] such that 
\[(g_A, E) \subseteq (h_A, E) \subseteq (f_A, E)\].

A IVFNS set \((f_A, E)\) in \( P(F_A, E) \) is an open IVFNS set if and only if \((f_A, E)\) is a neighbourhood of each IVFNS set \((g_A, E)\) contained in \((f_A, E)\).

Proof:
Let \((f_A, E)\) be an open IVFNS set and \((g_A, E)\) be any IVFNS set contained in \((f_A, E)\). Since we have \((g_A, E) \subseteq (h_A, E) \subseteq (f_A, E)\), it follows that \((f_A, E)\) is a neighborhood of \((g_A, E)\). Conversely let \((f_A, E)\) be a neighborhood for every IVFNS set contained in \((f_A, E)\). Since \((f_A, E) \subseteq (h_A, E) \subseteq (f_A, E)\) there exist an open IVFNS set \((h_A, E)\) such that \((f_A, E) \subseteq (h_A, E) \subseteq (f_A, E)\). Hence \((h_A, E) = (f_A, E)\) and \((f_A, E)\) is open.

Definition 3.17:
Let \((U, E), (F, E)\) be an interval valued fuzzy neutrosophic soft topological space on \((F_A, E)\) and \((f_A, E)\) be an IVFNS set in \( P(F_A, E)\). The family of all neighborhoods of \((f_A, E)\) is called the neighborhood system of \((f_A, E)\) up to topology and is denoted by \(N_{(f_A, E)}\).

Theorem 3.18:
Let \((U, E), (F, E)\) be an interval valued fuzzy neutrosophic soft topological space. If \(N_{(f_A, E)}\) is the neighborhood system of an IVFNS set \((f_A, E)\). Then
(i) Finite intersections of members of \(N_{(f_A, E)}\) belong to \(N_{(f_A, E)}\).
(ii) Each interval valued fuzzy neutrosophic soft set which contains a member of \(N_{(f_A, E)}\) belongs to \(N_{(f_A, E)}\).

Proof:
(i) Let \((g_A, E)\) and \((h_A, E)\) be two neighborhoods of \((f_A, E)\), so there exist two open sets \((g_A^*, E), (h_A^*, E)\) such that \((f_A, E) \subseteq (g_A^*, E) \subseteq (g_A, E)\) and \((f_A, E) \subseteq (h_A^*, E) \subseteq (h_A, E)\).

Hence \((f_A, E) \subseteq (g_A^*, E) \cap (h_A^*, E) \subseteq (g_A, E) \cap (h_A, E)\) and \((g_A^*, E) \cap (h_A^*, E)\) is open. Thus \((g_A, E) \cap (h_A, E)\) is a neighborhood of \((f_A, E)\).

(ii) Let \((g_A, E)\) be a neighborhood of \((f_A, E)\) and \((g_A, E) \subseteq (h_A, E)\), so there exist an open set \((g_A^*, E)\) such that \((f_A, E) \subseteq (g_A^*, E) \subseteq (g_A, E)\). By hypothesis \((g_A, E) \subseteq (h_A, E)\), so \((f_A, E) \subseteq (g_A^*, E) \subseteq (g_A, E) \subseteq (h_A, E)\) which implies that \((f_A, E) \subseteq (g_A^*, E) \subseteq (h_A, E)\) and hence \((h_A, E)\) is a neighborhood of \((f_A, E)\).

Definition 3.19:
Let \((F_{\alpha}, E), \tau\) be an interval valued fuzzy neutrosophic soft topological space on \((F_{\alpha}, E)\) and \((f_{\alpha}, E)\), \((g_{\alpha}, E)\) be IVFNS sets in \(P(F_{\alpha}, E)\) such that \((g_{\alpha}, E)\subseteq(f_{\alpha}, E)\). Then \((g_{\alpha}, E)\) is called an interior IVFNS set of \((f_{\alpha}, E)\) if and only if \((f_{\alpha}, E)\) is a neighbourhood of \((g_{\alpha}, E)\).

**Definition 3.20:**
Let \((F_{\alpha}, E), \tau\) be an interval valued fuzzy neutrosophic soft topological space on \((F_{\alpha}, E)\) and \((f_{\alpha}, E)\) be an IVFNS set in \(P(F_{\alpha}, E)\). Then the union of all interior IVFNS set of \((f_{\alpha}, E)\) is called the interior of \((f_{\alpha}, E)\) and is denoted by \(\text{int}(f_{\alpha}, E)\) and defined by \(\text{int}(f_{\alpha}, E) = \{g_{\alpha}, E\mid (g_{\alpha}, E)\) is a neighbourhood of \((g_{\alpha}, E)\}\).

Or equivalently \(\text{int}(f_{\alpha}, E) = \{g_{\alpha}, E \mid (g_{\alpha}, E)\) is an IVFNS open set contained in \((f_{\alpha}, E)\}\).

**Example 3.21:**
Let us consider the IVFNS topology \(\tau = \{(\varphi_{\alpha}, E), (F_{\alpha}, E), (f_{\alpha}, E), (f_{\alpha}, E), (f_{\alpha}, E)\}\) as in example 3.2 and let \((f_{\alpha}, E) = \{e_{1} = \{h_{1} [0.4, 0.5], [0.6, 0.7], [0.1, 0.2]\}, \{h_{2} [0.7, 0.8], [0.6, 0.7], [0.1, 0.2]\}, \{h_{3} [1, 1], [1, 1], [0, 0]\}\} \) \(\text{int}(f_{\alpha}, E) = \{g_{\alpha}, E\mid (g_{\alpha}, E)\) is an IVFNS open set contained in \((f_{\alpha}, E)\}\).

Since \((f_{\alpha}, E)\) is an interval valued fuzzy neutrosophic soft topological space on \((F_{\alpha}, E)\) and \((f_{\alpha}, E)\) be an IVFNS set in \(P(F_{\alpha}, E)\). Then the union of all interior IVFNS set of \((f_{\alpha}, E)\) is called the interior of \((f_{\alpha}, E)\) and is denoted by \(\text{int}(f_{\alpha}, E)\) and defined by \(\text{int}(f_{\alpha}, E) = \{g_{\alpha}, E\mid (g_{\alpha}, E)\) is a neighbourhood of \((g_{\alpha}, E)\}\).

Or equivalently \(\text{int}(f_{\alpha}, E) = \{g_{\alpha}, E \mid (g_{\alpha}, E)\) is an IVFNS open set contained in \((f_{\alpha}, E)\}\).

**Theorem 3.22:**
Let \((F_{\alpha}, E), \tau\) be an interval valued fuzzy neutrosophic soft topological space on \((F_{\alpha}, E)\) and \((f_{\alpha}, E)\) be an IVFNS set in \(P(F_{\alpha}, E)\). Then

(i) \(\text{int}(f_{\alpha}, E)\) is an open and \(\text{int}(f_{\alpha}, E)\) is the largest open IVFNS set contained in \((f_{\alpha}, E)\).

(ii) The IVFNS set \((f_{\alpha}, E)\) is open if and only if \((f_{\alpha}, E)\) is an interior IVFNS set contained in \((f_{\alpha}, E)\).

**Proof:** Proof follows from the definition.

**Proposition 3.23:**
For any two IVFNS sets \((f_{\alpha}, E)\) and \((g_{\alpha}, E)\) is an interval valued fuzzy neutrosophic soft topological space \(((F_{\alpha}, E), \tau)\) on \(P(F_{\alpha}, E)\) then

(i) \((g_{\alpha}, E)\subseteq(f_{\alpha}, E)\) implies \(\text{int}(g_{\alpha}, E)\subseteq\text{int}(f_{\alpha}, E)\).

(ii) \(\text{int}(\varphi_{\alpha}, E) = (\varphi_{\alpha}, E)\) and \(\text{int}(F_{\alpha}, E) = (F_{\alpha}, E)\).

(iii) \(\text{int}(f_{\alpha}, E) = \text{int}(f_{\alpha}, E)\).

(iv) \(\text{int}(g_{\alpha}, E)\cap(f_{\alpha}, E) = \text{int}(g_{\alpha}, E)\cap\text{int}(f_{\alpha}, E)\).

(v) \(\text{int}(g_{\alpha}, E)\cup(f_{\alpha}, E) = \text{int}(g_{\alpha}, E)\cup\text{int}(f_{\alpha}, E)\).

**Proof:**
Proofs are straight forward.
**Definition 3.27:**

Let \((F_A, E), \tau\) be an interval valued fuzzy neutrosophic soft topological space on \((F_A, E)\) and \((f_A, E)\) be an IVFNS set in \(P(F_A, E)\). Then the intersection of all closed IVFNS sets \((f_A, E)\) is denoted by \(\text{cl}(f_A, E)\) and defined by \(\text{cl}(f_A, E) = \cap \{ (g_A, E) \mid (g_A, E) \text{ is a IVFNS closed set containing } (f_A, E) \}\). Thus \((f_A, E)\) is the smallest IVFNS closed set containing \((f_A, E)\).

**Example 3.28:**

Let us consider an interval valued fuzzy neutrosophic soft topology \(\tau = \{(\varphi, A), (F_A, E), (f_A^1, E), (f_A^2, E)\}\) as in example 3.2 and let \((f_A, E) = \{e_1 = \{[0.1, 0.2], [0.3, 0.4], [0.5, 0.6]\}, e_2 = \{[0.0, 0.4, 0.5], [0.6, 0.7]\}, e_3 = \{[0, 0, 1.0], [1.0, 1.1]\}\} \) be an IVFNS set.

Then \(\text{cl}(f_A, E) = \cap \{ (g_A, E) \mid (g_A, E) \text{ is an IVFNS closed set containing } (f_A, E) \}\) and is denoted by \(\text{cl}(f_A, E)\).

**Proposition 3.29:**

For any two IVFNS sets \((f_A, E)\) and \((g_A, E)\) is an interval valued fuzzy neutrosophic soft topological space \((F_A, E), \tau\) on \(P(F_A, E)\) then

(i) \(\text{cl}(f_A, E)\) is the smallest IVFNS closed set containing \((f_A, E)\).

(ii) \((f_A, E)\) is IVFNS closed if and only if \((f_A, E) = \text{cl}(f_A, E)\).

(iii) \((g_A, E) \subseteq (f_A, E)\) implies \(\text{cl}(g_A, E) \subseteq \text{cl}(f_A, E)\).

(iv) \(\text{cl}(f_A, E) = \text{cl}(g_A, E)\).

(v) \(\text{cl}(\varphi, A, E) = (f_A, E)\) and \(\text{cl}(f_A, E) = (f_A, E)\).

(vi) \(\text{cl}(g_A, E) \cup (f_A, E) = \text{cl}(g_A, E) \cup \text{cl}(f_A, E)\).

Proof:

(i) and (ii) follows from the definition.

(iii) Since \((g_A, E) \subseteq (f_A, E)\) implies all the closed set containing \((f_A, E)\) also contain \((g_A, E)\).

Therefore \(\cap \{ (g_A^*, E) \mid (g_A^*, E) \text{ is an IVFNS closed set containing } (f_A, E) \}\) also contain \((g_A, E)\).

Hence \(\text{cl}(f_A, E)\). Therefore \(\text{cl}(f_A, E) = \text{cl}(g_A, E)\).

(v) Proof is obvious.

(vi) Since \(\text{cl}(g_A, E) \supseteq (f_A, E)\) and \(\text{cl}(f_A, E) \supseteq (f_A, E)\), we have \(\text{cl}(g_A, E) \cup \text{cl}(f_A, E) \supseteq (f_A, E)\). This implies \(\text{cl}(g_A, E) \cup \text{cl}(f_A, E) = (f_A, E)\). Therefore \(\text{cl}(g_A, E) \cup \text{cl}(f_A, E) = (f_A, E)\).

**Theorem 3.30:**

Let \((F_A, E), \tau\) be an interval valued fuzzy neutrosophic soft topological space on \((F_A, E)\) and \((f_A, E)\) be an IVFNS set in \(P(F_A, E)\). Then the collection \(\tau(f_A, E) = \{(f_A^1, E) \cap (g_A^*, E) \} / (g_A^*, E) \in \tau\) is an interval valued fuzzy neutrosophic soft topology on the interval valued fuzzy neutrosophic soft set \((f_A, E)\).

**Proof:**

(i) Since \((\varphi, A, E), (f_A, E) \in \tau\), \((f_A, E) = (f_A, E) \cap (\varphi, A, E)\).

(ii) Let \(\{ (f_A^i, E) \} / i = 1, 2, 3, \ldots, n\) be a finite family of IVFNS open sets in \(\tau(f_A, E)\) then for each \(i = 1, 2, 3, \ldots, n\), there exist \((g_A^i, E) \in \tau\) such that \((f_A^i, E) = (f_A, E) \cap (g_A^i, E)\).

Now let \(N = \bigcup_{i=1}^{n} (f_A^i, E)\). Since \(N = \bigcup_{i=1}^{n} (f_A^i, E) \in \tau\) so \(N = \bigcup_{i=1}^{n} (f_A^i, E) \in \tau(f_A, E)\).

(iii) Let \(\{ (f_A^k, E) / k \in K\} \) be an arbitrary family of interval valued fuzzy neutrosophic soft open sets in \(\tau(f_A, E)\) then for each \(k \in K\), there exist \((g_A^k, E) \in \tau(f_A, E)\) such that \((f_A^k, E) = (f_A, E) \cap (g_A^k, E)\).

Now let \(N = \bigcup_{k \in K} (f_A^k, E) \cap (g_A^k, E)\) and since \(N \subseteq \bigcup_{k \in K} (f_A^k, E) \in \tau\).
Let $(f_{A,E}, \tau)$ be an IVFNS topological space on $(F_{A,E})$ and $(f_{A,E})$ be an IVFNS set in $P(F_{A,E})$. Then the IVFNS topology.

$$\tau(f_{A,E}) = \{(f_{A,E}) \cap (g_{A,E}) | (g_{A,E}) \in \tau\}$$

is called interval valued fuzzy neutrosophic soft subspace topology (IVFNS subspace topology) and $(f_{A,E}, \tau(f_{A,E}))$ is called interval valued fuzzy neutrosophic soft subspace of $(F_{A,E}, \tau)$.

Example 3.32:

Let us consider the interval valued fuzzy neutrosophic soft topology $\tau = (\varphi_{E}, (F_{A,E}), (f_{A,E})^1, (f_{A,E})^2)$ as in the example 3.2 and an IVFNS-set $(f_{A,E}) = \varphi_{E}$.

$$(f_{A,E}) \cap (f_{A,E}) = (g_{A,E})$$

Thus $\tau(f_{A,E}) = \{(f_{A,E}) \cap (g_{A,E}) | (g_{A,E}) \in \tau\}$ is an interval valued fuzzy neutrosophic soft subspace topology for $\tau$ and $(f_{A,E}, \tau(f_{A,E}))$ is called interval valued fuzzy neutrosophic soft subspace of $(F_{A,E}, \tau)$.

Theorem 3.33:

Let $(\eta_{A,E}, \tau')$ be a IVFNS topological subspace of $(\xi_{A,E}, \tau)$ and let $(\xi_{A,E}, \tau')$ be a IVFNS topological subspace of $(F_{A,E}, \tau)$. Then $(\eta_{A,E}, \tau')$ is also an IVFNS topological subspace of $(F_{A,E}, \tau)$.

Proof:

Since $(\eta_{A,E}) \subseteq (\xi_{A,E}) \subseteq (F_{A,E})$, $(\eta_{A,E}, \tau')$ is an interval valued fuzzy neutrosophic soft topological subspace of $(F_{A,E}, \tau)$, if and only if $\tau(\eta_{A,E}) = \tau'$. Let $f_{A,E} \in \tau'$, now since $(\eta_{A,E}, \tau')$ is an IVFNS topological subspace of $(\xi_{A,E}, \tau')$, there exist $f_{A,E}^2 \in \tau'$ such that $f_{A,E}^1 \in \tau'$.

Now assume, $(g_{A,E}) \in \tau(\eta_{A,E})$ such that $(g_{A,E}) \in \tau(\eta_{A,E})$. But $(\xi_{A,E}) \in \tau(\eta_{A,E}) \cap (\eta_{A,E}, \tau')$. Hence $(\eta_{A,E}) \in \tau(\eta_{A,E}) \cap (\eta_{A,E}, \tau')$. We have
(g_A,E) \in \tau \implies \tau(g_A,E) \subseteq \tau \quad ---(2). From (1) and (2) \tau = \tau(g_A,E). Hence the proof.

**Theorem 3.34:**

Let \((F_A,E), \tau)\) be an IVFNS topological space of \((F_A,E), B\) be an basis for \(\tau\) and \((F_A,E)\) be an IVFNS set in \(P(F_A,E)\). Then the family \(B(F_A,E) = \{(F_A,E) \cap (g_A,E) | (g_A,E) \in B\}\) is an IVFNS basis for subspace topology \(\tau(F_A,E)\).

**Proof:**

Let \((h_A,E) \in \tau(F_A,E)\), then there exist an IVFNS set \((g_A,E) \in \tau\), such that \((h_A,E) = (F_A,E) \cap (g_A,E)\). Since \(B\) is a base for \(\tau\), there exist sub-collection \(\{(\psi_{i}^{A},E) | i \in I\}\) of \(B\), such that \((g_A,E) = \bigcup_{i \in I} (\psi_{i}^{A},E)\).

Therefore \((h_A,E) = (f_A,E) \cap (g_A,E) = (f_A,E) \cap (\bigcup_{i \in I} (\psi_{i}^{A},E)) = \bigcup_{i \in I} ((f_A,E) \cap (\psi_{i}^{A},E))\).

Since \((f_A,E) \cap (\psi_{i}^{A},E) \in B(F_A,E)\) implies \(B(F_A,E)\) is an IVFNS basis for the IVFNS subspace topology \(\tau(F_A,E)\).

**4. Conclusion**

In this paper the notion of topological space in interval valued fuzzy neutrosophic soft sets is introduced. Further, some of its operators and properties of topology in IVFNS set are established.

**References**


