Interval Valued Fuzzy Neutrosophic Soft Structure Spaces

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Abstract. In this paper we introduce the topological structure of interval valued fuzzy neutrosophic soft sets and obtain some of its properties. We also investigate some operators of interval valued fuzzy neutrosophic soft topological space.

Keywords: Fuzzy Neutrosophic soft set, Interval valued fuzzy neutrosophic soft set, Interval valued fuzzy neutrosophic soft topological space.

1 Introduction

In 1999,[9] Molodsov initiated the novel concept of soft set theory which is a completely new approach for modeling vagueness and uncertainty. In [6] Maji et al. intiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union , intersection, complement of fuzzy soft set. Moreover in [7,8] Maji et al extended soft sets to intuitionistic fuzzy soft sets and Neutrosophic soft sets.

Neutrosophic Logic has been proposed by Florentine Smarandache[14,15] which is based on non-standard analysis that was given by Abraham Robinson in 1960s. Neutrosophic Logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision undefined, incompleteness, inconsistency, redundancy, contradiction. The neutrosophic logic is a formal frame to measure truth, indeterminacy and falsehood. In Neutrosophic set, indeterminacy is quantified explicitly whereas the truth membership, indeterminacy membership and falsity membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors.

Yang et al.[16] presented the concept of interval valued fuzzy soft sets by combining the interval valued fuzzy set and soft set models. Jiang.Y et al.[5] introduced interval valued intuitionistic fuzzy soft set. In this paper we define interval valued fuzzy neutrosophic soft topological space and we discuss some of its properties.

2 Preliminaries

Definition 2.1[2]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as

$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X$

where $T, I, F : X \rightarrow [0, 1]$ and

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

.

Definition 2.2[3]:

An interval valued fuzzy neutrosophic set (IVFNS in short) on a universe X is an object of the form $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$ where $T_A(x) = X \rightarrow \text{Int}([0,1])$, $I_A(x) = X \rightarrow \text{Int}([0,1])$ and $F_A(x) = X \rightarrow \text{Int}([0,1])$ {Int([0,1]) stands for the set of all closed subinterval of [0,1] satisfies the condition $\forall x \in X$, $\sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$.

Definition 2.3[3]:

Let U be an initial universe and E be a set of parameters. IVFNS(U) denotes the set of all interval valued fuzzy neutrosophic sets of U. Let $A \subseteq E$. A pair (F,A) is an interval valued fuzzy neutrosophic soft set over U, where F is a mapping given by $F: A \rightarrow \text{IVFNS}(U)$.

Note : Interval valued fuzzy neutrosophic soft set is denoted by IVFNS set.

Definition 2.4[3]:

The complement of an INFNSS (F,A) is denoted by $(F,A)^c$ and is defined as $(F,A)^c = (F^c, \rceil A)$ where $F^c: \rceil A \rightarrow \text{IVFNSS}(U)$ is a mapping given by $F^c(e) = \langle x, F_{F(\neg e)}(x), (I_{F(\neg e)}(x))^c, F_{F(\neg e)}(x) \rangle$ for all $x \in U$ and $e \in \neg A$, $(I_{F(\neg e)}(x))^c = 1 - I_{F(\neg e)}(x) = [1 - \overline{I}_{F(e)}(x), 1 - \underline{I}_{F(e)}(x)]$.

Definition 2.5[3]:

The union of two IVFNSS (F,A) and (G,B) over a universe U is an IVFNSS (H,C) where $C = A \cup B, \forall e \in C$.

$$\tilde{H}_C(e) = \begin{cases} \left(\left\{ \dfrac{h}{T_{\tilde{F}(e)}(h), I_{\tilde{F}(e)}(h), F_{\tilde{F}(e)}(h)} \right\} \right), & \text{if } e \in A - B \\ \left(\left\{ \dfrac{h}{T_{\tilde{G}(e)}(h), I_{\tilde{G}(e)}(h), F_{\tilde{G}(e)}(h)} \right\} \right), & \text{if } e \in B - A \\ \left(\left\{ \dfrac{h}{T_{\tilde{H}(e)}(h), I_{\tilde{H}(e)}(h), F_{\tilde{H}(e)}(h)} \right\} \right), & \text{if } e \in A \cap B \end{cases}$$

where

$$T_{\tilde{H}(e)}(h) = \max\{T_{\tilde{F}(e)}(h), T_{\tilde{G}(e)}(h)\},$$
$$I_{\tilde{H}(e)}(h) = \max\{I_{\tilde{F}(e)}(h), I_{\tilde{G}(e)}(h)\},$$
$$F_{\tilde{H}(e)}(h) = \min\{F_{\tilde{F}(e)}(h), F_{\tilde{G}(e)}(h)\}$$

Definition 2.6[3]:

The intersection of two IVFNSS (F,A) and (G,B) over a universe U is an IVFNSS (H,C) where
C = A∪B,∀e∈C.

$$\tilde{H}_C(e) = \begin{cases} \left(\left\{\dfrac{h}{T_{\tilde{F}(e)}(h), I_{\tilde{F}(e)}(h), F_{\tilde{F}(e)}(h)}\right\}\right), & if\ e \in A - B \\ \left(\left\{\dfrac{h}{T_{\tilde{G}(e)}(h), I_{\tilde{G}(e)}(h), F_{\tilde{G}(e)}(h)}\right\}\right), & if\ e \in B - A \\ \left(\left\{\dfrac{h}{T_{\tilde{H}(e)}(h), I_{\tilde{H}(e)}(h), F_{\tilde{H}(e)}(h)}\right\}\right), & if\ e \in A \cap B \end{cases}$$

where

$$T_{\tilde{H}(e)}(h) = \min\{T_{\tilde{F}(e)}(h), T_{\tilde{G}(e)}(h)\},$$
$$I_{\tilde{H}(e)}(h) = \min\{I_{\tilde{F}(e)}(h), I_{\tilde{G}(e)}(h)\},$$
$$F_{\tilde{H}(e)}(h) = \max\{F_{\tilde{F}(e)}(h), F_{\tilde{G}(e)}(h)\}$$

3. INTERVAL VALUED FUZZY NEUTROSOPHIC SOFT TOPOLOGY

Definition 3.1:

Let (F_A, E) be an element of IVFNS set over (U,E),P(F_A, E) be the collection of all INFNS subsets of (F_A,E). A sub-family τ of P(F_A, E) is called an interval valued fuzzy neutrosophic soft topology (in short IVFNS-topology) on (F_A, E) if the following axioms are satisfied:

(i) (φ_A, E), (F_A, E) ∈τ.

(ii) $\{(f_A^k, E)/k \in K\} \subseteq \tau$ implies $\bigcup_{k\in K} (f_A^k, E) \in \tau$

(iii) If (f_A, E), (g_A, E) ∈τ then (f_A, E) ∩ (g_A, E) ∈τ.

Then the pair ((F_A, E),τ) is called interval valued fuzzy neutrosophic soft topological space (IVFNSTS). The members of τ are called τ-open IVFNS sets or open sets where φ_A:A→IVFNS(U) is defined as φ_A(e) = {<x, [0,0],[0,0],[1,1]>:x∈U, ∀e∈A} and F_A:A→IVFNS(U) is defined as F_A (e) = {<x, [1,1],[1,1],[0,0]>:x∈U, ∀e∈A}.

Example 3.2:

Let U={h_1, h_2, h_3}, E = {e_1, e_2, e_3, e_4}, A = {e_1, e_2, e_3}.
(F_A, E) = {e_1 ={<h_1 [1,1], [1,1], [0,0]>,
<h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>}
{e_2 ={<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,
<h_3 [1,1], [1,1], [0,0]>}
{e_3 ={<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,
<h_3 [1,1], [1,1], [0,0]>}
(φ_A, E) ={e_1 ={<h_1 [0,0], [0,0], [1,1]>,
<h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>}
{e_2 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A^1,E) = {e_1 ={<h_1 [0.5,0.6], [0.4,0.5], [0.2,0.3]>,
<h_2 [0.4,0.5], [0.5,0.6], [0.0,0.1]>, <h_3 [1,1], [1,1], [0,0]>}
{e_2 ={<h_1 [0.4,0.5], [0.5,0.6], [0.2,0.3]>,
<h_2 [0.4,0.5], [0.7,0.8], [0,0]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A^2,E) = {e_1 ={<h_1 [0.3,0.4], [0.5,0.6], [0.1,0.2]>,
<h_2 [0.6,0.7], [0.5,0.6], [0.2,0.3]>, <h_3 [1,1], [1,1], [0,0]>}
{e_2 ={<h_1 [0.2,0.3], [0.4,0.5], [0.0,0.1]>,
<h_2 [1,1], [1,1], [0,0]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A^3,E) = {e_1 ={<h_1 [0.3,0.4], [0.4,0.5], [0.2,0.3]>,
<h_2 [0.4,0.5], [0.5,0.6], [0.2,0.3]>, <h_3 [1,1], [1,1], [0,0]>
{e_2 ={<h_1 [0.2,0.3], [0.4,0.5], [0.2,0.3]>,
<h_2 [0.4,0.5], [0.7,0.8], [0,0]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A^4,E) = {e_1 ={<h_1 [0.5,0.6], [0.5,0.6], [0.1,0.2]>,
<h_2 [0.6,0.7], [0.5,0.6], [0.0,0.1]>, <h_3 [1,1], [1,1], [0,0]>}
{e_2 ={<h_1 [0.4,0.5], [0.5,0.6], [0.0,0.1]>,
<h_2 [1,1], [1,1], [0,0]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}.

Here τ={(φ_A, E), (F_A, E), (f_A^1,E), (f_A^2,E), (f_A^3,E) ,(f_A^4,E)} of P(F_A, E) is a IVFNS topology on (F_A, E) and ((F_A, E), τ) is an interval valued fuzzy neutrosophic soft topological space.

Note: The subfamily τ_1={(φ_A, E), (F_A, E), (f_A^1,E), (f_A^2,E), (f_A^3,E)} of P(F_A, E) is not an interval valued fuzzy neutrosophic soft topology on (F_A, E) since the union (f_A^1,E) ∪(f_A^2,E)= (f_A^4,E) does not belong to τ_1.

Definition 3.3:

As every IVFNS topology on (F_A, E) must contain the sets (φ_A, E) and (F_A, E), so the family τ={(φ_A, E), (F_A, E)} forms an IVFNS topology on (F_A, E). This topology is called indiscrete IVFNS- topology and the pair ((F_A, E), τ) is called an indiscrete interval valued fuzzy neutrosophic soft topological space.

Theorem 3.4:

Let {τ_i; i∈I} be any collection of IVFNS-topology on (F_A, E). Then their intersection $\bigcap_{i\in I} \tau_i$ is also a topology on (F_A, E).

Proof:

(i) Since $(\varphi_A, E), (F_A, E) \in \tau_i$ for each $i \in I$, hence $(\varphi_A, E), (F_A, E) \in \bigcap_{i\in I} \tau_i$.

(ii) Let $\{(f_A^k, E)/k \in K\}$ be an arbitrary family of interval valued fuzzy neutrosophic soft sets where $(f_A^k, E) \in \bigcap_{i\in I} \tau_i$ for each $k \in K$. Then for each $i \in I$, $(f_A^k, E) \in \tau_i$ for $k \in K$ and since for each $i \in I$, τ_i is an topology, therefore $\bigcup_{k\in K}(f_A^k, E) \in \tau_i$ for each $i \in I$. Hence $\bigcup_{k\in K}(f_A^k, E) \in \bigcap_{i\in I} \tau_i$.

(iii) Let $(f_A, E), (g_A, E) \in \bigcap_{i\in I} \tau_i$, then (f_A, E) and $(g_A, E) \in \tau_i$ for each $i \in I$ and since τ_i for each $i \in I$ is an topology, therefore $(f_A, E) \cap (g_A, E) \in \tau_i$ for each $i \in I$. Hnece $(f_A, E) \cap (g_A, E) \in \bigcap_{i\in I} \tau_i$ .Thus $\bigcap_{i\in I} \tau_i$ satisfies all the axioms of topology. Hence $\bigcap_{i\in I} \tau_i$ forms a topology.

But the union of topologies need not be a topology, which is shown in the following example.

Remark 3.5:

The union of two IVFNS – topology may not be a IVFNS- topology. If we consider the example 3.2 then the subfamilies $\tau_1 = \{(\varphi_A, E), (F_A, E), (f_A^1, E)\}$ and $\tau_2 = \{(\varphi_A, E), (F_A, E), (f_A^2, E)\}$ are the topologies in (F_A, E). But their union $\tau_1 \cup \tau_2 = \{(\varphi_A, E), (F_A, E), (f_A^1, E), (f_A^2, E)\}$ is not a topology on (F_A, E).

Definition 3.6:

Let $((F_A, E),\tau)$ be an IVFNS-topological space over (F_A, E). An IVFNS subset (f_A, E) of (F_A, E) is called interval valued fuzzy neutrosophic soft closed (IVFNS closed) if its complement $(f_A, E)^c$ is a member of τ.

Example 3.7:

Let us consider example 3.2, then the IVFNS closed sets in $((F_A, E),\tau)$ are

$(\varphi_A, E)^c = \{e_1 = \{<h_1 [1,1], [1,1], [0,0]>,$
$<h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>\}$
$\{e_2 = \{<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,$
$<h_3 [1,1], [1,1], [0,0]>\}$
$\{e_3 = \{<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,$
$<h_3 [1,1], [1,1], [0,0]>\}$

$(F_A, E)^c = \{e_1 = \{<h_1 [0,0], [0,0], [1,1]>,$
$<h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>\}$
$\{e_2 = \{<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,$
$<h_3 [0,0], [0,0], [1,1]>\}$
$\{e_3 = \{<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,$
$<h_3 [0,0], [0,0], [1,1]>\}$

$(f_A^1, E)^c = \{e_1 = \{<h_1 [0.2,0.3], [0.5,0.6], [0.5,0.6]>,$
$<h_2 [0.0,0.1], [0.4,0.5], [0.4,0.5]>, <h_3 [0,0], [0,0], [1,1]>\}$
$\{e_2 = \{<h_1 [0.2,0.3], [0.4,0.5], [0.4,0.5]>,$
$<h_2 [0,0], [0.2,0.3], [0.4,0.5]>, <h_3 [1,1], [1,1], [0,0]>\}$
$\{e_3 = \{<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,$
$<h_3 [1,1], [1,1], [0,0]>\}$

$(f_A^2, E)^c = \{e_1 = \{<h_1 [0.1,0.2], [0.4,0.5], [0.3,0.4]>,$
$<h_2 [0.2,0.3], [0.4,0.5], [0.6,0.7]>, <h_3 [0,0], [0,0], [1,1]>\}$
$\{e_2 = \{<h_1 [0.0,0.1], [0.5,0.6], [0.2,0.3]>,$
$<h_2 [0,0], [0,0], [1,1]>, <h_3 [1,1], [1,1], [0,0]>\}$
$\{e_3 = \{<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,$
$<h_3 [1,1], [1,1], [0,0]>\}$

$(f_A^3, E)^c = \{e_1 = \{<h_1 [0.2,0.3], [0.5,0.6], [0.3,0.4]>,$
$<h_2 [0.2,0.3], [0.4,0.5], [0.4,0.5]>, <h_3 [0,0], [0,0], [1,1]>\}$
$\{e_2 = \{<h_1 [0.2,0.3], [0.5,0.6], [0.2,0.3]>,$
$<h_2 [0,0], [0.2,0.3], [0.4,0.5]>, <h_3 [1,1], [1,1], [0,0]>\}$
$\{e_3 = \{<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,$
$<h_3 [1,1], [1,1], [0,0]>\}$

$(f_A^4, E)^c = \{e_1 = \{<h_1 [0.1,0.2], [0.4,0.5], [0.5,0.6]>,$
$<h_2 [0.0,0.1], [0.4,0.5], [0.6,0.7]>, <h_3 [0,0], [0,0], [1,1]>\}$
$\{e_2 = \{<h_1 [0.0,0.1], [0.4,0.5], [0.4,0.5]>,$
$<h_2 [0,0], [0,0], [1,1]>, <h_3 [1,1], [1,1], [0,0]>\}$
$\{e_3 = \{<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,$
$<h_3 [1,1], [1,1], [0,0]>\}$ are the interval valued fuzzy neutrosophic soft closed sets in $((F_A, E),\tau)$.

Theorem 3.8:

Let $((F_A, E),\tau)$ be an interval valued fuzzy neutrosophic soft topological space over (F_A, E). Then

(i) $(\varphi_A, E)^c$ $(F_A, E)^c$ are interval valued fuzzy neutrosophic soft closed sets.

(ii) The arbitrary intersection of interval valued fuzzy neutrosophic soft closed sets is interval valued fuzzy neutrosophic soft closed set.

(iii) The union of two interval valued fuzzy neutrosophic soft closed sets is an interval valued fuzzy neutrosophic closed set.

Proof:

(i) Since $(\varphi_A, E), (F_A, E) \in \tau$ implies $(\varphi_A, E)^c$ and $(F_A, E)^c$ are closed.

(ii) Let $\{(f_A^k, E)/k \in K\}$ be an arbitrary family of IVFNS closed sets in $((F_A, E),\tau)$ and let $(f_A, E) = \bigcap_{k\in K}(f_A^k, E)$. Now $(f_A, E)^c = \left(\bigcap_{k\in K}(f_A^k, E)\right)^c$ $= \bigcup_{k\in K}(f_A^k, E)^c$ and $(f_A^k, E)^c \in \tau$ for each $k\in K$, so $\bigcup_{k\in K}(f_A^k, E)^c \in\tau$. Hence $(f_A, E)^c\in\tau$. Thus (f_A, E) is IVFNS closed set.

(iii) Let $\{f_A^i, E)/i = 1,2,3,...n\}$ be a finite family of IVFNS closed sets in $((F_A, E),\tau)$ and let $(g_A, E) = \bigcup_{i=1}^{n}(f_A^i, E)$.

Now $(g_A, E)^c = \left(\bigcup_{i=1}^{n}(f_A^i, E)\right)^c = \bigcap_{i=1}^{n}(f_A^i, E)^c$ and $(f_A^i, E)^c \in \tau$. So $\bigcap_{i=1}^{n}(f_A^i, E)^c \in \tau$. Hence $(g_A, E)^c\in\tau$. Thus (g_A, E) is an IVFNS closed set.

Remark 3.9:

The intersection of an arbitrary family of IVFNS – open set may not be an IVFNS- open and the union of an arbitrary family of IVFNS closed set may not be an IVFNS closed.

Let us consider U= $\{h_1,h_2.h_3\}$; E = $\{e_1,e_2,e_3,e_4\}$, A = $\{e_1,e_2,e_3\}$ and let

(F_A, E) = {e_1 ={<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>}
{e_2 ={<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>}
{e_3 ={<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>}

(φ_A, E) = {e_1 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>}
{e_2 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>}

For each $n\in N$, we define

$$(f_A^n, E) = \left\{ e_1 = \left\{ \begin{array}{l} \left\langle h_1 \left[\frac{1}{4n}, \frac{1}{2n}\right], \left[\frac{1}{5n}, \frac{1}{2n}\right], \left[\frac{1}{2} - \frac{1}{2n}, \frac{1}{2} - \frac{1}{3n}\right] \right\rangle, \\ \left\langle h_2 [1,1], [1,1], [0,0] \right\rangle, \left\langle h_3 [0,0], [0,0], [1,1] \right\rangle \end{array} \right\} \right.$$

$$e_2 = \left\{ \begin{array}{l} \left\langle h_1 \left[\frac{1}{3n}, \frac{1}{2n}\right], \left[\frac{1}{4n}, \frac{1}{2n}\right], \left[\frac{1}{3} - \frac{1}{3n}, \frac{1}{3} - \frac{1}{4n}\right] \right\rangle, \\ \left\langle h_2 [0,0], [0,0], [1,1] \right\rangle, \left\langle h_3 [0,0], [0,0], [1,1] \right\rangle \end{array} \right\}$$

$$e_3 = \left\{ \begin{array}{l} \left\langle h_1 [0,0], [0,0], [1,1] \right\rangle, \left\langle h_2 [0,0], [0,0], [1,1] \right\rangle, \\ \left\langle h_3 [0,0], [0,0], [1,1] \right\rangle \end{array} \right\} \Bigg\}$$

We observe that $\tau = \{(F_A, E), (\varphi_A, E), (f_A^n, E)\}$ is a IVFNS topology on (F_A, E).

But $\bigcap_{n=1}^{\infty}(f_A^n, E)$ ={e_1 ={<h_1 [0,0], [0,0], [0.5,0.5]>, <h_2 [1,1],[1,1],[0,0]>, <h_3 [0,0], [0,0], [1,1]>}
{e_2 ={<h_1 [0,0], [0,0], [0.33,0.33]>, <h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>}} is not an IVFNS-open set in IVFNS topological space $((F_A, E),\tau)$, since $\bigcap_{n=1}^{\infty}(f_A^n, E) \notin\tau$.

The IVFNS closed sets in the IVFNS topological space $((F_A, E),\tau)$ are $(F_A, E)^c$,$(\varphi_A, E)^c$ and $(f_A^n, E)^c$ for (n= 1,2,3…).

But $\bigcup_{n=1}^{\infty}(f_A^n, E)$ ={e_1 ={<h_1 [0.5,0.5], [1,1], [0,0]>, <h_2 [0,0],[0,0],[1,1]>, <h_3 [1,1], [1,1], [0,0]>}
{e_2 ={<h_1[0.33,0.33], [1,1], [0,0] >, <h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>}
{e_3 ={<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>} is not an IVFNS-closed set in IVFNS topological space $((F_A, E),\tau)$, since $\bigcup_{n=1}^{\infty}(f_A^n, E) \notin\tau$.

Definition 3.10:

Let $((F_A, E), \tau_1)$ and $((F_A, E), \tau_2)$ be two IVFNS topological spaces. If each $(f_A,E)\in\tau_1$ implies $(f_A,E)\in\tau_2$, then τ_2 is called interval valued fuzzy neutrosophic soft finer topology than τ_1 and τ_1 is called interval valued fuzzy neutrosophic soft coarser topology than τ_2.

Example 3.11:

If we consider the topologies τ_1={(φ_A, E), (F_A, E), (f_A^1 ,E), (f_A^2 ,E), (f_A^3 ,E) ,(f_A^4 ,E)} as in example 3.2 and τ_2={(φ_A, E), (F_A, E), (f_A^1,E),(f_A^3,E)} on (F_A,E). Then τ_1 is interval valued fuzzy neutrosophic soft finer than τ_2 and τ_2 is interval valued fuzzy neutrosophic soft coarser topology than τ_1.

Definition 3.12:

Let ((F_A, E), τ) be an IVFNS topological space of (F_A, E) and $\mathscr{B}$ be a subfamily of τ. If every element of τ can be expressed as the arbitrary interval valued fuzzy neutrosophic soft union of some element of $\mathscr{B}$, then $\mathscr{B}$ is called an interval valued fuzzy neutrosophic soft basis for the interval valued fuzzy neutrosophic soft topology τ.

Example 3.13:

In example 3.2 for the topology τ={(φ_A, E), (F_A, E), (f_A^1,E), (f_A^2 ,E), (f_A^3,E) ,(f_A^4 ,E)} the subfamily $\mathscr{B}$ ={(φ_A, E), (F_A, E), (f_A^1,E), (f_A^2 ,E), (f_A^3 ,E) } of P(F_A,E) is a basis for the topology τ.

Definition 3.14:

Let τ be the IVFNS topology on (F_A,E) $\in$ IVFNS(U,E) and (f_A,E) be an IVFNS set in P(F_A,E) is a neighborhood of a IVFNS set (g_A,E) if and only if there exist an τ-open IVFNS set (h_A,E) ie., (h_A,E)$\in\tau$ such that (g_A,E)$\subseteq$ (h_A,E)$\subseteq$ (f_A,E).

Example 3.15:

Let U={h_1, h_2, h_3}, E = {e_1, e_2, e_3, e_4}, A = {e_1}.In an IVFNS topology τ={(φ_A, E), (F_A, E), (h_A,E)} where
(F_A, E) = {e_1 ={<h_1 [1,1], [1,1], [0,0]>,
<h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>}
(φ_A, E) = {e_1 ={<h_1 [0,0], [0,0], [1,1]>,
<h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>}
(h_A, E) = {e_1 ={<h_1 [0.4,0.5], [0.5,0.6], [0.4,0.5]>,
<h_2 [0.3,0.4], [0.4,0.5], [0.5,0.6]>,
<h_3 [0.4,0.5], [0.3,0.4], [0.1,0.2]>}}.
The IVFNS set
(f_A, E) = {e_1 ={<h_1 [0.5,0.6], [0.6,0.7], [0.2,0.3]>,
<h_2 [0.3,0.4], [0.4,0.5], [0.5,0.6]>,
<h_3 [0.4,0.5], [0.4,0.5], [0,0.1]>} is a neighbourhood of the IVFNS set
(g_A, E) = {e_1 ={<h_1 [0.3,0.4], [0.4,0.5], [0.4,0.5]>,
<h_2 [0.1,0.2], [0.2,0.3], [0.6,0.7]>,
<h_3 [0.4,0.5], [0.2,0.3], [0.3,0.4]>}}
because there exist an τ-open IVFNS set (h_A,E) such that (g_A,E)$\subseteq$ (h_A,E)$\subseteq$ (f_A,E).

Theorem 3.16:

A IVFNS set (f_A,E) in P(F_A,E) is an open IVFNS set if and only if (f_A,E) is a neighbourhood of each IVFNS set (g_A,E) contained in (f_A,E).

Proof:

Let (f_A,E) be an open IVFNS set and (g_A,E) be any IVFNS set contained in (f_A,E). Since we have (g_A,E)$\subseteq$ (h_A,E)$\subseteq$ (f_A,E) , it follows that (f_A,E) is a neighbourhood of (g_A,E). Conversely let (f_A,E) be a neighbourhood for every IVFNS set contained in it. Since (f_A,E)$\subseteq$(f_A,E) there exist an open IVFNS set (h_A,E) such that (f_A,E)$\subseteq$ (h_A,E)$\subseteq$ (f_A,E). Hence (h_A,E) = (f_A,E) and (f_A,E) is open.

Definition 3.17:

Let ((F_A, E), τ) be an interval valued fuzzy neutrosophic soft topological space on (F_A, E) and (f_A,E) be a IVFNS set in P(F_A,E). The family of all neighbourhoods of (f_A,E) is called the neighbourhood system of (f_A,E) up to topology and is denoted by $N_{(f_A,E)}$.

Theorem 3.18:

Let ((F_A, E), τ) be an interval valued fuzzy neutrosophic soft topological space. If $N_{(f_A,E)}$ is the neighbourhood system of an IVFNS set (f_A,E). Then

(i) Finite intersections of members of $N_{(f_A,E)}$ belong to $N_{(f_A,E)}$

(ii) Each interval valued fuzzy neutrosophic soft set which contains a member of $N_{(f_A,E)}$ belongs to $N_{(f_A,E)}$

Proof:

(i) Let (g_A,E) and (h_A,E) are two neighbourhoods of (f_A,E) , so there exist two open sets (g'_A, E) , (h'_A, E) such that (f_A,E)$\subseteq$ (g'_A, E) $\subseteq$(g_A,E) and (f_A,E)$\subseteq$ (h'_A, E) $\subseteq$(h_A,E).

Hence (f_A,E)$\subseteq$ $(g'_A, E) \cap (h'_A, E)$ $\subseteq$ (g_A,E)$\cap$ (h_A,E) and $(g'_A, E) \cap (h'_A, E)$ is open. Thus (g_A,E)$\cap$ (h_A,E) is a neighbourhood of (f_A,E).

(ii) Let (g_A,E) is a neighbourhood of (f_A,E) and (g_A,E)$\subseteq$ (h_A,E), so there exist an open set (g_A^*, E) , such that (f_A,E)$\subseteq$ (g_A^*, E) $\subseteq$(g_A,E). By hypothesis (g_A,E)$\subseteq$ (h_A,E), so (f_A,E)$\subseteq$ (g_A^*, E) $\subseteq$(g_A,E)$\subseteq$ (h_A,E) which implies that (f_A,E)$\subseteq$ (g_A^*, E) $\subseteq$(h_A,E) and hence (h_A,E) is a neighbourhood of (f_A,E).

Definition 3.19:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space on (F_A, E) and (f_A,E) , (g_A,E) be IVFNS sets in $P(F_A,E)$ such that $(g_A,E)\subseteq (f_A,E)$. Then (g_A,E) is called an interior IVFNS set of (f_A,E) if and only if (f_A,E) is a neighbourhood of (g_A,E).

Definition 3.20:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space on (F_A,E) and (f_A,E) be an IVFNS set in $P(F_A,E)$. Then the union of all interior IVFNS set of (f_A,E) is called the interior of (f_A,E) and is denoted by $int(f_A,E)$ and defined by $int(f_A,E) = \cup\{(g_A,E)/ (f_A,E)$ is a neighbourhood of $(g_A,E)\}$

Or equivalently $int(f_A,E)=\cup\{ (g_A,E)/ (g_A,E)$ is an IVFNS open set contained in $(f_A,E)\}$.

Example 3.21:

Let us consider the IVFNS topology $\tau=\{(\varphi_A, E), (F_A, E), (f_A^1,E), (f_A^2,E), (f_A^3,E), (f_A^4,E)\}$ as in example 3.2 and let

$(f_A, E) = \{e_1=\{<h_1 [0.4,0.5], [0.6,0.7], [0.1,0.2]>, <h_2 [0.7,0.8], [0.6,0.7], [0.1,0.2]>, <h_3 [1,1], [1,1], [0,0]>\}$
$\{e_2=\{<h_1 [0.3,0.4], [0.5,0.6], [0,0.1]>, <h_2 [1,1], [1,1], [0,0]>, <h_3 [0,0], [0,0], [1,1]>\}$
$\{e_3=\{<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>\}$ be an IVFNS set.

Then $int(f_A,E) = \cup\{ (g_A,E)/ (g_A,E)$ is an IVFNS open set contained in $(f_A,E)\} = (f_A^2,E) \cup (f_A^3,E) =(f_A^2,E)$.
Since $(f_A^2,E)\subseteq (f_A,E)$ and $(f_A^3,E) \subseteq(f_A,E)$.

Theorem 3.22:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space on (F_A,E) and (f_A,E) be an IVFNS set in $P(F_A,E)$. Then

(i) $int(f_A,E)$ is an open and $int(f_A,E)$ is the largest open IVFNS set contained in (f_A,E).
(ii) The IVFNS set (f_A,E) is open if and only if $(f_A,E) = int(f_A,E)$.

Proof: Proof follows form the definition.

Proposition 3.23:

For any two IVFNS sets (f_A,E) and (g_A,E) is an interval valued fuzzy neutrosophic soft topological space $((F_A, E), \tau)$ on $P(F_A, E)$ then

(i) $(g_A,E)\subseteq (f_A,E)$ implies $int(g_A,E)\subseteq int(f_A,E)$.
(ii) $int(\varphi_A,E) = (\varphi_A,E)$ and $int(F_A,E) = (F_A,E)$.
(iii) $int(int(f_A,E)) = int(f_A,E)$.
(iv) $int((g_A,E)\cap (f_A,E)) = int(g_A,E)\cap int(f_A,E)$.
(v) $int((g_A,E)\cup (f_A,E)) \supseteq int(g_A,E)\cup int(f_A,E)$.

Proof:

(i) Since $(g_A,E)\subseteq (f_A,E)$ implies all the IVFNS – open set contained in (g_A,E) also contained in (f_A,E). Therefore $\{(g_A^*,E) / (g_A^*,E)$ is an IVFNS open set contained in $(g_A,E)\}\subseteq \{(f_A^*,E) / (f_A^*,E)$ is an IVFNS open set contained in $(f_A,E)\}$. So $int(g_A,E)\subseteq int(f_A,E)$.
(ii) Proof is obvious.
(iii) $int(int (f_A,E)) = \cup\{ (g_A,E)/ (g_A,E)$ is an IVFNS open set contained in $int(f_A,E)\}$ and since $int(f_A,E)$ is the largest open IVFNS sset contained in $int(f_A,E)$, Therefore $int(int(f_A,E)) = int(f_A,E)$.
(iv) Since $int (g_A,E)\subseteq (g_A,E)$ and $int (f_A,E)\subseteq (f_A,E)$, we have $int (g_A,E) \cap int (f_A,E) \subseteq (g_A,E) \cap (f_A,E)$----(1) Again since $(g_A,E) \cap (f_A,E) \subseteq (g_A,E)$ and $(g_A,E) \cap (f_A,E)\subseteq (f_A,E)$ we have $int ((g_A,E) \cap (f_A,E))\subseteq int (g_A,E)$ and $int ((g_A,E)\cap (f_A,E))\subseteq int (f_A,E)$. Therefore $int((g_A,E)\cap (f_A,E))\subseteq int (g_A,E)\cap int (f_A,E)$ ------(2). From (1) and (2) $int((g_A,E)\cap (f_A,E))=int (g_A,E)\cap int (f_A,E)$.
(v) Since $(g_A,E)\subseteq (g_A,E)\cup (f_A,E)$ and $(f_A,E)\subseteq (g_A,E)\cup (f_A,E)$ so $int(g_A,E) \subseteq int((g_A,E)\cup(f_A,E))$ and $int (f_A,E)\subseteq int((g_A,E)\cup(f_A,E))$. Hence $int(g_A,E)\cup int (f_A,E)\subseteq int((g_A,E)\cup (f_A,E)$.

Definition 3.24:

Let $((F_A, E), \tau)$ be an IVFNS topological space on (F_A,E) and let (f_A,E), (g_A,E) be two IVFNS set in $P(F_A,E)$. Then (g_A,E) is called an exterior IVFNS set of (f_A,E) if and only if (g_A,E) is an interior IVFNS set of the complement (f_A,E).

Definition 3.25:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space on (F_A,E) and (f_A,E) be an IVFNS set in $P (F_A,E)$. Then the union of all exterior IVFNS set of (f_A,E) is called the exterior of (f_A,E) and is denoted by $ext (f_A,E)$ and is defined by $ext (f_A,E)=\cup\{ (g_A,E) / (f_A,E)^c$ is a neighbourhood of $(g_A,E)\}$. That is from definition $ext (f_A,E) = int((f_A,E)^c)$.

Proposition 3.26:

For any two IVFNS sets (f_A,E) and (g_A,E) in an interval valued fuzzy neutrosophic soft topological space $((F_A, E), \tau)$ on $P(F_A,E)$ then

(i) $ext (f_A,E)$ is open and is the largest open set contained in $(f_A,E)^c$.
(ii) $(f_A,E)^c$ is open if and only if $(f_A,E)^c = ext (f_A,E)$.
(iii) $(g_A,E)\subseteq (f_A,E)$ implies $ext (f_A,E)\subseteq ext (g_A,E)$.
(iv) $ext((g_A,E)\cap (f_A,E))\supseteq ext (g_A,E)\cup ext (f_A,E)$.
(v) $ext((g_A,E)\cup (f_A,E))=ext (g_A,E)\cap ext (f_A,E)$.

Proof:

Proofs are straight forward.

Definition 3.27:

Let ((F_A, E), τ) be an interval valued fuzzy neutrosophic soft topological space on (F_A,E) and (f_A,E) be an IVFNS set in P(F_A,E). Then the intersection of all closed IVFNS set containing (f_A,E) is called the closure of (f_A,E) and is denoted by cl (f_A,E) and defined by cl (f_A,E)=∩{ (g_A,E)/ (g_A,E) is a IVFNS closed set containing (f_A,E)}. Thus cl (f_A,E) is the smallest IVFNS closed set containing (f_A,E).

Example 3.28:

Let us consider an interval valued fuzzy neutrosophic soft topology τ={(φ_A, E), (F_A, E), (f_A^1,E), (f_A^2,E), (f_A^3,E) ,(f_A^4,E)} as in example 3.2 and let

(f_A, E) = {e_1={<h_1 [0.1,0.2], [0.3,0.4], [0.5,0.6]>, <h_2 [0,0.1], [0.4,0.5], [0.7,0.8]>, <h_3 [0,0], [0,0], [1,1]>}
{e_2 ={<h_1 [0,0], [0.4,0.5], [0.6,0.7]>, <h_2 [0,0], [0,0], [1,1]>, <h_3 [1,1], [1,1], [0,0]>}
{e_3={<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>} be an IVFNS set.

Then cl(f_A,E) =∩{ (g_A,E)/ (g_A,E) is a IVFNS closed set containing (f_A,E)} = (f_A^1,E)^c ∩(f_A^4,E)^c = (f_A^4,E)^c

Since (f_A,E)⊆ (f_A^1,E)^c and (f_A,E)⊆ (f_A^4,E)^c

Proposition 3.29:

For any two IVFNS sets (f_A,E) and (g_A,E) is an interval valued fuzzy neutrosophic soft topological space ((F_A, E), τ) on P(F_A, E) then

(i) cl (f_A,E) is the smallest IVFNS closed set containing (f_A,E).
(ii) (f_A,E) is IVFNS closed if and only if (f_A,E) = cl (f_A,E)
(iii) (g_A,E)⊆ (f_A,E) implies cl(g_A,E)⊆ cl(f_A,E).
(iv) cl(cl(f_A,E)) = cl(f_A,E).
(v) cl(φ_A,E) = (φ_A,E) and cl(F_A,E) = (F_A,E).
(vi) cl((g_A,E)∪ (f_A,E)) = cl(g_A,E)∪cl(f_A,E).
(vii) cl((g_A,E)∩ (f_A,E)) ⊆ cl(g_A,E)∩cl(f_A,E).

Proof:

(i) and (ii) follows from the definition.

(iii) Since (g_A,E)⊆ (f_A,E) implies all the closed set containing (f_A,E) also contain (g_A,E). Therefore ∩ { (g_A^*, E) / (g_A^*, E) is an IVFNS closed set containing (g_A,E)}⊆∩{ (f_A^*, E) / (f_A^*, E) is an IVFNS closed set containing (f_A,E)}. So cl (g_A,E)⊆cl (f_A,E).

(iv) cl(cl (f_A,E)) = ∩{ (g_A,E)/ (g_A,E) is an IVFNS closed set containing cl (f_A,E)} and since cl (f_A,E) is the smallest closed IVFNS set containing cl (f_A,E). Therefore cl(cl (f_A,E)) = cl (f_A,E).

(v) Proof is obvious.

(vi) Since cl(g_A,E) ⊇ (g_A,E) and cl(f_A,E)⊇ (f_A,E), we have cl(g_A,E)∪cl (f_A,E)⊇ (g_A,E)∪ (f_A,E). This implies cl(g_A,E)∪cl(f_A,E)⊇ cl((g_A,E)∪ (f_A,E)) -----(1). And since (g_A,E)∪ (f_A,E)⊇ (g_A,E) and (g_A,E)∪ (f_A,E) ⊇ (f_A,E) so cl((g_A,E)∪(f_A,E))⊇cl(g_A,E) and cl((g_A,E)∪(f_A,E))⊇cl(f_A,E). Therefore cl((g_A,E)∪(f_A,E))⊇cl(g_A,E)∪cl(f_A,E)-----(2).

Form (1) and (2) cl(g_A,E)∪cl(f_A,E) = cl(g_A,E)∪ cl(f_A,E).

(vii) Since (g_A,E) ⊇ (g_A,E)∩ (f_A,E) and (f_A,E)⊇ (g_A,E)∩ (f_A,E) so cl (g_A,E)⊇cl((g_A,E)∩(f_A,E)) and cl(f_A,E)⊇cl((g_A,E)∩(f_A,E)).

Hence cl ((g_A,E)∩cl(f_A,E)⊇cl((g_A,E)∩(f_A,E)).

Theorem 3.30:

Let ((F_A, E), τ) be an interval valued fuzzy neutrosophic soft topological space on (F_A,E) and (f_A,E) be an IVFNS set in P(F_A,E). Then the collection $\tau_{(f_A,E)} = \{(f_A, E) \cap (g_A, E) / (g_A, E) \in \tau\}$ is an interval valued fuzzy neutrosophic soft topology on the interval valued fuzzy neutrosophic soft set (f_A,E).

Proof:

(i) Since (φ_A, E), (F_A, E) ∈τ , (f_A, E)= (f_A, E)∩ (F_A, E) and (φ_A, E),= (f_A, E)∩ (φ_A, E). Therefore (φ_A, E), (f_A, E) ∈τ.

(ii) Let $\{(f_A^i, E) / i = 1,2,3,....n\}$ be a finite family of IVFNS open sets in $\tau_{(f_A,E)}$, then for each i=1,2,3..n there exist $(g_A^i, E) \in \tau$ such that (f_A^i, E) = (f_A, E)∩ (g_A^i, E) Now $\bigcap_{i=1}^{n} (f_A^i, E) =$

$\bigcap_{i=1}^{n} [(f_A, E) \cap (g_A^i, E)] = (f_A, E) \cap \left(\bigcap_{i=1}^{n} [g_A^i, E] \right)$ and since $\left(\bigcap_{i=1}^{n} [g_A^i, E] \right) \in \tau$ so $\bigcap_{i=1}^{n} (f_A^i, E) \in \tau_{(f_A,E)}$.

(iii) Let $\{(f_A^k, E) / k \in K\}$ be an arbitrary family of interval valued fuzzy neutrosophic soft open sets in $\tau_{(f_A,E)}$, then for each k∈K, there exist (g_A^k, E) ∈τ such that (f_A^k, E)=(f_A, E)∩ (g_A^k, E)

Now $\bigcup_{k \in K} (f_A^k, E) = \bigcup_{k \in K} ((f_A, E) \cap (g_A^k, E)) =$ $(f_A, E) \cap \bigcup_{k \in K} (g_A^k, E)$ and since $\bigcup_{k \in K} (g_A^k, E) \in \tau$.

So $\bigcup_{k\in K}(f_A^k, E) \in \tau_{(f_A,E)}$.

Definition 3.31:

Let ((F_A, E), τ) be an IVFNS topological space on (F_A, E) and (f_A, E) be an IVFNS set in P(F_A, E). Then the IVFNS topology.

$\tau_{(f_A,E)} = \{(f_A, E) \cap (g_A, E) / (g_A, E) \in \tau\}$ is called interval valued fuzzy neutrosophic soft subspace topology (IVFNS subspace topology) and ((f_A, E), $\tau_{(f_A,E)}$) is called interval valued fuzzy neutrosophic soft subspace of ((F_A, E), τ).

Example 3.32:

Let us consider the interval valued fuzzy neutrosophic soft topology τ={(φ_A, E), (F_A, E), (f_A^1,E), (f_A^2,E), (f_A^3,E) ,(f_A^4,E)} as in the example 3.2 and an IVFNS-set

(f_A,E)={e_1={<h_1[0.2,0.3],[0.3,0.4],[0,0.1]>,
<h_2[0.5,0.6],[0.4,0.5], [0.1,0.2]>,
<h_3 [0.2,0.3], [0.5,0.6], [0.6,0.7]>}
{e_2 ={<h_1 [0.3,0.4], [0.5,0.6], [0.1,0.2]>,
<h_2 [0.4,0.5], [0.6,0.7], [0.2,0.3]>,
<h_3 [0.4,0.5], [0.4,0.5], [0.2,0.3]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>} be an IVFNS set

(f_A, E)∩ (F_A, E) = (f_A, E)

(f_A, E)∩ (φ_A, E) = (φ_A, E)

(f_A, E)∩ (f_A^1,E) = (g_A^1,E) =
{e_1={<h_1[0.2,0.3],[0.3,0.4],[0.2,0.3]>,
<h_2[0.4,0.5],[0.4,0.5], [0.1,0.2]>,
<h_3 [0.2,0.3], [0.5,0.6], [0.6,0.7]>}
{e_2 ={<h_1 [0.3,0.4], [0.5,0.6], [0.2,0.3]>,
<h_2 [0.4,0.5], [0.6,0.7], [0.2,0.3]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A, E)∩ (f_A^2,E) = (g_A^2,E) =
{e_1={<h_1[0.2,0.3],[0.3,0.4],[0.1,0.2]>,
<h_2[0.5,0.6],[0.4,0.5], [0.2,0.3]>, <h_3 [0.2,0.3], [0.5,0.6], [0.6,0.7]>}
{e_2 ={<h_1 [0.2,0.3], [0.4,0.5], [0.1,0.2]>,
<h_2 [0.4,0.5], [0.6,0.7], [0.2,0.3]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A, E)∩ (f_A^3,E) = (g_A^3,E) =
{e_1={<h_1[0.2,0.3],[0.3,0.4],[0.2,0.3]>,
<h_2[0.4,0.5],[0.4,0.5], [0.2,0.3]>,
<h_3 [0.2,0.3], [0.5,0.6], [0.6,0.7]>}
{e_2 ={<h_1 [0.2,0.3], [0.4,0.5], [0.2,0.3]>,
<h_2 [0.4,0.5], [0.6,0.7], [0.2,0.3]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A, E)∩ (f_A^4,E) = (g_A^4,E) =
{e_1={<h_1[0.2,0.3],[0.3,0.4],[0.1,0.2]>,
<h_2[0.5,0.6],[0.4,0.5], [0.1,0.2]>,
<h_3 [0.2,0.3], [0.5,0.6], [0.6,0.7]>}
{e_2 ={<h_1 [0.3,0.4], [0.5,0.6], [0.1,0.2]>,
<h_2 [0.4,0.5], [0.6,0.7], [0.2,0.3]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

Thus $\tau_{(f_A,E)}$ = {(φ_A, E), (F_A, E), (g_A^1,E), (g_A^2,E), (g_A^3,E) ,(g_A^4,E)} is an interval valued fuzzy neutrosophic soft subspace topology for τ and ((f_A, E), $\tau_{(f_A,E)}$) is called interval valued fuzzy neutrosophic soft subspace of ((F_A, E), τ).

Theorem 3.33:

Let ((η_A,E), τ^1) be a IVFNS topological subspace of ((ξ_A ,E),τ^2) and let ((ξ_A ,E),τ^2) be a IVFNS topological subspace of ((F_A, E), τ). Then ((η_A,E), τ^1) is also an IVFNS topological subspace of ((F_A, E), τ)..

Proof:

Since (η_A,E) ⊆ (ξ_A ,E) ⊆ (F_A, E), ((η_A,E), τ^1) is an interval valued fuzzy neutrosophic soft topological subspace of ((F_A, E), τ). if and only if $\tau_{(\eta_A,E)}$ = τ^1. Let (f_A^1 ,E) $\in\tau^1$, now since ((η_A,E), τ^1) is an IVFNS topological subspace of ((ξ_A ,E),τ^2) ie., $\tau^2{}_{(\eta_A,E)}$ = τ^1, so there exist f_A^2 ,E) $\in\tau^2$ such that (f_A^1,E) =(η_A,E)∩ (f_A^2 ,E). But ((ξ_A ,E),τ^2) is an IVFNS topological subspace of ((F_A, E), τ).Therefore there exist (f_A, E) ∈τ such that (f_A^2 ,E) =(ξ_A,E)∩ (f_A,E). Thus (f_A^1,E) =(η_A,E)∩ (f_A^2 ,E) = (η_A,E)∩ (ξ_A ,E)∩ (f_A,E) =(η_A,E)∩ (f_A,E). So (f_A^1 ,E)∈ $\tau_{(\eta_A,E)}$ implies τ^1⊆ $\tau_{(\eta_A,E)}$ ---(1).

Now assume, (g_A, E) ∈ $\tau_{(\eta_A,E)}$ ie., there exist (h_A,E) ∈τ such that (g_A,E) = (η_A,E)∩ (h_A,E) . But (ξ_A, E)∩ (h_A,E) ∈ $\tau_{(\xi_A,E)}$ =τ^2.So (η_A,E)∩ ((ξ_A, E)∩ (h_A,E)) = (η_A,E)∩ (h_A,E) = (g_A,E). We have

$(g_A,E) \in \tau^1$ implies $\tau_{(\eta_A,E)} \subseteq \tau^1$--------(2). From (1) and (2) $\tau^1 = \tau_{(\eta_A,E)}$. Hence the proof.

Theorem 3.34:

Let $((F_A, E), \tau)$.be an IVFNS topological space of (F_A, E). B be an basis for τ and (f_A, E) be an IVFNS set in $P(F_A, E)$. Then the family $B_{(f_A,E)} = \{(f_A, E) \cap (g_A, E) / (g_A, E) \in B\}$ is an IVFNS basis for subspace topology $\tau_{(f_A,E)}$.

Proof:

Let $(h_A,E) \in \tau_{(f_A,E)}$, then there exist an IVFNS set $(g_A, E) \in \tau$, such that $(h_A,E) = (f_A, E) \cap (g_A, E)$. Since B is a base for τ, there exist sub-collection $\{\{(\psi^i_A, E) / i \in I\}$ of B, such that $(g_A, E) = \bigcup_{i\in I}(\psi^i_A, E)$. Therefore $(h_A,E) = (f_A, E) \cap (g_A,E) = (f_A, E) \cap (\bigcup_{i\in I}(\psi^i_A, E)) = \bigcup_{i\in I}((f_A, E) \cap (\psi^i_A, E))$. Since $(f_A, E) \cap (\psi^i_A, E) \in B_{(f_A,E)}$ implies $B_{(f_A,E)}$ is an IVFNS basis for the IVFNS subspace topology $\tau_{(f_A,E)}$.

4. Conclusion

In this paper the notion of topological space in interval valued fuzzy neutrosophic soft sets is introduced. Further, some of its operators and properties of topology in IVFNS set are established.

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